QUANTUM TGD: PART III

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0.1 PREFACE

Brief summary of TGD

Towards the end of the year 2023 I became convinced that it would be appropriate to prepare collections about books related to TGD and its applications. The finiteness of human lifetime was my first motivation. My second motivation was the deep conviction that TGD will mean a revolution of the scientific world view and I must do my best to make it easier.

The first collection would relate to the TGD proper and its applications to physics. Second collection would relate to TGD inspired theory of consciousness and the third collection to TGD based quantum biology. The books in these collections would focus on much more precise topics than the earlier books and would be shorter. This would make it much easier for the reader to understand what TGD is, when the time is finally mature for the TGD to be taken seriously. This particular book belongs to a collection of books about TGD proper.

The basic ideas of TGD

TGD can be regarded as a unified theory of fundamental interactions but is not the kind of unified theory as so called GUTs constructed by graduate students in the seventies and eighties using detailed recipes for how to reduce everything to group theory. Nowadays this activity has been completely computerized and it probably takes only a few hours to print out the predictions of this kind of unified theory as an article in the desired format. TGD is something different and I am not ashamed to confess that I have devoted the last 45 years of my life to this enterprise and am still unable to write The Rules.

If I remember correctly, I got the basic idea of Topological Geometrodynamics (TGD) during autumn 1977, perhaps it was October. What I realized was that the representability of physical space-times as 4-dimensional surfaces of some higher-dimensional space-time obtained by replacing the points of Minkowski space with some very small compact internal space could resolve the conceptual difficulties of general relativity related to the definition of the notion of energy. This belief was too optimistic and only with the advent of what I call zero energy ontology the understanding of the notion of Poincare invariance has become satisfactory. This required also the understanding of the relationship to General Relativity.

It soon became clear that the approach leads to a generalization of the notion of space-time with particles being represented by space-time surfaces with finite size so that TGD could be also seen as a generalization of the string model. Much later it became clear that this generalization is consistent with conformal invariance only if space-time is 4-dimensional and the Minkowski space factor of the embedding space is 4-dimensional. During last year it became clear that 4-D Minkowski space and 4-D complex projective space CP_2 are completely unique in the sense that they allow twistor space with Kähler structure.

It took some time to discover that also the geometrization of also gauge interactions and elementary particle quantum numbers could be possible in this framework: it took two years to find the unique internal space (CP_2) providing this geometrization involving also the realization that family replication phenomenon for fermions has a natural topological explanation in TGD framework and that the symmetries of the standard model symmetries are much more profound than pragmatic TOE builders have believed them to be. If TGD is correct, the mainstream particle physics chose the wrong track leading to the recent deep crisis when people decided that quarks and leptons belong to the same multiplet of the gauge group implying instability of the proton.

Instead of trying to describe in detail the path, which led to TGD as it is now with all its side tracks, it is better to summarize the recent view which of course need not be final.

TGD can be said to be a fusion of special and general relativities. The Relativity Principle (Poincare Invariance) of Special Relativity is combined with the General Coordinate Invariance and Equivalence Principle of General Relativity. TGD involves 3 views of physics: physics geometry, physics as number theory and physics as topological physics in some sense.

Physics as geometry

"Geometro-" in TGD refers to the idea about the geometrization of physics. The geometrization program of Einstein is extended to gauge fields allowing realization in terms of the geometry of surfaces so that Einsteinian space-time as abstract Riemann geometry is replaced with sub-manifold geometry. The basic motivation is the loss of classical conservation laws in General Relativity Theory (GRT)(see **Fig. 1**). Also the interpretation as a generalization of string models by replacing string with 3-D surface is natural.

- Standard model symmetries uniquely fix the choice of 8-D space in which space-time surfaces live to $H = M^4 \times CP_2$ [L85]. Also the notion of twistor is geometrized in terms of surface geometry and the existence of twistor lift fixes the choice of H completely so that TGD is unique [L24, L35](see **Fig. 2**). The geometrization applies even to the quantum theory itself and the space of space-time surfaces - "world of classical worlds" (WCW) - becomes the basic object endowed with Kähler geometry (see **Fig. 3**). The mere mathematical existence of WCW geometry requires that it has maximal isometries, which together twistor lift and number theoretic vision fixes it uniquely [L86].

- General Coordinate Invariance (GCI) for space-time surfaces has dramatic implications. A given 3-surface fixes the space-time surface almost completely as analog of Bohr orbit (preferred extremal). This implies holography and leads to zero energy ontology (ZEO) in which quantum states are superpositions of space-time surfaces [K89, L47].

- From the beginning it was clear that the theory predicts the presence of long ranged classical electro-weak and color gauge fields and that these fields necessarily accompany classical electromagnetic fields in all scales. It took about 26 years to gain the maturity to admit the obvious: these fields are classical correlates for long range color and weak interactions assignable to the phases of ordinary matter predicted by the number theoretic vision and behaving like dark matter but identifiable as matter explaining the missing baryon problem whereas the galactic dark matter would correspond to the dark energy assignable monopole flux tubes as deformations of cosmic strings. The only possible conclusion is that TGD physics is a fractal consisting of an entire hierarchy of fractal copies of standard model physics. Also the understanding of electro-weak massivation and screening of weak charges has been a long standing problem and p-adic physics solved this problem in terms of p-adic thermodynamics [K17, K42] [L81].

- One of the most recent discoveries of classical TGD is exact general solution of the field equations. Holography can be realized as a generalized holomorphy realized in terms of what I call Hamilton-Jacobi structure [L83]. Space-time surfaces correspond to holomorphic imbeddings of the space-time surface to H with a generalized complex structure defined by the vanishing of 2 analytic functions of 4 generalized complex coordinates of H. These surfaces are automatically minimal surfaces. This is true for any geneneral coordinate invariant action constructed in terms of the induced geometric structures so that the dynamics is universal. Different actions differ only in the sense that singularities at which the minimal surface property fails depend on the action. This affects the scattering amplitudes, which can be constructed in terms of the data related to the singularities [L91].

- Generalized conformal symmetries define an extension of conformal symmetries and one can assign to them Noether charges. Besides this the so called super-symplectic symmetries associated with $\delta M^4_+ \times CP_2$ define isometries of the "world of classical worlds" (WCW), which by holography is essentially the space of Bohr orbits of 3-surfaces as particles so that quantum TGD is expected to reduce to a generalization of wave mechanics.

Physics as number theory

During these years TGD led to a rather profound generalization of the space-time concept. Quite general properties of the theory led to the notion of many-sheeted space-time with sheets representing physical subsystems of various sizes. At the beginning of 90s I became dimly aware of the importance of p-adic number fields and soon ended up with the idea that p-adic thermodynamics for a conformally invariant system allows to understand elementary particle massivation with amazingly few input assumptions. The attempts to understand p-adicity from basic principles led gradually to the vision about physics as a generalized number theory as an approach complementary to the physics as an infinite-dimensional spinor geometry of WCW approach. One of its elements was a generalization of the number concept obtained by fusing real numbers and various p-adic numbers along common rationals. The number theoretical trinity involves besides p-adic number fields also quaternions and octonions and the notion of infinite prime.

Adelic physics [L22, L23] fusing real and various p-adic physics is part of the number theoretic vision, which provides a kind of dual description for the description based on space-time geometry and the geometry of "world of classical words". Adelic physics predicts two fractal length scale hierarchies: p-adic length scale hierarchy and the hierarchy of dark length scales labelled by $h_{eff} = nh_0$, where n is the dimension of extension of rational. The interpretation of the latter hierarchy is as phases of ordinary matter behaving like dark matter. Quantum coherence is possible in arbitarily long scales. These two hierarchies are closely related. p-Adic primes correspond to ramified primes for a polynomial, whose roots define the extension of rationals: for a given extension this polynomial is not unique.

M^8-H duality

The concrete realization of the number theoretic vision is based on M^8-H duality (see **Fig. 4**). What the precise form is this duality is, has been far from clear but the recent form is the simplest one and corresponds to the original view [L87]. M^8 corresponds to octonions O but with the number theoretic metric defined by $Re(o^2)$ rather than the standard norm and giving Minkowskian signature.

The physics in M^8 can be said to be algebraic whereas in H field equations are partial differential equations. The dark matter hierarchy corresponds to a hierarchy of algebraic extensions of rationals inducing that for adeles and has interpretation as an evolutionary hierarchy (see **Fig. 5**). p-Adic physics is an essential part of number theoretic vision and the space-time surfaces are such that at least their M^8 counterparts exists also in p-adic sense. This requires that the analytic function defining the space-time surfaces are polynomials with rational coefficients.

M^8-H duality relates two complementary visions about physics (see **Fig. 6**), and can be seen as a generalization of the momentum-position duality of wave mechanics, which fails to generalize to quantum field theories (QFTs). M^8-H duality applies to particles which are 3-surfaces instead of point-like particles.

p-Adic physics

The idea about p-adic physics as physics of cognition and intentionality emerged also rather naturally and implies perhaps the most dramatic generalization of the space-time concept in which most points of p-adic space-time sheets are infinite in real sense and the projection to the real imbedding space consists of discrete set of points. One of the most fascinating outcomes was the observation that the entropy based on p-adic norm can be negative. This observation led to the vision that life can be regarded as something in the intersection of real and p-adic worlds. Negentropic entanglement has interpretation as a correlate for various positively colored aspects of conscious experience and means also the possibility of strongly correlated states stable under state function reduction and different from the conventional bound states and perhaps playing key role in the energy metabolism of living matter.

If one requires consistency of Negentropy Mazimization Principle with standard measurement theory, negentropic entanglement defined in terms of number theoretic negentropy is necessarily associated with a density matrix proportional to unit matrix and is maximal and is characterized by the dimension n of the unit matrix. Negentropy is positive and maximal for a p-adic unique prime dividing n.

Hierarchy of Planck constants labelling phases ordinary matter dark matter behaving like dark matter

One of the latest threads in the evolution of ideas is not more than nine years old. Learning about the paper of Laurent Nottale about the possibility to identify planetary orbits as Bohr orbits with a gigantic value of gravitational Planck constant made once again possible to see the obvious. Dynamical quantized Planck constant is strongly suggested by quantum classical correspondence and the fact that space-time sheets identifiable as quantum coherence regions can have arbitrarily large sizes. Second motivation for the hierarchy of Planck constants comes from bio-electromagnetism suggesting that in living systems Planck constant could have large values making macroscopic quantum coherence possible. The interpretation of dark matter as a hierarchy of phases of ordinary matter characterized by the value of Planck constant is very natural.

During summer 2010 several new insights about the mathematical structure and interpretation of TGD emerged. One of these insights was the realization that the postulated hierarchy of Planck constants might follow from the basic structure of quantum TGD. The point is that due to the extreme non-linearity of the classical action principle the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is one-to-many and the natural description of the situation is in terms of local singular covering spaces of the imbedding space. One could speak about effective value of Planck constant $h_{eff} = n \times h$ coming as a multiple of minimal value of Planck constant. Quite recently it became clear that the non-determinism of Kähler action is indeed the fundamental justification for the hierarchy: the integer n can be also interpreted as the integer characterizing the dimension of unit matrix characterizing negentropic entanglement made possible by the many-sheeted character of the space-time surface.

Due to conformal invariance acting as gauge symmetry the n degenerate space-time sheets must be replaced with conformal equivalence classes of space-time sheets and conformal transformations correspond to quantum critical deformations leaving the ends of space-time surfaces invariant. Conformal invariance would be broken: only the sub-algebra for which conformal weights are divisible by n act as gauge symmetries. Thus deep connections between conformal invariance related to quantum criticality, hierarchy of Planck constants, negentropic entanglement, effective p-adic topology, and non-determinism of Kähler action perhaps reflecting p-adic non-determinism emerges.

The implications of the hierarchy of Planck constants are extremely far reaching so that the significance of the reduction of this hierarchy to the basic mathematical structure distinguishing between TGD and competing theories cannot be under-estimated.

TGD as an analog of topological QFT

Consider next the attribute "Topological". In condensed matter physical topological physics has become a standard topic. Typically one has fields having values in compact spaces, which are topologically non-trivial. In the TGD framework space-time topology itself is non-trivial as also the topology of $H = M^4 \times CP_2$. Since induced metric is involved with TGD, it is too much to say that TGD is topological QFT but one can for instance say, that space-time surfaces as preferred extremals define representatives for 4-D homological equivalence classes.

The space-time as 4-surface $X^4 \subset H$ has a non-trivial topology in all scales and this together with the notion of many-sheeted space-time brings in something completely new. Topologically trivial Einsteinian space-time emerges only at the QFT limit in which all information about topology is lost (see **Fig. 7**).

Any GCI action satisfying holography=holomorphy principle has the same universal basic extremals: CP_2 type extremals serving basic building bricks of elementary particles, cosmic strings and their thickenings to flux tubes defining a fractal hierarchy of structure extending from CP_2 scale to cosmic scales, and massless extremals (MEs) define space-time correletes for massless particles. World as a set or particles is replaced with a network having particles as nodes and flux tubes as bonds between them serving as correlates of quantum entanglement.

"Topological" could refer also to p-adic number fields obeying p-adic local topology differing radically from the real topology (see **Fig. 8**).

Zero energy ontology

TGD inspired theory of consciousness entered the scheme after 1995 as I started to write a book about consciousness. Gradually it became difficult to say where physics ends and consciousness theory begins since consciousness theory could be seen as a generalization of quantum measurement theory by identifying quantum jump as a moment of consciousness and by replacing the observer with the notion of self identified as a system which is conscious as long as it can avoid entanglement with environment. The somewhat cryptic statement "Everything is conscious and consciousness can be only lost" summarizes the basic philosophy neatly.

General coordinate invariance leads to the identification of space-time surfaces are analogous to Bohr orbits inside causal diamond (CD). CD obtained as intersection of future and past directed light-cones (with CP_2 factor included). By the already described hologamphy, 3-dimensional data replaces the boundary conditions at single 3-surface involving also normal derivatives with conditions involving no derivates.

In zero energy ontology (ZEO), the superpositions of space-time surfaces inside causal diamond (CD) having their ends at the opposite light-like boundaries of CD, define quantum states. CDs form a scale hierarchy (see **Fig. 9** and **Fig. 10**). Quantum states are modes of WCW spinor fields, essentially wave functions in the space WCW consisting of Bohr orbit-like 4-surfaces.

Quantum jumps occur between these and the basic problem of standard quantum measurement theory disappears. Ordinary state function reductions (SFRs) correspond to "big" SFRs (BSFRs) in which the arrow of time changes (see **Fig. 11**). This has profound thermodynamic implications and the question about the scale in which the transition from classical to quantum takes place becomes obsolete. BSFRs can occur in all scales but from the point of view of an observer with an opposite arrow of time they look like smooth time evolutions.

In "small" SFRs (SSFRs) as counterparts of "weak measurements" the arrow of time does not change and the passive boundary of CD and states at it remain unchanged (Zeno effect).

Equivalence Principle in TGD framework

There have been also longstanding problems related to the relationship between inertial mass and gravitational mass, whose identification has been far from obvious.

- Gravitational energy is well-defined in cosmological models but is not conserved. Hence the conservation of the inertial energy does not seem to be consistent with the Equivalence Principle. In this framework the quantum numbers are assigned with zero energy states located at the boundaries of CDs defined as intersections of future and past directed light-cones. The notion of energy-momentum becomes length scale dependent since one has a scale hierarchy for causal diamonds. This allows to understand the non-conservation of energy as apparent.

  Equivalence Principle in the form expressed by Einstein's equations follows from Poincare invariance once it is realized that GRT space-time is obtained from the many-sheeted space-time of TGD by lumping together the space-time sheets to a regionof Minkowski space and endowing it with an effective metric given as a sum of Minkowski metric and deviations of the metrices of space-time sheets from Minkowski metric. Similar description relates classical gauge potentials identified as components of induced spinor connection to Yang-Mills gauge potentials in GRT space-time. Various topological inhomogenities below resolution scale identified as particles are described using energy momentum tensor and gauge currents.

  At quantum level, the Equivalence Principle has a surprisingly strong content. In linear Minkowski coordinates, space-time projection of the M^4 spinor connection representing gravitational gauge potentials the coupling to induced spinor fields vanishes. Also the modified Dirac action for the solutions of the modified Dirac equation seems to vanish identically and in TGD perturbative approach separating interaction terms is not possible.

  The modified Dirac equation however fails at the singularities of the minimal surface representing space-time surface and Dirac action reduces to an integral over singularities for the trace of the second fundamental form slashed between the induced spinor field and its conjugate. Also the M^4 part of the trace is non-vanishing and gives rise to the gravitational coupling. The trace gives both standard model vertices and graviton emission vertices. One could say that at the quantum level gravitational and gauge interactions are eliminated everywhere except at the singularities identifiable as defects of the ordinary smooth structure. The exotic smooth structures [L75], possible only in dimension 4, are ordinary smooth structures apart from these defects serving as vertex representing a creation of a fermion-antifermion pair in the induced gauge potentials. The vertex is universal and essentially the trace of the second fundamental form as an analog of the Higgs field and the gravitational constant is proportional to the square of CP_2 radius.

- There is a delicate difference between inertial and gravitational masses. One can assume that the modes of the imbedding space spinor fields are solutions of massles Dirac equation in either $M^4 \times CP_2$ and therefore eigenstates of inertial momentum or in $CD = cd \times CP_2$: in this case they are only mass eigenstates. The mass spectra are identical for these options. Inertial momenta correspond naturally to the Poincare charges in the space of CDs. For the CD option the spinor modes correspond to mass squared eigenstates for which the mode for H^3 with a given value of light-proper time is a unitary irreducible $SO(1,3)$ representation rather than a representation of translation group. These two eigenmode basis correspond to gravitational basis for spinor modes.

Quantum TGD as a generalization of Einstein's geometrization program

I started the serious attempts to construct quantum TGD after my thesis around 1982. The original optimistic hope was that path integral formalism or canonical quantization might be enough to construct the quantum theory but it turned that this approach fails due to the extreme non-linearity of the theory.

It took some years to discover that the only working approach is based on the generalization of Einstein's program. Quantum physics involves the geometrization of the infinite-dimensional "world of classical worlds" (WCW) identified as the space of 3-dimensional surfaces. Later 3-surfaces where replaced with 4-surfaces satifying holography and therefore as analogs of Bohr orbits.

- If one assumes Bohr orbitology, then strong correlations between the 3-surfaces at the ends of CD follow and mean holography. It is natural to identify the quantum states of the Universe (and sub-Univeverses) as modes of a formally classical spinor field in WCW. WCW gamma matrices are expressible in terms of oscillator operators of free second quantized spinor fields of H. The induced spinor fields identified projections of H spinor fields to the space-time surfaces satisfy modified Dirac equation for the modified Dirac equation. Only quantum jump remains the genuinely quantal aspect of quantum physics.

- Quantum TGD can be seen as a theory for free spinor fields in WCW having maximal isometries and the generalization of the Super Virasoro conditions gives rise to the analog massless Dirac equation at the level of WCW.

The world of classical worlds and its symmetries

The notion of "World of Classical Worlds" (WCW) emerged around 1985 but found its basic form around 1990. Holography forced by the realization of General Coordinate Invariance forced/allowed to give up the attempts to make sense of the path integral.

A more concrete way to express this view is that WCW does not consist of 3-surfaces as particle-like entities but almost deterministic Bohr orbits assignable to them as preferred extremals of Kähler action so that quantum TGD becomes wave mechanics in WCW combined with Bohr orbitology. This view has profound implications, which can be formulated in terms of zero energy ontology (ZEO), solving among other things the basic paradox of quantum measurement theory. ZEO forms also the backbone of TGD inspired theory of consciousness and quantum biology.

WCW geometry exists only if it has maximal isometries: this statement is a generalization of the discovery of Freed for loop space geometries [A11]. I have proposed [K35, K19, K86, K66, L86] that WCW could be regarded as a union of generalized symmetric spaces labelled by zero modes which do not contribute to the metric. The induced Kähler field is invariant under symplectic transformations of CP_2 and would therefore define zero mode degrees of freedom if one assumes that WCW metric has symplectic transformations as isometries. In particular, Kähler magnetic fluxes would define zero modes and are quantized closed 2-surfaces. The induced metric appearing in Kähler action is however not zero mode degree of freedom. If the action contains volume term, the assumption about union of symmetric spaces is not well-motivated.

Symplectic transformations are not the only candidates for the isometries of WCW. The basic picture about what these maximal isometries could be, is partially inspired by string models.

- A weaker proposal is that the symplectomorphisms of H define only symplectomorphisms of WCW. Extended conformal symmetries define also a candidate for isometry group. Remarkably, light-like boundary has an infinite-dimensional group of isometries which are in 1-1 correspondence with conformal symmetries of $S^2 \subset S^2 \times R_+ = \delta M^4_+$.

- Extended Kac Moody symmetries induced by isometries of δM^4_+ are also natural candidates for isometries. The motivation for the proposal comes from physical intuition deriving from string models. Note they do not include Poincare symmetries, which act naturally as isometries in the moduli space of causal diamonds (CDs) forming the "spine" of WCW.

- The light-like orbits of partonic 2-surfaces might allow separate symmetry algebras. One must however notice that there is exchange of charges between interior degrees of freedom and partonic 2-surfaces. The essential point is that one can assign to these surface conserved charges when the dual light-like coordinate defines time coordinate. This picture also assumes a slicing of space-time surface by by the partonic orbits for which partonic orbits associated with wormrhole throats and boundaries of the space-time surface would be special. This slicing would correspond to Hamilton-Jacobi structure.

- Fractal hierarchy of symmetry algebras with conformal weights, which are non-negative integer multiples of fundamental conformal weights, is essential and distinguishes TGD from string models. Gauge conditions are true only the isomorphic subalgebra and its commutator with the entire algebra and the maximal gauge symmetry to a dynamical symmetry with generators having conformal weights below maximal value. This view also conforms with p-adic mass calculations.

- The realization of the symmetries for 3-surfaces at the boundaries of CD and for light-like orbits of partonic 2-surfaces is known. The problem is how to extend the symmetries to the interior of the space-time surface. It is natural to expect that the symmetries at partonic orbits and light-cone boundary extend to the same symmetries.

After the developments towards the end of 2023, it seems that the extension of conformal and Kac-Moody symmetries of string models to the TGD framework is understood. What about symplectic symmetries, which were originally proposed as isometries of WCW? In this article this question is discussed in detail and it will be found that these symmetries act naturally on 3-D holographic data and one can identify conserved charges. By holography this is in principle enough and might imply that the actions of holomorphic and symplectic symmetry algebras are dual. Holography=holomorphy hypothesis is discussed also in the case of the modified Dirac equation.

About the construction of scattering amplitudes

From the point of view of particle physics the ultimate goal is of course a practical construction recipe for the S-matrix of the theory. I have myself regarded this dream as quite too ambitious taking into account how far-reaching re-structuring and generalization of the basic mathematical structure of quantum physics is required. After having made several guesses for what the counterpart of S-matrix could be, it became clear that the dream about explicit formulas is unrealistic before one has understood what happens in quantum jump.

- In ZEO [K89, L47] one must distinguish between "small" state function reductions (SSFRs) and "big" SFRs (BSFRs). BSFR is the TGD counterpart of the ordinary SFRs and the arrow of the geometric time changes in it. SSFR follows the counterpart of a unitary time evolution and the arrow of the geometric time is preserved in SSFR. The sequence of SSFRs is the TGD counterpart for the sequence of repeated quantum measurements of the same observables in which nothing happens to the state. In TGD something happens in SSFRs and this gives rise to the flow of consciousness. When the set of the observables measured in SSFR does not commute with the previous set of measured observables, BSFR occurs.

The evolution by SSFRs means that also the causal diamond changes. At quantum level one has a wave function in the finite-dimensional moduli space of CDs which can be said to form a spine of WCW [L84]. CDs form a scale hierarchy. SSFRs are preceded by a dispersion in the moduli space of CDs and SSFR means localization in this space.

- There are several S-matrix like entities. One can assign an analog of the S-matrix to each analog of unitary time evolution preceding a given SSFR. One can also assign an analog S-matrix between the eigenstate basis of the previous set of observables and the eigenstate basis of new observers: this S-matrix characterizes BSFR. One can also assign to zero energy states an S-matrix like entity between the states assignable to the two boundaries of CD. These S-matrix like objects can be interpreted as a complex square root of the density matrix representable as a diagonal and positive square root of density matrix and unitary S-matrix so that quantum theory in ZEO can be said to define a square root of thermodynamics at least formally.

In standard QFTs Feynman diagrams provide the description of scattering amplitudes. The beauty of Feynman diagrams is that they realize unitarity automatically via the so-called Cutkosky rules. In contrast to Feynman's original beliefs, Feynman diagrams and virtual particles are taken only as a convenient mathematical tool in quantum field theories. The QFT approach is however plagued by UV and IR divergences and one must keep mind open for the possibility that a genuine progress might mean opening of the black box of the virtual particle.

In the TGD framework this generalization of Feynman diagrams indeed emerges unavoidably.

- The counterparts of elementary particles can be identified as closed monopole flux tubes connecting two parallel Minkowskian space-time sheets and have effective ends which are Euclidean wormhole contacts. The 3-D light-like boundaries of wormhole contacts as orbits of partonic 2-surfaces.

  The intuitive picture is that the 3-D light-like partonic orbits replace the lines of Feynman diagrams and vertices are replaced by 2-D partonic 2-surfaces. A stronger condition is that fermion number is carried by light-like fermion lines at the partonic orbits, which can be identified as boundaries string world sheets.

- The localization of the nodes of induced spinor fields to 2-D string world sheets (and possibly also to partonic 2-surfaces) implies a stringy formulation of the theory analogous to stringy variant of twistor formalism with string world sheets having interpretation as 2-braids. In the TGD framework, the fermionic variant of twistor Grassmann formalism combined with the number theoretic vision [L72, L73] led to a stringy variant of the twistor diagrammatics.

- Fundamental fermions are off-mass-shell in the sense that their momentum components are real algebraic integers in an extension of rationals associated with the space-time surfaces inside CD with a momentum unit determined by the CD size scale. Galois confinement states that the momentum components are integer valued for the physical states.

- The twistorial approach suggests also the generalization of the Yangian symmetry to infinite-dimensional super-conformal algebras, which would determine the vertices and scattering amplitudes in terms of poly-local symmetries.

The twistorial approach is however extremely abstract and lacks a concrete physical interpretation. The holography=holomorphy vision led to a breakthough in the construction of the scattering amplitudes by solving the problem of identifying interaction vertices [L91].

1. The basic prediction is that space-time surfaces as analogs of Bohr orbits are holomorphic in a generalized sense and are therefore minimal surfaces. The minimal surface property fails at lower-dimensional singularities and the trace of the second fundamental form (SFF) analogous to acceleration associated with the Bohr orbit of the particle as 3-surface has a delta function like singularity but vanishes elsewhere.

2. The minimal surface property expressess masslessness for both fields and particles as 3-surfaces. At singularities masslessness property fails and singularities can be said to serve as sources which also in QFT define scattering amplitudes.

3. The singularities are analogs of poles and cuts for the 4-D generalization of the ordinary holomorphic functions. Also for the ordinary holomorphic functions the Laplace equation as analog massless field equation and expressing analyticity fails. Complex analysis generalizes to dimension 4.

4. The conditions at the singularity give a generalization of Newton's "F=ma"! I ended up where I started more than 50 years ago!

5. In dimension 4, and only there, there is an infinite number of exotic diff structures [**?**], which differ from ordinary ones at singularities of measure zero analogous to defects. These defects correspond naturally to the singularities of minimal surfaces. One can say that for the exotic diff structure there is no singularity.

6. Group theoretically the trace of the SFF can be regarded as a generalization of the Higgs field, which is non-vanishing only at the vertices and this is enough. Singularities take the role of generalized particle vertices and determine the scattering amplitudes. The second fundamental form contracted with the embedding space gamma matrices and slashed between the second quantized induced spinor field and its conjugate gives the universal vertex involving only fermions (bosons are bound states of fermions in TGD). It contains both gauge and gravitational contributions to the scattering amplitudes and there is a complete symmetry between gravitational and gauge interactions. Gravitational couplings come out correctly as the radius squared of CP_2 as also in the classical picture.

7. The study of the modified Dirac equation leads to the conclusion that vertices as singularities and defects contain the standard electroweak gauge contribution coming from the induced spinor connection and a contribution from the M^4 spinor connection. M^4 part of the generalized Higgs can give rise to a graviton as an $L = 1$ rotational state of the flux tube representing the graviton. It is not clear whether M^4 Kähler gauge potential can give rise to a spin 1 particle. The vielbein part of M^4 spinor connection is pure gauge and could give rise to gravitational topological field theory.

Figures

What I have said above is strongly biased view about the recent situation in quantum TGD. This vision is single man's view and doomed to contain unrealistic elements as I know from experience. My dream is that young critical readers could take this vision seriously enough to try to demonstrate that some of its basic premises are wrong or to develop an alternative based on these or better premises. I must be however honest and tell that 45 years of TGD is a really vast bundle of thoughts and quite a challenge for anyone who is not able to cheat himself by taking the attitude of a blind believer or a light-hearted debunker trusting on the power of easy rhetoric tricks. The books provide a view of how TGD evolved rather than the final theory and there are archeological layers containing mammoth bones, which reflect earlier views not necessarily consistent with the recent view.

Karkkila, April 21, 2024, Finland

Matti Pitkänen

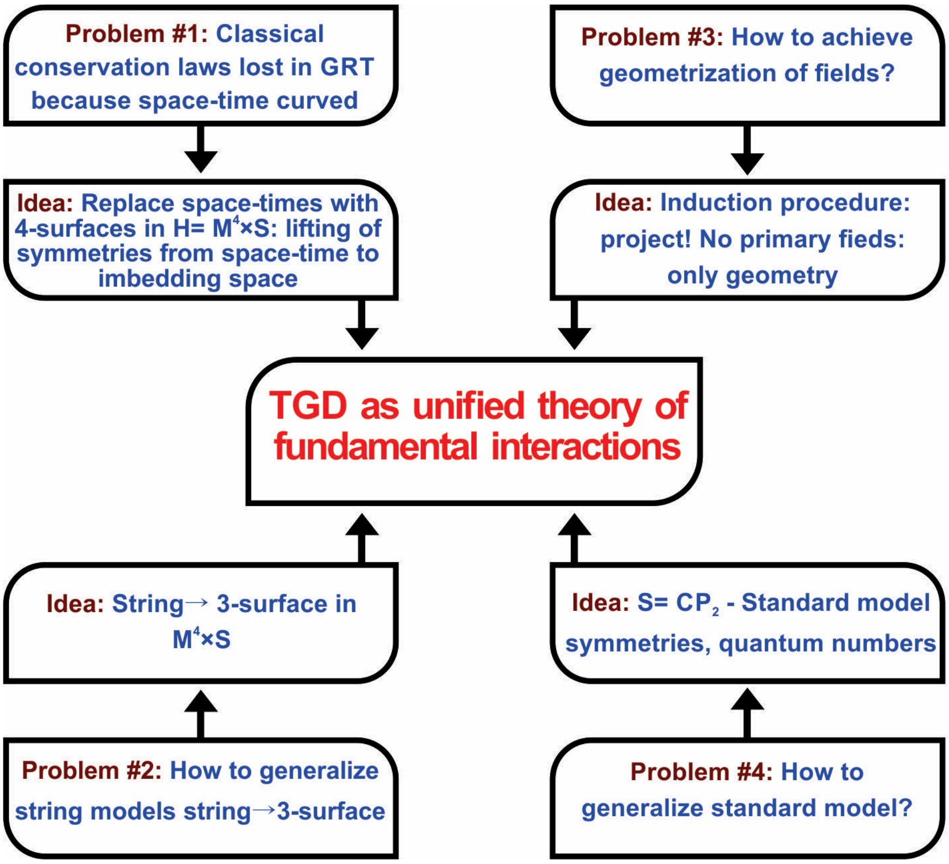


Figure 1: The problems leading to TGD as their solution.

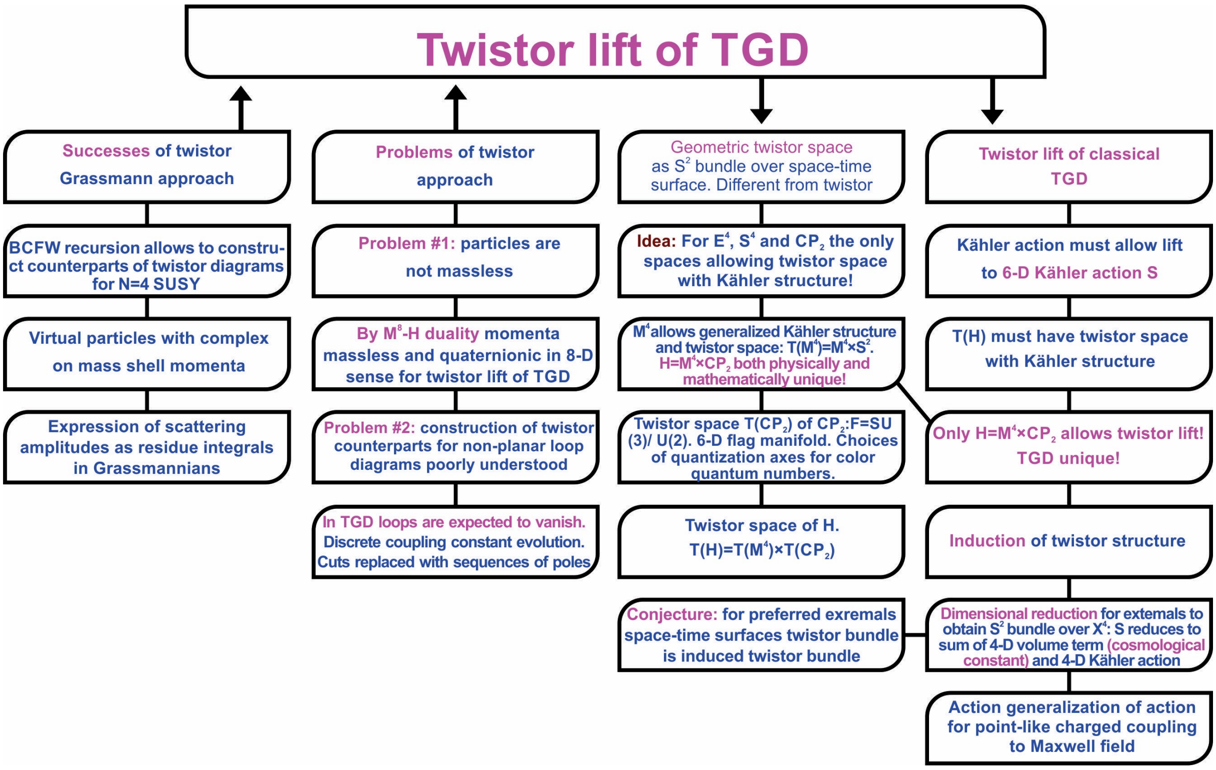


Figure 2: Twistor lift

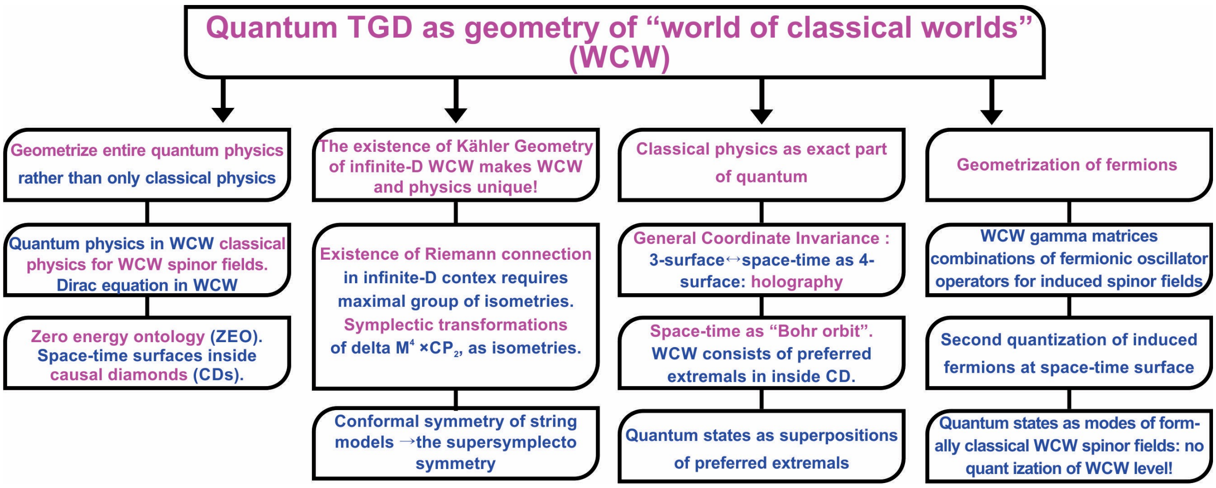


Figure 3: Geometrization of quantum physics in terms of WCW

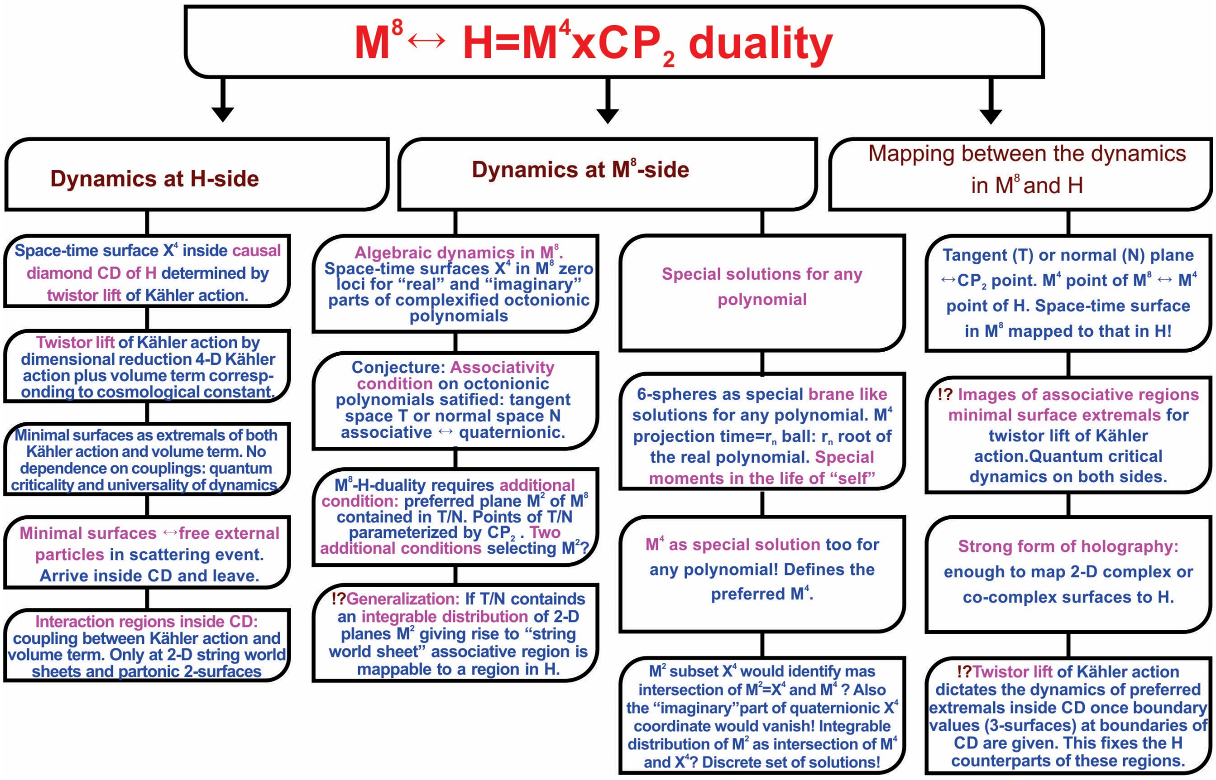


Figure 4: $M^8 - H$ duality

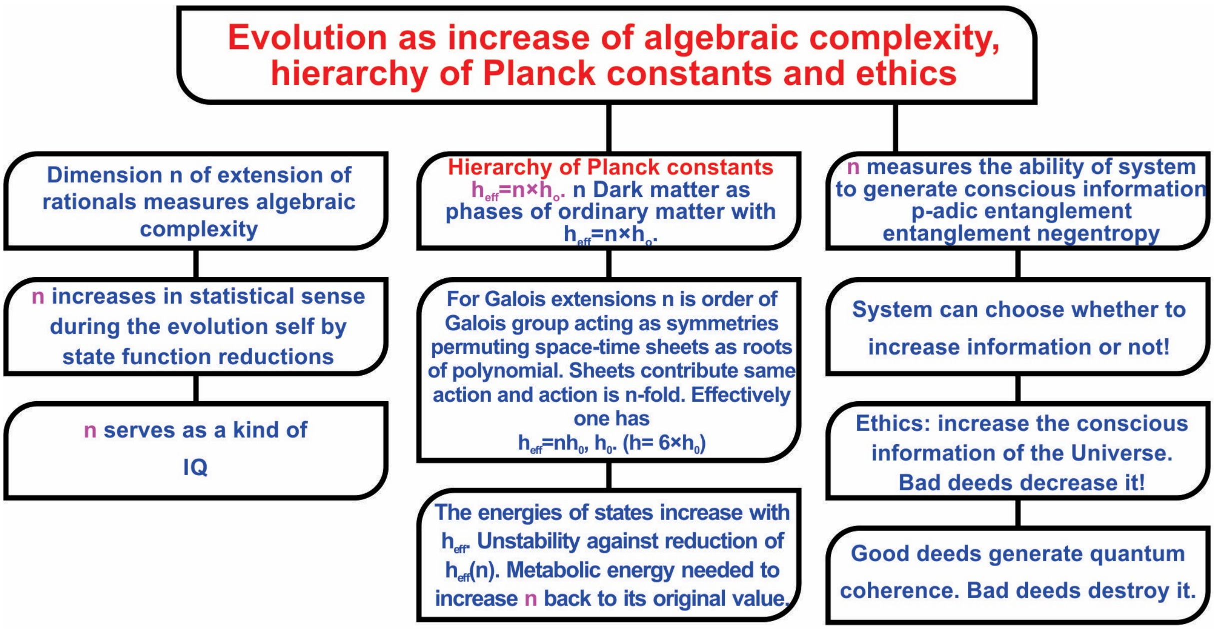


Figure 5: Number theoretic view of evolution

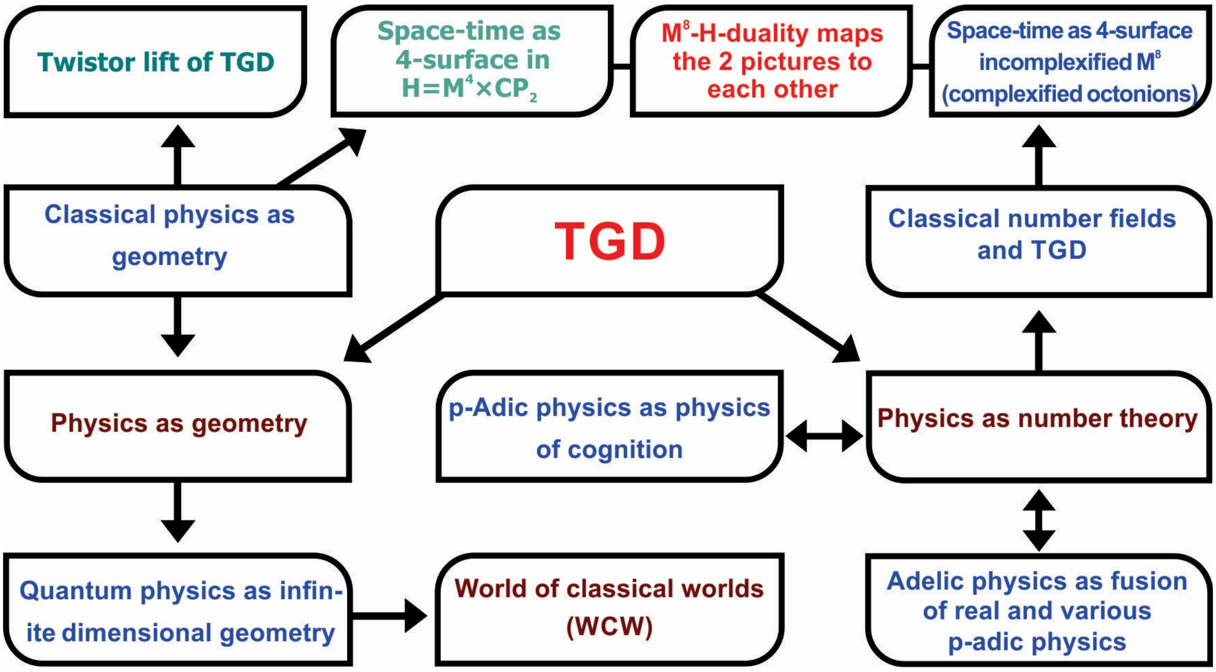


Figure 6: TGD is based on two complementary visions: physics as geometry and physics as number theory.

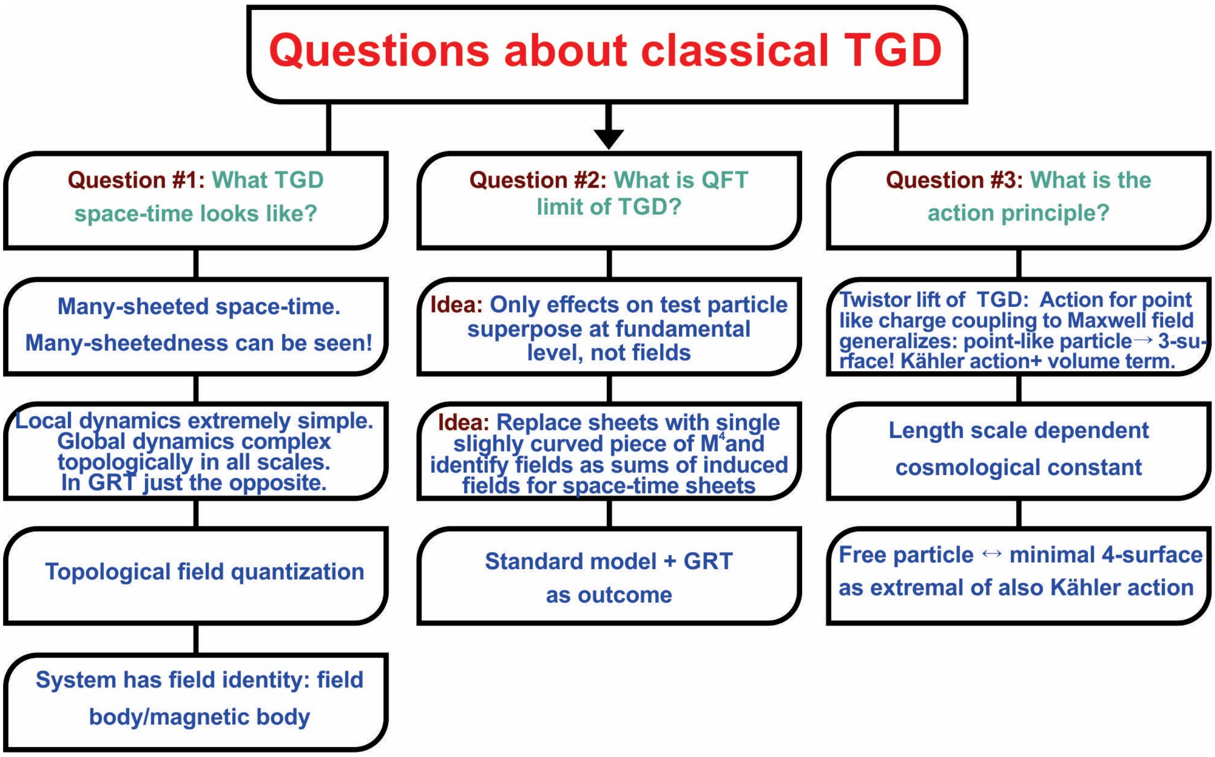


Figure 7: Questions about classical TGD.

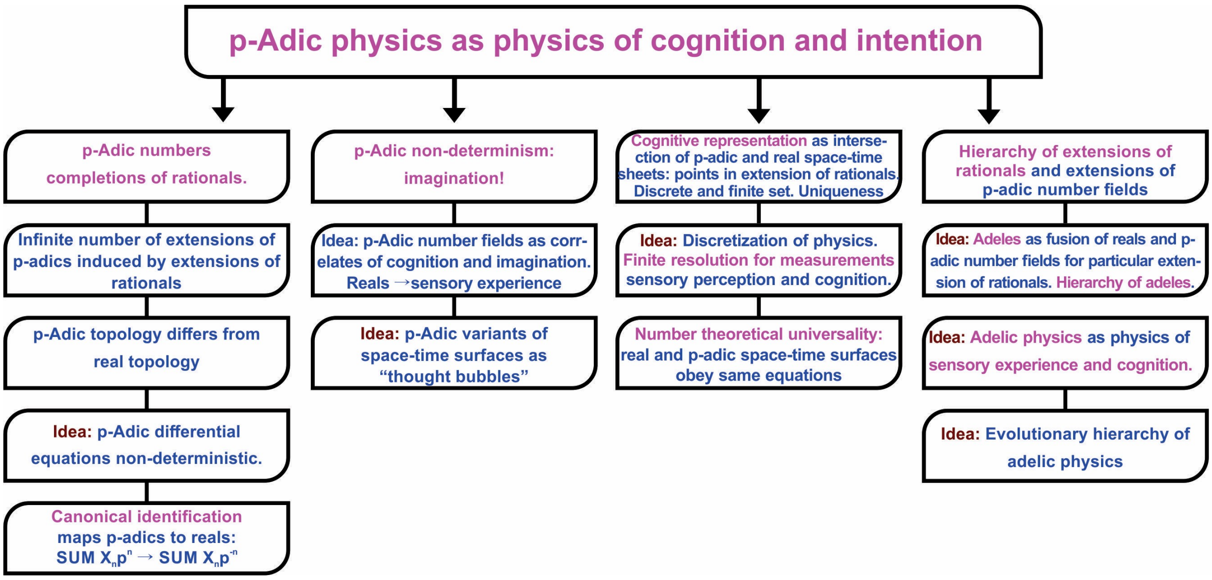


Figure 8: p-Adic physics as physics of cognition and imagination.

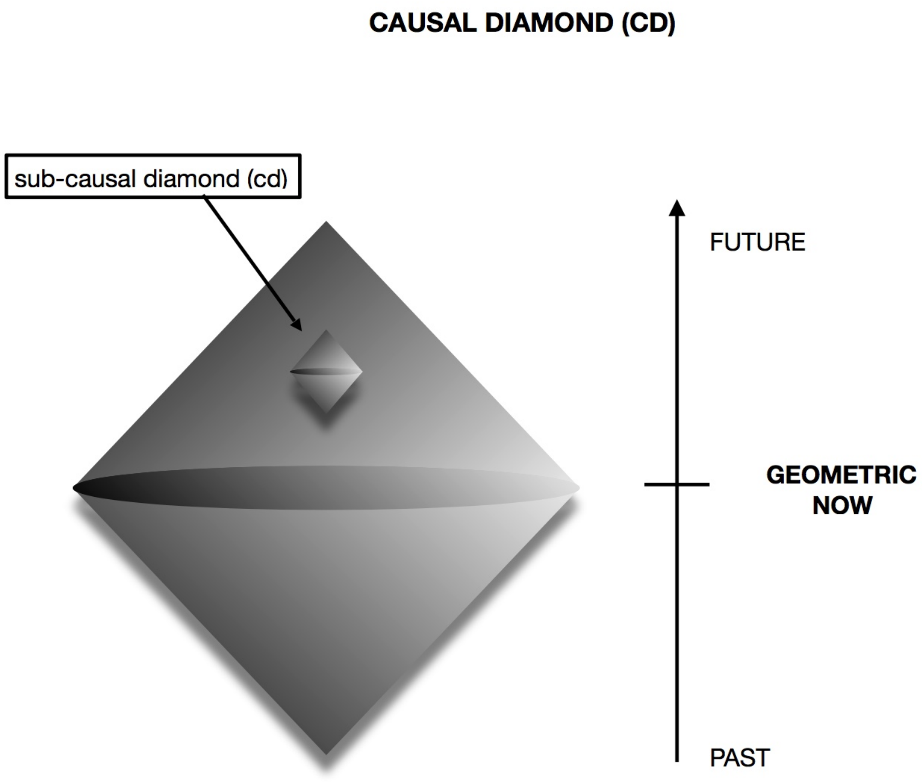
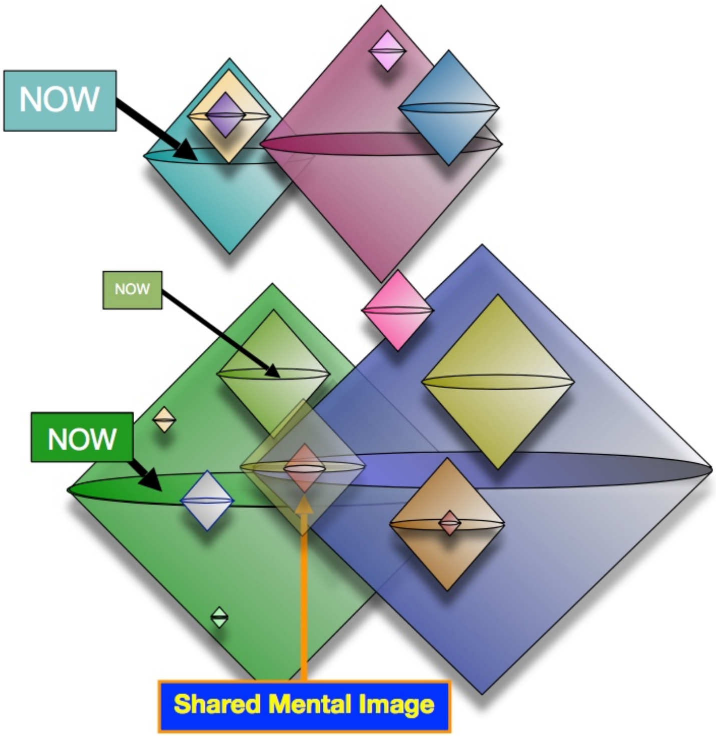


Figure 9: Causal diamond



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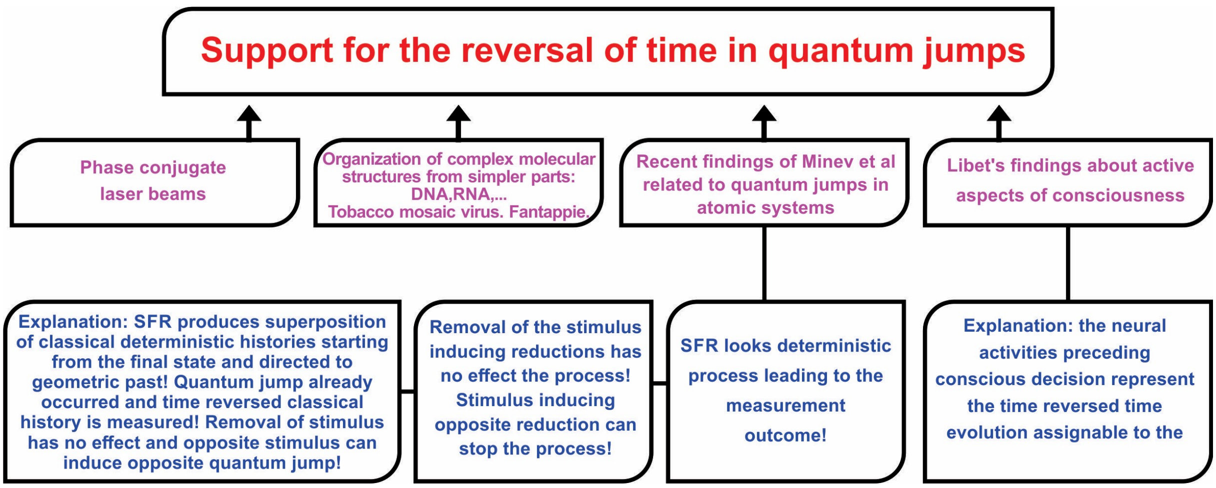

Figure 11: Time reversal occurs in BSFR

ACKNOWLEDGEMENTS

Neither TGD nor these books would exist without the help and encouragement of many people. The friendship with Heikki and Raija Haila and their family and Kalevi and Ritva Tikkanen and their family have been kept me in contact with the everyday world and without this friendship I would not have survived through these lonely 45 lonely years most of which I have remained unemployed as a scientific dissident. I am happy that my children have understood my difficult position and like my friends have believed that what I am doing is something valuable although I have not received any official recognition for it.

During the last decade Tapio Tammi has helped me quite concretely by providing the necessary computer facilities and being one of the few persons in Finland with whom to discuss my work. Pertti Kärkkäinen is my old physicist friend and has provided continued economic support for a long time. I have also had stimulating discussions with Samuli Penttinen who has also helped to get through the economical situations in which there seemed to be no hope. The continual updating of fifteen online books means quite a heavy bureaucracy at the level of bits and without a systemization one ends up with endless copying and pasting and internal consistency is soon lost. Tommi Ullgren has provided both economic support and encouragement during years. Pekka Rapinoja has offered his help in this respect and I am especially grateful to him for my Python skills.

During the last five years I have had inspiring discussions with many people in Finland interested in TGD. We have had video discussions with Sini Kunnas and had podcast discussions with Marko Manninen related to the TGD based view of physics and consciousness. Marko has also helped in the practical issues related to computers and quite recently he has done a lot of testing of chatGPT helping me to get an overall view of what it is. The discussions in a Zoom group involving Marko Manninen, Tuomas Sorakivi and Rode Majakka have given me the valuable opportunity to clarify my thoughts.

The collaboration with Lian Sidorov was extremely fruitful and she also helped me to survive economically through the hardest years. The participation in CASYS conferences in Liege has been an important window to the academic world and I am grateful for Daniel Dubois and Peter Marcer for making this participation possible. The discussions and collaboration with Eduardo de Luna and Istvan Dienes stimulated the hope that the communication of new vision might not be a mission impossible after all. Also blog discussions have been very useful. During these years I have received innumerable email contacts from people around the world. I am grateful to Mark McWilliams, Paul Kirsch, Gary Ehlenberg, and Ulla Matfolk and many others for providing links to possibly interesting websites and articles. We have collaborated with Peter Gariaev and Reza Rastmanesh. These contacts have helped me to avoid the depressive feeling of being some kind of Don Quixote of Science and helped me to widen my views: I am grateful for all these people.

In the situation in which the conventional scientific communication channels are strictly closed it is important to have some loop hole through which the information about the work done can at least in principle leak to the public through the iron wall of academic censorship. Without any exaggeration I can say that without the world wide web I would not have survived as a scientist nor as an individual. Homepage and blog are however not enough since only the formally published result is a result in recent day science. Publishing is however impossible without direct support from power holders- even in archives like arXiv.org.

Situation changed as Andrew Adamatsky proposed the writing of a book about TGD when I had already gotten used to the thought that my work would not be published during my lifetime. The Prespacetime Journal and two other journals related to quantum biology and consciousness - all of them founded by Huping Hu - have provided this kind of loophole. In particular, Dainis Zeps, Phil Gibbs, and Arkadiusz Jadczyk deserve my gratitude for their kind help in the preparation of an article series about TGD catalyzing a considerable progress in the understanding of quantum TGD. Also the viXra archive founded by Phil Gibbs and its predecessor Archive Freedom have been of great help: Victor Christianto deserves special thanks for doing the hard work needed to run Archive Freedom. Also the Neuroquantology Journal founded by Sultan Tarlaci deserves a special mention for its publication policy.

And last but not least: there are people who experience as a fascinating intellectual challenge to spoil the practical working conditions of a person working with something which might be called unified theory: I am grateful for the people who have helped me to survive through the virus attacks, an activity which has taken roughly one month per year during the last half decade and given a strong hue of grey to my hair.

For a person approaching his 73th birthday it is somewhat easier to overcome the hard feelings due to the loss of academic human rights than for an inpatient youngster. Unfortunately the economic situation has become increasingly difficult during the twenty years after the economic depression in Finland which in practice meant that Finland ceased to be a constitutional state in the strong sense of the word. It became possible to depose people like me from society without fear about public reactions and the classification as dropout became a convenient tool of ridicule to circumvent the ethical issues. During the period when the right wing held political power this trend was steadily strengthening and the situation is the same as I am writing this. In this kind of situation the concrete help from individuals has been and will be of utmost importance. Against this background it becomes obvious that this kind of work is not possible without the support from outside and I apologize for not being able to mention all the people who have helped me during these years.

Karkkila, August 30, 2023, Finland

Matti Pitkänen

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Chapter 1

Introduction

1.1 Basic Ideas of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged towards the end of 1977 - would emerge now it would be seen as an attempt to solve the difficulties of these approaches to unification.

The basic physical picture behind the geometric vision of TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model. After 1995 number theoretic vision started to develop and was initiated by the success of mass calculations based on p-adic thermodynamics. Number theoretic vision involves all number fields and is complementary to the geometric vision: one can say that this duality is analogous to momentum-position duality of wave mechanics. TGD can be also regarded as topological quantum theory in a very general sense as already the attribute "Topological" in "TGD" makes clear. Space-time surfaces as minimal surfaces can be regarded as representatives of homology equivalence classes and p-adic topologies generalize the notion of local topology and apply to the description of correlates of cognition.

1.1.1 Geometric Vision Very Briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K1].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional embedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).

2. Induction procedure (a standard procedure in fiber bundle theory, see Appendix) allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections of CP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface. Spinor connection is also projected. The interpretation is that distances are measured in embedding space metric and parallel translation using spinor connection of embedding space.

Twistor lift of TGD means that one can lift space-time surfaces in H to 6-D surfaces a analogs of twistor space of space-time surface in the Cartesian product of the twistor spaces of M^4 and CP_2, which are the only 4-manifolds allowing twistor space with Kähler structure [A18]. The twistor structure would be induced in some sense, and should coincide with that associated with the induced metric. Clearly, the 2-spheres defining the fibers of twistor spaces of M^4 and CP_2 must allow identification: this 2-sphere defines the S^2 fiber of the twistor space of the space-time surface. This poses a constraint on the embedding of the twistor space of space-time surfaces as sub-manifold in the Cartesian product of twistor spaces. The existence of Kähler structure allows to lift 4-D Kähler action to its 6-D counterparts and the 6-D counterpart of twistor space is obtained by its dimensional reduction so that one obtains a sphere bundle. This makes possible twistorialization for all space-time surfaces: in general relativity the general metric does not allow this.

3. A geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from the standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

   M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of embedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field-like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

   Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particles in space-time can be identified as a topological inhomogeneities in background space-time surface which looks like the space-time of general relativity in long length scales.

   One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distances of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

   This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which the standard model and general relativity follow as a topological simplification, however forcing a dramatic increase of the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. The basic problem is that one has long ranged classical electroweak gauge fields. The resolution of the problem is that the quantum averages of induced weak and color gauge fields vanish due to the fact that color rotations affect both space-time surfaces and induced weak and color fields. Only the averages of

electromagnetic fields are nonvanishing. The correlations functions for weak fields are nonvanishing below Compton lengths of weak bosons. In living matter large values of effective Planck constant labelling phases of ordinary matter identified as dark matter make possible long ranged weak fields and color fields.

6. General coordinate invariance requires holography so that space-time surfaces are analogous to Bohr orbits for particles identified as 3-surfaces. Bohr orbit property would be naturally realized by a 4-D generalization of holomorphy of string world sheets and implies that the space-time surfaces are minimal surfaces apart from singularities. This holds true for any action as long as it is general coordinate invariant and constructible in terms of the induced geometry. String world sheets and light-like orbits of partonic 2-surfaces correspond to singularities at which the minimal surface property of the space-time surfaces realizing the preferred extremal property fails. Preferred extremals are not completely deterministic, which implies what I call zero energy ontology (ZEO) meaning that the Bohr orbits are the fundamental objects. This leads to a solution of the basic paradox of quantum measurement theory. Also the mathematically ill-defined path integral disappears and leaves only the well-defined functional integral over the Bohr orbits.

7. A string model-like picture emerges from TGD and one ends up with a rather concrete view about the topological counterpart of Feynman diagrammatics. The natural stringy action would be given by the string world sheet area, which is present only in the space-time regions with Minkowskian signature. Gravitational constant could be present as a fundamental constant in string action and the ratio $\hbar/G/R^2$ would be determined by quantum criticality conditions. The hierarchy of Planck constants $h_{eff}/h = n$ assigned to dark matter in TGD framework would allow to circumvent the objection that only objects of length of order Planck length are possible since string tension given by $T = 1/\hbar_{eff}G$ apart from numerical factor could be arbitrary small. This would make possible gravitational bound states as partonic 2-surfaces as structures connected by strings and solve the basic problem of superstring theories. This option allows the natural interpretation of M^4 type vacuum extremals with CP_2 projection, which is Lagrange manifold as good approximations for space-time sheets at macroscopic length scales. String area does not contribute to the Kähler function at all.

   Whether induced spinor fields associated with Kähler-Dirac action and de-localized inside the entire space-time surface should be allowed remains an open question: super-conformal symmetry strongly suggests their presence. A possible interpretation for the corresponding spinor modes could be in terms of dark matter, sparticles, and hierarchy of Planck constants.

It is perhaps useful to make clear what TGD is not and also what new TGD can give to physics.

1. TGD is *not* just General Relativity made concrete by using embeddings: the 4-surface property is absolutely essential for unifying standard model physics with gravitation and to circumvent the incurable conceptual problems of General Relativity. The many-sheeted space-time of TGD gives rise only at the macroscopic limit to GRT space-time as a slightly curved Minkowski space. TGD is *not* a Kaluza-Klein theory although color gauge potentials are analogous to gauge potentials in these theories.

   TGD space-time is 4-D and its dimension is due to completely unique conformal properties of light-cone boundary and 3-D light-like surfaces implying enormous extension of the ordinary conformal symmetries. Light-like 3-surfaces represent orbits of partonic 2-surfaces and carry fundamental fermions at 1-D boundaries of string world sheets. TGD is *not* obtained by performing Poincare gauging of space-time to introduce gravitation and is plagued by profound conceptual problems.

2. TGD is *not* a particular string model although string world sheets emerge in TGD very naturally as loci for spinor modes: their 2-dimensionality makes among other things possible quantum deformation of quantization known to be physically realized in condensed matter, and conjectured in TGD framework to be crucial for understanding the notion of finite measurement resolution. Hierarchy of objects of dimension up to 4 emerge from TGD: this obviously means analogy with branes of super-string models.

TGD is *not* one more item in the collection of string models of quantum gravitation relying on Planck length mystics. Dark matter becomes an essential element of quantum gravitation and quantum coherence in astrophysical scales is predicted just from the assumption that strings connecting partonic 2-surfaces are responsible for gravitational bound states.

TGD is *not* a particular string model although AdS/CFT duality of super-string models generalizes due to the huge extension of conformal symmetries and by the identification of WCW gamma matrices as Noether super-charges of super-symplectic algebra having a natural conformal structure.

3. TGD is *not* a gauge theory. In TGD framework the counterparts of also ordinary gauge symmetries are assigned to super-symplectic algebra (and its Yangian [A4] [B22, B14, B15]), which is a generalization of Kac-Moody algebras rather than gauge algebra and suffers a fractal hierarchy of symmetry breakings defining hierarchy of criticalities. TGD is *not* one more quantum field theory like structure based on path integral formalism: path integral is replaced with functional integral over 3-surfaces, and the notion of classical space-time becomes an exact part of the theory. Quantum theory becomes formally a purely classical theory of WCW spinor fields: only state function reduction is something genuinely quantal.

4. TGD view about spinor fields is *not* the standard one. Spinor fields appear at three levels. Spinor modes of the embedding space are analogs of spinor modes characterizing incoming and outgoing states in quantum field theories. Induced second quantized spinor fields at space-time level are analogs of stringy spinor fields. Their modes are localized by the well-definedness of electro-magnetic charge and by number theoretic arguments at string world sheets. Kähler-Dirac action is fixed by supersymmetry implying that ordinary gamma matrices are replaced by what I call Kähler-Dirac gamma matrices - this something new. WCW spinor fields, which are classical in the sense that they are not second quantized, serve as analogs of fields of string field theory and imply a geometrization of quantum theory.

5. TGD is in some sense an extremely conservative geometrization of entire quantum physics: *no* additional structures such as gauge fields as independent dynamical degrees of freedom are introduced: Kähler geometry and associated spinor structure are enough. "Topological" in TGD should not be understood as an attempt to reduce physics to torsion (see for instance [B13]) or something similar. Rather, TGD space-time is topologically non-trivial in all scales and even the visible structures of the everyday world represent non-trivial topology of space-time in the TGD Universe.

6. Twistor space - or rather, a generalization of twistor approach replacing masslessness in 4-D sense with masslessness in 8-D sense and thus allowing description of also massive particles - emerged originally as a technical tool, and its Kähler structure is possible only for $H = M^4 \times CP_2$. It however turned out that much more than a technical tool is in question. What is genuinely new is the infinite-dimensional character of the Kähler geometry making it highly unique, and its generalization to p-adic number fields to describe correlates of cognition. Also the hierarchy of Planck constants $h_{eff} = n \times h$ reduces to the quantum criticality of the TGD Universe and p-adic length scales and Zero Energy Ontology represent something genuinely new.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last 45 years to the realization of this dream and this has resulted in 26 online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

A collection of 30 online books is now (August 2023) under preparation. The goal is to minimize overlap between the topics of the books and make the focus of a given book sharper.

1.1.2 Two Visions About TGD as Geometrization of Physics and Their Fusion

As already mentioned, TGD as a geometrization of physics can be interpreted both as a modification of general relativity and generalization of string models.

TGD as a Poincare Invariant Theory of Gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_\times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A14, A17, A9, A16].

The identification of the space-time as a sub-manifold [A15, A25] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

TGD as a Generalization of the Hadronic String Model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Quite recently, it has turned out that fermionic strings inside space-time surfaces define an exact part of quantum TGD and that this is essential for understanding gravitation in long length scales. Also the analog of AdS/CFT duality emerges in that the Kähler metric can be defined either in terms of Kähler function identifiable as Kähler action assignable to Euclidian space-time regions or Kähler action + string action assignable to Minkowskian regions.

The recent view about construction of scattering amplitudes is very "stringy". By strong form of holography string world sheets and partonic 2-surfaces provide the data needed to construct scattering amplitudes. Space-time surfaces are however needed to realize quantum-classical correspondence necessary to understand the classical correlates of quantum measurement. There is a huge generalization of the duality symmetry of hadronic string models.

The proposal is that scattering amplitudes can be regarded as sequences of computational operations for the Yangian of super-symplectic algebra. Product and co-product define the basic vertices and realized geometrically as partonic 2-surfaces and algebraically as multiplication for the elements of Yangian identified as super-symplectic Noether charges assignable to strings. Any computational sequences connecting given collections of algebraic objects at the opposite boundaries of causal diamond (CD) produce identical scattering amplitudes.

Fusion of the Two Approaches via a Generalization of the Space-Time Concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there could be "vapor phase" that is a "gas" of particle like 3-surfaces and string like objects (counterpart of the "baby universes" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possible existence vapour phase.

. What one obtains is what I have christened as many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. ??** in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory the physical system does not possess this kind of field identity. The notion of the magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology. The existence of monopole flux tubes requiring no current as a source of the magnetic field makes it possible to understand the existence of magnetic fields in cosmological and astrophysical scales.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces and identifiable as analogs of Bohr orbits. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification of the lines of generalized Feynman diagrams is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian . Also the Euclidian 4-D regions can have a similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about a strong form of holography.

The understanding of the super symplectic invariance leads to the proposal that super symplectic algebra and other Kac-Moody type algebras labelled by non-negative multiples of basic conformal weights allow a hierarchy of symmetry breakings in which the analog of gauge symmetry breaks down to a genuine dynamical symmetry. This gives rise to fractal hierarchies of algebras and symmetry breakings. This breaking can occur also for ordinary conformal algebras if one restricts the conformal weights to be non-negative integers.

1.1.3 Basic Objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four embedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particles topologically condense to several space-time sheets simultaneously and experience the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the embeddability to 8-D embedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation makes it possible to understand the relationship to GRT space-time and how the Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of the space-time sheets from Minkowski metric. Poincare invariance strongly suggests classical EP for the GRT limit in long length scales at least. One can also consider other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with a very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of the standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Topological Field Quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other thins this leads to models for cell membrane, nerve pulse, and EEG.

1.1.4 Quantum TGD as Spinor Geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones.

World of Classical Worlds

The notion of WCW reduces the interacting quantum theory to a theory of free WCW spinor fields.

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude identified as WCW spinor in the configuration space CH ("world of classical worlds", WCW) consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included.

2. 4-D general coordinate invariance forces holography and replaces the ill-defined path integral over all space-time surfaces with a discrete sum over 4-D analogs of Bohr orbits for particles identified as 3-surfaces. Holography means that basic objects are these analogs of Bohr orbits. Since there is no quantization at the level of WCW, one has an analog of wave mechanics with point-like particles replaced with 4-D Bohr orbits.

3. One must geometrize WCW as the space of Bohr orbits. In an infinite-dimensional situation the existence of geometry requires maximal symmetries already in the case of loop spaces. Physics is unique from its mathematical existence.

   WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operators, appearing in the field equations of the theory [1]

Identification of Kähler function

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

It took long time to realize that there is no discretization in 4-D sense - this would lead to difficulties with basic symmetries. Rather, the discretization occurs for the parameters characterizing co-dimension 2 objects representing the information about space-time surface so that they belong to some algebraic extension of rationals. These 2-surfaces - string world sheets and partonic 2-surfaces - are genuine physical objects rather than a computational approximation. Physics itself approximates itself, one might say! This is of course nothing but strong form of holography.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factorc coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

   Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The way to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this way almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

WCW spinor fields

Classical WCW spinor fields are analogous to Schrödinger amplitudes and the construction of WCW Kähler geometry reduces to the second quantization of free spinor fields of H.

[1]There are four kinds of Dirac operators in TGD. The geometrization of quantum theory requires Kähler metric definable either in terms of Kähler function identified as a the bosonic action for Euclidian space-time regions or as anti-commutators for WCW gamma matrices identified as conformal Noether super-charges associated with the second quantized modified Dirac action consisting of string world sheet term and possibly also modified Dirac action in Minkowskian space-time regions. These two possible definitions reflect a duality analogous to AdS/CFT duality.

1. The WCW metric is given by anticommutators of WCW gamma matrices which also have interpretation as supercharges assignable to the generators of WCW isometries and allowing expression as non-conserved Noether charges. Holography implies zero energy ontology (ZEO) meaning that zero energy states are superpositions of Bohr orbits connecting boundaries of causal diamond (CD). CDs form a fractal hierarchy and their space forming the spine of WCW is finite-dimensional and can be geometrized. The alternative interpretation is as a superposition of pairs of ordinary 3-D fermionic states assignable to the ends of the space-time surfaces.

2. There are several Dirac operators. WCW Dirac operator D_{WCW} appears in Super-symplectic gauge conditions analogous to Super Virasoro conditions. The algebraic variant of the H Dirac operator D_H appears in fermionic correlation functions: this is due to the fact that free fermions appearing as building bricks of WCW gamma matrices are modes of D_H. The modes of $_DH$ define the ground states of super-symplectic representations. There is also the modified Dirac operator D_{X^4} acting on the induced spinors at space-time surfaces and it is dictated by symmetry one the action fixing the space-time surfaces as Bohr orbits is fixed. D_H is needed since it determines the expressions of WCW gamma matrices as Noether charges assignable to 3-surfaces at the ends of WCW.

The role of modified Dirac action

1. By quantum classical correspondence, the construction of WCW spinor structure in sectors assignable to CDs reduces to the second quantization of the induced spinor fields of H. The basic action is so called modified Dirac action in which gamma matrices are replaced with the modified) gamma matrices defined as contractions of the canonical momentum currents of the bosonic action defining the space-time surfaces with the embedding space gamma matrices. In this way one achieves super-conformal symmetry and conservation of fermionic currents among other things and a consistent Dirac equation.

   Modified Dirac action is needed to define WCW gamma matrices as super charges assignable to WCW isometry generators identified as generators of symplectic transformations and by holography are needed only at the 3-surface at the boundaries of WCW. It is important to notice that the modified Dirac equation does not determine propagators since induced spinor fields are obtained from free second quantized spinor fields of H. This means enormous simplification and makes the theory calculable.

2. An important interpretational problem relates to the notion of the induced spinor connection. The presence of classical W boson fields is in conflict with the classical conservation of em charge since the coupling to classical W fields changes em charge.

   One way out of the problem is the fact that the quantum averages of weak and gluon fields vanish unlike the quantum average of the em field. This leads to a rather precise understanding of electroweak symmetry breaking as being due the fact that color symmetries rotate space-time surfaces and also affect the induced weak fields.

   One can also consider a stronger condition. If one requires that the spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrinos generating super-symmetries forms an exception. The vanishing of the Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that the string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

   The localization would simplify the mathematics enormously and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

   At the light-like 3-surfaces the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes. One can pose the condition that the algebraic analog of the massless Dirac equation is satisfied by the modes of the modified-Dirac action assignable to the Chern-Simons-Kähler action.

1.1.5 Construction of scattering amplitudes

Reduction of particle reactions to space-time topology

Particle reactions are identified as topology changes [A22, A27, A32]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B + C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

During years this naïve and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects un-expected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "configuration space". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

Construction of the counterparts of S-matrices

What does one mean with the counterpart of S-matrix in the TGD framework has been a long standing problem. The development of ZEO based quantum measurement theory has led to a rough overall view of the situation.

1. There are two kinds of state function reductions (SFRs). "Small" SFRs (SSFRs) following the TGD counterpart of a unitary time evolution defines a sequence of SFRs, which is analogous to a sequence of repeated quantum measurements associated with the Zeno effect. In wave mechanics nothing happens in these measurements. In quantum optics these measurements correspond to weak measurements. In TGD SSFR affects the zero energy state but leaves the 3-D state at the passive boundary of CD unaffected.

2. In TGD framework each SSFR is preceded by a counterpart of a unitary time evolution, which means dispersion in the space of CDs and unitary time evolution in fermionic degrees of freedom such that the passive boundary of CDs and 3-D states at it are unaffected but a superposition of CDs with varying active boundaries in the space of CDs is formed. In SSFR a localization in the space of CDs occurs such that the active is fixed. In a statistical sense the size of the CD increases and the increasing distance between the tips of the CD gives rise to the arrow of geometric time.

3. Also "big" SFRS (BSFRs) can occur and they correspond to ordinary SFRs. In BSFR the roles of the active and passive boundary are changed and this means that the arrow of time is changed. Big SFR occurs when the SSFR corresponds to a quantum measurement, which does not commute with the operators, which define the states at the passive boundary of CD as their eigenstates. This means a radical deviation from standard quantum measurement theory and has predictions in all scales.

4. One can assign the counterpart of S-matrix to the unitary time evolution between two subsequent SSFRs and also to the counterpart of S-matrix associated with BSFR. At least in the latter case the dimension of the state space can increase since at least BSFRs lead to the increase of the dimension of algebraic extension of rationals assignable to the space-time surface by $M^8 - H$ duality. Unitarity is therefore replaced with isometry.

5. I have also considered the possibility that unitary S-matrix could be replaced in the fermionic degrees of freedom with Kähler metric of the state space satisfying analogs of unitarity conditions but it seems that this is un-necessary and also too outlandish an idea.

The notion of M-matrix

1. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operators associated with WCWs associated with the spaces of CDs with fixed passive boundary: this would define an S-matrix assignable to SFR. Also the analog of S-matrix for the localizations of the states to the active boundary assignable to the BSFR changing the state at the passive boundary of CD is needed.

2. If one allows entanglement between positive and energy parts of the zero energy state but assumes that the states at the passive boundary are fixed, one must introduce the counterpart of the density matrix, or rather its square root. This classical free field theory would dictate what I have called M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. A biven M-matrix in turn would decompose to a product of a hermitian square root of density matrix and unitary S-matrix.

3. M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in a well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the M-matrices commuting with S-matrix means that they span infinite-dimensional Lie algebras acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in a well-defined sense.

4. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible and would correspond to a hierarchy of CDs with the temporal distances between tips coming as integer multiples of the CP_2 time.

   The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer n are naturally proportional to a representation matrix of scaling: $S(n) = S^n$, where S is unitary S-matrix associated with the minimal CD [K49]. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of S and represented as scaling with respect to the logarithm of the proper time distance between the tips of CD.

5. I have also considered the notion of U-matrix. U-matrix elements between M-matrices for various CDs are proportional to the inner products $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2} \lambda]$, where λ represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and H^i form an orthonormal basis of Hermitian square roots of density matrices. $\circ$ tells that S acts at the active boundary of CD only. I have proposed a general representation for the U-matrix, reducing its construction to that of the S-matrix.

1.1.6 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space ("world of classical worlds", WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name "TGD as a generalized number theory". It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of complexified counterparts of classical number fields, and the notion of infinite prime. Note that one can identify subrings such as hyper-quaternions and hyper-octonions as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product.

The Threads in the Development of Quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

2. The discussions with Tony Smith initiated a fourth thread which deserves the name "TGD as a generalized number theory". The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and rather fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the "physics as generalized number theory" thread.

3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called "physics as a generalized number theory". In the following I adopt this view. This reduces the number of threads to three corresponding to geometric, number theoretic and topological views of physics.

TGD forces the generalization of physics to a quantum theory of consciousness, and TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations.

Number theoretic vision very briefly

Number theoretic vision about quantum TGD involves notions like adelic physics, M^8-H duality and number theoretic universality. A short review of the basic ideas that have developed during years is in order.

1. The physical interpretation of M^8 is as an analog of momentum space and M^8-H duality is analogous to momentum-position duality of ordinary wave mechanics.

2. Adelic physics means that all classical number fields, all p-adic number fields and their extensions induced by extensions of rationals and defining adeles, and also finite number fields are basic mathematical building bricks of physics.

   The complexification of M^8, identified as complexified octonions, would provide a realization of this picture and M^8-H duality would map the algebraic physics in M^8 to the ordinary physics in $M^4\times CP_2$ described in terms of partial differential equations.

3. Negentropy Maximization Principle (NMP) states that the conscious information assignable with cognition representable measured in terms of p-adic negentropy increases in statistical sense.

   NMP is mathematically completely analogous to the second law of thermodynamics and number theoretic evolution as an unavoidable statistical increase of the dimension of the algebraic extension of rationals characterizing a given space-time region implies it. There is no paradox involved: the p-adic negentropy measures the conscious information assignable to the entanglement of two systems regarded as a conscious entity whereas ordinary entropy measures the lack of information about the quantums state of either entangled system.

4. Number theoretical universality requires that space-time surfaces or at least their M^8-H duals in M^8_c are defined for both reals and various p-adic number fields. This is true if they are defined by polynomials with integer coefficients as surfaces in M^8 obeying number theoretic holography realized as associativity of the normal space of 4-D surface using as holographic data 3-surfaces at mass shells identified in terms of roots of a polynomial. A physically motivated additional condition is that the coefficients of the polynomials are smaller than their degrees.

5. Galois confinement is a key piece of the number theoretic vision. It states that the momenta of physical states are algebraic integers in the extensions of rationals assignable to the space-time region considered. These numbers are in general complex and are not consistent with particle in box quantization. The proposal is that physical states satisfy Galois confinement being thus Galois singlets and having therefore total momenta, whose components are ordinary integers, when momentum unit defined by the scale of causal diamond (CD) is used.

6. The notion of p-adic prime was introduced in p-adic mass calculations that started the developments around 1995. p-Adic length scale hypothesis states that p-adic primes near powers of 2 have a special physical role (as possibly also the powers of other small primes such as $p=3$).

   The proposal is that p-adic primes correspond to ramified primes assignable to the extension and identified as divisors of the polynomial defined by the products of the root differences for the roots of the polynomial defining space-time space and having interpretation as values of, in general complex, virtual mass squared.

p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired “Universe as Computer” vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduces the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.

2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structure. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of embedding space and space-time concept and one can speak about real and p-adic space-time sheets. One can talk about adelic space-time, embedding space, and WCW.

The corresponds of real 4-surfaces with the p-adic ones is induced by number theoretical discretization using points of 4-surfaces $Y^4 \subset M^8_c$ identifiable as 8-momenta, whose components are assumed to be algebraic integers in an extension of rationals defined by the extension of rationals associated with a polynomial P with integer coefficients smaller than the degree of P. These points define a cognitive representation, which is universal in the sense that it exists also in the algebraic extensions of p-adic numbers. The points of the cognitive representations associated with the mass shells with mass squared values identified as roots of P are enough since $M^8 - H$ duality can be used at both M^8 and H sides and also in the p-adic context. The mass shells are special in that they allow for Minkowski coordinates very large cognitive representations unlike the interiors of the 4-surfaces determined by holography by using the data defined by the 3-surfaces at the mass shells. The higher the dimension of the algebraic extension associated with P, the better the accuracy of the cognitive representation.

Adelization providing number theoretical universality reduces to algebraic continuation for the amplitudes from this intersection of reality and various p-adicities - analogous to a back of a book - to various number fields. There are no problems with symmetries but canonical identification is needed: various group invariant of the amplitude are mapped by canonical identification to various p-adic number fields. This is nothing but a generalization of the mapping of the p-adic mass squared to its real counterpart in p-adic mass calculations.

This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book). One can also understand how preferred p-adic primes could emerge as so called ramified primes of algebraic extension of rationals in question and characterizing string world sheets and partonic 2-surfaces. Preferred p-adic primes would be ramified primes for extensions for which the number of p-adic continuations of two-surfaces to space-time surfaces (imaginations) allowing also real continuation (realization of imagination) would be especially large. These ramifications would be winners in the fight for number theoretical survival. Also a generalization of p-adic length scale hypothesis emerges from NMP [K46].

The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to "mind stuff", the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of complexified quaternions and octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

The infinite primes at the first level of hierarchy, which represent analogs of bound states, can be mapped to irreducible polynomials, which in turn characterize the algebraic extensions of rationals defining a hierarchy of algebraic physics continuable to real and p-adic number fields. The products of infinite primes in turn define more general algebraic extensions of rationals. The interesting question concerns the physical interpretation of the higher levels in the hierarchy of infinite primes and integers mappable to polynomials of $n > 1$ variables.

1.1.7 An explicit formula for $M^8 - H$ duality

$M^8 - H$ duality is a generalization of momentum-position duality relating the number theoretic and geometric views of physics in TGD and, despite that it still involves poorly understood aspects, it has become a fundamental building block of TGD. One has 4-D surfaces $Y^4 \subset M^8_c$, where M^8_c is complexified M^8 having interpretation as an analog of complex momentum space and 4-D spacetime surfaces $X^4 \subset H = M^4 \times CP_2$. M^8_c, equivalently E^8_c, can be regarded as complexified octonions. M^8_c has a subspace M^4_c containing M^4.

Comment: One should be very cautious with the meaning of "complex". Complexified octonions involve a complex imaginary unit i commuting with the octonionic imaginary units I_k. i is assumed to also appear as an imaginary unit also in complex algebraic numbers defined by the roots of polynomials P defining holographic data in M^8_c.

In the following $M^8 - H$ duality and its twistor lift are discussed and an explicit formula for the dualities are deduced. Also possible variants of the duality are discussed.

Holography in H

$X^4 \subset H$ satisfies holography and is analogous to the Bohr orbit of a particle identified as a 3-surface. The proposal is that holography reduces to a 4-D generalization of holomorphy so that X^4 is a simultaneous zero of two functions of complex CP_2 coordinates and of what I have called Hamilton-Jacobi coordinates of M^4 with a generalized Kähler structure.

The simplest choice of the Hamilton-Jacobi coordinates is defined by the decomposition $M^4 = M^2 \times E^2$, where M^2 is endowed with hypercomplex structure defined by light-like coordinates (u, v), which are analogous to z and $\overline{z}$. Any analytic map $u \to f(u)$ defines a new set of light-like coordinates and corresponds to a solution of the massless d'Alembert equation in M^2. E^2 has some complex coordinates with imaginary unit defined by i.

The conjecture is that also more general Hamilton-Jacobi structures for which the tangent space decomposition is local are possible. Therefore one would have $M^4 = M^2(x) \times E^2(x)$. These would correspond to non-equivalent complex and Kähler structures of M^4 analogous to those possessed by 2-D Riemann surfaces and parametrized by moduli space.

Number theoretic holography in M^8_c

$Y^4 \subset M^8_c$ satisfies number theoretic holography defining dynamics, which should reduce to associativity in some sense. The Euclidian complexified normal space $N^4(y)$ at a given point y of Y^4 is required to be associative, i.e. quaternionic. Besides this, $N^4(i)$ contains a preferred complex Euclidian 2-D subspace $Y^2(y)$. Also the spaces $Y^2(x)$ define an integrable distribution. I have assumed that $Y^2(x)$ can depend on the point y of Y^4.

These assumptions imply that the normal space $N(y)$ of Y^4 can be parameterized by a point of $CP_2 = SU(3)/U(2)$. This distribution is always integrable unlike quaternionic tangent space distributions. $M^8 - H$ duality assigns to the normal space $N(y)$ a point of CP_2. M^4_c point y is mapped to a point $x \in M^4 \subset M^4 \times CP_2$ defined by the real part of its inversion (conformal transformation): this formula involves effective Planck constant for dimensional reasons.

The 3-D holographic data, which partially fixes 4-surfaces Y^4 is partially determined by a polynomial P with real integer coefficients smaller than the degree of P. The roots define mass squared values which are in general complex algebraic numbers and define complex analogs of mass shells in $M^4_c \subset M^8_c$, which are analogs of hyperbolic spaces H^3. The 3-surfaces at these mass shells define 3-D holographic data continued to a surface Y^4 by requiring that the normal space of Y^4 is associative, i.e. quaternionic. These 3-surfaces are not completely fixed but an interesting conjecture is that they correspond to fundamental domains of tessellations of H^3.

