Quantum criticality

1. Motivations for the basic form of QC.
   (a) How to guarantee the uniqueness of TGD as unified theory?
   (b) Kähler action is unique candidate for the action. Its value for Euclidian regions of preferred extremal would define Kähler function of WCW. Its value for Minkowskian regions would define Morse function analogous to the action of quantum field theories giving rise to interference effects characteristic for QFTs.
   (c) Kähler action contains Kähler coupling strength α_K as a free parameter. How to fix its value to make TGD unique?
   (d) In ZEO TGD is "square root" of thermodynamics and vacuum functional defined by the exponent of Kähler action is analogous to square root of exponent of free energy.
   (e) α_K is analogous to temperature. Require that α_K is analogous to critical temperature. TGD Universe would be quantum critical. α_K would correspond to a fixed point of coupling constant evolution and renormalization group invariant.
   (f) Critical systems are characterized by long range fluctuations and allow maximal complexity being at the boundary of chaos and order. TGD Universe would allow maximal complexity.
2. Questions:
   (a) Could one allow several critical vales of α_K ? Could α_K depend on space-time sheet? Probably not.
   (b) Is there a minimal critical value of α_K different from zero?
   (c) If action contains other parameters, should one require them to be critical too.

   Comments:
   (a) α_K has trivial coupling constant evolution whereas other coupling constants evolve. They should be proportional to α_K .
   (b) The value of α_K does not affect preferred extremals. Vacuum functional however depends on α_K and affects quantum dynamics.
   (c) The constraint stating the equality of Kähler 4-momentum and Kähler-Dirac four-moment does not depend on α_K so that 4-momentum spectrum should not depend on α_K.
3. QC has also other meanings besides quantum criticality of Kähler coupling strength. These meanings are obtained by generalizing thermody- namical criticality to Quantum TGD identified as "square root of thermodynamics").
   (a) Thermodynamical analog defined by van der Waals type phase transi- tion. Free energy F is a function of particle number density n and has pressure p and temperature T as parameters. Van der Waals equation of state implied by extremization of F predicts that the states of the sys- tem define 2-D surface in (n,p,T) space.
   (b) In certain region n is 3-valued function of p and T. The middle branch is unstable in the sense that extrema of F are saddle points rather than maxima or minima. At the boundaries, where two branches co-incide a sudden phase transition is expected to happen and to change the value of n in discontinuous manner. This is actually quite not the situation: instead Maxwellrule holds true. Both stable phases are present.
   (c) This situation correspond in catastrophe theoretic description to cusp catastrophe. In this kind of systems free energy corresponds to potential function depending on behavior variables and control variables. Now behavior variable would be particle density n and control variables would be p and T.
4. It is possible to generalize the thermodynamical criticality in two different manners to quantum criticality in TGD framework.

(a) Criticality of Kähler/Morse function as function of quantum fluctua- ting WCW coordinates Q and zero modes Z. The number of 3-surfaces with same value of Kähler /Morse function would change in this transitions since several 3surfaces would co-incide (two roots of polynomial co-inciding).

   i. In catastrophe theory description Z corresponds to behavior variables and Q to control variables.
   ii. Quantum fluctuations inducing macroscopic phase transitions changing the values of zero modes defining classical variables.
   iii. This could happen in quantum measurement when microscopic state function reduction is amplified to macroscopic effect.

(b) Criticality of Kähler action with respect to variations leaving the 3-surfaces at the boundaries of CD invariant.

   i. At criticality the number of preferred extremals with same value of Kähler action would change. Hence the amount of amount of non-predictability characteristic for critical systems changes at criticality.
   ii. Q corresponds to behavior variables and Z to control variables now.
   iii. This could happen when microscopic quantum phase transition is induced at critical values of zero modes representing macroscopic degrees of freedom. Phase transitions changing the value of Planck constant might be an example of this.

(c) Quantum criticality and symmetries.

   i. 2-D conformal field theories describe critical 2-D 2-D systems. Conformal invariance can be identiified as local scaling invariance.
   ii. In TGD framework the generalization of conforformal symmetries from 2-D framework plays a key role. Both light-cone boundary and light-like 3-surfaces allow extended conformal symmetries.
   iii. One has effective 2-dimensionality implied by strong form of holography implied by strong form of general coordinate invariance.
   iv. String world sheets and partonic 2-surfaces are basic objects and there are good reasons to believe that 4-D analog of conformal invariance based on Yangian generalization of conformal invariance makes sense.

   This suggests a generalization of conformal invariance providing the mathematical tools for the description of quantum criticality.

   i. The hierarchies of sub-algebras of conformal algebras isomorphic to conformal algebra itself fits nicely with fractality of TGD Universe, and suggests a hierarchy of conformal symmetry breakings.
   ii. The sub-algebras would act as gauge transformations: that is transform to each other preferred extremals associated with given 3-surface but leaving it and physical states invariant.
   iii. The conformal algebra generators in the complement of the sub-algebra would transform 3-surfaces and quantum states non-trivially. In ordinary models this hierarchy is not present.