Cosmology and Astrophysics in Many-Sheeted Space-Time

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Abstract

This chapter is devoted to the applications of TGD to astrophysics and cosmology.

1. Many-sheeted cosmology

The many-sheeted space-time concept, the new view about the relationship between inertial and gravitational four-momenta, the basic properties of the paired cosmic strings, the existence of the limiting temperature, the assumption about the existence of the vapor phase dominated by cosmic strings, and quantum criticality imply a rather detailed picture of the cosmic evolution, which differs from that provided by the standard cosmology in several respects but has also strong resemblances with inflationary scenario.

It should be made clear that many-sheeted cosmology involves a vulnerable assumption. It is assumed that single-sheeted space-time surface is enough to model the cosmology. This need not to be the case. GRT limit of TGD is obtained by lumping together the sheets of many-sheeted space-time to a piece of Minkowski space and endowing it with an effective metric, which is sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Hence the proposed models make sense only if GRT limits allowing imbedding as a vacuum extremal of Kähler action have special physical role.

The most important differences are following.

1. Many-sheetedness implies cosmologies inside cosmologies Russian doll like structure with a spectrum of Hubble constants.
2. TGD cosmology is also genuinely quantal: each quantum jump in principle recreates each sub-cosmology in 4-dimensional sense: this makes possible a genuine evolution in cosmological length scales so that the use of anthropic principle to explain why fundamental constants are tuned for life is not necessary.
3. The new view about energy means provided by zero energy ontology (ZEO) means that the notion of energy and also other quantum numbers is length scale dependent. This allows to understand the apparent non-conservation of energy in cosmological scales although Poincare invariance is exact symmetry. In ZEO any cosmology is in principle creatable from vacuum and the problem of initial values of cosmology disappears. The density of matter near the initial moment is dominated by cosmic strings approaches to zero so that big bang is transformed to a silent whisper amplified to a relatively big bang.
4. Dark matter hierarchy with dynamical quantized Planck constant implies the presence of dark space-time sheets which differ from non-dark ones in that they define multiple coverings of M^4. Quantum coherence of dark matter in the length scale of space-time sheet involved implies that even in cosmological length scales Universe is more like a living organism than a thermal soup of particles.
5. Sub-critical and over-critical Robertson-Walker cosmologies are fixed completely from the imbeddability requirement apart from a single parameter characterizing the duration of the period after which transition to sub-critical cosmology necessarily occurs. The fluctuations of the microwave background reflect the quantum criticality of the critical period rather than amplification of primordial fluctuations by exponential expansion. This and also the finite size of the space-time sheets predicts deviations from the standard cosmology.

2. Cosmic strings

Cosmic strings belong to the basic extremals of the Kähler action. The string tension of the cosmic strings is $T \simeq .2 \times 10^{-6}/G$ and slightly smaller than the string tension of the GUT strings and this makes them very interesting cosmologically. Concerning the understanding of cosmic strings a decisive breakthrough came through the identification of gravitational four-momentum as the difference of inertial momenta associated with matter and antimatter and the realization that the net inertial energy of the Universe vanishes. This forced to conclude cosmological constant in TGD Universe is non-vanishing. p-Adic length fractality predicts that Λ scales as $1/L^2(k)$ as a function of the p-adic scale characterizing the space-time sheet. The recent value of the cosmological constant comes out correctly. The gravitational energy density described by the cosmological constant is identifiable as that associated with topologically condensed cosmic strings and of magnetic flux tubes to which they are gradually transformed during cosmological evolution.

p-Adic fractality and simple quantitative observations lead to the hypothesis that pairs of cosmic strings are responsible for the evolution of astrophysical structures in a very wide length scale range. Large voids with size of order 10^8 light years can be seen as structures containing knotted and linked cosmic string pairs wound around the boundaries of the void. Galaxies correspond to same structure with smaller size and linked around the supra-galactic strings. This conforms with the finding that galaxies tend to be grouped along linear structures. Simple quantitative estimates show that even stars and planets could be seen as structures formed around cosmic strings of appropriate size. Thus Universe could be seen as fractal cosmic necklace consisting of cosmic strings linked like pearls around longer cosmic strings linked like...

3. Dark matter and quantization of gravitational Planck constant

The notion of gravitational Planck constant having possibly gigantic values is perhaps the most radical idea related to the astrophysical applications of TGD. D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis comes from empirical data.

By Equivalence Principle and independence of the gravitational Compton length on particle mass m it is enough to assume g_{gr} only for flux tubes mediating interactions of microscopic objects with central mass M. In TGD framework h_{gr} relates to the hierarchy of Planck constants $h_{eff} = n \times h$ assumed to relate directly to the non-determinism and to the quantum criticality of Kähler action.

Dark matter can be identified as large h_{eff} phases at Kähler magnetic flux tubes and dark energy as the Kähler magnetic energy of these flux tubes carrying monopole magnetic fluxes. No currents are needed to create these magnetic fields, which explains the presence of magnetic fields in cosmological scales.

1 Introduction

This chapter is devoted to the applications of TGD to astrophysics and cosmology are discussed. It must be admitted that the development of the proper interpretation of the theory has been rather slow and involved rather weird twists motivated by conformist attitudes. Typically these attempts have brought into theory ad hoc identifications of say gravitational four-momentum although theory itself has from very beginning provided completely general formulas.

Perhaps the real problem has been that radically new views about ontology were necessary before it was possible to see what had been there all the time. Zero energy ontology (ZEO) states that all physical states have vanishing net quantum numbers. The hierarchy of dark matter identified as macroscopic quantum phases labeled by arbitrarily large values of Planck constant is second aspect of the new ontology.

1.1 Zero Energy Ontology

In zero energy ontology one replaces positive energy states with zero energy states with positive and negative energy parts of the state at the boundaries of future and past direct light-cones forming a causal diamond. All conserved quantum numbers of the positive and negative energy states are of opposite sign so that these states can be created from vacuum. "Any physical state is creatable from vacuum" becomes thus a basic principle of quantum TGD and together with the notion of quantum jump resolves several philosophical problems (What was the initial state of universe?, What are the values of conserved quantities for Universe, Is theory building completely useless if only single solution of field equations is realized?).

At the level of elementary particle physics positive and negative energy parts of zero energy state are interpreted as initial and final states of a particle reaction so that quantum states become physical events. Equivalence Principle would hold true in the sense that the classical gravitational four-momentum of the vacuum extremal whose small deformations appear as the argument of configuration space spinor field is equal to the positive energy of the positive energy part of the zero energy quantum state.

Robertson-Walker cosmologies correspond to vacua with respect to inertial energy and in fact with respect to all quantum numbers. They are not vacua with respect to gravitational charges defined as Noether charges associated with the curvature scalar. Also more general imbeddings of Einstein's equations are typically vacuum extremals with respect to Noether charges assignable to Kähler action since otherwise one ends up with conflict between imbeddability and dynamics. This suggests that physical states have vanishing net quantum numbers quite generally. The construction of quantum theory [K15, K7] indeed leads naturally to zero energy ontology stating that everything is creatable from vacuum.

Zero energy states decompose into positive and negative energy parts having identification as initial and final states of particle reaction in time scales of perception longer than the geometro-temporal separation T of positive and negative energy parts of the state. If the time scale of perception is smaller than T, the usual positive energy ontology applies.

In zero energy ontology inertial four-momentum is a quantity depending on the temporal time scale T used and in time scales longer than T the contribution of zero energy states with parameter $T_1 < T$ to four-momentum vanishes. This scale dependence alone implies that it does not make sense to speak about conservation of inertial four-momentum in cosmological scales. Hence it would be in principle possible to identify inertial and gravitational four-momenta and achieve strong form of Equivalence Principle. It however seems that this is not the correct approach to follow.

The the relationship between TGD and GRT was understood quite recently (2014). GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical form of Equivalence Principle (EP) for the GRT limit in long length scales at least expressed in terms of Einstein's equations in given resolution scale with space-time sheets with size smaller than resolution scale represented as external currents.

One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

The vacuum extremals are absolutely essential for the TGD based view about long length scale limit about gravitation. Effective GRT space time would be imbeddable as a vacuum extremal to H. This is just assumption albeit coming first in mind - especially so when one has not yet understood how GRT space-time emerges from TGD!

Already the Kähler action defined by CP_2 Kähler form J allows enormous vacuum degeneracy: any four-surface having Lagrangian sub-manifold of CP_2 as its CP_2 projection is a vacuum extremal. The dimension of these sub-manifolds is at most two. Robertson-Walker cosmologies correspond to vacua with respect to inertial energy and in fact with respect to all quantum numbers. They are not vacua with respect to gravitational charges defined as Noether charges associated with the curvature scalar. Also more general imbeddings of Einstein's equations are typically vacuum extremals with respect to Noether charges assignable to Kähler action since otherwise one ends up with conflict between imbeddability and dynamics. This suggests that physical states have vanishing net quantum numbers quite generally. The construction of quantum theory [K15, K7] indeed leads naturally to zero energy ontology stating that everything is creatable from vacuum.

In TGD framework topological field quantization leads to the hypothesis that quantum concepts should have geometric counterparts and also potential energy should have precise correlate at the level of description based on topological field quanta. This indeed seems to be the case. As already explained, TGD allows space-time sheets to have both positive and negative time orientations. This in turn implies that also the sign of energy can be also negative. This suggests that the generation of negative energy space-time sheets representing virtual gravitons together with energy conservation makes possible the generation of huge gravitationally induced kinetic energies and gravitational collapse. In this process inertial energy would be conserved since instead, of positive energy gravitons, the inertial energy would go to the energy of matter.

This picture has a direct correlate in quantum field theory where the exchange negative energy virtual bosons gives rise to the interaction potential. The gravitational red-shift of microwave background photons is the strongest support for the non-conservation of energy in General Relativity. In TGD it could have concrete explanation in terms of absorption of negative energy virtual gravitons by photons leading to gradual reduction of their energies. This explanation is consistent with the classical geometry based explanation of the red-shift based on the stretching of electromagnetic wave lengths. This explanation is also consistent with the intuition based on Feynman diagram description of gravitational acceleration in terms of graviton exchanges.

1.2 Dark Matter Hierarchy And Hierarchy Of Planck Constants

The idea about hierarchy of Planck constants relying on generalization of the imbedding space was inspired both by empirical input (Bohr quantization of planetary orbits and anomalies of biology) and by the mathematics of hyper-finite factors of type II_1 combined with the quantum classical correspondence. Consider first the mathematical structure in question.

1. The Clifford algebra of World of Classical Worlds (WCW) creating many fermion states is a standard example of an algebra expressible as a direct integral of copies of von Neumann algebras known as hyper-finite factor of type II_1 (HFFs). The inclusions of HFFs relate very intimately to the notion of finite measurement resolution. There is a canonical hierarchy of Jones inclusions [A1] labeled by finite subgroups of SU(2) [A4]. Quantum classical correspondence suggests that these inclusions have space-time correlates [K33, K13] and the generalization of imbedding space would provide these correlates.

2. The space $CD \times CP_2$, where $CD \subset M^4$ is so called causal diamond identified as the intersection of future and past directed light-cones defines the basic geometric structure in zero energy ontology. The positive (negative) energy part of the zero energy state is located to the lower (upper) light-like boundaries of $CD \times CP_2$ and has interpretation as the initial (final) state of the physical event in standard positive energy ontology. p-Adic length scale hypothesis follows if one assumes that the temporal distance between the tips of CD comes as an octave of fundamental time scale defined by the size of CP_2. The "world of classical worlds" (WCW) is union of sub-WCWs associated with spaces $CD \times CP_2$ with different locations in $M^4 \times CP_2$.

3. One can say that causal diamond CD and the space CP_2 appearing as factors in $CD \times CP_2$ forms the basic geometric structure in zero energy ontology, is replaced with a book like structure obtained by gluing together infinite number of singular coverings and factor spaces of CD *resp.* CP_2 together. The copies are glued together along a common "back" $M^2 \subset M^2$ of the book in the case of CD. In the case of CP_2 the most general option allows two backs corresponding to the two non-isometric geodesic spheres S^2_i, $i = I, II$, represented as sub-manifolds $\xi^1 = \overline{\xi}^2$ and $\xi^1 = \xi^2$ in complex coordinates transforming linearly under $U(2) \subset SU(3)$. Color rotations in CP_2 produce different choices of this pair.

4. The selection of S^2 and M^2 is an imbedding space correlate for the fixing of quantization axes and means symmetry breaking at the level of imbedding space geometry. WCW is union over all possible choices of CD and pairs of geodesic spheres so that at the level no symmetry breaking takes place. The points of M^2 and S^2 have a physical interpretation in terms of quantum criticality with respect to the phase transition changing Planck constant (leakage to another page of the book through the back of the book).

5. The pages of the singular coverings are characterized by finite subgroups G_a and G_b of $SU(2)$ and these groups act in covering or leave the points of factor space invariant. The pages are labeled by Planck constants $\hbar(CD) = n_a\hbar_0$ and $\hbar(CP_2) = n_b\hbar_0$, where n_a and n_b are integers characterizing the orders of maximal cyclic subgroups of G_a and G_b. For singular factor spaces one has $\hbar(CD) = \hbar_0/n_a$ and $\hbar(CP_2) = \hbar_0/n_b$. The observed Planck constant corresponds to $\hbar = (\hbar(CD)/\hbar(CP_2)) \times \hbar_0$. What is also important is that $(\hbar/\hbar_0)^2$ appears as a scaling factor of M^4 covariant metric so that Kähler action via its dependence on induced metric codes for radiative corrections coming in powers of ordinary Planck constant: therefore quantum criticality and vanishing of radiative corrections to functional integral over WCW does not mean vanishing of radiative corrections.

The interpretation in terms of dark matter comes as follows.

1. Large values of $\hbar$ make possible macroscopic quantum phase since all quantum scales are scaled upwards by $\hbar/\hbar_0$. Anyonic and charge fractionization effects allow to "measure" $\hbar(CD)$ and $\hbar(CP_2)$ rather than only their ratio. $\hbar(CD) = \hbar(CP_2) = \hbar_0$ corresponds to what might be called standard physics without any anyonic effects and visible matter is identified as this phase.

2. Particle states belonging to different pages of the book can interact via classical fields and by exchanging particles, such as photons, which leak between the pages of the book. This leakage means a scaling of frequency and wavelength in such a manner that energy and momentum of photon are conserved. Direct interactions in which particles from different pages appear in the same vertex of generalized Feynman diagram are impossible. This seems to be enough to explain what is known about dark matter. This picture differs in many respects from more conventional models of dark matter making much stronger assumptions and has far reaching implications for quantum biology, which also provides support for this view about dark matter.

This is the basic picture. One can imagine large number of speculative applications.

1. The number theoretically simple ruler-and-compass integers n having as factors only first powers of Fermat primes and power of 2 would define a physically preferred values of n_a and n_b and thus a sub-hierarchy of quantum criticality for which subsequent levels would correspond to powers of 2: a connection with p-adic length scale hypothesis suggests itself. Ruler and compass hypothesis implies that besides p-adic length scales also their 3- and 5-multiples should be important.

2. G_a could correspond directly to the observed symmetries of visible matter induced by the underlying dark matter if singular factor space is in question [K13]. For instance, in living matter molecules with 5- and 6-cycles could directly reflect the fact that free electron pairs associated with these cycles correspond to $n_a = 5$ and $n_a = 6$ dark matter possibly responsible for anomalous conductivity of DNA [K13, K6] and recently reported strange properties of graphene [D1]. Also the tetrahedral and icosahedral symmetries of water molecule clusters could have similar interpretation [K12]. [D2].

3. A further fascinating possibility is that the evidence for Bohr orbit quantization of planetary orbits [E14] could have interpretation in terms of gigantic Planck constant for underlying dark matter [K24] so that macroscopic and -temporal quantum coherence would be possible in astrophysical length scales manifesting itself in many manners: say as preferred directions of quantization axis (perhaps related to the CMB anomaly) or as anomalously low dissipation rates.

4. Since the gravitational Planck constant $\hbar_{gr} = GM_1m/v_0$, $v_0 = 2^{-11}$ for the inner planets, is proportional to the product of the gravitational masses of interacting systems, it must be assigned to the field body of the two systems and characterizes the interaction between systems rather than systems themselves. This observation applies quite generally and each field body of the system (em, weak, color, gravitational) is characterized by its own Planck constant.

1.3 Many-Sheeted Cosmology

The many-sheeted space-time concept, the new view about the relationship between inertial and gravitational four-momenta, the basic properties of the paired cosmic strings, the existence of the limiting temperature, the assumption about the existence of the vapor phase dominated by cosmic strings, and quantum criticality imply a rather detailed picture of the cosmic evolution, which differs from that provided by the standard cosmology in several respects but has also strong resemblances with inflationary scenario.

The most important differences are following.

1. Many-sheetedness implies cosmologies inside cosmologies Russian doll like structure with a spectrum of Hubble constants.

2. TGD cosmology is also genuinely quantal: each quantum jump in principle recreates each sub-cosmology in 4-dimensional sense: this makes possible a genuine evolution in cosmological length scales so that the use of anthropic principle to explain why fundamental constants are tuned for life is not necessary.

3. The new view about energy means that inertial energy is negative for space-time sheets with negative time orientation and that the density of inertial energy vanishes in cosmological length scales. Therefore any cosmology is in principle creatable from vacuum and the problem of initial values of cosmology disappears. The density of matter near the initial moment is dominated by cosmic strings approaches to zero so that big bang is transformed to a silent whisper amplified to a relatively big bang.

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1.4 Cosmic Strings

Cosmic strings belong to the basic extremals of the Kähler action. The string tension of the cosmic strings is $T \simeq .2 \times 10^{-6}/G$ and slightly smaller than the string tension of the GUT strings and this makes them very interesting cosmologically.

TGD predicts two basic types of strings.

1. The analogs of hadronic strings correspond to deformations of vacuum extremals carrying non-vanishing induced Kähler fields. p-Adic thermodynamics for super-symplectic quanta condensed on them with additivity of mass squared yields without further assumptions stringy mass formula. These strings are associated with various fractally scaled up variants of hadron physics.

2. Cosmic strings correspond to homologically non-trivial geodesic sphere of CP_2 (more generally to complex sub-manifolds of CP_2) and have a huge string tension. These strings are expected to have deformations with smaller string tension which look like magnetic flux tubes with finite thickness in M^4 degrees of freedom. The signature of these strings would be the homological non-triviality of the CP_2 projection of the transverse section of the string.

p-Adic fractality and simple quantitative observations lead to the hypothesis that pairs of cosmic strings are responsible for the evolution of astrophysical structures in a very wide length scale range. Large voids with size of order 10^8 light years can be seen as structures containing knotted and linked cosmic string pairs wound around the boundaries of the void. Galaxies correspond to same structure with smaller size and linked around the supra-galactic strings. This conforms with the finding that galaxies tend to be grouped along linear structures. Simple quantitative estimates show that even stars and planets could be seen as structures formed around cosmic strings of appropriate size. Thus Universe could be seen as fractal cosmic necklace consisting of cosmic strings linked like pearls around longer cosmic strings linked like...

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://tgdtheory.fi/cmaphtml.html` [L5]. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L6]. The topics relevant to this chapter are given by the following list.

- Classical TGD [L4]
- TGD inspired cosmology [L9]
- Astrophysics and TGD [L3]
- TGD and GRT [L8]
- Cosmic strings [L7]

2 Basic Principles Of General Relativity From TGD Point Of View

General Coordinate Invariance, Equivalence Principle are corner stones of general relativity and one expects that they hold true also in TGD some sense. The earlier attempts to understand the relationship between TGD and GRT have been in terms of solutions of Einstein's equations imbeddable to $M^4 \times CP_2$ instead of introducing GRT space-time as a fictive notion naturally emerging from TGD as a simplified concept replacing many-sheeted space-time. This resolves also the worries related to Equivalence Principle. TGD can be seen as a "microscopic" theory behind TGD and the understanding of the microscopic elements becomes the main focus of theoretical and hopefully also experimental work some day.

Objections against TGD have turned out to be the best route to the correct interpretation of the theory. A very general objection against TGD relies on the notion of induced gauge fields and metric implying extremely strong constraints between classical gauge fields for preferred extremals. These constraints cannot hold true for gauge fields in the usual sense. Also linear superposition is lost. The solution of the problem comes from simple observation: it is not fields which superpose but their effects on test particle topologically condensed to space-time sheets carrying the classical fields. Superposition is replaced with set theoretic union. This leads also naturally to explicit identification of the effective metric and gauge potentials defined in M^4 and defining GRT limit of TGD.

Finite length scale resolution is central notion in TGD and implies that the topological inhomogenities (space-time sheets and other topological inhomogenities) are treated as point-like objects and described in terms of energy momentum tensor of matter and various currents coupling to effective YM fields and effective metric important in length scales above the resolution scale. Einstein's equations with coupling to gauge fields and matter relate these currents to the Einstein tensor and metric tensor of the effective metric of M^4. The topological inhomogenities below cutoff scale serve determine the curvature of the effective metric.

The original proposal, which I called smoothed out space-time, took into account the topological inhomogenities but neglected many-sheetedness in length scales above resolution scale. I also identified the effective metric can be identified as induced metric: this is very strong assumption although the properties of vacuum extremals support this identification at least in some important special cases.

The attempts to understand Kähler-Dirac (or Kähler-Dirac-) action has provided very strong boost to the understanding of the basic problems related to GRT-TGD relationship, understanding of EP means at quantum level in TGD, and how the properties of induced electroweak gauge potentials can be consistent with what is known about electroweak interactions: for instance, if is far from clear how em charge can be well-defined for the modes of the induced spinor field and how the effective absence of weak bosons above weak scale is realized at classical level for Kähler-Dirac action.

2.1 General Coordinate Invariance

General Coordinate Invariance plays in the formulation of quantum TGD even deeper role than in that of GRT. Since the fundamental objects are 3-D surfaces, the construction of the geometry of the configuration space of 3-surfaces (the world of classical worlds, WCW) requires that the definition of the geometry assigns to a given 3-surface X^3 a unique space-time surface $X^4(X^3)$. This space-time surface is completely analogous to Bohr orbit, which means a completely unexpected connection with quantum theory.

General Coordinate Invariance is analogous to gauge symmetry and requires gauge fixing. The definition assigning $X^4(X^3)$ to given X^3 must be such that the outcome is same for all 4-diffeomorphs of X^3. This condition is highly non-trivial since $X^4(X^3) = X^4(Y^3)$ must hold true if X^3 and Y^3 are 4-diffeomorphs. One manner to satisfy this condition is by assuming quantum holography and weakened form of General Coordinate Invariance: there exists physically preferred 3-surfaces X^3 defining $X^4(X^3)$, and the 4-diffeomorphs Y^3 of X^3 at $X^4(X^3)$ provide classical holograms of X^3: $X^4(Y^3) = X^4(X^3)$ is trivially true. Zero energy ontology allows to realize this form of General Coordinate Invariance.

1. In ZEO WCW decomposes into a union of sub-WCWs associated with causal diamonds $CD \times CP_2$ (CD denotes the intersection of future and past directed light-cones of M^4), and the intersections of space-time surface with the light-light boundaries of $CD \times CP_2$ are excellent candidates for preferred space-like 3-surfaces X^3. The 3-surfaces at $\delta CD \times CP_2$ are indeed physically special since they carry the quantum numbers of positive and negative energy parts of the zero energy state.

2. Preferred 3-surfaces could be also identified as light-like 3-surfaces X_l^3 at which the Euclidian signature of the induced space-time metric changes to Minkowskian. Also light-like boundaries of X^4 can be considered. These 3-surfaces are assumed to carry elementary particle quantum numbers and their intersections with the space-like 3-surfaces X^3 are 2-dimensional partonic surfaces so that effective 2-dimensionality consistent with the conformal symmetries of X_l^3 results if the identifications of 3-surfaces are physically equivalent. Light-like 3-surfaces are identified as generalized Feynman diagrams and due to the presence of 2-D partonic 2-surfaces representing vertices fail to be 3-manifolds. Generalized Feynman diagrams could be also identified as Euclidian regions of space-time surface.

3. General Coordinate Invariance in minimal form requires that the slicing of $X^4(X_l^3)$ by light light 3-surfaces Y_l^3 "parallel" to X_l^3 predicted by number theoretic compactification gives rise to quantum holography in the sense that the data associated with any Y_l^3 allows an equivalent formulation of quantum TGD. This poses a strong condition on the spectra of the Kähler-Dirac operator at Y_l^3 and thus to the preferred extremals of Kähler action since the WCW Kähler functions defined by various choices of Y_l^3 can differ only by a sum of a holomorphic function and its conjugate [K34, K7] .

2.2 The Basic Objection Against TGD

The basic objection against TGD is that induced metrics for space-time surfaces in $M^4 \times CP_2$ form an extremely limited set in the space of all space-time metrics appearing in the path integral formulation of General Relativity. Even special metrics like the metric of a rotating black hole fail to be imbeddable as an induced metric. For instance, one can argue that TGD cannot reproduce the post-Newtonian approximation to General Relativity since it involves linear superposition of gravitational fields of massive objects. As a matter fact, Holger B. Nielsen- one of the very few colleagues who has shown interest in my work - made this objection for at least two decades ago in some conference and I remember vividly the discussion in which I tried to defend TGD with my poor English.

The objection generalizes also to induced gauge fields expressible solely in terms of CP_2 coordinates and their gradients. This argument is not so strong as one might think first since in standard model only classical electromagnetic field plays an important role.

1. Any electromagnetic gauge potential has in principle a local imbedding in some region. Preferred extremal property poses strong additional constraints and the linear superposition of massless modes possible in Maxwell's electrodynamics is not possible.

2. There are also global constraints leading to topological quantization playing a central role in the interpretation of TGD and leads to the notions of field body and magnetic body having non-trivial application even in non-perturbative hadron physics. For a very large class of

preferred extremals space-time sheets decompose into regions having interpretation as geometric counterparts for massless quanta characterized by local polarization and momentum directions. Therefore it seems that TGD space-time is very quantal. Is it possible to obtain from TGD what we have used to call classical physics at all?

The imbeddability constraint has actually highly desirable implications in cosmology. The enormously tight constraints from imbeddability imply that imbeddable Robertson-Walker cosmologies with infinite duration are sub-critical so that the most pressing problem of General Relativity disappears. Critical and over-critical cosmologies are unique apart from a parameter characterizing their duration and critical cosmology replaces both inflationary cosmology and cosmology characterized by accelerating expansion. In inflationary theories the situation is just the opposite of this: one ends up with fine tuning of inflaton potential in order to obtain recent day cosmology.

Despite these and many other nice implications of the induced field concept and of sub-manifold gravity the basic question remains. Is the imbeddability condition too strong physically? What about linear superposition of fields which is exact for Maxwell's electrodynamics in vacuum and a good approximation central also in gauge theories. Can one obtain linear superposition in some sense?

1. Linear superposition for small deformations of gauge fields makes sense also in TGD but for space-time sheets the field variables would be the deformations of CP_2 coordinates which are scalar fields. One could use preferred complex coordinates determined about SU(3) rotation to do perturbation theory but the idea about perturbations of metric and gauge fields would be lost. This does not look promising. Could linear superposition for fields be replaced with something more general but physically equivalent?

2. This is indeed possible. The basic observation is utterly simple: what we know is that the *effects* of gauge fields superpose. The assumption that fields superpose is un-necessary! This is a highly non-trivial lesson in what operationalism means for theoreticians tending to take these kind of considerations as mere "philosophy".

3. The hypothesis is that the superposition of effects of gauge fields occurs when the M^4 projections of space-time sheets carrying gauge and gravitational fields intersect so that the sheets are extremely near to each other and can touch each other (CP_2 size is the relevant scale).

A more detailed formulation goes as follows.

1. One can introduce common M^4 coordinates for the space-time sheets. A test particle (or real particle) is identifiable as a wormhole contact and is therefore point-like in excellent approximation. In the intersection region for M^4 projections of space-time sheets the particle forms topological sum contacts with all the space-time sheets for which M^4 projections intersect.

2. The test particle experiences the sum of various gauge potentials of space-time sheets involved. For Maxwellian gauge fields linear superposition is obtained. For non-Abelian gauge fields gauge fields contain interaction terms between gauge potentials associated with different space-time sheets. Also the quantum generalization is obvious. The sum of the fields induces quantum transitions for states of individual space time sheets in some sense stationary in their internal gauge potentials.

3. The linear superposition applies also in the case of gravitation. The induced metric for each space-time sheet can be expressed as a sum of Minkowski metric and CP_2 part having interpretation as gravitational field. The natural hypothesis that in the above kind of situation the effective metric is sum of Minkowski metric with the sum of the CP_2 contributions from various sheets. The effective metric for the system is well-defined and one can calculate a curvature tensor for it among other things and it contains naturally the interaction terms between different space-time sheets. At the Newtonian limit one obtains linear superposition of gravitational potentials. One can also postulate that test particles moving along geodesics in the effective metric. These geodesics are not geodesics in the metrics of the space-time sheets.

4. This picture makes it possible to interpret classical physics as the physics based on effective gauge and gravitational fields and applying in the regions where there are many space-time sheets which M^4 intersections are non-empty. The loss of quantum coherence would be due to the effective superposition of very many modes having random phases.

The effective superposition of the CP_2 parts of the induced metrics gives rise to an effective metric which is not in general imbeddable to $M^4 \times CP_2$. Therefore many-sheeted space-time makes possible a rather wide repertoire of 4-metrics realized as effective metrics as one might have expected and the basic objection can be circumvented In asymptotic regions where one can expect single sheetedness, only a rather narrow repertoire of "archetypal" field patterns of gauge fields and gravitational fields defined by topological field quanta is possible.

The skeptic can argue that this still need not make possible the imbedding of a rotating black hole metric as induced metric in any physically natural manner. This might be the case but need of course not be a catastrophe. We do not really know whether rotating blackhole metric is realized in Nature. I have indeed proposed that TGD predicts new physics new physics in rotating systems. Unfortunately, gravity probe B could not check whether this new physics is there since it was located at equator where the new effects vanish.

