

## Introduction to "Towards M-Matrix"

M. Pitkänen, December 8, 2012

Email: [matpitka@luukku.com](mailto:matpitka@luukku.com).

[http://tgdtheory.com/public\\_html/](http://tgdtheory.com/public_html/).

Recent postal address: Köydenpunojankatu 2 D 11, 10940, Hanko, Finland.

### Contents

<b>1</b>	<b>Basic Ideas of TGD</b>	<b>3</b>
1.1	Background . . . . .	3
1.2	TGD as a Poincare invariant theory of gravitation . . . . .	3
1.3	TGD as a generalization of the hadronic string model . . . . .	4
1.4	Fusion of the two approaches via a generalization of the space-time concept . . . . .	4
<b>2</b>	<b>The threads in the development of quantum TGD</b>	<b>5</b>
2.1	Quantum TGD as spinor geometry of World of Classical Worlds . . . . .	5
2.2	TGD as a generalized number theory . . . . .	7
2.2.1	p-Adic TGD and fusion of real and p-adic physics to single coherent whole . . . . .	7
2.2.2	The role of classical number fields . . . . .	8
2.2.3	Infinite primes . . . . .	9
2.3	Hierarchy of Planck constants and dark matter hierarchy . . . . .	10
2.3.1	Dark matter as large $\hbar$ phase . . . . .	10
2.3.2	Hierarchy of Planck constants from the anomalies of neuroscience biology . . . . .	10
2.3.3	Does the hierarchy of Planck constants reduce to the vacuum degeneracy of Kähler action? . . . . .	11
2.3.4	Dark matter as a source of long ranged weak and color fields . . . . .	11
2.4	TGD as a generalization of physics to a theory consciousness . . . . .	11
2.4.1	Quantum jump as a moment of consciousness . . . . .	11
2.4.2	The notion of self . . . . .	12
2.4.3	Relationship to quantum measurement theory . . . . .	13
2.4.4	Selves self-organize . . . . .	14
2.4.5	Classical non-determinism of Kähler action . . . . .	14
2.4.6	p-Adic physics as physics of cognition and intentionality . . . . .	14
2.4.7	p-Adic and dark matter hierarchies and hierarchy of moments of consciousness . . . . .	15

---

<b>3</b>	<b>Bird's eye of view about the topics of the book</b>	<b>17</b>
<b>4</b>	<b>The contents of the book</b>	<b>19</b>
4.1	Part I: The recent view about field equations . . . . .	19
4.1.1	Basic extremals of the Kähler action . . . . .	19
4.1.2	The recent vision about preferred extremals and solutions of the modified Dirac equation . . . . .	21
4.2	Part II: General Theory . . . . .	22
4.2.1	Construction of Quantum Theory: Symmetries . . . . .	22
4.2.2	Construction of Quantum Theory: M-matrix . . . . .	23
4.2.3	More about Matrices . . . . .	28
4.2.4	Category Theory and Quantum TGD . . . . .	28
4.2.5	Generalized Feynman Graphs as Generalized Braids . . . . .	29
4.3	Part III: Twistors, Bosonic Emergence, Space-time Supersymmetry . . . . .	29
4.3.1	Twistors, $N = 4$ Super-Conformal Symmetry, and Quantum TGD . . . . .	29
4.3.2	Yangian Symmetry, Twistors, and TGD . . . . .	30
4.3.3	Some Fresh Ideas about Twistorialization of TGD . . . . .	30
4.3.4	Quantum Field Theory Limit of TGD from Bosonic Emergence . . . . .	32
4.3.5	Does the QFT Limit of TGD Have Space-Time Super-Symmetry? . . . . .	32
4.4	Part IV: Hyper-Finite Factors of Type II and Hierarchy of Planck Constants . . . . .	34
4.4.1	What von Neumann Right After All? . . . . .	34
4.4.2	Does TGD predict spectrum of Planck constants? . . . . .	37
4.4.3	Mathematical speculations inspired by the hierarchy of Planck constants . . . . .	38

# 1 Basic Ideas of TGD

The basic physical picture behind TGD was formed as a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model.

## 1.1 Background

*T(opological) G(eometro)D(ynamics)* is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K1]. The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last twenty-three years for the realization of this dream and this has resulted in seven online books about TGD and eight online books about TGD inspired theory of consciousness and of quantum biology.

Quantum *T(opological) G(eometro)D(ynamics)* as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

For few years ago the discussions with Tony Smith initiated a fourth thread which deserves the name 'TGD as a generalized number theory'. The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the "physics as generalized number theory" th

A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and certainly possible in TGD framework. The identification of hierarchy of Planck constants whose values TGD "predicts" in terms of dark matter hierarchy would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

Every updating of the books makes me frustrated as I see how badly the structure of the representation reflects my bird's eye of view as it is at the moment of updating. At this time I realized that the chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called "physics as a generalized number theory". In the following I adopt this view. This reduces the number of threads to four! I am not even sure about the number of threads! Be patient!

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The seven online books [K24, K14, K12, K11, K15, K20, K19] about TGD and eight online books about TGD inspired theory of consciousness and of quantum biology [K23, K3, K13, K2, K7, K8, K10, K18] are warmly recommended to the interested reader.

## 1.2 TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space  $H = M^4 \times CP_2$ , where  $M^4$  denotes Minkowski space and  $CP_2 = SU(3)/U(2)$  is the complex projective space of two complex dimensions [A7, A3, A5, A2].

The identification of the space-time as a submanifold [A1, A6] of  $M^4 \times CP_2$  leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-

momentum in General Relativity.

It soon however turned out that submanifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of  $CP_2$  explains electro-weak and color quantum numbers. The different H-chiralities of  $H$ -spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the  $CP_2$  spinor connection, Killing vector fields of  $CP_2$  and of  $H$ -metric to four-surface define classical electro-weak, color gauge fields and metric in  $X^4$ .

### 1.3 TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

### 1.4 Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there could be "vapor phase" that is a "gas" of particle like 3-surfaces (counterpart of the "baby universes" of GRT) and the nonconservation of energy in GRT corresponds to the transfer of energy between the topological condensate and vapor phase.

What one obtains is what I have christened as many-sheeted space-time. One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory system does not possess this kind of field identity. The notion of magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond ( $CD$ ) identified as the Cartesian product of  $CP_2$  and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of  $CP_2$  size is fundamental.  $CD$ s form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of  $CD$  defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is in terms of initial and final states of a physical event, say particle reaction.

General Coordinate Invariance allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of  $CD$ : this means that space-time surface is analogous to Bohr orbit. An alternative identification is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian and interpreted as lines of

generalized Feynman diagrams. Also the Euclidian 4-D regions would have similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about strong form of holography.

There is a further generalization of the space-time concept inspired by p-adic physics forcing a generalization of the number concept through the fusion of real numbers and various p-adic number fields. Also the hierarchy of Planck constants forces a generalization of the notion of space-time.

A very concise manner to express how TGD differs from Special and General Relativities could be following. Relativity Principle (Poincare Invariance), General Coordinate Invariance, and Equivalence Principle remain true. What is new is the notion of sub-manifold geometry: this allows to realize Poincare Invariance and geometrize gravitation simultaneously. This notion also allows a geometrization of known fundamental interactions and is an essential element of all applications of TGD ranging from Planck length to cosmological scales. Sub-manifold geometry is also crucial in the applications of TGD to biology and consciousness theory.

The worst objection against TGD is the observation that all classical gauge fields are expressible in terms of four imbedding space coordinates only- essentially  $CP_2$  coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particle topologically condenses to several space-time sheets simultaneously and experiences the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

## 2 The threads in the development of quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

### 2.1 Quantum TGD as spinor geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones:

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space  $CH$  consisting of all possible 3-surfaces in  $H$ . "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [A4, A8, A9]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay  $A \rightarrow B + C$ . Classically this corresponds to a path of configuration space leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.
2. During years this naive and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects un-expected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of

manifold property. I have also started introduced the word "world of classical worlds" (WCW) instead of rather formal "configuration space". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

3. WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operator of WCW so that this classical free field theory would dictate M-matrices which form orthonormal rows of what I call U-matrix. Given M-matrix in turn would decompose to a product of a hermitian density matrix and unitary S-matrix.

M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the complex square roots of density matrices commuting with S-matrix means that they span infinite-dimensional Lie algebra acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in well-defined sense: its own symmetries would define the symmetries of the theory. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible.

4. By quantum classical correspondence the construction of WCW spinor structure reduces to the second quantization of the induced spinor fields at space-time surface. The basic action is so called modified Dirac action in which gamma matrices are replaced with the modified gamma matrices defined as contractions of the canonical momentum currents with the imbedding space gamma matrices. In this manner one achieves super-conformal symmetry and conservation of fermionic currents among other things and consistent Dirac equation. This modified gamma matrices define as anticommutators effective metric, which might provide geometrization for some basic observables of condensed matter physics. The conjecture is that Dirac determinant for the modified Dirac action gives the exponent of Kähler action for a preferred extremal as vacuum functional so that one might talk about bosonic emergence in accordance with the prediction that the gauge bosons and graviton are expressible in terms of bound states of fermion and antifermion.

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (*CD*)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the  $\sqrt{g_4}$  factor would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory. Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.
2. The manner to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulombic contribution to Kähler action is required and is true for all known extremals if one makes a

general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this manner almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

3. A further quite recent hypothesis inspired by effective 2-dimensionality is that Chern-Simons terms reduce to a sum of two 2-dimensional terms. An imaginary term proportional to the total area of Minkowskian string world sheets and a real term proportional to the total area of partonic 2-surfaces or equivalently strings world sheets in Euclidian space-time regions. Also the equality of the total areas of strings world sheets and partonic 2-surfaces is highly suggestive and would realize a duality between these two kinds of objects. String world sheets indeed emerge naturally for the proposed ansatz defining preferred extremals. Therefore Kähler action would have very stringy character apart from effects due to the failure of the strict determinism meaning that radiative corrections break the effective 2-dimensionality.

## 2.2 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name 'TGD as a generalized number theory'. It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of hyper-counterparts of classical number fields identified as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product, and the notion of infinite prime.

### 2.2.1 p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzlers and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired 'Universe as Computer' vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both,

how should one glue the physics in different number field together to get *The Physics*? Should one perform p-adicization also at the level of the configuration space of 3-surfaces? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.

2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite these frustrating uncertainties, the number of the applications of the poorly defined p-adic physics grew steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structures. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of imbedding space and space-time concept and one can speak about real and p-adic space-time sheets. The quantum dynamics should be such that it allows quantum transitions transforming space-time sheets belonging to different number fields to each other. The space-time sheets in the intersection of real and p-adic worlds are of special interest and the hypothesis is that living matter resides in this intersection. This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter.

The basic principle is number theoretic universality stating roughly that the physics in various number fields can be obtained as completion of rational number based physics to various number fields. Rational number based physics would in turn describe physics in finite measurement resolution and cognitive resolution. The notion of finite measurement resolution has become one of the basic principles of quantum TGD and leads to the notions of braids as representatives of 3-surfaces and inclusions of hyper-finite factors as a representation for finite measurement resolution.

### 2.2.2 The role of classical number fields

The vision about the physical role of the classical number fields relies on the notion of number theoretic compactification stating that space-time surfaces can be regarded as surfaces of either  $M^8$  or  $M^4 \times CP_2$ . As surfaces of  $M^8$  identifiable as space of hyper-octonions they are hyper-quaternionic or co-hyper-quaternionic- and thus maximally associative or co-associative. This means that their tangent space is either hyper-quaternionic plane of  $M^8$  or an orthogonal complement of such a plane. These surface can be mapped in natural manner to surfaces in  $M^4 \times CP_2$  [K22] provided one can assign to each point of tangent space a hyper-complex plane  $M^2(x) \subset M^4$ . One can also speak about  $M^8 - H$  duality.

This vision has very strong predictive power. It predicts that the extremals of Kähler action correspond to either hyper-quaternionic or co-hyper-quaternionic surfaces such that one can assign to tangent space at each point of space-time surface a hyper-complex plane  $M^2(x) \subset M^4$ . As a consequence, the  $M^4$  projection of space-time surface at each point contains  $M^2(x)$  and its orthogonal complement. These distributions are integrable implying that space-time surface allows dual slicings defined by string world sheets  $Y^2$  and partonic 2-surfaces  $X^2$ . The existence of this kind of slicing was earlier deduced from the study of extremals of Kähler action and christened as Hamilton-Jacobi structure. The physical interpretation of  $M^2(x)$  is as the space of non-physical polarizations and the plane of local 4-momentum.

One can fairly say, that number theoretical compactification is responsible for most of the understanding of quantum TGD that has emerged during last years. This includes the realization of Equivalence Principle at space-time level, dual formulations of TGD as Minkowskian and Euclidian string



model type theories, the precise identification of preferred extremals of Kähler action as extremals for which second variation vanishes (at least for deformations representing dynamical symmetries) and thus providing space-time correlate for quantum criticality, the notion of number theoretic braid implied by the basic dynamics of Kähler action and crucial for precise construction of quantum TGD as almost-topological QFT, the construction of configuration space metric and spinor structure in terms of second quantized induced spinor fields with modified Dirac action defined by Kähler action realizing automatically the notion of finite measurement resolution and a connection with inclusions of hyper-finite factors of type  $II_1$  about which Clifford algebra of configuration space represents an example.

The two most important number theoretic conjectures relate to the preferred extremals of Kähler action. The general idea is that classical dynamics for the preferred extremals of Kähler action should reduce to number theory: space-time surfaces should be either associative or co-associative in some sense.