What does the complexity of the mass shells mean? The simplest interpretation is that the space-like M^4 coordinates (3-momentum components) are real whereas the time-like coordinate (energy) is complex and determined by the mass shell condition. One would have $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3 when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as a time coordinate. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to an equation of mass shell when $\sqrt{(Re(E)^2 - Im(E)^2)}$, expressed in terms of $Re(E)$, is taken as new energy coordinate $E_{eff} = \sqrt{(Re(E)^2 - Im(E)^2)}$. Is this deformation of H^3 in imaginary time direction equivalent with a region of the hyperbolic 3-space H^3?

One can look at the formula in more detail. Mass shell condition gives $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3, when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as an effective energy. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to a dispersion relation for $Re(E)^2$.

$$Re(E)^2 = \frac{1}{2}(Re(m^2) - Im(m^2) + p^2)(1 \pm \sqrt{1 + \frac{2Im(m^2)^2}{(Re(m^2) - Im(m^2) + p^2)^2}} . \qquad (1.1.1)$$

Only the positive root gives a non-tachyonic result for $Re(m^2) - Im(m^2) > 0$. For real roots with $Im(m^2) = 0$ and at the high momentum limit the formula coincides with the standard formula. For $Re(m^2) = Im(m^2)$ one obtains $Re(E)^2 \to Im(m^2)/\sqrt{2}$ at the low momentum limit $p^2 \to 0$. Energy does not depend on momentum at all: the situation resembles that for plasma waves.

Can one find an explicit formula for $M^8 - H$ duality?

The dream is an explicit formula for the $M^8 - H$ duality mapping $Y^4 \subset M^8_c$ to $X^4 \subset H$. This formula should be consistent with the assumption that the generalized holomorphy holds true for X^4.

The following proposal is a more detailed variant of the earlier proposal for which Y^4 is determined by a map g of $M^4_c \to SU(3)_c \subset G_{2,c}$, where $G_{2,c}$ is the complexified automorphism group of octonions and $SU(3)_c$ is interpreted as a complexified color group.

This map defines a trivial $SU(3)_c$ gauge field. The real part of g however defines a non-trivial real color gauge field by the non-linearity of the non-abelian gauge field with respect to the gauge potential. The quadratic terms involving the imaginary part of the gauge potential give an additional condition to the real part in the complex situation and cancel it. If only the real part of g contributes, this contribution would be absent and the gauge field is non-vanishing.

How could the automorphism $g(x) \subset SU(3) \subset G_2$ give rise to M^8-H duality?

1. The interpretation is that $g(y)$ at given point y of Y^4 relates the normal space at y to a fixed quaternionic/associative normal space at point y_0, which corresponds is fixed by some subgroup $U(2)_0 \subset SU(3)$. The automorphism property of g guarantees that the normal space is quaternionic/associative at y. This simplifies the construction dramatically.

2. The quaternionic normal sub-space (which has Euclidian signature) contains a complex sub-space which corresponds to a point of sphere $S^2 = SO(3)/O(2)$, where $SO(3)$ is the quaternionic automorphism group. The interpretation could be in terms of a selection of spin quantization axes. The local choice of the preferred complex plane would not be unique and is analogous to the possibility of having non-trivial Hamilton Jacobi structures in M^4 characterized by the choice of $M^2(x)$ and equivalently its normal subspace $E^2(x)$.

   These two structures are independent apart from dependencies forced by the number theoretic dynamics. Hamilton-Jacobi structure means a selection of the quantization axis of spin and energy by fixing a distribution of light-like tangent vectors of M^4 and the choice of the quaternionic normal sub-space fixes a choice of preferred quaternionic imaginary unit defining a quantization axis of the weak isospin.

3. The real part $Re(g(y))$ defines a point of $SU(3)$ and the bundle projection $SU(3) \rightarrow CP_2$ in turn defines a point of $CP_2 = SU(3)/U(2)$. Hence one can assign to g a point of CP_2 as M^8-H duality requires and deduce an explicit formula for the point. This means a realization of the dream.

4. The construction requires a fixing of a quaternionic normal space N_0 at y_0 containing a preferred complex subspace at a single point of Y^4 plus a selection of the function g. If M^4 coordinates are possible for Y^4, the first guess is that g as a function of complexified M^4 coordinates obeys generalized holomorphy with respect to complexified M^4 coordinates in the same sense and in the case of X^4. This might guarantee that the M^8-H image of Y^4 satisfies the generalized holomorphy.

5. Also space-time surfaces X^4 with M^4 projection having a dimension smaller than 4 are allowed. I have proposed that they might correspond to singular cases for the above formula: a kind of blow-up would be involved. One can also consider a more general definition of Y^4 allowing it to have a M^4 projection with dimension smaller than 4 (say cosmic strings). Could one have implicit equations for the surface Y^4 in terms of the complex coordinates of $SU(3)_c$ and M^4? Could this give for instance cosmic strings with a 2-D M^4 projection and CP_2 type extremals with 4-D CP_2 projection and 1-D light-like M^4 projection?

What could the number theoretic holography mean physically?

What could be physical meaning of the number theoretic holography? The condition that has been assumed is that the CP_2 coordinates at the mass shells of $M^4_c \subset M^8_c$ mapped to mass shells H^3 of $M^4 \subset M^4 \times CP_2$ are constant at the H^3. This is true if the $g(y)$ defines the same CP_2 point for a given component X^3_i of the 3-surface at a given mass shell. g is therefore fixed apart from a local $U(2)$ transformation leaving the CP_2 point invariant. A stronger condition would be that the CP_2 point is the same for each component of X^3_i and even at each mass shell but this condition seems to be unnecessarily strong.

Comment: One can o criticize this condition as too strong and one can consider giving up this condition. The motivation for this condition is that the number of algebraic points at the 3-surfaces associated with H^3 explodes since the coordinates associated with normal directions vanish. Kind of cognitive explosion would be in question.

$SU(3)$ corresponds to a subgroup of G_2 and one can wonder what the fixing of this subgroup could mean physically. G_2 is 14-D and the coset space $G_2/SU(3)$ is 6-D and a good guess is that it is just the 6-D twistor space $SU(3)/U(1) \times U(1)$ of CP_2: at least the isometries are the same. The fixing of the $SU(3)$ subgroup means fixing of a CP_2 twistor. Physically this means the fixing of the quantization axis of color isospin and hypercharge.

Twistor lift of the holography

What is interesting is that by replacing $SU(3)$ with G_2, one obtains an explicit formula form the generalization of $M^8 - H$ duality to that for the twistorial lift of TGD!

One can also consider a twistorial generalization of the above proposal for the number theoretic holography by allowing local G_2 automorphisms interpreted as local choices of the color quantization axis. G_2 elements would be fixed apart from a local $SU(3)$ transformation at the components of 3-surfaces at mass shells. The choice of the color quantization axes for a connected 3-surface at a given mass shell would be the same everywhere. This choice is indeed very natural physically since 3-surface corresponds to a particle.

Is this proposal consistent with the boundary condition of the number theoretical holography mean in the case of 4-surfaces in M^8_c and $M^4 \times CP_2$?

1. The selection of $SU(3) \subset G_2$ for ordinary $M^8 - H$ duality means that the $G_{2,c}$ gauge field vanishes everywhere and the choice of color quantization axis is the same at all points of the 4-surface. The fixing of the CP_2 point to be constant at H^3 implies that the color gauge field at $H^3 \subset M^8_c$ and its image $H^3 \subset H$ vanish. One would have color confinement at the mass shells H^3_i, where the observations are made. Is this condition too strong?

2. The constancy of the G_2 element at mass shells makes sense physically and means a fixed color quantization axis. The selection of a fixed $SU(3) \subset G_2$ for entire space-time surface is in conflict with the non-constancy of G_2 element unless G_2 element differs at different points of 4-surface only by a multiplication of a local $SU(3)_0$ element, that is local $SU(3)$ transformation. This kind of variation of the G_2 element would mean a fixed color group but varying choice of color quantization axis.

3. Could one consider the possibility that the local $G_{2,c}$ element is free and defines the twistor lift of $M^8 - H$ duality as something more fundamental than the ordinary $M^8 - H$ duality based on $SU(3)_c$. This duality would make sense only at the mass shells so that only the spaces $H^3 \times CP_2$ assignable to mass shells would make sense physically? In the interior CP_2 would be replaced with the twistor space $SU(3)/U(1) \times U(1)$. Color gauge fields would be non-vanishing at the mass shells but outside the mass shells one would have G_2 gauge fields.

   There is also a physical objection against the G_2 option. The 14-D Lie algebra representation of G_2 acts on the imaginary octonions which decompose with respect to the color group to $1 \oplus 3 \oplus \overline{3}$. The automorphism property requires that 1 can be transformed to 3 or $\overline{3}$ to themselves: this requires that the decomposition contains $3 \oplus \overline{3}$. Furthermore, it must be possible to transform 3 and $\overline{3}$ to themselves, which requires the presence of 8. This leaves only the decomposition $8 \oplus 3 \oplus \overline{3}$. G_2 gluons would both color octet and triplets. In the TDG framework the only conceivable interpretation would be in terms of ordinary gluons and leptoquark-like gluons. This does not fit with the basic vision of TGD.

The choice of twistor as a selection of quantization axes should make sense also in the M^4 degrees of freedom. M^4 twistor corresponds to a choice of light-like direction at a given point of M^4. The spatial component of the light-like vector fixes the spin quantization axis. Its choice together with the light-likeness fixes the time direction and therefore the rest system and energy quantization axis. Light-like vector fixes also the choice of M^2 and of E^2 as its orthogonal complement. Therefore the fixing of M^4 twistor as a point of $SU(4)/SU(3) \times U(1)$ corresponds to a choice of the spin quantization axis and the time-like axis defining the rest system in which the energy is measured. This choice would naturally correspond to the Hamilton-Jacobi structure fixing the decompositions $M^2(x) \times E^2(x)$. At a given mass shell the choice of the quantization axis would be constant for a given X^3_i.

1.1.8 Hierarchy of Planck Constants and Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark Matter as Large $\hbar$ Phases

D. Da Rocha and Laurent Nottale [E1] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr}. Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K70].

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Certain experimental findings suggest the identification $h_{eff} = n\times = h_{gr}$. The large value of h_{gr} can be seen as a way to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description). The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n. Macroscopic quantum coherence in astrophysical scales is implied. If also Kähler-Dirac action is present, part of the interior degrees of freedom associated with the Kähler-Dirac part of conformal algebra become physical. A possible is that tfermionic oscillator operators generate super-symmetries and sparticles correspond almost by definition to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) of bunch of n low energy gravitons.

Hierarchy of Planck Constants from the Anomalies of Neuroscience and Biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K62, K63, K60]) support the view that dark matter might be a key player in living matter.

Dark Matter as a Source of Long Ranged Weak and Color Fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff}.

1.1.9 Twistors in TGD and connection with Veneziano duality

The twistorialization of TGD has two aspects. The attempt to generalize twistor Grassmannian approach emerged first. It was however followed by the realization that also the twistor lift of TGD at classical space-time level is needed. It turned out that the progress in the understanding of the classical twistor lift has been much faster - probably this is due to my rather limited technical QFT skills.

Twistor lift at space-time level

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [L3]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A18]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2. The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure coincides with the twistor structure defined by the induced metric.

In fact, it is enough to generalize the induction of spinor structure to that of twistor structure so that the induced twistor structure need not be identical with the ordinary twistor structure possibly assignable to the space-time surface. The induction procedure reduces to a dimensional reduction of 6-D Kähler action giving rise to 6-D surfaces having bundle structure with twistor sphere as fiber and space-time as base. The twistor sphere of this bundle is imbedded as sphere in the product of twistor spheres of twistor spaces of M^4 and CP_2.

This condition would define the dynamics, and the original conjecture was that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

It however turned out that twistor lift of TGD is much more than a mere technical tool. First of all, the dimensionally reduction of 6-D Kähler action contained besides 4-D Kähler action also a volume term having interpretation in terms of cosmological constant. This need not bring anything new, since all known extremals of Kähler action with non-vanishing induced Kähler form are minimal surfaces. There is however a large number of embeddings of twistor sphere of space-time surface to the product of twistor spheres. Cosmological constant has spectrum and depends on length scale, and the proposal is that coupling constant evolution reduces to that for cosmological constant playing the role of cutoff length. That cosmological constant could transform from a mere nuisance to a key element of fundamental physics was something totally new and unexpected.

1. The twistor lift of TGD at space-time level forces to replace 4-D Kähler action with 6-D dimensionally reduced Kähler action for 6-D surface in the 12-D Cartesian product of 6-D twistor spaces of M^4 and CP_2. The 6-D surface has bundle structure with twistor sphere as fiber and space-time surface as base.

   Twistor structure is obtained by inducing the twistor structure of 12-D twistor space using dimensional reduction. The dimensionally reduced 6-D Kähler action is sum of 4-D Kähler action and volume term having interpretation in terms of a dynamical cosmological constant depending on the size scale of space-time surface (or of causal diamond CD in zero energy ontology (ZEO)) and determined by the representation of twistor sphere of space-time surface in the Cartesian product of the twistor spheres of M^4 and CP_2.

2. The preferred extremal property as a representation of quantum criticality would naturally correspond to minimal surface property meaning that the space-time surface is separately an extremal of both Kähler action and volume term almost everywhere so that there is no coupling between them. This is the case for all known extremals of Kähler action with non-vanishing induced Kähler form.

   Minimal surface property could however fail at 2-D string world sheets, their boundaries and perhaps also at partonic 2-surfaces. The failure is realized in minimal sense if the 3-surface has 1-D edges/folds (strings) and 4-surface 2-D edges/folds (string world sheets) at which some partial derivatives of the embedding space coordinates are discontinuous but canonical momentum densities for the entire action are continuous.

   There would be no flow of canonical momentum between interior and string world sheet and minimal surface equations would be satisfied for the string world sheet, whose 4-D counterpart in twistor bundle is determined by the analog of 4-D Kähler action. These conditions allow the transfer of canonical momenta between Kähler- and volume degrees of freedom at string world sheets. These no-flow conditions could hold true at least asymptotically (near the boundaries of CD).

   M^8-H duality suggests that string world sheets (partonic 2-surfaces) correspond to images of complex 2-sub-manifolds of M^8 (having tangent (normal) space which is complex 2-plane of octonionic M^8).

3. Cosmological constant would depend on p-adic length scales and one ends up to a concrete model for the evolution of cosmological constant as a function of p-adic length scale and other number theoretic parameters (such as Planck constant as the order of Galois group): this conforms with the earlier picture.

   Inflation is replaced with its TGD counterpart in which the thickening of cosmic strings to flux tubes leads to a transformation of Kähler magnetic energy to ordinary and dark matter. Since the increase of volume increases volume energy, this leads rapidly to energy minimum at some flux tube thickness. The reduction of cosmological constant by a phase transition however leads to a new expansion phase. These jerks would replace smooth cosmic expansion of GRT. The discrete coupling constant evolution predicted by the number theoretical vision could be understood as being induced by that of cosmological constant taking the role of cutoff parameter in QFT picture [L34].

Twistor lift at the level of scattering amplitudes and connection with Veneziano duality

The classical part of twistor lift of TGD is rather well-understood. Concerning the twistorialization at the level of scattering amplitudes the situation is much more difficult conceptually - I already mentioned my limited QFT skills.

1. From the classical picture described above it is clear that one should construct the 8-D twistorial counterpart of theory involving space-time surfaces, string world sheets and their boundaries, plus partonic 2-surfaces and that this should lead to concrete expressions for the scattering amplitudes.

   The light-like boundaries of string world sheets as carriers of fermion numbers would correspond to twistors as they appear in twistor Grassmann approach and define the analog for the massless sector of string theories. The attempts to understand twistorialization have been restricted to this sector.

2. The beautiful basic prediction would be that particles massless in 8-D sense can be massive in 4-D sense. Also the infrared cutoff problematic in twistor approach emerges naturally and reduces basically to the dynamical cosmological constant provided by classical twistor lift.

   One can assign 4-momentum both to the spinor harmonics of the embedding space representing ground states of super-conformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this could be seen as a concretization of Equivalence Principle. Also a connection with string model emerges.

3. As far as symmetries are considered, the picture looks rather clear. Ordinary twistor Grassmannian approach boils down to the construction of scattering amplitudes in terms of Yangian invariants for conformal group of M^4. Therefore a generalization of super-symplectic symmetries to their Yangian counterpart seems necessary. These symmetries would be gigantic but how to deduce their implications?

4. The notion of positive Grassmannian is central in the twistor approach to the scattering amplitudes in $calN = 4$ SUSYs. TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings.

   Braid interpretation encourages the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

This is however not the only vision about a solution of non-planarity. Quantum criticality provides different view leading to a totally unexpected connection with string models, actually with the Veneziano duality, which was the starting point of dual resonance model in turn leading via dual resonance models to super string models.

1. Quantum criticality in TGD framework means that coupling constant evolution is discrete in the sense that coupling constants are piecewise constant functions of length scale replaced by dynamical cosmological constant. Loop corrections would vanish identically and the recursion formulas for the scattering amplitudes (allowing only planar diagrams) deduced in twistor Grassmann would involve no loop corrections. In particular, cuts would be replaced by sequences of poles mimicking them like sequences of point charge mimic line charges. In momentum discretization this picture follows automatically.

2. This would make sense in finite measurement resolution realized in number theoretical vision by number-theoretic discretization of the space-time surface (cognitive representation) as points with coordinates in the extension of rationals defining the adele [L23]. Similar discretization would take place for momenta. Loops would vanish at the level of discretization but what would happen at the possibly existing continuum limit: does the sequence of poles integrate to cuts? Or is representation as sum of resonances something much deeper?

3. Maybe it is! The basic idea of behind the original Veneziano amplitudes (see `http://tinyurl.com/yyhwvbqb`) was Veneziano duality. This 4-particle amplitude was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see `http://tinyurl.com/yyvkx7as`) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged. Later came superstring models and led to M-theory. Now it has become clear that something went wrong, and it seems that one must return to the roots. Could the return to the roots mean a careful reconsideration of the dual resonance model?

4. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

   The resonances had zero width, which was not consistent with unitarity. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of twistor Grassmann approach.

5. Veneziano duality is true only for s- and t- channels but not been s- and u-channel. Stringy description makes t-channel and s-channel pictures equivalent. Could it be that in fundamental description u-channels diagrams cannot be distinguished from s-channel diagrams or t-channel diagrams? Could the stringy representation of the scattering diagrams make u-channel twist somehow trivial if handles of string world sheet representing stringy loops in turn representing the analog of non-planarity of Feynman diagrams are absent? The permutation of external momenta for tree diagram in absence of loops in planar representation would be a twist of π in the representation of planar diagram as string world sheet and would not change the topology of the string world sheet and would not involve non-trivial world sheet topology.

   For string world sheets loops would correspond to handles. The presence of handle would give an edge with a loop at the level of 3-surface (self energy correction in QFT). Handles are not allowed if the induced metric for the string world sheet has Minkowskian signature. If the stringy counterparts of loops are absent, also the loops in scattering amplitudes should be absent.

   This argument applies only inside the Minkowskian space-time regions. If string world sheets are present also in Euclidian regions, they might have handles and loop corrections could emerge in this manner. In TGD framework strings (string world sheets) are identified to 1-D edges/folds of 3-surface at which minimal surface property and topological QFT property fails (minimal surfaces as calibrations). Could the interpretation of edge/fold as discontinuity of some partial derivatives exclude loopy edges: perhaps the branching points would be too singular?

A reduction to a sum over s-channel resonances is what the vanishing of loops would suggest. Could the presence of string world sheets make possible the vanishing of continuous cuts even at the continuum limit so that continuum cuts would emerge only in the approximation as the density of resonances is high enough?

The replacement of continuous cut with a sum of *infinitely* narrow resonances is certainly an approximation. Could it be that the stringy representation as a sum of resonances with *finite* width is an essential aspect of quantum physics allowing to get rid of infinities necessarily accompanying loops? Consider now the arguments against this idea.

1. How to get rid of the problems with unitarity caused by the zero width of resonances? Could *finite* resonance widths make unitarity possible? Ordinary twistor Grassmannian approach predicts that the virtual momenta are light-like but complex: obviously, the imaginary part of the energy in rest frame would have interpretation as resonance with.

   In TGD framework this generalizes for 8-D momenta. By quantum-classical correspondence (QCC) the classical Noether charges are equal to the eigenvalues of the fermionic charges in Cartan algebrable (maximal set of mutually commuting observables) and classical TGD indeed predicts complex momenta (Kähler coupling strength is naturally complex). QCC thus supports this proposal.

2. Sum over resonances/exchanges picture is in conflict with QFT picture about scattering of particles. Could *finite* resonance widths due to the complex momenta give rise to the QFT type scattering amplitudes as one develops the amplitudes in Taylor series with respect to the resonance width? Unitarity condition indeed gives the first estimate for the resonance width.

   QFT amplitudes should emerge in an approximation obtained by replacing the discrete set of finite width resonances with a cut as the distance between poles is shorter than the resolution for mass squared.

   In superstring models string tension has single very large value and one cannot obtain QFT type behavior at low energies (for instance, scattering amplitudes in hadronic string model are concentrated in forward direction). TGD however predicts an entire hierarchy of p-adic length scales with varying string tension. The hierarchy of mass scales corresponding roughly to the lengths and thickness of magnetic flux tubes as thickened cosmic strings and characterized by the value of cosmological constant predicted by twistor lift of TGD. Could this give rise to continuous QCT type cuts at the limit when measurement resolution cannot distinguish between resonances?

   The dominating term in the sum over sums of resonances in t-channel gives near forward direction approximately the lowest mass resonance for strings with the smallest string tension. This gives the behavior $1/(t - m_{min}^2)$, where m_{min} corresponds to the longest mass scale involved (the largest space-time sheet involved), approximating the $1/t$-behavior of massless theories. This also brings in IR cutoff, the lack of which is a problem of gauge theories. This should give rise to continuous QFT type cuts at the limit when measurement resolution cannot distinguish between resonances.

1.2 Bird's Eye of View about the Topics of "Quantum TGD: Part III"

The general ideas of the book "Quantum TGD" have been already described in its first part so that only the organization of the contents of "Quantum TGD:Part III" will be explained here.

The TGD view of twistors has evolved gradually over the years. The key idea is to replace the twistor space of M^4 as a sphere bundle over M^4 with the 12-D product of twistor spaces of M^4 and CP_2. These two spaces are completely unique in that their twistor spaces allow a Kähler structure. Therefore the existence of the twistor space of H with Kähler structure would fix TGD uniquely.

Twistor space for 4-geometry exists only if the Weyl tensor of the 4-geometry vanishes so that the geometry is conformally flat. The Weyl tensor does not vanish for CP_2 but the modified spinor structure solves this problem.

Since H allows a twistor structure, the natural proposal is that it induces twistor structure to 4-D space-time surfaces. They would be replaced with 6-D sphere bundles as 6-surfaces in 12-D twistor space of H and would be obtained by an analog of dimensional reduction for 6-D Kähler action for induced twistors. If the holography for space-time surfaces reduces to a 4-D analog of 2-D holomorphy, the twistor spaces of 4-surfaces would be highly uniquely determined.

The second key problem of the ordinary twistor theory is that the particles must be massless. In TGD, the interpretation of massive particles in 4-D sense as massless particles in 8-D sense would resolve the problem.

The evolution of the ideas about twistor lift of TGD have developed through many twists and turns and the chapters of this book should give an idea about this development. The last chapter is the recent vision about the construction of the twistor amplitudes and involves number theoretic notions, in particular Galois confinement.

1.3 Sources

The eight online books about TGD [K83, K79, K65, K54, K16, K50, K34, K73] and nine online books about TGD inspired theory of consciousness and quantum biology [K76, K13, K59, K12, K32, K41, K43, K72, K75] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (`http://tinyurl.com/ybv8dt4n`) contains a lot of material about TGD. In particular, a TGD glossary at `http://tinyurl.com/yd6jf3o7`).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (`http://tinyurl.com/ycyrxj4o` founded by Lian Sidorov and in *Prespacetime Journal* (`http://tinyurl.com/ycvktjhn`), *Journal of Consciousness Research and Exploration* (`http://tinyurl.com/yba4f672`), and *DNA Decipher Journal* (`http://tinyurl.com/y9z52khg`), all of them founded by Huping Hu. One can find the list about the articles published at `http://tinyurl.com/ybv8dt4n`. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

TGD variant of Twistor Story

Twistor Grassmannian formalism has made a breakthrough in $\mathcal{N} = 4$ supersymmetric gauge theories and the Yangian symmetry suggests that much more than mere technical breakthrough is in question. Twistors seem to be tailor made for TGD but it seems that the generalization of twistor structure to that for 8-D embedding space $H = M^4 \times CP_2$ is necessary. M^4 (and S^4 as its Euclidian counterpart) and CP_2 are indeed unique in the sense that they are the only 4-D spaces allowing twistor space with Kähler structure.

The Cartesian product of twistor spaces $P_3 = SU(2,2)/SU(2,1) \times U(1)$ and F_3 defines twistor space for the embedding space H and one can ask whether this generalized twistor structure could allow to understand both quantum TGD and classical TGD defined by the extremals of Kähler action. In the following I summarize the background and develop a proposal for how to construct extremals of Kähler action in terms of the generalized twistor structure. One ends up with a scenario in which space-time surfaces are lifted to twistor spaces by adding CP_1 fiber so that the twistor spaces give an alternative representation for generalized Feynman diagrams.

There is also a very closely analogy with superstring models. Twistor spaces replace Calabi-Yau manifolds and the modification recipe for Calabi-Yau manifolds by removal of singularities can be applied to remove self-intersections of twistor spaces and mirror symmetry emerges naturally. The overall important implication is that the methods of algebraic geometry used in super-string theories should apply in TGD framework.

The physical interpretation is totally different in TGD. The landscape is replaced with twistor spaces of space-time surfaces having interpretation as generalized Feynman diagrams and twistor spaces as sub-manifolds of $P_3 \times F_3$ replace Witten's twistor strings.

The classical view about twistorialization of TGD makes possible a more detailed formulation of the previous ideas about the relationship between TGD and Witten's theory and twistor Grassmann approach. Furthermore, one ends up to a formulation of the scattering amplitudes in terms of Yangian of the super-symplectic algebra relying on the idea that scattering amplitudes are sequences consisting of algebraic operations (product and co-product) having interpretation as vertices in the Yangian extension of super-symplectic algebra. These sequences connect given initial and final states and having minimal length. One can say that Universe performs calculations.

From Principles to Diagrams

The recent somewhat updated view about the road from general principles to diagrams is discussed. A more explicit realization of twistorialization as lifting of the preferred extremal X^4 of Kähler action to corresponding 6-D twistor space X^6 identified as surface in the 12-D product of twistor spaces of M^4 and CP_2 allowing Kähler structure suggests itself. Contrary to the original expectations, the twistorial approach is not mere reformulation but leads to a first principle identification of cosmological constant and perhaps also of gravitational constant and to a modification of the dynamics of Kähler action however preserving the known extremals and basic properties of Kähler action and allowing to interpret induced Kähler form in terms of preferred imaginary unit defining twistor structure.

Second new element is the fusion of twistorial approach with the vision that diagrams are representations for computations. This as also quantum criticality demands that the diagrams should allow huge symmetries allowing to transform them to braided generalizations of tree-diagrams. Several guiding principles are involved and what is new is the observation that they indeed seem to form a coherent whole.

About Twistor Lift of TGD

The twistor lift of classical TGD is attractive physically but it is still unclear whether it satisfies all constraints. The basic implication of twistor lift would be the understanding of gravitational and cosmological constants. Cosmological constant removes the infinite vacuum degeneracy of Kähler action but because of the extreme smallness of cosmological constant Λ playing the role of inverse of gauge coupling strength, the situation for nearly vacuum extremals of Kähler action in the recent cosmology is non-perturbative. Cosmological constant and thus twistor lift make sense only in zero energy ontology (ZEO) involving causal diamonds (CDs) in an essential manner.

One motivation for introducing the hierarchy of Planck constants was that the phase transition increasing Planck constant makes possible perturbation theory in strongly interacting system. Nature itself would take care about the converge of the perturbation theory by scaling Kähler coupling strength α_K to α_K/n, $n = h_{eff}/h$. This hierarchy might allow to construct gravitational perturbation theory as has been proposed already earlier. This would for gravitation to be quantum coherent in astrophysical and even cosmological scales.

In this chapter twistor lift is studied in detail.

1. The first working hypothesis is that the values of $\alpha_K(M^4)$ and $\alpha_K(CP_2)$ are widely different with $\alpha_K(M^4)$ being extremely large so that M^4 part of the 6-D Kähler action gives in dimensional reduction extremely small cosmological term. The first interesting finding is that allowing Kähler coupling strength $\alpha_K(CP_2)$ to correspond to zeros of zeta implies that for complex zeros the preferred extremals for $\alpha_K(M^4)$ having different phase are mimimal surface extremals of Kähler action so that the values of coupling constants do not matter and extremals depend on couplings only through the boundary conditions stating the vanishing of certain super-symplectic conserved charges.

2. The other working hypothesis is $\alpha_K(M^4) = \alpha_K(CP_2)$. The small effective value of cosmological constant is obtained if the Kähler action and volume term tend to cancel each other. In this case minimal surface extremals of Kähler action correspond naturally to asymptotic dynamics near the boundaries of CDs. This option looks more natural.

Both options lead to a generalization of Chladni mechanism to a "dynamics of avoidance" meaning that at least asymptotically the two dynamics decouple. This leads to an interpretation with profound implications for the views about what happens in particle physics experiment and in quantum measurement, for consciousness theory and for quantum biology.

A related observation is that a fundamental length scale of biology - size scale of neuron and axon - would correspond to the p-adic length scale assignable to vacuum energy density assignable to cosmological constant and be therefore a fundamental physics length scale.

Some Questions Related to the Twistor Lift of TGD

In this chapter I consider questions related to both classical and quantum aspects of twistorialization.

1. The first group of questions relates to the twistor lift of classical TGD. What does the induction of the twistor structure really mean? Can the analog of Kähler form assignable

to M^4 suggested by the symmetry between M^4 and CP_2 and by number theoretical vision appear in the theory. What would be the physical implications? How does gravitational coupling emerge at fundamental level? Could one regard the localization of spinor modes to string world sheets as a localization to Lagrangian sub-manifolds of space-time surface with vanishing induced Kähler form. Lagrangian sub-manifolds would be commutative in the sense of Poisson bracket. How this relates to the idea that string world sheets correspond complex (commutative) surfaces of quaternionic space-time surface in octonionic embedding space?

During the re-processing of the details related to twistor lift, it became clear that the earlier variant for the twistor lift can be criticized and allows an alternative. This option led to a much simpler view about twistor lift, to the conclusion that minimal surface extremals of Kähler action represent only asymptotic situation (external particles in scattering), and also to a re-interpretation for the p-adic evolution of the cosmological constant: cosmological term would correspond to the *entire* 4-D action and the cancellation of Kähler action and cosmological term would lead to the small value of the effective cosmological constant.

2. Second group of questions relates to the construction of scattering amplitudes. The idea is to generalize the usual construction for massless states. In TGD all single particle states are massless in 8-D sense and this gives excellent hopes about the applicability of 8-D twistor approach. M^8-H duality turns out to be the key to the construction. Also the holomorphy of twistor amplitudes in helicity spinors λ_i and independence on $\tilde{\lambda}_i$ is crucial. The basic vertex corresponds to 4-fermion vertex for which the simplest expression can be written immediately. $n>4$-fermion scattering amplitudes can be also written immediately.

   If scattering diagrams correspond to computations as number theoretic vision suggests, the diagrams should be reducible to tree diagrams by moves generalizing the old-fashioned hadronic duality. This condition reduces to the vanishing of loops which in terms of BCFW recursion formula states that the twistor diagrams correspond to closed objects in what might be called WCFW homology.

The Recent View about Twistorialization in TGD Framework

The recent view about the twistorialization in TGD framework is discussed.

1. A proposal made already earlier is that scattering diagrams as analogs of twistor diagrams are constructible as tree diagrams for CDs connected by free particle lines. Loop contributions are not even well-defined in zero energy ontology (ZEO) and are in conflict with number theoretic vision. The coupling constant evolution would be discrete and associated with the scale of CDs (p-adic coupling constant evolution) and with the hierarchy of extensions of rationals defining the hierarchy of adelic physics.

2. Logarithms appear in the coupling constant evolution in QFTs. The identification of their number theoretic versions as rational number valued functions required by number-theoretical universality for both the integer characterizing the size scale of CD and for the hierarchy of Galois groups leads to an answer to a long-standing question what makes small primes and primes near powers of them physically special. The primes $p\in\{2,3,5\}$ indeed turn out to be special from the point of view of number theoretic logarithm.

3. The reduction of the scattering amplitudes to tree diagrams is in conflict with unitarity in 4-D situation. The imaginary part of the scattering amplitude would have discontinuity proportional to the scattering rate only for many-particle states with light-like total momenta. Scattering rates would vanish identically for the physical momenta for many-particle states.

   In TGD framework the states would be however massless in 8-D sense. Massless pole corresponds now to a continuum for M^4 mass squared and one would obtain the unitary cuts from a pole at $P^2=0$! Scattering rates would be non-vanishing only for many-particle states having light-like 8-momentum, which would pose a powerful condition on the construction of many-particle states.

   This idea does not make sense for incoming/outgoing particles, which light-like momenta unless they are parallel: their total momentum cannot be light-like in the general case.

Rather, $P^2 = 0$ applies to the states formed inside CDs from groups of incoming and outgoing particles. BCFW deformation $p_i \rightarrow p_i + zr_i$ describes what happens for the single-particle momenta: they cease to be light-like but the total momenta for subgroups of particles in factorization channels are complex and light-like. This strong form of conformal symmetry has highly non-trivial implications concerning color confinement.

4. The key idea is number theoretical discretization in terms of "cognitive representations" as space-time time points with M^8-coordinates in an extension of rationals and therefore shared by both real and various p-adic sectors of the adele. Discretization realizes measurement resolution, which becomes an inherent aspect of physics rather than something forced by observed as outsider. This fixes the space-time surface completely as a zero locus of real or imaginary part of octonionic polynomial.

   This must imply the reduction of "world of classical worlds" (WCW) corresponding to a fixed number of points in the extension of rationals to a finite-dimensional discretized space with maximal symmetries and Kähler structure.

   The simplest identification for the reduced WCW would be as complex Grassmannian - a more general identification would be as a flag manifold. More complex options can of course be considered. The Yangian symmetries of the twistor Grassmann approach known to act as diffeomorphisms respecting the positivity of Grassmannian and emerging also in its TGD variant would have an interpretation as general coordinate invariance for the reduced WCW. This would give a completely unexpected connection with supersymmetric gauge theories and TGD.

5. M^8 picture implies the analog of SUSY realized in terms of polynomials of super-octonions whereas H picture suggests that supersymmetry is broken in the sense that many-fermion states as analogs of components of super-field at partonic 2-surfaces are not local. This requires breaking of SUSY. At M^8 level the breaking could be due to the reduction of Galois group to its subgroup G/H, where H is normal subgroup leaving the point of cognitive representation defining space-time surface invariant. As a consequence, local many-fermion composite in M^8 would be mapped to a non-local one in H by $M^8 - H$ correspondence.

About TGD counterparts of twistor amplitudes

The twistor lift of TGD, in which $H = M^4 \times CP_2$ is replaced with the product of twistor spaces $T(M^4)$ and $T(CP_2)$, and space-time surface $X^4 \subset H$ with its 6-D twistor space as 6-surface $X^6 \subset T(M^4) \times T(CP_2)$, is now a rather well-established notion and $M^8 - H$ duality predicts it at the level of M^8.

Number theoretical vision involves $M^8 - H$ duality. At the level of H the quark mass spectrum is determined by the Dirac equation in H. In M^8 mass squared spectrum is determined by the roots of the polynomial P determining space-time surface and are in general complex. By Galois confinement the momenta are integer valued when p-adic mass is used as a unit and mass squared spectrum is also integer valued. This raises hope about a generalization of the twistorial construction of scattering amplitudes to TGD context.

It is always best to start from a problem and the basic problem of the twistor approach is that physical particles are not massless.

1. The intuitive TGD based proposal has been that since quark spinors are massless in H, the masslessness in the 8-D sense could somehow solve the problems caused by the massivation in the construction of twistor scattering amplitudes. However, no obvious mechanism has been identified. One step in this direction was the realization that in H quarks propagate with well-defined chiralities and only the square of Dirac equation is satisfied. For a quark of given helicity the spinor can be identified as helicity spinor.

2. M^8 quark momenta are in general complex as algebraic integers. They are the counterparts of the area momenta x_i of momentum twistor space whereas H momenta are identified as ordinary momenta. Total momenta of Galois confined states have as components ordinary integers.

3. The M^8 counterpart of the 8-D massless condition in H is the restriction of momenta to mass shells $m^2 = r_n$ determined as roots of P. The M^8 counterpart of Dirac equation in H is octonionic Dirac equation, which is algebraic as everything in M^8 and analogous to massless Dirac equation. The solution is a helicity spinor $\tilde{\lambda}$ associated with the massive momentum.

The outcome is an extremely simple proposal for the scattering amplitudes.

1. Vertices correspond to trilinears of Galois confined many-quark states as states of super symplectic algebra acting as isometries of the "world of classical worlds" (WCW). Quarks are on-shell with H momentum p and M^8 momenta x_i, x_{i+1}, $p_i = x_{i+1} - x_i$. Dirac operator $x_i^k \gamma_k$ restricted to fixed helicity L, R appears as a vertex factor and has an interpretation as a residue of a pole from an on-mass-shell propagator D so that a correspondence with twistorial construction becomes obvious. D is expressible in terms of the helicity spinors of given chirality and gives two independent holomorphic factors as in case of massless theories.

2. The scattering amplitudes would be rational functions in accordance with the number theoretic vision. The absence of logarithmic radiative corrections is not a problem: the coupling constant evolution would be discrete and defined by the hierarchy of extensions of rationals.

3. The scattering amplitudes for a single 4-surface X^4 characterixzing interaction region are determined by a polynomial P. External particles are Galois singlets consisting of off-mass shell quarks with mass squared values coming as roots of the polynomial P characterizing the interaction region. External particles are characterized by polynomials P_i satisfying $P_i(0) = 0$. P is identified as the functional composite of P_i since it inherits the masses of incoming particles as their roots. This allows only cyclic permutations of P_i. The scattering event is essentially a re-combination of incoming Galois singlets to new Galois singlets and quarks propagate freely: hence OZI rule generalizes. Also a connection with the dual resonance models emerges.

4. The integration over WCW is replaced with a summation of polynomials characterized by rational coefficients. Monic polynomials are highly suggestive. A connection with p-adicization emerges via the identification of the p-adic prime as one of the ramified primes of P. Only (monic) polynomials having a common p-adic prime are allowed in the sum. The counterpart of the vacuum functional $exp(-K)$ is naturally identified as the discriminant D of the extension associated with P and p-adic coupling constant evolution emerges from the identification of $exp(-K)$ with D.

Unitary, locality, and the failure to find the twistorial counterparts of non-planar Feynman diagrams are the basic problems of the twistor Grassmannian approach. Also the non-existence of twistor spaces for most Riemannian manifolds is a problem in GRT framework but in TGD the existence of twistor spaces for M^4 and CP_2 solves this problem. In the TGD framework, the replacement of point-like particles with 3-surfaces leads to the loss of locality at the fundamental level. The analogs of non-planar diagrams are eliminated since only cyclic permutations of P_i are allowed.

This leaves only the problem with unitarity. The TGD counterpart of unitarity realized in terms of Kähler geometry of fermionic state space is very natural in the geometrization of quantum physics. Scattering probabilities are identified as products of covariant and contravariant matrix elements of the metric, and unitary conditions are replaced by the definition of the contravariant metric. Probabilities are complex but real and imaginary parts are separately conserved. The interpretation in terms of Fisher information is possible. Due to the infinite-D character of the state space, the Kähler geometry exists only if it has a maximal group of isometries and is a unique constant curvature geometry. Also the interpretation of this approach in zero energy ontology is discussed.

There are physical motivations for considering the number theoretic generalizations of the amplitudes. For an iterate of fixed P (say large number of gravitons), the roots of the iterate of P defined virtual mass squared values, approach to the Julia set of P. The construction of scattering amplitudes thus leads to chaos theory at the limit of an infinite number of identical particles.

The construction generalizes also to the surfaces defined by real analytic functions and the fermionic variant of Riemann zeta and elliptic functions are discussed as examples.

Twistors and holography= holomorphy vision

Twistorialization involves several problems. Mention only the identification of the twistor space, the googly problem meaning that only second massless M^4 chirality allows geometrization in this way, the problem that massive fields do not allow twistorialization, and the problem that in general relativity only space-times with vanishing Weyl tensor allow twistor structure.

In the TGD framework, twistorialization should be performed for $H = M^4 \times CP_2$. Now there are no primary bosonic fields since they are represented in terms of the induced spinor connection and metric and also classical color fields are obtained by induction. Twistor lift was based on the replacement of space-time surfaces in $H = M^4 \times CP_2$ with the analogs of their 6-D twistor spaces X^6 as sphere bundles as a surfaces in the twistor space $T(H)$ of H identified as the product $T(M^4) \times T(CP_2)$ of twistor spaces H. In TGD, the replacement of $T(M^4) = CP_3$ with $CP_{2,1}$ having one hypercomplex coordinate is natural. Dimensional reduction for the extremals of 6-D Kähler action and the identification of the fiber spheres CP_1 of $T(M^4)$ and $T(CP_2)$ was needed to product to produce the X^6 as a sphere bundle over X^4.

Holography= holomorphy (H-H) vision in turn allows to solve the field equations for any general coordinate invariant action expressible in terms of the induced geometry allows to solve the field equations, which are extremely nonlinear partial differential equations, exactly by reducing them to purely algebraic local equations. The independence of action means universality. H-H vision conforms with $T(H)$ view but one can ask whether one could twist TGD without the introduction of $T(H)$ by representing the twistor spheres of $T(M^4)$ and $T(CP_2)$ as homologically non-trivial spheres of the causal diamond CD (missing the line connecting its tips) and CP_2. The second condition involved with the H-H principle would represent the identification of the twistor spheres.

In this article various problems of the twistorialization are discussed in the TGD framework and the question whether the H-H principle is enough for twistorialization is discussed.

Chapter 2

TGD variant of Twistor Story

2.1 Introduction

Twistor Grassmannian formalism has made a breakthrough in $\mathcal{N} = 4$ supersymmetric gauge theories and the Yangian symmetry suggests that much more than mere technical breakthrough is in question. Twistors seem to be tailor made for TGD but it seems that the generalization of twistor structure to that for 8-D embedding space $H = M^4 \times CP_2$ is necessary. M^4 (and S^4 as its Euclidian counterpart) and CP_2 are indeed unique in the sense that they are the only 4-D spaces allowing twistor space with Kähler structure.

The Cartesian product of twistor spaces $P_3 = SU(2,2)/SU(2,1) \times U(1)$ and F_3 defines twistor space for the embedding space H and one can ask whether this generalized twistor structure could allow to understand both quantum TGD [K79, K54, K65] and classical TGD [K16] defined by the extremals of Kähler action.

In the following I summarize first the basic results and problems of the twistor approach. After that I describe some of the mathematical background and develop a proposal for how to construct extremals of Kähler action in terms of the generalized twistor structure. One ends up with a scenario in which space-time surfaces are lifted to twistor spaces by adding CP_1 fiber so that the twistor spaces give an alternative representation for generalized Feynman diagrams having as lines space-time surfaces with Euclidian signature of induced metric and having wormhole contacts as basic building bricks.

There is also a very close analogy with superstring models. Twistor spaces replace Calabi-Yau manifolds [A1, A30] and the modification recipe for Calabi-Yau manifolds by removal of singularities can be applied to remove self-intersections of twistor spaces and mirror symmetry [B7]emerges naturally. The overall important implication is that the methods of algebraic geometry used in super-string theories should apply in TGD framework.

The physical interpretation is totally different in TGD. Twistor space has space-time as base-space rather than forming with it Cartesian factors of a 10-D space-time. The Calabi-Yau landscape is replaced with the space of twistor spaces of space-time surfaces having interpretation as generalized Feynman diagrams and twistor spaces as sub-manifolds of $P_3 \times F_3$ replace Witten's twistor strings [B16]. The space of twistor spaces is the lift of the "world of classical worlds" (WCW) by adding the CP_1 fiber to the space-time surfaces so that the analog of landscape has beautiful geometrization.

The classical view about twistorialization of TGD makes possible a more detailed formulation of the previous ideas about the relationship between TGD and Witten's theory and twistor Grassmann approach.

1. The notion of quaternion analyticity extending the notion of ordinary analyticity to 4-D context is highly attractive but has remained one of the long-standing ideas difficult to take quite seriously but equally difficult to throw to paper basked. Four-manifolds possess almost quaternion structure. In twistor space context the formulation of quaternion analyticity becomes possible and relies on an old notion of tri-holomorphy about which I had not been aware earlier. The natural formulation for the preferred extremal property is as a condition stating that various charges associated with generalized conformal algebras vanish for preferred ex-

tremals. This leads to ask whether Euclidian space-time regions could be quaternion-Kähler manifolds for which twistor spaces are so called Fano spaces. In Minkowskian regions so called Hamilton-Jacobi property would apply.

2. The generalization of Witten's twistor theory to TGD framework is a natural challenge and the 2-surfaces studied defining scattering amplitudes in Witten's theory could correspond to partonic 2-surfaces identified as algebraic surfaces characterized by degree and genus. Besides this also string world sheets are needed. String worlds have 1-D lines at the light-like orbits of partonic 2-surfaces as their boundaries serving as carriers of fermions. This leads to a rather detailed generalization of Witten's approach using the generalization of twistors to 8-D context.

3. The generalization of the twistor Grassmannian approach to 8-D context is second fascinating challenge. If one requires that the basic formulas relating twistors and four-momentum generalize one must consider the situation in tangent space M^8 of embedding space (M^8-H duality) and replace the usual sigma matrices having interpretation in terms of complexified quaternions with octonionic sigma matrices.

   The condition that octonionic spinors are are equivalent with ordinary spinors has strong consequences. Induced spinors must be localized to 2-D string world sheets, which are (co-)commutative sub-manifolds of (co-)quaternionic space-time surface. Also the gauge fields should vanish since they induce a breaking of associativity even for quaternionic and complex surface so that CP_2 projection of string world sheet must be 1-D. If one requires also the vanishing of gauge potentials, the projection is geodesic circle of CP_2 so that string world sheets are restricted to Minkowskian space-time regions. Although the theory would be free in fermionic degrees of freedom, the scattering amplitudes are non-trivial since vertices correspond to partonic 2-surfaces at which partonic orbits are glued together along common ends. The classical light-like 8-momentum associated with the boundaries of string world sheets defines the gravitational dual for 4-D momentum and color quantum numbers associated with imbedding space spinor harmonics. This leads to a more detailed formulation of Equivalence Principle which would reduce to M^8-H duality basically.

   Number theoretic interpretation of the positivity of Grassmannians is highly suggestive since the canonical identification maps p-adic numbers to non-negative real numbers. A possible generalization is obtained by replacing positive real axis with upper half plane defining hyperbolic space having key role in the theory of Riemann surfaces. The interpretation of scattering amplitudes as representations of permutations generalizes to interpretation as braidings at surfaces formed by the generalized Feynman diagrams having as lines the light-like orbits of partonic surfaces. This because 2-fermion vertex is the only interaction vertex and induced by the non-continuity of the induced Dirac operator at partonic 2-surfaces. OZI rule generalizes and implies an interpretation in terms of braiding consistent with the TGD as almost topological QFT vision. This suggests that non-planar twistor amplitudes are constructible as analogs of knot and braid invariants by a recursive procedure giving as an outcome planar amplitudes.

4. Yangian symmetry is associated with twistor amplitudes and emerges in TGD from completely different idea interpreting scattering amplitudes as representations of algebraic manipulation sequences of minimal length (preferred extremal instead of path integral over space-time surfaces) connecting given initial and final states at boundaries of causal diamond. The algebraic manipulations are carried out in Yangian using product and co-product defining the basic 3-vertices analogous to gauge boson absorption and emission. 3-surface representing elementary particle splits into two or vice versa such that second copy carries quantum numbers of gauge boson or its super counterpart. This would fix the scattering amplitude for given 3-surface and leave only the functional integral over 3-surfaces.

2.2 Background And Motivations

In the following some background plus basic facts and definitions related to twistor spaces are summarized. Also reasons for why twistor are so relevant for TGD is considered at general level.

2.2.1 Basic Results And Problems Of Twistor Approach

The author describes both the basic ideas and results of twistor approach as well as the problems.

Basic results

There are three deep results of twistor approach besides the impressive results which have emerged after the twistor resolution.

1. Massless fields of arbitrary helicity in 4-D Minkowski space are in 1-1 correspondence with elements of Dolbeault cohomology in the twistor space CP_3. This was already the discovery of Penrose..The connection comes from Penrose transform. The light-like geodesics of M^4 correspond to points of 5-D sub-manifold of CP_3 analogous to light-cone boundary. The points of M^4 correspond to complex lines (Riemann spheres) of the twistor space CP_3: one can imagine that the point of M^4 corresponds to all light-like geodesics emanating from it and thus to a 2-D surface (sphere) of CP_3. Twistor transform represents the value of a massless field at point of M^4 as a weighted average of its values at sphere of CP_3. This correspondence is formulated between open sets of M^4 and of CP_3. This fits very nicely with the needs of TGD since causal diamonds which can be regarded as open sets of M^4 are the basic objects in zero energy ontology (ZEO).

2. Self-dual instantons of non-Abelian gauge theories for SU(n) gauge group are in one-one correspondence with holomorphic rank-N vector bundles in twistor space satisfying some additional conditions. This generalizes the correspondence of Penrose to the non-Abelian case. Instantons are also usually formulated using classical field theory at four-sphere S^4 having Euclidian signature.

3. Non-linear gravitons having self-dual geometry are in one-one correspondence with spaces obtained as complex deformations of twistor space satisfying certain additional conditions. This is a generalization of Penrose's discovery to the gravitational sector.

Complexification of M^4 emerges unavoidably in twistorial approach and Minkowski space identified as a particular real slice of complexified M^4 corresponds to the 5-D subspace of twistor space in which the quadratic form defined by the SU(2,2) invariant metric of the 8-dimensional space giving twistor space as its projectivization vanishes. The quadratic form has also positive and negative values with its sign defining a projective invariant, and this correspond to complex continuations of M^4 in which positive/negative energy parts of fields approach to zero for large values of imaginary part of M^4 time coordinate.

Interestgingly, this complexification of M^4 is also unavoidable in the number theoretic approach to TGD: what one must do is to replace 4-D Minkowski space with a 4-D slice of 8-D complexified quaternions. What is interesting is that real M^4 appears as a projective invariant consisting of light-like projective vectors of C^4 with metric signature (4,4). Equivalently, the points of M^4 represented as linear combinations of sigma matrices define hermitian matrices.

Basic problems of twistor approach

The best manner to learn something essential about a new idea is to learn about its problems. Difficulties are often put under the rug but the thesis is however an exception in this respect. It starts directly from the problems of twistor approach. There are two basic challenges.

1. Twistor approach works as such only in the case of Minkowski space. The basic condition for its applicability is that the Weyl tensor is self-dual. For Minkowskian signature this leaves only Minkowski space under consideration. For Euclidian signature the conditions are not quite so restrictive. This looks a fatal restriction if one wants to generalize the result of Penrose to a general space-time geometry. This difficulty is known as "googly" problem.

   According to the thesis MHV construction of tree amplitudes of $\mathcal{N}=4$ SYM based on topological twistor strings in CP_3 meant a breakthrough and one can indeed understand also have analogs of non-self-dual amplitudes. The problem is however that the gravitational theory assignable to topological twistor strings is conformal gravity, which is generally regarded as non-physical. There have been several attempts to construct the on-shell scattering amplitudes of Einstein's gravity theory as subset of amplitudes of conformal gravity and also thesis considers this problem.

2. The construction of quantum theory based on twistor approach represents second challenge. In this respect the development of twistor approach to $\mathcal{N}=4$ SYM meant a revolution and one can indeed construct twistorial scattering amplitudes in M^4.

2.2.2 Results About Twistors Relevant For TGD

First some background.

1. The twistors originally introduced by Penrose (1967) have made breakthrough during last decade. First came the twistor string theory of Edward Witten [B16] proposed twistor string theory and the work of Nima-Arkani Hamed and collaborators [B20] led to a revolution in the understanding of the scattering amplitudes of scattering amplitudes of gauge theories [B11, B9, B22]. Twistors do not only provide an extremely effective calculational method giving even hopes about explicit formulas for the scattering amplitudes of $\mathcal{N}=4$ supersymmetric gauge theories but also lead to an identification of a new symmetry: Yangian symmetry [A4], [B14, B15], which can be seen as multilocal generalization of local symmetries.

   This approach, if suitably generalized, is tailor-made also for the needs of TGD. This is why I got seriously interested on whether and how the twistor approach in empty Minkowski space M^4 could generalize to the case of $H=M^4\times CP_2$. The twistor space associated with H should be just the cartesian product of those associated with its Cartesian factors. Can one assign a twistor space with CP_2?

2. First a general result [A18] deserves to be mentioned: any oriented manifold X with Riemann metric allows 6-dimensional twistor space Z as an almost complex space. If this structure is integrable, Z becomes a complex manifold, whose geometry describes the conformal geometry of X. In general relativity framework the problem is that field equations do not imply conformal geometry and twistor Grassmann approach certainly requires conformal structure.

3. One can consider also a stronger condition: what if the twistor space allows also Kähler structure? The twistor space of empty Minkowski space M^4 (and its Euclidian counterpart S^4 is the Minkowskian variant of $P_3=SU(2,2)/SU(2,1)\times U(1)$ of 3-D complex projective space $CP_3=SU(4)/SU(3)\times U(1)$ and indeed allows Kähler structure.

   The points of the Euclidian twistor space $CP_3=SU(4)/SU(3)\times U(1)$ can be represented by any column of the 4×4 matrix representing element of $SU(4)$ with columns differing by phase multiplication being identified. One has four coordinate charts corresponding to four different choices of the column. The points of its Minkowskian variant $CP_{2,1}=SU(2,2)/SU(2,1)\times U(1)$ can be represented in similar manner as U(1) gauge equivalence classes for given column of SU(3,1) matrices, whose rows and columns satisfy orthonormality conditions with respect to the hermitian inner product defined by Minkowskian metric $\epsilon=(1,1,-1,-1)$.

   Rather remarkably, there are *no other space-times* with Minkowski signature allowing twistor space with Kähler structure [A18]. Does this mean that the empty Minkowski space of special relativity is much more than a limit at which space-time is empty?

   This also means a problem for GRT. Twistor space with Kähler structure seems to be needed but general relativity does not allow it. Besides twistor problem GRT also has energy problem: matter makes space-time curved and the conservation laws and even the definition of energy and momentum are lost since the underlying symmetries giving rise to the conservation laws through Noether's theorem are lost. GRT has therefore two bad mathematical problems which might explain why the quantization of GRT fails. This would not be surprising since quantum theory is to high extent representation theory for symmetries and symmetries are lost. Twistors would extend these symmetries to Yangian symmetry but GRT does not allow them.

4. What about twistor structure in CP_2? CP_2 allows complex structure (Weyl tensor is self-dual), Kähler structure plus accompanying symplectic structure, and also quaternion structure. One of the really big personal surprises of the last years has been that CP_2 twistor space indeed allows Kähler structure meaning the existence of antisymmetric tensor representing imaginary unit whose tensor square is the negative of metric in turn representing real unit.

   The article by Nigel Hitchin, a famous mathematical physicist, describes a detailed argument identifying S^4 and CP_2 as the only compact Riemann manifolds allowing Kählerian twistor space [A18]. Hitchin sent his discovery for publication 1979. An amusing coincidence is that I discovered CP_2 just this year after having worked with S^2 and found that it does not really allow to understand standard model quantum numbers and gauge fields. It is difficult to avoid thinking that maybe synchrony indeed a real phenomenon as TGD inspired theory of consciousness predicts to be possible but its creator cannot quite believe. Brains at different side of globe discover simultaneously something closely related to what some conscious self at the higher level of hierarchy using us as instruments of thinking just as we use nerve cells is intensely pondering.

   Although 4-sphere S^4 allows twistor space with Kähler structure, it does not allow Kähler structure and cannot serve as candidate for S in $H = M^4 \times S$. As a matter of fact, S^4 can be seen as a Wick rotation of M^4 and indeed its twistor space is CP_3.

   In TGD framework a slightly different interpretation suggests itself. The Cartesian products of the intersections of future and past light-cones - causal diamonds (CDs) - with CP_2 - play a key role in ZEO (ZEO) [K4]. Sectors of "world of classical worlds" (WCW) [K35, K19] correspond to 4-surfaces inside $CD \times CP_2$ defining a the region about which conscious observer can gain conscious information: state function reductions - quantum measurements - take place at its light-like boundaries in accordance with holography. To be more precise, wave functions in the moduli space of CDs are involved and in state function reductions come as sequences taking place at a given fixed boundary. This kind of sequence is identifiable as self and give rise to the experience about flow of time. When one replaces Minkowski metric with Euclidian metric, the light-like boundaries of CD are contracted to a point and one obtains topology of 4-sphere S^4.

5. Another really big personal surprise was that there are *no other* compact 4-manifolds with Euclidian signature of metric allowing twistor space with Kähler structure! The embedding space $H = M^4 \times CP_2$ is not only physically unique since it predicts the quantum number spectrum and classical gauge potentials consistent with standard model but also mathematically unique!

   After this I dared to predict that TGD will be the theory next to GRT since TGD generalizes string model by bringing in 4-D space-time. The reasons are many-fold: TGD is the only known solution to the two big problems of GRT: energy problem and twistor problem. TGD is consistent with standard model physics and leads to a revolution concerning the identification of space-time at microscopic level: at macroscopic level it leads to GRT but explains some of its anomalies for which there is empirical evidence (for instance, the observation that neutrinos arrived from SN1987A at two different speeds different from light velocity [**?**] has natural explanation in terms of many-sheeted space-time). TGD avoids the landscape problem of M-theory and anthropic non-sense. I could continue the list but I think that this is enough.

6. The twistor space of CP_2 is 3-complex dimensional flag manifold $F_3 = SU(3)/U(1) \times U(1)$ having interpretation as the space for the choices of quantization axes for the color hypercharge and isospin. This choice is made in quantum measurement of these quantum numbers and a means localization to single point in F_3. The localization in F_3 could be higher level measurement leading to the choice of quantizations for the measurement of color quantum numbers.

   F_3 is symmetric space meaning that besides being a coset space with $SU(3)$ invariant metric it also has involutions acting as a reflection at geodesics through a point remaining fixed under the involution. As a symmetric space with Fubini-Study metric F_3 is positive constant curvature space having thus positive constant sectional curvatures. This implies Einstein space property. This also conforms with the fact that F_3 is CP_1 bundle over CP_2 as base space (for more details see `http://tinyurl.com/ychdeqjz`).

The points of flag manifold $SU(3)/U(1)\times U(1)$ can be represented locally by identifying $SU(3)$ matrices for which columns differ by multiplication from left with exponential $e^{i(aY+bI_3)}$, a and b arbitrary real numbers. This transformation allows what might be called a "gauge choice". For instance, first two elements of the first row can be made real in this manner. These coordinates are not global.

7. Analogous interpretation could make sense for M^4 twistors represented as points of P_3. Twistor corresponds to a light-like line going through some point of M^4 being labelled by 4 position coordinates and 2 direction angles: what higher level quantum measurement could involve a choice of light-like line going through a point of M^4? Could the associated spatial direction specify spin quantization axes? Could the associated time direction specify preferred rest frame? Does the choice of position mean localization in the measurement of position? Do momentum twistors relate to the localization in momentum space? These questions remain fascinating open questions and I hope that they will lead to a considerable progress in the understanding of quantum TGD.

8. It must be added that the twistor space of CP_2 popped up much earlier in a rather unexpected context [K31]: I did not of course realize that it was twistor space. Topologist Barbara Shipman [A7] has proposed a model for the honeybee dance leading to the emergence of F_3. The model led her to propose that quarks and gluons might have something to do with biology. Because of her position and specialization the proposal was forgiven and forgotten by community. TGD however suggests both dark matter hierarchies and p-adic hierarchies of physics [K27, K21, K22, K23, K24]. For dark hierarchies the masses of particles would be the standard ones but the Compton scales would be scaled up by $h_{eff}/h = n$ [K21, K22, K23, K24]. Below the Compton scale one would have effectively massless gauge boson: this could mean free quarks and massless gluons even in cell length scales. For p-adic hierarchy mass scales would be scaled up or down from their standard values depending on the value of the p-adic prime.

2.2.3 Basic Definitions Related To Twistor Spaces

One can find from web several articles explaining the basic notions related to twistor spaces and Calabi-Yau manifolds. At the first look the notions of twistor as it appears in the writings of physicists and mathematicians don't seem to have much common with each other and it requires effort to build the bridge between these views. The bridge comes from the association of points of Minkowski space with the spheres of twistor space: this clearly corresponds to a bundle projection from the fiber to the base space, now Minkowski space. The connection of the mathematician's formulation with spinors remains still somewhat unclear to me although one can understand CP_1 as projective space associated with spinors with 2 complex components. Minkowski signature poses additional challenges. In the following I try my best to summarize the mathematician's view, which is very natural in classical TGD.

There are many variants of the notion of twistor depending on whether how powerful assumptions one is willing to make. The weakest definition of twistor space is as CP_1 bundle of almost complex structures in the tangent spaces of an orientable 4-manifold. Complex structure at given point means selection of antisymmetric form J whose natural action on vector rotates a vector in the plane defined by it by $\pi/2$ and thus represents the action of imaginary unit. One must perform this kind of choice also in normal plane and the direct sum of the two choices defines the full J. If one chooses J to be self-dual or anti-self-dual (eigenstate of Hodge star operation), one can fix J uniquely. Orientability makes possible the Hodge star operation involving 4-dimensional permutation tensor.

The condition $i^1 = -1$ is translated to the condition that the tensor square of J equals to $J^2 = -g$. The possible choices of J span sphere S^2 defining the fiber of the twistor spaces. This is not quite the complex sphere CP_1, which can be thought of as a projective space of spinors with two complex components. Complexification must be performed in both the tangent space of X^4 and of S^2. Note that in the standard approach to twistors the entire 6-D space is projective space P_3 associated with the C^8 having interpretation in terms of spinors with 4 complex components.

One can introduce almost complex structure also to the twistor space itself by extending the almost complex structure in the 6-D tangent space obtained by a preferred choice of J by identifying it as a point of S^2 and acting in other points of S^2 identified as antisymmetric tensors. If these points are interpreted as imaginary quaternion units, the action is commutator action divided by 2. The existence of quaternion structure of space-time surfaces in the sense as I have proposed in TGD framework might be closely related to the twistor structure.

Twistor structure as bundle of almost complex structures having itself almost complex structure is characterized by a hermitian Kähler form ω defining the almost complex structure of the twistor space. Three basic objects are involved: the hermitian form h, metric g and Kähler form ω satisfying $h = g + i\omega$, $g(X,Y) = \omega(X, JY)$.

In the base space the metric of twistor space is the metric of the base space and in the tangent space of fibre the natural metric in the space of antisymmetric tensors induced by the metric of the base space. Hence the properties of the twistor structure depend on the metric of the base space.

The relationship to the spinors requires clarification. For 2-spinors one has natural Lorentz invariant antisymmetric bilinear form and this seems to be the counterpart for J?

One can consider various additional conditions on the definition of twistor space.

1. Kähler form ω is not closed in general. If it is, it defines symplectic structure and Kähler structure. S^4 and CP_2 are the only compact spaces allowing twistor space with Kähler structure [A18].

2. Almost complex structure is not integrable in general. In the general case integrability requires that each point of space belongs to an open set in which vector fields of type (1, 0) or (0, 1) having basis ∂/∂_{z^k} and $\partial/\partial_{\overline{z}^k}$ expressible as linear combinations of real vector fields with complex coefficients commute to vector fields of same type. This is non-trivial conditions since the leading names for the vector field for the partial derivatives does not yet guarantee these conditions.

   This necessary condition is also enough for integrability as Newlander and Nirenberg have demonstrated. An explicit formulation for the integrability is as the vanishing of Nijenhuis tensor associated with the antisymmetric form J (see (`http://tinyurl.com/ybp9vsa5` and `http://tinyurl.com/y8j36p4m`). Nijenhuis tensor characterizes Nijenhuis bracket generalizing ordinary Lie bracket of vector fields (for detailed formula see `http://tinyurl.com/y83mbnso`).

3. In the case of twistor spaces there is an alternative formulation for the integrability. Curvature tensor maps in a natural manner 2-forms to 2-forms and one can decompose the Weyl tensor W identified as the traceless part of the curvature tensor to self-dual and anti-self-dual parts W^+ and W^-, whose actions are restricted to self-dual resp. antiself-dual forms (self-dual and anti-self-dual parts correspond to eigenvalue + 1 and -1 under the action of Hodge * operation: for more details see `http://tinyurl.com/ybkhj4np`). If W^+ or W^- vanishes - in other worlds W is self-dual or anti-self-dual - the assumption that J is self-dual or anti-self-dual guarantees integrability. One says that the metric is anti-self-dual (ASD). Note that the vanishing of Weyl tensor implies local conformal flatness (M^4 and sphere are obviously conformally flat). One might think that ASD condition guarantees that the parallel translation leaves J invariant.

   ASD property has a nice implication: the metric is balanced. In other words one has $d(\omega \wedge \omega) = 2\omega \wedge d\omega = 0$.

4. If the existence of complex structure is taken as a part of definition of twistor structure, one encounters difficulties in general relativity. The failure of spin structure to exist is similar difficulty: for CP_2 one must indeed generalize the spin structure by coupling Kähler gauge potential to the spinors suitably so that one obtains gauge group of electroweak interactions.

5. One could also give up the global existence of complex structure and require symplectic structure globally: this would give $d\omega = 0$. A general result is that hyperbolic 4-manifolds allow symplectic structure and ASD manifolds allow complex structure and hence balanced metric.

2.2.4 Why Twistor Spaces With Kähler Structure?

I have not yet even tried to answer an obvious question. Why the fact that M^4 and CP_2 have twistor spaces with Kähler structure could be so important that it could fix the entire physics? Let us consider a less general question. Why they would be so important for the classical TGD - exact part of quantum TGD - defined by the extremals of Kähler action [K10] ?

1. Properly generalized conformal symmetries are crucial for the mathematical structure of TGD [K19, K86, K18, L3]. Twistor spaces have almost complex structure and in these two special cases also complex, Kähler, and symplectic structures (note that the integrability of the almost complex structure to complex structure requires the self-duality of the Weyl tensor of the 4-D manifold).

   For years ago I considered the possibility that complex 3-manifolds of $CP_3 \times CP_3$ could have the structure of S^2 fiber space and have space-time surfaces as base space. I did not realize that these spaces could be twistor spaces nor did I realize that CP_2 allows twistor space with Kähler structure so that $CP_3 \times F_3$ looks a more plausible choice.

   The expectation was that the Cartesian product $CP_3 \times F_3$ of the two twistor spaces with Kähler structure is fundamental for TGD. The obvious wishful thought is that this space makes possible the construction of the extremals of Kähler action in terms of holomorphic surfaces defining 6-D twistor sub-spaces of $CP_3 \times F_3$ allowing to circumvent the technical problems due to the signature of M^4 encountered at the level of $M^4 \times CP_2$. It would also make the magnificent machinery of the algebraic geometry so powerful in string theories a tool of TGD. Here CP_3 could be replaced with its non-compact form and the problem is that one can have only compactification of M^4 for which metric is defined only modulo conformal scaling. There is however a problem: the compactified Minkowski space or its complexification has a metric defined only modulo conformal factor. This is not a problem in conformally invariant theories but becomes a problem if one wants to speak of induced metric.

   The next realization was that M^4 allows twistor bundle also in purely geometric sense and this bundle is just $T(M^4) = M^4 \times CP_2$. The two variants of twistor space would naturally apply at the level of momentum space and embedding space.

2. Every 4-D orientable Riemann manifold allows a twistor space as 6-D bundle with CP_1 as fiber and possessing almost complex structure. Metric and various gauge potentials are obtained by inducing the corresponding bundle structures. Hence the natural guess is that the twistor structure of space-time surface defined by the induced metric is obtained by induction from that for $T(M^4) \times F_3$ by restricting its twistor structure to a 6-D (in real sense) surface of $T(M^4) \times F_3$ with a structure of twistor space having at least almost complex structure with CP_1 as a fiber. For the embedding of the twistor space of space-time this requires the identification of S^2 fibers of $T(M^4)$ and F_3. If so then one can indeed identify the base space as 4-D space-time surface in $M^4 \times CP_2$ using bundle projections in the factors $T(M^4)$ and F_3.

3. There might be also a connection to the number theoretic vision about the extremals of Kähler action. At space-time level however complexified quaternions and octonions could allow alternative formulation. I have indeed proposed that space-time surfaces have associative of co-associative meaning that the tangent space or normal space at a given point belongs to quaternionic subspace of complexified octonions.

2.3 The Identification Of 6-D Twistor Spaces As Sub-Manifolds Of 12-D Twistor Space

How to identify the 6-D sub-manifolds with the structure of twistor space? Is this property all that is needed? Can one find a simple solution to this condition? What is the relationship of twistor spaces to the Calabi-Yau manifolds of super string models? In the following intuitive considerations of a simple minded physicist. Mathematician could probably make much more interesting comments.

2.3.1 Conditions For Twistor Spaces As Sub-Manifolds

Consider the conditions that must be satisfied using local trivializations of the twistor spaces. It will be assumed that the twistor space $T(M^4)$ is CP_3 or its Minkowskian variant. It has turned out that a more reasonable option $T(M^4) = M^4 \times CP_1$ is possible. The following consideration is however for CP_3 option. Before continuing let us introduce complex coordinates $z_i = x_i + iy_i$ resp. $w_i = u_i + iv_i$ for CP_3 resp. F_3.

1. 6 conditions are required and they must give rise by bundle projection to 4 conditions relating the coordinates in the Cartesian product of the base spaces of the two bundles involved and thus defining 4-D surface in the Cartesian product of compactified M^4 and CP_2.

2. One has Cartesian product of two fiber spaces with fiber CP_1 giving fiber space with fiber $CP_1^1 \times CP_1^2$. For the 6-D surface the fiber must be CP_1. It seems that one must identify the two spheres CP_1^i. Since holomorphy is essential, holomorphic identification $w_1 = f(z_1)$ or $z_1 = f(w_1)$ is the first guess. A stronger condition is that the function f is meromorphic having thus only finite numbers of poles and zeros of finite order so that a given point of CP_1^i is covered by CP_1^{i+1}. Even stronger and very natural condition is that the identification is bijection so that only Möbius transformations parametrized by SL(2, C) are possible.

3. Could the Möbius transformation $f : CP_1^1 \to CP_1^2$ depend parametrically on the coordinates z_2, z_3 so that one would have $w_1 = f_1(z_1, z_2, z_3)$, where the complex parameters a, b, c, d ($ad - bc = 1$) of Möbius transformation depend on z_2 and z_3 holomorphically? Does this mean the analog of local SL(2, C) gauge invariance posing additional conditions? Does this mean that the twistor space as surface is determined up to SL(2, C) gauge transformation?

   What conditions can one pose on the dependence of the parameters a, b, c, d of the Möbius transformation on (z_2, z_3)? The spheres CP_1 defined by the conditions $w_1 = f(z_1, z_2, z_3)$ and $z_1 = g(w_1, w_2, w_3)$ must be identical. Inverting the first condition one obtains $z_1 = f^{-1}(w_1, z_2, z_3)$. If one requires that his allows an expression as $z_1 = g(w_1, w_2, w_3)$, one must assume that z_2 and z_3 can be expressed as holomorphic functions of (w_2, w_3): $z_i = f_i(w_k)$, $i = 2, 3$, $k = 2, 3$. Of course, non-holomorphic correspondence cannot be excluded.

4. Further conditions are obtained by demanding that the known extremals - at least non-vacuum extremals - are allowed. The known extremals [K10] can be classified into CP_2 type vacuum extremals with 1-D light-like curve as M^4 projection, to vacuum extremals with CP_2 projection, which is Lagrangian sub-manifold and thus at most 2-dimensional, to massless extremals with 2-D CP_2 projection such that CP_2 coordinates depend on arbitrary manner on light-like coordinate defining local propagation direction and space-like coordinate defining a local polarization direction, and to string like objects with string world sheet as M^4 projection (minimal surface) and 2-D complex sub-manifold of CP_2 as CP_2 projection, . There are certainly also other extremals such as magnetic flux tubes resulting as deformations of string like objects. Number theoretic vision relying on classical number fields suggest a very general construction based on the notion of associativity of tangent space or co-tangent space.

5. The conditions coming from these extremals reduce to 4 conditions expressible in the holomorphic case in terms of the base space coordinates (z_2, z_3) and (w_2, w_3) and in the more general case in terms of the corresponding real coordinates. It seems that holomorphic ansatz is not consistent with the existence of vacuum extremals, which however give vanishing contribution to transition amplitudes since WCW ("world of classical worlds") metric is completely degenerate for them.

   The mere condition that one has CP_1 fiber bundle structure does not force field equations since it leaves the dependence between real coordinates of the base spaces free. Of course, CP_1 bundle structure alone does not imply twistor space structure. One can ask whether non-vacuum extremals could correspond to holomorphic constraints between (z_2, z_3) and (w_2, w_3).

6. The metric of twistor space is not Kähler in the general case. However, if it allows complex structure there is a Hermitian form ω, which defines what is called balanced Kähler form [A29]

satisfying $d(\omega \wedge \omega) = 2\omega \wedge d\omega = 0$: ordinary Kähler form satisfying $d\omega = 0$ is special case about this. The natural metric of compact 6-dimensional twistor space is therefore balanced. Clearly, mere CP_1 bundle structure is not enough for the twistor structure. If the Kähler and symplectic forms are induced from those of $CP_3 \times Y_3$, highly non-trivial conditions are obtained for the embedding of the twistor space, and one might hope that they are equivalent with those implied by Kähler action at the level of base space.

7. Pessimist could argue that field equations are additional conditions completely independent of the conditions realizing the bundle structure! One cannot exclude this possibility. Mathematician could easily answer the question about whether the proposed CP_1 bundle structure with some added conditions is enough to produce twistor space or not and whether field equations could be the additional condition and realized using the holomorphic ansatz.

2.3.2 Twistor Spaces By Adding CP_1 Fiber To Space-Time Surfaces

The physical picture behind TGD is the safest starting point in an attempt to gain some idea about what the twistor spaces look like.

1. Canonical embeddings of M^4 and CP_2 and their disjoint unions are certainly the natural starting point and correspond to canonical embeddings of CP_3 and F_3 to $CP_3 \times F_3$.

2. Deformations of M^4 correspond to space-time sheets with Minkowskian signature of the induced metric and those of CP_2 to the lines of generalized Feynman diagrams. The simplest deformations of M^4 are vacuum extremals with CP_2 projection which is Lagrangian manifold.

   Massless extremals represent non-vacuum deformations with 2-D CP_2 projection. CP_2 coordinates depend on local light-like direction defining the analog of wave vector and local polarization direction orthogonal to it.

   The simplest deformations of CP_2 are CP_2 type extremals with light-like curve as M^4 projection and have same Kähler form and metric as CP_2. These space-time regions have Euclidian signature of metric and light-like 3-surfaces separating Euclidian and Minkowskian regions define parton orbits.

   String like objects are extremals of type $X^2 \times Y^2$, X^2 minimal surface in M^4 and Y^2 a complex sub-manifold of CP_2. Magnetic flux tubes carrying monopole flux are deformations of these.

   Elementary particles are important piece of picture. They have as building bricks wormhole contacts connecting space-time sheets and the contacts carry monopole flux. This requires at least two wormhole contacts connected by flux tubes with opposite flux at the parallel sheets.

3. Space-time surfaces are constructed using as building bricks space-time sheets, in particular massless exrremals, deformed pieces of CP_2 defining lines of generalized Feynman diagrams as orbits of wormhole contacts, and magnetic flux tubes connecting the lines. Space-time surfaces have in the generic case discrete set of self intersections and it is natural to remove them by connected sum operation. Same applies to twistor spaces as sub-manifolds of $CP_3 \times F_3$ and this leads to a construction analogous to that used to remove singularities of Calabi-Yau spaces [A29].

Physical intuition suggests that it is possible to find twistor spaces associated with the basic building bricks and to lift this engineering procedure to the level of twistor space in the sense that the twistor projections of twistor spaces would give these structure. Lifting would essentially mean assigning CP_1 fiber to the space-time surfaces.

1. Twistor spaces should decompose to regions for which the metric induced from the $CP_3 \times F_3$ metric has different signature. In particular, light-like 5-surfaces should replace the light-like 3-surfaces as causal horizons. The signature of the Hermitian metric of 4-D (in complex sense) twistor space is (1, 1, -1, -1). Minkowskian variant of CP_3 is defined as projective space $SU(2,2)/SU(2,1) \times U(1)$. The causal diamond (CD) (intersection of future and past directed light-cones) is the key geometric object in ZEO (ZEO) and the generalization to the intersection of twistorial light-cones is suggestive.

2. Projective twistor space has regions of positive and negative projective norm, which are 3-D complex manifolds. It has also a 5-dimensional sub-space consisting of null twistors analogous to light-cone and has one null direction in the induced metric. This light-cone has conic singularity analogous to the tip of the light-cone of M^4.

   These conic singularities are important in the mathematical theory of Calabi-You manifolds since topology change of Calabi-Yau manifolds via the elimination of the singularity can be associated with them. The S^2 bundle character implies the structure of S^2 bundle for the base of the singularity (analogous to the base of the ordinary cone).

3. Null twistor space corresponds at the level of M^4 to the light-cone boundary (causal diamond has two light-like boundaries). What about the light-like orbits of partonic 2-surfaces whose light-likeness is due to the presence of CP_2 contribution in the induced metric? For them the determinant of induced 4-metric vanishes so that they are genuine singularities in metric sense. The deformations for the canonical embeddings of this sub-space (F_3 coordinates constant) leaving its metric degenerate should define the lifts of the light-like orbits of partonic 2-surface. The singularity in this case separates regions of different signature of induced metric.

   It would seem that if partonic 2-surface begins at the boundary of CD, conical singularity is not necessary. On the other hand the vertices of generalized Feynman diagrams are 3-surfaces at which 3-lines of generalized Feynman digram are glued together. This singularity is completely analogous to that of ordinary vertex of Feynman diagram. These singularities should correspond to gluing together 3 deformed F_3 along their ends.

4. These considerations suggest that the construction of twistor spaces is a lift of construction space-time surfaces and generalized Feynman diagrammatics should generalize to the level of twistor spaces. What is added is CP_1 fiber so that the correspondence would rather concrete.

5. For instance, elementary particles consisting of pairs of monopole throats connected buy flux tubes at the two space-time sheets involved should allow lifting to the twistor level. This means double connected sum and this double connected sum should appear also for deformations of F_3 associated with the lines of generalized Feynman diagrams. Lifts for the deformations of magnetic flux tubes to which one can assign CP_3 in turn would connect the two F_3s.

6. A natural conjecture inspired by number theoretic vision is that Minkowskian and Euclidian space-time regions correspond to associative and co-associative space-time regions. At the level of twistor space these two kinds of regions would correspond to deformations of CP_3 and F_3. The signature of the twistor norm would be different in this regions just as the signature of induced metric is different in corresponding space-time regions.

   These two regions of space-time surface should correspond to deformations for disjoint unions of CP_3s and F_3s and multiple connected sum form them should project to multiple connected sum (wormhole contacts with Euclidian signature of induced metric) for deformed CP_3s. Wormhole contacts could have deformed pieces of F_3 as counterparts.

There are interesting questions related to the detailed realization of the twistor spaces of space-time surfaces.

1. In the case of CP_2 J would naturally correspond to the Kähler form of CP_2. Could one identify J for the twistor space associated with space-time surface as the projection of J? For deformations of CP_2 type vacuum extremals the normalization of J would allow to satisfy the condition $J^2 = -g$. For general extremals this is not possible. Should one be ready to modify the notion of twistor space by allowing this?

2. Or could the associativity/co-associativity condition realized in terms of quaternionicity of the tangent or normal space of the space-time surface guaranteeing the existence of quaternion units solve the problem and J could be identified as a representation of unit quaternion? In this case J would be replaced with vielbein vector and the decomposition 1+3 of the tangent space implied by the quaternion structure allows to use 3-dimensional permutation symbol

to assign antisymmetric tensors to the vielbein vectors. Also the triviality of the tangent bundle of 3-D space allowing global choices of the 3 imaginary units could be essential.

3. Does associativity/co-associativity imply twistor space property or could it provide alternative manner to realize this notion? Or could one see quaternionic structure as an extension of almost complex structure. Instead of single J three orthogonal J: s (3 almost complex structures) are introduced and obey the multiplication table of quaternionic units? Instead of S^2 the fiber of the bundle would be $SO(3) = S^3$. This option is not attractive. A manifold with quaternionic tangent space with metric representing the real unit is known as quaternionic Riemann manifold and CP_2 with holonomy U(2) is example of it. A more restrictive condition is that all quaternion units define closed forms: one has quaternion Kähler manifold, which is Ricci flat and has in 4-D case Sp(1)=SU(2) holonomy. (see `http://tinyurl.com/y9qtoebe`).

4. Anti-self-dual property (ASD) of metric guaranteeing the integrability of almost complex structure of the twistor space implies the condition $\omega \wedge d\omega = 0$ for the twistor space. What does this condition mean physically for the twistor spaces associated with the extremals of Kähler action? For the 4-D base space this property is of course identically true. ASD property need of course not be realized.

2.3.3 Twistor Spaces As Analogs Of Calabi-Yau Spaces Of Super String Models

CP_3 is also a Calabi-Yau manifold in the strong sense that it allows Kähler structure and complex structure. Witten's twistor string theory considers 2-D (in real sense) complex surfaces in twistor space CP_3 or its Minkowskian variant. This choice does not however seem to be natural from the point of view of the induced geometry although it looks natural at the level of momentum space. It is less well-known that M^4 allows also second twistor space $T(M^4) = M^4 \times CP_1$, and this looks very natural concerning twistor lift of TGD replacing space-time surfaces in H with their twistor spaces in $T(H) = T(M^4) \times T(CP_2)$.

The original identification $T(M^4)$ with CP_3 or its Minkowskian variant has been given up bit it inspired some questions discussed in the sequel.

1. Could TGD in twistor space formulation be seen as a generalization of this theory?

2. General twistor space is not Calabi-Yau manifold because it does does not have Kähler structure. Do twistor spaces replace Calabi-Yaus in TGD framework?

3. Could twistor spaces be Calabi-Yau manifolds in some weaker sense so that one would have a closer connection with super string models.

Consider the last question.

1. One can indeed define non-Kähler Calabi-Yau manifolds by keeping the hermitian metric and giving up symplectic structure or by keeping the symplectic structure and giving up hermitian metric (almost complex structure is enough). Construction recipes for non-Kähler Calabi-Yau manifold are discussed in [A29]. It is shown that these two ways to give up Kähler structure correspond to duals under so called mirror symmetry [B7] which maps complex and symplectic structures to each other. This construction applies also to the twistor spaces.

2. For the modification giving up symplectic structure, one starts from a smooth Kähler Calabi-Yau 3-fold Y, such as CP_3. One assumes a discrete set of disjoint rational curves diffeomorphic to CP_1. In TGD framework work they would correspond to special fibers of twistor space.

   One has singularities in which some rational curves are contracted to point - in twistorial case the fiber of twistor space would contract to a point - this produces double point singularity which one can visualize as the vertex at which two cones meet (sundial should give an idea about what is involved). One deforms the singularity to a smooth complex manifold. One could interpret this as throwing away the common point and replacing it with connected sum contact: a tube connecting the holes drilled to the vertices of the two cones. In TGD one would talk about wormhole contact.

3. Suppose the topology looks locally like $S^3 \times S^2 \times R_\pm$ near the singularity, such that two copies analogous to the two halves of a cone (sundial) meet at single point defining double point singularity. In the recent case S^2 would correspond to the fiber of the twistor space. S^3 would correspond to 3-surface and $R_\pm$ would correspond to time coordinate in past/future direction. S^3 could be replaced with something else.

   The copies of $S^3 \times S^2$ contract to a point at the common end of R_+ and R_- so that both the based and fiber contracts to a point. Space-time surface would look like the pair of future and past directed light-cones meeting at their tips.

   For the first modification giving up symplectic structure only the fiber S^2 is contracted to a point and $S^2 \times D$ is therefore replaced with the smooth "bottom" of S^3. Instead of sundial one has two balls touching. Drill small holes two the two S^3s and connect them by connected sum contact (wormhole contact). Locally one obtains $S^3 \times S^3$ with k connected sum contacts.

   For the modification giving up Hermitian structure one contracts only S^3 to a point instead of S^2. In this case one has locally two CP_3: s touching (one can think that CP_n is obtained by replacing the points of C^n at infinity with the sphere CP_1). Again one drills holes and connects them by a connected sum contact to get k-connected sum of CP_3.

   For k CP_1s the outcome looks locally like to a k-connected sum of $S^3 \times S^3$ or CP_3 with $k \geq 2$. In the first case one loses symplectic structure and in the second case hermitian structure. The conjecture is that the two manifolds form a mirror pair.

   The general conjecture is that all Calabi-Yau manifolds are obtained using these two modifications. One can ask whether this conjecture could apply also the construction of twistor spaces representable as surfaces in $CP_3 \times F_3$ so that it would give mirror pairs of twistor spaces.

4. This smoothing out procedures isa actually unavoidable in TGD because twistor space is sub-manifold. The 6-D twistor spaces in 12-D $T(M^4) \times F_3$ have in the generic case self intersections consisting of discrete points. Since the fibers CP_1 cannot intersect and since the intersection is point, it seems that the fibers must contract to a point. In the similar manner the 4-D base spaces should have local foliation by spheres or some other 3-D objects with contract to a point. One has just the situation described above.

   One can remove these singularities by drilling small holes around the shared point at the two sheets of the twistor space and connected the resulting boundaries by connected sum contact. The preservation of fiber structure might force to perform the process in such a way that local modification of the topology contracts either the 3-D base (S^3 in previous example or fiber CP_1 to a point.

The interpretation of twistor spaces is of course totally different from the interpretation of Calabi-Yaus in superstring models. The landscape problem of superstring models is avoided and the multiverse of string models is replaced with generalized Feynman diagrams! Different twistor spaces correspond to different space-time surfaces and one can interpret them in terms of generalized Feynman diagrams since bundle projection gives the space-time picture. Mirror symmetry means that there are two different Calabi-Yaus giving the same physics. Also now twistor space for a given space-time surface can have several embeddings - perhaps mirror pairs define this kind of embeddings.

To sum up, the construction of space-times as surfaces of H lifted to those of (almost) complex sub-manifolds in $T(M^4) times F_3$ with induced twistor structure shares the spirit of the vision that induction procedure is the key element of classical and quantum TGD. It also gives deep connection with the mathematical methods applied in super string models and these methods should be of direct use in TGD.

2.4 Witten's Twistor String Approach And TGD

The twistor Grassmann approach has led to a phenomenal progress in the understanding of the scattering amplitudes of gauge theories, in particular the $\mathcal{N} = 4$ SUSY.

As a non-specialist I have been frustrated about the lack of concrete picture, which would help to see how twistorial amplitudes might generalize to TGD framework. A pleasant surprise in this respect was the proposal of a particle interpretation for the twistor amplitudes by Nima Arkani Hamed *et al* in the article "Unification of Residues and Grassmannian Dualities" [B23] (see `http://tinyurl.com/y86mad5n`)

In this interpretation incoming particles correspond to spheres CP_1 so that n-particle state corresponds to $(CP_1)^n/Gl(2)$ (the modding by $Gl(2)$ might be seen as a kind of formal generalization of particle identity by replacing permutation group S_2 with $Gl(2)$ of 2×2 matrices). If the number of "wrong" helicities in twistor diagram is k, this space is imbedded to $CP_{k-1}^n/Gl(k)$ as a surface having degree $k-1$ using Veronese map to achieve the embedding. The embedding space can be identified as Grassmannian $G(k,n)$. This surface defines the locus of the multiple residue integral defining the twistorial amplitude.

The particle interpretation brings in mind the extension of single particle configuration space E^3 to its Cartesian power E^{3n}/S_n for n-particle system in wave mechanics. This description could make sense when point-like particle is replaced with 3-surface or partonic 2-surface: one would have Cartesian product of WCWs divided my S_n. The generalization might be an excellent idea as far calculations are considered but is not in spirit with the very idea of string models and TGD that many-particle states correspond to unions of 3-surfaces in H (or light-like boundaries of causal diamond (CD) in Zero Energy Ontology (ZEO).

Witten's twistor string theory [B16] is more in spirit with TGD at fundamental level since it is based on the identification of generalization of vertices as 2-surfaces in twistor space.

1. There are several kinds of twistors involved. For massless external particles in eigenstates of momentum and helicity null twistors code the momentum and helicity and are pairs of 2-spinor and its conjugate. More general momenta correspond to two independent 2-spinors.

   One can perform twistor Fourier transform for the conjugate 2-spinor to obtain twistors coding for the points of compactified Minkowski space. Wave functions in this twistor space characterized by massless momentum and helicity appear in the construction of twistor amplitudes. BCFW recursion relation [B9] allows to construct more complex amplitudes assuming that intermediate states are on mass shells massless states with complex momenta.

   One can perform twistor Fourier transformation (there are some technical problems in Minkowski signature) also for the second 2-spinor to get what are called momentum twistors providing in some aspects simpler description of twistor amplitudes. These code for the four-momenta propagating between vertices at which the incoming particles arrive and the differences if two subsequent momenta are equal to massless external momenta.

2. In Witten's theory the interactions of incoming particles correspond to amplitudes in which the twistors appearing as arguments of the twistor space wave functions characterized by momentum and helicity are localized to complex curves X^2 of twistor space CP_3 or its Minkowskian counterpart. This can be seen as a non-local twistor space variant of local interactions in Minkowski space.

   The surfaces X^2 are characterized by their degree d (of the polynomial of complex coordinates defining the algebraic 2-surface) the genus g of the algebraic surface, by the number k of "wrong" (helicity violating) helicities, and by the number of loops of corresponding diagram of SUSY amplitude: one has $d = k - 1 + l$, $g \leq l$. The interaction vertex in twistor space is not anymore completely local but the n particles are at points of the twistorial surface X^2.

