Generalized Feynman Graphs as Generalized Braids

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November 7, 2015

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Contents

Abstract

The basic challenge of quantum TGD is to give a precise content to the notion of generalization Feynman diagram and the reduction to braids of some kind is very attractive possibility inspired by zero energy ontology. The point is that no $n > 2$-vertices at the level of braid strands are needed if bosonic emergence holds true.

1. For this purpose the notion of algebraic knot is introduced and the possibility that it could be applied to generalized Feynman diagrams is discussed. The algebraic structrures called kei, quandle, rack, and biquandle and their algebraic modifications as such are not enough. The lines of Feynman graphs are replaced by braids and in vertices braid strands redistribute. This poses several challenges: the crossing associated with braiding and crossing occurring in non-planar Feynman diagrams should be integrated to a more general notion; braids are replaced with sub-manifold braids; braids of braids ....of braids are possible; the redistribution of braid strands in vertices should be algebraized. In the following I try to abstract the basic operations which should be algebraized in the case of generalized Feynman diagrams.
2. One should be also able to concretely identify braids and 2-braids (string world sheets) as well as partonic 2-surfaces and I have discussed several identifications during last years. Legendrian braids turn out to be very natural candidates for braids and their duals for the partonic 2-surfaces. String world sheets in turn could correspond to the analogs of Lagrangian sub-manifolds or two minimal surfaces of space-time surface satisfying the weak form of electric-magnetic duality. The latter opion turns out to be more plausible. Finite measurement resolution would be realized as symplectic invariance with respect to the subgroup of the symplectic group leaving the end points of braid strands invariant. In accordance with the general vision TGD as almost topological QFT would mean symplectic QFT. The identification of braids, partonic 2-surfaces and string world sheets - if correct - would solve quantum TGD explicitly at string world sheet level in other words in finite measurement resolution.

   The breakthrough in this identification problem came as I realized that the well-definedness of em charge for the modes of induced spinor fields requires that the induced W fields vanish. This forces the region in which the spinor mode vanishes to have 2-D CP_2 projection (covariantly constant right-handed neutrino is exception). In the generic case this implies that spinor modes are localized to 2-D surfaces - string world sheets and possibly also partonic 2-surfaces. It is quite possible that this localization is consistent with Kähler-Dirac equation only in the Minkowskian regions where the effective metric defined by Kähler-Dirac gamma matrices can be effectively 2-dimensional and parallel to string world sheet.

   Above weak scale (proportional to h_{eff}) also classical Z^0 field is expected to vanish and can do so so that also the smallness of parity breaking effects for visible matter can be understood.
3. A brief summary of generalized Feynman rules in zero energy ontology is proposed. This requires the identification of vertices, propagators, and prescription for integrating over al 3-surfaces. It turns out that the basic building blocks of generalized Feynman diagrams are well-defined.
4. The notion of generalized Feynman diagram leads to a beautiful duality between the descriptions of hadronic reactions in terms of hadrons and partons analogous to gauge-gravity duality and AdS/CFT duality but requiring no additional assumptions. The model of quark gluon plasma as s strongly interacting phase is proposed. Color magnetic flux tubes are responsible for the long range correlations making the plasma phase more like a very large hadron rather than a gas of partons. One also ends up with a simple estimate for the viscosity/entropy ratio using black-hole analogy.

1 Introduction

The inspiration to this chapter came from an article by Sam Nelson about very interesting new-to-me notion known as algebraic knots [A6, A4], which has initiated a revolution in knot theory. This notion was introduced 1996 by Louis Kauffmann [A5] so that it is already 15 year old concept. While reading the article I realized that this notion fits perfectly the needs of TGD and leads to a progress in attempts to articulate more precisely what generalized Feynman diagrams are. It should be added that the knots and braids are not anything new in TGD framework and I have considered knot theory from TGD point of view already earlier [K7].

The basic challenge of quantum TGD is to give a precise content to the notion of generalized Feynman diagram and the reduction to braids of some kind is very attractive possibility inspired by zero energy ontology. The point is that no $n > 2$-vertices at the level of braid strands are needed if bosonic emergence holds true. In the following I will summarize briefly the vision about generalized Feynman diagrams, introduce the notion of algebraic knot, and after than discuss in more detail how the notion of algebraic knot could be applied to generalized Feynman diagrams.

1. The algebraic structrures kei, quandle, rack, and biquandle and their algebraic modifications as such are not enough. The lines of Feynman graphs are replaced by braids and in vertices braid strands redistribute. This poses several challenges: the crossing associated with braiding and crossing occurring in non-planar Feynman diagrams should be integrated to a more general notion; braids are replaced with sub-manifold braids; braids of braids....of braids are possible; the redistribution of braid strands in vertices should be algebraized. In the following I try to abstract the basic operations which should be algebraized in the case of generalized Feynman diagrams.

2. One should be also able to concretely identify braids and 2-braids (string world sheets) as well as partonic 2-surfaces and I have discussed several identifications during last years. Legendrian braids turn out to be very natural candidates for braids and their duals for the partonic 2-surfaces. String world sheets in turn could correspond to the analogs of Lagrangian sub-manifolds or to minimal surfaces of space-time surface satisfying the weak form of electric-magnetic duality. The latter option turns out to be more plausible. Finite measurement resolution would be realized as symplectic invariance with respect to the subgroup of the symplectic group leaving the end points of braid strands invariant. In accordance with the general vision TGD as almost topological QFT would mean symplectic QFT. The identification of braids, partonic 2-surfaces and string world sheets - if correct - would solve quantum TGD explicitly at string world sheet level in other words in finite measurement resolution.

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3. Irrespective of whether the algebraic knots are needed, the natural question is what generalized Feynman diagrams are. It seems that the basic building bricks can be identified so that one can write rather explicit Feynman rules already now. Of course, the rules are still far from something to be burned into the spine of the first year graduate student. A brief summary of generalized Feynman rules in zero energy ontology is proposed. This requires the identification of vertices, propagators, and prescription for integrating over al 3-surfaces. It turns out that the basic building blocks of generalized Feynman diagrams are well-defined.

4. The notion of generalized Feynman diagram leads to a beautiful duality between the descriptions of hadronic reactions in terms of hadrons and partons analogous to gauge-gravity duality and AdS/CFT duality but requiring no additional assumptions. The model of quark gluon plasma as s strongly interacting phase is proposed. Color magnetic flux tubes are responsible for the long range correlations making the plasma phase more like a very large hadron rather than a gas of partons. One also ends up with a simple estimate for the viscosity/entropy ratio using black-hole analogy.

The basic vision about generalized Feynman diagras

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://tgdtheory.fi/cmaphtml.html` [L2]. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L3].

- Quantum TGD [L5]
- Generalized Feynman diagrams [L4]
- The unique role of twistors in TGD [L6]
- Twistors and TGD [L7]

2 Algebraic Braids, Sub-Manifold Braid Theory, And Generalized Feynman diagrams

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In the following I will summarize briefly the vision about generalized Feynman diagrams, introduce the notion of algebraic knot, and after than discuss in more detail how the notion of algebraic knot could be applied to generalized Feynman diagrams. The algebraic structrures kei, quandle, rack, and biquandle and their algebraic modifications as such are not enough. The lines of Feynman graphs are replaced by braids and in vertices braid strands redistribute. This poses several challenges: the crossing associated with braiding and crossing occurring in non-planar Feynman diagrams should be integrated to a more general notion; braids are replaced with sub-manifold braids; braids of braids....of braids are possible; the redistribution of braid strands in vertices should be algebraized. In the following I try to abstract the basic operations which should be algebraized in the case of generalized Feynman diagrams.

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2.1 Generalized Feynman Diagrams, Feynman Diagrams, And Braid Diagrams

2.1.1 How knots and braids a la TGD differ from standard knots and braids?

TGD approach to knots and braids differs from the knot and braid theories in given abstract 3-manifold (4-manifold in case of 2-knots and 2-braids) is that space-time is in TGD framework identified as 4-D surface in $M^4 \times CP_2$ and preferred 3-surfaces correspond to light-like 3-surfaces defined by wormhole throats and space-like 3-surfaces defined by the ends of space-time sheets at the two light-like boundaries of causal diamond CD.

The notion of finite measurement resolution effectively replaces 3-surfaces of both kinds with braids and space-time surface with string world sheets having braids strands as their ends. The

4-dimensionality of space-time implies that string world sheets can be knotted and intersect at discrete points (counterpart of linking for ordinary knots). Also space-time surface can have self-intersections consisting of discrete points.

The ordinary knot theory in E^3 involves projection to a preferred 2-plane E^2 and one assigns to the crossing points of the projection an index distinguishing between two cases which are transformed to each other by violently taking the first piece of strand through another piece of strand. In TGD one must identify some physically preferred 2-dimensional manifold in imbedding space to which the braid strands are projected. There are many possibilities even when one requires maximal symmetries. An obvious requirement is however that this 2-manifold is large enough.

1. For the braids at the ends of space-time surface the 2-manifold could be large enough sphere S^2 of light-cone boundary in coordinates in which the line connecting the tips of CD defines a preferred time direction and therefore unique light-like radial coordinate. In very small knots it could be also the geodesic sphere of CP_2 (apart from the action of isometries there are two geodesic spheres in CP_2).

2. For light-like braids the preferred plane would be naturally M^2 for which time direction corresponds to the line connecting the tips of CD and spatial direction to the quantization axis of spin. Note that these axes are fixed uniquely and the choices of M^2 are labelled by the points of projective sphere P^2 telling the direction of space-like axis. Preferred plane M^2 emerges naturally also from number theoretic vision and corresponds in octonionic pictures to hyper-complex plane of hyper-octonions. It is also forced by the condition that the choice of quantization axes has a geometric correlate both at the level of imbedding space geometry and the geometry of the "world of classical worlds".

The braid theory in TGD framework could be called sub-manifold braid theory and certainly differs from the standard one.

1. If the first homology group of the 3-surface is non-trivial as it when the light-like 3-surfaces represents an orbit of partonic 2-surface with genus larger than zero, the winding of the braid strand (wrapping of branes in M-theory) meaning that it represents a homologically non-trivial curve brings in new effects not described by the ordinary knot theory. A typical new situation is the one in which 3-surface is locally a product of higher genus 2-surface and line segment so that knot strand can wind around the 2-surface. This gives rise to what are called non-planar braid diagrams for which the projection to plane produces non-standard crossings.

2. In the case of 2-knots similar exotic effects could be due to the non-trivial 2-homology of space-time surface. Wormhole throats assigned with elementary particle wormhole throats are homologically non-trivial 2-surfaces and might make this kind of effects possible for 2-knots if they are possible.

The challenge is to find a generalization of the usual knot and braid theories so that they apply in the case of braids (2-braids) imbedded in 3-D (4-D) surfaces with preferred highly symmetry sub-manifold of $M^4 \times CP_2$ defining the analog of plane to which the knots are projected. A proper description of exotic crossings due to non-trivial homology of 3-surface (4-surface) is needed.

2.1.2 Basic questions

The questions are following.

1. How the mathematical framework of standard knot theory should be modified in order to cope with the situation encountered in TGD? To my surprise I found that this kind of mathematical framework exists: so called algebraic knots [A6, A4] define a generalization of knot theory very probably able to cope with this kind of situation.

2. Second question is whether the generalized Feynman diagrams could be regarded as braid diagrams in generalized sense. Generalized Feynman diagrams are generalizations of ordinary Feynman diagrams. The lines of generalized Feynman diagrams correspond to the orbits

of wormhole throats and of wormhole contacts with throats carrying elementary particle quantum numbers.

The lines meet at vertices which are partonic 2-surfaces. Single wormhole throat can describe fermion whereas bosons have wormhole contacts with fermion and anti-fermion at the opposite throats as building bricks. It seems however that all fermions carry Kähler magnetic charge so that physical particles are string like objects with magnetic charges at their ends.

The short range of weak interactions results from the screening of the axial isospin by neutrinos at the other end of string like object and also color confinement could be understood in this manner. One cannot exclude the possibility that the length of magnetic flux tube is of order Compton length.

3. Vertices of the generalized Feynman diagrams correspond to the partonic 2-surfaces along which light-like 3-surfaces meet and this is certainly a challenge for the required generalization of braid theory. The basic objection against the reduction to algebraic braid diagrams is that reaction vertices for particles cannot be described by ordinary braid theory: the splitting of braid strands is needed.

   The notion of bosonic emergence [K8] however suggests that 3-vertex and possible higher vertices correspond to the splitting of braids rather than braid strands. By allowing braids which come from both past and future and identifying free fermions as wormhole throats and bosons as wormhole contacts consisting of a pair of wormhole throats carrying fermion and anti-fermion number, one can understand boson excanges as recombinations without anyneed to have splitting of braid strands. Strictly and technically speaking, one would have tangles like objects instead of braids. This would be an enormous simplification since $n > 2$-vertices which are the source of divergences in QFT: s would be absent.

4. Non-planar Feynman diagrams are the curse of the twistor approach and I have already earlier proposed that the generalized Feynman amplitudes and perhaps even twistorial amplitudes could be constructed as analogs of knot invariants by recursively transforming non-planar Feynman diagrams to planar ones for which one can write twistor amplitudes. This forces to answer two questions.

   (a) Does the non-nonplanarity of Feynman diagrams - completely combinatorial objects identified as diagrams in plane - have anything to do with the non-planarity of algebraic knot diagrams and with the non-planarity of generalized Feynman diagrams which are purely geometric objects?

   (b) Could these two kind of non-planarities be fused to together by identifying the projection 2-plane as preferred $M^2 \subset M^4$. This would mean that non-planarity in QFT sense is defined for entire braids: braid A can have virtual crossing with B. Non-planarity in the sense of knot theory would be defined for braid strands inside the braids. At vertices braid strands are redistributed between incoming lines and the analog of virtual crossing be identifiable as an exchange of braid strand between braids. Several kinds of non-planarities would be present and the idea about gradual unknotting of a non-planar diagram so that a planar diagram results as the final outcome might make sense and allow to generalize the recursion recipe for the twistorial amplitudes.

   (c) One might consider the possibility that inside orbits of wormhole throats defining the lines of Feynman diagrams the R-matrix for integrable QFT in M^2 (only permutations of momenta are allowed) describes the dynamics so that one obtains just a permutation of momenta assigned to the braid strands. Ordinary braiding would be described by existing braid theories. The core problem would be the representation of the exchange of a strand between braids algebraically.

It has become clear that there is different and much simpler general approach to the non-planarity problem. In twistor Grassmannian approach [K11] generalized Feynman diagrams correspond to TGD variants of stringy diagrams. In stringy approach one gets rid of non-planarity problem altogether.