1. The first meaning for associativity (co-associativity) would be that tangent (normal) spaces of space-time surfaces are quaternionic in some sense and thus associative. This can be formulated in terms of octonionic representation of the imbedding space gamma matrices possible in dimension  $D = 8$  and states that induced gamma matrices generate quaternionic sub-algebra at each space-time point. It seems that induced rather than modified gamma matrices must be in question.
2. Second meaning for associative (co-associativity) would be following. In the case of complex numbers the vanishing of the real part of real-analytic function defines a 1-D curve. In octonionic case one can decompose octonion to sum of quaternion and quaternion multiplied by an octonionic imaginary unit. Quaternionicity could mean that space-time surfaces correspond to the vanishing of the imaginary part of the octonion real-analytic function. Co-quaternionicity would be defined in an obvious manner. Octonionic real analytic functions form a function field closed also with respect to the composition of functions. Space-time surfaces would form the analog of function field with the composition of functions with all operations realized as algebraic operations for space-time surfaces. Co-associativity could be perhaps seen as an additional feature making the algebra in question also co-algebra.
3. The third conjecture is that these conjectures are equivalent.

### 2.2.3 Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory. The work with Riemann hypothesis led to further ideas.

After the realization that infinite primes can be mapped to polynomials representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially satisfying is that p-adic and real regions of the space-time surface could emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the imbedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to 'mind stuff', the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably an extremely brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

## 2.3 Hierarchy of Planck constants and dark matter hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

### 2.3.1 Dark matter as large $\hbar$ phase

D. Da Rocha and Laurent Nottale [E1] have proposed that Schrödinger equation with Planck constant  $\hbar$  replaced with what might be called gravitational Planck constant  $\hbar_{gr} = \frac{GmM}{v_0}$  ( $\hbar = c = 1$ ).  $v_0$  is a velocity parameter having the value  $v_0 = 144.7 \pm .7$  km/s giving  $v_0/c = 4.6 \times 10^{-4}$ . This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of  $v_0$  seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K17].

TGD predicts correctly the value of the parameter  $v_0$  assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of  $v_0$  can be understood as corresponding to perturbations replacing cosmic strings with their  $n$ -branched coverings so that tension becomes  $n^2$ -fold: much like the replacement of a closed orbit with an orbit closing only after  $n$  turns.  $1/n$ -sub-harmonic would result when a magnetic flux tube split into  $n$  disjoint magnetic flux tubes. Also a model for the formation of planetary system as a condensation of ordinary matter around quantum coherent dark matter emerges [K17].

The values of Planck constants postulated by Nottale are gigantic and it is natural to assign them to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta). The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

### 2.3.2 Hierarchy of Planck constants from the anomalies of neuroscience biology

The quantal effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about  $10^{-10}$  times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

Also the anomalies of biology support the view that dark matter might be a key player in living matter.

### 2.3.3 Does the hierarchy of Planck constants reduce to the vacuum degeneracy of Kähler action?

This starting point led gradually to the recent picture in which the hierarchy of Planck constants is postulated to come as integer multiples of the standard value of Planck constant. Given integer multiple  $\hbar = n\hbar_0$  of the ordinary Planck constant  $\hbar_0$  is assigned with a multiple singular covering of the imbedding space [K6]. One ends up to an identification of dark matter as phases with non-standard value of Planck constant having geometric interpretation in terms of these coverings providing generalized imbedding space with a book like structure with pages labelled by Planck constants or integers characterizing Planck constant. The phase transitions changing the value of Planck constant would correspond to leakage between different sectors of the extended imbedding space. The question is whether these coverings must be postulated separately or whether they are only a convenient auxiliary tool.

The simplest option is that the hierarchy of coverings of imbedding space is only effective. Many-sheeted coverings of the imbedding space indeed emerge naturally in TGD framework. The huge vacuum degeneracy of Kähler action implies that the relationship between gradients of the imbedding space coordinates and canonical momentum currents is many-to-one: this was the very fact forcing to give up all the standard quantization recipes and leading to the idea about physics as geometry of the "world of classical worlds". If one allows space-time surfaces for which all sheets corresponding to the same values of the canonical momentum currents are present, one obtains effectively many-sheeted covering of the imbedding space and the contributions from sheets to the Kähler action are identical. If all sheets are treated effectively as one and the same sheet, the value of Planck constant is an integer multiple of the ordinary one. A natural boundary condition would be that at the ends of space-time at future and past boundaries of causal diamond containing the space-time surface, various branches co-incide. This would raise the ends of space-time surface in special physical role.

### 2.3.4 Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken  $U(2)_{ew}$  invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics. A possible solution of the matter antimatter asymmetry is based on the identification of also antimatter as dark matter.

## 2.4 TGD as a generalization of physics to a theory consciousness

General coordinate invariance forces the identification of quantum jump as quantum jump between entire deterministic quantum histories rather than time=constant snapshots of single history. The new view about quantum jump forces a generalization of quantum measurement theory such that observer becomes part of the physical system. Thus a general theory of consciousness is unavoidable outcome. This theory is developed in detail in the books [K23, K3, K13, K2, K7, K8, K10, K18] .

### 2.4.1 Quantum jump as a moment of consciousness

The identification of quantum jump between deterministic quantum histories (configuration space spinor fields) as a moment of consciousness defines microscopic theory of consciousness. Quantum jump involves the steps

$$\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f ,$$

where  $U$  is informational "time development" operator, which is unitary like the S-matrix characterizing the unitary time evolution of quantum mechanics.  $U$  is however only formally analogous to Schrödinger time evolution of infinite duration although there is *no* real time evolution involved. It is not however clear whether one should regard U-matrix and S-matrix as two different things or not:  $U$ -matrix is a completely universal object characterizing the dynamics of evolution by self-organization whereas S-matrix is a highly context dependent concept in wave mechanics and in quantum field theories where it at least formally represents unitary time translation operator at the limit of an infinitely long interaction time. The S-matrix understood in the spirit of superstring models is however something very different and could correspond to U-matrix.

The requirement that quantum jump corresponds to a measurement in the sense of quantum field theories implies that each quantum jump involves localization in zero modes which parameterize also the possible choices of the quantization axes. Thus the selection of the quantization axes performed by the Cartesian outsider becomes now a part of quantum theory. Together these requirements imply that the final states of quantum jump correspond to quantum superpositions of space-time surfaces which are macroscopically equivalent. Hence the world of conscious experience looks classical. At least formally quantum jump can be interpreted also as a quantum computation in which matrix  $U$  represents unitary quantum computation which is however not identifiable as unitary translation in time direction and cannot be 'engineered'.

Can one say anything about the unitary process? Zero energy states correspond in positive energy ontology to physical events and break time reversal invariance. This because either the positive or negative energy part of the state is prepared whereas the second end of  $CD$  corresponds to a superposition of (negative/positive energy) states with varying particle numbers and single particle quantum numbers just as in ordinary particle physics experiment. State function reduction must change the roles of the ends of  $CD$ s. Therefore  $U$ -matrix should correspond to the unitary matrix relating zero energy state basis prepared at different ends of  $CD$  and state function reduction would be equivalent with state preparation.

The basic objection is that the arrow of geometric time alternates at imbedding space level but we know that arrow of time is universal. What one can say about the arrow of time at space-time level? Quantum classical correspondence requires that quantum mechanical irreversibility corresponds to irreversibility at space-time level. If the observer is analogous to an inhabitant of Flatland gaining information only about space-time surface, he or she is not able to discover that the arrow of time alternates at the level of imbedding space. The inhabitant of a folded bath towel is not able to observe the folding of the towel! Only by observing systems for which the imbedding space arrow of time is opposite, observer can discover the alternation. Living systems indeed behave as if they would contain space-time sheets with opposite arrow of geometric time (self-organization). Phase conjugate light beam is second example of this.

#### 2.4.2 The notion of self

The concept of self is absolutely essential for the understanding of the macroscopic and macro-temporal aspects of consciousness. Self corresponds to a subsystem able to remain un-entangled under the sequential informational 'time evolutions'  $U$ . Exactly vanishing entanglement is practically impossible in ordinary quantum mechanics and it might be that 'vanishing entanglement' in the condition for self-property should be replaced with 'subcritical entanglement'. On the other hand, if space-time decomposes into p-adic and real regions, and if entanglement between regions representing physics in different number fields vanishes, space-time indeed decomposes into selves in a natural manner.

It is assumed that the experiences of the self after the last 'wake-up' sum up to single average experience. This means that subjective memory is identifiable as conscious, immediate short term memory. Selves form an infinite hierarchy with the entire Universe at the top. Self can be also interpreted as mental images: our mental images are selves having mental images and also we represent mental images of a higher level self. A natural hypothesis is that self  $S$  experiences the experiences of its subselves as kind of abstracted experience: the experiences of subselves  $S_i$  are not experienced as such but represent kind of averages  $\langle S_{ij} \rangle$  of sub-subselves  $S_{ij}$ . Entanglement between selves, most naturally realized by the formation of join along boundaries bonds between cognitive or material space-time sheets, provides a possible a mechanism for the fusion of selves to larger selves (for instance, the fusion of the mental images representing separate right and left visual fields to single visual field) and forms wholes from parts at the level of mental images.

An attractive possibility suggested by zero energy ontology is that the notions of self and quantum jump reduce to each other and that a fractal hierarchy of quantum jumps within quantum jumps is enough. *CDs* would serve as imbedding space correlates of selves and quantum jumps would be followed by cascades of state function reductions beginning from given *CD* and proceeding downwards to the smaller scales (smaller *CDs*). State function reduction cascades could also take place in parallel branches of the quantum state. One ends up with concrete ideas about how the arrow of geometric time is induced from that of subjective time defined by the experiences induced by the sequences of quantum jumps for sub-selves of self. One ends also ends up with concrete ideas about how the localization of the contents of sensory experience and cognition to the upper boundaries of *CD* could take place.

### 2.4.3 Relationship to quantum measurement theory

The third basic element relates TGD inspired theory of consciousness to quantum measurement theory. The assumption that localization occurs in zero modes in each quantum jump implies that the world of conscious experience looks classical. It also implies the state function reduction of the standard quantum measurement theory as the following arguments demonstrate (it took incredibly long time to realize this almost obvious fact!).

1. The standard quantum measurement theory a la von Neumann involves the interaction of brain with the measurement apparatus. If this interaction corresponds to entanglement between microscopic degrees of freedom  $m$  with the macroscopic effectively classical degrees of freedom  $M$  characterizing the reading of the measurement apparatus coded to brain state, then the reduction of this entanglement in quantum jump reproduces standard quantum measurement theory provide the unitary time evolution operator  $U$  acts as flow in zero mode degrees of freedom and correlates completely some orthonormal basis of configuration space spinor fields in non-zero modes with the values of the zero modes. The flow property guarantees that the localization is consistent with unitarity: it also means 1-1 mapping of quantum state basis to classical variables (say, spin direction of the electron to its orbit in the external magnetic field).
2. Since zero modes represent classical information about the geometry of space-time surface (shape, size, classical Kähler field,...), they have interpretation as effectively classical degrees of freedom and are the TGD counterpart of the degrees of freedom  $M$  representing the reading of the measurement apparatus. The entanglement between quantum fluctuating non-zero modes and zero modes is the TGD counterpart for the  $m - M$  entanglement. Therefore the localization in zero modes is equivalent with a quantum jump leading to a final state where the measurement apparatus gives a definite reading.

This simple prediction is of utmost theoretical importance since the black box of the quantum measurement theory is reduced to a fundamental quantum theory. This reduction is implied by the replacement of the notion of a point like particle with particle as a 3-surface. Also the infinite-dimensionality of the zero mode sector of the configuration space of 3-surfaces is absolutely essential. Therefore the reduction is a triumph for quantum TGD and favors TGD against string models.

Standard quantum measurement theory involves also the notion of state preparation which reduces to the notion of self measurement. Each localization in zero modes is followed by a cascade of self measurements leading to a product state. This process is obviously equivalent with the state preparation process. Self measurement is governed by the so called Negentropy Maximization Principle (NMP) stating that the information content of conscious experience is maximized. In the self measurement the density matrix of some subsystem of a given self localized in zero modes (after ordinary quantum measurement) is measured. The self measurement takes place for that subsystem of self for which the reduction of the entanglement entropy is maximal in the measurement. In p-adic context NMP can be regarded as the variational principle defining the dynamics of cognition. In real context self measurement could be seen as a repair mechanism allowing the system to fight against quantum thermalization by reducing the entanglement for the subsystem for which it is largest (fill the largest hole first in a leaking boat).

#### 2.4.4 Selves self-organize

The fourth basic element is quantum theory of self-organization based on the identification of quantum jump as the basic step of self-organization [K16]. Quantum entanglement gives rise to the generation of long range order and the emergence of longer p-adic length scales corresponds to the emergence of larger and larger coherent dynamical units and generation of a slaving hierarchy. Energy (and quantum entanglement) feed implying entropy feed is a necessary prerequisite for quantum self-organization. Zero modes represent fundamental order parameters and localization in zero modes implies that the sequence of quantum jumps can be regarded as hopping in the zero modes so that Haken's classical theory of self organization applies almost as such. Spin glass analogy is a further important element: self-organization of self leads to some characteristic pattern selected by dissipation as some valley of the "energy" landscape.

Dissipation can be regarded as the ultimate Darwinian selector of both memes and genes. The mathematically ugly irreversible dissipative dynamics obtained by adding phenomenological dissipation terms to the reversible fundamental dynamical equations derivable from an action principle can be understood as a phenomenological description replacing in a well defined sense the series of reversible quantum histories with its envelope.

#### 2.4.5 Classical non-determinism of Kähler action

The fifth basic element are the concepts of association sequence and cognitive space-time sheet. The huge vacuum degeneracy of the Kähler action suggests strongly that the absolute minimum space-time is not always unique. For instance, a sequence of bifurcations can occur so that a given space-time branch can be fixed only by selecting a finite number of 3-surfaces with time like(!) separations on the orbit of 3-surface. Quantum classical correspondence suggest an alternative formulation. Space-time surface decomposes into maximal deterministic regions and their temporal sequences have interpretation a space-time correlate for a sequence of quantum states defined by the initial (or final) states of quantum jumps. This is consistent with the fact that the variational principle selects preferred extremals of Kähler action as generalized Bohr orbits.