2.3 How GRT And Equivalence Principle Emerge From TGD?

The question how TGD relates to General Relativity Theory (GRT) has been a rich source of problems during last 37 years. In the light of after-wisdom the problems have been due to my too limited perspective. I have tried to understand GRT limit in the TGD framework instead of introducing GRT space-time as a fictive notion naturally emerging from TGD as a simplified concept replacing many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. ??** in the appendix of this book) . This resolves also the worries related to Equivalence Principle.

TGD itself gains the status of "microscopic" theory of gravity and the experimental challenges relate to how make the microscopy of gravitation experimentally visible. This involves questions such as "How to make the presence of Euclidian space-time regions visible?",

> How to reveal many-sheeted character of space-time, topological field quantization, and the presence of magnetic flux tubes?,"How to reveal quantum gravity as understood in TGD involving in an essential manner gravitational Planck constant h_{gr} identifiable as h_{eff} inspired by anomalies of bio-electromagnetism?

[K22].

More technical questions relate to the Kähler-Dirac action, in particular to how conservation laws are realized. During all these years several questions have been lurking at the boarder of conscious and sub-conscious. How can one guarantee that em charge is well-defined for the spinor modes when classical W fields are present? How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induced vielbein rotation of weak fields? The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string picture a part of TGD.

2.3.1 *TGD and GRT*

Concerning GRT limit the basic questions are the following ones.

1. Is it really possible to obtain a realistic theory of gravitation if general space-time metric is replaced with induced metric depending on 8 imbedding space coordinates (actually only 4 by general coordinate invariance?

2. What happens to Einstein equations?

3. What about breaking of Poincare invariance, which seems to be real in cosmological scales? Can TGD cope with it?

4. What about Equivalence Principle (EP)

5. Can one predict the value of gravitational constant?
6. What about TGD counterpart of blackhole, which certainly represents the boundary of realm in which GRT applies?

Consider first possible answers to the first three questions.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig.** `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg` or **??** in the appendix of this book).
2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric in standard coordinates for the space-time sheets. One could replace flat metric of M^4 with effective metric as sum of metric and deviations associated with various space-time sheets "above" the M^4 point. This effective metric of M^4 regarded as independent space would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD. Also standard model gauge potentials can be defined as effective fields in the same manner and one expects that classical electroweak fields vanish in the length scales above weak scale.
3. This picture brings in mind the old intuitive notion of smoothed out quantum average space-time thought to be realized as surface in $M^4 \times CP_2$ rather than in terms of averages metric and gauge potentials in M^4. The problem of this approach was that it was not possible to imagine any quantitative recipe for the averaging and this was essentially dur to the sub-manifold assumption.
4. One could generalize this picture and consider effective metrics for CP_2 and $M^2 \times CP_2$ corresponding to CP_2 type vacuum extremals describing elementary particles and cosmic strings respectively.
5. Einstein's equations could hold true for the effective metric. The vanishing of the covariant divergence of energy momentum tensor would be a remnant of Poincare invariance actually still present in the sense of Zero Energy Ontology (ZEO) but having realization as global conservation laws.
6. The breaking of Poincare invariance at the level of effective metric could have interpretation as effective breaking due to zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

The following considerations are about answers to the fourth and fifth questions.

1. EP at classical level would hold true in local sense if Einstein's equations hold true for the effective metric. Underlying Poincare invariance suggests local covariant conservation laws.
2. The value of gravitational constant is in principle a prediction of theory containing only radius as fundamental scale and Kähler coupling strength as only coupling constant analogous to critical temperature. In GRT inspired quantum theory of gravitation Planck length scale given by $L_P = \sqrt{\hbar_{eff} \times G}$ is the fundamental length scale. In TGD size R defines it and it is independent of h_{eff}. The prediction for gravitational constant is prediction for the TGD counterpart of L_P: $L_P^2 = R^2/n$, n dimensionless constant. The prediction for G would be $G = R^2/(n \times \hbar_{eff})$ or $G = R^2/(n \times \hbar_{eff,min})$. The latter option is the natural one.

Interesting questions relate to the fate of blackholes in TGD framework.

1. Blackhole metric as such is quite possible as effective metric since there is no need to imbed it to imbedding space. One could however argue that blackhole metric is so simple that it must be realizable as single-sheeted space-time surface. This is indeed possible above some radius which can be smaller than Schwarschild radius. This is due to the compactness of CP_2 . A general result is that the embedding carriers non-vanishing gauge charge say em charge. This need not have physical significance if the metric of GRT corresponds to the effective metric obtained by the proposed recipe.

2. TGD forces to challenge the standard view about black holes. For instance, could it be that blackhole interior corresponds microscopically to Euclidian space time regions? For these CP_2 endowed with effective metric would be appropriate GRT type description. Reissner-Nordström metric with cosmological constant indeed allows CP_2 as solution [K30]. M^4 region and CP_2 region would be joined along boundaries at which determinant of four-metric vanishes. If the radial component of R-N metric is required to be finite, one indeed obtains metric with vanishing determinant at horizon and it is natural to assume that the metric inside is Euclidian. Similar picture would applied to the cosmic strings as spaces $M^2 \times S^2$ with effective metric.

3. Could holography hold true in the sense that blackhole horizon is replaced with a partonic 2-surface with astrophysical size and having light-like orbit as also black-hole horizon has.

4. The notion of gravitational Planck constant $\hbar_{gr} = GMm/v_0$, where v_0 is typical rotation velocity in the system consisting of masses M and m, has been one of the speculative aspects of TGD. h_{gr} would be assigned with "gravitational" magnetic flux tube connecting the systems in question and it has turned out that the identification $h_{gr} = h_{eff}$ makes sense in particle length scales. The gravitational Compton length is universal and given $\lambda_{gr} = GM/v_0$. This strongly suggests that quantum gravity becomes important already above Schwarschild radius $r_S = 2GMm$. The critical velocity at which gravitational Compton length becomes smaller than r_S is $v_0/c = 1/\sqrt{2}$. All astrophysical objects would be genuinely quantal objects in TGD Universe point and blackholes would lose their unique role. An experimental support for these findings comes from experiments of Tajmar et al [E20, E26] [K22].

For few ago entropic gravity [**?**, **?**] was a buzzword in blogs. The idea was that gravity would have a purely thermodynamical origin. I have commented the notion of entropic gravity from the point of view of TGD earlier [K30].

The basic objection is standard QM against the entropic gravity is that gravitational interaction of neutrons with Earth's gravitational field is describable by Schrödinger equation and this does not fit with thermodynamical description.

Although the idea as such does not look promising TGD indeed suggests that the correlates for thermodynamical quantities at space-time level make sense in ZEO leading to the view that quantum TGD is square root of thermodynamics.

Th interesting question is whether temperature has space-time correlate.

1. In Zero Energy Ontology quantum theory can be seen as a square root of thermodynamics formally and this raises the question whether ordinary temperature could parametrize wave functions having interpretation as square roots of thermal distributions in ZEO. The quantum model for cell membrane [K11] having the usual thermodynamical model as limit gives support for this idea. If this were the case, temperature would have by quantum classical correspondence direct space-time correlate.

2. A less radical view is that temperature can be assigned with the effective space-time metric only. The effective metric associated with M^4 defining GRT limit of TGD is defined statistically in terms of metric of many-sheeted space-time and would naturally contain in its geometry thermodynamical parameters. The averaging over the WCW spinors fields involving integral over 3-surfaces is also involved.

2.3.2 *Equivalence Principle*

Equivalence Principle has several interpretations.

1. The global form form of Equivalence Principle (EP) realized in Newtonian gravity states that inertial mass = gravitational mass (mass is replaces with four-momentum in the possible relativistic generalization). This form does not make sense in general relativity since four-momentum is not well-defined: this problem is the starting point TGD.

2. The local form of EP can be expressed in terms of Einstein's equations. Local covariant conservation law does not imply global conservation law since energy momentum tensor is

indeed tensor. One can try to define gravitational mass as something making sense in special cases. The basic problem is that there is no unique identification of empty space Minkowski coordinates. Gravitational mass could be identified as a a parameter appearing in asymptotic expression of solutions of Einstein's equations.

In TGD framework EP need not be problem of principle.

1. In TGD gravitational interaction couples to inertial four-momentum, which is well-defined as classical Noether charge associated with Kähler action. The very close analogy of TGD with string models suggest the same.

2. Only if one assumes that gravitational and inertial exist separately and are forced to be identical, one ends up with potential problems in TGD. This procedure might have sound physical basis in TGD but one should identify it in convincing manner.

3. In cosmology mass is not conserved, which in positive energy ontology would suggests breaking of Poincare invariance. In Zero Energy Ontology (ZEO) this is not the case. The conserved four-momentum assignable to either positive or negative energy part of the states in the basis of zero energy states depends on the scale of causal diamond (CD). Note that in ZEO zero energy states can be also superpositions of states with different four-momenta and even fermion numbers as in case of coherent state formed by Cooper pairs.

Consider now EP in quantum TGD.

1. Inertial momentum is defined as Noether charge for Kähler action.

2. One can assign to Kähler-Dirac action quantal four-momentum (I will use "Kähler-Dirac" instead of "modified" used in earlier work) [K34]. Its conservation is however not at all all trivial since imbedding space coordinates appear in KD action like external fields. It however seems that at least for the modes localized at string world sheets the four-momentum conservation could be guaranteed by an assumption motivated by holomorphy [K34]. The assumption states that the variation of holomorphic/antiholomorphic Kähler-Dirac gamma matrices induced by isometry is superposition of K-D gamma matrices of same type.

3. Quantum Classical Correspondence (QCC) suggests that the eigenvalues of quantal four-momentum are equal to those of Kähler four-momentum. If this is the case, QCC would imply EP and force conservation of antal four-momenta even if the assumption about variations of gamma matrices fails! This could be realized in terms of Lagrange multiplier terms added to Kähler action and localized at the ends of CD and analogous to constraint terms in ordinary thermodynamics.

4. QCC generalizes to Cartan sub-algebra of symmetries and would give a correlation between geometry of space-time sheet and conserved quantum numbers. One can consider even stronger form of QCC stating that classical correlation functions at space-time surface are same as the quantal once.

The understanding of EP at classical level has been a long standing head-ache in TGD framework. What seems to be the eventual solution looks disappointingly trivial in the sense that its discovery requires only some common sense.

The trivial but important observation is that the GRT limit of TGD does *not* require that the space-times of GRT limit are imbeddable to the imbedding space $M^4 \times CP_2$. The most elegant understanding of EP at classical level relies on following argument suggesting how GRT space-time emerges from TGD as an effective notion.

1. Particle experiences the sum of the effects caused by gravitational forces. The linear superposition for gravitational fields is replaced with the sum of effects describable in terms of effective metric in GRT framework. Hence it is natural to identify the metric of the effective space-time as the sum of M^4 metric and the deviations of various space-time sheets to which particle has topological sum contacts. This metric is defined for the M^4 serving as coordinate space and is not in general expressible as induced metric.

2. Underlying Poincare invariance is not lost but global conservation laws are lost for the effective space-time. A natural assumption is that that global energy-momentum conservation translates to the vanishing of covariant divergence of energy momentum tensor.

3. By standard argument this implies Einstein's equations with cosmological constant Λ: this at least in statistical sense. Λ would parametrize the presence of topologically condensed magnetic flux tubes. Both gravitational constant and cosmological constant would come out as predictions.

This picture is in principle all that is needed. TGD is in this framework a "microscopic" theory of gravitation and GRT describes statistically the many-sheetedness in terms of single sheeted space-time identified as M^4 as manifold. All notions related to many-sheeted space-time - such as cosmic strings, magnetic flux tubes, generalized Feynman diagrams representing deviations from GRT. The theoretical and experimental challenge is discover what these deviations are and how to make them experimentally visible.

One can of course ask whether EP or something akin to it could be realized for preferred extremals of Kähler action.

1. In cosmological and astrophysical models vacuum extremals play a key role. Could small deformations of them provide realistic enough models for astrophysical and cosmological scales in statistical sense?

2. Could preferred extremals satisfy something akin to Einstein's equations? Maybe! The mere condition that the covariant divergence of energy momentum tensor for Kähler action vanishes, is satisfied if Einsteins equations with cosmological terms are satisfied. One can however consider also argue that this condition can be satisfied also in other manners. For instance, four-momentum currents associated with them be given by Einstein's equations involving several cosmological "constants". The vanishing of covariant divergence would however give a justification for why energy momentum tensor is locally conserved for the effective metric and thus gives rise to Einstein's equations.

2.3.3 *EP as quantum classical correspondence*

Quite recently I returned to an old question concerning the meaning of Equivalence Principle (EP) in TGD framework.

Heretic would of course ask whether the question about whether EP is true or not is a pseudo problem due to uncritical assumption there really are two different four-momenta which must be identified. If even the identification of these two different momenta is difficult, the pondering of this kind of problem might be waste of time.

At operational level EP means that the scattering amplitudes mediated by graviton exchange are proportional to the product of four-momenta of particles and that the proportionality constant does not depend on any other parameters characterizing particle (except spin). The are excellent reasons to expect that the stringy picture for interactions predicts this.

1. The old idea is that EP reduces to the coset construction for Super Virasoro algebra using the algebras associated with G and H. The four-momenta assignable to these algebras would be identical from the condition that the differences of the generators annihilate physical states and identifiable as inertial and gravitational momenta. The objection is that for the preferred 3-surface H by definition acts trivially so that time-like translations leading out from the boundary of CD cannot be contained by H unlike G. Hence four-momentum is not associated with the Super-Virasoro representations assignable to H and the idea about assigning EP to coset representations does not look promising.

2. Another possibility is that EP corresponds to quantum classical correspondence (QCC) stating that the classical momentum assignable to Kähler action is identical with gravitational momentum assignable to Super Virasoro representations. This view might be equivalent with coset space view. This forced to reconsider the questions about the precise identification of the Kac-Moody algebra and about how to obtain the magic five tensor factors required by p-adic mass calculations [K30].

A more precise formulation for EP as QCC comes from the observation that one indeed obtains two four-momenta in TGD approach. The classical four-momentum assignable to the Kähler action and that assignable to the Kähler-Dirac action. This four-momentum is an operator and QCC would state that given eigenvalue of this operator must be equal to the value of classical four-momentum for the space-time surfaces assignable to the zero energy state in question. In this form EP would be highly non-trivial. It would be justified by the Abelian character of four-momentum so that all momentum components are well-defined also quantum mechanically. One can also consider the splitting of four-momentum to longitudinal and transversal parts as done in the parton model for hadrons: this kind of splitting would be very natural at the boundary of CD. The objection is that this correspondence is nothing more than QCC.

3. A further possibility is that duality of light-like 3-surfaces and space-like 3-surfaces holds true. This is the case if the action of symplectic algebra can be defined at light-like 3-surfaces or even for the entire space-time surfaces. This could be achieved by parallel translation of light-cone boundary providing slicing of CD. The four-momenta associated with the two representations of super-symplectic algebra would be naturally identical and the interpretation would be in terms of EP.

2.4 The Recent View About Kähler-Dirac Action

The understanding of Kähler-Dirac action and equation have provided very strong boost to the understanding of the basic problems related to GRT-TGD relationship, understanding of howe EP means at quantum level in TGD, and how the properties of induced electroweak gauge potentials can be consistent with what is known about electroweak interactions.

The understanding of Kähler Dirac action has been second long term project. How can one guarantee that em charge is well-defined for the spinor modes when classical W fields are present? How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induced vielbein rotation of weak fields? The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string picture a part of TGD.

2.4.1 *Kähler-Dirac action*

2.5 Kähler-Dirac Action

2.5.1 *Kähler-Dirac equation*

2.6 Kähler-Dirac Equation In The Interior Of Space-Time Surface

The solution of K-D equation at string world sheets is very much analogous to that in string models and holomorphy (actually, its Minkowskian counterpart) plays a key role. Note however the K-D gamma matrices might not necessarily define effective metric with Minkowskian signature even for string world sheets. Second point to notice is that one can consider also solutions restricted to partonic 2-surfaces. Physical intuition suggests that they are very important because wormhole throats carry particle quantum numbers and because wormhole contacts mediat the interaction between space-time sheets. Whether partonic 2-surfaces are somehow dual to string world sheets remains an open question.

1. Conformal invariance/its Minkowskian variant based on hyper-complex numbers realized at string world sheets suggests a general solution of Kähler-Dirac equation. The solution ansatz is essentially similar to that in string models.

2. Second half of complexified Kähler-Dirac gamma matrices annihilates the spinors which are either holomorphic or anti-holomorphic functions of complex (hyper-complex) coordinate.

3. What about possible modes delocalized into entire 4-D space-time sheet possible if there are preferred extremals for which induced gauge field has only em part. What suggests itself

is global slicing by string world sheets and obtain the solutions as integrals over localized modes over the slices.

The understanding of symmetries (isometries of imbedding space) of K-D equation has turned out to be highly non-trivial challenge. The problem is that imbedding space coordinates appear in the role of external fields in K-D equation. One cannot require the vanishing of the variations of the K-D action with respect to the imbedding space-time coordinates since the action itself is second quantized object. Is it possible to have conservation laws associated with the imbedding space isometries?

1. Quantum classical correspondence (QCC) suggests the conserved Noether charges for Kähler action are equal to the eigenvalues of the Noether charges for Kähler-Dirac action. The quantal charge conservation would be forced by hand. This condition would realize also Equivalence Principle.

2. Second possibility is that the current following from the vanishing of second variation of Kähler action and the modification of Kähler gamma matrices defined by the deformation are linear combinations of holomorphic or anti-holomorphic gammas just like the gamma matrix itself so that K-D remains true. Conformal symmetry would therefore play a fundamental role. Isometry currents would be conserved although variations with respect to imbedding space coordinates would not vanish in general.

3. The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see **Fig. ??** also in the Appendix).

2.7 Boundary Terms For Kähler-Dirac Action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naive guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

2.7.1 *What one wants?*

It is could to make first clear what one really wants.

1. What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$$

at the space-like ends of space-time surface. This condition makes sense also at partonic orbits although they are not boundaries in the usual sense of the word. Here however delicacies since g_4 vanishes at them. The localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

The general idea is that the space-time geometry near the fermion line would *define* the four-momentum propagating along the line and quantum classical correspondence would be realized. The integral over four-momenta would be included to the functional integral over 3-surfaces.

The basic condition is that $\sqrt{g_4}\Gamma^n$ is constant at the boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant.

2. If p^k is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

3. One can wonder what the spectrum of p_k could be. If the identification as virtual momenta is correct, continuous mass spectrum suggests itself. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation. Note however that the string curves along the space-like ends of space-time surface are also internal lines and expected to carry virtual momentum: classical picture suggests that p^k tends to be space-like.

2.7.2 *Chern-Simons Dirac action from mathematical consistency*

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of imbedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \tag{2.1}$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = \gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. If Chern-Simons term is present one obtains also Chern-Simons term in this condition but also now fermion would be massless in global sense. The physical interpretation would be as incoming massless fermions.

2.8 About The Notion Of Four-Momentum In TGD Framework

The starting point of TGD was the energy problem of General Relativity [K30]. The solution of the problem was proposed in terms of sub-manifold gravity and based on the lifting of the isometries of space-time surface to those of $M^4 \times CP_2$ in which space-times are realized as 4-surfaces so that Poincare transformations act on space-time surface as an 4-D analog of rigid body rather than moving points at space-time surface. It however turned out that the situation is not at all so simple.

There are several conceptual hurdles and I have considered several solutions for them. The basic source of problems has been Equivalence Principle (EP): what does EP mean in TGD framework [K30, K38]? A related problem has been the interpretation of gravitational and inertial masses, or more generally the corresponding 4-momenta. In General Relativity based cosmology gravitational mass is not conserved and this seems to be in conflict with the conservation of Noether charges. The resolution is in terms of ZEO (ZEO), which however forces to modify slightly the original view about the action of Poincare transformations.

A further problem has been quantum classical correspondence (QCC): are quantal four-momenta associated with super conformal representations and classical four-momenta associated as Noether charges with Kähler action for preferred extremals identical? Could inertial-gravitational duality - that is EP - be actually equivalent with QCC? Or are EP and QCC independent dualities. A powerful experimental input comes p-adic mass calculations [K37] giving excellent predictions provided the number of tensor factors of super-Virasoro representations is five, and this input together with Occam's razor strongly favors QCC=EP identification.

There is also the question about classical realization of EP and more generally, TGD-GRT correspondence.

Twistor Grassmannian approach has meant a technical revolution in quantum field theory (for attempts to understand and generalize the approach in TGD framework see [K28]. This approach seems to be extremely well suited to TGD and I have considered a generalization of this approach from $\mathcal{N} = 4$ SUSY to TGD framework by replacing point like particles with string world sheets in TGD sense and super-conformal algebra with its TGD version: the fundamental objects are now massless fermions which can be regarded as on mass shell particles also in internal lines (but with unphysical helicity). The approach solves old problems related to the realization of stringy amplitudes in TGD framework, and avoids some problems of twistorial QFT (IR divergences and the problems due to non-planar diagrams). The Yangian variant of 4-D conformal symmetry is crucial for the approach in $\mathcal{N} = 4$ SUSY, and implies the recently introduced notion of amplituhedron [?]. A Yangian generalization of various super-conformal algebras seems more or less a "must" in TGD framework. As a consequence, four-momentum is expected to have characteristic multilocal contributions identifiable as multipart on contributions now and possibly relevant for the understanding of bound states such as hadrons.

2.8.1 *Scale dependent notion of four-momentum in zero energy ontology*

Quite generally, General Relativity does not allow to identify four-momentum as Noether charges but in GRT based cosmology one can speak of non-conserved mass [K25], which seems to be in conflict with the conservation of four-momentum in TGD framework. The solution of the problem comes in terms of ZEO (ZEO) [K4, K36], which transforms four-momentum to a scale dependent notion: to each causal diamond (CD) one can assign four-momentum assigned with say positive energy part of the quantum state defined as a quantum superposition of 4-surfaces inside CD.

ZEO is necessary also for the fusion of real and various p-adic physics to single coherent whole. ZEO also allows maximal "free will" in quantum jump since every zero energy state can be created from vacuum and at the same time allows consistency with the conservation laws. ZEO has rather dramatic implications: in particular the arrow of thermodynamical time is predicted to vary so that second law must be generalized. This has especially important implications in living matter, where this kind of variation is observed.

More precisely, this superposition corresponds to a spinor field in the "world of classical worlds" (WCW) [K36]: its components - WCW spinors - correspond to elements of fermionic Fock basis for a given 4-surface - or by holography implied by general coordinate invariance (GCI) - for 3-surface having components at both ends of CD. Strong form of GGI implies strong form of holography

(SH) so that partonic 2-surfaces at the ends of space-time surface plus their 4-D tangent space data are enough to fix the quantum state. The classical dynamics in the interior is necessary for the translation of the outcomes of quantum measurements to the language of physics based on classical fields, which in turn is reduced to sub-manifold geometry in the extension of the geometrization program of physics provided by TGD.

Holography is very much reminiscent of QCC suggesting trinity: GCI-holography-QCC. Strong form of holography has strongly stringy flavor: string world sheets connecting the wormhole throats appearing as basic building bricks of particles emerge from the dynamics of induced spinor fields if one requires that the fermionic mode carries well-defined electromagnetic charge [K34].

2.8.2 *Are the classical and quantal four-momenta identical?*

One key question concerns the classical and quantum counterparts of four-momentum. In TGD framework classical theory is an exact part of quantum theory. Classical four-momentum corresponds to Noether charge for preferred extremals of Kähler action. Quantal four-momentum in turn is assigned with the quantum superposition of space-time sheets assigned with CD - actually WCW spinor field analogous to ordinary spinor field carrying fermionic degrees of freedom as analogs of spin. Quantal four-momentum emerges just as it does in super string models - that is as a parameter associated with the representations of super-conformal algebras. The precise action of translations in the representation remains poorly specified. Note that quantal four-momentum does not emerge as Noether charge: at at least it is not at all obvious that this could be the case.

Are these classical and quantal four-momenta identical as QCC would suggest? If so, the Noether four-momentum should be same for all space-time surfaces in the superposition. QCC suggests that also the classical correlation functions for various general coordinate invariant local quantities are same as corresponding quantal correlation functions and thus same for all 4-surfaces in quantum superposition - this at least in the measurement resolution used. This would be an extremely powerful constraint on the quantum states and to a high extend could determined the U-, M-, and S-matrices.

QCC seems to be more or less equivalent with SH stating that in some respects the descriptions based on classical physics defined by Kähler action in the interior of space-time surface and the quantal description in terms of quantum states assignable to the intersections of space-like 3-surfaces at the boundaries of CD and light-like 3-surfaces at which the signature of induced metric changes. SH means effective 2-dimensionality since the four-dimensional tangent space data at partonic 2-surfaces matters. SH could be interpreted as Kac-Mody and symplectic symmetries meaning that apart from central extension they act almost like gauge symmetries in the interiors of space-like 3-surfaces at the ends of CD and in the interiors of light-like 3-surfaces representing orbits of partonic 2-surfaces. Gauge conditions are replaced with Super Virasoro conditions. The word "almost" is of course extremely important.

2.8.3 *What Equivalence Principle (EP) means in quantum TGD?*

EP states the equivalence of gravitational and inertial masses in Newtonian theory. A possible generalization would be equivalence of gravitational and inertial four-momenta. In GRT this correspondence cannot be realized in mathematically rigorous manner since these notions are poorly defined and EP reduces to a purely local statement in terms of Einstein's equations.

What about TGD? What could EP mean in TGD framework?

1. Is EP realized at both quantum and space-time level? This option requires the identification of inertial and gravitational four-momenta at both quantum and classical level. It is now clear that at classical level EP follows from very simple assumption that GRT space-time is obtained by lumping together the space-time sheets of the many-sheeted space-time and by the identification the effective metric as sum of M^4 metric and deviations of the induced metrics of space-time sheets from M^2 metric: the deviations indeed define the gravitational field defined by multiply topologically condensed test particle. Similar description applies to gauge fields. EP as expressed by Einstein's equations would follow from Poincare invariance at microscopic level defined by TGD space-time. The effective fields have as sources the energy momentum tensor and YM currents defined by topological inhomogenities smaller than the resolution scale.

2. QCC would require the identification of quantal and classical counterparts of both gravitational and inertial four-momenta. This would give three independent equivalences, say $P_{I,class} = P_{I,quant}$, $P_{gr,class} = P_{gr,quant}$, $P_{gr,class} = P_{I,quant}$, which imply the remaining ones.

   Consider the condition $P_{gr,class} = P_{I,class}$. At classical level the condition that the standard energy momentum tensor associated with Kähler action has a vanishing divergence is guaranteed if Einstein's equations with cosmological term are satisfied. If preferred extremals satisfy this condition they are constant curvature spaces for non-vanishing cosmological constant. A more general solution ansatz involves several functions analogous to cosmological constant corresponding to the decomposition of energy momentum tensor to terms proportional to Einstein tensor and several lower-dimensional projection operators [K38]. It must be emphasized that field equations are extremely non-linear and one must also consider preferred extremals (which could be identified in terms of space-time regions having so called Hamilton-Jacobi structure): hence these proposals are guesses motivated by what is known about exact solutions of field equations.

   Consider next $P_{gr,class} = P_{I,class}$. At quantum level I have proposed coset representations for the pair of super conformal algebras g and $h \subset g$ which correspond to the coset space decomposition of a given sector of WCW with constant values of zero modes. The coset construction would state that the differences of super-Virasoro generators associated with g resp. h annhilate physical states.

   The identification of the algebras g and h is not straightforward. The algebra g could be formed by the direct sum of super-symplectic and super Kac-Moody algebras and its sub-algebra h for which the generators vanish at partonic 2-surface considered. This would correspond to the idea about WCW as a coset space G/H of corresponding groups (consider as a model $CP_2 = SU(3)/U(2)$ with $U(2)$ leaving preferred point invariant). The sub-algebra h in question includes or equals to the algebra of Kac-Moody generators vanishing at the partonic 2-surface. A natural choice for the preferred WCW point would be as maximum of Kähler function in Euclidian regions: positive definiteness of Kähler function allows only single maximum for fixed values of zero modes). Coset construction states that differences of super Virasoro generators associated with g and h annihilate physical states. This implies that corresponding four-momenta are identical that is Equivalence Principle.

   The objection against the identification h in the decomposition $g = t + h$ of the symplectic algebra as Kac-Moody algebra is that this does not make sense mathematically. The strong form of holography implied by strong form of General Coordinate Invariance however implies that the action of Kac-Moody algebra for the maxima of Kähler function induces unique action of sub-algebra of symplectic algebra so that the identification makes sense after all [K8].

3. Does EP reduce to one aspect of QCC? This would require that classical Noether four-momentum identified as inertial momentum equals to the quantal four-momentum assignable to the states of super-conformal representations and identifiable as gravitational four-momentum. There would be only one independent condition: $P_{class} \equiv P_{I,class} = P_{gr,quant} \equiv P_{quant}$.

   Holography realized as AdS/CFT correspondence states the equivalence of descriptions in terms of gravitation realized in terms of strings in 10-D space-time and gauge fields at the boundary of AdS. What is disturbing is that this picture is not completely equivalent with the proposed one. In this case the super-conformal algebra would be direct sum of super-symplectic and super Kac-Moody parts.

Which of the options looks more plausible? The success of p-adic mass calculations [K37] have motivated the use of them as a guideline in attempts to understand TGD. The basic outcome was that elementary particle spectrum can be understood if Super Virasoro algebra has five tensor factors. Can one decide the fate of the two approaches to EP using this number as an input?

This is not the case. For both options the number of tensor factors is five as required. Four tensor factors come from Super Kac-Moody and correspond to translational Kac-Moody type degrees of freedom in M^4, to color degrees of freedom and to electroweak degrees of freedom ($SU(2) \times U(1)$). One tensor factor comes from the symplectic degrees of freedom in $\Delta CD \times CP_2$

(note that Hamiltonians include also products of δCD and CP_2 Hamiltonians so that one does not have direct sum!).

