Classical TGD

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Contents

Abstract

In this chapter the classical field equations associated with the Kähler action are studied.

1. Are all extremals actually "preferred"?

The notion of preferred extremal has been central concept in TGD but is there really compelling need to pose any condition to select preferred extremals in zero energy ontology (ZEO) as there would be in positive energy ontology? In ZEO the union of the space-like ends of space-time surfaces at the boundaries of causal diamond (CD) are the first guess for 3-surface. If one includes to this 3-surface also the light-like partonic orbits at which the signature of the induced metric changes to get analog of Wilson loop, one has good reasons to expect that the preferred extremal is highly unique without any additional conditions apart from non-determinism of Kähler action proposed to correspond to sub-algebra of conformal algebra acting on the light-like 3-surface and respecting light-likeness. One expects that there are finite number n of conformal equivalence classes and n corresponds to n in $h_{eff} = nh$. These objects would allow also to understand the assignment of discrete physical degrees of freedom to the partonic orbits as required by the assignment of hierarchy of Planck constants to the non-determinism of Kähler action.

2. Preferred extremals and quantum criticality

The identification of preferred extremals of Kähler action defining counterparts of Bohr orbits has been one of the basic challenges of quantum TGD. By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence.

The space-time representation for dissipation comes from the interpretation of regions of space-time surface with Euclidian signature of induced metric as generalized Feynman diagrams (or equivalently the light-like 3-surfaces defining boundaries between Euclidian and Minkowskian regions). Dissipation would be represented in terms of Feynman graphs representing irreversible dynamics and expressed in the structure of zero energy state in which positive energy part corresponds to the initial state and negative energy part to the final state. Outside Euclidian regions classical dissipation should be absent and this indeed the case for the known extremals.

The non-determinism should also give rose to space-time correlate for quantum criticality. The study of Kähler-Dirac equations suggests how to define quantum criticality. Noether currents assignable to the Kähler-Dirac equation are conserved only if the first variation of Kähler-Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action - at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries.

It became later clear that the well-definedness of em charge forces in the generic case the localization of the spinor modes to 2-D surfaces - string world sheets. This would suggest that the equations stating the vanishing of the second variation of Kähler action hold true only at string world sheets.

The vanishing of second variations of preferred extremals suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In zero energy ontology (ZEO) catastrophe theory would be generalized to infinite-dimensional context. Finite number of sheets for catastrophe would be replaced with finite number of conformal equivalence classes of space-time surfaces connecting given space-like 3-surfaces at the boundaries causal diamond (CD).

3. Hamilton-Jacobi structure

Most known extremals share very general properties. One of them is Hamilton-Jacobi structure meaning the possibility to assign to the extremal so called Hamilton-Jacobi coordinates. This means dual slicings of M^4 by string world sheets and partonic 2-surfaces. Number theoretic compactification led years later to the same condition. This slicing allows a dimensional reduction of quantum TGD to Minkowskian and Euclidian variants of string model. Also holography in the sense that the dynamics of 3-dimensional space-time surfaces reduces to that for 2-D partonic surfaces in a given measurement resolution follows. The construction of quantum TGD relies in essential manner to this property. CP_2 type vacuum extremals do not possess Hamilton-Jaboci structure but have holomorphic structure.

4. Specific extremals of Kähler action

The study of extremals of Kähler action represents more than decade old layer in the development of TGD.

1. The huge vacuum degeneracy is the most characteristic feature of Kähler action (any 4-surface having CP_2 projection which is Legendre sub-manifold is vacuum extremal, Legendre sub-manifolds of CP_2 are in general 2-dimensional). This vacuum degeneracy is behind the spin glass analogy and leads to the p-adic TGD. As found in the second part of the book, various particle like vacuum extremals also play an important role in the understanding of the quantum TGD.
2. The so called CP_2 type vacuum extremals have finite, negative action and are therefore an excellent candidate for real particles whereas vacuum extremals with vanishing Kähler action are candidates for the virtual particles. These extremals have one dimensional M^4 projection, which is light like curve but not necessarily geodesic and locally the metric of the extremal is that of CP_2: the quantization of this motion leads to Virasoro algebra. Space-times with topology $CP_2\#CP_2\#...CP_2$ are identified as the generalized Feynmann diagrams with lines thickened to 4-manifolds of "thickness" of the order of CP_2 radius. The quantization of the random motion with light velocity associated with the CP_2 type extremals in fact led to the discovery of Super Virasoro invariance, which through the construction of the WCW geometry, becomes a basic symmetry of quantum TGD.
3. There are also various non-vacuum extremals.
   (a) String like objects, with string tension of same order of magnitude as possessed by the cosmic strings of GUTs, have a crucial role in TGD inspired model for the galaxy formation and in the TGD based cosmology.
   (b) The so called massless extremals describe non-linear plane waves propagating with the velocity of light such that the polarization is fixed in given point of the space-time surface. The purely TGD:eish feature is the light like Kähler current: in the ordinary Maxwell theory vacuum gauge currents are not possible. This current serves as a source of coherent photons, which might play an important role in the quantum model of bio-system as a macroscopic quantum system.

1 Introduction

A brief summary of what might be called basic principles is in order to facilitate the reader to assimilate the basic tools and rules of intuitive thinking involved.

1.1 Quantum-Classical Correspondence

The fundamental meta level guiding principle is quantum-classical correspondence (classical physics is an exact part of quantum TGD). The principle states that all quantum aspects of the theory, which means also various aspects of consciousness such as volition, cognition, and intentionality, should have space-time correlates [K25] . Real space-time sheets provide kind of symbolic representations whereas p-adic space-time sheets provide correlates for cognition. All that we can symbolically communicate about conscious experience relies on quantal space-time engineering to build these representations.

The progress in the understanding of quantum TGD has demonstrated that quantum classical correspondence is more or less equivalent with holography, quantum criticality, and criticality as the principle selecting the preferred extremals of Kähler action. It also guarantees 1-1 correspondence between quantum states and classical states essential for quantum measurement theory.

1.2 Classical Physics As Exact Part Of Quantum Theory

Classical physics corresponds to the dynamics of space-time surfaces determined by the criticality in the sense that extremals allow an infinite number of deformations giving rise to a vanishing second variation of the Kähler action [K24] . This dynamics have several unconventional features basically due to the possibility to interpret the Kähler action as a Maxwell action expressible in terms of the induced metric defining classical gravitational field and induced Kähler form defining a non-linear Maxwell field not as such identifiable as electromagnetic field however.

1.2.1 *Classical long ranged weak and color fields as signature for a fractal hierarchy of copies standard model physics*

The geometrization of classical fields means that various classical fields are expressible in terms of imbedding space-coordinates and are thus not primary dynamical variables. This predicts the presence of long range weak and color (gluon) fields not possible in standard physics context. It took 26 years to end up with a convincing interpretation for this puzzling prediction.

What seems to be the correct interpretation is in terms of an infinite fractal hierarchy of copies of standard models physics with appropriately scaled down mass spectra for quarks, leptons, and gauge bosons. Both p-adic length scales and the values of Planck constant predicted by TGD [K28] label various physics in this hierarchy. Also other quantum numbers are predicted as labels. This means that universe would be analogous to an inverted Mandelbrot fractal with each bird's eye of view revealing new long length scale structures serving also as correlates for higher levels of self hierarchy.

Exotic dark weak forces and their dark variants are consistent with the experimental widths for ordinary weak gauge bosons since the particles belonging to different levels of the hierarchy do not have direct couplings at Feynman diagram level although they have indirect classical interactions and also the de-coherence reducing the value of $\hbar$ is possible. Classical long ranged weak fields play a key role in quantum control and communications in living matter [K8, K5] . Long ranged classical color force in turn is the backbone in the model of color vision [K11] : colors correspond to the increments of color quantum numbers in this model. The increments of weak isospin in turn could define the basic color like quale associated with hearing (black-white $\leftrightarrow$ to silence-sound [K11, K20, K21]).

1.2.2 *Topological field quantization and the notion of many-sheeted space-time*

The compactness of CP_2 implies the notions of many-sheeted space-time and topological field quantization. Topological field quantization means that various classical field configurations decompose into topological field quanta. One can see space-time as a gigantic Feynman diagram with lines thickened to 4-surfaces. Criticality of the preferred extremals implies that only selected field configurations analogous to Bohr's orbits are realized physically so that quantum-classical correspondence becomes very predictive. An interpretation as a 4-D quantum hologram is a further very useful picture [K13] but will not be discussed in this chapter in any detail.

Topological field quantization implies that the field patterns associated with material objects form extremely complex topological structures which can be said to belong to the material objects. The notion of field body , in particular magnetic body, typically much larger than the material system, differentiates between TGD and Maxwell's electrodynamics, and has turned out to be of fundamental importance in the TGD inspired theory of consciousness. One can say that field body provides an abstract representation of the material body.

One implication of many-sheetedness is the possibility of macroscopic quantum coherence. By quantum classical correspondence large space-time sheets as quantum coherence regions are macroscopic quantum systems and therefore ideal sites of the quantum control in living matter.

1. The original argument was that each space-time sheet carrying matter has a temperature determined by its size and the mass of the particles residing at it via de Broglie wave length $\lambda_{dB} = \sqrt{2mE}$ assumed to define the p-adic length scale by the condition $L(k) < \lambda_{dB} < L(k_>)$. This would give very low temperatures when the size of the space-time sheet becomes large enough. The original belief indeed was that the large space-time sheets can be very cold because they are not in thermal equilibrium with the smaller space-time sheets at higher temperature.

2. The assumption about thermal isolation is not needed if one accepts the possibility that Planck constant is dynamical and quantized and that dark matter corresponds to a hierarchy of phases characterized by increasing values of Planck constant [K28, K4] . From $E = hf$ relationship it is clear that arbitrarily low frequency dark photons (say EEG photons) can have energies above thermal energy which would explain the correlation of EEG with consciousness. This vision allows to formulate more precisely the basic notions of TGD inspired

theory of consciousness and leads to a model of living matter giving precise quantitative predictions. Also the ability of this vision to generate new insights to quantum biology provides strong support for it [K5] .

Many-sheeted space-time predicts also fundamental mechanisms of metabolism based on the dropping of particles between space-time sheets with an ensuing liberation of the quantized zero point kinetic energy. Also the notion of many-sheeted laser follows naturally and population inverted many-sheeted lasers serve as storages of metabolic energy [K14] .

Space-time sheets topologically condense to larger space-time sheets by wormhole contacts which have Euclidian signature of metric. This implies causal horizon (or elementary particle horizon) at which the signature of the induced metric changes from Minkowskian to Euclidian. This forces to modify the notion of sub-system. What is new is that two systems represented by space-time sheets can be unentangled although their sub-systems bound state entangle with the mediation of the join along boundaries bonds connecting the boundaries of sub-system space-time sheets. This is not allowed by the notion of sub-system in ordinary quantum mechanics. This notion in turn implies the central concept of fusion and sharing of mental images by entanglement [K25] .

1.2.3 *Zero energy ontology*

The notion of zero energy ontology emerged implicitly in cosmological context from the observation that the imbeddings of Robertson-Walker metrics are always vacuum extremals. In fact, practically all solutions of Einstein's equations have this property very naturally. The explicit formulation emerged with the progress in the formulation of quantum TGD. In zero energy ontology physical states are creatable from vacuum and have vanishing net quantum numbers, in particular energy. Zero energy states can be decomposed to positive and negative energy parts with definite geometro-temporal separation, call it T, and having interpretation in terms of initial and final states of particle reactions. Zero energy ontology is consistent with ordinary positive energy ontology at the limit when the time scale of the perception of observer is much shorter than T. One of the implications is a new view about fermions and bosons allowing to understand Higgs mechanism among other things.

Zero energy ontology leads to the view about S-matrix as a characterizer of time-like entanglement associated with the zero energy state and a generalization of S-matrix to what might be called M-matrix emerges. M-matrix is complex square root of density matrix expressible as a product of real valued "modulus" and unitary matrix representing phase and can be seen as a matrix valued generalization of Schrödinger amplitude. Also thermodynamics becomes an inherent element of quantum theory in this approach.

1.2.4 *TGD Universe is quantum spin glass*

Since Kähler action is Maxwell action with Maxwell field and induced metric expressed in terms of $M^4_+ \times CP_2$ coordinates, the gauge invariance of Maxwell action as a symmetry of the vacuum extremals (this implies is a gigantic vacuum degeneracy) but not of non-vacuum extremals. Gauge symmetry related space-time surfaces are not physically equivalent and gauge degeneracy transforms to a huge spin glass degeneracy. Spin glass degeneracy provides a universal mechanism of macro-temporal quantum coherence and predicts degrees of freedom called zero modes not possible in quantum field theories describing particles as point-like objects. Zero modes not contributing to the configuration space line element are identifiable as effectively classical variables characterizing the size and shape of the 3-surface as well as the induced Kähler field. Spin glass degeneracy as mechanism of macroscopic quantum coherence should be equivalent with dark matter hierarchy as a source of the coherence [K13] .

1.2.5 *Classical and p-adic non-determinism*

The vacuum degeneracy of Kähler action implies classical non-determinism, which means that space-like 3-surface is not enough to fix the space-time surface associated with it uniquely as an absolute minimum of action, and one must generalize the notion of 3-surface by allowing sequences of 3-surfaces with time like separations to achieve determinism in a generalized sense. These "association sequences" can be seen as symbolic representations for the sequences of quantum jumps defining selves and thus for contents of consciousness. Not only speech and written language define symbolic representations but all real space-time sheets of the space-time surfaces can be seen in a very general sense as symbolic representations of not only quantum states but also of quantum jump sequences. An important implication of the classical non-determinism is the possibility to have conscious experiences with contents localized with respect to geometric time. Without this non-determinism conscious experience would have no correlates localized at space-time surface, and there would be no psychological time.

p-Adic non-determinism follows from the inherent non-determinism of p-adic differential equations for any action principle and is due to the fact that integration constants, which by definition are functions with vanishing derivatives, are not constants but functions of the pinary cutoffs x_N defined as $x = \sum_k x_k p^k \rightarrow x_N = \sum_{k<N} x_k p^k$ of the arguments of the function. In p-adic topology one can therefore fix the behavior of the space-time surface at discrete set of space-time points *above* some length scale defined by p-adic concept of nearness by fixing the integration constants. In the real context this corresponds to the fixing the behavior *below* some time/length scales since points p-adically near to each other are in real sense faraway. This is a natural correlate for the possibility to plan the behavior and p-adic non-determinism is assumed to be a classical correlate for the non-determinism of intentionality, and perhaps also imagination and cognition.

These two non-determinisms allow to understand the self-referentiality of consciousness at a very general level. In a given quantum jump a space-time surface can be created with the property that it represents symbolically or cognitively something about the contents of consciousness before the quantum jump. Thus it becomes possible to become conscious about being conscious of something. This is very much like mathematician expressing her thoughts as symbol sequences which provides feedback to go the next abstraction level.

Classical and p-adic non-determinisms force also the generalization of the notion of quantum entanglement. Time-like entanglement, crucial for understanding long term memory and precognition becomes possible. The notion of many-sheeted space-time forces also to modify the notion of sub-system, which implies that unentangled systems can have entangled sub-systems. One can partially understand this in terms of length scale dependent notion of entanglement (the entanglement of sub-systems is not seen in the length scale resolution defined by the size of unentangled systems) but only partially. The formation of join along boundaries bonds between sub-system space-time sheets and the fact that topologically condensed space-time sheets are separated by elementary particle horizons from larger space-time sheets, provide the deeper topological motivation for the generalization of sub-system concept.

1.2.6 *Dark matter hierarchy and hierarchy of Planck constants*

Dark matter revolution with levels of the hierarchy labeled by values of Planck constant forces a further generalization of the notion of imbedding space and thus of space-time. One can say, that imbedding space is a book like structure obtained by gluing together infinite number of copies of the imbedding space like pages of a book: two copies characterized by singular discrete bundle structure are glued together along 4-dimensional set of common points. These points have physical interpretation in terms of quantum criticality. Particle states belonging to different sectors (pages of the book) can interact via field bodies representing space-time sheets which have parts belonging to two pages of this book.

The hierarchy of Planck constants can be reduced to the quantum criticality of Kähler action due to the non-determinism of Kähler action and the generalization of imbedding space is only a useful auxiliary tool to describe the situation mathematically.

The hierarchy of Planck constants is necessary in order to understand the formation of gravitational and in fact all bound states if one assumes that this is due to the fermionic strings connecting partonic 2-surfaces as AdS/CFT correspondence suggests. The generalization of AdS/CFT duality to TGD framework suggests strongly that Kähler action can be expressed as string area action for string world sheets with effective covariant metric defined by Kähler-Dirac gamma matrices proportional to $\alpha_K^2 \propto 1/h_{eff}^2$. Macroscopic quantum coherence even in astrophysical scales is unavoidable prediction and it becomes also clear that super string models cannot describe the formation of gravitational bound states.

Of course, all this is a work in progress and there are many uncertainties involved. Despite this it seems that it is good to sum up the recent view in order to make easier to refer to the new developments in the existing chapters.

1.2.7 *p-Adic fractality of life and consciousness*

p-Adic fractality of biology and consciousness has become an increasingly important guide line in the construction of the theory. This notion allows to relate phenomena occurring in the molecular level to phenomena like remote viewing and psychokinesis and it leads also to the view that topological field quanta of various fields of astrophysical size are crucial for the functioning of bio-systems. If one accepts p-adic fractality, the theory can be tested in unexpected manners, in particular in molecular and cellular length scales where the systems are much simpler. Sensory perception, long term memory, remote mental interactions, metabolism: all these phenomena rely on the same basic mechanisms. p-Adic length scale hypothesis allows to quantify the hypothesis with testable quantitative predictions.

1.2.8 *Double slit experiment and classical non-determinism*

Bohr's complementarity principle is the basic element of Copenhagen interpretation and at the same time one of the most poorly defined aspects of this interpretation. If the possibility of macroscopic quantum entanglement between measurement instrument and quantum system is accepted, complementary principle becomes un-necessary. This is however not all that is needed. If classical non-determinism makes it possible to represent quantum jump sequences at space-time level, a revision of space-time description of quantum measurement is necessary. This sounds very logical but to be honest, I write these lines only after having learned about the remarkable experiment done by Shahriar Afshar [J2] .

The variant of double slit experiment by Shahriar Afshar seems to contradict the Copenhagen interpretation which states that the particle and field aspects are complementarity and thus mutually exclusive. In the case of double slit experiment complementarity predicts that the measurement of whether the photon came to the detector through slit 1 or 2 should destroy the interference pattern of electromagnetic fields in the region behind the screen.

The experimental arrangement of Afshar differs from the standard double slit experiment in that a lens was added behind the screen. The lens transmitted the photons coming from slits 1 and 2 via mirrors to detectors A and B so that in particle picture a photon detected by A (B) could be regarded as coming from slit 1 (2). In the first step both slits were open and the detectors represented interference patterns representing diffraction through single slit. The other slit was then closed and metal wires at the positions of dark interference rings were added. These wires degraded somewhat the image in the second detector. After this the second slit was opened again. Surprisingly, the resulting interference pattern was the original one.

The measurement certainly measures the particle aspect of photons. On the other hand, the preservation of the detected patterns means that no photons did enter in the regions containing the wires so that also interference pattern is there. Hence wave and particle aspects seem to be mutually consistent.

This finding is difficult to understand in Copenhagen interpretation and also in the many-worlds interpretation of quantum mechanics. Afshar himself suggest that the very notion of photon must be questioned. It is however difficult to accept this view since the photon absorption quite concretely corresponds to a click in the detector and also because the mathematical formalism of second quantization works so fantastically.

The conclusion can be criticized. What is primarily measured is not basically through which slit the photons came but whether the direction of the momentum of the photon emerging from the lens was in the angle range characterizing the detector or not. One can however argue that in deterministic physics for fields the two measurements are equivalent so that the problem remains.