2.2 Brief Summary Of Algebraic Knot Theory

2.2.1 Basic ideas of algebraic knot theory

In ordinary knot theory one takes as a starting point the representation of knots of E^3 by their plane plane projections to which one attach a "color" to each crossing telling whether the strand goes over or under the strand it crosses in planar projection. These numbers are fixed uniquely as one traverses through the entire knot in given direction.

The so called Reidermeister moves are the fundamental modifications of knot leaving its isotopy equivalence class unchanged and correspond to continuous deformations of the knot. Any algebraic invariant assignable to the knot must remain unaffected under these moves. Reidermeister moves as such look completely trivial and the non-trivial point is that they represent the minimum number of independent moves which are represented algebraically.

In algebraic knot theory topological knots are replaced by typographical knots resulting as planar projections. This is a mapping of topology to algebra. It turns out that the existing knot invariants generalize and ordinary knot theory can be seen as a special case of the algebraic knot theory. In a loose sense one can say that the algebraic knots are to the classical knot theory what algebraic numbers are to rational numbers.

Virtual crossing is the key notion of the algebraic knot theory. Virtual crossing and their rules of interaction were introduced 1996 by Louis Kauffman as basic notions [A1]. For instance, a strand with only virtual crossings should be replaceable by any strand with the same number of virtual crossings and same end points. Reidermeister moves generalize to virtual moves. One can say that in this case crossing is self-intersection rather than going under or above. I cannot be eliminated by a small deformation of the knot. There are actually several kinds of non-standard crossings: examples listed in figure 7 of [A6]) are virtual, flat, singular, and twist bar crossings.

Algebraic knots have a concrete geometric interpretation.

(a) Virtual knots are obtained if one replaces E^3 as imbedding space with a space which has non-trivial first homology group. This implies that knot can represent a homologically non-trivial curve giving an additional flavor to the unknottedness since homologically non-trivial curve cannot be transformed to a curve which is homologically non-trivial by any continuous deformation.

(b) The violent projection to plane leads to the emergence of virtual crossings. The product $(S^1 \times S^1) \times D$, where $(S^1 \times S^1)$ is torus D is finite line segment, provides the simplest example. Torus can be identified as a rectangle with opposite sides identified and homologically non-trivial knots correspond to curves winding n_1 times around the first S^1 and n_2 times around the second S^1. These curves are not continuous in the representation where $S^1 \times S^1$ is rectangle in plane.

(c) A simple geometric visualization of virtual crossing is obtained by adding to the plane a handle along which the second strand traverses and in this manner avoids intersection. This visualization allows to understand the geometric motivation for the the virtual moves.

This geometric interpretation is natural in TGD framework where the plane to which the projection occurs corresponds to $M^2 \subset M^4$ or is replaced with the sphere at the boundary of S^2 and 3-surfaces can have arbitrary topology and partonic 2-surfaces defining as their orbits light-like 3-surfaces can have arbitrary genus.

In TGD framework the situation is however more general than represented by sub-manifold braid theory. Single braid represents the line of generalized Feynman diagram. Vertices represent something new: in the vertex the lines meet and the braid strands are redistributed but do not disappear or pop up from anywhere. That the braid strands can come both from the future and past is also an important generalization. There are physical argments suggesting that there are only 3-vertices for braids but not higher ones [K3]. The challenge is to represent algebraically the vertices of generalized Feynman diagrams.

2.2.2 Algebraic knots

The basic idea in the algebraization of knots is rather simple. If x and y are the crossing portions of knot, the basic algebraic operation is binary operation giving "the result of x going under y", call it $x \triangleright y$ telling what happens to x. "Portion of knot" means the piece of knot between two crossings and $x \triangleright y$ denotes the portion of knot next to x. The definition is asymmetrical in x and y and the dual of the operation would be $y \triangleleft x$ would be "the result of y going above x". One can of course ask, why not to define the outcome of the operation as a pair $(x \triangleleft y, y \triangleright x)$. This operation would be bi-local in a well-defined sense. One can of course do this: in this case one has binary operation from $X \times X \rightarrow X \times X$ mapping pairs of portions to pairs of portions. In the first case one has binary operation $X \times X \rightarrow X$.

The idea is to abstract this basic idea and replace X with a set endowed with operation $\triangleright$ or $\triangleleft$ or both and formukate the Reidermeister conditions given as conditions satisfied by the algebra. One ends up to four basic algebraic structures kei, quandle, rack, and biquandle.

(a) In the case of non-oriented knots the kei is the algebraic structure. Kei - or invontary quandle-is a set X with a map $X \times X \rightarrow X$ satisfying the conditions

   i. $x \triangleright x = x$ (idenpotency, one of the Reidemeister moves)
   ii. $(x \triangleright y) \triangleright y$ =x (operation is its own right inverse having also interpretation as Reidemeister move)
   iii. $(x \triangleright y) \triangleright z = (x \triangleright z) \triangleright (y \triangleright z)$ (self-distributivity)

   $Z([t])/(t^2)$ module with $x \triangleright y = tx + (1-t)y$ is a kei.

(b) For orientable knot diagram there is preferred direction of travel along knot and one can distinguish between $\triangleright$ and its right inverse $\triangleright^{-1}$. This gives quandle satisfying the axios

   i. $x \triangleright x = x$
   ii. $(x \triangleright y) \triangleright^{-1} y = (x \triangleright^{-1} y) \triangleright y = x$
   iii. $(x \triangleright y) \triangleright z = (x \triangleright z) \triangleright (y \triangleright z)$

   $Z[t^{\pm 1}]$ nodule with $x \triangleright y = tx + (1-t)y$ is a quandle.

(c) One can also introduce framed knots: intuitively one attaches to a knot very near to it. More precise formulation in terms of a section of normal bundle of the knot. This makes possible to speak about self-linking. Reidermeister moves must be modified appropriately. In this case rack is the appropriate structure. It satisfied the axioms of quandle except the first axiom since corresponding operation is not a move anymore. Rack axioms are eqivalent with the requirement that functions $f_y : X \rightarrow X$ defined by $f_y(x)x \triangleright y)$ are automorphisms of the structure. Therefore the elements of rack represent its morphisms. The modules over $Z[t^{\pm 1}, s]/s(t+s-1)$ are racks. Coxeter racks are inner product spaces with $x \triangleright y$ obtained by reflecting x across y.

(d) Biquandle consists of arcs connecting the subsequent crossings (both under- and over-) of oriented knot diagram. Biquandle operation is a map $B : X \times X \rightarrow X \times X$ of order pairs satisfying certain invertibility conditions together with set theoretic Yang-Baxter equation:

$$(B \times I)(I \times B)(B \times I) = (I \times B)(B \times I)(I \times B) \ .$$

   Here $I : X \rightarrow X$ is the identity map. The three conditions to which Yang-Baxter equation decomposes gives the counterparts of the above discussed axioms. Alexander biquandle is the module $Z(t^{\pm 1}, s^{\pm}1$ with $B(x,y) = (ty + (1-ts)x, sx)$ where one has $s \neq 1$. If one includes virtual, flat and singular crossings one obtains virtual/singular aundles and semiquandles.

2.3 Generalized Feynman Diagrams As Generalized Braid Diagrams?

Zero energy ontology suggests the interpretation of the generalized Feynman diagrams as generalized braid diagrams so that there would be no need for vertices at the fundamental braid strand level. The notion of algebraic braid (or tangle) might allow to formulate this idea more precisely.

2.3.1 Could one fuse the notions of braid diagram and Feynman diagram?

The challenge is to fuse the notions of braid diagram and Feynman diagram having quite different origin.

(a) All generalized Feynman diagrams are reduced to sub-manifold braid diagrams at microscopic level by bosonic emergence (bosons as pairs of fermionic wormhole throats). Three-vertices appear only for entire braids and are purely topological whereas braid strands carrying quantum numbers are just re-distributed in vertices. No 3-vertices at the really microscopic level! This is an additional nail to the coffin of divergences in TGD Universe.

(b) By projecting the braid strands of generalized Feynman diagrams to preferred plane $M^2 \subset M^4$ (or rather 2-D causal diamond), one could achieve a unified description of non-planar Feynman diagrams and braid diagrams. For Feynman diagrams the intersections have a purely combinatorial origin coming from representations as 2-D diagrams.

For braid diagrams the intersections have different origin and non-planarity has different meaning. The crossings of entire braids analogous to those appearing in non-planar Feynman diagrams should define one particular exotic crossing besides virtual crossings of braid strands due to non-trivial first homology of 3-surfaces.

(c) The necessity to choose preferred plane M^2 looks strange from QFT point of view. In TGD framework it is forced by the number theoretic vision in which M^2 represents hyper-complex plane of sub-space of hyper-octonions which is subspace of complexified octonions. The choice of M^2 is also forced by the condition that the choice of quantization axes has a geometric correlate both at the level of imbedding space geometry and the geometry of the "world of classical worlds".

(d) Also 2-braid diagrams defined as projections of string world sheets are suggestive and would be defined by a projections to the 3-D boundary of CD or to $M^3 \subset M^4$. They would provide a more concrete stringy illustration about generalized Feynman diagram as analog of string diagram. Another attractive illustration is in terms of dance metaphor with the boundary of CD defining the 3-D space-like parquette. The duality between space-like and light-like braids is expected to be of importance.

The obvious conjecture is that Feynman amplitudes are a analogous to knot invariants constructible by gradually reducing non-planar Feynman diagrams to planar ones after which the already existing twistor theoretical machinery of $\mathcal{N} = 4$ SYMs would apply [K12].

2.3.2 Does 2-D integrable QFT dictate the scattering inside the lines of generalized Feynman diagrams

The preferred plane M^2 (more precisely, 2-D causal diamond having also interpretation as Penrose diagram) plays a key role as also the preferred sphere S^2 at the boundary of CD. It is perhaps not accident that a generalization of braiding was discovered in integrable quantum field theories in M^2. The S-matrix of this theory is rather trivial looking: particle moving with different velocities cross each other and suffer a phase lag and permutation of 2-momenta which has physical effects only in the case of non-identical particles. The R-matrix describing this process reduces to the R-matrix describing the basic braiding operation in braid theories at the static limit.

I have already earlier conjectured that this kind of integrable QFT is part of quantum TGD [K4]. The natural guess is that it describes what happens for the projections of 4-momenta in

M^2 in scattering process inside lines of generalized Feynman diagrams. If integrable theories in M^2 control this scattering, it would cause only phase changes and permutation of the M^2 projections of the 4-momenta. The most plausible guess is that M^2 QFT characterized by R-matrix describes what happens to the braid momenta during the free propagation and the remaining challenge would be to understand what happens in the vertices defined by 2-D partonic surfaces at which re-distribution of braid strands takes place.

2.3.3 How quantum TGD as almost topological QFT differs from topological QFT for braids and 3-manifolds

One must distinguish between two topological QFTs. These correspond to topological QFT defining braid invariants and invariants of 3-manifolds respectively. The reason is that knots are an essential element in the procedure yielding 3-manifolds. Both 3-manifold invariants and knot invariants would be defined as Wilson loops involving path integral over gauge connections for a given 3-manifold with exponent o non-Abelkian f Chern-Simons action defining the weight.

(a) In TGD framework the topological QFT producing braid invariants for a given 3-manifold is replaced with sub-manifold braid theory. Kähler action reduces Chern-Simons terms for preferred extremals and only these contribute to the functional integral. What is the counterpart of topological invariance in this framework? Are general isotopies allowed or should one allow only sub-group of symplectic group of CD boundary leaving the end points of braids invariant? For this option Reidermeister moves are undetectable in the finite measurement resolution defined by the subgroup of the symplectic group. Symplectic transformations would not affect 3-surfaces as the analogs of abstract contact manifold since induced Kähler form would not be affected and only the imbedding would be changed.

    In the approach based on inclusions of HFFs gauge invariance or its generalizations would represent finite measurement resolution (the action of included algebra would generate states not distiguishable from the original one).

(b) There is also ordinary topological QFT allowing to construct topological invariants for 3-manifold. In TGD framework the analog of topological QFT is defined by Chern-Simons-Kähler action in the space of preferred 3-surfaces. Now one sums over small deformations of 3-surface instead of gauge potentials. If extremals of Chern-Simons-Kähler action are in question, symplectic invariance is the most that one can hope for and this might be the situation quite generally. If all light-like 3-surfaces are allowed so that only weak form of electric-magnetic duality at them would bring metric into the theory, it might be possible to have topological invariance at 3-D level but not at 4-D level. It however seems that symplectic invariance with respect to subgroup leaving end points of braids invariant is the realistic expectation.

2.3.4 Could the allowed braids define Legendrian sub-manifolds of contact manifolds?

The basic questions concern the identification of braids and 2-braids. In quantum TGD they cannot be arbitrary but determined by dynamics providing space-time correlates for quantum dynamics. The deformations of braids should mean also deformations of 3-surfaces which as topological manifolds would however remain as such. Therefore topological QFT for given 3-manifold with path integral over gauge connections would in TGD correspond to functional integral of 3-surfaces corresponding to same topology even symplectic structure. The quantum fluctuating degrees of freedom indeed correspond to symplectic group divided by its subgroup defining measurement resolution.

What is the dynamics defining the braids strands? What selects them? I have considered this problem several times. Just two examples is enough here.

(a) Could they be some special light-like curves? Could the condition that the end points of the curves correspond to rational points in some preferred coordinates allow to select these light-like curves? But what about light-like curves associated with the ends of the space-time surface?

(b) The solutions of Kähler-Dirac equation [K13] are localized to curves by using the analog of periodic boundary conditions: the length of the curve is quantized in the effective metric defined by the Kähler-Dirac gamma matrices. Here one however introcuced a coordinate along light-like 3-surface and it is not clear how one should fix this preferred coordinate.

1. Legendrian and Lagrangian sub-manifolds

A hint about what is missing comes from the observation that a non-vanishing Chern-Simons-Kähler form A defines a contact structure [A2] at light-like 3-surfaces if one has $A \wedge dA \neq 0$. This condition states complete non-intebrability of the distribution of 2-planes defined by the condition $A_\mu t^\mu = 0$, where t is tangent vector in the tangent bundle of light-like 3-surface. It also states that the flow lines of A do *not* define global coordinate varying along them.

(a) It is however possible to have 1-dimensional curves for which $A_\mu t^\mu = 0$ holds true at each point. These curves are known as Legendrian sub-manifolds to be distinguished from Lagrangian manifolds for which the projection of symplectic form expressible locally as $J = dA$ vanishes. The set of this curves is discrete so that one obtains braids. Legendrian knots are the simplest example of Legendrian sub-manifolds and the question is whether braid strands could be identified as Legendrian knots. For Legendrian braids symplectic invariance replaces topological invariance and Legendrian knots and braids can be trivial in topological sense. In some situations the property of being Legendrian implies unknottedness.