In the case that non-determinism is located to a finite time interval and is microscopic, this sequence of 3-surfaces has interpretation as a simulation of a classical history, a geometric correlate for contents of consciousness. When non-determinism has long lasting and macroscopic effect one can identify it as volitional non-determinism associated with our choices. Association sequences relate closely with the cognitive space-time sheets defined as space-time sheets having finite time duration and psychological time can be identified as a temporal center of mass coordinate of the cognitive space-time sheet. The gradual drift of the cognitive space-time sheets to the direction of future force by the geometry of the future light cone explains the arrow of psychological time.

#### 2.4.6 p-Adic physics as physics of cognition and intentionality

The sixth basic element adds a physical theory of cognition to this vision. TGD space-time decomposes into regions obeying real and p-adic topologies labelled by primes  $p = 2, 3, 5, \dots$ . p-Adic regions obey the same field equations as the real regions but are characterized by p-adic non-determinism since the functions having vanishing p-adic derivative are pseudo constants which are piecewise constant functions. Pseudo constants depend on a finite number of positive binary digits of arguments just like numerical predictions of any theory always involve decimal cutoff. This means that p-adic space-time regions are obtained by gluing together regions for which integration constants are genuine constants. The natural interpretation of the p-adic regions is as cognitive representations of real physics. The freedom of imagination is due to the p-adic non-determinism. p-Adic regions perform mimicry and make possible for the Universe to form cognitive representations about itself. p-Adic physics space-time sheets serve also as correlates for intentional action.

A more more precise formulation of this vision requires a generalization of the number concept obtained by fusing reals and p-adic number fields along common rationals (in the case of algebraic extensions among common algebraic numbers). This picture is discussed in [K21]. The application this notion at the level of the imbedding space implies that imbedding space has a book like structure with various variants of the imbedding space glued together along common rationals (algebraics). The implication is that genuinely p-adic numbers (non-rationals) are strictly infinite as real numbers so that most points of p-adic space-time sheets are at real infinity, outside the cosmos, and that the

projection to the real imbedding space is discrete set of rationals (algebraics). Hence cognition and intentionality are almost completely outside the real cosmos and touch it at a discrete set of points only.

This view implies also that purely local p-adic physics codes for the p-adic fractality characterizing long range real physics and provides an explanation for p-adic length scale hypothesis stating that the primes  $p \simeq 2^k$ ,  $k$  integer are especially interesting. It also explains the long range correlations and short term chaos characterizing intentional behavior and explains why the physical realizations of cognition are always discrete (say in the case of numerical computations). Furthermore, a concrete quantum model for how intentions are transformed to actions emerges.

The discrete real projections of p-adic space-time sheets serve also space-time correlate for a logical thought. It is very natural to assign to p-adic binary digits a  $p$ -valued logic but as such this kind of logic does not have any reasonable identification. p-Adic length scale hypothesis suggest that the  $p = 2^k - n$  binary digits represent a Boolean logic  $B^k$  with  $k$  elementary statements (the points of the  $k$ -element set in the set theoretic realization) with  $n$  taboos which are constrained to be identically true.

#### 2.4.7 p-Adic and dark matter hierarchies and hierarchy of moments of consciousness

Dark matter hierarchy assigned to a spectrum of Planck constant having arbitrarily large values brings additional elements to the TGD inspired theory of consciousness.

1. Macroscopic quantum coherence can be understood since a particle with a given mass can in principle appear as arbitrarily large scaled up copies (Compton length scales as  $\hbar$ ). The phase transition to this kind of phase implies that space-time sheets of particles overlap and this makes possible macroscopic quantum coherence.
2. The space-time sheets with large Planck constant can be in thermal equilibrium with ordinary ones without the loss of quantum coherence. For instance, the cyclotron energy scale associated with EEG turns out to be above thermal energy at room temperature for the level of dark matter hierarchy corresponding to magnetic flux quanta of the Earth's magnetic field with the size scale of Earth and a successful quantitative model for EEG results [K5].

Dark matter hierarchy leads to detailed quantitative view about quantum biology with several testable predictions [K5]. The general prediction is that Universe is a kind of inverted Mandelbrot fractal for which each bird's eye of view reveals new structures in long length and time scales representing scaled down copies of standard physics and their dark variants. These structures would correspond to higher levels in self hierarchy. This prediction is consistent with the belief that 75 per cent of matter in the universe is dark.

##### 1. *Living matter and dark matter*

Living matter as ordinary matter quantum controlled by the dark matter hierarchy has turned out to be a particularly successful idea. The hypothesis has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of EEG [K5]. Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [K9, K5]. A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [K5].

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally. That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of  $\hbar$  at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

##### 2. *Dark matter hierarchy and the notion of self*

The vision about dark matter hierarchy leads to a more refined view about self hierarchy and hierarchy of moments of consciousness [K4, K5]. The larger the value of Planck constant, the longer the subjectively experienced duration and the average geometric duration  $T(k) \propto \hbar$  of the quantum jump.

Quantum jumps form also a hierarchy with respect to p-adic and dark hierarchies and the geometric durations of quantum jumps scale like  $\hbar$ . Dark matter hierarchy suggests also a slight modification of the notion of self. Each self involves a hierarchy of dark matter levels, and one is led to ask whether the highest level in this hierarchy corresponds to single quantum jump rather than a sequence of quantum jumps. The averaging of conscious experience over quantum jumps would occur only for sub-selves at lower levels of dark matter hierarchy and these mental images would be ordered, and single moment of consciousness would be experienced as a history of events. The quantum parallel dissipation at the lower levels would give rise to the experience of flow of time. For instance, hadron as a macro-temporal quantum system in the characteristic time scale of hadron is a dissipating system at quark and gluon level corresponding to shorter p-adic time scales. One can ask whether even entire life cycle could be regarded as a single quantum jump at the highest level so that consciousness would not be completely lost even during deep sleep. This would allow to understand why we seem to know directly that this biological body of mine existed yesterday.

The fact that we can remember phone numbers with 5 to 9 digits supports the view that self corresponds at the highest dark matter level to single moment of consciousness. Self would experience the average over the sequence of moments of consciousness associated with each sub-self but there would be no averaging over the separate mental images of this kind, be their parallel or serial. These mental images correspond to sub-selves having shorter wake-up periods than self and would be experienced as being time ordered. Hence the digits in the phone number are experienced as separate mental images and ordered with respect to experienced time.

### 3. The time span of long term memories as signature for the level of dark matter hierarchy

The basic question is what time scale can one assign to the geometric duration of quantum jump measured naturally as the size scale of the space-time region about which quantum jump gives conscious information. This scale is naturally the size scale in which the non-determinism of quantum jump is localized. During years I have made several guesses about this time scales but zero energy ontology and the vision about fractal hierarchy of quantum jumps within quantum jumps leads to a unique identification.

Causal diamond as an imbedding space correlate of self defines the time scale  $\tau$  for the space-time region about which the consciousness experience is about. The temporal distances between the tips of  $CD$  as come as integer multiples of  $CP_2$  length scales and for prime multiples correspond to what I have christened as secondary p-adic time scales. A reasonable guess is that secondary p-adic time scales are selected during evolution and the primes near powers of two are especially favored. For electron, which corresponds to Mersenne prime  $M_{127} = 2^{127} - 1$  this scale corresponds to .1 seconds defining the fundamental time scale of living matter via 10 Hz biorhythm (alpha rhythm). The unexpected prediction is that all elementary particles correspond to time scales possibly relevant to living matter.

Dark matter hierarchy brings additional finesse. For the higher levels of dark matter hierarchy  $\tau$  is scaled up by  $\hbar/\hbar_0$ . One could understand evolutionary leaps as the emergence of higher levels at the level of individual organism making possible intentionality and memory in the time scale defined  $\tau$ .

Higher levels of dark matter hierarchy provide a neat quantitative view about self hierarchy and its evolution. Various levels of dark matter hierarchy would naturally correspond to higher levels in the hierarchy of consciousness and the typical duration of life cycle would give an idea about the level in question. The level would determine also the time span of long term memories as discussed in [K5]. The emergence of these levels must have meant evolutionary leap since long term memory is also accompanied by ability to anticipate future in the same time scale. This picture would suggest that the basic difference between us and our cousins is not at the level of genome as it is usually understood but at the level of the hierarchy of magnetic bodies [K9, K5]. In fact, higher levels of dark matter hierarchy motivate the introduction of the notions of super-genome and hyper-genome. The genomes of entire organ can join to form super-genome expressing genes coherently. Hyper-genomes would result from the fusion of genomes of different organisms and collective levels of consciousness would express themselves via hyper-genome and make possible social rules and moral.



### 3 Bird's eye of view about the topics of the book

This book is devoted to a detailed representation of what quantum TGD in its recent form. Quantum TGD relies on two different views about physics: physics as an infinite-dimensional spinor geometry and physics as a generalized number theory. The most important guiding principle is quantum classical correspondence whose most profound implications follow almost trivially from the basic structure of the classical theory forming an exact part of quantum theory. A further mathematical guideline is the mathematics associated with hyper-finite factors of type  $II_1$  about which the spinors of the world of classical worlds represent a canonical example.

#### 1. Quantum classical correspondence

Quantum classical correspondence has turned out to be the most important guiding principle concerning the interpretation of the theory.

1. Quantum classical correspondence and the properties of the simplest extremals of Kähler action have served as the basic guideline in the attempts to understand the new physics predicted by TGD. The most dramatic predictions follow without even considering field equations in detail by using quantum classical correspondence and form the backbone of TGD and TGD inspired theory of living matter in particular.

The notions of many-sheeted space-time, topological field quantization and the notion of field/magnetic body, follow from simple topological considerations. The observation that space-time sheets can have arbitrarily large sizes and their interpretation as quantum coherence regions forces to conclude that in TGD Universe macroscopic and macro-temporal quantum coherence are possible in arbitrarily long scales.

2. Also long ranged classical color and electro-weak fields are an unavoidable prediction. It however took a considerable time to make the obvious conclusion: TGD Universe is fractal containing fractal copies of standard model physics at various space-time sheets and labeled by the collection of p-adic primes assignable to elementary particles and by the level of dark matter hierarchy characterized partially by the value of Planck constant labeling the pages of the book like structure formed by singular covering spaces of the imbedding space  $M^4 \times CP_2$  glued together along a four-dimensional back. Particles at different pages are dark relative to each other since purely local interactions defined in terms of the vertices of Feynman diagram involve only particles at the same page.
3. The new view about energy and time finding a justification in the framework of zero energy ontology means that the sign of the inertial energy depends on the time orientation of the space-time sheet and that negative energy space-time sheets serve as correlates for communications to the geometric future. This alone leads to profoundly new views about metabolism, long term memory, and realization of intentional action.
4. The general properties of Kähler action, in particular its vacuum degeneracy and the failure of the classical determinism in the conventional sense, have also strong implications. Space-time surface as a generalization of Bohr orbit provides not only a representation of quantum states but also of sequences of quantum jumps and thus contents of consciousness. Vacuum degeneracy implies spin glass degeneracy in 4-D sense reflecting quantum criticality which is the fundamental characteristic of TGD Universe.
5. The detailed study of the simplest extremals of Kähler action interpreted as correlates for asymptotic self organization patterns provides additional insights.  $CP_2$  type extremals representing elementary particles, cosmic strings, vacuum extremals, topological light rays ("massless extremal", ME), flux quanta of magnetic and electric fields represent the basic extremals. Pairs of wormhole throats identifiable as parton pairs define a completely new kind of particle carrying only color quantum numbers in ideal case and I have proposed their interpretation as quantum correlates for Boolean cognition. MEs and flux quanta of magnetic and electric fields are of special importance in living matter.

Topological light rays have interpretation as space-time correlates of "laser beams" of ordinary or dark photons or their electro-weak and gluonic counterparts. Neutral MEs carrying em and

$Z^0$  fields are ideal for communication purposes and charged  $W$  MEs ideal for quantum control. Magnetic flux quanta containing dark matter are identified as intentional agents quantum controlling the behavior of the corresponding biological body parts utilizing negative energy  $W$  MEs. Bio-system in turn is populated by electrets identifiable as electric flux quanta.

### 2. *Physics as infinite-dimensional geometry in the "world of classical worlds"*

Physics as infinite-dimensional Kähler geometry of the "world of classical worlds" with classical spinor fields representing the quantum states of the universe and gamma matrix algebra geometrizing fermionic statistics is the first vision.

The mere existence of infinite-dimensional non-flat Kähler geometry has impressive implications. Configuration space must decompose to a union of infinite-dimensional symmetric spaces labelled by zero modes having interpretation as classical dynamical degrees of freedom assumed in quantum measurement theory. Infinite-dimensional symmetric space has maximal isometry group identifiable as a generalization of Kac Moody group obtained by replacing finite-dimensional group with the group of canonical transformations of  $\delta M_+^4 \times CP_2$ , where  $\delta M_+^4$  is the boundary of 4-dimensional future light-cone. The infinite-dimensional Clifford algebra of configuration space gamma matrices in turn can be expressed as direct sum of von Neumann algebras known as hyper-finite factors of type  $II_1$  having very close connections with conformal field theories, quantum and braid groups, and topological quantum field theories.

### 3. *Physics as a generalized number theory*

Second vision is physics as a generalized number theory. This vision forces to fuse real physics and various p-adic physics to a single coherent whole having rational physics as their intersection and poses extremely strong conditions on real physics.

A further aspect of this vision is the reduction of the classical dynamics of space-time sheets to number theory with space-time sheets identified as what I have christened hyper-quaternionic sub-manifolds of hyper-octonionic imbedding space. Field equations would state that space-time surfaces are Kähler calibrations with Kähler action density reducing to a closed 4-form at space-time surfaces. Hence TGD would define a generalized topological quantum field theory with conserved Noether charges (in particular rest energy) serving as generalized topological invariants having extremum in the set of topologically equivalent 3-surfaces.

Infinite primes, integers, and rationals define the third aspect of this vision. The construction of infinite primes is structurally similar to a repeated second quantization of an arithmetic quantum field theory and involves also bound states. Infinite rationals can be also represented as space-time surfaces somewhat like finite numbers can be represented as space-time points.

### 4. *The organization of the book*

The first part of the book describes basic quantum TGD in its recent form.