In TGD framework the classical physics is not completely deterministic and this has led to a generalization of the notion of quantum classical correspondence. Space-time surface provides a classical (unfaithful) representation not only for quantum states but for quantum jump sequences or equivalently, for sequences of quantum states. The most obvious identification for the quantum states is as the maximal non-deterministic regions of a given space-time sheet.

In the recent context this would mean that the fields in the region between the screen and lens represent the state before the state function reduction and thus the interference pattern, whereas the fields in the region between lens and detectors represent the situation after the state function reduction. The interaction with lens involves classical non-determinism.

This picture conforms also with the notion of topological field quantization. The space-time decomposes into space-time sheets interpreted, topological field quanta (topological light rays containing photons, flux quanta of magnetic field, etc..). Topological field quanta correspond to the coherence regions for classical fields with spinor fields included. De-coherence corresponds to the splitting of space-time sheet to smaller, possibly parallel space-time sheets. Topological field quantum carries classical fields inside it but behaves as a whole like particle. Hence particle and wave aspects are consistent in the sense that below the size scale L of the topological field quantum (say the thickness of a magnetic flux tube or topological light ray) the description as a wave applies and above L particle description makes sense. In the recent case the coherence is lost at the lens space-time sheet where the space-time sheet representing interference pattern decomposes to two sheets representing photon beams going to the two detectors.

1.3 Some Basic Ideas Of TGD Inspired Theory Of Consciousness And Quantum Biology

The following ideas of TGD inspired theory of consciousness and of quantum biology are the most relevant ones for what will follow.

“Everything is conscious and consciousness can be only lost” is the briefest manner to summarize TGD inspired theory of consciousness. Quantum jump as moment of consciousness and the notion of self are key concepts of the theory. Self is a system able to avoid bound state entanglement with environment and can be formally seen as an ensemble of quantum jumps. The contents of consciousness of self are defined by the averaged increments of quantum numbers and zero modes (sensory and geometric qualia). Moment of consciousness can be said to be the counterpart of elementary particle and self the counterpart of many-particle state, either bound and free. The selves formed by macro-temporal quantum coherence are in turn the counterparts of atoms, molecules and larger structures. Macro-temporal quantum coherence effectively binds a sequence of quantum jumps to a single quantum jump as far as conscious experience is considered. The idea that conscious experience is about changes amplified to macroscopic quantum phase transitions, is the key philosophical guideline in the construction of various models, such as the model of qualia, the capacitor model of sensory receptor, the model of cognitive representations, and declarative memories.

2. Macro-temporal quantum coherence is a second consequence of the spin glass degeneracy [K13] . It is essentially due to the formation of bound states and has as a topological correlate the formation of join along boundaries bonds connecting the boundaries of the component systems. During macro-temporal coherence quantum jumps integrate effectively to single long-lasting quantum jump and one can say that system is in a state of oneness, eternal now, outside time. Macro-temporal quantum coherence makes possible stable non-entropic mental images. Negative energy MEs are one particular mechanism making possible macro-temporal quantum coherence via the formation of bound states, and remote metabolism and sharing of mental images are other facets of this mechanism. The real understanding of the origin of macroscopic quantum coherence requires the generalization of quantum theory allowing dynamical and quantized Planck constant [K4, K5] .

3. p-Adic physics as physics of intentionality and of cognition is a further key idea of TGD inspired theory of consciousness. p-Adic space-time sheets as correlates for intentions and p-adic-to-real transformations of them as correlates for the transformation of intentions to actions allow deeper understanding of also psychological time as a front of p-adic-to-real transition propagating to the direction of the geometric future. Negative energy MEs are absolutely essential for the understanding of how precisely targeted intentionality is realized.

1.4 About Preferred Extremals

The understanding of preferred extremals of Kähler action is the basic challenge of classical TGD. The field equations are known locally but the key problem is to give a precise meaning to the "preferred". Various attempts in this direction are discussed in [K3, K32]. These options give different perspectives to the properties of preferred extremals but provide no magic formula.

Before continuing, it must be emphasized that the notion of preferred extremal originated in positive energy ontology. In ZEO 3-surfaces are pairs of space-like 3-surfaces at the boundaries of CD. Also the light-like partonic orbits at which the induced metric changes its signature could be included to get a closed 3-surface analogous to Wilson loop. In deterministic theory one would expect that the extremals are unique so that "preferred" would become obsolete. Kähler action is non-deterministic and quantum criticality suggests that the preferred extremals have Kac-Moody type symmetries are gauge symmetries deforming partonic orbits and preserving their light-likeness. The number of gauge equivalence classes would be finite and correspond to the integer n defining the value of effective Planck constant $h_{eff} = n \times h$. The conformal subalgebra with conformal weights coming as multiples of n would act as gauge symmetries. The most that one might expect that above measurement resolution the attribute "preferred" is un-necessary in ZEO.

The idea about Bohr orbitology would require that "preferred" is not an empty attribute even in ZEO. There would be strong correlations between the space-like 3-surfaces at the oppposite boundaries of CD: the pairs would be like point pairs at Bohr orbits connecting the boundaries of CD.

It is good to summarize the attempts to give meaning to "preferred". The properties assigned to preferred extremals characterize also the known extremals.

1. The original proposal was that preferred extremals correspond to absolute minima of Kähler action. This makes sense only for Euclidian regions of space-time surfaces representing lines of generalized Feynman diagrams. This option is not number theoretically attractive since the very notion of minimum is p-adically poorly defined unless one can reduce absolute minimization to purely algebraic conditions making sense also p-adically.

2. Later I ended up with the idea that preferred extremals are critical being analogous to saddle points of potential function: what this exactly means is not obvious [K3, K30]. This option is not consistent with absolute minimization. It took a long time to realize that Minkowskian and Euclidian regions give imaginary *resp.* real contributions to the exponent of the vacuum functional having interpretation as Morse function *resp.* Kähler function of WCW. One might ask whether absolute minimization works in Euclidian regions and criticality in Minkowskian regions.

3. I have proposed the characterization of preferred extremal property in terms of Hamilton-Jacobi structure generalizing the notion of complex structure and being motivated by the huge super-conformal symmetries of "world of classical worlds" [K3]. This picture is consistent with the identification of Minkowskian regions consisting of massless and interaction topological field quanta since one can speak about local light direction and local polarization directions. This picture is very quantal since linear superposition is not possible and the counterpart of it is set theoretic union of space-time sheets representing quanta. Number theoretic vision based on classical number fields supports similar picture.

4. Almost topological QFT property requires that Kähler action reduces to "boundary" terms transformable to Chern-Simons terms. This is guaranteed if the Kähler current is orthogonal to Kähler gauge potential ($j \cdot A = 0$) and the weak form of electric-magnetic duality holds true.

5. The recent view emerged from the realization that the extended super-conformal invariances of TGD define naturally a hierarchy of broken conformal gauge symmetries with sub-algebras with conformal weights coming as multiples of some integer n defining gauge symmetries. At classical level this means that the corresponding Noether charges associated with 3-surfaces at the ends of CD vanish. These conditions realize the strong form of holography implied by the strong form of General Coordinate Invariance stating that partonic 2-surfaces and their tangent space data (or more or less equivalently string world sheets) fix the quantum states.

   The vanishing of conformal charges is extremely powerful condition and the proposal is that the space-time sheets connecting given 3-surfaces at the ends of CD form $n = h_{eff}/n$

conformal equivalence classes. It seems that this provides the long sought for realization of preferred extremal property.

The physical interpretation for the sequences of conformal symmetry breakings with $n_{i+1} = \sum_{k<i+1} m_k$ is in terms of hierarchy of criticalities. At each phase transition new gauge generators are transformed to generators of genuine physical charges. These hierarchies also correspond to the gradual transformation of degrees of freedom below measurement resolution to physical degrees of freedom as the measurement resolution increases. At the level of biology the increase of h_{eff} corresponds to evolution leading to improved sensory and cognitive resolutions.

1.5 TGD Space-Time Viz. Space-Time Of GRT

It took decades to realize that GRT space-time is only an effective space-time obtained by replacing the sheets of many-sheeted space-time with single piece of Minkowski space with effective metric defined by the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Equivalence Principle as it is expressed by Einstein's equations would follow from Poincare invariance in long length scales. Therefore it is not necessary that Einstein's equations are satisfy in TGD and the vanishing of the divergence of energy momentum tensor for Kähler action could be enough.

I have however considered also other alternatives before ending up this view.

1. Vacuum extremals of Kähler action seem to be excellent candidates for defining cosmological models. In light of what was said above this would mean that GRT space-times having imbedding as vacuum extremals are in favored position. This would not be surprising since the addition of string world sheets carrying non-vanishing Kähler form to vacuum extremals should in good approximation give non-vacuum extremals serving as building bricks of the many-sheeted space-time.

2. An obvious question is whether Einstein equations could be true for preferred extremals [K3]. The condition that Kähler 4-force vanishes implies vanishing of the divergence of the energy momentum tensor. In general relativity this leads to Einstein's equations with cosmological constant term since the linear combination of Einstein tensor and metric tensor is automatically divergenceless. In TGD framework Einstein equations would have powerful consequences: for instance, curvature scalar would be constant so that mathematically highly interesting constant curvature spaces allowing to classify manifolds topologically, could emerge naturally. A weaker condition would be that energy momentum tensor is divergenceless only asymptotically when dissipation characterize by Lorentz force classically is absent.

3. In TGD framework one can also consider a weaker form of Einstein equations with cosmological constant: several (at most two) cosmological "constants" would appear in the counterpart of Einstein equations [K31].

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

2 Many-Sheeted Space-Time, Magnetic Flux Quanta, Electrets And MEs

TGD inspired theory of consciousness and of living matter relies on space-time sheets carrying ordinary matter, topological light rays (massless extremals, MEs), and magnetic and electric flux quanta. There are some new results which motivate a separate discussion of them.

2.1 Dynamical Quantized Planck Constant And Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that

Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

2.1.1 *Dark matter as phase with non-standard value of h_{eff}?*

D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also sub-harmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K22] .

It has been found in CERN (see`http://tinyurl.com/jbvwmrp3`) that matter and antimatter atoms have no differences in the energies of their excited states. This is predicted by CPT symmetry. Notice however that CP and T can be separately broken and that this is indeed the case. Kaon is classical example of this in particle physics. Neutral kaon and anti-kaon behave slightly differently.

This finding forces to repeat an old question. Where does the antimatter reside? Or does it exist at all?

In TGD framework one possibility is that antimatter corresponds to dark matter in TGD sense that is a phase with $h_{eff} = n \times h$, $n = 1, 2, 3, ...$ such that the value of n for antimatter is different from that for visible matter. Matter and antimatter would not have direct interactions and would interact only via classical fields or by emission of say photons by matter (antimatter) suffering a phase transition changing the value of h_{eff} before absorbtion by antimatter (matter). This could be rather rare process. Bio-photons could be produced from dark photons by this process and this is assumed in TGD based model of living matter.

What the value of n for ordinary visible matter is? The naive guess is that it is $n = 1$, the smallest possible value. Randell Mills [D4] has however claimed the existence of scaled down hydrogen atoms - Mills calls them hydrinos - with ground state binding energy considerably higher than for hydrogen atom. The experimental support for the claim is published in respected journals and the company of Mills is developing a new energy technology based on the energy liberated in the transition to hydrino state (see `http://tinyurl.com/hajyqo6`).

These findings can be understood in TGD framework if one has actually $n = 6$ for visible atoms and $n = 1, 2$, or 3 for hydrinos. Hydrino states would be stabilized in the presence of some catalysts [L5] (see `http://tinyurl.com/goruuzm`). This also suggest a universal catalyst action. Among other things catalyst action requires that reacting molecule gets energy to overcome the potential barrier making reaction very slow. If an atom - say hydrogen - in catalyst suffers a phase transition to hydrogen like state, it liberates binding energy, and if one of the reactant molecules receives it it can overcome the barrier. After the reaction the energy can be sent back and catalyst hydrino returns to the ordinary hydrogen state.

So: could it be that one has n=6 for stable matter and n is different from this for stable antimatter? Could the small CP breaking cause this?

2.1.2 *Dark matter as a source of long ranged weak and color fields*

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long ranged classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics. An attractive solution of the matter antimatter asymmetry is based on the identification of also antimatter as dark matter.

2.1.3 *Dark matter hierarchy and consciousness*

The emergence of the vision about dark matter hierarchy has meant a revolution in TGD inspired theory of consciousness. Dark matter hierarchy means also a hierarchy of long term memories with the span of the memory identifiable as a typical geometric duration of moment of consciousness at the highest level of dark matter hierarchy associated with given self so that even human life cycle represents at this highest level single moment of consciousness.

Dark matter hierarchy leads to detailed quantitative view about quantum biology with several testable predictions [K5] . The applications to living matter suggests that there is a hierarchy a hierarchy of Planck constants h_{eff} coming as integer multiples of ordinary Planck constant. The original - too limited - proposal was that in living matter there would be preferred values for the integer coming as power of 2^{11} $\hbar(k) = \lambda^k\hbar_0$, $\lambda = 2^{11}$ for $k = 0, 1, 2, ...$ [K5]. Also integer valued sub-harmonics and integer valued sub-harmonics of λ were found to be possible. The hypothesis $h_{eff} = n \times h$ turned out to follow naturally from TGD. Each p-adic length scale corresponds to this kind of hierarchy [K7]. One can also ask whether number theoretically very simple integers could define the values of h_{eff}/h. One candidate for this class of integers characterize polygons constructible using only compass and ruler. The sine and cosine of the angle $2\pi/n$ characterizing the polygon reduces to expression involving only square root operations applied on rationals. These integers are products of power of two with a subset of distinct Fermat primes.

The general prediction is that Universe is a kind of inverted Mandelbrot fractal for which each bird's eye of view reveals new structures in long length and time scales representing scaled down copies of standard physics and their dark variants. These structures would correspond to higher levels in self hierarchy. This prediction is consistent with the belief that 75 per cent of matter in the universe is dark.

1. Living matter and dark matter

Living matter as ordinary matter quantum controlled by the dark matter hierarchy has turned out to be a particularly successful idea. The hypothesis has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of EEG [K5] . Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [K15, K5] . A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [K5] .

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally. That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of $\hbar$ at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

2. Dark matter hierarchy and the notion of self

The vision about dark matter hierarchy leads to a more refined view about self hierarchy and hierarchy of moments of consciousness [K4, K5] . The larger the value of Planck constant, the longer the subjectively experienced duration. The naive guess for the average geometric duration of self was $T(n) \propto h_{eff}/h = n$. The identification of self as a sequence of state function reductions at the same boundary of CD at which state does not change gives an identification of the lifetime of self as the increase of the temporal distance between the tips of CD during this period [K27, K1]. The order of magnitude could be however proportional to h_{eff}/h.

Dark matter hierarchy suggests also a slight modification of the notion of self. Each self involves a hierarchy of dark matter levels, and one is led to ask whether the highest level in this hierarchy corresponds to a single quantum jump rather than a sequence of quantum jumps. The averaging of conscious experience over quantum jumps would occur only for sub-selves at lower levels of dark matter hierarchy and these mental images would be ordered, and single moment of consciousness would be experienced as a history of events. One can ask whether even entire life cycle could be regarded as a single quantum jump at the highest level so that consciousness would not be completely lost even during deep sleep. This would allow to understand why we seem to know directly that this biological body of mine existed yesterday.

The fact that we can remember phone numbers with 5 to 9 digits supports the view that self corresponds at the highest dark matter level to single moment of consciousness. Self would experience the average over the sequence of moments of consciousness associated with each sub-self but there would be no averaging over the separate mental images of this kind, be their parallel or serial. These mental images correspond to sub-selves having shorter wake-up periods than self and would be experienced as being time ordered. Hence the digits in the phone number are experienced as separate mental images and ordered with respect to experienced time.

2.2 P-Adic Length Scale Hypothesis And The Connection Between Thermal De Broglie Wave Length And Size Of The Space-Time Sheet

Also real space-time sheets are assumed to be characterized by p-adic prime p and assumed to have a size determined by primary p-adic length scale L_p or possibly n-ary p-adic length scale $L_p(n)$.

The possibility to assign a p-adic prime to the real space-time sheets is required by the success of the elementary particle mass calculations and various applications of the p-adic length scale hypothesis. The original idea was that the non-determinism of Kähler action corresponds to p-adic determinism for some primes. It has been however difficult to make this more concrete.

Rational numbers are common to reals and all p-adic number fields. One can actually assign to any algebraic extension of rationals extensions of p-adic numbers and construct corresponding adeles. These extensions can be arranged according to the complexity and I have already earlier proposed that this hierarchy defines an evolutionary hierarchy.

How the existence of preferred p-adic primes characterizing space-time surfaces emerge was solved only quite recently. The solution relies on p-adicization based on strong holography and the idea that string world sheets and partonic surfaces with parameters in algebraic extensions of rationals define the intersection of reality and various p-adicities. The algebraic extension possess preferred primes as primes, which are ramified meaning that their decomposition to a product of primes of the extension contains higher than first powers of its primes (prime ideals to be more rigorous). These primes are obviously natural candidates for primes characterizing string world sheets number theoretically.

In strong form of holography p-adic continuations of 2-surfaces to preferred extremals identifiable as imaginations would be easy due to the existence of p-adic pseudo-constants. The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K17]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes. Whether these primes correspond to p-adic lengths scale hypothesis or its generalization to small primes, is an open question.

Weak form of NMP indeed allows also to justify a generalization of p-adic length scale hypothesis [K16]. Primes near but below powers of primes are favoured since they allow exceptionally large negentropy gain so that state function reductions to tend to select them.

Parallel space-time sheets with distance about 10^4 Planck lengths form a hierarchy. Each material object (...,atom, molecule, ..., cell,...) would correspond to this kind of space-time sheet. The p-adic primes $p \simeq 2^k$, k prime or power of prime, characterize the size scales of the space-time sheets in the hierarchy. The p-adic length scale $L(k)$ can be expressed in terms of cell membrane thickness as

$$L(k) = 2^{(k-151)/2} \times L(151) \ , \tag{2.1}$$

$L(151) \simeq 10$ nm. These are so called primary p-adic length scales but there are also n-ary p-adic length scales related by a scaling of power of $\sqrt{p}$ to the primary p-adic length scale. Quite recent model for photosynthesis [K14] gives additional support for the importance of also n-ary p-adic length scales so that the relevant p-adic length scales would come as half-octaves in a good approximation but prime and power of prime values of k would be especially important.

2.3 Topological Light Rays (Massless Extremals, Mes)

I have described MEs, or "topological light rays", in detail in [L1] and in [K19] newphys, and describe here only very briefly the basic characteristics of MEs and concentrate on new idea about their possible role for consciousness and life.

2.3.1 *What MEs are?*

MEs (massless extremals, topological light rays) can be regarded as topological field quanta of classical radiation fields [K19, K2] . They are typically tubular space-time sheets inside which radiation fields propagate with light velocity in single direction without dispersion. The simplest case corresponds to a straight cylindrical ME but also curved MEs, kind of curved light rays, are possible. The initial values for a given moment of time are arbitrary by light likeness. Therefore MEs are ideal for precisely targeted communications. What distinguishes MEs from Maxwellian radiation fields in empty space is that light like vacuum 4-current is possible: ordinary Maxwell's equations would state that this current vanishes. Quite generally, purely geometric vacuum charge densities and 3-currents are purely TGD based prediction and could be seen as a classical correlate of the vacuum polarization predicted by quantum field theories.

