p-Adic number fields

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1. are completions of rational numbers just as real numbers being labelled by primes and algebraic numbers characterizing their algebraic extensions, which can have arbitrarily high algebraic dimension.

2. obey p-adic topology in which the norm of rational $p^k \times (m/n)$, is p^{-k}, when m and n are not divisible by p that is are p-adic integers of p-adic norm 1 p-adic units. Integers m and n can be also infinite as real integers.

   (a) p-Adic topology is ultrametric and totally disconneted implying that p-adic balls are either disjoint or nested and that the definition of p-adic manifold in purely p-adic context is problematic suggesting that p-adic and real number fields should be combined to a larger structure.

   (b) p-Adic norm implies that real integers proportional to p^k and ap- proaching infinity approach zero as p-adic integers and that most p-adic numbers can be said to be infinite as real numbers.

3. contain rationals as numbers with possibly infinite number of pinary digits but having periodic pinary expansion above some pinary digit and p-adic transcendentals as numbers for which pinary series is infinite and non-periodic.

4. allow differential calculus obeying same rules as real calculus so that bit sequences including also infinitely long sequences form space of 2-adic integers in which one can define differential equations and therfore the notion of integral function whereas the definition of definite integral is highly problematic in purely p-adic context suggesting also that reals and p-adic should be fused to a larger structure.