1. The properties of the preferred extremals of Kähler action are crucial for the construction and the discussion of known extremals is therefore included.
2. General coordinate invariance and generalized super-conformal symmetries - the latter present only for 4-dimensional space-time surfaces and for 4-D Minkowski space - define the basic symmetries of quantum TGD. A generalization of Equivalence Principle can be formulated as a generalized coset construction.
3. In zero energy ontology S-matrix is replaced with M-matrix and identified as time-like entanglement coefficients between positive and negative energy parts of zero energy states assignable to the past and future boundaries of 4-surfaces inside causal diamond defined as intersection of future and past directed light-cones. M-matrix is a product of diagonal density matrix and unitary S-matrix and there are reasons to believe that S-matrix is universal. Generalized Feynman rules based on the generalization of Feynman diagrams obtained by replacing lines with light-like 3-surfaces and vertices with 2-D surfaces at which the lines meet.
4. A category theoretical formulation of quantum TGD is considered. Finite measurement resolution realized in terms of a fractal hierarchy of causal diamonds inside causal diamonds leads

to a stringy formulation of quantum TGD involving effective replacement of the 3-D light-like surface with a collection of braid strands representing the ends of strings. A formulation in terms of category theoretic concepts is proposed and leads to a hierarchy of algebras forming what is known as operads.

5. Twistors emerge naturally in TGD framework and could allow the formulation of low energy limit of the theory in the approximation that particles are massless. The replacement of massless plane waves with states for which amplitudes are localized are light-rays is suggestive in twistor theoretic framework. Twistors could allow also a dual representation of space-time surfaces in terms of surfaces of  $X \times CP_2$ , where  $X$  is 8-D twistor space or its 6-D projective variant. These surfaces would have dimension higher than four in non-perturbative phases meaning an analogy with branes. In full theory a massive particles must be included but represent a problem in approach based on standard twistors. The interpretation of massive particles in 4-D sense as massless particles in 8-D sense would resolve the problem and requires a generalization of twistor concept involving in essential manner the triality of vector and spinor representations of  $SO(7, 1)$ .
6. In TGD Universe bosons are in well-defined sense bound states of fermion and anti-fermion. This leads to the notion of bosonic emergence meaning that the fundamental action is just Dirac action coupled to gauge potentials and bosonic action emerges as part of effective action as one functionally integrates over the spinor fields. This kind of approach predicts the evolution of all coupling constants if one is able to fix the necessary UV cutoffs of mass and hyperbolic angle in loop integrations. The guess for the hyperbolic cutoff motivated by the geometric view about finite measurement resolution predicts coupling constant evolution which is consistent with that predicted by standard model. The condition that all N-vertices defined by fermionic loops vanish for  $N > 3$  when incoming particles are massless gives hopes of fixing completely the hyperbolic cutoff from fundamental principles.

Second part of the book is devoted to hyper-finite factors and hierarchy of Planck constants.

1. Configuration space spinors indeed define a canonical example about hyper-finite factor of type  $II_1$ . The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors of type  $II_1$  could provide the mathematics needed to develop a more explicit view about the construction of M-matrix. This has turned out to be the case to the extent that a general master formula for M-matrix with interactions described as a deformation of ordinary tensor product to Connes tensor products emerges.
2. The idea about hierarchy of Planck constants emerged from anomalies of biology and the strange finding that planetary orbits could be regarded as Bohr orbits but with a gigantic value of Planck constant. This led to the vision that dark matter corresponds to ordinary particles but with non-standard value of Planck constant and to a generalization of the 8-D imbedding space to a book like structure with pages partially characterized by the value of Planck constant. Using the intuition provided by the inclusions of hyper-finite factors of type  $II_1$  one ends up to a prediction for the spectrum of Planck constants associated with  $M^4$  and  $CP_2$  degrees of freedom. This inspires the proposal that dark matter could be in quantum Hall like phase localized at light-like 3-surfaces with macroscopic size and behaving in many respects like black hole horizons.

The seven online books about TGD [K24, K14, K15, K20, K12, K11, K19] and eight online books about TGD inspired theory of consciousness and quantum biology [K23, K3, K13, K2, K7, K8, K10, K18] are warmly recommended for the reader willing to get overall view about what is involved.

## 4 The contents of the book

### 4.1 Part I: The recent view about field equations

#### 4.1.1 Basic extremals of the Kähler action

The physical interpretation of the Kähler function and the TGD based space-time concept are the basic themes of this book. The aim is to develop what might be called classical TGD at fundamental

level. The strategy is simple: try to guess the general physical consequences of the configuration space geometry and of the TGD based gauge field concept and study the simplest extremals of Kähler action and try to abstract general truths from their properties.

The fundamental underlying assumptions are the following:

1. The 4-surface associated with given 3-surface defined by Kähler function  $K$  as a preferred extremal of the Kähler action is identifiable as a classical space-time. Number theoretically preferred extremals would decompose to hyper-quaternionic and co-hyper-quaternionic regions. The reduction of the classical theory to the level of the modified Dirac action implies that the preferred extremals are critical in the sense of allowing infinite number of deformations for which the second variation of Kähler action vanishes [?] It is not clear whether criticality and hyper-quaternionicity are consistent with each other.

Due to the preferred extremal property classical space-time can be also regarded as a generalized Bohr orbit so that the quantization of the various parameters associated with a typical extremal of the Kähler action is expected to take place in general. In TGD quantum states corresponds to quantum superpositions of these classical space-times so that this classical space-time is certainly not some kind of effective quantum average space-time.

2. The bosonic vacuum functional of the theory is the exponent of the Kähler function  $\Omega_B = \exp(K)$ . This assumption is the only assumption about the dynamics of the theory and is necessitated by the requirement of divergence cancellation in perturbative approach.
3. Renormalization group invariance and spin glass analogy. The value of the Kähler coupling strength is such that the vacuum functional  $\exp(K)$  is analogous to the exponent  $\exp(H/T)$  defining the partition function of a statistical system at critical temperature. This allows Kähler coupling strength to depend on zero modes of the configuration space metric and as already found there is very attractive hypothesis determining completely the dependence of the Kähler coupling strength on the zero modes based on p-adic considerations motivated by the spin glass analogy.
4. In spin degrees of freedom the massless Dirac equation for the induced spinor fields with modified Dirac action defines classical theory: this is in complete accordance with the proposed definition of the configuration space spinor structure.

The geometrization of the classical gauge fields in terms of the induced gauge field concept is also important concerning the physical interpretation. Electro-weak gauge potentials correspond to the space-time projections of the spinor connection of  $CP_2$ , gluonic gauge potentials to the projections of the Killing vector fields of  $CP_2$  and gravitational field to the induced metric. The topics to be discussed in this part of the book are summarized briefly in the following.

What the selection of preferred extremals of Kähler action might mean has remained a long standing problem and real progress occurred only quite recently (I am writing this towards the end of year 2003).

1. The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that all preferred extremals of Kähler action satisfy the condition. The vanishing of the Lorentz 4-force in turn implies local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. The condition implies that vacuum currents can be non-vanishing only provided the dimension  $D_{CP_2}$  of the  $CP_2$  projection of the space-time surface is less than four so that in the regions with  $D_{CP_2} = 4$ , Maxwell's vacuum equations are satisfied.
2. The hypothesis that Kähler current is proportional to a product of an arbitrary function  $\psi$  of  $CP_2$  coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Instanton current has a vanishing divergence for  $D_{CP_2} < 4$ , and Lorentz 4-force indeed vanishes. Four 4-dimensional projection the scalar function multiplying the instanton current can make it divergenceless. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.

3. By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that preferred extremals of Kähler action correspond to space-time sheets which at least asymptotically satisfy the generalized Beltrami conditions so that one can indeed assign to the final 3-surface a unique 4-surface apart from effects related to non-determinism. Preferred extremal property abstracted to purely algebraic generalized Beltrami conditions makes sense also in the p-adic context.

This chapter is mainly devoted to the study of the basic extremals of the Kähler action besides the detailed arguments supporting the view that the preferred extrema satisfy generalized Beltrami conditions at least asymptotically.

The newest results discussed in the last section about the weak form of electric-magnetic duality suggest strongly that Beltrami property is general and together with the weak form of electric-magnetic duality allows a reduction of quantum TGD to almost topological field theory with Kähler function allowing expression as a Chern-Simons term.

The surprising implication of the duality is that Kähler form of  $CP_2$  must be replaced with that for  $S^2 \times CP_2$  in order to obtain a configuration space metric which is non-trivial in  $M^4$  degrees of freedom. This modification implies much richer vacuum structure than the original Kähler action which is a good news as far as the description of classical gravitational fields in terms of small deformations of vacuum extremals with the four-momentum density of the topologically condensed matter given by Einstein's equations is considered. The breaking of Lorentz invariance from  $SO(3,1)$  to  $SO(3)$  is implied already by the geometry of  $CD$  but is extremely small for a given causal diamond ( $CD$ ). Since a wave function over the Lorentz boosts and translates of  $CD$  is allowed, there is no actual breaking of Poincaré invariance at the level of the basic theory. Beltrami property leads to a rather explicit construction of the general solution of field equations based on the hydrodynamic picture implying that single particle quantum numbers are conserved along flow lines defined by the instanton current. The construction generalizes also to the fermionic sector.

#### 4.1.2 The recent vision about preferred extremals and solutions of the modified Dirac equation

During years several approaches to what preferred extremals of Kähler action and solutions of the modified Dirac equation could be have been proposed and the challenge is to see whether at least some of these approaches are consistent with each other. It is good to list various approaches first.

1. For preferred extremals generalization of conformal invariance to 4-D situation is very attractive approach and leads to concrete conditions formally similar to those encountered in string model. The approach based on basic heuristics for massless equations, on effective 3-dimensionality, and weak form of electric magnetic duality is also promising. An alternative approach is inspired by number theoretical considerations and identifies space-time surfaces as associative or co-associative sub-manifolds of octonionic imbedding space.
2. There are also several approaches for solving the modified Dirac equation. The most promising approach is assumes that the solutions are restricted on 2-D stringy world sheets and/or partonic 2-surfaces. This strange looking view is a rather natural consequence of number theoretic vision. The conditions stating that electric charge is conserved for preferred extremals is an alternative very promising approach.

In this chapter the question whether these various approaches are mutually consistent is discussed. It indeed turns out that the approach based on the conservation of electric charge leads under rather general assumptions to the proposal that solutions of the modified Dirac equation are localized on 2-dimensional string world sheets and/or partonic 2-surfaces. Einstein's equations are satisfied for the preferred extremals and this implies that the earlier proposal for the realization of Equivalence Principle is not needed. This leads to a considerable progress in the understanding of super Virasoro representations for super-symplectic and super-Kac-Moody algebra. In particular, the proposal is that super-Kac-Moody currents assignable to string world sheets define duals of gauge potentials and their generalization for gravitons: in the approximation that gauge group is Abelian - motivated by the

notion of finite measurement resolution - the exponents for the sum of KM charges would define non-integrable phase factors. One can also identify Yangian as the algebra generated by these charges. The approach allows also to understand the special role of the right handed neutrino in SUSY according to TGD.

## 4.2 Part II: General Theory

### 4.2.1 Construction of Quantum Theory: Symmetries

This chapter provides a summary about the role of symmetries in the construction of quantum TGD. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the "world of the classical worlds" identified as the infinite-dimensional configuration space of light-like 3-surfaces of  $H = M^4 \times CP_2$  (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis of this vision.

TGD relies heavily on geometric ideas, which have gradually generalized during the years. Symmetries play a key role as one might expect on basis of general definition of geometry as a structure characterized by a given symmetry.

#### 1. *Physics as infinite-dimensional Kähler geometry*

1. The basic idea is that it is possible to reduce quantum theory to configuration space geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes configuration space Kähler geometry uniquely. Accordingly, configuration space can be regarded as a union of infinite-dimensional symmetric spaces labeled by zero modes labeling classical non-quantum fluctuating degrees of freedom.

The huge symmetries of the configuration space geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. Configuration space spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the configuration space. Configuration space gamma matrices contracted with Killing vector fields give rise to a super-symplectic algebra which together with Hamiltonians of the configuration space forms what I have used to call super-symplectic algebra.

Super-symplectic degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-symplectic quanta correspond to what has been identified as non-perturbative sector of QCD: they define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

3. Besides super-symplectic symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD. Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.
4. Modified Dirac equation gives also rise to a hierarchy super-conformal algebras assignable to zero modes. These algebras follow from the existence of conserved fermionic currents. The corresponding deformations of the space-time surface correspond to vanishing second variations of Kähler action and provide a realization of quantum criticality. This led to a breakthrough in the understanding of the modified Dirac action via the addition of a measurement interaction term to the action allowing to obtain among other things stringy propagator and the coding

of quantum numbers of super-conformal representations to the geometry of space-time surfaces required by quantum classical correspondence.

### 2. *p-adic physics and p-adic variants of basic symmetries*

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both configuration space geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no adhoc elements and is inherent to the physics of TGD.

### 3. *Hierarchy of Planck constants and dark matter hierarchy*

The realization for the hierarchy of Planck constants proposed as a solution to the dark matter puzzles leads to a profound generalization of quantum TGD through a generalization of the notion of imbedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of the imbedding space representing the pages of the book meeting at quantum critical sub-manifolds. A particular page of the book can be seen as an n-fold singular covering or factor space of  $CP_2$  or of a causal diamond ( $CD$ ) of  $M^4$  defined as an intersection of the future and past directed light-cones. Therefore the cyclic groups  $Z_n$  appear as discrete symmetry groups.

### 4. *Number theoretical symmetries*

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

1. There are good reasons to believe that the strands of number theoretical braids can be assigned with the roots of a polynomial which suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group  $S_\infty$  of infinitely many objects acting as the Galois group of algebraic numbers. The group algebra of  $S_\infty$  is HFF which can be mapped to the HFF defined by configuration space spinors. This picture suggests a number theoretical gauge invariance stating that  $S_\infty$  acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of  $G \times G \times \dots$  of the completion of  $S_\infty$ .
2. HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular,  $SU(3)$  acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit. If space-time surfaces are hyper-quaternionic (meaning that the octonionic counterparts of the modified gamma matrices span complex quaternionic sub-algebra of octonions) and contain at each point a preferred plane  $M^2$  of  $M^4$ , one ends up with  $M^8 - H$  duality stating that space-time surfaces can be equivalently regarded as surfaces in  $M^8$  or  $M^4 \times CP_2$ . One can actually generalize  $M^2$  to a two-dimensional Minkowskian sub-manifold of  $M^4$ . One ends up with quantum TGD by considering associative sub-algebras of the local octonionic Clifford algebra of  $M^8$  or  $H$ . so that TGD could be seen as a generalized number theory.