MEs are fractal structures containing MEs within MEs. The so called scaling law of homeopathy predicts that the high frequency MEs inside low frequency MEs are in a ratio having discrete values [K12] . One can indeed justify this relationship. As ions drop from smaller space-time sheets to magnetic flux tubes, zero point kinetic energy is liberated as high frequency MEs, and the ions dropped to magnetic flux tubes generate cyclotron radiation, and the ratio of the fundamental frequencies is constant not depending on particle mass and being determined solely by p-adic length scale hypothesis. The model for the radio waves induced by the irradiation of DNA by laser light [I1] gives support for this picture [K13] .

2.3.2 *Two basic types of MEs*

MEs have 2-dimensional CP_2 projection which means that electro-weak holonomy group is Abelian (color holonomy is always Abelian which suggests that physical states in TGD Universe correspond to states of color multiplets with vanishing color hypercharge and isospin rather than color singlets). If CP_2 projection belongs to a homologically non-trivial geodesic sphere, only em and Z^0 fields and Abelian color gauge fields are present. In the homologically trivial case only classical W fields are non-vanishing.

1. Neutral MEs can be assigned to various kinds of communications from biological body to the magnetic body and fractal hierarchy of EEGs and ZEGs represent the basic example in this respect [K5] .

2. Dark W MEs serving as correlate for dark W exchanges induce an exotic ionization of atomic nuclei [K23, K6, K5] . This induces charge entanglement between magnetic body and biological body generating dark plasma oscillation patterns inducing nerve pulse patterns and ion waves at the space-time sheets occupied by the ordinary matter. The mechanism is based on many-sheeted Faraday law inducing electromagnetic fields at ordinary space-time sheet in turn giving rise to ohmic currents. State function reduction selects one of the exotically ionized configurations. This mechanism is the most plausible candidate for how magnetic body as an intentional agent controls biological body.

2.3.3 *Negative energy MEs*

MEs can have either positive or negative energy depending on the time orientation. The understanding of negative energy MEs has increased considerably. Phase conjugate laser beams [D2] are the most plausible standard physics counterparts of negative energy MEs since they can be interpreted as time reversed laser beams and do not possess direct Maxwellian analog. By quantum-classical correspondence one can interpret the frequencies associated with negative energy MEs as energies. One can also assume that the Bose-Einstein condensed photons associated with negative energy MEs and with the coherent light generated by the light like vacuum current have negative energies.

For frequencies for which energy is above the thermal energy there is no system which could interact with negative energy MEs or absorb negative energy photons. Therefore negative energy MEs and corresponding photons should propagate through matter practically without any interaction. Feinberg has demonstrated that phase conjugate laser beams behave similarly: for instance, one can see through chickens using these laser beams [D1] . This means that negative energy MEs do not respect Faraday cages and thus represent an attractive candidate for the hypothetical Psi field.

Negative energy MEs have many applications.

1. Negative energy MEs ideal for generating time like entanglement. Since negative energies are involved, this entanglement can be seen as a correlate for the bound state entanglement leading to a macro-temporal quantum coherence. Negative energy MEs make thus possible telepathic sharing of mental images. Negative energy MEs are involved with both sensory perception, long term memory, and motor action. In the model for living matter [K5] The charge entanglement generated by W MEs inducing exotic weak charge and electromagnetic charge is assumed to be responsible for bio-control whereas neutral MEs in general carrying both em and Z^0 fields are responsible for communications.

2. Negative energy MEs are ideal for a precisely targeted realization of intentions. p-Adic ME having a large number of common rational points with negative energy ME is generated and transformed to a real ME in quantum jump. The system receives positive energy and momentum as a recoil effect and the transition is not masked by ordinary spontaneously occurring quantum transitions since the energy of the system increases. One can say that negative energy ME represents the desires communicated to the geometric past and inducing as a reaction the desired action realized as say neuronal activity and generation of positive energy MEs.

3. The generation of negative energy MEs is also in a key role in remote metabolism and MEs serve as quantum credit cards implying an extreme flexibility of the metabolism. If the system receiving negative energy MEs is a population inverted laser or its many-sheeted counterpart, then quite a small field intensity associated with negative energy MEs (intensity of negative energy photons) can lead to the amplification of the time reflected positive energy signal. The reason is that the rate for the induced emission is proportional to the number of particles dropped to the ground state from the excited state. Therefore even negative energy bio-photons might serve as quantum controllers of metabolism and induce much more intense beams of positive energy photons, say when interacting with mitochondria.

2.4 Magnetic Flux Quanta And Electrets

Magnetic flux tubes and electrets are extremals of Kähler action dual to each other. Also layer like magnetic flux quanta and their electric counterparts are possible. The magnetic/electric field is in a good approximation of constant magnitude but has varying direction.

2.4.1 *Magnetic fields and life*

The magnetic field associated with any material system is topologically quantized, and one can assign to any system a magnetic body. An attractive idea is that the relationship of the magnetic body to the material system is to some degree that of the manual to an electronic instrument. Quantitative arguments related to the dark matter hierarchy assuming that magnetic bodies are dark suggest that cognitions and emotions are regarded as somatosensory qualia of the magnetic body [K11, K5] . Magnetic body would in this case serve as a kind of computer screen at which the data items processes in say brain are communicated either classically (positive energy MEs) or by sharing of mental images (negative energy MEs).

Magnetic body is also an active intentional agent: motor actions are controlled from magnetic body and proceed as cascade like processes from long to short length and time scales as quantum communications of desires at various levels of hierarchy of magnetic bodies. Communication occurs backwards in geometric time by negative energy MEs. Motor action as a response to these desires occurs by classical communications by positive energy MEs and as neural activities. This explains the coherence and synchrony of motor actions difficult to understand in neuroscience framework. The sizes of flux quanta are astrophysical: for instance, EEG frequency of 7.8 Hz corresponds to a wave length defined by Earth's circumference. The non-locality in the length scale of magnetosphere, and even in length scales up to light life, is forced by Uncertainty Principle alone, if taken seriously in macroscopic length scales.

The leakage of supra currents of ions and their Cooper pairs from magnetic flux tubes of the Earth's magnetic field to smaller space-time sheets and their dropping back involving liberation of the zero point kinetic energy defines one particular metabolic “Karma's cycle”.

In many-sheeted space-time particles topologically condense at all space-time sheets having projection to given region of space-time so that this option makes sense only near the boundaries of space-time sheet of a given system. Also p-adic phase transition increasing the size of the space-time sheet could take place and the liberated energy would correspond to the reduction of zero point kinetic energy. Particles could be transferred from a portion of magnetic flux tube portion to another one with different value of magnetic field and possibly also of Planck constant h_{eff} so that cyclotron energy would be liberated.

The dropping of protons from $k = 137$ atomic space-time sheet involved with the utilization of ATP molecules is only a special instance of the general mechanism involving an entire hierarchy of zero point kinetic energies defining universal metabolic currencies. This leads to the idea that the topologically quantized magnetic field of Earth defines the analog of central nervous system and blood circulation present already during the pre-biotic evolution and making possible primitive metabolism. This has far reaching implications for the understanding of how pre-biotic evolution led to living matter as we understand it [K8] .

For instance, it has recently become clear that the dropping of atoms and molecules from space-time sheets labelled by p-adic prime $p \simeq 2^k$, $k = 131$, liberates photons at visible and near infrared wave lengths. The hot $k = 131$ space-time sheets (with temperatures above 1000 K) could have served as a source of metabolic energy for life-forms at cool $k = 137$ sheets. Photosynthesis could have developed in the circumstances where solar radiation was replaced with these photons. The correct prediction is that chlorophylls should be especially sensitive to these wave lengths. In particular, it is predicted that also IR wave lengths 700-1000 nm should have been utilized. There indeed are bacteria using only this portion of solar radiation. This leads to a scenario making sense only in TGD universe. Pre-biotic life could have developed at the cool space-time sheets in the hot interior of Earth below crust, where $k = 131$ space-time sheets are possible and this life could still be there [K8] . Also the life as we know it, could involve hot spots generated by the cavitation of water inside cell. The classical repulsive Z^0 force causes a strong acceleration during final stages of bubble collapse creating high temperatures, and could explain also sono-luminescence [D3] , [D3] as suggested in [K6] .

Magnetic Mother Gaia could also form sensory and other representations receiving input from several brains via negative energy EEG MEs entangling magnetosphere with brains. The multi-brained magnetospheric selves could be responsible for the third person aspect of consciousness and for the evolution of social structures. For instance, the successful healing by prayer and meditation groups [J1] , and the experiments of Mark Germine [J3] provide support for the notion of multi-brained magnetospheric selves are involved. Magnetic flux tubes could function as wave guides for MEs and this aspect is crucial in the model of long term memory.

2.4.2 *Electrets and bio-systems*

Bio-systems are known to be full of electrets and liquid crystals [I2] . Perhaps the most fundamental electret structure is cell membrane. In particular, the water inside cells tends to be in gel phase which is liquid crystal phase. There are many good reasons for why water should be in ordered phase. One very fundamental reason is that bio-polymers are stable in liquid crystal/ordered water phase since there are no free water molecules available for the de-polymerization by hydration. In fact, only a couple of years ago it was experimentally discovered that bio-polymers can be stabilized around ice.

The capacitor model for sensory receptor is one very important application of the electret concept [K11] , [L2] . Sensory qualia result in the flow of particles with given quantum numbers from the plate to another one in quantum discharge. This kind of amplification of quantum number *resp.* zero mode increments would give rise to both geometric *resp.* non-geometric qualia [K11] .

Also micro-tubuli are electrets. Sol-gel transition, as any phase transition, is an good candidate for the representation of a conscious bit and controlled local sol-gel transitions between ordinary and liquid crystal water could be a basic control tool making possible cellular locomotion, changes of protein conformations, etc... The tubulin dimers of micro-tubuli could induce sol-gel transformations by generating negative energy MEs, and micro-tubular surface could provide bit maps of their environment somewhat like sensory areas of brain provide maps of body. If gel→sol transition around tubulin inducing conformational change induces sol→gel transformation in some point of environment as would be the case for the seesaw mechanism to be discussed below, a one-one correspondence would result. By this one-one correspondence micro-tubules would automatically generate kind of conscious log files about the control activities which could have evolved to micro-tubular declarative memory representations about what happens inside cell [K14] .

3 General View About Field Equations

In this section field equations are deduced and discussed in general level. The fact that the divergence of the energy momentum tensor, Lorentz 4-force, does not vanish in general, in principle makes possible the mimicry of even dissipation and of the second law. For asymptotic self organization patterns for which dissipation is absent the Lorentz 4-force must vanish. This condition is guaranteed if Kähler current is proportional to the instanton current in the case that CP_2 projection of the space-time sheet is smaller than four and vanishes otherwise. An attractive identification for the vanishing of Lorentz 4-force is as a condition equivalent with the selection of preferred extremal of Kähler action. If preferred extremals correspond to absolute minima this principle would be essentially equivalent with the second law of thermodynamics.

3.1 Field Equations

The requirement that Kähler action is stationary leads to the following field equations in the interior of the four-surface

$$\begin{aligned} D_\beta(T^{\alpha\beta}h^k_\alpha) &- j^\alpha J^k_{\ l}\partial_\alpha h^l = 0 \ , \\ T^{\alpha\beta} &= J^{\nu\alpha}J_\nu^{\ \beta} - \frac{1}{4}g^{\alpha\beta}J^{\mu\nu}J_{\mu\nu} \ . \end{aligned} \tag{3.1}$$

Here $T^{\alpha\beta}$ denotes the traceless canonical energy momentum tensor associated with the Kähler action. An equivalent form for the first equation is

$$\begin{aligned} T^{\alpha\beta}H^k_{\alpha\beta} &- j^\alpha(J_\alpha^{\ \beta}h^k_{\ \beta} + J^k_{\ l}\partial_\alpha h^l) = 0 \ . \\ H^k_{\alpha\beta} &= D_\beta\partial_\alpha h^k \ . \end{aligned} \tag{3.2}$$

$H^k_{\alpha\beta}$ denotes the components of the

second fundamental form and $j^\alpha = D_\beta J^{\alpha\beta}$ is the gauge current associated with the Kähler field.

On the boundaries of X^4 and at wormhole throats the field equations are given by the expression

$$\frac{\partial L_K}{\partial_n h^k} = T^{n\beta}\partial_\beta h^k - J^{n\alpha}(J_\alpha^{\ \beta}\partial_\beta h^k + J^k_{\ l})\partial_\alpha h^k) = 0 \ . \tag{3.3}$$

At wormhole throats problems are caused by the vanishing of metric determinant implying that contravariant metric is singular.

For M^4 coordinates boundary conditions are satisfied if one assumes

$$T^{n\beta} = 0 \tag{3.4}$$

stating that there is no flow of four-momentum through the boundary component or wormhole throat. This means that there is no energy exchange between Euclidian and Minkowskian regions so that Euclidian regions provide representations for particles as autonomous units. This is in accordance with the general picture [K10] . Note that momentum transfer with external world necessarily involves generalized Feynman diagrams also at classical level.

For CP_2 coordinates the boundary conditions are more delicate. The construction of WCW spinor structure [K29] led to the conditions

$$g_{ni} = 0 \ , \ J_{ni} = 0 \ . \tag{3.5}$$

$J^{ni} = 0$ does not and should not follow from this condition since contravariant metric is singular. It seems that limiting procedure is necessary in order to see what comes out.

The condition that Kähler electric charge defined as a gauge flux is non-vanishing would require that the quantity $J^{nr}\sqrt{g}$ is finite (here r refers to the light-like coordinate of X^3_l). Also $g^{nr}\sqrt{g_4}$ which is analogous to gravitational flux if n is interpreted as time coordinate could be non-vanishing. These conditions are consistent with the above condition if one has

$$\begin{array}{llll} J_{ni} = 0 \ , & g_{ni} = 0 \ , & J_{ir} = 0 \ , & g_{ir} = 0 \ , \\ J^{nk} = 0 \ \ k \neq r \ , & g^{nk} = 0 \ \ k \neq r \ , & J^{nr}\sqrt{g_4} \neq 0 \ , & g^{nr}\sqrt{g_4} \neq 0 \ . \end{array} \tag{3.6}$$

The interpretation of this conditions is rather transparent.

1. The first two conditions state that covariant form of the induced Kähler electric field is in direction normal to X^3_l and metric separate into direct sum of normal and tangential contributions. Fifth and sixth condition state the same in contravariant form for $k \neq n$.

2. Third and fourth condition state that the induced Kähler field at X^3_l is purely magnetic and that the metric of x^3_l reduces to a block diagonal form. The reduction to purely magnetic field is of obvious importance as far as the understanding of the generalized eigen modes of the Kähler-Dirac operator is considered [K29] .

3. The last two conditions must be understood as a limit and $\neq$ means only the possibility of non-vanishing Kähler gauge flux or analog of gravitational flux through X^3_l.

4. The vision inspired by number theoretical compactification allows to identify r and n in terms of the light-like coordinates assignable to an integrable distribution of planes $M^2(x)$ assumed to be assignable to M^4 projection of $X^4(X^3_l)$. Later it will be found that Hamilton-Jacobi structure assignable to the extremals indeed means the existence of this kind of distribution meaning slicing of $X^4(X^3_l)$ both by string world sheets and dual partonic 2-surfaces as well as by light-like 3-surfaces Y^3_l.

5. The physical analogy for the situation is the surface of an ideal conductor. It would not be surprising that these conditions are satisfied by all induced gauge fields.

3.2 Topologization And Light-Likeness Of The Kähler Current AsAlternative Manners To Guarantee Vanishing Of Lorentz 4-Force

The general solution of 4-dimensional Einstein-Yang Mills equations in Euclidian 4-metric relies on self-duality of the gauge field, which topologizes gauge charge. This topologization can be achieved by a weaker condition, which can be regarded as a dynamical generalization of the Beltrami condition. An alternative manner to achieve vanishing of the Lorentz 4-force is light-likeness of the Kähler 4-current. This does not require topologization.

3.2.1 *Topologization of the Kähler current for* $D_{CP_2} = 3$*: covariant formulation*

The condition states that Kähler 4-current is proportional to the instanton current whose divergence is instanton density and vanishes when the dimension of CP_2 projection is smaller than four: $D_{CP_2} < 4$. For $D_{CP_2} = 2$ the instanton 4-current vanishes identically and topologization is equivalent with the vanishing of the Kähler current.

If the simplest vision about light-like 3-surfaces as basic dynamical objects is accepted $D_{CP_2} = 2$, corresponds to a non-physical situation and only the deformations of these surfaces - most naturally resulting by gluing of CP_2 type vacuum extremals on them - can represent preferred extremals of Kähler action. One can however speak about $D_{CP_2} = 2$ phase if 4-surfaces are obtained are obtained in this manner.

$$j^\alpha \equiv D_\beta J^{\alpha\beta} = \psi \times j_I^\alpha = \psi \times \epsilon^{\alpha\beta\gamma\delta} J_{\beta\gamma} A_\delta \ . \tag{3.7}$$

Here the function ψ is an arbitrary function $\psi(s^k)$ of CP_2 coordinates s^k regarded as functions of space-time coordinates. It is essential that ψ depends on the space-time coordinates through the CP_2 coordinates only. Hence the representation as an imbedded gauge field is crucial element of the solution ansatz.

The field equations state the vanishing of the divergence of the 4-current. This is trivially true for instanton current for $D_{CP_2} < 4$. Also the contraction of $\nabla\psi$ (depending on space-time coordinates through CP_2 coordinates only) with the instanton current is proportional to the winding number density and therefore vanishes for $D_{CP_2} < 4$.

The topologization of the Kähler current guarantees the vanishing of the Lorentz 4-force. Indeed, using the self-duality condition for the current, the expression for the Lorentz 4-force reduces to a term proportional to the instanton density:

$$\begin{aligned} j^\alpha J_{\alpha\beta} &= \psi \times j_I^\alpha J_{\alpha\beta} \\ &= \psi \times \epsilon^{\alpha\mu\nu\delta} J_{\mu\nu} A_\delta J_{\alpha\beta} \ . \end{aligned} \tag{3.8}$$

Since all vector quantities appearing in the contraction with the four-dimensional permutation tensor are proportional to the gradients of CP_2 coordinates, the expression is proportional to the instanton density, and thus winding number density, and vanishes for $D_{CP_2} < 4$.

Remarkably, the topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{k_l} \partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M_+^4 and CP_2 degrees of freedom to

$$T^{\alpha\beta} H^k_{\alpha\beta} = 0 \ . \tag{3.9}$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The earlier proposal that quaternion conformal invariance in a suitable sense might provide a general solution of the field equations could be seen as a generalization of the ordinary conformal invariance of string models. If the topologization of the Kähler current implying effective dimensional reduction in CP_2 degrees of freedom is consistent with quaternion conformal invariance, the quaternion conformal structures must differ for the different dimensions of CP_2 projection.

3.2.2 *Topologization of the Kähler current for* $D_{CP_2} = 3$*: non-covariant formulation*

In order to gain a concrete understanding about what is involved it is useful to repeat these arguments using the 3-dimensional notation. The components of the instanton 4-current read in three-dimensional notation as

$$\overline{j}_I = \overline{E} \times \overline{A} + \phi \overline{B} \ , \quad \rho_I = \overline{B} \cdot \overline{A} \ . \tag{3.10}$$

The self duality conditions for the current can be written explicitly using 3-dimensional notation and read

$$\begin{aligned} \nabla\times\overline{B}-\partial_t\overline{E} &= \overline{j}=\psi\overline{j}_I=\psi\left(\phi\overline{B}+\overline{E}\times\overline{A}\right) \ , \\ \nabla\cdot E &= \rho=\psi\rho_I \ . \end{aligned} \tag{3.11}$$

For a vanishing electric field the self-duality condition for Kähler current reduces to the Beltrami condition

$$\nabla\times\overline{B}=\alpha\overline{B} \ , \quad \alpha=\psi\phi \ . \tag{3.12}$$

The vanishing of the divergence of the magnetic field implies that α is constant along the field lines of the flow. When ϕ is constant and $\overline{A}$ is time independent, the condition reduces to the Beltrami condition with $\alpha=\phi=constant$, which allows an explicit solution [B1].