The reduction of EP to the coset structure of WCW sectors would be extremely beautiful property. But also the reduction of EP to QCC looks very nice and deep, and it seems that the coset option is definitely wrong: the reason is that for H in G/H decomposition the four-momentum vanishes.

2.8.4 *TGD-GRT correspondence and Equivalence Principle*

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig. ??** in the Appendix).

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Khler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in ZEO (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

2.8.5 *How translations are represented at the level of WCW?*

The four-momentum components appearing in the formulas of super conformal generators correspond to infinitesimal translations. In TGD framework one must be able to identify these infinitesimal translations precisely. As a matter of fact, finite measurement resolution implies that it is probably too much to assume infinitesimal translations. Rather, finite exponentials of translation generators are involved and translations are discretized. This does not have practical signficance since for optimal resolution the discretization step is about CP_2 length scale.

Where and how do these translations act at the level of WCW? ZEO provides a possible answer to this question.

1. Discrete Lorentz transformations and time translations act in the space of CDs: inertial four-momentum

Quantum state corresponds also to wave function in moduli space of CDs. The moduli space is obtained from given CD by making all boosts for its non-fixed boundary: boosts correspond to a discrete subgroup of Lorentz group and define a lattice-like structure at the hyperboloid for which proper time distance from the second tip of CD is fixed to $T_n = n \times T(CP_2)$. The quantization of cosmic redshift for which there is evidence, could relate to this lattice generalizing ordinary 3-D lattices from Euclidian to hyperbolic space by replacing translations with boosts (velocities).

The additional degree of freedom comes from the fact that the integer $n > 0$ obtains all positive values. One has wave functions in the moduli space defined as a pile of these lattices defined at the hyperboloid with constant value of $T(CP_2)$: one can say that the points of this pile of lattices correspond to Lorentz boosts and scalings of CDs defining sub-WCW:s.

The interpretation in terms of group which is product of the group of shifts $T_n(CP_2) \rightarrow T_{n+m}(CP_2)$ and discrete Lorentz boosts is natural. This group has same Cartesian product structure as Galilean group of Newtonian mechanics. This would give a discrete rest energy and by

Lorentz boosts discrete set of four-momenta giving a contribution to the four-momentum appearing in the super-conformal representation.

What is important that each state function reduction would mean localisation of either boundary of CD (that is its tip). This localization is analogous to the localization of particle in position measurement in E^3 but now discrete Lorentz boosts and discrete translations $T_n - - > T_{n+m}$ replace translations. Since the second end of CD is necessary del-ocalized in moduli space, one has kind of flip-flop: localization at second end implies de-localization at the second end. Could the localization of the second end (tip) of CD in moduli space correspond to our experience that momentum and position can be measured simultaneously? This apparent classicality would be an illusion made possible by ZEO.

The flip-flop character of state function reduction process implies also the alternation of the direction of the thermodynamical time: the asymmetry between the two ends of CDs would induce the quantum arrow of time. This picture also allows to understand what the experience growth of geometric time means in terms of CDs.

2. The action of translations at space-time sheets

The action of imbedding space translations on space-time surfaces possibly becoming trivial at partonic 2-surfaces or reducing to action at δCD induces action on space-time sheet which becomes ordinary translation far enough from end end of space-time surface. The four-momentum in question is very naturally that associated with Kähler action and would therefore correspond to inertial momentum for $P_{I,class} = P_{quant,gr}$ option. Indeed, one cannot assign quantal four-momentum to Kähler action as an operator since canonical quantization badly fails. In finite measurement infinitesimal translations are replaced with their exponentials for $P_{I,class} = P_{quant,gr}$ option.

What looks like a problem is that ordinary translations in the general case lead out from given CD near its boundaries. In the interior one expects that the translation acts like ordinary translation. The Lie-algebra structure of Poincare algebra including sums of translation generators with positive coefficient for time translation is preserved if only time-like superpositions if generators are allowed also the commutators of time-like translation generators with boost generators give time like translations. This defines a Lie-algebraic formulation for the arrow of geometric time. The action of time translation on preferred etxremal would be ordinary translation plus continuation of the translated preferred extremal backwards in time to the boundary of CD. The transversal space-like translations could be made Kac-Moody algebra by multiplying them with functions which vanish at δCD.

A possible interpretation would be that $P_{quant,gr}$ corresponds to the momentum assignable to the moduli degrees of freedom and $P_{cl,I}$ to that assignable to the time like translations. $P_{quant,gr} = P_{cl,I}$ would code for QCC. Geometrically quantum classical correspondence would state that time-like translation shift both the interior of space-time surface and second boundary of CD to the geometric future/past while keeping the second boundary of space-time surface and CD fixed.

2.8.6 *Yangian and four-momentum*

Yangian symmetry implies the marvellous results of twistor Grassmannian approach to $\mathcal{N} = 4$ SUSY culminating in the notion of amplituhedron which promises to give a nice projective geometry interpretation for the scattering amplitudes [?]. Yangian symmetry is a multilocal generalization of ordinary symmetry based on the notion of co-product and implies that Lie algebra generates receive also multilocal contributions. I have discussed these topics from slightly different point of view in [K28], where also references to the work of pioneers can be found.

1. Yangian symmetry

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [K28] . Besides ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant and and concrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [?] . Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N} = 4$ SUSY). One of the conditions conditions is that the tensor product $R \otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n = 0$ and $n = 1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n = 1$ generators with themselves are however something different for a non-vanishing deformation parameter h. Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n + 1$-local in the sense that they involve $n + 1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

2. How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, it is not much to say. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

1. The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A2] and Virasoro algebras [A3] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.

2. The formal generalization looks surprisingly straightforward at the formal level. In ZEO one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in ZEO and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.

3. This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with

Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context)?

1. At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N}=4$ SUSY would allow only the adjoint.

2. Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of the partonic 2-surface. This picture also justifies p-adic thermodynamics applied to either symplectic or isometry Super-Virasoro and giving thermal contribution to the vacuum conformal and thus to mass squared.

3. The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.

4. Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

3.Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n=0$ and $n=1$ levels of Yangian algebra commute. Since the co-product Δ maps $n=0$ generators to $n=1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n=1$ level and give $n=2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n=2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

3 TGD Inspired Cosmology

TGD Universe consists of quantum counterparts of a statistical system at critical temperature. As a consequence, topological condensate is expected to possess hierarchical, fractal like structure containing topologically condensed 3-surfaces with all possible sizes. Both Kähler magnetized and Kähler electric 3-surfaces ought to be important and string like objects indeed provide a good example of Kähler magnetic structures important in TGD inspired cosmology. In particular space-time is expected to be many-sheeted even at cosmological scales and ordinary cosmology must be replaced with many-sheeted cosmology. The presence of vapor phase consisting of free cosmic strings and possibly also elementary particles is second crucial aspects of TGD inspired cosmology.

It should be made clear from beginning that many-sheeted cosmology involves a vulnerable asumption. It is assumed that single-sheeted space-time surface is enough to model the cosmology. This need not to be the case. GRT limit of TGD is obtained by lumping together the sheets of many-sheeted space-time to a piece of Minkowski space and endowing it with an effective metric, which is sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Hence the proposed models make sense only if GRT limits allowing imbedding as a vacuum extremal of Kähler action have special physical role.

Quantum criticality of TGD Universe (Kähler coupling strength is analogous to critical temperature) supports the view that many-sheeted cosmology is in some sense critical. Criticality in turn suggests fractality. Phase transitions, in particular the topological phase transitions giving rise to new space-time sheets, are (quantum) critical phenomena involving no scales. If the curvature of the 3-space does not vanish, it defines scale: hence the flatness of the cosmic time=constant section of the cosmology implied by the criticality is consistent with the scale invariance of the critical phenomena. This motivates the assumption that the new space-time sheets created in topological phase transitions are in good approximation modellable as critical Robertson-Walker cosmologies for some period of time at least.

Any one-dimensional sub-manifold allows global imbeddings of subcritical cosmologies whereas for a given 2-dimensional Lagrange manifold of CP_2 critical and overcritical cosmologies allow only one-parameter family of partial imbeddings. The infinite size of the horizon for the imbeddable critical cosmologies is in accordance with the presence of arbitrarily long range quantum fluctuations at criticality and guarantees the average *isotropy* of the cosmology. Imbedding is possible for some critical duration of time. The parameter labelling these cosmologies is a scale factor characterizing the duration of the critical period. These cosmologies have the same optical properties as inflationary cosmologies but exponential expansion is replaced with logarithmic one. Critical cosmology can be regarded as a "Silent Whisper amplified to Bang" rather than "Big Bang" and transformed to hyperbolic cosmology before its imbedding fails. Split strings decay to elementary particles in this transition and give rise to seeds of galaxies. In some later stage the hyperbolic cosmology can decompose to disjoint 3-surfaces. Thus each sub-cosmology is analogous to biological growth process leading eventually to death.

The critical cosmologies can be used as a building blocks of a fractal cosmology containing cosmologies containing ... cosmologies. p-Adic length scale hypothesis allows a quantitative formulation of the fractality [K24] . Fractal cosmology predicts cosmos to have essentially same optical properties as inflationary scenario. Fractal cosmology explains the paradoxical result that the observed density of the matter is much lower than the critical density associated with the largest space-time sheet of the fractal cosmology. Also the observation that some astrophysical objects seem to be older than the Universe, finds a nice explanation.

Absolutely essential element of the considerations (and longstanding puzzle of TGD inspired cosmology) is the conservation of energy implied by Poincare invariance which seems to be in conflict with the non-conservation of gravitational energy. It took long time to discover the natural resolution of the paradox. In TGD Universe matter and antimatter have opposite energies and gravitational four-momentum is identified as difference of the four momenta of matter and antimatter (or vice versa, so that gravitational energy is positive). The assumption that the net inertial energy density vanishes in cosmological length scales is the proper interpretation for the fact that Robertson-Walker cosmologies correspond to vacuum extremals of Kähler action.

Tightly bound, possibly coiled pairs of cosmic strings are the basic building block of TGD inspired cosmology and all al structures including large voids, galaxies, stars, and even planets can be seen as pearls in a cosmic fractal necklace consisting of cosmic strings containing smaller cosmic strings linked around them containing... During cosmological evolution the cosmic strings are transformed to magnetic flux tubes and these structures are also key players in TGD inspired quantum biology.

Negative energy virtual gravitons represented by topological quanta having negative time orientation and hence also negative energy. The absorption of negative energy gravitons by photons could explain gradual red-shifting of the microwave background radiation at particle level. Negative energy virtual gravitons give also rise to a negative gravitational potential energy. Quite generally, negative energy virtual bosons build up the negative interaction potential energy. An important constraint to TGD inspired cosmology is the requirement that Hagedorn temperature $T_H \sim 1/R$, where R is CP_2 size, is the limiting temperature of radiation dominated phase.

3.1 Robertson-Walker Cosmologies

Robertson-Walker cosmologies are the basic building block of standard cosmologies and sub-critical R-W cosmologies have a very natural place in TGD framework as Lorentz invariant cosmologies. Inflationary cosmologies are replaced with critical cosmologies being parameterized by a single parameter telling the duration of the critical cosmology. Over-critical cosmologies are not possible at all.

3.1.1 *Why Robertson-Walker cosmologies?*

One can hope Robertson Walker cosmology represented as a vacuum extremal of the Kähler action to be a reasonable idealization only in the length scales, where the density of the Kähler charge vanishes. Since (visible) matter and antimatter carry Kähler charges of opposite sign this means that Kähler charge density vanishes in length scales, where matter-antimatter asymmetry disappears on the average. This length scale is certainly very large in present day cosmology: in the proposed model for cosmology its present value is of the order of 10^8 light years: the size of the observed regions containing visible matter predominantly on their boundaries [E32] . That only matter is observed can be understood from the fact that fermions reside dominantly at future oriented space-time sheets and anti-fermions on past-oriented space-time sheets.

Robertson Walker cosmology is expected to apply in the description of the condensate locally at each condensate level and it is assumed that the GRT based criteria for the formation of "structures" apply. In particular, the Jeans criterion stating that density fluctuations with size between Jeans length and horizon size can lead to the development of the "structures" will be applied.

3.1.2 *Imbeddability requirement for RW cosmologies*

Standard Robertson-Walker cosmology is characterized by the line element [E25]

$$ds^2 = f(a)da^2 - a^2(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2) \ , \tag{3.1}$$

where the values $k = 0, \pm 1$ of k are possible.

The line element of the light cone is given by the expression

$$ds^2 = da^2 - a^2(\frac{dr^2}{1+r^2} + r^2 d\Omega^2) \ . \tag{3.2}$$

Here the variables a and r are defined in terms of standard Minkowksi coordinates as

$$\begin{aligned} a &= \sqrt{(m^0)^2 - r_M^2} \ , \\ r_M &= ar \ . \end{aligned} \tag{3.3}$$

Light cone clearly corresponds to mass density zero cosmology with $k = -1$ and this makes the case $k = -1$ is rather special as far imbeddings are considered since any Lorentz invariant map $M^4_+ \to CP_2$ defines imbedding

$$s^k = f^k(a) \ . \tag{3.4}$$

Here f^k are arbitrary functions of a.

$k = -1$ requirement guarantees imbeddability if the matter density is positive as is easy to see. The matter density is given by the expression

$$\rho = \frac{3}{8\pi G a^2}(\frac{1}{g_{aa}} + k) \ . \qquad (3.5)$$

A typical imbedding of $k = -1$ cosmology is given by

$$\begin{aligned} \phi &= f(a) \ , \\ g_{aa} &= 1 - \frac{R^2}{4}(\partial_a f)^2 \ . \end{aligned} \qquad (3.6)$$

where ϕ can be chosen to be the angular coordinate associated with a geodesic sphere of CP_2 (any one-dimensional sub-manifold of CP_2 works equally well). The square root term is always positive by the positivity of the mass density and the imbedding is indeed well defined. Since g_{aa} is smaller than one, the matter density is necessarily positive.

3.1.3 *Critical and over-critical cosmologies*

TGD allows vacuum extremal imbeddings of a one-parameter family of critical over-critical cosmologies. Critical cosmologies are however not inflationary in the sense that they would involve the presence of scalar fields. Exponential expansion is replaced with a logarithmic one so that the cosmologies are in this sense exact opposites of each other. Critical cosmology has been used hitherto as a possible model for the very early cosmology. What is remarkable that this cosmology becomes vacuum at the moment of "Big Bang" since mass density behaves as $1/a^2$ as function of the light cone proper time. Instead of "Big Bang" one could talk about "Small Whisper" amplified to bang gradually. This is consistent with the idea that space-time sheet begins as a vacuum space-time sheet for some moment of cosmic time. As an imbedded 4-surface this cosmology would correspond to a deformed future light cone having its tip inside the future light cone. The interpretation of the tip as a seed of a phase transition is possible. The imbedding makes sense up to some moment of cosmic time after which the cosmology becomes necessarily hyperbolic. At later time hyperbolic cosmology stops expanding and decomposes to disjoint 3-surfaces behaving as particle like objects co-moving at larger cosmological space-time sheet. These 3-surfaces topologically condense on larger space-time sheets representing new critical cosmologies.

Consider now in more detail the imbeddings of the critical and overcritical cosmologies. For $k = 0, 1$ the imbeddability requirement fixes the cosmology almost uniquely. To see this, consider as an example of $k = 0/1$ imbedding the map from the light cone to S^2, where S^2 is a geodesic sphere of CP_2 with a vanishing Kähler form (any Lagrage manifold of CP_2 would do instead of S^2). In the standard coordinates (Θ, Φ) for S^2 and Robertson-Walker coordinates (a, r, θ, ϕ) for future light cone (, which can be regarded as empty hyperbolic cosmology), the imbedding is given as

$$\begin{aligned} sin(\Theta) &= \frac{a}{a_1} \ , \\ (\partial_r \Phi)^2 &= \frac{1}{K_0}\left[\frac{1}{1 - kr^2} - \frac{1}{1 + r^2}\right] \ , \\ K_0 &= \frac{R^2}{4a_1^2} \ , \ k = 0, 1 \ , \end{aligned} \qquad (3.7)$$

when Robertson-Walker coordinates are used for both the future light cone and space-time surface.

The differential equation for Φ can be written as

$$\partial_r \Phi = \pm\sqrt{\frac{1}{K_0}\left[\frac{1}{1 - kr^2} - \frac{1}{1 + r^2}\right]} \ . \qquad (3.8)$$

For $k=0$ case the solution exists for all values of r. For $k=1$ the solution extends only to $r=1$, which corresponds to a 4-surface $r_M = m^0/\sqrt{2}$ identifiable as a ball expanding with the velocity $v = c/\sqrt{2}$. For $r \rightarrow 1$ Φ approaches constant Φ_0 as $\Phi - \Phi_0 \propto \sqrt{1-r}$. The space-time sheets corresponding to the two signs in the previous equation can be glued together at $r=1$ to obtain sphere S^3.

The expression of the induced metric follows from the line element of future light cone

$$ds^2 = da^2 - a^2(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2) \ . \tag{3.9}$$

The imbeddability requirement fixes almost uniquely the dependence of the S^2 coordinates a and r and the g_{aa} component of the metric is given by the same expression for both $k=0$ and $k=1$.

$$\begin{aligned} g_{aa} &= 1-K \ , \\ K &\equiv K_0 \frac{1}{(1-u^2)} \ , \\ u &\equiv \frac{a}{a_1} \ . \end{aligned} \tag{3.10}$$

The imbedding fails for $a \geq a_1$. For $a_1 \gg R$ the cosmology is essentially flat up to immediate vicinity of $a = a_1$. Energy density and "pressure" follow from the general equation of Einstein tensor and are given by the expressions

$$\begin{aligned} \rho &= \frac{3}{8\pi G a^2}(\frac{1}{g_{aa}} + k) \ , \ k = 0,1 \ , \\ \frac{1}{g_{aa}} &= \frac{1}{1-K} \ , \\ p &= -(\rho + \frac{a\partial_a \rho}{3}) = -\frac{\rho}{3} + \frac{2}{3}K_0 u^2 \frac{1}{(1-K)(1-u^2)^2}\rho_{cr} \ , \\ u &\equiv \frac{a}{a_1} \ . \end{aligned} \tag{3.11}$$

Here the subscript "cr" refers to $k=0$ case. Since the time component g_{aa} of the metric approaches constant for very small values of the cosmic time, there are no horizons associated with this metric. This is clear from the formula

$$r(a) = \int_0^a \sqrt{g_{aa}} \frac{da}{a}$$

for the horizon radius.

The mass density associated with these cosmologies behaves as $\rho \propto 1/a^2$ for very small values of the M^4_+ proper time. The mass in a co-moving volume is proportional to $a/(1-K)$ and goes to zero at the limit $a \rightarrow 0$. Thus, instead of Big Bang one has "Silent Whisper" gradually amplifying to Big Bang. The imbedding fails at the limit $a \rightarrow a_1$. At this limit energy density becomes infinite. This cosmology can be regarded as a cosmology for which co-moving strings ($\rho \propto 1/a^2$) dominate the mass density as is clear also from the fact that the "pressure" becomes negative at big bang ($p \rightarrow -\rho/3$) reflecting the presence of the string tension. The natural interpretation is that cosmic strings condense on the space-time sheet which is originally empty.

The facts that the imbedding fails and gravitational energy density diverges for $a = a_1$ necessitates a transition to a hyperbolic cosmology. For instance, a transition to radiation or matter dominated hyperbolic cosmology can occur at the limit $\theta \rightarrow \pi/2$. At this limit $\phi(r)$ must transform to a function $\phi(a)$. The fact, that vacuum extremals of Kähler action are in question, allows large flexibility for the modelling of what happens in this transition. Quantum criticality and p-adic fractality suggest the presence of an entire fractal hierarchy of space-time sheets representing critical cosmologies created at certain values of cosmic time and having as their light cone projection sub-light cone with its tip at some a=constant hyperboloid.

3.1.4 *More general imbeddings of critical and over-critical cosmologies as vacuum extremals*

In order to obtain imbeddings as more general vacuum extremals, one must pose the condition guaranteeing the vanishing of corresponding the induced Kähler form (see the Appendix of this book). Using coordinates $(r, u = cos(\Theta), \Psi, \Phi)$ for CP_2 the surfaces in question can be expressed as

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} \ , \\ X &= D|k+u| \ , \\ u &\equiv cos(\Theta) \ , \ \ D = \frac{r_0^2}{1+r_0^2} \times \frac{1}{C} \ , \quad C = |k + cos(\Theta_0)| \ . \end{aligned} \tag{3.12}$$

Here C and D are integration constants.

These imbeddings generalize to imbeddings to $M^4 \times Y^2$, where Y^2 belongs to a family of Lagrange manifolds described in the Appendix of this book with induced metric

$$\begin{aligned} ds^2_{eff} &= \frac{R^2}{4}[s^{eff}_{\Theta\Theta} d\Theta^2 + s^{eff}_{\Phi\Phi} d\Phi^2] \ , \\ s^{eff}_{\Theta\Theta} &= X \times \left[\frac{(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\ s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ . \end{aligned} \tag{3.13}$$

For $k \neq 1$ $u = \pm 1$ corresponds in general to circle rather than single point as is clear from the fact that $s^{eff}_{\Phi\Phi}$ is non-vanishing at $u = \pm 1$ so that u and Φ parameterize a piece of cylinder. The generalization of the previous imbedding is as

$$sin(\Theta) = ka \quad \rightarrow \quad \sqrt{s^{eff}_{\Phi\Phi}} = ka \ . \tag{3.14}$$

For Φ the expression is as in the previous case and determined by the requirement that g_{rr} corresponds to $k = 0, 1$.

The time component of the metric can be expressed as

$$g_{aa} = 1 - \frac{R^2 k^2}{4} \frac{s^{eff}_{\Theta\Theta}}{\frac{d\sqrt{s^{eff}_{\Phi\Phi}}}{d\Theta}} \tag{3.15}$$

In this case the $1/(1 - k^2a^2)$ singularity of the density of gravitational mass at $\Theta = \pi/2$ is shifted to the maximum of $s^{eff}_{\Phi\Phi}$ as function of Θ defining the maximal value a_{max} of a for which the imbedding exists at all. Already for $a_0 < a_{max}$ the vanishing of g_{aa} implies the non-physicality of the imbedding since gravitational mass density becomes infinite.

The geometric properties of critical cosmology change radically in the transition to the radiation dominated cosmology: before the transition the CP_2 projection of the critical cosmology is two-dimensional. After the transition it is one-dimensional. Also the isometry group of the cosmology changes from $SO(3) \times E^3$ to $SO(3, 1)$ in the transition. One could say that critical cosmology represents Galilean Universe whereas hyperbolic cosmology represents Lorentzian Universe.

3.1.5 *String dominated cosmology*

A particularly interesting cosmology is string dominated cosmology with very nearly critical mass density. Assuming that strings are co-moving the mass density of this cosmology is proportional to $1/a^2$ instead of the $1/a^3$ behavior characteristic to the standard matter dominated cosmology. The line element of this metric is very simple: the time component of the metric is simply constant smaller than 1:

$$g_{aa} = K < 1 \ . \tag{3.16}$$

The Hubble constant for this cosmology is given by

$$H = \frac{1}{\sqrt{K}a} \ , \tag{3.17}$$

and the so called acceleration parameter [E25] k_0 proportional to the second derivative $\ddot{a}$ therefore vanishes. Mass density and pressure are given by the expression

$$\rho = \frac{3}{8\pi G K a^2}(1-K) = -3p \ . \tag{3.18}$$

What makes this cosmology so interesting is the absence of the horizons. The comparison with the critical cosmology shows that these two cosmologies resemble each other very closely and both could be used as a model for the very early cosmology.

3.1.6 *Stationary cosmology*

An interesting candidate for the asymptotic cosmology is stationary cosmology for which gravitational four-momentum currents (and also gravitational color currents) are conserved. This cosmology extremizes the Einstein-Hilbert action with cosmological term given by $\int (kR+\lambda)\sqrt{g}d^4x + \lambda$ and is obtained as a sub-manifold $X^4 \subset M^4_+ \times S^1$, where S^1 is the geodesic circle of CP_2 (note that imbedding is now unique apart from isometries by variational principle).

For a vanishing cosmological constant, field equations reduce to the conservation law for the isometry associated with S^1 and read

$$\partial_a(G^{aa}\partial_a\phi\sqrt{g}) = 0 \ , \tag{3.19}$$

where ϕ denotes the angle coordinate associated with S^1. From this one finds for the relevant component of the metric the expression

$$\begin{aligned} g_{aa} &= \frac{(1-2x)}{(1-x)} \ , \\ x &= (\frac{C}{a})^{2/3} \ . \end{aligned} \tag{3.20}$$

The mass density and "pressure" of this cosmology are given by the expressions

$$\begin{aligned} \rho &= \frac{3}{8\pi G a^2}\frac{x}{(1-2x)} \ , \\ p &= -(\rho + \frac{a\partial_a\rho}{3}) = -\frac{\rho}{9}\left[3 - \frac{2}{(1-2x)}\right] \ . \end{aligned} \tag{3.21}$$

The asymptotic behavior of the energy density is $\rho \propto a^{-8/3}$. "Pressure" becomes negative indicating that this cosmology is dominated by the string like objects, whose string tension gives negative contribution to to the "pressure". Also this cosmology is horizon free as are all string dominated cosmologies: this is of crucial importance in TGD inspired cosmology.

It should be noticed that energy density for this cosmology becomes infinite for $x = (C/a)^{2/3} = 1/2$ implying that this cosmology doesn't make sense at very early times so that the non-conservation of gravitational energy is necessary during the early stages of the cosmology.

3.1.7 *Non-conservation of gravitational energy in RW cosmologies*

In *RW* cosmology the gravitational energy in a given co-moving sphere of radius r in local light cone coordinates (a, r, θ, ϕ) is given by

$$E = \int \rho g^{aa} \partial_a m^0 \sqrt{g} dV \ . \tag{3.22}$$

The rate characterizing the non-conservation of gravitational energy is determined by the parameter X defined as

$$X \equiv \frac{(dE/da)_{vap}}{E} = \frac{(dE/da + \int |g^{rr}| p \partial_r m^0 \sqrt{g} d\Omega)}{E} \ , \tag{3.23}$$

where p denotes the pressure and $d\Omega$ denotes angular integration over a sphere with radius r. The latter term subtracts the energy flow through the boundary of the sphere.

The generation of the pairs of positive and negative (inertial) energy space-time sheets leads to non-conservation of gravitational energy. The generation of pairs of positive and negative energy cosmic strings would be involved with the generation of a critical sub-cosmology.

For *RW* cosmology with subcritical mass density the calculation gives

$$X = \frac{\partial_a(\rho a^3/\sqrt{g_{aa}})}{(\rho a^3/\sqrt{g_{aa}})} + \frac{3pg_{aa}}{\rho a} \ . \tag{3.24}$$

This formula applies to any infinitesimal volume. The rate doesn't depend on the details of the imbedding (recall that practically any one-dimensional sub-manifold of CP_2 defines a huge family of subcritical cosmologies). Apart from the numerical factors, the rate behaves as $1/a$ in the most physically interesting *RW* cosmologies. In the radiation dominated and matter dominated cosmologies one has $X = -1/a$ and $X = -1/2a$ respectively so that gravitational energy decreases in radiation and matter dominated cosmologies. For the string dominated cosmology with $k = -1$ having $g_{aa} = K$ one has $X = 2/a$ so that gravitational energy increases: this might be due to the generation of dark matter due to pairs of cosmic strings with vanishing net inertial energy.

For the cosmology with exactly critical mass density Lorentz invariance is broken and the contribution of the rate from 3-volume depends on the position of the co-moving volume. Taking the limit of infinitesimal volume one obtains for the parameter X the expression

$$\begin{aligned} X &= X_1 + X_2 \ , \\ X_1 &= \frac{\partial_a(\rho a^3/\sqrt{g_{aa}})}{(\rho a^3/\sqrt{g_{aa}})} \ , \\ X_2 &= \frac{pg_{aa}}{\rho a} \times \frac{3+2r^2}{(1+r^2)^{3/2}} \ . \end{aligned} \tag{3.25}$$

Here r refers to the position of the infinitesimal volume. Simple calculation gives

$$\begin{aligned} &X = X_1 + X_2 \ , \\ &X_1 = \tfrac{1}{a}\left[1 + 3K_0 u^2 \tfrac{1}{1-K}\right] \ , \\ &X_2 = -\tfrac{1}{3a}\left[1 - K - \tfrac{2K_0u^2}{(1-u^2)^2}\right] \times \tfrac{3+2r^2}{(1+r^2)^{3/2}} \ , \\ &K = \tfrac{K_0}{1-u^2} \ , \quad u = \tfrac{a}{a_0} \ , \quad K_0 = \tfrac{R^2}{4a_0^2} \ . \end{aligned} \tag{3.26}$$

The positive density term X_1 corresponds to increase of gravitational energy which is gradually amplified whereas pressure term ($p < 0$) corresponds to a decrease of gravitational energy changing however its sign at the limit $a \rightarrow a_0$.

The interpretation is in terms of creation of pairs of positive and negative energy particles contributing nothing to the inertial energy. Also pairs of positive energy gravitons and negative anti-gravitons are involved. The contributions of all particle species are determined by thermal arguments so that gravitons should not play any special role as thought originally.

Pressure term is negligible at the limit $r \to \infty$ so that topological condensation occurs all the time at this limit. For $a \to 0, r \to 0$ one has $X > 0 \to 0$ so that condensation starts from zero at $r = 0$. For $a \to 0, r \to \infty$ one has $X = 1/a$ which means that topological condensation is present already at the limit $a \to 0$.

Both the existence of the finite limiting temperature and of the critical mass density imply separately finite energy per co-moving volume for the condensate at the very early stages of the cosmic evolution. In fact, the mere requirement that the energy per co-moving volume in the vapor phase remains finite and non-vanishing at the limit $a \to 0$ implies string dominance as the following argument shows.