#### 4.2.2 Construction of Quantum Theory: M-matrix

The construction of M-matrix has remained the key challenge of quantum TGD from the very beginning when it had become clear that path integral approach and canonical quantization make no sense in TGD framework. My intuitive feeling that the problems are not merely technical has turned out to be correct.

The rapid evolution of a bundle of new ideas has taken place during last five years (zero energy ontology, the notion of finite measurement resolution, the role of hyper-finite factors of type  $II_1$ , the

hierarchy of Planck constants, the construction of configuration space geometry in terms of second quantized induced spinor fields, number theoretic compactification,...). These ideas are now converging to an overall view in which various approaches to quantum TGD (physics as infinite dimensional geometry, physics as generalized number theory, physics from number theoretical universality, physics from finite measurement resolution implying effective discretization, TGD as almost topological QFT) neatly fuse together to single coherent overall view. Many ideas have been of course thrown away because they have not produced anything useful.

In this chapter the overall view about the construction of the TGD counterpart of  $S$ -matrix -  $M$ -matrix - is discussed. It is perhaps wise to summarize briefly the vision about  $M$ -matrix.

1. *Zero energy ontology and interpretation of light-like 3-surfaces as generalized Feynman diagrams*

1. Zero energy ontology is the cornerstone of the construction. Zero energy states have vanishing net quantum numbers and consist of positive and negative energy parts, which can be thought of as being localized at the boundaries of light-like 3-surface  $X_l^3$  connecting the light-like boundaries of a causal diamond  $CD$  identified as intersection of future and past directed light-cones. There is entire hierarchy of  $CD$ s, whose scales are suggested to come as powers of 2. A more general proposal is that prime powers of fundamental size scale are possible and would conform with the most general form of p-adic length scale hypothesis. The hierarchy of size scales assignable to  $CD$ s corresponds to a hierarchy of length scales and code for a hierarchy of radiative corrections to generalized Feynman diagrams.
2. Light-like 3-surfaces are the basic dynamical objects of quantum TGD and have interpretation as generalized Feynman diagrams having light-like 3-surfaces as lines glued together along their ends defining vertices as 2-surfaces. By effective 2-dimensionality (holography) of light-like 3-surfaces the interiors of light-like 3-surfaces are analogous to gauge degrees of freedom and partially parameterized by Kac-Moody group respecting the light-likeness of 3-surfaces. This picture differs dramatically from that of string models since light-like 3-surfaces replacing stringy diagrams are singular as manifolds whereas 2-surfaces representing vertices are not.

2. *Identification of the counterpart of  $S$ -matrix as time-like entanglement coefficients*

1. The TGD counterpart of  $S$ -matrix -call it  $M$ -matrix- defines time-like entanglement coefficients between positive and negative energy parts of zero energy state located at the light-like boundaries of  $CD$ . One can also assign to quantum jump between zero energy states a matrix- call it  $U$ -matrix - which is unitary and assumed to be expressible in terms of  $M$ -matrices.  $M$ -matrix need not be unitary unlike the  $U$ -matrix characterizing the unitary process forming part of quantum jump. There are several good arguments suggesting that that  $M$ -matrix cannot be unitary but can be regarded as thermal  $S$ -matrix so that thermodynamics would become an essential part of quantum theory. In fact,  $M$ -matrix can be decomposed to a product of positive diagonal matrix identifiable as square root of density matrix and unitary matrix so that quantum theory would be kind of square root of thermodynamics. Path integral formalism is given up although functional integral over the 3-surfaces is present.
2. In the general case only thermal  $M$ -matrix defines a normalizable zero energy state so that thermodynamics becomes part of quantum theory. One can assign to  $M$ -matrix a complex parameter whose real part has interpretation as interaction time and imaginary part as the inverse temperature.

3. *Hyper-finite factors and  $M$ -matrix*

HFFs of type  $III_1$  provide a general vision about  $M$ -matrix.

1. The factors of type III allow unique modular automorphism  $\Delta^{it}$  (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the  $M$ -matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.



2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.
3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.
4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW.

#### 4. Connes tensor product as a realization of finite measurement resolution

The inclusions  $\mathcal{N} \subset \mathcal{M}$  of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In zero energy ontology  $\mathcal{N}$  would create states experimentally indistinguishable from the original one. Therefore  $\mathcal{N}$  takes the role of complex numbers in non-commutative quantum theory. The space  $\mathcal{M}/\mathcal{N}$  would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative  $\mathcal{N}$ -valued coordinates.
2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their  $\mathcal{N}$  "averaged" counterparts. The "averaging" would be in terms of the complex square root of  $\mathcal{N}$ -state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.
3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that  $\mathcal{N}$  acts like complex numbers on M-matrix elements as far as  $\mathcal{N}$ -"averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in  $\mathcal{M}(\mathcal{N}$  interpreted as finite-dimensional space with a projection operator to  $\mathcal{N}$ ). The condition that  $\mathcal{N}$  averaging in terms of a complex square root of  $\mathcal{N}$  state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

#### 5. Input from the construction of configuration space spinor structure

The construction of configuration space spinor structure in terms of second quantized induced spinor fields is certainly the most important step made hitherto towards explicit formulas for M-matrix elements.

1. Number theoretical compactification ( $M^8 - H$  duality) states that space-time surfaces can be equivalently regarded as 4-dimensional surfaces of either  $H = M^4 \times CP_2$  or of 8-D Minkowski space  $M^8$ , and consisting of hyper-quaternionic and co-hyper-quaternionic regions identified as regions with Minkowskian and Euclidian signatures of induced metric. Duality preserves induced metric and Kähler form. This duality poses very strong constraints on the geometry of the preferred extremals of Kähler action implying dual slicings of the space-time surface by string worlds sheets and partonic 2-surfaces as also by light-like 1-surfaces and light-like 3-surfaces. These predictions are consistent with what is known about the extremals of Kähler action. The predictions of number theoretical compactification lead to dramatic progress in the construction of configurations space spinor structure and geometry. One consequence is dimensional reduction of space-time surface to string world sheet allowing to understand how the space-time correlate for Equivalence Principle is realized in TGD framework (its quantum counterpart emerges from coset construction for super-symplectic and super Kac-Moody algebras).
2. The construction of configuration space geometry and spinor structure in terms of induced spinor fields leads to the conclusion that finite measurement resolution is an intrinsic property of quantum states basically due to the vacuum degeneracy of Kähler action. This gives a justification for the notion of number theoretic braid effectively replacing light-like 3-surfaces. Hence the infinite-dimensional configuration space is replaced with a finite-dimensional space  $(\delta M_{\pm}^4 \times CP_2)^n / S_n$ . A possible interpretation is that the finite fermionic oscillator algebra for given partonic 2-surface  $X^2$  represents the factor space  $\mathcal{M}/\mathcal{N}$  identifiable as quantum variant of Clifford algebra.  $(\delta M_{\pm}^4 \times CP_2)^n / S_n$  would represent its bosonic analog.
3. The isometries of the configuration space corresponds to  $X^2$  local symplectic transformations  $\delta M_{\pm}^4 \times CP_2$  depending only on the value of the invariant  $\epsilon^{\mu\nu} J_{\mu\nu}$ , where  $J_{\mu\nu}$  can correspond to the Kähler form induced from  $\delta M_{\pm}^4$  or  $CP_2$ . This group parameterizes quantum fluctuating degrees of freedom. Zero modes correspond to coordinates which cannot be made complex, in particular to the values of the induced symplectic form which thus behaves as a classical field so that configuration space allows a slicing by the classical field patterns  $J_{\mu\nu}(x)$  representing zero modes.
4. By the effective 2-dimensionality of light-like 3-surfaces  $X_l^3$  (holography) the interiors of light-like 3-surfaces are analogous to gauge degrees of freedom and partially parameterized by Kac-Moody group respecting the light-likeness of 3-surfaces. Quantum classical correspondence suggests that gauge fixing in Kac-Moody degrees of freedom takes place and implies correlation between the quantum numbers of the physical state and  $X_l^3$ . There would be no path integral over  $X_l^3$  and only functional integral defined by configuration space geometry over partonic 2-surfaces.
5. The condition that the Noether currents assignable to the modified Dirac equation are conserved requires that space-time surfaces correspond to extremals for which second variation of Kähler action vanishes. A milder condition is that the rank of the matrix defined by the second variation of Kähler action is less than maximal. Preferred extremals of Kähler action can be identified as this kind of 4-surface and the interpretation is in terms of quantum criticality.

#### 6. Conformal symmetries and stringy diagrammatics

The modified Dirac equation has rich super-conformal symmetries helping to achieve concrete vision about the structure of  $M$ -matrix in terms of generalized Feynman diagrammatics

1. Both super-conformal symmetries, the slicing of space-time surface by string worlds sheets, and the reduction of space-time sheet to string world sheet as a consequence of finite measurement resolution suggest that the generalized Feynman diagrams have as vertices  $N$ -point functions of a conformal field theory assignable to the partonic 2-surfaces at which the lines of Feynman diagram meet. Finite measurement resolution means that this conformal theory is defined in the discrete set defined by the number theoretic braid. The presence of symplectic invariants in turn suggest a symplectic variant of conformal field theory leading to a concrete construction of symplectic fusion rules relying in crucial manner to discretization.

2. The effective 3-dimensionality implied by the modified Dirac operator associated with Kähler action plays crucial role in the construction of both configuration space geometry (Kähler function is identified as Dirac determinant assignable to the modified Dirac operator) and of  $M$ -matrix. By effective 3-dimensionality the propagators reduce to the propagators assignable the light-like 3-surfaces. This does not give stringy propagators and massive stringy excitations would not appear at all in propagators. This does not conform with what p-adic mass calculations and conformal symmetries suggest.
3. The solution of the problem is provided by the addition of measurement interaction term to the modified Dirac action and assignable to wormhole throats or equivalently any light-like 3-surface parallel to them into the slicing of space-time sheet: this condition defines additional symmetry. Measurement interaction term implies that the preferred extremals of Kähler action depend on quantum numbers of the states of super-conformal representations as quantum classical correspondence requires. The coupling constants appearing in the measurement interaction term are fixed by the condition that Kähler function transforms only by a real part of a holomorphic function of complex coordinates of WCW depending also on zero modes so that Kähler metric of WCW remains unchanged. This realizes also the effective 2-dimensionality of space-like 3-surfaces but only in finite regions where the slicing by light-like 3-surfaces makes sense.

#### 7. TGD as almost topological QFT

The idea that TGD could be regarded as almost topological QFT has been very fruitful although the hypothesis that Chern-Simons term for induced Kähler gauge potential assignable to light-like 3-surfaces identified as regions of space-time where the Euclidian signature of induced metric assignable to the interior or generalized Feynman diagram changes to Minkowskian one turned out to be too strong. The reduction of configuration space and its Clifford algebra to finite dimensional structures due to finite measurement resolution however realizes this idea but in different manner.

1. There is functional integral over the small deformations of Feynman cobordisms corresponding to the maxima of Kähler function which is finite-dimensional if finite measurement resolution is taken into account. Almost topological QFT property of quantum suggests the identification of  $M$ -matrix as a functor from the category of generalized Feynman cobordisms (generalized Feynman diagrams) to the category of operators mapping the Hilbert space of positive energy states to that for negative energy states: these Hilbert spaces are assignable to partonic 2-surfaces.
2. The limit at which momenta vanish is well-defined for  $M$ -matrix since the modified Dirac action contains measurement interaction term and at this limit one indeed obtains topological QFT.
3. Almost TQFT property suggests that braiding  $S$ -matrices should have important role in the construction. It is indeed possible to assign the with the lines of the generalized Feynman diagram. The reduction of quantum TGD to topological QFT should occur at quantum criticality with respect to the change of Planck constant since in this situation the  $M$ -matrix should not depend at all on Planck constant. Factoring QFTs in 1+1 dimensions give examples of this kind of theories.

#### 8. Bosonic emergence

The construction of QFT limit of quantum TGD based on the notion of bosonic emergence led to the most concrete picture about  $M$ -matrix achieved hitherto.

1. An "almost stringy" fermion propagator arises as one adds to the modified Dirac action a term coupling the charges in a Cartan algebra of the isometry group of  $H = M^4 \times CP_2$  to conserved fermionic currents (there are several of them). Also more general observables allow this kind of coupling and the interpretation in terms of measurement interaction. This term also realizes quantum classical correspondence by feeding information about quantum numbers of partons to the geometry of space-time sheet so that quantum numbers entangle with the geometry of space-time sheet as holography requires. This measurement interaction was the last piece in the puzzle "What are the basic equations of quantum TGD" and unified several visions about the

physics predicted by quantum TGD. "Almost stringy" means that the on mass shell fermions obey stringy mass formulas dictated by super-conformal symmetry but that propagator itself -although it depends on four-momentum- is not the inverse of super-Virasoro generator  $G_0$  as it would be in string models.

2. The identification of bosons as wormhole contacts means that bosonic propagation reduces to a propagation of fermion and antifermion at opposite throats of the wormhole throat. In this framework bosonic  $n$ -vertex would correspond to the decay of bosons to fermion-antifermion pairs in the loop. Purely bosonic gauge boson couplings would be generated radiatively from triangle and box diagrams involving only fermion-boson couplings. In particular, bosonic propagator would be generated as a self-energy loop: bosons would propagate by decaying to fermion-antifermion pair and then fusing back to the boson. TGD counterpart for gauge theory dynamics would be emergent and bosonic couplings would have form factors with IR and UV behaviors allowing finiteness of the loops constructed from them since the constraint that virtual fermion pair corresponds to wormhole contact poses strong constraint on virtual momenta of fermion and antifermion.