One can check also the vanishing of the Lorentz 4-force by using 3-dimensional notation. Lorentz 3-force can be written as

$$\rho_I\overline{E}+\overline{j}\times\overline{B}=\psi\overline{B}\cdot\overline{AE}+\psi\left(\overline{E}\times\overline{A}+\phi\overline{B}\right)\times\overline{B}=0 \ . \tag{3.13}$$

The fourth component of the Lorentz force reads as

$$\overline{j}\cdot\overline{E}=\psi\overline{B}\cdot\overline{E}+\psi\left(\overline{E}\times\overline{A}+\phi\overline{B}\right)\cdot\overline{E}=0 \ . \tag{3.14}$$

The remaining conditions come from the induction law of Faraday and could be guaranteed by expressing $\overline{E}$ and $\overline{B}$ in terms of scalar and vector potentials.

The density of the Kähler electric charge of the vacuum is proportional to the the helicity density of the so called helicity charge $\rho=\psi\rho_I=\psi B\cdot A$. This charge is topological charge in the sense that it does not depend on the induced metric at all. Note the presence of arbitrary function ψ of CP_2 coordinates.

Further conditions on the functions appearing in the solution ansatz come from the 3 independent field equations for CP_2 coordinates. What is remarkable that the generalized self-duality condition for the Kähler current allows to understand the general features of the solution ansatz to very high degree without any detailed knowledge about the detailed solution. The question whether field equations allow solutions consistent with the self duality conditions of the current will be dealt later. The optimistic guess is that the field equations and topologization of the Kähler current relate to each other very intimately.

3.2.3 *Vanishing or light likeness of the Kähler current guarantees vanishing of the Lorentz 4-force for* $D_{CP_2}=2$

For $D_{CP_2}=2$ one can always take two CP_2 coordinates as space-time coordinates and from this it is clear that instanton current vanishes so that topologization gives a vanishing Kähler current. In particular, the Beltrami condition $\nabla\times\overline{B}=\alpha\overline{B}$ is not consistent with the topologization of the instanton current for $D_{CP_2}=2$.

$D_{CP_2}=2$ case can be treated in a coordinate invariant manner by using the two coordinates of CP_2 projection as space-time coordinates so that only a magnetic or electric field is present depending on whether the gauge current is time-like or space-like. Light-likeness of the gauge current provides a second manner to achieve the vanishing of the Lorentz force and is realized in case of massless extremals having $D_{CP_2}=2$: this current is in the direction of propagation whereas magnetic and electric fields are orthogonal to it so that Beltrami conditions is certainly not satisfied.

3.2.4 *Under what conditions topologization of Kähler current yields Beltrami conditions?*

Topologization of the Kähler 4-current gives rise to magnetic Beltrami fields if either of the following conditions is satisfied.

1. The $\overline{E} \times \overline{A}$ term contributing besides $\phi\overline{B}$ term to the topological current vanishes. This requires that $\overline{E}$ and $\overline{A}$ are parallel to each other

$$\overline{E} = \nabla\Phi - \partial_t\overline{A} = \beta\overline{A} \quad (3.15)$$

   This condition is analogous to the Beltrami condition. Now only the 3-space has as its coordinates time coordinate and two spatial coordinatesand$\overline{B}$ is replaced with $\overline{A}$. Since E and B are orthogonal, this condition implies $\overline{B} \cdot \overline{A} = 0$ so that Kähler charge density is vanishing.

2. The vector $\overline{E} \times \overline{A}$ is parallel to $\overline{B}$.

$$\overline{E} \times \overline{A} = \beta\overline{B} \quad (3.16)$$

   The condition is consistent with the orthogonality of $\overline{E}$ and $\overline{B}$ but implies the orthogonality of $\overline{A}$ and $\overline{B}$ so that electric charge density vanishes

In both cases vector potential fails to define a contact structure since $B \cdot A$ vanishes (contact structures are discussed briefly below), and there exists a global coordinate along the field lines of $\overline{A}$ and the full contact structure is lost again. Note however that the Beltrami condition for magnetic field means that magnetic field defines a contact structure irrespective of whether $\overline{B} \cdot \overline{A}$ vanishes or not. The transition from the general case to Beltrami field would thus involve the replacement

$$(\overline{A}, \overline{B}) \rightarrow_{\nabla\times} (\overline{B}, \overline{j})$$

induced by the rotor.

One must of course take these considerations somewhat cautiously since the inner product depends on the induced 4-metric and it might be that induced metric could allow small vacuum charge density and make possible genuine contact structure.

3.2.5 *Hydrodynamic analogy*

The field equations of TGD are basically hydrodynamic equations stating the local conservation of the currents associated with the isometries of the imbedding space. Therefore it is intriguing that Beltrami fields appear also as solutions of ideal magnetohydrodynamics equations and as steady solutions of non-viscous incompressible flow described by Euler equations [B4] .

In hydrodynamics the role of the magnetic field is taken by the velocity field. TGD based models for nuclei [K9] and condensed matter [K6] involve in an essential manner valence quarks having large $\hbar$ and exotic quarks giving nucleons anomalous color and weak charges creating long ranged color and weak forces. Weak forces have a range of order atomic radius and could be responsible for the repulsive core in van der Waals potential.

This raises the idea that the incompressible flow could occur along the field lines of the Z^0 magnetic field so that the velocity field would be proportional to the Z^0 magnetic field and the Beltrami condition for the velocity field would reduce to that for Z^0 magnetic field. Thus the flow lines of hydrodynamic flow would directly correspond to those of Z^0 magnetic field. The generalized Beltrami flow based on the topologization of the Z^0 current would allow to model also the non-stationary incompressible non-viscous hydrodynamical flows.

It would seem that one cannot describe viscous flows using flows satisfying generalized Beltrami conditions since the vanishing of the Lorentz 4-force says that there is no local dissipation of the classical field energy. One might claim that this is not a problem since in TGD framework viscous flow could be seen as a practical description of a quantum jump sequence by replacing the corresponding sequence of space-time surfaces with a single space-time surface.

One the other hand, quantum classical correspondence requires that also dissipative effects have space-time correlates. Kähler fields, which are dissipative, and thus correspond to a non-vanishing Lorentz 4-force, represent one candidate for correlates of this kind. If this is the case, then the fields satisfying the generalized Beltrami condition provide space-time correlates only for the asymptotic self organization patterns for which the viscous effects are negligible, and also the solutions of field equations describing effects of viscosity should be possible.

One must however take this argument with a grain of salt. Dissipation, that is the transfer of conserved quantities to degrees of freedom corresponding to shorter scales, could correspond to a transfer of these quantities between different space-time sheets of the many-sheeted space-time. Here the opponent could however argue that larger space-time sheets mimic the dissipative dynamics in shorter scales and that classical currents represent "symbolically" averaged currents in shorter length scales, and that the local non-conservation of energy momentum tensor consistent with local conservation of isometry currents provides a unique manner to mimic the dissipative dynamics. This view will be developed in more detail below.

3.2.6 *The stability of generalized Beltrami fields*

The stability of generalized Beltrami fields is of high interest since unstable points of space-time sheets are those around which macroscopic changes induced by quantum jumps are expected to be localized.

1. Contact forms and contact structures

The stability of Beltrami flows has been studied using the theory of contact forms in three-dimensional Riemann manifolds [B2] . Contact form is a one-form A (that is covariant vector field A_α) with the property $A \wedge dA \neq 0$. In the recent case the induced Kähler gauge potential A_α and corresponding induced Kähler form $J_{\alpha\beta}$ for any 3-sub-manifold of space-time surface define a contact form so that the vector field $A^\alpha = g^{\alpha\beta}A_\beta$ is not orthogonal with the magnetic field $B^\alpha = \epsilon^{\alpha\beta\delta}J_{\beta\gamma}$. This requires that magnetic field has a helical structure. Induced metric in turn defines the Riemann structure.

If the vector potential defines a contact form, the charge density associated with the topologized Kähler current must be non-vanishing. This can be seen as follows.

1. The requirement that the flow lines of a one-form X_μ defined by the vector field X^μ as its dual allows to define a global coordinate x varying along the flow lines implies that there is an integrating factor ϕ such that $\phi X = dx$ and therefore $d(\phi X) = 0$. This implies $dlog(\phi) \wedge X = -dX$. From this the necessary condition for the existence of the coordinate x is $X \wedge dX = 0$. In the three-dimensional case this gives $\overline{X} \cdot (\nabla \times \overline{X}) = 0$.

2. This condition is by definition not satisfied by the vector potential defining a contact form so that one cannot identify a global coordinate varying along the flow lines of the vector potential. The condition $\overline{B} \cdot \overline{A} \neq 0$ states that the charge density for the topologized Kähler current is non-vanishing. The condition that the field lines of the magnetic field allow a global coordinate requires $\overline{B} \cdot \nabla \times \overline{B} = 0$. The condition is not satisfied by Beltrami fields with $\alpha \neq 0$. Note that in this case magnetic field defines a contact structure.

Contact structure requires the existence of a vector ξ satisfying the condition $A(\xi) = 0$. The vector field ξ defines a plane field, which is orthogonal to the vector field A^α. Reeb field in turn is a vector field for which $A(X) = 1$ and $dA(X;) = 0$ hold true. The latter condition states the vanishing of the cross product $X \times B$ so that X is parallel to the Kähler magnetic field B^α and has unit projection in the direction of the vector field A^α. Any Beltrami field defines a Reeb field irrespective of the Riemannian structure.

2. Stability of the Beltrami flow and contact structures

Contact structures are used in the study of the topology and stability of the hydrodynamical flows [B2] , and one might expect that the notion of contact structure and its proper generalization to the four-dimensional context could be useful in TGD framework also. An example giving some idea about the complexity of the flows defined by Beltrami fields is the Beltrami field in R^3 possessing closed orbits with all possible knot and link types simultaneously [B2] !

Beltrami flows associated with Euler equations are known to be unstable [B2] . Since the flow is volume preserving, the stationary points of the Beltrami flow are saddle points at which also vorticity vanishes and linear instabilities of Navier-Stokes equations can develop. From the point of view of biology it is interesting that the flow is stabilized by vorticity which implies also helical structures. The stationary points of the Beltrami flow correspond in TGD framework to points at which the induced Kähler magnetic field vanishes. They can be unstable by the vacuum degeneracy of Kähler action implying classical non-determinism. For generalized Beltrami fields velocity and vorticity (both divergence free) are replaced by Kähler current and instanton current.

More generally, the points at which the Kähler 4-current vanishes are expected to represent potential instabilities. The instanton current is linear in Kähler field and can vanish in a gauge invariant manner only if the induced Kähler field vanishes so that the instability would be due to the vacuum degeneracy also now. Note that the vanishing of the Kähler current allows also the generation of region with $D_{CP_2} = 4$. The instability of the points at which induce Kähler field vanish is manifested in quantum jumps replacing the generalized Beltrami field with a new one such that something new is generated around unstable points. Thus the regions in which induced Kähler field becomes weak are the most interesting ones. For example, unwinding of DNA could be initiated by an instability of this kind.

3.3 How To Satisfy Field Equations?

The topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{k_l}\partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M^4_+ and CP_2 degrees of freedom to

$$T^{\alpha\beta}H^k_{\alpha\beta} = 0 \ . \tag{3.17}$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The following approach utilizes the properties of Hamilton Jacobi structures of M^4_+ introduced in the study of massless extremals and contact structures of CP_2 emerging naturally in the case of generalized Beltrami fields.

3.3.1 *String model as a starting point*

String model serves as a starting point.

1. In the case of Minkowskian minimal surfaces representing string orbit the field equations reduce to purely algebraic conditions in light cone coordinates (u, v) since the induced metric has only the component g_{uv}, whereas the second fundamental form has only diagonal components H^k_{uu} and H^k_{vv}.

2. For Euclidian minimal surfaces (u, v) is replaced by complex coordinates $(w, \overline{w})$ and field equations are satisfied because the metric has only the component $g^{w\overline{w}}$ and second fundamental form has only components of type H^k_{ww} and $H^{\overline{k}}_{\overline{w}\overline{w}}$. The mechanism should generalize to the recent case.

3.3.2 *The general form of energy momentum tensor as a guideline for the choice of coordinates*

Any 3-dimensional Riemann manifold allows always a orthogonal coordinate system for which the metric is diagonal. Any 4-dimensional Riemann manifold in turn allows a coordinate system for which 3-metric is diagonal and the only non-diagonal components of the metric are of form g^{ti}. This kind of coordinates might be natural also now. When $\overline{E}$ and $\overline{B}$ are orthogonal, energy momentum tensor has the form

$$T = \begin{pmatrix} \frac{E^2+B^2}{2} & 0 & 0 & EB \\ 0 & \frac{E^2+B^2}{2} & 0 & 0 \\ 0 & 0 & \frac{-E^2+B^2}{2} & 0 \\ EB & 0 & 0 & \frac{E^2-B^2}{2} \end{pmatrix} \tag{3.18}$$

in the tangent space basis defined by time direction and longitudinal direction $\overline{E}\times\overline{B}$, and transversal directions $\overline{E}$ and $\overline{B}$. Note that T is traceless.

The optimistic guess would be that the directions defined by these vectors integrate to three orthogonal coordinates of X^4 and together with time coordinate define a coordinate system containing only g^{ti} as non-diagonal components of the metric. This however requires that the fields in question allow an integrating factor and, as already found, this requires $\nabla \times X \cdot X = 0$ and this is not the case in general.

Physical intuition suggests however that X^4 coordinates allow a decomposition into longitudinal and transversal degrees freedom. This would mean the existence of a time coordinate t and longitudinal coordinate z the plane defined by time coordinate and vector $\overline{E}\times\overline{B}$ such that the coordinates $u = t-z$ and $v = t+z$ are light like coordinates so that the induced metric would have only the component g^{uv} whereas g^{vv} and g^{uu} would vanish in these coordinates. In the transversal space-time directions complex space-time coordinate coordinate w could be introduced. Metric could have also non-diagonal components besides the components $g^{w\overline{w}}$ and g^{uv}.

3.3.3 *Hamilton Jacobi structures in* M^4_+

Hamilton Jacobi structure in M^4_+ can understood as a generalized complex structure combing transversal complex structure and longitudinal hyper-complex structure so that notion of holomorphy and Kähler structure generalize.

1. Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct, not necessarily orthogonal, sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has an interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray. Assume that E^2 allows complex coordinates $w = E^1 + iE^2$ and $\overline{w} = E^1 - iE^2$. The simplest decomposition of this kind corresponds to the decomposition $(S^+ \equiv u = t+z, S^- \equiv v = t-z, w = x+iy, \overline{w} = x-iy)$.

2. In accordance with this physical picture, S^+ and S^- define light-like curves which are normals to light-like surfaces and thus satisfy the equation:

$$(\nabla S_\pm)^2 = 0 \quad .$$

   The gradients of $S_\pm$ are obviously analogous to local light like velocity vectors $v = (1, \overline{v})$ and $\tilde{v} = (1, -\overline{v})$. These equations are also obtained in geometric optics from Hamilton Jacobi equation by replacing photon's four-velocity with the gradient ∇S: this is consistent with the interpretation of massless extremals as Bohr orbits of em field. $S_\pm = constant$ surfaces can be interpreted as expanding light fronts. The interpretation of $S_\pm$ as Hamilton Jacobi functions justifies the term Hamilton Jacobi structure.

   The simplest surfaces of this kind correspond to $t = z$ and $t = -z$ light fronts which are planes. They are dual to each other by hyper complex conjugation $u = t-z \rightarrow v = t+z$. One should somehow generalize this conjugation operation. The simplest candidate for the conjugation $S^+ \rightarrow S^-$ is as a conjugation induced by the conjugation for the arguments: $S^+(t-z, t+z, x, y) \rightarrow S^-(t-z, t+z, x, y) = S^+(t+z, t-z, x, -y)$ so that a dual pair is mapped to a dual pair. In transversal degrees of freedom complex conjugation would be involved.

3. The coordinates $(S_\pm, w, \overline{w})$ define local light cone coordinates with the line element having the form

$$\begin{aligned} ds^2 &= g_{+-}dS^+dS^- + g_{w\overline{w}}dwd\overline{w} \\ &+ g_{+w}dS^+dw + g_{+\overline{w}}dS^+d\overline{w} \\ &+ g_{-w}dS^-dw + g_{-\overline{w}}dS^-d\overline{w} \ . \end{aligned} \tag{3.19}$$

Conformal transformations of M^4_+ leave the general form of this decomposition invariant. Also the transformations which reduces to analytic transformations $w \rightarrow f(w)$ in transversal degrees of freedom and hyper-analytic transformations $S^+ \rightarrow f(S^+), S^- \rightarrow f(S^-)$ in longitudinal degrees of freedom preserve this structure.

4. The basic idea is that of generalized Kähler structure meaning that the notion of Kähler function generalizes so that the non-vanishing components of metric are expressible as

$$\begin{aligned} g_{w\overline{w}} &= \partial_w \partial_{\overline{w}} K \ , \quad g_{+-} = \partial_{S^+} \partial_{S^-} K \ , \\ g_{w\pm} &= \partial_w \partial_{S^\pm} K \ , \quad g_{\overline{w}\pm} = \partial_{\overline{w}} \partial_{S^\pm} K \ . \end{aligned} \tag{3.20}$$

for the components of the metric. The expression in terms of Kähler function is coordinate invariant for the same reason as in case of ordinary Kähler metric. In the standard light-cone coordinates the Kähler function is given by

$$K = w_0 \overline{w}_0 + uv \ , \quad w_0 = x + iy \ , \quad u = t - z \ , \quad v = t + z \ . \tag{3.21}$$

The Christoffel symbols satisfy the conditions

$$\left\{ {}_{w\ \overline{w}}^{\ k} \right\} = 0 \ , \quad \left\{ {}_{+-}^{\ k} \right\} = 0 \ . \tag{3.22}$$

If energy momentum tensor has only the components $T^{w\overline{w}}$ and T^{+-}, field equations are satisfied in M^4_+ degrees of freedom.

5. The Hamilton Jacobi structures related by these transformations can be regarded as being equivalent. Since light-like 3- surface is, as the dynamical evolution defined by the light front, fixed by the 2-surface serving as the light source, these structures should be in one-one correspondence with 2-dimensional surfaces with two surfaces regarded as equivalent if they correspond to different time=constant snapshots of the same light front, or are related by a conformal transformation of M^4_+. Obviously there should be quite large number of them. Note that the generating two-dimensional surfaces relate also naturally to quaternion conformal invariance and corresponding Kac Moody invariance for which deformations defined by the M^4 coordinates as functions of the light-cone coordinates of the light front evolution define Kac Moody algebra, which thus seems to appear naturally also at the level of solutions of field equations.

The task is to find all possible local light cone coordinates defining one-parameter families 2-surfaces defined by the condition $S_i = constant$, $i = +\ or\ = -$, dual to each other and expanding with light velocity. The basic open questions are whether the generalized Kähler function indeed makes sense and whether the physical intuition about 2-surfaces as light sources parameterizing the set of all possible Hamilton Jacobi structures makes sense.

Hamilton Jacobi structure means the existence of foliations of the M^4 projection of X^4 by 2-D surfaces analogous to string word sheets labeled by w and the dual of this foliation defined by partonic 2-surfaces labeled by the values of S_i. Also the foliation by light-like 3-surfaces Y^3_l labeled by $S_\pm$ with $S_\mp$ serving as light-like coordinate for Y^3_l is implied. This is what number theoretic compactification and $M^8 - H$ duality predict when space-time surface corresponds to hyper-quaternionic surface of M^8 [K10, K24] .