(b) For Legendrian braid strands the Kähler gauge potential vanishes. Since the solutions of the Kähler-Dirac equation are localized to braid strands, this means that the coupling to Kähler gauge potential vanishes. From physics point of view a generalization of Legendre braid strand by allowing gauge transformations $A \rightarrow A + d\Phi$ looks natural since it means that the coupling of induced spinors is pure gauge terms and can be eliminated by a gauge transformation.

2. 2-D duals of Legendrian sub-manifolds

One can consider also what might be called 2-dimensional duals of Legendrian sub-manifolds.

(a) Also the one-form obtained from the dual of Kähler magnetic field defined as $B^\mu = \epsilon^{\mu\nu\gamma} J_{\nu\nu}$ defines a distribution of 2-planes. This vector field is ill-defined for light-like surfaces since contravariant metric is ill-defined. One can however multiply B with the square root of metric determining formally so that metric would disappear completely just as it disappears from Chern-Simons action. This looks however somewhat tricky mathematically. At the 3-D space-like ends of space-time sheets at boundaries of CD B^μ is however well-defined as such.

(b) The distribution of 2-planes is integrable if one has $B \wedge dB = 0$ stating that one has Beltrami field: physically the conditions states that the current dB feels no Lorentz force. The geometric content is that B defines a global coordinate varying along its flow lines. For the preferred extremals of Kähler action Beltrami condition is satisfied by isometry currents and Kähler current in the interior of space-time sheets. If this condition holds at 3-surfaces, one would have an global time coordinate and integrable distribution of 2-planes defining a slicing of the 2-surface. This would realize the conjecture that space-time surface has a slicing by partonic 2-surfaces. One could say that the 2-surfaces defined by the distribution are orthogonal to B. This need not however mean that the projection of J to these 2-surfaces vanishes. The condition $B \wedge dB = 0$ on the space-like 3-surfaces could be interpreted in terms of effective 2-dimensionality. The simplest option posing no additional conditions would allow two types of braids at space-like 3-surfaces and only Legendrian braids at light-like 3-surfaces.

These observations inspire a question. Could it be that the conjectured dual slicings of space-time sheets by space-like partonic 2-surfaces and by string world sheets are defined by A_μ and B^μ respectively associated with slicings by light-like 3-surfaces and space-like 3-surfaces? Could partonic 2-surfaces be identified as 2-D duals of 1-D Legendrian sub-manifolds?

The identification of braids as Legendrian braids for light-like 3-surfaces and with Legendrian braids or their duals for space-like 3-surfaces would in turn imply that topological braid theory is replaced with a symplectic braid theory in accordance with the view about TGD as almost topological QFT. If finite measurement resolution corresponds to the replacement of symplectic group with the coset space obtained by dividing by a subgroup, symplectic subgroup would take the role of isotopies in knot theory. This symplectic subgroup could be simply the symplectic group leaving the end points of braids invariant.

2.3.5 An attempt to identify the constraints on the braid algebra

The basic problems in understanding of quantum TGD are conceptual. One must proceed by trying to define various concepts precisely to remove the many possible sources of confusion. With this in mind I try collect essential points about generalized Feynman diagrams and their relation to braid diagrams and Feynman diagrams and discuss also the most obvious constraints on algebraization.

Let us first summarize what generalized Feynman diagrams are.

(a) Generalized Feynman diagrams are 3-D (or 4-D, depends on taste) objects inside $CD \times CP_2$. Ordinary Feynman diagrams are in plane. If finite measurement resolution has as a space-time correlate discretization at the level of partonic 2-surfaces, both space-like and light-like 3-surfaces reduce to braids and the lines of generalized Feynman diagrams correspond to braids. It is possible to obtain the analogs of ordinary Feynman diagrams by projection to $M^2 \subset M^4$ defined uniquely for given CD. The resulting apparent intersections would represent ne particular kind of exotic intersection.

(b) Light-like 3-surfaces define the lines of generalized Feynman diagrams and the braiding results naturally. Non-trivial first homology for the orbits of partonic 2-surfaces with genus $g > 0$ could be called homological virtual intersections.

(c) It zero energy ontology braids must be characterized by time orientation. Also it seems that one must distinguish in zero energy ontology between on mass shell braids and off mass shell braid pairs which decompose to pairs of braids with positive and negative energy massless on mass shell states. In order to avoid confusion one should perhaps speak about tangles insie CD rather than braids. The operations of the algebra are same except that the braids can end either to the upper or lower light-like boundary of CD. The projection to M^2 effectively reduces the CD to a 2-dimensional causal diamond.

(d) The vertices of generalized Feynman diagrams are partonic 2-surfaces at which the light-like 3-surfaces meet. This is a new element. If the notion of bosonic emergence is accepted no $n > 2$-vertices are needed so that braid strands are redistributed in the reaction vertices. The redistribution of braid strands in vertices must be introduced as an additional operation somewhat analogous to $\triangleright$ and the challenge is to reduce this operation to something simple. Perhaps the basic operation reduces to an exchange of braid strand between braids. The process can be seen as a decay of of braid with the conservation of braid strands with strands from future and past having opposite strand numbers. Also for this operation the analogs of Reidermeister moves should be identified. In dance metaphor this operation corresponds to a situation in which the dancer leaves the group to which it belongs and goes to a new one.

(e) A fusion of Feynman diagrammatic non-planarity and braid theoretic non-planarity is needed and the projection to M^2 could provide this fusion when at least two kinds of virtual crossings are allowed. The choice of M^2 could be global. An open question is whether the choice of M^2 could characterize separately each line of generalized Feynman diagram characterized by the four-momentum associated with it in the rest system defined by the tips of CD. Somehow the theory should be able to fuse the braiding matrix for integrable QFT in M^2 applying to entire braids with the braiding matrix for braid theory applying at the level of single braid.

Both integral QFTs in M^2 and braid theories suggest that biquandle structure is the structure that one should try to generalized.

(a) The representations of resulting bi-quandle like structure could allow abstract interesting information about generalized Feynman diagrams themselves but the dream is to construct generalized Feynman diagrams as analogs of knot invariants by a recursive procedure analogous to un-knotting of a knot.

(b) The analog of bi-quandle algebra should have a hierarchical structure containing braid strands at the lowest level, braids at next level, and braids of braids...of braids at higher levels. The notion of operad would be ideal for formulating this hierarchy and I have already proposed that this notion must be essential for the generalized Feynman diagrammatics. An essential element is the vanishing of total strand number in the vertex (completely analogous to conserved charged such as fermion number). Again a convenient visualization is in terms of dancers forming dynamical groups, forming groups of groups forming .....

I have already earlier suggested [K4] that the notion of operad [A3] relying on permutation group and its subgroups acting in tensor products of linear spaces is central for understanding generalized Feynman diagrams. $n \rightarrow n_1 + n_2$ decay vertex for n-braid would correspond to "symmetry breaking" $S_n \rightarrow S_{n_1} \times S_{n_2}$. Braid group represents the covering of permutation group so that braid group and its subgroups permuting braids would suggest itself as the basic group theoretical notion. One could assign to each strand of n-braid decaying to n_1 and n_2 braids a two-valued color telling whether it becomes a strand of n_1-braid or n_2-braid. Could also this "color" be interpreted as a particular kind of exotic crossing?

(c) What could be the analogs of Reidermaster moves for braid strands?

   i. If the braid strands are dynamically determined, arbitrary deformations are not possible. If however all isotopy classes are allowed, the interpretation would be that a kind of gauge choice selecting one preferred representation of strand among all possible ones obtained by continuous deformations is in question.

   ii. Second option is that braid strands are dynamically determined within finite measurement resolution so that one would have braid theory in given length scale resolution.

   iii. Third option is that topological QFT is replaced with symplectic QFT: this option is suggested by the possibility to identify braid strands as Legendrian knots or their duals. Subgroup of the symplectic group leaving the end points of braids invariant would act as the analog of continous transformations and play also the role of gauge group. The new element is that symplectic transformations affect partonic 2-surfaces and space-time surfaces except at the end points of braid.

(d) Also 2-braids and perhaps also 2-knots could be useful and would provide string theory like approach to TGD. In this case the projections could be performed to the ends of CD or to M^3, which can be identified uniquely for a given CD.

(e) There are of course many additional subtleties involved. One should not forget loop corrections, which naturally correspond to sub-CDs. The hierarchy of Planck constants and number theoretical universality bring in additional complexities.

All this looks perhaps hopelessly complex but the Universe around is complex even if the basic principles could be very simple.

2.4 About String World Sheets, Partonic 2-Surfaces, And Two-Knots

String world sheets and partonic 2-surfaces provide a beatiful visualization of generalized Feynman diagrams as braids and also support for the duality of string world sheets and partonic 2-surfaces as duality of light-like and space-like braids. Dance metaphor is very helpful here.

(a) The projection of string world sheets and partonic 2-surfaces to 3-D space replaces knot projection. In TGD context this 3-D of space could correspond to the 3-D light-like boundary of CD and 2-knot projection would correspond to the projection of the braids associated with the lines of generalized Feynman diagram. Another identification would be as $M^1 \times E^2$, where M^1 is the line connecting the tips of CD and E^2 the orthogonal complement of M^2.

(b) Using dance metaphor for light-like braiding, braids assignable to the lines of generalized Feynman diagrams would correspond to groups of dancers. At vertices the dancing groups would exchange members and completely new groups would be formed by the dancers. The number of dancers (negative for those dancing in the reverse time direction) would be conserved. Dancers would be connected by threads representing strings having braid points at their ends. During the dance the light-like braiding would induce space-like braiding as the threads connecting the dancers would get entangled. This would suggest that the light-like braids and space-like braidings are equivalent in accordance with the conjectured duality between string-world sheets and partonic 2-surfaces. The presence of genuine 2-knottedness could spoil this equivalence unless it is completely local.

Can string world sheets and partonic 2-surfaces get knotted?

(a) Since partonic 2-surfaces (wormhole throats) are imbedded in light-cone boundary, the preferred 3-D manifolds to which one can project them is light-cone boundary (boundary of CD). Since the projection reduces to inclusion these surfaces cannot get knotted. Only if the partonic 2-surfaces contains in its interior the tip of the light-cone something non-trivial identifiable as virtual 2-knottedness is obtained.

(b) One might argue that the conjectured duality between the descriptions provided by partonic 2-surfaces and string world sheets requires that also string world sheets represent trivial 2-braids. I have shown earlier that nontrivial local knots glued to the string world sheet require that M^4 time coordinate has a local maximum. Does this mean that 2-knots are excluded? This is not obvious: TGD allows also regions of space-time surface with Euclidian signature and generalized Feynman graphs as 4-D space-time regions are indeed Euclidian. In these regions string world sheets could get knotted.

What happens for knot diagrams when the dimension of knot is increased to two? According to the articles of Nelson [A6] and Carter [A4] the crossings for the projections of braid strands are replaced with more complex singularities for the projections of 2-knots. One can decompose the 2-knots to regions surrounded by boxes. Box can contain just single piece of 2-D surface; it can contain two intersection pieces of 2-surfaces as the counterpart of intersecting knot strands and one can tell which of them is above which; the box can contain also a discrete point in the intersection of projections of three disjoint regions of knot which consists of discrete points; and there is also a box containing so called cone point. Unfortunately, I failed to understand the meaning of the cone point.

For 2-knots Reidemeister moves are replaced with Roseman moves. The generalization would allow virtual self intersections for the projection and induced by the non-trivial second homology of 4-D imbedding space. In TGD framework elementary particles have homologically non-trivial partonic 2-surfaces (magnetic monpoles) as their building bricks so that even if 2-knotting in standard sense might be not allowed, virtual 2-knotting would be possible. In TGD framework one works with a subgroup of symplectic transformations defining measurement resolution instead of isotopies and this might reduce the number of allowed mov

2.4.1 The dynamics of string world sheets and the expression for Kähler action

The dynamics of string world sheets is an open question. Effective 2-dimensionality suggests that Kähler action for the preferred extremal should be expressible using 2-D data but there are several guesses for what the explicit expression could be, and one can only make only guesses at this moment and apply internal consistency conditions in attempts to kill various options.

1. Could weak form of electric-magnetic duality hold true for string world sheets?

If one believes on duality between string world sheets and partonic 2-surfaces, one can argue that string world sheets are most naturally 2-surfaces at which the weak form of electric magnetic duality holds true. One can even consider the possibility that the weak form of electric-magnetic duality holds true only at the the string world sheets and partonic 2-surfaces but not at the preferred 3-surfaces.

(a) The weak form of electric magnetic duality would mean that induced Kähler form is non-vanishing at them and Kähler magnetic flux over string world sheet is proportional to Kähler electric flux.

(b) The flux of the induced Kähler form of CP_2 over string world sheet would define a dimensionless "area". Could Kähler action for preferred extremals reduces to this flux apart from a proportionality constant. This "area" would have trivially extremum with respect to symplectic variations if the braid strands are Legendrian sub-manifolds since in this case the projection of Kähler gauge potential on them vanishes. This is a highly non-trivial point and favors weak form of electric-magnetic duality and the identification of Kähler action as Kähler magnetic flux. This option is also in spirit with the vision about TGD as almost topological QFT meaning that induced metric appears in the theory only via electric-magnetic duality.

(c) Kähler magnetic flux over string world sheet has a continuous spectrum so that the identification as Kähler action could make sense. For partonic 2-surfaces the magnetic flux would be quantized and give constant term to the action perhaps identifiable as the contribution of CP_2 type vacuum extremals giving this kind of contribution.

The change of space-time orientation by changing the sign of permutation symbol would change the sign in electric-magnetic duality condition and would not be a symmetry. For a given magnetic charge the sign of electric charge changes when orientation is changed. The value of Kähler action does not depend on space-time orientation but weak form of electric-magnetic duality as boundary condition implies dependence of the Kähler action on space-time orientation. The change of the sign of Kähler electric charge suggests the interpretation of orientation change as one aspect of charge conjugation. Could this orientation dependence be responsible for matter antimatter asymmetry?

2. Could string world sheets be Lagrangian sub-manifolds in generalized sense?

Legendrian sub-manifolds can be lifted to Lagrangian sub-manifolds [A2] Could one generalize this by replacing Lagrangian sub-manifold with 2-D sub-manifold of space-times surface for which the projection of the induced Kähler form vanishes? Could string world sheets be Lagrangian sub-manifolds?

I have also proposed that the inverse image of homologically non-trivial sphere of CP_2 under imbedding map could define counterparts of string world sheets or partonic 2-surfaces. This conjecture does not work as such for cosmic strings, massless extremals having 2-D projection since the inverse image is in this case 4-dimensional. The option based on homologically non-trivial geodesic sphere is not consistent with the identification as analog of Lagrangian manifold but the identification as the inverse image of homologically trivial geodesic sphere is.