This picture leads to generalized Feynman rules for  $M$ -matrix. The QFT limit based on this picture is able to reproduce the  $p$ -adic length scale evolution of various gauge coupling strengths with simple cutoffs on mass squared and hyperbolic angle characterizing the state of fermion in the rest system of virtual boson. The presence of these cutoffs is dictated by geometric picture about loops provided by zero energy ontology. The condition that the bosonic  $N > 3$ -vertices vanish when incoming states are on mass shell gives an infinite number of conditions which could fix the cutoffs uniquely.

### 4.2.3 More about Matrices

This chapter is a second part of chapter representing material related to the construction of  $U$ -,  $M$ -, and  $S$ -matrices. The general philosophy is discussed in the first part of the chapter and I will not repeat the discussion.

The views about  $M$ -matrix as a characterizer of time-like entanglement and  $M$ -matrix as a functor are analyzed. The role of hyper-finite factors in the construction of  $M$ -matrix is considered. One section is devoted to the possibility that Connes tensor product could define fundamental vertices. The last section is devoted to the construction of unitary  $U$ -matrix characterizing the unitary process forming part of quantum jump.

The last section is about the anatomy of quantum jump. The first part of the chapter began with a similar piece of text. This reflects the fact that the ideas are developing all the time so that the vision about the matrices is by no means top-down view beginning from precisely state assumption and proceeding to conclusions.

### 4.2.4 Category Theory and Quantum TGD

Possible applications of category theory to quantum TGD are discussed. The so called 2-plectic structure generalizing the ordinary symplectic structure by replacing symplectic 2-form with 3-form and Hamiltonians with Hamiltonian 1-forms has a natural place in TGD since the dynamics of the light-like 3-surfaces is characterized by Chern-Simons type action. The notion of planar operad was developed for the classification of hyper-finite factors of type  $II_1$  and its mild generalization allows to understand the combinatorics of the generalized Feynman diagrams obtained by gluing 3-D light-like surfaces representing the lines of Feynman diagrams along their 2-D ends representing the vertices.

The fusion rules for the symplectic variant of conformal field theory, whose existence is strongly suggested by quantum TGD, allow rather precise description using the basic notions of category theory and one can identify a series of finite-dimensional nilpotent algebras as discretized versions of field algebras defined by the fusion rules. These primitive fusion algebras can be used to construct more complex algebras by replacing any algebra element by a primitive fusion algebra. Trees with arbitrary numbers of branches in any node characterize the resulting collection of fusion algebras forming an operad. One can say that an exact solution of symplectic scalar field theory is obtained.

Conformal fields and symplectic scalar field can be combined to form symplecto-formal fields. The combination of symplectic operad and Feynman graph operad leads to a construction of Feynman diagrams in terms of  $n$ -point functions of conformal field theory.  $M$ -matrix elements with a finite

measurement resolution are expressed in terms of a hierarchy of symplecto-conformal  $n$ -point functions such that the improvement of measurement resolution corresponds to an algebra homomorphism mapping conformal fields in given resolution to composite conformal fields in improved resolution. This expresses the idea that composites behave as independent conformal fields. Also other applications are briefly discussed.

#### 4.2.5 Generalized Feynman Graphs as Generalized Braids

The basic challenge of quantum TGD is to give a precise content to the notion of generalization Feynman diagram and the reduction to braids of some kind is very attractive possibility inspired by zero energy ontology. The point is that no  $n > 2$ -vertices at the level of braid strands are needed if bosonic emergence holds true.

1. For this purpose the notion of algebraic knot is introduced and the possibility that it could be applied to generalized Feynman diagrams is discussed. The algebraic structures *kei*, quandle, rack, and biquandle and their algebraic modifications as such are not enough. The lines of Feynman graphs are replaced by braids and in vertices braid strands redistribute. This poses several challenges: the crossing associated with braiding and crossing occurring in non-planar Feynman diagrams should be integrated to a more general notion; braids are replaced with sub-manifold braids; braids of braids ...of braids are possible; the redistribution of braid strands in vertices should be algebraized. In the following I try to abstract the basic operations which should be algebraized in the case of generalized Feynman diagrams.
2. One should be also able to concretely identify braids and 2-braids (string world sheets) as well as partonic 2-surfaces and I have discussed several identifications during last years. Legendrian braids turn out to be very natural candidates for braids and their duals for the partonic 2-surfaces. String world sheets in turn could correspond to the analogs of Lagrangian sub-manifolds or two minimal surfaces of space-time surface satisfying the weak form of electric-magnetic duality. The latter option turns out to be more plausible. Finite measurement resolution would be realized as symplectic invariance with respect to the subgroup of the symplectic group leaving the end points of braid strands invariant. In accordance with the general vision TGD as almost topological QFT would mean symplectic QFT. The identification of braids, partonic 2-surfaces and string world sheets - if correct - would solve quantum TGD explicitly at string world sheet level in other words in finite measurement resolution.
3. A brief summary of generalized Feynman rules in zero energy ontology is proposed. This requires the identification of vertices, propagators, and prescription for integrating over all 3-surfaces. It turns out that the basic building blocks of generalized Feynman diagrams are well-defined.
4. The notion of generalized Feynman diagram leads to a beautiful duality between the descriptions of hadronic reactions in terms of hadrons and partons analogous to gauge-gravity duality and AdS/CFT duality but requiring no additional assumptions. The model of quark gluon plasma as a strongly interacting phase is proposed. Color magnetic flux tubes are responsible for the long range correlations making the plasma phase more like a very large hadron rather than a gas of partons. One also ends up with a simple estimate for the viscosity/entropy ratio using black-hole analogy.

### 4.3 Part III: Twistors, Bosonic Emergence, Space-time Supersymmetry

#### 4.3.1 Twistors, $N = 4$ Super-Conformal Symmetry, and Quantum TGD

Twistors - a notion discovered by Penrose - have provided a fresh approach to the construction of perturbative scattering amplitudes in Yang-Mills theories and in  $N = 4$  supersymmetric Yang-Mills theory. This approach was pioneered by Witten. The latest step in the progress was the proposal by Nima Arkani-Hamed and collaborators that super Yang Mills and super gravity amplitudes might be formulated in 8-D twistor space possessing real metric signature  $(4, 4)$ . The questions considered in this chapter are following.

1. Could twistor space could provide a natural realization of  $N = 4$  super-conformal theory requiring critical dimension  $D = 8$  and signature metric  $(4, 4)$ ? Could string like objects in TGD sense be understood as strings in twistor space? More concretely, could one in some sense lift quantum TGD from  $M^4 \times CP_2$  to 8-D twistor space  $T$  so that one would have three equivalent descriptions of quantum TGD.
2. Could one construct the preferred extremals of Kähler action in terms of twistors -may be by mimicking the construction of hyper-quaternionic *resp.* co-hyper-quaternionic surfaces in  $M^8$  as surfaces having hyper-quaternionic tangent space *resp.* normal space at each point with the additional property that one can assign to each point  $x$  a plane  $M^2(x) \subset M^4$  as sub-space or as sub-space defined by light-like tangent vector in  $M^4$ . Could one mimic this construction by assigning to each point of  $X^4$  regarded as a 4-surface in  $T$  a 4-D plane of twistor space satisfying some conditions making possible the interpretation as a tangent plane and guaranteeing the existence of a map of  $X^4$  to a surface in  $M^4 \times CP_2$ . Could twistor formalism help to resolve the integrability conditions involved?
3. Could one define 8-D counterpart of twistors in order to avoid the problems posed by the description of massive states by regarding them as massless states in 8-D context. Could the octonionic realization of 8-D gamma matrices allow to define twistors in 8-D framework? Could associativity constraint reducing twistors to quaternionic twistors locally imply effective reduction to four-dimensional twistors.

The arguments of this chapter suggest that some these questions might have affirmative answers.

### 4.3.2 Yangian Symmetry, Twistors, and TGD

There has been impressive steps in the understanding of  $\mathcal{N} = 4$  maximally supersymmetric YM theory possessing 4-D super-conformal symmetry. This theory is related by AdS/CFT duality to certain string theory in  $AdS_5 \times S^5$  background. Second stringy representation was discovered by Witten and is based on 6-D Calabi-Yau manifold defined by twistors. The unifying proposal is that so called Yangian symmetry is behind the mathematical miracles involved.

In the following I will discuss briefly the notion of Yangian symmetry and suggest its generalization in TGD framework by replacing conformal algebra with appropriate super-conformal algebras. Also a possible realization of twistor approach and the construction of scattering amplitudes in terms of Yangian invariants defined by Grassmannian integrals is considered in TGD framework and based on the idea that in zero energy ontology one can represent massive states as bound states of massless particles. There is also a proposal for a physical interpretation of the Cartan algebra of Yangian algebra allowing to understand at the fundamental level how the mass spectrum of n-particle bound states could be understood in terms of the n-local charges of the Yangian algebra.

Twistors were originally introduced by Penrose to characterize the solutions of Maxwell's equations. Kähler action is Maxwell action for the induced Kähler form of  $CP_2$ . The preferred extremals allow a very concrete interpretation in terms of modes of massless non-linear field. Both conformally compactified Minkowski space identifiable as so called causal diamond and  $CP_2$  allow a description in terms of twistors. These observations inspire the proposal that a generalization of Witten's twistor string theory relying on the identification of twistor string world sheets with certain holomorphic surfaces assigned with Feynman diagrams could allow a formulation of quantum TGD in terms of 3-dimensional holomorphic surfaces of  $CP_3 \times CP_3$  mapped to 6-surfaces dual  $CP_3 \times CP_3$ , which are sphere bundles so that they are projected in a natural manner to 4-D space-time surfaces. Very general physical and mathematical arguments lead to a highly unique proposal for the holomorphic differential equations defining the complex 3-surfaces conjectured to correspond to the preferred extremals of Kähler action.

### 4.3.3 Some Fresh Ideas about Twistorialization of TGD

The reading of the article of Tim Adamo and the recent work of Nima Arkani Hamed and Jaroslav Trnka has inspired a fresh look on twistors and a possible answer to several questions (I have written two chapters about twistors and TGD giving a view about development of ideas).

Both  $M^4$  and  $CP_2$  are highly unique in that they allow twistor structure and in TGD one can overcome the fundamental "googly" problem of the standard twistor program preventing twistorialization

in general space-time metric by lifting twistorialization to the level of the imbedding space containing  $M^4$  as a Cartesian factor. Also  $CP_2$  allows twistor space identifiable as flag manifold  $SU(3)/U(1) \times U(1)$  as the self-duality of Weyl tensor indeed suggests. This provides an additional "must" in favor of sub-manifold gravity in  $M^4 \times CP_2$ . Both octonionic interpretation of  $M^8$  and triality possible in dimension 8 play a crucial role in the proposed twistorialization of  $H = M^4 \times CP_2$ . It also turns out that  $M^4 \times CP_2$  allows a natural twistorialization respecting Cartesian product: this is far from obvious since it means that one considers space-like geodesics of  $H$  with light-like  $M^4$  projection as basic objects. p-Adic mass calculations however require tachyonic ground states and in generalized Feynman diagrams fermions propagate as massless particles in  $M^4$  sense. Furthermore, light-like H-geodesics lead to non-compact candidates for the twistor space of  $H$ . Hence the twistor space would be 12-dimensional manifold  $CP_3 \times SU(3)/U(1) \times U(1)$ .

Generalisation of 2-D conformal invariance extending to infinite-D variant of Yangian symmetry; light-like 3-surfaces as basic objects of TGD Universe and as generalised light-like geodesics; light-likeness condition for momentum generalized to the infinite-dimensional context via super-conformal algebras. These are the facts inspiring the question whether also the "world of classical worlds" (WCW) could allow twistorialization. It turns out that center of mass degrees of freedom (imbedding space) allow natural twistorialization: twistor space for  $M^4 \times CP_2$  serves as moduli space for choice of quantization axes in Super Virasoro conditions. Contrary to the original optimistic expectations it turns out that although the analog of incidence relations holds true for Kac-Moody algebra, twistorialization in vibrational degrees of freedom does not look like a good idea since incidence relations force an effective reduction of vibrational degrees of freedom to four.

The Grassmannian formalism for scattering amplitudes is expected to generalize for generalized Feynman diagrams: the basic modification is due to the possible presence of  $CP_2$  twistorialization and the fact that 4-fermion vertex -rather than 3-boson vertex- and its super counterparts define now the fundamental vertices. Both QFT type BFCW and stringy BFCW can be considered.

1. For QFT type BFCW BFF and BBB vertices would be an outcome of bosonic emergence (bosons idealized as wormhole contacts) and 4-fermion vertex is proportional to factor with dimensions of inverse mass squared and naturally identifiable as proportional to the factor  $1/p^2$  assignable to each boson line. This predicts a correct form for the bosonic propagators for which mass squared is in general non-vanishing unlike for fermion lines. The usual BFCW construction would emerge naturally in this picture. There is however a problem: the emergent bosonic propagator diverges or vanishes depending on whether one assumes SUSY at the level of single wormhole throat or not. By the special properties of SUSY generated by right handed neutrino the SUSY cannot be applied to single wormhole throat but only to a pair of wormhole throats.
2. This as also the fact that physical particles are necessarily pairs of wormhole contacts connected by fermionic strings forces stringy variant of BFCW avoiding the problems caused by non-planar diagrams. Now boson line BFCW cuts are replaced with stringy cuts and loops with stringy loops. By generalizing the earlier QFT twistor Grassmannian rules one ends up with their stringy variants in which super Virasoro generators  $G, G^\dagger$  and  $L$  bringing in  $CP_2$  scale appear in propagator lines: most importantly, the fact that  $G$  and  $G^\dagger$  carry fermion number in TGD framework ceases to be a problem since a string world sheet carrying fermion number has  $1/G$  and  $1/G^\dagger$  at its ends. Twistorialization applies because all fermion lines are light-like.
3. A more detailed analysis of the properties of right-handed neutrino demonstrates that modified gamma matrices in the modified Dirac action mix right and left handed neutrinos but that this happens markedly only in very short length scales comparable to  $CP_2$  scale. This makes neutrino massive and also strongly suggests that SUSY generated by right-handed neutrino emerges as a symmetry at very short length scales so that spartners would be very massive and effectively absent at low energies. Accepting  $CP_2$  scale as cutoff in order to avoid divergent gauge boson propagators QFT type BFCW makes sense. The outcome is consistent with conservative expectations about how QFT emerges from string model type description.