3.3.4 *Contact structure and generalized Kähler structure of CP_2 projection*

In the case of 3-dimensional CP_2 projection it is assumed that one can introduce complex coordinates $(\xi, \overline{\xi})$ and the third coordinate s. These coordinates would correspond to a contact structure in 3-dimensional CP_2 projection defining transversal symplectic and Kähler structures. In these coordinates the transversal parts of the induced CP_2 Kähler form and metric would contain only components of type $g_{w\overline{w}}$ and $J_{w\overline{w}}$. The transversal Kähler field $J_{w\overline{w}}$ would induce the Kähler magnetic field and the components J_{sw} and $J_{s\overline{w}}$ the Kähler electric field.

It must be emphasized that the non-integrability of the contact structure implies that J cannot be parallel to the tangent planes of $s = constant$ surfaces, s cannot be parallel to neither A nor the dual of J, and ξ cannot vary in the tangent plane defined by J. A further important conclusion is that for the solutions with 3-dimensional CP_2 projection topologized Kähler charge density is necessarily non-vanishing by $A \wedge J \neq 0$ whereas for the solutions with $D_{CP_2} = 2$ topologized Kähler current vanishes.

Also the CP_2 projection is assumed to possess a generalized Kähler structure in the sense that all components of the metric except s_{ss} are derivable from a Kähler function by formulas similar to M^4_+ case.

$$s_{w\overline{w}} = \partial_w \partial_{\overline{w}} K \ , \quad s_{ws} = \partial_w \partial_s K \ , \quad s_{\overline{w}s} = \partial_{\overline{w}} \partial_s K \ . \tag{3.23}$$

Generalized Kähler property guarantees that the vanishing of the Christoffel symbols of CP_2 (rather than those of 3-dimensional projection), which are of type $\{{}^{\ k}_{\xi \ \overline{\xi}}\}$.

$$\{{}^{\ k}_{\xi \ \overline{\xi}}\} = 0 \ . \tag{3.24}$$

Here the coordinates of CP_2 have been chosen in such a manner that three of them correspond to the coordinates of the projection and fourth coordinate is constant at the projection. The upper index k refers also to the CP_2 coordinate, which is constant for the CP_2 projection. If energy momentum tensor has only components of type T^{+-} and $T^{w\overline{w}}$, field equations are satisfied even when if non-diagonal Christoffel symbols of CP_2 are present. The challenge is to discover solution ansatz, which guarantees this property of the energy momentum tensor.

A stronger variant of Kähler property would be that also s_{ss} vanishes so that the coordinate lines defined by s would define light like curves in CP_2. The topologization of the Kähler current however implies that CP_2 projection is a projection of a 3-surface with strong Kähler property. Using $(s, \xi, \overline{\xi}, S^-)$ as coordinates for the space-time surface defined by the ansatz $(w = w(\xi, s), S^+ = S^+(s))$ one finds that g_{ss} must be vanishing so that stronger variant of the Kähler property holds true for $S^- = constant$ 3-surfaces.

The topologization condition for the Kähler current can be solved completely generally in terms of the induced metric using $(\xi, \overline{\xi}, s)$ and some coordinate of M^4_+, call it x^4, as space-time coordinates. Topologization boils down to the conditions

$$\begin{aligned} \partial_\beta (J^{\alpha\beta} \sqrt{g}) &= 0 \text{ for } \alpha \in \{\xi, \overline{\xi}, s\} \ , \\ g^{4i} &\neq 0 \ . \end{aligned} \tag{3.25}$$

Thus 3-dimensional empty space Maxwell equations and the non-orthogonality of X^4 coordinate lines and the 3-surfaces defined by the lift of the CP_2 projection.

3.3.5 *A solution ansatz yielding light-like current in* $D_{CP_2} = 3$ *case*

The basic idea is that of generalized Kähler structure and solutions of field equations as maps or deformations of canonically imbedded M^4_+ respecting this structure and guaranteeing that the only non-vanishing components of the energy momentum tensor are $T^{\xi\xi}$ and T^{s-} in the coordinates $(\xi, \overline{\xi}, s, S^-)$.

1. The coordinates (w, S^+) are assumed to holomorphic functions of the CP_2 coordinates (s, ξ)

$$S^+ = S^+(s) \ , \quad w = w(\xi, s) \ . \tag{3.26}$$

Obviously S^+ could be replaced with S^-. The ansatz is completely symmetric with respect to the exchange of the roles of (s, w) and (S^+, ξ) since it maps longitudinal degrees of freedom to longitudinal ones and transverse degrees of freedom to transverse ones.

2. Field equations are satisfied if the only non-vanishing components of the energy momentum tensor are of type $T^{\xi\overline{\xi}}$ and T^{s-}. The reason is that the CP_2 Christoffel symbols for projection and projections of M^4_+ Christoffel symbols are vanishing for these lower index pairs.

3. By a straightforward calculation one can verify that the only manner to achieve the required structure of energy momentum tensor is to assume that the induced metric in the coordinates $(\xi, \overline{\xi}, s, S^-)$ has as non-vanishing components only $g_{\xi\overline{\xi}}$ and g_{s-}

$$g_{ss} = 0 \ , \quad g_{\xi s} = 0 \ , \quad g_{\overline{\xi} s} = 0 \ . \tag{3.27}$$

   Obviously the space-time surface must factorize into an orthogonal product of longitudinal and transversal spaces.

4. The condition guaranteeing the product structure of the metric is

$$\begin{aligned} s_{ss} &= m_{+w}\partial_s w(\xi, s)\partial_s S^+(s) + m_{+\overline{w}}\partial_s \overline{w}(\xi, s)\partial_s S^+(s) \ , \\ s_{s\xi} &= m_{+w}\partial_\xi w(\xi)\partial_s S^+(s) \ , \\ s_{s\overline{\xi}} &= m_{+w}\partial_{\overline{\xi}} w(\overline{\xi})\partial_s S^+(s) \ . \end{aligned} \tag{3.28}$$

   Thus the function of dynamics is to diagonalize the metric and provide it with strong Kähler property. Obviously the CP_2 projection corresponds to a light-like surface for all values of S^- so that space-time surface is foliated by light-like surfaces and the notion of generalized conformal invariance makes sense for the entire space-time surface rather than only for its boundary or elementary particle horizons.

5. The requirement that the Kähler current is proportional to the instanton current means that only the j^- component of the current is non-vanishing. This gives the following conditions

$$\begin{aligned} & j^\xi \sqrt{g} = \partial_\beta (J^{\xi\beta}\sqrt{g}) = 0 \ , \quad j^{\overline{\xi}} \sqrt{g} = \partial_\beta (J^{\overline{\xi}\beta}\sqrt{g}) = 0 \ , \\ & j^+ \sqrt{g} = \partial_\beta (J^{+\beta}\sqrt{g}) = 0 \ . \end{aligned} \tag{3.29}$$

   Since $J^{+\beta}$ vanishes, the condition

$$\sqrt{g} j^+ = \partial_\beta (J^{+\beta}\sqrt{g}) \quad = \quad 0 \tag{3.30}$$

   is identically satisfied. Therefore the number of field equations reduces to three.

The physical interpretation of the solution ansatz deserves some comments.

1. The light-like character of the Kähler current brings in mind CP_2 extremals for which CP_2 projection is light like. This suggests that the topological condensation of CP_2 type extremal occurs on $D_{CP_2} = 3$ helical space-time sheet representing zitterbewegung. In the case of many-body system light-likeness of the current does not require that particles are massless if particles of opposite charges can be present. Field tensor has the form $(J^{\xi\overline{\xi}}, J^{\xi-}, J^{\overline{\xi}-})$. Both helical magnetic field and electric field present as is clear when one replaces the coordinates (S^+, S^-) with time-like and space-like coordinate. Magnetic field dominates but the presence of electric field means that genuine Beltrami field is not in question.

2. Since the induced metric is product metric, 3-surface is metrically product of 2-dimensional surface X^2 and line or circle and obeys product topology. If absolute minima correspond to asymptotic self-organization patterns, the appearance of the product topology and even metric is not so surprising. Thus the solutions can be classified by the genus of X^2. An interesting question is how closely the explanation of family replication phenomenon in terms

of the topology of the boundary component of elementary particle like 3-surface relates to this. The heaviness and instability of particles which correspond to genera $g > 2$ (sphere with more than two handles) might have simple explanation as absence of (stable) $D_{CP_2} = 3$ solutions of field equations with genus $g > 2$.

3. The solution ansatz need not be the most general. Kähler current is light-like and already this is enough to reduce the field equations to the form involving only energy momentum tensor. One might hope of finding also solution ansätze for which Kähler current is time-like or space-like. Space-likeness of the Kähler current might be achieved if the complex coordinates $(\xi, \overline{\xi})$ and hyper-complex coordinates (S^+, S^-) change the role. For this solution ansatz electric field would dominate. Note that the possibility that Kähler current is always light-like cannot be excluded.

4. Suppose that CP_2 projection quite generally defines a foliation of the space-time surface by light-like 3-surfaces, as is suggested by the conformal invariance. If the induced metric has Minkowskian signature, the fourth coordinate x^4 and thus also Kähler current must be time-like or light-like so that magnetic field dominates. Already the requirement that the metric is non-degenerate implies $g_{s4} \neq 0$ so that the metric for the $\xi = constant$ 2-surfaces has a Minkowskian signature. Thus space-like Kähler current does not allow the lift of the CP_2 projection to be light-like.

3.3.6 *Are solutions with time-like or space-like Kähler current possible in* $D_{CP_2} = 3$ *case?*

As noticed in the section about number theoretical compactification, the flow of gauge currents along slices Y^3_l of $X^4(X^3_l)$ "parallel" to X^3_l requires only that gauge currents are parallel to Y^3_l and can thus space-like. The following ansatz gives good hopes for obtaining solutions with space-like and perhaps also time-like Kähler currents.

1. Assign to light-like coordinates coordinates (T, Z) by the formula $T = S^+ + S^-$ and $Z = S^+ - S^-$. Space-time coordinates are taken to be $(\xi, \overline{\xi}, s)$ and coordinate Z. The solution ansatz with time-like Kähler current results when the roles of T and Z are changed. It will however found that same solution ansatz can give rise to both space-like and time-like Kähler current.

2. The solution ansatz giving rise to a space-like Kähler current is defined by the equations

$$T = T(Z, s) \ , \quad w = w(\xi, s) \ . \tag{3.31}$$

If T depends strongly on Z, the g_{ZZ} component of the induced metric becomes positive and Kähler current time-like.

3. The components of the induced metric are

$$\begin{aligned} g_{ZZ} &= m_{ZZ} + m_{TT}\partial_Z T \partial_s T \ , & g_{Zs} &= m_{TT}\partial_Z T \partial_s T \ , \\ g_{ss} &= s_{ss} + m_{TT}\partial_s T \partial_s T \ , & g_{w\overline{w}} &= s_{w\overline{w}} + m_{w\overline{w}}\partial_\xi w \partial_{\overline{\xi}} \overline{w} \ , \\ g_{s\xi} &= s_{s\xi} \ , & g_{s\overline{\xi}} &= s_{s\overline{\xi}} \ . \end{aligned} \tag{3.32}$$

Topologized Kähler current has only Z-component and 3-dimensional empty space Maxwell's equations guarantee the topologization.

In CP_2 degrees of freedom the contractions of the energy momentum tensor with Christoffel symbols vanish if T^{ss}, $T^{\xi s}$ and $T^{\xi\xi}$ vanish as required by internal consistency. This is guaranteed if the condition

$$J^{\xi s} = 0 \quad (3.33)$$

holds true. Note however that $J^{\xi Z}$ is non-vanishing. Therefore only the components $T^{\xi\overline{\xi}}$ and $T^{Z\xi}$, $T^{Z\overline{\xi}}$ of energy momentum tensor are non-vanishing, and field equations reduce to the conditions

$$\begin{aligned} \partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g}) + \partial_Z(J^{\xi Z}\sqrt{g}) &= 0 \ , \\ \partial_{\xi}(J^{\overline{\xi}\xi}\sqrt{g}) + \partial_Z(J^{\overline{\xi} Z}\sqrt{g}) &= 0 \ . \end{aligned} \quad (3.34)$$

In the special case that the induced metric does not depend on z-coordinate equations reduce to holomorphicity conditions. This is achieve if T depends linearly on Z: $T = aZ$.

The contractions with M^4_+ Christoffel symbols come from the non-vanishing of $T^{Z\xi}$ and vanish if the Hamilton Jacobi structure satisfies the conditions

$$\begin{aligned} \left\{ {}^{\ k}_{T\ w} \right\} = 0 \ , \quad \left\{ {}^{\ k}_{T\ \overline{w}} \right\} = 0 \ , \\ \left\{ {}^{\ k}_{Z\ w} \right\} = 0 \ , \quad \left\{ {}^{\ k}_{Z\ \overline{w}} \right\} = 0 \end{aligned} \quad (3.35)$$

hold true. The conditions are equivalent with the conditions

$$\left\{ {}^{\ k}_{\pm\ w} \right\} = 0 \ , \quad \left\{ {}^{\ k}_{\pm\ \overline{w}} \right\} = 0 \ . \quad (3.36)$$

These conditions possess solutions (standard light cone coordinates are the simplest example). Also the second derivatives of $T(s, Z)$ contribute to the second fundamental form but they do not give rise to non-vanishing contractions with the energy momentum tensor. The cautious conclusion is that also solutions with time-like or space-like Kähler current are possible.

3.3.7 $D_{CP_2} = 4$ *case*

The preceding discussion was for $D_{CP_2} = 3$ and one should generalize the discussion to $D_{CP_2} = 4$ case.

1. Hamilton Jacobi structure for M^4_+ is expected to be crucial also now.

2. One might hope that for $D_{CP_2} = 4$ the Kähler structure of CP_2 defines a foliation of CP_2 by 3-dimensional contact structures. This requires that there is a coordinate varying along the field lines of the normal vector field X defined as the dual of the three-form $A \wedge dA = A \wedge J$. By the previous considerations the condition for this reads as $dX = d(log\phi) \wedge X$ and implies $X \wedge dX = 0$. Using the self duality of the Kähler form one can express X as $X^k = J^{kl}A_l$. By a brief calculation one finds that $X \wedge dX \propto X$ holds true so that (somewhat disappointingly) a foliation of CP_2 by contact structures does not exist.

For $D_{CP_2} = 4$ case Kähler current vanishes and this case corresponds to what I have called earlier Maxwellian phase since empty space Maxwell's equations would be indeed satisfied, provided this phase exists at all. It however seems that Maxwell phase is probably realized differently.

1. Solution ansatz with a 3-dimensional M^4_+ projection

The basic idea is that the complex structure of CP_2 is preserved so that one can use complex coordinates (ξ^1, ξ^2) for CP_2 in which CP_2 Christoffel symbols and energy momentum tensor have automatically the desired properties. This is achieved the second light like coordinate, say v, is non-dynamical so that the induced metric does not receive any contribution from the longitudinal degrees of freedom. In this case one has

$$S^+ = S^+(\xi^1, \xi^2) \ , \quad w = w(\xi^1, \xi^2) \ , \quad S^- = constant \ . \quad (3.37)$$

The induced metric does possesses only components of type $g_{i\bar{j}}$ if the conditions

$$g_{+w} = 0 \ , \quad g_{+\overline{w}} = 0 \ . \tag{3.38}$$

This guarantees that energy momentum tensor has only components of type $T^{i\bar{j}}$ in coordinates (ξ^1, ξ^2) and their contractions with the Christoffel symbols of CP_2 vanish identically. In M^4_+ degrees of freedom one must pose the conditions

$$\left\{ {k \atop w+} \right\} = 0 \ , \quad \left\{ {k \atop \overline{w}+} \right\} = 0 \ , \quad \left\{ {k \atop ++} \right\} = 0 \ . \tag{3.39}$$

on Christoffel symbols. These conditions are satisfied if the the M^4_+ metric does not depend on S^+:

$$\partial_+ m_{kl} \quad = \quad 0 \ . \tag{3.40}$$

This means that m_{-w} and $m_{-\overline{w}}$ can be non-vanishing but like m_{+-} they cannot depend on S^+.

The second derivatives of S^+ appearing in the second fundamental form are also a source of trouble unless they vanish. Hence S^+ must be a linear function of the coordinates ξ^k:

$$S^+ \quad = \quad a_k \xi^k + \overline{a}_k \overline{\xi}^k \ . \tag{3.41}$$

Field equations are the counterparts of empty space Maxwell equations $j^\alpha = 0$ but with M^4_+ coordinates (u, w) appearing as dynamical variables and entering only through the induced metric. By holomorphy the field equations can be written as

$$\partial_j (J^{j\bar{i}} \sqrt{g}) = 0 \ , \quad \partial_{\bar{j}} (J^{\bar{j}i} \sqrt{g}) = 0 \ , \tag{3.42}$$

and can be interpreted as conditions stating the holomorphy of the contravariant Kähler form.

What is remarkable is that the M^4_+ projection of the solution is 3-dimensional light like surface and that the induced metric has Euclidian signature. Light front would become a concrete geometric object with one compactified dimension rather than being a mere conceptualization. One could see this as topological quantization for the notion of light front or of electromagnetic shock wave, or perhaps even as the realization of the particle aspect of gauge fields at classical level.

If the latter interpretation is correct, quantum classical correspondence would be realized very concretely. Wave and particle aspects would both be present. One could understand the interactions of charged particles with electromagnetic fields both in terms of absorption and emission of topological field quanta and in terms of the interaction with a classical field as particle topologically condenses at the photonic light front.

For CP_2 type extremals for which M^4_+ projection is a light like curve correspond to a special case of this solution ansatz: transversal M^4_+ coordinates are constant and S^+ is now arbitrary function of CP_2 coordinates. This is possible since M^4_+ projection is 1-dimensional.

2. Are solutions with a 4-dimensional M^4_+ projection possible?

The most natural solution ansatz is the one for which CP_2 complex structure is preserved so that energy momentum tensor has desired properties. For four-dimensional M^4_+ projection this ansatz does not seem to make promising since the contribution of the longitudinal degrees of freedom implies that the induced metric is not anymore of desired form since the components $g_{ij} = m_{+-}(\partial_{\xi^i} S^+ \partial_{\xi^j} S^- + m_{+-} \partial_{\xi^i} S^- \partial_{\xi^j} S^+)$ are non-vanishing.

1. The natural dynamical variables are still Minkowski coordinates $(w, \overline{w}, S^+, S^-)$ for some Hamilton Jacobi structure. Since the complex structure of CP_2 must be given up, CP_2 coordinates can be written as (ξ, s, r) to stress the fact that only "one half" of the Kähler structure of CP_2 is respected by the solution ansatz.

2. The solution ansatz has the same general form as in $D_{CP_2} = 3$ case and must be symmetric with respect to the exchange of M^4_+ and CP_2 coordinates. Transverse coordinates are mapped to transverse ones and longitudinal coordinates to longitudinal ones:

$$(S^+, S^-) = (S^+(s,r), S^-(s,r)) \quad , \quad w = w(\xi) \ . \tag{3.43}$$

This ansatz would describe ordinary Maxwell field in M^4_+ since the roles of M^4_+ coordinates and CP_2 coordinates are interchangeable.

It is however far from obvious whether there are any solutions with a 4-dimensional M^4_+ projection. That empty space Maxwell's equations would allow only the topologically quantized light fronts as its solutions would realize quantum classical correspondence very concretely.

The recent view conforms with this intuition. The Maxwell phase is certainly physical notion but would correspond effective fields experience by particle in many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. ??** in the appendix of this book). Test particle topological condenses to all the space-time sheets with projection to a given region of Minkowski space and experiences essentially the sum of the effects caused by the induced gauge fields at different sheets. This applies also to gravitational fields interpreted as deviations from Minkowski metric.