In the following a proposal generalizing Witten's approach to TGD is discussed.

1. The fundamental challenge is the generalization of the notion of twistor associated with massless particle to 8-D context, first for $M^4 = M^4 \times E^4$ and then for $H = M^4 \times CP_2$. The notion of twistor space solves this question at geometric level. As far as construction of the TGD variant of Witten's twistor string is considered, this might be quite enough.

2. $M^8 - H$ duality and quantum-classical correspondence however suggest that M^8 twistors might allow tangent space description of four-momentum, spin, color quantum numbers and electroweak numbers and that this is needed. What comes in mind is the identification of fermion lines as light-like geodesics possessing M^8 valued 8-momentum, which would define the long sought gravitational counterparts of four-momentum and color quantum numbers at classical point-particle level. The M^8 part of this four-momentum would be equal to that associated with embedding space spinor mode characterizing the ground state of super-conformal representation for fundamental fermion.

   Hence one might also think of starting from the 4-D condition relating Minkowski coordinates to twistors and looking what it could mean in the case of M^8. The generalization is indeed possible in $M^8 = M^4 \times E^4$ by its flatness if one replaces gamma matrices with octonionic gamma matrices.

   In the case of $M^4 \times CP_2$ situation is different since for octonionic gamma matrices $SO(1,7)$ is replaced with G_2, and the induced gauge fields have different holonomy structure than for ordinary gamma matrices and octonionic sigma matrides appearing as charge matrices bring in also an additional source of non-associativity. Certainly the notion of the twistor Fourier transform fails since CP_2 Dirac operator cannot be algebraized.

   Algebraic twistorialization however works for the light-like fermion lines at which the ordinary and octonionic representations for the induced Dirac operator are equivalent. One can indeed assign 8-D counterpart of twistor to the particle classically as a representation of light-like hyper-octonionic four-momentum having massive M^4 and CP_2 projections and CP_2 part perhaps having interpretation in terms of classical tangent space representation for color and electroweak quantum numbers at fermionic lines.

   If all induced electroweak gauge fields - rather than only charged ones as assumed hitherto - vanish at string world sheets, the octonionic representation is equivalent with the ordinary one. The CP_2 projection of string world sheet should be 1-dimensional: inside CP_2 type vacuum extremals this is impossible, and one could even consider the possibility that the projection corresponds to CP_2 geodesic circle. This would be enormous technical simplification. What is important that this would not prevent obtaining non-trivial scattering amplitudes at elementary particle level since vertices would correspond to re-arrangement of fermion lines between the generalized lines of Feynman diagram meeting at the vertices (partonic 2-surfaces).

3. In the fermionic sector one is forced to reconsider the notion of the induced spinor field. The modes of the embedding space spinor field should coincide in some region of the space-time surface with those of the induced spinor fields. The light-like fermionic lines defined by the boundaries of string world sheets or their ends are the obvious candidates in this respect. String world sheets is perhaps too much to require.

   The only reasonable identification of string world sheet gamma matrices is as induced gamma matrices and super-conformal symmetry requires that the action contains string world sheet area as an additional term just as in string models. String tension would correspond to gravitational constant and its value - that is ratio to the CP_2 radius squared, would be fixed by quantum criticality.

4. The generalization of the Witten's geometric construction of scattering amplitudes relying on the induction of the twistor structure of the embedding space to that associated with space-time surface looks surprisingly straight-forward and would provide more precise formulation of the notion of generalized Feynman diagrams forcing to correct some wrong details. One of the nice outcomes is that the genus appearing in Witten's formulation naturally corresponds to family replication in TGD framework.

2.4.1 Basic Ideas About Twistorialization Of TGD

The recent advances in understanding of TGD motive the attempt to look again for how twistor amplitudes could be realized in TGD framework. There have been several highly non-trivial steps of progress leading to a new more profound understanding of basic TGD.

1. $M^4 \times CP_2$ is twistorially unique [L3] in the sense that its factors are the only 4-D geometries allowing twistor space with Kähler structure (M^4 corresponds to S^4 in Euclidian signature) [A18]. The twistor spaces in question are CP_3 for S^4 and its Minkowskian variant for M^4 (I will use P^3 as short hand for both twistor spaces) and the flag manifold $F = SU(3)/U(1) \times U(1)$ parametrizing the choices of quantization axes for color group $SU(3)$ in the case of CP_2. Recall that twistor spaces are S^2 bundles over the base space and that all orientable four-manifolds have twistor space in this sense. Note that space-time surfaces allow always almost quaternionic structure and that preferred extremals are suggested to be quaternionic [L3].

2. The light-likeness condition for twistors in M^4 is fundamental in the ordinary twistor approach. In 8-D context light-likeness holds in generalized sense for the spinor harmonics of H: the square of the Dirac operator annihilates spinor modes. In the case M^8 one can indeed define twistors by generalizing the standard approach by replacing ordinary gamma matrices with octonionic ones [?] For H octonionic and ordinary gamma matrices are equivalent at the fermionic lines defined by the light-like boundaries of string world sheets and at string world sheets if they carry vanishing induced electro-weak gauge fields that is have 1-D CP_2 projection.

3. Twistor spaces emerge in TGD framework as lifts of space-time surfaces to corresponding twistor spaces realized as 6-surfaces in the lift of $M^4 \times CP_2$ to $T(H) = P^3 \times F$ having as base spaces space-time surfaces. This implies that that generalized Feynman diagrams and also generalized twistor diagrams can be lifted to diagrams in T and that the construction of twistor spaces as surfaces of T has very concrete particle interpretation.

   The modes of the embedding space spinor field defining ground states of the extended conformal algebras for which classical conformal charges vanish at the ends of the space-time surface (this defines gauge conditions realizing strong form of holography [K86]) are lifted to the products of modes of spinor fields in $T(H)$ characterized by four-momentum and helicity in M^4 degrees of freedom and by color quantum numbers and electroweak quantum numbers in F degrees of freedom. Thus twistorialization provides a purely geometric representation of spin and electro-weak spin and it seems that twistorialization allows to a formulation without H-spinors.

   What is especially nice, that twistorialization extends to from spin to include also electroweak spin. These two spins correspond correspond to M^4 and CP_2 helicities for the twistor space amplitude, and are non-local properties of these amplitudes. In TGD framework only twistor amplitudes for which helicities correspond to that for massless fermion and antifermion are possible and by fermion number conservation the numbers of positive and negative helicities are identical and equal to the fermion number (or antifermion number). Separate lepton and baryon number conservation realizing 8-D chiral symmetry implies that M^4 and CP_2 helicities are completely correlated.

   For massless fermions in M^4 sense helicity is opposite for fermion and antifermion and conserved. The contributions of initial and final states to k are same and equal to $k_i = k_f = 2(n(F) - n(\overline{F})$. This means restriction to amplitudes with $k = 2(n(F) - n(\overline{F})$. If fermions are massless only in M^8 sense, chirality mixing occurs and this rule does not hold anymore. This holds true in quark and lepton sector separately.

4. All generalized Feynman graphs defined in terms of Euclidian regions of space-time surface are lifted to twistor spaces [K18]. Incoming particles correspond quantum mechanically to twistor space amplitudes defined by their momenta and helicities and and classically to the entire twistor space of space-time surface as a surface in the twistor space of H. Of course, also the Minkowskian regions have this lift. The vertices of Feynman diagrams correspond to regions of twistor space in which the incoming twistor spaces meet along their 5-D ends having also S^2 bundle structure over space-like 3-surfaces. These space-like 3-surfaces correspond to ends of Euclidian and Minkowskian space-time regions separated from each other by light-like 3-surfaces at which the signature of the metric changes from Minkowskian to Euclidian. These "partonic orbits" have as their ends 2-D partonic surfaces. By strong form of General Coordinate Invariance implying strong of holography, these 2-D partonic surfaces and their

4-D tangent space data should code for quantum physics. Their lifts to twistor space are 4-D S^2 bundles having partonic 2-surface X^2 as base.

5. The well-definedness of em charge for the spinor modes demands that they are localized at 2-D string world sheets [K86] and also these world sheets are lifted to sub-spaces of twistor space of space-time surface. If one demands that octonionic Dirac operator makes sense at string world sheets, they must carry vanishing induced electro-weak gauge fields and string world sheets could be minimal surfaces in $M^4 \times S^1$, $S^1 \subset CP_2$ a geodesic circle.

   The boundaries of string world sheets at partonic orbits define light-like curves identifiable as carriers of fermion number and they define the analogs of lines of Feynman diagrams in ordinary sense. The only purely fermionic vertices are 2-fermion vertices at the partonic 2-surfaces at which the end of space-time surface meet. As already explained, the string world sheets can be seen as correlates for the correlations between fermion vertices at different wormhole throats giving rise to the counterpart of bosonic propagator in quantum field theories.

   The localization of spinor fields to 2-D string world sheets corresponds to the localization of twistor amplitudes to their 4-D lifts, which are S^2 bundles and the boundaries of string world sheets are lifted to 3-D twistor lifts of fermion lines. Clearly, the localization of spinors to string world sheets would be absolutely essential for the emergence of twistor description.

6. All elementary particles are many particle bound states of massless fundamental fermions: the non-collinearity (and possible complex character) of massless momenta explains massivation. The fundamental fermions are localized at wormhole throats defining the light-like orbits of partonic 2-surfaces. Throats are associated with wormhole contacts connecting two space-time sheets. Stability of the contact is guaranteed by non-vanishing monopole magnetic flux through it and this requires the presence of second wormhole contact so that a closed magnetic flux tube carrying monopole flux and involving the two space-time sheets is formed. The net fermionic quantum numbers of the second throat correspond to particle's quantum numbers and above weak scale the weak isospins of the throats sum up to zero.

7. Fermionic 2-vertex is the only *local* many-fermion vertex [K18] being analogous to a mass insertion. The non-triviality of fundamental 4-fermion vertex is due to classical interactions between fermions at opposite throats of worm-hole. The non-triviality of the theory involves also the analog of OZI mechanism: the fermionic lines inside partonic orbits are redistributed in vertices. Lines can also turn around in time direction which corresponds to creation or annihilation of a pair. 3-particle vertices are obtained only in topological sense as 3 space-time surfaces are glued together at their ends. The interaction between fermions at different wormhole throats is described in terms of string world sheets.

8. The earlier proposal was that the fermions in the internal fermion lines are massless in M^4 sense but have non-physical helicity so that the algebraic M^4 Dirac operator emerging from the residue integration over internal four-momentum does not annihilate the state at the end of the propagator line. Now the algebraic induced Dirac operator defines the propagator at fermion lines. Should one assume generalization of non-physical helicity also now?

9. All this stuff must be lifted to twistorial level and one expects that the lift to S^2 bundle allows an alternative description of fermions and spinor structure so that one can speak of induced twistor structure instead of induced spinor structure. This approach allows also a realization of M^4 conformal symmetries in terms of globally well-defined linear transformations so that it might be that twistorialization is not a mere reformulation but provides a profound unification of bosonic and fermionic degrees of freedom.

2.4.2 The Emergence Of The Fundamental 4-Fermion Vertex And Of Boson Exchanges

The emergence of the fundamental 4-fermion vertex and of boson exchanges deserves a more detailed discussion.

1. I have proposed that the discontinuity of the Dirac operator at partonic two-surface (corner of fermion line) defines both the fermion boson vertex and TGD analog of mass insertion (not scalar but embedding space vector) giving rise to mass parameter having interpretation as Higgs vacuum expectation and various fermionic mixing parameters at QFT limit of TGD obtained by approximating many-sheeted space-time of TGD with the single sheeted region of M^4 such that gravitational field and gauge potentials are obtained as sums of those associated with the sheets.

2. Non-trivial scattering requires also correlations between fermions at different partonic 2-surfaces. Both partonic 2-surfaces and string world sheets are needed to describe these correlations. Therefore the string world sheets and partonic 2-surfaces cannot be dual: both are needed and this means deviation from Witten's theory. Fermion vertex corresponds to a "corner" of a fermion line at partonic 2-surface at which generalized 4-D lines of Feynman diagram meet and light-like fermion line changes to space-like one. String world sheet with its corners at partonic 2-surfaces (wormhole throats) describes the momentum exchange between fermions. The space-like string curve connecting two wormhole throats serves as the analog of the exchanged gauge boson.

3. Two kinds of 4-fermion amplitudes can be considered depending on whether the string connects throats of single wormhole contact (CP_2 scale) or of two wormhole contacts (p-adic length scale - typically of order elementary particle Compton length). If string worlds sheets have 1-D CP_2 projection, only Minkowskian string world sheets are possible. The exchange in Compton scale should be assignable to the TGD counterpart of gauge boson exchange and the fundamental 4-fermion amplitude should correspond to single wormhole contact: string need not to be involved now. Interaction is basically classical interaction assignable to single wormhole contact generalizing the point like vertex.

4. The possible TGD counterparts of BCFW recursion relations [B9] should use the twistorial representations of fundamental 4-fermion scattering amplitude as seeds. Yangian invariance poses very strong conditions on the form of these amplitudes and the exchange of massless bosons is suggestive for the general form of amplitude.

   The 4-fermion amplitude assignable to two wormhole throats defines the analog of gauge boson exchange and is expressible as fusion of two fundamental 4-fermion amplitudes such that the 8-momenta assignable to the fermion and anti-fermion at the opposite throats of exchanged wormhole contact are complex by BCFW shift acting on them to make the exchanged momenta massless but complex. This entity could be called fundamental boson (not elementary particle).

5. Can one assume that the fundamental 4-fermion amplitude allows a purely formal composition to a product of $F\overline{F}B_v$ amplitudes, B_v a purely fictive boson? Two 8-momenta at both $F\overline{F}B_v$ vertices must be complex so that at least two external fermionic momenta must be complex. These external momenta are naturally associated with the throats of the a wormhole contact defining virtual fundamental boson. Rather remarkably, without the assumption about product representation one would have general four-fermion vertex rather than boson exchange. Hence gauge theory structure is not put in by hand but emerges.

2.4.3 What About SUSY In TGD?

Extended super-conformal symmetry is crucial for TGD and extends to quaternion-super-conformal symmetry giving excellent hopes about calculability of the theory. $\mathcal{N} = 4$ space-time supersymmetry is in the key role in the approach of Witten and others.

In TGD framework space-time SUSY could be present as an approximate symmetry.

1. The many fermion states at partonic surfaces are created by oscillator operators of fermionic Clifford algebra having also interpretation as a supersymmetric algebra but in principle having $\mathcal{N} = \infty$. This SUSY is broken since the generators of SUSY carry four-momentum.

2. More concrete picture would be that various SUSY multiplets correspond to collinear many-fermion states at the same wormhole throat. By fermionic statistics only the collinear states

for which internal quantum numbers are different are realized: other states should have antisymmetric wave function in spatial degrees of freedom implying wiggling in CP_2 scale so that the mass of the state would be very high. In both quark and lepton sectors one would have $\mathcal{N} = 4$ SUSY so that one would have the analog $\mathcal{N} = \forall$ SUSY (color is not spin-like quantum number in TGD).

At the level of diagrammatics single line would be replaced with "line bundle" representing the fermions making the many-fermion state at the light-like orbit of the partonic 2-surface. The fusion of neighboring collinear multifermion stats in the twistor diagrams could correspond to the process in which partonic 2-surfaces and single and many-fermion states fuse.

3. Right handed neutrino modes, which are not covariantly constant, are also localized at the fermionic lines and defines the least broken $\mathcal{N} = 2$ SUSY. The covariantly constant mode seems to be a pure gauge degree of freedom since it carriers no quantum numbers and the SUSY norm associated with it vanishes. The breaking would be smallest for $\mathcal{N} = 2$ variant assignable to right-handed neutrino having no weak and color interactions with other particles but whose mixing with left-handed neutrino already induces SUSY breaking.

Why this SUSY has not been observed? One can consider two scenarios [K68].

1. The first scenario relies on the absence of weak and color interactions: one can argue that the bound states of fermions with right-handed neutrino are highly unstable since only gravitational interaction so that sparticle decays very rapidly to particle and right-handed or left-handed neutrino. By Uncertainty Principle this makes sparticle very massive, maybe having mass of order CP_2 mass. Neutrino mixing caused by the mixing of M^4 and CP_2 gamma matrices in induced gamma matrices is the weak point of this argument.

2. The mixing of left and right-handed neutrinos could be characterized by the p-adic mass scale of neutrinos and be long. Sparticles would have same p-adic mass scale as particles and would be dark having non-standard value of Planck constant $h_{eff} = n \times h$: this would scale up the lifetime by factor n and correlate with breaking of conformal symmetry assignable to the mixing [K68].

What implications the approximate SUSY would have for scattering amplitudes?

1. $k = 2(n(F) - n(\overline{F})$ condition reduces the number of amplitudes dramatically if the fermions are massless in M^4 sense but the presence of weak iso-spin implies that the number of amplitudes is 2^n as in non-supersymmetric gauge theories. One however expects broken SUSY with generators consisting of fermionic oscillator operators at partonic 2-surfaces with symmetry breaking taking place only at the level of physical particles identifiable as many particle bound states of massless (in 8-D sense) particles. This motivates the guess that the formal $F\overline{F}B_v$ amplitudes defining fundamental 4-fermion vertex are expressible as those associated with $\mathcal{N} = 4$ SUSY in quark and lepton sectors respectively. This would reduce the number of independent amplitudes to just one.

2. Since SUSY and its breaking emerge automatically in TGD framework, super-space can provide a useful technical tool but is not fundamental.

Side note: The number of external fermions is always even suggesting that the super-conformal anomalies plaguing the amplitudes with odd n (`http://tinyurl.com/yb85tnvc`) [B38] are absent.

2.4.4 What Does One Really Mean With The Induction Of Embedding Space Spinors?

The induction of spinor structure is a central notion of TGD but its detailed definition has remained somewhat obscure. The attempt to generalize Witten's approach has made it clear that the mere restriction of spinor fields to space-time surfaces is not enough and that one must understand in detail the correspondence between the modes of embedding space spinor fields and those of induced spinor fields.

Even the identification of space-time gamma matrices is far from obvious at string world sheets.

1. The simplest notion of the space-time gamma matrices is as projections of embedding space gamma matrices to the space-time surface - induced gamma matrices. If one assumes that induced spinor fields are defined at the entire space-time surfaces, this notion fails to be consistent with fermionic super-conformal symmetry unless one replaces Kähler action by space-time volume. This option is certainly unphysical.

2. The notion of Kähler-Dirac matrices in the interior of space as gamma matrices defined as contractions of canonical momentum densities of Kähler with embedding space gamma matrices allows to have conformal super-symmetry with fermionic super charges assignable to the modes of the induced spinor field. Also Chern-Simons action could define gamma matrices in the same manner at the light-like 3-surfaces between Minkowskian and Euclidian space-time regions and at space-like 3-surfaces at the ends of space-time surface. Chern-Simons-Dirac matrices would involve only CP_2 gamma matrices.

It is however not quite clear whether the spinor fields in the interior of space-time surface are needed at all in the twistorial approach and they seem to be only an un-necessary complication. For instance, their modes would have well-defined electromagnetic charge only when induced W gauge fields vanish, which implies that CP_2 projection is 2-dimensional. This forces to consider very seriously the possibility that induced spinor fields reside at string world sheets only (their ends are at partonic 2-surfaces). This option supported also by strong form of holography and number theoretic universality.

What about the space-time gamma matrices at string world sheets and their boundaries?

1. The first option would be reduction of Kähler-Dirac gamma matrices by requiring that they are parallel to the string world sheets. This however poses additional conditions besides the vanishing of W fields already restricting the dimension to two in the generic case. The conditions state that the embedding space 1-forms defined by the canonical momentum densities of Kähler action involve only 2 linearly independent ones and that they are proportional to embedding space coordinate gradients: this gives Frobenius conditions. These conditions look first too strong but one can also think that one fixes first string world sheets, partonic 2-surfaces, and perhaps also their light-like orbits, requires that the normal components of canonical momentum currents at string world sheets vanish, and deduces space-time surface from this data. This would be nothing but strong form of holography!

   For this option the string world sheets could emerge in the sense that it would be possible to express Kähler action as an area of string world sheet in the effective metric defined by the anticommutator of K-D gamma matrices appearing also in the expressions for the matrix elements of WCW metric. Gravitational constant would be a prediction of the theory.

2. Second possibility is to use induced gamma matrices automatically parallel to the string world sheet so that no additional conditions would result. This would also conform with the ordinary view about string world sheets and spinors.

   Supersymmetry would require the addition of the area of string world sheet to the action defining Kähler function in Euclidian regions and its counterpart in Minkowskian regions. This would bring in gravitational constant, which otherwise remains a prediction. Quantum criticality could fix the ratio of $\hbar G/R^2$ (R is CP_2 radius). The vanishing of induced weak gauge fields requires that string world sheets have 1-D CP_2 projection and are thus restricted to Minkowskian regions with at most 3-D CP_2 projection. Even stronger condition would be that string world sheets are minimal surfaces in $M^4 \times S^1$, S^1 a geodesic sphere of CP_2.

   There are however grave objections. The presence of a dimensional parameter G as fundamental coupling parameter does not encourage hopes about the renomalizibility of the theory. The idea that strings connecting partonic 2-surfaces gives rise to the formation of gravitationally bound states is suggested by AdS/CFT correspondence. The problem is that the string tension defined by gravitational constant is so large that only Planck length sized bound states are feasible. Even the replacement $\hbar \rightarrow \hbar_{eff}$ fails to allow gravitationally bound states with length scale of order Schwartschild radius. For the K-D option the string tension behaves like $1/\hbar^2$ and there are no problems in this respect.

At this moment my feeling is that the first option - essentially the original view - is the correct one. The short belief that the second option is the correct choice was a sidetrack, which however helped to become convinced that the original vision is indeed correct, and to understand the general mechanism for the formation of bound states in terms of strings connection partonic 2-surfaces (in the earlier picture I talked about magnetic flux tubes carrying monopole flux: the views are equivalent).

Both options have the following nice features.

1. Induced gammas at the light-like string boundaries would be light-like. Massless Dirac equation would assign to spinors at these lines a light-like space-time four-momentum and twistorialize it. This four-momentum would be essentially the tangent vector of the light-like curve and would not have a constant direction. Light-likeness for it means light-likeness in 8-D sense since light-like curves in H correspond to non-like momenta in M^4. Both M^4 mass squared and CP_2 mass would be conserved. Even four-momentum could be conserved if M^4 projection of stringy curve is geodesic line of M^4.

2. A new connection with Equivalence Principle (EP) would emerge. One could interpret the induced four-momentum as gravitational four-momentum which would be massless in 4-D sense and correspond to inertial 8-momentum. EP wold state in the weakest form that only the M^4 masses associated with the two momenta are identical. Stronger condition would be that the Minkowski parts of the two momenta coincide at the ends of fermion lines at partonic 2-surfaces. Even stronger condition is that the 8-momentum is 8-momentum is conserved along fermion line. This is certainly consistent with the ordinary view about Feynman graphs. This is guaranteed if the light-like curve is light-like geodesic of embedding space.

The induction of spinor fields has also remained somewhat imprecise notion. It how seems that quantum-classical correspondence forces a unique picture.

1. Does the induced spinor field coincide with embedding space spinor harmonic in some domain? This domain would certainly include the ends of fermionic lines at partonic 2-surfaces associated with the incoming particles and vertices. Could it include also the boundaries of string world sheets and perhaps also the string world sheets? The Kähler-Dirac equation certainly does not allow this for entire space-time surface.

2. Strong form of holography suggest that the light-like momenta for the Dirac equation at the ends of light-like string boundaries has interpretation as 8-D light-like momentum has M^4 projection equal to that of H spinor-harmonic. The mass squared of M^4 momentum would be same as the CP_2 momentum squared in both senses. Unless the gravitational four-momentum assignable to the induced Dirac operato r is conserved one cannot pose stronger condition.

3. If the induced spinor mode equals to embedding space-spinor mode also at fermion line, the light like momentum is conserved. The fermion line would be also light-like geodesic of the embedding space so that twistor polygons would have very concrete interpretation. This condition would be clearly analogous to the conditions in Witten's twistor string theory. A stronger condition would be that the mode of the embedding space spinor field coincides with induced spinor field at the string world sheet.

4. p-Adic mass calculations require that the massive excitations of embedding space spinor fields with CP_2 mass scale are involved. The thermodynamics could be for fermion line, wormhole throat carrying possible several fermions, or wormhole contact carrying fermion at both throats. In the case of fermions physical intuition suggests that p-adic thermodynamics must be associated with single fermionic line. The massive excitations would correspond to light-like geodesics of the embedding space.

To minimize confusion I must confess that until recently I have considered a different options for the momenta associated with fermionic lines.

1. The action of the Kähler-Dirac operator on the induced spinor field at the fermionic line equals to that of 4-D Dirac operator $p^k\gamma_k$ for a massless momentum identified as M^4 momentum [K18].

   Now the action reduces to that of 8-D massless algebraic Dirac operator for light-like string boundaries in the case of induced gamma matrices. Explicit calculation shows that in case of K-D gamma matrices and for light-like string boundaries it can happen that the 8-momentum of the mode can be tachyonic. Intriguingly, p-adic mass calculations require a tachyonic ground state?

2. For this option the helicities for virtual fermions were assumed to be non-physical in order to get non-vanishing fermion lines by residue integration: momentum integration for Dirac operator would replace Dirac propagators with Dirac operators. This would be the counterpart for the disappearance of bosonic propagators in residue integration.

3. This option has problems: quantum classical correspondence is not realized satisfactorily and the interpretation of p-adic thermodynamics is problematic.

2.4.5 About The Twistorial Description Of Light-Likeness In 8-D Sense Using Octonionic Spinors

The twistor approach to TGD [L3] require that the expression of light-likeness of M^4 momenta in terms of twistors generalizes to 8-D case. The light-likeness condition for twistors states that the 2×2 matrix representing M^4 momentum annihilates a 2-spinor defining the second half of the twistor. The determinant of the matrix reduces to momentum squared and its vanishing implies the light-likeness. This should be generalized to a situation in one has M^4 and CP_2 twistor which are not light-like separately but light-likeness in 8-D sense holds true.

The case of $M^8 = M^4 \times E^4$

$M^8 - H$ duality [K74] suggests that it might be useful to consider first the twistorialiation of 8-D light-likeness first the simpler case of M^8 for which CP_2 corresponds to E^4. It turns out that octonionic representation of gamma matrices provide the most promising formulation.

In order to obtain quadratic dispersion relation, one must have 2×2 matrix unless the determinant for the 4×4 matrix reduces to the square of the generalized light-likeness condition.

1. The first approach relies on the observation that the 2×2 matrices characterizing four-momenta can be regarded as hyper-quaternions with imaginary units multiplied by a commuting imaginary unit. Why not identify space-like sigma matrices with hyper-octonion units?

2. The square of hyper-octonionic norm would be defined as the determinant of 4×4 matrix and reduce to the square of hyper-octonionic momentum. The light-likeness for pairs formed by M^4 and E^4 momenta would make sense.

3. One can generalize the sigma matrices representing hyper-quaternion units so that they become the 8 hyper-octonion units. Hyper-octonionic representation of gamma matrices exists ($\gamma_0 = \sigma_z \times 1$, $\gamma_k = \sigma_y \times I_k$) but the octonionic sigma matrices represented by octonions span the Lie algebra of G_2 rather than that of $SO(1,7)$. This dramatically modifies the physical picture and brings in also an additional source of non-associativity. Fortunately, the flatness of M^8 saves the situation.

4. One obtains the square of $p^2 = 0$ condition from the massless octonionic Dirac equation as vanishing of the determinant much like in the 4-D case. Since the spinor connection is flat for M^8 the hyper-octonionic generalization indeed works.

This is not the only possibility that I have by-passingly considered [K18].

1. Is it enough to allow the four-momentum to be complex? One would still have 2×2 matrix and vanishing of complex momentum squared meaning that the squares of real and imaginary parts are same (light-likeness in 8-D sense) and that real and imaginary parts are orthogonal to each other. Could E^4 momentum correspond to the imaginary part of four-momentum?

2. The signature causes the first problem: M^8 must be replaced with complexified Minkowski space M^4_c for to make sense but this is not an attractive idea although M^4_c appears as subspace of complexified octonions.

3. For the extremals of Kähler action Euclidian regions (wormhole contacts identifiable as deformations of CP_2 type vacuum extremals) give imaginary contribution to the four-momentum. Massless complex momenta and also color quantum numbers appear also in the standard twistor approach. Also this suggest that complexification occurs also in 8-D situation and is not the solution of the problem.

The case of $M^8 = M^4 \times CP_2$

What about twistorialization in the case of $M^4 \times CP_2$? The introduction of wave functions in the twistor space of CP_2 seems to be enough to generalize Witten's construction to TGD framework and that algebraic variant of twistors might be needed only to realize quantum classical correspondence. It should correspond to tangent space counterpart of the induced twistor structure of space-time surface, which should reduce effectively to 4-D one by quaternionicity of the space-time surface.

1. For $H = M^4 \times CP_2$ the spinor connection of CP_2 is not trivial and the G_2 sigma matrices are proportional to M^4 sigma matrices and act in the normal space of CP_2 and to M^4 parts of octonionic embedding space spinors, which brings in mind co-associativity. The octonionic charge matrices are also an additional potential source of non-associativity even when one has associativity for gamma matrices.

   Therefore the octonionic representation of gamma matrices in entire H cannot be physical. It is however equivalent with ordinary one at the boundaries of string world sheets, where induced gauge fields vanish. Gauge potentials are in general non-vanishing but can be gauge transformed away. Here one must be of course cautious since it can happen that gauge fields vanish but gauge potentials cannot be gauge transformed to zero globally: topological quantum field theories represent basic example of this.

2. Clearly, the vanishing of the induced gauge fields is needed to obtain equivalence with ordinary induced Dirac equation. Therefore also string world sheets in Minkowskian regions should have 1-D CP_2 projection rather than only having vanishing W fields if one requires that octonionic representation is equivalent with the ordinary one. For CP_2 type vacuum extremals electroweak charge matrices correspond to quaternions, and one might hope that one can avoid problems due to non-associativity in the octonionic Dirac equation. Unless this is the case, one must assume that string world sheets are restricted to Minkowskian regions. Also embedding space spinors can be regarded as octonionic (possibly quaternionic or co-quaternionic at space-time surfaces): this might force vanishing 1-D CP_2 projection.

   (a) Induced spinor fields would be localized at 2-surfaces at which they have no interaction with weak gauge fields: of course, also this is an interaction albeit very implicit one! This would not prevent the construction of non-trivial electroweak scattering amplitudes since boson emission vertices are essentially due to re-groupings of fermions and based on topology change.

   (b) One could even consider the possibility that the projection of string world sheet to CP_2 corresponds to CP_2 geodesic circle so that one could assign light-like 8-momentum to entire string world sheet, which would be minimal surface in $M^4 \times S^1$. This would mean enormous technical simplification in the structure of the theory. Whether the spinor harmonics of embedding space with well-defined M^4 and color quantum numbers can coincide with the solutions of the induced Dirac operator at string world sheets defined by minimal surfaces remains an open problem.

(c) String world sheets cannot be present inside wormhole contacts which have 4-D CP_2 projection so that string world sheets cannot carry vanishing induced gauge fields.

2.4.6 How To Generalize Witten's Twistor String Theory To TGD Framework?

The challenge is to lift the geometric description of particle like aspects of twistorial amplitudes involving only algebraic curves (2-surfaces) in twistor space to TGD framework.

1. External particles correspond to the lifts of H-spinor harmonics to spinor harmonics in the twistor space and are labeled by four-momentum, helicity, color, and weak helicity (isospin) so that there should be no need to included fermions explicitly. The twistorial wave functions would be superpositions of the eigenstates of helicity operator which would become a non-local property in twistor space. Light-likeness would hold true in 8-D sense for spinor harmonics as well as for the corresponding twistorial harmonics.

2. The surfaces X^2 in Witten's theory would be replaced with the lifts of partonic 2-surfaces X^2 to 4-D surfaces with bundle structure with X^2 as base and S^2 as fiber. S^2 would be non-dynamical. Whether X^2 or its lift to 4-surface allows identification as algebraic surface is not quite clear but it seems that X^2 could be the relevant object identifiable as surface of the base space of $T(X^4)$. If X^2 is the basic object one would have the additional constraint (not present in Witten's theory) that it belongs to the base space X^4. The genus of the lift of X^2 would be that of its base space X^2. One obtains a union of partonic 2-surfaces rather than single surface and lines connecting them as boundaries of string world sheets.

   The n points of given X^2 would correspond to the ends of boundaries of string world sheets at the partonic 2-surface X^2 carrying fermion number so that the helicities of twistorial spinor modes would be highly fixed unless M^4 chiralities mix making fermions massive in M^4 sense. This picture is in accordance with the fact that the lines of fundamental fermions should correspond to QFT limit of TGD.

3. In TGD genus g of the partonic 2-surface labels various fermion families and $g < 3$ holds true for physical fermions. The explanation could be that Z^2 acts as global conformal symmetry (hyper-ellipticity) for $g < 3$ surfaces irrespective of their conformal moduli but for $g > 3$ only in for special moduli. Physically for $g > 2$ the additional handles would make the partonic 2-surface to behave like many-particle state of free particles defined by the handles.

   This assumption suggests that assigns to the partonic surface what I have called modular invariant elementary particle vacuum functional (EVPF) in modular degrees of freedom such that for a particle characterized by genus g one has $l \geq g$ and $l > g$ amplitudes are possible because the EPVF leaks partially to higher genera [K17]. This would also induce a mixing of boundary topologies explaining CKM mixing and its leptonic counterpart. In this framework it would be perhaps more appropriate to define the number of loops as $l_1 = l - g$.

   A more precise picture is as follows. Elementary particles have actually four wormhole throats corresponding to the two wormhole contacts. In the case of fermions the wormhole throat carrying the electroweak quantum numbers would have minimum value g of genus characterized by the fermion family. Furthermore, the universality of the standard model physics requires that the couplings of elementary fermions to gauge bosons do not depend on genus. This is the case if one has quantum superposition of the wormhole contacts carrying the quantum numbers of observed gauge bosons at their opposite throats over the three lowest genera $g = 0, 1, 2$ with identical coefficients. This meas SU(3) singlets for the dynamical SU(3) associated with genus degeneracy. Also their exotic variants - say octets - are in principle possible.

4. This description is not complete although already twistor string description involves integration over the conformal moduli of the partonic 2-surface. One must integrate over the "world of classical worlds" (WCW) -roughly over the generalized Feynman diagrams and their complements consisting of Minkowskian and Euclidian regions. TGD as almost topological QFT reduces this integration to that of the boundaries of space-time regions.

By quaternion conformal invariance [L3] this functional integral could reduce to ordinary integration over the quaternionic-conformal moduli space of space-time surfaces for which the moduli space of partonic 2-surfaces should be contained (note that strong form of holography suggests that only the modular invariants associated with the tangent space data should enter the description). One might hope that twistor space approach allows an elegant description of the moduli assignable to the tangent space data.

2.4.7 Yangian Symmetry

One of the victories of the twistor Grassmannian approach is the discovery of Yangian symmetry [A4], [B15, B22], [L3], whose variant associated with extended super-conformal symmetries is expected to be in key role in TGD.

1. The very nature of the residue integral implies that the complex surface serving as the locus of the integrand of the twistor amplitude is highly non-unique. Indeed, the Yangian symmetry [L3] acting as multi-local symmetry and implying dual of ordinary conformal invariance acting on momentum twistors, has been found to reduce to diffeomorphisms of $G(k,n)$ respecting positivity property of the decomposition of $G(k,n)$ to polyhedrons. It is quite possible that this symmetry corresponds to quaternion conformal symmetries in TGD framework.

2. Positivity of a given regions means parameterization by non-negative coordinates in TGD framework a possible interpretation is based on the observation that canonical identification mapping reals to p-adic number and vice versa is well-defined only for non-negative real numbers. Number theoretical Universality of spinor amplitudes so that they make sense in all number fields, would therefore be implied.

3. Could the crucial Yangian invariance generalizing the extended conformal invariance of TGD could reduce to the diffeomorphisms of the extended twistor space $T(H)$ respecting positivity. In the case of CP_2 all coordinates could be regarded as angle coordinates and be replaced by phase factors coding for the angles which do not make sense p-adically. The number theoretical existence of phase factors in p-adic case is guaranteed if they belong to some algebraic extension of rationals and thus also p-adics containing these phases as roots of unity. This implies discretization of CP_2.

   ZEO allows to reduce the consideration to causal diamond CD defined as an intersection of future and past directed light-cones and having two light-like boundaries. CD is indeed a natural counterpart for S^4. One could use as coordinates light-cone proper time a invariant under Lorentz transformations of either boundary of CD, hyperbolic angle η and two spherical angles (θ,ϕ). The angle variables allow representation in terms of finite algebraic extension. The coordinate a is naturally non-negative and would correspond to positivity. The diffeomorphisms perhaps realizing Yangian symmetry would respect causality in the sense that they do not lead outside CD.

   Quaternionic conformal symmetries the boundaries of $CD \times CP_2$ continued to the interior by translation of the light-cones serve as a good candidates for the diffeomorphisms in question since they do not change the value of the Minkowski time coordinate associated with the line connecting the tips of CD.

2.4.8 Does BCFW Recursion Have Counterpart In TGD?

Could BCFW recursion for tree diagrams and its generalization to diagrams with loops have a generalization in TGD framework? Could the possible TGD counterpart of BCFW recursion have a representation at the level of the TGD twistor space allowing interpretation in terms of geometry of partonic 2-surfaces and associated string world sheets? Supersymmetry is essential ingredient in obtaining this formula and the proposed SUSY realized at the level of amplitudes and broken at the level of states gives hopes for it. One could however worry about the fact that spinors are Dirac spinors in TGD framework and that Majorana property might be essential element.

How to produce Yangian invariants

Nima Arkani-Hamed *et al* [B22] (`http://tinyurl.com/y97rlzqb`) describe in detail various ways to form Yangian invariants defining the singular parts of the integrands of the amplitudes allowing to construct the full amplitudes. The following is only a rough sketch about what is involved using particle picture and I cannot claim of having understood the details.

1. One can *add* particle $((k,n) \rightarrow (k+1,n+1))$ to the amplitude by deforming the momentum twistors of two neighboring particles in a way depending on the momentum twistor of the added particle. One inserts the new particle between n-1:th and 1st particle, modifies their momentum twistors without changing their four-momenta, and multiplying the resulting amplitude by a twistor invariant known as $[n-2,n-1,n,1,2]$ so that there is dependence on the added n:th momentum twistor.

2. One can *remove* particle $((k,n) \rightarrow (k-1,n-1))$ by contour integrating over the momentum twistor variable of one particle.

3. One can *fuse* invariants simply by multiplying them.

4. One can *merge* invariants by identifying momentum twistors appearing in the two invariants. The integration over the common twistor leads to an elimination of particle.

5. One can form a *BCFW bridge* between n_1+1-particle invariant and n_2+1-particle invariant to get $n = n_1+n_2$-particle invariant using the operations listed. One starts with the *fusion* giving the product $I_1(1,...,n_1,I)I_2(n_1+1,..n,I)$ of Yangian invariants followed by *addition* of n_1+1 to I_1 between n_1 and I and 1 to I_2 between I and n_1+1 (see the first item for details). After that follows the *merging* of lines labelled by I next to n_1 in I_1 and the precedessor of n_1+1 in I_2 reducing particle number by one unit and followed by residue integration over Z_I reducing particle number further by one unit so that the resulting amplitude is n-particle amplitude.

6. One can perform *entangled removal* of two particles. One could remove them one-by-one by independent contour integrations but one can also perform the contour integrations in such a way that one first integrates over two twistors at the same complex line and then over the lines: this operation adds to n-particle amplitude loop.

BCFW recursion formula

BCFW recursion formula allows to express n-particle amplitudes with l loops in terms of amplitudes with amplitudes having at most $l-1$ loops. The basic philosophy is that singularities serve as data allowing to deduce the full integrands of the amplitudes by generalized unitarity and other kinds of arguments.

Consider first the arguments behind the BCFW formula.

1. BCFW formula is derived by performing the canonical momentum twistor deformation $Z_n \rightarrow z_n + zZ_{n-1}$, multiplying by $1/z$ and performing integration along small curve around origin so that one obtains original amplitude from the residue inside the curve. One obtains also and alternative of the residue integral expression as sum of residues from its complement. The singularities emerge by residue integral from poles of scattering amplitudes and eliminate two lines so that the recursion formula for n-particle amplitude can involve at most $n+2$-particle amplitudes.

   It seems that one must combine all n-particle amplitudes to form a single entity defining the full amplitude. I do not quite understand what how this is done. In ZEO zero energy state involving different particle numbers for the final state and expressible in terms of S-matrix (actually its generalization to what I call M-matrix) might allow to understand this.

2. In the general formula for the BCFW bridge of the "left" and "right" amplitudes one has $n_L + n_R = n+2$, $k_L + k_R = k-1$, and $l_L + l_R = l$.

3. The singularities are easy to understand in the case of tree amplitudes: they emerge from the poles of the conformally invariant quantities in the denominators of amplitudes. Physically this means that the sum of the momenta for a subset of particles corresponds to a complex pole (BCFW deformation makes two neighboring momenta complex). Hence one obtains sum over products of $j+1$-particle amplitudes BCFW bridged with $n-j$-particle amplitude to give n-particle amplitude by the merging process.

4. This is not all that is needed since the diagrams could be reduced to products of 1 loop 3-particle amplitudes which vanish by the triviality of coupling constant evolution in $\mathcal{N}=4$ SUSY. Loop amplitudes serving as a kind of source in the recursion relation save the situation. There is indeed also a second set of poles coming from loop amplitudes.

   One-loop case is the simplest one. One begins from $n+2$ particle amplitude with $l-1$ loops. At momentum space level the momenta the neighboring particles have opposite light-like momenta: one of the particles is not scattered at all. This is called forward limit. This limit suffers from collinear divergences in a generic gauge theory but in supersymmetric theories the limit is well-defined. This forward limit defines also a Yangian invariant at the level of twistor space. It can be regarded as being obtained by entangled removal of two particles combined with merge operation of two additional particles. This operation leads from $(n+2, l-1)$ amplitude to (n, l) amplitude.

Does BCFW formula make sense in TGD framework?

In TGD framework the four-fermion amplitude but restricted so that two outgoing particles have (in general) complex massless 8-momenta is the basic building brick. This changes the character of BCFW recursion relations although the four-fermion vertex effectively reduces to $F\overline{F}B$ vertex with boson identified as wormhole contact carrying fermion and antifermion at its throats.

The fundamental 4-fermion vertices assignable to wormhole contact could be formally expressed in terms of the product of two $F\overline{F}B_v$ vertices (MHV expression), where B_v is purely formal gauge boson, using the analog of MHV expression and taking into account that the second $F\overline{F}$ pair is associated with wormhole contact analogous to exchanged gauge boson.

If the fermions at fermion lines of the same partonic 2-surface can be assumed to be collinear and thus to form single coherent particle like unit, the description as superspace amplitude seems appropriate. Consequently, the effective $F\overline{F}B_v$ vertices could be assumed to have supersymmetry defined by the fermionic oscillator operator algebra at the partonic 2-surface (Clifford algebra). A good approximation is to restrict this algebra to that generating various spinor components of embedding space spinors so that $\mathcal{N}=4$ SUSY is obtained in leptonic and quark sector. Together these give rise to $\mathcal{N}=8$ SUSY at the level of vertices broken however at the level of states.

Side note: The number of external fermions is always even suggesting that the super-conformal anomalies plaguing the SUSY amplitudes with odd n (`http://tinyurl.com/yb85tnvc`) [B38] are absent in TGD: this would be basically due to the decomposition of gauge bosons to fermion pairs.

The leading singularities of scattering amplitudes would naturally correspond to the boundaries of the moduli space for the unions of partonic 2-surfaces and string world sheets.

1. The tree contribution to the gauge boson scattering amplitudes with $k=0,1$ vanish as found by Parke and Taylor who also found the simple twistorial form for the $k=2$ case (`http://tinyurl.com/y7nas26b`). In TGD framework, where lowest amplitude is 4-fermion amplitude, this situation is not encountered. According to Wikipedia article the so called CSW rules inspired by Witten's twistor theory have a problem due to the vanishing of $++-$ vertex which is not MHV form unless one changes the definition of what it is to be "wrong helicity". $++-$ is needed to construct $++++$ amplitude at one loop which does not vanish in YM theory. In SUSY it however vanishes.

   In TGD framework one does not encounter these problems since 4-fermion amplitudes are the basic building bricks. Fermion number conservation and the assumption that helicities do not mix (light-likeness in M^4 rather than only M^8-sense) implies $k=2(n(F)-n(\overline{F})$.

   In the general formula for the BCFW bridge of the "left" and "right" amplitudes one has $n_L+n_R=n+2$, $k_L+k_R=k-1$. If the TGD counterpart of the bridge eliminates two antifermions with the same "wrong" helicity -1/2, and one indeed has $k_L + k_R = k - 1$ if fermions have well-defined M^4 helicity rather than being in superposition in completely correlated M^4 and CP_2 helicities.

2. In string theory loops correspond to handles of a string world sheet. Now one has partonic 2-surfaces and string world sheets and both can in principle have handles. The condition $l \geq g$ of Witten's theory suggests that $l - g$ defines the handle number for string world sheet so that l is the total number of handles.

   The identification of loop number as the genus of partonic 2-surface is second alternative: one would have $l = g$ and string world sheets would not contain handles. This might be forced by the Minkowskian signature of the induced metric at string world sheet. The signature of the induced metric would be presumably Euclidian in some region of string world sheet since the M^4 projection of either homology generator assignable with the handle would presumably define time loop in M^4 since the derivative of M^4 time coordinate with respect to string world sheet time should vanish at the turning points for M^4 time. Minimal surface property might eliminate Euclidian regions of the string world sheet. In any case, the area of string world sheet would become complex.

3. In the moduli space of partonic 2-surfaces first kind of leading singularities could correspond to pinches formed as n partonic 2-surfaces decomposes to two 2-surfaces having at least single common point so that moduli space factors into a Cartesian product. This kind of singularities could serve as counterparts for the merge singularities appearing in the BCFW bridging of amplitudes. The numbers of loops must be additive and this is consistent with both interpretations for l.

4. What about forward limit? One particle should go through without scattering and is eliminated by entangled removal. In ZEO one can ask whether there is also quantum entanglement between the positive and negative energy parts of this single particle state and state function reduction does not occur. The addition of particle and merging it with another one could correspond to a situation in which two points of partonic 2-surface touch. This means addition of one handle so that loop number l increases.

   It seems that analytically the loop is added by the entangled removal but at the level of partonic surface it is added by the merging. Also now both $l > g$ and $l = g$ options make sense.

2.4.9 Possible Connections Of TGD Approach With The Twistor Grassmannian Approach

For a non-specialist lacking the technical skills, the work related to twistors is a garden of mysteries and there are a lot of questions to be answered: most of them of course trivial for the specialist. The basic questions are following.

How the twistor string approach of Witten and its possible TGD generalization relate to the approach involving residue integration over projective sub-manifolds of Grassmannians $G(k,n)$?

1. In [B23] Nima *et al* argue that one can transform Grassmannian representation to twistor string representation for tree amplitudes. The integration over $G(k,n)$ translates to integration over the moduli space of complex curves of degree $d = k - 1 + l$, $l \geq g$ is the number of loops. The moduli correspond to complex coefficients of the polynomial of degree d and they form naturally a projective space since an overall scaling of coefficients does not change the surfaces. One can expect also in the general case that moduli space of the partonic 2-surfaces can be represented as a projective sub-manifold of some projective space. Loop corrections would correspond to the inclusion of higher degree surfaces.

2. This connection gives hopes for understanding the integration contours in $G(k,n)$ at deeper level in terms of the moduli spaces of partonic 2-surfaces possibly restricted by conformal gauge conditions.

Below I try to understand and relate the work of Nima Arkani Hamed *et al* with twistor Grassmannian approach to TGD.

The notion of positive Grassmannian

The notion of positive Grassmannian is one of the central notions introduced by Nima et al.

1. The claim is that the sub-spaces of the real Grassmannian $G(k,n)$ contributing to the amplitudes for $++--$ signature are such that the determinants of the $k \times k$ minors associated with ordered columns of the $k \times n$ matrix C representing point of $G(k,n)$ are positive. To be precise, the signs of all minors are positive or negative simultaneously: only the ratios of the determinants defining projective invariants are positive.

2. At the boundaries of positive regions some of the determinants vanish. Some k-volumes degenerate to a lower-dimensional volume. Boundaries are responsible for the leading singularities of the scattering amplitudes and the integration measure associated with $G(k,n)$ has a logarithmic singularity at the boundaries. These boundaries would naturally correspond to the boundaries of the moduli space for the partonic 2-surfaces. Here also string world sheets could contribute to singularities.

3. This condition has a partial generalization to the complex case: the determinants whose ratios serve as projectively invariant coordinates are non-vanishing. A possible further manner to generalize this condition would be that the determinants have positive real part so that apart from rotation by $\pi/2$ they would reside in the upper half plane of complex plane. Upper half plane is the hyperbolic space playing key role in complex analysis and in the theory of hyperbolic 2-manifolds for which it serves as universal covering space by a finite discrete subgroup of Lorentz group $SL(2,C)$. The upper half-plane having a deep meaning in the theory of Riemann surfaces might play also a key role in the moduli spaces of partonic 2-surfaces. The projective space would be based - not on projectivization of C^n but that of H^n, H the upper half plane.

Could positivity have some even deeper meaning?

1. In TGD framework the number theoretical universality of amplitudes suggests this. Canonical identification maps $\sum x_n p^n \rightarrow \sum x_n p^{-n}$ p-adic number to non-negative reals. p-Adicization is possible for angle variables by replacing them by discrete phases, which are roots of unity. For non-angle like variables, which are non-negative one uses some variant of canonical identification involving cutoffs [K87]. The positivity should hold true for all structures involved, the $G(2,n)$ points defined by the twistors characterizing momenta and helicities of particles (actually pairs of orthogonal planes defined by twistors and their conjugates), the moduli space of partonic 2-surfaces, etc...

2. p-Adicization requires discretization of phases replacing angles so that they come as roots of unity associated with the algebraic extension used. The p-adic valued counterpart of Riemann or Lebesque integral does not make sense p-adically. Residue integrals can however allow to define p-adic integrals by analytic continuation of the integral and discretization of the phase factor along the integration contour does not matter (not however the p-adically troublesome factor 2π!).

3. TGD suggests that the generalization of positive real projectively invariant coordinates to complex coordinates of the hyperbolic space representable as upper half plane, or equivalently as unit disk obtained from the upper half plane by exponential mapping $w = exp(iz)$: positive coordinate α would correspond to the radial coordinate for the unit disk (Poincare hyperbolic disk appearing in Escher's paintings). The real measure $d\alpha/\alpha$ would correspond to $dz = dw/w$ restricted to a radial line from origin to the boundary of the unit disk. This integral should correspond to integral over a closed contour in complex case. This is the case if the integrand is discontinuity over a radial cut and equivalent with an integral over curve including also the boundary of the unit disk. This integral would reduce to the sum of the residues of poles inside the unit disk.

The notion of amplituhedron

The notion of amplituhedron is the latest step of progress in the twistor Grassmann approach [B6, B5]. What is so remarkable, is the simplicity of the expressions for all-loop amplitudes and the fact that positivity implies locality and unitarity for $\mathcal{N}=4$ SUSY.

Consider first tree amplitudes with general value of k.

1. The notion of amplituhedron relies on the mapping of $G(k,n)$ to $G_+(k,k+m)$ $n \geq k+m$. $G_+(k,k+m)$ is positive Grassmannian characterized by the condition that all $k\times k$- minors $k\times(k+m)$ matrix representing the point of $G_+(k,k+m)$ are non-negative and vanish at the boundaries $G_+(k,k+m)$. The value of m is $m=4$ and follows from the conditions that amplitudes come out correctly. The constraint $Y=C\cdot Z$, where Y corresponds to point of $G_+(k,k+4)$ and Z to the point of $G(k,n)$ performs this mapping, which is clearly many-to one. One can decompose $G_+(k,k+4)$ to positive regions intersecting only along their common boundary portions. The decomposition of a convex polygon in plane represent the basic example of this kind of decomposition.

2. Each decomposition defines a sum of contributions to the scattering amplitudes involving integration of a projectively invariant volume form over the positive region in question. The form has a logarithmic singularity at the boundaries of the integration region but spurious singularities cancel so that only the contribution of the genuine boundary of $G_+(k,k+4)$ remains. There are additional delta function constraints fixing the integral completely in real case.

3. In complex case one has residue integral. The proposed generalization to the complex case is by analytic continuation. TGD inspired proposal is that the positivity condition in the real case is generalized to the condition that the positive coordinates are replaced by complex coordinates of hyperbolic space representable as upper half plane or equivalently as the unit disk obtained from upper half plane by exponential mapping $w=exp(iz)$. The measure $d\alpha/\alpha$ would correspond to $dz=dw/w$. If taken over boundary circle labelled by discrete phase factors $exp(i\phi)$ given by roots of unity the integral would be numerically a discrete Riemann sum making no sense p-adically but residue theorem could allow to avoid the discretizaton and to define the p-adic variant of the integral by analytic continuation. These conditions would be completely general conditions on various projectively invariant moduli involved.

4. One must extend the bosonic twistors Z_a of external particles by adding k coordinates. Somewhat surprisingly, these coordinates are anticommutative super-coordinates expressible as linear combinations of fermionic parts of super-twistor using coefficients, which are also Grassmann numbers. Integrating over these one ends up with the standard expression of the amplitude using canonical integration measure for the regions in the decomposition of amplituhedron.

   What looks to me intriguing is that there is only super-integration involved over the additional k degrees of freedom. In Witten's approach $k-1$ corresponds to the minimum degree of the polynomial defining the string world sheet representing tree diagram. In TGD framework $k+1$ (rather than $k-1$) could correspond to the minimum degree of partonic 2-surface. In TGD approximate SUSY would correspond to Grassmann algebra of fermionic oscillator operators defined by the spinor basis for embedding space spinors. The interpretation could be that each fermion whose helicity differs from that allowed by light-likeness in M^4 sense (this requires non-vanishing M^4 mass), contributes $\Delta k=1$ to the degree of corresponding partonic 2-surface. Since the partonic 2-surface is common for all particles, one must have $d=k+1$ at least. The k-fold super integration would be basically integral over the moduli characterizing the polynomials of degree k realizing quantum classical correspondence in fermionic degrees of freedom.

BFCW recursion formula involves also loop amplitudes for which amplituhedron provides also a very nice representation.

1. The basic operation is the addition of a loop to get (n,k,l) amplitude from $(n+2,k,l-1)$ amplitude. That 2 particles must be removed for each loop is not obvious in $\mathcal{N}=4$ SUSY

but follows from the condition that positivity of the integration domain is preserved. This procedure removes from $(n+2,k,l-1)$-amplitude 2 particles with opposite four-momenta so that (n,k,l) amplitude is obtained. In the case of L-loops one extends $G(k,n)$ by adding its "complement" as a Cartesian factor $G(n-k,n)$ and imbeds to $G(n-k,n)$ 2-plane for each loop. Positivity conditions can be generalized so that they apply to $(k+2l)\times(k+2l)$-minors associated with matrices having as rows $0\leq l\leq L$ ordered D_{i_k}:s and of C. The general expressions of the loop contributions are of the same form as for tree contributions: only the number of integration variables is $4\times(k+L)$.

2. As already explained, in TGD framework the addition of loop would correspond to the formation of a handle to the partonic surface by fusing two points of partonic 2-surface and thus creating a surface intermediate between topologies with g and g+1 handles. g would correspond to the genus characterizing fermion family and one would have $L\geq g$. In elementary particle wave functionals loop [K17] contributions would correspond to higher genus contributions $l_1=l-g>0$ with basic contribution coming from genus g. For scattering amplitudes loop contributions would involve the change of the genus of the incoming wormhole throat so that they correspond to singular surfaces at the boundaries of their moduli space identifiable as loop corrections. $l_1=l-g>0$ would represent the number of pinches of the partonic 2-surface.

What about non-planar amplitudes?

Non-planar Feynman diagrams have remained a challenge for the twistor approach. The problem is simple: there is no canonical ordering of the extrenal particles and the loop integrand involving tricky shifts in integrations to get finite outcome is not unique and well-defined so that twistor Grasmann approach encounters difficulties.

Recently Nima Arkani-Hamed *et al* have considered also non-planar MHV diagrams [B25] (having minimal number of "wrong" helicities) of N=4 SUSY, and shown that they can be reduced to non-planar diagrams for different permutations of vertices of planar diagrams ordered naturally. There are several integration regions identified as positive Grassmannians corresponding to different orderings of the external lines inducing non-planarity. This does not however hold true generally.

At the QFT limit the crossings of lines emerges purely combinatorially since Feynman diagrams are purely combinatorial objects with the ordering of vertices determining the topological properties of the diagram. Non-planar diagrams correspond to diagrams, which do not allow crossing-free embedding to plane but require higher genus surface to get rid of crossings.

1. The number of the vertices of the diagram and identification of lines connecting them determines the diagram as a graph. This defines also in TGD framework Feynman diagram like structure as a graph for the fermion lines and should be behind non-planarity in QFT sense.

2. Could 2-D Feynman graphs exists also at geometric rather than only combinatorial level? Octonionization at embedding space level requires identification of preferred $M^2\subset M^4$ defining a preferred hyper-complex sub-space. Could the projection of the Fermion lines defined concrete geometric representation of Feynman diagrams?

3. Despite their purely combinatorial character Feynman diagrams are analogous to knots and braids. For years ago [K36] I proposed the generalization of the construction of knot invariants in which one gradually eliminates the crossings of the knot projection to end up with a trivial knot is highly suggestive as a procedure for constructing the amplitudes associated with the non-planar diagrams. The outcome should be a collection of planar diagrams calculable using twistor Grassmannian methods. Scattering amplitudes could be seen as analogs of knot invariants. The reduction of MHV diagrams to planar diagrams could be an example of this procedure.

One can imagine also analogs of non-planarity, which are geometric and topological rather than combinatorial and not visible at the QFT limit of TGD.

1. The fermion lines representing boundaries of string world sheets at the light-like orbits of partonic 2-surfaces can get braided. The same can happen also for the string boundaries at

space-like 3-surfaces at the ends of the space-time surface. The projections of these braids to partonic 2-surfaces are analogs of non-planar diagrams. If the fermion lines at single wormhole throat are regarded effectively as a line representing one member of super-multiplet, this kind of braiding remains below the resolution used and cannot correspond to the braiding at QFT limit.

2. 2-knotting and 2-braiding are possible for partonic 2-surfaces and string world sheets as 2-surfaces in 4-D space-time surfaces and have no counterpart at QFT limit.

2.4.10 Permutations, Braidings, And Amplitudes

In [B19] Nima Arkani-Hamed demonstrates that various twistorially represented on-mass-shell amplitudes (allowing light-like complex momenta) constructible by taking products of the 3-particle amplitude and its conjugate can be assigned with unique permutations of the incoming lines. The article describes the graphical representation of the amplitudes and its generalization. For 3-particle amplitudes, which correspond to $++-$ and $+--$ twistor amplitudes, the corresponding permutations are cyclic permutations, which are inverses of each other. One actually introduces double cover for the labels of the particles and speaks of decorated permutations meaning that permutation is always a right shift in the integer and in the range $[1, 2\times n]$.

Amplitudes as representation of permutations

It is shown that for on mass shell twistor amplitudes the definition using on-mass-shell 3-vertices as building bricks is highly reducible: there are two moves for squares defining 4-particle sub-amplitudes allowing to reduce the graph to a simpler one. The first ove is topologically like the s-t duality of the old-fashioned string models and second one corresponds to the transformation black $\leftrightarrow$ white for a square sub-diagram with lines of same color at the ends of the two diagonals and built from 3-vertices.

One can define the permutation characterizing the general on mass shell amplitude by a simple rule. Start from an external particle a and go through the graph turning in in white (black) vertex to left (right). Eventually this leads to a vertex containing an external particle and identified as the image $P(a)$ of the a in the permutation. If permutations are taken as right shifts, one ends up with double covering of permutation group with $2\times n!$ elements - decorated permutations. In this manner one can assign to any any line of the diagram two lines. This brings in mind 2-D integrable theories where scattering reduces to braiding and also topological QFTs where braiding defines the unitary S-matrix. In TGD parton lines involve braidings of the fermion lines so that an assignment of permutation also to vertex would be rather nice.

BCFW bridge has an interpretation as a transposition of two neighboring vertices affecting the lines of the permutation defining the diagram. One can construct all permutations as products of transpositions and therefore by building BCFW bridges. BCFW bridge can be constructed also between disjoint diagrams as done in the BCFW recursion formula.

Can one generalize this picture in TGD framework? There are several questions to be answered.

(a) What should one assume about the states at the light-like boundaries of string world sheets? What is the precise meaning of the supersymmetry: is it dynamical or gauge symmetry or both?

(b) What does one mean with particle: partonic 2-surface or boundary line of string world sheet? How the fundamental vertices are identified: 4 incoming boundaries of string world sheets or 3 incoming partonic orbits or are both aspects involved?

(c) How the 8-D generalization of twistors bringing in second helicity and doubling the M^4 helicity states assignable to fermions does affect the situation?

(d) Does the crucial right-left rule relying heavily on the possibility of only 2 3-particle vertices generalize? Does M^4 massivation imply more than 2 3-particle vertices implying many-to-one correspondence between on-mass-shell diagrams and permutations? Or should one generalize the right-left rule in TGD framework?

Fermion lines for fermions massless in 8-D sense

What does one mean with particle line at the level of fermions?

(a) How the addition of CP_2 helicity and complete correlation between M^4 and CP_2 chiralities does affect the rules of $\mathcal{N}=4$ SUSY? Chiral invariance in 8-D sense guarantees fermion number conservation for quarks and leptons separately and means conservation of the product of M^4 and CP_2 chiralities for 2-fermion vertices. Hence only M^4 chirality need to be considered. M^4 massivation allows more 4-fermion vertices than $\mathcal{N}=4$ SUSY.

(b) One can assign to a given partonic orbit several lines as boundaries of string world sheets connecting the orbit to other partonic orbits. Supersymmetry could be understoond in two ways.

   i. The fermions generating the state of super-multiplet correspond to boundaries of different string world sheets which need not connect the string world sheet to same partonic orbit. This SUSY is dynamical and broken. The breaking is mildest breaking for line groups connected by string world sheets to same partonic orbit. Right handed neutrinos generated the least broken $\mathcal{N}=2$ SUSY.

   ii. Also single line carrying several fermions would provide realization of generalized SUSY since the multi-fermion state would be characterized by single 8-momentum and helicity. One would have $\mathcal{N}=4$ SUSY for quarks and leptons separately and $\mathcal{N}=8$ if both quarks and leptons are allowed. Conserved total for quark and antiquarks and leptons and antileptons characterize the lines as well.

   What would be the propagator associated with many-fermion line? The first guess is that it is just a tensor power of single fermion propagator applied to the tensor power of single fermion states at the end of the line. This gives power of $1/p^{2n}$ to the denominator, which suggests that residue integral in momentum space gives zero unless one as just single fermion state unless the vertices give compensating powers of p. The reduction of fermion number to 0 or 1 would simplify the diagrammatics enormously and one would have only 0 or 1 fermions per given string boundary line. Multi-fermion lines would represent gauge degrees of freedom and SUSY would be realized as gauge invariance. This view about SUSY clearly gives the simplest picture, which is also consistent with the earlier one, and will be assumed in the sequel

(c) The multiline containing n fermion oscillator operators can transform by chirality mixing in 2^n ways at 4-fermion vertex so that there is quite a large number of options for incoming lines with n_i fermions.

(d) In 4-D Dirac equation light-likeness implies a complete correlation between fermion number and chirality. In 8-D case light-likeness should imply the same: now chirality correspond to fermion number. Does this mean that one must assume just superposition of different M^4 chiralities at the fermion lines as 8-D Dirac equation requires. Or should one assume that virtual fermions at the end of the line have wrong chirality so that massless Dirac operator does not annihilate them?

Fundamental vertices

One can consider two candidates for fundamental vertices depending on whether one identifies the lines of Feynman diagram as fermion lines or as light-like orbits of partonic 2-surfaces. The latter vertices reduces microscopically to the fermionic 4-vertices.

(a) If many-fermion lines are identified as fundamental lines, 4-fermion vertex is the fundamental vertex assignable to single wormhole contact in the topological vertex defined by common partonic 2-surface at the ends of incoming light-like 3-surfaces. The discontinuity is what makes the vertex non-trivial.

(b) In the vertices generalization of OZI rule applies for many-fermion lines since there are no higher vertices at this level and interactions are mediated by classical induced gauge fields and chirality mixing. Classical induced gauge fields vanish if CP_2 projection is 1-dimensional for string world sheets and even gauge potentials vanish if the projection is to geodesic circle. Hence only the chirality mixing due to the mixing of M^4 and CP_2 gamma matrices is possible and changes the fermionic M^4 chiralities. This would dictate what vertices are possible.

(c) The possibility of two helicity states for fermions suggests that the number of amplitudes is considerably larger than in $\mathcal{N} = 4$ SUSY. One would have 5 independent fermion amplitudes and at each 4-fermion vertex one should be able to choose between 3 options if the right-left rule generalizes. Hence the number of amplitudes is larger than the number of permutations possibly obtained using a generalization of right-left rule to right-middle-left rule.

(d) Note however that for massless particles in M^4 sense the reduction of helicity combinations for the fermion and antifermion making virtual gauge boson happens. The fermion and antifermion at the opposite wormhole throats have parallel four-momenta in good approximation. In M^4 they would have opposite chiralities and opposite helicities so that the boson would be M^4 scalar. No vector bosons would be obtained in this manner.

    In 8-D context it is possible to have also vector bosons since the M^4 chiralities can be same for fermion and anti-fermion. The bosons are however massive, and even photon is predicted to have small mass given by p-adic thermodynamics [K42]. Massivation brings in also the M^4 helicity 0 state. Only if zero helicity state is absent, the fundamental four-fermion vertex vanishes for $+++ +$ and $- - --$ combinations and one extend the right-left rule to right-middle-left rule. There is however no good reason for he reduction in the number of 4-fermion amplitudes to take place.

Partonic surfaces as 3-vertices

At space-time level one could identify vertices as partonic 2-surfaces.

(a) At space-time level the fundamental vertices are 3-particle vertices with particle identified as wormhole contact carrying many-fermion states at both wormhole throats. Each line of BCFW diagram would be doubled. This brings in mind the representation of permutations and leads to ask whether this representation could be re-interpreted in TGD framework. For this option the generalization of the decomposition of diagram to 3-particle vertices is very natural. If the states at throats consist of bound states of fermions as SUSY suggests, one could characterize them by total 8-momentum and helicity in good approximation. Both helicities would be however possible also for fermions by chirality mixing.

(b) A genuine decomposition to 3-vertices and lines connecting them takes place if two of the fermions reside at opposite throats of wormhole contact identified as fundamental gauge boson (physical elementary particles involve two wormhole contacts).

    The 3-vertex can be seen as fundamental and 4-fermion vertex becomes its microscopic representation. Since the 3-vertices are at fermion level 4-vertices their number is greater than two and there is no hope about the generalization of right-left rule.

OZI rule implies correspondence between permutations and amplitudes

The realization of the permutation in the same manner as for $\mathcal{N} = 4$ amplitudes does not work in TGD. OZI rule following from the absence of 4-fermion vertices however implies much simpler and physically quite a concrete manner to define the permutation for external fermion lines and also generalizes it to include braidings along partonic orbits.

(a) Already $\mathcal{N} = 4$ approach assumes decorated permutations meaning that each external fermion has effectively two states corresponding to labels k and $k + n$ (permutations are shifts to the right). For decorated permutations the number of external states is effectively 2^n and the number of decorated permutations is $2 \times n!$. The number of different helicity configurations in TGD framework is 2^n for incoming fermions at the vertex defined by the partonic 2-surface. By looking the values of these numbers for lowest integers one finds $2n \geq 2^n$: for $n = 2$ the equation is saturated. The inequality $log(n!) > nlog(n)/e) + 1$ (see `http://tinyurl.com/2bjk5h`). gives

$$\frac{log(2n!)}{log(2^n)} \geq \frac{log(2) + 1 + nlog(n/e)}{nlog(2)} = log(n/e)/log(2) + O(1/n)$$

so that the desired inequality holds for all interesting values of n.

(b) If OZI rule holds true, the permutation has very natural physical definition. One just follows the fermion line which must eventually end up to some external fermion since the only fermion vertex is 2-fermion vertex. The helicity flip would map $k \to k + n$ or vice versa.

(c) The labelling of diagrams by permutations generalizes to the case of diagrams involving partonic surfaces at the boundaries of causal diamond containing the external fermions and the partonic 2-surfaces in the interior of CD identified as vertices. Permutations generalize to braidings since also the braidings along the light-like partonic 2-surfaces are allowed. A quite concrete generalization of the analogs of braid diagrams in integrable 2-D theories emerges.

(d) BCFW bridge would be completely analogous to the fundamental braiding operation permuting two neighboring braid strands. The almost reduction to braid theory - apart from the presence of vertices conforms with the vision about reduction of TGD to almost topological QFT.

To sum up, the simplest option assumes SUSY as both gauge symmetry and broken dynamical symmetry. The gauge symmetry relates string boundaries with different fermion numbers and only fermion number 0 or 1 gives rise to a non-vanishing outcome in the residue integration and one obtains the picture used hitherto. If OZI rule applies, the decorated permutation symmetry generalizes to include braidings at the parton orbits and $k \to k \pm n$ corresponds to a helicity flip for a fermion going through the 4-vertex. OZI rules follows from the absence of non-linearities in Dirac action and means that 4-fermion vertices in the usual sense making theory non-renormalizable are absent. Theory is essentially free field theory in fermionic degrees of freedom and interactions in the sense of QFT are transformed to non-trivial topology of space-time surfaces.

3. If one can approximate space-time sheets by maps from M^4 to CP_2, one expects General Relativity and QFT description to be good approximations. GRT space-time is obtained by replacing space-time sheets with single sheet - a piece of slightly deformed Minkowski space but without assupmtion about embedding to H. Induced classical gravitational field and gauge fields are sums of those associated with the sheets. The generalized Feynman diagrams with lines at various sheets and going also between sheets are projected to single piece of M^4. Many-sheetedness makes 1-homology non-trivial and implies analog of braiding, which should be however invisible at QFT limit.

A concrete manner to eliminate line crossing in non-planar amplitude to get nearer to non-planar amplitude could proceed roughly as follows. This is of course a pure guess motivated only by topological considerations. Professional might kill it in few seconds.

1. If the lines carry no quantum numbers, reconnection allows to eliminate the crossings. Consider the crossing line pair connecting AB in the initial state to CD in final state. The crossing lines are AD and BC. Reconnection can take place in two ways: AD and BC transform either to AB and CD or to AC and BD: neither line pair has crossing. The final state of the braid would be quantum superposition of the resulting more planar braids.

2. The crossed lines however carry different quantum numbers in the generic situation: for instance, they can be fermionic and bosonic. In this particular case the reconnection does not make sense since a line carrying fermion number would transform to a line carrying boson.

   In TGD framework all lines are fermion lines at fundamental level but the constraint due to different quantum numbers still remains and it is easy to see that mere reconnection is not enough. Fermion number conservation allows only one of the two alternatives to be realized. Conservation of quantum numbers forces to restrict gives an additional constraint which for simplest non-planar diagram with two crossed fermion lines forces the quantum numbers of fermions to be identical.

   It seems also more natural to consider pairs of wormhole contacts defining elementary particles as "lines" in turn consisting of fermion lines. Yangian symmetry allows to develop a more detailed view about what this decomposition could mean.

   Quantum number conservation demands that reconnection is followed by a formation of an additional internal line connecting the non-crossing lines obtained by reconnection. The additional line representing a quantum number exchange between the resulting non-crossing lines would guarantee the conservation of quantum numbers. This would bring in two additional vertices and one additional internal line. This would be enough to reduce planarity. The repeated application of this transformation should produced a sum of non-planar diagrams.

3. What could go wrong with this proposal? In the case of gauge theory the order of diagram increases by g^2 since two new vertices are generated. Should a multiplication by $1/g^2$ accompany this process? Or is this observation enough to kill the hypothesis in gauge theory framework? In TGD framework the situation is not understood well enough to say anything. Certainly the critical value of α_K implies that one cannot regard it as a free parameter and cannot treat the contributions from various orders as independent ones.

2.5 Could The Universe Be Doing Yangian Arithmetics?

One of the old TGD inspired really crazy ideas about scattering amplitudes is that Universe is doing some sort of arithmetics so that scattering amplitude are representations for computational sequences of minimum length. The idea is so crazy that I have even given up its original form, which led to an attempt to assimilate the basic ideas about bi-algebras, quantum groups [K9], Yangians [L3], and related exotic things. The work with twistor Grassmannian approach inspired a reconsideration of the original idea seriously with the idea that super-symplectic Yangian could define the arithmetics. I try to describe the background, motivation, and the ensuing reckless speculations in the following.

2.5.1 Do Scattering Amplitudes Represent Quantal Algebraic Manipulations?

I seems that tensor product $\otimes$ and direct sum $\oplus$ - very much analogous to product and sum but defined between Hilbert spaces rather than numbers - are naturally associated with the basic vertices of TGD. I have written about this a highly speculative chapter - both mathematically and physically [K56]. The chapter [K9] is a remnant of earlier similar speculations.

1. In $\otimes$ vertex 3-surface splits to two 3-surfaces meaning that the 2 "incoming" 4-surfaces meet at single common 3-surface and become the outgoing 3-surface: 3 lines of Feynman diagram meeting at their ends. This has a lower-dimensional shadow realized for partonic 2-surfaces. This topological 3-particle vertex would be higher-D variant of 3-vertex for Feynman diagrams.

2. The second vertex is trouser vertex for strings generalized so that it applies to 3-surfaces. It does not represent particle decay as in string models but the branching of the particle wave function so that particle can be said to propagate along two different paths simultaneously. In double slit experiment this would occur for the photon space-time sheets.

3. The idea is that Universe is doing arithmetics of some kind in the sense that particle 3-vertex in the above topological sense represents either multiplication or its time-reversal co-multiplication.

The product, call it $\circ$, can be something very general, say algebraic operation assignable to some algebraic structure. The algebraic structure could be almost anything: a random list of structures popping into mind consists of group, Lie-algebra, super-conformal algebra quantum algebra, Yangian, etc.... The algebraic operation $\circ$ can be group multiplication, Lie-bracket, its generalization to super-algebra level, etc...). Tensor product and thus linear (Hilbert) spaces are involved always, and in product operation tensor product $\otimes$ is replaced with $\circ$.

1. The product $A_k \otimes A_l \rightarrow C = A_k \circ A_l$ is analogous to a particle reaction in which particles A_k and A_l fuse to particle $A_k \otimes A_l \rightarrow C = A_k \circ A_l$. One can say that $\otimes$ between reactants is transformed to $\circ$ in the particle reaction: kind of bound state is formed.

2. There are very many pairs A_k, A_l giving the same product C just as given integer can be divided in many ways to a product of two integers if it is not prime. This of course suggests that elementary particles are primes of the algebra if this notion is defined for it! One can use some basis for the algebra and in this basis one has $C = A_k \circ A_l = f_{klm}A_m$, f_{klm} are the structure constants of the algebra and satisfy constraints. For instance, associativity $A(BC) = (AB)C$ is a constraint making the life of algebraist more tolerable and is almost routinely assumed.

   For instance, in the number theoretic approach to TGD associativity is proposed to serve as fundamental law of physics and allows to identify space-time surfaces as 4-surfaces with associative (quaternionic) tangent space or normal space at each point of octonionic embedding space $M^4 \times CP_2$. Lie algebras are not associative but Jacobi-identities following from the associativity of Lie group product replace associativity.

3. Co-product can be said to be time reversal of the algebraic operation $\circ$. Co-product can be defined as $C = A_k \rightarrow \sum_{lm} f_k^{lm} A_l \otimes A_m$, where f_k^{lm} are the structure constants of the algebra. The outcome is quantum superposition of final states, which can fuse to C (the "reaction" $A_k \otimes A_l \rightarrow C = A_k \circ A_l$ is possible). One can say that $\circ$ is replaced with $\otimes$: bound state decays to a superposition of all pairs, which can form the bound states by product vertex.

There are motivations for representing scattering amplitudes as sequences of algebraic operations performed for the incoming set of particles leading to an outgoing set of particles with particles identified as algebraic objects acting on vacuum state. The outcome would be analogous to Feynman diagrams but only the diagram with minimal length to which a preferred extremal can be assigned is needed. Larger ones must be equivalent with it.

The question is whether it could be indeed possible to characterize particle reactions as computations involving transformation of tensor products to products in vertices and co-products to tensor products in co-vertices (time reversals of the vertices). A couple of examples gives some idea about what is involved.

1. The simplest operations would preserve particle number and to just permute the particles: the permutation generalizes to a braiding and the scattering matrix would be basically unitary braiding matrix utilized in topological quantum computation.

2. A more complex situation occurs, when the number of particles is preserved but quantum numbers for the final state are not same as for the initial state so that particles must interact. This requires both product and co-product vertices. For instance, $A_k \otimes A_l \rightarrow f_{kl}^m A_m$ followed by $A_m \rightarrow f_m^{rs} A_r \otimes A_s$ giving $A_k \rightarrow f_{kl}^m f_m^{rs} A_r \otimes A_s$ representing 2-particle scattering. State function reduction in the final state can select any pair $A_r \otimes A_s$ in the final state. This reaction is characterized by the ordinary tree diagram in which two lines fuse to single line and defuse back to two lines. Note also that there is a non-deterministic element involved. A given final state can be achieved from a given initial state after large enough number of trials. The analogy with problem solving and mathematical theorem proving is obvious. If the interpretation is correct, Universe would be problem solver and theorem prover!

3. More complex reactions affect also the particle number. 3-vertex and its co-vertex are the simplest examples and generate more complex particle number changing vertices. For instance, on twistor Grassmann approach on can construct all diagrams using two 3-vertices. This encourages the restriction to 3-vertice (recall that fermions have only 2-vertices)

4. Intuitively it is clear that the final collection of algebraic objects can be reached by a large - maybe infinite - number of ways. It seems also clear that there is the shortest manner to end up to the final state from a given initial state. Of course, it can happen that there is no way to achieve it! For instance, if $\circ$ corresponds to group multiplication the co-vertex can lead only to a pair of particles for which the product of final state group elements equals to the initial state group element.

5. Quantum theorists of course worry about unitarity. How can avoid the situation in which the product gives zero if the outcome is element of linear space. Somehow the product should be such that this can be avoided. For instance, if product is Lie-algebra commutator, Cartan algebra would give zero as outcome.

2.5.2 Generalized Feynman Diagram As Shortest Possible Algebraic Manipulation Connecting Initial And Final Algebraic Objects

There is a strong motivation for the interpretation of generalized Feynman diagrams as shortest possible algebraic operations connecting initial and final states. The reason is that in TGD one does not have path integral over all possible space-time surfaces connecting the 3-surfaces at the ends of CD. Rather, one has in the optimal situation a space-time surface unique apart from conformal gauge degeneracy connecting the 3-surfaces at the ends of CD (they can have disjoint components).

Path integral is replaced with integral over 3-surfaces. There is therefore only single minimal generalized Feynman diagram (or twistor diagram, or whatever is the appropriate term). It would be nice if this diagram had interpretation as the shortest possible computation leading from the initial state to the final state specified by 3-surfaces and basically fermionic states at them. This would of course simplify enormously the theory and the connection to the twistor Grassmann approach is very suggestive. A further motivation comes from the observation that the state basis created by the fermionic Clifford algebra has an interpretation in terms of Boolean quantum logic and that in ZEO the fermionic states would have interpretation as analogs of Boolean statements $A \rightarrow B$.

To see whether and how this idea could be realized in TGD framework, let us try to find counterparts for the basic operations $\otimes$ and $\circ$ and identify the algebra involved. Consider first the basic geometric objects.

1. Tensor product could correspond geometrically to two disjoint 3-surfaces representing 3-particles. Partonic 2-surfaces associated with a given 3-surface represent second possibility. The splitting of a partonic 2-surface to two could be the geometric counterpart for co-product.

2. Partonic 2-surfaces are however connected to each other and possibly even to themselves by strings. It seems that partonic 2-surface cannot be the basic unit. Indeed, elementary particles are identified as pairs of wormhole throats (partonic 2-surfaces) with magnetic monopole flux flowing from throat to another at first space-time sheet, then through throat

to another sheet, then back along second sheet to the lower throat of the first contact and then back to the thirst throat. This unit seems to be the natural basic object to consider. The flux tubes at both sheets are accompanied by fermionic strings. Whether also wormhole throats contain strings so that one would have single closed string rather than two open ones, is an open question.

3. The connecting strings give rise to the formation of gravitationally bound states and the hierarchy of Planck constants is crucially involved. For elementary particle there are just two wormhole contacts each involving two wormhole throats connected by wormhole contact. Wormhole throats are connected by one or more strings, which define space-like boundaries of corresponding string world sheets at the boundaries of CD. These strings are responsible for the formation of bound states, even macroscopic gravitational bound states.

2.5.3 Does Super-Symplectic Yangian Define The Arithmetics?

Super-symplectic Yangian would be a reasonable guess for the algebra involved.

1. The 2-local generators of Yangian would be of form $T_1^A = f^A_{BC} T^B \otimes T^C$, where f^A_{BC} are the structure constants of the super-symplectic algebra. n-local generators would be obtained by iterating this rule. Note that the generator T_1^A creates an entangled state of T^B and T^C with f^A_{BC} the entanglement coefficients. T_n^A is entangled state of T^B and T^C_{n-1} with the same coefficients. A kind replication of T^A_{n-1} is clearly involved, and the fundamental replication is that of T^A. Note that one can start from any irreducible representation with well defined symplectic quantum numbers and form similar hierarchy by using T^A and the representation as a starting point.

   That the hierarchy T_n^A and hierarchies irreducible representations would define a hierarchy of states associated with the partonic 2-surface is a highly non-trivial and powerful hypothesis about the formation of many-fermion bound states inside partonic 2-surfaces.

2. The charges T^A correspond to fermionic and bosonic super-symplectic generators. The geometric counterpart for the replication at the lowest level could correspond to a fermionic/bosonic string carrying super-symplectic generator splitting to fermionic/bosonic string and a string carrying bosonic symplectic generator T^A. This splitting of string brings in mind the basic gauge boson-gauge boson or gauge boson-fermion vertex.

   The vision about emission of virtual particle suggests that the entire wormhole contact pair replicates. Second wormhole throat would carry the string corresponding to T^A assignable to gauge boson naturally. T^A should involve pairs of fermionic creation and annihilation operators as well as fermionic and anti-fermionic creation operator (and annihilation operators) as in quantum field theory.

3. Bosonic emergence suggests that bosonic generators are constructed from fermion pairs with fermion and anti-fermion at opposite wormhole throats: this would allow to avoid the problems with the singular character of purely local fermion current. Fermionic and anti-fermionic string would reside at opposite space-time sheets and the whole structure would correspond to a closed magnetic tube carrying monopole flux. Fermions would correspond to superpositions of states in which string is located at either half of the closed flux tube.

4. The basic arithmetic operation in co-vertex would be co-multiplication transforming T_n^A to $T^A_{n+1} = f^A_{BC} T^B_n \otimes T^C$. In vertex the transformation of T^A_{n+1} to T^A_n would take place. The interpretations would be as emission/absorption of gauge boson. One must include also emission of fermion and this means replacement of T^A with corresponding fermionic generators F^A, so that the fermion number of the second part of the state is reduced by one unit. Particle reactions would be more than mere braidings and re-grouping of fermions and anti-fermions inside partonic 2-surfaces, which can split.

5. Inside the light-like orbits of the partonic 2-surfaces there is also a braiding affecting the M-matrix. The arithmetics involved would be therefore essentially that of measuring and "co-measuring" symplectic charges.

Generalized Feynman diagrams (preferred extremals) connecting given 3-surfaces and many-fermion states (bosons are counted as fermion-anti-fermion states) would have a minimum number of vertices and co-vertices. The splitting of string lines implies creation of pairs of fermion lines. Whether regroupings are part of the story is not quite clear. In any case, without the replication of 3-surfaces it would not be possible to understand processes like e-e scattering by photon exchange in the proposed picture.

It is easy to hear the comments of the skeptic listener in the back row.

1. The attribute "minimal" - , which could translate to minimal value of Kähler function - is dangerous. It might be very difficult to determine what the minimal diagram is - consider only travelling salesman problem or the task of finding the shortest proof of theorem. It would be much nicer to have simple calculational rules.

   The original proposal might help here. The generalization of string model duality was in question. It stated that it is possible to move the positions of the vertices of the diagrams just as one does to transform s-channel resonances to t-channel exchange. All loops of generalized diagrams could be be eliminated by transforming the to tadpoles and snipped away so that only tree diagrams would be left. The variants of the diagram were identified as different continuation paths between different paths connecting sectors of WCW corresponding to different 3-topologies. Each step in the continuation procedure would involve product or co-product defining what continuation between two sectors means for WCW spinors. The continuations between two states require some minimal number of steps. If this is true, all computations connecting identical states are also physically equivalent. The value of the vacuum functional be same for all of them. This looks very natural.

   That the Kähler action should be same for all computational sequences connecting the same initial and final states looks strange but might be understood in terms of the vacuum degeneracy of Kähler action.

2. QFT perturbation theory requires that should have superposition of computations/continuations. What could the superposition of QFT diagrams correspond to in TGD framework?

   Could it correspond to a superposition of generators of the Yangian creating the physical state? After all, already quantum computer perform superpositions of computations. The fermionic state would not be the simplest one that one can imagine. Could AdS/CFT analogy allow to identify the vacuum state as a superposition of multi-string states so that single super-symplectic generator would be replaced with a superposition of its Yangian counterparts with same total quantum numbers but with a varying number of strings? The weight of a given superposition would be given by the total effective string world sheet area. The sum of diagrams would emerge from this superposition and would basically correspond to functional integration in WCW using exponent of Kähler action as weight. The stringy functional integral ("functional" if also wormhole contacts contain string portion, otherwise path integral) would give the perturbation theory around given string world sheet. One would have effective reduction of string theory.

2.5.4 How Does This Relate To The Ordinary Perturbation Theory?

One can of course worry about how to understand the basic results of the usual perturbation theory in this picture. How does one obtain a perturbation theory in powers of coupling constant, what does running coupling constant mean, etc...? I have already discussed how the superposition of diagrams could be understood in the new picture.

1. The QFT picture with running coupling constant is expected at QFT limit, when many-sheeted space-time is replaced with a slightly curved region of M^4 and gravitational field and gauge potentials are identified as sums of the deviations of induced metric from M^4 metric and classical induced gauge potentials associated with the sheets of the many-sheeted space-time. The running coupling constant would be due to the dependence of the size scale of CD, and p-adic coupling constant evolution would be behind the continuous one.

2. The notion of running coupling constant is very physical concept and should have a description also at the fundamental level and be due to a finite computational resolution, which indeed has very concrete description in terms of Noether charges of super-symplectic Yangian creating the states at the ends of space-time surface at the boundaries of CD. The space-time surface and the diagram associated with a given pair of 3-surfaces and stringy Noether charges associated with them can be characterized by a complexity measured in terms of the number of vertices (3-surface at which three 3-surfaces meet).

   For instance, 3-particle scattering can be possible only by using the simplest 3-vertex defined by product or co-product for pairs of 3-surfaces. In the generic case one has more complex diagram and what looks first 3-particle vertex has complex substructure rather than being simple product or co-product.

3. Complexity seems to have two separate aspects: the complexities of the positive and negative parts of zero energy state as many-fermion states and the complexity of associated 3-surfaces. The generalization of AdS/CFT however suggests that once the string world sheets and partonic 2-surfaces appearing in the diagram have been fixed, the space-time surface itself is fixed. The principle also suggests that the fixing partonic 2-surface and the strings connecting them at the boundaries of CD fixes the 3-surface apart from the action of sub-algebra of Yangian acting as gauge algebra (vanishing classical Noether charges). If one can determine the minimal sequence of allowed algebraic operation of Yangian connecting initial and final fermion states, one knows the minimum number of vertices and therefore the topological structure of the connecting minimal space-time surface.

4. In QFT spirit one could describe the finite measurement resolution by introducing effective 3-point vertex, which is need not be product/co-produce anymore. 3-point scattering amplitudes in general involve microscopic algebraic structure involving several vertices. One can however give up the nice algebraic interpretation and just talk about effective 3-vertex for practical purposes. Just as the QFT vertex described by running coupling constant decomposes to sum of diagrams, product/co-product in TGD could be replaced with effective product/co-product expressible as a longer computation. This would imply coupling constant evolution.

   Fermion lines could however remain as such since they are massless in 8-D sense and mass renormalization does not make sense.

   Similar practical simplification could be done the initial and final states to get rid of superposition of the Yangian generators with different numbers of strings ("cloud of virtual particles"). This would correspond to wave function renormalization.

5. The number of vertices and wormhole contact orbits serves as a measure for the complexity of the diagram. Since fermion lines are associated with wormhole throats assignable with wormhole contacts identifiable as deformations CP_2 type vacuum extremals, one expects that the exponent of the Kähler function defining vacuum functional is in the first approximation the total CP_2 volume of wormhole contacts giving a measure for the importance of the contribution in functional integral. If it converges very rapidly only Gaussian approximation around maximum is needed.

6. Convergence depends on how large the fraction of volume of CP_2 is associated with a given wormhole contact. The volume is proportional to the length of the wormhole contact orbit. One expects exponential convergence with the number of fermion lines and their lengths for long lines. For short distances the exponential damping is small so that diagrams with microscopic structure of diagrams are needed and are possible. This looks like adding small scale details to the algebraic manipulations.