Assuming that the mass density of the condensate behaves as $\rho \propto 1/a^{2(1+\alpha)}$ one finds from the expression

$$\rho \propto \frac{(\frac{1}{g_{aa}} - 1)}{a^2} ,$$

that the time component of the metric behaves as $g_{aa} \propto a^{\alpha}$. Unless the condition $\alpha < 1/3$ is satisfied or equivalently the condition

$$\rho \quad < \quad \frac{k}{a^{2+2/3}} \tag{3.27}$$

is satisfied, gravitational energy density is reduced. In fact, the limiting behavior corresponds to the stationary cosmology, which is not imbeddable for the small values of the cosmic time. For stationary cosmology gravitational energy density is conserved which suggests that the reduction of the density of cosmic strings is solely due to the cosmic expansion.

3.2 Free Cosmic Strings

The free cosmic strings correspond to four-surfaces of type $X^2 \times S^2$, where S^2 is the homologically nontrivial geodesic sphere of CP_2 [L1] , [L1] and X^2 is minimal surface in M^4_+. As a matter fact, any complex manifold $Y^2 \subset CP_2$ is possible. In this section, a co-moving cosmic string solution inside the light cone $M^4_+(m)$ associated with a given m point of M^4_+ will be constructed.

Recall that the line element of the light cone in co-moving coordinates inside the light cone is given by

$$ds^2 \quad = \quad da^2 - a^2(\frac{dr^2}{1+r^2} + r^2 d\Omega^2) \ . \tag{3.28}$$

Outside the light cone the line element is given

$$ds^2 \quad = \quad -da^2 - a^2(-\frac{dr^2}{1-r^2} + r^2 d\Omega^2) \ , \tag{3.29}$$

and is obtained from the line element inside the light cone by replacements $a \to ia$ and $r \to -ir$.

3.2.1 *Simplest solutions*

Using the coordinates $(a = \sqrt{(m^0)^2 - r_M^2}, ar = r_M)$ for X^2 the orbit of the cosmic string is given by

$$\begin{aligned} \theta &= \frac{\pi}{2} , \\ \phi &= f(r) \ . \end{aligned} \tag{3.30}$$

Inside the light cone the line element of the induced metric of X^2 is given by

$$ds^2 = da^2 - a^2(\frac{1}{1+r^2} + r^2 f_{,r}^2)dr^2 \ . \tag{3.31}$$

The equations stating the minimal surface property of X^2 can be expressed as a differential conservation law for energy or equivalently for the component of the angular momentum in the direction orthogonal to the plane of the string. The conservation of the energy current T^α gives

$$\begin{aligned} T^\alpha_{,\alpha} &= 0 \ , \\ T^\alpha &= T g^{\alpha\beta} m^0_{,\beta}\sqrt{g} \ , \\ T &= \frac{1}{8\alpha_K R^2} \simeq .52\times 10^{-6}\frac{1}{G} \ . \end{aligned} \tag{3.32}$$

The numerical estimate $TG \simeq .52\times 10^{-6}$ for the string tension is upper bound and corresponds to a situation in which the entire area of S^2 contributes to the tension. It has been obtained using $\alpha_K/104$ and $R^2/G = 2.5\times 10^7 G$ given by the most recent version of p-adic mass calculations (the earlier estimate was roughly by a factor 1/2 too small due to error in the calculation [K15, K3]). The string tension belongs to the range $TG \in [10^{-6} - 10^{-7}]$ predicted for GUT strings [E28] . WMAP data give the upper bound $TG \in [10^{-6} - 10^{-7}]$, which does not however hold true in the recent case since criticality predicts adiabatic spectrum of perturbations as in the inflationary scenarios.

The non-vanishing components of energy current are given by

$$\begin{aligned} T^a &= TUa \ , \\ T^r &= -T\frac{r}{U} \ , \\ U &= \sqrt{1+r^2(1+r^2)f_{,r}^2} \ . \end{aligned} \tag{3.33}$$

The equations of motion give

$$U = \frac{r}{\sqrt{r^2-r_0^2}} \ , \tag{3.34}$$

or equivalently

$$\phi_{,r} = \frac{r_0}{r\sqrt{(r^2-r_0^2)(1+r^2)}} \ , \tag{3.35}$$

where r_0 is an integration constant to be determined later. Outside the light cone the solution has the form

$$\phi_{,r} = \frac{r_0}{\sqrt{r^2+r_0^2}r\sqrt{1-r^2}} \ . \tag{3.36}$$

In the region inside the light cone, where the conditions

$$r_0 << r << 1 \tag{3.37}$$

hold, the solution has the form

$$\begin{aligned} \phi(r) &\simeq \phi_0 + \frac{v}{r} \ , \\ v &= \frac{r_0}{\sqrt{1+r_0^2}} \ , \end{aligned} \tag{3.38}$$

corresponding to the linearized equations of motion

$$f_{,rr} + \frac{2f_{,r}}{r} = 0 \ , \qquad (3.39)$$

obtained most nicely from the angular momentum conservation condition.

3.2.2 *Cosmic string is stationary in comoving coordinates*

In co-moving coordinates (in general the co-moving coordinates of sub-light-cone M^4_+!) the string is stationary. In Minkowski coordinates string rotates with an angular velocity inversely proportional to the distance from the origin

$$\omega \simeq \frac{v}{r_M} \qquad (3.40)$$

so that the orbital velocity of the string becomes essentially constant in this region. For very large values of r the orbital velocity of the string vanishes as $1/r$. Outside the light cone the variable r is in the role of time and for a given value of the time variable r strings are straight and one can regard the string as a rigidly rotating straight string in this region.

Inside the light cone, the solution becomes ill defined for the values of r smaller than the critical value r_0. Although the derivative $\phi_{,r}$ becomes infinite at this limit, the limiting value of ϕ is finite so that strings winds through a finite angle. The normal component T^r of the energy momentum current vanishes at $r = r_0$ identically, which means that no energy flows out at the end of the string. The coordinate variable r becomes however bad at $r = r_0$ (string resembles a circle at r_0) and this conclusion must be checked using ϕ as coordinate instead of r. The result is that the normal component of the energy current indeed vanishes.

Field equations are not however satisfied at the end of the string since the normal component of the angular momentum current (in z- direction) is non-vanishing at the boundary and given by

$$J^r = Tr^2a \ . \qquad (3.41)$$

This means that the string loses angular momentum through its ends although the angular momentum density of the string is vanishing. The angular momentum lost at moment a is given by

$$J = \frac{Tr^2a^2}{2} = \frac{Tr_M^2}{2} \ . \qquad (3.42)$$

This angular momentum is of the same order of magnitude as the angular momentum of a typical galaxy [E30] .

In M^4 coordinates singularity corresponds to a disk in the plane of string growing with a constant velocity, when the coordinate m^0 is positive

$$\begin{aligned} r_M &= vm^0 \ , \\ v &= \frac{r_0}{\sqrt{1+r_0^2}} \ . \end{aligned} \qquad (3.43)$$

From the expression of the energy density of the string

$$\begin{aligned} T^a &= T\frac{ar}{\sqrt{r^2-r_0^2}} \ , \\ T &= \frac{1}{8\alpha_K R^2} \ , \end{aligned} \qquad (3.44)$$

it is clear that energy density diverges at the singularity.

3.2.3 *Energy of the cosmic string*

As already noticed, the string tension is by a factor of order 10^{-6} smaller than the critical string tension $T_{cr} = 1/4G$ implying angle deficit of 2π in GRT so that there seems to be no conflict with General Relativity (unlike in the original scenario, in which the CP_2 radius was of order Planck length).

The energy of the string portion ranging from r_0 to r_1 is given by

$$E = T\sqrt{(r_1^2 - r_0^2)a} = T\sqrt{\delta r_M^2} \ . \tag{3.45}$$

It should be noticed that M^4 time development of the string can be regarded as a scaling: each point of the string moves to radial direction with a constant velocity v.

One can calculate the total change of the angle ϕ from the integral

$$\Delta\phi = \sqrt{\frac{r_0^2}{1+r_0^2}} \int_{r_0}^{\infty} dr \frac{1}{r\sqrt{(r^2 - r_0^2)(1+r^2)}} \ . \tag{3.46}$$

The upper bound of this quantity is obtained at the limit $r_0 \to 0$ and equals to $\Delta\phi = \pi/2$.

3.3 Cosmic Strings And Cosmology

The model for cosmic strings has forced to question all cherished assumptions including positive energy ontology, Equivalence Principle, and positivity of gravitational mass. The final outcome turned out to be rather conservative. ZEO is unavoidable, Equivalence Principle holds true universally but its general relativistic formulation makes sense only in long length scales, and gravitational mass has definite sign for positive/negative energy states. As a matter fact, all problems were created by the failure to realize that the expression of gravitational energy in terms of Einstein's tensor does not hold true in short length scales and must be replaced with the stringy expression resulting naturally by dimensional reduction of quantum TGD to string model like theory [K34, K15, K3].

The realization that GRT is only an effective description of many-sheeted space-time as Minkowski space M^4 endowed with effective metric whose deviation from flat metric is the sum of the corresponding deviations for space-time sheets in the region of M^4 considered resolved finally the problems and allowed to reduced Equivalence Principle to its form in GRT. Similar description applies also to gauge interactions.

TGD is therefore a microscopic theory and the physics for single space-time sheet is expected to be extremely simpler, much simpler than in gauge theory and general relativity already due to the fact that only four bosonic variables (4 imbedding space coordinates) defined the dynamics at this level.

3.3.1 *ZEO and cosmic strings*

There are two kinds of cosmic strings: free and topological condensed ones and both are important in TGD inspired cosmology.

1. Free cosmic strings are not absolute minima of the Kähler action (the action has wrong sign). In the original identification of preferred extremals as absolute minima of Kähler action this was a problem. In the new formulation preferred extremals correspond to quantum criticality identified as the vanishing of the second variation of Kähler action at least for the deformations defining symmetries of Kähler action [K34, K15]. The symmetries very probably correspond to conformal symmetries acting as or almost as gauge symmetries. The number of conformal equivalence classes of space-time sheets with same Kähler action and conserved charges is expected to be finite and correspond to n in $h_{eff} = n \times h$ defining the hierarchy of Planck constants labelling phases of dark matter (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg` or **Fig. ??**) in the appendix of this book).

   Criticality guarantees the conservation of the Noether charges assignable to the Kähler-Dirac action. Ideal cosmic strings are excluded because they fail to satisfy the conditions characterizing the preferred extremal as a space-time surface containing regions with both Euclidian and Minkowskian signature of the induced metric with light-like 3-surface separating them identified as orbits of partonic 2-surfaces carrying elementary particle quantum numbers. The topological condensation of CP_2 type vacuum extremals representing fermons generates negative contribution to the action and reduces the string tension and leaves cosmic strings still free.

2. If the topologically condensate of fermions has net Kähler charges as the model for matter antimatter asymmetry suggests, the repulsive interaction of the particles tends to thicken the cosmic string by increasing the thickness of its infinitely thin M^4 projection so that Kähler magnetic flux tubes result. These flux tubes are ideal candidates for the carriers of dark matter with a large value of Planck constant. The criterion for the phase transition increasing $\hbar$ is indeed the presence of a sufficiently dense plasma implying that perturbation theory in terms of $Z^2\alpha_{em}$ (Z is the effective number of charges with interacting with each other without screening effects) fails for the standard value of Planck constant. The phase transition $h \to h_{eff}$ reduces the value of $\alpha_{em} = e^2/2 \times h_{eff}$ so that perturbation theory works. This phase transition scales up also the transversal size of the cosmic string. Similar criterion works also for other charges. The resulting phase is anyonic if the resulting 2-surfaces containing almost spherical portions connected by flux tubes to each other encloses the tip of the causal diamond (CD). The proposal is that dark matter resides on complex anyonic 2-surfaces surrounding the tips of CDs.

3. The topological condensation of cosmic strings generates wormhole contacts represented as pieces of CP_2 type vacuum extremals identified as bosons composed of fermion-anti-fermion pairs. Also this generates negative action and can make cosmic string a preferred extremal of Kähler action. The earliest picture was based on dynamical cancelation mechanism involving generation of strong Kähler electric fields in the condensation whose action compensated for Kähler magnetic action [K1] . Also this mechanism might be at work. Cosmic strings could also form bound states by the formation graviton like flux tubes connecting them and having wormhole contacts at their ends so that again action is reduced.

4. One can argue that in long enough length and time scales Kähler action per volume must vanish so that the idealization of cosmology as a vacuum extremal becomes possible and there must be some mechanism compensating the positive action of the free cosmic strings. The general mechanism could be topological condensation of fermions and creation of bosons by topological condensation of cosmic strings to space-time sheets.

In this framework zero energy states correspond to cosmologies leading from big bang to big crunch separated by some time interval T of geometric time. Quantum jumps can gradually increase the value T and TGD inspired theory of consciousness suggests that the increase of T might relate to the shift for the contents of conscious experience towards geometric future. In particular, what is usually regarded as cosmology could have started from zero energy state with a small value of T.

3.3.2 *Topological condensation of cosmic strings*

In the original vision about topological condensation of cosmic strings I assumed that large voids represented by space-time sheets contain "big" cosmic string in their interior and galactic strings near their boundaries. The recent much simpler view is that there are just galactic strings which carry net fermion numbers (matter antimatter asymmetry). If they have also net em charge they have a repulsive interaction and tend to end up to the boundaries of the large void. Since this slows down the expansive motion of strings, the repulsive interaction energy increases and a phase transition increasing Planck constant and scaling up the size of the void occurs after which cosmic strings are again driven towards the boundary of the resulting larger void.

One cannot assume that the exterior metric of the galactic strings is the one predicted by assuming General Relativity in the exterior region. This would mean that metric decomposes as $g = g_2(X^2) + g_2(Y^2)$. $g(X^2)$ would be flat as also $g_2(Y^2)$ expect at the position of string. The resulting angle defect due to the replacement of plane Y^2 with cone would be large and give rise to lense effect of same magnitude as in the case of GUT cosmic strings. Lense effect has not been observed.

This suggests that General Relativity fails in the length scale of large void as far as the description of topologically condensed cosmic strings is considered. The constant velocity spectrum for distant stars of galaxies and the fact that galaxies are organized along strings suggests that these string generate in a good approximation Newtonian potential. This potential predicts constant velocity spectrum with a correct value velocity.

In the stationary situation one expects that the exterior metric of galactic string corresponds to a small deformation of vacuum extremal of Kähler action which is also extremal of the curvature scalar in the induced metric. This allows a solution ansatz which conforms with Newtonian intuitions and for which metric decomposes as $g = g_1 + g_3$, where g_1 corresponds to axis in the direction of string and g_3 remaining $1 + 2$ directions.

3.3.3 *Dark energy is replaced with dark matter in TGD framework*

The observed accelerating expansion of the Universe has forced to introduce the notion of cosmological constant in the GRT based cosmology. In TGD framework the situation is different.

1. The gigantic value of gravitational Planck constant implies that dark matter makes TGD Universe a macroscopic quantum system even in cosmological length scales. Astrophysical systems become stationary quantum systems which participate in cosmic expansion only via quantum phase transitions increasing the value of gravitational Planck constant.

2. Critical cosmologies, which are determined apart from a single parameter in TGD Universe, are natural during all quantum phase transitions, in particular the phase transition periods increasing the size of large voids and having interpretation in terms of an increase of gravitational Planck constant. Cosmic expansion is predicted to be accelerating during these periods. The mere criticality requires that besides ordinary matter there is a contribution $\Omega_\Lambda \simeq .74$ to the mass density besides visible matter and dark matter. In fact, also for the over-critical cosmologies expansion is accelerating.

3. In GRT framework the essential characteristic of dark energy is its negative pressure. In TGD framework critical and over-critical cosmologies have automatically effective negative pressure. This is essentially due to the constraint that Lorentz invariant vacuum extremal of Kähler action is in question. The mysterious negative pressure would be thus a signal about the representability of space-time as 4-surface in H and there is no need for any microscopic description in terms of exotic thermodynamics.

3.3.4 *The values for the TGD counterpart of cosmological constant*

One can introduce a parameter characterizing the contribution of dark mass to the mass density during critical periods and call it cosmological constant recalling however that the contribution does not correspond to dark energy. The value of this parameter is same as in the standard cosmology from mere criticality assumption.

What is new that p-adic fractality predicts that Λ scales as $1/L^2(k)$ as a function of the p-adic scale characterizing the space-time sheet implying a series of phase transitions reducing Λ. The order of magnitude for the recent value of the cosmological constant comes out correctly. The gravitational energy density assignable to the cosmological constant is identifiable as that associated with topologically condensed cosmic strings and magnetic flux tubes to which they are gradually transformed during cosmological evolution.

The naive expectation would be the density of cosmic strings would behave as $1/a^2$ as function of M^4_+ proper time. The vision about dark matter as a phase characterized by gigantic Planck constant however implies that large voids do not expand in continuous manner during cosmic evolution but in discrete quantum jumps increasing the value of the gravitational Planck constant and thus increasing the size of the large void as a quantum state. Since the set of preferred values of Planck constant is closed under multiplication by powers of 2, p-adic length scales L_p, $p \simeq 2^k$ form a preferred set of sizes scales for the large voids.

3.3.5 *TGD cosmic strings are consistent with the fluctuations of CMB*

GUT cosmic strings were excluded by the fluctuation spectrum of the CMB background [E1] . In GRT framework these fluctuations can be classified to adiabatic density perturbations and isocurvature density perturbations. Adiabatic density perturbations correspond to overall scaling of various densities and do not affect the vanishing curvature scalar. For isocurvature density fluctuations the net energy density remains invariant. GUT cosmic strings predict isocurvature density perturbations while inflationary scenario predicts adiabatic density fluctuations.

In TGD framework inflation is replaced with quantum criticality of the phase transition period leading from the cosmic string dominated phase to matter dominated phase. Since curvature scalar vanishes during this period, the density perturbations are indeed adiabatic.

3.3.6 *Matter-antimatter asymmetry and cosmic strings*

Despite huge amount of work done during last decades (during the GUT era the problem was regarded as being solved!) matter-antimatter asymmetry remains still an unresolved problem of cosmology. A possible resolution of the problem is matter-antimatter asymmetry in the sense that cosmic strings contain antimatter and their exteriors matter. The challenge would be to understand the mechanism generating this asymmetry. The vanishing of the net gauge charges of cosmic string allows this symmetry since electro-weak charges of quarks and leptons can cancel each other.

The challenge is to identify the mechanism inducing the CP breaking necessary for the matter-antimatter asymmetry. Quite a small CP breaking inside cosmic strings would be enough.

1. The key observation is that vacuum extremals as such are not physically acceptable: small deformations of vacuum extremals to non-vacua are required. This applies also to cosmic strings since as such they do note present preferred extremals. The reason is that the preferred extremals involve necessary regions with Euclidian signature providing four-dimensional representations of generalized Feynman diagrams with particle quantum numbers at the light-like 3-surfaces at which the induced metric is degenerate.

2. The simplest deformation of vacuum extremals and cosmic strings would be induced by the topological condensation of CP_2 type vacuum extremals representing fermions. The topological condensation at larger space-time surface in turn creates bosons as wormhole contacts.

3. This process induces a Kähler electric fields and could induce a small Kähler electric charge inside cosmic string. This in turn would induce CP breaking inside cosmic string inducing matter antimatter asymmetry by the minimization of the ground state energy. Conservation of Kähler charge in turn would induce asymmetry outside cosmic string and the annihilation of matter and antimatter would then lead to a situation in which there is only matter.

4. Either galactic cosmic strings or big cosmic strings (in the sense of having large string tension) at the centers of galactic voids or both could generate the asymmetry and in the recent scenario big strings are not necessary. One might argue that the photon to baryon ratio $r \sim 10^{-9}$ characterizing matter asymmetry quantitatively must be expressible in terms of some fundamental constant possibly characterizing cosmic strings. The ratio $\epsilon = G/\hbar R^2 \simeq 4 \times 10^{-8}$ is certainly a fundamental constant in TGD Universe. By replacing R with $2\pi R$ would give $\epsilon = G/(2\pi R)^2 \simeq 1.0 \times 10^{-9}$. It would not be surprising if this parameter would determine the value of r.

The model can be criticized.

1. The model suggest only a mechanism and one can argue that the Kähler electric fields created by topological condensates could be random and would not generate any Kähler electric charge. Also the sign of the asymmetry could depend on cosmic string. A CP breaking at the fundamental level might be necessary to fix the sign of the breaking locally.

2. The model is not the only one that one can imagine. It is only required that antimatter is somewhere else. Antimatter could reside also at other p-adic space-time sheets and at the dark space-time sheets with different values of Planck constant.

The needed CP breaking is indeed predicted by the fundamental formulation of quantum TGD in terms of the Kähler-Dirac action associated with Kähler action and its generalization allowing include instanton term as imaginary part of Kähler action inducing CP breaking [K34, K21] .

1. The key idea in the formulation of quantum TGD in terms of modified Dirac equation associated with Kähler action is that the Dirac determinant defined by the generalized eigenvalues assignable to the Dirac operator D_K equals to the vacuum functional defined as the exponent of Kähler function in turn identifiable as Kähler action for a preferred extremal, whose proper identification becomes a challenge. In ZEO (ZEO) 3-surfaces are pairs of space-like 3-surfaces assignable to the boundaries of causal diamond (CD) and for deterministic action principle this suggests that the extremals are unique. In presence of non-determinism the situation changes.

2. The huge vacuum degeneracy of Kähler action suggests that for given pair of 3-surfaces at the boundaries of CD there is a continuum of extremals with the same Kähler action and conserved charges obtained from each other by conformal transformations acting as gauge symmetries and respecting the light-likeness of wormhole throats (as well as the vanishing of the determinant of space-time metric at them). The interpretation is in terms of quantum criticality with the hierarchy of symmetries defining a hierarchy of criticalities analogous to the hierarchy defined by the rank of the matrix defined by the second derivatives of potential function in Thom's catastrophe theory.

3. The number of gauge equivalence classes is expected to be finite integer n and the proposal is that it corresponds to the value of the effective Planck constant $h_{eff} = n \times h$ so that a connection with dark matter hierarchy labelled by values of n emerges [K13].

4. This representation generalizes - at least formally. One could add an imaginary instanton term to the Kähler function and corresponding Kähler-Dirac operator D_K so that the generalized eigenvalues assignable to D_K become complex. The generalized eigenvalues correspond to the square roots of the eigenvalues of the operator $DD^\dagger = (p^k\gamma_k + \Gamma^n)(p^k\gamma_k + \Gamma^n)^\dagger$ acting at the boundaries of string world sheets carrying fermion modes and it seems that only space-like 3-surfaces contribute. Γ^n is the normal component of the vector defined by Kähler-Dirac gamma matrices. One can define Dirac determinant formally as the product of the eigenvalues of $DD^\dagger$.

   The conjecture is that the resulting Dirac determinant equals to the exponent of Kähler action and imaginary instanton term for the preferred extremal. The instanton term does not contribute to the WCW metric but could provide a first principle description for CP breaking and anyonic effects. It also predicts the dependence of these effects on the page of the book like structure defined by the generalized imbedding space realizing the dark matter hierarchy with levels labeled by the value of Planck constant.

5. In the case of cosmic strings CP breaking could be especially significant and force the generation of Kähler electric charge. Instanton term is proportional to $1/h_{eff}$ so that CP breaking would be small for the gigantic values of h_{eff} characterizing dark matter. For small values of h_{eff} the breaking is large provided that the topological condensation is able to make the CP_2 projection of cosmic string four-dimensional so that the instanton contribution to the complexified Kähler action is non-vanishing and large enough. Since instanton contribution as a local divergence reduces to the contributions assignable to the light-like 3-surfaces X_l^3 representing topologically condensed particles, CP breaking is large if the density of topologically condensed fermions and wormhole contacts generated by the condensation of cosmic strings is high enough.

3.3.7 *CP breaking at the level of CKM matrix*

The CKM matrix for quarks contains CP breaking phase factors and this could lead to different evaporation rates for baryons and anti-baryons are different (quark cannot appear as vapor phase particle since vapor phase particle must have vanishing color gauge charges and in the recent vision about quantum TGD CP_2 type vacuum extremal which has not suffered topological condensation represents vacuum). The CP breaking at the level of CKM matrix would be implied by the instanton term present in the complexified Kähler action and Kähler-Dirac operator. The mechanism might rely on hadronic Kähler electric fields which are accompanied by color electric gauge fields proportional to induced Kähler form.

The topological condensation of quarks on hadronic strings containing weak color electric fields proportional to Kähler electric fields should be responsible for its string tension and this should in turn generate CP breaking. At the parton level the presence of CP breaking phase factor $exp(ikS_{CS})$, where $S_{CS} = \int_{X^4} J \wedge J + boundary\ term$ is purely topological Chern Simons term and naturally associated with the boundaries of space-time sheets with at most $D = 3$-dimensional CP_2 projection, could have something to do with the matter antimatter asymmetry. Note however that TGD predicts no strong CP breaking as QCD does [K3] .

3.3.8 *Development of strings in the string dominated cosmology*

The development of the string perturbations in the Robertson Walker cosmology has been studied [E9] and the general conclusion seems to be that that all the details smaller than horizon are rapidly smoothed out. One must of course take very cautiously the application of these result in TGD framework.

In present case, the horizon has an infinite size so that details in all scales should die away. To see what actually happens consider small perturbations of a static string along z-axis. Restrict the consideration to a perturbation in the y-direction. Using instead of the proper time coordinate t the "conformal time coordinate" τ defined by $d\tau = dt/a$ the equations of motion read [E9]

$$\begin{aligned} (\partial_\tau + \frac{2\dot{a}}{a})(\dot{y}U) &= \partial_z(y'U) \ , \\ U &= \frac{1}{\sqrt{1+(y')^2-\dot{y}^2}} \ . \end{aligned} \tag{3.47}$$

Rest Restrict the consideration to small perturbations for which the condition $U \simeq 1$ holds. For the string dominated cosmology the quantity $\dot{a}/a = 1/\sqrt{K}$ is constant and the equations of motion reduce to a very simple approximate form

$$\ddot{y} + \frac{2}{\sqrt{K}}\dot{y} - y'' = 0 \ . \tag{3.48}$$

The separable solutions of this equation are of type

$$\begin{aligned} y &= g(a)(C\sin(kz) + D\cos(kz)) \ , \\ g(a) &= (\frac{a}{a_0})^r \ . \end{aligned} \tag{3.49}$$

where r is a solution of the characteristic equation $r^2 + 2r/\sqrt{K} + k^2 = 0$:

$$r = -\frac{1}{\sqrt{K}}(1 \pm \sqrt{1-k^2K}) \ . \tag{3.50}$$

For perturbations of small wavelength $k > 1/\sqrt{K}$, an extremely rapid attenuation occurs; $1/\sqrt{K} \simeq 10^{27}$! For the long wavelength perturbations with $k << 1/\sqrt{K}$ (physical wavelength is larger than t) the attenuation is milder for the second root of above equation: attenuation takes place as $(a/a_0)^{\sqrt{K}k^2/2}$. The conclusion is that irregularities in all scales are smoothed away but that attenuation is much slower for the long wave length perturbations.

The absence of horizons in the string dominated phase has a rather interesting consequence. According to the well known Jeans criterion the size L of density fluctuations leading to the formation of structures [E9] must satisfy the following conditions

$$l_J < L < l_H \ , \tag{3.51}$$

where l_H denotes the size of horizon and l_J denotes the Jeans length related to the sound velocity v_s and cosmic proper time as [E9]

$$l_J \simeq 10 v_s t \ . \tag{3.52}$$

For a string dominated cosmology the size of the horizon is infinite so that no upper bound for the size of the possible structures results. These structures of course, correspond to string like objects of various sizes in the microscopic description. This suggests that primordial fluctuations create structures of arbitrary large size, which become visible at much later time, when cosmology becomes string dominated again.

3.3.9 *Limiting temperature*

Since particles are extended objects in TGD, one expects the existence of the limiting temperature T_H (Hagedorn temperature as it is called in string models) so that the primordial cosmology is in Hagedorn temperature. A special consequence is that the contribution of the light particles to the energy density becomes negligible: this is in accordance with the string dominance of the critical mass cosmology. The value of T_H is of order $T_H \sim \hbar/R$, where R is CP_2 radius of order $R \sim 10^{3.5}\sqrt{G}$ and thus considerably smaller than Planck temperature. Note that T_H increases with Planck constant and one can wonder whether this increase continues only up to $T_H = \hbar_{cr}/R = \sqrt{\hbar_{cr}/G}$, which corresponds to the critical value $\hbar_{cr} = R^2/G$. The value $R^2/G = 3 \times 20^{23}\hbar_0$ is consistent with p-adic mass calculations and is favored by by number theoretical arguments [K15, K3] .

The existence of limiting temperature gives strong constraint to the value of the light cone proper time a_F when radiation dominance must have established itself in the critical cosmology which gave rise to our sub-cosmology. Before the moment of transition to hyperbolic cosmology critical cosmology is string dominated and the generation of negative energy virtual gravitons builds up gradually the huge energy density density, which can lead to gravitational collapse, splitting of the strings and establishment of thermal equilibrium with gradually rising temperature. This temperature cannot however become higher than Hagedorn temperature T_H, which serves thus as the highest possible temperature of the effectively radiation dominated cosmology following the critical period. The decay of the split strings generates elementary particles providing the seeds of galaxies.

If most strings decay to light particles then energy density is certainly of the form $1/a^4$ of radiation dominated cosmology. This is not the only manner to obtain effective radiation dominance. Part of the thermal energy goes to the kinetic energy of the vibrational motion of strings and energy density $\rho \propto 1/a^2$ cannot hold anymore. The strings of the condensate is expected to obey the scaling law $\rho \propto 1/a^4$, $p = \rho/3$ [E9] . The simulations with string networks suggest that the energy density of the string network behaves as $\rho \propto 1/a^{2(1+v^2)}$, where v^2 is the mean square velocity of the point of the string [E12] . Therefore, if the value of the mean square velocity approaches light velocity, effective radiation dominance results even when strings dominate [E27] . In radiation dominated cosmology the velocity of sound is $v = 1/\sqrt{3}$. When v lowers to sound velocity one obtains stationary cosmology which is string dominated.