The transition to GRT and QFT picture means the replacement of many-sheeted space-time with piece of Minkowski space with effective metric defined as the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Effective gauge potentials are sums of the induced gauge potentials. Hence the rather simple topologically quantized induced gauge fields associated with space-time sheets become the classical fields in the sense of Maxwell's theory and gauge theories.

3.3.8 $D_{CP_2} = 2$ *case*

Hamilton Jacobi structure for M^4_+ is assumed also for $D_{CP_2} = 2$, whereas the contact structure for CP_2 is in $D_{CP_2} = 2$ case replaced by the induced Kähler structure. Topologization yields vanishing Kähler current. Light-likeness provides a second manner to achieve vanishing Lorentz force but one cannot exclude the possibility of time- and space-like Kähler current.

1. Solutions with vanishing Kähler current

1. String like objects, which are products $X^2 \times Y^2 \subset M^4_+ \times CP_2$ of minimal surfaces Y^2 of M^4_+ with geodesic spheres S^2 of CP_2 and carry vanishing gauge current. String like objects allow considerable generalization from simple Cartesian products of $X^2 \times Y^2 \subset M^4 \times S^2$. Let $(w, \overline{w}, S^+, S^-)$ define the Hamilton Jacobi structure for M^4_+. $w = constant$ surfaces define minimal surfaces X^2 of M^4_+. Let ξ denote complex coordinate for a sub-manifold of CP_2 such that the imbedding to CP_2 is holomorphic: $(\xi^1, \xi^2) = (f^1(\xi), f^2(\xi))$. The resulting surface $Y^2 \subset CP_2$ is a minimal surface and field equations reduce to the requirement that the Kähler current vanishes: $\partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g_2}) = 0$. One-dimensional strings are deformed to 3-dimensional cylinders representing magnetic flux tubes. The oscillations of string correspond to waves moving along string with light velocity, and for more general solutions they become TGD counterparts of Alfven waves associated with magnetic flux tubes regarded as oscillations of magnetic flux lines behaving effectively like strings. It must be emphasized that Alfven waves are a phenomenological notion not really justified by the properties of Maxwell's equations.

2. Also electret type solutions with the role of the magnetic field taken by the electric field are possible. $(\xi, \overline{\xi}, u, v)$ would provide the natural coordinates and the solution ansatz would be of the form

$$(s,r) = (s(u,v), r(u,v)) \quad , \quad \xi = constant \ , \tag{3.44}$$

and corresponds to a vanishing Kähler current.

3. Both magnetic and electric fields are necessarily present only for the solutions carrying non-vanishing electric charge density (proportional to $\overline{B} \cdot \overline{A}$). Thus one can ask whether more general solutions carrying both magnetic and electric field are possible. As a matter fact, one must first answer the question what one really means with the magnetic field. By choosing the coordinates of 2-dimensional CP_2 projection as space-time coordinates one can define what one means with magnetic and electric field in a coordinate invariant manner. Since the CP_2 Kähler form for the CP_2 projection with $D_{CP_2} = 2$ can be regarded as a pure Kähler magnetic field, the induced Kähler field is either magnetic field or electric field.

The form of the ansatz would be

$$(s, r) = (s, r)\,(u, v, w, \overline{w}) \ \ , \quad \xi = constant \ \ . \tag{3.45}$$

As a matter fact, CP_2 coordinates depend on two properly chosen M^4 coordinates only.

1. Solutions with light-like Kähler current

There are large classes of solutions of field equations with a light-like Kähler current and 2-dimensional CP_2 projection.

1. Massless extremals for which CP_2 coordinates are arbitrary functions of one transversal coordinate $e = f(w, \overline{w})$ defining local polarization direction and light like coordinate u of M^4_+ and carrying in the general case a light like current. In this case the holomorphy does not play any role.

2. The string like solutions thickened to magnetic flux tubes carrying TGD counterparts of Alfven waves generalize to solutions allowing also light-like Kähler current. Also now Kähler metric is allowed to develop a component between longitudinal and transversal degrees of freedom so that Kähler current develops a light-like component. The ansatz is of the form

$$\xi^i = f^i(\xi) \ \ , \ \ w = w(\xi) \ \ , \ \ S^- = s^- \ \ , \ \ S^+ = s^+ + f(\xi, \overline{\xi}) \ \ . \tag{3.46}$$

   Only the components $g_{+\xi}$ and $g_{+\overline{\xi}}$ of the induced metric receive contributions from the modification of the solution ansatz. The contravariant metric receives contributions to $g^{-\xi}$ and $g^{-\overline{\xi}}$ whereas $g^{+\xi}$ and $g^{+\overline{\xi}}$ remain zero. Since the partial derivatives $\partial_\xi \partial_+ h^k$ and $\partial_{\overline{\xi}} \partial_+ h^k$ and corresponding projections of Christoffel symbols vanish, field equations are satisfied. Kähler current develops a non-vanishing component j^-. Apart from the presence of the electric field, these solutions are highly analogous to Beltrami fields.

3.3.9 *Could $D_{CP_2} = 2 \rightarrow 3$ transition occur in rotating magnetic systems?*

I have studied the imbeddings of simple cylindrical and helical magnetic fields in various applications of TGD to condensed matter systems, in particular in attempts to understand the strange findings about rotating magnetic systems [K26] .

Let S^2 be the homologically non-trivial geodesic sphere of CP_2 with standard spherical coordinates ($U \equiv cos(\theta), \Phi$) and let (t, ρ, ϕ, z) denote cylindrical coordinates for a cylindrical space-time sheet. The simplest possible space-time surfaces $X^4 \subset M^4_+ \times S^2$ carrying helical Kähler magnetic field depending on the radial cylindrical coordinate ρ, are given by:

$$\begin{aligned} U &= U(\rho) \ \ , & \Phi &= n\phi + kz \ \ , \\ J_{\rho\phi} &= n\partial_\rho U \ \ , & J_{\rho z} &= k\partial_\rho U \ \ . \end{aligned} \tag{3.47}$$

This helical field is not Beltrami field as one can easily find. A more general ansatz corresponding defined by

$$\Phi = \omega t + kz + n\phi$$

would in cylindrical coordinates give rise to both helical magnetic field and radial electric field depending on ρ only. This field can be obtained by simply replacing the vector potential with its rotated version and provides the natural first approximation for the fields associated with rotating magnetic systems.

A non-vanishing vacuum charge density is however generated when a constant magnetic field is put into rotation and is implied by the condition $\overline{E} = \overline{v} \times \overline{B}$ stating vanishing of the Lorentz force. This condition does not follow from the induction law of Faraday although Faraday observed this effect first. This is also clear from the fact that the sign of the charge density depends on the direction of rotation.

The non-vanishing charge density is not consistent with the vanishing of the Kähler 4-current and requires a 3-dimensional CP_2 projection and topologization of the Kähler current. Beltrami condition cannot hold true exactly for the rotating system. The conclusion is that rotation induces a phase transition $D_{CP_2} = 2 \rightarrow 3$. This could help to understand various strange effects related to the rotating magnetic systems [K26] . For instance, the increase of the dimension of CP_2 projection could generate join along boundaries contacts and wormhole contacts leading to the transfer of charge between different space-time sheets. The possibly resulting flow of gravitational flux to larger space-time sheets might help to explain the claimed antigravity effects.

3.4 $D_{CP_2} = 3$ Phase Allows Infinite Number Of Topological Charges Characterizing The Linking Of Magnetic Field Lines

When space-time sheet possesses a $D = 3$-dimensional CP_2 projection, one can assign to it a non-vanishing and conserved topological charge characterizing the linking of the magnetic field lines defined by Chern-Simons action density $A \wedge dA/4\pi$ for induced Kähler form. This charge can be seen as classical topological invariant of the linked structure formed by magnetic field lines.

The topological charge can also vanish for $D_{CP_2} = 3$ space-time sheets. In Darboux coordinates for which Kähler gauge potential reads as $A = P_k dQ^k$, the surfaces of this kind result if one has $Q^2 = f(Q^1)$ implying $A = f dQ^1$, $f = P_1 + P_2 \partial_{Q_1} Q^2$, which implies the condition $A \wedge dA = 0$. For these space-time sheets one can introduce Q^1 as a global coordinate along field lines of A and define the phase factor $exp(i \int A_\mu dx^\mu)$ as a wave function defined for the entire space-time sheet. This function could be interpreted as a phase of an order order parameter of super-conductor like state and there is a high temptation to assume that quantum coherence in this sense is lost for more general $D_{CP_2} = 3$ solutions.

Chern-Simons action is known as helicity in electrodynamics [B3] . Helicity indeed describes the linking of magnetic flux lines as is easy to see by interpreting magnetic field as incompressible fluid flow having A as vector potential: $B = \nabla \times A$. One can write A using the inverse of $\nabla\times$ as $A = (1/\nabla\times)B$. The inverse is non-local operator expressible as

$$\frac{1}{\nabla\times} B(r) = \int dV' \frac{(r - r')}{|r - r'|^3} \times B(r') \ ,$$

as a little calculation shows. This allows to write $\int A \cdot B$ as

$$\int dV A \cdot B = \int dV dV' B(r) \cdot \left(\frac{(r - r')}{|r - r'|^3} \times B(r') \right) \ ,$$

which is completely analogous to the Gauss formula for linking number when linked curves are replaced by a distribution of linked curves and an average is taken.

For $D_{CP_2} = 3$ field equations imply that Kähler current is proportional to the helicity current by a factor which depends on CP_2 coordinates, which implies that the current is automatically divergence free and defines a conserved charge for $D = 3$-dimensional CP_2 projection for which the instanton density vanishes identically. Kähler charge is not equal to the helicity defined by the inner product of magnetic field and vector potential but to a more general topological charge.

The number of conserved topological charges is infinite since the product of any function of CP_2 coordinates with the helicity current has vanishing divergence and defines a topological charge. A very natural function basis is provided by the scalar spherical harmonics of $SU(3)$ defining Hamiltonians of CP_2 canonical transformations and possessing well defined color quantum numbers. These functions define and infinite number of conserved charges which are also classical knot invariants in the sense that they are not affected at all when the 3-surface interpreted as a map from CP_2 projection to M^4_+ is deformed in M^4_+ degrees of freedom. Also canonical transformations induced by Hamiltonians in irreducible representations of color group affect these invariants via Poisson bracket action when the $U(1)$ gauge transformation induced by the canonical transformation corresponds to a single valued scalar function. These link invariants are additive in union whereas the quantum invariants defined by topological quantum field theories are multiplicative.

Also non-Abelian topological charges are well-defined. One can generalize the topological current associated with the Kähler form to a corresponding current associated with the induced electro-weak gauge fields whereas for classical color gauge fields the Chern-Simons form vanishes identically. Also in this case one can multiply the current by CP_2 color harmonics to obtain an infinite number of invariants in $D_{CP_2} = 3$ case. The only difference is that $A \wedge dA$ is replaced by $Tr(A \wedge (dA + 2A \wedge A/3))$.

There is a strong temptation to assume that these conserved charges characterize colored quantum states of the conformally invariant quantum theory as a functional of the light-like 3-surface defining boundary of space-time sheet or elementary particle horizon surrounding wormhole contacts. They would be TGD analogs of the states of the topological quantum field theory defined by Chern-Simons action as highest weight states associated with corresponding Wess-Zumino-Witten theory. These charges could be interpreted as topological counterparts of the isometry charges of WCW defined by the algebra of canonical transformations of CP_2.

The interpretation of these charges as contributions of light-like boundaries to WCW Hamiltonians would be natural. The dynamics of the induced second quantized spinor fields relates to that of Kähler action by a super-symmetry, so that it should define super-symmetric counterparts of these knot invariants. The anti-commutators of these super charges cannot however contribute to WCW Kähler metric so that topological zero modes are in question. These Hamiltonians and their super-charge counterparts would be responsible for the topological sector of quantum TGD.

3.5 Preferred Extremal Property And The Topologization And Light-Likeness Of Kähler Current?

The basic question is under what conditions the Kähler current is either topologized or light-like so that the Lorentz force vanishes. Does this hold for all preferred extremals of Kähler action? Or only asymptotically as suggested by the fact that generalized Beltrami fields can be interpreted as asymptotic self-organization patterns, when dissipation has become insignificant. Or does topologization take place in regions of space-time surface having Minkowskian signature of the induced metric? And what asymptotia actually means? Do absolute minima of Kähler action correspond to preferred extremals?

One can challenge the interpretation in terms of asymptotic self organization patterns assigned to the Minkowskian regions of space-time surface.

1. ZEO challenges the notion of approach to asymptotia in Minkowskian sense since the dynamics of light-like 3-surfaces is restricted inside finite volume $CD \subset M^4$ since the partonic 2-surfaces representing their ends are at the light-like boundaries of causal diamond in a given p-adic time scale.

2. One can argue that generic non-asymptotic field configurations have $D_{CP_2} = 4$, and would thus carry a vanishing Kähler four-current if Beltrami conditions were satisfied universally rather than only asymptotically. $j^\alpha = 0$ would obviously hold true also for the asymptotic configurations, in particular those with $D_{CP_2} < 4$ so that empty space Maxwell's field equations would be universally satisfied for asymptotic field configurations with $D_{CP_2} < 4$. The weak point of this argument is that it is 3-D light-like 3-surfaces rather than space-time surfaces which are the basic dynamical objects so that the generic and only possible case corresponds to $D_{CP_2} = 3$ for X^3_l. It is quite possible that preferred extremal property implies that $D_{CP_2} = 3$ holds true in the Minkowskian regions since these regions indeed represent empty space. Geometrically this would mean that the CP_2 projection does not change as the light-like coordinate labeling Y^3_l varies. This conforms nicely with the notion of quantum gravitational holography.

3. The failure of the generalized Beltrami conditions would mean that Kähler field is completely analogous to a dissipative Maxwell field for which also Lorentz force vanishes since $\overline{j} \cdot \overline{E}$ is non-vanishing (note that isometry currents are conserved although energy momentum tensor is not). Quantum classical correspondence states that classical space-time dynamics is by its classical non-determinism able to mimic the non-deterministic sequence of quantum jumps at space-time level, in particular dissipation in various length scales defined by the hierarchy of space-time sheets. Classical fields would represent "symbolically" the average dynamics, in particular dissipation, in shorter length scales. For instance, vacuum 4-current would be a symbolic representation for the average of the currents consisting of elementary particles. This would seem to support the view that $D_{CP_2} = 4$ Minkowskian regions are present. The weak point of this argument is that there is fractal hierarchy of length scales represented by the hierarchy of causal diamonds (CDs) and that the resulting hierarchy of generalized Feynman graphs might be enough to represent dissipation classically.

4. One objection to the idea is that second law realized as an asymptotic vanishing of Lorentz-Kähler force implies that all space-like 3-surfaces approaching same asymptotic state have the same value of Kähler function assuming that the Kähler function assignable to space-like 3-surface is same for all space-like sections of $X^4(X_l^3)$ (assuming that one can realize general coordinate invariance also in this sense). This need not be the case. In any case, this need not be a problem since it would mean an additional symmetry extending general coordinate invariance. The exponent of Kähler function would be highly analogous to a partition function defined as an exponent of Hamiltonian with Kähler coupling strength playing the role of temperature.

It seems that asymptotic self-organization pattern need not be correct interpretation for non-dissipating regions, and the identification of light-like 3-surfaces as generalized Feynman diagrams encourages an alternative interpretation.

1. $M^8 - H$ duality states that also the H counterparts of co-hyper-hyperquaternionic surfaces of M^8 are preferred extremals of Kähler action. CP_2 type vacuum extremals represent the basic example of these and a plausible conjecture is that the regions of space-time with Euclidian signature of the induced metric represent this kind of regions. If this conjecture is correct, dissipation could be assigned with regions having Euclidian signature of the induced metric. This makes sense since dissipation has quantum description in terms of Feynman graphs and regions of Euclidian signature indeed correspond to generalized Feynman graphs. This argument would suggest that generalized Beltrami conditions or light-likeness hold true inside Minkowskian regions rather than only asymptotically.

2. One could of course play language games and argue that asymptotia is with respect to the Euclidian time coordinate inside generalized Feynman graps and is achieved exactly when the signature of the induced metric becomes Minkowskian. This is somewhat artificial attempt to save the notion of asymptotic self-organization pattern since the regions outside Feynman diagrams represent empty space providing a holographic representations for the matter at X_l^3 so that the vanishing of $j^\alpha F_{\alpha\beta}$ is very natural.

3. What is then the correct identification of asymptotic self-organization pattern. Could correspond to the negative energy part of the zero energy state at the upper light-like boundary δM^4_- of CD? Or in the case of phase conjugate state to the positive energy part of the state at δM^4_+? An identification consistent with the fractal structure of ZEO and TGD inspired theory of consciousness is that the entire zero energy state reached by a sequence of quantum jumps represents asymptotic self-organization pattern represented by the asymptotic generalized Feynman diagram or their superposition. Biological systems represent basic examples about self-organization, and one cannot avoid the questions relating to the relationship between experience and geometric time. A detailed discussion of these points can be found in [L3] .

Absolute minimization of Kähler action was the first guess for the criterion selecting preferred extremals. Absolute minimization in a strict sense of the word does not make sense in the p-adic context since p-adic numbers are not well-ordered, and one cannot even define the action integral as a p-adic number. The generalized Beltrami conditions and the boundary conditions defining the preferred extremals are however local and purely algebraic and make sense also p-adically. If absolute minimization reduces to these algebraic conditions, it would make sense.

3.6 Generalized Beltrami Fields And Biological Systems

The following arguments support the view that generalized Beltrami fields play a key role in living systems, and that $D_{CP_2} = 2$ corresponds to ordered phase, $D_{CP_2} = 3$ to spin glass phase and $D_{CP_2} = 4$ to chaos, with $D_{CP_2} = 3$ defining life as a phenomenon at the boundary between order and chaos. If the criteria suggested by the number theoretic compactification are accepted, it is not clear whether D_{CP_2} extremals can define preferred extremals of Kähler action. For instance, cosmic strings are not preferred extremals and the Y_l^3 associated with MEs allow only covariantly constant right handed neutrino eigenmode of $D_K(X^2)$. The topological condensation of CP_2 type vacuum extremals around $D_{CP_2} = 2$ type extremals is however expected to give preferred extremals and if the density of the condensate is low enough one can still speak about $D_{CP_2} = 2$ phase. A natural guess is also that the deformation of $D_{CP_2} = 2$ extremals transforms light-like gauge currents to space-like topological currents allowed by $D_{CP_2} = 3$ phase.

3.6.1 *Why generalized Beltrami fields are important for living systems?*

Chirality, complexity, and high level of organization make $D_{CP_2} = 3$ generalized Beltrami fields excellent candidates for the magnetic bodies of living systems.

1. Chirality selection is one of the basic signatures of living systems. Beltrami field is characterized by a chirality defined by the relative sign of the current and magnetic field, which means parity breaking. Chirality reduces to the sign of the function ψ appearing in the topologization condition and makes sense also for the generalized Beltrami fields.

2. Although Beltrami fields can be extremely complex, they are also extremely organized. The reason is that the function α is constant along flux lines so that flux lines must in the case of compact Riemann 3-manifold belong to 2-dimensional $\alpha = constant$ closed surfaces, in fact two-dimensional invariant tori [B4] .

For generalized Beltrami fields the function ψ is constant along the flow lines of the Kähler current. Space-time sheets with 3-dimensional CP_2 projection serve as an illustrative example. One can use the coordinates for the CP_2 projection as space-time coordinates so that one space-time coordinate disappears totally from consideration. Hence the situation reduces to a flow in a 3-dimensional sub-manifold of CP_2. One can distinguish between three types of flow lines corresponding to space-like, light-like and time-like topological current. The 2-dimensional $\psi = constant$ invariant manifolds are sub-manifolds of CP_2. Ordinary Beltrami fields are a special case of space-like flow with flow lines belonging to the 2-dimensional invariant tori of CP_2. Time-like and light-like situations are more complex since the flow lines need not be closed so that the 2-dimensional $\psi = constant$ surfaces can have boundaries.