The most general option suggested is that string world sheet is mapped to 2-D Lagrangian sub-manifold of CP_2 in the imbedding map. This would mean that theory is exactly solvable at string world sheet level. Vacuum extremals with a vanishing induced Kähler form would be exceptional in this framework since they would be mapped as a whole to Lagrangian sub-manifolds of CP_2. The boundary condition would be that the boundaries of string world sheets defined by braids at preferred 3-surfaces are Legendrian sub-manifolds. The generalization would mean that Legendrian braid strands could be continued to Lagrangian string world sheets for which induced Kähler form vanishes. The physical interpretation would be that if particle moves along this kind of string world sheet, it feels no covariant Lorentz-Kähler force and contra variant Lorentz forces is orthogonal to the string world sheet.

There are however serious objections.

(a) This proposal does not respect the proposed duality between string world sheets and partonic 2-surfaces which as carries of Kähler magnetic charges cannot be Lagrangian 2-manifolds.

(b) One loses the elegant identification of Kähler action as Kähler magnetic flux since Kähler magnetic flux vanishes. Apart from proportionality constant Kähler electric flux

$$\int_{Y^2} *J$$

is as a dimensionless scaling invariant a natural candidate for Kähler action but need not be extremum if braids are Legendrian sub-manifolds whereas for Kähler magnetic flux this is the case. There is however an explicit dependence on metric which does not conform with the idea that almost topological QFT is symplectic QFT.

(c) The sign factor of the dual flux which depends on the orientation of the string world sheet and thus changes sign when the orientation of space-time sheet is changed by changing that of the string world sheet. This is in conflict with the independence of Kähler action on orientation. One can however argue that the orientation makes itself actually physically visible via the weak form of electric-magnetic duality. If the above discussed duality holds true, the net contribution to Kähler action would vanish as the total Kähler magnetic flux for partonic 2-surfaces. Therefore the duality cannot hold true if Kähler action reduces to dual flux.

(d) There is also a purely formal counter argument. The inverse images of Lagrangian sub-manifolds of CP_2 can be 4-dimensional (cosmic strings and massless extremals) whereas string world sheets are 2-dimensonal.

2.4.2 String world sheets as minimal surfaces

Effective 2-dimensionality suggests a reduction of Kähler action to Chern-Simons terms to the area of minimal surfaces defined by string world sheets holds true [K6]. Skeptic could argue that the expressibility of Kähler action involving no dimensional parameters except CP_2 scaled does not favor this proposal. The connection of minimal surface property with holomorphy and conformal invariance however forces to take the proposal seriously and it is easy to imagine how string tension emerges since the size scale of CP_2 appears in the induced metric [K6].

One can ask whether the mimimal surface property conforms with the proposal that string worlds sheets obey the weak form of electric-magnetic duality and with the proposal that they are generalized Lagrangian sub-manifolds.

(a) The basic answer is simple: minimal surface property and possible additional conditions (Lagrangian sub-manifold property or the weak form of electric magnetic duality) poses only additional conditions forcing the space-time sheet to be such that the imbedded string world sheet is a minimal surface of space-time surface: minimal surface property is a condition on space-time sheet rather than string world sheet. The weak form of electric-magnetic duality is favored because it poses conditions on the first derivatives in the normal direction unlike Lagrangian sub-manifold property.

(b) Any proposal for 2-D expression of Kähler action should be consistent with the proposed real-octonion analytic solution ansatz for the preferred extremals [K2]. The ansatz is based on real-octonion analytic map of imbedding space to itself obtained by algebraically continuing real-complex analytic map of 2-D sub-manifold of imbedding space to another such 2-D sub-manifold. Space-time surface is obtained by requiring that the "imaginary" part of the map vanishes so that image point is hyper-quaternion valued. Wick rotation allows to formulate the conditions using octonions and quaternions. Minimal surfaces (of space-time surface) are indeed objects for which the imbedding maps are holomorphic and the real-octonion analyticity could be perhaps seen as algebraic continuation of this property.

(c) Does Kähler action for the preferred exremals reduce to the area of the string world sheet or to Kähler magnetic flux or are the representations equivalent so that the induced Kähler form would effectively define area form? If the Kähler form form associated with the induced metric on string world sheet is proportional to the induced Kähler form the Kähler magnetic flux is proportional to the area and Kähler action reduces to genuine area. Could one pose this condition as an additional constraint on string world sheets? For Lagrangian sub-manifolds Kähler electric field should be proportional to the area form and the condition involves information about space-time surface and is therefore more complex and does not look plausible.

2.4.3 Explicit conditions expressing the minimal surface property of the string world sheet

It is instructive to write explicitly the condition for the minimal surface property of the string world sheet and for the reduction of the area Kähler form to the induced Kähler form. For string world sheets with Minkowskian signature of the induced metric Kähler structure must be replaced by its hyper-complex analog involving hyper-complex unit e satisfying $e^2 = 1$ but replaced with real unit at the level hyper-complex coordinates. e can be represented as antisymmetric Kähler form J_g associated with the induced metric but now one has $J_g^2 = g$ instead of $J_g^2 = -g$. The condition that the signed area reduces to Kähler electric flux means that J_g must be proportional to the induced Kähler form: $J_g = kJ$, $k = constant$ in a given space-time region.

One should make an educated guess for the imbedding of the string world sheet into a preferred extremal of Kähler action. To achieve this it is natural to interpret the minimal surface property as a condition for the preferred Kähler extremal in the vicinity of the string world sheet guaranteeing that the sheet is a minimal surface satisfying $J_g = kJ$. By the weak form of electric-magnetic duality partonic 2-surfaces represent both electric and magnetic monopoles. The weak form of electric-magnetic duality requires for string world sheets that the Kähler magnetic field at string world sheet is proportional to the component of the Kähler electric field parallel to the string world sheet. Kähler electric field is assumed to have component only in the direction of string world sheet.

1. Minkowskian string world sheets

Let us try to formulate explicitly the conditions for the reduction of the signed area to Kähler electric flux in the case of Minkowskian string world sheets.

(a) Let us assume that the space-time surface in Minkowskian regions has coordinates coordinates $(u, v, w, \overline{w})$ [K2]. The pair (u, v) defines light-like coordinates at the string world sheet having identification as hyper-complex coordinates with hyper-complex unit satisfying $e = 1$. u and v need not - nor cannot as it turns out - be light-like with respect to the metric of the space-time surface. One can use (u, v) as coordinates for string world sheet and assume that $w = x^1 + ix^2$ and $\overline{w}$ are constant for the string world sheet. Without a loss of generality one can assume $w = \overline{w} = 0$ at string world sheet.

(b) The induced Kähler structure must be consistent with the metric. This implies that the induced metric satisfies the conditions

$$g_{uu} = g_{vv} \quad = \quad 0 \ . \tag{2.1}$$

The analogs of these conditions in regions with Euclidian signature would be $g_{zz} = g_{\overline{z}\overline{z}} = 0$.

(c) Assume that the imbedding map for space-time surface has the form

$$s^m \quad = \quad s^m(u,v) + f^m(u,v,x^m)_{kl}x^k x^l \ , \tag{2.2}$$

so that the conditions

$$\partial_l k s^m \quad = \quad 0 \ , \ \ \partial_k\partial_u s^m = 0, \ \ \partial_k\partial_v s^m = 0 \tag{2.3}$$

are satisfies at string world sheet. These conditions imply that the only non-vanishing components of the induced CP_2 Kähler form at string world sheet are J_{uv} and $J_{w\overline{w}}$. Same applies to the induced metric if the metric of M^4 satisfies these conditions (no non-vanishing components of form m_{uk} or m_{vk}).

(d) Also the following conditions hold true for the induced metric of the space-time surface

$$\partial_k g_{uv} = 0 \ , \ \ \partial_u g_{kv} = 0 \ , \ \ \partial_v g_{ku} = 0 \ . \tag{2.4}$$

at string world sheet as is easy to see by using the ansatz.

Consider now the minimal surface conditions stating that the trace of the four components of the second fundamental form whose components are labelled by the coordinates $\{x^\alpha\} \equiv (u,v,w,\overline{w})$ vanish for string world sheet.

(a) Since only g_{uv} is non-vanishing, only the components H^k_{uv} of the second fundamental form appear in the minimal surface equations. They are given by the general formula

$$\begin{aligned} H^\alpha_{uv} \quad &= \quad H^\gamma P^\alpha_\gamma \ , \\ H^\alpha \quad &= \quad \left(\partial_u\partial_v x^\alpha + \left({}_\beta{}^\alpha{}_\gamma\right)\partial_u x^\beta \partial_v x^\gamma\right) \ . \end{aligned} \tag{2.5}$$

Here P^α_γ is the projector to the normal space of the string world sheet. Formula contains also Christoffel symbols $\left({}_\beta{}^\alpha{}_\gamma\right)$.

(b) Since the imbedding map is simply $(u,v) \to (u,v,0,0)$ all second derivatives in the formula vanish. Also $H^k = 0$, $k \in \{w,\overline{w}\}$ holds true. One has also $\partial_u x^\alpha = \delta^\alpha_u$ and $\partial_v x^\beta = \delta^\beta_v$. This gives

$$H^\alpha \quad = \quad \left({}_u{}^\alpha{}_v\right) \ . \tag{2.6}$$

All these Christoffel symbols however vanish if the assumption $g_{uu} = g_{vv} = 0$ and the assumptions about imbedding ansatz hold true. Hence a minimal surface is in question.

Consider now the conditions on the induced metric of the string world sheet

(a) The conditions reduce to

$$g_{uu} = g_{vv} \quad = \quad 0 \ . \tag{2.7}$$

The conditions on the diagonal components of the metric are the analogs of Virasoro conditions fixing the coordinate choices in string models. The conditions state that the coordinate lines for u and v are light-like curves in the induced metric.

(b) The conditions can be expressed directly in terms of the induced metric and read

$$\begin{aligned} m_{uu} + s_{kl}\partial_u s^k \partial_u s^l &= 0 \ , \\ m_{vv} + s_{kl}\partial_v s^k \partial_v s^l &= 0 \ . \end{aligned} \tag{2.8}$$

The CP_2 contribution is negative for both equations. The conditions make sense only for $(m_{uu} > 0, m_{vv} > 0)$. Note that the determinant condition $m_{uu}m_{vv} - m_{uv}m_{vu} < 0$ expresses the Minkowskian signature of the (u, v) coordinate plane in M^4.

The additional condition states

$$J^g_{uv} = kJ_{uv} \ . \tag{2.9}$$

It reduces signed area to Kähler electric flux. If the weak form of electric-magnetic duality holds true one can interpret the area as magnetic flux defined as the flux of the dual of induced Kähler form over space-like surface and defining electric charge. A further condition is that the boundary of string world sheet is Legendrean manifold so that the flux and thus area is extremized also at the boundaries.

2.Conditions for the Euclidian string world sheets

One can do the same calculation for string world sheet with Euclidian signature. The only difference is that (u, v) is replaced with $(z, \overline{z})$. The imbedding map has the same form assuming that space-time sheet with Euclidian signature allows coordinates $(z, \overline{z}, w, \overline{w})$ and the local conditions on the imbedding are a direct generalization of the above described conditions. In this case the vanishing for the diagonal components of the string world sheet metric reads as

$$\begin{aligned} h_{kl}\partial_z s^k \partial_z s^l &= 0 \ , \\ h_{kl}\partial_{\overline{z}} s^k \partial_{\overline{z}} s^l &= 0 \ . \end{aligned} \tag{2.10}$$

The natural ansatz is that complex CP_2 coordinates are holomorphic functions of the complex coordinates of the space-time sheet.

3. Wick rotation for Minkowskian string world sheets leads to a more detailed solution ansatz

Wick rotation is a standard trick used in string models to map Minkowskian string world sheets to Euclidian ones. Wick rotation indeed allows to define what one means with real-octonion analyticity. Could one identify string world sheets in Minkowskian regions by using Wick rotation and does this give the same result as the direct approach?

Wick rotation transforms space-time surfaces in $M^4 \times CP_2$ to those in $E^4 \times CP_2$. In $E^4 \times CP_2$ octonion real-analyticity is a well-defined notion and one can identify the space-time surfaces surfaces at which the imaginary part of of octonion real-analytic function vanishes: imaginary part is defined via the decomposition of octonion to two quaternions as $o = q_1 + Iq_2$ where I is a preferred octonion unit. The reverse of the Wick rotation maps the quaternionic surfaces to what might be called hyper-quaternionic surfaces in $M^4 \times CP_2$.

In this picture string world sheets would be hyper-complex surfaces defined as inverse imagines of complex surfaces of quaternionic space-time surface obtained by the inverse of Wick rotation. For this approach to be equivalent with the above one it seems necessary to require that the the treatment of the conditions on metric should be equivalent to that for which hyper-complex unit e is not put equal to 1. This would mean that the conditions reduce to independent conditions for the real and imaginary parts of the real number formally represented as hyper-complex number with $e = 1$.

Wick rotation allows to guess the form of the ansatz for CP_2 coordinates as functions of space-time coordinates In Euclidian context holomorphich functions of space-time coordinates are the natural ansatz. Therefore the natural guess is that one can map the hypercomplex number $t \pm ez$ to complex coordinate $t \pm iz$ by the analog of Wick rotation and assume that CP_2 complex coordinates are analytic functions of the complex space-time coordinates obtained in this manner.

The resulting induced metric could be obtained directly using real coordinates (t, z) for string world sheet or by calculating the induced metric in complex coordinates $t \pm iz$ and by mapping the expressions to hyper-complex numbers by Wick rotation (by replacing i with $e = 1$). If the diagonal components of the induced metric vanish for $t \pm iz$ they vanish also for hyper-complex coordinates so that this approach seem to make sense.

2.4.4 Electric-magnetic duality for flux Hamiltonians and the existence of Wilson sheets

One must distinguish between two conjectured dualities. The weak form of electric-magnetic duality and the duality between string world sheets and partonic 2-surfaces. Could the first duality imply equivalence of not only electric and magnetic flux Hamiltonians but also electric and magnetic Wilson sheets? Could the latter duality allow two different representations of flux Hamiltonians?

(a) For electric-magnetic duality holding true at string world sheets one would have non-vanishing Kähler form and the fluxes would be non-vanishing. The Hamiltonian fluxes

$$Q_{m,A} = \int_{X^2} JH_A dx^1 dx^2 = \int_{X^2} H_A J_{\alpha\beta} dx^\alpha \wedge dx^\beta \quad (2.11)$$

for partonic 2-surfaces X^2 define WCW Hamiltonians playing a key role in the definition of WCW Kähler geometry. They have also interpretation as a generalization of Wilson loops to Wilson 2-surfaces.

(b) Weak form of electric magnetic duality would imply both at partonic 2-surfaces and string world sheets the proportionality

$$Q_{m,A} = \int_{X^2} JH_A dx^1 \wedge dx^2 \propto Q^*_{m,A} = \int_{X^2} H_A * J_{\alpha\beta} dx^\alpha \wedge dx^\beta \ . \quad (2.12)$$

Therefore the electric-magnetic duality would have a concrete meaning also at the level of WCW geometry.