7. One must be of course be very cautious in making conclusions. The presence of $1/\alpha_K \propto h_{eff}$ in the exponent of Kähler function would suggest that for large values of h_{eff} only the 3-surfaces with smallest possible number of wormhole contact orbits contribute. On the other hand, the generalization of AdS/CFT duality suggests that Kähler action reducible to area of string world sheet in the effective metric defined by canonical momentum currents of Kähler action behaves as $\alpha_K^2 \propto 1/h_{eff}^2$. What does this mean?

To sum up, the identification of vertex as a product or co-product in Yangian looks highly promising approach. The Nother charges of the super-symplectic Yangian are associated with strings and are either linear or bilinear in the fermion field. The fermion fields associated with the partonic 2-surface defining the vertex are contracted with fermion fields associated with other partonic 2-surfaces using the same rule as in Wick expansion in quantum field theories. The contraction gives fermion propagator for each leg pair associated with two vertices. Vertex factor is proportional to the contraction of spinor modes with the operators defining the Noether charge or super charge and essentially Kähler-Dirac gamma matrix and the representation of the action of the symplectic generator on fermion realizable in terms of sigma matrices. This is very much like the corresponding expression in gauge theories but with gauge algebra replaced with symplectic algebra. The possibility of contractions of creation and annihilation operator for fermion lines associated with opposite wormhole throats at the same partonic 2-surface (for Noether charge bilinear in fermion field) gives bosonic exchanges as lines in which the fermion lines turns in time direction: otherwise only regroupings of fermions would take place.

2.5.5 This Was Not The Whole Story Yet

The proposed amplitude represents only the value of WCW spinor field for single pair of 3-surfaces at the opposite boundaries of given CD. Hence Yangian construction does not tell the whole story.

1. Yangian algebra would give only the vertices of the scattering amplitudes. On basis of previous considerations, one expects that each fermion line carries propagator defined by 8-momentum. The structure would resemble that of super-symmetric YM theory. Fermionic propagators should emerge from summing over intermediate fermion states in various vertices and one would have integrations over virtual momenta which are carried as residue integrations in twistor Grassmann approach. 8-D counterpart of twistorialization would apply.

2. Super-symplectic Yangian would give the scattering amplitudes for single space-time surface and the purely group theoretical form of these amplitudes gives hopes about the independence of the scattering amplitude on the pair of 3-surfaces at the ends of CD near the maximum of Kähler function. This is perhaps too much to hope except approximately but if true, the integration over WCW would give only exponent of Kähler action since metric and poorly defined Gaussian and determinants would cancel by the basic properties of Kähler metric. Exponent would give a non-analytic dependence on α_K.

   The Yangian supercharges are proportional to $1/\alpha_K$ since covariant Kähler-Dirac gamma matrices are proportional to canonical momentum currents of Kähler action and thus to $1/\alpha_K$. Perturbation theory in powers of $\alpha_K = g_K^2/4\pi\hbar_{eff}$ is possible after factorizing out the exponent of vacuum functional at the maximum of Kähler function and the factors $1/\alpha_K$ multiplying super-symplectic charges.

   The additional complication is that the characteristics of preferred extremals contributing significantly to the scattering amplitudes are expected to depend on the value of α_K by quantum interference effects. Kähler action is proportional to $1/\alpha_K$. The analogy of AdS/CFT correspondence states the expressibility of Kähler function in terms of string area in the effective metric defined by the anti-commutators of K-D matrices. Interference effects eliminate string length for which the area action has a value considerably larger than one so that the string length and thus also the minimal size of CD containing it scales as h_{eff}. Quantum interference effects therefore give an additional dependence of Yangian super-charges on h_{eff} leading to a perturbative expansion in powers of α_K although the basic expression for scattering amplitude would not suggest this.

3. Non-planar diagrams of quantum field theories should have natural counterpart and linking and knotting for braids defines it naturally. This suggests that the amplitudes can be interpreted as generalizations of braid diagrams defining braid invariants: braid strands would appear as legs of 3-vertices representing product and co-product. Amplitudes could be constructed as generalized braid invariants transforming recursively braided tree diagram to an un-braided diagram using same operations as for braids. In [L7] I considered a possible breaking of associativity occurring in weak sense for conformal field theories and was led

to the vision that there is a fractal hierarchy of braids such that braid strands themselves correspond to braids. This hierarchy would define an operad with subgroups of permutation group in key role. Hence it seems that various approaches to the construction of amplitudes converge.

2.6 Appendix: Some Mathematical Details About Grasmannian Formalism

In the following I try to summarize my amateurish understanding about the mathematical structure behind the Grassmann integral approach. The representation summarizes what I have gathered from the articles of Arkani-Hamed and collaborators [B20, B22]. These articles are rather sketchy and the article of Bullimore provides additional details [B37] related to soft factors. The article of Mason and Skinner provides excellent introduction to super-twistors [B15] and dual super-conformal invariance. I apologize for unavoidable errors.

Before continuing a brief summary about the history leading to the articles of Arkani-Hamed and others is in order. This summary covers only those aspects which I am at least somewhat familiar with and leaves out many topics about existence which I am only half-conscious.

1. It is convenient to start by summarizing the basic facts about bi-spinors and their conjugates allowing to express massless momenta as $p^{aa'} = \lambda^a \tilde{\lambda}^{a'}$ with $\tilde{\lambda}$ defined as complex conjugate of λ and having opposite chirality. When λ is scaled by a complex number $\tilde{\lambda}$ suffers an opposite scaling. The bi-spinors allow the definition of various inner products

$$\begin{aligned} \langle \lambda, \mu \rangle &= \epsilon_{ab} \lambda^a \mu^b \ , \\ \left[\tilde{\lambda}, \tilde{\mu} \right] &= \epsilon_{a'b'} \lambda^{a'} \mu^{b'} \ , \\ p \cdot q &= \langle \lambda, \mu \rangle \left[\tilde{\lambda}, \tilde{\mu} \right] \ , \ (q_{aa'} = \mu_a \tilde{\mu}_{a'}) \ . \end{aligned} \quad (2.6.1)$$

If the particle has spin one can assign it a positive or negative helicity $h = \pm 1$. Positive helicity can be represented by introducing artitrary negative (positive) helicity bispinor μ_a ($\mu_{a'}$) not parallel to λ_a ($\mu_{a'}$) so that one can write for the polarization vector

$$\begin{aligned} \epsilon_{aa'} &= \frac{\mu_a \tilde{\lambda}_{a'}}{\langle \mu, \lambda \rangle} \ , \ \text{positive helicity} \ , \\ \epsilon_{aa'} &= \frac{\lambda_a \tilde{\mu}_{a'}}{\left[\tilde{\mu}, \tilde{\lambda} \right]} \ , \ \text{negative helicity} \ . \end{aligned} \quad (2.6.2)$$

In the case of momentum twistors the μ part is determined by different criterion to be discussed later.

2. Tree amplitudes are considered and it is convenient to drop the group theory factor $Tr(T_1 T_2 \cdots T_n)$. The starting point is the observation that tree amplitude for which more than $n-2$ gluons have the same helicity vanish. MHV amplitudes have exactly $n-2$ gluons of same helicity-taken by a convention to be negative- have extremely simple form in terms of the spinors and reads as

$$A_n = \frac{\langle \lambda_x, \lambda_y \rangle^4}{\prod_{i=1}^n \langle \lambda_i, \lambda_{i+1} \rangle} \quad (2.6.3)$$

When the sign of the helicities is changed $\langle .. \rangle$ is replaced with [..].

3. The article of Witten [B16] proposed that twistor approach could be formulated as a twistor string theory with string world sheets "living" in 6-dimensional CP_3 possessing Calabi-Yau structure and defining twistor space. In this article Witten introduced what is known as half Fourier transform allowing to transform momentum integrals over light-cone to twistor integrals. This operation makes sense only in space-time signature $(2,2)$. Witten also demonstrated that maximal helicity violating (MHV) twistor amplitudes (two gluons with negative helicity) with n particles with $k+2$ negative helicities and l loops correspond in this approach to holomorphic 2-surfaces defined by polynomials defined by polynomials of degree $D = k-1+l$, where the genus of the surface satisfies $g \leq l$. AdS/CFT duality provides a second stringy approach to $\mathcal{N}=4$ theory allowing to understand the scattering amplitudes in terms of Wilson loops with light-like edges: about this I have nothing to say. In any case, the generalization of twistor string theory to TGD context is highly attractive idea and will be considered later.

4. In the article [B11] Cachazo, Svrcek, and Witten propose the analog of Feynman diagrammatics in which MHV amplitudes can be used as analogs of vertices and ordinary $1/P^2$ propagator as propagator to construct tree diagrams with arbitrary number of negative helicity gluons. This approach is not symmetric with respect to the change of the sign of helicities since the amplitudes with two positive helicities are constructed as tree diagrams. The construction is non-trivial because one must analytically continue the on mass shell tree amplitudes to off mass shell momenta. The problem is how to assign a twistor to these momenta. This is achieved by introducing an arbitrary twistor $\eta^{a'}$ and defining λ_a as $\lambda_a = p_{aa'}\eta^{a'}$. This works for both massless and massive case. It however leads to a loss of the manifest Lorentz invariance. The paper however argues and the later paper [B9, B9] shows rigorously that the loss is only apparent. In this paper also BCFW recursion formula is introduced allowing to construct tree amplitudes recursively by starting from vertices with 2 negative helicity gluons. Also the notion which has become known as BCFW bridge representing the massless exchange in these diagrams is introduced. The tree amplitudes are not tree amplitudes in gauge theory sense where correspond to leading singularities for which 4 or more lines of the loop are massless and therefore collinear. What is important that the very simple MHV amplitudes become the building blocks of more complex amplitudes.

5. The nex step in the progress was the attempt to understand how the loop corrections could be taken into account in the construction BCFW formula. The calculation of loop contributions to the tree amplitudes revealed the existence of dual super-conformal symmetry which was found to be possessed also by BCFW tree amplitudes besides conformal symmetry. Together these symmetries generate infinite-dimensional Yangian symmetry [B15].

6. The basic vision of Arkani-Hamed and collaborators is that the scattering amplitudes of $\mathcal{N}=4$ SYM are constructible in terms of leading order singularities of loop diagrams. These singularities are obtained by putting maximum number of momenta propagating in the lines of the loop on mass shell. The non-leading singularities would be induced by the leading singularities by putting smaller number of momenta on mass shell are dictated by these terms. A related idea serving as a starting point in [B20] is that one can define loop integrals as residue integrals in momentum space. If I have understood correctly, this means that one an imagine the possibility that the loop integral reduces to a lower dimensional integral for on mass shell particles in the loops: this would resemble the approach to loop integrals based on unitarity and analyticity. In twistor approach these momentum integrals defined as residue integrals transform to residue integrals in twistor space with twistors representing massless particles. The basic discovery is that one can construct leading order singularities for n particle scattering amplitude with $k+2$ negative helicities as Yangian invariants $Y_{n,k}$ for momentum twistors and invariants constructed from them by canonical operations changing n and k. The correspondence $k = l$ does not hold true for the more general amplitudes anymore.

2.6.1 Yangian Algebra And Its Super Counterpart

The article of Witten [B14] gives a nice discussion of the Yangian algebra and its super counterpart. Here only basic formulas can be listed and the formulas relevant to the super-conformal case are given.

Yangian algebra

Yangian algebra $Y(G)$ is associative Hopf algebra. The elements of Yangian algebra are labelled by non-negative integers so that there is a close analogy with the algebra spanned by the generators of Virasoro algebra with non-negative conformal weight. The Yangian symmetry algebra is defined by the following relations for the generators labeled by integers $n=0$ and $n=1$. The first half of these relations discussed in very clear manner in [B14] follows uniquely from the fact that adjoint representation of the Lie algebra is in question

$$\left[J^A,J^B\right]=f_C^{AB}J^C \quad , \quad \left[J^A,J^{(1)B}\right]=f_C^{AB}J^{(1)C} \quad . \tag{2.6.4}$$

Besides this Serre relations are satisfied. These have more complex and read as

$$\begin{aligned}
&\left[J^{(1)A},\left[J^{(1)B},J^C\right]\right]+\left[J^{(1)B},\left[J^{(1)C},J^A\right]\right]+\left[J^{(1)C},\left[J^{(1)A},J^B\right]\right]\\
&\qquad =\frac{1}{24}f^{ADK}f^{BEL}f^{CFM}f_{KLM}\{J_D,J_E,J_F\} \ ,\\
&\left[\left[J^{(1)A},J^{(1)B}\right],\left[J^C,J^{(1)D}\right]\right]+\left[\left[J^{(1)C},J^{(1)D}\right],\left[J^A,J^{(1)B}\right]\right]\\
&\qquad =\frac{1}{24}(f^{AGL}f^{BEM}f_K^{CD}\\
&\qquad +f^{CGL}f^{DEM}f_K^{AB})f^{KFN}f_{LMN}\{J_G,J_E,J_F\} \ .
\end{aligned} \tag{2.6.5}$$

The indices of the Lie algebra generators are raised by invariant, non-degenerate metric tensor g_{AB} or g^{AB}. $\{A,B,C\}$ denotes the symmetrized product of three generators.

Repeated commutators allow to generate the entire algebra whose elements are labeled by non-negative integer n. The generators obtain in this manner are n-local operators arising in $(n-1)$-commutator of $J^{(1)}$: s. For $SU(2)$ the Serre relations are trivial. For other cases the first Serre relation implies the second one so the relations are redundant. Why Witten includes it is for the purposed of demonstrating the conditions for the existence of Yangians associated with discrete one-dimensional lattices (Yangians exists also for continuum one-dimensional index).

Discrete one-dimensional lattice provides under certain consistency conditions a representation for the Yangian algebra. One assumes that each lattice point allows a representation R of J^A so that one has $J^A=\sum_i J_i^A$ acting on the infinite tensor power of the representation considered. The expressions for the generators J^{1A} are given as

$$J^{(1)A} \quad = \quad f_{BC}^A\sum_{i<j}J_i^BJ_j^C \quad . \tag{2.6.6}$$

This formula gives the generators in the case of conformal algebra. This representation exists if the adjoint representation of G appears only one in the decomposition of $R\otimes R$. This is the case for SU(N) if R is the fundamental representation or is the representation of by k^{th} rank completely antisymmetric tensors.

This discussion does not apply as such to $\mathcal{N}=4$ case the number of lattice points is finite and corresponds to the number of external particles so that cyclic boundary conditions are needed guarantee that the number of lattice points reduces effectively to a finite number. Note that the Yangian in color degrees of freedom does not exist for SU(N) SYM.

As noticed, Yangian algebra is a Hopf algebra and therefore allows co-product. The co-product Δ is given by

$$\begin{aligned} \Delta(J^A) &= J^A \otimes 1 + 1 \otimes J^A \ , \\ \Delta(J^{(1)A}) &= J^{(1)A} \otimes 1 + 1 \otimes J^{(1)A} + f^A_{BC} J^B \otimes J^C \end{aligned} \tag{2.6.7}$$

Δ allows to imbed Lie algebra to the tensor product in non-trivial manner and the non-triviality comes from the addition of the dual generator to the trivial co-product. In the case that the single spin representation of $J^{(1)A}$ is trivial, the co-product gives just the expression of the dual generator using the ordinary generators as a non-local generator. This is assumed in the recent case and also for the generators of the conformal Yangian.

Super-Yangian

Also the Yangian extensions of Lie super-algebras make sense. From the point of physics especially interesting Lie super-algebras are $SU(m|m)$ and $U(m|m)$. The reason is that $PSU(2,2|4)$ (P refers to "projective") acting as super-conformal symmetries of $\mathcal{N}=4$ SYM and this super group is a real form of $PSU(4|4)$. The main point of interest is whether this algebra allows Yangian representation and Witten demonstrated that this is indeed the case [B14].

These algebras are Z_2 graded and decompose to bosonic and fermionic parts which in general correspond to n- and m-dimensional representations of $U(n)$. The representation associated with the fermionic part dictates the commutation relations between bosonic and fermionic generators. The anti-commutator of fermionic generators can contain besides identity also bosonic generators if the symmetrized tensor product in question contains adjoint representation. This is the case if fermions are in the fundamental representation and its conjugate. For $SU(3)$ the symmetrize tensor product of adjoint representations contains adjoint (the completely symmetric structure constants d_{abc}) and this might have some relevance for the super $SU(3)$ symmetry.

The elements of these algebras in the matrix representation (no Grassmann parameters involved) can be written in the form

$$x = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \ .$$

a and d representing the bosonic part of the algebra are $n\times n$ matrices and $m\times m$ matrices corresponding to the dimensions of bosonic and fermionic representations. b and c are fermionic matrices are $n\times m$ and $m\times n$ matrices, whose anti-commutator is the direct sum of $n\times n$ and $n\times n$ matrices. For $n=m$ bosonic generators transform like Lie algebra generators of $SU(n)\times SU(n)$ whereas fermionic generators transform like $n\otimes \overline{n}\oplus \overline{n}\otimes n$ under $SU(n)\times SU(n)$. Supertrace is defined as $Str(x)=Tr(a)-Tr(b)$. The vanishing of Str defines $SU(n|m)$. For $n\neq m$ the super trace condition removes identity matrix and $PU(n|m)$ and $SU(n|m)$ are same. That this does not happen for $n=m$ is an important delicacy since this case corresponds to $\mathcal{N}=4$ SYM. If any two matrices differing by an additive scalar are identified (projective scaling as now physical effect) one obtains $PSU(n|n)$ and this is what one is interested in.

Witten shows that the condition that adjoint is contained only once in the tensor product $R\otimes \overline{R}$ holds true for the physically interesting representations of $PSU(2,2|4)$ so that the generalization of the bilinear formula can be used to define the generators of $J^{(1)A}$ of super Yangian of $PU(2,2|4)$. The defining formula for the generators of the Super Yangian reads as

$$\begin{aligned} J^{(1)}_C &= g_{CC'}J^{(1)C'} = g_{CC'}f^{C'}_{AB}\sum_{i<j} J^A_i J^B_j \\ &= g_{CC'}f^{C'}_{AB}g^{AA'}g^{BB'}\sum_{i<j} J^i_{A'}J^j_{B'} \ . \end{aligned} \tag{2.6.8}$$

Here $g_{AB}=Str(J_AJ_B)$ is the metric defined by super trace and distinguishes between $PSU(4|4)$ and $PSU(2,2|4)$. In this formula both generators and super generators appear.

Generators of super-conformal Yangian symmetries

The explicit formula for the generators of super-conformal Yangian symmetries in terms of ordinary twistors is given by

$$\begin{aligned} j_B^A &= \sum_{i=1}^{n} Z_i^A \partial_{Z_i^B} \ , \\ j_B^{(1)A} &= \sum_{i<j} (-1)^C \left[Z_i^A \partial_{Z_j^C} Z_j^C \partial_{Z_j^B} \right] \ . \end{aligned} \tag{2.6.9}$$

This formula follows from completely general formulas for the Yangian algebra discussed above and allowing to express the dual generators $j_N^{(1)}$ as quadratic expression of j_N involving structures constants. In this rather sketchy formula twistors are ordinary twistors. Note however that in the recent case the lattice is replaced with its finite cutoff corresponding to the external particles of the scattering amplitude. This probably corresponds to the assumption that for the representations considered only finite number of lattice points correspond to non-trivial quantum numbers or to cyclic symmetry of the representations.

In the expression for the amplitudes the action of transformations is on the delta functions and by partial integration one finds that a total divergence results. This is easy to see for the linear generators but not so for the quadratic generators of the dual super-conformal symmetries. A similar formula but with j_B^A and $j_B^{(1)A}$ interchanged applies in the representation of the amplitudes as Grassmann integrals using ordinary twistors. The verification of the generalization of Serre formula is also straightforward.

2.6.2 Twistors And Momentum Twistors And Super-Symmetrization

In [B15] the basics of twistor geometry are summarized. Despite this it is perhaps good to collect the basic formulas here.

Conformally compactified Minkowski space

Conformally compactified Minkowski space can be described as $SO(2,4)$ invariant (Klein) quadric

$$T^2 + V^2 - W^2 - X^2 - Y^2 - Z^2 = 0 \ . \tag{2.6.10}$$

The coordinates (T, V, W, X, Y, Z) define homogenous coordinates for the real projective space RP^5. One can introduce the projective coordinates $X_{\alpha\beta} = -X_{\beta\alpha}$ through the formulas

$$\begin{array}{lll} X_{01} = W - V \ , & X_{02} = Y + iX \ , & X_{03} = \frac{i}{\sqrt{2}} T - Z \ , \\ X_{12} = -\frac{i}{\sqrt{2}}(T + Z) \ , & X_{13} = Y - iX \ , & X_{23} = \frac{1}{2}(V + W) \ . \end{array} \tag{2.6.11}$$

The motivation is that the equations for the quadric defining the conformally compactified Minkowski space can be written in a form which is manifestly conformally invariant:

$$\epsilon^{\alpha\beta\gamma\delta} X_{\alpha\beta} X_{\gamma\delta} = 0 \ per. \tag{2.6.12}$$

The points of the conformally compactified Minkowski space are null separated if and only if the condition

$$\epsilon^{\alpha\beta\gamma\delta} X_{\alpha\beta} Y_{\gamma\delta} = 0 \tag{2.6.13}$$

holds true.

Correspondence with twistors and infinity twistor

One ends up with the correspondence with twistors by noticing that the condition is equivalent with the possibility to expression $X_{\alpha\beta}$ as

$$X_{\alpha\beta} = A_{[\alpha}B_{\beta]} \ , \tag{2.6.14}$$

where brackets refer to antisymmetrization. The complex vectors A and B define a point in twistor space and are defined only modulo scaling and therefore define a point of twistor space CP_3 defining a covering of 6-D Minkowski space with metric signature $(2,4)$. This corresponds to the fact that the Lie algebras of $SO(2,4)$ and $SU(2,2)$ are identical. Therefore the points of conformally compactified Minkowski space correspond to lines of the twistor space defining spheres CP_1 in CP_3.

One can introduce a preferred scale for the projective coordinates by introducing what is called infinity twistor (actually a pair of twistors is in question) defined by

$$I_{\alpha\beta} = \begin{pmatrix} \epsilon^{A'B'} & 0 \\ 0 & 0 \end{pmatrix} \ . \tag{2.6.15}$$

Infinity twistor represents the projective line for which only the coordinate X_{01} is non-vanishing and chosen to have value $X_{01} = 1$.

One can define the contravariant form of the infinite twistor as

$$I^{\alpha\beta} = \epsilon^{\alpha\beta\gamma\delta} I_{\gamma\delta} = \begin{pmatrix} 0 & 0 \\ 0 & \epsilon^{AB} \end{pmatrix} \ . \tag{2.6.16}$$

Infinity twistor defines a representative for the conformal equivalence class of metrics at the Klein quadric and one can express Minkowski distance as

$$(x-y)^2 = \frac{X^{\alpha\beta}Y_{\alpha\beta}}{I_{\alpha\beta}X^{\alpha\beta}I_{\mu\nu}Y^{\mu\nu}} \ . \tag{2.6.17}$$

Note that the metric is necessary only in the denominator. In twistor notation the distance can be expressed as

$$(x-y)^2 = \frac{\epsilon(A,B,C,D)}{\langle AB\rangle\langle CD\rangle} \ . \tag{2.6.18}$$

Infinite twistor $I_{\alpha\beta}$ and its contravariant counterpart project the twistor to its primed and unprimed parts usually denoted by $\mu^{A'}$ and λ^A and defined spinors with opposite chiralities.

Relationship between points of M^4 and twistors

In the coordinates obtained by putting $X_{01} = 1$ the relationship between space-time coordinates $x^{AA'}$ and $X^{\alpha\beta}$ is

$$X_{\alpha\beta} = \begin{pmatrix} -\frac{1}{2}\epsilon^{A'B'}x^2 & -ix^{A'}_{\ B} \\ ix_{\ A}^{B'} & \epsilon_{A,B} \end{pmatrix} \ , \quad X^{\alpha\beta} = \begin{pmatrix} \epsilon_{A'B\ '}x^2 & -ix_{\ A'}^{B} \\ ix^A_{B'} & -\frac{1}{2}\epsilon^{AB}x^2 \end{pmatrix} \ , \tag{2.6.19}$$

If the point of Minkowski space represents a line defined by twistors (μ_U, λ_U) and (μ_V, λ_V), one has

$$x^{AC'} = i\frac{(\mu_V\lambda_U - \mu_U\lambda_V)^{AC'}}{\langle UV\rangle} \tag{2.6.20}$$

The twistor μ for a given point of Minkowski space in turn is obtained from λ by the twistor formula by

$$\mu^{A'} = -ix^{AA'}\lambda_A \ . \tag{2.6.21}$$

Generalization to the super-symmetric case

This formalism has a straightforward generalization to the super-symmetric case. CP_3 is replaced with $CP_{3|4}$ so that Grassmann parameters have four components. At the level of coordinates this means the replacement $[W_I] = [W_\alpha, \chi_\alpha]$. Twistor formula generalizes to

$$\mu^{A'} = -ix^{AA'}\lambda_A \ , \quad \chi_\alpha = \theta^A_\alpha \lambda_A \ . \tag{2.6.22}$$

The relationship between the coordinates of chiral super-space and super-twistors generalizes to

$$(x,\theta) \quad = \quad \left(i\frac{(\mu_V\lambda_U - \mu_U\lambda_V)}{\langle UV\rangle}, \frac{(\chi_V\lambda_U - \chi_U\lambda_V)}{\langle UV\rangle}\right) \tag{2.6.23}$$

The above formulas can be applied to super-symmetric variants of momentum twistors to deduce the relationship between region momenta x assigned with edges of polygons and twistors assigned with the ends of the light-like edges. The explicit formulas are represented in [B15]. The geometric picture is following. The twistors at the ends of the edge define the twistor pair representing the region momentum as a line in twistor space and the intersection of the twistor lines assigned with the region momenta define twistor representing the external momenta of the graph in the intersection of the edges.

Basic kinematics for momentum twistors

The super-symmetrization involves replacement of multiplets with super-multiplets

$$\Phi(\lambda, \tilde{\lambda}, \eta) = G^+(\lambda, \tilde{\lambda}) + \eta_i\Gamma^a\lambda, \tilde{\lambda}) + \cdots + \epsilon_{abcd}\eta^a\eta^b\eta^c\eta^d G^-(\lambda, \tilde{\lambda}) \ . \tag{2.6.24}$$

Momentum twistors are dual to ordinary twistors and were introduced by Hodges. The light-like momentum of external particle a is expressed in terms of the vertices of the closed polygon defining the twistor diagram as

$$p_i^\mu = x_i^\mu - x_{i+1}^\mu = \lambda_i\tilde{\lambda}_i \ , \quad \theta_i - \theta_{i+1} = \lambda_i\eta_i \ . \tag{2.6.25}$$

One can say that massless momenta have a conserved super-part given by $\lambda_i\eta_i$. The dual of the super-conformal group acts on the region momenta exactly as the ordinary conformal group acts on space-time and one can construct twistor space for dua region momenta.

Super-momentum conservation gives the constraints

$$\sum p_i = 0 \ , \quad \sum \lambda_i\eta_i = 0 \ . \tag{2.6.26}$$

The twistor diagrams correspond to polygons with edges with lines carrying region momenta and external massless momenta emitted at the vertices.

This formula is invariant under overall shift of the region momenta x_a^μ. A natural interpretation for x_a^μ is as the momentum entering to the vertex where p_a is emitted. Overall shift would have interpretation as a shift in the loop momentum. x_a^μ in the dual coordinate space is associated with the line $Z_{a-1}Z_a$ in the momentum twistor space. The lines $Z_{a-1}Z_a$ and Z_aZ_{a+1} intersect at Z_a representing a light-like momentum vector p_a^μ.

The brackets $\langle abcd\rangle \equiv \epsilon_{IJKL}Z_a^IZ_b^JZ_c^KZ_d^L$ define fundamental bosonic conformal invariants appearing in the tree amplitudes as basic building blocks. Note that Z_a define points of 4-D complex twistor space to be distinguished from the projective twistor space CP_3. Z_a define projective coordinates for CP_3 and one of the four complex components of Z_a is redundant and one can take $Z_a^0 = 1$ without a loss of generality.

2.6.3 Brief Summary Of The Work Of Arkani-Hamed And Collaborators

The following comments are an attempt to summarize my far from complete understanding about what is involved with the representation as contour integrals. After that I shall describe in more detail my impressions about what has been done.

Limitations of the approach

Consider first the limitations of the approach.

1. The basis idea is that the representation for tree amplitudes generalizes to loop amplitudes. On other words, the amplitude defined as a sum of Yangian invariants expressed in terms of Grassmann integrals represents the sum of loops up to some maximum loop number. The problem is here that shifts of the loop momenta are essential in the UV regularization procedure. Fixing the coordinates $x_1, \cdots, x_n$ having interpretation as momenta associated with lines in the dual coordinate space allows to eliminate the non-uniqueness due to the common shift of these coordinates.

2. It is not however not possible to identify loop momentum as a loop momentum common to different loop integrals unless one restricts to planar loops. Non-planar diagrams are obtained from a planar diagram by permuting the coordinates x_i but this means that the unique coordinate assignment is lost. Therefore the representation of loop integrands as Grassmann integrals makes sense only for planar diagrams. From TGD point of view one could argue that this is one good reason for restricting the loops so that they are for on mass shell particles with non-parallel on mass shell four-momenta and possibly different sign of energies for given wormhole contact representing virtual particle.

3. IR regularization is needed even in $\mathcal{N} = 4$ for SYM given by "moving out on the Coulomb branch theory" so that IR singularities remain the problem of the theory.

What has been done?

The article proposes a generalization of the BCFW recursion relation for tree diagrams of $\mathcal{N} = 4$ for SYM so that it applies to planar diagrams with a summation over an arbitrary number of loops.

1. The basic goal of the article is to generalize the recursion relations of tree amplitudes so that they would apply to loop amplitudes. The key idea is following. One can formally represent loop integrand as a contour integral in complex plane whose coordinate parameterizes the deformations $Z_n \to Z_n + \epsilon Z_{n-1}$ and re-interpret the integral as a contour integral with oppositely oriented contour surrounding the rest of the complex plane which can be imagined also as being mapped to Riemann sphere. What happens only the poles which correspond to lower number of loops contribute this integral. One obtains a recursion relation with respect to loop number. This recursion seems to be the counterpart for the recursive construction of the loops corrections in terms of absorptive parts of amplitudes with smaller number of loop using unitarity and analyticity.

2. The basic challenge is to deduce the Grassmann integrands as Yangian invariants. From these one can deduce loop integrals by integration over the four momenta associated with the lines of the polygonal graph identifiable as the dual coordinate variables x_a. The integration over loop momenta can induce infrared divergences breaking Yangian symmetry. The big idea here is that the operations described above allow to construct loop amplitudes from the Yangian invariants defining tree amplitudes for a larger number of particles by removing external particles by fusing them to form propagator lines and by using the BCFW bridge to fuse lower-dimensional invariants. Hence the usual iterative procedure (bottom-up) used to construct scattering amplitudes is replaced with a recursive procedure (top-down). Of course, once lower amplitudes has been constructed they can be used to construct amplitudes with higher particle number.

3. The first guess is that the recursion formula involves the same lower order contributions as in the case of tree amplitudes. These contributions have interpretation as factorization of

channels involving single particle intermediate states. This would however allow to reduce loop amplitudes to 3-particle loop amplitudes which vanish in $\mathcal{N}=4$ SYM by the vanishing of coupling constant renormalization. The additional contribution is necessary and corresponds to a source term identifiable as a "forward limit" of lower loop integrand. These terms are obtained by taking an amplitude with two additional particles with opposite four-momenta and forming a state in which these particles are entangled with respect to momentum and other quantum numbers. Entanglement means integral over the massless momenta on one hand. The insertion brings in two momenta x_a and x_b and one can imagine that the loop is represented by a branching of propagator line. The line representing the entanglement of the massless states with massless momentum define the second branch of the loop. One can of course ask whether only massless momentum in the second branch. A possible interpretation is that this state is expressible by unitarity in terms of the integral over light-like momentum.

4. The recursion formula for the loop amplitude $M_{n,k,l}$ involves two terms when one neglects the possibility that particles can also suffer trivial scattering (cluster decomposition). This term basically corresponds to the Yangian invariance of n arguments identified as Yangian invariant of $n-1$ arguments with the same value of k.

    (a) The first term corresponds to single particle exchange between particle groups obtained by splitting the polygon at two vertices and corresponds to the so called BCFW bridge for tree diagrams. There is a summation over different splittings as well as a sum over loop numbers and dimensions k for the Grassmann planes. The helicities in the two groups are opposite.

    (b) Second term is obtained from an amplitude obtained by adding of two massless particles with opposite momenta and corresponds to $n+2, k+1, l-1$. The integration over the light-like momentum together with other operations implies the reduction $n+2 \rightarrow n$. Note that the recursion indeed converges. Certainly the allowance of added zero energy states with a finite number of particles is necessary for the convergence of the procedure.

2.6.4 The General Form Of Grassmannian Integrals

If the recursion formula proposed in [B22] is correct, the calculations reduce to the construction of N^kMHV (super) amplitudes. MHV refers to maximal helicity violating amplitudes with 2 negative helicity gluons. For N^kMHV amplitude the number of negative helicities is by definition $k+2$ [B20]. Note that the total right handed R-charge assignable to 4 super-coordinates η_i of negative helicity gluons can be identified as $R=4k$. BCFW recursion formula [B9, B9] allows to construct from MHV amplitudes with arbitrary number of negative helicities.

The basic object of study are the leading singularities of color-stripped n-particle N^kMHV amplitudes. The discovery is that these singularities are expressible in terms Yangian invariants $Y_{n,k}(Z_1,\cdots,Z_n)$, where Z_i are momentum super-twistors. These invariants are defined by residue integrals over the compact $nk-1$-dimensional complex space $G(n,k)=U(n)/U(k)\times U(n-k)$ of k-planes of complex n-dimensional space. n is the number of external massless particles, k is the number negative helicity gluons in the case of N^kMHV amplitudes, and Z_a, $i=1,\cdots,n$ denotes the projective 4-coordinate of the super-variant $CP^{3|4}$ of the momentum twistor space CP_3 assigned to the massless external particles is following. $Gl(n)$ acts as linear transformations in the n-fold Cartesian power of twistor space. Yangian invariant $Y_{n,k}$ is a function of twistor variables Z^a having values in super-variant $CP_{3|3}$ of momentum twistor space CP_3 assigned to the massless external particles being simple algebraic functions of the external momenta.

It is also possible to define N^kMHV amplitudes in terms of Yangian invariants $L_{n,k+2}(W_1,\cdots,W_n)$ by using ordinary twistors W_a and identical defining formula. The two invariants are related by the formula $L_{n,k+2}(W_1,\cdots,W_n)=M^{tree}_{MHV}\times Y_{n,k}(Z_1,\cdots,Z_n)$. Here M^{tree}_{MHV} is the tree contribution to the maximally helicity violating amplitude for the scattering of n particles: recall that these amplitudes contain two negative helicity gluons whereas the amplitudes containing a smaller number of them vanish [B11]. One can speak of a factorization to a product of n-particle amplitudes with $k-2$ and 2 negative helicities as the origin of the duality. The equivalence between the descriptions based on ordinary and momentum twistors states the dual conformal invariance of the amplitudes implying Yangian symmetry. It has been conjectured that Grassmannian integrals generate all Yangian invariants.

The formulas for the Grassmann integrals for twistors and momentum twistors appearing in the expressions of N^kMHV amplitudes are given by following expressions.

1. The integrals $L_{n,k}(W_1,\cdots,W_n)$ associated with $N^{k-2}MHV$ amplitudes in the description based on ordinary twistors correspond to k negative helicities and are given by

$$\begin{aligned} L_{n,k}(W_1,\cdots,W_n) &= \frac{1}{Vol(GL(2))}\int \frac{d^{k\times n}C_{\alpha a}}{(1\cdots k)(2\cdots k+1)\cdots(n1\cdots k-1)}\times \\ &\times \prod_{\alpha=1}^{k} d^{4|4}Y_\alpha \prod_{i=1}^{n}\delta^{4|4}(W_i - C_{\alpha i}Y_\alpha) \ . \end{aligned} \tag{2.6.27}$$

   Here $C_{\alpha a}$ denote the $n\times k$ coordinates used to parametrize the points of $G_{k,n}$.

2. The integrals $Y_{n,k}(W_1,\cdots,W_n)$ associated with N^kMHV amplitudes in the description based on momentum twistors are defined as

$$Y_{n,k}(Z_1,\cdots,Z_n) = \frac{1}{Vol(GL(k))}\times\int \frac{d^{k\times n}C_{\alpha a}}{(1\cdots k)(2\cdots k+1)\cdots(n1\cdots k-1)}\times\prod_{\alpha=1}^{k}\delta^{4|4}(C_{\alpha a}Z_a) \ . \tag{2.6.28}$$

   The possibility to select $Z_a^0 = 1$ implies $\sum_k C_{\alpha k} = 0$ allowing to eliminate $C_{\alpha n}$ so that the actual number of coordinates Grassman coordinates is $nk-1$. As already noticed, $L_{n,k+2}(W_1,\cdots,W_n) = M_{MHV}^{tree}\times Y_{n,k}(Z_1,\cdots,Z_n)$. Momentum twistors are obviously calculationally easier since the value of k is smaller by two units.

The $4k$ delta functions reduce the number of integration variables of contour integrals from nk to $(n-4)k$ in the bosonic sector (the definition of delta functions involves some delicacies not discussed here). The n quantities $(m,\cdots m+k)$ are $k\times k$-determinants defined by subsequent columns from m to $m+k-1$ of the $k\times n$ matrix defined by the coordinates $C_{\alpha a}$ and correspond geometrically to the k-volumes of the k-dimensional parallel-pipeds defined by these column vectors. The fact that the scalings of twistor space coordinates Z_a can be compensated by scalings of $C_{\alpha a}$ deforming integration contour but leaving the residue integral invariant so that the integral depends on projective twistor coordinates only.

Since the integrand is a rational function, a multi-dimensional residue calculus allows to deduce the values of these integrals as residues associated with the poles of the integrand in a recursive manner. The poles correspond to the zeros of the $k\times k$ determinants appearing in the integrand or equivalently to singular lower-dimensional parallel-pipeds. It can be shown that local residues are determined by $(k-2)(n-k-2)$ conditions on the determinants in both cases. The value of the integral depends on the explicit choice of the integration contour for each variable $C_{\alpha a}$ left when delta functions are taken into account. The condition that a correct form of tree amplitudes is obtained fixes the choice of the integration contours.

For the ordinary twistors W the residues correspond to projective configurations in CP_{k-1}, or more precisely in the space $CP_{k-1}^n/Gl(k)$, which is $(k-1)n-k^2$-dimensional space defining the support for the residues integral. $Gl(k)$ relates to each other different complex coordinate frames for k-plane and since the choice of frame does not affect the plane itself, one has $Gl(k)$ gauge symmetry as well as the dual $Gl(n-k)$ gauge symmetry.

CP_{k-1} comes from the fact that C_{α_k} are projective coordinates: the amplitudes are indeed invariant under the scalings $W_i\rightarrow t_iW_i$, $C_{\alpha i}\rightarrow tC_{\alpha i}$. The coset space structure comes from the fact that $Gl(k)$ is a symmetry of the integrand acting as $C_{\alpha i}\rightarrow \Lambda_\alpha^{\ \beta}C_{\beta_i}$. This analog of gauge symmetry allows to fix k arbitarily chosen frame vectors $C_{\alpha i}$ to orthogonal unit vectors. For instance, one can have $C_{\alpha i} = \delta_{\alpha_i}$ for $\alpha = i \in 1, \cdots, k$. This choice is discussed in detail in [B20]. The reduction to CP_{k-1} implies the reduction of the support of the integral to line in the case of MHV amplitudes and to plane in the case of NMHV as one sees from the expression $d\mu = \prod_\alpha d^{4|4}Y_\alpha \prod_{i=1}^n \delta^{4|4}(W_i - C_{\alpha i}Y_\alpha)$. For $(i_1, \cdots, i_k) = 0$ the vectors $i_1, ..i_k$ belong to $k-2$-dimensional plane of CP_{k-1}. In the case of $NMHV$ (N^2MHV) amplitudes this translates at the level of twistors to the condition that the corresponding twistors $\{i_1, i_2, i_3\}$ ($\{i_1, i_2, i_3, i_4\}$) are collinear (in the same plane) in twistor space. This can be understood from the fact that the delta functions in $d\mu$ allow to express W_i in terms of $k-1$ Y_α: s in this case.

The action of conformal transformations in twistor space reduces to the linear action of $SU(2,2)$ leaving invariant Hermitian sesquilinear form of signature $(2,2)$. Therefore the conformal invariance of the Grassmannian integral and its dual variant follows from the possibility to perform a compensating coordinate change for $C_{\alpha a}$ and from the fact that residue integral is invariant under small deformations of the integration contour. The above described relationship between representations based on twistors and momentum twistors implies the full Yangian invariance.

2.6.5 Canonical Operations For Yangian Invariants

General l-loop amplitudes can be constructed from the basic Yangian invariants defined by N^kMHV amplitudes by various operations respecting Yangian invariance apart from possible IR anomalies. There are several operations that one can perform for Yangian invariants $Y_{n,k}$ and all these operations appear in the recursion formula for planar all loop amplitudes. These operations are described in [B22] much better than I could do it so that I will not go to any details. It is possible to add and remove particles, to fuse two Yangian invariants, to merge particles, and to construct from two Yangian invariants a higher invariant containing so called BCFW bridge representing single particle exchange using only twistorial methods.

Inverse soft factors

Inverse soft factors add to the diagram a massless collinear particles between particles a and b and by definition one has

$$O_{n+1}(a, c, b, \cdots) = \frac{\langle ab\rangle}{\langle ac\rangle\langle cb\rangle} O_n(a'b') \ . \tag{2.6.29}$$

At the limit when the momentum of the added particle vanishes both sides approach the original amplitude. The right-handed spinors and Grassmann parameters are shifted

$$\begin{aligned} \tilde{\lambda}'_a &= \tilde{\lambda}_a + \tfrac{\langle cb\rangle}{\langle ab\rangle}\tilde{\lambda}_c \ , \quad \tilde{\lambda}'_b = \tilde{\lambda}_b + \tfrac{\langle ca\rangle}{\langle ba\rangle}\tilde{\lambda}_c \ , \\ \eta'_a &= \eta_a + \tfrac{\langle cb\rangle}{\langle ab\rangle}\eta_c \ , \quad \eta'_b = \eta_b + \tfrac{\langle ca\rangle}{\langle ba\rangle}\eta_c \ . \end{aligned} \tag{2.6.30}$$

There are two kinds of inverse soft factors.

1. The addition of particle leaving the value k of negative helicity gluons unchanged means just the re-interpretation

$$Y'_{n,k}(Z_1, \cdots, Z_{n-1}, Z_n) = Y_{n-1,k}(Z_1, \cdots, Z_{n-1}) \tag{2.6.31}$$

without actual dependence on Z_n. There is however a dependence on the momentum of the added particle since the relationship between momenta and momentum twistors is modified by the addition obtained by applying the basic rules relating region super momenta and momentum twistors (light-like momentum determines λ_i and twistor equations for x_i and λ_i, η_i determine (μ_i, χ_i)) is expressible assigned to the external particles [B37]. Modifications are needed only for the new vertex and its neighbors.

2. The addition of a particle increasing k with single unit is a more complex operation which can be understood in terms of a residue of $Y_{n,k}$ proportional to $Y_{n-1,k-1}$ and Yangian invariant $[z_1 \cdots z_5]$ with five arguments constructed from basic Yangian invariants with four arguments. The relationship between the amplitudes is now

$$Y'_{n,k}(.., Z_{n-1}Z_n, Z_1 \cdots) = [n-2\ n-1\ n\ 1\ 2] \times Y_{n-1,k-1}(\cdots \hat{Z}_{n-1}, \hat{Z}_1, \cdots) \quad (2.6.32)$$

Here

$$[abcde] = \frac{\delta^{0|4}(\eta_a \langle bcde\rangle + cyclic)}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle} . \quad (2.6.33)$$

denoted also by $R(a, b, c, d, e)$ is the fundamental R-invariant appearing in one loop corrections of MHV amplitudes and will appears also in the recursion formulas. $\langle abcd\rangle$ is the fundamental super-conformal invariant associated with four super twistors defined in terms of the permutation symbol.

$\hat{Z}_{n-1}, \hat{Z}_1$ are deformed momentum twistor variables. The deformation is determined from the relationship between external momenta, region momenta and momentum twistor variables. $\hat{Z}^1$ is the intersection $\hat{Z}^1 = (n-2\ n-1\ 2) \cap (12)$ of the line (12) with the plane $(n-2\ n-1\ 2)$ and $\hat{Z}^{n-1}$ the intersection $\hat{Z}^1 = (12n) \cap (n-2\ n-1)$ of the line $(n-2\ n-1)$ with the plane $(12n)$. The interpretation for the intersections at the level of ordinary Feynman diagrams is in terms of the collinearity of the four-momenta involved with the underlying box diagram with parallel on mass shell particles. These result from unitarity conditions obtained by putting maximal number of loop momenta on mass shell to give the leading singularities.

The explicit expressions for the momenta are

$$\begin{aligned} \hat{Z}^1 &\equiv (n-2\ n-1\ 2) \cap (12)Z_1 = \langle 2\ n-2\ n-1\ n\rangle + Z_2\langle n-2\ n-1\ n\ 1\rangle , \\ \hat{Z}^{n-1} &\equiv (12n) \cap (n-2\ n-1) = Z_{n-2}\langle n-2\ n-1\ n\ 2\rangle + Z_{n-1}\langle n\ 1\ 2\ n-2\rangle . \end{aligned} \quad (2.6.34)$$

These intersections also appear in the expressions defining the recursion formula.

Removal of particles and merge operation

Particles can be also removed. The first manner to remove particle is by integrating over the twistor variable characterizing the particle. This reduces k by one unit. Merge operation preserves the number of loops but removes a particle particle by identifying the twistor variables of neighboring particles. This operation corresponds to an integral over on mass shell loop momentum at the level of tree diagrams and by Witten's half Fourier transform can be transformed to twistor integral.

The product

$$Y'(Z_1, \cdots Z_n) = Y_1(Z_1, \cdots Z_m) \times Y_2(Z_{m+1}, \cdots Z_n) \quad (2.6.35)$$

of two Yangian invariants is again a Yangian invariant. This is not quite trivial since the dependence of region momenta and momentum twistors on the momenta of external particles makes the operation non-trivial.

Merge operation allows to construct more interesting invariants from the products of Yangian invariants. One begins from a product of Yangian invariants (Yangian invariant trivially) represented cyclically as points of circle and identifies the last twistor argument of given invariant with the first twistor argument of the next invariant and performs integrals over the momentum twistor variables appearing twice. The soft k-increasing and preserving operations can be described also in terms of this operation for Yangian invariants such that the second invariant corresponds to 3-vertex. The cyclic merge operation applied to four MHV amplitudes gives NMHV amplitudes associated with on mass shell momenta in box diagrams. By applying similar operation to NMHV amplitudes and MHV amplitudes one obtains 2-loop amplitudes. In [B22] examples about these operations are described.

BCFW bridge

BCFW bridge allows to build general tree diagrams from MHV tree diagrams [B9, B9] and recursion formula of [B22] generalizes this to arbitrary diagrams. At the level of Feynman diagrams it corresponds to a box diagram containing general diagrams labeled by L and R and MHV and $\overline{MHV}$ 3-vertices ($\overline{MHV}$ 3-vertex allows expression in terms of MHV diagrams) with the lines of the box on mass shell so that the three momenta emanating from the vertices are parallel and give rise to a one-loop leading singularity.

At the level of Feynman diagrams BCFW bridge corresponds to so called "two-mass hard" leading singularities associated with box diagrams with light-like momenta at the four lines of the diagram [B20]. The motivation for the study of these diagrams comes from the hypothesis the leading order singularities obtained by putting as many particles as possible on mass shell contain the data needed to construct scattering amplitudes of $\mathcal{N}=4$ SYM completely. This representation of the leading singularities generalizes to arbitrary loops. The recent article is a continuation of this program to planar amplitudes.

Also BCFW bridge allows an interpretation as a particular kind fusion for Yang invariants and involves all the basic operations. One starts from the amplitudes $Y^L_{n_L,k_L}$ and $Y^R_{n_R,k_R}$ and constructs an amplitude $Y'_{n_L+n_R,k_L+k_R+1}$ representing the amplitude which would correspond to a generalization of the MHV diagrams with the two tree diagrams connected by the MHV propagator (BCFW bridge) replaced with arbitrary loop diagrams. Particle "1" *resp.* "j+1" is added by the soft k-increasing factor to Y_{n_L+1,k_L+1} *resp.* Y_{n_R+1,k_R+1} giving amplitude with $n+2$ particles and with k-charge equal to k_L+k_R+2. The subsequent operations must reduce k-charge by one unit. First repeated "1" and "j+1" are identified with their copies by k conserving merge operation, and after that one performs an integral over the twistor variable Z^I associated with the internal line obtained and reducing k by one unit. The soft k-increasing factors bring in the invariants $[n-1\ n\ 1\ I\ j+2]$ associated with Y_L and $[1\ I\ j+1\ j\ j-1]$ associated with Y_R. The integration contour is chosen so that it selects the pole defined by $\angle n-1\ n\ 1\ I\rangle$ in the denominator of $[n-1\ n\ 1\ I\ j+2]$ and the pole defined by $\langle 1\ I\ j+1\ j\rangle$ in the denominator of $[1\ I\ j+1\ j\ j-1]$.

The explicit expression for the BCFW bridge is very simple:

$$\begin{aligned}(Y_L \otimes_{BCFW} Y_R)(1,\cdots,n) &= [n-1\ n\ 1\ j\ j+1]\times Y_R(1,\cdots,j,I)Y_L(I,j+1,\cdots,n-1,\hat{n}) \ , \\ \hat{n} &= (n-1\ n)\cap(j\ j+1\ 1) \ , \ \ I=(j\ j+1)\cap(n-1\ n\ 1) \ . \end{aligned} \quad (2.6.36)$$

Single cuts and forward limit

Forward limit operation is used to increase the number of loops by one unit. The physical picture is that one starts from say 1-loop amplitude and cuts one line by assigning to the pieces of the line opposite light-like momenta having interpretation as incoming and outgoing particles. The resulting amplitude is called forward limit. The only reasonable interpretation seems to be that the loop integration is expressed by unitarity as forward limit meaning cutting of the line carrying the loop momentum. This operation can be expressed in a manifestly Yangian invariant way as entangled removal of two particles with the merge operation meaning the replacement $Z_n \rightarrow Z_{n-1}$. Particle $n+1$ is added adjacent to A, B as a k-increasing inverse soft factor and then A and B are removed by entangled integration, and after this merge operation identifies $n+1$ and 1.

Forward limit is crucial for the existence of loops and for Yangian invariants it corresponds to the poles arising from $\langle (AB)_q Z_n(z) Z_1)\rangle$ the integration contour $Z_n + zZ_{n-1}$ around Z_b in the basic formula $M=\oint(dz/z)M_n$ leading to the recursion formula. A and B denote the momentum twistors associated with opposite light-like momenta. In the generalized unitarity conditions the singularity corresponds to the cutting of line between particles n and 1 with momenta q and $-q$, summing over the multiplet of stats running around the loop. Between particles n_2 and 1 one has particles $n-1$, n with momenta $q, -q$. $q = x_1 - x_n = -x_n + x_{n-1}$ giving $x_1 = x_{n-1}$. Light-likeness of q means that the lines $(71) = (76)$ and (15) intersect. At the forward limit giving rise to the pole Z_6 and Z_7 approach to the intersection point $(76) \cap (15)$. In a generic gauge theories the forward limits are ill-defined but in super-symmetric gauge theories situation changes.

The corresponding Yangian operation removes two external particles with opposite four-momenta and involves integration over two twistor variables Z_a and Z_b and gives rise to the following expression

$$\int_{GL(2)} Y(\cdots, Z_n, Z_A, Z_B, Z_1, \cdots) \ . \tag{2.6.37}$$

The integration over $GL(2)$ corresponds to integration over twistor variables associated Z_A and Z_B. This operation allows addition of a loop to a given amplitude. The line $Z_a Z_b$ represents loop momentum on one hand and the dual x-coordinate identified as momentum propagating along the line on the other hand.

The integration over these variables is equivalent to an integration over loop momentum as the explicit calculation of [B22] (see pages 12-13) demonstrates. If the integration contours are products in the product of twistor spaces associated with a and b the and gives lower order Yangian invariant as answer. It is however also possible to choose the integration contour to be entangled in the sense that it cannot be reduced to a product of integration contours in the Cartesian product of twistor spaces. In this case the integration gives a loop integral. In the removal operation Yangian invariance can be broken by IR singularities associated with the integration contour and the procedure does not produce genuine Yangian invariant always.

What is highly interesting from TGD point of view is that this integral can be expressed as a contour integral over $CP_1 \times CP_1$ combined with integral over loop momentum. If TGD vision about generalized Feynman graps in zero energy ontology is correct, the loop momentum integral is discretized to an an integral over discrete mass shells and perhaps also to a sum over discretized momenta and one can therefore avoid IR singularities.

2.6.6 Explicit Formula For The Recursion Relation

Recall that the recursion formula is obtained by considering super-symmetric momentum-twistor deformation $Z_n \rightarrow Z_n + zZ_{n-1}$ and by integrating over z to get the identity

$$M_{n,k,l} \ = \ \oint \frac{dz}{z} \hat{M}_{n,k,l}(z) \ . \tag{2.6.38}$$

This integral equals to integral with reversed integration contour enclosing the exterior of the contour. The challenge is to deduce the residues contributing to the residue integral and the claim of [B22] is that these residues reduce to simple basic types.

1. The first residue corresponds to a pole at infinity and reduces the particle number by one giving a contribution $M_{n-1,k,l}(1, \cdots, n-1)$ to $M_{n,k,l}(1, \cdots, n-1, n)$. This is not totally trivial since the twistor variables are related to momenta in different manner for the two amplitudes. This gives the first contribution to the right hand side of the formula below.

2. Second pole corresponds to the vanishing of $\langle Z_n(z) Z_1 Z_j Z_{j+1} \rangle$ and corresponds to the factorization of channels. This gives the second BCFW contribution to the right hand side of the formula below. These terms are however not enough since the recursion formula would imply the reduction to expressions involving only loop corrections to 3-loop vertex which vanish in $\mathcal{N} = 4$ SYM.

3. The third kind of pole results when $\langle (AB)_q Z_n(z) Z_1 \rangle$ vanishes in momentum twistor space. $(AB)_q$ denotes the line in momentum twistor space associated with q: th loop variable.

The explicit formula for the recursion relation yielding planar all loop amplitudes is obtained by putting all these pieces together and reads as

$$\begin{aligned} M_{n,k,l}(1,\cdots,n) &= M_{n-1,k,l}(1,\cdots,n-1) \\ &+ \sum_{n_L,k_L,l_L;j} [j\ j+1\ n-1\ n\ 1]\, M^R_{n_R,k_R,l_R}(1,\cdots,j,I_j)\times M^L_{n_L,k_L,l_L}(I_j,j+1,\cdots,\hat{n}_j) \\ &+ \int_{GL(2)} [AB\ n-1\ n\ 1]\, M_{n+2,k+1,n,k-1}(1,\cdots,\hat{n}_{AB},\hat{A},B)\ , \\ n_L &+ n_R = n+2\ ,\ \ k_L+k_R=k-1\ ,\ \ l_R+l_L=l\ . \end{aligned} \tag{2.6.39}$$

The momentum super-twistors are given by

$$\begin{aligned} \hat{n}_j &= (n-1\ n)\cap(j\ j+1\ 1)\ ,\quad I_j=(j\ j+1\ 1)\cap(n-1\ n\ 1)\ , \\ \hat{n}_{AB} &= (n-1\ n)\cap(AB\ 1)\ ,\quad \hat{A}=(AB)\cap(n-1\ n\ 1)\ . \end{aligned} \tag{2.6.40}$$

The index l labels loops in $n+2$-particle amplitude and the expression is fully symmetrized with equal weight for all loop integration variables $(AB)_l$. A and B are removed by entangled integration meaning that $GL(2)$ contour is chosen to encircle points where both points A,B on the line (AB) are located at the intersection of the line (AB) with the plane $(n-1\ n\ 1)$. GL(2) integral can be done purely algebraically in terms of residues.

In [B22] and [B37] explicit calculations for N^kMHV amplitudes are carried out to make the formulas more concrete. For N^1MHV amplitudes second line of the formula vanishes and the integrals are rather simple since the determinants are 1×1 determinants.

Chapter 3

From Principles to Diagrams

3.1 Introduction

The generalization of twistor diagrams to TGD framework has been very inspiring (and also frightening) mission impossible and allowed to gain deep insights about what TGD diagrams could be mathematically. I of course cannot provide explicit formulas but the general structure for the construction of twistorial amplitudes in $\mathcal{N} = 4$ SUSY suggests an analogous construction in TGD thanks to huge symmetries of TGD and unique twistorial properties of $M^4 \times CP_2$. The twistor program in TGD framework has been summarized in [L3].

Contrary to the original expectations, the twistorial approach is not a mere reformulation but leads to a first principle identification of cosmological constant and perhaps also of gravitational constant and to a modification of the dynamics of Kähler action however preserving the known extremals and basic properties of Kähler action and allowing to interpret induced Kähler form in terms of preferred imaginary unit defining twistor structure.

There are some new results forcing a profound modification of the recent view about TGD but consistent with the general picture. A more explicit realization of twistorialization as lifting of the preferred extremal X^4 of Kähler action to corresponding 6-D twistor space X^6 identified as surface in the 12-D product of twistor spaces of M^4 and CP_2 allowing Kähler structure suggests itself. The fiber F of Minkowskian twistor space must be identified with sphere S^2 with signature $(-1,-1)$ and would be a variant of the complex space with complex coordinates associated with S^2 and transversal space E^2 in the decomposition $M^4 = M^2 \times E^2$ and one hyper-complex coordinate associated with M^2.

The action principle in 6-D context is also Kähler action, which dimensionally reduces to Kähler action plus cosmological term. This brings in the radii of spheres $S^2(M^4)$ and $S^2(CP_2)$ associated with the twistors space of M^4 and CP_2. For $S(CP_2)$ the radius is of order CP_2 radius R. $R(S^2(M^4))$ could be of the order of Planck length l_P, which would thus become purely classical parameter contrary the expectations. An alternative option is $R(S^2(M^4)) = R$ The radius of S^2 associated with space-time surface is determined by the induced metric and is emergent length scale. The normalization of 6-D Kähler action by a scale factor $1/L^2$ with dimension, which is inverse length squared brings in a further length scale closely related to cosmological constant which is also dynamical and has correct sign to explain accelerated expansion of the Universe. The order of magnitude for L must be radius of the $S^2(X^4)$ and therefore small. This could mean a gigantic cosmological constant. Just as in GRT based cosmology!

This issue can be solved by using the observation that thanks to the decomposition $H = M^4 \times CP_2$ 6-D Kähler action is a sum of two independent terms. The first term corresponds to the 6-D lift of the ordinary Kähler action and for it the contribution from $S^2(CP_2)$ fiber is assumed to be absent: this could be due to the imbedding of $S^2(X^4)$ reducing to identification $S^2(M^4)$ and is not true generally. Second term in action is assumed to come from the $S^2(M^4)$ fiber of twistor space $T(M^4)$. The independency implies that couplings strengths are independent for them.

The analog for Kähler coupling strength (analogous to critical temperature) associated with $S^2(M^4)$ must be extremely large - so large that one has $\alpha_K(M^4) \times R(M^4)^2 \sim L^2$, L size scale of the recent Universe. This makes possible the small value of cosmological constant assignable to the volume term given by this part of the dimensionally reduced action. Both Kähler coupling strengths are assumed to have a spectrum determined by quantum criticality and the spectrum of $\alpha_K(M^4)$ comes essentially as p-adic primes satisfying p-adic length scale hypothesis $p \simeq 2^k$, k prime. In fact, it turns that one can assumed that the entire 6-D Kähler action contributes if one assumes that the winding numbers (w_1, w_2) for the map $S^2(X^4) \to S^2(M^4) \times S^2(CP_2)$ satisfy $(w_1, w_2) = (n, 0)$ in cosmological scales. The identification of w_1 as $h_{eff}/h = n$ is highly suggestive.

The dimensionally reduced dynamics is a highly non-trivial modification of the dynamics of Kähler action however preserving the known extremals and basic properties of Kähler action and allowing to interpret induced Kähler form in terms of preferred imaginary unit defining twistor structure. Strong constraints come also from the condition that induced spinor structure coming from that for twistor space $T(H)$ is essentially that coming from that of H.

Second new element is the fusion of the twistorial approach with the vision that diagrams are representations for computations. This as also quantum criticality demands that the diagrams should allow huge symmetries allowing to transform them to braided generalizations of tree-diagrams. Several guiding principles are involved and what is new is the observation that they indeed seem to form a coherent whole.

In the sequel I will discuss the recent understanding of twistorizalization, which is considerably improved from that in the earlier formulation. I formulate the dimensional reduction of 6-D Kähler action and consider the physical interpretation. There are considerable uncertainties at the level of details I dare believe that basically the situation is understood. After that I proceed to discuss the basic principles behind the recent view about scattering amplitudes as generalized Feynman diagrams.

3.2 Twistor lift of Kähler action

First I will try to clarify the mathematical details related to the twistor spaces and how they emerge in the recent context. I do not regard myself as a mathematician in technical sense and I can only hope that the representation based on physical intuition does not contain serious mistakes.

3.2.1 Embedding space is twistorially unique

It took roughly 36 years to learn that M^4 and CP_2 are twistorially unique. Space-times are surfaces in $H = M^4 \times CP_2$. M^4 and CP_2 are unique 4-manifolds in the sense that both allow twistor space with Kähler structure: Kähler structure is the crucial concept. Strictly speaking, it is E^4 and S^4 allow twistor space with Kähler structure [A18] : in the case of M^4 signature could cause problems. The standard identification for the twistor space of M^4 would be Minkowskian variant $PT = P_3 = SU(2,2)/SU(2,1) \times U(1)$ of 6-D twistor space $PT = CP_3 = SU(4)/SU(3) \times U(1)$ of E^4. The twistor space of CP_2 is 6-D $T(CP_2) = SU(3)/U(1) \times U(1)$, the space for the choices of quantization axes of color hypercharge and isospin.

The case of M^4 is however problematic. It is often stated that the twistor space is $PT = CP_3 = SU(4)/SU(3) \times U(1)$. The metric of twistor space does not appear in the construction of twistor amplitudes. Already the basic structure of PT suggests that this identification cannot be correct.

As if the situation were not complicated enough, there are two notions of twistor space: the twistor space identified as P_3 and as a trivial sphere bundle $M^4 \times CP_1$ having Kähler structure - what Kähler structure actually means in case of M^4 is hower not quite clear.

These considerations lead to a proposal - just a proposal - for the formulation of TGD in which space-time surfaces X^4 in H are lifted to twistor spaces X^6, which are sphere bundles over X^4 and such that they are surfaces in 12-D product space $T(M^4) \times T(CP_2)$ such the twistor structure of X^4 are in some sense induced from that of $T(M^4) \times T(CP_2)$. In the following $T(M^4)$ therefore denotes the trivial sphere bundle $M^4 \times CP_1$ over M^4 and twistorialization of scattering amplitudes would involve the projection from $T(M^4)$ to P_3. What is nice in this formulation is that one could use all the machinery of algebraic geometry so powerful in superstring theory (Calabi-Yau manifolds).

3.2.2 Some basic definitions

What twistor structure in Minkowskian signature does really mean geometrically has remained a confusing question for me. The problems associated with the Minkowskian signature of the metric are encountered also in twistor Grassmann approach to the scattering amplitudes but are circumvented by performing Wick rotation that is using E^4 or S^4 instead of M^4 and applying algebraic continuation. Also complexification of Minkowksi space for momenta is used. These tricks do not apply now.

To make this more concrete, let us sum up the basic definitions.

1. Bi-spinors in representations (1/2,0) and (0,1/2) of Lorentz group are the building bricks of twistors. Bi-spinors v^a and their conjugates $v^{a'}$ have the following inner products:

$$\begin{aligned} \langle vw\rangle = \epsilon_{ab}v^a w^b \ \ , \quad [vw] = \epsilon_{a'b'}v^{a'}w^{b'} \ \ , \\ \epsilon_{ab} = (0,1;-1,0) \ \ , \quad \epsilon_{a'b'} = (0,1;-1,0) \ \ . \end{aligned} \tag{3.2.1}$$

   Unprimed spinor and its primed variant of the spinor are related by complex conjugation. Index raising is by the inverse ϵ^{ab} of ϵ_{ab}.

2. Twistors are identified as pairs of 2-spinor and its conjugate

$$Z^\alpha = (\lambda_a, \mu^{a'}) \ \ , \quad \overline{Z}_\alpha = (\overline{\mu}^a, \lambda_{a'}) \tag{3.2.2}$$

   The norm for Z^α is defined as

$$Z^\alpha \overline{Z}^\alpha = \langle \lambda\overline{\mu}\rangle + [\overline{\lambda}\mu] \ \ . \tag{3.2.3}$$

   One can write the metric explicitly as direct sum of terms of form $dudv$ (metric of M^2) and each of the can be taken to diagonal form (1,-1). Hence the metric can be written as $diag(1,1,1,1,-1,-1,-1,-1)$.

3. This norm allows to decompose PT to 3 parts PT_+,PT_- and PN in a projectively invariant manner depending on whether the sign of the norm is negative, positive, or whether it vanishes. PT_+ and PT_- serve as loci for the twistor lifts of positive and negative energy modes of massless fields. PN corresponds to the 5-D boundary of the lightcone of $M(2,4)$. By projective identification along light-like radial coordinate it reduces to what is known as conformal compactification of M^4, whose metric is defined only apart from a conformal factor. The natural metric of $PT = P_3$ does not seem to play any role in the construction of the amplitudes relying on projective invariants. The signature of M^4 metric however makes itself visible in the structure of PT: for the Euclidian variant of twistor space one would not have this decomposition to three parts.

Another definition of twistor space - to be used in the geometrization of twistor approach to be proposed - is as a trivial S^2 bundle $M^4 \times CP_1$ over M^4. Since the twistor spheres associated with the points of M^4 with light-like separation intersect, these two definitions cannot be equivalent. In fact, the proper definition of twistor space relies on double fibration involving both views about twistor space discussed in [B42] (see `http://tinyurl.com/yb4bt74l`).

1. The twistor bundle denoted as PS is the product $M^4 \times CP_1$ with CP_1 realized as projective space and having coordinates $(x^{aa'}, \lambda_a)$, $\{x^{aa'}\} \leftrightarrow x^\mu \sigma_\mu$, where the spinor λ_a is projective 2-spinor in $(1/2, 0)$ representation.

2. The twistors defined in this manner have a trivial projection q to M^4 and non-trivial projection p to P_3 with local projective coordinates $(\lambda_a, \mu^{a'})$. The projection p is defined by the projectively invariant incidence relation

$$\mu^{a'} = ix^{aa'}\lambda_a$$

If $y^{aa'}$ and $a^{aa'}$ differ by light-like vector there exists spinor λ annihilated by the difference vector and there exists twistor $(\lambda_a, \mu^{a\prime})$ to which both (x, λ) and (y, λ) are mapped by the incidence relation. Thus the images of twistor spheres associated for points with light-like separation intersect so that one does not have a proper CP_1 bundle structure.

3. The trivial twistor bundle $T(M^4) = M^4 \times CP_1$ would define the twistor space of M^4 in geometric sense. For this space the metric matters and the radius of CP_1 turns out to allow identification in terms of Planck length. Gravitational interaction would bring in Planck length as a basic scale in this manner. PT in turn would define the twistor space in which the twistor lifts of embedding space-spinor fields are defined. For this space the metric, which is degenerate and seems to be only projectively defined should not be relevant as the construction of twistorial amplitudes suggests. Note however that the identification as the Minkowskian variant of P_3 allows also the introduction of metric.

This picture has an important immediate implication for the construction of quantum TGD. Positive and negative energy parts of zero energy states are defined at light-like boundaries of $CD \times CP_2$, where CD is the intersection of future and past directed light-cones. The twistor lifts of the amplitudes from $\delta CD \times CP_2$ must be single valued. The strongest condition guaranteeing this is that they do not depend on the radial light-like coordinate at δCD. Super-symplectic symmetry implying the analog of conformal gauge symmetry for the radial light-like coordinate could guarantee this. There is however a hierarchy of conformal gauge symmetry breakings corresponding to the inclusion hierarchy of isomorphic sub-algebras so that this condition is too strong. A weaker condition is that the amplitude $F(m, \lambda)$ in $T(M^4)$ is constant along the light-like ray for the λ associated with the m along this ray. An even stronger condition is that $F(m, \lambda)$ vanishes along the ray. Particle would not propagate along δCD and would avoid remaining at the boundary of CD, a condition which is perfectly sensible physically.

3.2.3 What does twistor structure in Minkowskian signature really mean?

The following considerations relate to $T(M^4)$ identified as trivial bundle $M^4 \times CP_1$ with natural coordinates $(m^{aa'}, \lambda_a)$, where λ_a is projective spinor. The challenge is to generalize the complex structure of twistor space of E^4 to that for M^4. It turns out that the assumption that twistor space has ordinary complex structure fails. The first guess was that the fiber of twistor space is hyperbolic sphere with metric signature $(1, -1)$ having infinite area so that the 6-D Kähler action would be infinite. This makes no sense. The only alternative, which comes in mind is a hypercomplex generalization of the Kähler structure for M^4 lifted to twistor space, which locally means only adding of S^2 fiber with metric signature $(-1, -1)$.

1. To proceed one must make an explicit the definition of twistor space. The 2-D fiber S^2 consists of antisymmetric tensors of X^4 which can be taken to be self-dual or anti-self-dual by taking any antisymmetric form and by adding to its plus/minus its dual. Each tensor of this kind defines a direction - point of S^2. These points can be also regarded as quaternionic imaginary units. One has a natural metric in S^2 defined by the X^4 inner product for antisymmetric tensors: this inner product depends on space-time metric. Kähler action density is example of a norm defined by this inner product in the special case that the antisymmetric tensor is induced Kähler form. Induced Kähler form defines a preferred imaginary unit and is needed to define the imaginary part $\omega(X, Y) = ig(X, -JY)$ of hermitian form $h = h + i\omega$.

2. To define the analog of Kähler structure for M^4, one must start from a decomposition of $M^4 = M^2 \times E^2$ (M^2 is generated by light-like vector and its dual) and E^2 is orthogonal to it. M^2 allows hypercomplex structure, which light-like coordinates $(u = t - z, v = t + z)$ and E^2 complex structure and the metric has form $ds^2 = dudv + dzd\bar{z}$. Hypercomplex numbers can

be represented as $h = t + iez$, $i^2 = -1, e^2 = -1$ $i^2 = -1, e^2 = -1$. Hyper-complex numbers do not define number field since for light-like hypercomplex numbers $t + iez$, $t = \pm z$ do not have finite inverse. Hypercomplex numbers allow a generalization of analytic functions used routinely in physics. Kähler form representing hypercomplex imaginary unit would be replaced with eJ. One would consider sub-spaces of complexified quaternions spanned by real unit and units eI_k, $k = 1, 2, 3$ as representation of the tangent space of space-time surfaces in Minkowskian regions. This is familiar already from M^8 duality [K84].

$M^4 = M^2 \times E^2$ decomposition can depend on point of M^4 (polarization plane and light-like momentum direction depend on point of M^4. The condition that this structure allows global coordinates analogous to $(u, v, z, \overline{z})$ requires that the distributions for M^2 and E^2 are integrable and thus define 2-D surfaces. I have christened this structure Hamilton-Jacobi structure. It emerges naturally in the construction of extremals of Kähler action that I have christened massless extremals (MEs, [K10]) and also in the proposal for the generalization of complex structure to Minkowskian signature.

One can define the analog of Kähler form by taking sum of induced Kähler form J and its dual $*J$ defined in terms of permutation tensor. The normalization condition is that this form integrates to the negative of metric $(J \pm *J)^2 = -g$. This condition is possible to satisfy.

3. How to lift the Hamilton Jacobi structure of M^4 to Kähler structure of its twistor space? The basic definition of twistors assumes that their exists a field of time-like directions, and that one considers projections of 4-D antisymmetric tensors to the 3-space orthogonal to the time-like direction at given point. One can say that the projection yields magnetic part of the antisymmetric tensor (say induced Kähler form J) with positive norm with respect to natural metric induced to the twistor fiber from the inner product between two-forms. This unique time direction would be defined the light-like vector defining M^2 and its dual. Therefore the signature of the metric of S^2 would be $(-1, -1)$. In quaternionic picture this direction corresponds to real quaternionic unit.

4. To sum up, the metric of the Minkowskian twistor space has signature $(-1, -1, 1, -1, -1, -1)$. The Minkowskian variant of the twistor space would give 2 complex coordinates and one hyper-complex coordinate. Cosmological term would be finite and the sign of the cosmological term in the dimensionally reduced action would be positive as required. Also metric determinant would be imaginary as required. At this moment I cannot invent any killer objection against this option.

It must be made clear that the proposed definition of twistor space of M^4 does not seem to be equivalent with the twistor space assignable to conformally compactified M^4. One has trivial S^2 bundle and Hamilton-Jacobi structure, which is hybrid of complex and hyper-complex structure.

3.2.4 What does the induction of the twistor structure to space-time surface really mean?

Consider now what the induction of the twistor structure to space-time surface X^4 could mean.

1. The induction procedure for Kähler structure of 12-D twistor space T requires that the induced metric and Kähler form of the base space X^4 of X^6 obtained from T is the same as that obtained by inducing from $H = M^4 \times CP_2$. Since the Kähler structure and metric of T is lift from H this seems obvious. Projection would compensate the lift.

2. This is not yet enough. The Kähler structure and metric of S^2 projected from T must be same as those lifted from X^4. The connection between metric and ω implies that this condition for Kähler form is enough. The antisymmetric Kähler forms in fiber obtained in these two ways coincide. Since Kähler form has only one component in 2-D case, one obtains single constraint condition giving a commutative diagram stating that the direct projection to S^2 equals with the projection to the base followed by a lift to fiber. The resulting induced Kähler form is not covariantly constant but in fiber S^2 one has $J^2 = -g$.

As a matter of fact, this condition might be trivially satisfied as a consequence of the bundle structure of twistor space. The Kähler form from $S^2 \times S^2$ can be projected to S^2 associated with X^4 and by bundle projection to a two-form in X^4. The intuitive guess - which might be of course wrong - is that this 2-form must be same as that obtained by projecting the Kähler form of CP_2 to X^4. If so then the bundle structure would be essential but what does it really mean?

3. Intuitively it seems clear that X^6 must decompose locally to a product $X^4 \times S^2$ in some sense. This is true if the metric and Kähler form reduce to direct sums of contributions from the tangent spaces of X^4 and S^2. This guarantees that 6-D Kähler action decomposes to a sum of 4-D Kähler action and Kähler action for S^2.

   This could be however too strong a condition. Dimensional reduction occurs in Kaluza-Klein theories and in this case the metric can have also components between tangent spaces of the fiber and base being interpreted as gauge potentials. This suggests that one should formulate the condition in terms of the matrix $T \leftrightarrow g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu}$ defining the norm of the induced Kähler form giving rise to Kähler action. T maps Kähler form $J \leftrightarrow J_{\alpha\beta}$ to a contravariant tensor $J_c \leftrightarrow J^{\alpha\beta}$ and should have the property that $J_c(X^4)$ $(J_c(S^2))$ does not depend on $J(S^2)$ $(J(X^4))$.

   One should take into account also the self-duality of the form defining the imaginary unit. In X^4 the form $S = J \pm *J$ is self-dual/anti-self dual and would define twistorial imaginary unit since its square equals to $-g$ representing the negative of the real unit. This would suggest that 4-D Kähler action is effectively replaced with $(J \pm *J) \wedge (J \pm *J) = J^*J \pm J \wedge J$, where $*J$ is the Hodge dual defined in terms of 4-D permutation tensor ϵ. The second term is topological term (Abelian instanton term) and does not contribute to field equations. This in turn would mean that it is the tensor $T \pm \epsilon$ for which one can demand that $S_c(X^4)$ $(S_c(S^2))$ does not depend on $S(S^2)$ $(S(X^4))$.

4. The preferred quaternionic imaginary unit should be represented as a projection of Kähler form of 12-D twistor space $T(H)$. The preferred imaginary unit defining twistor structure as sum of projections of both $T(CP_2)$ and $T(M^4)$ Kähler forms would guarantee that vacuum extremals like canonically imbedded M^4 for which $T(CP_2)$ Kähler form contributes nothing have well-defined twistor structure. $T(M^4)$ or $T(CP_2)$ are treated completely symmetrically but the maps of $S^2(X^4)$ to $S^2(M^4)$ and $S^2(CP_2)$ characterized by winding numbers induce symmetry breaking.

   For Kähler action $M^4 - CP_2$ symmetry does not make sense. 4-D Kähler action to which 6-D Kähler action dimensionally reduces can depend on CP_2 Kähler form only. I have also considered the possibility of covariantly constant self-dual M^4 term in Kähler action but given it up because of problems with Lorentz invariance. One should couple the gauge potential of M^4 Kähler form to induced spinors. This would mean the existence of vacuum gauge fields coupling to sigma matrices of M^4 so that the gauge grop would be non-compact $SO(3,1)$ leading to a breakdown of unitarity.

There is still one difficulty to be solved.

1. The normalization of 6-D Kähler action by a scale factor $1/L^2$ with dimension, which is inverse length squared, brings in a further length scale. The first guess is that $1/L^2$ is closely related to cosmological constant, which is also dynamical and $1/L^2$ has indeed correct sign to explain accelerated expansion of the Universe. Unfortunately, if $1/L^2$ is of order cosmological constant, the value of the ordinary Kähler coupling strength α_K would be enormous. As a matter of fact, the order of magnitude for L^2 must be equal to the area of $S^2(X^4)$ and in good approximation equal to $L^2 = 4\pi R^2(S^2(M^4))$ and therefore in the same range as Planck length l_P and CP_2 radius R. This would imply a gigantic value of cosmological constant. Just as in GRT based cosmology!

2. This issue can be solved by using the observation that thanks to the decomposition $H = M^4 \times CP_2$, 6-D Kähler action is sum of two independent terms. The first term corresponds to the 6-D lift of the ordinary Kähler action. For it the contribution from $S^2(CP_2)$ fiber is absent if the embedding of $S^2(X^4)$ to $S^2(M^4) \times S^2(CP_2)$ reduces to identification with $S^2(M^4)$ so that $S^2(CP_2)$ is effectively absent: this is not true generally. Second term in the

action is assumed to come from the $S^2(M^4)$ fiber of twistor space $T(M^4)$, which can indeed contribute without breaking of Lorentz symmetry. In fact, one can assume that also the Kähler form of M^4 contributes as will be found.

3. The independency implies that Kähler couplings strengths are independent for them. If one wants that cosmological constant has a reasonable order of magnitude, $L \sim R(S^2(M^4))$ must hold true and the analog $\alpha_K(S^2(M^4))$ of the ordinary Kähler coupling strength (analogous to critical temperature) must be extremely large - so large that one has

$$\alpha_K(M^4) \times 4\pi R(M^4)^2 \sim L^2 \quad ,$$

where L is the size scale of the recent Universe.

This makes possible the small value of cosmological constant assignable to the volume term given by this part of dimensionally reduced action. Both Kähler coupling strengths are assumed to have a spectrum determined by quantum criticality and the spectrum of $\alpha_K(M^4)$ would be essentially as p-adic primes satisfying p-adic length scale hypothesis $p \simeq 2^k$, k prime. One can criticize this identification of 6-D Kähler action as artificial but it seems to be the only option that works. Interestingly also the contribution from M^4 Kähler form can be allowed since it is also extremely small. For canonically imbedded M^4 this contribution vanishes by self-duality of M^4 Kähler form and is extremely small for the vacuum extremals of Kähler action.

4. For general winding numbers of the map $S^2(X^4) \to S^2(M^4) \times S^2(CP_2)$ also $S^2(CP_2)$ Kähler form contributes and cosmological constant is gigantic. It would seem that only the winding numbers $(w_1, w_2) = (n, 0)$ are consistent with the observed value of cosmological constant. Hence it seems that there is no need to pose any additional conditions to the Kähler action if one uses the fact that $T(M^4)$ and $T(CP_2)$ parts are independent!

It is good to list the possible open issues related to the precise definition of the twistor structure and of M^4 Kähler action.

1. The proposed definition of M^4 twistor space a Cartesian product of M^4 and $S^2(M^4)$ parts involving Hamilton-Jacobi structure does not seem to be equivalent with the twistor identification as $SU(2,2)/SU(2,1) \times U(1)$ having conformally compactified M^4 as base space. There exists an entire moduli space of Hamilton-Jacobi structures. If the M^4 part of Kähler form participates in dynamics, one must include the specification of the Hamilton-Jacobi structure to the definition of CD and integrate over Hamilton Jacobi-structures as part of integral over WCW in order to gain Lorentz invariance. Note that Hamilton-Jacobi structure enters to dynamics also through the construction of massless extremals [K10].

2. The presence of M^4 part of Kähler form in action implies breaking of Lorentz invariance for extremals of lifted Kähler action. The same happens at the level of induced spinors if this Kähler form couples to embedding space spinors. If $T(M^4)$ is trivial bundle, one can include only the $T(S^2(M^4))$ part of Kähler form to Kähler action and couple only this to the spinors of $T(H)$. The integration over Hamilton-Jacobi structures becomes un-necessary.

3. If one includes M^4 part of Kähler form to 6-D Kähler action, one has several options. One can have sum of the Kähler actions for $T(M^4)$ and $T(CP_2)$ or Kähler action defined by the sum $J(T(M^4)/g_K$ and $J(T(CP_2)/\alpha_K$ with $\alpha_K(M^4) = g_K^2(M^4)/4\pi\hbar$ and $\alpha_K = g_K^2/4\pi\hbar$ with a proper normalization to guarantee that the squares of induced Kähler forms give sum of Kähler actions as in the first option. In this case one obtains interference term proportional to $Tr(J(M^4)J(CP_2)$. For the proposed value of α_K also the interference term is extremely small as compared to Kähler action in recent cosmology.

3.2.5 Could M^4 Kähler form introduce new gravitational physics?

The introduction of M^4 Kähler form could bring in new gravitational physics.

1. As found, the twistorial formulation of TGD assigns to M^4 a self dual Kähler form whose square gives Minkowski metric. It can (but need not if M^4 twistor space is trivial as bundle) contribute to the 6-D twistor counterpart of Kähler action inducing M^4 term to 4-D Kähler action vanishing for canonically imbedded M^4.

2. Self-dual Kähler form in empty Minkowski space satisfies automatically Maxwell equations and has by Minkowskian signature and self-duality a vanishing action density. Energy momentum tensor is proportional to the metric so that Einstein Maxwell equations are satisfied for a non-vanishing cosmological constant! M^4 indeed allows a large number of self dual Kähler fields (I have christened them as Hamilton-Jacobi structures). These are probably the simplest solutions of Einstein-Maxwell equations that one can imagine!

3. There however exist quite a many Hamilton-Jacobi structures. However, if this structure is to be assigned with a causal diamond (CD) it must satisfy additional conditions, say $SO(3)$ symmetry and invariance under time translations assignable to CD. Alternatively, covariant constancy and $SO(2) \subset SO(3)$ symmetry might be required.

This raises several questions. Could M^4 Kähler form replace CP_2 Kähler form in the picture for how gravitational interaction is mediated at quantal level? Could one speak of flux tubes of the magnetic part of this Kähler form? Or should one consider the Kähler field as a sum of the two Kähler forms weighted by the inverses $1/g_K$ of corresponding Kähler couplings. If so then M^4 contribution would be negligible except for canonically imbedded M^4 in the recent cosmology. Note that α_K and $\alpha_K(M^4)$ have interpretation as analogs of quantum critical temperatures but can depend on the p-adic lengths scale defining the cosmology.

1. The natural expectation is that Kähler form characterizes CD having preferred time direction suggested strongly by number theoretical considerations involving quaternionic structure with preferred direction of time axis assignable to real unit quaternion.

   Self-duality gives rise to Kähler magnetic and electric fields in the same spatial direction identifiable as a local quantization axis for spin assignable to CD assignable to observer. CD indeed serves as a correlate for conscious entity in TGD inspired theory of consciousness. Flux tube would connect mass M to mass m assignable to observer and flux tube direction would define spin quantization axes for the CD of the observer. Spin quantization axis would be naturally in the direction of magnetic field, which is direction of the flux tube.

2. The self-dual Kähler form could be spherically symmetric for CDs and represent self dual magnetic monopole field (dyon) with monopole charge at the line connecting the tips of CD and have non-vanishing components $J^{tr} = \epsilon^{tr\theta\phi} J_{\theta\phi}$, $J_{\theta\phi} = sin(\theta)$. One would have genuine monopole, which is somewhat questionable feature. Only the entire radial flux would be quantized. CD could be associated with the mass M of the central object. The gauge potential associated with J could be chosen to be $A_\mu \leftrightarrow (1/r, 0, 0, cos(\theta)$. I have considered this kind of possibility earlier in context of TGD inspired model of anyons but gave up the idea.

   The moduli space for CDs with second tip fixed would be hyperbolic space $H^3 = SO(3,1)/SO(3)$ or a space obtained by identifying points at the orbits of some discrete subgroup of $SO(3,1)$ as suggested by number theoretic considerations. This induced Kähler field could make the blackholes with center at this line to behave like M^4 magnetic monopoles if the M^4 part of Kähler form is induced into the 6-D lift of Kähler action with extremely small coefficients of order of magnitude of cosmological constant. Cosmological constant and the possibility of CD monopoles would thus relate to each other.

3. The self-dual M^4 Kähler form could be also covariantly constant ($J_{tz} = J_{xy} = 1$) and represent electric and magnetic fluxes in a fixed direction identifiable as a quantization axes for spin and characterizing CD. In this case the CD would be associated with the mass m of observer. The moduli space of CDs would be now $SO(3,1)/SO(1,1) \times SO(2)$ which is completely analogous to the twistor space $SU(3)/U(1) \times U(1)$.

4. Boundary conditions (allowing no boundaries!) demand that the flux tubes have closed cross section - say sphere S^2 - rather than disk: stability is guaranteed if the S^2 cross section is

mapped to homologically non-trivial surface of CP_2 or is projection of it. This would give monopole flux also for CP_2 Kähler form so that the original hypothesis would be correct.

5. Radial flux tubes are possible both spherically symmetric and covariantly constant Kähler form possibly mediating gravitational interaction but the flux is not quantized unless preferred extremal property implies this: in any case M^4 flux would be very small unless one has large value of gravitational Planck constant implying n-sheeted covering of M^4 and flux is scale up by n since every sheet gives a contribution. For spherically symmetric M^4 Kähler form the flux tubes would have naturally conical structure spanning a constant solid angle. For covariantly constant Kähler form the flux tubes would be cylindrical.

There are further interpretational problems.

1. The classical coupling of M^4 Kähler gauge potential to induced spinors is not small. Can one really tolerate this kind of coupling equivalent to a coupling to a self dual monopole field carrying electric and magnetic charges? One could of course consider the condition that the string world sheets carrying spinor modes are such that the induced M^4 Kähler form vanishes and gauge potential become pure gauge. M^4 projection would be 2-D Lagrange manifold whereas CP_2 projection would carry vanishing induce W and possibly also Z^0 field in order that em charge is well defined for the modes. These conditions would fix the string world sheets to a very high degree in terms of maps between this kind of 2-D sub-manifolds of M^4 and CP_2. Spinor dynamics would be determined by the avoidance of interaction!

   Recall that one could interpret the localization of spinor modes to 2-surfaces in the sense of strong form of holography: one can continued induced spinor fields to the space-time interior as indeed assumed but the continuation is completely determined by the data at 2-D string world sheets.

   It must be emphasized that the embedding space spinor modes characterizing the ground states of super-symplectic representations would not couple to the monopole field so that at this level Poincare invariance is not broken. The coupling would be only at the space-time level and force spinor modes to Lagrangian sub-manifolds.

2. At the static limit of GRT and for $g_{ij} \simeq \delta_{ij}$ implying $SO(3)$ symmetry there is very close analogy with Maxwell's equations and one can speak of gravi-electricity and gravi-magnetism with 4-D vector potential given by the components of $g_{t\alpha}$. The genuine $U(1)$ gauge potential does not however relate to the gravimagnetism in GRT sense. Situation would be analogous to that for CP_2, where one must add to the spinor connection $U(1)$ term to obtain respectable spinor structure. Now the $U(1)$ term would be added to trivial spinor connection of flat M^4: its presence would be justified by twistor space Kähler structure. If the induced M^4 Kähler form is present as a classical physical field it means genuinely new contribution to U(1) electroweak of standard model. If string world sheets carry vanishing M^4 Kähler form, this contribution vanishes classically.

3.2.6 A connection with the hierarchy of Planck constants?

A connection with the hierarchy of Planck constants is highly suggestive. Since also a connection with the p-adic length scale hierarchy suggests itself for the hierarchy of p-adic length scales it seems that both length scale hierarchies might find first principle explanation in terms of twistor lift of Kähler action.

1. Cosmological considerations encourage to think that $R_1 \simeq l_P$ and $R_2 \simeq R$ hold true. One would have in early cosmology $(w_1, w_2) = (1, 0)$ and later $(w_1, w_2) = (0, 1)$ guaranteeing R_D grows from l_P to R during cosmological evolution. These situations would correspond the solutions $(w_1 = n, 0)$ and $(0, w_2 = n)$ one has $A = n4\pi R_1^2$ and $A = n \times 4\pi R_2^2$ and both Kähler coupling strengths are scaled down to α_K/n. For $\hbar_{eff}/h = n$ exactly the same thing happens!

   There are further intriguing similarities. $h_{eff}/h = n$ is assumed to correspond *multi-sheeted* (to be distinguished from *many-sheeted*!) covering space structure for space-time surface.

Now one has covering space defined by the lift $S^2(X^4) \rightarrow S^2(M4) \times S^2(CP_2)$. These lifts define also lifts of space-time surfaces.