For periodic self-organization patterns flow lines are closed and $\psi = constant$ surfaces of CP_2 must be invariant tori. The dynamics of the periodic flow is obtained from that of a steady flow by replacing one spatial coordinate with effectively periodic time coordinate. Therefore topological notions like helix structure, linking, and knotting have a dynamical meaning at the level of CP_2 projection. The periodic generalized Beltrami fields are highly organized also in the temporal domain despite the potentiality for extreme topological complexity.

For these reasons topologically quantized generalized Beltrami fields provide an excellent candidate for a generic model for the dynamics of biological self-organization patterns. A natural guess is that many-sheeted magnetic and Z^0 magnetic fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chiral selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

The intricate topological structures of DNA, RNA, and protein molecules are known to have a deep significance besides their chemical structure, and they could even define something analogous to the genetic code. Usually the topology and geometry of bio-molecules is believed to reduce to chemistry. TGD suggests that space-like generalized Beltrami fields serve as templates for the formation of bio-molecules and bio-structures in general. The dynamics of bio-systems would in turn utilize the time-like Beltrami fields as templates. There could even exist a mapping from the topology of magnetic flux tube structures serving as templates for bio-molecules to the templates of self-organized dynamics. The helical structures, knotting, and linking of bio-molecules would thus define a symbolic representation, and even coding for the dynamics of the bio-system analogous to written language.

3.6.2 $D_{CP_2} = 3$ *systems as boundary between* $D_{CP_2} = 2$ *order and* $D_{CP_2} = 4$ *chaos*

The dimension of CP_2 projection is basic classifier for the asymptotic self-organization patterns.

1. $D_{CP_2} = 4$ phase, dead matter, and chaos

$D_{CP_2} = 4$ phase - if present at all- would correspond to the ordinary Maxwellian phase in which Kähler current and charge density vanish and there is no topologization of Kähler current. By its maximal dimension this phase would naturally correspond to disordered phase, ordinary "dead matter". If one assumes that Kähler charge corresponds to either em charge or Z^0 charge then the signature of this state of matter would be em neutrality or Z^0 neutrality. As already found, Maxwell phase is very probably not realized in this manner but is essentially outcome of many-sheeted space-time concept.

2. $D_{CP_2} = 2$ phase as ordered phase

By the low dimension of CP_2 projection $D_{CP_2} = 2$ phase is the least stable phase possible only at cold space-time sheets. Kähler current is either vanishing or light-like, and Beltrami fields are not possible. This phase is highly ordered and much like a topological quantized version of ferro-magnet. In particular, it is possible to have a global coordinate varying along the field lines of the vector potential also now. The magnetic and Z^0 magnetic body of any system is a candidate for this kind of system. Z^0 field is indeed always present for vacuum extremals having $D_{CP_2} = 2$ and the vanishing of em field requires that that $sin^2(\theta_W)$ (θ_W is Weinberg angle) vanishes.

3. $D_{CP_2} = 3$ corresponds to living matter

$D_{CP_2} = 3$ corresponds to highly organized phase characterized in the case of space-like Kähler current by complex helical structures necessarily accompanied by topologized Kähler charge density $\propto \overline{A} \cdot \overline{B} \neq 0$ and Kähler current $\overline{E} \times \overline{A} + \phi \overline{B}$. For time like Kähler currents the helical structures are replaced by periodic oscillation patterns for the state of the system. By the non-maximal dimension of CP_2 projection this phase must be unstable against too strong external perturbations and cannot survive at too high temperatures. Living matter is thus excellent candidate for this phase and it might be that the interaction of the magnetic body with living matter makes possible the transition from $D_{CP_2} = 2$ phase to the self-organizing $D_{CP_2} = 3$ phase.

Living matter which is indeed populated by helical structures providing examples of space-like Kähler current. Strongly charged lipid layers of cell membrane might provide example of time-like Kähler current. Cell membrane, micro-tubuli, DNA, and proteins are known to be electrically charged and Z^0 charge plays key role in TGD based model of catalysis discussed in [K8] . For instance, denaturing of DNA destroying its helical structure could be interpreted as a transition leading from $D_{CP_2} = 3$ phase to $D_{CP_2} = 4$ phase. The prediction is that the denatured phase should be electromagnetically (or Z^0) neutral.

Beltrami fields result when Kähler charge density vanishes. For these configurations magnetic field and current density take the role of the vector potential and magnetic field as far as the contact structure is considered. For Beltrami fields there exist a global coordinate along the field lines of the vector potential but not along those of the magnetic field. As a consequence, the covariant consistency condition $(\partial_s - qeA_s)\Psi = 0$ frequently appearing in the physics of super conducting systems would make sense along the flow lines of the vector potential for the order parameter of Bose-Einstein condensate. If Beltrami phase is super-conducting, then the state of the system must change in the transition to a more general phase. It is impossible to assign slicing of 4-surface by 3-D surfaces labeled by a coordinate t varying along the flow lines. This means that one cannot speak about a continuous evolution of Schrödinger amplitude with t playing the role of time coordinate. One could perhaps say that the entire space-time sheet represents single quantum event which cannot be decomposed to evolution. This would conform with the assignment of macroscopic and macro-temporal quantum coherence with living matter.

The existence of these three phases brings in mind systems allowing chaotic de-magnetized phase above critical temperature T_c, spin glass phase at the critical point, and ferromagnetic phase below T_c. Similar analogy is provided by liquid phase, liquid crystal phase possible in the vicinity of the critical point for liquid to solid transition, and solid phase. Perhaps one could regard $D_{CP_2} = 3$ phase and life as a boundary region between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos. This would naturally explain why life as it is known is possible in relatively narrow temperature interval.

3.6.3 *Can one assign a continuous Schrödinger time evolution to light-like 3-surfaces?*

Alain Connes wrote [A1] about factors of various types using as an example Schrödinger equation for various kinds of foliations of space-time to time=constant slices. If this kind of foliation does not exist, one cannot speak about time evolution of Schrödinger equation at all. Depending on the character of the foliation one can have factor of type I, II, or III. For instance, torus with slicing $dx = ady$ in flat coordinates, gives a factor of type I for rational values of a and factor of type II for irrational values of a.

1. 3-D foliations and type III factors

Connes mentioned 3-D foliations V which give rise to type III factors. Foliation property requires a slicing of V by a one-form v to which slices are orthogonal (this requires metric).

1. The foliation property requires that v multiplied by suitable scalar is gradient. This gives the integrability conditions $dv = w \wedge v$, $w = -d\psi/\psi = -dlog(\psi)$. Something proportional to $log(\psi)$ can be taken as a third coordinate varying along flow lines of v: the flow defines a continuous sequence of maps of 2-dimensional slice to itself.

2. If the so called Godbillon-Vey invariant defined as the integral of $dw \wedge w$ over V is non-vanishing, factor of type III is obtained using Schrödinger amplitudes for which the flow lines of foliation define the time evolution. The operators of the algebra in question are transversal operators acting on Schrödinger amplitudes at each slice. Essentially Schrödinger equation in 3-D space-time would be in question with factor of type III resulting from the exotic choice of the time coordinate defining the slicing.

2. What happens in case of light-like 3-surfaces?

In TGD light-like 3-surfaces are natural candidates for V and it is interesting to look what happens in this case. Light-likeness is of course a disturbing complication since orthogonality condition and thus contravariant metric is involved with the definition of the slicing. Light-likeness is not however involved with the basic conditions.

1. The one-form v defined by the induced Kähler gauge potential A defining also a braiding is a unique identification for v. If foliation exists, the braiding flow defines a continuous sequence of maps of partonic 2-surface to itself.

2. Physically this means the possibility of a super-conducting phase with order parameter satisfying covariant constancy equation $D\psi = (d/dt - ieA)\psi = 0$. This would describe a supra current flowing along flow lines of A.

3. If the integrability fails to be true, one *cannot* assign Schrödinger time evolution with the flow lines of v. One might perhaps say that 3-surface behaves like single quantum event not allowing slicing into a continuous Schrödinger time evolution.

4. In TGD Schrödinger amplitudes are replaced by second quantized induced spinor fields. Hence one does not face the problem whether it makes sense to speak about Schrödinger time evolution of complex order parameter along the flow lines of a foliation or not. Also the fact that the "time evolution" for the Kähler-Dirac operator corresponds to single position dependent generalized eigenvalue identified as Higgs expectation same for all transversal modes (essentially z^n labeled by conformal weight) is crucial since it saves from the problems caused by the possible non-existence of Schrödinger evolution.

4. Extremals of Kähler action

Some comments relating to the interpretation of the classification of the extremals of Kähler action by the dimension of their CP_2 projection are in order. It has been already found that the extremals can be classified according to the dimension D of the CP_2 projection of space-time sheet in the case that $A_a = 0$ holds true.

1. For $D_{CP_2} = 2$ integrability conditions for the vector potential can be satisfied for $A_a = 0$ so that one has generalized Beltrami flow and one can speak about Schrödinger time evolution associated with the flow lines of vector potential defined by covariant constancy condition $D\psi = 0$ makes sense. Kähler current is vanishing or light-like. This phase is analogous to a super-conductor or a ferromagnetic phase. For non-vanishing A_a the Beltrami flow property is lost but the analogy with ferromagnetism makes sense still.
2. For $D_{CP_2} = 3$ foliations are lost. The phase is dominated by helical structures. This phase is analogous to spin glass phase around phase transition point from ferromagnetic to non-magnetized phase and expected to be important in living matter systems.
3. $D_{CP_2} = 4$ is analogous to a chaotic phase with vanishing Kähler current and to a phase without magnetization. The interpretation in terms of non-quantum coherent "dead" matter is suggestive.

An interesting question is whether the ordinary 8-D imbedding space which defines one sector of the generalized imbedding space could correspond to $A_a = 0$ phase. If so, then all states for this sector would be vacua with respect to M^4 quantum numbers. M^4-trivial zero energy states in this sector could be transformed to non-trivial zero energy states by a leakage to other sectors.

4 Basic Extremals Of Kähler Action

The solutions of field equations can be divided to vacuum extremals and non-vacuum extremal. Vacuum extremals come as two basic types: CP_2 type vacuum extremals for which the induced Kähler field and Kähler action are non-vanishing and the extremals for which the induced Kähler field vanishes. The deformations of both extremals are expected to be of fundamental importance in TGD universe.

4.1 CP_2 Type Vacuum Extremals

These extremals correspond to various isometric imbeddings of CP_2 to $M^4_+ \times CP_2$. One can also drill holes to CP_2. Using the coordinates of CP_2 as coordinates for X^4 the imbedding is given by the formula

$$\begin{aligned} m^k &= m^k(u) \ , \\ m_{kl}\dot{m}^k\dot{m}^l &= 0 \ , \end{aligned} \tag{4.1}$$

where $u(s^k)$ is an arbitrary function of CP_2 coordinates. The latter condition tells that the curve representing the projection of X^4 to M^4 is light like curve. One can choose the functions $m^i, i = 1,2,3$ freely and solve m^0 from the condition expressing light likeness so that the number of this kind of extremals is very large.

The induced metric and Kähler field are just those of CP_2 and energy momentum tensor $T^{\alpha\beta}$ vanishes identically by the self duality of the Kähler form of CP_2. Also the canonical current $j^\alpha = D_\beta J^{\alpha\beta}$ associated with the Kähler form vanishes identically. Therefore the field equations in the interior of X^4 are satisfied. The field equations are also satisfied on the boundary components of CP_2 type extremal because the non-vanishing boundary term is, besides the normal component of Kähler electric field, also proportional to the projection operator to the normal space and vanishes identically since the induced metric and Kähler form are identical with the metric and Kähler form of CP_2.

As a special case one obtains solutions for which M^4 projection is light like geodesic. The projection of $m^0 = constant$ surfaces to CP_2 is $u = constant$ 3-sub-manifold of CP_2. Geometrically these solutions correspond to a propagation of a massless particle. In a more general case the interpretation as an orbit of a massless particle is not the only possibility. For example, one can imagine a situation, where the center of mass of the particle is at rest and motion occurs along a circle at say (m^1, m^2) plane. The interpretation as a massive particle is natural. Amusingly, there is nice analogy with the classical theory of Dirac electron: massive Dirac fermion moves also with the velocity of light (zitterbewegung). The quantization of this random motion with light velocity leads to Virasoro conditions and this led to a breakthrough in the understanding of the symmetries of TGD. Super Virasoro invariance is a general symmetry of WCW geometry and quantum TGD.

The action for all extremals is same and given by the Kähler action for the imbedding of CP_2. The value of the action is given by

$$S = -\frac{\pi}{8\alpha_K} . \tag{4.2}$$

To derive this expression we have used the result that the value of Lagrangian is constant: $L = 4/R^4$, the volume of CP_2 is $V(CP_2) = \pi^2 R^4/2$ and the definition of the Kähler coupling strength $k_1 = 1/16\pi\alpha_K$ (by definition, πR is the length of CP_2 geodesics). Four-momentum vanishes for these extremals so that they can be regarded as vacuum extremals. The value of the action is negative so that these vacuum extremals are indeed favored by the minimization of the Kähler action. The principle selecting preferred extremals of Kähler action suggests that ordinary vacuums with vanishing Kähler action density are unstable against the generation of CP_2 type extremals. There are even reasons to expect that CP_2 type extremals are for TGD what black holes are for GRT. Indeed, the nice generalization of the area law for the entropy of black hole [K18] supports this view.

In accordance with the basic ideas of TGD topologically condensed vacuum extremals should somehow correspond to massive particles. The properties of the CP_2 type vacuum extremals are in accordance with this interpretation. Although these objects move with a velocity of light, the motion can be transformed to a mere so that the center of mass motion is trivial. Even the generation of the rest mass could might be understood classically as a consequence of the minimization of action. Long range Kähler fields generate negative action for the topologically condensed vacuum extremal (momentum zero massless particle) and Kähler field energy in turn is identifiable as the rest mass of the topologically condensed particle.

An interesting feature of these objects is that they can be regarded as gravitational instantons [A2] . A further interesting feature of CP_2 type extremals is that they carry nontrivial classical color charges. The possible relationship of this feature to color confinement raises interesting questions. Could one model classically the formation of the color singlets to take place through the emission of "colorons": states with zero momentum but non-vanishing color? Could these peculiar states reflect the infrared properties of the color interactions?

4.2 Vacuum Extremals With Vanishing Kähler Field

Vacuum extremals correspond to 4-surfaces with vanishing Kähler field and therefore to gauge field zero configurations of gauge field theory. These surfaces have CP_2 projection, which is Lagrange manifold. The condition expressing Lagrange manifold property is obtained in the following manner. Kähler potential of CP_2 can be expressed in terms of the canonical coordinates (P_i, Q_i) for CP_2 as

$$A = \sum_k P_k dQ^k . \tag{4.3}$$

The conditions

$$P_k = \partial_{Q^k} f(Q^i) , \tag{4.4}$$

where $f(Q^i)$ is arbitrary function of its arguments, guarantee that Kähler potential is pure gauge. It is clear that canonical transformations, which act as local $U(1)$ gauge transformations, transform different vacuum configurations to each other so that vacuum degeneracy is enormous. Also M^4_+ diffeomorphisms act as the dynamical symmetries of the vacuum extremals. Some sub-group of these symmetries extends to the isometry group of the WCW in the proposed construction of the WCW metric. The vacuum degeneracy is still enhanced by the fact that the topology of the four-surface is practically free.

Vacuum extremals are certainly not absolute minima of the action. For the induced metric having Minkowski signature the generation of Kähler electric fields lowers the action. For Euclidian signature both electric and magnetic fields tend to reduce the action. Therefore the generation of Euclidian regions of space-time is expected to occur. CP_2 type extremals, identifiable as real (as contrast to virtual) elementary particles, can be indeed regarded as these Euclidian regions.

Particle like vacuum extremals can be classified roughly by the number of the compactified dimensions D having size given by CP_2 length. Thus one has $D = 3$ for CP_2 type extremals, $D = 2$ for string like objects, $D = 1$ for membranes and $D = 0$ for pieces of M^4. As already mentioned, the rule $h_{vac} = -D$ relating the vacuum weight of the Super Virasoro representation to the number of compactified dimensions of the vacuum extremal is very suggestive. $D < 3$ vacuum extremals would correspond in this picture to virtual particles, whose contribution to the generalized Feynman diagram is not suppressed by the exponential of Kähler action unlike that associated with the virtual CP_2 type lines.

M^4 type vacuum extremals (representable as maps $M^4_+ \rightarrow CP_2$ by definition) are also expected to be natural idealizations of the space-time at long length scales obtained by smoothing out small scale topological inhomogenuities (particles) and therefore they should correspond to space-time of GRT in a reasonable approximation.

The reason would be "Yin-Yang principle" discussed in [K3] .

1. Consider first the option for which Kähler function corresponds to an absolute minimum of Kähler action. Vacuum functional as an exponent of Kähler function is expected to concentrate on those 3-surfaces for which the Kähler action is non-negative. On the other hand, the requirement that Kähler action is absolute minimum for the space-time associated with a given 3-surface, tends to make the action negative. Therefore the vacuum functional is expected to differ considerably from zero only for 3-surfaces with a vanishing Kähler action per volume. It could also occur that the degeneracy of 3-surfaces with same large negative action compensates the exponent of Kähler function.

2. If preferred extrema correspond to Kähler calibrations or their duals [K24] , Yin-Yang principle is modified to a more local principle. For Kähler calibrations (their duals) the absolute value of action in given region is minimized (maximized). A given region with a positive (negative sign) of action density favors Kähler electric (magnetic) fields. In long length scales the average density of Kähler action per four-volume tends to vanish so that Kähler function of the entire universe is expected to be very nearly zero. This regularizes the theory automatically and implies that average Kähler action per volume vanishes. Positive and finite values of Kähler function are of course favored.

In both cases the vanishing of Kähler action per volume in long length scales makes vacuum extremals excellent idealizations for the smoothed out space-time surface. Robertson-Walker cosmologies provide a good example in this respect. As a matter fact the smoothed out space-time is not a mere fictive concept since larger space-time sheets realize it as a essential part of the Universe.

Several absolute minima could be possible and the non-determinism of the vacuum extremals is not expected to be reduced completely. The remaining degeneracy could be even infinite. A good example is provided by the vacuum extremals representable as maps $M^4_+ \rightarrow D^1$, where D^1 is one-dimensional curve of CP_2. This degeneracy could be interpreted as a space-time correlate for the non-determinism of quantum jumps with maximal deterministic regions representing quantum states in a sequence of quantum jumps.

4.3 Cosmic Strings

Cosmic strings are extremals of type $X^2 \times S^2$, where X^2 is minimal surface in M^4_+ (analogous to the orbit of a bosonic string) and S^2 is the homologically non-trivial geodesic sphere of CP_2.

The action of these extremals is positive and thus absolute minima are certainly not in question. One can however consider the possibility that these extremals are building blocks of the absolute minimum space-time surfaces since the principle selecting preferred extremals of the Kähler action is global rather than a local. Cosmic strings can contain also Kähler charged matter in the form of small holes containing elementary particle quantum numbers on their boundaries and the negative Kähler electric action for a topologically condensed cosmic string could cancel the Kähler magnetic action.

The string tension of the cosmic strings is given by

$$T = \frac{1}{8\alpha_K R^2} \simeq .2210^{-6}\frac{1}{G} \ , \quad (4.5)$$

where $\alpha_K \simeq \alpha_{em}$ has been used to get the numerical estimate. The string tension is of the same order of magnitude as the string tension of the cosmic strings of GUTs and this leads to the model of the galaxy formation providing a solution to the dark matter puzzle as well as to a model for large voids as caused by the presence of a strongly Kähler charged cosmic string. Cosmic strings play also fundamental role in the TGD inspired very early cosmology.