(c) If string world sheets are Lagrangian sub-manifolds Hamiltonian fluxes would vanish identically so that the identification as Wilson sheets does not make sense. One would lose electric-magnetic duality for flux sheets. The dual fluxes

$$*Q_A = \int_{Y^2} *JH_A dx^1 \wedge dx^2 = \int_{Y^2} \epsilon_{\alpha\beta}{}^{\gamma\delta} J_{\gamma\delta} = \int_{Y^2} \frac{\sqrt{det(g_4)}}{det(g_2^\perp)} J_{34}^\perp dx^1 \wedge dx^2$$

for string world sheets Y^2 are however non-vanishing. Unlike fluxes, the dual fluxes depend on the induced metric although they are scaling invariant.

Under what conditions the conjectured duality between partonic 2-surface and string world sheets hold true at the level of WCW Hamiltonians?

(a) For the weak form of electric-magnetic duality at string world sheets the duality would mean that the sum of the fluxes for partonic 2-surfaces and sum of the fluxes for string world sheets are identical apart from a proportionality constant:

$$\sum_i Q_A(X_i^2) \quad \propto \quad \sum_i Q_A(Y_i^2) \ . \tag{2.13}$$

Note that in zero ontology it seems necessary to sum over all the partonic surfaces (at both ends of the space-time sheet) and over all string world sheets.

(b) For Lagrangian sub-manifold option the duality can hold true only in the form

$$\sum_i Q_A(X_i^2) \quad \propto \quad \sum_i Q_A^*(Y_i^2) \ . \tag{2.14}$$

Obviously this option is less symmetric and elegant.

2.4.5 Summary

There are several arguments favoring weak form of electric-magnetic duality for both string world sheets and partonic 2-surfaces. Legendrian sub-manifold property for braid strands follows from the assumption that Kähler action for preferred extremals is proportional to the Kähler magnetic flux associated with preferred 2-surfaces and is stationary with respect to the variations of the boundary. What is especially nice is that Legendrian sub-manifold property implies automatically unique braids. The minimal option favored by the idea that 3-surfaces are basic dynamical objects is the one for which weak form of electric-magnetic duality holds true only at partonic 2-surfaces and string world sheets. A stronger option assumes it at preferred 3-surfaces. Duality between string world sheets and partonic 2-surfaces suggests that WCW Hamiltonians can be defined as sums of Kähler magnetic fluxes for either partonic 2-surfaces or string world sheets.

2.5 What Generalized Feynman Rules Could Be?

After all these explanations the skeptic reader might ask whether this lengthy discussion gives any idea about what the generalized Feynman rules might look like. The attempt to answer this question is a good manner to make a map about what is understood and what is not understood. The basic questions are simple. What constraints does zero energy ontology (ZEO) pose? What does the necessity to projecti the four-momenta to a preferred plane M^2 mean? What mathematical expressions one should assign to the propagator lines and vertices? How does one perform the functional integral over 3-surfaces in finite measurement resolution? The following represents tentatative answers to these questions but does not say much about exact role of algebraic knots.

2.5.1 Zero energy ontology

Zero energy ontology (ZEO) poses very powerful constraints on generalized Feynman diagrams and gives hopes that both UV and IR divergences cancel.

(a) ZEO predicts that the fermions assigned with braid strands associated with the virtual particles are on mass shell massless particles for which the sign of energy can be also negative: in the case of wormhole throats this can give rise to a tachyonic exchange.

(b) The on mass shell conditions for each wormhole throat in the diagram involving loops are very stringent and expected to eliminate very large classes of diagrams. If however given diagonal diagram leading from n-particle state to the same n-particle state -completely analogous to self energy diagram- is possible then the ladders form by these diagrams are also possible and one one obtains infinite of this kind of diagrams as generalized self energy correction and is excellent hopes that geometric series gives a closed algebraic function.

(c) IR divergences plaguing massless theories are cancelled if the incoming and outgoing particles are massive bound states of massless on mass shell particles. In the simplest manner this is achieved when the 3-momenta are in opposite direction. For internal lines the massive on-mass shell-condition is not needed at all. Therefore there is an almost complete separation of the problem how bound state masses are determined from the problem of constructing the scattering amplitudes.

(d) What looks like a problematic aspect ZEO is that the massless on-mass-shell propagators would diverge for wormhole throats. The solution comes from the projection of 4-momenta to M^2. In the generic the projection is time-like and one avoids the singularity. The study of solutions of the Kähler-Dirac equation [K13] and number theoretic vision [K10] indeed suggests that the four-momenta are obtained by rotating massless M^2 momenta and their projections to M^2 are in general integer multiples of hyper-complex primes or light-like. The light-like momenta would be treated like in the case of ordinary Feynman diagrams using $i\epsilon$-prescription of the propagator and would also give a finite contributions corresponding to integral over physical on mass shell states. This guarantees also the vanishing of the possible IR divergences coming from the summation over different M^2 momenta.

There is a strong temptation to identify - or at least relate - the M^2 momenta labeling the solutions of the Kähler-Dirac equation with the region momenta of twistor approach [K11]. The reduction of the region momenta to M^2 momenta could dramatically simplify the twistorial description. It does not seem however plausible that $\mathcal{N} = 4$ super-symmetric gauge theory could allow the identification of M^2 projections of 4-momenta as region momenta. On the other hand, there is no reason to expect the reduction of TGD certainly to a gauge theory containing QCD as part. For instance, color magnetic flux tubes in many-sheeted space-time are central for understanding jets, quark gluon plasma, hadronization and fragmentation [L1] but cannot be deduced from QCD. Note also that the splitting of parton momenta to their M^2 projections and transversal parts is an ad hoc assumption motivated by parton model rather than first principle implication of QCD: in TGD framework this splitting would emerge from first principles.

(e) ZEO strongly suggests that all particles (including photons, gluons, and gravitons) have mass which can be arbitrarily small and could be perhaps seen as being due to the fact that particle "eats" Higgs like states giving it the otherwise lacking polarization states. This would mean a generalization of the notion of Higgs particle to a Higgs like particle with spin. It would also mean rearrangmenet of massless states at wormhole throat level to massives physical states. The slight massication of photon by p-adic thermodynamics does not however mean disappearance of Higgs from spectrum, and one can indeed construct a model for Higgs like states [K15].

The projection of the momenta to M^2 is consistent with this vision. The natural generalization of the gauge condition $p \cdot \epsilon = 0$ is obtained by replacing p with the projection of the total momentum of the boson to M^2 and ϵ with its polarization so that one has $p_{||} \cdot \epsilon$. If the projection to M^2 is light-like, three polarization states are possible in the generic case, so that massivation is required by internal consistency. Note that if intermediate states in the unitary condition were states with light-like M^2-momentum one could have a problematic situation.

(f) A further assumption vulnerable to criticism is that the M^2 projections of all momenta assignable to braid strands are parallel. Only the projections of the momenta to the orthogonal complement E^2 of M^2 can be non-parallel and for massive wormhole throats they must be non-parallel. This assumption does not break Lorentz invariance since in the full amplitude one must integrate over possible choices of M^2. It also interpret the gauge conditions either at the level of braid strands or of partons. Quantum classical correspondence in strong form would actually suggests that quantum 4-momenta should co-incide with the classical ones. The restriction to M^2 projections is however necessary and seems also natural. For instance, for massless extremals only M^2 projection of wave-vector can be well-defined: in transversal degrees of freedom there is a superposition over Fourier components with diffrent transversal wave-vectors. Also the partonic description of hadrons gives for the M^2 projections of the parton momenta a preferred role. It is highly encouraging that this picture emerged first from the Kähler-Dirac equation and purely number theoretic vision based on the identification of M^2 momenta in terms of hyper-complex primes.

The number theoretical approach also suggests a number theoretical quantization of the transversal parts of the momenta [K10]: four-momenta would be obtained by rotating massless M^2 momenta in M^4 in such a manner that the components of the resulting 3-momenta are integer valued. This leads to a classical problem of number theory which is to deduce the number of 3-vectors of fixed length with integer valued components. One encounters the n-dimensional generalization of this problem in the construction of discrete analogs of quantum groups (these "classical" groups are analogous to Bohr orbits) and emerge in quantum arithmetics [K14], which is a deformation of ordinary arithmetics characterized by p-adic prime and giving rigorous justification for the notion of canonical identification mapping p-adic numbers to reals.

(g) The real beauty of Feynman rules is that they guarantee unitarity automatically. In fact, unitarity reduces to Cutkosky rules which can be formulated in terms of cut obtained by putting certain subset of interal lines on mass shell so that it represents on mass shell state. Cut analyticity implies the usual $iDisc(T) = TT^\dagger$. In the recent context the cutting of the internal lines by putting them on-mass-shell requires a generalization.

   i. The first guess is that on mass shell property means that M^2 projection for the momenta is light-like. This would mean that also these momenta contribute to the amplitude but the contribution is finite just like in the usual case. In this formulation the real particles would be the massless wormhole throats.

   ii. Second possibility is that the internal lines on on mass shell states corresponding to massive on mass-shell-particles. This would correspond to the experimental meaning of the unitary conditions if real particles are the massive on mass shell particles. Mathematically it seems possible to pick up from the amplitude the states which correspond to massive on mass shell states but one should understand why the discontinuity should be associated with physical net masses for wormhole contacts or many-particle states formed by them. General connection with unitarity and analyticity might allow to understand this.

(h) CDs are labelled by various moduli and one must integrate over them. Once the tips of the CD and therefore a preferred M^1 is selected, the choice of angular momentum quantization axis orthogonal to M^1 remains: this choice means fixing M^2. These choices are parameterized by sphere S^2. It seems that an integration over different choices of M^2 is needed to achieve Poincare invariance.

2.5.2 How the propagators are determined?

In accordance with previous sections it will be assumed that the braid are Legendrian braids and therefore completely well-defined. One should assign propagator to the braid. A good guess is that the propagator reduces to a product of three terms.

(a) A multi-particle propagator which is a product of collinear massless propagators for braid strands with fermionin number $F = 0, 1 - 1$. The constraint on the momenta is $p_i = \lambda_i p$ with $\sum_i \lambda_i = 1$. So that the fermionic propagator is $\frac{1}{\prod_i \lambda_i} p^k \gamma_k$. If one gas $p = nP$, where P is hyper-complex prime, one must sum over combinations of $\lambda_i = n_i$ satisfying $\sum_i n_i = n$.

(b) A unitary S-matrix for integrable QFT in M^2 in which the velocities of particles assignable to braid strands appear for which fixed by R-matrix defines the basic 2-vertex representing the process in which a particle passes through another one. For this S-matrix braids are the basic units. To each crossing appearing in non-planar Feynman diagram one would have an R-matrix representing the effect of a reconnection the ends of the lines coming to the crossing point. In this manner one could gradually transform the non-planar diagram to a planar diagram. One can ask whether a formulation in terms of a suitable R-matrix could allow to generalize twistor program to apply in the case of non-planar diagrams.

(c) An S-matrix predicted by topological QFT for a given braid. This S-matrix should be constructible in terms of Chern-Simons term defining a sympletic QFT.

There are several questions about quantum numbers assignable to the braid strands.

(a) Can braid strands be only fermionic or can they also carry purely bosonic quantum numbers corresponding to WCW Hamiltonians and therefore to Hamiltonians of $\delta M^4_\pm \times CP_2$? Nothing is lost if one assumes that both purely bosonic and purely fermionic lines are possible and looks whether this leads to inconsistencies. If virtual fermions correspond to single wormhole throat they can have only time-like M^2-momenta. If virtual fermions correspond to pairs of wormhole throats with second throat carrying purely bosonic quantum numbers, also fermionic can have space-like net momenta. The interpretation would be in terms of topological condensation. This is however not possible if all strands are fermionic. Situation changes if one identifies physical fermions wormhole throats at the ends of Kähler magnetic flux tube as one indeed does: in this case virtual net momentum can be space-like if the sign of energy is opposite for the ends of the flux tube.

(b) Are the 3-momenta associated with the wormholes of wormhole contact parallel so that only the sign of energy could distinguish between them for space-like total momentum and M^2 mass squared would be the same? This assumption simplifies the situation but is not absolutely necessary.

(c) What about the momentum components orthogonal to M^2? Are they restricted only by the massless mass shell conditions on internal lines and quantization of the M^2 projection of 4-momentum?

(d) What kind of braids do elementary particles correspond? The braids assigned to the wormhole throat lines can have arbitrary number n of strands and for $n = 1, 2$ the treatment of braiding is almost trivial. A natural assumption is that propagator is simply a product of massless collinear propagators for M^2 projection of momentum [K5]. Collinearity means that propagator is product of a multifermion propagator $\frac{1}{\lambda_i p_k \gamma_k}$, znd multiboson propagator $\frac{1}{\mu_i p_k \gamma_k}$, $\sum \lambda_i + \sum_i \mu_i = 1$. There are also quantization conditions on M^2 projections of momenta from Kähler-Dirac equation implying that multiplies of hyper-complex prime are in question in suitable units. Note however that it is not clear whether purely bosonic strands are present.

(e) For ordinary elementary particles with propagators behaving like $\prod_i \lambda_i^{-1} 1p^{-n}$, only $n \leq 2$ is possible. The topologically really interesting states with more than two braid strands are something else than what we have used to call elementary particles. The proposed interpretation is in terms of anyonic states [K9]. One important implication is that $\mathcal{N} = 1$ SUSY generated by right-handed neutrino or its antineutrino is SUSY for which all members of the multiplet assigned to a wormhole throat have braid number smaller than 3. For $\mathcal{N} = 2$ SUSY generated by right-handed neutrino and its antiparticle the states containing fermion and neutrino-antineutrino pair have three braid strands and SUSY breaking is expected to be strong.

2.5.3 Vertices

Conformal invariance raises the hope that vertices can be deduced from super-conformal invariance as n-point functions. Therefore lines would come from integrable QFT in M^2 and topological braid theory and vertices from confofmal field theory: both theories are integrable.

The basic questions is how the vertices are defined by the 2-D partonic surfaces at which the ends of lines meet. Finite measurement resolution reduces the lines to braids so that the vertices reduces to the intersection of braid strands with the partonic 2-surface.

(a) Conformal invariance is the basic symmetry of quantum TGD. Does this mean that the vertices can be identified as n-point functions for points of the partonic 2-surface defined by the incoming and outgoing braid strands? How strong constraints can one pose on this conformal field theory? Is this field theory free and fixed by anti-commutation relations of induced spinor fields so that correlation function would reduce to product of fermionic two points functions with standard operator in the vertices represented by strand ends. If purely bosonic vertices are present, their correlation functions must result from the functional integral over WCW .