Perhaps it is not exaggeration to say that the architecture of generalized Feynman diagrams and their connection to twistor approach is now reasonably well-understood. There are of course several problems to be solved. One must feed in p-adic thermodynamics for external particles (here zero energy ontology might be highly relevant). Also the description of elementary particle families in terms of

elementary particle functionals in the space of conformal equivalence classes of partonic 2-surface must be achieved.

#### 4.3.4 Quantum Field Theory Limit of TGD from Bosonic Emergence

This chapter summarizes the basic mathematical realization of the modified Feynman rules hoped to give rise to a unitary M-matrix (recall that M-matrix is product of a positive square root of density matrix and unitary S-matrix in TGD framework and need not be unitary in the general case). The basic idea is that bosonic propagators emerge as fermionic loops. The approach is bottom up and leads to a precise general formulation for how the counterpart of YM action emerges from Dirac action coupled to gauge bosons and to modified Feynman rules. An essential element of the approach is a physical formulation for UV cutoff. Actually cutoff in both mass squared and hyperbolic angle is needed since Wick rotation does not make sense in TGD framework. This approach predicts all gauge couplings and assuming a geometrically very natural hyperbolic UV cutoff motivated by zero energy ontology one can understand the evolution of standard model gauge couplings and reproduce correctly the values of fine structure constant at electron and intermediate boson length scales. Also asymptotic freedom follows as a basic prediction. The UV cutoff for the hyperbolic angle as a function of p-adic length scale is somewhat ad hoc element of the model and a quantitative model for how this function could follow from the requirement of quantum criticality is formulated and discussed.

These considerations and numerical calculations lead to a general vision about how real and p-adic variants of TGD relate to each other and how p-adic fractalization takes place. As in case of twistorialization Cutkosky rules allowing unitarization of the tree amplitudes in terms of  $TT^\dagger$  contribution involving only light-like momenta seems to be the only working option and requires that  $TT^\dagger$  makes sense p-adically. The vanishing of the fermionic loops defining bosonic vertices for the incoming massless momenta emerges as a consistency condition suggested also by quantum criticality and by the fact that only BFF vertex is fundamental vertex if bosonic emergence is accepted. The vanishing of on mass shell N-vertices gives an infinite number of conditions on the hyperbolic cutoff as function of the integer  $k$  labeling p-adic length scale at the limit when bosons are massless and IR cutoff for the loop mass scale is taken to zero. It is not yet clear whether dynamical symmetries, in particular super-conformal symmetries, are involved with the realization of the vanishing conditions or whether hyperbolic cutoff is all that is needed.

#### 4.3.5 Does the QFT Limit of TGD Have Space-Time Super-Symmetry?

Contrary to the original expectations, TGD seems to allow a generalization of the space-time super-symmetry. This became clear with the increased understanding of the modified Dirac action. The introduction of a measurement interaction term to the action allows to understand how stringy propagator results and provides profound insights about physics predicted by TGD.

The appearance of the momentum and color quantum numbers in the measurement interaction couples space-time degrees of freedom to quantum numbers and allows also to define SUSY algebra at fundamental level as anti-commutation relations of fermionic oscillator operators. Depending on the situation a finite-dimensional SUSY algebra or the fermionic part of super-conformal algebra with an infinite number of oscillator operators results. The addition of a fermion in particular mode would define particular super-symmetry. Zero energy ontology implies that fermions as wormhole throats correspond to chiral super-fields assignable to positive or negative energy SUSY algebra whereas bosons as wormhole contacts with two throats correspond to the direct sum of positive and negative energy algebra and fields which are chiral or antichiral with respect to both positive and negative energy theta parameters. This super-symmetry is badly broken due to the dynamics of the modified Dirac operator which also mixes  $M^4$  chiralities inducing massivation. Since righthanded neutrino has no electro-weak couplings the breaking of the corresponding super-symmetry should be weakest.

The question is whether this SUSY has a realization as a SUSY algebra at space-time level and whether the QFT limit of TGD could be formulated as a generalization of SUSY QFT. There are several problems involved.

1. In TGD framework super-symmetry means addition of fermion to the state and since the number of spinor modes is larger states with large spin and fermion numbers are obtained. This picture does not fit to the standard view about super-symmetry. In particular, the identification of theta parameters as Majorana spinors and super-charges as Hermitian operators is not possible.



2. The belief that Majorana spinors are somehow an intrinsic aspect of super-symmetry is however only a belief. Weyl spinors meaning complex theta parameters are also possible. Theta parameters can also carry fermion number meaning only the supercharges carry fermion number and are non-hermitian. The the general classification of super-symmetric theories indeed demonstrates that for  $D = 8$  Weyl spinors and complex and non-hermitian super-charges are possible. The original motivation for Majorana spinors might come from MSSM assuming that right handed neutrino does not exist. This belief might have also led to string theories in  $D=10$  and  $D=11$  as the only possible candidates for TOE after it turned out that chiral anomalies cancel.
3. The massivation of particles is basic problem of both SUSYs and twistor approach. The fact that particles which are massive in  $M^4$  sense can be interpreted as massless particles in  $M^4 \times CP_2$  suggests a manner to understand super-symmetry breaking and massivation in TGD framework. The octonionic realization of twistors is a very attractive possibility in this framework and quaternionicity condition guaranteing associativity leads to twistors which are almost equivalent with ordinary 4-D twistors.
4. The first approach is based on an approximation assuming only the super-multiplets generated by right-handed neutrino or both right-handed neutrino and its antineutrino. The assumption that right-handed neutrino has fermion number opposite to that of the fermion associated with the wormhole throat implies that bosons correspond to  $\mathcal{N} = (1, 1)$  SUSY and fermions to  $\mathcal{N} = 1$  SUSY identifiable also as a short representation of  $\mathcal{N} = (1, 1)$  SUSY algebra trivial with respect to positive or negative energy algebra. This means a deviation from the standard view but the standard SUSY gauge theory formalism seems to apply in this case.
5. A more ambitious approach would put the modes of induced spinor fields up to some cutoff into super-multiplets. At the level next to the one described above the lowest modes of the induced spinor fields would be included. The very large value of  $\mathcal{N}$  means that  $\mathcal{N} \leq \exists \in$  SUSY cannot define the QFT limit of TGD for higher cutoffs. One must generalize SUSYs gauge theories to arbitrary value of  $\mathcal{N}$  but there are reasons to expect that the formalism becomes rather complex. More ambitious approach working at TGD however suggest a more general manner to avoid this problem.
  - (a) One of the key predictions of TGD is that gauge bosons and Higgs can be regarded as bound states of fermion and antifermion located at opposite throats of a wormhole contact. This implies bosonic emergence meaning that its QFT limit can be defined in terms of Dirac action. The resulting theory was discussed in detail in [?] and it was shown that bosonic propagators and vertices can be constructed as fermionic loops so that all coupling constants follow as predictions. One must however pose cutoffs in mass squared and hyperbolic angle assignable to the momenta of fermions appearing in the loops in order to obtain finite theory and to avoid massivation of bosons. The resulting coupling constant evolution is consistent with low energy phenomenology if the cutoffs in hyperbolic angle as a function of p-adic length scale is chosen suitably.
  - (b) The generalization of bosonic emergence that the TGD counterpart of SUSY is obtained by the replacement of Dirac action with action for chiral super-field coupled to vector field as the action defining the theory so that the propagators of bosons and all their super-counterparts would emerge as fermionic loops.
  - (c) The huge super-symmetries give excellent hopes about the cancelation of infinities so that this approach would work even without the cutoffs in mass squared and hyperbolic angle assignable to the momenta of fermions appearing in the loops. Cutoffs have a physical motivation in zero energy ontology but it could be an excellent approximation to take them to infinity. Alternatively, super-symmetric dynamics provides cutoffs dynamically.
6. The condition that  $\mathcal{N} = \infty$  variants for chiral and vector superfields exist fixes completely the identification of these fields in zero energy ontology.
  - (a) In this framework chiral fields are generalizations of induced spinor fields and vector fields those of gauge potentials obtained by replacing them with their super-space counterparts. Chiral condition reduces to analyticity in theta parameters thanks to the different definition

of hermitian conjugation in zero energy ontology ( $\theta$  is mapped to a derivative with respect to theta rather than to  $\bar{\theta}$ ) and conjugated super-field acts on the product of all theta parameters.

- (b) Chiral action is a straightforward generalization of the Dirac action coupled to gauge potentials. The counterpart of YM action can emerge only radiatively as an effective action so that the notion emergence is now unavoidable and indeed basic prediction of TGD.
- (c) The propagators associated with the monomials of  $n$  theta parameters behave as  $1/p^n$  so that only  $J = 0, 1/2, 1$  states propagate in normal manner and correspond to normal particles. The presence of monomials with number of thetas higher than 2 is necessary for the propagation of bosons since by the standard argument fermion and scalar loops cancel each other by super-symmetry. This picture conforms with the identification of graviton as a bound state of wormhole throats at opposite ends of string like object.
- (d) This formulation allows also to use modified gamma matrices in the measurement interaction defining the counterpart of super variant of Dirac operator. Poincare invariance is not lost since momenta and color charges act on the tip of  $CD$  rather than the coordinates of the space-time sheet. Hence what is usually regarded as a quantum theory in the background defined by classical fields follows as exact theory. This feeds all data about space-time sheet associated with the maximum of Kähler function. In this approach WCW as a Kähler manifold is replaced by a cartesian power of  $CP_2$ , which is indeed quaternionic Kähler manifold. The replacement of light-like 3-surfaces with number theoretic braids when finite measurement resolution is introduced, leads to a similar replacement.
- (e) Quantum TGD as a "complex square root" of thermodynamics approach suggests that one should take a superposition of the amplitudes defined by the points of a coherence region (identified in terms of the slicing associated with a given wormhole throat) by weighting the points with the Kähler action density. The situation would be highly analogous to a spin glass system since the modified gamma matrices defining the propagators would be analogous to the parameters of spin glass Hamiltonian allowed to have a spatial dependence. This would predict the proportionality of the coupling strengths to Kähler coupling strength and bring in the dependence on the size of  $CD$  coming as a power of 2 and give rise to p-adic coupling constant evolution. Since TGD Universe is analogous to 4-D spin glass, also a sum over different preferred extremals assignable to a given coherence regions and weighted by  $\exp(K)$  is probably needed.
- (f) In TGD Universe graviton is necessarily a bi-local object and the emission and absorption of graviton are bi-local processes involving two wormhole contacts: a pair of particles rather than single particle emits graviton. This is definitely something new and defies a description in terms of QFT limit using point like particles. Graviton like states would be entangled states of vector bosons at both ends of stringy curve so that gravitation could be regarded as a square of YM interactions in rather concrete sense. The notion of emergence would suggest that graviton propagator is defined by a bosonic loop. Since bosonic loop is dimensionless, IR cutoff defined by the largest  $CD$  present must be actively involved. At QFT limit one can hope a description as a bi-local process using a bi-local generalization of the QFT limit. It turns out that surprisingly simple candidate for the bi-local action exists.

## 4.4 Part IV: Hyper-Finite Factors of Type II and Hierarchy of Planck Constants

### 4.4.1 What von Neumann Right After All?

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors, could provide the mathematics needed to develop a more explicit view about the construction of M-matrix generalizing the notion of S-matrix in zero energy ontology. In this chapter I will discuss various aspects of hyper-finite factors and their possible physical interpretation in TGD framework. The original discussion has transformed during years from free speculation reflecting in many aspects my ignorance about the mathematics involved to a more realistic view about the role of these algebras in quantum TGD.

### 1. Hyper-finite factors in quantum TGD

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type III<sub>1</sub> appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

1. The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type II<sub>1</sub>. There also the Clifford algebra at a given point (light-like 3-surface) of world of classical worlds (WCW) is therefore HFF of type II<sub>1</sub>. If the fermionic Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type II<sub>1</sub>. Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type II<sub>∞</sub> results.
2. WCW is a union of sub-WCWs associated with causal diamonds (*CD*) defined as intersections of future and past directed light-cones. One can allow also unions of *CD*s and the proposal is that *CD*s within *CD*s are possible. Whether *CD*s can intersect is not clear.
3. The assumption that the  $M^4$  proper distance  $a$  between the tips of *CD* is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that  $a$  can have all possible values. Since  $SO(3)$  is the isotropy group of *CD*, the *CD*s associated with a given value of  $a$  and with fixed lower tip are parameterized by the Lobatchevski space  $L(a) = SO(3, 1)/SO(3)$ . Therefore the *CD*s with a free position of lower tip are parameterized by  $M^4 \times L(a)$ . A possible interpretation is in terms of quantum cosmology with  $a$  identified as cosmic time [?] Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type III<sub>1</sub>. If one allows all values of  $a$ , one ends up with  $M^4 \times M^4_{\pm}$  as the space of moduli for WCW.
4. An interesting special aspect of 8-dimensional Clifford algebra with Minkowski signature is that it allows an octonionic representation of gamma matrices obtained as tensor products of unit matrix 1 and 7-D gamma matrices  $\gamma_k$  and Pauli sigma matrices by replacing 1 and  $\gamma_k$  by octonions. This inspires the idea that it might be possible to end up with quantum TGD from purely number theoretical arguments. This seems to be the case. One can start from a local octonionic Clifford algebra in  $M^8$ . Associativity condition is satisfied if one restricts the octonionic algebra to a subalgebra associated with any hyper-quaternionic and thus 4-D sub-manifold of  $M^8$ . This means that the modified gamma matrices associated with the Kähler action span a complex quaternionic sub-space at each point of the sub-manifold. This associative sub-algebra can be mapped a matrix algebra. Together with  $M^8 - H$  duality [?]his leads automatically to quantum TGD and therefore also to the notion of WCW and its Clifford algebra which is however only mappable to an associative algebra and thus to HFF of type II<sub>1</sub>.