An estimate for a_F is obtained from the requirement that the temperature of the radiation dominated cosmology, when extrapolated from its value $T_R \simeq .3$eV at the time about $a_R \sim 3 \times 10^7$ years for the decoupling of radiation and matter to $a = a_F$ using the scaling law $T \propto 1/a$, corresponds to Hagedorn temperature. This gives

$$\begin{aligned} a_F &= a_R \frac{T_R}{T_H} \ , \\ T_H &= \frac{n}{R} \ , \qquad a_R \sim 3 \times 10^7 \ y \ , \quad T_R = .27 \ eV \ . \end{aligned} \tag{3.53}$$

This gives a rough estimate $a_F \sim 3 \times 10^{-10}$ seconds, which corresponds to length scale of order 7.7×10^{-2} meters. The value of a_F is quite large.

The result does not mean that radiation dominated sub-cosmologies might have not developed before $a = a_F$. In fact, entire series of critical sub-cosmologies could have developed to radiation dominated phase before the final one leading to our sub-cosmology is actually possible. The contribution of sub-cosmology i to the total energy density of recent cosmology is in the first approximation equal to the fraction $(a_F(i)/a_F)^4$. This ratio is multiplied by a ratio of numerical factors telling the number of effectively massless particle species present in the condensate if elementary particles dominate the mass density. If strings dominate the mass density (as expected) the numerical factor is absent.

For some reason the later critical cosmologies have not evolved to the radiation dominated phase. This might be due to the reduced density of cosmic strings in the vapor phase caused by the formation of the earlier cosmologies which does not allow sufficiently strong gravitational collapse to develop and implies that critical cosmology transforms directly to stationary cosmology without the intervening effectively radiation dominated phase. Indeed, condensed cosmic strings develop Kähler electric field compensating the huge positive Kähler action of free string and can survive the decay to light particles if they are not split. The density of split strings yielding light particles is presumably the proper parameter in this respect.

p-Adic length scale hypothesis allows rather predictive quantitative model for the series of sub-cosmologies [K24] predicting the number of them and allowing to estimate the moments of their birth, the durations of the critical periods and also the durations of radiation dominated phases. p-Adic length scale hypothesis allows also to estimate the maximum temperature achieved during the critical period: this temperature depends on the duration of the critical period a_1 as $T \sim n/a_1$, where n turns out to be of order 10^{30}. This means that if the duration of the critical period is long enough, transition to string dominated asymptotic cosmology occurs with the intervening decay of cosmic strings leading to the radiation dominated phase.

The existence of the limiting temperature has radical consequences concerning the properties of the very early cosmology. The contribution of a given massless particle to the energy density becomes constant. So, unless the number of the effectively massless particle families $N(a)$ increases too fast the contribution of the effectively massless particles to the energy density becomes negligible. The massive excitations of large size (string like objects) are indeed expected to become dominant in the mass density.

3.3.10 *What about thermodynamical implications of dark matter hierarchy?*

The previous discussion has not mentioned dark matter hierarchy labeled by increasing values of Planck constants and predicted macroscopic quantum coherence in arbitrarily long scales. In TGD Universe dark matter hierarchy means also a hierarchy of conscious entities with increasingly long span of memory and higher intelligence [K27, K11] .

This forces to ask whether the second law is really a fundamental law and whether it could reflect a wrong view about existence resulting resulting when all these dark matter levels and information associated with conscious experiences at these levels is neglected. For instance, biological evolution difficult to understand in a universe obeying second law relies crucially on evolution as gradual progress in which sudden leaps occur as new dark matter levels emerge.

TGD inspired consciousness suggests that Second Law holds true only for the mental images of a given self (a system able to avoid bound state entanglement with environment [K27]) rather than being a universal physical law. Besides these mental images there is irreducible basic awareness of self and second law does not apply to it. Also the hierarchy of higher level conscious entities is there. In this framework second law would basically reflect the exclusion of conscious observers from the physical model of the Universe.

3.4 Mechanism Of Accelerated Expansion In TGD Universe

In TGD framework the most plausible identification for the accelerated periods of cosmic expansion is in terms of phase transitions increasing gravitational Planck constant. These phase transitions would in average sense provide quantum counterpart for smooth cosmic expansion. These phase transitions might be initiated by the repulsive Coulomb interaction between cosmic strings driven to the boundaries of the large voids. It is interesting to see how this view relates with the assumption of positive cosmological constant.

3.4.1 *How accelerated expansion results in standard cosmology?*

The accelerated of cosmic expansion means that the deceleration parameter

$$q = -(ad^2a/ds^2)/(da/ds)^2$$

is negative. For Robertson-Walker cosmologies one has

$$\begin{aligned} H^2 &\equiv (\frac{da/ds}{a})^2 = \frac{8\pi G\rho + \Lambda}{3} - K/a^2, \quad K = 0, \pm 1 \ , \\ 3\frac{d^2a/ds^2}{a} &= \Lambda - 4\pi G(\rho + 3p) \equiv -4\pi G(1+3w)\rho \ . \end{aligned} \tag{3.54}$$

It is clear that the accelerated expansion requires positive value of Λ.

The deceleration parameter can be expressed as $q = \frac{1}{2}(1+3w)(1+K/(aH)^2)$. $K =, 0, 1, -1$ tells whether the cosmology is flat, hyper-spherical, or hyperbolic. The rate for the change of Hubble constant can be expressed as $(dH/ds)/H^2 = (1+q)$ and the acceleration of cosmic expansion means $q < -1$. All particle models predict $q \geq -1$.

On basis of modified Einstein's equations written for the recent metric convention (+,-,-.-) (note that opposite signature changes the sign of the left hand side)

$$-G^{\alpha\beta} - \Lambda g^{\alpha\beta} = 8\pi G T^{\alpha\beta} \tag{3.55}$$

it is clear that the introduction of a positive cosmological constant could be interpreted by saying that for gravitational vacuum carries energy density equal to $\Lambda/8\pi$ and negative pressure. The negative gravitational pressure would induce the acceleration.

Cosmological term at the level of field equations could be also interpreted by saying that Einstein's equations hold true in the original sense but that energy momentum tensor contains besides the density of inertial mass also a positive density of purely gravitational mass: $T \rightarrow T + \Lambda g$ so that Equivalence Principle fails. Since cosmological constant means effectively negative pressure $p = -\Lambda/8\pi$ the introduction of the cosmological constant means the effective replacement $\rho+3p \rightarrow \rho+3p-2\Lambda/8\pi$). In the so called $\Lambda-CDM$ model [E4] the densities of dark energy, ordinary matter, and dark matter are assumed to sum up to critical mass density $\rho_{cr} = 3/(8\pi g_{aa} G a^2)$. The fraction of dark matter density is deduced to be $\Omega_\Lambda = .74$ from mere criticality.

3.4.2 *Critical cosmology predicts accelerated expansion*

In order to get clue about the mechanism of accelerated cosmic expansion in TGD framework it is useful to study the deceleration parameter for various cosmologies in TGD framework.

In standard Friedmann cosmology with non-vanishing cosmological constant one has

$$3\frac{d^2a/ds^2}{a} = \Lambda - 4\pi G(\rho + 3p) \ . \tag{3.56}$$

From this form it is obvious why $\Lambda > 0$ is required in order to obtain accelerating expansion.

Deceleration parameter is a purely geometric property of cosmology and defined as

$$q \equiv -a\frac{d^2a/ds^2}{(da/ds)^2} \ . \tag{3.57}$$

During radiation and matter dominated phases the value of q is positive. In TGD framework there are several metrics which are independent of details of dynamics.

1. String dominated cosmology

String dominated cosmology is hyperbolic cosmology and might serve as a model for very early cosmology corresponds to the metric

$$g_{aa} \equiv (ds/da)^2 = 1 - K_0 \ . \tag{3.58}$$

In this case one has $q = 0$.

2. Critical cosmology

Critical cosmology with flat 3-space corresponds to

$$\begin{aligned} g_{aa} &= 1 - K \ , \\ K &\equiv \frac{K_0}{1-u^2} \ , \\ u &\equiv \frac{a}{a_1} \ . \end{aligned} \quad (3.59)$$

g_{aa} has the same form also for over-critical cosmologies. Both cosmologies have finite duration. In this case q is given by

$$q = -K_0 \frac{K_0 u^2}{1 - u^2 - K_0} < 0 \ , \quad (3.60)$$

and is negative. The rate of change for Hubble constant is

$$\frac{dH/ds}{H^2} = -(1+q) \ , \quad (3.61)$$

so that one must have $q < -1$ in order to have acceleration. This holds true for $a > \sqrt{(1-K_0)/(1+K_0)}a_1$.

Quantum critical cosmology could be seen as a universal characteristic of quantum critical phases associated with phase transition like phenomena. No assumptions about the mechanism behind the transition are made. There is great temptation to assign this cosmology to the phase transitions increasing the size of large voids occurring during late cosmology. The observed jerk assumed to lead from de-accelerated to accelerated expansion for about 13 billion years ago might have interpretation as a transition of this kind.

3. Stationary cosmology

TGD predicts a one-parameter family of stationary cosmologies from the requirement that the density of gravitational 4-momentum is conserved. This is guaranteed if curvature scalar is extremized. These cosmologies are expected to define asymptotic cosmologies or at least characterize the stationary phases between quantum phase transitions. The metric is given by

$$\begin{aligned} g_{aa} &= \frac{1-2x}{1-x} \ , \\ x &= (\frac{a_0}{a})^{2/3} \ . \end{aligned} \quad (3.62)$$

The deceleration parameter

$$q = \frac{1}{3} \frac{x}{(1-2x)(1-x)} \ . \quad (3.63)$$

is positive so that it seems that TGD does not lead to a continual acceleration which might be regarded as tearing galaxies into pieces.

If quantum critical phases correspond to the expansion of large voids induced by the accelerated radial motion of galactic strings as they reach the boundaries of the voids, one can consider a series of phase transitions between stationary cosmologies in which the value of gravitational Planck constant and the parameter a_0 characterizing the stationary cosmology increase by some even power of two as the ruler-and-compass integer hypothesis [K15, K13] and p-adic length scale hypothesis suggests.

4. Summary

One can safely conclude that TGD predict accelerated cosmic expansion during critical periods and that dark energy is replaced with dark matter in TGD framework. There is also a rather clear view about detailed mechanism leading to the accelerated expansion at "microscopic" level. Some summarizing remarks are in order.

1. Accelerated expansion is predicted only during periods of over-critical and critical cosmologies parameterized essentially by their duration. The microscopic description would be in terms of phase transitions increasing the size scale of large void. This phase transition is basically a quantum jump increasing gravitational Planck constant and thus the size of the large void. p-Adic length scales are favored sizes of the large voids. A large piece of 4-D cosmological history would be replaced by a new one in this transition so that quite a dramatic event would be in question.

2. p-Adic fractality forces to ask whether there is a fractal hierarchy of time scales in which Equivalence Principle in the formulation provided by General Relativity sense fails locally (no failure in stringy sense). This would predict a fractal hierarchy of large voids and phase transitions during which accelerated expansion occurs.

3. Cosmological constant can be said to be vanishing in TGD framework and the description of accelerated expansion in terms of a positive cosmological constant is not equivalent with TGD description since only effective pressure is negative. TGD description has some resemblance to the description in terms of quintessence [E6] , a hypothetical form of matter for which equation of state is of form $p = -w\rho$, $w < -1/3$, so that one has $\rho + 3p = 1 - w < 0$ and deceleration parameter can be negative. The energy density of quintessence is however positive. TGD does not predict endlessly accelerated acceleration tearing galaxies into pieces if the total purely gravitational energy of large voids is assumed to vanish so that Equivalence Principle holds above this length scale.

3.4.3 *TGD counterpart of Λ as a density of dark matter rather than dark energy*

The value of Λ is expressed usually as a fraction of vacuum energy density from the critical mass density. Combining the data about acceleration of cosmic expansion with the data about cosmic microwave background gives $\Omega_\Lambda \simeq .74$.

1. Critical mass density requires also in TGD framework the presence of dark contribution since visible matter contribute only a few percent of the total mass density and $\Omega_\Lambda \simeq .-74$ characterizes this contribution. Since the acceleration mechanism has nothing to do with dark energy, dark energy can be replaced with dark matter in TGD framework.

2. The dark matter hierarchy labeled by the values of Planck constant suggests itself. The $1/a^2$ behavior of dark matter density suggests an interpretation as dark matter topologically condensed on cosmic strings. Besides ordinary particles also super-symplectic bosons and their super partners playing a key role in the model of hadrons and black holes suggest themselves.

3. Stationary cosmology predicts that the density of stringy matter and thus dark matter decreases like $1/a^2$ as a function of M^4_+ proper time. This behavior is very natural in cosmic string dominated cosmology and one expects that the TGD counterpart of cosmological constant should behave as $\Lambda \propto 1/a^2$ in average sense. At primordial period cosmological constant would be gigantic but its recent value would be extremely small and naturally of correct order of magnitude if the fraction of positive gravitational energy is few per cent about negative gravitational energy. Hence the basic problem of the standard cosmology would find an elegant solution.

3.4.4 *Piecewise constancy of TGD counterpart of Λ and p-adic length scale hypothesis*

There are good reasons to believe that TGD counterpart of Λ is piecewise constant. Classical picture suggests that the sizes of large voids increase in discrete jumps. The transitions increasing the size of the void would occur when the galactic strings end up to the boundary of the large void and large repulsive Coulomb energy forces the phase transition increasing Planck constant.

Also the quantum astrophysics based on the notion of gravitational Planck constant strongly suggests that astrophysical systems are analogous to stationary states of atoms so that the sizes of astrophysical systems remain constant during the cosmological expansion, and can change only in quantum jumps increasing the value of Planck constant and therefore increasing the radius of the large void regarded as dark matter bound state.

Since the set of preferred values of Planck constant is closed under multiplication by powers of 2, p-adic length scales L_p, $p \simeq 2^k$ form a preferred set of sizes scales for the large voids with phase transitions increasing k by even integer. What values of k are realized depends on the time scale of the dynamics driving the galactic strings to the boundaries of expanded large void. Even if all values of k are realized the transitions becomes very rare for large values of a.

p-Adic fractality predicts that the effective cosmological constant Λ scales as $1/L^2(k)$ as a function of the p-adic scale characterizing the space-time sheet implying a series of phase transitions reducing the value of effective cosmological constant Λ. As noticed, the allowed values of k would be of form $k = k_0 + 2n$, where however all integer value need not be realized. By p-adic length scale hypothesis primes are candidates for k. The recent value of the effective cosmological constant can be understood. The gravitational energy density usually assigned to the cosmological constant is identifiable as that associated with topologically condensed cosmic strings and magnetic flux tubes to which they are gradually transformed during cosmological evolution.

p-Adic prediction is consistent with the recent study [E29] according to which cosmological constant has not changed during the last 8 billion years: the conclusion comes from the reshifts of supernovae of type I_a. If p-adic length scales $L_e(k) = p \simeq 2^k$, k any positive integer, are allowed, the finding gives the lower bound $T_N > \sqrt(2)/(\sqrt{2}-1)) \times 8 = 27.3$ billion years for the recent age of the universe.

Brad Shaefer from Lousiana University has studied the red shifts of gamma ray bursters up to a red shift $z = 6.3$, which corresponds to a distance of 13 billion light years [E11] , and claims that the fit to the data is not consistent with the time independence of the cosmological constant. In TGD framework this would mean that a phase transition changing the value of the cosmological constant must have occurred during last 13 billion years. In principle the phase transitions increasing the size of large voids could be observed as sudden changes of sign for the deceleration parameter.

3.4.5 *The reported cosmic jerk as an accelerated period of cosmic expansion*

There is an objection against the hypothesis that cosmological constant has been gradually decreasing during the cosmic evolution. Type Ia supernovae at red shift $z \sim .45$ are fainter than expected, and the interpretation is in terms of an accelerated cosmic expansion [E10] . If a period of an accelerated expansion has been preceded by a decelerated one, one would naively expect that for older supernovae from the period of decelerating expansion, say at redshifts about $z > 1$, the effect should be opposite. The team led by Adam Riess [E19] has identified 16 type Ia supernovae at redshifts $z > 1.25$ and concluded that these supernovae are indeed brighter. The conclusion is that about about 5 billion years ago corresponding to $z \simeq .48$, the expansion of the Universe has suffered a cosmic jerk and transformed from a decelerated to an accelerated expansion.

The apparent dimming/brightening of supernovae at the period of accelerated/decelerated expansion the follows from the luminosity distance relation

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi d_L^2} \ , \tag{3.64}$$

where $\mathcal{L}$ is actual luminosity and $\mathcal{F}$ measured luminosity, and from the expression for the distance d_L in flat cosmology in terms of red shift z in a flat Universe

$$\begin{aligned} d_L &= (1+z)\int_0^z \frac{du}{H(u)} \\ &= (1+z)H_0^{-1}\int_0^z exp\left[-\int_0^u du\,[1+q(u)]\,d(ln(1+u)\right] du \ , \end{aligned} \tag{3.65}$$

where one has

$$H(z) = \frac{dln(a)}{ds} ,$$
$$q \equiv -\frac{d^2a/ds^2}{aH^2} = \frac{dH^{-1}}{ds} - 1 . \quad (3.66)$$

In TGD framework a corresponds to the light-cone proper time and s to the proper time of Robertson-Walker cosmology. Depending on the sign of the deceleration parameter q, the distance d_L is larger or smaller and accordingly the object looks dimmer or brighter.

The natural interpretation for the jerk would be as a period of accelerated cosmic expansion due to a phase transition increasing the value of gravitational Planck constant.

4 Microscopic Description Of Black-Holes In TGD Universe

In TGD framework the imbedding of the metric for the interior of Schwartshild black-hole fails below some critical radius. This strongly suggests that only the exterior metric of black-hole makes sense in TGD framework and that TGD must provide a microscopic description of black-holes. Somewhat unexpectedly, I ended up with this description from a model of hadrons.

Super-symplectic algebra is a generalization of Kac-Moody algebra obtained by replacing the finite-dimensional group G with the group of symplectic transformations of $\delta M^4_\pm \times CP_2$. This algebra defines the group of isometries for the "world of classical worlds" and together with the Kac-Moody algebra assignable to the deformations of light-like 3-surfaces representing orbits of 2-D partonic surfaces it defines the mathematical backbone of quantum TGD as almost topological QFT.

From the point of view of experimentalist the basic question is how these super-symplectic degrees of freedom reflect themselves in existing physics and the pleasant surprise was that super-symplectic bosons explain what might be called the missing hadronic mass and spin. The point is that quarks explain only about 170 MeV of proton mass. Also the spin puzzle of proton is known for years. Also precise mass formulas for hadrons emerge.

Super-symplectic degrees of freedom represent dark matter in electro-weak sense and highly entangled hadronic strings in Hagedorn temperature are very much analogous to black-holes. This indeed generalizes to a microscopic model for black-holes created when hadronic strings fuse together in high density.

4.1 Super-Symplectic Bosons

TGD predicts also exotic bosons which are analogous to fermion in the sense that they correspond to single wormhole throat associated with CP_2 type vacuum extremal whereas ordinary gauge bosons corresponds to a pair of wormhole contacts assignable to wormhole contact connecting positive and negative energy space-time sheets. These bosons have super-conformal partners with quantum numbers of right handed neutrino and thus having no electro-weak couplings. The bosons are created by the purely bosonic part of super-symplectic algebra [K8, K34] , whose generators belong to the representations of the color group and 3-D rotation group but have vanishing electro-weak quantum numbers. Their spin is analogous to orbital angular momentum whereas the spin of ordinary gauge bosons reduces to fermionic spin. Recall that super-symplectic algebra is crucial for the construction of WCW Kähler geometry. If one assumes that super-symplectic gluons suffer topological mixing identical with that suffered by say U type quarks, the conformal weights would be (5,6,58) for the three lowest generations. The application of super-symplectic bosons in TGD based model of hadron masses is discussed in [K18] and here only a brief summary is given.

As explained in [K18] , the assignment of these bosons to hadronic space-time sheet is an attractive idea.

1. Quarks explain only a small fraction of the baryon mass and that there is an additional contribution which in a good approximation does not depend on baryon. This contribution should correspond to the non-perturbative aspects of QCD. A possible identification of this

contribution is in terms of super-symplectic gluons. Baryonic space-time sheet with $k = 107$ would contain a many-particle state of super-symplectic gluons with net conformal weight of 16 units. This leads to a model of baryons masses in which masses are predicted with an accuracy better than 1 per cent.

2. Hadronic string model provides a phenomenological description of non-perturbative aspects of QCD and a connection with the hadronic string model indeed emerges. Hadronic string tension is predicted correctly from the additivity of mass squared for $J = 2$ bound states of super-symplectic quanta. If the topological mixing for super-symplectic bosons is equal to that for U type quarks then a 3-particle state formed by 2 super-symplectic quanta from the first generation and 1 quantum from the second generation would define baryonic ground state with 16 units of conformal weight. A very precise prediction for hadron masses results by assuming that the spin of hadron correlates with its super-symplectic particle content.

3. Also the baryonic spin puzzle caused by the fact that quarks give only a small contribution to the spin of baryons, could find a natural solution since these bosons could give to the spin of baryon an angular momentum like contribution having nothing to do with the angular momentum of quarks.

4. Super-symplectic bosons suggest a solution to several other anomalies related to hadron physics. The events observed for a couple of years ago in RHIC [C1] suggest a creation of a black-hole like state in the collision of heavy nuclei and inspire the notion of color glass condensate of gluons, whose natural identification in TGD framework would be in terms of a fusion of hadronic space-time sheets containing super-symplectic matter materialized also from the collision energy. In the collision, valence quarks connected together by color bonds to form separate units would evaporate from their hadronic space-time sheets in the collision, and would define TGD counterpart of Pomeron, which experienced a reincarnation for few years ago [C2]. The strange features of the events related to the collisions of high energy cosmic rays with hadrons of atmosphere (the particles in question are hadron like but the penetration length is anomalously long and the rate for the production of hadrons increases as one approaches surface of Earth) could be also understood in terms of the same general mechanism.

4.2 Are Ordinary Black-Holes Replaced With Super-Symplectic Black-Holes In TGD Universe?

Some variants of super string model predict the production of small black-holes at LHC. I have never taken this idea seriously but in a well-defined sense TGD predicts black-hole like states associated with super-symplectic gravitons with strong gravitational constant defined by the hadronic string tension. The proposal is that super-symplectic black-holes have been already seen in Hera, RHIC, and the strange cosmic ray events.

Baryonic super-symplectic black-holes of the ordinary M_{107} hadron physics would have mass 934.2 MeV, very near to proton mass. The mass of their M_{89} counterparts would be 512 times higher, about 478 GeV. "Ionization energy" for Pomeron, the structure formed by valence quarks connected by color bonds separating from the space-time sheet of super-symplectic black-hole in the production process, corresponds to the total quark mass and is about 170 MeV for ordinary proton and 87 GeV for M_{89} proton. This kind of picture about black-hole formation expected to occur in LHC differs from the stringy picture since a fusion of the hadronic mini black-holes to a larger black-hole is in question.

An interesting question is whether the ultrahigh energy cosmic rays having energies larger than the GZK cut-off of 5×10^{10} GeV are baryons, which have lost their valence quarks in a collision with hadron and therefore have no interactions with the microwave background so that they are able to propagate through long distances.

In neutron stars the hadronic space-time sheets could form a gigantic super-symplectic black-hole and ordinary black-holes would be naturally replaced with super-symplectic black-holes in TGD framework (only a small part of black-hole interior metric is representable as an induced metric). This obviously means a profound difference between TGD and string models.

1. Hawking-Bekenstein black-hole entropy would be replaced with its p-adic counterpart given by

$$S_p = (\frac{M}{m(CP_2)})^2 \times log(p) \ , \quad (4.1)$$

where $m(CP_2)$ is CP_2 mass, which is roughly 10^{-4} times Planck mass. M is the contribution of p-adic thermodynamics to the mass. This contribution is extremely small for gauge bosons but for fermions and super-symplectic particles it gives the entire mass.

2. If p-adic length scale hypothesis $p \simeq 2^k$ holds true, one obtains

$$S_p = klog(2) \times (\frac{M}{m(CP_2)})^2, \quad (4.2)$$

$m(CP_2) = \hbar/R$, R the "radius" of CP_2, corresponds to the standard value of h for all values of h_{eff}.

3. Hawking-Bekenstein area law gives in the case of Schwartschild black-hole

$$S = \frac{A}{4G} \times \hbar = \pi G M^2 \times \hbar \ . \quad (4.3)$$

For the p-adic variant of the law Planck mass is replaced with CP_2 mass and $klog(2) \simeq log(p)$ appears as an additional factor. Area law is obtained in the case of elementary particles if k is prime and wormhole throats have M^4 radius given by p-adic length scale $L_k = \sqrt{k}R$ which is exponentially smaller than L_p. For macroscopic super-symplectic black-holes modified area law results if the radius of the large wormhole throat equals to Schwartschild radius. Schwartschild radius is indeed natural: a simple deformation of the Schwartschild exterior metric to a metric representing rotating star transforms Schwartschild horizon to a light-like 3-surface at which the signature of the induced metric is transformed from Minkowskian to Euclidian.

4. The formula for the gravitational Planck constant appearing in the Bohr quantization of planetary orbits and characterizing the gravitational field body mediating gravitational interaction between masses M and m [K24] reads as

$$\hbar_{gr} = \frac{GMm}{v_0}\hbar_0 \ .$$

$v_0 = 2^{-11}$ is the preferred value of v_0. One could argue that the value of gravitational Planck constant is such that the Compton length $\hbar_{gr}/M$ of the black-hole equals to its Schwartshild radius. This would give

$$\hbar_{gr} = \frac{GM^2}{v_0}\hbar_0 \ , \ v_0 = 1/2 \ . \quad (4.4)$$

The requirement that $\hbar_{gr}$ is a ratio of ruler-and-compass integers expressible as a product of distinct Fermat primes (only four of them are known) and power of 2 would quantize the mass spectrum of black hole [K24] . Even without this constraint M^2 is integer valued using p-adic mass squared unit and if p-adic length scale hypothesis holds true this unit is in an excellent approximation power of two.

5. The gravitational collapse of a star would correspond to a process in which the initial value of v_0 , say $v_0 = 2^{-11}$, increases in a stepwise manner to some value $v_0 \leq 1/2$. For a supernova with solar mass with radius of 9 km the final value of v_0 would be $v_0 = 1/6$. The star could have an onion like structure with largest values of v_0 at the core as suggested by the model of planetary system. Powers of two would be favored values of v_0. If the formula holds true also for Sun one obtains $1/v_0 = 3 \times 17 \times 2^{13}$ with 10 per cent error.

6. Black-hole evaporation could be seen as means for the super-symplectic black-hole to get rid of its electro-weak charges and fermion numbers (except right handed neutrino number) as the antiparticles of the emitted particles annihilate with the particles inside super-symplectic black-hole. This kind of minimally interacting state is a natural final state of star. Ideal super-symplectic black-hole would have only angular momentum and right handed neutrino number.

7. In TGD light-like partonic 3-surfaces are the fundamental objects and space-time interior defines only the classical correlates of quantum physics. The space-time sheet containing the highly entangled cosmic string might be separated from environment by a wormhole contact with size of black-hole horizon.

This looks the most plausible option but one can of course ask whether the large partonic 3-surface defining the horizon of the black-hole actually contains all super-symplectic particles so that super-symplectic black-hole would be single gigantic super-symplectic parton. The interior of super-symplectic black-hole would be a space-like region of space-time, perhaps resulting as a large deformation of CP_2 type vacuum extremal. Black-hole sized wormhole contact would define a gauge boson like variant of the black-hole connecting two space-time sheets and getting its mass through Higgs mechanism. A good guess is that these states are extremely light.

4.3 Anyonic View About Blackholes

A new element to the model of black hole comes from the vision that black hole horizon as a light-like 3-surface corresponds to a light-like orbit of light-like partonic 2-surface. This allows two kinds of black holes. Fermion like black hole would correspond to a deformed CP_2 type extremal which Euclidian signature of metric and topologically condensed at a space-time sheet with a Minkowskian signature. Boson like black hole would correspond to a wormhole contact connecting two space-time sheets with Minkowskian signature. Wormhole contact would be a piece deformed CP_2 type extremal possessing two light-like throats defining two black hole horizons very near to each other. It does not seem absolutely necessary to assume that the interior metric of the black-hole is realized in another space-time sheet with Minkowskian signature.

Second new element relates to the value of Planck constant. For $\hbar_{gr} = 4GM^2$ the Planck length $L_P(\hbar) = \sqrt{\hbar G}$ equals to Schwartschild radius and Planck mass equals to $M_P(\hbar) = \sqrt{\hbar/G} = 2M$. If the mass of the system is below the ordinary Planck mass: $M \leq m_P(\hbar_0)/2 = \sqrt{\hbar_0/4G}$, gravitational Planck constant is smaller than the ordinary Planck constant.

Black hole surface contains ultra dense matter so that perturbation theory is not expected to converge for the standard value of Planck constant but do so for gravitational Planck constant. If the phase transition increasing Planck constant is a friendly gesture of Nature making perturbation theory convergent, one expects that only the black holes for which Planck constant is such that $GM^2/4\pi\hbar < 1$ holds true are formed. Black hole entropy - being proportional to $1/\hbar$- is of order unity so that TGD black holes are not very entropic. $\hbar = GM^2/v_0$, $v_0 = 1/4$, would hold true for an ideal black hole with Planck length $(\hbar G)^{1/2}$ equal to Schwartshild radius $2GM$. Since black hole entropy is inversely proportional to $\hbar$, this would predict black hole entropy to be of order single bit. This of course looks totally non-sensible if one believes in standard thermodynamics. For the star with mass equal to 10^{40} Planck masses the entropy associated with the initial state of the star would be roughly the number of atoms in star equal to about 10^{60}. Black hole entropy proportional to $GM^2/\hbar$ would be of order 10^{80} provided the standard value of $\hbar$ is used as unit. This stimulates some questions.