(b) For the fermionic fields associated with each incoming braid the anti-commutators of fermions and anti-fermions are trivial just as the usual equal time anti-commutation relations. This means that the vertex reduces to sum of products of fermionic correlation functions with arguments belonging to different incoming and outgoing lines. How can one calculate the correlators?

   i. Should one perform standard second quantization of fermions at light-like 3-surface allowing infinite number of spinor modes, apply a finite measurement resolution to obtain braids, for each partonic 2-surface, and use the full fermion fields to calculate the correlators? In this case braid strands would be discontinuous in vertices. A possible problem might be that the cutoff in spinor modes seems to come from the theory itself: finite measurement resolution is a property of quantum state itself.

   ii. Could finite measurement resolution allow to approximate the braid strands with continuous ones so that the correlators between strands belonging to different lines are given by anti-commutation relations? This would simplify enormously the situation and would conform with the idea of finite measurement resolution and the vision that interaction vertices reduce to braids. This vision is encouraged by the previous considerations and would mean that replication of braid strands analogous to replication of DNA strands can be seen as a fundamental process of Nature. This of course represents an important deviation from the standard picture.

(c) Suppose that one accepts the latter option. What can happen in the vertex, where line goes from one braid to another one?

   i. Can the direction of momentum changed as visual intuition suggests? Is the total braid momentum conservation the only constraint so that the velocities assignable braid strands in each line would be constrained by the total momentum of the line.

   ii. What kind of operators appear in the vertex? To get some idea about this one can look for the simplest possible vertex, namely FFB vertex which could in fact be the only fundamental vertex as the arguments of [K3] suggest. The propagator of spin one boson decomposes to product of a projection operator to the polarization states divited by p^2 factor. The projection operator sum over products $\epsilon_i^k\gamma_k$ at both ends where γ_k acts in the spinor space defined by fermions. Also fermion lines have spinor and its conjugate at their ends. This gives rise to $p^k\gamma_k/p^2$. $p^k\gamma_k$ is the analog of the bosonic polarization tensor factorizing into a sum over products of fermionic spinors and their conjugates. This gives the BFF vertex $\epsilon_i^k\gamma_k$ slashed between the fermionic propagators which are effectively 2-dimensional.

   iii. Note that if H-chiralities are same at the throats of the wormhole contact, only spin one states are possible. Scalars would be leptoquarks in accordance with general view about lepton and quark number conservation. One particular implication is that Higgs in the standard sense is not possible in TGD framework. It can appear only as a state with a polarization which is in CP_2 direction. In any case, Higgs like states would be eaten by massless state so that all particles would have at least a small mass.

2.5.4 Functional integral over 3-surfaces

The basic question is how one can functionally integrate over light-like 3-surfaces or space-like 3-surfaces.

(a) Does effective 2-dimensionality allow to reduce the functional integration to that over partonic 2-surfaces assigned with space-time sheet inside CD plus radiative corrections from the hierarchy of sub-CDs?

(b) Does finite measurement resolution reduce the functional integral to a ordinary integral over the positions of the end points of braids and could this integral reduce to a sum? Symplectic group of $\delta M^4_\pm \times CP_2$ basically parametrizes the quantum fluctuating degrees of freedom in WCW . Could finite measurement resolution reduce the symplectic group of $\delta M^4_\pm \times CP_2$ to a coset space obtained by dividing with symplectic transformations leaving the end points invariant and could the outcome be a discrete group as proposed? Functional integral would reduce to sum.

(c) If Kähler action reduces to Chern-Simons-Kähler terms to surface area terms in the proposed manner, the integration over WCW would be very much analogous to a functional integral over string world sheets and the wisdom gained in string models might be of considerable help.

2.5.5 Summary

What can one conclude from these argument? To my view the situation gives rise to a considerable optimism. I believe that on basis of the proposed picture it should be possible to build a concrete mathematical models for the generalized Feynman graphics and the idea about reduction to generalized braid diagrams having algebraic representations could pose additional powerful constraints on the construction. Braid invariants could also be building bricks of the generalized Feynman diagrams. In particular, the treatment of the non-planarity of Feynman diagrams in terms of M^2 braiding matrix would be something new and therefore can be questioned.

Few years after writing these lines a view about generalized Feynman diagrams as a stringy generalization of twistor Grassmannian diagrams has emerged [K11]. This approach relies heavily on the localization of spinor modes on 2-D string world sheets (covariantly constant right-handed neutrino is an exception) [K13]. This approach can be regarded as an effective QFT (or rather, effective string theory) approach: all information about the microscopic character of the fundamental particle like entities has been integrated out so that a string model type description at the level of imbdding space emerges. The presence of gigantic symmetries, in particular, the Yangian generalization of super-conformal symmetries, raises hopes that this approach could work. The approach to generalized Feynman diagrams considered above is obviously microscopic.

3 Proposal For A Twistorial Description Of Generalized Feynman Graphs

Listening of the lectures of Nima Arkani-Hamed is always an inspiring experience and so also at this time [B2]. The first recorded lectures was mostly about the basic "philosophical" ideas behind the approach and the second lecture continued discussion of the key points about twistor kinematics which I should already have in my backbone but do not. The lectures stimulated again the feeling that the generalized Feynman diagrammatics has all the needed elements to allow a twistorial description. It should be possible to interpret the diagrams as the analogs of twistorial diagrams.

A couple of new ideas emerged as a result of concentrate effort to build bridge to the twistorial approach.

(a) Generalized Feynman diagrams involve only massless states at wormhole throats so that twistorial description makes sense for the kinematical variables. One should identify the counterparts of the lines and vertices of the twistor diagrams constructed from planar polygons and counterparts of the region momenta.

(b) $M^2 \subset M^4$ appears as a central element of TGD based Feynman diagrammatics and M^2 projection of the four momentum appears in propagator and also in the Kähler-Dirac equation. I realized that p-adic mass calculations must give the thermal expectation value of the M^2 mass squared. Since the throats are massless this means that the transversal momentum squared equal to CP_2 contribution plus conformal weight contribution to mass squared.

(c) It is not too surprising that a very beautiful interpretation in terms of the analogs of twistorial diagrams becomes possible. The idea is to interpret wormhole contacts as pairs of lines of twistor diagrams carrying on mass shell momenta. In this manner triangles with truncated apexes with double line representing the wormhole throats become the basic objects in generalized Feynman diagrammatics. The somewhat mysterious region momenta of twistor approach correspond to momentum exchanges at the wormhole contacts defining the vertices. A reasonable expectation is that the Yangian invariants used to construct the amplitudes of $\mathcal{N} = 4$ SUSY can be used as basic building bricks also now.

(d) Renormalization group is not understood in the usual twistor approach and p-adic considerations and quantization of the size of causal diamond (CD) suggests that the old proposal about discretization of coupling constant evolution to p-adic length scale evolution makes sense. A very concrete realization of the evolution indeed suggest itself and would mean the replacement of each triangle with the quantum superposition of amplitudes associated with triangles with smaller size scale and contained with the original triangle characterized by the size scale of corresponding CD containing it. In fact the incoming and outgoing particles of of vertex could be located at the light-like boundaries of CD.

(e) The approach should be also number theoretically universal and this suggests that the amplitudes should be expressible in terms of quantum rationals and rational functions having quantum rationals as coefficients of powers of the arguments. Quantum rationals are characterized by p-adic prime p and p-adic momentum with mass squared interpreted as p-adic integer appears in the propagator. This means that the propagator proportional to $1/P^2$ is proportional to $1/p$ when mass squared is divisible by p, which means that one has pole like contribution. The real counterpart of propagator in canonical identification is proportional to p. This would select the all CD characterized by n divisible by p as analogs of poles.

3.1 What Generalized Feynman Diagrams Could Be?

Let us first list briefly what these generalized Feynman diagrams emerge and what they should be.

(a) Zero energy ontology and the closely related notion of causal diamond (CD are absolutely essential for the whole approach. U-matrix between zero energy states is unitary but does not correspond to the S-matrix. Rather, U-matrix has as its orthonormal rows M-matrices which are "complex" square roots of density matrices representable as a product of a Hermitian square root of density matrix and unitary and universal S-matrix commuting with it so that the Lie algebra of these Hermitian matrices acts as symmetries of S-matrix. One can allow all M-matrices obtained by allowing integer powers of S-matrix and obtains the analog of Kac-Moody algebra. The powers of S correspond to CD with temporal distance between its tips coming as integer multiple of CP_2 size. The goal is to construct M-matrices and these could be non-unitary because of the presence of the hermitian square root of density matrix.

(b) If is assumed that M-matrix elements can be constructed in terms of generalized Feynman diagrams. What generalized Feynman diagrams strictly speaking are is left open. The basic properties of generalized Feynman diagrams - in particular the property that only massless on mass shell states but with both signs of energy appear- however suggest strongly that they are much more like twistor diagrams and that twistorial method used to sum up Feynman diagrams apply.

3.1.1 The lines of the generalized Feynman diagrams

Generalized Feynman diagrams are constructed using solely diagrams containing on mass shell massless particles in both external and internal lines. Massless-ness could mean also massless-ness in $M^4 \times CP_2$ sense, and p-adic thermodynamics indeed suggests that this is true in some sense.

(a) For massless-ness in $M^4 \times CP_2$ sense the standard twistor description should fail for massive excitations having mass scale of order 10^4 Planck masses. At external lines massless states form massive on mass shell particles. In the following this possible difficulty will be neglected. Stringy picture suggests that this problem cannot be fatal.

(b) Second possibility is that massless states form composites which in the case of fermions have the mass spectrum determined by CP_2 Dirac operator and and that that physical states correspond to states of super-conformal representations with ground states weight determined by the sum of vacuum conformal weight and the contribution of CP_2 mass squared. In this case, one would have massless-ness in M^4 sense but composite would be massless in $M^4 \times CP_2$ sense. In this case twistorial description would work.

(c) The third and the most attractive option is based on the fact that its is M^2 momentum that appears in the propagators. The picture behind p-adic mass calculations is string picture inspired by hadronic string model and in hadron physics one can assign M^2 to longitudinal parts of the parton momenta.

One can therefore consider the possibility that M^2 momentum square obeys p-adic thermodynamics. M^2 momentum appears also in the solutions of the Kähler-Dirac equation so that this identification looks physically very natural. M^2 momentum characterizes naturally also massless extremals (topological light rays) and is in this case massless. Therefore throats could be massless but M^2 momentum identifiable as the physical momentum would be predicted by p-adic thermodynamics and its p-adic norm could correspond to the scale of CD.

Mathematically this option is certainly the most attractive one and it might be also physically acceptable since integration over moduli characterizing M^2 is performed to get the full amplitude so that there is no breaking of Poincare invariance.

There are also other complications.

(a) Massless wormhole throats carry magnetic charges bind to form magnetically neutral composite particles consisting of wormholes connected by magnetic flux tubes. The wormhole throat at the other end of the wormhole carries opposite magnetic charge and neutrino pair cancelling the electro-weak isospin of the physical particle. This complication is completely analogous to the appearance of the color magnetic flux tubes in TGD description of hadrons and will be neglected for a moment.

(b) Free fermions correspond to single wormhole throats and the ground state is massless for them. Topologically condensed fermions carry mass and the ground states has developed mass by p-adic thermodynamics. Above considerations suggests that the correct interpretation of p-adic thermal mass squared is as M^2 mass squared and that the free fermions are still massless! Bosons are always pairs of wormhole throats. It is convenient to denote bosons and topologically condensed fermions by a pair of parallel lines very close to each other and free fermion by single line.

(c) Each wormhole throat carries a braid and braid strands are carriers of four-momentum.

   i. The four momenta are parallel and only the M^2 projection of the momentum appears in the fermionic propagator. To obtain Lorentz invariance one must integrate over boosts of M^2 and this corresponds to integrating over the moduli space of causal diamond (CD) inside which the generalized Feynman diagrams reside.

   ii. Each line gives rise to a propagator. The sign of the energy for the wormhole throat can be negative so that one obtains also space-like momentum exchanges.

iii. It is not quite clear whether one can allow also purely bosonic braid strands. The dependence of the over all propagator factor on longitudinal momentum is $1/p^{2n}$ so that throats carrying 1 or 2 fermionic strands (or single purely bosonic strand) are in preferred position and braid strand numbers larger than 2 give rise to something different than ordinary elementary particle. It is probably not an accident that quantum phases $q = exp(i2\pi/n)$ give rise to bosonic and fermionic statistics for $n = 1, 2$ and to braid statistics for $n > 2$. States with $n \geq 3$ are expected to be anyonic. This also reduces the large super symmetry generated by fermionic oscillator operators at the partonic 2-surfaces effectively to $\mathcal{N} = 1$ SUSY.

In the following It will be assume that all braid strands appearing in the lines are massless and have parallel four-momenta and that M^2 momentum squared is given by p-adic thermodynamics and actually mass squared vanishes. It is also assumed that M^2 momenta of the throats of the wormhole throats are paralleI in accordance with the classical idea that wormhole throats move in parallel. It is convenient to denote graphically the wormhole throat by a pair of parallel lines very close to each other.

3.1.2 Vertices

The following proposal for vertices neglects the fact that physical elementary particles are constructed from wormhole throat pairs connected by magnetic flux tubes. It is however easy to generalizes the proposal to that case.

(a) Conservation of momentum holds in each vertex but only for the total momentum assignable to the wormhole contact rather than for each throat. The latter condition would force all partons to have parallel massless four-momenta and the S-matrix would be more or less trivial. Conservation of four-momentum, the massless on mass shell conditions for 4-momenta of wormhole throats and on mass shell conditions M^2 momentum squared given by stringy mass squared spectrum are extremely powerful and it is quite possible that one obtains in a given resolution defined by the largest and smallest causal diamonds finite number of diagrams.

(b) I have already earlier developed argments strongly suggesting that that only three-vertices are fundamental [K3]. The three vertex at the level of wormhole throats means gluing of the ends of the generalized line along 2-D partonic two surface defining their ends so that diagrams are generalization of Feynman diagrams rather than 4-D generalizations of string diagrams so that a generalization of a a trouser diagram does not describe particle decay). The vertex can be BFF or BBB vertex or a variant of this kind of vertex obtained by replacing some B: s and F: s with their super-partners obtained by adding right handed neutrino or antineutrino on the wormhole throat carrying fermion number. Massless on mass shell conditions hold true for wormhole throats in internal lines but they are not on mass shell as a massive particles like external lines.