Could the hierarchy of Planck constants correspond to the twistorial surfaces for which $S^2(M^4)$ is n-fold covering of $S^2(X^4)$? The assumption has been that the n-fold multi-sheeted coverings of space-time surface for $h_{eff}/h = n$ are singular at the ends of space-time surfaces at upper and lower boundaries if causal diamond (CD). Could one consider a more precise definition of twistor space in such a way that CD replaces M^4 and the covering becomes singular at the light-like boundaries of CD - the branches of space-time surface would collapse to single one.

Does this collapse have a clear geometric meaning? Are the projections of various branches of the S^2 lift automatically identical so that one would have the original picture in which one has n identical copies of the same space-time surface? Or can one require identical projections only at the light-like boundaries of CD?

2. $w_1 = w_2 = w$ is essentially the first proposal for conditions associated with the lifting of twistor space structure. $w_1 = w_2 = n$ gives $ds^2 = (R_1^2 + R_2^2)(d\theta^2 + w^2 d\phi^2)$ and $A = n \times 4\pi(R_1^2 + R_2^2)$. Also now Kähler coupling strength is scaled down to α/n. Again a connection with the hierarchy of Planck constants suggests itself.

3. One can consider also the option $R_1 = R_2$ option giving $ds^2 = R_1^2(2d\theta^2 + (w_1^2 + w_2^2)d\phi^2$. If the integers w_i define Pythagorean square one has $w_1^2 + w_2^2 = n^2$ and one has $R_1 = R_2$ option that one has $A = n \times 4\pi R^2$. Also now the connection with the hierarchy of Planck constants might make sense.

3.2.7 Twistorial variant for the embedding space spinor structure

The induction of the spinor structure of embedding space is in key role in quantum TGD. The question arises whether one should lift also spinor structure to the level of twistor space. If so one must understand how spinors for $T(M^4)$ and $T(CP_2)$ are defined and how the induced spinor structure is induced.

1. In the case of CP_2 the definition of spinor structure is rather delicate and one must add to the ordinary spinor connection U(1) part, which corresponds physically to the addition of classical U(1) gauge potential and indeed produces correct electroweak couplings to quarks and leptons. It is assumed that the situation does not change in any essential manner: that is the projections of gauge potentials of spinor connection to the space-time surface give those induced from $M^4 \times CP_2$ spinor connection plus possible other parts coming as a projection from the fiber $S^2(M^2) \times S^2(CP_2)$. As a matter of fact, these other parts should vanish if dimensional reduction is what it is meant to be.

2. The key question is whether the complications due to the fact that the geometries of twistor spaces $T(M^4)$ and $T(CP_2)$ are not quite Cartesian products (in the sense that metric could be reduced to a direct sum of metrics for the base and fiber) can be neglected so that one can treat the sphere bundles approximately as Cartesian products $M^4 \times S^2$ and $CP_2 \times S^2$. This will be assumed in the following but should be carefully proven.

3. Locally the spinors of the twistorspace $T(H)$ are tensor products of embedding spinors and those for of $S^2(M^4) \times S^2(CP_2)$ expressible also as tensor products of spinors for $S^2(M^4)$ and $S^2(CP_2)$. Obviously, the number of spinor components increases by factor $2 \times 2 = 4$ unless one poses some additional conditions taking care that one has dimensional reduction without the emergence of any new spin like degrees of freedom for which there is no physical evidence. The only possible manner to achieve this is to pose covariant constancy conditions already at the level of twistor spaces $T(M^4)$ and $T(CP_2)$ leaving only single spin state in these degrees of freedom.

4. In CP_2 covariant constancy is possible for right-handed neutrino so that CP_2 spinor structure can be taken as a model. In the case of CP_2 spinors covariant constancy is possible for right-handed neutrino and is essentially due to the presence of U(1) part in spinor connection

forced by the fact that the spinor structure does not exist otherwise. Ordinary S^2 spinor connection defined by vielbein exists always. One can however add a coupling to a suitable multiple of Kähler potential satisfying the quantization of magnetic charge (the magnetic flux defined by $U(1)$ connection is multiple of 2π so that its imaginary exponential is unity).

S^2 spinor connections must must have besides ordinary vielbein part determined by S^2 metric also $U(1)$ part defined by Kähler form coupled with correct coupling so that the curvature form annihilates the second spin state for both $S^2(M^4)$ and $S^2(CP_2)$. $U(1)$ part of the spinor curvature is proportional to Kähler form $J \propto sin(\theta)d\theta d\phi$ so that this is possible. The vielbein and $U(1)$ parts of the spinor curvature ear proportional Pauli spin matrix $\sigma_z = (1,0;0,-1)/2$ and unit matrix $(1,0;0,1)$ respectively so that the covariant constancy is possible to satisfy and fixes the spin state uniquely.

5. The covariant derivative for the induced spinors is defined by the sum of projections of spinor gauge potentials for $T(M^4)$ and $T(CP_2)$. With above assumptions the contributions gauge potentials from $T(M^4)$ and $T(CP_2)$ separately annihilate single spinor component. As a consequence there are no constraints on the winding numbers w_i, $i = 1,2$ of the maps $S^2(X^4) \rightarrow S^2(M^4)$ and $S^2(X^4) \rightarrow S^2(CP_2)$. Winding number w_i corresponds to the embedding map $(\Theta_i = \theta, \Phi_i = w_i\phi)$.

6. If the square of the Kähler form in fiber degrees of freedom gives metric to that its square is metric, one obtains just the area of S^2 from the fiber part of action. This is given by the area $A = 4\pi\sqrt{2(w_1^2R_1^2 + w_2^2R_2^2)}$ since the induced metric is given by $ds^2 = (R_1^2 + R_2^2)d\theta^2 + (w_1^2R_1^2 + w_2^2R_2^2)d\phi^2$ for $(\Theta_1 = \theta, \Phi = n_1\phi, \Phi_2 = n_2\phi)$.

3.2.8 Twistor googly problem transforms from a curse to blessing in TGD framework

There was a nice story with title "Michael Atiyah's Imaginative State of Mind" about mathematician Michael Atyiah in Quanta Magazine (see `http://tinyurl.com/jta2va8`). The works of Atyiah have affected profoundly the development of theoretical physics. What was pleasant to hear that Atyiah belongs to those scientists who do not care what others think. As he tells, he can afford this since he has got all possible prices. This is consoling and encouraging even for those who have not cared what others think and for this reason have not earned any prizes. Nor even a single coin from what they have been busily doing their whole lifetime!

In the beginning of the story "twistor googly problem" was mentioned. I had to refresh my understanding about googly problem. In twistorial description the modes of massless fields (rather than entire massless fields) in space-time are lifted to the modes in its 6-D twistor-space and dynamics reduces to holomorphy. The analog of this takes place also in string models by conformal invariance and in TGD by its extension.

One however encounters what is known as googly problem: one can have twistorial description for circular polarizations with well-defined helicity +1/-1 but not for general polarization states - say linear polarizations, which are superposition of circular polarizations. This reflects itself in the construction of twistorial amplitudes in twistor Grassmann program for gauge fields but rather implicitly: the amplitudes are constructed only for fixed helicity states of scattered particles. For gravitons the situation gets really bad because of non-linearity.

Mathematically the most elegant solution would be to have only +1 or -1 helicity but not their superpositions implying very strong parity breaking and chirality selection. Parity parity breaking occurs in physics but is very small and linear polarizations are certainly possible! The discusion of Penrose with Atyiah has inspired a possible solution to the problem known as "palatial twistor theory" (see `http://tinyurl.com/hr7hmh2`). Unfortunately, the article is behind paywall too high for me so that I cannot say anything about it.

What happens to the googly problem in TGD framework? There is twistorialization at space-time level and embedding space level.

1. One replaces space-time with 4-surface in $H = M^4 \times CP_2$ and lifts this 4-surface to its 6-D twistor space represented as a 6-surface in 12-D twistor space $T(H) = T(M^4) \times T(CP_2)$. The twistor space has Kähler structure only for M^4 and CP_2 so that TGD is unique. This

Kähler structure is needed to lift the dynamics of Kähler action to twistor context and the lift leads to the a dramatic increase in the understanding of TGD: in particular, Planck length and cosmological constant with correct sign emerge automatically as dimensional constants besides CP_2 size.

2. Twistorialization at embedding space level means that spinor modes in H representing ground states of super-symplectic representations are lifted to spinor modes in T(H). M^4 chirality is in TGD framework replaced with H-chirality, and the two chiralities correspond to quarks and leptons. But one cannot superpose quarks and leptons! "Googly problem" is just what the superselection rule preventing superposition of quarks and leptons requires in TGD!

One can look this in more detail.

1. Chiral invariance makes possible for the modes of massless fields to have definite chirality: these modes correspond to holomorphic or antiholomorphic amplitudes in twistor space and holomorphy (antiholomorphy is holomorphy with respect to conjugates of complex coordinates) does not allow their superposition so that massless bosons should have well-defined helicities in conflict with experimental facts. Second basic problem of conformally invariant field theories and of twistor approach relates to the fact that physical particles are massive in 4-D sense. Masslessness in 4-D sense also implies infrared divergences for the scattering amplitudes. Physically natural cutoff is required but would break conformal symmetry.

2. The solution of problems is masslessness in 8-D sense allowing particles to be massive in 4-D sense. Fermions have a well-defined 8-D chirality - they are either quarks or leptons depending on the sign of chirality. 8-D spinors are constructible as superpositions of tensor products of M^4 spinors and of CP_2 spinors with both having well-defined chirality so that tensor product has chiralities (ϵ_1, ϵ_2), $\epsilon_i = \pm 1$, $i = 1,2$. H-chirality equals to $\epsilon = \epsilon_1\epsilon_2$. For quarks one has $\epsilon = 1$ (a convention) and for leptons $\epsilon = -1$. For quark states massless in M^4 sense one has either $(\epsilon_1, \epsilon_2) = (1,1)$ or $(\epsilon_1, \epsilon_2) = (-1,-1)$ and for massive states superposition of these. For leptons one has either $(\epsilon_1, \epsilon_2) = (1,-1)$ or $(\epsilon_1, \epsilon_2) = (-1,1)$ in massless case and superposition of these in massive case.

3. The twistor lift to $T(M^4) \times T(CP_2)$ of the ground states of super-symplectic representations represented in terms of tensor products formed from H-spinor modes involves only quark and lepton type spinor modes with well-defined H-chirality. Superpositions of amplitudes in which different M^4 helicities appear but M^4 chirality is always paired with completely correlating CP_2 chirality to give either $\epsilon = 1$ or $\epsilon = -1$. One has never a superposition of of different chiralities in either M^4 or CP_2 tensor factor. I see no reason forbidding this kind of mixing of holomorphicities and this is enough to avoid googly problem. Linear polarizations and massive states represent states with entanglement between M^4 and CP_2 degrees of freedom. For massless and circularly polarized states the entanglement is absent.

4. This has interesting implications for the massivation. Higgs field cannot be scalar in 8-D sense since this would make particles massive in 8-D sense and separate conservation of B and L would be lost. Theory would also contain a dimensional coupling. TGD counterpart of Higgs boson is actually CP_2 vector, and one can say that gauge bosons and Higgs combine to form 8-D vector. This correctly predicts the quantum numbers of Higgs. Ordinary massivation by constant vacuum expectation value of vector Higgs is not an attractive idea since no covariantly constant CP_2 vector field exists so that Higgsy massivation is not promising except at QFT limit of TGD formulated in M^4. p-Adic thermodynamics gives rise to 4-D massivation but keeps particles massless in 8-D sense. It also leads to powerful and correct predictions in terms of p-adic length scale hypothesis.

Anonymous reader gave me a link to the paper of Penrose and this inspired further more detailed considerations of googly problem.

1. After the first reading I must say that I could not understand how the proposed elimination of conjugate twistor by quantization of twistors solves the googly problem, which means that both helicities are present (twistor Z and its conjugate) in linearly polarized classical modes so that holomorphy is broken classically.

2. I am also very skeptic about quantizing of either space-time coordinates or twistor space coordinates. To me quantization is natural only for linear objects like spinors. For bosonic objects one must go to higher abstraction level and replace superpositions in space-time with superpositions in field space. Construction of "World of Classical Worlds" (WCW) in TGD means just this.

3. One could however think that circular polarizations are fundamental and quantal linear combination of the states carrying circularly polarized modes give rise to linear and elliptic polarizations. Linear combination would be possible only at the level of field space (WCW in TGD), not for classical fields in space-time. If so, then the elimination of conjugate of Z by quantization suggested by Penrose would work.

4. Unfortunately, Maxwell's equations allow classically linear polarisations! In order to achieve classical-quantum consistency, one should modify classical Maxwell's equations somehow so that linear polarizations are not possible. Googly problem is still there!

What about TGD?

1. Massless extremals representing massless modes are very "quantal": they cannot be superposed classically unless both momentum and polarisation directions for them (they can depend space-time point) are exactly parallel. Optimist would guess that the classical local classical polarisations are circular. No, they are linear! Superposition of classical linear polarizations at the level of WCW can give rise to local linear but not local circular polarization! Something more is needed.

2. The only sensible conclusion is that only gauge boson quanta (not classical modes) represented as pairs of fundamental fermion and antifermion in TGD framework can have circular polarization! And indeed, massless bosons - in fact, all elementary particles- are constructed from fundamental fermions and they allow only two M^4, CP_2 and $M^4 \times CP_2$ helicities/-chiralities analogous to circular polarisations. B and L conservation would transform googly problem to a superselection rule as already described.

To sum up, both the extreme non-linearity of Kähler action, the representability of all elementary particles in terms of fundamental fermions and antifermions, and the generalization of conserved M^4 chirality to conservation of H-chirality would be essential for solving the googly problem in TGD framework.

3.3 Surprise: Twistorial Dynamics Does Not Reduce to a Trivial Reformulation of the Dynamics of Kähler Action

I have thought that twistorialization classically means only an alternative formulation of TGD. This is definitely not the case as the explicit study demonstrated. Twistor formulation of TGD is in terms of of 6-D twistor spaces $T(X^4)$ of space-time surfaces $X^4 \subset M^4 \times CP_2$ in 12-dimensional product $T = T(M^4) \times T(CP_2)$ of 6-D twistor spaces of $T(M^4)$ of M^4 and $T(CP_2)$ of CP_2. The induced Kähler form in X^4 defines the quaternionic imaginary unit defining twistor structure: how stupid that I realized it only now! I experienced during single night many other "How stupid I have been" experiences.

Classical dynamics is determined by 6-D variant of Kähler action with coefficient $1/L^2$ having dimensions of inverse length squared. Since twistor space is bundle, a dimensional reduction of 6-D Kähler action to 4-D Kähler action plus a term analogous to cosmological term - space-time volume - takes place so that dynamics reduces to 4-D dynamics also now. Here one must be careful: this happens provided the radius of $S2$ associated with X^4 does not depend on point of X^4. The emergence of cosmological term was however completely unexpected: again "How stupid I have been" experience. The scales of the spheres and the condition that the 6-D action is dimensionless bring in 3 fundamental length scales!

3.3.1 New scales emerge

The twistorial dynamics gives to several new scales with rather obvious interpretation. The new fundamental constants that emerge are the radii of the spheres associated with $T(M^4)$ and $T(CP_2)$. The radius of the sphere associated with X^4 is not a fundamental constant but determined by the induced metric. By above argument the fiber is sphere for both Euclidian signature and Minkowskian signatures.

1. For CP_2 twistor space the radius of $S^2(CP_2)$ must be apart from numerical constant equal to CP_2 radius R. For $S^2(M^4)$ one an consider two options. The first option is that also now the radius for $S^2(M^4)$ equals to $R(M^4) = R$ so that Planck length would not emerge from fundamental theory classically as assumed hitherto. Second imaginable option is that it does and one has $R(M^4) = l_P$.

2. If the signature of $S^2(M^4)$ is $(-1,-1)$ both Minkowskian and Euclidian regions have $S^2(X^4)$ with the same signature $(-1,-1)$. The radius R_D of $S^2(X^4)$ is dynamically determined.

Recall first how the cosmological constant emerges from TGD framework. The key point is that the 6-D Kähler action contains two terms.

1. The first term is essentially the ordinary Kähler action multiplied by the area of $S^2(X^4)$ which is compensated by the length scale, which can be taken to be the area $4\pi R^2(M^4)$ of $S^2(M^4)$. This makes sense for winding numbers $(w_1, w_2) = (1, 0)$ meaning that $S^2(CP_2)$ is effectively absent but $S^2(M^4)$ is present.

2. Second term is the analog of Kähler action assignable assignable to the projection of $S^2(M^4)$ Kähler form. The corresponding Kähler coupling strength $\alpha_K(M^4)$ is huge - so huge that one has $\alpha_K(M^4)4\pi R^2(M^4) \equiv L^2$, where $1/L^2$ is of the order of cosmological constant and thus of the order of the size of the recent Universe. $\alpha_K(M^4)$ is also analogous to critical temperature and the earlier hypothesis that the values of L correspond to p-adic length scales implies that the values of come as $\alpha_K(M^4) \propto p \simeq 2^k$, p prime, k prime.

   The assignment of different value of α_K to M^4 and CP_2 degrees of freedom can be criticized as ad hoc assumption. In [L24] a scenario in which the value of α_K is universal. This option has very nice properties and one can overcome the problem associated with cosmological constant by assuming that it the *entire* 4-D action corresponds to the effective cosmological constant. The cancellation between Kähler action and volume term would give rise to very small cosmological constant and also its p-adic evolution could be understood.

3. One can get an estimate for the relative magnitude of the Kähler action $S(CP_2) = \pi/8\alpha_K$ assignable to CP_2 type vacuum extremal and the corresponding cosmological term. The magnitude of the volume term is of order $1/4\pi\alpha_K(M^4)$ with $\alpha_K(M^4)$ given by $\alpha_K(M^4) = L^2/4\pi R^2(M^4)$. The sequel the magnitude of L is estimated to be $L = (2^{3/2}\pi l_P/R_D) \times R_U$, where R_U is the recent size of the Universe. This estimate follows from the identification of the volume term as cosmological constant term.

   For $R_D = R_M = l_P$ this gives $\alpha_K(M^4) = 2\pi(R_U/l_P)^2 \sim 2 \times 10^{18}$. For $\alpha_K \simeq 1/137$ the ratio of the two terms is of order 10^{-20}. The cosmological terms is completely negligible in elementary particle scales. For vacuum extremals the situation changes and the overall effect is presumably the transformation of 4-D spin glass degeneracy so that the potentials wells in the analog spin glass energy landscape do not correspond to vacuum extremal anymore and perturbation theory around them is in principle possible. The huge value of $\alpha_K(M^4)$ implies that the system corresponds mathematically to an extremely strongly interacting system so that perturbation theory fails to converge. The geometry of "world of classical worlds" (WCW) provides the needed non-perturbative approach and leads to strong form of holography.

4. One could argue that the Kähler form assignable to M^4 cannot contribute to the action since it does not contribute to spinor connection of M^4 - an assumption that can be challenged. For canonically imbedded M^4 self-duality implies that this contribution to action vanishes. For vacuum extremals of ordinary Kähler action the contribution to the action density is

proportional to the CP_2 part of induced metric and to $1/\alpha_K(M^4)$, and therefore extremely small.

The breaking of Lorentz invariance can be seen as a possible problem for the induced spinor fields coupling to the self-dual Kähler potential. This corresponds to coupling to constant magnetic field and constant electric field, which are duals of each other. This would give rise to the analogs of cyclotron energy states in transversal directions and to the analogs of states in constant electric field in longitudinal directions. Could this extremely small effect serve as a seed for the generation of Kähler magnetic flux tubes carrying longitudinal electric fields in various scales? Note also that the value of $\alpha_K(M^4)$ is predicted to decrease as p-adic length scale so that the effect would be larger in early cosmology and in short length scales.

Hence one can consider the possibility that the action is just the sum of full 6-D Kähler actions assignable to $T(M^4)$ and $T(CP_2)$ but with different values of α_K if one has $(w_1, w_2) = (n, 0)$. Also other $w_2 \neq 0$ is possible but corresponds to gigantic cosmological constant.

Given the parameter L^2 as it is defined above, one can deduce an expression for cosmological constant Λ and show that it is positive.

1. 6-D Kähler action has dimensions of length squared and one must scale it by a dimensional constant: call it $1/L^2$. L is a fundamental scale and in dimensional reduction it gives rise to cosmological constant. Cosmological constant Λ is defined in terms of vacuum energy density as $\Lambda = 8\pi G\rho_{vac}$ can have two interpretations. Λ can correspond to a modification of Einstein-Hilbert action or - as now - to an additional term in the action for matter. In the latter case positive Λ means negative pressure explaining the observed accelerating expansion. It is actually easy to deduce the sign of Λ.

   $1/L^2$ multiplies both Kähler action - $F^{ij}F_{ij}$ ($\propto E^2 - B^2$ in Minkowskian signature). The energy density is positive. For Kähler action the sign of the multiplier must be positive so that $1/L^2$ is positive. The volume term is fiber space part of action having same form as Kähler action. It gives a positive contribution to the energy density and negative contribution to the pressure.

   In $\Lambda = 8\pi G\rho_{vac}$ one would have $\rho_{vac} = \pi/L^2R_D^2$ as integral of the $-F^{ij}F_{ij}$ over S^2 given the π/R_D^2 (no guarantee about correctness of numerical constants). This gives $\Lambda = 8\pi^2 G/L^2R_D^2$. Λ is positive and the sign is same as as required by accelerated cosmic expansion. Note that super string models predict wrong sign for Λ. Λ is also dynamical since it depends on R_D, which is dynamical. One has $1/L^2 = k\Lambda$, $k = 8\pi^2 G/R_D^2$ apart from numerical factors.

   The value of L of deduced from Euclidian and Minkowskian regions in this formal manner need not be same. Since the GRT limit of TGD describes space-time sheets with Minkowskian signature, the formula seems to be applicable only in Minkowskian regions. Again one can argue that one cannot exclude Euclidian space-time sheets of even macroscopic size and blackholes and even ordinary concept matter would represent this kind of structures.

2. L is not size scale of any fundamental geometric object. This suggests that L is analogous to α_K and has value spectrum dictated by p-adic length scale hypothesis. In fact, one can introduce the ratio of $\epsilon = R^2/L^2$ as a dimensionless parameter analogous to coupling strength what it indeed is in field equations. If so, L could have different values in Minkowskian and Euclidian regions.

3. I have earlier proposed that $R_U \equiv 1/\sqrt{1/\Lambda}$ is essentially the p-adic length scale $L_p \propto \sqrt{p} = 2^{k/2}$, $p \simeq 2^k$, k prime, characterizing the cosmology at given time and satisfies $R_U \propto a$ meaning that vacuum energy density is piecewise constant but on the average decreases as $1/a^2$, a cosmic time defined by light-cone proper time. A more natural hypothesis is that L satisfies this condition and in turn implies similar behavior or R_U. p-Adic length scales would be the critical values of L so that also p-adic length scale hypothesis would emerge from quantum critical dynamics! This conforms with the hypothesis about the value spectrum of α_K labelled in the same manner [L6].

4. At GRT limit the magnetic energy of the flux tubes gives rise to an average contribution to energy momentum tensor, which effectively corresponds to negative pressure for which the

expansion of the Universe accelerates. It would seem that both contributions could explain accelerating expansion. If the dynamics for Kähler action and volume term are coupled, one would expect same orders of magnitude for negative pressure and energy density - kind of equipartition of energy.

Consider first the basic scales emerging also from GRT picture. $R_U \sim \sqrt{1/\Lambda} \sim 10^{26}$ m = 10 Gly is not far from the recent size of the Universe defined as $c \times t \sim 13.8$ Gly. The derived size scale $L_1 \equiv (R_U \times l_P)^{1/2}$ is of the order of $L_1 = .5 \times 10^{-4}$ meters, the size of neuron. Perhaps this is not an accident. To make life of the reader easier I have collected the basic numbers to the following table.

$$\begin{array}{lll} m(CP_2) \simeq 5.7 \times 10^{14}\ GeV \ , & m_P = 2.435 \times 10^{18}\ GeV \ , & \frac{R(CP_2)}{l_P} \simeq 4.1 \times 10^3 \ , \\ R_U = 10\ Gy \ , & t = 13.8\ Gy \ , & L_1 = \sqrt{l_P R_U} = .5 \times 10^{-4}\ m \ . \end{array} \tag{3.3.1}$$

Let us consider now some quantitative estimates. $R(X^4)$ depends on homotopy equivalence classes of the maps from $S^2(X^4) \to S^2(M^4)$ and $S^2(X^4) \to S^2(CP_2)$ - that is winding numbers w_i, $i = 1,2$ for these maps. The simplest situations correspond to the winding numbers $(w_1, w_2) = (1,0)$ and $(w_1, w_2) = (0,1)$. For $(w_1, w_2) = (1,0)$ M^4 contribution to the metric of $S^2(X^4)$ dominates and one has $R(X^4) \simeq R(M^4)$. For $R(M^4) = l_P$ so Planck length would define a fundamental length and Planck mass and Newton's constant would be quantal parameters. For $(w_1, w_2) = (0,1)$ the radius of sphere would satisfy $R_D \simeq R$ (CP_2 size): now also Planck length would be quantal parameter.

Consider next additional scales emerging from TGD picture.

1. One has $L = (2^{3/2}\pi l_P/R_D) \times R_U$. In Minkowskian regions with $R_D = l_P$ this would give $L = 8.9 \times R_U$: there is no obvious interpretation for this number in recent cosmology. For $(R_D = R)$ one obtains the estimate $L = 29$ Mly. The size scale of large voids varies from about 36 Mly to 450 Mly (see `http://tinyurl.com/jyqcjhl`).

2. Consider next the derived size scale $L_2 = (L \times l_P)^{1/2} = \sqrt{L/R_U} \times L_1 = \sqrt{2^{3/2}\pi l_P/R_D} \times L_1$. For $R_D = l_P$ one has $L_2 \simeq 3L_1$. For $R_D = R$ making sense in Euclidian regions, this is of the order of size of neutrino Compton length: 3 μm, the size of cellular nucleus and rather near to the p-adic length scale $L(167) = 2.6$ m, corresponds to the largest miracle Gaussian Mersennes associated with $k = 151, 157, 163, 167$ defining length scales in the range between cell membrane thickness and the size of cellular nucleus. Perhaps these are coincidences are not accidental. Biology is something so fundamental that fundamental length scale of biology should appear in the fundamental physics.

The formulas and predictions for different options are summarized by the following table.

$$\begin{array}{llll} Option & L = \frac{2^{3/2}\pi l_P}{R_D} \times R_U & L_2 = \sqrt{L l_P} = \sqrt{\frac{2^{3/2}\pi l_P}{R_D}} \times L_1 \\ R_D = R \ , & 29\ Mly \ , & \simeq 3\ \mu m \ , \\ R_D = l_P \ , & 8.9 R_U \ , & \simeq 3L_1 = 1.5 \times 10^{-4}\ m \ , \end{array} \tag{3.3.2}$$

In the case of M^4 the radius of S^2 cannot be fixed it remains unclear whether Planck length scale is fundamental constant or whether it emerges.

3.3.2 Estimate for the cosmic evolution of R_D

One can actually get estimate for the evolution of R_D as function of cosmic time if one accepts Friedman cosmology as an approximation of TGD cosmology.

1. Assume critical mass density so that one has

$$\rho_{cr} = \frac{3H^2}{8\pi G} \ .$$

2. Assume that the contribution of cosmological constant term to the mass mass density dominates. This gives $\rho \simeq \rho_{vac} = \Lambda/8\pi G$. From $\rho_{cr} = \rho_{vac}$ one obtains

$$\Lambda = 3H^2 \ .$$

3. From Friedman equations one has $H^2 = ((da/dt)/a)^2$, where a corresponds to light-cone proper time and t to cosmic time defined as proper time along geodesic lines of space-time surface approximated as Friedmann cosmology. One has

$$\Lambda = \frac{3}{g_{aa}a^2}$$

in Robertson-Walker cosmology with $ds^2 = g_{aa}da^2 - a^2d\sigma_3^2$.

4. Combining this equations with the TGD based equation

$$\Lambda = \frac{8\pi^2 G}{L^2R_D^2}$$

one obtains

$$\frac{8\pi^2 G}{L^2R_D^2} = \frac{3}{g_{aa}a^2} \ . \tag{3.3.3}$$

5. Assume that quantum criticality applies so that L has spectrum given by p-adic length scale hypothesis so that one discrete p-adic length scale evolution for the values of L. There are two options to consider depending on whether p-adic length scales are assigned with light-cone proper time a or with cosmic time t

$$T = a \ \ (Option \ \ I) \ \ , \quad T = t \ \ (Option \ \ II) \tag{3.3.4}$$

Both options give the same general formula for the p-adic evolution of $L(k)$ but with different interpretation of $T(k)$.

$$\frac{L(k)}{L_{now}} = \frac{T(k)}{T_{now}} \ \ , \quad T(k) = L(k) = 2^{(k-151)/2} \times L(151) \ \ , \quad L(151) \simeq 10 \ \ nm \ . \tag{3.3.5}$$

Here $T(k)$ is assumed to correspond to primary p-adic length scale. An alternative - less plausible - option is that $T(k)$ corresponds to secondary p-adic length scale $L_2(k) = 2^{k/2}L(k)$ so that $T(k)$ would correspond to the size scale of causal diamond. In any case one has $L \propto L(k)$. One has a discretized version of smooth evolution

$$L(a) = L_{now} \times \frac{T}{T_{now}} \ . \tag{3.3.6}$$

6. Feeding into this to Eq. 3.3.3 one obtains an expression for $R_D(a)$

$$\frac{R_D}{l_P} = (\frac{8}{3})^{1/2}\pi \times \frac{a}{L(a)} \times g_{aa}^{1/2} \quad . \tag{3.3.7}$$

Unless the dependences on cosmic time compensate each other, R_D is dynamical and becomes very small at very early times since g_{aa} becomes very small. $R(M^4) = l_P$ however poses a lower boundary since either of the maps $S^2(X^4) \to S^2(M^4)$ and $S^2(X^4) \to S^2(CP_2)$ must be homotopically non-trivial. For $R(M^4) = l_P$ one would obtain $R_D/l_P = 1$ at this limit giving also lower bound for g_{aa}. For $T = t$ option $a/L(a)$ becomes large and g_{aa} small.

As a matter of fact, in very early cosmic string dominated cosmology g_{aa} would be extremely small constant [K71]. In late cosmology $g_{aa} \to 1$ holds true and one obtains at this limit

$$\frac{R_D(now)}{l_P} = (\frac{8}{3})^{1/2}\pi \times \frac{a_{now}}{L_{now}} \times l_P \simeq 4.4\frac{a_{now}}{L_{now}} \quad . \tag{3.3.8}$$

7. For $T = t$ option R_D/l_P remains constant during both matter dominated cosmology, radiation dominated cosmology, and string dominated cosmology since one has $a \propto t^n$ with $n = 1/2$ during radiation dominated era, $n = 2/3$ during matter dominated era, and $n = 1$ during string dominated era [K71]. This gives

$$\frac{R_D}{l_P} = (\frac{8}{3})^{1/2}\pi \times \frac{a}{t}\sqrt{g_{aa}}\frac{t(end)}{L(end)} = (\frac{8}{3})^{1/2}\frac{\pi}{n}\frac{t(end)}{L(end)} \quad .$$

Here "end" refers the end of the string or radiation dominated period or to the recent time in the case of matter dominated era. The value of n would have evolved as $R_D/l_P \propto (1/n)(t_{end}/L_{end})$, $n \in \{1, 3/2, 2\}$. During radiation dominated cosmology $R_D \propto a^{1/2}$ holds true. The value of R_D would be very nearly equal to $R(M^4)$ and $R(M^4)$ would be of the same order of magnitude as Planck length. In matter dominated cosmology would would have $R_D \simeq 2.2(t(now)/L(now)) \times l_P$.

8. For $R_D(now) = l_P$ one would have

$$\frac{L_{now}}{a_{now}} = (\frac{8}{3})^{1/2}\pi \simeq 4.4 \quad .$$

In matter dominated cosmology $g_{aa} = 1$ gives $t_{now} = (2/3) \times a_{now}$ so that predictions differ only by this factor for options I and II. The winding number for the map $S^2(X^4) \to S^2(CP_2)$ must clearly vanish since otherwise the radius would be of order R.

9. For $R_D(now) = R$ one would obtain

$$\frac{a_{now}}{L_{now}} = (\frac{8}{3})^{1/2} \times \frac{R}{l_P} \simeq 2.1 \times 10^4 \quad .$$

One has $L_{now} = 10^6$ ly: this is roughly the average distance scale between galaxies. The size of Milky Way is in the range $1 - 1.8 \times 10^5$ ly and of an order of magnitude smaller.

10. An interesting possibility is that $R_D(a)$ evolves from $R_D \sim R(M^4) \sim l_P$ to $R_D \sim R$. This could happen if the winding number pair $(w_1, w_2) = (1, 0)$ transforms to $(w_1, w_2) = (0, 1)$ during transition to from radiation (string) dominance to matter (radiation) dominance. R_D/l_P radiation dominated cosmology would be related by a factor

$$\frac{R_D(rad)}{R_D(mat)} = (3/4)\frac{t(rad, end)}{L(rad, end)} \times \frac{L(now)}{t(now)}$$

to that in matter dominated cosmology. Similar factor would relate the values of R_D/l_P in string dominated and radiation dominated cosmologies. The condition $R_D(rad)/R_D(mat) = l_P/R$ expressing the transformation of winding numbers would give

$$\frac{L(now)}{L(rad,end)} = \frac{4}{3}\frac{l_P}{R}\frac{t(now)}{t(rad,end)} \quad .$$

One has $t(now)/t(rad,end) \simeq .5 \times 10^6$ and $l_P/R = 2.5 \times 10^{-4}$ giving $L(now)/L(rad,end) \simeq 125$, which happens to be near fine structure constant.

11. For the twistor lifts of space-time surfaces for which cosmological constant has a reasonable value , the winding numbers are equal to $(w_1, w_2) = (n, 0)$ so that $R_D = \sqrt{n}R(S^2(M^4))$ holds true in good approximation. This conforms with the observed constancy of R_D during various cosmological eras, and would suggest that the ratio $\frac{t(end)}{L(end)}$ characterizing these periods is same for all periods. This determines the evolution for the values of $\alpha_K(M^4)$.

$R(M^4) \sim l_P$ seems rather plausible option so that Planck length would be fundamental classical length scale emerging naturally in twistor approach. Cosmological constant would be coupling constant like parameter with a spectrum of critical values given by p-adic length scales.

3.3.3 What about the extremals of the dimensionally reduced 6-D Kähler action?

It seems that the basic wisdom about extremals of Kähler action remains unaffected and the motivations for WCW are not lost in the case that M^4 Kähler form does not contribute to 6-D Kähler action (the case to be considered below): otherwise the predicted effects are extremely small in the recent Universe. What is new is that the removal of vacuum degeneracy is forced by twistorial action.

1. All extremals, which are minimal surfaces remain extremals. In fact, all the known extremals except vacuum extremals. For minimal surfaces the dynamics of the volume term and 4-D Kähler action separate and field equations for them are separately satisfied. The vacuum degeneracy motivating the introduction of WCW is preserved. The induced Kähler form vanishes for vacuum extremals and the imaginary unit of twistor space is ill-defined. Hence vacuum extremals cannot belong to WCW. This correspond to the vanishing of WCW metric for vacuum extremals.

2. For non-minimal surfaces Kähler coupling strength does not disappear from the field equations and appears as a genuine coupling very much like in classical field theories. Minimal surface equations are a generalization of wave equation and Kähler action would define analogs of source terms. Field equations would state that the total isometry currents are conserved. It is not clear whether other than minimal surfaces are possible, I have even conjectured that all preferred extremals are always minimal surfaces having the property that being holomorphic they are almost universal extremals for general coordinate invariant actions.

3. Thermodynamical analogy might help in the attempts to interpret. Quantum TGD in zero energy ontology (ZEO) corresponds formally to a complex square root of thermodynamics. Kähler action can be identified as a complexified analog of free energy. Complexification follows both from the fact that $\sqrt{g}$ is real/imaginary in Euclidian/Minkowskian space-time regions. Complex values are also implied by the proposed identification of the values of Kähler coupling strength in terms of zeros and pole of Riemann zeta in turn identifiable as poles of the so called fermionic zeta defining number theoretic partition function for fermions [K84] [L6, L8]. The thermodynamical for Kähler action with volume term is Gibbs free energy $G = F - TS = E - TS + PV$ playing key role in chemistry.

4. The boundary conditions at the ends of space-time surfaces at boundaries of CD generalize appropriately and symmetries of WCW remain as such. At light-like boundaries between

Minkowskian and Euclidian regions boundary conditions must be generalized. In Minkowkian regions volume can be very large but only the Euclidian regions contribute to Kähler function so that vacuum functional can be non-vanishing for arbitrarily large space-time surfaces since exponent of Minkowskian Kähler action is a phase factor.

5. One can worry about almost topological QFT property. Although Kähler action from Minkowskian regions at least would reduce to Chern-Simons terms with rather general assumptions about preferred extremals, the extremely small cosmological term does not. Could one say that cosmological constant term is responsible for “almost”?

   It is interesting that the volume of manifold serves in algebraic geometry as topological invariant for hyperbolic manifolds, which look locally like hyperbolic spaces $H_n = SO(n,1)/SO(n)$ [A5] [K45]. See also the article “Volumes of hyperbolic manifolds and mixed Tate motives” (see `http://tinyurl.com/yargy3uw`). Now one would have $n = 4$. It is probably too much to hope that space-time surfaces would be hyperbolic manifolds. In any case, by the extreme uniqueness of the preferred extremal property expressed by strong form of holography the volume of space-time surface could also now serve as topological invariant in some sense as I have earlier proposed. What is intriguing is that AdS_n appearing in AdS/CFT correspondence is Lorentzian analogue H_n.

6. $\alpha(M^4)$ is extremely large so that there is no hope of quantum perturbation theory around canonically imbedded M^4 although the propagator for CP_2 coordinate exists. In the new framework WCW can be seen as a solution to how to construct non-perturbative quantum TGD.

To sum up, I have the feeling that the final formulation of TGD has now emerged and it is clear that TGD is indeed a quantum theory of gravitation allowing to understand standard model symmetries. The existence of twistorial formulation is all that is needed to fix the theory completely. It makes possible gravitation and predicts standard model symmetries. This cannot be said about any competitor of TGD.

3.4 Basic Principles Behind Construction of Amplitudes

Basic principles of the construction summarized in this section could be seen as axioms trying to abstract the essentials. The explicit construction of amplitudes is too heavy challenge at this stage and at least for me.

3.4.1 Embedding space is twistorially unique

It took roughly 36 years to learn that M^4 and CP_2 are twistorially unique.

1. As already explained, M^4 and CP_2 are unique 4-manifolds in the sense that both allow twistor space with Kähler structure: Kähler structure is the crucial concept as one might guess from the fact that the projection of Kähler form naturally defines the preferred quaternionic imaginary unit defining the twistor structure for space-time surface. Both M^4 and its Euclidian variant E^4 allow twistor space. The first guess is that the twistor space of M^4 is Minkowskian variant $T(M^4) = SU(2,2)/SU(2,1)\times U(1)$ of 6-D twistor space $CP_3 = SU(4)/SU(3)\times U(1)$ of E^4. This is sensible assumption at the level of momentum space but the second candidate, which is simply $T(M^4) = M^4 \times CP_1$, is the only sensible option at space-time level. The twistor space of CP_2 is 6-D $T(CP_2) = SU(3)/U(1) \times U(1)$, the space for the choices of quantization axes of color hypercharge and isospin.

2. This leads to a proposal for the formulation of TGD in which space-time surfaces X^4 in H are lifted to twistor spaces X^6, which are sphere bundles over X^4 and such that they are surfaces in 12-D product space $T(M^4)\times T(CP_2)$ such the twistor structure of X^4 are in some sense induced from that of $T(M^4) \times T(CP_2)$.

   What is nice in this formulation is that one might be able to use all the machinery of algebraic geometry so powerful in superstring theory (Calabi-Yau manifolds) provided one can generalize the notion of Kähler structure from Euclidian to Minkowskian signature. It has been already described how this approach leads to a profound understanding of the relationship between TGD and GRT. Planck length emerges whereas fundamental constant as also cosmological constant emerges dynamically from the length scale parameter appearing in 6-D Kähler action. One can say, that twistor extension is absolutely essential for really understanding the gravitational interactions although the modification of Kähler action is extremely small due to the huge value of length scale defined by cosmological constant.

3. Masslessness (masslessness in complex sense for virtual particles in twistorialization) is essential condition for twistorialization. In TGD massless is masslessness in 8-D sense for the representations of superconformal algebras. This suggests that 8-D variant of twistors makes sense. 8-dimensionality indeed allows octonionic structure in the tangent space of embedding space. One can also define octonionic gamma matrices and this allows a possible generalization of 4-D twistors to 8-D ones using generalization of sigma matrices representing quaternionic units to octonionic sigma "matrices" essential for the notion of twistors. These octonion units do not of course allow matrix representation unless one restricts to units in some quaternionic subspace of octonions. Space-time surfaces would be associative and thus have quaternionic tangent space at each point satisfying some additional conditions.

3.4.2 Strong form of holography

Strong form of holography (SH) following from general coordinate invariance (GCI) for space-times as surfaces states that the data assignable to string world sheets and partonic 2-surfaces allows to code for scattering amplitudes. The boundaries of string world sheets at the space-like 3-surfaces defining the ends of space-time surfaces at boundaries of causal diamonds (CDs) and the fermionic lines along light-like orbits of partonic 2-surfaces representing lines of generalized Feynman diagrams become the basic elements in the generalization of twistor diagrams (I will not use the attribute "Feynman" in precise sense, one could replace it with "twistor" or even drop away). One can assign fermionic lines massless in 8-D sense to flux tubes, which can also be braided. One obtains a fractal hierarchy of braids with strands, which are braids themselves. At the lowest level one has braids for which fermionic lines are braided. This fractal hierarchy is unavoidable and means generalization of the ordinary Feynman diagram. I have considered some implications of this hierarchy in [L7].

The precise formulation of strong form of holography (SH) is one of the technical problems in TGD. A comment in FB page of Gareth Lee Meredith led to the observation that besides the purely number theoretical formulation based on commutativity also a symplectic formulation in the spirit of non-commutativity of embedding space coordinates can be considered. One can however use only the notion of Lagrangian manifold and avoids making coordinates operators leading to a loss of General Coordinate Invariance (GCI).

3.4.3 The existence of WCW demands maximal symmetries

Quantum TGD reduces to the construction of Kähler geometry of infinite-D "world of classical worlds" (WCW), of associated spinor structure, and of modes of WCW spinor fields which are purely classical entities and quantum jump remains the only genuinely quantal element of quantum TGD. Quantization without quantization, would Wheeler say.

By its infinite-dimensionality, the mere mathematical existence of the Kähler geometry of WCW requires maximal isometries. Physics is completely fixed by the mere condition that its mathematical description exists. Super-symplectic and other symmetries of "world of classical worlds" (WCW) are in decisive role. These symmetry algebras have conformal structure and generalize and extend the conformal symmetries of string models (Kac-Moody algebras in particular). These symmetries give also rise to the hierarchy of Planck constants. The super-symplectic symmetries extend to a Yangian algebra, whose generators are polylocal in the sense that they involve products of generators associated with different partonic surfaces. These symmetries leave scattering amplitudes invariant. This is an immensely powerful constraint, which remains to be understood.

3.4.4 Quantum criticality

Quantum criticality (QC) of TGD Universe is a further principle. QC implies that Kähler coupling strength is mathematically analogous to critical temperature and has a discrete spectrum. Coupling constant evolution is replaced with a discrete evolution as function of p-adic length scale: sequence of jumps from criticality to a more refined criticality or vice versa (in spin glass energy landscape you at bottom of well containing smaller wells and you go to the bottom of smaller well). This implies that either all radiative corrections (loops) sum up to zero (QFT limit) or that diagrams containing loops correspond to the same scattering amplitude as tree diagrams so that loops can eliminated by transforming them to arbitrary small ones and snipping away moving the end points of internal lines along the lines of diagram (fundamental description).

Quantum criticality at the level of super-conformal symmetries leads to the hierarchy of Planck constants $h_{eff} = n \times h$ labelling a hierarchy of sub-algebras of super-symplectic and other conformal algebras isomorphic to the full algebra. Physical interpretation is in terms of dark matter hierarchy. One has conformal symmetry breaking without conformal symmetry breaking as Wheeler would put it.

3.4.5 Physics as generalized number theory, number theoretical universality

Physics as generalized number theory vision has important implications. Adelic physics is one of them. Adelic physics implied by number theoretic universality (NTU) requires that physics in real and various p-adic numbers fields and their extensions can be obtained from the physics in their intersection corresponding to an extension of rationals. This is also enormously powerful condition and the success of p-adic length scale hypothesis and p-adic mass calculations can be understood in the adelic context.

In TGD inspired theory of consciousness various p-adic physics serve as correlates of cognition and p-adic space-time sheets can be seen as cognitive representations, "thought bubbles". NTU is closely related to SH. String world sheets and partonic 2-surfaces with parameters (WCW coordinates) characterizing them in the intersection of rationals can be continued to space-time surfaces by preferred extremal property but not always. In p-adic context the fact that p-adic integration constants depend on finite number of pinary digits makes the continuation easy but in real context this need not be possible always. It is always possible to imagine something but not always actualize it!

3.4.6 Scattering diagrams as computations

Quantum criticality as possibility to eliminate loops has a number theoretic interpretation. Generalized Feynman diagram can be interpreted as a representation of a computation connecting given set X of algebraic objects to second set Y of them (initial and final states in scattering) (trivial example: $X = \{3, 4\} \rightarrow 3 \times 4 = 12 \rightarrow 2 \times 6 \rightarrow \{2, 6\} = Y$. The 3-vertices ($a \times b = c$) and their time-reversals represent algebraic product and co-product.

There is a huge symmetry: all diagrams representing computation connecting given X and Y must produce the same amplitude and there must exist minimal computation. This generalization of string model duality implies an infinite number of dualities unless the finite size of CD allows only a finite number of equivalent computations. These dualities are analogous to the dualities of super-string model, in particular mirror symmetry stating that same quantum physical situation does not correspond to a unique space-time geometry and topology (Calabi-Yau and its mirror represent the same situation). The task of finding this computation is like finding the simplest representation for the formula X=Y and the noble purpose of math teachers is that we should learn to find it during our school days. This generalizes the duality symmetry of old fashioned string models: one can transform any diagram to a tree diagram without loops. This corresponds to quantum criticality in TGD: coupling constants do not evolve. The evolution is actually there but discrete and corresponds to infinite number critical values for Kahler coupling strength analogous to temperature.

3.4.7 Reduction of diagrams with loops to braided tree-diagrams

1. In TGD pointlike particles are replaced with 3-surfaces and by SH by partonic 2-surfaces. The important implication of 3-dimensionality is braiding. The fermionic lines inside light-like orbits of partonic 2-surfaces can be knotted and linked - that is braided (this is dynamical braiding analogous to dance). Also the fermionic strings connecting partonic 2-surfaces at space-like 3-surfaces at boundaries of causal diamonds (CDs) are braided (space-like braiding).

   Therefore ordinary Feynman diagrams are not enough and one must allow braiding for tree diagrams. One can also imagine of starting from braids and allowing 3-vertices for their strands (product and co-product above). It is difficult to imagine what this braiding could mean. It is better to imagine braid and allow the strands to fuse and split (annihilation and pair creation vertices).

2. This braiding gives rise in the planar projection representation of braids to a generalization of non-planar Feynman diagrams. Non-planar diagrams are the basic unsolved problem of twistor approach and have prevented its development to a full theory allowing to construct exact expressions for the full scattering amplitudes (I remember however that Nima Arkani-Hamed *et al* have conjectured that non-planar amplitudes could be constructed by some procedure: they notice the role of permutation group and talk also about braidings (describable using covering groups of permutation groups)). In TGD framework the non-planar Feynman diagrams correspond to non-trivial braids for which the projection of braid to plane has crossing lines, say a and b, and one must decide whether the line a goes over b or vice versa.

3. An interesting open question is whether one must sum over all braidings or whether one can choose only single braiding. Choice of single braiding might be possible and reflect the failure of string determinism for Kähler action and it would be favored by TGD as almost topological quantum field theory (TQFT) vision in which Kähler action for preferred extremal is topological invariant.

3.4.8 Scattering amplitudes as generalized braid invariants

The last big idea is the reduction of quantum TGD to generalized knot/braid theory (I have talked also about TGD as almost TQFT). The scattering amplitude can be identified as a generalized braid invariant and could be constructed by the generalization of the recursive procedure transforming in a step-by-step manner given braided tree diagram to a non-braided tree diagram: essentially what Alexander the Great did for Gordian knot but tying the pieces together after cutting. At each step one must express amplitude as superposition of amplitudes associated with the different outcomes of splitting followed by reconnection. This procedure transforms braided tree diagram to a non-braided tree diagrams and the outcome is the scattering amplitude!

3.5 Tensor Networks and S-matrices

The concrete construction of scattering amplitudes has been the toughest challenge of TGD and the slow progress has occurred by identification of general principles with many side tracks. One of the key problems has been unitarity. The intuitive expectation is that unitarity should reduce to a local notion somewhat like classical field equations reduce the time evolution to a local variational principle. The presence of propagators have been however the obstacle for locally realized unitarity in which each vertex would correspond to unitary map in some sense.

TGD suggests two approaches to the construction of S-matrix.

1. The first approach is generalization of twistor program [L3]. What is new is that one does not sum over diagrams but there is a large number of equivalent diagrams giving the same outcome. The complexity of the scattering amplitude is characterized by the minimal diagram. Diagrams correspond to space-time surfaces so that several space-time surfaces give rise to the same scattering amplitude. This would correspond to the fact that the dynamics breaks

classical determinism. Also quantum criticality is expected to be accompanied by quantum critical fluctuations breaking classical determinism. The strong form of holography would not be unique: there would be several space-time surfaces assignable as preferred extremals to given string world sheets and partonic 2-surfaces defining "space-time genes".

2. Second approach relies on the number theoretic vision and interprets scattering amplitudes as representations for computations with each 3-vertex identifiable as a basic algebraic operation [L3]. There is an infinite number of equivalent computations connecting the set of initial algebraic objects to the set of final algebraic objects. There is a huge symmetry involved: one can eliminate all loops moving the end of line so that it transforms to a vacuum tadpole and can be snipped away. A braided tree diagram is left with braiding meaning that the fermion lines inside the line defined by light-like orbit are braided. This kind of braiding can occur also for space-like fermion lines inside magnetic flux tubes and defining correlate for entanglement. Braiding is the TGD counterpart for the problematic non-planarity in twistor approach.

Third approach involving local unitary as an additional key element is suggested by tensor networks relying on the notion of perfect entanglement discussed by Preskill *et al* [B30].

1. Tensor networks provide an elegant representation of holography mapping interior states isometrically (in Hilbert space sense) to boundary states or vice versa for selected subsets of states defining the code subspace for holographic quantum error correcting code. Again the tensor net is highly non-unique but there is some minimal tensor net characterizing the complexity of the entangled boundary state.

2. Tensor networks have two key properties, which might be abstracted and applied to the construction of S-matrix in zero energy ontology (ZEO): perfect tensors define isometry for any subspace defined by the index subset of perfect tensor to its complement and the non-unique graph representing the network. As far as the construction of Hilbert space isometry between local interior states and highly non-local entangled boundary states is considered, these properties are enough.

One cannot avoid the question whether these three constructions could be different aspects of one and same construction and that tensor net construction with perfect tensors representing vertices could provide and additional strong constraint to the long sought for explicit recipe for the construction of scattering amplitudes.

3.5.1 Objections

It is certainly clear from the beginning that the possibly existing description of S-matrix in terms of tensor networks cannot correspond to the perturbative QFT description in terms of Feynman diagrams.

1. Tensor network description relates interior and boundary degrees in holography by a isometry. Now however unitary matrix has quite different role. It could correspond to U-matrix relating zero energy states to each other or to the S-matrix relating to each other the states at boundary of CD and at the shifted boundary obtained by scaling. These scalings shifting the second boundary of CD and increasing the distance between the tips of CD define the analog of unitary time evolution in ZEO. The U-matrix for transitions associated with the state function reductions at fixed boundary of CD effectively reduces to S-matrix since the other boundary of CD is not affected.

   The only manner one could see this as holography type description would be in terms of ZEO in which zero energy states are at boundaries of CD and U-matrix is a representation for them in terms of holography involving the interior states representing scattering diagram in generalized sense.

2. The appearance of small gauge coupling constant tells that the entanglement between "states" in state spaces whose coordinates formally correspond to quantum fields is weak and just

opposite to that defined by a perfect tensor. Quite generally, coupling constant might be the fatal aspect of the vertices preventing the formulation in terms of perfect entanglement.

One should understand how coupling constant emerges from this kind of description - or disappears from standard QFT description. One can think of including the coupling constant to the definition of gauge potentails: in TGD framework this is indeed true for induced gauge fields. There is no sensical manner to bring in the classical coupling constants in the classical framework and the inverse of Kähler coupling strength appears only as multiplier of the Kähler action analogous to critical temperature.

More concretely, there are WCW spin degrees of freedom (fermionic degrees of freedom) and WCW orbital degrees of freedom involving functional integral over WCW. Fermionic contribution would not involve coupling constants whereas the functional integral over WCW involving exponential of vacuum functional could give rise to the coupling constants assignable to the vertices in the minimal tree diagram.

3. The decomposition $S = 1 + iT$ of unitary S-matrix giving unitarity as the condition $-i(T - T^\dagger) + T^\dagger T = 0$ reflects the perturbative thinking. If one has only isometry instead of unitary transformation, this decomposition becomes problematic since T and $T^\dagger$ whose some appears in the formula act in different spaces. One should have the generalization of Id as a "trivial" isometry. Alternatively, one should be able to extend the state space H_{in} by adding a tensor factor mapped trivially in isometry.

4. There are 3- and 4-vertices rather than only -say, 3-vertices as in tensor networks. For non-Abelian Chern-Simons term for simple Lie group one would have besides kinetic term only 3-vertex $Tr(A \wedge A \wedge A)$ defining the analog of perfect tensor entanglement when interpreted as co-product involving 3-D permutation symbol and structure constants of Lie algebra. Note also that for twistor Grassmannian approach the fundamental vertices are 3-vertices. It must be however emphasized that QFT description emerges from TGD only at the limit when one identifies gauge potentials as sums of induced gauge potentials assignable to the space-time sheets, which are replaced with single piece of Minkowski space.

5. Tensor network description does not contain propagators since the contractions are between perfect tensors. It is to make sense propagators must be eliminated. The twistorial factorization of massless fermion propagator suggest that this might be possible by absorbing the twistors to the vertices.

These reasons make it clear that the proposed idea is just a speculative question. Perhaps the best strategy is to look this crazy idea from different view points: the overly optimistic view developing big picture and the approach trying to debunk the idea.

3.5.2 The overly optimistic vision

With these prerequisites on one can follow the optimistic strategy and ask how tensor networks could the allow to generalize the notion of unitary S-matrix in TGD framework.

1. Tensor networks suggests the replacement of unitary correspondence with the more general notion of Hilbert space isometry. This generalization is very natural in TGD since one must allow phase transitions increasing the state space and it is quite possible that S-matrix represents only isometry: this would mean that $S^\dagger S = Id_{in}$ holds true but $SS^\dagger = Id_{out}$ does not even make sense. This conforms with the idea that state function reduction sequences at fixed boundary of causal diamonds defining conscious entities give rise evolution implying that the size of the state space increases gradually as the system becomes more complex. Note that this gives rise to irreversibility understandandable in terms of NMP [K46]. It might be even impossible to formally restore unitary by introducing formal additional tensor factor to the space of incoming states if the isometric map of the incoming state space to outgoing state space is inclusion of hyperfinite factors.

2. If the huge generalization of the duality of old fashioned string models makes sense, the minimal diagram representing scattering is expected to be a tree diagram with braiding

and should allow a representation as a tensor network. The generalization of the tensor network concept to include braiding is trivial in principle: assign to the legs connecting the nodes defined by perfect tensors unitary matrices representing the braiding - here topological QFT allows realization of the unitary matrix. Besides fermionic degrees of freedom having interpretation as spin degrees of freedom at the level of "World of Classical Worlds" (WCW) there are also WCW orbital degrees of freedom. These two degrees of freedom factorize in the generalized unitarity conditions and the description seems much simpler in WCW orbital degrees of freedom than in WCW spin degrees of freedom.

3. Concerning the concrete construction there are two levels involved, which are analogous to descriptions in terms of boundary and interior degrees of freedom in holography. The level of fundamental fermions assignable to string world sheets and their boundaries and the level of physical particles with particles assigned to sets of partonic 2-surface connected by magnetic flux tubes and associated fermionic strings. One could also see the ends of causal diamonds as analogous to boundary degrees of freedom and the space-time surface as interior degrees of freedom.

The description at the level of fundamental fermions corresponds to conformal field theory at string world sheets.

1. The construction of the analogs of boundary states reduces to the construction of N-point functions for fundamental fermions assignable to the boundaries of string world sheets. These boundaries reside at 3-surfaces at the space-like space-time ends at CDs and at light-like 3-surfaces at which the signature of the induced space-time metric changes.

2. In accordance with holography, the fermionic N-point functions with points at partonic 2-surfaces at the ends of CD are those assignable to a conformal field theory associated with the union of string world sheets involved. The perfect tensor is assignable to the fundamental 4-fermion scattering which defines the microscopy for the geometric 3-particle vertices having twistorial interpretation and also interpretation as algebraic operation.

   What is important is that fundamental fermion modes at string world sheets are labelled by conformal weights and standard model quantum numbers. No four-momenta nor color quantum numbers are involved at this level. Instead of propagator one has just unitary matrix describing the braiding.

3. Note that four-momenta emerging in somewhat mysterious manner to stringy scattering amplitudes and mean the possibility to interpret the amplitudes at the particle level.

Twistorial and number theoretic constructions should correspond to particle level construction and also now tensor network description might work.

1. The 3-surfaces are labelled by four-momenta besides other standard model quantum numbers but the possibility of reducing diagram to that involving only 3-vertices means that momentum degrees of freedom effectively disappear. In ordinary twistor approach this would mean allowance of only forward scattering unless one allows massless but complex virtual momenta in twistor diagrams. Also vertices with larger number of legs are possible by organizing large blocks of vertices to single effective vertex and would allow descriptions analogous to effective QFTs.

2. It is highly non-trivial that the crucial factorization to perfect tensors at 3-vertices with unitary braiding matrices associated with legs connecting them occurs also now. It allows to split the inverses of fermion propagators into sum of products of two parts and absorb the halves to the perfect tensors at the ends of the line. The reason is that the inverse of massless fermion propagator (also when masslessness is understood in 8-D sense allowing M^4 mass to be non-vanishing) to be express as bilinear of the bi-spinors defining the twistor representing the four-momentum. It seems that this is absolutely crucial property and fails for massive (in 8-D sense) fermions.

3.5.3 Twistorial and number theoretic visions

Both twistorial and number theoretical ideas have given a strong boost to the development of ideas.

1. With experience coming from twistor Grassmannian approach, twistor approach is conjectured to allow an extension of super-symplectic and other superconformal symmetry algebras to Yangian algebras by adding a hierarchy of multilocal generators [L3]. The twistorial diagrams for $\mathcal{N} = 4$ SUSY can be reduced to a finite number and there is large number of equivalent diagrams. One expects that this is true also in TGD framework.

   Twistorial approach is extremely general and quite too demanding to my technical skills but its is a useful guideline. An important outcome of twistor approach is that the intermediate states are massless on-mass-shell states but with complex momenta. Does this generalize and could each vertex define unitary scattering event with complex four-momenta in possibly complexified Minkowski space? Or could even real momenta be possible for massive particles, which would be massless in 8-D sense thanks to the existence of octonionic tangent space structure of 8-D embedding space? And what is the role of the unique twistorial properties of M^4 and CP_2?

2. Number theoretical vision suggests that the scattering amplitudes correspond to sequences of algebraic operations taking inputs and producing outputs, which in turn serve as inputs for a neighboring node [L3]. The vertices form a diagram defining a network like structure defining kind of distributed computations leading from given inputs to given outputs. A computation leading from given inputs to given outputs is suggestive. There exists an infinite number of this kind of computations and there must be the minimal one which defines the complexity of the scattering. The maximally simplifying guess is that this diagram would correspond to a braided tree diagram. At space-time level these diagrams would correspond to different space-time surfaces defining same physics: this is because of holography meaning that only the ends of space-time surfaces at boundaries of CD matter.

   This vision generalizes of the old-fashioned stringy duality. It states that all diagrams can be reduced to minimal diagrams. This is achieved by by moving the ends of internal lines so that loops becomes vacuum tadpoles and can be snipped off. Tree diagrams must be however allowed to braid and outside the vertices the diagrams look like braids. Braids for which threads can split and glue together is the proper description for what the diagrams could be. Braiding would provide the counterpart for the non-planar twistor diagrams.

   The fermion lines inside the light-like 3-surfaces can get braided. Smaller partonic 2-surfaces can topologically condense at given bigger partonic 2-surface (electronic parton surface can topologically condense to nano-scopic parton surface) and the orbits of the condensed partonic 2-surfaces at the light-like orbit of the parton surface can get braided. This gives rise to a hierarchy of braids with braids.

3.5.4 Generalization of the notion of unitarity

The understanding of unitarity has been the most difficult issue in my attempts to understand S-matrix in TGD framework. When something turns out to be very difficult to understand, it might make sense to ask whether the definition of this something involves un-necessary assumptions. Could unitarity be this kind of notion?

The notion of tensor network suggests that unitarity can generalized and that this generalization allows the realization of unitarity in extremely simple manner using perfect tensors as building bricks of diagrams.

1. Both twistorial and number theoretical approaches define M-matrix and associated S-matrix as a map between the state spaces H_{in} and H_{out} assignable to the opposite boundaries of CD - say positive and negative energy parts of zero energy state. In QFT one has $H_{in} = H_{out}$ and the map would be Hilbert space unitary transformation satisfying $SS^\dagger = S^\dagger S = Id$.

2. The basic structure of TGD (NMP favoring generation of negentropic entanglement, the hierarchy of Planck constants, length scale hierarchies, and hierarchy of space-time sheets)

suggests that the time evolution leads to an increasingly complex systems with higher-dimensional Hilbert space so that $H_{in} = H_{out}$ need not hold true but is replaced with $H_{in} \subset H_{out}$. This view is very natural since one must allow quantum phase transitions increasing the value of h_{eff} and the value of p-adic prime defining p-adic length scale.

S-matrix would thus define isometric map $H_{in} \subset H_{out}$. Isometry property requires $U^\dagger U = Id_{in}$. If the inclusion of H_{in} to H_{out} is a genuine subspace of H_{out}, the condition $UU^\dagger = Id_{out}$ does not make sense anymore. This means breaking of reversibility and is indeed implied by the quantum measurement theory based on ZEO.

3. It would be at least formally possible to fuse all state spaces to single very large state space by replacing isometry $H_{in} \subset H_{out}$ with unitary map $H_{out} \to H_{out}$ by adding a tensor factor in which the map acts as identity transformation. This is not practical since huge amounts of redundant information would be introduced. Also the information about hierarchical structure essential for the idea of evolution would be lost. This hierarchical of inclusions should also be crucial for understanding the construction of S-matrix or rather, the hierarchy of S-matrices of isometric inclusions including as a special case unitary S-matrices.

4. There is also a further intricacy, which might prevent the formal unitarization by the addition of an inert tensor factor. I have talked a lot about HFFs referring to hyper-finite factors of type II_1 (possibly also of type III_1) and their inclusions [K85]. The reason is that WCW spinors form a canonical representation for these von Neumann algebras.

   Could the isometries replacing unitary S-matrix correspond to inclusions of HFFs? In the recent interpretation the included factor (now H_{in}) corresponds to the degrees of freedom below measurement resolution. Certainly this does not make sense now. The interpretation in terms of finite measurement resolution need not however be the only possible interpretation and the interpretation in terms of measurement resolution might of course be wrong. Therefore one can ask whether the relation between H_{in} and H_{out} could be more complex than just $H_{out} = H_{in} \otimes H_1$ so that formal unitarization would fail.

3.5.5 Scattering diagrams as tensor networks constructed from perfect tensors

Preskill's tensor network construction [B30] realizes isometric maps as representations of holography and as models for quantum error correcting codes. These tensor networks have remarkable similarities with twistorial and number theoretical visions, which suggests that it could be used to construct scattering amplitudes. A further idea inspired by holography is that the description of scattering amplitudes in terms of fundamental fermions and physical particles are dual to each other.

1. In the construction of quantum error codes tensor network defines an isometric embedding of local states in the interior to strongly entangled non-local states at boundary. Their vertices correspond to tensors, which in the proposal of Preskill *et al* [B30] are perfect tensors such that one can take any m legs of the vertex and the tensor defines isometry from the state space of m legs to that of $n - m$ legs. When the number of indices is $2n$, the entanglement defined by perfect tensor between any n-dimensional subspace and its complement is maximal

   TGD framework maximal entanglement corresponds to negentropic entanglement with density matrix proportional to identity matrix. What is important that the isometry is constructed by composing local isometries associated with a network. Given isometry can be constructed in very many ways but there is some minimal realization.

2. The tensor networks considered in [B30] are very special since they are determined by tessellations of hyperbolic space H_2. This kind of tessellations of H_3 could be crucial for understanding the analog of condensed matter physics for dark matter and could appear in biology [K33]. What is crucial is that only the graph property and perfect tensor property matter as far as isometricity is considered so that it is possible to construct very general isometries by using tensor networks.

3.5.6 Eigenstates of Yangian co-algebra generators as a way to generate maximal entanglement?

Negentropically entangled objects are key entities in TGD inspired theory of consciousness and also of tensor networks, and the challenge is to understand how these could be constructed and what their properties could be. These states are diametrically opposite to unentangled eigenstates of single particle operators, usually elements of Cartan algebra of symmetry group. The entangled states should result as eigenstates of poly-local operators. Yangian algebras involve a hierarchy of poly-local operators, and twistorial considerations inspire the conjecture that Yangian counterparts of super-symplectic and other algebras made poly-local with respect to partonic 2-surfaces or end-points of boundaries of string world sheet at them are symmetries of quantum TGD [L11]. Could Yangians allow to understand maximal entanglement in terms of symmetries?

1. In this respect the construction of maximally entangled states using bi-local operator $Q^z = J_x \otimes J_y - J_x \otimes J_y$ is highly interesting since entangled states would result by state function. Single particle operator like J_z would generate un-entangled states. The states obtained as eigenstates of this operator have permutation symmetries. The operator can be expressed as $Q^z = f^z_{ij} J^i \otimes J^j$, where f^A_{BC} are structure constants of SU(2) and could be interpreted as co-product associated with the Lie algebra generator J^z. Thus it would seem that unentangled states correspond to eigenstates of J^z and the maximally entangled state to eigenstates of co-generator Q^z. Kind of duality would be in question.

2. Could one generalize this construction to n-fold tensor products? What about other representations of SU(2)? Could one generalize from SU(2) to arbitrary Lie algebra by replacing Cartan generators with suitably defined co-generators and spin 1/2 representation with fundamental representation? The optimistic guess would be that the resulting states are maximally entangled and excellent candidates for states for which negentropic entanglement is maximized by NMP [K46].

3. Co-product is needed and there exists a rich spectrum of algebras with co-product (quantum groups, bialgebras, Hopf algebras, Yangian algebras). In particular, Yangians of Lie algebras are generated by ordinary Lie algebra generators and their co-generators subject to constraints. The outcome is an infinite-dimensional algebra analogous to one half of Kac-Moody algebra with the analog of conformal weight N counting the number of tensor factors. Witten gives a nice concrete explanation of Yangian [B14] for which co-generators of T^A are given as $Q^A = \sum_{i<j} f^A_{BC} T^B_i \otimes T^C_j$, where the summation is over discrete ordered points, which could now label partonic 2-surfaces or points of them or points of string like object (see `http://tinyurl.com/y727n8ua`). For a practically totally incomprehensible description of Yangian one can look at the Wikipedia article (see `http://tinyurl.com/y7heufjh`).

4. This would suggest that the eigenstates of Cartan algebra co-generators of Yangian could define an eigen basis of Yangian algebra dual to the basis defined by the totally unentangled eigenstates of generators and that the quantum measurement of poly-local observables defined by co-generators creates entangled and perhaps even maximally entangled states. A duality between totally unentangled and completely entangled situations is suggestive and analogous to that encountered in twistor Grassmann approach where conformal symmetry and its dual are involved. A beautiful connection between generalization of Lie algebras, quantum measurement theory and quantum information theory would emerge.

3.5.7 Two different tensor network descriptions

The obvious question is whether also unitary S-matrix of TGD could be constructed using tensor network built from perfect tensors. In ZEO the role of boundary would be taken by the ends of the space-time at upper and lower light-like boundaries of CD carrying the particles characterized by standard model quantum numbers. Strong form of holography would suggest that partonic surfaces and strings at the ends of CD provide information for the description of zero energy states and therefore of scattering amplitudes. The role of interior would be taken by the space-time surface - in particular the light-like orbits of partonic surfaces carrying the fermion lines identified as boundaries of string world sheets. Conformal field theory description would apply to fermions residing at string world sheets with boundaries at light-like orbits of partonic 2-surfaces.

In QFT Feynman diagrammatics one obtains a sum over diagrams with arbitrary numbers of loops. In both twistorial and number theoretic approach however only a finite number of diagrams with possibly complex on mass shell massless momenta are needed. If the vertices are however such that particles remain on-mass-shell but are allowed to have complex four-momenta then the integration over internal momenta (loops) is not present and tensor network description could make sense. This encourages the conjecture that tensor networks could be used to construct the scattering amplitudes in TGD framework.

What could perfect tensor property mean for the vertices identified as nodes of a tensor network? There are two levels to be considered: the geometric level identifying particles as 3-surfaces with net quantum numbers and the fermion level identifying particles as fundamental fermions at the boundaries of string world sheets.

1. At the geometric level vertices corresponds to light-like orbits of partonic 2-surfaces meeting at common end which is partonic 2-surface. This is 3-D generalization of Feynman diagram as a geometric entity. At the level of fermion lines associated with the light-like 3-surfaces one the basic interaction corresponds to the scattering of 2-fermions leading to re-sharing of fermion lines between outgoing light-like 3-surfaces, which include also representations for virtual particles. One has 4-fermion vertex but not in the sense that it appears in the interaction of weak interactions at low energies.

   Geometrically the basic vertex could be 3-vertex: $n > 3$-vertices are unstable against deformation to lower vertices. For 3-vertex perfect tensor property means that the tensor defining the vertex maps any 1-particle subspaces to 2-particle subspace isometrically. The geometric vertices define a network consisting of 3-D "lines" and 2-D vertices but one cannot tell what is within the 3-D lines and what happens in the 2-D nodes. The lines would consist of braided fundamental fermion lines and in nodes the basic process would be 2+2 scattering for fermions. In the case of 3-vertex momentum conservation would effectively eliminate the four-momentum and the state spaces associated with vertex would be effectively discrete. This is p-adically of utmost importance.

2. At the level of fundamental fermion lines in the interior of particle lines one would have 4-vertices and if a perfect tensor describes it, it gives rise to a unitary map of any 2-fermion subspace to its complement plus isometric maps of 1-fermion subspaces to 3-fermion subspaces. In this case momenta cannot act as labels of fermion lines for rather obvious reasons: the solution of the problem is that conformal weights label fundamental fermion lines

   The conservation of discrete quark and lepton numbers allows only vertices of type $qL \to qL$ and its variants obtained by crossing. In this case the isometries might allow realization. The isometries must be defined to take into account quark and lepton number conservation by crossing replacing fermion with antifermion. By allowing the states of Hilbert space in node to be both quarks and leptons, difficulties can be avoided.

Tensor network description in terms of fundamental fermions and CFT

Consider first fundamental fermions. What are the labels characterizing the states of fundamental fermions fermions propagating along the lines? There are two options: the labels are either conformal weights or four-momenta.

1. Since fermions corresponds to strings defining the boundaries of string world sheets and since strong form of holography implies effective 2-dimensionality also in fermion sector, the natural guess is that the conformal weights plus some discrete quantum numbers - standard model quantum numbers at least - are in question. The situation would be well-defined also p-adically for this option. In this case one can hope that conformal field theory at partonic 2-surface could define the fermionic 4-vertex more or less completely. There would be no need to assign propagators between different four-fermion vertices. The scattering diagram would define a composite formed from light-like 3-surfaces and one would have single isometry build from 4-fermion perfect tensors. There would be no integrations over internal momenta.

2. Second option is that fundamental fermions are labelled by four-momenta. The outgoing four-momenta in 4-vertices would not be completely fixed by the values of the incoming momenta and this extends the state space. Concerning p-adicization this integral is not desirable and this forces to consider seriously discrete labelling. The unitarity condition for 2+2 scattering would involve integral over 2-sphere. Four-fermion scattering must be unitary process in QFT so that this condition might be possible to satisfy. The problem would be how to fix this fundamental scattering matrix uniquely. This option does not look attractive number theoretically.

The most plausible option is that holography means that conformal field theory describes the scattering of fundamental fermions and QFT type description analogous to twistorial approach describes the scattering of physical fermions. If only 3-vertices are allowed, and if masslessness corresponds to masslessness in 8-D sense, one obtains non-trivial scattering vertices (for ordinary twistor approach all massless momenta would be collinear if real).

Tensor network description for physical particles

Could the twistorial description expected to correspond to the description in terms of particles allow tensor network description?

1. Certainly one must assign four-momenta to incoming *physical* particles - also fermions - but they correspond to pairs of wormhole contacts rather than fundamental fermions at the boundaries of string world sheets. It would be natural to assign four-momenta also to the virtual *physical* fermions appearing in the diagram and the geometric view about scattering would allow only 3-vertices so that momentum conservation would eliminate momentum degrees of freedom effectively. This would be a p-adically good news.

2. At the level of fundamental fermions entanglement is described as a tensor contraction of the CFT vertices. This locality is natural since the vertices are at null distance from each other. At QFT limit the entanglement between the ends of the line is characterized the propagator.

   One must get rid of propagators in order to have tensor network description. The inclusion of propagators to the fundamental tensor diagrams would break the symmetry between the legs of vertex since the propagator cannot be included to its both ends. Situation changes if one can represent the propagator as a bilinear of something more primitive and include the halves to the opposite ends of the line. Twistor representation of four-momentum indeed defines this kind of representation as a bilinear $p^{a\tilde{b}} = \lambda\tilde{\mu}^{\tilde{b}}$ of twistors λ and $\tilde{\mu}$. There is problem due to the diverging $1/p^2$ factor but residue integral eliminates this factor and one can write directly the fermionic propagator factors as $p^{a\tilde{b}}$.

3. In QFT description the perturbative expansion is in powers of coupling constant. If the reduction to braided tree diagrams analogous to twistor diagrams occurs, power g^{N-2} of coupling constant is expected to factorize as a multiplier of a tree diagram with N external legs. One should understand this aspect in the tensor net-work picture.

   For $\mathcal{N} = 4$ SUSY there is coupling constant renormalization. Similar prediction is expected from TGD. Coupling constant evolution is expected to be discrete and induced by the discrete evolution of Kähler coupling strength defined by the spectrum of its critical values. The conjecture is that critical values are naturally labelled by p-adic primes $p \simeq 2^k$, k prime, labelling p-adic length scales. Therefore one might hope that problems could be avoided.

These observations encourage the expectation that twistorial approach involving only 3-vertices allows to realize tensor network idea also at the level of physical particles. It might be essential that twistors can be generalized to 8-D twistors. Octonionic representation of gamma matrices might make this possible. Also the fact twistorial uniqueness of M^4 and CP_2 might be crucial.

Gauge theory follows as QFT limit of TGD so that one cannot in principle require that gauge theory vertices satisfy the isometricity conditions. Nothing however prevents from checking whether gauge theory limit might inherit this property.

1. For instance, could 3-vertices of Yang-Mills theory define isometric embedding of 1-particle states to 2 particle states? For a given gauge boson there should exist always a pair of gauge bosons, which can fuse to it. Consider a basis for Lie-algebra generators of the gauge group. If the generator T is such that there exists no pair $[A, B]$ with the property $[A, B] = T$, Jacobi identity implies that T must commute with all generators and one has direct sum of Lie algebras generated by T and remaining generators.

2. In the case of weak algebra $SU(2) \times U(1)$ the weak mixing of Y and I_3 might allow the isometric embeddings of type $1 \to 2$. Does this mean that Weinberg angle must be non-vanishing in order to have consistent theory? A realistic manner to get rid of the problem is to allow at QFT limit the lines to be also fermions so that also U(1) gauge boson can be constructed as fermion pair.

How the two tensor network descriptions would be related?

There are two descriptions for the zero energy states providing representation of scattering amplitudes: the CFT description in terms of fundamental fermions at the boundaries of string world sheets, and the description in terms of physical particles to which one can assign light-like 3-surfaces as virtual lines and total quantum numbers.

1. CFT description in terms of fundamental fermions in some aspects very simple because of its 2-dimensionality and conformal invariance. The description is in terms of physical particles having light-like 3-surfaces carrying some total quantum numbers as correlates and is simpler in different sense. These descriptions should be related by an Hilbert space isometry.

2. The perfect tensor property for 4-fermion vertices makes fundamental fermion states analogous to physical states realizing logical qubits as highly entangled structures. Geometric description in terms of 3-surfaces is in turn analogous to the description in terms of logical qubits.

3. Holography-like correspondence between these descriptions of zero energy states (scattering diagrams) should exist. Physical particles should correspond to the level, at which resolution is smaller and which should be isometrically mapped to the strongly entangled level defined by fundamental fermions and analogous to boundary degrees of freedom (fundamental fermions *are* at the boundaries of string world sheets!).

   The map relating the two descriptions seems to exist. One can assign four-momenta to the legs of conformal four-point function as parameters so that one obtains a mapping from the states labelled by conformal weights to the states labelled by four-momenta! The appearance of 4-momenta from conformal theory is somewhat mysterious looking phenomenon but this duality makes it rather natural.

3.5.8 Taking into account braiding and WCW degrees of freedom

One must also take intro account braiding and orbital degrees of freedom of WCW. The generalization of tensor network to braided tensor network is trivial. Thanks to the properties of tensor network orbital and spinor degrees of freedom factorize so that also the treatment of WCW degrees of freedom seems to be possible.

What about braiding?

The scattering diagrams would be tree diagrams with braiding of fermionic lines along light-like 3-surfaces - dance of fundamental quarks and leptons at parquette defined by the partonic 2-surface one might say. Also space-like braiding at magnetic flux tubes at the ends of CD is possible and its time evolution between the ends of space-time surfaces defines 2-braiding which is generalization of the ordinary braiding but will not be discussed here. This gives rise to a hierarchy of braidings. One can talk about flux tubes within flux tubes and about light-like 3-surface within light-like 3-surfaces. The smaller light-like 3-surface would be glued by a wormhole contact to the larger one and contact could have Euclidian signature of induced metric.

How can one treat the braiding in the tensor network picture? The answer is simple. Braiding corresponds to an element of braid group and one can represent it by a unitary matrix as one does in topological QFT as one constructs knot invariants. In particular, the trace of this unitary matrix defines a knot invariant. The generalization of the tensor network is simple. One attaches to the links connecting two nodes unitary transformation defining a representation of the braid involved. Local variant of unitarity would mean isometricity at nodes and unitarity at links.

What about WCW degrees of freedom?

The above considerations are about fermions that its WCW spinor degrees of freedom and the space-time surface itself has been regarded as a fixed background. How can one take into account WCW degrees of freedom?

The scattering amplitude involves a functional integral over the 3-surfaces at the ends of CD. The functional integration over WCW degrees of freedom gives an expression depending on Kähler coupling strength α_K and determines the dependence on various gauge coupling strengths expressible in terms of α_K. This makes it possible to have the tensor network description in fermionic degrees of freedom without losing completely the dependence of the scattering amplitudes on gauge couplings. By strong form of holography the functional integral should reduce to that over partonic 2-surfaces and strings connecting them. Number theoretic discretization with a cutoff determined by measurement resolution forces the parameters characterizing the 2-surfaces to belong to an algebraic extension of rationals and is expected to reduce functional integral to a sum over discretized WCW so that it makes sense also in p-adic sectors [K66, K84].

A brief summary of quantum measurement theory in ZEO is necessary. The repeated state function reduction shifts active boundary A of CD and affects the states at it. The passive boundary of CD- call it P - and the states at it - remain unaffected. The repeated state function reductions leaving P unaffected and giving usually rise to Zeno effect, correspond now to the TGD counterpart of unitary time evolution by shifts between subsequent state function reductions. Call A and its shifted version A_{in} and A_{out} and the corresponding state spaces H_{in} and H_{out}. The unitary (or more generally isometric) S matrix represents this shift. This is the TGD counterpart of a unitary evolution of QFTs. S forms a building brick of a more general unitary matrix U acting in the space of zero energy states but U is not considered now.

Consider now the isometricity conditions.

1. Unitarity conditions generalized to isometricity conditions apply to S. Isometricity conditions $S^\dagger S = Id_{in}$ can be applied at A_{in}. The states appearing in the isometry conditions as initial and final states correspond to A_{in} and A_{out}. There is a trace over WCW spin indices (labels for many-fermion states) of H_{out} in the conditions $S^\dagger S = Id_{in}$. Isometricity conditions involve also an integral over WCW orbital degrees of freedom at both ends: these degrees of freedom are strongly correlated and for a strict classical determinism the correlation between the ends is complete. If the tensor network idea works, the summation over spinor degrees of freedom at A_{out} gives just a unit matrix in the spinor indices at A_{in} and leaves only the WCW orbital degrees of freedom in consideration. This factorization of spinor and orbital WCW degrees of freedom simplifies the situation dramatically.

2. One can express isometricity conditions for modes with $\Psi_{in,M}$ and $\Psi_{out,N}$ at A_{in} and A_{out}: this requires functional integration over 3-surfaces WCW at A_{in} and A_{out}. The conditions are formulated in terms of the labels - call them M_{in}, N_{in} - of WCW spinor modes at A_{in} including standard model quantum numbers and labels characterizing the states of supersymplectic and super-conformal representations. The trace is over the corresponding indices R_{out} at A_{out}. The WCW functional integrals in the generalized unitarity conditions are therefore over A_{in} and A_{out} and should give Kronecker delta $\sum_{R_{out}} S^\dagger_{M_{in}R_{out}} S_{R_{out}N_{in}} = \delta_{M_{in},N_{in}}$.

3. The simplest view would be that Kähler action with boundary conditions implies completely deterministic dynamics. The conditions expressing strong form of holography state that sub-algebras of super-symplectic algebra and related conformal algebras isomorphic to the entire algebra give rise to vanishing Noether charges. Suppose that these conditions posed at the ends of CD are so strong that they fix the time evolution of the space-time surface as preferred extremal completely when posed at either boundary. In this case the isometricity conditions

would be so strong that the double functional integration appearing in the matrix product reduces to that at A_{in} and the isometricity conditions would state just the orthonormality of the basis of WCW spinor modes at A_{in}.

4. Quantum criticality and in particular, the hierarchy of Planck constants providing a geometric description for non-deterministic long range fluctuations, does not support this view. Also the fact that string world sheets connect the boundaries of CD suggests that determinism must be broken. The inner product defining the completeness of the WCW state basis in orbital degrees of freedom can be however generalized to a bi-local inner product involving functional integration over 3-surfaces at both A_{in} and A_{out}. There is however a very strong correlation so that integration volume at A_{out} is expected to be small. This also suggests that one can have only isometricity conditions.

3.5.9 How do the gauge couplings appear in the vertices?

Reader is probably still confused and wondering how the gauge couplings appear in the vertices from the functional integral over WCW degrees of freedom. In twistorial approach, the vanishing of loops in $\mathcal{N}=4$ SYM theory gives just g^N, N the number of 3-vertices. Each vertex should give gauge coupling. Or equivalently, each propagator line connecting vertices should give α_K. The functional integral should give this factor for each propagator line. Generalization of conformal invariance is expected to give this picture.

To proceed some basic facts about N-point functions of CFTs are needed.

1. In conformal field theory the functional form of two-point function is completely fixed by conformal symmetry:

$$\begin{aligned}
G^{(2)}(z_i,\overline{z}_i) &= \frac{C_{12}}{z_{12}^{2h}\overline{z}_{12}^{2\overline{h}}} \ , \\
z_{ij} &= z_i - z_j \ , \ \overline{z}_{ij} = \overline{z}_i - \overline{z}_j \ , \\
h_1 = h_2 = h &= h_a + ih_b \ , \ \overline{h} = \overline{h}_a + i\overline{h}_b \ .
\end{aligned} \tag{3.5.1}$$

$h_1 = h_2 \equiv h$ and its conjugate $\overline{h}$ are conformal weights of conformal field and its conjugate. Note that the conformal weights of conformal fields Φ_1 and Φ_2 must be same. In TGD context C_{12} is expected to be proportional to α_K and this would give to each vertex g_K when couplings are absorbed into vertices.

2. The 3-point function for 3 conformal fields Φ_i, $i=1,2,3$ is dictated by conformal symmetries apart from constant C_{123}:

$$G^{(3)}(z_i,\overline{z}_i) = C_{123} \times \frac{1}{z_{12}^{h_1+h_2-h_3} z_{23}^{h_2+h_3-h_1} z_{31}^{h_3+h_1-h_2}} \times \frac{1}{\overline{z}_{12}^{\overline{h}_1+\overline{h}_2-\overline{h}_3} \overline{z}_{23}^{\overline{h}_2+\overline{h}_3-\overline{h}_1} \overline{z}_{31}^{\overline{h}_3+\overline{h}_1-\overline{h}_2}} \ . \tag{3.5.2}$$

Here C_{123} should bef fixed by super-symplectic and related symmetries and determined the numerical coefficients various couplings when expressed in terms of g_K.

3. 4-point functions have analogous form

$$\begin{aligned}
G^{(4)}(z_i,\overline{z}_i) &= f_{1234}(x,\overline{x}) \prod_{i<j} z_{ij}^{-(h_i+h_j)+h/3} \prod_{i<j} \overline{z}_{ij}^{-(\overline{h}_i+\overline{h}_j)+\overline{h}/3} \ , \\
h &= \sum_i h_i \ ,
\end{aligned} \tag{3.5.3}$$

but are proportional to an arbitrary function f_{1234} of conformal invariant $x = z_{12}z_{34}/z_{13}z_{24}$ and its conjugate.

If only 3-vertices appear/are needed for physical particles - as both twistorial and number theoretic approaches strongly suggest - the conformal propagators and vertices are fixed apart from constants C_{ijk}, which in turn should be fixed by the huge generalization of conformal symmetries. α_K emerges in the expected manner.

This picture seems to follow from first principles.

1. One can fix the partonic 2-surfaces at the boundaries of CD but there is a functional integral over partonic 2-surfaces defining the vertices: their deformations induce deformations of the legs. One can expand the exponent of Kähler action and in the lowest order the perturbation term is trilinear and non-local in the perturbations. This gives rise to 3-point function of CFT nonlocal in z_i. The functional integral over perturbations gives the propagators in legs proportional to α_K in terms of two point function of CFT. Note that the external propagator legs can be eliminated in S-matrix.

2. The cancellation of higher order perturbative corrections in WCW functional integral is required by the quantum criticality and means trivial coupling constant evolution for α_K and other coupling constants. Coupling constant evolution is discretized with values of α_K analogous to critical temperatures and should correspond to p-adic coupling constant evolution [L6].

3. This picture leaves a lot of details open. An integration over the values of z_i is needed and means a kind of Fourier analysis leading from complex domain. The analog of Fourier analysis would be for deformations of partonic 2-surface labelled by some natural labels. Conformal weights could be natural labels of this kind.

It is easy to get confused since there are several diagrammatics involved: the topological diagrammatics of 3-surfacse assignable to the physical particles with partonic 2-surfaces as vertices, the diagrammatics associated with the perturbative functional integral for the Kähler action, and the fermionic diagrammatics suggested to reduce to tensor network. The conjectures are as follows.

1. The "primary" vertices $G^{(n)}$, $n > 3$ assignable to single partonic 2-surface and coming from a functional integral for Kähler action vanishes. This corresponds to quantum criticality and trivial RG evolution.

2. $G^{(n)}$, $n > 3$ in the sense of topological diagrammatics without loops and involving n partonic 2-surfaces do not vanish. One can construct the analog of $G^{(4)}$ from two $G^{(3)}$:s at different partonic 2-surfaces and propagator defined by 2-point function connecting them as string diagram.

   Also topological variant of $G^{(4)}$ assignable to single partonic 2-surface can be constructed by allowing the 3-D propagator "line" to return back to the partonic 2-surface. This would correspond to an analog of loop. Similar construction applies to "primary" $G^{(n)}$,$n > 4$. In number theoretic vision these loops are eliminated as redundant representations so that one has only braided tree diagrams. Also twistor Grassmann approach supports this view.

To sum up, the tensor network description would apply to fermionic degrees of freedom. In bosonic degrees of freedom functional integral would give CFT picture with 3-vertex as the only "primary" vertex and from this twistorial and number theoretic visions follow via the super-symplectic symmetries of the vertex coefficients C_{ijk} extended to Yangian symmetries.

Chapter 4

About Twistor Lift of TGD

4.1 Introduction

The twistor lift of classical TGD [L11] is attractive physically but it is still unclear whether it satisfies all constraints. The basic implication of twistor lift would be the understanding of gravitational and cosmological constants. Volume term in action removes the infinite vacuum degeneracy of Kähler action but because of the extreme smallness of cosmological constant Λ playing the role of inverse of gauge coupling strength, the situation for nearly vacuum extremals of Kähler action in the recent cosmology is non-perturbative.

What is remarkable that twistor lift is possible only in zero energy ontology (ZEO) since the volume term would be infinite by infinite volume of space-time surface in ordinary ontology: by the finite size of causal diamond (CD) the space-time volume is however finite in ZEO. Furthermore, the condition that the destructive interference does not cancel vacuum functional implies Bohr quantization for the action in ZEO. The scale of CD corresponds naturally to the length scale $L_\Lambda = \sqrt{8\pi/\Lambda}$ defined by the cosmological constant.