1. Does second law pose an upper bound on the value of $\hbar$ of dark black hole from the requirement that black hole has at least the entropy of the initial state. The maximum value of $\hbar$

would be given by the ratio of black hole entropy to the entropy of the initial state and about 10^{20} in the example consider to be compared with $GM^2/v_0 \sim 10^{80}$.

2. Or should one generalize thermodynamics in a manner suggested by ZEO by making explicit distinction between subjective time (sequence of quantum jumps) and geometric time? The arrow of geometric time would correlate with that of subjective time. One can argue that the geometric time has opposite direction for the positive and negative energy parts of the zero energy state interpreted in standard ontology as initial and final states of quantum event. If second law would hold true with respect to subjective time, the formation of ideal dark black hole would destroy entropy only from the point of view of observer with standard arrow of geometric time. The behavior of phase conjugate laser light would be a more mundane example. Do self assembly processes serve as example of non-standard arrow of geometric time in biological systems? In fact, zero energy state is geometrically analogous to a big bang followed by big crunch. One can however criticize the basic assumption as ad hoc guess. One should really understand the the arrow of geometric time. This is discussed in detail in [L2] .

If the partonic 2-surface surrounds the tip of causal diamond CD, the matter at its surface is in anyonic state with fractional charges. Anyonic black hole can be seen as single gigantic elementary particle stabilized by fractional quantum numbers of the constituents preventing them from escaping from the system and transforming to ordinary visible matter.

One can imagine that the partonic surface is not exact sphere except for ideal black holes but contains large number of magnetic flux tubes giving rise to handles. Also a pair of spheres with different radii can be considered with surfaces of spheres connected by braided flux tubes. The braiding of these handles can represent information and one can even consider the possibility that black hole can act as a topological quantum computer. There would be no sharp difference between the dark parts of black holes and those of ordinary stars. Only the volume containing the complex flux tube structures associated with the orbits of planets and various objects around star would become very small for black hole so that the black hole might code for the topological information of the matter collapsed into it.

5 A Quantum Model For The Formation Of Astrophysical Structures And Dark Matter?

D. Da Rocha and Laurent Nottale, the developer of Scale Relativity, have ended up with an highly interesting quantum theory like model for the evolution of astrophysical systems [E14] (I am grateful for Victor Christianito for informing me about the article). In particular, this model applies to planetary orbits. I learned later that also A. Rubric and J. Rubric have proposed a Bohr model for planetary orbits [E24] already 1998.

The model is simply Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant

$$\hbar \quad \rightarrow \quad \hbar_{gr} = \frac{GmM}{v_0} \quad . \tag{5.1}$$

Here I have used units $\hbar = c = 1$. v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. The peak orbital velocity of stars in galactic halos is 142 ± 2 km/s whereas the average velocity is 156 ± 2 km/s. Also sub-harmonics and harmonics of v_0 seem to appear.

The model makes fascinating predictions which hold true. For instance, the radii of planetary orbits fit nicely with the prediction of the hydrogen atom like model. The inner solar system (planets up to Mars) corresponds to v_0 and outer solar system to $v_0/5$.

The predictions for the distribution of major axis and eccentricities have been tested successfully also for exoplanets. Also the periods of 3 planets around pulsar PSR B1257+12 fit with the predictions with a relative accuracy of few hours/per several months. Also predictions for the distribution of stars in the regions where morphogenesis occurs follow from the gravitational Schödinger equation.

What is important is that there are no free parameters besides v_0. In [E14] a wide variety of astrophysical data is discussed and it seem that the model works and has already now made predictions which have been later verified. In the following I shall discuss Nottale's model from the point of view of TGD.

5.1 TGD Prediction For The Parameter v_0

One of the basic questions is the origin of the parameter v_0, which according to a rich amount of experimental data discussed in [E14] seems to play a role of a constant of Nature. One of the first applications of cosmic strings in TGD sense was an explanation of the velocity spectrum of stars in the galactic halo in terms of dark matter which could consists of cosmic strings. Cosmic strings could be orthogonal to the galactic plane going through the nucleus (jets) or they could be in galactic plane in which case the strings and their decay products would explain dark matter assuming that the length of cosmic string inside a sphere of radius R is or has been roughly R [K9] . The predicted value of the string tension is determined by the CP_2 radius whose ratio to Planck length is fixed by electron mass via p-adic mass calculations. The resulting prediction for the v_0 is correct and provides a working model for the constant orbital velocity of stars in the galactic halo.

The parameter $v_0 \simeq 2^{-11}$, which has actually the dimension of velocity unless on puts $c = 1$, and also its harmonics and sub-harmonics appear in the scaling of $\hbar$. v_0 corresponds to the velocity of distant stars in the model of galactic dark matter. TGD allows to identify this parameter as the parameter

$$\begin{aligned} v_0 &= 2\sqrt{TG} = \sqrt{\frac{1}{2\alpha_K}}\sqrt{\frac{G}{R^2}} \ , \\ T &= \frac{1}{8\alpha_K}\frac{\hbar_0}{R^2} \ . \end{aligned} \quad (5.2)$$

Here T is the string tension of cosmic strings, R denotes the "radius" of CP_2 ($2R$ is the radius of geodesic sphere of CP_2). α_K is Kähler coupling strength, the basic coupling constant strength of TGD, whose evolution as a function of p-adic length scale is fixed by quantum criticality. The condition that G is invariant in the p-adic coupling constant evolution and number theoretical arguments predict

$$\begin{aligned} \alpha_K(p) &= k\frac{1}{log(p)+log(K)} \ , \\ K &= \frac{R^2}{\hbar_0 G} = 2\times 3\times 5\times 7\times 11\times 13\times 17\times 19\times 23 \ , \quad k\simeq \pi/4 \ . \end{aligned} \quad (5.3)$$

The predicted value of v_0 depends logarithmically on the p-adic length scale and for $p \simeq 2^{127}-1$ (electron's p-adic length scale) one has $v_0 \simeq 2^{-11}$.

5.2 Model For Planetary Orbits Without $v_0 \Rightarrow V_0/5$Scaling

Also harmonics and sub-harmonics of v_0 appear in the model of Nottale and Da Rocha. For instance, the outer planets (Jupiter, Saturn,...) correspond to $v_0/5$ whereas inner planets correspond to v_0. Quite generally, it is found that the values seem to come as harmonics and sub-harmonics of v_0: $v_n = nv_0$ and v_0/n, and the argument [E14] is that the different values of n relate to fractality. This scaling is not necessary for the planetary orbits in TGD based model.

Effectively a multiplication $n \rightarrow 5n$ of the principal quantum number is in question in the case of outer planets. If one accepts the interpretation that visible matter has concentrated around dark matter, which is in macroscopic quantum phase around Bohr orbits, this allows to consider also the possibility that $\hbar_{gr}$ has the same value for all planets.

1. Some gravitational perturbation has kicked dark matter from the region of the asteroid belt to $n \simeq 5k$, $k = 2,..,6$, orbits. The best fit is obtained by using values of n deviating somewhat from multiples of 5 which suggests that the scaling of v_0 is not needed. Gravitational

perturbations might have caused the same for the visible matter. The fact that the tilt angles of Earth and outer planets other than Pluto are nearly the same suggests that the orbits of these planets might be an outcome of some violent quantum process for dark matter preserving the orbital plane in a good approximation. Pluto might in turn have experienced some violent collision changing its orbital plane.

2. There could exist at least small amounts of dark matter at all orbits but visible matter is concentrated only around orbits containing some critical amount of dark matter.

Table 1 gives the radii of planet orbits predicted by Bohr orbit model and by Titius-Bode law.

Table 1: Table represents the experimental average orbital radii of planets, the predictions of Titius-Bode law (note the failure for Neptune), and the predictions of Bohr orbit model assuming a) that the principal quantum number n corresponds to best possible fit, b) the scaling $v_0 \to v_0/5$ for outer planets. Option a) gives the best fit with errors being considerably smaller than the maximal error $|\Delta R|/R \simeq 1/n$ except for Uranus. R_M denotes the orbital radius of Mercury. T-B refers to Titius-Bode law.

	Exp.	T-B	$Bohr_1$	$Bohr_2$
Planet	R/R_M	R/R_M	$[n, R/R_M]$	$[n, R/R_M]$
Mercury	1	1	[3, 1]	
Venus	1.89	1.75	[4, 1.8]	
Earth	2.6	2.5	[5, 2.8]	
Mars	3.9	4	[6, 4]	
Asteroids	6.1-8.7	7	[(7, 8, 9), (5.4, 7.1, 9)]	
Jupiter	13.7	13	[11, 13.4]	[2×5,11.1]
Saturn	25.0	25	[3×5, 25]	[3×5, 25]
Uranus	51.5	49	[22, 53.8]	[4×5,44.4]
Neptune	78.9	97	[27 , 81]	[5×5, 69.4]
Pluto	105.2	97	[31, 106.7]	[6×5,100]

5.2.1 *How to understand the harmonics and sub-harmonics of v_0 in TGD framework?*

Also harmonics and sub-harmonics of v_0 appear in the model of Nottale and Da Rocha. In particular, the outer planets (Jupiter, Saturn,...) correspond to $v_0/5$ whereas inner planets correspond to v_0 in this model. As already found, TGD allows also an alternative explanation.

Quite generally, it is found that the values seem to come as harmonics and sub-harmonics of v_0: $v_n = nv_0$ and v_0/n, and the argument [E14] is that the different values of n relate to fractality. This quantization is a challenge for TGD since v_0 certainly defines a fundamental constant in TGD Universe.

1. Consider first the harmonics of v_0. Besides cosmic strings of type $X^2 \times S^2 \subset M^4 \times CP_2$ one can consider also deformations of these strings defining their multiple coverings so that the deformation is n-valued as a function of S^2-coordinates (Θ, Φ) and the projection to S^2 is thus an $n \to 1$ map. The solutions are higher dimensional analogs of originally closed orbits which after perturbation close only after n turns. This kind of surfaces emerge in the TGD inspired model of quantum Hall effect naturally [K32] and $n \to \infty$ limit has an interpretation as an approach to chaos [K29] .

   Using the coordinates (x, y, θ, ϕ) of $X^2 \times S^2$ and coordinates m^k for M^4 of the unperturbed solution the space-time surface the deformation can be expressed as

$$\begin{aligned} m^k &= m^k(x, y, \theta, \phi) \ , \\ (\Theta, \Phi) &= (\theta, n\phi) \ . \end{aligned} \tag{5.4}$$

The value of the string tension would be indeed n^2-fold in the first approximation since the induced Kähler form defining the Kähler magnetic field would be $J_{\theta\phi} = nsin(\Theta)$ and one would have $v_n = nv_0$. At the limit $m^k = m^k(x,y)$ different branches for these solutions collapse together.

2. Consider next how sub-harmonics appear in TGD framework. Suppose that cosmic strings decay to magnetic flux tube structures. This could the counterpart for cosmic expansion. The Kähler magnetic flux $\Phi = BS$ is conserved in the process but the thickness of the M^4 projection of the cosmic string increases field strength is reduced. This means that string tension, which is proportional to B^2S, is reduced (so that also Kähler action is reduced). The fact that space-time surface is Bohr orbit in generalized sense means that the reduced string tension (magnetic energy per unit length) is quantized.

The task is to guess how the quantization occurs. There are two options.

1. The simplest explanation for the reduction of v_0 is based on the decay of a flux tube resembling a disk with a hole to n identical flux tubes so that $v_0 \to v_0/n$ results for the resulting flux tubes. It turns out that this mechanism is favored and explains elegantly the value of $\hbar_{gr}$ for outer planetary system. One can also consider small-p p-adicity so that n would be prime.

2. Second explanation is more intricate. Consider a magnetic flux tube. Since magnetic flux is quantized, the magnetic field strengths are quantized in integer multiples of basic strength: $B = nB_0$ and would rather naturally correspond to the multiple coverings of the original magnetic flux tube with magnetic energy quantized in multiples of n^2. The idea is to require internal consistency in the sense that the allowed reduced field strengths are such that the spectrum associated with B_0 is contained to the spectrum associated with the quantized field strengths $B_1 > B_0$. This would allow only field strengths $B = B_S/n^2$, where B_S denotes the field strength of the fundamental cosmic string and one would have $v_n = v_0/n$. Flux conservation requires that the area of the flux tube scales as n^2.

Sub-harmonics might appear in the outer planetary system and there are indications for the higher harmonics below the inner planetary system [E14] : for instance, solar radius corresponds to $n = 1$ orbital for $v_3 = 3v_0$. This would suggest that Sun and also planets have an onion like structure with highest harmonics of v_0 and strongest string tensions appearing in the solar core and highest sub-harmonics appearing in the outer regions. If the matter results as decay remnants of cosmic strings this means that the mass density inside Sun should correlate strongly with the local value of n characterizing the multiple covering of cosmic strings.

One can ask whether the very process of the formation of the structures could have excited the higher values of n just like closed orbits in a perturbed system become closed only after n turns. The energy density of the cosmic string is about one Planck mass per $\sim 10^7$ Planck lengths so that $n > 1$ excitation increasing this density by a factor of n^2 is obviously impossible except under the primordial cosmic string dominated period of cosmology during which the net inertial energy density must have vanished. The structure of the future solar system would have been dictated already during the primordial phase of cosmology when negative energy cosmic string suffered a time reflection to positive energy cosmic strings.

5.2.2 *Nottale equation is consistent with the TGD based model for dark matter*

TGD allows two models of dark matter. The first one is spherically symmetric and the second one cylindrically symmetric. The first thing to do is to check whether these models are consistent with the gravitational Schrödinger equation/Bohr quantization.

1. Spherically symmetric model for the dark matter

The following argument based on Bohr orbit quantization demonstrates that this is indeed the case for the spherically symmetric model for dark matter. The argument generalizes in a trivial manner to the cylindrically symmetric case.

1. The gravitational potential energy $V(r)$ for a mass distribution $M(r) = xTr$ (T denotes string tension) is given by

$$V(r) = Gm\int_r^{R_0} \frac{M(r)}{r^2}dr = GmxTlog(\frac{r}{R_0}) \ . \quad (5.5)$$

Here R_0 corresponds to a large radius so that the potential is negative as it should in the region where binding energy is negative.

2. The Newton equation $\frac{mv^2}{r} = \frac{GmxT}{r}$ for circular orbits gives

$$v = xGT \ . \quad (5.6)$$

3. Bohr quantization condition for angular momentum by replacing $\hbar$ with $\hbar_{gr}$ reads as $mvr = n\hbar_{gr}$ and gives

$$\begin{aligned} r_n &= \frac{n\hbar_{gr}}{mv} = nr_1 \ , \\ r_1 &= \frac{GM}{vv_0} \ . \end{aligned} \quad (5.7)$$

Here v is rather near to v_0.

4. Bound state energies are given by

$$E_n = \frac{mv^2}{2} - xTlog(\frac{r_1}{R_0}) + xTlog(n) \ . \quad (5.8)$$

The energies depend only weakly on the radius of the orbit.

5. The centrifugal potential $l(l+1)/r^2$ in the Schrödinger equation is negligible as compared to the potential term at large distances so that one expects that degeneracies of orbits with small values of l do not depend on the radius. This would mean that each orbit is occupied with same probability irrespective of value of its radius. If the mass distribution for the starts does not depend on r, the number of stars rotating around galactic nucleus is simply the number of orbits inside sphere of radius R and thus given by $N(R) \propto R/r_0$ so that one has $M(R) \propto R$. Hence the model is self consistent in the sense that one can regard the orbiting stars as remnants of cosmic strings and thus obeying same mass distribution.

2. Cylindrically symmetric model for the galactic dark matter

TGD allows also a model of the dark matter based on cylindrical symmetry. In this case the dark matter would correspond to the mass of a cosmic string orthogonal to the galactic plane and traversing through the galactic nucleus. The string tension would the one predicted by TGD. In the directions orthogonal to the plane of galaxy the motion would be free motion so that the orbits would be helical, and this should make it possible to test the model. The quantization of radii of the orbits would be exactly the same as in the spherically symmetric model. Also the quantization of inclinations predicted by the spherically symmetric model could serve as a sensitive test. In this kind of situation general theory of relativity would predict only an angle deficit giving rise to a lens effect. TGD predicts a Newtonian $1/\rho$ potential in a good approximation.

Spiral galaxies are accompanied by jets orthogonal to the galactic plane and a good guess is that they are associated with the cosmic strings. The two models need not exclude each other. The vision about astrophysical structures as pearls of a fractal necklace would suggest that the visible matter has resulted in the decay of cosmic strings originally linked around the cosmic string going through the galactic plane and creating $M(R) \propto R$ for the density of the visible matter in the galactic bulge. The finding that galaxies are organized along linear structures [E32] fits nicely with this picture.

5.2.3 *MOND and TGD*

TGD based model explains also the MOND (Modified Newton Dynamics) model of Milgrom [E23] for the dark matter. Instead of dark matter the model assumes a modification of Newton's laws. The model is based on the observation that the transition to a constant velocity spectrum seems in the galactic halos seems to occur at a constant value of the stellar acceleration equal to $a_0 \simeq 10^{-11}g$, where g is the gravitational acceleration at the Earth. MOND theory assumes that Newtonian laws are modified below a_0.

The explanation relies on Bohr quantization. Since the stellar radii in the halo are quantized in integer multiples of a basic radius and since also rotation velocity v_0 is constant, the values of the acceleration are quantized as $a(n) = v_0^2/r(n)$ and a_0 correspond to the radius $r(n)$ of the smallest Bohr orbit for which the velocity is still constant. For larger orbital radii the acceleration would indeed be below a_0. a_0 would correspond to the distance above which the density of the visible matter does not appreciably perturb the gravitational potential of the straight string. This of course requires that gravitational potential is that given by Newton's theory and is indeed allowed by TGD.

The MOND theory [E23] and its variants predict that there is a critical acceleration below which Newtonian gravity fails. This would mean that Newtonian gravitation is modified at large distances. String models and also TGD predict just the opposite since in this regime General Relativity should be a good approximation.

1. The $1/r^2$ force would transform to $1/r$ force at some critical acceleration of about $a = 10^{-10}$ m/s^2: this is a fraction of 10^{-11} about the gravitational acceleration at the Earth's surface.
2. The recent empirical study [E21] giving support for this kind of transition in the dynamics of stars at large distances and therefore breakdown of Newtonian gravity in MOND like theories.

In TGD framework critical acceleration is predicted but the recent experiment does not force to modify Newton's laws. Since Big Science is like market economy in the sense that funding is more important than truth, the attempts to communicate TGD based view about dark matter [K13, K24, K20, K25, K9] have turned out to be hopeless. Serious Scientist does not read anything not written on silk paper.

1. One manner to produce this spectrum is to assume density of dark matter such that the mass inside sphere of radius R is proportional to R at last distances [K9]. Decay products of and ideal cosmic strings would predict this. The value of the string tension predicted correctly by TGD using the constraint that p-adic mass calculations give electron mass correctly [K17].
2. One could also assume that galaxies are distributed along cosmic string like pearls in necklace. The mass of the cosmic string would predict correct value for the velocity of distant stars. In the ideal case there would be no dark matter outside these cosmic strings.
   (a) The difference with respect to the first mechanism is that this case gravitational acceleration would vanish along the direction of string and motion would be free motion. The prediction is that this kind of motions take place along observed linear structures formed by galaxies and also along larger structures.
   (b) An attractive assumption is that dark matter corresponds to phases with large value of Planck constant is concentrated on magnetic flux tubes. Holography would suggest that the density of the magnetic energy is just the density of the matter condensed at wormhole throats associated with the topologically condensed cosmic string.
   (c) Cosmic evolution modifies the ideal cosmic strings and their Minkowski space projection gets gradually thicker and thicker and their energy density - magnetic energy - characterized by string tension could be affected

TGD option differs from MOND in some respects and it is possible to test empirically which option is nearer to the truth.

1. The transition at same critical acceleration is predicted universally by this option for all systems-now stars- with given mass scale if they are distributed along cosmic strings like like pearls in necklace. The gravitational acceleration due the necklace simply wins the gravitational acceleration due to the pearl. Fractality encourages to think like this.
2. The critical acceleration predicted by TGDr depends on the mass scale as $a \propto GT^2/M$, where M is the mass of the object- now star. Since the recent study considers only stars with solar mass it does not allow to choose between MOND and TGD and Newton can continue to rest in peace in TGD Universe. Only a study using stars with different masses would allow to compare the predictions of MOND and TGD and kill either option or both. Second test distinguishing between MOND and TGD is the prediction of large scale free motions by TGD option.

TGD option explains also other strange findings of cosmology.

1. The basic prediction is the large scale motions of dark matter along cosmic strings. The characteristic length and time scale of dynamics is scaled up by the scaling factor of $\hbar$. This could explain the observed large scale motion of galaxy clusters -dark flow [E2]- assigned with dark matter in conflict with the expectations of standard cosmology.
2. Cosmic strings could also relate to the strange relativistic jet like structures [E7] meaning correlations between very distant objects. Universe would be a spaghetti of cosmic strings around which matter is concentrated.
3. The TGD based model for the final state of star [K30] actually predicts the presence of string like object defining preferred rotation axis. The beams of light emerging from supernovae would be preferentially directed along this lines- actually magnetic flux tubes. Same would apply to the gamma ray bursts [E3] from quasars, which would not be distributed evenly in all directions but would be like laser beams along cosmic strings.

5.3 The Interpretation Of $\hbar_{gr}$ And Pre-Planetary Period

$\hbar_{gr}$ could corresponds to a unit of angular momentum for quantum coherent states at magnetic flux tubes or walls containing macroscopic quantum states. Quantitative estimate demonstrates that $\hbar_{gr}$ for astrophysical objects cannot correspond to spin angular momentum. For Sun-Earth system one would have $\hbar_{gr} \simeq 10^{77}\hbar$. This amount of angular momentum realized as a mere spin would require 10^{77} particles! Hence the only possible interpretation is as a unit of orbital angular momentum. The linear dependence of $\hbar_{gr}$ on m is consistent with the additivity of angular momenta in the fusion of magnetic flux tubes to larger units if the angular momentum associated with the tubes is proportional to both m and M.

Just as the gravitational acceleration is a more natural concept than gravitational force, also $\hbar_{gr}/m = GM/v_0$ could be more natural unit than $\hbar_{gr}$. It would define a universal unit for the circulation $\oint v \cdot dl$, which is apart from $1/m$-factor equal to the phase integral $\oint p_\phi d\phi$ appearing in Bohr rules for angular momentum. The circulation could be associated with the flow associated with outer boundaries of magnetic flux tubes surrounding the orbit of mass m around the central mass $M \gg m$ and defining light like 3-D CDs analogous to black hole horizons.

The expression of $\hbar_{gr}$ depends on masses M and m and can apply only in space-time regions carrying information about the space-time sheets of M and and the orbit of m. Quantum gravitational holography suggests that the formula applies at 3-D light like causal determinant (CD) X^3_l defined by the wormhole contacts gluing the space-time sheet X^3_l of the planet to that of Sun. More generally, X^3_l could be the space-time sheet containing the planet, most naturally the magnetic flux tube surrounding the orbit of the planet and possibly containing dark matter in super-conducting state. This would give a precise meaning for $\hbar_{gr}$ and explain why $\hbar_{gr}$ does not depend on the masses of other planets.

The simplest option consistent with the quantization rules and with the explanatory role of magnetic flux structures is perhaps the following one.

1. X^3_l is a torus like surface around the orbit of the planet containing de-localized dark matter. The key role of magnetic flux quantization in understanding the values of v_0 suggests the

interpretation of the torus as a magnetic or Z^0 magnetic flux tube. At pre-planetary period the dark matter formed a torus like quantum object. The conditions defining the radii of Bohr orbits follow from the requirement that the torus-like object is in an eigen state of angular momentum in the center of mass rotational degrees of freedom. The requirement that rotations do not leave the torus-like object invariant is obviously satisfied. Newton's law required by the quantum-classical correspondence stating that the orbit corresponds to a geodesic line in general relativistic framework gives the additional condition implying Bohr quantization.

2. A simple mechanism leading to the localization of the matter would have been the pinching of the torus causing kind of a traffic jam leading to the formation of the planet. This process could quite well have involved a flow of matter to a smaller planet space-time sheet Y^3_l topologically condensed at X^3_l. Most of the angular momentum associated with torus like object would have transformed to that of planet and situation would have become effectively classical.

3. The conservation of magnetic flux means that the splitting of the orbital torus would generate a pair of Kähler magnetic charges. It is not clear whether this is possible dynamically and hence the torus could still be there. In fact, TGD explanation for the tritium beta decay anomaly citeTroitsk,Mainz in terms of classical Z^0 force [K26] requires the existence of this kind of torus containing neutrino cloud whose density varies along the torus. This picture suggests that the lacking $n = 1$ and $n = 2$ orbits in the region between Sun and Mercury are still in magnetic flux tube state containing mostly dark matter.

4. The fact that $\hbar_{gr}$ is proportional to m means that it could have varied continuously during the accumulation of the planetary mass without any effect in the planetary motion: this is of course nothing but a manifestation of Equivalence Principle.

5. It is interesting to look for the scaled up versions of Planck mass $m_{Pl} = \sqrt{\hbar_{gr}/\hbar} \times \sqrt{\hbar/G} = \sqrt{M_1M_2/v_0}$ and Planck length $L_{Pl} = \sqrt{\hbar_{gr}/\hbar} \times \sqrt{\hbar/G} = G\sqrt{M_1M_2/v_0}$. For $M_1 = M_2 = M$ this gives $m_{Pl} = M/\sqrt{v_0} \simeq 45.6 \times M$ and $L_{Pl} = r_S/2\sqrt{v_0} \simeq 22.8 \times r_S$, where r_S is Schwartshild radius. For Sun r_S is about 2.9 km so that one has $L_{Pl} \simeq 66$ km. For a few years ago it was found that Sun contains "inner-inner" core of radius about $R = 300$ km [F1] which is about $4.5 \times L_{Pl}$.

5.4 Inclinations For The Planetary Orbits And The Quantum Evolution Of The Planetary System

The inclinations of planetary orbits provide a test bed for the theory. The semiclassical quantization of angular momentum gives the directions of angular momentum from the formula

$$cos(\theta) = \frac{m}{\sqrt{j(j+1)}} \quad , \quad |m| \leq j \ . \tag{5.9}$$

where θ is the angle between angular momentum and quantization axis and thus also that between orbital plane and (x,y)-plane. This angle defines the angle of tilt between the orbital plane and (x,y)-plane.

$m = j = n$ gives minimal value of angle of tilt for a given value of n of the principal quantum number as

$$cos(\theta) = \frac{n}{\sqrt{n(n+1)}} \ . \tag{5.10}$$

For $n = 3, 4, 5$ (Mercury, Venus, Earth) this gives $\theta = 30.0$, 26.6, and 24.0 degrees respectively.

Only the relative tilt angles can be compared with the experimental data. Taking as usual the Earth's orbital plane as the reference the relative tilt angles give what are known as inclinations. The predicted inclinations are 6 degrees for Mercury and 2.6 degrees for Venus. The observed values [E8] are 7.0 and 3.4 degrees so that the agreement is satisfactory. If one allows half-odd integer spin the fit is improved. For $j = m = n - 1/2$ the predictions are 7.1 and 2.9 degrees for Mercury and Venus respectively. For Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto the inclinations are 1.9, 1.3, 2.5, 0.8, 1.8, 17.1 degrees. For Mars and outer planets the tilt angles are predicted to have wrong sign for $m = j$. In a good approximation the inclinations vanish for outer planets except Pluto and this would allow to determine m as $m \simeq \sqrt{5n(n+1)/6}$: the fit is not good.

The assumption that matter has condensed from a matter rotating in (x,y)-plane orthogonal to the quantization axis suggests that the directions of the planetary rotation axes are more or less the same and by angular momentum conservation have not changed appreciably. The prediction for the tilt of the rotation axis of the Earth is 24 degrees of freedom in the limit that the Earth's spin can be treated completely classically, that is for $m = j >> 1$ in the units used for the quantization of the Earth's angular momentum. What is the value of $\hbar_{gr}$ for Earth is not obvious (using the unit $\hbar_{gr} = GM^2/v_0$ the Earth's angular momentum would be much smaller than one). The tilt of the rotation axis of Earth with respect to the orbit plane is 23.5 degrees so that the agreement is again satisfactory. This prediction is essentially quantal: in purely classical theory the most natural guess for the tilt angle for planetary spins is 0 degrees.

The observation that the inner planets Mercury, Venus, and Earth have in a reasonable approximation the predicted inclinations suggest that they originate from a primordial period during which they formed spherical cells of dark matter and had thus full rotational degrees of freedom and were in eigen states of angular momentum corresponding to a full rotational symmetry. The subsequent $SO(3) \rightarrow SO(2)$ symmetry breaking leading to the formation of torus like configurations did not destroy the information about this period since the information about the value of j and m was coded by the inclination of the planetary orbit.

In contrast to this, the dark matter associated with Earth and outer planets up to Neptune formed a flattened magnetic or Z^0 magnetic flux tube resembling a disk with a hole and the subsequent symmetry breaking broke it to separate flux tubes. Earth's spherical disk was joined to the disk formed by the outer planets. The spherical disk could be still present and contain super-conducting dark matter. The presence of this "heavenly sphere" might closely relate to the fact that Earth is a living planet. The time scale $T = 2\pi R/c$ is very nearly equal to 5 minutes and defines a candidate for a bio-rhythm.

If this flux tube carried the same magnetic flux as the flux tubes associated with the inner planets, the decomposition of the disk with a hole to 5 flux tubes corresponding to Earth and to the outer planets Mars, Jupiter, Saturn and Neptune, would explain the value of v_0 correctly and also the small inclinations of outer planets. That Pluto would not originate from this structure, is consistent with its anomalously large values of inclination $i = 17.1$ degrees, small value of eccentricity $e = .248$, and anomalously large value of inclination of equator to orbit about 122 degrees as compared to 23.5 degrees in the case of Earth [E8] .