(c) What happens in the vertex is momentum exchange between different wormhole throats regarded as braids with strands carrying parallel momenta. This momentum exchange in general corresponds to a non-vanishing mass squared and can be graphically described as a line connecting two vertices of a triangle defined by the particles emerging into the vertex. To each vertex of the triangle either massless fermion line or pair of lines describing topologically condensed fermion or boson enters. The lines connecting the vertices of the triangle carry the analogs of region momenta [K11], which are in general massive but the differences of two adjacent region momenta are massless. The outcome is nothing but the analog of the twistor diagram. 3- vertices are fundamental and one would obtain only 3-gons and the Feynman graph would be a collection of 3-gons such that from each line emerges an internal or external line.

(d) A more detailed graphical description utilizes double lines. For FFB vertices with free fermions one would have 4-gon containing a pair of vertices very near to each other corresponding to the outgoing boson wormhole described by double line. This is obtained by truncating the bosonic vertex of 3-gon and attaching bosonic double line to it. For topologically condensed fermions and BBB vertex one would have 6-gon obtained by truncating all apices of a 3-gon.

Some comments about the diagrammatics is in order.

(a) On mass shell conditions and momentum conservation conditions are extremely powerful so that one has excellent reasons to expect that in a given resolution defined by the largest and smallest CD involves the number of contributing diagrams is finite.

(b) The resulting diagrams are very much like twistor diagrams in $\mathcal{N} = 4$ D=4 SYM for which also three-vertex and its conjugate are the fundamental building bricks from which tree amplitudes are constructed: from tree amplitudes one in turn obtains loop amplitudes by using the recursion formulas. Since all momenta are massless, one can indeed use twistor formalism. For topologically condensed fermions one just forms all possible diagrams consisting of 6-gons for which the truncated apices are connected by double lines and takes care that n lines are taken to be incoming lines.

(c) The lines can cross, and this corresponds to the analog of non-planar diagram. I have proposed a knot-theoretic description of this situation based on the generalized braiding matrix appearing in integrable QFTs defined in M^2. By using a representation for the braiding operation which can be used to eliminate the crossings of the lines one could transform all diagrams to planar diagrams for which one could apply existing construction recipe.

(d) The basic conjecture is that the basic building bricks are Yangian invariants. Not only for the conformal group of M^4 but also for the super-conformal algebra should have an extension to Yangian. This Yangian should be related to the symmetry algebra generated by the M-matrices and analogous to Kac-Moody algebra. For this Yangian points as vertices of the momentum polygon are replaced with partonic 2-surfaces.

3.1.3 Generalization of the diagrammatics to apply to the physical particles

The previous discussion has neglected the fact that the physical particles are not wormhole contacts. Topologically condensed elementary fermions and bosons indeed correspond to magnetic flux pairs at different space-time sheets with wormhole contacts at the ends. How could one describe this situation in terms of the generalization Feynman diagrams?

The natural guess is that one just puts two copies of diagrams above each other so that the triangles are replaced with small cylinders with cross section given by the triangle and the edges of this triangular cylinder representing magnetic flux tubes. It is natural to allow momentum exchanges also at the other end of the cylinder: for ordinary elementary particle these ends carry only neutrino pairs so that the contribution to interactions is screening at small momenta. Also momentum exchanges long the direction of the cylinder should be allowed and would correspond to the non-perturbative low energy degrees of freedom in the case of hadrons. This momentum exchange assignable to flux tube would be between the truncated triangle rather than separately along the three vertical edges of the triangular cylinder.

3.2 Number Theoretical Universality And Quantum Arithmetics

The approach should be also number theoretically universal meaning that amplitudes should make sense also in p-adic number fields or perhaps in adelic sense in the tensor product of p-adic numbers fields. Quantum arithmetics is characterized by p-adic prime and canonical identification mapping p-adic amplitudes to real amplitudes is expected to make number theoretical universality possible.

This is achieved if the amplitudes should be expressible in terms of quantum rationals and rational functions having quantum rationals as coefficients of powers of the arguments. This would be achieved by simply mapping ordinary rationals to quantum rationals if they appear as coefficients of polynomials appearing in rational functions.

Quantum rationals are characterized by p-adic prime p and p-adic momentum with mass squared interpreted as p-adic integer appears in the propagator. If M^2 mass squared is proportional to this p-adic prime p, propagator behaves as $1/P^2 \propto 1/p$, which means that one has pole like contribution for these on mass shell longitudinal masses. p-Adic mass calculations indeed give mass squared proportional to p. The real counterpart of propagator in canonical identification is proportional to p. This would select the all CD characterized by n divisible by p as analogs of propagator poles. Note that the infrared singularity is moved and the largest p-adic prime appearing as divisor of integer characterizing the largest CD indeed serves as a physical IR cufoff.

It would seem that one must allow different p-adic primes in the generalized Feynman diagram since physical particles are in general characterized by different p-adic primes. This would require the analog of tensor product for different quantum rationals analogous to adeles. These numbers would be mapped to real (or complex) numbers by canonical identification.

3.2.1 How to get only finite number of diagrams in a given IR and UV resolution?

In gauge theory one obtains infinite number of diagrams. In zero energy ontology the overall important additional constraint comes from on mass shell conditions at internal lines and external lines and from the requirement that the M^2 momentum squared is quantized for super-conformal representation in terms of stringy mass squared spectrum.

This condition alone does not however imply that the number of diagrams is finite. If forward scattering diagram is non-vanishing also scattering without on mass shell massive conditions on final state lines is possible. One can construct diagrams representing a repeated $n \to n$ scattering and combining these amplitudes with non-forward scattering amplitude one obtains infinite number of scattering diagrams with fixed initial and final states. Number theoretic universality however requires that the number of the contributing diagrams must be finite unless some analytic miracles happens.

The finite number of diagrams could be achieved if one gives for the vision about CDs within CDs a more concrete metric meaning. In spirit of Uncertainty Principle, the size scale of the CD defined by the temporal distance between its tips could correspond to the inverse of the momentum scale defined as its inverse. A further condition would be that the sub-CDs and their Lorentz boosts are indeed within the CD and do not overlap. Obviously the number of diagrams representing repeated $n-n$ scattering forward scattering is finite if these assumptions are made. This would also suggest a scale hierarchy in powers of 2 for CDs: the reason is that given CD with scale $T = nT(CP_2)$ can contain two non-overlapping sub-CDs with the same rest frame only if sub-CD has size scale smaller than $nT_{CP_2}/2$. This applies also to the Lorentz boosts of the sub-CDs.

Amplitudes would be constructed by labeling the CDs by integer n defining its size scale. p-Adicity suggests that the factorization of n to primes must be important and if $n = p$ condition holds true, a new resonant like contribution appears corresponding to p-adic diagrams involving propagator.

Should one allow all M^2 momenta in the loops in all scales or should one restrict the M^2 momenta to have a particular mass squared scale determined somehow by the size of CD involved? If this kind of constraint is posed it must be posed in mathematically elegant manner and it is not clear how to to this.

Is this kind of constraint really necessary? Quantum arithmetics for the length scale characterized by p-adic prime p would make M^2 mass squared values divisible by p to almost poles of the propagators, and this might be enough to effectively select the particular p and corresponding momentum scale and CD scale. Consider only the Mersenne prime $M_{127} = 2^{127} - 1$ as a concrete example.

3.2.2 How to realize the number theoretic universality?

One should be able to realized the p-adicity in some elegant manner. One must certainly allow different p-adic primes in the same diagram and here adelic structure seems unavoidable as tensor product of amplitudes in different p-adic number fields or rather - their quantum arithmetic counterparts characterized by a preferred prime p and mapped to reals by the substitution $p \rightarrow 1/p$. What does this demand?

(a) One must be able to glue amplitudes in different p-adic number fields together so that the lines in some case must have dual interpretation as lines of two p-adic number fields. It also seems that one must be able to assign p-adic prime and quantum arithmetics characterized by a given prime p to to a given propagator line. This prime is probably not arbitrarily and it will be found that it should not be larger than the largest prime dividing n characterizing the CD considered.

(b) Should one assign p-adic prime to a given vertex?

   i. Suppose first that bare 3-vertices reduce to algebraic numbers containing no rational factors. This would guarantee that they are same in both real and p-adic sense. Propagators would be however quantum rationals and depend on p and have almost pole when the integer valued mass squared is proportional to p.

   ii. The radiative corrections to the vertex would involve propagators and this suggests that they bring in the dependence on p giving rise to p-adic coupling constant evolution for the real counterparts of the amplitudes obtained by canonical identification.

      A. Should also vertices obey p-adic quantum arithmetics for some p? What about a vertex in which particles characterized by different p-adic primes enter? Which prime defines the vertex or should the vertex somehow be multi-p p-adic? It seems that vertex cannot contain any prime as such although it could depend on incoming p-adic primes in algebraic or transcendental manner.

      B. Could the radiative corrections sum up to algebraic number depending on the incoming p-adic primes? Or are the corrections transcendental as ordinary perturbation theory suggests and involve powers of π and logarithm of mass squared and basically logarithms of some primes requiring infinite-dimensional transcendental extension of p-adic numbers? If radiative corrections depend only on the logarithms of these primes p-adic coupling constant evolution would be obtained. The requirement that radiative vertex corrections vanish does not look physically plausible.

   iii. Only sub-CDs corresponding to integers $m < n$ would be possible as sub-CD. A geometrically attractive possibility is that CD characterized by integer n allows only propagator lines which correspond to prime factors of integers not larger than the largest prime dividing n in their quantum arithmetics. Bare vertices in turn could contain only primes larger than the maximal prime dividing n. This would simplify the situation considerably- This could give rise to coupling constant evolution even in the case that the radiative corrections are vanishing since the rational factors possibly present in vertices would drop away as n would increase.

   iv. Integers $n = 2^k$ give rise to an objection. They would allow only 2-adic propagators and vertices containing no powers of 2. For $p = 2$ the quantum arithmetics reduces to ordinary arithmetics and ordinary rationals correspond to $p = 2$ apart from the fact that powers of 2 mapped to their inverses in the canonical identification. This is not a problem and might relate to the fact that primes near powers of 2 are physically preferred. Indeed, the CDs with $n = 2^k$ would be in a unique position number theoretically. This would conform with the original - and as such wrong - hypothesis that only these time scales are possible for CDs. The preferred role of powers of two supports also p-adic length scale hypothesis.

These observations give rather strong clues concerning the construction of the amplitudes. Consider a CD with time scale characterized by integer n.

(a) For given CD all sub-CDs with $m < n$ are allowed and all p-adicities corresponding to the primes appearing as prime factors of given m are possible. $m = 2^k$ are in a preferred position since $p = 2$ quantum rationals not containing 2 reduce to ordinary rationals.

(b) The geometric condition that sub-CDs and their boosts remain inside CD and do not overlap together with momentum conservation and on-mass-shell conditions on internal lines implies that only a finite number of generalized Feynman diagrams are possible for given CD. This is essential for number theoretical universality. To each sub-CD one must assign its moduli spaces including its not-too-large boosts. Also the planes M^2 associated with sub-CDs should be regarded as independent and one should integrate over their moduli.

(c) The construction of amplitudes with a given resolution would be a process involving a finite number of steps. The notion of renormalization group evolution suggests a generalization as a change of the amplitude induced by adding CDs with size smaller than smallest CDs and their boosts in a given resolution.

(d) It is not clear whether increase of the upper length scale interpreted as IR cutoff makes sense in the similar manner although physical intuition would encourage this expectation.

3.3 How To Understand Renormalization Flow In Twistor Context?

In twistor context the notion of mass renormalization is not straightforward since everything is massless. In TGD framework p-adic mass scale hypothesis suggests a solution to the problem.

(a) At the fundamental level all elementary particles are massless and only their composites forming physical particles are massive.

(b) M^2 mass squared is given by p-adic mass calculations and should correspond to the mass squared of the physical particle. There are contributions from magnetic flux tubes and in the case of baryons this contribution dominates.

(c) p-Adic physics discretizes coupling constant flow. Once the p-adic length scale of the particle is fixed its M^2 momentum squared is fixed and massless takes care of the rest.

Consider now how renormalization flow would emerge in this picture. At the level of generalized Feynman diagrams the change of the IR (UV) resolution scale means that the maximal size of the CDs involve increases (the minimal size of the sides decreases).

Concerning the question what CD scales should be allowed, the situation is not completely clear.

(a) The most general assumption allows integer multiples of CP_2 scale and would guarantee that the products of hermitian matrices and powers of S-matrix commuting with them define Kac-Moody type algebra assignable to M-matrices. If one uses in renormalization group evolution equation CDs corresponding to integer multiples of CP_2 length scale, the equation would become a difference equation for integer valued variable.

(b) p-Adicity would suggest that the scales of CDs come as prime multiples of CP_2 scale. The proposed realization of p-adicity indeed puts CDs characterized by p-adic primes p in a special position since they correspond to the emergence of a vertex corresponding to p-adic prime p which depends on p in the sense that the radiative corrections to 3-vertex can give it a dependence on $log(p)$. This requires infinite-D transcendental extension of p-adic numbers.

   As far as coupling constant evolution in strict sense is considered, a natural looking choice is evolution of vertices as a function of p-adic primes of the particles arriving to the vertex since radiative corresponds are expected to depend on their logarithms.

(c) p-Adic length scale hypothesis would allow only p-adic length scales near powers of two. There are excellent reasons to expect that these scales are selected by a kind of evolutionary process favoring those scales for CDs for which particles are maximally stable. The fact that quantum arithmetics for $p = 2$ reduces to ordinary arithmetics when quantum integers do not contain 2 raises with size scales coming as powers of 2 in a special position and also supports p-adic length scale hypothesis.

Renormalization group equations are based on studying what happens in an infinitesimal reduction of UV resolution scale would mean. Now the change cannot be infinitesimal but must correspond to a change in the scale of CD by one unit defined by CP_2 size scale.

(a) The decrease of UV cutoff means addition of new details represented as bare 3-vertices represented by truncated triangle having size below the earlier length scale resolution. The addition can be done inside the original CD and inside any sub-CD would be in question taking care that the details remain inside CD. The hope is that this addition of details allows a recursive definition. Typically addition would involve attaching two sub-CDs to propagator line or two propagator lines and connecting them with propagator. The vertex in question would correspond to a p-adic prime dividing the integer characterizing the sub-CDs. Also the increase of the shortest length scale makes sense and means just the deletion of the corresponding sub-CDs. Note that also the positions of $sub - CD$s inside CD manner since the number of allowed boosts depends on the position. This would mean an additional complication.

(b) The increase of IR cutoff length means that the size of the largest CD increases. The physical interpretation would be in terms of the time scale in which one observes the process. If this time scale is too long, the process is not visible. For instances, the study of strong interactions between quarks requires short enough scale for CD. At long scales one only observes hadrons and in even longer scales atomic nuclei and atoms.

(c) One could also allow the UV scale to depend on the particle. This scale should correspond to the p-adic mass scales assignable to the stable particle. In hadron physics this kind of renormalization is standard operation.