One motivation for introducing the hierarchy of Planck constants [K27, **?**] was that the phase transition increasing Planck constant makes possible perturbation theory in strongly interacting system. Nature itself would take care about the converge of the perturbation theory by scaling Kähler coupling strength α_K to α_K/n, $n = h_{eff}/h$. This hierarchy might allow to construct gravitational perturbation theory as has been proposed already earlier. This would for gravitation to be quantum coherent in astrophysical and even cosmological scales.

In this chapter two options for the twistor lift are studied in detail.

1. Option I (the original option): The values of $\alpha_K(M^4)$ and $\alpha_K(CP_2)$ are widely different with $\alpha_K(M^4)$ being extremely large so that M^4 part of the 6-D Kähler action gives in dimensional reduction extremely small cosmological term. Allowing Kähler coupling strength $\alpha_K(CP_2)$ to correspond to zeros of zeta implies that for complex zeros the preferred extremals for $\alpha_K(M^4)$ having different phase are mimimal surface extremals of Kähler action so that the values of coupling constants do not matter and extremals depend on couplings only through the boundary conditions stating the vanishing of certain super-symplectic conserved charges.

   It has turned out that this option has several shortcomings. First of all, $\alpha_K(M^4) \neq \alpha_K(CP_2)$ looks like ad hoc assumption tailored to make cosmological constant small. Secondly, the decoupling between Kähler action and volume term implies separately conserved Noether charges which looks strange. Thirdly, for $\sqrt{g_4}$ instead of $\sqrt{|g_4|}$ in the volume element assumed hitherto, there is no charge transfer between Minkowski and Euclidian regions.

2. Option II: $\alpha_K(M^4) = \alpha_K(CP_2)$ is satisfied. Now entire action is identified as the cosmological term. A small effective value of cosmological constant is obtained if the Kähler action and volume term tend to cancel each other. Minimal surface extremals of Kähler action correspond naturally to asymptotic dynamics near the boundaries of CDs, where the analog of free geodesic motion as minimal surfaces is expected. For $\sqrt{|g_4|}$ option there is charge transfer between Minkowski and Euclidian regions.

The two options provide different generalizations of Chladni mechanism [K44] [L15, L16] (see "An Amazing Resonance Experiment" at `http://tinyurl.com/kcbmrzz`)to a "dynamics of avoidance". Both options have profound implications for the views about what happens in particle physics experiment and in quantum measurement, and for consciousness theory and for quantum biology. It is however clear that Option II is the favored one.

The need to understand the twistor lift leads to a critics of the formulation of the basic action principle and the outcome is a more elegant formulation with non-trivial physical consequences.

1. Dimensionless gauge field is obtained from dimension 2 induced Kähler form by division with constant R_1^2 with dimension two. This parameter defines a hidden coupling parameter in the action and the identification in terms of CP_2 radius made hitherto rather implicitly is probably reasonable but ad hoc. The simple idea is to use the induced Kähler form as basic object and formulate the action principle accordingly. This brings in the dimensional parameter $1/R_1^4$ compensating for the dimension of $\sqrt{g_4}$ in the action.

2. One ends up to a general formulation of both bosonic and fermionic action principles showing that the overall scaling factor of fermionic and bosonic actions - call it X, disappears from classical dynamics so that extremals have no explicit independence on X. This is crucial for number theoretical universality.

   Quantum Classical Correspondence (QCC) realized as the condition that classical Noether charges in Cartan algebra correspond to eigenvalues of quantal fermionic charges however breaks the invariance with respect to scalings of action via fermionic anticommutation relations which depend on the scaling factor. The new formulation leads to a unique guesses for the 6-D actions, their 4-D dimensionally reduced variants, and 2-D effective actions.

3. The formulation helps to realize that Number Theoretical Universality (NTU) requires that $\sqrt{|g_4|}$ option is the only possible one. Physically the need to have charge transfer between Euclidian and Minkowskian space-time regions implies the same result.

This leads to two different views about cosmological constant.

1. For Option I the explanation for dark energy is in terms of volume term of the action and small value of cosmological constant obeying p-adic coupling constant evolution as function of p-adic length scale. For Option II the cancellation of Kähler action and volume term would give rise to a small value of cosmological constant and its p-adic evolution.

2. Either $L_\lambda = \sqrt{8\pi/\Lambda}$ or the length L characterizing vacuum energy density as $\rho_{vac} = \hbar/L^4$ or both can obey p-adic length scale hypothesis as analogs of coupling constant parameters. The third option makes sense if the ratio R/l_P of CP_2 radius and Planck length is power of two: it can be indeed chosen to be $R/l_P = 2^{12}$ within measurement uncertainties. $L(now)$ corresponds to the p-adic length scale $L(k) \propto 2^{k/2}$ for $k = 175$, size scale of neuron and axon.

3. A microscopic explanation for the vacuum energy realizing strong form of holography (SH) is in terms of vacuum energy for radial flux tubes emanating from the source of gravitational field. The independence of energy from the value of $h_{eff}/h = n$ implies analog of Uncertainty Principle: the product Nn for the number N of flux tubes and the value of n defining the number of sheets of the covering associated with $h_{eff} = n \times h$ is constant. This picture suggests that holography is realized in biology in terms of pixels whose size scale is characterized by L rather than Planck length.

4. A interesting observation is that a fundamental length scale of biology - size scale of neuron and axon - would correspond to the p-adic length scale assignable to vacuum energy density characterized by cosmological constant and be therefore a fundamental physics length scale. An especially interesting result is that in the recent cosmology the size scale of a large neuron would be fundamental physical length scale determined by cosmological constant. This gives additional boost to the idea that biology and fundamental physics could relate closely to each other: the size scale of neuron would not be an accident but "determined in stars" and even beyond them!

4.2 More about twistor lift of Kähler action

The following piece of text was motivated by some observations relating to the twistor lift of Kähler action forcing a criticism of the earlier view about twistor lift.

The first observation was that the correct formulation of 6-D Kähler action in the framework of adelic physics implies that the *classical* physics of TGD does not depend on the overall scaling of Kähler action. This implies that the preferred extremals need not be minimal surface extremals of Kähler action. It is enough that they are so asymptotically - near the boundaries of CDs where they behave like free particles. This also nicely conforms with the physical idea that they are 4-D generalizations for orbits of particles in induced Kähler field.

The independence of the classical physics on the scale of the action inspires a detailed discussion of the number theoretic vision. Quantum Classical Correspondence (QCC) breaks the invariance with respect to the scalings via fermionic anti-commutation relations and Number Theoretical Universality (NTU) can fix the spectrum of values of the over-all scaling parameter of the action. One ends up to a condition guaranteeing NTU of the action exponential and finds an answer to the nagging question whether one should use $\sqrt{g_4}$ (imaginary in Minkowskian regions) or $\sqrt{|g_4|}$ in the action. Complex α_K allows $\sqrt{|g_4|}$ and NTU assuming that $1/\alpha_K = s$, $s = 1/2 + iy$ zero of Riemann zeta, implies $y = q\pi$, q rational as proposed also in [L6].

Second observation relates to cosmological constant. The proposed vision for the p-adic evolution of cosmological constant assumes that $\alpha_K(M^4)$ and $\alpha_K(CP_2)$ are different for the twistor lift. One however finds that single value of α_K is the natural choice. This destroys the original proposal for the p-adic length scale evolution of cosmological constant explaining why it is so small in cosmological scale.

The solution to the problem of the cosmological constant would be that the *entire* 6-D action decomposing to 4-D Kähler action and volume term is identified in terms of cosmological constant. The cancellation of Kähler electric contribution and remaining contributions would explain why the cosmological constant is so small in cosmological scales and also allows to understand p-adic coupling constant evolution of cosmological constant. One must however remain cautious: also the original proposal can be defended.

4.2.1 Kähler action contains overall scale as a hidden coupling parameter

The first observation leads to a more precise understanding of 6-D Kähler action relates to the induction procedure.

1. Kähler form has dimension two since its square gives metric: $J^2 = -g$. Gauge fields are however 2-forms, which are usually taken to be dimensionless (this requires that coupling constant g is included as multiplicative factor to gauge potential). Accordingly, I have assumed that induced Kähler form is obtained by diving Kähler form by $1/R^2$, R the radius of CP_2 identified as the radius of its geodesic sphere. One can however argue that the identification of the scaling factor is ad hoc since its value does not affect classical field equations.

2. What would happen if one induces the dimensional Kähler form as such? Kähler action density $L_K\sqrt{g_4}$ would have dimension of volume so that $1/\alpha_K$ must be replaced with $1/8\pi\alpha_K R_1^4$, where R_1 a fundamental coupling constant with dimension of length. This coupling however disappears from the classical field equations and in the recent adelic formulation also from quantum theory [L24].

3. For the 6-D twistor lift of Kähler action one must introduce an additional dimensional factor to get a dimensionless action. One has $R_1^4 \rightarrow R_1^4 R_0^2$, where R_0^2 has dimensions of area. The 4-D action density obtained from dimensional reduction for twistor sphere $S^2(X^4)$ assuming that the induced Kähler form for the sphere satisfies $J^4 = -g$ for $S^2(X^4)$ is proportional to

$$L = X \times (J \cdot J - 2)\sqrt{g_4} \quad , \quad X = \frac{1}{2\alpha_K}\frac{Area(S^2(X^4))}{S_0}\frac{1}{R_1^4} \quad , \quad S_0 = 4\pi R_0^2 \quad . \qquad (4.2.1)$$

The shift of Kähler action density by -2 comes from $S^2(X^4)$ part of 6-D Kähler action.

4. From this form one can immediately see that the factor X in Eq. 4.2.1 disappears from field equations, and the functional form of preferred extremals has no dependence on coupling parameters! The quantum classical correspondence (QCC) stating that fermionic Noether charges in Cartan algebra have eigenvalues equals to their classical counterparts however implies this dependence.

Modified Dirac action and string world sheet action in the new formalism

What about the modified Dirac action related super-symmetrically to Kähler action in the new formalism? The 6-D formalism for the induced spinors doubles the number of spinor components and dimensional reduction must eliminate half of them to give something equivalent with the ordinary induced spinor structure. Chirality condition is the most plausible manner to achieve this. This answers the old question whether one could assume only leptonic spinors as fundamental spinors and construct quarks as some of anyonic leptons. This would require two chirality conditions and this is very probably not possible. The 6-D modified Dirac action can be written using the same rules as applied in 4-D case. The possible delicacies of the fermionic dimensional reduction require a separate discussion.

The 4-D dimensionally reduced part of 6-D modified Dirac action must reduce to the 4-D modified Dirac action associated with the full bosonic action. The modified gamma matrices Γ^α are expressible as contractions of the canonical momentum currents with embedding space gamma matrices (this applies also in $D = 6$). Therefore they are proportional to the dimensionless quantity $X\sqrt{g_4}$. Γ^α has dimension $1/L$ so that induced spinors must have dimension $L^{1/2}$. In the usual approach the dimension would be $1/L^{3/2}$.

With these conventions X apparently drop from the equations stating QCC as identity of eigenvalues of fermionic Noether charges and corresponding classical Noether charges in Cartan algebra. This not true. The anti-commutations for Ψ and time component J^0 of the canonical momentum density $J^\alpha = \partial L/\partial(\partial_\alpha\Psi) = \overline{\Psi}\Gamma^\alpha$ involve X and affect the scale of anti-commutation relations and therefore QCC. That the anti-commutations can be indeed realized under these dimensional constraints, requires a proof.

What about the spinors restricted to 2-D string world sheets and corresponding space-time action? Perhaps the most plausible option is that they do not appear at the fundamental level and appear only as the effective action suggested by SH. If this is the case, it is rather easy to guess the form of the bosonic and fermion 2-D effective actions. Their forms could be exactly the same as the form of 4-D actions. The only modification would be in the bosonic case the replacement of $1/R_1^4$ with $1/R_1^2$ to get the dimensions correctly! The bosonic action would dictate the fermionic action by above rules.

The bosonic string world sheet action would differ from the area action. The action density would be $XR_1^2(J \cdot J - 2)\sqrt{g}$ in complete analogy with the 4-D case. Two special cases deserve to be mentioned.

1. This action vanishes for string world sheets with $J \cdot J = 2$. This is the case if one has $J = M(M^4)$ and J is self-dual. This is true if string world sheet is the preferred plane M^2 defining the symplectic structure of M^4 (there is moduli space form them in order to gain Lorentz invariance and giving rise to sectors of WCW).

   Small deformations of this plane would give rise to strings with small string tension and be naturally relating to the small value of the cosmological constant. These strings should accompany long strings mediating gravitational interaction in long length scales. The small action would require large value of $h_{eff}/h = n = h_{gr}$ for the perturbation theory to work.

2. Second special case corresponds to Lagrangian surfaces for which $J(M^4) + J(CP_2)$ induced to string world sheet vanishes. One would have ordinary strings with area action. String tension would be determined by CP_2 size scale. The appearance of also light strings would distinguish between TGD and super string models.

Kähler action can contain also a topological instanton term affecting the field equations only via boundary condition. This term could induce to the string world sheet action a magnetic flux term reducing to a boundary term at the boundaries of string world sheets adding an interaction term to the usual action defined by word-line length. The outcome would be equation of motion for a point-like particle experiencing Kähler force. These topological terms give additional terms to corresponding modified Dirac equations.

It would seem that the new approach to action principle allows a more unified approach to the details of the variational principle in dimension $D = 4$ and allows also to deduce the general form of 6-D and 2-D effective action. It must be however made clear that one could have brane like hierarchy of structures already at fundamental level. Also in this case the new approach applies.

Action principle, quantum classical correspondence, and number theoretical universality

The above observations force to reconsider the interpretation of the action principle. Here the adelic physics based vision can be used as a guideline.

1. It is good to list the geometric parameters and coupling constant like parameters of TGD. CP_2 scale $R(CP_2)$ certainly appears in the theory. The radius of $S^2(M^4)$ makes l_P^2 a natural scale factor of M^4 metric. One can re-scale $J(M^4)$ and the M^4 part of the metric of $T(M^4)$ but not the entire metric.

2. $r = R_1/R(CP_2)$ can be seen as a dimensionless coupling constant like parameter and in principle quantum criticality allows it to have a spectrum values determined by the extension of rationals defining adeles. The QCC condition stating the quantized values of the fermionic Noether charges are equal to their classical counterparts having non-local expressions forces to consider the possibility that the value of R_1 can indeed vary and has value guaranteeing that QCC holds true. Also α_K has spectrum of values: one possible spectrum corresponds to the zeros of Riemann zeta [L6]. Even the number theoretically problematic exponent of action could belong to the extension with a suitable choice of R_1.

   This would allow to speak about the exponent of action and of Kähler function making sense also p-adically in the intersection of real and p-adic WCWs. Both action and its exponent should exist in the extension. This is true if the action is of form $q_1 + q_2\pi$, q_i rational numbers. One might hope that a suitable choice of R_1 could make possible to realize QCC and this condition.

QCC and the value spectrum of R_1

Classical field equations do not depend at all on the value on the overall coefficient X of the action in Eq. 4.2.1. Also boundary conditions are independent of the scaling of X. Does this mean that one has projective invariance in the sense that the value of R_1 does not matter at all? No!

1. QCC for the Cartan algebra of fermionic and classical Noether charges gives meaning for the scale R_1. QCC states that the eigenvalues of the Cartan algebra charges are equal to the corresponding values of classical Noether charges. Since the normalization of quantal charges is fixed by the value of $\hbar$, this fixes the normalization of classical charges and thus the parameter R_1. If Ψ is taken dimensionless, the modified Dirac action can be taken to be proportional to factor $1/R_1^3$. Therefore R_1 has physical meaning. The above argument suggests that R_1 is fixed by quantum criticality and characterizes the extension of rationals.

2. Could one require that the values of classical charges belong to the extension of rationals defining the adeles in question? This condition involves in real context integral over 3-surface and is thus a non-local operation. How can one know, which 3-surfaces satisfy the condition? Is the choice of R_1 dictated by this condition so that it depends on the extension of rationals involved and obeys number theoretic coupling constant evolution?

   Note that classical Noether charges serve as WCW coordinates, and the interpretation would be the same as at space-time level: these special 3-surfaces would form a kind of cognitive representation analogous to that formed by the points of space-time surface with coordinates in extension. The quantization of these WCW coordinates would give a cognitive representation!

3. The action would be same for the symmetry related 3-surfaces and one could have WCW wave functions at the orbits of symmetries with coordinates which are conjugate variables for the quantized Noether charges. For the orbits of symmetry groups the allowed points in WCW would correspond to values of group parameters in the extension. Besides isometries and corresponding Kac-Moody algebras supersymplectic symmetry gives rise to this kind of wave functions. In case of four-momentum, the basic number theoretic conditions would be for rest masses.

   Strong form of holography (SH) could be realized by the reduction of both bosonic and fermionic action to an effective action restricted to string world sheets and partonic 2-surfaces. This option looks more attractive from the point of view of SH than fundamental action containing terms located at lower-dimensional surfaces.

Number theoretical universality and action exponential

In adelic physics number theoretical universality plays a key role.

1. Adelic physics leads to the proposal that the action exponentials appearing in the scattering amplitudes disappear. The normalization factor defined by functional integral of action exponential to which also the scattering amplitude is proportional would cancel them as in QFTs [L24].

   This would require that each maximum of Kähler function with respect to variations of 3-surface and having fixed topological scattering diagram defined by light-like partonic orbits and same action defines its own zero energy state as functional integral and these states can be freely superposed. One would not functionally integrate over different topological scattering diagrams: this would allow to interpret topological scattering diagram as a representation of computation.

2. At the level of scattering amplitudes - but not at the level of WCW geometry - the absence of exponents would allow to get rid of the grave difficulty posed by the fact that the exponent of Kähler action belongs to an extension of rationals only when powerful additional conditions are satisfied. The cancellation of exponents of action from scattering amplitudes looks compelling if one requires number theoretical universality since there are no practical means for checking that the exponent of action is in the extension of rationals for an arbitrary preferred extremal. Also the definition of the action as integral is problematic in p-adic context and the only possible means to define it seems to be in terms of algebraic continuations from the real sector.

   One can however argue against number theoretical extremism. Action exponentials are needed for the interpretation of the theory. Maxima of Kähler function, which also correspond to stationary phase correspond to the most probable 3-surfaces. Hence one can argue that the exponents should appear in the scattering amplitudes. Number theoretical cognition theorist could however argue that the points of WCW, which correspond to maxima have WCW coordinates in an extension of rationals and thus define cognitive representation at the level of WCW. Furthermore, one can argue that scattering amplitudes are not the entire physics. Kähler action and its exponent have real meaning independent of scattering amplitudes.

3. On the other hand, if the value of R_1 adjusts to guarantee that the action is of form

   $$S = q_1 + iq_2\pi \quad . \tag{4.2.2}$$

   exponents can appear in the amplitudes and the standard approach allowing functional integral giving sum of several exponents makes sense. In this case the scattering amplitudes are proportional to X_i/X, $X = \sum_i X_i$, where X_i denotes action exponent for a particular maximum of action as function of WCW coordinates. Note however that the action itself is not number theoretically universal: only its exponent. This is completely analogous with the fact that angles do not make sense p-adically and one can speak about corresponding phases identified as roots of unity.

Number theoretical universality (NTU) allows two options to consider depending on whether the action exponentials can appear in the scattering amplitudes or not. In WCW geometry action and also its exponent certainly appear.

1. The elimination of exponents of 6-D action from the scattering amplitudes would be a huge simplification and make practical calculations possible. This kind of assumption is in practice made also in standard path integral approach as approximation. ZEO allows this and the interpretation is in terms of the notion of quantum phase of matter: different topologies for partonic 2-surfaces correspond to different phases and the localization to single phase for zero energy states is possible: space-time would be much more classical object than without localization. One must however remain critical: the value of R_1 depending on extension of rationals could allow to achieve QCC conditions.

2. If something is gained, something is also lost. The earlier arguments involving exponent of Kähler function are lost if the exponentials do not appear in scattering amplitudes. In particular, the estimate for the value of gravitational coupling strength in terms of exponent of Kähler function and α_K (see the last section of [L34]) is lost if exponents do not appear anywhere. One can argue that this argument was actually lost already when the twistor lift was introduced and Planck length was transformed to a fundamental parameter appearing as scaling factor of M^4 Kähler form and metric.

There is a further challenge for the adelic physics. What could fix the value of the fundamental parameter $l_P^2/R^2(CP_2)$ (of order $10-7$)? It seems that quantum criticality cannot help here. Both l_P^2 and R^2 appear in the induced metric of space-time surface and number theoretical universality for field equations demands that $l_P^2/R^2(CP_2)$ is a rational number. The p-adic evolution scenario of cosmological constant and empirical input for the cosmological constant gives $l_P^2/R^2(CP_2) = 2^{-12}$ [L12]. Why power of 2 which having unit p-adic norm for all odd primes and why just this power?

To sum up, a more precise adelic formulation of the classical action has allowed to detect a hitherto hidden scaling parameter in the action appearing as an additional coupling parameter depending on the extension of rationals, to understand better the number theoretical role of QCC, and allowed to answer a nagging question about whether to use metric determinant or its absolute value in the action assuming NTU for the exponential of action, and deduce the earlier conjecture for the zeros of zeta.

Answer to an old nagging question

Eq. 5.4.1 can be applied to the situation in which the extremal is known. For CP_2 type extremals volume and Kähler action (-4 times volume) are indeed known. Quite surprisingly, this suggest a solution to an old problem whether one should use $\sqrt{g_4}$ giving imaginary volume element in Minkowskian space-time regions or $\sqrt{|g_4|}$ used usually.

1. The action exponent

$$e^{\frac{x}{2\alpha_K}} \quad , \quad x = \frac{6Vol(CP_2)}{R_1^4}$$

is a number in an extension of rationals guaranteed if one has

$$(1/2)Re(\frac{1}{\alpha_K}) \times x = q_1 \quad , \quad (1/2)Im(\frac{1}{\alpha_K}) \times x = q_2\pi \ .$$

.

2. Suppose that the volume integral uses volume element $\sqrt{g_4}$, which is imaginary in Minkowskian space-time regions and real in Euclidian regions. The motivation is that for real α_K the action exponential from Minkowskian space-time regions is phase as QFT picture demands.

   For $1/\alpha_K = is = i/2 + y$, s a complex zero of zeta, the phase of the action exponential coming from Minkowskian regions is proportional to iy and in a good approximation equal to $1/Re(\alpha_K)$. The conditions give $Vol(CP_2)/R_1^4 \propto \pi$ and $y = q$. Note that $Vol(CP_2)$ is proportional to π^2 so that the normalization volume R_1^4 would be proportional to π. Since $R_1^4 = q \times Vol(CP_2)$ is natural normalization factor one would have expected x to be rational. This does not look promising.

That the zeros of zeta should be complex rationals is totally unexpected but would conform with the number theoretical universality. This would be of course very nice from TGD point of view strongly suggesting that zeros belong to some extension of rationals. I have proposed that the zeros of zeta appear as conformal weights in TGD framework [L6].

3. Suppose that the volume element is given by $\sqrt{|g_4|}$ as was done originally. If α_K is complex, the phase factor is obtained in any case. This option favours $1/\alpha_K = s$, s a complex zero of zeta. Eq. 5.4.1 would predict $Vol(CP_2)/R_1^4 = q$ and $y = q\pi$. These predictions conform with the physical intuition. I have proposed earlier [L6] that the exponents of imaginary parts for the zeros of zeta could correspond to roots of unity. Only the exponents of zeros of zeta would be number theoretically universal and continuable to the p-adic sectors.

To sum up, a more precise adelic formulation of the classical action has allowed to detect a hitherto hidden scaling parameter in the action appearing as an additional coupling parameter depending on the extension of rationals, to understand better the number theoretical role of QCC, and allowed to answer a nagging question about whether to use metric determinant or its absolute value in the action assuming NTU for the exponential of action, and deduce the earlier conjecture for the zeros of zeta.

There is however a further challenge for the adelic physics. What could fix the value of the fundamental parameter $l_P^2/R^2(CP_2)$ (of order 10−7)? It seems that quantum criticality cannot help here. Both l_P^2 and R^2 appear in the induced metric of space-time surface and number theoretical universality for field equations demands that $l_P^2/R^2(CP_2)$ is a rational number. The p-adic evolution scenario of cosmological constant and empirical input for the cosmological constant gives $l_P^2/R^2(CP_2) = 2^{-12}$ [L12]. Why power of 2 which having unit p-adic norm for all odd primes and why just this power?

4.2.2 The problem with cosmological constant

Second (unpleasant) observation was that the previous proposal for the twistor lift of Kähler action has an ad hoc feature.

Can the original proposal for the twistor lift of Kähler action be correct?

Consider first the unpleasant observation about cosmological constant.

1. α_K is also assumed to be complex and the conjecture [L6] has been that its values correspond to zeros of Riemann zeta. In the earlier proposal for twistor lift cosmological constant and α_K are assumed to obey independent p-adic evolutions, and cosmological constant was assumed to be real and to behave like $1/p$ as function of p-adic prime in p-adic length scale evolution so that its extreme smallness in cosmological scales could be understood [L11, L12].

   The motivation for the proposal was the decomposition $T(H) = T(M^4) \times T(CP_2)$ of the twistor space of H. It was argued that this allows to decompose the Kähler action of $T(H)$ to a sum of two parts with *different* values of α_K. For M^4 part the value of α_K, call it $\alpha_K(M^4)$, would be enormous and the resulting volume term in the dimensionally reduced 6-D Kähler action would have cosmological constant $\hbar/l_D^4$ as its coefficient: l_D would be of the order of the size about 10^{-4} meters of a large neuron in cosmological length scales.

2. If the value of $\alpha_K(M^4)$ is real or has different phase than $1/\alpha_K$, whose spectrum is proposed to correspond to zeros of zeta [L6], the action is complex, and one has separate field equations for real and imaginary part of action. The extremals would be minimal surface extremals of Kähler action. That all known extremals of Kähler action have this property was seen as a support for the hypothesis.

   The physically problematic aspect is that Kähler action and volume term effectively decouple. This would make sense asymptotically but looks strange as a general property [?] On the other hand, the independence of the extremals on coupling constants is a highly desirable outcome from the point of view of number theoretical universality.

3. The assumption about different Kähler coupling strengths admittedly looks somewhat ad hoc. If one assumes that also M^4 possesses Kähler form $J(M^4)$ [L26], and induced Kähler form corresponds to the sum $J(M^4) + J(M^2)$, universal value of α_K is the natural option. This assumption however allowed to understand the smallness of cosmological term in 4-D action and also the p-adic coupling constant evolution for the cosmological constant.

4. Also boundary conditions are problematic for this option. It would be highly desirable to have flow of classical Noether charges between Euclidian and Minkowskian space-time regions as a correlate for classical interactions between physical objects having Euclidian regions as space-time correlates (analogous to lines of scattering diagrams). The conditions stating the conservation of sums of complex Kähler and volume charges from Minkowskian and Euclidian regions however give 2+2 conditions if the phases of Kähler action and volume term are different and the metric determinant $\sqrt{g_4}$ is imaginary for Minkowskian regions. It is easy to see that Kähler and volume charges are conserved separately and that there is no charge transfer between Euclidian and Minkowskian regions. The alternative $\sqrt{|g_4|}$ allows the flow of real and imaginary charges between the two regions. One can however insist that the existence of two separate conserved energies should have been discovered long time ago.

What if one gives up the assumption $\alpha_K(M^4) \neq \alpha_K(CP_2)$?

1. The volume term would be also proportional to $1/\alpha_K$ so that the phases of both Kähler action and volume term would be identical. The pleasant surprise is that coupling constants disappear from the field equations altogether! It is not necessary to postulate minimal surface property of the preferred extremals anymore to guarantee number theoretical universality.

   Minimal surface property could be however asymptotic so that there would be no exchange of conserved quantities between these degrees of freedom. This would conform with the idea that incoming and outgoing particles are free and thus minimal surfaces as 4-D generalization of a geodesic line resulting when 4-D generalization of Abelian Maxwell force vanishes. Causal diamond (CD) would represent a region with the property that the extremals approach minimal surfaces at its boundary. One can loosely say that interactions are coupled on and off near the opposite boundaries of CD: CD corresponds to scattering volume.

   The vertices of topological diagrams defined by as 2-D intersections of the ends of orbits of partonic 2-surfaces - analogous to vertices of Feynman diagrams - would be also accompanied by transient regions, where there the motion of 3-surface is not geodesic. The results are extremely nice from the point of view of number theoretical universality.

2. Also in this case the charge transfer between Euclidian and Minkowskian regions is impossible if $\sqrt{g_4}$ defines volume element (imaginary in Minkowskian regions). $\sqrt{|g_4|}$ this is not the case. As found, also NTU favors this option.

3. The above result is extremely nice. What makes the shower cold is that one ends up with problems with cosmological constant since Kähler and volume terms in the action are of same order of magnitude. Also the proposed p-adic evolution scenario for the cosmological constant is lost. The only cure that I can imagine is that the entire 4-D action has interpretation as a cosmological term, and that a cancellation between Kähler action and volume term take place giving rise to a very small effective value of cosmological constant.

Can one understand the p-adic evolution of cosmological constant?

The above findings lead to a problem with cosmological constant.

1. If the cosmological constant corresponds to the volume term in the dimensionally reduced 6-D Kähler action with scaling factor $X = 1/2\alpha_K R_1^2 S_0$, one has from Eq. 4.2.1

$$\rho_{vac} = \frac{1}{l_D^4} = \frac{2}{\alpha_K R_1^4}\frac{Area(S^2(X^4))}{S_0} = \frac{\Lambda}{8\pi l_P^2} \quad . \tag{4.2.3}$$

Here l_D corresponds to a length scale which is roughly the size 10^{-4} meters of large neuron for cosmological constant in cosmic scales. Also Kähler action would be extremely small. It would however seem that the ratio of these actions should be extremely small. The simplest solution corresponds to $\frac{Area(S^2(X^4))}{S_0} = 1$.

2. The Kähler action for CP_2 type extremal with light-like geodesic as M^4 projection the action would be

$$S = -3\frac{Vol(CP_2)}{l_D^4} \quad .$$

The action has totally different order of magnitude than assumed earlier if R_1 corresponds to the value of cosmological constant. If one assumes $R_1 = R(CP_2)$, cosmological constant is enormous. Something seems to go wrong.

How could one overcome this problem?

1. Could l_D be small and imply large cosmological constant? Could the parameter $X = \frac{Area(S^2(X^4)}{S_0}$ be small and increase the effective size of l_D? Could the time-like signature for $S^2(M^4)$ allow this by reducing the value of $Area(S^2(X^4)$?

   One can study the embedding of $S^2(X^4)$ to $S^2(M^4)$ and $S^2(CP_2)$ characterized by winding numbers n_1 and n_2. One can choose S_0 to be the area for the embedding with $n_1 = n_2 = 1$. This gives $\frac{Area(S^2(X^4)}{S_0} = (n_1X^2 - n_2)(X^2 - 1)$, $X^2 = (R^2(CP_2)/l_P^2)$ for time-like signature for $S^2(M^4)$. The condition $\frac{Area(S^2(X^4)}{S_0} = 1/p$ would give p-adic length scales but could be satisfied for finite number of primes p only. Second problem is that this would *not* affect the ratio of Kähler and volume contributions to the action.

2. Could effective cosmological constant correspond to the entire action so that Kähler would cancel the real cosmological term in cosmological scales?

   Could $J \cdot J - 2$ should become small in Minkowskian regions and be necessarily large in Euclidian regions? The positive Kähler electric contribution to the action should sum up to almost zero with the negative magnetic contribution and cosmological term. This cancellation should take place in cosmic scales at least and require long range induced Kähler electric fields. They are assumed to be present in the model for large voids. If M^4 Kähler form is present as CP breaking and some other arguments suggest [L26] [L12], it could give a large Kähler electric contribution in long scales if CP_2 contribution becomes small as one might expect.

   The values of 6-D Kähler action should have tendency to concentrate around values inversely proportional to prime p near power of 2 (also other small primes can be considered). The values of Kähler action for the maxima of Kähler function could have this property. This conjecture was made earlier in an attempt to understand gravitational constant in terms of p-adic length scale hypothesis and the exponent of Kähler action for CP_2 type extremals (see the last section of [L34]).

3. This interpretation would mean that for strings like objects having both vanishing induced M^4 and CP_2 parts of induced Kähler fields the action would be large and coming from cosmological constant in CP_2 scale, and one could at least formally say that the situation is perturbative. Strings could however carry non-vanishing and large M^4 parts of Kähler electric fields and the action could be small in this case.

4. I must be added that the interpretation of cosmological constant has varied during years. For the 4-D Kähler action the proposal was that cosmological constant corresponds to the magnetic part of Kähler action with magnetic tension responsible for the negative pressure. The twistor lift in turn led to ask whether Kähler action and volume term could provide alternative, dual ways to understand cosmological constant. For the recent option the small effective cosmological constant results from the cancellation of Kähler action and volume term.

The cautious conclusion would be following. If the 6-D Kähler action contains only single α_K, the cosmological constant is very large at short scales and for Euclidian space-time regions. The cancellation of Minkowskian Kähler electric contribution and Kähler magnetic action in 6-D sense however makes the effective value of cosmological very small. The solution of the problem of cosmological constant would be dynamical. The previous option for which Kähler action decomposes to M^4 and CP_2 parts with different values of $\alpha_K(M^4)$ and $\alpha_K(CP_2) \leq\leq \alpha_K(M^4)$ cannot be however excluded.

4.3 Twistor lift of TGD, hierarchy of Planck constant, quantum criticality, and p-adic length scale hypothesis

Kähler action is characterized by enormous vacuum degeneracy: any four-surface, whose CP_2 projection is Lagrangian sub-manifold of CP_2 having therefore vanishing induced Kähker form, defines a vacuum extremal. The perturbation theory around canonically imbedded M^4 in $M^4 \times CP_2$ defined in terms of path integral fails completely as also canonical quantization. This led to the construction of quantum theory in "world of classical worlds" (WCW) and to identification of quantum theory as classical physics for the spinor fields of WCW: WCW spinors correspond to fermionic Fock states. The outcome is 4-D spin glass degeneracy realizing non-determinism at classical space-time level [K35, K19, K86, K66].

The twistor lift of TGD is based on unique properties of the twistor spaces of M^4 and CP_2. Note that M^4 allows two notions of twistor space. The first one involves conformal compactification allowing only conformal equivalence class of metrics. Second one is equal to Cartesian product $M^4 \times S^2$ [B42] (see `http://tinyurl.com/yb4bt74l`). CP_2 has flag manifold $SU(3)/U(1) \times U(1)$ as twistor space having interpretation as the space for the choices for quantization axis of color hypercharge and isospin. Both these spaces Kähler structure (strictly speaking E^4 and S^4 allow it but the notion generalizes to M^4) and there are no others. Therefore TGD is unique both from standard model symmetries and twistorial considerations.

The existence of Kähler structure is a unique hint for how to proceed in the twistorial formulation of classical TGD. One must lift Kähler action to that in the twistor space of space-time surface having also S^2 as a fiber and identify the preferred extremals of this 6-D Kähler action as those of dimensionally reduced Kähler action, which is 4-D Kähler action plus volume term identifiable in terms of cosmological constant. As found, there are two options to consider.

1. Option I: The values of $\alpha_K(M^4)$ and $\alpha_K(CP_2)$ are widely different with $\alpha_K(M^4)$ being extremely large so that M^4 part of the 6-D Kähler action gives in dimensional reduction extremely small cosmological term. Allowing Kähler coupling strength $\alpha_K(CP_2)$ to correspond to zeros of zeta implies that for complex zeros the preferred extremals for $\alpha_K(M^4)$ having different phase are mimimal surface extremals of Kähler action so that the values of coupling constants do not matter and extremals depend on couplings only through the boundary conditions stating the vanishing of certain super-symplectic conserved charges. In this case the cosmological constant would correspond to running $\alpha_K(M^4)$ and would behave like $1/p$, p p-adic prime. This was the original proposal.

2. Option II: $\alpha_K(M^4) = \alpha_K(CP_2)$ is satisfied. A small effective value of cosmological constant is obtained if the Kähler action and volume term tend to cancel each other. In this case minimal surface extremals of Kähler action correspond naturally to asymptotic dynamics near the boundaries of CDs, where the analog of free geodesic motion as minimal surfaces is expected. In this case effective cosmological constant would correspond to the *entire action*: volume term and Kähler action receiving also M^4 contribution would cancel almost completely in cosmic scales.

   One can in fact argue that one cannot distinguish between Kähler and volume contributions to the action so that Option II remains the only possible one. Option I also breaks the symmetry between Kähler forms of M^4 and CP_2. It is natural that the induced Kähler form is the sum of both and appears in the Kähler action: hence $\alpha_K(M^4) = \alpha_K(CP_2)$.

Option I might be argued to be adhoc but at this moment it is not yet wise to select between these two options. The most conservative assumption is that the twistorial approach is only an alternative for the space-time formulation: in this formulation preferred extremal property might reduce to twistor space property.

Kähler action gives as fundamental constants the radius $R \simeq 2^{12} l_P$ of CP_2 serving as the TGD counterpart of the unification scale of GUTs and Kähler coupling strength α_K in terms of which gauge coupling strengths can be expressed. Twistor lift gives 2 additional dimensional constants. The radius of S^2 fiber of M^4 twistor space $M^4 \times S^2$ is essentially Planck length $l_P = \sqrt{G/\hbar}$, and the cosmological constant $\Lambda = 8\pi G\rho_{vac}$ defining vacuum energy density is dynamical in the sense that it allows p-adic coupling constant evolution as does also α_K.

For both Option I and II one can imagine two options for the p-adic coupling constant evolution of cosmological constant.

1. $\rho_{vac} = k_1 \times \hbar/L_p^4$, where $p \simeq 2^k$ characterizes a given level in the p-adic length scale hierarchy for space-time sheets. Here one can in principle allow $k_1 \neq 1$.

2. $\Lambda/8\pi = k_2/L_\Lambda^2 \propto \frac{1}{p_\Lambda^2}$. Also k_2 could differ from unity. Number theoretical universality suggests $k_1 = k_2 = 1$. The that here secondary p-adic length scale is assumed.

The first option seems more natural physically. During very early cosmology $\Lambda R^2/8\pi$ approaches l_P^2/R^2 for the first option, where $R \simeq 2^{12} l_P$ is the size scale of CP_2 so that one has $\Lambda R^2/8\pi \simeq 2^{-24} \simeq 6 \times 10^{-8}$ at this limit. Therefore perturbation theory would fail for Option I also in early cosmology near vacuum extremals. In the recent cosmology Λ is extremely small. Note that vacuum energy density would be always smaller than $\hbar/R^4$ and thus by a factor $(l_P/R)^4 \simeq 2^{-48} \simeq 3.6 \times 10^{-15}$ lower than in GRT based cosmology.

It it is good go recall that the earlier identification of the cosmological constant was in terms of the effective description for the magnetic energy density of the magnetic flux tubes. Magnetic tension would give rise to effective negative pressure. For Option II the cosmological constant would correspond to the entire action with magnetic and volume contributions slightly larger than Kähler electric contribution. For Option I it wold correspond to the volume term.

4.3.1 Twistor lift brings volume term back

Concerning volume term the situation changed as I introduced twistor lift of TGD. One could say that twistor lift forces cosmological constant. As already described, there are two options: Option I and Option II. The following arguments developed for Option I apply with small modifications also to Option II. The only difference is that the volume term has complex phase for complex α_K [L6] and effective cosmological constant follows from the compensation of Kähler and volume contributions.

1. The twistor lift of Kähler action is 6-D Kähler action for the twistor space $T(X^4)$ of space-time surface X^4. The analog of twistor structure would be induced from the product $T(M^4) \times T(CP_2)$, of twistor spaces $T(M^4) = M^4 \times S^2$ of M^4 [B42] and $T(CP_2) = SU(3)/U(1) \times U(1)$ of CP_2 having Kähler structure so that the induction of Kähler structure to $T(X^4)$ makes sense. Besides M^4 and CP_2 only the spaces E^4 and the S^4, which are variants of M^4 have twistor space with Kähler structure or analog of it. The induction conditions would imply dimensional reduction so that the 6-D Kähler action for the twistor lift would reduce to 4-D Kähler action plus volume term identifiable in terms of cosmological constant Λ.

2. 4-D Kähler action has Kähler coupling strength α_K as coupling parameter and volume term has coefficient $1/L^4$ identifiable in terms of cosmological constant

$$\frac{1}{L^4} \equiv \frac{\Lambda}{8\pi l_P^2} \quad .$$

$l_P = \sqrt{G/\hbar}$ would correspond to the radius of twistor sphere for M^4 and thus becomes fundamental length scale of twistorially lifted TGD besides radius of CP_2. Note that the radius of twistor sphere of CP_2 is naturally CP_2 radius.

L is in the role of coupling constant and expected to obey discrete p-adic coupling constant evolution $L \propto \sqrt{p}$, prime or prime near power of two if p-adic length scale hypothesis is accepted. In the recent cosmology L could correspond to the p-adic length scale $L(175) \simeq 40$ μm, the size of large neuron.

L $\simeq 40\mu$m corresponds to the energy scale $E = 1/L \simeq .031$ eV, which is thermal energy at temperature of 310 K (40 C) - the physiological temperature. A deep connection with quantum biology is suggestive. Also the energy scale defined by cell membrane potential is in this energy scale. This energy scale about 10 times smaller than the mass scale of neutrinos.

Also $L_\Lambda = \sqrt{8\pi/\Lambda}$ would satisfy p-adic coupling constant evolution as already discussed. Now the p-adic length scale would be secondary p-adic length scale $L_\Lambda = L(2,p) = \sqrt{p} \times (R/l_P)$, l_P Planck length. p-Adic length scale hypothesis demands that R/l_P - the ratio for the radii of CP_2 and twistor sphere is power of 2. p-Adic mass calculations indeed allow this ratio can be indeed chosen to be equal to $R/l_P = 2^{12}$.

4.3.2 ZEO and twistor lift

The volume term, which I gave up 38 years ago, has creeped back to the theory! The infinite value of volume for space-time surfaces of infinite duration? This would not make the notion of vacuum functional poorly defined. Should one forget twistor lift because of this? No! ZEO saves the situation.

In ZEO given CD defines a sub-WCW consisting of space-time surfaces inside CD. This implies that the volumes for the M^4 projections of allowed space-time surfaces are smaller than CD volume having the order of magnitude $L^4(CD)$, $L(CD)$ is the temporal distance between the tips of CD (one has $c = 1$). I have also proposed that $L(CD)$ is quantized in multiples of integers, primes or primes near power of two so that the identification might make sense. $L(CD) = L$ is not possible due to the small value 40 μm of L but $L(CD) = L_\Lambda$ could make sense.

Stationary phase condition and ZEO

The preferred extremal property realizing SH poses extremely strong constraints on the value of total action and it should force the phase defined by action to be stationary so that interference effects would be practically absent. This argument assumes that the action exponentials indeed appear in the scattering amplitudes defined by the WCW spinor fields in ZEO. NTU however forces to challenge this assumption unless one assumes that action is quantized as $q_1 + iq_2\pi$: this might be achieved by the quantization of the overall scale factor X of the action. The construction of twistor scattering amplitude suggests that the cancellation of action exponentials might be indeed achieved. If the exponents are present, the question is how the stationarity of phase could be achieved.

1. The most general possibility is that the phase of the vacuum functional can be large but is localized around very narrow range of values. The imaginary part of the action S_{Im} for preferred extremals should be around values $S_{Im} = A_0 + n2\pi$. Standard Bohr orbitology indeed assumes the quantization of action in this manner. One could also argue that just the absence of destructive interference demands Bohr quantization of the action in the vacuum functional. Whether preferred extremal property indeed gives rise to this kind of Bohr quantization, is an open problem. The real exponent of the vacuum functional should in turn be large enough and positive values are favored. They are however bounded in ZEO because of the finite size of CDs.

2. To proceed further one must say something about the value spectrum of α_K. In the most general situation α_K is complex number: the proposal of [L6] is that the discrete p-adic coupling constant evolution for $1/\alpha_K$ corresponds to a complex zero $s = 1/2 + iy$ of Riemann zeta: also the trivial real zeros can be considered. For large values of y the imaginary part of y would determine $1/\alpha_K$ and $Re(s) = 1/2$ would be responsible for complex value of α_K. This makes sense since quantum TGD can be regarded formally as a complex square root of thermodynamics.

3. Denote by $S = S_{Re} + iS_{Im}$ the exponent of vacuum functional. For complex values of $1/\alpha_K$ S_{Im} and S_{Re} receive a contribution from both Euclidian and Minkowskian regions and a

contribution also from the Minkowskian regions. For S_{Im} the contributions should obey the condition

$$S_{Im} = S_{Im}(M) + S_{Im}(E) \simeq A_0 + n2\pi \quad (4.3.1)$$

to achieve constructive interference.

For real parts the condition $S_{Re} = S_{Re}(M) + S_{Re}(E)$ must be small if negative. Large positive values of S_{Re} are favored. S_{Re} automatically selects the configurations, which contribute most and among these configurations the phase $exp(iS_{Im})$ must be stationary. The conditions for S_{Im} relate the values of action in the Euclidian and Minkowskian regions. If α_K is real, one has $S_{Im}(M) \simeq A_0 + n2\pi$ and $S_{Re}(E)$ small if negative and Euclidian and Minkowskian regions effectively decouple in the conditions. It seems that complex values of α_K are indeed needed.

4. $S_{Re}(E) = S_{Re}(M) + S_{Re}(E)$ receives a positive contribution from Euclidian regions. Minkowskian regions a contributions for complex value α_K. Both positive and negative contributions are present and the character of these contributions depends on sign of the imaginary part of α_K. Depending on the sign factor ± 1 of $Im(1/\alpha_K)$ Minkowskian regions give negative (positive) contribution from the space-time regions dominated by Kähler electric fields and positive (negative) contribution from the volume term and the regions dominated by Kähler magnetic field.

   The option "+" for which Kähler magnetic action and volume term give positive contribution to $S_{Re}(M)$ looks physically attractive. "+" option would have no problems in ZEO since the contribution to S_{Re} would be automatically positive but bounded by the finite size of CD: this would give a deep reason for the notion of CD (also the realization of super-symplectic symmetries gives it). For "-" option Minkowskian regions containing Kähler electric fields would be essential in order to obtain $S_{Re} > 0$: Kähler magnetic fields would not be favored and the unavoidable volume term would give wrong sign contribution to $S_{Re} > 0$.

The condition $S_{Im} \leq \pi/2$ is not realistic

One can look what the mere volume term contributes to S_{Im} assuming $S_{Im} \leq \pi/2$. Volume term dominates for near to vacuum extremals with a small Kähler action: in particular, for string like objects $X^2 \times S^2$, S^2 a homologically trivial geodesic sphere with vanishing induced Kähler form. It turns out that these conditions are not physically plausible and that $S_{Im} \simeq A_0 + n2\pi$ is the only realistic option.

1. Cosmological constant (parametrizable using the scale L) together with the finite size of CD gives a very stringent upper bound for the volume term of the action: $A = vol(X^4)/L^4$. The rough estimate is that for the largest CDs involved the volume action is not much larger than $L^4\pi/2$ in the recent cosmology. In the recent cosmology L would be only about 40 μm so that the bound is extremely strong! and suggests that $S_{Im} < \pi/2$ is not a realistic condition.

2. $L(CD) = L$ is certainly excluded. Can one have $L(CD) = L_\Lambda$? How can one achieve space-time volume not much larger that L^4 for space-time surfaces with duration $L(CD)$? Could magnetic flux tubes help! For the simplest string like objects $X^2 \times Y^2$, where $X^2 \subset M^4$ is minimal surface and Y^2 a 2-D surface (complex sub-manifold of CP_2) the volume action is essentially

$$Action = \frac{V}{l_P^2 L_\Lambda^2} = \frac{Area(X^2)}{L_\Lambda^2} \times \frac{Area(Y^2)}{l_P^2} \quad . \quad (4.3.2)$$

   The conservative condition for the absence of destructive interference is roughly $Action < \pi/2$.

3. To get a more concrete idea about the situation one can use the parameterization

$$Area(string) = L(CD) \times L(string) \ , \quad Area(Y^2) = x \times 4\pi R^2 \ . \tag{4.3.3}$$

x is a numerical parameter, which can be quite large for deformations of cosmic strings with thick transversal M^4 projection. The condition for the absence of destructive interference is roughly

$$\frac{L(CD)\times L(string))}{L_\Lambda^2} \times x \times \frac{4\pi R^2}{l_P^2} < \frac{\pi}{2} \ . \tag{4.3.4}$$

For $L(string) \ll L(CD)$ one can have space-time surfaces of temporal duration $L(CD) = L_\Lambda$. For these the condition reduces to

$$\begin{aligned} y \times x &< \pi \frac{l_P^2}{4\pi R^2} = 2^{-13}\pi \ , \\ y &\equiv \frac{L(string)}{L_\Lambda} \ . \end{aligned} \tag{4.3.5}$$

For deformations the transversal area of string like object can be also chosen to be considerably larger than the area of geodesic sphere. For flux tubes of length of order 1 AU the one have $y \sim 10^{-16}$. This would require $x \leq 10^{13}$. This would correspond to a radius $L(Y^2)$ about $10^6 R$ much smaller than required.

For $L(string) \sim L$ this would give $y \sim 10^{-31}$ giving $x \leq 10^{28}$ $L(Y^2) \leq 10^{14} R$, which corresponds to elementary particle scale. Still this fails to fit with intuitive expectations, which are of course inspired by the standard positive energy ontology.

4. One could try to invent mechanisms making volume term small. The required reduction would be enormous. This does look sensible. One can have vacuum extremals of Kähler action for which CP_2 projection is a geodesic line: $\Phi = \omega t$. The time component $g_{tt} = 1 - R^2\omega^2$ of the flat metric can be arbitrarily small so that the volume proportional to $\sqrt{gtt}$ can be arbitrarily small. One expects that this happens in early cosmology but as a general mechanism this is not plausible. Also very rapidly rotating string like objects with small area of string world sheet are in principle possible but do not represent a realistic option.

The cautious conclusion is that Bohr quantization $S_{Im} \simeq A_0 + n2\pi$ is the only sensible option. The hypothesis that the coupling constant evolution for $1/\alpha_K$ is given in terms of zeros of Riemann zeta seems to be consistent with this picture and correlates the values of actions in Minkowskian and Euclidian regions.

4.3.3 Hierarchy of Planck constants

One motivation (besides motivations from bio-electromagnetism and Nottale's work [E1]) for the hierarchy of Planck constants $h_{eff} = n \times h$ identified as gravitational Planck constants $\hbar_{gr} = GMm/v_0$ at the magnetic flux tubes mediating the gravitational interaction was that it effectively replaces the large coupling parameter GMm with dimensionless coupling $v_0/c < 1$. This assumes quantum coherence in even astrophysical length and time scales. For gauge interaction corresponding to gauge coupling g one $\hbar_g = Q_1Q_2\alpha/v_0$. Also Kähler coupling strength α_K to α_K/n and makes perturbation theory converging for large enough value of n.

The geometric interpretation for $h_{eff} = n \times h$ emerges if one asks how to make the action large for very large value of coupling parameter to guarantee convergence of functional integral.

1. The answer is simple: space-time surfaces are replaced with n-fold coverings of a space-space giving n-fold action and effectively scaling h to $h_{eff} = n \times h$ so that coupling strength scale down by $1/n$. The coverings would be singular in the sense that at the 3-D ends of space-time surface at the boundaries of causal diamond (CD) the sheets coincide.

2. The branches of the space-time surface would be related by discrete symmetries. The symmetry group could be Galois group in number theoretic vision about finite measurement resolution realized in terms of what I call monadic or adelic geometries [L14] [K38].

   On the other hand, the twistor lift suggests that covering could be induced by the covering of the fiber $S^2(X^6)$ by the spheres $S^2(M^4 \times S^2)$ and the twistor space $S^2(SU(3)/U(1) \times U(1))$ defining fibers of twistor spaces of M^4 and CP_2. There would be gauge transformations transforming the light-like parton orbits to each other and the discrete set would consists of gauge equivalence classes. These two identifications for the symmetries could be equivalent.

$h_{eff} = h_{gr} = n \times h$ would make perturbation theory possible for the space-time surfaces near vacuum extremals. For far from vacuum extremals Kähler action dominates and one would have $h_{eff} = h_{g_K} = n \times h$. This picture would conform with the idea that gravitational interactions are mediated by massless extremals (MEs) topologically condensed at magnetic flux tubes obtained as deformations of string like objects $X^2 \times S^2_I$, S^2_I a homologically trivial geodesic sphere of CP_2. The other interactions could be mediated in the similar manner. The flux tubes would be deformations of $X^2 \times S^2_{II}$, S^2_{II} a homologically non-trivial sphere so that the flux tubes would carry monopole flux.

The enormously small value of cosmological constant would require large value of $h_{eff}/h = n$ explaining the huge value of h_{gr} whereas for other interactions the value of n would be much smaller. Since only the size of the action matters, this is true for both Option I and Option II. One can consider also variants of this working hypothesis. For instance, all long range interactions mediated by massless quanta could correspond to extremals for which cosmological constant is small.

What smallness requires depends on option. For Option I the reason is that very long homologically non-trivial magnetic flux tubes tend to have large energy (the energy goes as $1/S$) so that homologically trivial flux tubes having only vacuum energy are favored. For Option II the cancellation of Kähler action and volume term is necessary. The compensating Kähler electric action could come from the M^4 Kähler from $J(M^4)$. These flux tubes could be also homologically non-trivial

Quantum criticality would suggest that both homologically trivial and non-trivial phases are important. In TGD inspired quantum biology [K40] I have considered the possibility that structures with size scaled by $h_{eff}/h = n$ can transform to structures with $n = 1$ but p-adic length scale scaled up by n. Here n would be power of two by p-adic length scale hypothesis.

This would have interpretation in terms of quantum criticality. Homologically non-trivial string like objects with given string tension determined by Kähler action would be transformed to homologically trivial string like objects with the same string tension but determined by the cosmological constant term. This would give a condition on the value of the cosmological constant and thickness of flux tubes to be discussed later.

4.3.4 Magnetic flux tubes as mediators of interactions

The gravitational Planck constant $\hbar_{gr} = GMm/v_0$ [K70, K57, K58, K21, K22, K23, K24] introduced originally by Nottale [E1] depends on the large central mass M and small mass m. This makes sense only if h_{gr} characterizes a magnetic flux tube connecting the two masses. Similar conclusion holds true for h_g. This leads to a picture in which mass M has involves a collection of radial flux tubes emanating radially from it. This assumption makes sense in many-sheeted space-time since the fluxes can go to the another space-time sheets through wormhole contacts associated also with elementary particles. For single-sheeted space-time one should have genuine magnetic charges.

This picture encourages a strongly simplified vision about how holography is realized. From center mass flux tubes emanate and in given size scale of the space-time sheet from by the flux tubes having say spherical boundary, the boundary is decomposed of pixels representing finite number of qubits. Each pixel receives one flux tube.

Vacuum energy for Options I and II

For Option I and magnetic flux tubes with vanishing Kähler form carry mere vacuum energy and are candidates for the mediators of long range interactions including gravitation. The homologically trivial flux tubes carry vacuum energy, which by flux conservation is proportional to $1/S$, where S is surface area. Long flux tubes are necessarily thick.

For Option II the thin magnetic flux tubes with vanishing induced Kähler form have very large tension and could be perturbative so that there would be no need for large values of $h_{eff}/h = n$. These flux tubes are expected to be short. The string world sheets mediating gravitational interaction should be long and have small string tension. They would naturally carry non-vanishing Kähler electric field in the direction of string (and flux tube).

1. Gravitational action (interaction energy from $J(M^4)$) and volume action (energy) would compensate to give a small cosmological constant forcing $h_{eff}/h = n$ hierarchy describing dark matter. Thus $J(M^4)$ crucial for understanding CP breaking and matter antimatter asymetry would be also crucial for the smallness of cosmological constant. This option looks physically rather attractive.

2. For flux tubes with vanishing induced $J(CP_2)$ the condition for cancellation would be $J \cdot J - 2 \simeq 0$. The compensating Kähler field would be electric and would naturally due to $J(M^4)$ and also responsible for the gravitational field along flux tube at QFT limit. Compensation of actions giving a small and scale dependent cosmological constant requiring large $h_{eff}/h = n = h_{gr}/h$ is possible.

3. For flux tubes with Kähler magnetic tube carrying magnetic monopole flux the cancellation condition would $J(M^4) \cdot J(M^4) - 2 - J(CP_2) \cdot J(CP_2) \simeq 0$. The thickening of flux tubes weakening the value of $J(CP_2)$ behaving from flux conservation like $J(CP_2) \propto 1/S$, S the cross sectional area of the flux tube, should make approximate cancellation possible. Elementary particles would represent an example of structures formed by closed monopole flux tubes assignable with a pair of space-time sheets. Homologically non-trivial magnetic flux tubes with small string tension could explain the mysterious cosmic magnetic fields: homological non-triviality implies that no current is needed to create the fields.

Magnetic flux tubes as carriers of magnetic energy

The holographic picture leads to a picture about vacuum energy. The following arguments developed originally for Option I should apply to both options since it is enough that magnetic flux tubes have only low vacuum energy density. Possible delicacies relate to the fact that small Kähler action $(E^2 - B^2)$ does not necessarily mean small Kähler energy. For Option II this situation is however not encountered.

1. Vacuum energy can be expressed as a sum of energies assignable to the flux tubes. Same applies to Kähler interaction energy. The contribution of individual flux tube is proportional to its length given by radius r of the large sphere considered. The total vacuum energy must be proportional to r^3 so that the number of flux tubes must be proportional to r^2. This implies that single flux tube correponds to constant area ΔS of the boundary sphere for given value of cosmological constant. The natural guess is that ΔS is of the same order of magnitude as the area defined by the length scale defined L by the vacuum energy density $\rho_{vac} = \Lambda/8\pi G$ allowing parameterization $\rho_{vac} = k_1\hbar/L^4$.

2. In the recent cosmology one has $\hbar/L(now) \simeq .029$ eV, which equals roughly to $M/10$, where $M = \sum m(\nu_i) \simeq .032 \pm 0.081$ eV is the sum of the three neutrino masses. L is given as a geometric mean

$$L = \sqrt{L_\Lambda l_P} \simeq .42 \times 10^{-4}$$

meters of length scales $l_P = \sqrt{G/\hbar}$ and $L_\Lambda = (8\pi/\Lambda)^{1/2}$. $L(now)$ corresponds to the size scale of large neuron. This is perhaps not an accident.

The area of pixel must be of order $L^2(now)$ suggesting strongly a p-adic length scale assignable with neuron: maybe neuronal system wold realize holography. $L(151) = 10$ nm (cell length scale thickness) and $L(k) \propto \sqrt{p} \simeq 2^{k/2}$ gives the estimate $p \simeq 2^k$, $k = 175$: the p-adic length scale is 4 per cent smaller than $L(now)$.

3. The pixel area would be by a factor $L^2(now)/l_P^2$ larger than Planck length squared usually assumed to define the pixel size but would conform with the p-adic variant of Hawking-Bekenstein law in which p-adic length scale replaces Planck length [K53].

The value of the vacuum energy density for a given flux tube is proportional to the value of $h_{eff}/h = n$ by the multi-sheeted covering property. Vacuum energy cannot however depend on n. There are two ways to achieve this: local and global.

1. For the local option the energy of each flux tube would remain invariant under $h \to n \times h$ as would also the number N of flux tubes. This requires that the cross section S of the radial gravitational flux tube to which energy is proportional, scales down as S/n. This looks strange.

2. For the global option flux tubes are not changed but the number N of the radial flux tubes scales down as $N \propto 1/n$: one has $Nn = constant$. In the situation in which Kähler magnetic energy dominant local option demands $S \propto n$ and global option $N \propto 1/n$. Nn constant conditions brings in mind something analogous to Uncertainty Principle. The resolutions characterized by N and n are associated with complementary variables.

The global option applies to both homologically trivial and non-trivial options and is more promising.

Could the value of endogenous dark magnetic field relate to cosmological constant?

TGD development of inspired model for quantum biology was initiated by the observation [J2] that ELF em fields have non-trivial effects on the brain physiology and behavior of vertebrates [K61, K64]. Since the energies of ELF photons (with frequencies in EEG range) are many orders of magnitude below thermal energy, the proposal was that one has dark photons having $h_{eff}/h = n$ increasing the value of the energy $E = h_{eff}f$ of ELF photons above thermal energy, possibly even to the energies of bio-photons in visible and UV range identified as resulting in a phase transition reducing h_{eff} to its value for visible matter.

The effects appear at multiples of cyclotron frequencies of biologically important ions in endogenous ("dark") magnetic field of $B_{end} \simeq .2$ Gauss. This corresponds to magnetic length $1/\sqrt{eB}$ not far from the size of large neuron. Could this field strength correspond to the Kähler magnetic field assignable to the flux tubes carrying monopole magnetic field, whose strength is determined by the value of cosmological constant? This would give a direct connection between cosmology and biology!

1. In recent cosmology the value of B_K (more precisely, $g_K B_K$ using ordinary conventions) at criticality would be

$$B_K = \frac{\Phi_0}{4\pi}\frac{1}{L^2(175)} \quad .$$

B_K corresponds to the U(1) magnetic field in standard model and is therefore as such not the ordinary magnetic field. For S^2_{II} Kähler magnetic field is non-vanishing. If Z^0 field vanishes, classical em field (with e included as normalization factor) equals to $\gamma = 3J$, where J is K"ahler induced Kähler form (see [L2]. One has

$$B_K = \frac{eB_{em}}{3} \quad . \tag{4.3.6}$$

2. An interesting question is whether one could identify physically the ordinary magnetic field assignable to the critical Kähler magnetic field.

   Earth's magnetic field $B_E = .5$ Gauss corresponds to magnetic length $L_B = \sqrt{\hbar e B} = 5\mu$m. Endogenous magnetic field $B_{end} \simeq 2B_E/5$ explaining the findings of Blackman [J2] about the effects of ELF em fields on vertebrate brain in terms of cyclotron transitions corresponds to $L_B = 12.5$ μm to be compared with the p-adic length scale $L(175) = 40$ μm. Also these findings served as inspiration of $h_{eff} = n \times h$ hypothesis [K61, K60].

   I have assigned large Planck constant phases with the flux tubes of B_{end}, which have however remained somewhat mysterious entity. Could B_{end} correspond to quantum critical value of B_K and therefore relate directly to cosmology?

   One can check whether $B_K = eB_{end}/3$ holds true. The hypothesis would give

$$eB_{end} = \frac{1}{L_B^2} = 3 \times \frac{\Phi_0}{4\pi\hbar}\frac{1}{L^2(175)} \quad .$$

   implying

$$r = \frac{L^2(175)}{L_B^2} = \frac{3\Phi_0}{4\pi\hbar} \quad .$$

   The left hand side gives $r = 10.24$. For $\Phi_0 = 8\pi\hbar$ the right hand side gives $r = 6$. $B_E = .34$ Gauss left and right hand sides of the formula are identical.

3. One can wonder the proposed formulas might be exact for preferred extremals satisfying extremely powerful conditions to guarantee strong form of holography. This would require in both cases bundle structure with transversal cross section action as fiber. In the case of extremals of Kähler this would require that induce Kähler magnetic field is covariantly constant.

4.3.5 Two variants for p-adic length scale hypothesis for cosmological constant

There are two options for the dependence string tension Tandarea S of the cross section of the flux tube on p-adic length scale: either $L_\Lambda = \sqrt{8\pi/\Lambda}$ or $L = (\hbar/\rho_{vac})^{1/4}$ satisfies p-adic length scale hypothesis. The "boundary condition" is that the radius of flux tubes would be of the order of neutron size scale in recent cosmology.

1. $L(now) = L_p$ scaling gives

$$S = S(now)\frac{p(now)}{p} \tag{4.3.7}$$

   with $p_{now} \simeq 2^{175}$ by p-adic length scale hypothesis. $L(175)$ is by about 4 per cent smaller than the Compton length assignable to $\hbar/L(now) = .029$ eV.

   If one wants $L(now) = L(175)$ exactly, one must increase R by 4 per cent, which is allowed by p-adic mass calculations fixing the value of R only with 10 per cent accuracy. Indeed, the second order contribution in p-adic mass calculations is uncertain and the ratio of maximal and minimal values of R is $R_{max}/R_{min} = \sqrt{6/5} \simeq 1.1$.

   As already noticed, $L(now)$ corresponds to neutron size scale, which conforms with p-adic mass calculations since the radius of flux tubes would correspond to p-adic length scale. This option looks more natural and suggest a profound connection with biology and fundamental physics.

2. $L_\lambda \equiv \sqrt{8\pi/\Lambda}$ could be proportional to secondary p-adic length scale $L(2,p_\Lambda) \equiv \sqrt{p_\Lambda} L_{p_\Lambda}$. The scaling law

$$L_\Lambda \propto \frac{p_\Lambda(now)}{p_\Lambda} \quad (4.3.8)$$

gives

$$L_\Lambda^2(now) = \frac{8\pi}{\Lambda(now)} = (\frac{p}{p(now)})^2 \times \frac{L^4(now)}{l_P^2} \quad . \quad (4.3.9)$$

$L_\Lambda(now) \sim 50$ Gly (roughly the age of the Universe) holds true. Note that one has $S \propto \sqrt{p_{now}/p} S(now)$ and $T = T_{now}\sqrt{p/p_{now}}$.

$1/p$-dependence for the string tension T looks more natural in light of p-adic mass calculations. One must however notice that the $L = L(175)$ is 4 per cent small than $L(now)$.

The density of dark energy is uncertain by few per cent at least and one can ask whether $L(now) = L(175)$ could fix it. The change induced to ρ_{vac} by that of $L(now)$ is

$$\frac{\Delta\rho_{vac}}{\rho_{vac}} = -4\frac{\Delta L(now)}{L(now)}$$

and the reduction L by 4 per cent would reduce vacuum density by 16 per cent, which looks rather large change. The value of R can be determined by 10 per cent accuracy and the increase of R by four per cent is another manner to achieve $L(now) = L(175)$.

One can of course ask, whether both variants of p-adic length scale hypothesis could be correct. The reader night protest that this leads to the murky waters of p-adic numerology.

1. Could L_Λ be proportional to the secondary p-adic length scale $L(p,2) = \sqrt{p} L_p = 2^{k/2} \times L(k)$ associated with p characterizing L such that the proportionality constant is power of $\sqrt{2}$. The application of the condition defining L in terms of $L_\Lambda^2 = 8\pi/\Lambda$ gives

$$L_\Lambda^2 = \frac{L^4}{l_P^2} \quad .$$

Using $L_\Lambda = \sqrt{p_\Lambda} R$ and taking square roots, this gives

$$\sqrt{p_\Lambda} = pk^2 \quad , \quad k = \frac{R_{CP_2}}{l_P} \quad . \quad (4.3.10)$$

This conforms with the p-adic length scales hypothesis in its simplest form if k is power of $\sqrt{2}$.

2. The estimate from p-adic mass calculations for $r \equiv R(CP_2)/l_P$ is $r = 4.167 \times 10^3$ and is 2 per cent larger than 2^{12}. Could the $R(CP_2)/l_P = 2^{12}$ for the radii of CP_2 and M^4 twistorial sphere be an exact formula between fundamental length scales? As noticed, the second order contribution in p-adic mass calculations is uncertain by 10 per cent. This would allow the reduction of $R(CP_2)$ by 2 percent.

This looks an attractive option. The bad news is that the *increase* of $R(CP_2)$ by about 4 per cent to achieve $L(now) = L(175)$ is in conflict with its *reduction* by 2 per cent to achieve $R(CP_2)/l_P = 2^{12}$: this would reduce $L(175)$ by 2 per cent and increase ρ_{vac} by about 8 per cent. ρ_{vac} is however an experimental parameter depending on theoretical assumption and it value could allow this tuning. Therefore

$$\begin{aligned} \frac{R_{CP_2}}{l_P} &= 2^{12} \ , \\ p_\Lambda &= 2^{48}\times p^2 \ . \end{aligned} \quad (4.3.10)$$

is an attractive option fixing completely the value of $R(CP_2)/l_P$ and predicting relation between cosmological scale L_Λ and a fundamental scale in recent biology, which could be assigned to magnetic flux tubes assignable to axons. Note that for $k_{now}=175$ the value of $k_\Lambda = k_{now}+48$ is $k_\Lambda = 175+48 = 223$ which corresponds to p-adic length scale of 64 m.

3. Needless to say that one must be take these estimates with a big grain of salt. Number theoretical universality suggests that one might apply number theoretical constraints to fundamental constants like R, l_P, and Λ but one should be very critical concerning the values of empirical parameters such as ρ_{vac} depending on theoretical assumptions. Furthermore, p-adic length scale hypothesis is applied at the level of embedding space metric and one can ask whether it actually applies for the induced metric (Robertson-Walker metric now).

4.4 What happens for the extremals of Kähler action in twistor lift

As I started to work with TGD around 1977, I adopted path integral and canonical quantization as the first approaches. One of the first guesses for the action principle was 4-volume in induced metric giving minimal surfaces as preferred extremals. The field equations are a generalization of massless field equation and at least in the case of string models Hamiltonian formalism and second quantization is possible. The reason why for giving up this option was that for space-time surfaces of infinite duration the volume is infinite. This is not pleasant news concerning quantization since subtraction of exponent of infinite volume factor looked really ugly thing to do. At that time I did of course have no idea about ZEO and CDs.

For Kähler action there is however infinite vacuum degeneracy. All space-time surfaces with CP_2 projection, which is Lagrangian manifold (at most 2-dimensional) are vacuum extremals and canonical quantization fails completely. This implies classical non-determinism also for non-vacuum extremals obtained as small deformations of vacuum extremals. This feature seems to have nice implications such as 4-D spin glass degeneracy. It would however make WCW metric singular for nearly vacuum extremals.

The twistor lift brings volume term to the action. For option II there is also coupling between Kähler action and volume term but asymptotically one expects minimal surface extremals as analogs for free geodesic motion. The question is what happens to the known extremals of Kähler action, most of which are minimal surfaces.

4.4.1 The coupling between Kähler action and volume term

The addition of the volume term to Kähler action has very nice interpretation as a generalization of equations of motion for a world-line extended to a 4-D space-time surface. The field equations generalize in the same manner for 3-D light-like surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian, for 2-D string world sheets, and for their 1-D boundaries defining world lines at the light-like 3-surfaces. For 3-D light-like surfaces the volume term is absent. Either light-like 3-surface is freely choosable in which case one would have Kac-Moody symmetry as gauge symmetry or that the extremal property for Chern-Simons term fixes the gauge.

The condition that the dynamics based on Kähler action and volume term is number theoretically universal demands that coupling constants do not appear in it. This leaves only Option I ($\alpha_K(M^4)\neq\alpha_K(CP_2)$ with different phases) and option II ($\alpha_K(M^4)=\alpha_K(CP_2)$ with the same phase). This condition is taken as granted in the following.

The dynamics of twistor lift as a generalization of the dynamics of point like particle coupling to Maxwell field

Almost all the known non-vacuum extremals are minimal surface extremals of Kähler action [K10, K5] and it might well be that the preferred extremal property realizing SH quite generally demands this. CP_2 type vacuum extremals are also minimal surfaces if one assumes that the M^4 projection is light-like geodesic rather than only geodesic line.

The addition of the volume term could however make Kähler coupling strength a manifest coupling parameter also classically when the phases of Λ and α_K are same. Therefore quantum criticality for Λ and α_K would have a precise local meaning also classically in the interior of space-time surface. The equations of motion for a world line of U(1) charged particle would generalize to field equations for a "world line" of 3-D extended particle.

This is an attractive idea consistent with standard wisdom but for Option I one can invent strong objections against it.

1. The conjecture is that α_K has zeros of zeta as its spectrum of critical values [L6]. If so then all preferred extremals are minimal surface extremals of Kähler action for a real value of cosmological constant Λ possible for Option I ($\alpha_K(M^2)$ would be real). Hence the two actions decouple: this does not look nice. For Option II the phase is same and there is interaction between these degrees of freedom. One could of course force also the phase for Option I to be same.

2. All known non-vacuum extremals of Kähler action are minimal surfaces and the minimal surface vacuum extremals of Kähler action become non-vacuum extremals. This allows to consider the possibility that preferred extremals are minimal surface extremals of Kähler action so that the two dynamics apparently decouple. For Option II this makes sense since the solutions do not depend at all on the common over-all scaling factor of Kähler action and volume term. Minimal surface extremals are analogs for geodesics in the case of point-like particles: one might say that one has only gravitational interaction. This conforms with SH stating that gauge interactions at boundaries (orbits of partonic 2-surfaces and 2-surfaces at the ends of CD) correspond classically to the gravitational dynamics in the space-time interior.

   Note that at the boundaries of the string world sheets at light-like 3-surfaces the situation is different: one has equations of motion for geodesic line coupled to induce Kähler gauge potential and gauge coupling indeed appears classically as one might expect! For string world sheets one has only the topological magnetic flux term and minimal surface equation in string world sheet. Magnetic flux term gives the Kähler coupling at the boundary.

3. For Option I decoupling implied by extremal property of both real and imaginary parts of action would allow to realize number theoretical universality [K84] since the field equations would not depend on coupling parameters at all. For Option II same is achieved even without decoupling.

4. One can argue that the decoupling for Option I makes it impossible to understand coupling constant evolution. This need not be the case. The point is that the classical charges assignable to super-symplectic algebra are sums over contributions from Kähler action and volume term and therefore depend on the coupling parameters. Their vanishing conditions for sub-algebra and its commutator with entire algebra give boundary conditions on preferred extremals so that coupling constant evolution creeps in classically!

   Quantum classical correspondence realized as the condition that the eigenvalues of fermionic charge operators are equal to the classical charges brings in the dependence of quantum charges on coupling parameters. Since the elements of scattering matrix are expected to involve as building bricks the matrix elements of super-symplectic algebra and Kac-Moody algebra of isometry charges, one expects that discrete coupling constant evolution creeps in also quantally via the boundary conditions for preferred extremals.

Options I and II and Chladni mechanism

One can compare Options I and II.

1. For Option I the coupling between the two dynamics could be induced just by the condition that the space-time surface becomes an analog of geodesic line by arranging its interior so that the U(1) force vanishes! This would generalize Chladni mechanism (see `http://tinyurl.com/j9rsyqd`)!

   The interaction would be present but be based on going to the nodal surfaces! Also the dynamics of string world sheets is similar: if the string sheets carry vanishing W boson classical fields, em charge is well-defined and conserved. One would also avoid the problems produced by large coupling constant between the two-dynamics present already at the classical level. At quantum level the fixed point property of quantum critical couplings would be the counterparts for decoupling. This option however seems to be missing the transient phase preceding the Chladni configuration.

2. For Option II the coupling would be present during transient periods leading to decoupling. The alternative view is that the deviation from minimal surface and can act as a controller of the dynamics defined by the volume term providing a small push or pull now and then. Could this sensitivity relate to quantum criticality and to the view about morphogenesis relying on Chladni mechanism in which field patterns control the dynamics with charged flux tubes ending up to the nodal surfaces of (Kähler) electric field [L15]? Magnetic flux tubes containing dark matter would in turn control and serve as template for the dynamics of ordinary matter.

   Chladni mechanism would not be instantaneous but lead via transient phase to minimal surface extremals near either or both boundaries of CDs analogous to external particles in particle reaction. The space-time regions assignable to particle interaction vertices identified as 2-surfaces at which the ends of three 3-D light-like partonic orbits meet, would correspond to transient regions, where the coupling is present. This option looks clearly more realistic.

Admittedly Option II looks more attractive.

As an example one can consider a typical particle physics experiment. There are incoming and outgoing free particles moving along geodesics, these particles interact, and emanate as free particles from the interaction volume. This phenomenological picture does not follow from QFT but is put in by hand, in particular the idea about interaction couplings becoming non-zero is involved. Also the role of the observer remains poorly understood.

The motion of incoming and outgoing particles is analogous to free motion along geodesic lines with particles generalized to 3-D extended objects. For both options these would correspond to the preferred extremals in the complement of CD within larger CD representing observer or measurement instrument. Decoupling would take place. In interaction volume interactions are "coupled on" and particles interact inside the volume characterized by causal diamond (CD). What could be the TGD view translation of this picture?

1. For Option I one would still have decoupling and the interpretation would be in terms of twistor picture in which one always has also in the internal lines on mass shell particles but with complex four-momenta. In TGD framework the momenta would be always complex due to the contribution of Euclidian regions defining the lines of generalized scattering diagrams. Note however that the real and imaginary parts of the conserved charges are predicted to be proportional to each other. This result is obtained also in twistor approach from 8-D light-likeness and is crucial for twistorialization in TGD sense [L24]. As explained, coupling constant evolution can be understood also in this case and also classical dynamics depends on coupling parameters via the boundary conditions. There would be no counterpart for transitory period (interaction on) leading to the decoupled situation so that Option I is not attractive.

2. For Option II the transitory period would correspond to the coupling between the two classical dynamics in regions assignable to the vertices of topological scattering diagrams at which the ends of the parton orbits meet. Near the ends the dynamics would decouple and one would have the analog of free geodesic motion.

Second example comes from biology. The free geodesic line dynamics with vanishing $U(1)$ Kähler force indeed brings in mind the proposed generalization of Chladni mechanism generating nodal surfaces at which charged magnetic flux tubes are driven [K44] [L15, L16] . Chlandi mechanism could be seen as a basic mechanism behind morphogenesis.

1. For Option I the interiors of all space-time surfaces would be analogous to nodal surfaces and "big" state function reductions would correspond to transition periods between different nodal surfaces. The decoupling would be dynamics of avoidance and could highly analogous to Chladni mechanism.

2. For Option II transition period would correspond to a period during which nodal surfaces are formed.

It seems that Option II is favored by both SH, number theoretical universality, and generalization of Chladni mechanism to a dynamics of avoidance.

4.4.2 Twistor lift and the extremals of Kähler action

The addition of the volume term makes Kähler coupling strength a genuine coupling parameter also classically when the variation of Kähler action is non-vanishing. Therefore quantum criticality for Λ and α_K gets precise meaning also classically. The equations of motion for a worldline of U(1) charged particle generalize to field equations for a "world line" of 3-D extended particle.

The field equations generalize in the same manner for 3-D light-like surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian, for 2-D string world sheets, and for their 1-D boundaries defining world lines at the light-like 3-surfaces. For 3-D light-like surfaces the volume term is absent. Either light-like 3-surface is freely choosable in which case one would have Kac-Moody symmetry as gauge symmetry or that the extremal property for Chern-Simons term fixes the gauge.

What happens to the extremals of Kähler action?

What happens to the extremals of Kähler action when volume term is introduced?

1. The known non-vacuum extremals [K10, K5] such as massless extremals (topological light rays) and cosmic strings are minimal surfaces.

2. For $J(M^4) = 0$ these extremals remain extremals for both Option I and II and only the classical Noether charges receive an additional volume term. In particular, string tension is modified by the volume term. Homologically non-trivial cosmic strings are of form $X^2 \times Y^2$, where $X^2 \subset M^4$ is minimal surface and $Y^2 \subset CP_2$ is complex 2-surface and therefore also minimal surface.

3. For $J(M^4) \neq 0$ essential for obtaining small cosmological constant for Option II, the situation changes and minimal surface property is possible only under additional conditions. For instance, one can have minimal surfaces of form $X^2 \times Y^2 \subset M^4 \times Y^2$, where Y^2 is minimal surface in CP_2. X^2 cane be $M^2 \subset N^2 \times E^2$ defining the $J(M^4)$ giving $J(M^4) \cdot J(M^4) - 2 = 0$. X^2 can be also minimal surface, which is an analog of Lagrangian manifold for $J(M^4)$.

4. Vacuum degeneracy is lifted for both options. For $J(M^4) = 0$ vacuum extremals, which are minimal surfaces survive as extremals for both options. For $J(M^4) \neq 0$ the situation is more complex.

Vacuum extremals

For CP_2 type vacuum extremals [K10, K5] the roles of M^4 and CP_2 are changed. M^4 projection is light-like curve, and can be expressed as $m^k = f^k(s)$ with light-likeness conditions reducing to Virasoro conditions. These surfaces are isometric to CP_2 and have same Kähler and symplectic structures as CP_2 itself. What is new as compared to GRT is that the induced metric has Euclidian signature. The interpretation is as lines of generalized scattering diagrams. The addition of the volume term forces the random light-like curve to be light-like geodesic and the action becomes the volume of CP_2 in the normalization provided by cosmological constant. What looks strange is that the volume of any CP_2 type vacuum extremals equals to CP_2 volume but only the extremal with light-like geodesic as M^4 projection is extremal of volume term. A little calculation shows that for CP_2 type extremals the contribution of the volume term to the action would be completely negligible as compared to the Kähler action.

Consider next vacuum extremals, which have vanishing induced Kähler form and are thus have CP_2 projection belonging to at most 2-D Lagrangian manifold of CP_2 [K10, K5].

1. Vacuum extremals with 2-D projections to CP_2 and M^4 are possible and are of form $X^2 \times Y^2$, X^2 arbitrary 2-surface and Y^2 a Lagrangian manifold. Volume term forces X^2 to be a minimal surface and Y^2 is Lagrangian minimal surface unless the minimal surface property destroys the Lagrangian character.

   If the Lagrangian sub-manifold is homologically trivial geodesic sphere, one obtains string like objects with string tension determined by the cosmological constant alone.

   Do more general 2-D Lagrangian minimal surfaces than geodesic sphere exist? For general Kähler manifold there are obstructions but for Kähler-Einstein manifolds such as CP_2, these obstructions vanish (see `http://tinyurl.com/gtkpya6`). The case of CP_2 is also discussed in the slides "On Lagrangian minimal surfaces on the complex projective plane" (see `http://tinyurl.com/jrhl6gy`). The discussion is very technical and demonstrates that Lagrangian minimal surfaces with all genera exist. In some cases these surfaces can be also lifted to twistor space of CP_2.

2. More general vacuum extremals have 4-D M^4 projection. Could the minimal surface condition for 4-D M^4 projection force a deformation spoiling the Lagrangian property? The physically motivated expectation is that string like objects give as deformations magnetic flux tubes for which string is thicknened so that it has a 2-D cross section. This would suggest that the deformations of string like objects $X^2 \times Y^2$, where Y^2 is Lagrangian minimal surface, give rise to homologically trivial magnetic flux tubes. In this case Kähler magnetic field would vanish but the spinor connection of CP_2 would give rise to induced magnetic field reducing to some U(1) subgroup of U(2). In particular, electromagnetic magnetic field could be present.

3. p-Adically Λ behaves like $1/p$ as also string tension. Could hadronic string tension be understood also in terms of cosmological constant in hadronic p-adic length scale for strings if one assumes that cosmological constant for given space-time sheet is determined by its p-adic length scale?

Maxwell phase

What might be called Maxwell phase which would correspond to small perturbations of M^4 is also possible for 4-D Kähler action. For the twistor lift the volume term makes this phase possible. Maxwell phase is highly interesting since it corresponds to the intuitive view about what QFT limit of TGD could be. The following arguments apply only for $J(M^4) = 0$.

1. The field equations are a generalization of massless field equations for fields identifiable as CP_2 coordinates and with a coupling to the deviation of the induced metric from M^4 metric. It representes very weak perturbation. Hence the linearized field equations are expected to be an excellent approximation. The general challenge would be however the construction of exact solutions. One should also understand the conditions defining preferred extremals and stating that most of symplectic Noether charges vanish at the ends of space-time surface about boundaries of CD.

2. Maxwell phase is the TGD analog for the perturbative phase of gauge theories. The smallness of the cosmological constant in cosmic length scales would make the perturbative approach useless in the path integral formulation. In TGD approach the path integral is replaced by functional integral involving also a phase but also now the small value of cosmological constant is a problem in long length scales. As proposed, the hierarchy of Planck constants would provide the solution to the problem.

3. The value of cosmological constant behaving like $\Lambda \propto 1/p$ as the function of p-adic prime could be in short p-adic length scales large enough to allow a converging perturbative expansion in Maxwellian phase. This would conform with the idea that Planck constant has its ordinary value in short p-adic length scales.

4. Does Maxwell phase allow extremals for which the CP_2 projection is 2-D Lagrangian manifold - say a perturbation of a minimal Lagrangian manifold? This perturbation could be seen also as an alternative view about thickened minimal Lagrangian string allowing also M^4 coordinates as local coordinates. If the projection is homologically trivial geodesic sphere this is the case. Note that solutions representable as maps $M^4 \rightarrow CP_2$ are also possible for homologically non-trivial geodesic sphere and involve now also the induced Kähler form.

5. The simplest deformations of canonically imbedded M^4 are of form $\Phi = k \cdot m$, where Φ is an angle coordinate of geodesic sphere. The induced metric in M^4 coordinates reads as $g_{kl} = m_{kl} - R^2 k_k k_l$ and is flat and in suitably scaled space-time coordinates reduces to Minkowski metric or its Euclidian counterpart. k_k is proportional to classical four-momentum assignable to the dark energy. The four-momentum is given by

$$P^k = A \times \hbar k^k \quad , \quad A = \frac{Vol(X^3)}{L_\Lambda^4} \times \frac{1+2x}{1+x} \quad , \quad x = R^2 k^2 \quad .$$

   Here k^k is dimensionless since the the coordinates m^k are regarded as dimensionless.

6. There are interesting questions related to the singularities forced by the compactness of CP_2. Eguchi-Hanson coordinates (r, θ, Φ, Ψ) [L2] (see `http://tinyurl.com/z86o5qk`) allow to get grasp about what could happen.

   For the cyclic coordinates Ψ and Φ periodicity conditions allow to get rid of singularities. One can however have n-fold coverings of M^4 also now.

   (r, θ) correspond to canonical momentum type canonical coordinates. Both of them correspond to angle variables ($r/\sqrt{1+r^2}$ is essentially sine function). It is convenient to express the solution in terms of trigonometric functions of these angle variables. The value of the trigonometric function can go out of its range $[-1, 1]$ at certain 3-surface so that the solution ceases to be well-defined. The intersections of these surfaces for r and θ are 2-D surfaces. Many-sheeted space-time suggests a possible manner to circumvent the problem by gluing two solutions along the 3-D surfaces at which the singularities for either variable appear. These surfaces could also correspond to the ends of the space-time surface at the boundaries of CD or to the light-like orbits of the partonic 2-surfaces.

   Could string world sheets and partonic 2-surfaces correspond to the singular 2-surfaces at which both angle variables go out of their allowed range. If so, 2-D singularities would code for data as assumed in strong form of holography (SH). SH brings strongly in mind analytic functions for which also singularities code for the data. Quaternionic analyticity which makes sense would indeed suggest that co-dimension 2 singularities code for the functions in absence of 3-D counterpart of cuts (light-like 3-surfaces?) [L11].

7. A more general picture might look like follows. Basic objects come in two classes. Surfaces $X^2 \times Y^2$, for which Y^2 is either homologically non-trivial complex minimal 2-surface of CP_2 of Lagrangian minimal surface. The perturbations of these two surfaces would also produce preferred extremals, which look locally like perturbations of M^4. Quaternionic analyticity might be shared by both solution types. Singularities force many-sheetedness and strong form of holography.

Astrophysical and cosmological solutions

Cosmological constant is expected to obey p-adic evolution and in very early cosmology the volume term becomes large. What are the implications for the vacuum extremals representing Robertson-Walker metrics having arbitrary 1-D CP_2 projection? [K10, K5, K71]. One can also ask what is the fate of spherically symmetric solutions of GRT providing a model of star.

Already the existing physical picture explaining $h_{gr}/hh_{eff}/h = n$ in terms of flux tubes mediating gravitational interactions suggests that Robertson-Walker metrics and spherically symmetric metrics are possible only at QFT limit. The presence of covariantly constant $J(M^4)$ breaking Lorentz symmetry and rotational symmetry makes this obvious. One could consider variants of $J(M^4)$ invariant under Lorentz group or some subgroup of Lorentz group but $J(M^4)$ would not be covariantly constant anymore. It is not clear when it makes sense to extend the moduli space for $J(M^4)$.

1. The TGD inspired cosmology involves primordial phase during a gas of cosmic strings in M^4 with 2-D M^4 projection dominates. The value of cosmological constant at that period could be fixed from the condition that homologically trivial and non-trivial cosmic strings have the same value of string tension. After this period follows the analog of inflationary period when cosmic strings condense are the emerging 4-D space-time surfaces with 4-D M^4 projection and the M^4 projections of cosmic strings are thickened. A fractal structure with cosmic strings topologically condensed at thicker cosmic strings suggests itself.

2. GRT cosmology is obtained as an approximation of the many-sheeted cosmology as the sheets of the many-sheeted space-time are replaced with region of M^4, whose metric is replaced with Minkowski metric plus the sum of deformations from Minkowski metric for the sheet. The vacuum extremals with 4-D M^4 projection and arbitrary 1-D projection could serve as an approximation for this GRT cosmology. Note however that this representability is not required by basic principles.

3. For cosmological solutions with 1-D CP_2 projection minimal surface property forces the CP_2 projection to belong to a geodesic circle S^1. Denote the angle coordinate of S^1 by Φ and its radius by R. For the future directed light-cone M^4_+ use the Robertson-Walker coordinates $(a = \sqrt{m_0^2 - r_M^2}, r = ar_M, \theta, \phi)$, where (m^0, r_M, θ, ϕ) are spherical Minkowski coordinates. The metric of M^4_+ is that of empty cosmology and given by $ds^2 = da^2 - a^2 d\Omega^2$, where Ω^2 denotes the line element of hyperbolic 3-space identifiable as the surface $a = constant$.

   One can can write the ansatz as a map from M^4_+ to S^1 given by $\Phi = f(a)$. One has $g_{aa} = 1 \rightarrow g_{aa} = 1 - R^2(df/da)^2$. The field equations are minimal surface equations and the only non-trivial equation is associated with Φ and reads $d^2f/da^2 = 0$ giving $\Phi = \omega a$, where ω is analogous to angular velocity. The metric corresponds to a cosmology for which mass density goes as $1/a^2$ and the gravitational mass of comoving volume (in GRT sense) behaves is proportional to a and vanishes at the limit of Big Bang smoothed to "Silent whisper amplified to rather big bang" for the critical cosmology for which the 3-curvature vanishes. This cosmology is proposed to results at the limit when the cosmic temperature approaches Hagedorn temperature [K71].

