

Infinite primes

1. Background.
 - (a) Original motivation from TGD inspired theory of consciousness.
 - (b) Generalizes the notion of infinite number by introducing the notion of divisibility besides the notion of follower ($n \rightarrow n + 1$) used by Cantor to construct infinite numbers.
 - (c) Construction has interpretation as repeated second quantization of super-symmetric arithmetic quantum field theory.
 - (d) Construction can be also used to define infinite hierarchy of real units as ratios of infinite rationals but having arbitrarily complex number theoretic anatomy. This leads to a generalization of the notions of point and space.
 - (e) Space-time point with number theoretic anatomy would be infinite-dimensional space and able to represent quantum states of entire Universe! Number theoretic Brahman=Atman or algebraic holography!
2. Basic observation.
 - (a) Form the product X of all finite primes. It is divisible by any prime
 - (b) Add to $X + 1$ or -1 to get number $P = X + 1$ or $X - 1$.
 - (c) The number $P \bmod p = +1$ or -1 for any finite prime p . P is prime albeit infinite!
 - (d) Useful analogy: P is like Dirac sea for which all states labelled by finite primes are filled.
3. Construction generalizes to give representation of many-fermion states.
 - (a) Divide X by any square free integer (product of first powers of primes) to get $X_1 = X/n$.
 - (b) Form the number $P_1 = X_1 + n$. $P_1 \bmod p$ is nonvanishing for any finite prime p so that P_1 is infinite prime.
 - (c) This number is analogous to many fermion states obtained by kicking the negative energy fermions labelled by primes p dividing n to positive energy states. P_1 represents many-fermion state!
4. Infinite primes can represent also states containing bosons labelled by finite primes!
 - (a) Take integer m having not common factors with n . k :th power of prime p represents state consisting of k bosons labeled by prime p . Multiply X_1 with it m to get $P_2 = m \times X_1 + n$. P_2 is infinite prime.
 - (b) Take an integer r having only prime factors appearing in n and multiply n with r to get $P_3 = m \times X_1 + r \times n$. P_3 is prime and represents a state in which there are bosons labelled by primes appearing in X_1 and in n .
 - (c) This is the most general infinite prime representing Fock state of free bosons and fermions.
5. Physical fermions and bosons form also bound states. Could infinite primes represent also them?
 - (a) X is formally like coordinate variable. One can form sums and products of infinite primes and interpret them as polynomials.
 - (b) The non-divisibility by infinite primes (not only finite) boils down the notion of prime polynomial property and one constructs infinite hierarchy of this kind of states having interpretation as infinite primes representing bound states.
6. One can iterate the construction.
 - (a) Take finite and infinite primes obtained and define Dirac sea as product of them. Repeat the procedure. Now bound states correspond to polynomials of two variables.
 - (b) One can repeat the construction again and again.
 - (c) Repeated second quantization of arithmetic QFT is in question. Many-sheeted space-time strongly suggests that this procedure is physical. Even galaxy sized objects would at some scale represent elementary particles.