Kähler-Dirac equation

1. Kähler-Dirac (KD) equation is algebraically very much like Dirac equation in string models. Holomorphy algebraizes KD equation. This happens only for Kähler-Dirac action so that the dynamics of TGD is unique.

   (a) Kähler-Dirac (KD) equation is obtained by variation of Kähler-Dirac action with respect to induced spinor field. KD equation is well defined only if an extremal of Kähler action is in question: this supersymmetry guarantees that the vector field defined by KD gammas has vanishing covariant divergence. This guarantees existence of conserved super currents labelled by the modes of KD operator. For the ordinary Dirac equation this is automatically true.

   (b) The condition that the modes are localized to partonic 2surfaces requires that KD gammas have not components orthogonal to the string world sheet. Otherwise they would contribute to KD equation. This requires that KD gammas are in 11 correspondence with tangent vectors of string world sheet or partonic 2surface.

   (c) As in string models KD equation reduces to holomorphy conditions. KD gammas are holomorphic (antiholomorphic) functions of comp lex coordinate and spinor are holomorphic (antiholomorphic) and annihilated by holomorpic (antiholomorphic) KD gamma Γ^z (Γ^{z*}).

   (d) This generalizes trivially to hypercomplex case corresponding to Minkowskian signature of the effective metric defined by KD gammas.

   (e) The localization to 2D surfaces makes sense only for KD action. For the counterpart of ordinary Dirac equation assignable to the action de fined by four-volume all four gammas are linearly independent. Therefore KD action and Käh ler action define a unique choice.

2. Well-definedness of em charge highly nontrivial condition. One can vary KD action also with respect to imbedding space coordinates.

   (a) Since induced spinor fields are second quantized operators, it does not makes sense to consider the sum of Kähler action and KD action as action principle. The general variation of KD action cannot vanish and in general situation there is no hope about conservation laws for isometries leaving δCD invariant.

   (b) In the recent case variation can however vanish for deformations of space-time surface which at string world sheets are such that hey respect the holomorphy properties of KD gammas. Also the holomorphic gauge potentials must suffer only a gauge transformation, which can be compensated by a gauge transformation for the induced spinor field.

   (c) This kind of situation might be true for much more general transformations and one can ask whether symplectic transformations of $\delta CD \times CP_2$ satisfy these conditions so that one could assign both bosonic and fermionic charges to them and these contribute to WCW gamma matrices a contribution coming from the interior of 3-surface. Also holomorphic transformations of $\delta CD \times CP_2$ could define this kind of transformations.

   (d) Also now the holomorphy and localization of the modes to 2D surfaces due to the spe cial properties of Kähler action are essential.

   (e) One can consider also the possibility that preferred extremal has 2D projection with vanishing classical W and possibly also Z^0 fields being homologically nontrivial geodesic sphere or complex 2-surface in CP_2. In this case one expects continuous slicing by string world sheets so that the situation would redu ce to that already considered.

3. KD equation contains also boundary terms expressing the conservation of fermion number.

   (a) The variation of KD action gives also boundary terms proportional to the normal component of fermion current involving Γ^n. At par tonic orbits these terms differ by a multiplication with imaginary unit coming from the square root of metric determinant in Minkowskian space-time regions. Conservation of fermion number requires that these terms vanish. This condition reduces to condition at the orbits of string ends.

(b) KD action reduces to algebraic measure ment interaction term $p^k\gamma_k$ assumed to be present at light-like three surfaces and possibly also at space-like 3-surfaces at the boundaries of CD, where p^k is four-momentum-at the fermion line defined by string end. The outcome is massless propagator in perturbation theory which in twistor Grassmann approach interpreting integration over the momentum of internal lines as residue integral gives contribution involving only light-like momenta p^k. One obtains a contraction of $p^k\gamma_k$, with spinors at the ends of line. This is non-vanishing for nonphysical helicities.

(c) At the space-like 3-surfaces one obtains sum of $p^k\gamma_k$ and Γ^n, which must annihilate the spinor mode. Γ^n contains contribution from both M^4 and CP_2 gamma matrices, which means breaking of M^4 chiral symmetry and is signature for massivation. Fermion number conservation however implies that the sum of terms from upper and lower boundary of CD sum up to zero.