4. The TGD counterpart for inflationary cosmology corresponds to a cosmology for which CP_2 projection is homologically trivial geodesic sphere S^2 (presumably also more general Lagrangian (minimal) manifolds are allowed). This cosmology is vacuum extremal of Kähler action. The metric is unique apart from a parameter defining the duration of this period serving as the TGD counterpart for inflationary period during which the gas of string like objects condensed at space-time surfaces with 4-D M^4 projection. This cosmology could serve as an approximate representation for the corresponding GRT cosmology.

   The form of this solution is completely fixed from the condition that the induced metric of $a = constant$ section is transformed from hyperbolic metric to Euclidian metric. It should be easy to check whether this condition is consistent with the minimal surface property. It seems that one cannot satisfy minimal surface equations.

5. For $J(M^4) \neq 0$ the spherical and Lorentz symmetries are lost and the only cosmological solution are light-cones $M^4_\pm$. Also the existence of stationary spherically symmetric minimal surface extremals is impossible for $J(M^4) \neq 0$. Spherically symmetric metrics and Robertson-Walker metric would serve only as long length scale approximations providing a statistical description of the gravitational interaction described microscopically in terms of a flux tube network.

4.4.3 Are minimal surface extremals of Kähler action holomorphic surfaces in some sense?

If the spectrum for the critical value of Kähler coupling strength is complex - say given by the complex zeros of zeta [L6] - the preferred extremals of Kähler action are minimal surfaces for Option I. For Option II they correspond to asymptotic solutions.

I have considered several ansätze for the general solutions of the field equations for the preferred extremals. One proposal is that preferred extremals as 4-surfaces of embedding space with octonionic tangent space structure have quaternionic tangent space or normal space (so called M^8-H duality [K74]). Second proposal is that preferred extremals can be seen as quaternion analytic [A33] surfaces [K66, L3] [L4]. Third proposal relies on a fusion of complex and hyper-complex structures to what I call Hamilton-Jacobi structure [K82, K5]. In Euclidian regions this would correspond to complex structure. Twistor approach [L11] suggests that the condition that the twistor lift of the space-time surface to a 6-D surface in the product of twistor spaces of M^4 and CP_2 equals to the twistor space of CP_2. This proposal is highly interesting since twistor lift works only fr $M^4 \times CP_2$. The intuitive picture is that the field equations are integrable and all these views might be consistent.

Preferred extremals of Kähler action as minimal surfaces would be a further proposal. Can one make conclusions about general form of solutions assuming that one has minimal surface extremals of Kähler action?

In $D = 2$ case minimal surfaces are holomorphic surfaces or they hyper-complex variants and the embedding space coordinates can be expressed as complex-analytic functions of complex coordinate or a hypercomplex analog of this. Field equations stating the vanishing of the trace $g\alpha\beta H^k_{\alpha\beta}$ if the second fundamental form $H^k_{\alpha\beta} \equiv D_\alpha \partial_\beta h^k$ are satisfied because the metric is tensor of type $(1,1)$ and second fundamental form of type $(2,0) \oplus (2,0)$. Field equations reduce to an algebraic identity and functions involved are otherwise arbitrary functions. The constraint comes from the condition that metric is of form $(1,1)$ as holomorphic tensor.

This raises the question whether this finding generalizes to the level of 4-D space-time surfaces and perhaps allows to solve the field equations exactly in coordinates generalizing the hypercomplex coordinates for string world sheet and complex coordinates for the partonic 2-surface.

Almost all the known non-vacuum extremals are minimal surface extremals of Kähler action [K10, K5] and it might well be that the preferred extremal property realizing SH quite generally demands this. CP_2 type vacuum extremals are also minimal surfaces if one assumes that the M^4 projection is light-like geodesic rather than only geodesic line. The common feature suggested already earlier to be common for all preferred extremals is the existence of generalization of complex structure.

1. For Minkowskian regions this structure would correspond to what I have called Hamilton-Jacobi structure [K82, K5]. The tangent space of the space-time surface X^4 decomposes to local direct sum $T(X^4) = T(X^2) \oplus T(Y^2)$, where the 2-D tangent places $T(X^2)$ and $T(Y^2)$ define an integrable distribution integrating to a decomposition $X^4 = X^2 \times Y^2$. The complex structure is generalized to a direct some of hyper-complex structure in X^2 meaning that there is a local light-like direction defining light-like coordinate u and its dual v. $Y2$ has complex complex coordinate $(w, \overline{w})$. Minkowski space M^4 has similar structure. It is still an open question whether metric decomposes to a direct sum of orthogonal metrics assignable to X^2 and Y^2 or is the most general analog of complex metric in question. g_{uv} and $g_{w\overline{w}}$ are certainly non-vanishing components of the induced metric. Metric could allow as non-vanishing components also g_{uw} and $g_{v\overline{w}}$. This slicing by pairs of surfaces would correspond to decomposition to a product of string world sheet and partonic 2-surface everywhere.

   In Euclidian regions ne would have 4-D complex structure with two complex coordinates (z, w) and their conjugates and completely analogous decompositions. In CP_2 one has similar complex structure and actually Kähler structure extending to quaternionic structure. I have actually proposed that quaternion analyticity could provide the general solution of field equations.

2. Assuming minimal surface property the field equations for Kähler action reduce to the vanishing of a sum of two terms. The first term comes from the variation with respect to the induced metric and is proportional to the contraction

$$A = J^{\alpha}_{\ \gamma} J^{\gamma\beta} H^k_{\alpha\beta} \quad . \tag{4.4.1}$$

Second term comes from the variation with respect to induced Kähler form and is proportional to

$$B = j^{\alpha} P^k_{\ s} J^s_{\ l} \partial_{\alpha} h^l \quad . \tag{4.4.2}$$

Here P^k_l is projector to the normal space of space-time surface and $j^{\alpha} = D_{\beta} J^{\alpha\beta}$ is the conserved Kähler current.

For the known extremals j vanishes or is light-like (for massless extremals) in which case A and B vanish separately.

3. An attractive manner to satisfy field equations would be by assuming that the situation for 2-D minimal surface generalizes so that minimal surface equations are identically satisfied. Extremal property for Kähler action could be achieved by requiring that energy momentum tensor also for Kähler action is of type $(1,1)$ so that one would have $A = 0$. This implies $j^{\alpha}\partial_{\alpha}s^k = 0$. This is true if j vanishes or is light-like as it is for the known extremals. In Euclidian regions one would have $j = 0$.

4. The proposed generalization is especially interesting in the case of cosmic string extremals of form $X^2 \times Y^2$, where $X^2 \subset M^4$ is minimal surface (string world sheet) and Y^2 is complex homologically non-trivial sub-manifold of CP_2 carrying Kähler magnetic charge. The generalization would be that the two transversal coordinates $(w, \overline{w})$ in the plane orthogonal to the string world sheet defining polarization plane depend holomorphically on the complex coordinates of complex surface of CP_2. This would transform cosmic string to flux tube.

5. There are also solutions of form $X^2 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2 with vanishing Kähler magnetic charge and their deformations with $(w, \overline{w})$ depending on the complex coordinates of Y^2 (see the slides "On Lagrangian minimal surfaces on the complex projective plane" at `http://tinyurl.com/jrhl6gy`). In this case Y^2 is not complex sub-manifold of CP_2 with arbitrary genus and induced Kähler form vanishes. The simplest choice for Y^2 would be as homologically trivial geodesic sphere. Because of its 2-dimensionality Y^2 has a complex structure defined by its induced metric so that solution ansatz makes sense also now.

4.5 About string like objects

String like objects and partonic 2-surfaces carry the information about quantum states and about space-time surfaces as preferred extremals if strong form of holography (SH) holds true. SH has of course some variants. The weakest variant states that fundamental information carrying objects are metrically 2-D. The light-like 3-surfaces separating space-time regions with Minkowskian and Euclidian signature of the induced metric are indeed metrically 2-D, and could thus carry information about quantum state.

The original observation was that string world sheets should carry vanishing W boson fields in order that the em charge for the modes of the induced spinor field is well-defined. This condition can be satisfied in certain situations also for the entire space-time surface. This raises several questions. What is the fundamental condition forcing the restriction of the spinor modes to string world sheets - or more generally, to a surface of given dimension?

Can one have an analog of brane hierarchy in which also higher-D objects can carry modes of induced spinor field [K68]. Or should one identify 2-surfaces in terms of effective action, which by SH allows to describe the dynamics in terms of 2-D data? Both options have their nice features.

4.5.1 Two options for fundamental variational principle

String world sheets and partonic 2-surfaces seems to be fundamental for TGD - especially so in the fermionic sector - but also the 4-D action seems to necessary and supersymmetry forces 4-D modified Dirac action too. The interpretation of the situation is far from obvious. One ends up to two options for the fundamental variational principle.

Option A: The *fundamental* action principle for space-time surfaces contains besides 4-D action also 2-D action assignable to string world sheets, whose topological part (magnetic flux) gives rise to a coupling term to Kähler gauge potentials assignable to the 1-D boundaries of string world sheets containing also geodesic length part. Super-symplectic symmetry demands that modified Dirac action has 1-, 2-, and 4-D parts: spinor modes would exist at both string boundaries, string world sheets, and space-time interior. A possible interpretation for the interior modes would be as generators of space-time super-symmetries [K68].

This option is not quite in the spirit of SH and string tension appears as an additional parameter. Also the conservation of em charge forces 2-D string world sheets carrying vanishing induced W fields and this is in conflict with the existence of 4-D spinor modes unless they satisfy the same condition. This looks strange.

Option B: Stringy action and its fermionic counterpart are effective actions only and justified by SH. In this case there are no problems of interpretation. SH requires only that the induced spinor fields at string world sheets determine them in the interior much like the values of analytic function at curve determine it in an open set of complex plane. At the level of quantum theory the scattering amplitudes should be determined by the data at string world sheets. If the induced W fields at string world sheets are vanishing, the mixing of different charge states in the interior of X^4 would not make itself visible at the level of scattering amplitudes!

If string world sheets are generalized Lagrangian sub-manifolds, only the induced em field would be non-vanishing and electroweak symmetry breaking would be a fundamental prediction. This however requires that M^4 has the analog of symplectic structure suggested also by twistorialization. This in turn provides a possible explanation of CP breaking and matter-antimatter asymmetry. In this case 4-D spinor modes do not define space-time super-symmetries.

The latter option conforms with number theoretically broken SH and would mean that the theory is amazingly simple. String world sheets together with number theoretical space-time discretization meaning small breaking of SH would provide the basic data determining classical and quantum dynamics. The Galois group of the extension of rationals defining the number-theoretic space-time discretization would act as a covering group of the covering defined by the discretization of the space-time surface, and the value of $h_{eff}/h = n$ would correspond to the dimension of the extension dividing the order of its Galois group. The phase transitions reducing $ord(G) \geq n$ would correspond to spontaneous symmetry breaking leading from Galois group to a subgroup H so that $ord(H)$ would divide $ord(G)$ and the new value of n would divide n.

The ramified primes of the extension would be preferred primes of given extension. The extensions for which the number of p-adic space-time surfaces representable also as a real algebraic continuation of string world sheets to preferred extrenal is especially large would be physically favored as also corresponding ramified primes. In other words, maximal number of p-adic imaginations would be realizable so that these extensions and corresponding ramified primes would be winners in the number-theoretic fight for survival. Whether this conforms with p-adic length scale hypothesis, remains an open question.

An attractive possibility is that this information is basically topological. For instance, the value of Planck constant $h_{eff} = n \times h$ would tell the number sheets of the singular covering defining this surface such that the sheets coincide at partonic 2-surfaces at the ends of space-time surface at boundaries of CD. In the following some questions related to string world sheets are considered. The information could be also number theoretical. Galois group for the algebraic extension of rationals defining particular adelic physics would transform to each other the number theoretic discretizations of light-like 3-surfaces and give rise to covering space structure. The action is trivial at partonic 2-surfaces should be trivial if one wants singular covering: this would mean that discretizations of partonic 2-surfaces consist of rational points. $h_{eff}/h = n$ could in this case be a factor of the order of Galois group.

4.5.2 How to achieve low value of string tension?

String tension should be low for string world sheets in long scales. If string actions are effective actions (Option B), the same should be true for the string tensions of the magnetic flux tubes accompanying strings. Minimal surface property for string world sheets is natural. Let us consider only Option B in the following.

1. Could the analogs of Lagrangian sub-manifolds of $X^4 \subset M^4 \times CP_2$ satisfying $J(M^4) + J(CP_2) = 0$ define string world sheets and their variants with varying dimension? For Option I ($\alpha_K(M^4) \neq \alpha_K(CP_2)$) this could make sense if the flux tubes are homologically trivial. Homologically non-trivial (monopole) flux tubes should be thick enough to have small enough string tension, which is inversely proportional to the cross sectional area of the flux tube.

2. For Option II ($\alpha_K(M^4) = \alpha_K(CP_2)$) the action density is proportional to $J \cdot J - 2$ also for stringy action and this does not seem to make sense. Could the additional condition be $J(M^4) \cdot J(M^4) - 2 \sim 0$ holding true in 4-D sense for space-time regions with a small value of cosmological constant behaving like $1/p$, p preferred p-adic prime near power of 2. That low string tension and small cosmological constant would have the same origin, would be nice.

   The cancellation mechanism involving in an essential manner $J(M^4)$ would give rise to low mass strings and light hadron like particles and small cosmological constant instead of only high mass strings as in super string models. p-Adic thermodynamic for CP_2-mass excitations assignable to wormhole throats would determine elementary particle masses and long monopole flux tubes with small string tension connecting pairs of wormhole contacts would give stringy contribution to particle masses. In the case of hadrons this contribution from color magnetic flux tubes would dominate over quark masses. Clearly, Option II seems to conform with the existing picture about masses of elementary particles and hadrons.

4.5.3 How does the gravitational coupling emerge?

The appearance of $G = l_P^2$ has coupling constant remained for a long time actually somewhat of a mystery in TGD. l_P defines the radius of the twistor sphere of M^4 replaced with its geometric twistor space $M^4 \times S^2$ in twistor lift. G makes itself visible via the coefficients $\rho_{vac} = 8\pi\Lambda/G$ volume term but not directly and if preferred extremals are minimal surface extremals of Kähler action ρ_{vac} makes itself visible only via boundary conditions. How G appears as coupling constant?

Somehow the M^4 Kähler form should appear in field equations. $1/G$ could naturally appear in the string tension for string world sheets as string models suggest. p-Adic mass calculations identify the analog of string tension as something of order of magnitude of $1/R^2$ [K42]. This identification comes from the fact that the ground states of super-conformal representations correspond to embedding space spinor modes, which are solutions of Dirac equation in $M^4 \times CP_2$. This argument is rather convincing and allows to expect that the p-adic mass scale is not determined by string tension.

The problem is that the length of string like objects would be given by Planck length or CP_2 length if either of these pictures is the whole truth. One expects long gravitational flux tubes mediating gravitational interactions. The hypothesis $\hbar_{eff} = n\hbar = \hbar_{gr} = GMm/v_0$, where $v_0 < c$ is a parameter with dimensions of velocity, suggests that the string tension assignable to the flux tubes mediating gravitational interaction between masses M and m is apart from a numerical factor equal to Λ_{gr}^{-2}, where gravitational Compton length is $\Lambda_{gr} = h_{gr}/m = GM/v_0$ so that the length of the flux tubes is of order Λ_{gr}.

The problem is that the length of string like objects would be given by Planck length or CP_2 length if either of these pictures is the whole truth. One would like to have long gravitational flux tubes mediating gravitational interactions. Strong form of holography (SH) indeed suggests that stringy action appears as effective action expressing 4-D space-time action and modified Dirac action as 2-D actions assignable to string world sheets [L21] (see `http://tinyurl.com/zylrd7w`). This view would allow to understand the localization of spinor modes to string world sheets carrying vanishing W fields in terms as an effective description implying well-defineness of classical em charge and conservation of em charge at the level of scattering amplitudes. In fact that the introduction of the Kähler form $J(M^4)$ would allow to understand string world sheets as analogs of Lagrangian sub-manifolds.

4.5.4 Non-commutative embedding space and strong form of holography

Quantum group theorists have studied the idea that space-time coordinates are non-commutative and tried to construct quantum field theories with non-commutative space-time coordinates (see `http://tinyurl.com/z3m8sny`). My impression is that this approach has not been very successful. The non-commutativity is introduced by postulating the Minkowskian analog of symplectic form and $J(M^4)$ forced by Option II indeed is symplectic form. The loss of Lorentz invariance induced by $J(M^4)$ is the basic stumbling block. In TGD framework the moduli space for $J(M^4)$ emerges already when one introduces the moduli space for CDs. $J(M^4)$ would define quantization axis of energy (rest system) and quantization axis of spin. The nice features of $J(M^4)$ is that it could allow to understand CP breaking and matter antimatter asymmetry at fundamental level.

The analog of non-commutative space-time in TGD framework

In Minkowski space one introduces antisymmetry tensor J_{kl} and uncertainty relation in linear M^4 coordinates m^k would look something like $[m^k, m^l] = l_P^2 J^{kl}$, where l_P is Planck length. This would be a direct generalization of non-commutativity for momenta and coordinates expressed in terms of symplectic form J^{kl}.

1+1-D case serves as a simple example. The non-commutativity of p and q forces to use either p or q. Non-commutativity condition reads as $[p, q] = \hbar J^{pq}$ and is quantum counterpart for classical Poisson bracket. Non-commutativity forces the restriction of the wave function to be a function of p or of q but not both. More geometrically: one selects Lagrangian sub-manifold to which the projection of J_{pq} vanishes: coordinates become commutative in this sub-manifold. This condition can be formulated purely classically: wave function is defined in Lagrangian sub-manifolds to which the projection of J vanishes. Lagrangian manifolds are however not unique and this leads to problems in this kind of quantization. In TGD framework the notion of "World of Classical Worlds" (WCW) allows to circumvent this kind of problems and one can say that quantum theory is purely classical field theory for WCW spinor fields. "Quantization without quantization" would have Wheeler stated it.

General Coordinate Invariance (GCI) poses however a problem if one wants to generalize quantum group approach from M^4 to general space-time: linear M^4 coordinates assignable to Lie-algebra of translations as isometries do not generalize. In TGD space-time is surface in embedding space $H = M^4 \times CP_2$: this changes the situation since one can use 4 embedding space coordinates (preferred by isometries of H) also as space-time coordinates. The analog of symplectic structure J for M^4 makes sense and number theoretic vision involving octonions and quaternions leads to its introduction. Note that CP_2 has naturally symplectic form.

Could it be that the coordinates for space-time surface are in some sense analogous to symplectic coordinates (p_1, p_2, q_1, q_2) so that one must use either (p_1, p_2) or (q_1, q_2) providing coordinates for a Lagrangian sub-manifold. This would mean selecting a Lagrangian sub-manifold of space-time surface? Could one require that the sum $J_{\mu\nu}(M^4) + J_{\mu\nu}(CP_2)$ for the projections of symplectic forms vanishes and forces in the generic case localization to string world sheets and partonic 2-surfaces. In special case also higher-D surfaces - even 4-D surfaces as products of Lagrangian 2-manifolds for M^4 and CP_2 are possible: they would correspond to homologically trivial cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$, which are not anymore vacuum extremals but minimal surfaces if the action contains besides Käction also volume term.

But why this kind of restriction? In TGD one has strong form of holography (SH): 2-D string world sheets and partonic 2-surfaces code for data determining classical and quantum evolution. Could this projection of $M^4 \times CP_2$ symplectic structure to space-time surface allow an elegant mathematical realization of SH and bring in the Planck length l_P defining the radius of twistor sphere associated with the twistor space of M^4 in twistor lift of TGD? Note that this can be done without introducing embedding space coordinates as operators so that one avoids the problems with general coordinate invariance. Note also that the non-uniqueness would not be a problem as in quantization since it would correspond to the dynamics of 2-D surfaces.

The analog of brane hierarchy at fundamental level or from SH?

The analog of brane hierarchy for the localization of spinors - space-time surfaces; string world sheets and partonic 2-surfaces; boundaries of string world sheets - is suggestive (note however that SH does not favour it). Could this hierarchy correspond to a hierarchy of Lagrangian sub-manifolds of space-time in the sense that $J(M^4) + J(CP_2) = 0$ is true at them? Boundaries of string world sheets would be trivially Lagrangian manifolds. String world sheets allowing spinor modes should have $J(M^4)+J(CP_2) = 0$ at them. The vanishing of induced W boson fields is needed to guarantee well-defined em charge at string world sheets and that also this condition allow also 4-D solutions besides 2-D generic solutions. As already found, for the physically favoured Option II the more plausible option is $J(M^4) \cdot J(M^4) - 2 \sim 0$ for space-time regions with small cosmological constant. Despite this one can discuss this idea.

This condition is physically obvious but mathematically not well-understood: could the condition $J(M^4) + J(CP_2) = 0$ force the vanishing of induced W boson fields? Lagrangian cosmic string type minimal surfaces $X^2 \times Y^2$ would allow 4-D spinor modes. If the light-like 3-surface defining boundary between Minkowskian and Euclidian space-time regions is Lagrangian surface, the total induced Kähler form Chern-Simons term would vanish. The 4-D canonical momentum currents would however have non-vanishing normal component at these surfaces. I have considered the possibility that TGD counterparts of space-time super-symmetries could be interpreted as addition of higher-D right-handed neutrino modes to the 1-fermion states assigned with the boundaries of string world sheets [K68].

Induced spinor fields at string world sheets could obey the "dynamics of avoidance" in the sense that *both* the induced weak gauge fields W, Z^0 and induced Kähler form (to achieve this U(1) gauge potential must be sum of M^4 and CP_2 parts) would vanish for the regions carrying induced spinor fields. They would couple only to the *induced em field (!)* given by the R_{12} part of CP_2 spinor curvature [L2] for $D = 2, 4$. For $D = 1$ at boundaries of string world sheets the coupling to gauge potentials would be non-trivial since gauge potentials need *not* vanish there. Spinorial dynamics would be extremely simple and would conform with the vision about symmetry breaking of electro-weak group to electromagnetic gauge group.

It seems relatively easy to construct am infinite family of Lagrangian string world sheets satisfying $J(M^4) + J(CP_2) = 0$ using generalized symplectic transformations of M^4 and CP_2 as Hamiltonian flows to generate new ones from a given Lagrangian string world sheets. One must pose minimal surface property as a separate condition. Consider a piece of M^2 with coordinates (t, z) and homologically non-trivial geodesic sphere S^2 of CP_2 with coordinates $(u = cos(\Theta), \Phi)$. One has $J(M^4)_{tz} = 1$ and $J_{u\Phi} = 1$. Identify string world sheet via map $(u, \Phi) = (kz, \omega t)$ from M^2 to S^2. The induced CP_2 Kahler form is $J(CP_2)_{tz} = k\omega$. $k\omega = -1$ guarantees $J(M^4)+J(CP_2) = 0$. The strings have necessarily finite length from $L = 1/k \leq z \leq L$. One can perform symplectic transformations of CP_2 and symplectic transformations of M^4 to obtain new string world sheets. In general these are not minimal surfaces and this condition would select some preferred string world sheets.

Number theoretic vision about the analog of brane hierarchy

An alternative - but of course not necessarily equivalent - attempt to formulate SH would be in terms of number theoretic vision. Space-time surfaces would be associative or co-associative depending on whether tangent space or normal space in embedding space is associative - that is quaternionic. These two conditions would reduce space-time dynamics to associativity and commutativity conditions. String world sheets and partonic 2-surfaces would correspond to maximal commutative or co-commutative sub-manifolds of embedding space. Commutativity (co-commutativity) would mean that tangent space (normal space as a sub-manifold of space-time surface) has complex tangent space at each point and that these tangent spaces integrate to 2-surface. SH would mean that data at these 2-surfaces plus number theoretic discretization of space-time surface would be enough to construct quantum states. Therefore SH would be thus slightly broken. String world sheet boundaries would in turn correspond to real curves of the complex 2-surfaces intersecting partonic 2-surfaces at points so that the hierarchy of classical number fields would have nice realization at the level of the classical dynamics of quantum TGD.

To sum up, one cannot exclude the possibility that $J(M^4)$ is present implying a universal transversal localization of embedding space spinor harmonics and the modes of spinor fields in the interior of X^4: this could perhaps relate to somewhat mysterious de-coherence interaction producing locality and to CP breaking and matter-antimatter asymmetry. The moduli space for M^4 Kähler structures proposed by number theoretic considerations would save from the loss of Poincare invariance and the number theoretic vision based on quaternionic and octonionic structure would have rather concrete realization. This moduli space would only extend the notion of WCW.

Chapter 5

Some Questions Related to the Twistor Lift of TGD

5.1 Introduction

During last couple years (I am writing this in the beginning of 2017) a kind of palace revolution has taken place in the formulation and interpretation of TGD. The notion of twistor lift and 8-D generalization of twistorialization have dramatically simplified and also modified the view about what classical TGD and quantum TGD are.

The notion of adelic physics suggests the interpretation of scattering diagrams as representations of algebraic computations with diagrams producing the same output from given input are equivalent. The simplest possible way to perform the computation corresponds to a tree diagram [L11]. As will be found, it is now possible to even propose explicit twistorial formulas for scattering formulas since the horrible problems related to the integration over WCW might be circumvented altogether.

From the interpretation of p-adic physics as physics of cognition, $h_{eff}/h = n$ could be interpreted dimension of extension dividing the the order of its Galois group. Discrete coupling constant evolution would correspond to phase transitions changing the extension of rationals and its Galois group. TGD inspired theory of consciousness is an essential part of TGD and the crucial Negentropy Maximization Principle in statistical sense follows from number theoretic evolution as increase of the order of Galois group for extension of rationals defining adeles.

In the sequel I consider the questions related to both classical and quantum aspects of twistorialization.

5.1.1 Questions related to the classical aspects of twistorialization

Classical aspects are related to the twistor lift of classical TGD replacing space-time surfaces with their twistor spaces realized as extremals of 6-D analog of Kähler action in the product $T(M^4) \times T(CP_2)$ of twistor space of M^4 and CP_2 such that twistor structure is induced. The outcome is 4-D Kähler action with volume term having interpretation in terms of cosmological constant. Hence the twistorialization has profound physical content rather than being mere alternative formulation for TGD.

1. What does the induction of the twistor structure really mean? What is meant with twistor space. For instance, is the twistor sphere for M^4 time-like or space-like. The induction procedure involves dimensional reduction forced by the condition that the projection of the sum of Kähler forms for the twistor spaces $T(M^4)$ and $T(CP_2)$ gives Kähler form for the twistor sphere of X^4. Better understanding of the details is required.

2. Can the analog of Kähler form $J(M^4)$ assignable to M^4 suggested by the symmetry between M^4 and CP_2 and by number theoretical vision appear in the theory? What would be the physical implications?

The basic objection is the loss of Poincare invariance. This can be however avoided by introducing the moduli space for Kähler forms. This moduli space is actually the moduli space of causal diamonds (CDs) forced in any case by zero energy ontology (ZEO) and playing central role in the generalization of quantum measurement theory to a theory of consciousness and in the explanation of the relationship between geometric and subjective time [K46].

Why $J(M^4)$ would be needed? $J(M^4)$ corresponds to parallel constant electric and magnetic fields in given direction. Constant E and $B = E$ fix directions of quantization axes for energy (rest system) and spin. One implication is transversal localization of embedding space spinor modes: embedding space spinor modes are products of harmonic oscillator Gaussians in transversal degrees of freedom very much like quarks inside hadrons.

Also CP breaking is implied by the electric field and the question is whether this could explain the observed CP breaking as appearing already at the level of embedding space $M^4 \times CP_2$. The estimate for the mass splitting of neutral kaon and anti-kaon is of correct order of magnitude.

Whether stationary spherically symmetric metric as minimal surface allows a sensible physical generalization is a killer test for the hypothesis that $J(M^4)$ is covariantly constant. The question is basically about how large the moduli space of forms $J(M^4)$ can be allowed to be. The mere self duality and closedness condition outside the line connecting the tips of CD allows also variants which are spherically symmetric in either Minkowski coorinates or Robertson-Walker coordinates for light-cone.

3. How does gravitational coupling emerge at fundamental level? The first naive guess is obvious: string area action is scaled by $1/G$ as in string models. The objection is that p-adic mass calculations suggest that string tension is determined by CP_2 size R: the analog of string tension appearing in mass formula given by p-adic mass calculations would be by a factor about 10^{-8} smaller than that estimated from string tension. The discrepancy evaporates by noticing that p-adic mass calculations rely on p-adic thermodynamics at embedding space level whereas string world sheets appear at space-time level. Furthermore, if the action assignable to string world sheets is effective action expressing 4-D action in 2-D form as strong form of holography (SH) suggests string tension is expected to be function of the parameters appearing in the 4-D action.

4. Could one regard the localization of spinor modes to string world sheets as a localization to Lagrangian sub-manifolds of space-time surface having by definition vanishing induced Kähler form: $J(M^4)+J(CP_2) = 0$. Lagrangian sub-manifolds would be commutative in the sense of Poisson bracket? Could string world sheets be minimal surfaces satisfying $J(M^4)+J(CP_2) = 0$. The Lagrangian condition allows also more general solutions - even 4-D space-time surfaces and one obtains analog of brane hierarchy. Could one allow spinor modes also at these analogs of branes. Is Lagrangian condition equivalent with the original condition that induced W boson fields making the em charge of induced spinor modes ill-defined vanish and allowing also solution with other dimensions. How Lagrangian property relates to the idea that string world sheets correspond to complex (commutative) surfaces of quaternionic space-time surface in octonionic embedding space.

During the re-processing of the details related to twistor lift, it became clear that the earlier variant for the twistor lift [L12] contained an error. This led to much simpler view about twistor lift, to the conclusion that minimal surface extremals of Kähler action represent only asymptotic situation (external particles in scattering), and also to a re-interpretation for the p-adic evolution of the cosmological constant.

5.1.2 Questions related to the quantum aspects of twistorialization

Also the questions related to the quantum aspects of twistorialization of TGD are discussed.

1. There are several notions of twistor. Twistor space for M^4 is $T(M^4) = M^4 \times S^2$ [B42] (see `http://arxiv.org/pdf/1308.2820.pdf`) having projections to both M^4 and to the standard twistor space $T_1(M^4)$ often identified as CP_3. $T(M^4) = M^4 \times S^2$ is necessary for

the twistor lift of space-time dynamics. CP_2 gives the factor $T(CP_2) = SU(3)/U(1) \times U(1)$ to the classical twistor space $T(H)$. The quantal twistor space $T(M^8) = T_1(M^4) \times T(CP_2)$ assignable to momenta. The possible way out is $M^8 - H$ duality relating the momentum space M^8 (isomorphic to the tangent space H) and H by mapping space-time associative and co-associative surfaces in M^8 to the surfaces which correspond to the base spaces of in H: they construction would reduce to holomorphy in complete analogy with the original idea of Penrose in the case of massless fields.

2. The standard twistor approach has problems. Twistor Fourier transform reduces to ordinary Fourier transform only in signature (2,2) for Minkowski space: in this case twistor space is real RP_3 but can be complexified to CP_3. Otherwise the transform requires residue integral to define the transform (in fact, p-adically multiple residue calculus could provide a nice way to define integrals and could make sense even at space-time level making possible to define action).

   Also the positive Grassmannian requires (2,2) signature. In $M^8 - H$ relies on the existence of the decomposition $M^2 \subset M^2 = M^2 \times E^2 \subset M^8$. M^2 could even depend on position but $M^2(x)$ should define an integrable distribution. There always exists a preferred M^2, call it M_0^2, where 8-momentum reduces to light-like M^2 momentum. Hence one can apply 2-D variant of twistor approach. Now the signature is (1,1) and spinor basis can be chosen to be real! Twistor space is RP_3 allowing complexification to CP_3 if light-like complex momenta are allowed as classical TGD suggests!

3. A further problem of the standard twistor approach is that in M^4 twistor approach does not work for massive particles. In TGD all particles are massless in 8-D sense. In M^8 M^4-mass squared corresponds to transversal momentum squared coming from $E^4 \subset M^4 \times E^4$ (from CP_2 in H). In particular, Dirac action cannot contain anyo mass term since it would break chiral invariance.

   Furthermore, the ordinary twistor amplitudes are holomorphic functions of the helicity spinors λ_i and have no dependence on $\tilde{\lambda}_i$: no information about particle masses! Only the momentum conserving delta function gives the dependence on masses. These amplitudes would define as such the M^4 parts of twistor amplitudes for particles massive in TGD sense. The simplest 4-fermion amplitude is unique.

Twistor approach gives excellent hopes about the construction of the scattering amplitudes in ZEO. The construction would split into two pieces corresponding to the orbital degrees of freedom in "world of classical worlds" (WCW) and to spin degrees of freedom in WCW: that is spinors, which correspond to second quantized induced spinor fields at space-time surface (actually string world sheets- either at fundamental level or for effective action implied by strong form of holography (SH)).

1. At WCW level there is a perturbative functional integral over small deformations of the 3-surface to which space-time surface is associated. The strongest assumption is that this 3-surface corresponds to maximum for the real part of action and to a stationary phase for its imaginary part: minimal surface extremal of Kähler action would be in question. A more general but number theoretically problematic option is that an extremal for the sum of Kähler action and volume term is in question.

   By Kähler geometry of WCW the functional integral reduces to a sum over contributions from preferred extremals with the fermionic scattering amplitude multiplied by the ration X_i/X, where $X = \sum_i X_i$ is the sum of the action exponentials for the maxima. The ratios of exponents are however number theoretically problematic.

   Number theoretical universality is satisfied if one assigns to each maximum independent zero energy states: with this assumption $\sum X_i$ reduces to single X_i and the dependence on action exponentials becomes trivial! ZEO allow this. The dependence on coupling parameters of the action essential for the discretized coupling constant evolution is only via boundary conditions at the ends of the space-time surface at the boundaries of CD.

   Quantum criticality of TGD [?, K66, K84] demands that the sum over loops associated with the functional integral over WCW vanishes and strong form of holography (SH) suggests that the integral over 4-surfaces reduces to that over string world sheets and partonic 2-surfaces corresponding to preferred extremals for which the WCW coordinates parametrizing them belong to the extension of rationals defining the adele [L21]. Also the intersections of the real and various p-adic space-time surfaces belong to this extension.

2. Second piece corresponds to the construction of twistor amplitude from fundamental 4-fermion amplitudes. The diagrams consists of networks of light-like orbits of partonic two surfaces, whose union with the 3-surfaces at the ends of CD is connected and defines a boundary condition for preferred extremals and at the same time the topological scattering diagram.

   Fermionic lines correspond to boundaries of string world sheets. Fermion scattering at partonic 2-surfaces at which 3 partonic orbits meet are analogs of 3-vertices in the sense of Feynman and fermions scatter classically. There is no local 4-vertex. This scattering is assumed to be described by simplest 4-fermion twistor diagram. These can be fused to form more complex diagrams. Fermionic lines runs along the partonic orbits defining the topological diagram.

3. Number theoretic universality [K84] suggests that scattering amplitudes have interpretation as representations for computations. All space-time surfaces giving rise to the same computation wold be equivalent and tree diagrams corresponds to the simplest computation. If the action exponentials do not appear in the amplitudes as weights this could make sense but would require huge symmetry based on two moves. One could glide the 4-vertex at the end of internal fermion line along the fermion line so that one would eventually get the analog of self energy loop, which should allow snipping away. An argument is developed stating that this symmetry is possible if the preferred M_0^2 for which 8-D momentum reduces to light-like M^2-momentum having unique direction is same along entire fermion line, which can wander along the topological graph.

   The vanishing of topological loops would correspond to the closedness of the diagrams in what might be called BCFW homology. Boundary operation involves removal of BCFW bridge and entangled removal of fermion pair. The latter operation forces loops. There would be no BCFW bridges and entangled removal should give zero. Indeed, applied to the proposed four fermion vertex entangled removal forces it to correspond to forward scattering for which the proposed twistor amplitude vanishes.

To sum up, the twistorial approach leads to a proposal for an explicit construction of scattering amplitudes for the fundamental fermions. Bosons and fermions as elementary particles are bound states of fundamental fermions assignable to pairs of wormhole contacts carrying fundamental fermions at the throats. Clearly, this description is analogous to a quark level description of hadron. Yangian symmetry with multilocal generators is expected to crucial for the construction of the many-fermion states giving rise to elementary particles. The problems of the standard twistor approach find a nice solution in terms of M^8-H duality, 8-D masslessness, and holomorphy of twistor amplitudes in λ_i and their indepence on $\tilde{\lambda}_i$.

5.2 More details about the induction of twistor structure

The notion of twistor lift of TGD [L11] [L26] has turned out to have powerful implications concerning the understanding of the relationship of TGD to general relativity. The meaning of the twistor lift really has remained somewhat obscure. There are several questions to be answered. What does one mean with twistor space? What does the induction of twistor structure of $H = M^4 \times CP_2$ to that of space-time surface realized as its twistor space mean?

5.2.1 What does one mean with twistor space?

The notion of twistor space has been discussed in [L11] from TGD point of view.

1. In the case of twistor space of M^4 the starting point of Penrose was the isomorphism between the conformal group of Spin(4,2) of 6-D Minkowski space $M^{4,2}$ and the group SU(2,2) acting on 2+2 complex spinors.

6-D twistor space could be identified as 6-D coset space $SU(2,2)/SU(2,1)\times U(1)$. For E^6 this would give projective space $CP_3 = SU(4)/SU(3)\times U(1)$ and in twistor Grassmann approach this definition is indeed used. It is thought that the problems caused by Euclidization are not serious.

2. One can think $SU(2,2)$ as 4×4 complex matrices with orthogonal complex row vector $Z_i = (Z_{i1},...,Z_{i4})$, and norms $(1,1,-1-1)$ in the metric $s^2 = \sum \epsilon_i|z_i|^2$, $\epsilon_i \leftrightarrow (1,1,-1,-1)$. The sub-matrices defined by (Z_{k2}, Z_{k3}, Z_{k4}), $k = 2,3,4$, can be regarded apart from normalization elements of $SU(1,2)$. The column vector with components Z_{i1} with $Z_{11} = \sqrt{1+\rho^2}$, $\rho^2 = |Z_{21}|^2 - |Z_{31}|^2 - |Z_{41}|^2$ corresponds to a point of the twistor space. The S^2 fiber for given values of ρ and (Z_{31}, Z_{41}) could be identified as the space spanned by the values of Z_{21}. Note that S^2 would have time-like signature and the signature of twistor space would be (3,3), which conforms with the existence of complex structure. There would be dimensional democracy at this level.

3. The identification of 4-D base of the twistor space is unclear to me. The base space of the this twistor space should correspond to the conformal compactification M^4_c of M^4 having metric defined only apart from conformal scaling. The concrete realization M^4_c would be in terms of $M^{4,2}$ light-cone with points projectively identified. As a metric object this space is ill-defined and can appear only at the level of scattering amplitudes in conformally invariant quantum field theories in M^4.

4. Mathematicians define also a second variant of twistor space with S^2 fiber and this space is just $M^4\times S^2$ [B42] (see `http://tinyurl.com/yb4bt74l`). This space has a well-defined metric and seems to be the only possible one for the twistor lift of classical TGD replacing space-time surfaces with their twistor spaces. Whether the signature of S^2 is time-like or space-like has remained an open question but time-like signature looks natural. The radius R_P of S^2 has been proposed to be apart from a numerical constant equal to Planck length l_P. Note that the isometry group is 9-D $SO(3,1)\times SU(2)$ rather than 15-D $SU(2,2)$. In TGD light-likeness in 8-D sense replaces light-likeness in 4-D sense: does this somehow replace the conformal symmetry group $SO(4,2)$ with $SO(3,1)\times SO(3)$? Could $SU(2)$ rotate the direction of spin quantization axis.

I must confess that I have found the notions of twistor and twistor sphere very difficult to understand. Perhaps this is not solely due to my restricted mathematical skills. Also the physics of twistors looks confusing to me.

The twistor space assignable to Minkowski space and corresponding twistor sphere have several meanings. Consider first the situation in standard framework.

1. One can define twistor space as complex 8-D space C^4. Given four-momentum corresponds however to projective line so that one can argue that twistor space is 6-D space $T_1(M^4) = CP_3 = SU(4)/SU(3)\times U(1)$ of projective lines of C^4 in C^4. One could also argue that one must take the signature of Minkowski space into account. $SU(2,2)$ acts as symmetries of twistor bilinear form and one would have $T_1(M^4) = SU(2,2)/SU(2,1)\times U(1)$. In this case twistor sphere could correspond to the projective line in C^4.

2. Incidence relations $\mu^{\dot a} = m^{a\dot a}\lambda_a$ relate M^4 points to those of twistor space. In the usual twistor formalism twistor sphere corresponds to the projective line of 8-D C^4. When m is not light-like, it corresponds to a matrix which is invertible and one can solve μ from λ and vice versa. The twistor spheres associated with m_1 and m_2 are said to intersect if $m_1 - m_2$ is a complex light-like vector defining a complexified light ray. One could identify twistor sphere of $T_1(M^4)$ as the Riemann sphere defined by these complex points and going to CP_3 one actually eliminates it altogether, which is somewhat unsatisfactory.

3. When m is light-like and thus expressible as $\mu = \lambda\otimes\tilde\lambda$ one has $\mu = \mu_0 + t\tilde\lambda$, t a complex number. One can say that one has a full Riemann sphere S^2 of solutions. There is also additional degeneracy due to the scaling of both λ and μ. For light-like M^4 points (say momenta) one obtains a Riemann sphere in 6-D twistor space. Which twistor sphere is the correct one: the sphere associated with all points of M^4 and 8-D twistor space or the sphere associated with light-like points of M^4 and 6-D twistor space?

Consider now the situation in TGD.

1. For the twistor lift of Kähler action lifting the dynamics of space-time surfaces to the dynamics of their twistor spaces, the twistor lift of M^4 corresponds to $T(M^4) = M^4 \times CP_2$. This might look strange but the proper mathematical definition of twistor space relies on double fibration involving both views about twistor space discussed in [B42] (see `http://tinyurl.com/yb4bt74l`). This double fibration would be crucially involved with M^8-H duality. The fiber space is $T(M^4) = M^4 \times CP_1$, where CP_1 corresponds to the projective sphere assignable to complex spinors λ. This fiber is trivially projected both to M^4 and less trivially to a subset of 6-dimensional complex projective space $T_!(M^4) = CP_3$.

   At space-time level $T(M^4)$ is the only correct choice since twistor space must have isometries of M^4. This choices brings into the dynamics Planck length essentially as the radius of S^2 and cosmological constant as volume term resulting in the dimensional reduction of 6-D Kähler action forced by twistor space property of 6-surface.

   At the level of momentum space - perhaps the M^8 appearing in $M^8 - H$ duality identifiable as tangent space of H - the twistor space would correspond to twistor space assignable to momentum space and should relate to the ordinary twistor space $T_1(M^4)$ - whatever it is!

2. In M^8 picture the twistor space is naturally associated with preferred $M^2 \subset M^4$, where M^4 is quaternionic space. The moduli space of $M^2 \subset M^4$ for time direction assigned with real octonion, is parametrized by S^2 and a possible interpretation is as twistor sphere of $M^2 \times CP_1$. Interestingly, $M^2 \subset M^4$ is characterized by light-like vector together with its unique dual light-like vector.

   By restricting 4-D conformal invariance to 2-D situation, one finds that the twistor space becomes RP_3 but can be complexified to CP_3 to allowing complexified M^2 momenta. The signature (1,1) of M^2 and reality of spinor basis gives hopes of resolving the conceptual problems of the ordinary twistor approach. For the real spinor spinor pair (λ, μ) the solutions to the coincidence relations real M^2 spinors but one can allowing their complex multiples.

3. $M^8 - H$ correspondence allows to map M^4 points to each other: this involves a choice of $M^4 \subset M^8$. $M^8 - H$ correspondence maps quaternionic (and co-quaternionic) surfaces in M^8 to preferred extremals of Kähler in H proposed to correspond to the base bases of of twistor bundles $T(X^4) \subset T(M^4) \times T(CP_2)$ constructible using holomorphic maps. One can thus argue that there should be also a correspondence between the twistor spaces $T(M^4)$ and $T_1(M^4)$ - the correspondence between the twistor spheres would be enough.

   The two M^4:s correspond to each other naturally. What is required is a map of twistorial spheres S^2 to each other. Suppose that the twistorial sphere of H corresponds to that assignable to the choice of $M^2 \subset M^8$ by a choice of quaternionic imaginary unit in M^4 of equivalently by a choice of a light-like vector n of M^2 plane. But by incidence relations the light-like vector n has twistor sphere CP_1 as a pre-image in complexified $T_1(M^2) = CP_3$ characterized by the shifts $\mu \rightarrow \mu + \tilde{\lambda}$. Therefore the two twistor spheres can be identified by mapping n of $S^2(T(M^4)$ to its counterpart of $T_1(M^2)$ isometrically.

It therefore seems that the double fibration is essential in TGD framework and the usual twistor space is assignable to the M^8 interpreted asthe space of complexified octonion momenta subject to the quaternionicity condition. Sharply defined transversed quaternionic momentum eigenstates in $E^2 \times E^4$ are replaced with wave functions in $T(CP_2)$ reducing locally to $CP_2 \times U(2)/U(1) \times U(1)$ with em charge identifiable as the analog of angular momentum for the wave functions in $CP_1 = U(2)/U(1) \times U(1)$. In $M^4 \times CP_2$ picture one has spinor modes labelled by electroweak quantum numbers.

5.2.2 Twistor lift of TGD

In TGD one replaces embedding space $H = M^4 \times CP_2$ with the product $T = T(M^4) \times T(CP_2)$ of their 6-D twistor spaces, and calls $T(H)$ the twistor space of H. For CP_2 the twistor space is the flag manifold $T(CP_2) = SU(3)/U(1) \times U(1)$ consisting of all possible choices of quantization axis of color isospin and hypercharge.

1. The basic idea is to generalize Penrose's twistor program by lifting the dynamics of space-time surfaces as preferred extremals of Kähler action to those of 6-D Kähler action in twistor space $T(H)$. The conjecture is that field equations reduce to the condition that the twistor structure of space-time surface as 4-manifold is the twistor structure induced from $T(H)$.

   Induction requires that dimensional reduction occurs effectively eliminating twistor fiber $S^2(X^4)$ from the dynamics. Space-time surfaces would be preferred extremals of 4-D Kähler action plus volume term having interpretation in terms of cosmological constant. Twistor lift would be more than an mere alternative formulation of TGD.

2. The reduction would take place as follows. The 6-D twistor space $T(X^4)$ has S^2 as fiber and can be expressed locally as a Cartesian product of 4-D region of space-time and of S^2. The signature of the induced metric of S^2 should be space-like or time-like depending on whether the space-time region is Euclidian or Minkowskian. This suggests that the twistor sphere of M^4 is time-like as also standard picture suggests.

3. Twistor structure of space-time surface is induced to the allowed 6-D surfaces of $T(H)$, which as twistor spaces $T(X^4)$ must have fiber space structure with S^2 as fiber and space-time surface X^4 as base. The Kähler form of $T(H)$ expressible as a direct sum

$$J(T(H)) = J(T(M^4)) \oplus J(T(CP_2))$$

   induces as its projection the analog of Kähler form in the region of $T(X^4)$ considered.

   There are physical motivations (CP breaking, matter antimatter symmetry, the well-definedness of em charge) to consider the possibility that also M^4 has a non-trivial symplectic/Kähler form of M^4 obtained as a generalization of ordinary symplectic/Kähler form [L26]. This requires the decomposition $M^4 = M^2 \times E^2$ such that M^2 has hypercomplex structure and E^2 complex structures.

   This decomposition might be even local with the tangent spaces $M^2(x)$ and $E^2(x)$ integrating to locally orthogonal 2-surfaces. These decomposition would define what I have called Hamilton-Jacobi structure [K82]. This would give rise to a moduli space of M^4 Kähler forms allowing besides covariantly constant self-dual Kähler forms with decomposition (m^0, m^3) and (m^1, m^2) also more general self-dual closed Kähler forms assignable to integrable local decompositions. One example is spherically symmetric stationary self-dual Kähler form corresponding to the decomposition (m^0, r_M) and (θ, ϕ) suggested by the need to get spherically symmetric minimal surface solutions of field equations. Also the decomposition of Robertson-Walker coordinates to (a, r) and (θ, π) assignable to light-cone M^4_+ can be considered.

   The moduli space giving rise to the decomposition of WCW to sectors would be finite-dimensional if the integrable 2-surfaces defined by the decompositions correspond to orbits of subgroups of the isometry group of M^4 or CD. This would allow planes of M^4, and radial half-planes and spheres of M^4 in spherical Minkowski coordinates and of M^4_+ in Robertson-Walker coordinates. These decomposition could relate to the choices of measured quantum numbers inducing symmetry breaking to the subgroups in question. These choices would chose a sector of WCW [K46] and would define quantum counterpart for a choice of quantization axes as distinct from ordinary state function reduction with chosen quantization axes.

4. The induced Kähler form of S^2 fiber of $T(X^4)$ is assumed to reduce to the sum of the induced Kähler forms from S^2 fibers of $T(M^4)$ and $T(CP_2)$. This requires that the projections of the Kähler forms of M^4 and CP_2 to $S^2(X^4)$ are trivial. Also the induced metric is assumed to be direct sum and similar conditions holds true.These conditions are analogous to those occurring in dimensional reduction.

   Denote the radii of the spheres associated with M^4 and CP_2 as $R_P = kl_P$ and R and the ratio R_P/R by ϵ. Both the Kähler form and metric are proportional to R_p^2 resp. R^2 and satisfy the defining condition $J_{kr}g^{rs}J_{sl} = -g_{kl}$. This condition is assumed to be true also for the induced Kähler form of $J(S^2(X^4)$.

   Let us introduce the following shorthand notations

$$\begin{array}{lll} S_1^2 = S^2(X^4) \ , & S_2^2 = S^2(CP_2) \ , & S_3^2 = S^2(M^4) \ , \\ J_i = \frac{J(S_i^2)}{R^2} \ , & g_i = \frac{g(S_i^2))}{R^2} \ . & \end{array} \tag{5.2.1}$$

This gives the following equations.

$$J_1 = J_2 + \epsilon J_3 \ , \quad g_1 = g_2 + \epsilon g_3 \ , \quad J_1 g_1 J_1 = -g_1 \ . \tag{5.2.2}$$

Projections to $S_1^2 = S^2(X^4)$ are assumed at r.h.s.. The product of the third equation is defined as tensor contraction and involves contravariant form of g.

5.2.3 Solutions to the conditions defining the twistor lift

Consider now solutions to the conditions defining the twistor lift.

1. The simplest solution type corresponds to the situation in which either S_2^2 (S_3^2) equals to S_1^2) and S_3^2 (S_2^2) projection of $T(X^4)$ is single point. In this case the conditions of Eq. are trivially satisfied. These two solutions could correspond to Euclidian and Minkowskian space-time regions. Also the solution for which twistor sphere degenerates to a point must be considered and form $J(M^4) = 0$ this would correspond to the reduction of dimensionally reduced action to Kähler action defining the original variant of TGD. Note that preferred extremals are conjectured to be minimal surfaces extremals of Kähler action always [L9].

2. One can consider also more general solutions. Depending on situation, one can use for $S^2(X^4)$ either the coordinates of S_2^2 or S_3^2. Let us choose S_2^2. One can of course change the roles of the spheres.

   Consider an ansatz for which the projections of J_2 and J_3 to S_1^2 are in constant proportionality to each other. This is guaranteed if the spherical coordinates $(u = cos(\Theta), \Phi)$ of S_2^2 and S_3^2 are related by $(u(M^4), \Phi(M^4)) = (u(CP_2), n\Phi(CP_2))$ so that the map between the two spheres has winding number n. With this assumption one has

$$\begin{array}{l} J_1 = (1 + \epsilon n) J_2 \ , \\ g_1 = (1 + \epsilon n^2) g_2 \ , \end{array} \tag{5.2.3}$$

   The third condition of Eq. 1 equation gives

$$(1 + n\epsilon)^2 \quad = \quad (1 + n^2\epsilon)^2 \ . \tag{5.2.4}$$

   This in turn gives

$$1 + n\epsilon = \delta(1 + n^2\epsilon) \ , \quad \delta = \pm 1 \ . \tag{5.2.5}$$

   The only solution for $\delta = +1$ is $n = 0$ or $n = 1$. For $\delta = -1$ there are no solutions.

   One has 3+1 different solutions corresponding to the degenerate solution $(n_1, n_2) = (0, 0)$ and 3 solutions with (n_1, n_2) equal $(1, 0)$, $(0, 1)$ or $(1, 1)$. The conditions are very stringent and it is not clear whether there are any other solutions.

3. The further conditions implying locally direct sum for g and J pose strong restrictions on space-time surfaces. The conjecture that the solutions of these conditions correspond to preferred extremals of 6-D Kähler action leads by dimensional reduction to the conclusion that the 4-D action contains besides 4-D Kähler action also a volume term coming from S^2 Kähler actions and giving rise to cosmological constant.

   What is of special interest is that for the degenerate solution the volume term vanishes, and one has mere 4-D Kähler action with induced Kähler form possibly containing also $J(M^4)$, which leads to a rather sensible cosmology having interpretation as infinite volume limit for causal diamond (CD) inside which space-time surfaces exist. This limit could be appropriate for QFT limit of TGD, which indeed corresponds to infinite-volume limit at which cosmological constant approaches zero.

What could be the physical interpretation of the solutions?

1. Physical intuition suggests that S_1^2 must be space-like for Euclidian signature of space-time region $[(n_1, n_2) = (1, 0)]$ and time-like for Minkowskian signature $[(n_1, n_2) = (0, 1)]$.

2. By quantum classical correspondence one can argue that the non-vanishing of space-time projection of $J(M^4)$ resp. $J(CP_2)$ is necessary to fix local quantization axis of spin resp. weak isospin. If so, then $n_1 = 1/0$ resp. $n_2 = 1/0$ would tell that the projection of $J(CP_2)$ resp. $J(M^2)$ is non-vanishing/vanishes. If both contributions vanish $[(n_1, n_2) = (0, 0)]$ one has generalized Lagrangian 4-surface, which would be vacuum extremal. The products of 2-D Lagrangian manifolds for M^4 and CP_2 would be vacuum extremals. One can wonder whether there exist 4-surfaces representable as a graph of a map $M^4 \rightarrow CP_2$ such that the induced Kähler form vanishes. This picture allows only the embeddings of trivial Robertson-Walker cosmology as vacuum extremal of Kähler action since both M^4 contribution to Kähler action and volume term would be non-vanishing $[(n_1, n_2) = (0, 1)]$.

5.2.4 Twistor lift and the reduction of field equations and SH to holomorphy

It has become clear that twistorialization has very nice physical consequences. But what is the deep mathematical reason for twistorialization? Understanding this might allow to gain new insights about construction of scattering amplitudes with space-time surface serving as analogs of twistor diatrams.

Penrose's original motivation for twistorilization was to reduce field equations for massless fields to holomorphy conditions for their lifts to the twistor bundle. Very roughly, one can say that the value of massless field in space-time is determined by the values of the twistor lift of the field over the twistor sphere and helicity of the massless modes reduces to cohomology and the values of conformal weights of the field mode so that the description applies to all spins.

I want to find the general solution of field equations associated with the Kähler action lifted to 6-D Kähler action. Also one would like to understand strong form of holography (SH). In TGD fields in space-time are are replaced with the embedding of space-time as 4-surface to H. Twistor lift imbeds the twistor space of the space-time surface as 6-surface into the product of twistor spaces of M^4 and CP_2. Following Penrose, these embeddings should be holomorphic in some sense.

Twistor lift $T(H)$ means that M^4 and CP_2 are replaced with their 6-D twistor spaces.

1. If S^2 for M^4 has 2 time-like dimensions one has 3+3 dimensions, and one can speak about hyper-complex variants of holomorphic functions with time-like and space-like coordinate paired for all three hypercomplex coordinates. For the Minkowskian regions of the space-time surface X^4 the situation is the same.

2. For $T(CP_2)$ Euclidian signature of twistor sphere guarantees this and one has 3 complex coordinates corresponding to those of S^2 and CP_2. One can also now also pair two real coordinates of S^2 with two coordinates of CP_2 to get two complex coordinates. For the Euclidian regions of the space-time surface the situation is the same.

Consider now what the general solution could look like. Let us continue to use the shorthand notations $S_1^2 = S^2(X^4); S_2^2 = S^2(CP_2); S_3^2 = S^2(M^4)$.

1. Consider first solution of type $(1,0)$ so that coordinates of S^2_2 are constant. One has holomorphy in hypercomplex sense (light-like coordinate $t-z$ and $t+z$ correspond to hypercomplex coordinates).

   (a) The general map $T(X^4)$ to $T(M^4)$ should be holomorphic in hyper-complex sense. S^2_1 is in turn identified with S^2_3 by isometry realized in real coordinates. This could be also seen as holomorphy but with different imaginary unit. One has analytical continuation of the map $S^2_1 \rightarrow S^2_3$ to a holomorphic map. Holomorphy might allows to achieve this rather uniquely. The continued coordinates of S^2_1 correspond to the coordinates assignable with the integrable surface defined by $E^2(x)$ for local $M^2(x) \times E^2(x)$ decomposition of the local tangent space of X^4. Similar condition holds true for $T(M^4)$. This leaves only $M^2(x)$ as dynamical degrees of freedom. Therefore one has only one holomorphic function defined by 1-D data at the surface determined by the integrable distribution of $M^2(x)$ remains. The 1-D data could correspond to the boundary of the string world sheet.

   (b) The general map $T(X^4)$ to $T(CP_2)$ cannot satisfy holomorphy in hyper-complex sense. One can however provide the integrable distribution of $E^2(x)$ with complex structure and map it holomorphically to CP_2. The map is defined by 1-D data.

   (c) Altogether, 2-D data determine the map determining space-time surface. These two 1-D data correspond to 2-D data given at string world sheet: one would have SH.

2. What about solutions of type $(0,1)$ making sense in Euclidian region of space-time. One has ordinary holomorphy in CP_2 sector.

   (a) The simplest picture is a direct translation of that for Minkowskian regions. The map $S^2_1 \rightarrow S^2_2$ is an isometry regarded as an identification of real coordinates but could be also regarded as holomorphy with different imaginary unit. The real coordinates can be analytically continued to complex coordinates on both sides, and their imaginary parts define coordinates for a distribution of transversal Euclidian spaces $E^2_2(x)$ on X^4 side and $E^2(x)$ on M^4 side. This leaves 1-D data.

   (b) What about the map to $T(M^4)$? It is possible to map the integrable distribution $E^2_2(x)$ to the corresponding distribution for $T(M^4)$ holomorphically in the ordinary sense of the word. One has 1-D data. Altogether one has 2-D data and SH and partonic 2-surfaces could carry these data. One has SH again.

3. The above construction works also for the solutions of type $(1,1)$, which might make sense in Euclidian regions of space-time. It is however essential that the spheres S^2_2 and S^2_3 have real coordinates.

SH thus would thus emerge automatically from the twistor lift and holomorphy in the proposed sense.

1. Two possible complex units appear in the process. This suggests a connection with quaternion analytic functions [L11] suggested as an alternative manner to solve the field equations. Space-time surface as associative (quaterionic) or co-associate (co-quaternionic) surface is a further solution ansatz.

   Also the integrable decompositions $M^2(x) \times E^2(x)$ resp. $E^2_1(x) \times E^2_2(x)$ for Minkowskian resp. Euclidian space-time regions are highly suggestive and would correspond to a foliation by string wold sheets and partonic 2-surfaces. This expectation conforms with the number theoretically motivated conjectures [K84].

2. The foliation gives good hopes that the action indeed reduces to an effective action consisting of an area term plus topological magnetic flux term for a suitably chosen stringy 2-surfaces and partonic 2-surfaces. One should understand whether one must choose the string world sheets to be Lagrangian surfaces for the Kähler form including also M^4 term. Minimal surface

condition could select the Lagrangian string world sheet, which should also carry vanishing classical W fields in order that spinors modes can be eigenstates of em charge.

The points representing intersections of string world sheets with partonic 2-surfaces defining punctures would represent positions of fermions at partonic 2-surfaces at the boundaries of CD and these positions should be able to vary. Should one allow also non-Lagrangian string world sheets or does the space-time surface depend on the choice of the punctures carrying fermion number (quantum classical correspondence)?

3. The alternative option is that any choice produces of the preferred 2-surfaces produces the same scattering amplitudes. Does this mean that the string world sheet area is a constant for the foliation - perhaps too strong a condition - or could the topological flux term compensate for the change of the area?

   The selection of string world sheets and partonic 2-surfaces could indeed be also only a gauge choice. I have considered this option earlier and proposed that it reduces to a symmetry identifiable as U(1) gauge symmetry for Kähler function of WCW allowing addition to it of a real part of complex function of WCW complex coordinates to Kähler action. The additional term in the Kähler action would compensate for the change if string world sheet action in SH. For complex Kähler action it could mean the addition of the entire complex function.

A couple of questions remain to be pondered.

1. In TGD the induced spinor structure need not be equivalent with the ordinary spinor structure. For instance, induced gamma matrices are not covariantly constant and spinors are embedding space spinors. Induced spinor structure saves also from problems. Induced spinor structure exists even when standard twistor structure fails to do so. Induced spinor structure is also unique unlike the ordinary spinor structure. A practical example relates to the difficulty of the lattice QCD as thermodynamics with periodic boundary conditions in a box: there are $2^4 = 16$ spinor structures.

   In the same way, there is no need to expect or require that the induced twistor structure reduces to ordinary one: it is enough to require that the S^2 bundle structure implied by the proposed dimensional reduction of 6-D surfaces to S^2 bundles having space-time surface as a base space takes place. This would simplify the construction in an essential manner.

2. Space-time surface can be identified as a section of twistor bundle. For physical reasons this section should not only exist but be global and unique. For general bundles this need not be the case. For non-trivial principal bundles one cannot find any sections. The tangent bundle of sphere does not allow a global everywhere non-vanishing section. Could some additional condition guarantee that the section exists and is unique? In algebraic geometry additional conditions such as holomorphy can fix the global section highly uniquely.

   Now the variational principle reducing the construction to finding of space-time surfaces as an extremal of dimensionally reduced Kähler action guarantees both the existence and uniqueness. This also gives the reason why for the twistor lift of Kähler action: one cannot only assume that the 6-surface equals to ordinary twistor bundle of some 4-surface since in this case the section need not be unique.

5.2.5 What about 2-D objects and fermions?

TGD involves also 2-D objects - partonic 2-surfaces and string world sheets in an essential manner and strong form of holography (SH) states that these objects carry the information about quantum states. This does not mean that the dynamics would reduce to that for string like objects since it is essential that these objects are sub-manifolds of space-time surface. String world sheets carry induced spinor fields and it seems that these are crucial for understanding elementary particles. There are several questions to be answered.

1. Are fermionic fields localized to 2-surfaces? The generalization superconformal symmetry fixing both the bosonic and fermion parts of the action requires that also the interior of space-time carries induced spinor field. Their interpretation is not quite clear: could they

perhaps give rise to an additional supersymmetry induced by addition of interior fermions to the state?

The condition of super-symmetry at the level of action fixes the analog of massless Dirac action uniquely for both string world sheets, partonic 2-surfaces in the interior of causal-diamond (CD), and for the interior of space-time surface. There is an infinite number of conserved super currents associated with the modes of the modified Dirac operator defining fermionic super generators. This leads to quantum classical correspondence stating that the eigenvalues of Cartan generators for the fermionic representations of Noether charges are equal to corresponding classical Noether charges defined by the space-time dynamics.

2. A long-standing question has been whether stringlike objects and partonic 2-surfaces are fundamental dynamical objects or whether they emerge only at the level of effective action. M^8-H duality [L20] suggests answer to this question.

   M^8-H duality states that space-time surfaces M^8 picture are associative in the sense that either tangent or normal space of space-time surface at any point is associative and therefore quaternionic. Number theoretic vision suggests that also 2-D objects are fundamental. Commutative sub-manifolds of space-time surfaces having induced quaternionic structure reducing to commutative (complex) structure are number theoretically very natural. Either the tangent space or normal space of 2-surface can be commutative and this gives rise to string world sheets and partonic 2-surfaces as duals of each other just as space-time surfaces have regions for which either tangent spaces or normal spaces are associative (these correspond to regions of space-time with Minkowskian *resp.* Euclidian signatures of the induced metric).

   Note that the reduction of the theory to mere string theory is not possible since partonic 2-surfaces have commutative normal space (partonic 2-surfaces) as part of the tangent space of space-time surface.

3. What action one should assign with the 2-D objects? The action should be assigned to string world sheets and partonic 2-surfaces representing vertices but the assignent of action with partonic 2-surfaces at the ends of CD does not look natural since they are in the role of initial values. The naïve first guess for the action is as area action. Fermionic action would be fixed uniquely in terms of modified gamma matrices reducing to induced gamma matrices.

   Also space-time surfaces in the simplest scenario are minimal surfaces except for a discrete set of singular points at which there is energy transfer between Kähler action and volume term. Something similar should occur also in 2-D case: there must also second part in the action and transfer of Noether changes between the two parts in this set of points.

   The singular points have an identification as point-like particles carrying fermion number and located at partonic 2-surfaces at boundaries of causal diamond (CD) or defining topological vertices so that a classical space-time correlates for twistor diagrams emerge.

   Since particles in twistor approaches are associated with the ends of string boundaries at the ends of light-like orbits of partonic 2-surfaces at boundaries of causal diamond (CD), the exceptional points for both space-time surface and string world sheets would correspond to the intersections of string world sheets and partonic 2-surfaces defining also topological vertices.

Twistor lift provides a first principle approach to the action assignable to the 2-D surfaces.

1. The simplest possibility is that one has also now a Kähler action but now for 4-D space-time surface in the product of twistor spaces of M^4 and CP_2 dimensionally reduced to Cartesian product of twistor sphere S^2 and 2-D surface. The assignment of action to partonic 2-surface at the boundary of CD does not look feasible. 4-D Kähler action would be dimensionally reduced to 2-D form and area term.

2. Field equations contain two terms coming from the variation with respect to the induced metric and Kähler form respectively. The terms coming from the variation with respect to the metric vanish for minimal surfaces since energy momentum tensor is proportional to the induced metric. The term coming from the variation with respect to the induced Kähler form need not vanish for minimal surfaces unless there are additional conditions.

The term is of the same form as in 4-D case, which case this term vanishes for holomorphic solutions and also for all known extremals. There are excellent reasons to expect that this is true also in 2-D case. It therefore seems that minimal surfaces are in question except for a discrete set of points as in 4-D case: this conforms with universality forced by quantum criticality stating that Kähler coupling constant disappears from dynamics except in this discrete set of points.

In accordance with SH, this set of points at which the minimal surface property fails would define also the corresponding points for space-time surface itself. This singularity could mean breakdown of holomorphy, perhaps analogs of poles for analytic functions are in question. One cannot exclude the possibility that the boundaries of string world sheets defining orbits of fundamental fermions are analogous to cuts for holomorphic functions.

3. One might guess that 2-D minimal surfaces in space-time are also minimal surfaces in embedding space since the induction from space-time surface to 2-surface can be also thought of as an induction from embedding space. The variations for minimal surfaces inside space-time surface are more restricted so that this need not be the case. For holomorphic solutions the situation might change. SH in strongest form would therefore suggest that space-time as 4-D surface is determined by fixing the 2-D minimal surfaces in H and finding space-time surface containing them. A weaker condition would force to fix also the normal space of the minimal surface in space-time.

   This space-time surface need not always exist, and one of the key ideas about cognition [K52] is that in p-adic case the possibility of p-adic pseudo-constants allows the existence of p-adic space-time surfaces always but that in real case this is not always the case: what is imaginable is not necessarily realizable.

   At the level of M^8 the condition that the coefficients of a polynomial determining the space-time surface are in a fixed extension of rationals is very powerful requirement and might prevent SH. As a matter fact, SH becomes at the level of M^8 even stronger: discrete set of points naturally identifiable as the set of singular points and thus as poles and zeros of analytic function would determine the space-time surface. If fermion lines correspond to cuts, this super-strong form of SH would weaken. For polynomials considered in [L20] cuts are however not possible and they should be generated in the map from H to $M^4 \times CP_2$ for by allowing analytic functions instead of polynomials: this is quite possible in which case polynomials could define a a hierarchy of resolutions.

5.3 How does the twistorialization at embedding space level emerge?

An objection against twistorialization at embedding space level is that M^4-twistorialization requires 4-D conformal invariance and massless fields. In TGD one has towers of particle with massless particles as the lightest states. The intuitive expectation is that the resolution of the problem is that particles are massless in 8-D sense as also the modes of the embedding space spinor fields are.

To explain the idea, let us select a fixed decomposition $M^8 = M_0^4 \times E_0^4$ and assume that the momenta are complex - for motivations see below.

1. With inspiration coming from M^8-H duality [K74] suppose that for the allowed compositions $M^8 = M^4 \times E^4$ one has $M^4 = M_0^2 \times E^2$ with M_0^2 fixed, and corresponding to real octonionic unit and preferred imaginary unit. Obviously 8-D light-likeness for $M^8 = M_0^4 \times E_0^4$ reduces to 4-D light-likeness for a preferred choice of $M^8 = M^4 \times CP_2$ decomposition.

2. This suggests that in the case of massive M_0^4 momenta one can apply twistorialization to the light-like M^4-momentum and code the information about preferred M^4 by a point of CP_2 and about 8-momentum in $M^8 = M_0^4 \times E_0^4$ by an $SU(3)$ transformation taking M_0^4 to M^4. Pairs of twistors and $SU(3)$ transformations would characterize arbitrary quaternionic 8-momenta. 8-D masslessness gives however 2 additional conditions for the complex 8-momenta probably reducing $SU(3)$ to $SU(3)/U(1) \times U(1)$ - the twistor space of CP_2! This would also solve the basic problem of twistor approach created by the existence of massive particles.

The assumption of complex momenta in previous considerations might raise some worries. The space-time action of TGD is however complex if Kähler coupling strength is complex, and there are reasons to believe that this is the case. Both four-momenta and color quantum numbers - all Noether charges in fact - could be complex. A possible physical interpretation for complex momenta could be in terms of the natural width of states induced by the finite size of CD. Also in twistor Grassmannian approach one encounters complex but light-like four-momenta. Note that complex light-like space-time momenta correspond in general to massive real momenta. It is not clear whether it makes sense to speak about width of color quantum numbers: their reality would give additional constraint. The emergence of M^4 mass in this manner could be involved with the classical description for the emergence of the third helicity.

The observation that octonionic twistors make sense and their restriction to quaternionic twistors produce ordinary M^4 twistors provides an alternative view point to the problem. Also $M^8 - H$ duality proposed to map quaternionic 4-D surfaces in octonionic M^8 to (possibly quaternionic) 4-D surfaces in $M^4 \times CP_2$ is expected to be relevant. The twistor lift of $M^8 - H$ duality would give $T(M^8) - T(H)$ duality.

Twistor Grassmann approach [B16, B11, B9, B20, B22, B6] uses as twistor space the space $T_1(M^4) = SU(2,2)/SU(2,1) \times U(1)$ whereas the twistor lift of classical TGD uses $M^4 \times S^2$. The formulation of the twistor amplitudes in terms of SH using the data assignable to the 2-D surfaces - string world sheets and partonic 2-surfaces perhaps - identified as surfaces in $T(M^4) \times T(CP_2)$ requires the mapping of these twistor spaces to each other - the incidence relations of Penrose indeed realize this map.

5.3.1 $M^8 - H$ duality at space-time level

Twistors emerge as a description of massless particles with spin [B41] but are not needed for spin zero particles. Therefore one can consider first mere momenta.

1. Consider first space-time surfaces of M^8 with Minkowskian signature of the induced metric so that the tangent space is M^4. $M^8 - H$ duality [K74] implies that CP_2 points parameterize quaternionic sub-spaces M^4 of octonions containing fixed $M_0^2 \subset M^4$. Using the decomposition $1+1+3+\overline{3}$ of complexified octonions to representations of $SU(3)$, it is easy to see that this space is indeed CP_2. M^4 correspond to the sub-space $1+1+2$ where 2 is $SU(2) \subset SU(3)$ doublet.

   CP_2 spinor mode would be spinor mode in the space of quaternionic sub-spaces $M^4 \subset M^8$ with $M_0^2 \subset M^4$ with real octonionic unit defining preferred time like direction and imaginary unit defining preferred spin quantization axis. $M^8 - H$ duality allows to map quaternionic 4-surfaces of $M^4 \supset M_0^2$ to 4-surfaces in H. The latter could be quaternionic but need not to.

2. For Euclidian signature of the induced metric tangent space is E^4. In this case co-associative surfaces are needed since the above correspondence make sense only if the tangent space corresponds to M^4. For instance, for CP_2 type exremals tangent space corresponds to E^4. M^4 and E^4 change roles. Also now the space of co-associative tangent spaces is CP_2 since co-associative tangent space is the octonionic orthogonal complement of the associative tangent space. One would have Euclidian variant of the associative case.

$M^8 - H$ correspondence raises the question whether the octonionic M^8 or $M^4 \times CP_2$ represents the level, which deserves to be called fundamental. Or are they just alternative descriptions made possible by the quaternionicity of space-time surface in M^8 and quaternionic momentum space necessitating quaternionicity of the tangent space of X^4? In any case, one should demonstrate that the spectrum of states with $M^4 \times E^4$ with quaternionic light-like 8-momenta is equivalent with the spectrum of states for $M^4 \times CP_2$

5.3.2 Parametrization of light-like quaternionic 8-momenta in terms of $T(CP_2)$

The following argument shows that the twistor space $T(CP_2)$ emerges naturally from $M^8 - H$ correspondence for quaternionic light-like M^8 momenta.

1. Continue to assume a fixed decomposition $M^8 = M_0^4 \times E_0^4$, and that for the allowed compositions $M^8 = M^4 \times E^4$ one has $M^4 = M_0^2 \times E^2$ with M_0^2 fixed. Light-like quaternionic 8-momentum in $M^8 = M_0^4 \times E_0^4$ can be reduced to light-like M^4 momentum and vanishing E^4 momentum for some preferred $M^8 = M^4 \times E^4$ decomposition.

   One can therefore describe the situation in terms of light-like M^4-momentum and $U(2)$ transformation (as it turns out) mapping this momentum to 8-D momentum in given frame and giving the M_0^4 and E_0^4 momenta. The alternative description is in terms M_0^4 massive momentum and the E_0^4 momentum. The space of light-like complex M^4 momenta with fixed M_0^2 part and non-vanishing E^2 part is given by CP_2 as also the space of quaternionic planes. Given quaternionic plane is in turn characterized by massless M^4-momentum.

2. The description of M^4-massive momentum should be based on twistor associated with the light-like M^4 momentum plus something describing the $SU(3)$ transformation leaving the preferred imaginary unit of M_0^2 un-affected. The transformations leaving unaffected the M^4 part of M^8-momentum coded by the $SU(2)$ doublet 2 of color triplet 3 in the color decomposition of complex 8-momentum $1+1+3+\overline{3}$ but acting on E^4 part $1+\overline{3}$ non-trivially correspond to $U(2)$ subgroup. $U(2)$ element thus codes for the E^4 part of the light-like momentum and $SU(3)$ code for quaternionic 8-momenta, which can be also massive. Massless and complex M^4 momenta are coded by $SU(3)/U(2) = CP_2$ as also the tangent spaces of Minkowskian space-time regions (by $M^8 - H$ duality).

   The complexity of particle 8-momenta -and more generally Noether charges - is not in conflict with the hermiticity of quantal Noether charges if total classical and quantal Noether charges are real (and equal by QCC). This would give rise to a kind of confinement condition applying to many-particle states. I have earlier proposed that single particle conformal weights are complex but that conformal confinement holds in the sense that the total conformal weights are real.

3. General complex quaternionic momenta with fixed M^4 part are parameterized by $SU(3)$. Complex light-like 8-momenta satisfy two additional constraints from light-likenes condition, and one expects the reduction of $SU(3)$ to $SU(3)/U(1) \times U(1)$ - the twistor space of CP_2. Therefore the light-like 8-momentum is coded by a twistor assignable to massless M^4-momentum by an point of $SU(3)/U(1) \times U(1)$ giving $T(M^4) \times T(CP_2)$.

By the previous arguments, the inclusion of helicities and electroweak charges gives twistor lift of $M^8 - H$ correspondence.

1. In the case of E^4 the helicities would correspond to two $SO(4)$ spins to be mapped to right and left-handed electroweak spins or weak spin and weak charges. Twistor space $T(CP_2)$ gives hopes about a unified description of color - and electro-weak quantum numbers in terms of partial waves in the space $SU(3)/U(1) \times U(1)$ for selections of quantization axes for color quantum numbers.

2. A possible problem relates to the particles massive in M^4 sense having more helicity states than massless particles. How can one describe the presence of additional helicities. Should one introduce the analog of Higgs mechanism providing the missing massless helicities? Quantum view about twistors describes helicity as a quantum number - conformal weight - of a wave function in the twistor sphere S^2. In the case of massive gauge bosons which would require the introduction of zero helicity as a spin 0 wave function in twistor space.

3. One should relate the description in terms of M^8 momenta to the description in terms of $M^4 \times CP_2$ color partial waves massless in 8-D sense. The number of partial waves for given CP_2 mass squared is finite and this should be the case for quaternionic E^4 momenta. How color quantum numbers determining the M^4 mass relate to complex E^4 momenta parameterized by $U(2)$ plus two constraints coming from complex light-likeness. The number of degrees of freedom is 2 for given $U(2)$ orbit and the quantization suggests dramatic reduction in the number of 8-momenta. This strongly suggests that it is only possible to talk about wave functions in the space of allowed E^4 momenta - that is in the twistor space $T(CP_2)$. Fixing the M^4-part of 8-momentum parameterized by a point of CP_2 leaves only a wave function in the fiber S^2.

The discussion leaves some questions to ponder.

1. M^8-H correspondence raises the question whether the octonionic M^8 or $M^4 \times CP_2$ represents the fundamental level. Or are they just alternative descriptions made possible by the quaternionicity of space-time surface in M^8 and quaternionic momentum space necessitating quaternionicity of the tangent space of X^4?

2. What about more general $SO(1,7)$ transformations? Are they needed? One could consider the possibility that $SO(1,7)$ acts in the moduli space of octonion structures of M^8. If so, then these additional moduli must be included. Otherwise given 8-D momenta have M_0^2 part fixed and orbit of given M^4 momentum is the smaller, the smaller the E^2 part of M^4 momentum is. It reduces to point if M^4 momentum reduces to M_0^2.

5.3.3 A new view about color, color confinement, and twistors

To my humble opinion twistor approach to the scattering amplitudes is plagued by some mathematical problems. Whether this is only my personal problem is not clear.

1. As Witten shows in [B16], the twistor transform is problematic in signature (1,3) for Minkowski space since the the bi-spinor μ playing the role of momentum is complex. Instead of defining the twistor transform as ordinary Fourier integral, one must define it as a residue integral. In signature (2,2) for space-time the problem disappears since the spinors μ can be taken to be real.

2. The twistor Grassmannian approach works also nicely for (2,2) signature, and one ends up with the notion of positive Grassmannians. Could it be that something is wrong with the ordinary view about twistorialization rather than only my understanding of it?

3. For M^4 the twistor space should be non-compact $SU(2,2)/SU(2,1) \times U(1)$ rather than $CP_3 = SU(4)/SU(3) \times U(1)$, which is taken to be. I do not know whether this is only about short-hand notation or a signal about a deeper problem.

4. Twistorilizations does not force SUSY but strongly suggests it. The super-space formalism allows to treat all helicities at the same time and this is very elegant. This however forces Majorana spinors in M^4 and breaks fermion number conservation in $D=4$. LHC does not support $\mathcal{N}=1$ SUSY. Could the interpretation of SUSY be somehow wrong? TGD seems to allow broken SUSY but with separate conservation of baryon and lepton numbers.

In number theoretic vision something rather unexpected emerges and I will propose that this unexpected might allow to solve the above problems and even more, to understand color and even color confinement number theoretically. First of all, a new view about color degrees of freedom emerges at the level of M^8.

1. One can always find a decomposition $M^8 = M_0^2 \times E^6$ so that the possibly complex light-like quaternionic 8-momentum restricts to M_0^2. The preferred octonionic imaginary unit represent the direction of imaginary part of quaternionic 8-momentum. The action of G_2 to this momentum is trivial. Number theoretic color disappears with this choice. For instance, this could take place for hadron but not for partons which have transversal momenta.

2. One can consider also the situation in which one has localized the 8-momenta only to $M^4 = M_0^2 \times E^2$. The distribution for the choices of $E^2 \subset M_0^2 \times E^2 = M^4$ is a wave function in CP_2. Octonionic $SU(3)$ partial waves in the space CP_2 for the choices for $M_0^2 \times E^2$ would correspond ot color partial waves in H. The same interpretation is also behind M^8-H correspondence.

3. The transversal quaternionic light-like momenta in $E^2 \subset M_0^2 \times E^2$ give rise to a wave function in transversal momenta. Intriguingly, the partons in the quark model of hadrons have only precisely defined longitudinal momenta and only the size scale of transversal momenta can be specified. This would of course be a profound and completely unexpected connection! The introduction of twistor sphere of $T(CP_2)$ allows to describe electroweak charges and brings in

CP_2 helicity identifiable as em charge giving to the mass squared a contribution proportional to Q_{em}^2 so that one could understand electromagnetic mass splitting geometrically.

The physically motivated assumption is that string world sheets at which the data determining the modes of induced spinor fields carry vanishing W fields and also vanishing generalized Kähler form $J(M^4) + J(CP_2)$. Em charge is the only remaining electroweak degree of freedom. The identification as the helicity assignable to $T(CP_2)$ twistor sphere is natural.

4. In general case the M^2 component of momentum would be massive and mass would be equal to the mass assignable to the E^6 degrees of freedom. One can however always find $M_0^2 \times E^6$ decomposition in which M^2 momentum is light-like. The naïve expectation is that the twistorialization in terms of M^2 works only if M^2 momentum is light-like, possibly in complex sense. This however allows only forward scattering: this is true for complex M^2 momenta and even in M^4 case.

   The twistorial 4-fermion scattering amplitude is however *holomorphic* in the helicity spinors λ_i and has no dependence on $\tilde{\lambda}_i$. Therefore carries no information about M^2 mass! Could M^2 momenta be allowed to be massive? If so, twistorialization might make sense for massive fermions!

M_0^2 momentum deserves a separate discussion.

1. A sharp localization of 8-momentum to M_0^2 means vanishing E^2 momentum so that the action of $U(2)$ would becomes trivial: electroweak degree of freedom would simply disappear, which is not the same thing as having vanishing em charge (wave function in $T(CP_2)$ twistorial sphere S^2 would be constant). Neither M_0^2 localization nor localization to single M^4 (localization in CP_2) looks plausible physically - consider only the size scale of CP_2. For the generic CP_2 spinors this is impossible but covariantly constant right-handed neutrino spinor mode has no electro-weak quantum numbers: this would most naturally mean constant wave function in CP_2 twistorial sphere.

   For the preferred extremals of twistor lift of TGD either M^4 or CP_2 twistor sphere can effectively collapse to a point. This would mean disappearence of the degrees of freedom associated with M^4 helicity or electroweak quantum numbers.

2. The localization to $M^4 \supset M_0^2$ is possible for the tangent space of quaternionic space-time surface in M^8. This could correlate with the fact that neither leptonic nor quark-like induced spinors carry color as a spin like quantum number. Color would emerge only at the level of H and M^8 as color partial waves in WCW and would require de-localization in the CP_2 cm coordinate for partonic 2-surface. Note that also the integrable local decompositions $M^4 = M^2(x) \times E^2(x)$ suggested by the general solution ansätze for field equations are possible.

3. Could it be possible to perform a measurement localization the state precisely in fixed M_0^2 always so that the complex momentum is light-like but color degrees of freedom disappear? This does not mean that the state corresponds to color singlet wave function! Can one say that the measurement eliminating color degrees of freedom corresponds to color confinement. Note that the subsystems of the system need not be color singlets since their momenta need not be complex massless momenta in M_0^2. Classically this makes sense in many-sheeted space-time. Colored states would be always partons in color singlet state.

4. At the level of H also leptons carry color partial waves neutralized by Kac-Moody generators, and I have proposed that the pion like bound states of color octet excitations of leptons explain so called lepto-hadrons [K80]. Only right-handed covariantly constant neutrino is an exception as the only color singlet fermionic state carrying vanishing 4-momentum and living in all possible M_0^2:s, and might have a special role as a generator of supersymmetry acting on states in all quaternionic subs-spaces M^4.

5. Actually, already p-adic mass calculations performed for more than two decades ago [K42, K17, K51], forced to seriously consider the possibility that particle momenta correspond to their projections o $M_0^2 \subset M^4$. This choice does not break Poincare invariance if one

introduces moduli space for the choices of $M_0^2 \subset M^4$ and the selection of M_0^2 could define quantization axis of energy and spin. If the tips of CD are fixed, they define a preferred time direction assignable to preferred octonionic real unit and the moduli space is just S^2. The analog of twistor space at space-time level could be understood as $T(M^4) = M^4 \times S^2$ and this one must assume since otherwise the induction of metric does not make sense.

What happens to the twistorialization at the level of M^8 if one accepts that only M_0^2 momentum is sharply defined?

1. What happens to the conformal group $SO(4,2)$ and its covering $SU(2,2)$ when M^4 is replaced with $M_0^2 \subset M^8$? Translations and special conformational transformation span both 2 dimensions, boosts and scalings define 1-D groups $SO(1,1)$ and R respectively. Clearly, the group is 6-D group $SO(2,2)$ as one might have guessed. Is this the conformal group acting at the level of M^8 so that conformal symmetry would be broken? One can of course ask whether the 2-D conformal symmetry extends to conformal symmetries characterized by hyper-complex Virasoro algebra.

2. Sigma matrices are by 2-dimensionality real (σ_0 and σ_3 - essentially representations of real and imaginary octonionic units) so that spinors can be chosen to be real. Reality is also crucial in signature $(2,2)$, where standard twistor approach works nicely and leads to 3-D real twistor space.

   Now the twistor space is replaced with the real variant of $SU(2,2)/SU(2,1) \times U(1)$ equal to $SO(2,2)/SO(2,1)$, which is 3-D projective space RP^3 - the real variant of twistor space CP_3, which leads to the notion of positive Grassmannian: whether the complex Grassmannian really allows the analog of positivity is not clear to me. For complex momenta predicted by TGD one can consider the complexification of this space to CP_3 rather than $SU(2,2)/SU(2,1) \times U(1)$. For some reason the possible problems associated with the signature of $SU(2,2)/SU(2,1) \times U(1)$ are not discussed in literature and people talk always about CP_3. Is there a real problem or is this indeed something totally trivial?

3. SUSY is strongly suggested by the twistorial approach. The problem is that this requires Majorana spinors leading to a loss of fermion number conservation. If one has $D = 2$ only effectively, the situation changes. Since spinors in M^2 can be chosen to be real, one can have SUSY in this sense without loss of fermion number conservation! As proposed earlier, covariantly constant right-handed neutrino modes could generate the SUSY but it could be also possible to have SUSY generated by all fermionic helicity states. This SUSY would be however broken.

   There is an delicacy involved. If $J(M^4)$ is present, the action of the gauge commutator $[D_k, D_l] = J_{kl}(M^4)$ on right-handed neutrino is non-vanishing and gives rise to the constant term $J^{kl}(M^4)\Sigma_{kl}$ appearing in the square of Dirac equation at embedding space level. Neutrino would become massive at embedding space level and also other states receive an additional contribution to mass squared. String world sheets can be however analogs of Lagrangian sub-manifolds so that $J(M^4)$ projected to them vanishes, and one can have massless right-handed neutrino. Also the right- or left M^4-handedness of operator $J^{kl}(M^4)\Sigma_{kl}$ makes it possible to annihilate the spinor mode at string world sheet. The physical interpretation of this picture is still unclear.

4. The selection of M_0^2 could correspond at space-time level to a localization of spinor modes to string world sheets. Could the condition that the modes of induced spinors at string world sheets are expressible using real spinor basis imply the localization? Whether this localization takes place at fundamental level or only for effective action being due to SH, is a question to be settled. The latter options looks more plausible.

To sum up, these observation suggest a profound re-evaluqtion of the beliefs related to color degrees of freedom, to color confinement, and to what twistors really are.

5.3.4 How do the two twistor spaces assignable to M^4 relate to each other?

Twistor Grassmann approach [B16, B11, B9, B20, B22, B6] uses as twistor space the space $T_1(M^4) = SU(2,2)/SU(2,1) \times U(1)$. Twistor lift of classical TGD uses $M^4 \times S^2$: this seems to be necessary since $T_1(M^4)$ does not allow M^4 as space-space. The formulation of the twistor amplitudes in terms of SH using the data assignable to the 2-D surfaces - string world sheets and partonic 2-surfaces perhaps - identifed as surfaces in $T(M^4) \times T(CP_2)$ is an attractive idea suggesting a very close correspondence with twistor string theory of Witten and construction of scattering amplitudes in twistor Grassmann approach.

One should be able to relate these two twistor spaces and map the twistor spaces $T(X^4)$ identified as surfaces in $T(H) = T(M^4) \times T(CP_2)$ to those in $T_1(H) = T_1(M^4) \times T(CP_2)$. This map is strongly suggested also by twistor string theory. This map raises hopes about the analogs of twistor Grassmann amplitudes based on introduction of $T(CP_2)$.

At least the projections of 2-surfaces to $T(M^4)$ should be mappable to those in $T_1(M^4)$. A stronger condition is that $T(M^4)$ is mappable to $T_1(M^4)$. Incidence relations for twistors $Z = (\lambda, \mu)$ assigning to given M^4 coordinates twistor sphere, are given by

$$\mu_{\dot{\alpha}} = m_{\alpha\dot{\alpha}} \lambda^{\alpha} \quad .$$

This condition determines a 2-D sub-space - complex light ray - of complexified Minkowski space M^4_c. Also complex scaling of Z determines the same sub-space. Therefore twistor sphere corresponds to a complex light ray M^4_c, whose points differ by a shift by a complex light-like vector (λ is null bi-spinor annihilated by light-like m).