5.5 Eccentricities And Comets

Bohr-Sommerfeld quantization allows also to deduce the eccentricities of the planetary and comet orbits. One can write the quantization of energy as

$$\begin{aligned} \frac{p_r^2}{2m_1} + \frac{p_\theta^2}{2m_1r^2} + \frac{p_\phi^2}{2m_1r^2sin^2(\theta)} - \frac{k}{r} &= -\frac{E_1}{n^2} \ , \\ E_1 = \frac{k^2}{2\hbar_{gr}^2} \times m_1 &= \frac{v_0^2}{2} \times m_1 \ . \end{aligned} \tag{5.11}$$

Here one has $k = GMm_1$. E_1 is the binding energy of $n = 1$ state. In the orbital plane ($\theta = \pi/2, p_\theta = 0$) the conditions are simplified. Bohr quantization gives $p_\phi = m\hbar_{gr}$ implying

$$\frac{p_r^2}{2m_1} + \frac{k^2\hbar_{gr}^2}{2m_1r^2} - \frac{k}{r} = -\frac{E_1}{n^2} \ . \tag{5.12}$$

For $p_r = 0$ the formula gives maximum and minimum radii $r_\pm$ and eccentricity is given by

$$e^2 = \frac{r_+ - r_-}{r_+} = \frac{2\sqrt{1-\frac{m^2}{n^2}}}{1+\sqrt{1-\frac{m^2}{n^2}}} \quad . \tag{5.13}$$

For small values of n the eccentricities are very large except for $m = n$. For instance, for $(m = n-1, n)$ for $n = 3, 4, 5$ gives $e = (.93, .89, .86)$ to be compared with the experimental values (.206, .007, .0167). Thus the planetary eccentricities with Pluto included ($e = .248$) must vanish in the lowest order approximation and must result as a perturbation of the magnetic flux tube.

The large eccentricities of comet orbits might however have an interpretation in terms of $m < n$ states. The prediction is that comets with small eccentricities have very large orbital radius. Oort's cloud is a system weakly bound to a solar system extending up to 3 light years. This gives the upper bound $n \leq 700$ if the comets of the cloud belong to the same family as Mercury, otherwise the bound is smaller. This gives a lower bound to the eccentricity of not nearly circular orbits in the Oort cloud as $e > .32$.

5.6 Why The Quantum Coherent Dark Matter Is Not Visible?

The obvious objection against quantal astrophysics is that astrophysical systems look extremely classical. Quantal dark matter in many-sheeted space-time resolves this counter argument. As already explained, the sequence of symmetry breakings of the rotational symmetry would explain nicely why astral Bohr rules work. The prediction is however that de-localized quantal dark matter is probably still present at (the boundaries of) magnetic flux tubes and spherical shells. It is however the entire structure defined by the orbit which behaves like a single extended particle so that the localization in quantum measurement does not mean a localization to a point of the orbit. Planet itself corresponds to a smaller localized space-time sheet condensed at the flux tube.

One should however understand why this dark matter with a gigantic Planck constant is not visible. The simplest explanation is that there cannot be any direct quantum interactions between ordinary and dark matter in the sense that particles with different values of Planck constant could appear in the same particle vertex. This would allow also a fractal hierarchy copies of standard model physics to exist with different p-adic mass scales.

There is also second argument. The inability to observe dark matter could mean inability to perform state function reduction localizing the dark matter. The probability for this should be proportional to the strength of the measurement interaction. For photons the strength of the interaction is characterized by the fine structure constant. In the case of dark matter the fine structure constant is replaced with

$$\alpha_{em,gr} = \alpha_{em} \times \frac{\hbar}{\hbar_{gr}} = \alpha_{em} \times \frac{v_0}{GMm} \quad . \tag{5.14}$$

For $M = m = m_{Pl} \simeq 10^{-8}$ kg the value of the fine structure constant is smaller than $\alpha_{em} v_0$ and completely negligible for astrophysical masses. However, for processes for which the lowest order classical rates are non-vanishing, rates are not affected in the lowest order since the increase of the Compton length compensates the reduction of α. Higher order corrections become however small. What makes dark matter invisible is not the smallness of α_{em} but the fact that the binding energies of say hydrogen atom proportional to $\alpha^2 m_e$ are scaled as $1/\hbar^2$ so that the spectrum is scaled down.

5.7 Quantum Interpretation Of Gravitational Schrödinger Equation

Schrödinger equation - or even Bohr rules - in astrophysical length scales with a gigantic value of Planck constant looks sheer madness from the standard physics point of view. In TGD Universe situation is different. TGD predicts infinite hierarchy of effective values of Planck constants $h_{eff} = n \times h$ and $h_{gr} = h_{eff}$ is a natural assumption. The high values of Planck constant is effective but it implies macroscopic quantum coherence in scales proportional to h_{eff}. The hierarchy of effective

Planck constants labels the levels of a hierarchy of quantum criticalities, which is basic prediction of TGD. The hierarchy of Planck constants is associated with dark matter.

The special feature of gravitational interaction is that h_{gr} characterizing its strength is proportional to the product of the interacting masses. Hence gravitational Compton length $\hbar_{gr}/m = GM/v_0$ is independent of the smaller mass and same for all particles. The predictions for the quantal behavior of massive bodies follow from the mere assumption that microscopic particles couple to the large central mass via magnetic flux tubes with large value of h_{gr}. What the situation actually is remains open. Interestingly, in the model of bio-photons as decay products of dark photons with $h_{gr} = h_{eff}$ the energy spectrum of dark cyclotron photons is universal and co-incides with the spectrum of bio-photons [K40, K39].

5.7.1 *Bohr quantization of planetary orbits and prediction for Planck constant*

The predictions of the generalization of the p-adic length scale hypothesis are consistent with the TGD based model for the Bohr quantization of planetary orbits and some new non-trivial predictions follow.

1. Generalization of the p-adic length scale hypothesis

The evolution in phase resolution in p-adic degrees of freedom corresponds to emergence of algebraic extensions allowing increasing variety of phases $exp(i\pi/n)$ expressible p-adically. This evolution can be assigned to the emergence of increasingly complex quantum phases and the increase of Planck constant.

One expects that quantum phases $q = exp(i\pi/n)$ which are expressible using only square roots of rationals are number theoretically special since they correspond to algebraic extensions of p-adic numbers involving only square roots which should emerge first and therefore systems involving these values of q should be especially abundant in Nature.

These polygons are obtained by ruler and compass construction and Gauss showed that these polygons, which could be called Fermat polygons, have $n_F = 2^k \prod_s F_{n_s}$ sides/vertices: all Fermat primes F_{n_s} in this expression must be different. The analog of the p-adic length scale hypothesis emerges since larger Fermat primes are near a power of 2. The known Fermat primes $F_n = 2^{2^n} + 1$ correspond to $n = 0, 1, 2, 3, 4$ with $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65537$. It is not known whether there are higher Fermat primes. $n = 3, 5, 15$-multiples of p-adic length scales clearly distinguishable from them are also predicted and this prediction is testable in living matter. I have already earlier considered the possibility that Fermat polygons could be of special importance for cognition and for biological information processing [K19] .

This condition could be interpreted as a kind of resonance condition guaranteeing that scaled up sizes for space-time sheets have sizes given by p-adic length scales. The numbers n_F could take the same role in the evolution of Planck constants assignable with the phase resolution as Mersenne primes have in the evolution assignable to the p-adic length scale resolution. The conjecture would be that $h_{gr}/h = n_F$ holds true.

2. Can one really identify gravitational and inertial Planck constants?

The original unconsciously performed identification of the gravitational and inertial Planck constants leads to some confusing conclusions but it seems that the new view about the quantization of Planck constants resolves these problems and allows to see $\hbar_{gr}$ as a special case of $\hbar_{eff} = n \times h$.

1. $\hbar_{gr}$ is proportional to the product of masses of interacting systems and not a universal constant like $\hbar$. One can however express the gravitational Bohr conditions as a quantization of circulation $\oint v \cdot dl = n(GM/v_0)\hbar_0$ so that the dependence on the planet mass disappears as required by Equivalence Principle. This suggests that gravitational Bohr rules relate to velocity rather than inertial momentum as is indeed natural. The quantization of circulation is consistent with the basic prediction that space-time surfaces are analogous to Bohr orbits.

2. $\hbar_{gr}$ seems to characterize a relationship between planet and central mass and quite generally between two systems with the property that smaller system is topologically condensed at the space-time sheet of the larger system. Thus it would seem that $\hbar_{gr}$ is not a universal constant and cannot correspond to a special value of h_{eff}. Due to the large masses the identification $h_{gr} = h_{eff} = n \times h$ can be made without experimental uncertainties.

The recent view about the quantization of Planck constant in terms of coverings of space-time seems to resolve these problems.

1. One can also make the identification $\hbar_{gr} = \hbar_{eff} = n \times \hbar_0$ and associate it with the space-time sheet along which the masses interact provided each pair (M, m_i) of masses is characterized by its own sheets. These sheets would correspond to flux tube like structures carrying the gravitational flux of dark matter. If these sheets correspond to n-fold covering of M^4, one can understand $\hbar_{gr} = n \times \hbar_0$ as a particular instance of the $\hbar_{eff}$. Note that v_0 could depend on planet in this case.

2. The integer quantization of Planck constants is consistent with the huge values of gravitational Planck constant $\hbar_{gr=}h_{eff} = n \times h$ within experimental resolution. A stronger prediction would follow from that v_0 is constant for inner *resp.* outer planets and $\hbar_{gr}/\hbar_0 = n_F$. The ratios of planetary masses would be ratios of Fermat integers in this case. The accuracy is about 10 per cent and the discrepancy could be explained in terms of the variation of v_0. One can imagine also other preferred values of n. In particular, $n = p^k$, p prime, is favored by the generalized p-adic length scale hypothesis following from number theoretical arguments and NMP [K41].

5.7.2 *Quantization as a means of avoiding gravitational collapse*

Schrödinger equation provided a solution to the infrared catastrophe of the classical model of atom: the classical prediction was that electron would radiate its energy as brehmstrahlung and would be captured by the nucleus. The gravitational variant of this process would be the capture of the planet by a black hole, and more generally, a collapse of the star to a black hole. Gravitational Schrödinger equation could obviously prevent the catastrophe.

For $1/r$ gravitation potential the Bohr radius is given by $a_{gr} = GM/v_0^2 = r_S/2v_0^2$, where $r_S = 2GM$ is the Schwartchild radius of the mass creating the gravitational potential: obviously Bohr radius is much larger than the Schwartschild radius. That the gravitational Bohr radius does not depend on m conforms with Equivalence Principle, and the proportionality $\hbar_{gr} \propto Mm$ can be deduced from it. Gravitational Bohr radius is by a factor $1/2v_0^2$ larger than black hole radius so that black hole can swallow the piece of matter with a considerable rate only if it is in the ground state and also in this state the rate is proportional to the black hole volume to the volume defined by the black hole radius given by $2^3v_0^6 \sim 10^{-20}$.

The $\hbar_{gr} \to \infty$ limit for $1/r$ gravitational potential means that the exponential factor $exp(-r/a_0)$ of the wave function becomes constant: on the other hand, also Schwartshild and Bohr radii become infinite at this limit. The gravitational Compton length associated with mass m does not depend on m and is given by GM/v_0 and the time $T = E_{gr}/\hbar_{gr}$ defined by the gravitational binding energy is twice the time taken to travel a distance defined by the radius of the orbit with velocity v_0 which suggests that signals travelling with a maximal velocity v_0 are involved with the quantum dynamics.

In the case of planetary system the proportionality $\hbar_{gr} \propto mM$ creates problems of principle since the influence of the other planets is not taken account. One might argue that the generalization of the formula should be such that M is determined by the gravitational field experienced by mass m and thus contains also the effect of other planets. The problem is that this field depends on the position of m which would mean that $\hbar_{gr}$ itself would become kind of field quantity.

5.7.3 *Does the transition to non-perturbative phase correspond to a change in the value of* $\hbar$*?*

Nature is populated by systems for which perturbative quantum theory does not work. Examples are atoms with $Z_1Z_2e^2/4\pi\hbar > 1$ for which the binding energy becomes larger than rest mass, non-perturbative QCD resulting for $Q_{s,1}Q_{s,2}g_s^2/4\pi\hbar > 1$, and gravitational systems satisfying $GM_1M_2/4\pi\hbar > 1$. Quite generally, the condition guaranteeing troubles is of the form $Q_1Q_2g^2/4\pi\hbar > 1$. There is no general mathematical approach for solving the quantum physics of these systems but it is believed that a phase transition to a new phase of some kind occurs.

The gravitational Schrödinger equation forces to ask whether Nature herself takes care of the problem so that this phase transition would involve a change of the value of the Planck constant to guarantee that the perturbative approach works. The values of $\hbar$ would vary in a stepwise manner from $\hbar(\infty)$ to $\hbar(3) = \hbar(\infty)/4$. The non-perturbative phase transition would correspond to transition to the value of

$$\frac{\hbar}{\hbar_0} \rightarrow \left[\frac{Q_1 Q_2 g^2}{v}\right] \tag{5.15}$$

where $[x]$ is the integer nearest to x, inducing

$$\frac{Q_1 Q_2 g^2}{4\pi\hbar} \rightarrow \frac{v}{4\pi} \ . \tag{5.16}$$

The simplest (and of course ad hoc) assumption making sense in TGD Universe is that v is a harmonic or subharmonic of v_0 appearing in the gravitational Schrödinger equation. For instance, for the Kepler problem the spectrum of binding energies would be universal (independent of the values of charges) and given by $E_n = v^2 m/2n^2$ with v playing the role of small coupling. Bohr radius would be $g^2 Q_2/v^2$ for $Q_2 \gg Q_1$.

This provides a new insight to the problems encountered in quantizing gravity. QED started from the model of atom solving the infrared catastrophe. In quantum gravity theories one has started directly from the quantum field theory level and the recent decline of the M-theory shows that we are still practically where we started. If the gravitational Schrödinger equation indeed allows quantum interpretation, one could be more modest and start from the solution of the gravitational IR catastrophe by assuming a dynamical spectrum of $\hbar$ comes as integer multiples of ordinary Planck constant. The implications would be profound: the whole program of quantum gravity would have been misled as far as the quantization of systems with $GM_1M_2/\hbar > 1$ is considered. In practice, these systems are the most interesting ones and the prejudice that their quantization is a mere academic exercise would have been completely wrong.

An alternative formulation for the occurrence of a transition increasing the value of $\hbar$ could rely on the requirement that classical bound states have reasonable quantum counterparts. In the gravitational case one would have $r_n = n^2\hbar_{gr}^2/GM_1^2M$, for $M_1 \ll M$, which is extremely small distance for $\hbar_{gr} = \hbar$ and reasonable values of n. Hence, either n is so large that the system is classical or $\hbar_{gr}/\hbar$ is very large. Equivalence Principle requires the independence of r_n on M_1, which gives $\hbar = kGM_1M_2$ giving $r_n = n^2 kGM$. The requirement that the radius is above Schwartshild radius gives $k \geq 2$. In the case of Dirac equation the solutions cease to exist for $Z \geq 137$ and which suggests that $\hbar$ is large for hypothetical atoms having $Z \geq 137$.

5.8 How Do The Magnetic Flux Tube Structures And Quantum Gravitational Bound States Relate?

In the case of stars in galactic halo the appearance of the parameter v_0 characterizing cosmic strings as orbital rotation velocity can be understood classically. That v_0 appears also in the gravitational dynamics of planetary orbits could relate to the dark matter at magnetic flux tubes. The argument explaining the harmonics and sub-harmonics of v_0 in terms of properties of cosmic strings and magnetic flux tubes identifiable as their descendants strengthens this expectation.

5.8.1 *The notion of magnetic body*

In TGD inspired theory of consciousness the notion of magnetic body plays a key role: magnetic body is the ultimate intentional agent, experiencer, and performer of bio-control and can have astrophysical size: this does not sound so counter-intuitive if one takes seriously the idea that cognition has p-adic space-time sheets as space-time correlates and that rational points are common to real and p-adic number fields. The point is that infinitesimal in p-adic topology corresponds to infinite in real sense so that cognitive structures would have literally infinite size.

The magnetic flux tubes carrying various supra phases can be interpreted as special instance of dark energy and dark matter. This suggests a correlation between gravitational self-organization and quantum phases at the magnetic flux tubes and that the gravitational Schrödinger equation somehow relates to the ordinary Schrödinger equation satisfied by the macroscopic quantum phases at magnetic flux tubes. Interestingly, the transition to large Planck constant phase should occur when the masses of interacting is above Planck mass since gravitational self-interaction energy is $V \sim GM^2/R$. For the density of water about 10^3 kg/m^3 the volume carrying a Planck mass correspond to a cube with side 2.8×10^{-4} meters. This corresponds to a volume of a large neuron, which suggests that this phase transition might play an important role in neuronal dynamics.

5.8.2 *Could gravitational Schrödinger equation relate to a quantum control at magnetic flux tubes?*

An infinite self hierarchy is the basic prediction of TGD inspired theory of consciousness ("everything is conscious and consciousness can be only lost"). Topological quantization allows to assign to any material system a field body as the topologically quantized field pattern created by the system [K31, K14] . This field body can have an astrophysical size and would utilize the material body as a sensory receptor and motor instrument.

Magnetic flux tube and flux wall structures are natural candidates for the field bodies. Various empirical inputs have led to the hypothesis that the magnetic flux tube structures define a hierarchy of magnetic bodies, and that even Earth and larger astrophysical systems possess magnetic body which makes them conscious self-organizing living systems. In particular, life at Earth would have developed first as a self-organization of the super-conducting dark matter at magnetic flux tubes [K14] .

For instance, EEG frequencies corresponds to wavelengths of order Earth size scale and the strange findings of Libet about time delays of conscious experience [J2, J1] find an elegant explanation in terms of time taken for signals propagate from brain to the magnetic body [K31] . Cyclotron frequencies, various cavity frequencies, and the frequencies associated with various p-adic frequency scales are in a key role in the model of bio-control performed by the magnetic body. The cyclotron frequency scale is given by $f = eB/m$ and rather low as are also cavity frequencies such as Schumann frequencies: the lowest Schumann frequency is in a good approximation given by $f = 1/2\pi R$ for Earth and equals to 7.8 Hz.

1. Quantum time scales as "bio-rhythms" in solar system?

To get some idea about the possible connection of the quantum control possibly performed by the dark matter with gravitational Schrödinger equation, it is useful to look for the values of the periods defined by the gravitational binding energies of test particles in the fields of Sun and Earth and look whether they correspond to some natural time scales. For instance, the period $T = 2GM_S n^2/v_0^3$ defined by the energy of n^{th} planetary orbit depends only on the mass of Sun and defines thus an ideal candidate for a universal "bio-rhythm".

For Sun black hole radius is about 2.9 km. The period defined by the binding energy of lowest state in the gravitational field of Sun is given $T_S = 2GM_S/v_0^3$ and equals to 23.979 hours for $v_0/c = 4.8233 \times 10^{-4}$. Within experimental limits for v_0/c the prediction is consistent with 24 hours! The value of v_0 corresponding to exactly 24 hours would be $v_0 = 144.6578$ km/s (as a matter fact, the rotational period of Earth is 23.9345 hours). As if as the frequency defined by the lowest energy state would define a "biological" clock at Earth! Mars is now a strong candidate for a seat of life and the day in Mars lasts 24hr 37m 23s! $n = 1$ and $n = 2$ are orbitals are not realized in solar system as planets but there is evidence for the $n = 1$ orbital as being realized as a peak in the density of IR-dust [E14] . One can of course consider the possibility that these levels are populated by small dark matter planets with matter at larger space-time sheets. Bet as it may, the result supports the notion of quantum gravitational entrainment in the solar system.

The slower rhythms would become as n^2 sub-harmonics of this time scale. Earth itself corresponds to $n = 5$ state and to a rhythm of .96 hours: perhaps the choice of 1 hour to serve as a fundamental time unit is not merely accidental. The magnetic field with a typical ionic cyclotron frequency around 24 hours would be very weak: for 10 Hz cyclotron frequency in Earth's magnetic field the field strength would about 10^{-11} T. However, $T = 24$ hours corresponds with 6 per cent accuracy to the p-adic time scale $T(k = 280) = 2^{13}T(2, 127)$, where $T(2, 127)$ corresponds to the secondary p-adic time scale of .1 s associated with the Mersenne prime $M_{127} = 2^{127} - 1$ characterizing electron and defining a fundamental bio-rhythm and the duration of memetic codon [K16] .

Comorosan effect [K35] , [I2, I1] demonstrates rather peculiar looking facts about the interaction of organic molecules with visible laser light at wavelength $\lambda = 546\ nm$. As a result of irradiation molecules seem to undergo a transition $S \rightarrow S^*$. S^* state has anomalously long lifetime and stability in solution. $S \rightarrow S^*$ transition has been detected through the interaction of S^* molecules with different biological macromolecules, like enzymes and cellular receptors. Later Comorosan found that the effect occurs also in non-living matter. The basic time scale is $\tau = 5$ seconds. p-Adic length scale hypothesis does not explain τ, and it does not correspond to any obvious astrophysical time scale and has remained a mystery.

The idea about astro-quantal dark matter as a fundamental bio-controller inspires the guess that τ could correspond to some Bohr radius R for a solar system via the correspondence $\tau = R/c$. As observed by Nottale, $n = 1$ orbit for $v_0 \rightarrow 3v_0$ corresponds in a good approximation to the solar radius and to $\tau = 2.18$ seconds. For $v_0 \rightarrow 2v_0$ $n = 1$ orbit corresponds to $\tau = AU/(4 \times 25) = 4.992$ seconds: here $R = AU$ is the astronomical unit equal to the average distance of Earth from Sun. The deviation from τ_C is only one per cent and of the same order of magnitude as the variation of the radius for the orbit due to orbital eccentricity $(a - b)/a = .0167$ [E8] .

2. Earth-Moon system

For Earth serving as the central mass the Bohr radius is about 18.7 km, much smaller than Earth radius so that Moon would correspond to $n = 147.47$ for v_0 and $n = 1.02$ for the sub-harmonic $v_0/12$ of v_0. For an afficionado of cosmic jokes or a numerologist the presence of the number of months in this formula might be of some interest. Those knowing that the Mayan calendar had 11 months and that Moon is receding from Earth might rush to check whether a transition from $v/11$ to $v/12$ state has occurred after the Mayan culture ceased to exist: the increase of the orbital radius by about 3 per cent would be required! Returning to a more serious mode, an interesting question is whether light satellites of Earth consisting of dark matter at larger space-time sheets could be present. For instance, in [K14] I have discussed the possibility that the larger space-time sheets of Earth could carry some kind of intelligent life crucial for the bio-control in the Earth's length scale.

The period corresponding to the lowest energy state is from the ratio of the masses of Earth and Sun given by $M_E/M_S = (5.974/1.989) \times 10^{-6}$ given by $T_E = (M_E/M_S) \times T_S = .2595$ s. The corresponding frequency $f_E = 3.8535$ Hz frequency is at the lower end of the theta band in EEG and is by 10 per cent higher than the p-adic frequency $f(251) = 3.5355$ Hz associated with the p-adic prime $p \simeq 2^k$, $k = 251$. The corresponding wavelength is 2.02 times Earth's circumference. Note that the cyclotron frequencies of Nn, Fe, Co, Ni, and Cu are $5.5, 5.0, 5.2, 4.8$ Hz in the magnetic field of $.5 \times 10^{-4}$ Tesla, which is the nominal value of the Earth's magnetic field. In [K23] I have proposed that the cyclotron frequencies of Fe and Co could define biological rhythms important for brain functioning. For $v_0/12$ associated with Moon orbit the period would be 7.47 s: I do not know whether this corresponds to some bio-rhythm.

It is better to leave for the reader to decide whether these findings support the idea that the super conducting cold dark matter at the magnetic flux tubes could perform bio-control and whether the gravitational quantum states and ordinary quantum states associated with the magnetic flux tubes couple to each other and are synchronized.

5.9 About The Interpretation Of The Parameter v_0

The formula for the gravitational Planck constant contains the parameter $v_0/c = 2^{-11}$. This velocity defines the rotation velocities of distant stars around galaxies. It can be seen also as a charasteristic velocity scale for inner planets. The presence of a parameter with dimensions of velocity should carry some important information about the geometry of dark matter space-time sheets.

Velocity like parameters appear also in other contexts. There is evidence for the Tifft's quantization of cosmic redshifts in multiples of $v_0/c = 2.68 \times 10^{-5}/3$: also other units of quantization have been proposed but they are multiples of v_0 [E31] .

The strange behavior of graphene includes high conductivity with conduction electrons behaving like massless particles with light velocity replaced with $v_0/c = 1/300$. The TGD inspired model [K6] explains the high conductivity as being due to the Planck constant $\hbar(M^4) = 6\hbar_0$ increasing the delocalization length scale of electron pairs associated with hexagonal rings of mono-atomic graphene layer by a factor 6 and thus making possible overlap of electron orbitals. This explains also the anomalous conductivity of DNA containing 5- and 6-cycles [K6] .

5.9.1 *p-Adic length scale hypothesis and* $v_0 \rightarrow v_0/5$ *transition at inner-outer border for planetary system*

$v_0 \rightarrow v_0/5$ transition would allow to interpret the orbits of outer planets as $n \geq 1$ orbits. The obvious question is whether inner to outer zone as $v_0 \rightarrow v_0/5$ transition could be interpreted in terms of the p-adic length scale hierarchy.

1. The most important p-adic length scale are given by primary p-adic length scales $L_e(k) = 2^{(k-151)/2} \times 10$ nm and secondary p-adic length scales $L_e(2,k) = 2^{k-151} \times 10$ nm, k prime.

2. The p-adic scale $L_e(2,139) = 114$ Mkm is slightly above the orbital radius 109.4 Mkm of Venus. The p-adic length scale $L_e(2,137) \simeq 28.5$ Mkm is roughly one half of Mercury's orbital radius 57.9 Mkm. Thus strong form of p-adic length scale hypothesis could explain why the transition $v_0 \rightarrow v_0/5$ occurs in the region between Venus and Earth ($n = 5$ orbit for v_0 layer and $n = 1$ orbit for $v_0/5$ layer).

3. Interestingly, the *primary* p-adic length scales $L_e(137)$ and $L_e(139)$ correspond to fundamental atomic length scales which suggests that solar system be seen as a fractally scaled up "secondary" version of atomic system.

4. Planetary radii have been fitted also using Titius-Bode law predicting $r(n) = r_0 + r_1 \times 2^n$. Hence on can ask whether planets are in one-one correspondence with primary and secondary p-adic length scales $L_e(k)$. For the orbital radii $58, 110, 150, 228$ Mkm of Mercury, Venus, Earth, and Mars indeed correspond approximately to k= $276, 278, 279, 281$: note the special position of Earth with respect to its predecessor. For Jupiter, Saturn, Uranus, Neptune, and Pluto the radii are 52,95,191,301,395 Mkm and would correspond to p-adic length scales $L_e(280 + 2n))$, $n = 0, ..., 3$. Obviously the transition $v_0 \rightarrow v_0/5$ could occur in order to make the planet–p-adic length scale one-one correspondence possible.

5. It is interesting to look whether the p-adic length scale hierarchy applies also to the solar structure. In a good approximation solar radius .696 Mkm corresponds to $L_e(270)$, the lower radius .496 Mkm of the convective zone corresponds to $L_e(269)$, and the lower radius .174 Mkm of the radiative zone (radius of the solar core) corresponds to $L_e(266)$. This encourages the hypothesis that solar core has an onion like sub-structure corresponding to various p-adic length scales. In particular, $L_e(2,127)$ ($L_e(127)$ corresponds to electron) would correspond to 28 Mm. The core is believed to contain a structure with radius of about 10 km: this would correspond to $L_e(231)$. This picture would suggest universality of star structure in the sense that stars would differ basically by the number of the onion like shells having standard sizes.

Quite generally, in TGD Universe the formation of join along boundaries bonds is the space-time correlate for the formation of bound states. This encourages to think that (Z^0) magnetic flux tubes are involved with the formation of gravitational bound states and that for $v_0 \rightarrow v_0/k$ corresponds either to a splitting of a flux tube resembling a disk with a whole to k pieces, or to the scaling down $B \rightarrow B/k^2$ so that the magnetic energy for the flux tube thickened and stretched by the same factor k^2 would not change.

After decade of developing this model, it has become clear that TGD favors generalization of p-adic length scale hypothesis: primes near but below powers of prime are favored. This could explain the factor five scaling of $1/v_0$

5.9.2 *Is dark matter warped?*

The reduced light velocity could be due to the warping of the space-time sheet associated with dark electrons. TGD predicts besides gravitational red-shift a non-gravitational red-shift due to the warping of space-time sheets possible because space-time is 4-surface rather than abstract 4-manifold. A simple example of everyday life is the warping of a paper sheet: it bends but is not stretched, which means that the induced metric remains flat although one of its component scales (distance becomes longer along direction of bending). For instance, empty Minkowski space represented canonically as a surface of $M^4 \times CP_2$ with constant CP_2 coordinates can become periodically warped in time direction because of the bending in CP_2 direction. As a consequence, the distance in time direction shortens and effective light-velocity decreases when determined from the comparison of the time taken for signal to propagate from A to B along warped space-time sheet with propagation time along a non-warped space-time sheet.

The simplest warped imbedding defined by the map $M^4 \to S^1$, S^1 a geodesic circle of CP_2. Let the angle coordinate of S^1 depend linearly on time: $\Phi = \omega t$. g_{tt} component of metric becomes $1 - R^2\omega^2$ so that the light velocity is reduced to $v_0/c = \sqrt{1 - R^2\omega^2}$. No gravitational field is present.

The fact that M^4 Planck constant $n_a\hbar_0$ defines the scaling factor n_a^2 of CP_2 metric could explain why dark matter resides around strongly warped imbeddings of M^4. The quantization of the scaling factor of CP_2 by $R^2 \to n_a^2 R^2$ implies that the initial small warping in the time direction given by $g_{tt} = 1-\epsilon$, $\epsilon = R^2\omega^2$, will be amplified to $g_{tt} = 1-n_a^2\epsilon$ if ω is not affected in the transition to dark matter phase. $n_a = 6$ in the case of graphene would give $1 - x \simeq 1 - 1/36$ so that only a one per cent reduction of light velocity is enough to explain the strong reduction of light velocity for dark matter.