4 Still About Non-Planar Twistor Diagrams

4.1 Background

A question Krzysztof Bielas about how non-planar Feynman diagrams could be represented in twistor Grassmannian approach inspired a re-reading of the recent article by recent article by Nima Arkani-Hamed et al [B1].

This inspired the conjecture that non-planar twistor diagrams correspond to non-planar Feynman diagrams and a concrete proposal for realizing the earlier proposal [K7] that the contribution of non-planar diagrams could be calculated by transforming them to planar ones by using the procedure applied in knot theories to eliminate crossings by reducing the knot diagram with crossing to a combination of two diagrams for which the crossing is replaced with reconnection. The Wikipedia article about magnetic reconnection explains what reconnection means. More explicitly, the two reconnections for crossing line pair (AB, CD) correspond to the non-crossing line pairs (AD, BC) and (AC, BD).

In the article of Nima et al [B1] the twistor Grassmann program is discussed at rather detailed level and I found that I had moments of "I understand" feeling. A good test for whether this was just an illusion is to try to sum up up some basic ideas involved.

(a) The crucial observation is that the on mass shell condition for n-particle vertex containing massless particles characterized by bi-spinors λ and $\tilde{\lambda}$ can be satisfied if either λ: s or $\tilde{\lambda}$: s are parallel. In the case of 3-vertices this dictates completely the dependence of the vertex on twistor variables for arbitrary helicities. There are therefore two vertices depending on the two manners to satisfy momentum conservation conditions.

In $\mathcal{N} = 4$ theory different helicities belong to the same super multiplet and the dependence on helicities disappears from the amplitude. There are only two twistor 3-vertices : "black" and "white". From on mass shell 3-particle scattering amplitudes one can construct arbitrary planar scattering amplitudes. All virtual particles are on mass shell but complex momenta must be allowed. The physical interpretation of complex momenta in TGD framework is not quite clear: one possibility is that Euclidian regions of space-time surface (lines of generalized Feynman diagram indeed give imaginary contribution to four-momentum as the reality of $\sqrt{g_4}$ as compared to its imaginary value in Minkowskian regions suggests. Euclidian regions are indeed responsible for dissipation.

(b) The diagrams have two basic symmetries. So called mergers and square moves generate twistor diagrams equivalent with the original one. Merger allows to transform a diagram involving n incoming particles and only black or white vertices to single n-vertex. The diagrams can be transformed to bipartite form in which black vertices *resp.* white vertices are lumped to single vertex are connected to each other. Square move rotates 4-particle twistor box diagram (counterpart of tree 4-particle tree diagrams) in which only black and white vertices are connected so that white and black vertices change positions. These equivalences reduce enormously the number of independent diagrams.

These moves imply that in the case of $\mathcal{N} = 4$ SYM the amplitude assignable to the diagram is completely determined by the permutation assignable to it by the so called left-right rule stating that one starts from an external particle, call it "a", and moves along the diagram turning to the left if the vertex is white and to the right if it is black. Eventually one ends up to an external line - call it "b". The fate of "a" in permutation is $\sigma(a) = b$. It is difficult to exaggerate the importance of this result.

These moves are analogous to something, which I proposed long time ago in [K1]. I however concluded that this is too crazy idea even from me and removed the chapter for several years from my homepage. During last year (2012) I returned to this idea from different point of view. The idea was that generalized Feynman diagrams could be seen as a sequences of algebraic operations in the generalization of arithmetic system including besides tensor product and direct sum also their inverse operations. Any fan of the Universe as quantum computer idea would be fascinated by this idea. Given sequence of arithmetic operations has infinite number of different representations: this would be the counterpart for the equivalence for infinite number of twistor diagrams.

(c) The situation in $\mathcal{N} = 4$ theories is analogous to that in 1+1-D integral quantum field theories. Here the basic scattering event is $2 \rightarrow 2$ scattering: 4-vertex instead of 3-vertex. The sole effect of the scattering is permutation of the momenta and quantum numbers of the particle and phase lag. One can say that particle stops for a moment in the scattering vertex. The number of particles is conserved in the scattering. Yang-Baxter equations states that the scattering amplitude is characterized by a permutation (actually braiding that is element in the braid group defining the covering group of permutations).

4.2 Does TGD Generalize $N = 4$ Sym Or 1+1-D Integrable QFT?

What happens in TGD? To what alternative TGD corresponds to: $\mathcal{N} = 4$ SYM or 1+1-D integrable QFT?

(a) Effective 2-dimensionality suggests that 1+1-D integrable QFTs might be the natural analog for TGD. In zero energy ontology fermions are the only fundamental particles and bosons emerge as fermion-anti-fermion pairs at opposite wormhole throat. This implies that 2+2-fermion vertex is the fundamental vertex. This vertex involves wormhole contact and throats as an additional topological ingredient. In TGD framework the conservation of particle numbers is replaced by fermion-number conservation which allows creation of pairs of fundamental fermions, in particular bosons. The essentially new element is the formation of bound states of massless bound states of fermions and anti-fermions which allows to solve the problems related to IR singularities since the theory itself generates the infrared cutoff in terms of mass scales of the bound states identifiable as p-adic mass scales.

(b) Braiding is the key element of 1+1-D integrable QFTs and also in TGD generalized Feynman diagrams can be regarded as generalizations of braid diagrams allowing braids of braids. 3-D light-like orbits of wormhole throats carry braid strands carrying fermion number.

(c) The proposal is that the 2-D plane M^2 carrying Feynman diagram - interpreted usually as a purely combinatorial auxiliary notion - is realized quite concretely as plane $M^2 \subset M^4$ to which the lines of the generalized Feynman diagram are projected. M^2 has several interpretations.

   In quantum measurement theory it corresponds to a plane spanned by the time axis of the rest system and spin quantization axis and characterizes given causal diamond (CD): note that quantum measurement has geometrization at the level of WCW ("world of classical worlds" defined as the space of 3-surfaces).

   At particle physics level M^2 corresponds to the plane of non-physical polarizations.

   M^2 has also number theoretic interpretation as (hyper)-complex plane of complexified octonions spanned by real unit and preferred imaginary unit. If TGD indeed relates closely to 1+1-D integrable QFT, one can ask whether the scattering is such that it represents just a permutation of incoming lines which in ZEO have either positive or negative energy: just this makes possible particle creation since particle number conservation is reduced to fermion number conservation.

(d) One conjecture is that only the M^2 projections of massless 4-momenta of fermions appear as inverses of propagators assignable to the lines of generalized Feynman diagrams if they are actually twistor diagrams as ZEO strongly suggests (virtual fundamental fermions are on mass shell massless particles). Another possibility is that the virtual fermions have non-physical helicities so that the inverse of the massless propagator would not annihilate them.

(e) Knotting and intersections associated with non-planarity would be both described in terms of generalized knot diagrams which are braids of braids... Crossings would result as one projects the lines of generalized Feynman diagram to M^2. The conjecture is that generalized Feynman diagrams allow a generalization of the recursion process used to construct knot invariants to transform the diagrams to sums of planar diagrams to which twistor Grassmannian approach modified so that it applies to fermions applies. In algebraic knot theory one indeed allows also knot diagrams in which the intersection of the lines can be real rather than apparent (strand goes over or below the other one).

4.3 Could One Understand Non-Planar Diagrams In Twistor Approach?

Non-planar Feynman diagrams remain the technical challenge for the twistor Grassmannian approach (I have written something about this earlier in my blog). In ZEO all particles can be seen as bound states of massless fundamental fermions (leptons and quarks assignable to single generation with family replications described topologically). Hence twistor description is very natural in TGD framework

The vague idea that I try to make more precise in sequel is that non-planar diagrams could be reduced to planar ones by a procedure similar to construct knot invariants [K7]. Knots are generalized so that one allows also vertices. The crossings of lines could be reduced by to a combination of non-crossing lines (by reconnecting the four lines in crossing in two different non-crossing manners) and in this manner one would obtain eventually only planar diagrams.

In algebraic knot theory one considers also genuine crossings besides strand going over or below another one. I have discussed this from TGD point of view [K7] (see also the blog posting). One should somehow eliminate the crossing. One could imagine of adding at each crossing a handle to the plane M^2 containing the diagram to obtain an imbedding to a higher genus surface. Knot theoretic approach suggests that the non-planar crossed amplitude is equal to a quantum superposition of two amplitudes without crossing obtained by reconnecting lines.

Also non-planar massless twistor diagrams make sense although only planar ones are discussed in the article by Nima et al [B1]. This raises some questions.

(a) Could the non-planar twistor diagrams represent the contribution of the non-planar Feynman diagrams? This would mean an enormous simplification and perhaps the possibility to calculate the non-planar contribution to the scattering amplitudes. That this should be the case is strongly suggested by the power and elegance of the twistor formalism itself.

(b) Could the identification of the permutation associated with planar diagrams in terms of left-right paths generalize? The hope is that suitably defined right-left paths define a permutation also in the presence of crossings. The basic question is what happens at crossings? Should one go straight through or turn to the right or left in a given crossing? The straight-through option is the most natural one and allows to assign with each right-left path an arrow so that 2^n different arrow combinations are obtained: more details are discussed below.

(c) Knot theory approach suggests that one recursively reduces non-planar amplitude to a superposition of planar amplitudes by replacing at each crossing the amplitude with a superposition of two "more planar" amplitudes obtained by reconnecting the crossing lines in two different manners. The simplest assumption is that one obtains either the sum or difference of the "more planar" amplitudes associated with the resulting two diagrams. How to choose between "+" and " -" ?

It is known that non-planar contributions are negligible at large $\mathcal{N}$ limit for SUSYs. If the relative for "more planar" amplitudes is $-'$, and the two reconnected amplitudes approach asymptotically the same amplitude, one can understand the dominance of the planar amplitudes at this limit. This suggests that "-" is the correct option. But which "more planar" amplitude corresponds to "+" and which to "-" ?

In knot theory the overall sign would be fixed by whether the line that one is traversing goes over or below the crossing line. Now this option does not work. There is however an alternative possibility to fix the signs if one can assign to a given non-planar diagram the 2^n coverings with fixed arrows of right-left paths. Depending on how the reconnection is carried out, the right-left path continues in the same or opposite direction as the arrow assigned with the crossing line. It is natural to assign a positive sign with the "parallel" reconnected diagram and negative sign to the "antiparallel" one.

One might of course argue that the arrows must be consistent so that one should actually allow only the "parallel" option. This would however produce only positive signs so that it does not look promising.

If this procedure works, it reduces non-planar twistor diagrams to a superposition of non-planar ones. One must however check that the procedure is well-defined. Consider first the problem of assigning right-left paths to a non-planar twistor diagram.

(a) One must decide what happens in the crossings and the simplest rule is that one just continues straight forward.

(b) The possibility to assign freely an arrow with two possible directions to right-left path beginning from any external line is essential. Suppose that the notion of right-left paths based on the straight-through rule defines always a permutation also for non-planar diagrams. Suppose that one assign freely two possible arrows to right-left paths beginning from any external line. This would give 2^n assignments altogether.

(c) If the right-left paths $a \to b$ and $b \to a$ are identical, this rule leads to inconsistency since the choice of the arrow for $a \to b$ would fix the arrow for $b \to a$ and the total number of independent choices would be reduced. Fortunately, this situation cannot occur since the right-left path beginning for b leaves the path coming from a at the first vertex.

(d) Note that the notion of decorated permutation introduced by Nima et al also brings in 2^n-fold degeneracy by replacing the set of n external lines with its 2-fold covering space containing $2n$ lines and allowing besides permutation $a \to \sigma(a)$ also $a \to n + \sigma(a)$. Presumably these two descriptions are equivalent. A possible interpretation of the covering would be in terms of braid group representations defining a 2-fold covering of the permutation group.

The recursive elimination of crossings would proceed in the following manner.

(a) One proceeds along right-left path in the direction of its arrow. If the movement in direction opposite to the arrow were allowed the resulting "more planar" amplitudes would sum up to zero. As one changes the direction of arrow, the elimination process begins from $\sigma(a)$ instead of a and proceeds along different path.

(b) When a particular crossing on a given right-left path is eliminated the diagram with a superposition $A - B$ of "more planar" diagrams obtained by reconnection. The rule is that A corresponds to the reconnection for which the directions of the arrows are same and B to that for which they are opposite. One can continue for both resulting reconnected diagrams along the left-right path repeat the procedure at each crossing. k steps produces 2^k planar amplitudes with varying sign factors.

(c) Eventually one ends up to an external line $b = \sigma(a)$: b is expected to depend on the particular "more planar" diagram that one is considering. The diagrams obtained in this manner can still contain crossings. One must continue to some direction and the natural choice is to turn around. The next turning point would be $c = \sigma(\sigma(a))$, where c again depends on the resulting "more planar" diagram. One can repeat the process and eventually end up to a situation in which one has returned back to a and there is no point to continue anymore since the process would repeat itselfw without eliminating crossings anymore.

(d) Crossings could however still be present. What one can do is to repeat the reduction process by starting from some other external line not belonging to the path traversed. The hope is that eventually one has only planar twistor amplitudes reducible to their minimal form using the left-right rule assigning a unique permutation to each resulting planar diagram. One can also hope that the outcome is independent of the order in which one performs these wanderings around the diagrams rise to new diagrams. The similarity of the elimination process to that applied to knots gives hopes that the outcome does not depend on the order in which the right-left paths associated with external particles are treated in the process.

   Permutations can be decomposed to products of cycles in commuting cyclic subgroups Z_{n_i}, $\prod_{n_i} Z_{n_i} \subset S_n$ and $\sum_i n_i = n$. Therefore each cycle for a given final planar diagram defines one step in this process needed to obtain that particular planar diagram.

4.4 How Stringy Diagrams Could Relate To The Planar And Non-Planar Twistor Diagrams?

What also popped up to my innocent mind was a question which any string theorist could probably answer immediately. Could it be that string world sheets with g handles could correspond in QFT description to non-planar diagrams imbeddable to a surface of genus g?

In TGD framework this would have a concrete meaning. In TGD Universe all fermions except right-handed neutrino are localized at string world sheets (sub-manifolds of the 4-surface of $M^4 \times CP_2$ representing space-time). The localization is forced by the condition that the modes of the induced spinor field are eigenstates of electric charge. The generalized Feynman diagrams involves a functional integration over WCW giving an expansion in terms of fermion propagators for fundamental fermions. By symmetry considerations the outcome is expected to give twistorial diagrams but with fermions as fundamental particles rather than super-symmetrized gauge fields. The conjecture is that Yangian symmetry forces twistorial Grassmann amplitudes.

In this framework the non-planar twistor diagrams could indeed correspond to the contributions of space-time surfaces for which string world sheets have handles. In Euclidian regions defining the lines of the generalized Feynman diagram higher genera should be possible although one does not have path integral but functional integral over preferred extremals of Kähler action [K13] for which also the dynamics of the string world sheets is fixed.

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