### 4. Hyper-finite factors and M-matrix

HFFs of type III<sub>1</sub> provide a general vision about M-matrix.

1. The factors of type III allow unique modular automorphism  $\Delta^{it}$  (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.
2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.
4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW.

The concrete construction of M-matrix utilizing the idea of bosonic emergence (bosons as fermion anti-fermion pairs at opposite throats of wormhole contact) meaning that bosonic propagators reduce to fermionic loops identifiable as wormhole contacts leads to generalized Feynman rules for M-matrix in which modified Dirac action containing measurement interaction term defines stringy propagators. This M-matrix should be consistent with the above proposal.

#### 5. Connes tensor product as a realization of finite measurement resolution

The inclusions  $\mathcal{N} \subset \mathcal{M}$  of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In zero energy ontology  $\mathcal{N}$  would create states experimentally indistinguishable from the original one. Therefore  $\mathcal{N}$  takes the role of complex numbers in non-commutative quantum theory. The space  $\mathcal{M}/\mathcal{N}$  would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative  $\mathcal{N}$ -valued coordinates.
2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their  $\mathcal{N}$  "averaged" counterparts. The "averaging" would be in terms of the complex square root of  $\mathcal{N}$ -state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.
3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that  $\mathcal{N}$  acts like complex numbers on M-matrix elements as far as  $\mathcal{N}$ -"averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in  $\mathcal{M}(\mathcal{N}$  interpreted as finite-dimensional space with a projection operator to  $\mathcal{N}$ ). The condition that  $\mathcal{N}$  averaging in terms of a complex square root of  $\mathcal{N}$  state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

#### 6. Quantum spinors and fuzzy quantum mechanics

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to  $q = 1$ . The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with 'true' and 'false'. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to  $q=1$  phase and decoherence is not a problem as long as it does not induce this transition.

#### 4.4.2 Does TGD predict spectrum of Planck constants?

The quantization of Planck constant has been the basic theme of TGD since 2005. The basic idea was stimulated by the finding of Nottale that planetary orbits could be seen as Bohr orbits with enormous value of Planck constant given by  $\hbar_{gr} = GM_1M_2/v_0$ , where the velocity parameter  $v_0$  has the approximate value  $v_0 \simeq 2^{-11}$  for the inner planets. This inspired the ideas that quantization is due to a condensation of ordinary matter around dark matter concentrated near Bohr orbits and that dark matter is in macroscopic quantum phase in astrophysical scales. The second crucial empirical input were the anomalies associated with living matter. The recent version of the chapter represents the evolution of ideas about quantization of Planck constants from a perspective given by seven years's work with the idea. A very concise summary about the situation is as follows.

##### *Basic physical ideas*

The basic phenomenological rules are simple and there is no need to modify them.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Imbedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies.
2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order  $CP_2$  size.

This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain:  $E = hf$  implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of imbedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

In astrophysics and cosmology the implications are even more dramatic. It was who first introduced the notion of gravitational Planck constant as  $\hbar_{gr} = GMm/v_0$ ,  $v_0 < 1$  has interpretation as velocity light parameter in units  $c = 1$ . This would be true for  $GMm/v_0 \geq 1$ . The interpretation of  $\hbar_{gr}$  in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses  $M$  and  $m$ . The huge value of  $\hbar_{gr}$  means that the integer  $\hbar_{gr}/\hbar_0$  interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This changes the view about gravitons and suggests that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

3. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge

couplings strengths  $\alpha = g^2/4\pi\hbar$ . If the effective value of  $\hbar$  replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle,  $\alpha$  is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter  $GMm/\hbar$  has gigantic value. Replacing  $\hbar$  with  $\hbar_{gr} = GMm/v_0$  the coupling strength becomes  $v_0 < 1$ .

#### *Space-time correlates for the hierarchy of Planck constants*

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of imbedding spaces defined as Cartesian products of singular coverings of  $M^4$  and  $CP_2$  with numbers of sheets given by integers  $n_a$  and  $n_b$  and  $\hbar = n\hbar_0$ .  $n = n_a n_b$ .

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the imbedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded  $M^4$  in  $M^4 \times CP_2$  have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of  $CP_2$  coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents  $\partial L_K/\partial(\partial_\alpha h^k)$  defining the modified gamma matrices and gradients  $\partial_\alpha h^k$  is not one-to-one. Same canonical momentum current corresponds to several values of gradients of imbedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of  $CD$  carrying the elementary particle quantum numbers this implies that the two normal derivatives of  $h^k$  are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system. What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to  $N$  branches  $b_i$  of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches  $b_i$  and  $b_j$  of multi-furcation.  $N$ -particle state would correspond to  $N$ -sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches co-incide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization  $N = n_a n_b$  occurs but now  $n_a$  and  $n_b$  would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than  $M^4$  and  $CP_2$  as in the original hypothesis.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless one poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single  $n$ -sub-furcations of  $N$ -furcation is selected. The most general state of this kind involves superposition of various  $n$ -sub-furcations.

#### **4.4.3 Mathematical speculations inspired by the hierarchy of Planck constants**

This chapter contains the purely mathematical speculations about the hierarchy of Planck constants (actually only effective hierarchy if the recent interpretation is correct) as separate from the material describing the physical ideas, key mathematical concepts, and the basic applications. These mathematical speculations emerged during the first stormy years in the evolution of the ideas about Planck constant and must be taken with a big grain of salt. I feel myself rather conservative as compared to the fellow who produced this stuff for 7 years ago. This all is of course very relative. Many readers might experience this recent me as a reckless speculator.

The first speculative question is about possible relationship between Jones inclusions of hyperfinite factors of type  $II_1$  (hyper-finite factors are von Neuman algebras emerging naturally in TGD framework). The basic idea is that the discrete groups assignable to inclusions could correspond to

discrete groups acting in the effective covering spaces of imbedding space assignable to the hierarchy of Planck constants.

There are also speculations relating to the hierarchy of Planck constants, Mc-Kay correspondence, and Jones inclusions. Even Farey sequences, Riemann hypothesis and N-tangles are discussed. Depending on reader these speculations might be experienced as irritating or entertaining. It would be interesting to go this stuff through in the light of recent understanding of the effective hierarchy of Planck constants to see what portion of its survives.

## REFERENCES

### Mathematics

- [A1] Eisenhart. *Riemannian Geometry*. Princeton University Press, 1964.
- [A2] C. N. Pope G. W. Gibbons.  $CP_2$  as gravitational instanton. *Comm. Math. Phys.*, 55, 1977.
- [A3] W. Hawking, S. and N. Pope, C. Generalized Spin Structures in Quantum Gravity. *Phys. Lett.*, (1), 1978.
- [A4] J. Milnor. *Topology form Differential Point of View*. The University Press of Virginia, Virginia, 1965.
- [A5] N. Pope, C. Eigenfunctions and  $Spin^c$  Structures on  $CP_2$ , 1980.
- [A6] M. Spivak. *Differential Geometry I,II,III,IV*. Publish or Perish, Boston, 1970.
- [A7] J. Hanson T. Eguchi, B. Gilkey. *Phys. Rep.*, 66:1980, 1980.
- [A8] R. Thom. *Commentarii Math. Helvet.*, 28, 1954.
- [A9] Wallace. *Differential Topology*. W. A. Benjamin, New York, 1968.

### Cosmology and Astro-Physics

- [E1] D. Da Roacha and L. Nottale. Gravitational Structure Formation in Scale Relativity. <http://arxiv.org/abs/astro-ph/0310036>, 2003.

### Books related to TGD

- [K1] M. Pitkänen. 1983.
- [K2] M. Pitkänen. *Bio-Systems as Conscious Holograms*. Onlinebook. [http://tgdtheory.com/public\\_html/hologram/hologram.html](http://tgdtheory.com/public_html/hologram/hologram.html), 2006.
- [K3] M. Pitkänen. *Bio-Systems as Self-Organizing Quantum Systems*. Onlinebook. [http://tgdtheory.com/public\\_html/bioselforg/bioselforg.html](http://tgdtheory.com/public_html/bioselforg/bioselforg.html), 2006.
- [K4] M. Pitkänen. Dark Forces and Living Matter. In *p-Adic Length Scale Hypothesis and Dark Matter Hierarchy*. Onlinebook. [http://tgdtheory.com/public\\_html/paddark/paddark.html#darkforces](http://tgdtheory.com/public_html/paddark/paddark.html#darkforces), 2006.
- [K5] M. Pitkänen. Dark Matter Hierarchy and Hierarchy of EEGs. In *TGD and EEG*. Onlinebook. [http://tgdtheory.com/public\\_html/tgdeeg/tgdeeg.html#eegdark](http://tgdtheory.com/public_html/tgdeeg/tgdeeg.html#eegdark), 2006.
- [K6] M. Pitkänen. Does TGD Predict the Spectrum of Planck Constants? In *Towards M-Matrix*. Onlinebook. [http://tgdtheory.com/public\\_html/tgdquant/tgdquant.html#Planck](http://tgdtheory.com/public_html/tgdquant/tgdquant.html#Planck), 2006.
- [K7] M. Pitkänen. *Genes and Memes*. Onlinebook. [http://tgdtheory.com/public\\_html/genememe/genememe.html](http://tgdtheory.com/public_html/genememe/genememe.html), 2006.

- [K8] M. Pitkänen. *Magnetospheric Consciousness*. Onlinebook. [http://tgdtheory.com/public\\_html/magnconsc/magnconsc.html](http://tgdtheory.com/public_html/magnconsc/magnconsc.html), 2006.
- [K9] M. Pitkänen. Many-Sheeted DNA. In *Genes and Memes*. Onlinebook. [http://tgdtheory.com/public\\_html/genememe/genememe.html#genecodec](http://tgdtheory.com/public_html/genememe/genememe.html#genecodec), 2006.
- [K10] M. Pitkänen. *Mathematical Aspects of Consciousness Theory*. Onlinebook. [http://tgdtheory.com/public\\_html/mathconsc/mathconsc.html](http://tgdtheory.com/public_html/mathconsc/mathconsc.html), 2006.
- [K11] M. Pitkänen. *p-Adic length Scale Hypothesis and Dark Matter Hierarchy*. Onlinebook. [http://tgdtheory.com/public\\_html/paddark/paddark.html](http://tgdtheory.com/public_html/paddark/paddark.html), 2006.
- [K12] M. Pitkänen. *Physics in Many-Sheeted Space-Time*. Onlinebook. [http://tgdtheory.com/public\\_html/tgdclass/tgdclass.html](http://tgdtheory.com/public_html/tgdclass/tgdclass.html), 2006.
- [K13] M. Pitkänen. *Quantum Hardware of Living Matter*. Onlinebook. [http://tgdtheory.com/public\\_html/bioware/bioware.html](http://tgdtheory.com/public_html/bioware/bioware.html), 2006.
- [K14] M. Pitkänen. *Quantum Physics as Infinite-Dimensional Geometry*. Onlinebook. [http://tgdtheory.com/public\\_html/tgdgeom/tgdgeom.html](http://tgdtheory.com/public_html/tgdgeom/tgdgeom.html), 2006.
- [K15] M. Pitkänen. *Quantum TGD*. Onlinebook. [http://tgdtheory.com/public\\_html/tgdquant/tgdquant.html](http://tgdtheory.com/public_html/tgdquant/tgdquant.html), 2006.
- [K16] M. Pitkänen. Quantum Theory of Self-Organization. In *Bio-Systems as Self-Organizing Quantum Systems*. Onlinebook. [http://tgdtheory.com/public\\_html/bioselforg/bioselforg.html#selforgac](http://tgdtheory.com/public_html/bioselforg/bioselforg.html#selforgac), 2006.
- [K17] M. Pitkänen. TGD and Astrophysics. In *Physics in Many-Sheeted Space-Time*. Onlinebook. [http://tgdtheory.com/public\\_html/tgdclass/tgdclass.html#astro](http://tgdtheory.com/public_html/tgdclass/tgdclass.html#astro), 2006.
- [K18] M. Pitkänen. *TGD and EEG*. Onlinebook. [http://tgdtheory.com/public\\_html/tgdeeg/tgdeeg.html](http://tgdtheory.com/public_html/tgdeeg/tgdeeg.html), 2006.
- [K19] M. Pitkänen. *TGD and Fringe Physics*. Onlinebook. [http://tgdtheory.com/public\\_html/freenergy/freenergy.html](http://tgdtheory.com/public_html/freenergy/freenergy.html), 2006.
- [K20] M. Pitkänen. *TGD as a Generalized Number Theory*. Onlinebook. [http://tgdtheory.com/public\\_html/tgdnumber/tgdnumber.html](http://tgdtheory.com/public_html/tgdnumber/tgdnumber.html), 2006.
- [K21] M. Pitkänen. TGD as a Generalized Number Theory: p-Adicization Program. In *TGD as a Generalized Number Theory*. Onlinebook. [http://tgdtheory.com/public\\_html/tgdnumber/tgdnumber.html#visiona](http://tgdtheory.com/public_html/tgdnumber/tgdnumber.html#visiona), 2006.
- [K22] M. Pitkänen. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In *TGD as a Generalized Number Theory*. Onlinebook. [http://tgdtheory.com/public\\_html/tgdnumber/tgdnumber.html#visionb](http://tgdtheory.com/public_html/tgdnumber/tgdnumber.html#visionb), 2006.
- [K23] M. Pitkänen. *TGD Inspired Theory of Consciousness*. Onlinebook. [http://tgdtheory.com/public\\_html/tgdconsc/tgdconsc.html](http://tgdtheory.com/public_html/tgdconsc/tgdconsc.html), 2006.
- [K24] M. Pitkänen. *Topological Geometro-dynamics: an Overview*. Onlinebook. [http://tgdtheory.com/public\\_html/tgdview/tgdview.html](http://tgdtheory.com/public_html/tgdview/tgdview.html), 2006.