5.9.3 *Is c/v_0 quantized in terms of ruler and compass rationals?*

The known cases suggests that c/v_0 is always a rational number expressible as a ratio of integers associated with n-polygons constructible using only ruler and compass.

1. $c/v_0 = 300$ would explain graphene. The nearest rational satisfying the ruler and compass constraint would be $q = 5 \times 2^{10}/17 \simeq 301.18$.

2. If dark matter space-time sheets are warped with $c_0/v = 2^{11}$ one can understand Nottale's quantization for the radii of the inner planets. For dark matter space-time sheets associated with outer planets one would have $c/v_0 = 5 \times 2^{11}$.

3. If Tifft's red-shifts relate to the warping of dark matter space-time sheets, warping would correspond to $v_0/c = 2.68 \times 10^{-5}/3$. $c/v_0 = 2^5 \times 17 \times 257/5$ holds true with an error smaller than .1 per cent.

5.9.4 *Tifft's quantization and cosmic quantum coherence*

An explanation for Tifft's quantization in terms of Jones inclusions could be that the subgroup G of Lorentz group defining the inclusion consists of boosts defined by multiples $\eta = n\eta_0$ of the hyperbolic angle $\eta_0 \simeq v_0/c$. This would give $v/c = sinh(n\eta_0) \simeq nv_0/c$. Thus the dark matter systems around which visible matter is condensed would be exact copies of each other in cosmic length scales since G would be an exact symmetry. The property of being an exact copy applies of course only in single level in the dark matter hierarchy. This would mean a de-localization of elementary particles in cosmological length scales made possible by the huge values of Planck constant. A precise cosmic analog for the de-localization of electron pairs in benzene ring would be in question.

Why then η_0 should be quantized as ruler and compass rationals? In the case of Planck constants the quantum phases $q = exp(im\pi/n_F)$ are number theoretically simple for n_F a ruler and compass integer. If the boost $exp(\eta)$ is represented as a unitary phase $exp(im\eta)$ at the level of discretely de-localized dark matter wave functions, the quantization $\eta_0 = n/n_F$ would give rise to number theoretically simple phases. Note that this quantization is more general than $\eta_0 = n_{F,1}/n_{F,2}$.

6 Some Examples About Gravitational Anomalies In TGD Universe

The many-sheeted space-time and the hierarchy of Planck constants predict new physics which should be seen as anomalies in the models based on general relativity. In the following some examples about these anomalies are discussed.

6.1 SN1987A And Many-Sheeted Space-Time

Lubos Motl has written a highly rhetoric, polemic, and adrenaline rich posting (see http://tinyurl.com/px4hzdc) about the media buzz related to supernova SN1987A. The target of Lubos Motl is the explanation proposed by James Franson from the University of Maryland for the findings discussed in Physics Archive Blog (see http://tinyurl.com/mde7jat). I do not have any strong attitude to Franson's explanation but the buzz is about very real thing: unfortunately Lubos Motl tends to forget the facts in his extreme orthodoxy.

What happened was following. Two separate neutrino bursts arrived from SN 1987 A. At 7.35 AM Kamionakande detected 11 antineutrons, IMB 8 antineutrinos, and Baksan 5 antineutrinos. Approximately 3 hours later Mont Blanc liquid scintillator detected 5 antineutrinos. Optical signal came 4.7 hours later.

The are several very real problems as one can get convinced by going to Wikipedia (http://en.wikipedia.org/wiki/SN_1987A):

1. If neutrinos and photons are emitted simultaneously and propagate with the same speed, they should arrive simultaneously. I am not specialist enough to try to explain this difference in terms of standard astrophysics. Franson however sees this difference as something not easy to explain and tries to explain it in his own model.

2. There are two neutrino bursts rather than one. A modification of the model of supernova explosion allowing two bursts of neutrinos would be needed but this would suggest also two photon bursts.

These problems have been put under the carpet. Those who are labelled as crackpots often are much more aware about real problems than the academic career builders.

In TGD framework the explanation would be in terms of many-sheeted space-time. In GRT limit of TGD the sheets of the many-sheeted space-time (see **Fig.** http://tgdtheory.fi/appfigures/manysheeted.jpg or **Fig. ??** in the appendix of this book) are lumped to single sheet: Minkowski space with effective metric defined by the sum of Minkowski metric and deviations of the metrics of the various sheets from Minkowski metric. The same recipe gives effective gauge potentials in terms of induced gauge potentials.

Different arrival times for neutrinos and photons would be however a direct signature of the many-sheeted space-time since the propagation velocity along space-time sheets depends on the induced metric. The larger the deviation from the flat metric is, the slower the propagation velocity and thus longer the arrival time is. Two neutrino bursts would have explanation as arrivals along two different space-time sheets. Different velocity for photons and neutrinos could be explained if they arrive along different space-time sheets. I proposed for more than two decades ago this mechanism as an explanation for the finding of cosmologists that there are two different Hubble constants: they would correspond to different space-time sheets.

The distance of SN1987A is 168, 000 light- years. This means that the difference between velocities is $\Delta c/c \simeq \Delta T/T \simeq 3hours/168\times 10^3 \simeq 2\times 10^{-9}$. The long distance is what makes the effect visible.

I proposed earlier sub-manifold gravity as an explanation for the claimed super-luminality of the neutrinos coming to Gran Sasso from CERN. In this case the effect would have been $\Delta c/c \simeq 2.5\times 10^{-5}$ and thus four orders of magnitude larger than four supernova neutrinos. It however turned out that the effect was not real.

Towards the end of 2014 Lubos Motl Motl had a posting about galactic blackhole Sagittarius A as neutrino factory (see http://tinyurl.com/pvzrqoz). Chandra X-ray observatory (see http://chandra.harvard.edu) and also Nustar (http://www.nustar.caltech.edu) and Swift Gamma-Ray Burst Mission (see https://en.wikipedia.org/wiki/Swift_Gamma-Ray_Burst_Mission) detected some X-ray flares from Sagittarius A. 2-3 hours earlier IceCube (see http://en.wikipedia.org/wiki/Ice_Cube) detected high energy neutrinos by IceCube on the South Pole.

Could neutrinos arrive from the galactic center? If they move with the same (actually somewhat lower) velocity than photons, this cannot be the case. The neutrinos did the same trick as SN1987A neutrinos and arrived 2-3 hours before the X-rays! What if one takes TGD seriously and estimates $\Delta c/c$ for this event? The result is $\Delta c/c \sim (1.25-1.40)\times 10^{-8}$ for 3 hours lapse using the estimate $r = 25,900\pm 1,400$ light years (see http://en.wikipedia.org/wiki/Sagittarius_A). $\Delta c/c$ is by a factor 4 larger than for SN1987A at distance about 168, 000 light years (see http://en.wikipedia.org/wiki/SN_1987A). This distance is roughly 8 times longer. This would suggests that the smaller the space-time sheets the nearer the velocity of neutrinos is to its maximal value. For photons the reduction from the maximal signal velocity is larger.

6.2 Pioneer And Flyby Anomalies For Almost Decade Later

The article [E15] (see http://tinyurl.com/avmndwa) is about two old anomalies discovered in the solar system: Pioneer anomaly [E5] and Flyby anomaly [E17, E16, E13, E22] with which I worked for years ago.

I remember only the general idea that dark matter concentrations at orbits of planets or at spheres with radii equal that of orbit could cause the anomalies. So I try to reconstruct all from scratch and during reconstruction become aware of something new and elegant that I could not discover for years ago.

The popular article [E15] claims that Pioneer anomaly is understood. I am not at all convinced about the solution of Pioneer anomaly. Several "no new physics" solutions have been tailored during years but later it has been found that they do not work.

Suppose that dark matter is at the surface of sphere so that by a well-known text book theorem it does not create gravitational force inside it. This is an overall important fact, which I did not use earlier. The model explains both anomalies and also allow to calculate the total amount of dark matter at the sphere.

1. Consider first the Pioneer anomaly.

   (a) Inside the dark matter sphere with radius of Jupiter's orbit the gravitational force caused by dark matter vanishes. Outside the sphere also dark matter contributes to the gravitational attraction and Pioneer's acceleration becomes a little bit smaller since the dark matter at the sphere containing the orbit radius of Jupiter or Saturn also attracts the space-craft after the passby. A simple test for spherical model is the prediction that the mass of Jupiter effectively increases by the amount of dark matter at the sphere after passby.

   (b) The magnitude of the Pioneer anomaly is about $\Delta a/a = 1.3\times 10^{-4}$ [K24] and translates to $M_{dark}/M \simeq 1.3\times 10^{-4}$. What is highly non-trivial is that the anomalous acceleration is given by Hubble constant suggesting that there is a connection with cosmology fixing the value of dark mass once the area of the sphere containing it is fixed. This follows as a prediction if the surface mass density is universal and proportional to the Hubble constant.

   Could one interpret the equality of the two accelerations as an equilibrium condition? The Hubble acceleration H associated with the cosmic expansion (expansion velocity increases with distance) would be compensated by the acceleration due to the gravitational force of dark matter. The formula for surface density of dark matter is from Newton's law $GM_{dark} = H$ given by $\sigma_{dark} = H/4\pi G$. The approximate value of dark matter surface density is from $Hc = 6.7 \times 10^{-10}$ m/s^2 equal to $\sigma = .8$ kg/m^2 and surprisingly large.

   (c) The value of acceleration is $a = .8 \times 10^{-10} \times g$, $g = 9.81$ m/s^2 whereas the MOND model (see http://tinyurl.com/32t9wt) finds the optimal value for the postulated minimal gravitational acceleration to be $a_0 = 1.2 \times 10^{-10}$ m/s^2. In TGD framework it would be assignable to the traversal through the dark matter shell. The ratio of the two accelerations is $a/a_0 = 6.54$.

   (d) TGD inspired quantum biology requiring that the universal cyclotron energy spectrum of dark photons $h_{eff} = h_{gr}$ transforming to to bio-photons is in visible and UV range for charged particles gives the estimate $M_{dark}/M_E \simeq 2 \times 10^{-4}$ [K40] and is of the same order of magnitude smaller than for Jupiter. The minimum value of the magnetic field at flux tubes has been assumed to be $B_E = .2$ Gauss, which is the value of endogenous magnetic field explaining the effects of ELF em radiation on vertebrate brain. The two estimates are clearly consistent.

2. In Flyby anomaly spacecraft goes past Earth to gain momentum (Earth acts as a sling) for its travel towards Jupiter. During flyby a sudden acceleration occurs but this force is on only during the flyby but not before or after that. The basic point is that the spacecraft visits near Earth, and this is enough to explain the anomaly.

   The space-craft enters from a region outside the orbit of Earth containing dark matter and thus experiences also the dark force created by the sphere. After that the space craft enters inside the dark matter region, and sees a weaker gravitational force since the dark matter sphere is outside it and does not contribute. This causes a change in its velocity. After flyby the spacecraft experiences the forces caused by both Earth and dark matter sphere and the situation is the same as before flyby. The net effect is a change in the velocity as observed. From this the total amount of dark matter can be estimated. Also biology based argument gives an estimate for the fraction of dark matter in Earth.

This model supports the option in which the dark matter is concentrated on sphere. The other option is that it is concentrated at flux tube around orbit: quantitative calculations would be required to see whether this option can work. One can consider of course also more complex distributions: say $1/r$ distribution outside the sphere giving rise to constant change in acceleration outside the sphere.

A possible very simple TGD model for the sphere containing dark matter could be in terms of a boundary defined by a gigantic wormhole contact with large $h_{eff} = h_{gr}$ (at its space-time sheet representing "line of generalized Feynman diagram" one has deformation of CP_2 type vacuum extremal with Euclidian signature of induced metric) with radius given by the radius of Bohr orbit with gravitational Planck constant equal to $\hbar_{gr} = GMm/v_0$, where v_0 is a parameter with dimensions of velocity. This radius does not depend on the mass of the particle involved and is given by $r_n = GM/v_0^3$ where $r_S = 2GM$ is Schwartschild radius equal to 3 km for Sun [K24]. One has $v_0/c \simeq 2^{-11}$ for three inner planets. For outer planets v_0 is scaled down by a factor $1/5$.

The sphere should also correspond to a magnetic flux sheet with field line topology of dipole field. By flux conservation the flux must arrive along flux tube parallel to a preferred axis presumably orthogonal to the plane of planets and flux conservation should must true. This kind of structure is predicted also by the TGD model in terms of cylindrically symmetric candidate for an extremal of Kähler action representing astrophysical object [K5].

An interesting possibility is that also Earth-Moon system contains a spherical shell of dark matter at distance given by the radius of Moon's orbit (about 60 Earth's radii). If so the analogs of the two effects could be observed also in Earth Moon system and the testing of the effects would become much easier. This would also mean understanding of the formation of Moon. Also interior of Earth (and also Sun) could contain spherical shells containing dark matter as the TGD inspired model for the spherically symmetric orbit constructed for more than two decades ago [K5] suggests. One can raise interesting questions. Could also the matter in small scale systems be accompanied by dark matter shells at radii equal to Bohr radii in the first approximation and could these effects be tested? Note that a universal surface density for dark matter predicts that the change of acceleration universally be given by Hubble constant H.

6.3 Further Progress In The Understanding Of Dark Matter And Energy In TGD Framework

The remarks below were inspired by an extremely interesting link to a popular article about a possible explanation of dark matter in terms of vacuum polarization associated with gravitation. The model can make sense only if the sign of the gravitational energy of antimatter is opposite to that of matter and whether this is the case is not known. Since the inertial energies of matter and antimatter are positive, one might expect that this is the case also for gravitational energies by Equivalence Principle but one might also consider alternative and also I have done this in TGD framework.

The popular article lists four observations related to dark matter that neither cold dark matter (CMD) model nor modified gravitation model (MOND) can explain, and the claim is that the vacuum energy model is able to cope with them.

Consider first the TGD based model.

1. The model assumes that galaxies are like pearls along strings defined by cosmic strings expended to flux tubes during cosmic expansion survives also these tests. This is true also in longer scales due to the fractality if TGD inspired cosmology: for instance, galaxy clusters would be organized in a similar manner.

2. The dark magnetic energy of the string like object (flux tube) is identifiable as dark energy and the pearls would correspond to dark matter shells with a universal mass density of.8 kg/m^2 estimated from Pioneer and Flyby anomalies assuming to be caused by spherical dark matter shells assignable to the orbits of planets. This value follows from the condition that the anomalous acceleration is identical with Hubble acceleration. Even Moon could be accompanied by this kind of shell: if so, the analog of Pioneer anomaly is predicted.

3. The dark matter shell around galactic core could have decayed to smaller shells by h_{eff} reducing phase transition. This phase transition would have created smaller surfaces with smaller values of $h_{eff} = h_{gr}$. One can consider also the possibility that it contains all the galactic matter as dark matter. There would be nothing inside the surface of the gigantic wormhole throat: this would conform with holography oriented thinking.

I checked the four observations listed in the popular article some of which CMD (cold dark matter) scenario and MOND fail to to explain. TGD explains all of them.

1. It has been found that the effective surface mass density $\sigma = \rho_0 R_0/3$ (volume density times volume of ball equals to effective surface density times surface area of the ball for constant volume density) of galactic core region containing possible halo is universal and its value is.9 kg/m^2 (see the article). Pioneer and Flyby anomalies fix the surface density to.8 kg/m^2. The difference is about 10 per cent! One must of course be cautious here: even the correct order of magnitude would be fine since Hubble acceleration parameter might be different for the cluster than for the solar system now.

   Note that in the article the effective surface density is defined as $\sigma = \rho_0 r_0$, where r_0 is the radius of the region and ρ_0 is density in its center. The correct definition for a constant 3-D density inside ball is $\sigma = \rho_0 r_0/3$.

2. The dark matter has been found to be inside core region within few hundred parsecs. This is just what TGD predicts since the velocity spectrum of distant stars is due to the gravitational field created by dark energy identifiable as magnetic energy of cosmic string like object - the thread containing galaxies as pearls.

3. It has been observed that there is no dark matter halo in the galactic disk. Also this is an obvious prediction of TGD model.

4. The separation of matter - now plasma clouds between galaxies - and dark matter in the collisions of galaxy clusters (observed for instance for bullet cluster consisting of two colliding clusters) is also explained qualitatively by TGD. The explanation is qualitatively similar to that in the CMD model of the phenomenon. Stars of galaxies are not affected except from gravitational slow-down much but the plasma phase interacts electromagnetically and is slowed down much more in the collision. The dominating dark matter component making itself visible by gravitational lensing separates from the plasma phase and this is indeed observed: the explanation in TGD framework would be that it is macroscopically quantum coherent ($h_{eff} = h_{gr}$) and does not dissipate so that the thermodynamical description does not apply.

   In the case of galaxy clusters also the dark energy of cosmic strings is involved besides the galactic matter and this complicates the situation but the basic point is that dark matter component does not slow down as plasma phase does.

   CMD model has the problem that the velocity of dark matter bullet (smaller cluster of bullet cluster) is higher than predicted by CMD scenario. Attractive fifth force acting between dark matter particles becoming effective at short distances has been proposed as an explanation: intuitively this adds to the potential energy negative component so that kinetic energy is increased. I have proposed that gravitational constant might vary and be roughly twice the standard value: I do not believe this explanation now.

The most feasible explanation is that the anomaly relates to the presence of thickened cosmic strings carrying dark energy as magnetic energy and dark matter shells instead of 3-D cold dark matter halos. This additional component would contribute to gravitational potential experienced by the smaller cluster and explain the higher velocity.

6.4 Variation Of Newston's Constant And Of Length Of Day

J. D. Anderson et al [E18] have published an article discussing the observations suggesting a periodic variation of the measured value of Newton constant and variation of length of day.

According to the article, about a dozen measurements of Newton's gravitational constant, G, since 1962 have yielded values that differ by far more than their reported random plus systematic errors. Authors find that these values for G are oscillatory in nature, with a period of $P = 5.899 \pm 0.062$ yr , an amplitude of , $S = 1.619 \pm 0.103 \times 10^{-14}$ m^{3}kg^{-1} s^{-2} and mean-value crossings in 1994 and 1997. The relative variation $\Delta G/G \sim 2.4 \times 10^{-4}$. Authors suggest that the actual values of G does not vary but some unidentified factor in the measurement process is responsible for an apparent variations.

According to the article, of other recently reported results, the only measurement with the same period and phase is the Length of Day (LOD defined as a frequency measurement such that a positive increase in LOD values means slower Earth rotation rates and therefore longer days). The period is also about half of a solar activity cycle, but the correlation is far less convincing. The 5.9 year periodic signal in LOD has previously been interpreted as due to fluid core motions and inner-core coupling. We report the G/LOD correlation, whose statistical significance is 0.99764 assuming no difference in phase, without claiming to have any satisfactory explanation for it. Least unlikely, perhaps, are currents in the Earth's fluid core that change both its moment of inertia (affecting LOD) and the circumstances in which the Earth-based experiments measure G. In this case, there might be correlations with terrestrial-magnetic-field measurements.

In the popular article "Why do measurements of the gravitational constant vary so much?" (see `http://tinyurl.com/k5onwoe`) Anderson states that there is also a possible connection with Flyby anomaly [E17], which also shows periodic variation.

In the following TGD inspired model for the findings is developed. The gravitational coupling would be in radial scaling degree of freedom and rigid body rotational degrees of freedom. In rotational degrees of freedom the model is in the lowest order approximation mathematically equivalent with Kepler model. The model for the formation of planets around Sun suggests that the dark matter shell has radius equal to that of Moon's orbit. This leads to a prediction for the oscillation period of Earth radius: the prediction is consistent with the observed 5.9 years period. The dark matter shell would correspond to $n = 1$ Bohr orbit in the earlier model for quantum gravitational bound states based on large value of Planck constant if the velocity parameter $v_0$appearing in $\hbar_{gr} = GM_EM_D/v_0$ equals to the rotation velocity of Moon. Also $n > 1$ orbits are suggestive and their existence would provide additional support for TGD view about quantum gravitation. There are further amazing co-incidences. The gravitational Compton length GM/v_0 of particle is very near to to the Earth's radius in case Earth if central mass is Earth mass. For the mass of dark matter shell it is the variation ΔR_E. This strongly suggest that quantum coherence in astrophysical scales has been and perhaps still is present.

6.4.1 Coupled oscillations of radii of Earth and dark matter shell as an explanation for the variations

A possible TGD explanation for the variation emerges from the following arguments.

1. By angular momentum conservation requiring $I\omega = L = constant$ the oscillation of the length of day (LOD) can be explained by the variation of the radius R_E of Earth since the moment of inertia is proportional to R_E^2. This gives $\Delta LOD/LOD = 2\Delta R/R$. This explains also the apparent variation of G since the gravitational acceleration at the surface of Earth is $g = GM/R_E^2$ so that one has $\Delta g/g = 2\Delta R/R$. Note that the variations have opposite phase.

2. Flyby and Pioneer anomalies [K10] relies on the existience of dark matter shell with a universal surface mass density, whose value is such that in the case of Earth the total mass in the shell would be $M_D \sim 10^{-4} M_E$. The value $M_D/M_E \simeq 1.3 \times 10^{-4}$ suggested by TGD is of the same order of magnitude as $\Delta R/R$. Even galactic dark matter around galactic core could correspond to a shell with this surfaces density of mass [K10]. This plus the claim that also Flyby anomaly has oscillatory character suggest a connection. Earth and dark mass shell are in a collective pulsation with a frequency of Earth pulsation about 6 years and the interaction is gravitational attraction. Note that the frequencies need not be the same. Momentum conservation in radial direction indeed requires that both of them participate in oscillation.

6.4.2 A detailed model

One can construct a model for the situation.

1. Earth and dark matter shell are modelled as rigid bodies with spatially constant density except that their radii can change. Earth and dark matter shell are characterized by moments of inertia $I_E = (3/5) \times M_E r_E^3$ and $I_D = (2/3) \times M_D r_D^2$. If one restricts the consideration to a rigid body rotation around fixed axis (call it z-axis), one has effective point masses $M_1 = 3M_E/5$ and $M_2 = 2M_D/3$ and the problem is mathematically very similar to a motion point like particles with these effective masses in plane subject to the mutual gravitational force obtained by averaging the gravitational $1/r$ potential over the volumes of the two mass distributions. In the lowest order the problem is very similar to a central force problem with $1/r$-potential plus corrections coming as series in r_E/r_D. This problem can be solved by using angular momentum conservation and energy conservation.

2. In the lowest order approximation $r_E/r_D = 0$ one has just Kepler problem in $1/r_D$ force between masses M_1 and M_2 for M_D and one obtains the analogs of elliptic orbit in the analog of plane defined by r_D and ϕ. Kepler's law $T_D^2 \propto r_D^3$ fixes the average value of r_D, call this value R_D.

3. In the next approximation one feeds this solution to the equations for r_E by replacing r_D with its average value R_D to obtain the interaction potential depending on the radius r_E. It must be harmonic oscillator potential and the elastic constant determines the oscillation period of r_E. The value of this period should be about 6 yr.

The Lagrangian is sum of kinetic terms plus potential term

$$\begin{aligned}
L &= T_E + T_D + V_{gr} \ , \\
T_E &= \tfrac{1}{2} M_E (\tfrac{dR_E}{dt})^2 + \tfrac{1}{2} I_E (\tfrac{d\Phi_E}{dt})^2 \ , \quad T_D = \tfrac{1}{2} M_D (\tfrac{dR_D}{dt})^2 + \tfrac{1}{2} I_D (\tfrac{d\Phi_D}{dt})^2 \ .
\end{aligned} \tag{6.1}$$

One could criticize the choice of the coefficients of the kinetic terms for radial coordinates R_E and R_D as masses and one could indeed consider a more general choices. One can also argue, that the rigid bodies cannot be completely spherically since in this case it would not be possible to talk about rotation - at least in quantum mechanical sense.

Gravitational interaction potential is given by

$$\begin{aligned}
V_{gr} &= -G \int dV_E \int dA_D \rho_E \sigma_D \tfrac{1}{r_{D,E}} \ , \quad & r_{D,E} &= |\overline{r}_D - \overline{r_E}| \ , \\
dA_D &= r_D^2 d\Omega_D & dV_E &= r_E^2 dr_E d\Omega_E \ , \\
\rho_E &= \tfrac{3M_E}{4\pi R_E^3} \ , & \sigma_D &= \tfrac{M_D}{4\pi R_D^2} \ .
\end{aligned} \tag{6.2}$$

The integration measures are the standard integration measures in spherical coordinates.

One can extract the r_D factor from $r_{D,E}$ (completely standard step) to get

$$\frac{1}{r_{D,E}} = \frac{1}{r_D}X \ ,$$
$$X = \frac{1}{|\overline{n}_D - x\overline{n}_E|} = \frac{1}{[1+x^2-2xcos(\theta)]^{1/2}} = \frac{1}{(1+x^2)^{1/2}}\frac{1}{(1-2xcos(\theta)/(1+x^2))^{1/2}} \ ,$$
$$x = \frac{r_E}{r_D} \ , \ cos(\theta) = \overline{n}_D \cdot \overline{n}_E \ . \tag{6.3}$$

Angular integration over θ is trivial and only the integration over r_E remains.

$$V_{gr} = -GM_DM_E\frac{3r_D^2}{r_E^3}\int_0^{r_E/r_D} F(\epsilon(x))\frac{x^2}{(1-x^2)^{1/2}}dx \ ,$$
$$F(\epsilon) = \frac{(1+\epsilon)^{1/2}-(1-\epsilon)^{1/2})}{\epsilon} \simeq 1 - \frac{\epsilon}{8} \ ,$$
$$\epsilon = \frac{2x}{1+x^2} \ , \ x = \frac{r_E}{r_D} \ . \tag{6.4}$$

In the approximation $F(\epsilon) = 1$ introducing error of few per cent the outcome is

$$V_{gr} = -\frac{3GM_DM_E}{r_D} \times \left[arcsin(x) - x\sqrt{1-x^2}\right] = \frac{3GM_DM_E}{r_D}\left[\frac{2}{3} + \frac{x^2}{5} + O(x^3) + ...\right] \ ,$$
$$x = \frac{r_E}{r_D} \ . \tag{6.5}$$

The physical interpretation of the outcome is clear.

1. The first term in the series gives the gravitational potential between point like particles depending on r_D only giving rise to the Kepler problem. The orbit is closed - an ellipse whose eccentricity determines the amplitude of $\Delta R_D/R_D$. In higher orders one expects that the strict periodicity is lost in the general case. From the central force condition $M_2\omega_d^2 r_D = GM_DM_E/r_D^2$ one has

$$T_D = \sqrt{\frac{2}{3}} \times \sqrt{\frac{R_D}{r_{S,E}}}\frac{2\pi R_D}{c} \ , \ r_{S,E} = 2GM_E \ . \tag{6.6}$$

$r_{S,E} \simeq 8.87$ mm is the Earth's Schwartschild radius. The first guess is that the dark matter shell has the radius of Moon orbit $R_{Moon} \simeq 60.33 \times R_E$, $R_E = 6.731 \times 10^6$ m. This would give $T_D = T_{Moon} \simeq 30$ days.

2. Second term gives harmonic oscillator potential $k_ER_E^2/2$, $k_E = 6GM_DM_E/5R_D^3$ in the approximation that r_D is constant. Oscillator frequency is

$$T\omega_E^2 = \frac{k_E}{M_E} \times \frac{6GM_D}{5R_D^3} \ . \tag{6.7}$$

The oscillator period is given by

$$T_E = 2\pi \times \sqrt{\frac{5R_D^3}{6GM_D}} = 2\pi \times \sqrt{53} \times \sqrt{\frac{R_D}{R_{S,D}}} \times \frac{R_D}{c} \quad . \tag{6.8}$$

In this approximation the amplitude of oscillation cannot be fixed but the non-linearity relates the amplitude to the amplitude of r_D.

3. One can estimate the period of oscillation by feeding in the basic numbers. One has $R_D \sim R_{Moon} = 60.34R_E$, $R_E = 6.371 \times 10^6$ m. A rough earlier estimate for M_D is given by $M_D/M_E \simeq 1.3 \times 10^{-4}$. The relative amplitude of the oscillation is $\Delta G/G = 2\Delta R/R \simeq 2.4 \times 10^{-4}$, which suggests $\Delta R/R \simeq M_D/M_E$.

   The outcome is $T_E \simeq 6.1$ yr whereas the observed period is $T_E \simeq 5.9$ yr. The discrepancy could be due to non-linear effects making the frequency continuous classically.

An interesting question is whether macroscopic quantal effects might be involved.

1. The applicability of Bohr rules to the planetary motion [K24] first proposed by Nottale [E14] encourages to ask whether one could apply also to the effective Kepler problem Bohr rules with gravitational Planck constant $\hbar_{gr} = GM_EM_D/v_0$, where v_0 is a parameter with dimensions of velocity. The rotation velocity of Moon $v_0/c = 10^{-5}/3$ is the first order of magnitude guess. Also one can ask whether also $n > 1$ other dark matter layers are possible at Bohr orbits so that one would have the analog of atomic spectroscopy.

2. From angular momentum quantization requires $L = m\omega^2R = n\hbar_{gr}$ and from central force condition one obtains the standard formula for the radius of Bohr orbit $r_n = n^2GM_E/v_0^2$. For $n = 1$ the radius of the orbit would be radius of the orbit of Moon with accuracy of 3 per cent. Note that the mass of Moon is about 1 per cent of the Earth's mass and thus roughly by a factor 100 higher than the mass of the spherical dark matter shell.

Clearly, the model might have caught something essential about the situation. What remains to be understood is the amplitude $\Delta R/R$. It seems that $\Delta R/R \simeq M_D/M_E$ holds true. This is not too surprising but one should understand how this follows from the basic equations.

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