4.4 Massless Extremals

Massless extremals are characterized by massless wave vector p and polarization vector ε orthogonal to this wave vector. Using the coordinates of M^4 as coordinates for X^4 the solution is given as

$$\begin{array}{ll} s^k = f^k(u,v) \ , & \\ u = p\cdot m \ , & v = \varepsilon\cdot m \ , \\ p\cdot\varepsilon = 0 \ , & p^2 = 0 \ . \end{array} \quad (4.6)$$

CP_2 coordinates are arbitrary functions of $p\cdot m$ and $\varepsilon\cdot m$. Clearly these solutions correspond to plane wave solutions of gauge field theories. It is important to notice however that linear super position doesn't hold as it holds in Maxwell phase. Gauge current is proportional to wave vector and its divergence vanishes as a consequence. Also cylindrically symmetric solutions for which the transverse coordinate is replaced with the radial coordinate $\rho = \sqrt{m_1^2+m_2^2}$ are possible. In fact, v can be *any* function of the coordinates m^1, m^2 transversal to the light like vector p.

Boundary conditions on the boundaries of the massless extremal are satisfied provided the normal component of the energy momentum tensor vanishes. Since energy momentum tensor is of the form $T^{\alpha\beta}\propto p^\alpha p^\beta$ the conditions $T^{n\beta}=0$ are satisfied if the M^4 projection of the boundary is given by the equations of form

$$\begin{array}{lll} H(p\cdot m,\varepsilon\cdot m,\varepsilon_1\cdot m) = 0 \ , & & \\ \varepsilon\cdot p = 0 \ , & \varepsilon_1\cdot p = 0 \ , & \varepsilon\cdot\varepsilon_1 = 0 \ . \end{array} \quad (4.7)$$

where H is arbitrary function of its arguments. Recall that for M^4 type extremals the boundary conditions are also satisfied if Kähler field vanishes identically on the boundary.

The following argument suggests that there are not very many manners to satisfy boundary conditions in case of M^4 type extremals. The boundary conditions, when applied to M^4 coordinates imply the vanishing of the normal component of energy momentum tensor. Using coordinates, where energy momentum tensor is diagonal, the requirement boils down to the condition that at least one of the eigen values of $T^{\alpha\beta}$ vanishes so that the determinant $det(T^{\alpha\beta})$ must vanish on the boundary: this condition defines 3-dimensional surface in X^4. In addition, the normal of this surface must have same direction as the eigen vector associated with the vanishing eigen value: this means that three additional conditions must be satisfied and this is in general true in single point only. The boundary conditions in CP_2 coordinates are satisfied provided that the conditions

$$J^{n\beta}J^k_{\ l}\partial_\beta s^l = 0$$

are satisfied. The identical vanishing of the normal components of Kähler electric and magnetic fields on the boundary of massless extremal property provides a manner to satisfy all boundary conditions but it is not clear whether there are any other manners to satisfy them.

The characteristic feature of the massless extremals is that in general the Kähler gauge current is non-vanishing. In ordinary Maxwell electrodynamics this is not possible. This means that these extremals are accompanied by vacuum current, which contains in general case both weak and electromagnetic terms as well as color part.

A possible interpretation of the solution is as the exterior space-time to a topologically condensed particle with vanishing mass described by massless CP_2 type extremal, say photon or neutrino. In general the surfaces in question have boundaries since the coordinates s^k are are bounded: this is in accordance with the general ideas about topological condensation. The fact that massless plane wave is associated with CP_2 type extremal combines neatly the wave and particle aspects at geometrical level.

The fractal hierarchy of space-time sheets implies that massless extremals should interesting also in long length scales. The presence of a light like electromagnetic vacuum current implies the generation of coherent photons and also coherent gravitons are generated since the Einstein tensor is also non-vanishing and light like (proportional to $k^\alpha k^\beta$). Massless extremals play an important role in the TGD based model of bio-system as a macroscopic quantum system. The possibility of vacuum currents is what makes possible the generation of the highly desired coherent photon states.

4.5 Does GRT really allow gravitational radiation: could cosmological constant save the situation?

In Facebook discussion Nils Grebäck mentioned Weyl tensor and I learned something that I should have noticed long time ago. Wikipedia article (see `http://tinyurl.com/y7fsnzk8`) lists the basic properties of Weyl tensor as the traceless part of curvature tensor, call it R. Weyl tensor C is vanishing for conformally flat space-times. In dimensions D=2,3 Weyl tensor vanishes identically so that they are always conformally flat: this obviously makes the dimension $D = 3$ for space very special. Interestingly, one can have non-flat space-times with nonvanishing Weyl tensor but the vanishing Schouten/Ricci/Einstein tensor and thus also with vanishing energy momentum tensor.

The rest of curvature tensor R can be expressed in terms of so called Kulkarni-Nomizu product $P \cdot g$ of Schouten tensor P and metric tensor g: $R = C + P \cdot g$, which can be also transformed to a definition of Weyl tensor using the definition of curvature tensor in terms of Christoffel symbols as the fundamental definition. Kulkarni-Nomizu product $\cdot$ is defined as tensor product of two 2-tensors with symmetrization with respect to first and second index pairs plus antisymmetrization with respect to second and fourth indices.

Schouten tensor P is expressible as a combination of Ricci tensor Ric defined by the trace of R with respect to the first two indices and metric tensor g multiplied by curvature scalar s (rather than R in order to use index free notation without confusion with the curvature tensor). The expression reads as

$$P = \frac{1}{D-2}\left[Ric - \frac{s}{2(D-1)}g\right] \ .$$

Note that the coefficients of Ric and g differ from those for Einstein tensor. Ricci tensor and Einstein tensor are proportional to energy momentum tensor by Einstein equations relate to the part.

Weyl tensor is assigned with gravitational radiation in GRT. What I see as a serious interpretational problem is that by Einstein's equations gravitational radiation would carry no energy and momentum in absence of matter. One could argue that there are no free gravitons in GRT if this interpretation is adopted! This could be seen as a further argument against GRT besides the problems with the notions of energy and momentum: I had not realized this earlier.

Interestingly, in TGD framework so called massless extremals (MEs) [K3, K33, K19] are four-surfaces, which are extremals of Kähler action, have Weyl tensor equal to curvature tensor and therefore would have interpretation in terms of gravitons. Now these extremals are however non-vacuum extremals.

1. Massless extremals correspond to graphs of possibly multi-valued maps from M^4 to CP_2. CP_2 coordinates are arbitrary functions of variables $u = k \cot m$ and $w = \epsilon \cdot m$. k is light-like wave vector and ϵ space-like polarization vector orthogonal to k so that the interpretation in terms of massless particle with polarization is possible. ME describes in the most general case a wave packet preserving its shape and propagating with maximal signal velocity along a kind of tube analogous to wave guide so that they are ideal for precisely targeted communications and central in TGD inspired quantum biology. MEs do not have Maxwellian counterparts. For instance, MEs can carry light-like gauge currents parallel to them: this is not possible in Maxwell's theory.

2. I have discussed a generalization of this solution ansatz so that the directions defined by light-like vector k and polarization vector ϵ orthogonal to it are not constant anymore but define a slicing of M^4 by orthogonal curved surfaces (analogs of string world sheets and space-like surfaces orthogonal to them). MEs in their simplest form at least are minimal surfaces and actually extremals of practically any general coordinate invariance action principle. For instance, this is the case if the volume term suggested by the twistor lift of Kähler action [K34] and identifiable in terms of cosmological constant is added to Kähler action.

3. MEs carry non-trivial induced gauge fields and gravitational fields identified in terms of the induced metric. I have identified them as correlates for particles, which correspond to pairs of wormhole contacts between two space-times such that at least one of them is ME. MEs would accompany to both gravitational radiation and other forms or radiation classically and serve as their correlates. For massless extremals the metric tensor is of form

$$g = m + a\epsilon \otimes \epsilon + bk \otimes k + c(\epsilon \otimes kv + k \otimes \epsilon) \ ,$$

   where m is the metric of empty Minkowski space. The curvature tensor is necessarily quadrilinear in polarization vector ϵ and light-like wave vector k (light-like ifor both M^4 and ME metric) and from the general expression of Weyl tensor C in terms of R and g it is equal to curvature tensor: $C = R$.

   Hence the interpretation as graviton solution conforms with the GRT interpretation. Now however the energy momentum tensor for the induced Kähler form is non-vanishing and bilinear in velocity vector k and the interpretational problem is avoided.

What is interesting that also at GRT limit cosmological constant saves gravitons from reducing to vacuum solutions. The deviation of the energy density given by cosmological term from that for Minkowski metric is identifiable as gravitonic energy density. The mysterious cosmological constant would be necessary for making gravitons non-vacuum solutions. The value of graviton amplitude would be determined by the continuity conditions for Einstein's equations with cosmological term. The p-adic evolution of cosmological term predicted by TGD is however difficult to understand in GRT framework.

4.6 Generalization Of The Solution Ansatz Defining Massless Extremals (MEs)

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

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4.6.1 *Local light cone coordinates*

The solution involves a decomposition of M^4_+ tangent space localizing the decomposition of Minkowski space to an orthogonal direct sum $M^2 \oplus E^2$ defined by light-like wave vector and polarization vector orthogonal to it. This decomposition defines what might be called local light cone coordinates.

1. Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct *orthogonal* sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray.

2. With these assumptions the coordinates $(S_\pm, E^i)$ define local light cone coordinates with the metric element having the form

$$ds^2 = 2g_{+-}dS^+dS^- + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \quad . \tag{4.8}$$

   If complex coordinates are used in transversal degrees of freedom one has $g_{11} = g_{22}$.

3. This family of light cone coordinates is not the most general family since longitudinal and transversal spaces are orthogonal. One can also consider light-cone coordinates for which one non-diagonal component, say m_{1+}, is non-vanishing if the solution ansatz is such that longitudinal and transversal spaces are orthogonal for the induced metric.

4.6.2 *A conformally invariant family of local light cone coordinates*

The simplest solutions to the equations defining local light cone coordinates are of form $S_\pm = k \cdot m$ giving as a special case $S_\pm = m^0 \pm m^3$. For more general solutions of from

$$S_\pm = m^0 \pm f(m^1, m^2, m^3) \ , \ (\nabla_3 f)^2 = 1 \ , \tag{4.9}$$

where f is an otherwise arbitrary function, this relationship reads as

$$S^+ + S^- = 2m^0 \quad . \tag{4.10}$$

This condition defines a natural rest frame. One can integrate f from its initial data at some two-dimensional $f = constant$ surface and solution describes curvilinear light rays emanating from this surface and orthogonal to it. The flow velocity field $\overline{v} = \nabla f$ is irrotational so that closed flow lines are not possible in a connected region of space and the condition $\overline{v}^2 = 1$ excludes also closed flow line configuration with singularity at origin such as $v = 1/\rho$ rotational flow around axis.

One can identify E^2 as a local tangent space spanned by polarization vectors and orthogonal to the flow lines of the velocity field $\overline{v} = \nabla f(m^1, m^2, m^3)$. Since the metric tensor of any 3-dimensional space allows always diagonalization in suitable coordinates, one can always find coordinates (E^1, E^2) such that (f, E^1, E^2) form orthogonal coordinates for $m^0 = constant$ hyperplane. Obviously one can select the coordinates E^1 and E^2 in infinitely many manners.

4.6.3 *Closer inspection of the conditions defining local light cone coordinates*

Whether the conformal transforms of the local light cone coordinates $\{S_\pm = m^0 \pm f(m^1, m^2, m^3), E^i\}$ define the only possible compositions $M^2 \oplus E^2$ with the required properties, remains an open question. The best that one might hope is that any function S^+ defining a family of light-like curves defines a local decomposition $M^4 = M^2 \oplus E^2$ with required properties.

1. Suppose that S^+ and S^- define light-like vector fields which are not orthogonal (proportional to each other). Suppose that the polarization vector fields $\epsilon_i = \nabla E^i$ tangential to local E^2 satisfy the conditions $\epsilon_i \cdot \nabla S^+ = 0$. One can formally integrate the functions E^i from these condition since the initial values of E^i are given at $m^0 = constant$ slice.

2. The solution to the condition $\nabla S_+ \cdot \epsilon_i = 0$ is determined only modulo the replacement

$$\epsilon_i \rightarrow \hat{\epsilon}_i = \epsilon_i + k\nabla S_+ \quad , \tag{4.11}$$

where k is any function. With the choice

$$k = -\frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-} \tag{4.12}$$

one can satisfy also the condition $\hat{\epsilon}_i \cdot \nabla S^- = 0$.

3. The requirement that also $\hat{\epsilon}_i$ is gradient is satisfied if the integrability condition

$$k = k(S^+) \tag{4.13}$$

is satisfied: in this case $\hat{\epsilon}_i$ is obtained by a gauge transformation from ϵ_i. The integrability condition can be regarded as an additional, and obviously very strong, condition for S^- once S^+ and E^i are known.

4. The problem boils down to that of finding local momentum and polarization directions defined by the functions S^+, S^- and E^1 and E^2 satisfying the orthogonality and integrability conditions

$$\begin{array}{ll} (\nabla S^+)^2 = (\nabla S^-)^2 = 0 \ , & \nabla S^+ \cdot \nabla S^- \neq 0 \ , \\ \nabla S^+ \cdot \nabla E^i = 0 \ , & \frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-} = k_i(S^+) \ . \end{array} \tag{4.14}$$

The number of integrability conditions is 3+3 (all derivatives of k_i except the one with respect to S^+ vanish): thus it seems that there are not much hopes of finding a solution unless some discrete symmetry relating S^+ and S^- eliminates the integrability conditions altogether.

A generalization of the spatial reflection $f \rightarrow -f$ working for the separable Hamilton Jacobi function $S_\pm = m^0 \pm f$ ansatz could relate S^+ and S^- to each other and trivialize the integrability conditions. The symmetry transformation of M^4_+ must perform the permutation $S^+ \leftrightarrow S^-$, preserve the light-likeness property, map E^2 to E^2, and multiply the inner products between M^2 and E^2 vectors by a mere conformal factor. This encourages the conjecture that all solutions are obtained by conformal transformations from the solutions $S_\pm = m^0 \pm f$.

4.6.4 *General solution ansatz for MEs for given choice of local light cone coordinates*

Consider now the general solution ansatz assuming that a local wave-vector-polarization decomposition of M^4_+ tangent space has been found.

1. Let $E(S^+, E^1, E^2)$ be an arbitrary function of its arguments: the gradient ∇E defines at each point of E^2 an S^+-dependent (and thus time dependent) polarization direction orthogonal to the direction of local wave vector defined by ∇S^+. Polarization vector depends on E^2 position only.

2. Quite a general family of MEs corresponds to the solution family of the field equations having the general form

$$s^k = f^k(S^+, E) \quad , \tag{4.15}$$

where s^k denotes CP_2 coordinates and f^k is an arbitrary function of S^+ and E. The solution represents a wave propagating with light velocity and having definite S^+ dependent polarization in the direction of ∇E. By replacing S^+ with S^- one obtains a dual solution. Field equations are satisfied because energy momentum tensor and Kähler current are light-like so that all tensor contractions involved with the field equations vanish: the orthogonality of M^2 and E^2 is essential for the light-likeness of energy momentum tensor and Kähler current.

3. The simplest solutions of the form $S_\pm = m^0 \pm m^3$, $(E^1, E^2) = (m^1, m^2)$ and correspond to a cylindrical MEs representing waves propagating in the direction of the cylinder axis with light velocity and having polarization which depends on point (E^1, E^2) and S^+ (and thus time). For these solutions four-momentum is light-like: for more general solutions this cannot be the case. Polarization is in general case time dependent so that both linearly and circularly polarized waves are possible. If m^3 varies in a finite range of length L, then "free" solution represents geometrically a cylinder of length L moving with a light velocity. Of course, ends could be also anchored to the emitting or absorbing space-time surfaces.

4. For the general solution the cylinder is replaced by a three-dimensional family of light like curves and in this case the rectilinear motion of the ends of the cylinder is replaced with a curvilinear motion with light velocity unless the ends are anchored to emitting/absorbing space-time surfaces. The non-rotational character of the velocity flow suggests that the freely moving particle like 3-surface defined by ME cannot remain in a infinite spatial volume. The most general ansatz for MEs should be useful in the intermediate and nearby regions of a radiating object whereas in the far away region radiation solution is excepted to decompose to cylindrical ray like MEs for which the function $f(m^1, m^2, m^2)$ is a linear function of m^i.

5. One can try to generalize the solution ansatz further by allowing the metric of M^4_+ to have components of type g_{i+} or g_{i-} in the light cone coordinates used. The vanishing of T^{11}, T^{+1}, and T^{--} is achieved if $g_{i\pm} = 0$ holds true for the induced metric. For $s^k = s^k(S^+, E^1)$ ansatz neither $g_{2\pm}$ nor g_{1-} is affected by the imbedding so that these components of the metric must vanish for the Hamilton Jacobi structure:

$$ds^2 = 2g_{+-}dS^+dS^- + 2g_{1+}dE^1dS^+ + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \quad . \tag{4.16}$$

$g_{1+} = 0$ can be achieved by an additional condition

$$m_{1+} = s_{kl}\partial_1 s^k \partial_+ s^k \quad . \tag{4.17}$$

The diagonalization of the metric seems to be a general aspect of absolute minima. The absence of metric correlations between space-time degrees of freedom for asymptotic self-organization patterns is somewhat analogous to the minimization of non-bound entanglement in the final state of the quantum jump.

4.6.5 *Are the boundaries of space-time sheets quite generally light like surfaces with Hamilton Jacobi structure?*

Quantum holography principle naturally generalizes to an approximate principle expected to hold true also in non-cosmological length and time scales.

1. The most general ansatz for topological light rays or massless extremals (MEs) inspired by the quantum holographic thinking relies on the introduction of the notion of local light cone coordinates S_+, S_-, E_1, E_2. The gradients ∇S_+ and ∇S_- define two light like directions just like Hamilton Jacobi functions define the direction of propagation of wave in geometric optics. The two polarization vector fields ∇E_1 and ∇E_2 are orthogonal to the direction of propagation defined by either S_+ or S_-. Since also E_1 and E_2 can be chosen to be orthogonal, the metric of M^4_+ can be written locally as $ds^2 = g_{+-}dS_+dS_- + g_{11}dE_1^2 + g_{22}dE_2^2$. In the earlier ansatz S_+ and S_- where restricted to the variables $k \cdot m$ and $\tilde{k} \cdot m$, where k and $\tilde{k}$ correspond to light like momentum and its mirror image and m denotes linear M^4 coordinates: these MEs describe cylindrical structures with constant direction of wave propagation expected to be most important in regions faraway from the source of radiation.

2. Boundary conditions are satisfied if the 3-dimensional boundaries of MEs have one light like direction (S_+ or S_- is constant). This means that the boundary of ME has metric dimension $d = 2$ and is characterized by an infinite-dimensional super-symplectic and super-conformal symmetries just like the boundary of the imbedding space $M^4_+ \times CP_2$: The boundaries are like moments for mini big bangs (in TGD based fractal cosmology big bang is replaced with a silent whisper amplified to not necessarily so big bang).

3. These observations inspire the conjecture that boundary conditions for M^4 like space-time sheets fixed by the variational principle selecting preferred extremals of Kähler action quite generally require that space-time boundaries correspond to light like 3-surfaces with metric dimension equal to $d = 2$. This does not yet imply that light like surfaces of imbedding space would take the role of the light cone boundary: these light like surface could be seen only as a special case of causal determinants analogous to event horizons.

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