Since twistor line (projective sphere) determines a point of M^4_c, two points of twistor sphere labelled by A and B are needed to determined m:

$$m_{\alpha\dot{\alpha}} = \frac{\lambda_{A,\alpha}\mu_{B,\dot{\alpha}}}{\langle\lambda_A\lambda_B\rangle\rangle} + \frac{\lambda_{B,\alpha}\mu_{A,\dot{\alpha}}}{\langle\lambda_B\lambda_A\rangle} \quad .$$

The solutions are invariant under complex scalings $(\lambda, \mu) \to k(\lambda, \mu)$. Therefore coincidence relations allow to assign projective line - sphere S^2 - to a point of M^4 in $T(M^4)$. This sphere naturally corresponds to S^2 in $T(M^4) = M^4 \times S^2$. This allows to assign pairs $(m \times S^2)$ in $T(M^4)$ to spheres of $T_1(M^4)$ and one can map the projections of 2-surfaces to $T(M^4)$ to $T_1(M^4)$.

Thus one cannot assign M^4 point to single twistor but can map any pair of points at twistor sphere of $T_1(M^4)$ to the same point of M^4 in $T(M^4) = M^4 \times S^2$ and also identify the twistor sphere with S^2. Twistor spheres are labelled by the base space of $T_1(M^4)$ and therefore base space can be mapped to M^4.

Two M^4 points separated by light-like distance correspond to twistor spheres intersecting at one point as is clear from the fact that the difference $m_1 - m_2$ of the points annihilates the twistor λ. $T_1(M^4)$ is singular as fiber bundle over M^4 since the same point of fiber is projected to two different points of M^4.

Could one replace $T(M^4)$ with $T_1(M^4)$ by modifying the induction procedure suitable?

1. $T_1(M^4) = SU(2,2)/SU(2,1) \times U(1)$ has $SU(2,2)$ invariant metric and $SU(2,2)$ corresponds to the 15-D spin covering group of $SO(4,2)$ having $SO(3,1)$ as sub-group. What does one obtain if one induces the metric of the base space of $T_1(M^4)$ to M^4 via the above identification?

   The induced metric would depend on the choice of the base space, and one would have analog of gauge invariance since for a given point of the base the point of the fiber sphere can be chosen freely. A reasonable guess is that the induced metric is determined apart from conformal scaling. One could fix the gauge by - say - assuming that the S^2 point is constant but it is not clear whether this allows to get the flat M^4 metric with any choice.

2. If the twistor sphere of $T_1(M^4)$ has radius of order Planck length l_P, the overall scaling factor of the metric of $T_1(M^4)$ is of order l_P^2. Also the induced M^4 metric would have this scaling factor. For $T_1(M^4)$ one could not perform this scaling. This need not be a problem in $T(M^4)$ since one scale up the flat metric of M^4 by scaling the coordinates. This kind of

scaling would in fact smooth out the possible deviations from flat M^4 metric very effectively. In any case, it seems that one must assume that embedding space corresponds to $T(M^4)$.

5.3.5 How could Planck length be actually equal to much larger CP_2 radius?!

The following argument stating that Planck length l_P equals to CP_2 radius R: $l_P = R$ and Newton's constant can be identified $G = R^2/\hbar_{eff}$. This idea looking non-sensical at first glance was inspired by an FB discussion with Stephen Paul King.

First some background.

1. I believed for long time that Planck length l_P would be CP$_2$ length scale R squared multiplied by a numerical constant of order $10^{-3.5}$. Quantum criticality would have fixed the value of l_P and therefore $G = l_P^2/\hbar$.

2. Twistor lift of TGD [L3, L12, L24, L35] led to the conclusion that that Planck length l_P is essentially the radius of twistor sphere of M^4 so that in TGD the situation seemed to be settled since l_P would be purely geometric parameter rather than genuine coupling constant. But it is not! One should be able to understand why the ratio l_P/R but here quantum criticality, which should determine only the values of genuine coupling parameters, does not seem to help.

   Remark: M^4 has twistor space as the usual conformal sense with metric determined only apart from a conformal factor and in geometric sense as $M^4 \times S^2$: these two twistor spaces are part of double fibering.

Could CP_2 radius R be the radius of M^4 twistor sphere, and could one say that Planck length l_P is actually equal to R: $l_P = R$? One might get $G = l_P^2/\hbar$ from $G = R^2/\hbar_{eff}$!

1. It is indeed important to notice that one has $G = l_P^2/\hbar$. $\hbar$ is in TGD replaced with a spectrum of $\hbar_{eff} = n\hbar_0$, where $\hbar = 6\hbar_0$ is a good guess [L13, L29]. At flux tubes mediating gravitational interactions one has

$$\hbar_{eff} = \hbar_{gr} = \frac{GMm}{v_0} \ ,$$

   where v_0 is a parameter with dimensions of velocity. I recently proposed a concrete physical interpretation for v_0 [L27] (see `http://tinyurl.com/yclefxb2`). The value $v_0 = 2^{-12}$ is suggestive on basis of the proposed applications but the parameter can in principle depend on the system considered.

2. Could one consider the possibility that twistor sphere radius for M^4 has CP_2 radius R: $l_P = R$ after all? This would allow to circumvent introduction of Planck length as new fundamental length and would mean a partial return to the original picture. One would $l_P = R$ and $G = R^2/\hbar_{eff}$. $\hbar_{eff}/\hbar$ would be of $10^7 - 10^8$!

The problem is that $\hbar_{eff}$ varies in large limits so that also G would vary. This does not seem to make sense at all. Or does it?!

To get some perspective, consider first the phase transition replacing $\hbar$ and more generally $\hbar_{eff,i}$ with $\hbar_{eff,f} = h_{gr}$.

1. Fine structure constant is what matters in electrodynamics. For a pair of interacting systems with charges Z_1 and Z_2 one has coupling strength $Z_1Z_2e^2/4\pi\hbar = Z_1Z_2\alpha$, $\alpha \simeq 1/137$.

2. As shown in [K70, K57, K58, K21, K22, K23, K24] one can also define gravitational fine structure constant α_{gr}. Only α_{gr} should matter in quantum gravitational scattering amplitudes. α_{gr} wold be given by

$$\alpha_{gr} = \frac{GMm}{4\pi\hbar_{gr}} = \frac{v_0}{4\pi} \ . \tag{5.3.1}$$

$v_0/4\pi$ would appear as a small expansion parameter in the scattering amplitudes. This in fact suggests that v_0 is analogous to α and a universal coupling constant which could however be subject to discrete number theoretic coupling constant evolution.

3. The proposed physical interpretation is that a phase transition $\hbar_{eff,i} \rightarrow \hbar_{eff,f} = h_{gr}$ at the flux tubes mediating gravitational interaction between M and m occurs if the perturbation series in $\alpha_{gr} = GMm/4\pi/\hbar$ fails to converge ($Mm \sim m_{Pl}^2$ is the naïve first guess for this value). Nature would be theoretician friendly and increase h_{eff} and reducing α_{gr} so that perturbation series converges again.

   Number theoretically this means the increase of algebraic complexity as the dimension $n = h_{eff}/h_0$ of the extension of rationals involved increases fron n_i to n_f [L20] and the number n sheets in the covering defined by space-time surfaces increases correspondingly. Also the scale of the sheets would increase by the ratio n_f/n_i.

   This phase transition can also occur for gauge interactions. For electromagnetism the criterion is that $Z_1Z_2\alpha$ is so large that perturbation theory fails. The replacement $\hbar \rightarrow Z_1Z_2e^2/v_0$ makes $v_0/4\pi$ the coupling constant strength. The phase transition could occur for atoms having $Z \geq 137$, which are indeed problematic for Dirac equation. For color interactions the criterion would mean that $v_0/4\pi$ becomes coupling strength of color interactions when α_s is above some critical value. Hadronization would naturally correspond to the emergence of this phase.

   One can raise interesting questions. Is v_0 (presumably depending on the extension of rationals) a completely universal coupling strength characterizing any quantum critical system independent of the interaction making it critical? Can for instance gravitation and electromagnetism are mediated by the same flux tubes? I have assumed that this is not the case. It it could be the case, one could have for $GMm < m_{Pl}^2$ a situation in which effective coupling strength is of form $(GmMm/Z_1Z_2e^2)(v_0/4\pi)$.

The possibility of the proposed phase transition has rather dramatic implications for both quantum and classical gravitation.

1. Consider first quantum gravitation. v_0 does not depend on the value of G at all! The dependence of G on $\hbar_{eff}$ could be therefore allowed and one could have $l_P = R$. At quantum level scattering amplitudes would not depend on G but on v_0. I was of course very happy after having found the small expansion parameter v_0 but did not realize the enormous importance of the independence on G! Quantum gravitation would be like any gauge interaction with dimensionless coupling, which is even small! This might relate closely to the speculated TGD counterpart of AdS/CFT duality between gauge theories and gravitational theories.

2. What about classical gravitation? Here G should appear. What could the proportionality of classical gravitational force on $1/\hbar_{eff}$ mean? The invariance of Newton's equation

$$\frac{d\overline{v}}{dt} = -\frac{GM\overline{r}}{r^3} \tag{5.3.2}$$

   under $h_{eff} \rightarrow xh_{eff}$ would be achieved by scaling $\overline{r} \rightarrow \overline{r}/x$ and $t \rightarrow t/x$. Note that these transformations have general coordinate invariant meaning as scalings of Minkowski coordinates of M^4 in $M^4 \times CP_2$. This scaling means the zooming up of size of space-time sheet by x, which indeed is expected to happen in $h_{eff} \rightarrow xh_{eff}$!

What is so intriguing that this connects to an old problem that I pondered a lot during the period 1980-1990 as I attempted to construct to the field equations for Kähler action approximate spherically symmetric stationary solutions [K82]. The naïve arguments based on the asymptotic behavior of the solution ansatz suggested that the one should have $G = R^2/\hbar$. For a long time indeed assumed $R = l_P$ but p-adic mass calculations [K42] and work with cosmic strings [K20] forced to conclude that this cannot be the case. The mystery was how $G = R^2/\hbar$ could be normalized to $G = l_P^2/\hbar$: the solution of the mystery is $\hbar \rightarrow \hbar_{eff}$ as I have now - decades later - realized!

5.3.6 Can the Kähler form of M^4 appear in Kähler action?

I have already earlier considered the question whether the analog of Kähler form assignable to M^4 could appear in Kähler action. Could one replace the induced Kähler form $J(CP_2)$ with the sum $J = J(M^4) + J(CP_2)$ such that the latter term would give rise to a new component of Kähler form both in space-time interior at the boundaries of string world sheets regarded as point-like particles? This could be done both in the Kähler action for the interior of X^4 and also in the topological magnetic flux term $\int J$ associated with string world sheet and reducing to a boundary term giving couplings to U(1) gauge potentials $A_\mu(CP_2)$ and $A_\mu(M^4)$ associated with $J(CP_2)$ and $J(M^4)$. The interpretation of this coupling is an interesting challenge.

Conditions on $J(M^4)$

What conditions one can pose on $J(M^4)$?

1. The simplest possibility is that $J(M^4)$ is covariantly constant and self-dual and satisfies $J^2(M^4) = -g(M^4)$ meaning that $J(M^4)$ *resp.* $g(M^4)$ represents imaginary *resp.* real unit. Hypercomplexity for M^2 would suggest the restriction $J^2(M^2) = g(M^2)$ and $J^2(E^2) = -g(E^2)$. Since complexified octonions are used, it is convenient to include imaginary unit to $J(M^2)$ so that one indeed obtains $J^2(M^4) = -g(M^4)$. $J(M^4)$ would define a global decomposition $M^4 = M^2 \times E^2$ in terms of parallel constant electric and magnetic fields of equal magnitude. CD with this variant of $J(M^4)$ would be naturally associated with planewave like radiative solutions.

2. One could however give up the covariant constancy. In this case spherically symmetric variants of $J(M^4)$ naturally associated with spherically symmetric stationary metric and possible analogs of Robertson-Walker metrics. $J(M^4)$ would be closed except at the world line connecting the tips of CD and carry identical magnetic and electric charges.

3. $J(M^4)$ would define Hamilton Jacobi-structure and an attractive idea is that the orthogonal 2-surfaces associated with the foliation of M^4 are orbits of a subgroup of Poincare group. This structure would characterize quantum measurement at the level of WCW and quantum measurement would involve selection of a sector of WCW characterized by $J(M^4)$ [K46].

The most plausible assumption is that $J(M^4)$ is covariantly constant.

Objections against $J(M^4)$

Consider now the objections against introducing $J(M^4)$ to the Kähler action at embedding space level.

1. $J(M^4)$ would would break translational and Lorentz symmetries at the level of embedding space since $J(M^4)$ cannot be Lorentz invariant. For embedding space spinor modes this term would bring in coupling to the self-dual Kähler form in M^4. The simplest choice is $A = (A_t = z, A_z = 0, A_x = y, A_y = 0)$ defining decomposition $M^4 = M^2 \times E^2$. For Dirac equation in M^4 one would have free motion in preferred time-like (t,z)-plane plane M^2 in whereas in x- and y-directions (E^2 plane) would one have harmonic oscillator potentials due to the gauge potentials of electric and magnetic fields. One would have something very similar to quark model of hadron: quark momenta would have conserved longitudinal part and non-conserved transversal part. The solution spectrum has scaling invariance $\Psi(m^k) \rightarrow \Psi(\lambda m^k)$ so that there is no preferred scale and the transversal scales scale as $1/E$ and $1/k_x$.

2. Since $J(M^4)$ is not Lorentz invariant, Lorentz boosts would produce new $M^2 \times E^2$ decomposition (or its local variant). If one assumes above kind of linear gauge as gauge invariance suggests, the choices with fixed second tip of causal diamond (CD) define finite-dimensional moduli space $SO(3,1)/SO(1,1) \times SO(2)$ having in number theoretic vision an interpretation as a choice of preferred hypercomplex plane and its orthogonal complement. This is the moduli space for hypercomplex structures in M^4 with the choices of origins parameterized by M^4. The introduction of the moduli space would allow to preserve Poincare invariance.

3. If one generalizes the condition for Kähler metric to $J^2(M^4) = -g(M^4)$ fixing the scaling of J, the coupling to $A(M^4)$ is also large and suggests problems with the large breaking of Poincare symmetry for the spinor modes of the embedding space for given moduli. The transversal localization by the self-dual magnetic and electric fields for $J(M^4)$ would produce wave packets in transversal degrees of freedom: is this physical?

   This moduli space is actually the moduli space introduced for causal diamonds (CDs) in zero energy ontology (ZEO) forced by the finite value of volume action: fixing of the line connecting the tips of CD the Lorentz boost fixing the position for the second tip of CD parametrizes this moduli space apart from division with the group of transformations leaving the planes M^2 and E^2 having interpretation a plane defined by light-like momentum and polarization plane associated with a given CD invariant.

4. Why this kind of symmetry breaking for Poincare invariance? A possible explanation proposed already earlier is that quantum measurement involves a selection of quantization axis. This choice necessarily breaks the symmetries and $J(M^4)$ would be an embedding space correlate for the selection of rest frame and quantization axis of spin. This conforms with the fact that CD is interpreted as the perceptive field of conscious entity at embedding space level: the contents of consciousness would be determined by the superposition of space-time surfaces inside CD. The choice of $J(M^4)$ for CD would select preferred rest system (quantization axis for energy as a line connecting tips of CD) via electric part of $J(M^4)$ and quantization axis of spin (via magnetic part of $J(M^4)$. The moduli space for CDs would be the space for choices of these particular quantization axis and in each state function reduction would mean a localization in this moduli space. Clearly, this reduction would be higher level reduction and correspond to a decision of experimenter.

To summarize, for $J(M^4) = 0$ Poincare symmetries are realized at the level of embedding space but obviously broken slightly by the geometry of CD. The allowance of $J(M^4) \neq 0$ implies that both translational and rotational symmetries are reduced for a given CD: the interpretation would be in terms of a choice of quantization axis in state function reduction. They are however lifted to the level of moduli space of CDs and exact in this more abstract sense. This is nothing new: already the introduction of ZEO and CDs force by volume term in action forced by twistor lift of TGD implies the same. Also the view about state function reduction requires wave functions in the moduli space of CDs. This is also essential for understanding how the arrow of geometric time is inherited from that of subjective time in TGD inspired theory of consciousness [K4, K39].

Situation at space-time level

What about the situation at space-time level?

1. The introduction of $J(M^4)$ part to Kähler action has nice number theoretic aspects. In particular, J selects the preferred complex and quaternionic sub-space of octonionic space of embedding space. The simplest possibility is that the Kähler action is defined by the Kähler form $J(M^4) + J(CP_2)$.

   Since M^4 and CP_2 Kähler geometries decouple it should be possible to take the counterpart of Kähler coupling strength in M^4 to be much larger than in CP_2 degrees of freedom so that M^4 Kähler action is a small perturbation and slowly varying as a functional of preferred extremal. This option is however not in accordance with the idea that entire Kähler form is induced.

2. Whether the proposed ansätze for general solutions make still sense is not clear. In particular, can one still assume that preferred extremals are minimal surfaces? Number theoretical vision strongly suggests - one could even say demands - the effective decoupling of Kähler action and volume term. This would imply the universality of quantum critical dynamics. The solutions would not depend at all on the coupling parameters except through the dependence on boundary conditions. The coupling between the dynamics of Kähler action and volume term would come also from the conservation conditions at light-like 3-surfaces at which the signature of the induced metric changes.

3. At space-time level the field equations get more complex if the M^4 projection has dimension $D(M^4) > 2$ and also for $D(M^4) = 2$ if it carries non-vanishing induced $J(M^4)$. One would obtain cosmic strings of form $X^2 \times Y^2$ as minimal surface extremals of ordinary Kähler action or X^2 Lagrangian manifold of M^4 as also CP_2 type vacuum extremals and their deformations with M^4 projection Lagrangian manifold. Thus the differences would not be seen for elementary particle and string like objects. Simplest string worlds sheet for which $J(M^4)$ vanishes would correspond to a piece of plane M^2.

   M^4 is the simplest minimal surface extremal of Kähler action necessarily involving also $J(M^4)$. The action in this case vanishes identically by self-duality (in Euclidian signature self-duality does not imply this). For perturbations of M^4 such as spherically symmetric stationary metric the contribution of M^4 Kähler term to the action is expected to be small and the come mainly from cross term mostly and be proportional to the deviation from flat metric. The interpretation in terms of gravitational contribution from M^4 degrees of freedom could make sense.

4. What about massless extremals (MEs)? How the induced metric affects the situation and what properties second fundamental form has? Is it possible to obtain a situation in which the energy momentum tensor T^α and second fundamental form $H^k_{\alpha\beta}$ have in common components which are proportional to light-like vector so that the contraction $T^{\alpha\beta}H^k_{\alpha\beta}$ vanishes?

   Minimal surface property would help to satisfy the conditions. By conformal invariance one would expect that the total Kähler action vanishes and that one has $J^\alpha_{\ \gamma}J^{\gamma\beta} \propto ag^{\alpha\beta} + bk^\alpha k^\beta$. These conditions together with light-likeness of Kähler current guarantee that field equations are satisfied.

   In fact, one ends up to consider a generalization of MEs by starting from a generalization of holomorphy. Complex CP_2 coordinates ξ^i would be functions of light-like M^2 coordinate $u_+ = k \cdot m$, k light-like vector, and of complex coordinate w for E^2 orthogonal to M^2. Therefore the CP_2 projection would 3-D rather than 2-D now.

   The second fundamental form has only components of form $H^k_{u_+w}$, $H^k_{u_+\overline{w}}$ and H^k_{ww}, $H^k_{\overline{w}\overline{w}}$. The CP_2 contribution to the induced metric has only components of form Δg_{u_+w}, $\Delta g_{+\overline{w}}$, and $g_{\overline{w}w}$. There is also contribution $g_{u_+u_-} = 1$, where v is the light-like dual of u in plane M^2. Contravariant metric can be expanded as a power series for in the deviation (Δg_{u_+w}, $\Delta g_{u_+\overline{w}}$) of the metric from ($g_{u_+u_-}, g_{w\overline{w}}$). Only components of form g^{u_+,u_i} and $g^{w,\overline{w}}$ are obtained and their contractions with the second fundamental form vanish identically since there are no common index pairs with simultaneously non-vanishing components. Hence it seems that MEs generalize!

   I have asked earlier whether this construction might generalize for ordinary MEs. One can introduce what I have called Hamilton-Jacobi structure for M^4 consisting of locally orthogonal slicings by integrable 2-surfaces having tangent space having local decomposition $M^2_x \times E^2_x$ with light-like direction depending on point x. An objection is that the direction of light-like momentum depends on position: this need not be inconsistent with momentum conservation but would imply that the total four-momentum is not light-like anymore. Topological condensation for MEs and at MEs could imply this kind modification.

5. There is also a topological magnetic flux type term for string world sheet. Topological term can be transformed to a boundary term coupling classical particles at the boundary of string world sheet to CP_2 Kähler gauge potential (added to the equation for a light-like geodesic line). Now also the coupling to M^4 gauge potential would be obtained. The condition $J(M^4) + J(CP_2) = 0$ at string world sheets [L11] is very attractive manner to identify string world sheets as analogs of Lagrangian manifolds but does not imply the vanishing of the net $U(1)$ couplings at boundary since the induce gauge potentials are in general different.

   Also topological term including also M^4 Kähler magnetic flux for string world sheet contributes also to the modified Dirac equation since the gamma matrices are modified gamma matrices required by super-conformal symmetries and defined as contractions of canonical momentum densities with embedding space gamma matrices [K86]. This is true both in space-time interior, at string world sheets and at their boundaries. CP_2 (M^4) term gives a contribution proportional to CP_2 (M^4) gamma matrices.

At embedding space level transversal localization would be the outcome and a good guess is that the same happens also now. This is indeed the case for M^4 defining the simplest extremal. The general interpretation of M^4 Kähler form could be as a quantum tool for transversal dynamical localization of wave packets in Kähler magnetic and electric fields of M^4. Analog for decoherence occurring in transversal degrees of freedom would be in question. Hadron physics could be one application.

Testing the existence of $J(M^4)$

How to test the idea about $J(M^4)$?

1. It might be possible to kill the assumption that $J(M^4)$ is covariantly constant by showing that one does not obtain spherically symmetric Schwartschild type metric as a minimal surface extremal of generalized Kähler action: these extremals are possible for ordinary Kähler action [L9] [K14]. For the canonical embedding of M^4 field equations are satisfied since energy momentum tensor vanishes identically. For the small deformations the presence of $J(M^4)$ would reduce rotational symmetry to cylindrical symmetry.

   The question is basically about how large the moduli space of forms $J(M^4)$ can be allowed to be. The mere self duality and closedness condition outside the line connecting the tips of CD allows also variants which are spherically symmetric in either Minkowski coorinates or Robertson-Walker coordinates for light-cone.An attractive proposal is that the pairs of orthogonal 2-surface correspond to Hamilton-Jacobi structures for which the two surfaces are orbits of subgroups of Poincare group.

2. $J(M^4)$ could make its presence manifest in the physics of right-handed neutrino having no direct couplings to electroweak gauge fields. Mixing with left handed neutrino is however induced by mixing of M^4 and CP_2 gamma matrices. The transversal localization of right-handed neutrino in a background, which is a small deformation of M^4 could serve as an experimental signature.

3. CP breaking in hadronic systems is one of the poorly understood aspects of fundamental physics and relates closely to the mysterious matter-antimatter asymmetry. The constant electric part of self dual $J(M^4)$ implies CP breaking. I have earlier consider that Kähler electric fields could cause this breaking but now the electric field is not constant. Second possibility is that matter and antimatter correspond to different values of h_{eff} and are dark relative to each other. The question is whether $J(M^4)$ could explain the observed CP breaking as appearing already at the level of embedding space $M^4 \times CP_2$ and whether this breaking could explain hadronic CP breaking and matter anti-matter asymmetry. Could M^4 part of Kähler electric field induce different $h_{eff}/h = n$ for particles and antiparticles.

Kerr effect, breaking of T symmetry, and Kähler form of M^4

I encountered in Facebook a link to a very interesting article [D1] (see `http://tinyurl.com/h5lmplw`). Here is the abstract of the article.

We prove an instance of the Reciprocity Theorem that demonstrates that Kerr rotation, also known as the magneto-optical Kerr effect, may only arise in materials that break microscopic time reversal symmetry. This argument applies in the linear response regime, and only fails for nonlinear effects. Recent measurements with a modified Sagnac Interferometer have found finite Kerr rotation in a variety of superconductors. The Sagnac Interferometer is a probe for nonreciprocity, so it must be that time reversal symmetry is broken in these materials.

Magneto-optic Kerr effect (see `http://tinyurl.com/hef8xgv`) occurs when a circularly polarized light beam (plane wave) (often with normal incidence) reflects from a sample. For instance, reflected circular polarized beams suffers a phase change in the reflection: as if they would spend some time at the surface before reflecting. Linearly polarized light reflects as elliptically polarized light. In magneto-optic Kerr effect there are many options depending on the relative directions of the reflection plane (incidence is not normal in the general case so that one can talk about reflection plane) and magnetization.

Kerr angle θ_K is defined as 1/2 of the difference of these phase angle increments caused by reflection for oppositely circularly polarized plane wave beams. As the name tells, magneto-optic Kerr effect is often associated with magnetic materials. Kerr effect has been however observed also for high Tc superconductors and this has raised controversy. As a layman in these issues I can safely wonder whether the controversy is created by the expectation that there are no magnetic fields inside the super-conductor. Anti-ferromagnetism is however important for high Tc superconductivity. In TGD based model for high Tc superconductors the supracurrents would flow along pairs of flux tubes with the members of $S = 0$ ($S = 1$) Cooper pairs at parallel flux tubes carrying magnetic fields with opposite (parallel) magnetic fluxes. Therefore magneto-optic Kerr effect could be in question after all.

The author claims to have proven that Kerr effect in general requires breaking of microscopic time reversal symmetry. Time reversal symmetry breaking (TRSB) caused by the presence of magnetic field and in the case of unconventional superconductors is explained nicely at `http://tinyurl.com/jbabcjt`. Magnetic field is required. Magnetic field is generated by a rotating current and by right-hand rule time reversal changes the direction of the current and also of magnetic field. For spin 1 Cooper pairs the analog of magnetization is generated, and this leads to T breaking.

This result is very interesting from the point of TGD. The reason is that twistorial lift of TGD requires that embedding space $M^4 \times CP_2$ has Kähler structure in generalized sense [L12, L24]. M^4 has the analog of Kähler form, call it $J(M^4)$. $J(M^4)$ is assumed to be self-dual and covariantly constant as also CP_2 Kähler form, and contributes to the Abelian electroweak U(1) gauge field (electroweak hypercharge) and therefore also to electromagnetic field. By definition it satisfies $J^2(M^4) = -g(M^4)$ saying that it represents imaginary unit geometrically.

$J(M^4)$ implies breaking of Lorentz invariance since it defines decomposition $M^4 = M^2 \times E^2$ implying preferred rest frame and preferred spatial direction identifiable as direction of spin quantization axis. In zero energy ontology (ZEO) one has moduli space of causal diamonds (CDs) and therefore also moduli space of Kähler forms and the breaking of Lorentz invariance cancels. Note that a similar Kähler form is conjectured in quantum group inspired non-commutative quantum field theories and the problem is the breaking of Lorentz invariance.

What is interesting that the action of P,CP, and T on Kähler form transforms it from self-dual to anti-self-dual form and vice versa. If $J(M^4)$ is self-dual as also $J(CP_2)$, all these 3 discrete symmetries are broken in arbitrarily long length scales. On basis of tensor property of $J(M^4)$ one expects P: $(J(M^2), J(E^2) \rightarrow (J(M^2), -J(E^2)$ and T: $(J(M^2), J(E^2) \rightarrow (-J(M^2), J(E^2)$. Under C one has $(J(M^2), J(E^2) \rightarrow (-J(M^2), -J(E^2)$. This gives CPT: $(J(M^2), J(E^2) \rightarrow (J(M^2), J(E^2)$ as expected.

One can imagine several consequences at the level of fundamental physics.

1. One implication is a first principle explanation for the mysterious CP violation and matter antimatter asymmetry not predicted by standard model (see below).

2. A new kind of parity breaking is predicted. This breaking is separate from electroweak parity breaking and perhaps closely related to the chiral selection in living matter.

3. The breaking of T might in turn relate to Kerr effect if the argument of authors is correct. It could occur in high Tc superconductors in macroscopic scales. Also large $h_{eff}/h = n$ scaling up quantum scales in high Tc superconductors could be involved as with the breaking of chiral symmetry in living matter. Strontium ruthenate for which Cooper pairs are in $S = 1$ state is is indeed found to exhibit TRSB (for references and explanation see `http://tinyurl.com/jbabcjt`).

   In TGD based model of high Tc superconductivity [K62, K63] the members of the Cooper pair are at parallel magnetic flux tubes with the same spin direction of magnetic field. The magnetic fields and thus the direction of spin component in this direction changes under T causing TRSB. The breaking of T for $S = 1$ Cooper pairs is not spontaneous but would occur at the level of physics laws: the time reversed system finds itself experiences in the original self-dual $J(M^4)$) rather than in $(-J(M^2), J(E^2))$ demanded by T symmetry.

5.3.7 What causes CP violation?

CP violation and matter antimatter asymmetry involving it represent white regions in the map provided by recent day physics. Standard model does not predict CP violation necessarily accompanied by the violation of time reflection symmetry T by CPT symmetry assumed to be exact. The violation of T must be distinguished from the emergence of time arrow implies by the randomness associated with state function reduction.

CP violation was originally observed for mesons via the mixing of neutral kaon and antikaon having quark content $n\overline{s}$ and $\overline{n}s$. The lifetimes of kaon and antikaon are different and they transform to each other. CP violation has been also observed for neutral mesons of type $n\overline{b}$. Now it has been observed also for baryons Λ_b with quark composition u-d-b and its antiparticle (see `http://tinyurl.com/zyk8w44`). Standard model gives the Feynman graphs describing the mixing in standard model in terms of CKM matrix (see `http://tinyurl.com/hvpz2su`).

The CKM mixing matrix associated with weak interactions codes for the CP violation. More precisely, the small imaginary part for the determinant of CKM matrix defines the invariant coding for the CP violation. The standard model description of CP violation involves box diagrams in which the coupling to heavy quarks takes place. b quark gives rise to anomalously large CP violation effect also for mesons and this is not quite understood. Possible new heavy fermions in the loops could explain the anomaly.

Quite generally, the origin of CP violation has remained a mystery as also CKM mixing. In TGD framework CKM mixing has topological explanation in terms of genus of partonic 2-surface assignable to quark (sphere, torus or sphere with two handles). Topological mixings of U and D type quarks are different and the difference is not same for quarks and antiquarks. But this explains only CKM mixing, not CP violation.

Classical electric field - not necessary electromagnetic - prevailing inside hadrons could cause CP violation. So called instantons are basic prediction of gauge field theories and could cause strong CP violation since self-dual gauge field is involved with electric and magnetic fields having same strength and direction. That this strong CP violation is not observed is a problem of QCD. There are however proposals that instantons in vacuum could explain the CP violation of hadron physics (see `http://tinyurl.com/zptbd4j`).

What says TGD? I have considered this in [L26] and earlier blog posting (see `http://tinyurl.com/hvzqjua`).

1. M^4 and CP_2 are unique in allowing twistor space with Kähler structure (in generalized sense for M^4) [A18]. If the twistor space $T(M^4) = M^4 \times S^2$ having bundle projections to both M^4 and to the conventional twistor space CP_3, or rather its non-compact version) allows Kähler structure then also M^4 allow the generalized Kähler structure and the analog symplectic structure.

   This boils down to the existence of self-dual and covariantly constant U(1) gauge field $J(M^4)$ for which electric and magnetic fields E and B are equal and constant and have the same direction. This field is not dynamical like gauge fields but would characterize the geometry of M^4. $J(M^4)$ implies violation Lorentz invariance. TGD however leads to a moduli space for causal diamonds (CDs) effectively labelled by different choices of direction for these self-dual Maxwell fields. The common direction of E and B could correspond to that for spin quantization axis. $J(M^4)$ has nothing to do with instanton field. It should be noticed that also the quantum group inspired attempts to build quantum field theories for which space-time geometry is non-commutative introduce the analog of Kähler form in M^4, and are indeed plagued by the breaking of Lorentz invariance. Here there is no moduli space saving the situation.

2. The choice of quantization axis would therefore have a correlate at the level of "world of classical worlds" (WCW). Different choices would correspond to different sectors of WCW. The moduli space for the choices of preferred point of CP_2 and color quantization axis corresponds to the twistor space $T(CP_2) = SU(3)/U(1) \times U(1)$ of WCW. One could interpret also the twistor space $T(M^4) = M^4 \times S^2$ as the space with given point representing the position of the tip of CD and the direction of the quantization axis of angular momentum. This choice requires a characterization of a unique rest system and the directions of quantization axis and time axes defines plane M^2 playing a key role in TGD approach to twstorialization [L12, L24].

3. The prediction would be CP violation for a given choice of $J(M^4)$. Usually this violation would be averaged out in the average over the moduli space for the choices of M^2 but in some situation this would not happen. Why the CP violation does not average out when there is CKM mixing of quarks? Why the parity violation due to the preferred direction is not compensated by C violation meaning that the directions of E and B fields would be exactly opposite for quarks and antiquarks. Could the fact that quarks are not free but inside hadron induce CP violation? Could a more abstract formulation say that the wave function in the moduli space for $J(M^4)$ (wave function for the choices of spin quantization axis!) is not CP symmetric and this is reflected in the CKM matrix.

4. An important delicacy is that $J(M^4)$ can be both self-dual and anti-self-dual depending on whether the magnetic and electric field have same or opposite directions. It will be found that reflection P and CP transform self-dual $J(M^4)$ to anti-self-dual one. If only self-dual $J(M^4)$ is allowed, one has both parity breaking and CP violations.

Can one understand the emergence of CP violation in TGD framework?

1. Zero energy state is pair of two positive and negative energy parts. Let us assume that positive energy part is fixed - one can call corresponding boundary of CD passive. This state corresponds to the outcome of state function reduction fixing the direction of quantization axes and producing eigenstates of measured observables, for instance spin. Single system at passive boundary is by definition unentangled with the other systems. It can consists of entangled subsystems hadrons are basic example of systems having entanglement in spin degrees of freedom of quarks: only the total spin of hadron is precisely defined.

   The states at the active boundary of CD evolve by repeated unitary steps by the action of the analog of S-matrix and are not anymore eigenstates of single particle observables but entangled. There is a sequence of trivial state function reductions at passive boundary inducing sequence of unitary time evolutions to the state at the active boundary of CD and shifting it. This gives rise to self as a generalized Zeno effect.

   Classically the time evolution of hadron corresponds to a superposition of space-time surfaces inside CD. The passive ends of the space-time surface or rather, the quantum superposition of them - is fixed. At the active end one has a superposition of 3-surfaces defining classical correlates for quantum states at the active end: this superposition changes in each unitary step during repeated measurements not affecting the passive end. Also time flows, which means that the distance between the tips of CD defining clock-time increases as the active boundary of CD shifts farther away.

2. The classical field equations for space-time surface follow from an action, which at space-time level is sum of Kähler action and volume term. If Kähler form at space-time surface is induced (projected to space-time surface) from $J = J(M^4) + J(CP_2)$, the classical time evolution is CP violating. CKM mixing is induced by different topological mixings for U and D type quarks (recall that 3 particle generations correspond to different genera for partonic 2-surfaces: sphere, torus, and sphere with two handles). $J(M^4) + J(CP_2)$ defines the electroweak $U(1)$ component of electric field so that $J(M^4)$ contributes to $U(1)$ part of em field and is thus physically observable.

3. Topological mixing of quarks corresponds to a superposition of time evolutions for the partonic 2-surfaces, which can also change the genus of partonic 2-surface defined as the number of handles attached to 2-sphere. For instance, sphere can transform to torus or torus to a sphere with two handles. This induces mixing of quantum states. For instance, one can say that a spherical partonic 2-surface containing quark would develop to quantum superposition of sphere, torus, and sphere with two handles. The sequence of state function reductions leaving the passive boundary of CD unaffected (generalized Zeno effect) by shifting the active boundary from its position after the first state function reduction to the passive boundary could but need not give rise to a further evolution of CKM matrix.

4. The determinant of CKM matrix is equal to phase factor by unitarity ($UU^\dagger = 1$) and its imaginary part characterizes CP breaking. The imaginary part of the determinant should be

proportional to the Jarlskog invariant $J = \pm Im(V_{us}V_{cb}\overline{V}_{ub}\overline{V}_{cs})$ characterizing CP breaking of CKM matrix (see `http://tinyurl.com/kakxwl8`).

If the topological mixings are different for U and D type quarks, one obtains CKM mixing. How could the classical time evolution for quarks and for antiquarks as their CP transforms differ? To answer the question one must look how $J(M^4)$ transforms under C, P, T and CP.

1. $J(M^4) = (J_{0z}, J_{xy} = \epsilon J_{0z})$, $\epsilon = \pm 1$, characterizes hadronic space-time sheet (all space-time sheets in fact). Since $J(M^4)$ is tensor, P changes only the sign of J_{0z} giving $J(M^4) \rightarrow (-J_{0z}, J_{xy})$. Since C changes the signs of charges and therefore the signs of fields created by them, one expects $J(M^4) \rightarrow -J(M4)$ under C. CP would give $J(M^4) \rightarrow (J_{0z}, -J_{xy})$ transforming selfdual $J(M^4)$ to anti-selfdual $J(M^4)$. If WCW has no anti-self-dual sector, CP is violated at the level of WCW.

2. If CPT leaves $J(M^4)$ invariant, one must have $J(M^4) \rightarrow (J_{0z}, -J_{xy})$ under T rather than $J(M^4) \rightarrow (-J_{0z}, J_{xy})$. The anti-unitary character of T could correspond for additional change of sign under T. Otherwise CPT should act as $J(M^4) \rightarrow -J(M^4)$ and only $(CPT)^2$ would correspond to unity.

3. Same considerations apply to $J(CP_2)$ but the difference would be that induced $J(M^4)$ for space-time surfaces, which are small deformations of M^4 covariantly constant in good approximation. Also for string world sheets corresponding to small cosmological constant $J(M^4) \times J(M^4) - 2 \simeq 0$ holds true in good approximation and induced $J(M^4)$ at string world sheet is in good approximation covariantly constant. If the string world sheet is just M^2 characterizing $J(M^4)$ the condition is exact and was has Kähler electric field induced by $J(M^4)$ but no corresponding magnetic field. This would make the CP breaking effect large.

If CP is not violated, particles and their CP transforms correspond to different sectors of WCW with self dual and anti-self dual $J(M^4)$. If only self-dual sector of WCW is present then CP is violated. Also P is violated at the level of WCW and this parity breaking is different from that associated with weak interactions and could relate to the geometric parity breaking manifesting itself via chiral selection in living matter. Classical time evolutions induce different CKM mixings for quarks and antiquarks reflecting itself in the small imaginary part of the determinant of CKM matrix. CP breaking at the level of WCW could explain also matter-antimatter asymmetry. For instance, antimatter could be dark with different value of $h_{eff}/h = n$.

What is interesting that P is badly broken in long length scales as also CP. The same could be true for T. Could this relate to the thermodynamical arrow of time? In ZEO state function reductions to the opposite boundary change the direction of clock time. Most physicist believe that the arrow of thermodynamical time and thus also clock time is always the same. There is evidence that in living matter both arrows are possible. For instance, Fantappie has introduced the notion of syntropy as time reversed entropy [J3]. This suggests that thermodynamical arrow of time could correspond to the dominance of the second arrow of time and be due to self-duality of $J(M^4)$ leading to breaking of T. For instance, the clock time spend in time reversed phase could be considerably shorter than in the dominant phase. A quantitative estimate for the ratio of these times might be given some power of the ratio $X = l_P/R$.

5.3.8 Quantitative picture about CP breaking in TGD

One must specify the value of α_1 and the scaling factor transforming $J(CD)$ having dimension length squared as tensor square root of metric to dimensionless $U(1)$ gauge field $F = J(CD)/S$. This leads to a series of questions.

How to fix the scaling parameter S?

1. The scaling parameter relating $J(CD)$ and F is fixed by flux quantization implying that the flux of $J(CD)$ is the area of sphere S^2 for the twistor space $M^4 \times S^2$. The gauge field is obtained as $F = J/S$, where $S = 4\pi R^2(S^2)$ is the area of S^2.

2. Note that in Minkowski coordinates the length dimension is by convention shifted from the metric to linear Minkowski coordinates so that the magnetic field B_1 has dimension of inverse

length squared and corresponds to $J(CD)/SL^2$, where L is naturally be taken to the size scale of CD defining the unit length in Minkowski coordinates. The $U(1)$ magnetic flux would the signed area using L^2 as a unit.

How $R(S^2)$ relates to Planck length l_P? l_P is either the radius $l_P = R$ of the twistor sphere S^2 of the twistor space $T = M^4 \times S^2$ or the circumference $l_P = 2\pi R(S^2)$ of the geodesic of S^2. Circumference is a more natural identification since it can be measured in Riemann geometry whereas the operational definition of the radius requires embedding to Euclidian 3-space.

How can one fix the value of $U(1)$ coupling strength α_1? As a guideline one can use CP breaking in K and B meson systems and the parameter characterizing matter-antimatter symmetry.

1. The recent experimental estimate for so called Jarlskog parameter characterizing the CP breaking in kaon system is $J \simeq 3.0 \times 10^{-5}$. For B mesons CP breading is about 50 times larger than for kaons and it is clear that Jarlskog invariant does not distinguish between different meson so that it is better to talk about orders of magnitude only.

2. Matter-antimatter asymmetry is characterized by the number $r = n_B/n_\gamma \sim 10^{-10}$ telling the ratio of the baryon density after annihilation to the original density. There is about one baryon 10 billion photons of CMB left in the recent Universe.

Consider now the identification of α_1.

1. Since the action is obtained by dimensional reduction from the 6-D Kähler action, one could argue $\alpha_1 = \alpha_K$. This proposal leads to unphysical predictions in atomic physics since neutron-electron $U(1)$ interaction scales up binding energies dramatically.

   $U(1)$ part of action can be however regarded a small perturbation characterized by the parameter $\epsilon = R^2(S^2)/R^2(CP_2)$, the ratio of the areas of twistor spheres of $T(M^4)$ and $T(CP_2)$. One can however argue that since the relative magnitude of $U(1)$ term and ordinary Kähler action is given by ϵ, one has $\alpha_1 = \epsilon \times \alpha_K$ so that the coupling constant evolution for α_1 and α_K would be identical.

2. ϵ indeed serves in the role of coupling constant strength at classical level. α_K disappears from classical field equations at the space-time level and appears only in the conditions for the super-symplectic algebra but ϵ appears in field equations since the Kähler forms of J resp. CP_2 Kähler form is proportional to $R^2(S^2)$ resp. $R^2(CP_2)$ times the corresponding $U(1)$ gauge field. $R(S^2)$ appears in the definition of 2-bein for $R^2(S^2)$ and therefore in the modified gamma matrices and modified Dirac equation. Therefore $\sqrt{\epsilon} = R(S^2)/R(CP_2)$ appears in modified Dirac equation as required by CP breaking manifesting itself in CKM matrix.

   NTU for the field equations in the regions, where the volume term and Kähler action couple to each other demands that ϵ and $\sqrt{\epsilon}$ are rational numbers, hopefully as simple as possible. Otherwise there is no hope about extremals with parameters of the polynomials appearing in the solution in an arbitrary extension of rationals and NTU is lost. Transcendental values of ϵ are definitely excluded. The most stringent condition $\epsilon = 1$ is also unphysical. $\epsilon = 2^{2r}$ is favoured number theoretically.

Concerning the estimate for ϵ it is best to use the constraints coming from p-adic mass calculations.

1. p-Adic mass calculations [K42] predict electron mass as

$$m_e = \frac{\hbar}{R(CP_2)\sqrt{5+Y}} \quad .$$

   Expressing m_e in terms of Planck mass m_P and assuming $Y = 0$ ($Y \in (0,1)$) gives an estimate for $l_P/R(CP_2)$ as

$$\frac{l_P}{R(CP_2)} \simeq 2.0 \times 10^{-4} \quad .$$

2. From $l_P = 2\pi R(S^2)$ one obtains estimate for ϵ, α_1, $g_1 = \sqrt{4\pi\alpha_1}$ assuming $\alpha_K \simeq \alpha \simeq 1/137$ in electron length scale.

$$\begin{aligned} \epsilon &= 2^{-30} \simeq 1.0 \times 10^{-9} \quad , \\ \alpha_1 &= \epsilon\alpha_K \simeq 6.8 \times 10^{-12} \quad , \\ g_1 &= \sqrt{4\pi\alpha_1} \simeq 9.24 \times 10^{-6} \quad . \end{aligned}$$

There are two options corresponding to $l_P = R(S^2)$ and $l_P = 2\pi R(S^2)$. Only the length of the geodesic of S^2 has meaning in the Riemann geometry of S^2 whereas the radius of S^2 has operational meaning only if S^2 is imbedded to E^3. Hence $l_P = 2\pi R(S^2)$ is more plausible option.

For $\epsilon = 2^{-30}$ the value of $l_P^2/R^2(CP_2)$ is $l_P^2/R^2(CP_2) = (2\pi)^2 \times R^2(S^2)/R^2(CP_2) \simeq 3.7 \times 10^{-8}$. $l_P/R(S^2)$ would be a transcendental number but since it would not be a fundamental constant but appear only at the QFT-GRT limit of TGD, this would not be a problem.

One can make order of magnitude estimates for the Jarlskog parameter J and the fraction $r = n(B)/n(\gamma)$. Here it is not however clear whether one should use ϵ or α_1 as the basis of the estimate

1. The estimate based on ϵ gives

$$J \sim \sqrt{\epsilon} \simeq 3.2 \times 10^{-5} \quad , \quad r \sim \epsilon \simeq 1.0 \times 10^{-9} \quad .$$

The estimate for J happens to be very near to the recent experimental value $J \simeq 3.0 \times 10^{-5}$. The estimate for r is by order of magnitude smaller than the empirical value.

2. The estimate based on α_1 gives

$$J \sim g_1 \simeq 0.92 \times 10^{-5} \quad , \quad r \sim \alpha_1 \simeq .68 \times 10^{-11} \quad .$$

The estimate for J is excellent but the estimate for r by more than order of magnitude smaller than the empirical value. One explanation is that α_K has discrete coupling constant evolution and increases in short scales and could have been considerably larger in the scale characterizing the situation in which matter-antimatter asymmetry was generated.

There is an intriguing numerical coincidence involved. $h_{eff} = \hbar_{gr} = GMm/v_0$ in solar system corresponds to $v_0 \simeq 2^{-11}$ and appears as coupling constant parameter in the perturbative theory obtained in this manner [K70]. What is intriguing that one has $\alpha_1 = v_0^2/4\pi^2$ in this case. Where does the troublesome factor $(1/2\pi)^2$ come from? Could the p-adic coupling constant evolutions for v_0 and α_1 correspond to each other and could they actually be one and the same thing? Can one treat gravitational force perturbatively either in terms of gravitational field or $J(CD)$? Is there somekind of duality involved?

Atomic nuclei have baryon number equal the sum $B = Z+N$ of proton and neutron numbers and neutral atoms have $B = N$. Only hydrogen atom would be also $U(1)$ neutral. The dramatic prediction of $U(1)$ force is that neutrinos might not be so weakly interacting particles as has been thought. If the quanta of $U(1)$ force are not massive, a new long range force is in question. $U(1)$ quanta could become massive via $U(1)$ super-conductivity causing Meissner effect. As found, $U(1)$ part of action can be however regarded a small perturbation characterized by the parameter $\epsilon = R^2(S^2)/R^2(CP_2)$. One can however argue that since the relative magnitude of $U(1)$ term and ordinary Kähler action is given by ϵ, one has $\alpha_1 = \epsilon \times \alpha_K$.

Quantal $U(1)$ force must be also consistent with atomic physics. The value of the parameter α_1 consistent with the size of CP breaking of K mesons and with matter antimatter asymmetry is $\alpha_1 = \epsilon\alpha_K = 2^{-30}\alpha_K$.

1. Electrons and baryons would have attractive interaction, which effectively transforms the em charge Z of atom $Z_{eff} = rZ$, $r = 1 + (N/Z)\epsilon_1$, $\epsilon_1 = \alpha_1/\alpha = \epsilon \times \alpha_K/\alpha \simeq \epsilon$ for $\alpha_K \simeq \alpha$ predicted to hold true in electron length scale. The parameter

$$s = (1 + (N/Z)\epsilon)^2 - 1 = 2(N/Z)\epsilon + (N/Z)^2\epsilon^2$$

would characterize the isotope dependent relative shift of the binding energy scale.

The comparison of the binding energies of hydrogen isotopes could provide a stringent bounds of the value of α_1. For $l_P = 2\pi R(S^2)$ option one would have $\alpha_1 = 2^{-30}\alpha_K \simeq .68 \times 10^{-11}$ and $s \simeq 1.4 \times 10^{-10}$. s is by order of magnitude smaller than $\alpha^4 \simeq 2.9 \times 10^{-9}$ corrections from QED (see http://tinyurl.com/kk9u4rh). The predicted differences between the binding energy scales of isotopes of hydrogen might allow to test the proposal.

2. $B = N$ would be neutralized by the neutrinos of the cosmic background. Could this occur even at the level of single atom or does one have a plasma like state? The ground state binding energy of neutrino atoms would be $\alpha_1^2 m_\nu/2 \sim 10^{-24}$ eV for $m_\nu = .1$ eV! This is many many orders of magnitude below the thermal energy of cosmic neutrino background estimated to be about 1.95×10^{-4} eV (see http://tinyurl.com/ldu95o9). The Bohr radius would be $\hbar/(\alpha_1 m_\nu) \sim 10^6$ meters and same order of magnitude as Earth radius. Matter should be $U(1)$ plasma. $U(1)$ superconductor would be second option.

5.4 About the interpretation of the duality assignable to Yangian symmetry

The $D = 4$ conformal generators acting on twistors have a dual representation in which they act on momentum twistors: one has dual conformal symmetry, which becomes manifest in this representation. These two separate symmetries extend to Yangian symmetry providing a powerful constraint on the scattering amplitudes.

In TGD the conformal Yangian extends to super-symplectic Yangian - actually, all symmetry algebras have a Yangian generalization with multi-locality generalized to multi-locality with respect to partonic 2-surfaces. The generalization of the dual conformal symmetry has remained obscure. In the following I describe what the generalization of the two conformal symmetries and Yangian symmetry would mean in TGD framework. I also propose an information theoretic duality between Euclidian and Minkowskian regions of space-time surface. I am not algebraist and apologize for the unavoidable inaccuracies.

5.4.1 Formal definition associated with Yangian

The notion of Yangian appears as two very different looking variants. The first variant can be found from Wikipedia (see goo.gl/q1twRZ) and second variant assignable to gauge theories can be found from [B14, B15].

Consider first the Wikipedia definition. The definition is in terms of quantum group notion in which the elements of matrix representing group element are made non-commuting operators.

1. The generators of Yangian algebra are labelled by an integer $n \geq -1$ with $n = -1$ generator identified as unit matrix. $n \geq 1$ generators generate the algebra and commutators with $n = 1$ generators preserving the weight allow to assign quantum numbers to them. From the Wikipedia article one learns that Yangian is generated by elements $t_{ij}^{(p)}$, $1 \leq i,j \leq N$, $p \geq 0$ of quantum matrices satisfy the relations

$$\left[t_{ij}^{(p+1)}, t_{kl}^{(q)}\right] - \left[t_{ij}^{(p)}, t_{kl}^{(q+1)}\right] = -(t_{kj}^{(p)} t_{il}^{(q)} - t_{kj}^{(q)} t_{il}^{(p)}) \quad . \tag{5.4.1}$$

Note there are two operations involved: commutator and operator product. The formula here is not consistent with the formula used in Yang-Mills theories for the commutators between $m = 0$ generators and generators with generators having $n \in \{0, 1\}$, and it seems that this formula suggesting $m, n \to m + n - 1$ in commutator cannot hold true for the commutators with $m = 0$ generators.

By defining $t_{ij}^{(-1)} = \delta_{ij}$ and setting

$$T(z) \quad = \quad \sum_{p\geq -1} t_{ij}^{(p)} z^{-p+1} \quad . \tag{5.4.2}$$

$T(z)$ is thus a quantum matrix depending on the point of 2-D space.

2. Introduce R-matrix $R(z) = 1 + z^{-1}P$ acting on $C^N \otimes C^N$, where P is the operator permuting the tensor factors. This allows to write the defining relations as Yang-Baxter equation (see `http://tinyurl.com/gogn75s`):

$$R_{12}(z-w)T_1(z)T_2(w) = T_2(w)T_1(z)R_{12}(z-w) \quad . \tag{5.4.3}$$

R_{12}, which depends only on the difference $z-w$, performs the permutation of the generators $T_1(z)$ and $T_2(w)$.

Yangian is a Hopf algebra with co-multiplication Δ mapping $T(z)$ acting in V to operator acting in $V \otimes V$, co-unit ϵ and antipode s given by

$$(\Delta \otimes id)T(z) = T_{12}(z)T_{13}(z) \quad , \quad (\epsilon \otimes id)T(z) = I \quad , \quad (s \otimes id)T(z) = T(z)^{-1} \quad . \tag{5.4.4}$$

Δ taking generator $T(z)$ acting in V to generator $\Delta(T) = T_{12}(z)$ acting in $V \otimes V$. Δ transforms a generator acting on single-particle states to a generator acting on 2-particles states.

3. The Yangian weight of the commutator of elements with weights m and n is $m+n-1$ rather than $m+n$ as for Virasoro and Kac-Moody algebras. This means that generators with conformal weight 1 do not affect the conformal weight and Cartan algebra elements defining quantum numbers of generators have weight 1. For conformal algebras the Cartan algebra defining quantum numbers has conformal weight 0.

   For Virasoro algebra having integer valued conformal weights the scaling $L_0 = zd/dz$ appears as basic derivative operation and generators are products $L_n = z^n zd/dz$. By taking translation operator $T = d/dz$ as the derivative operator and writing $K_n = z^n d/dz$, the weight of commutator becomes $m+n-1$. This is a trivial change. The map $u = exp(z)$ relates these two representations. That $n \leq 2$ appear in generators distinguishes the representations from Virasoro and Kac-Moody representations - note however that also for these algebras the generators with positive weight generate physical states.

What bothers me in this definition is that only the action of the generators with $p=1$ leaves the weight unaffected whereas for the dual conformal symmetry generators with both $p=0$ and $p=1$ do this and define conformal symmetry and its dual.

5.4.2 Dual conformal symmetry in $\mathcal{N}=4$ SUSY

Yangian symmetry appears also in gauge theories and the definition looks very different from the Wikipedia definition. In $\mathcal{N}=4$ SUSY conformal symmetry (in 4-D sense) has two representations. There is a duality between two representations of conformal generators crucial for twistor Grassmannian approach [B14, B15] (see `http://tinyurl.com/n22lwuy`).

1. In the first representation conformal symmetry generators $J_a^{(0)}$ are local and act in the space of external momenta. This induces a local and linear action in twistor space.

2. The generators $J_a^{(1)}$ of the dual conformal symmetry act in a local manner in the space of region momenta and associated momentum twistor space whereas the action of $J_a^{(1)}$ is bi-local in the momentum space and corresponding twistor space.

   Region momenta can be assigned with a twistor diagram defined by a closed polygon of Minkowski space having region momenta (, which need not be light-like) as edges having external light-like momenta emitted at the corners. The dual of this representation is the representation in which the light-like external momenta summing up to zero form a closed polygon.

Yangian is generated by ordinary generators $J_a^{(0)}$ and bi-local dual generators $J_a^{(1)}$.

1. They satisfy the commutations

$$\left[J_a^{(0)}, J_b^{(1)}\right] = f_{ab}{}^c J_c^{(1)} \quad . \tag{5.4.5}$$

   This condition is perfectly sensible physically but is not consistent with the above general consistency condition pf Eq. 5.4.1 from R-matrix requiring that the commutator has vanishing weight. Now the weights are additive in commutator.

2. The generators $J_a^{(1)}$ have an easy-to-guess representation:

$$J_a^{(1)} = f_a{}^{cb} \sum_{0\leq i<j\leq n} J_{ib}^{(0)} J_{jc}^{(0)} \tag{5.4.6}$$

   making explicit the bi-locality. The commutators of these generators have also weight 1. This is consistent with the above general formula unlike the formula the commutators of generators with vanishing weight. Both generators form a closed sub-algebra of Yangian and this must be behind the possibility to represent $J_a^{(1)}$ locally.

3. Also so called Serre relations are satisfied. They look rather complex and look different from the relations associated with R-matrix.

$$\begin{aligned} X(a,b,c) &+ \epsilon(a,b,c)X(b,c,a) + \epsilon(c,a,b)X(c,a,b) = h\epsilon_{rm,tn}Y(l,m,n)f^l_{ar}f^m_{bs}f^n_{ct}f^{rst} \quad , \\ X(a,b,c) &= \left[J_a^{(1)}, \left[J_b^{(1)}, J_c^{(0)}\right\}\right\} \quad , \quad Y(l,m,n) = \left\{J_l^{(0)}, J_m^{(0)}, J_n^{(0)}\right] \\ \epsilon(a,b,c) &= (-1)^{|a|(|b|+|c|)} \quad , \quad \epsilon_{rm,tn} = (-1)^{|r|m|+|t|n|} \quad . \end{aligned} \tag{5.4.7}$$

   Here the mixed brackets the $[.,\}$ denote the graded commutator, and $\{.,..]$ denotes the graded symmetrizer. h is a parameter characterizing the Yangian and should correspond to the parameter characterizing quantum group.

   These conditions are sufficient to give a representation of graded Yangian if the tensor product $\mathcal{R}\otimes\overline{\mathcal{R}}$ of the representation $\mathcal{R}$ and its conjugate $\overline{\mathcal{R}}$ contains adjoint representation only once. The higher generators can be generate by applying co-product operation to the generators.

4. Both local and bi-local generators form two closed sub-algebras. This is not consistent with the consistency conditions of appearing in Wikipedia definition. The Wikipedia definition seems to be wrong for commutators of generators $[J_A^{(m)}, J_B^{(n)}]$ with weights $(m,n) \in \{(0,0),(0,1),(1,0)\}$.

5. Co-product Δ has representation

$$\begin{aligned}\Delta(J^A) &= J^A \otimes 1 + 1 \otimes J^A \ , \\ \Delta(Q^A) &= Q^A \otimes 1 + 1 \otimes Q^A + f^A_{BC} J^B \otimes J^C \ .\end{aligned} \quad (5.4.8)$$

The first formula is obvious. Single particle generator lifted to a tensor product is sum of the single particle generators acting on the tensor factors. When Q^A annihilates single spin representations, one obtains just the defining formula for the bi-local generators.

One could have a situation in which single particle states are actually many-particle states annihilated by Q^A and satisfying the condition that adjoint is contained only once in $\mathcal{R}\otimes\overline{\mathcal{R}}$. In TGD framework one might argue that this kind of effective single particle states could quite generally define bound states behaving like single particle states physically. One would obtain infinite hierarchy of this kind of states realizing concretely the vision about fractal hierarchy.

5.4.3 Possible TGD based interpretation of Yangian symmetries

In TGD partonic 2-surfaces replace point-like objects and multi-locality is with respect to these. The proposal is that the TGD counterpart of the Yangian algebra [B15] of gauge theories could act as symmetries of many-parton states characterized by n partonic 2-surfaces assignable to the same 3-D surface at the boundary of causal diamond (CD). What is remarkable that this symmetry would relate particle states with different particle numbers to each other unlike the usual single particle symmetries.

1. This condition forces the partons to form a bound state with partonic 2-surfaces having *space-like* separations. Note that the separations along orbits of wormhole throats at opposite ends of CD are space-like or light-like. This must be taken into account when correlation functions are calculated. In QFT there is no description of this kind and this could explain the general failure of QFT in the description of bound states already in QED, where Bethe-Salpeter equation predicts large numbers of non-existing states.

2. Yangian algebra involves complex (hypercomplex) coordinate z which could be associated with the boundaries of string world sheets connecting partonic surfaces at the same boundary (at opposite boundaries) of CD. One can also assign complex coordinate with partonic 2-surfaces and the braiding of fermionic lines would be described by the matrix R assignable to the Yangian. The Cartan algebra of local and bi-local string like operators define quantum numbers for states. That point-like and string-like operators generate the algebra conforms with the idea about tensor networks with nodes connected by edges.

   On can think that partonic 2-surfaces form a single connected unit consisting of partonic surfaces connected by boundaries of string world sheets assignable to the topological Feynman diagram defined by the light-like 3-surface defining the boundary between Euclidian and Minkowskian regions of the space-time surface.

3. The operation Δ for Yangian would assign to the generators acting on single parton states generators acting on 2-parton states. R_{12} would act as an exchange operation for parton states, which could reduces to many-fermion states at partonic 2-surfaces.

4. R_{12} can appear in many contexts in TGD. It can be associated with braiding of fermionic lines inside partonic orbits or magnetic flux tubes at the ends of space-time surfaces. It can be also associated with the fermionic lines in the preferred plane M^2 associated with twistor scattering amplitudes.

   From the twistorial point of view the preferred M^2 defined by light-like quaterionic 8-momentum is of special interest. M^2 identified as octonionic complex plane and its complexification brings in mind integrable field theories in M^2 allowing Yangian symmetry characterized by R-matrix. The scattering matrix is trivial for these field theories: scattering involves only a phase shift. In twistorial approach to TGD scattering is non-trivial. The R-matrix would be present also now and exchange the momentum projections in preferred M^2 plane. If the entire scattering diagram -apart from external lines corresponds to the same M^2, the braiding operation permutes also fermions at different partonic 2-surfaces located at the ends of string.

The possibility to localize the action of generators $J^{(1)}$ in momentum twistor representation leads to ask whether the stringy generators appearing TGD framework could allow local action using the analog of the space of region momenta. Could M^8-H duality [K74, K66] make this possible? At M^8 level the light-like momenta (in 8-D sense) would correspond to differences of region momenta assignable to strings connecting the partonic 2-surfaces. The 8-D region momenta should be quaternionic. They cannot be light-like as is easy to see.

The notion of region momentum and thus localization would make sense only in M^8, where the wave functions are completely localizable to quaternionic light-like momenta in M^8, whereas in H one has localization to light-like momenta only in preferred M^2 plus wave functions in the space of planes M^4 and in the space of transverse momenta in $E^2 \subset M^4$. This would suggest that M^8-H duality corresponds to the duality of twistor and momentum twistor representations.

What would be new that this duality would be realized also at the level of space-time surfaces. One would have associative/quaternionic space-time surfaces in M^8 and preferred extremals of dimensionally reduced Kähler action in H identifiable as 6-D holomorphic surfaces representing twistor spaces of space-time surfaces.

Note that M^8-H duality could be seen as a number-theoretic analog of spontaneous compactification. Non-perturbative effects would force a delocalization in the space of light-like 8-momenta in M^8 to give states having interpretation as wave functions in H. Nothing would happen to the topology of M^8. Only the state space would be compactified.

5.4.4 A new kind of duality of old duality from a new perspective?

M^8-H duality [K74, K66] maps the preferred extremals in H to those $M^4\times CP_2$ and vice versa. The tangent spaces of an associative space-time surface in M^8 would be quaternionic (Minkowski) spaces.

In M^8 one can consider also co-associative space-time surfaces having associative *normal* space [K74]. Could the co-associative normal spaces of associative space-time surfaces in the case of preferred extremals form an integrable distribution therefore defining a space-time surface in M^8 mappable to H by M^8-H duality? This might be possible but the associative tangent space and the normal space correspond to the same CP_2 point so that associative space-time surface in M^8 and its possibly existing co-associative companion would be mapped to the same surface of H.

This dead idea however inspires an idea about a duality mapping Minkowskian space-time regions to Euclidian ones. This duality would be analogous to inversion with respect to the surface of sphere, which is conformal symmetry. Maybe this inversion could be seen as the TGD counterpart of finite-D conformal inversion at the level of space-time surfaces. There is also an analogy with the method of images used in some 2-D electrostatic problems used to reflect the charge distribution outside conducting surface to its virtual image inside the surface. The 2-D conformal invariance would generalize to its 4-D quaterionic counterpart. Euclidian/Minkowskian regions would be kind of Leibniz monads, mirror images of each other.

1. If strong form of holography (SH) holds true, it would be enough to have this duality at the informational level relating only 2-D surfaces carrying the holographic information. For instance, Minkowskian string world sheets would have duals at the level of space-time surfaces in the sense that their 2-D normal spaces in X^4 form an integrable distribution defining tangent spaces of a 2-D surface. This 2-D surface would have induced metric with Euclidian signature.

   The duality could relate either a) Minkowskian and Euclidian string world sheets or b) Minkowskian/Euclidian string world sheets and partonic 2-surfaces common to Minkowskian and Euclidian space-time regions. a) and b) is apparently the most powerful option information theoretically but is actually implied by b) due to the reflexivity of the equivalence relation. Minkowskian string world sheets are dual with partonic 2-surfaces which in turn are dual with Euclidian string world sheets.

(a) Option a): The dual of Minkowskian string world sheet would be Euclidian string world sheet in an Euclidian region of space-time surface, most naturally in the Euclidian "wall neighbour" of the Minkowskian region. At parton orbits defining the light-like boundaries between the Minkowskian and Euclidian regions the signature of 4-metric is $(0,-1,-1,-1)$ and the induced 3-metric has signature $(0,-1,-1)$ allowing light-like curves. Minkowskian and Euclidian string world sheets would naturally share these light-like curves aas common parts of boundary.

(b) Option b): Minkowskian/Euclidian string world sheets would have partonic 2-surfaces as duals. The normal space of the partonic 2-surface at the intersection of string world sheet and partonic 2-surface would be the tangent space of string world sheets so that this duality could make sense locally. The different topologies for string world sheets and partonic 2-surfaces force to challenge this option as global option but it might hold in some finite region near the partonic 2-surface. The weak form of electric-magnetic duality [K88] could closely relate to this duality.

In the case of elementary particles regarded as pairs of wormhole contacts connected by flux tubes and associated strings this would give a rather concrete space-time view about stringy structure of elementary particle. One would have a pair of relatively long (Compton length) Minkowskian string sheets at parallel space-time sheets completed to a parallelepiped by adding Euclidian string world sheets connecting the two space-time sheets at two extremely short (CP_2 size scale) Euclidian wormhole contacts. These parallelepipeds would define lines of scattering diagrams analogous to the lines of Feynman diagrams.

This duality looks like new but as already noticed is actually just the old electric-magnetic duality [?]een from number-theoretic perspective.

5.5 TGD view about construction of twistor amplitudes

In the following TGD view about twistorialization and its relation to other visions about TGD is discussed. I start with a brief summary of twistor approach to scattering amplitudes and then describe the application of this approach TGD.

5.5.1 Some key ideas of the twistor Grassmann approach

In the following I summarize the basic technical ideas of twistor Grassmann approach. I am not a specialist. On the other hand, my views about twistorialization of TGD differ in many aspects about those applied in the twistorialization of gauge theories, and my own attention is directed towards the physical interpretation and mathematical consistency rather than calculational techniques.

Variants of twistor formalism

The reader can find details about twistors in the article of Witten [B16] and in the thesis of Trnka [B43] (see `http://tinyurl.com/zbj9ad7`).

1. Helicity spinor formalism assigns to light-like momentum pair of conjugate spinors $(\lambda_a, \tilde{\lambda}_{\dot{a}})$ transforming in conjugate representations of Lorentz group $SL(2,C)$. Light-like momentum is expressible as $p^k\sigma_k$ using Pauli sigma matrices and this gives the representation as matrix components $p^{a\dot{a}} = \lambda^a\lambda^{\dot{a}}$. The determinant of the matrix equals to $p^kp_k = 0$ since its rows are linearly dependent.

   One can introduce the bilinears $[\tilde{\lambda}_1, \tilde{\lambda}_2] = -[\tilde{\lambda}_2, \tilde{\lambda}_1]$ and $\langle\lambda_1, \lambda_2\rangle = -\langle\lambda_2, \lambda_1\rangle$ using the antisymmetric Lorentz invariant bilinear defined by permutation symbols ϵ^{ab} and $\epsilon^{\dot{a}\dot{b}}$. The inner product $p_1 \cdot p_2$ is expressible as $p_1 \cdot p_2 = \langle\lambda_1, \lambda_2\rangle[\tilde{\lambda}_1, \tilde{\lambda}_2]$.

   One could express also polarization vectors of massless bosons using pair $(\lambda, \tilde{\mu})$ of helicity spinors. There is however a more elegant approach available. The spinors $(t\lambda, \tilde{\lambda}/t)$ correspond to same momentum for all non-vanishing complex values of t. t represents an element of little group of Lorentz group leaving the helicity state invariant. The helicity dependence of the scattering amplitude is fixed by the transformation property under little group and coded to the weight under the scalings by t: $A(t_a\lambda, t_a^{-1}\tilde{\lambda}_a) = t_a^{-2h_a}A(\lambda,\tilde{\lambda})$. Thus the formalism allows very elegant description of spin and can be applied in SUSYs.

For Minkowski signature (2,2) the spinors are real and this makes this signature preferred. Personally I see this as a basic problem of twistorialization. A possible TGD inspired solution of the problem is provided by the effective replacement of M^4 with M^2 with signature $(1,1)$ and thus allowing real spinors.

2. Twistors $(\lambda_a, \mu_{\dot{a}})$ are obtained by performing a twistor Fourier transform of scattering amplitude $A(\lambda,\tilde{\lambda})$ with respect to $\tilde{\lambda}$.

   At local level [B16] the twistor transform corresponds to Fourier transform

$$\begin{array}{c} \tilde{\lambda}_{\dot{a}} \rightarrow i\frac{\partial}{\mu^{\dot{a}}} \ , \\ -i\frac{\partial}{\tilde{\lambda}^{\dot{a}}} \rightarrow \mu_{\dot{a}} \ . \end{array}$$

   The action of little group corresponds now to the scaling $(\lambda,\mu) \rightarrow t(\lambda,\mu)$ and does not affect the helicity state. For this reason twistors differing by complex scaling can be identified. The proper twistor space is CP_3 rather than C^4.

   The twistor transform of the amplitude transforms as $A(t_a\lambda, t_a\tilde{\lambda}_a) = t_a^{-2h_a-2}A(\lambda,\mu)$.

   In signature (2,2) the helicity spinors $(\lambda,\tilde{\lambda})$ are real so that the twistor Fourier transform reduces to an ordinary Fourier transform. In signature (1,3) the rigorous definition is rather challenging and is discussed by Penrose [B41]. One manner to define the transform is by using residue integral. Residue integral is also p-adically attractive.

   The incidence relation of Penrose given by

$$\mu_{\dot{a}} = -x_{a\dot{a}}\lambda^a$$

   relates M^4 coordinates to λ,μ. By little group invariance entire complex twistor line corresponds to a given point of M^4.

   The twistor transform of plane wave allows to construct the twistor transform of momentum space wave function, and is given by $\delta^2(\mu_{\dot{a}} + x_{a\dot{a}}\lambda^a)$, which is non-vanishing at complex light ray. Twistor Fourier transform in real Minkowski space is therefore non-vanishing at light ray and maps light rays to twistors.

   If the incidence relation for given (λ,μ) is satisfied at two space-time points m_1, m_2, the difference $m_1 - m_2$ is a light-like vector since corresponding matrix has vanishing determinant. Two intersecting twistor lines correspond to M^4 points with light-like distance. This allows to develop geometric picture about twistor diagrams in which the external light-like momenta correspond to intersections of twistor lines assignable to the internal lines of graph.

3. Momentum twistors define a third basic notion. It is convenient to describe particle scattering with external light-like momenta in terms of a diagram in which the external momenta are assigned with the vertices of a polygon such that the lines carry possibly complex momenta. Clearly, the polygon like object is obtained by repeatedly adding light-like momenta to the polygon and since the sum of the external momenta vanishes, the polygon closes.

   The vertices of polygon correspond to intersections of twistor lines defining light-like momenta as differences of the momenta associated with the lines meeting at the vertex. One can assign to the complex momenta of internal lines twistors known as momentum twistors.

   Dual momentum twistor is a further variant of twistor concept being defined in terms of three adjacent momentum twistors contracting them with the 4-D permutation symbol defined in the representation of twistor as a point of C^4 [B43].

Leading singularities

Twistor Grassmann approach to planar loop amplitudes relies on the idea that the discontinuities associated with the singularities of the scattering amplitudes carry all information about the amplitudes. This of course holds true already for the tree diagrams having only poles as singularities.

The idea is same as in the case of analytic continuation: 1-D data at poles and cuts allows to construct the functions. This idea generalizes to functions of several variables and leads to a generalization of residue calculus. At space-time level strong form of holography (SH) relies on the same idea: the 3-D data determine 4-D dynamics and in TGD allowing strong form of holography 2-D data is almost enough.

The discontinuities assignable to singularities can have lower-dimensional singularities so that a hierarchical structure is obtained. The leading singularities are those for which maximal number of propagators are on mass shell and the diagram decomposes to a product of diagrams with virtual particle on mass shell. For one loop diagrams the maximal number of propagators is $N = 4$ corresponding to the fixing of four components of loop momentum. For L loops it is $N = 4L$.

Non-leading singularities have less than the maximal number of propagators on shell and this leaves integral over a subset of loop momenta. If the number of propagator is larger than $4L$, one can have kinematical singularities for some combinations of external momenta.

In the case of scattering amplitudes in twistor Grassmann formulation one encounters a similar situation. In twistor Grassmann approach one defines also the loop integrals in momentum space as residue integrals in the space of complexified momenta. If the functions involved are rational functions the residue integrals are well-defined.

One of the surprising findings is that the leading singularities of MHV loop amplitudes always proportional to tree amplitudes. Second finding is that for $\mathcal{N} = 4$ theory the leading singularities determine completely the scattering amplitudes [B43].

In TGD framework quantum criticality suggests that locally all loop corrections vanish and coupling constant evolution is discrete. This would mean that the only singularities correspond to poles of propagators and this indeed leads to diagrams in which internal lines have complex on mass shell momenta. If this vision is correct, this part of twistor Grassmann approach does not look relevant from TGD point of view.

BCFW recursion formula

The original form of BCFW recursion formula [B9] was derived for tree diagrams. The finding was that the diagrams can be decomposed to two pieces containing with a propagator line connecting them.

1. The proof of this result was rather simple in spinor helicity formalism and based on modification of two momenta p_k and p_n by BCFW shift:

$$\begin{aligned} p_k(z) &= \lambda_k(\tilde{\lambda}_k - z\tilde{\lambda}_n) \ , \\ p_n(z) &= (\lambda_n + z\lambda_k)\tilde{\lambda}_n) \ , \end{aligned} \tag{5.5.1}$$

   Obviously, the modification is induced by modifications $\tilde{\lambda}_k$ and λ_n. With some assumptions about asymptotic behaviour of scattering amplitude A, one can express the original amplitude $A = A(z = 0)$ as residue integral

$$A(z=0) = \frac{1}{2\pi}\oint_C dz \frac{A(z)}{z} \ . \tag{5.5.2}$$

   Here C does not close any other poles than $z = 0$. This integral is the negative of the residue integral around the complement of the region closed by C.

2. It is assumed that poles are the only singularities in this region. Hence one can express $A(z)$ as sum of its poles

$$A(z) = \sum_i \frac{c_i}{z-z_i} \quad . \tag{5.5.3}$$

3. With these assumptions the residue integral gives

$$A = A(0) = \frac{1}{2\pi}\sum_i \frac{c_i}{z_i} \quad . \tag{5.5.4}$$

This leads to the desired factorization with c_i reducing to a product of amplitudes and z_i identifiable as a complex pole for the propagator connecting the sub-diagrams in the decomposition.

In [B18] details of the BCFW shift in the general case are given. One assumes a more general shift $p_i \to \hat{p}_i = p_i + zr_i$ such that r_i are light-like, mutually orthogonal, orthogonal to p_i, and sum up to zero. The modified momenta are complex massless and sum up to zero. One can define $P_I = \sum_{i<I} p_i$ and $R_I = \sum_{i<I} r_i$. The shifted variant $\hat{P_I}^2 = P_I^2 + 2zP\cdot R_I$ is linear in z and vanishes for $z = z_I = -P_I^2/P_I\cdot R_I$. Z_I define the counterparts z_i. Performing the residue integral one obtains $A(0) = \frac{1}{2\pi}\sum_I \frac{c_I}{z_I}$.

This formula allows a recursive construction of tree diagrams by starting from the basic vertices of YM theory. BCFW recursion formula was later generalized to a recursion for the sum planar loops diagrams in terms of diagrams with lower number of loops [B18, B43].

Scattering amplitudes in terms of Yangian invariants defined as multiple residue integrals in Grassmannian manifolds

The generators of Yangian are ordinary conformal generators with conformal weight 0 and dual generators with conformal weight 1. The latter generators act in simple manner in momentum twistor space.

Twistor Grassmannian approach utilizing either twistors or momentum twistors allows to demonstrate that these both conformal symmetry and its dual are present.

The construction of Yangian invariants is summarize in [B43]. Grassmannian residues are Yangian invariants. Yangian transformation introduces total divergence and is exact if its integral vanishes. The operations producing new Yangian invariant can change n or k or both.

1. There are several relatively trivial ways to construct Yangian invariants. One can take the integrand of n-1-D invariant and formally interpret it as integrand of n-D invariant. One can integrate over one twistor variable so that n decreases by one unit.

   Invariants can be multiplied. One can a merge invariants by identifying the twistors in the factors of the product. For instance, one can take the fundamental invariants defining 3-vertices and multiply them to build twistor box giving rise to four particles. One can also merge invariants by integrating over the identified invariants.

2. Inverse soft factor [B37] adds to the diagram expressed in terms of spinor helicity formalism one new particle but keeps k constant. Therefore this operation does cannot be applied in TGD where one has only fermions as external particles. The operation can be formulated as a linear shift for $\tilde{\lambda}_a$ and $\tilde{\lambda}_b$.

3. One can prove the BCFW recursion formula for tree diagrams [B9] by using a deformation of the twistor amplitude in helicity spinor formalism allowing to deduce the factorized formula of the amplitude, two adjacent external lines and deform the twistors λ and $\tilde{\lambda}$ in helicity spinor representation by performing the BCFW shift [B40].

This deformation describes interaction between the external lines, and is essential in the construction of the scattering amplitudes using BCFW recursion. One takes the sum over the products of diagrams with left and right helicities obtained by putting internal particle on mass shell and adds BCFW bridge. BCFW allows to construct all tree amplitudes by starting from fundamental 3-particle amplitudes.

4. Entangled removal [B20, B43, B18] removing two external particles producing a loop in the sense of Feynman diagrammatics but residue of the pole of the propagator is possible and appears as part of the boundary operation for the diagrams. The resulting recursion formula allows to deduce loop corrections.

Twistor Grassmann diagrams are known to allow “moves” [B43, B19]. For instance, moves can be used to remove boxes: it is known that apart from scaling factors depending on momenta the diagrams are reducible to ordinary tree diagrams [B43] (`http://tinyurl.com/zbj9ad7`). This allows to consider the possibility that twistor trees could allow to construct all diagrams. Note however that the moves reducing the twistor diagram to a counterpart of tree diagram gives an overall multiplicative factor depending on momenta and helicities.

From TGD point the definition of loop integrals and Grassmannian integrals as residue integrals is of great potential importance. Scattering amplitudes should be number theoretically universal but in p-adic context the definition of definite integral is very difficult. Residue integral provides however a manner to define multiple residue integrals using only holomorphy and the notion of pole. This could be the deep reason for why one should be able to reduce loop integrals to residue integrals.

There is however a potential problem involved related to number theoretic universality. 2π does not exist p-adically in any reasonable sense (if one wants to define it one must introduce infinite-D extension of rationals by powers of 2π. One might hope that 2π cancels from the scattering amplitudes by normalization. Another possibility is that for an extension containing $exp(i2\pi/N)$ as the highest root of unity, one can define π approximately as $i\pi \equiv N\times(exp(i\pi/N)-1)$. An alternative option is that only the analogs of tree diagrams having only poles as singularities are possible

Linearization of the twistorial representation of overall momentum delta function

An little but not insignificant technical detail [B20] is the linearization of the constraint expressing the overall momentum conservation by interpreting it as a condition in Grassmannian $G(k,n)$, where k is the number of negative helicities and n is the number of particles, and allowing to reduce integrations over $G(k,n)$ to those over $G(k-2,n-4)$.

Spinor helicity diagrams and twistor diagrams are proportional to a delta function expressing overall momentum conservation. Dropping twistor indices this delta function one reads as $\delta(\sum_k P_k) = \delta(\lambda_i\tilde{\lambda}_i)$. One can combine the 2 components of λ_i and $\tilde{\lambda}_i$ to form 2+2 n-component vectors and interpret momentum conservation as orthogonality conditions for the 2-planes spanned by λ_a and $\tilde{\lambda}_{\dot{a}}$ for $k>2$. These plane spanned by 2 n-component λ vectors can be interpreted as 2 vectors in $G(k,n-k)$ defining rows of $G(k,n-k)$ matrix. $\tilde{\lambda}$ defines a similar plane in $G(n-k,k)$.

These conditions are equivalent with the condition that there exists in $G(k,n)$ a 2-D C and its $n-k$-dimensional orthogonal complement $\tilde{C}$ such that the 2-plane spanned by λ_a is orthogonal to $\tilde{C}$ and the two-plane spanned by $\tilde{\lambda}_{\dot{a}}$ is orthogonal to C. These conditions can be expressed as a product of delta functions $\delta(C\cdot\tilde{\lambda})$ and $\delta(\tilde{C}\cdot\lambda)$.

Since $G(k)$ acts as a "gauge symmetry" for $G(k,n)$, the first $k\times k$ block of the $k\times n$ matrix representing a point of C can be transformed to a unit matrix so that $k\times(n-k)$ variables remain.. Same can be carried out for the last $n\times(n-k)$ block of $\tilde{C}$ by $G(n)$ "gauge invariance" so that $(n-k)\times n$ variables remain. With these gauge choices the orthogonality conditions can be solved explicitly and corresponding integrations can be carried out. The integration over delta functions leaves $(k-2)(n-k-2)$ variables, the dimension of $G(k-2,n-4)$. $G(k,n)$ reduces to $G(k-2,n-4)$ by momentum conservation.

5.5.2 Basic vision behind scattering amplitudes

It is good to summarize the basic vision about TGD first.

Separation of WCW functional integral and fermionic dynamics

The works of Penrose and Witten have served as inspiration in the attempts to twistorialize TGD and led to the conjecture that the twistor lift of TGD is possible and means that space-time surfaces are replaced with their twistor spaces representable as 6-D surfaces in 12-D product of twistor spaces of M^4 and CP_2. What makes this idea so attractive is that S^4 and CP_2 are the only 4-D compact manifolds with Euclidian signature having twistor space with Kähler structure [A18]. TGD would be unique both from the existence of the lift of Kähler action to the product of twistor spaces of M^4 and CP_2!

What the twistor space of M^4 is, is however not at all clear. It can be defined in two ways: as the usual CP_3 very natural at the level of momentum space or as the trivial bundle $T(M^4) = M^4 \times S^2$ natural in the twistorialization at classical space-time level. Standard twistorialization has however problems.

1. There is problem associated with the signature. Twistorialization works best at signature $(2, 2)$ for Minkowski space and gives rise to real projective space P^3.
2. Second problem is that CP_3 should be actually $SU(2, 2)/SU(2, 1) \times U(1)$. There is clearly something not so well understood.

In the number theoretic vision about TGD twistor space would be replaced with commutative hyper-complex $M_2 \subset M^4 \subset M^8$ and this space is just RP^3 and problems wth signature disappear since 2-D spinors can be chosen to have real basis. For complex momenta this extends to CP_3. Number theory would also justify the identification of geometric twistor sphere as $M^4 \times S^2$.

In TGD the dynamics of fields is replaced with that for 4-surfaces. Penrose's idea about generalization of holomorphy of field modes in twistor space generalizes to the holomorphy of the representation of 6-surface representing twistor bundle of space-time leads to a concrete ansatz for space-time surfaces as preferred extremals [L12] [L26].

SH leads to the proposal that the data determining space-surfaces are preferred extremals is given at 2-D surfaces and these 2-D surfaces bring in mind Witten's twistor strings [B16]. By SH the functional integral over them would correspond to that over WCW and twistor amplitudes asignable to given space-time surface would be constructed at fermionic level by the analog of twistor Grassmannian approach. This integral over 2-surfaces corresponds to the deviation of TGD from QFT in fixed background and cannot be equivalent with the introduction of twistor strings.

Adelic physics and scattering diagram as a representation of computation

Adelic physics [L21] suggested to provide quantum physical correlates also for cognition is in a central role. Adelic physics predicts the hierarchy $h_{eff} = n \times h$, where n as dimension of the extension is divisor of the order its Galois group identified in terms of dark matter regarded as a phase of ordinary matter. p-Adic physics and p-adic length scale hypothesis could be also understood.

The number theoretic universality of scattering amplitudes suggests that all loops vanish identically and the evolution of various couplings constants is discrete occurring by phase transitions changing the extension of rationals and values of various coupling parameters.

1. The vanishing of loops at the level of space-time action would mean that the loops associated with the functional integral defined by the action, which is sum of Kähler action and volume term. This vanishing would state essentially local quantum criticality as invariance of coupling parameters under local renormalization group evolution. One would obtain only a sum of action exponentials since Gaussian and metric determinants cancel each other in Kähler metric.
2. Exponents of Kähler action represent a number theoretical nightmare.
   (a) The functional integral expressions for scattering amplitudes are normalized by a functional integral for for the vacuum state. This implies that only the ratios X_i/X of the exponents X_i for the extrema and sum $\sum X_i$ appear in the amplitudes [L21] so that there are slightly better hopes of achieving number theoretic universality.

   (b) Number theoretical universality forces to imagine even more attractive option making sense in ZEO but not in standard ontology. If the amplitude is sum over the contributions normalized by corresponding exponentials X_i rather than $\sum X_i$, exponentials cancel altogether and the couplings constants appear only in boundary conditions. In this case one could speak of a basis of zero energy states assignable to various extrema of the action. The real part of the action is maximum and the the imaginary part of the action saddle point if preferred extrema are minimal surface extremals of Kähler action [L12]. Number-theoretical universality more or less forces this option.

3. An even stronger proposal is based on the idea that the TGD analogs of stringy diagrams. The lines of these diagrams correspond to light-like parton orbits carrying fermion lines and meeting at vertices which are partonic 2-surfaces. The proposal is that the topological diagrams involving analogs of loops represent algebraic computations so that all diagrams with given initial and final collection of algebraic objects are equivalent.

   If this is the case, all topological diagrams should reduce to topological tree diagrams by a generalization of the duality symmetry of the old-fashioned hadronic string model stating that the sum of s-channel resonances equals to the sum of t-channel exchanges and that these diagrams can be constructed as twistor Grassmann diagrams by allowing on mass shell fermions with complex momenta at internal lines. For external particles the momenta could be real and light-like in 8-D sense. A weaker condition is that real and imaginary parts of complex momenta 8-D momenta are separately light-like and orthogonal.

   One could indeed argue that one cannot allow loops of this kind since it would be impossible to decide which kind graph experimental scattering situation corresponds if all these graphs are different since one observes only the initial and final states. Therefore all scattering diagrams with same real particles in the final states correspond to identical scattering amplitudes.

   These diagrams would correspond to the same amplitude but it might be possible to perform a localization to any of them. p-Adically however the corresponding space-time surface would be different by p-adic non-determinism (the number theoretic discretization - cognitive representation - defined by the common points of reality and p-adicities as space-time surfaces would be different): one might say that the tree representation involves smallest cognitive representation and is therefore the shortest one.

   If the action exponentials X_i cancel from the scattering amplitudes, this option can indeed make sense. Otherwise it is extremely implausible since different contributions would have different vacuum weights.

4. If only the twistor analogs from tree diagrams in Feynman sense are allowed, the scattering amplitudes are rational functions of external momenta as strongly suggested by the number theoretic universality and by the requirement that the diagrams can be interpreted in terms of algebraic computations so that the simplest manner to do the computation corresponds to a tree diagram. Even tree diagrams in Feynman sense are planar so that one would get rid of the basic problem of the twistor approach to SUSY.

Quantum classical correspondence (QCC) states that scattering diagrams have classical counterparts in the sense that fermion lines correspond to the boundaries of string worlds sheets assignable to the light-like orbits of partonic 2-surfaces and topological 3.vertices correspond to 2-surfaces at which the ends of light-like orbits meet. This correlation is extremely restrictive and it is not at all clear whether it leaves room for loops.

In the most general case one would have a superposition of allowed space-time surfaces realizing scattering diagram with given initial and final quantum numbers identified as corresponding classical charges.

The idea about diagram as computation suggests that the simplest possible diagram - tree diagram - is realized together with the corresponding space-time topology. If diagrams with topological loops are possible this requires the existence of moves transforming diagrams to each other. This condition might be not consistent with the condition that the move acts on the space-time surface too. Very simple diagrammatics - even twistor tree diagrammatics - could follow from mere QCC.

Classical number fields and M^8-H duality

Quaternionicity and octonionicity is second central aspect of number theoretical vision.

1. The key concept is $M^8-M^4\times CP_2$ duality allowing to see space-time surfaces quaternionic surface in M^8 or as holomorphic surfaces in the twistor space $T(M^4)\times T(CP_2)$. This would realize SH. Physical states are characterized by quaternionic (possibly complexified-) octonion valued 8-momenta in accordance with the vision that tangent space Minkowskian region of space-time surface is quaternionic and contains preferred hyper-complex M^2, which can depend on point provided that tangent spaces $M^2(x)$ integrate to 2-D surface. This view leads to a new view about QCD color as octonionic color.

2. Twistor space reduces to that associated with M^2 and 2-D variant of conformal invariance corresponds to SO(2,2) and leads to the identification real projective space P^3 as twistor space. One can however complexify it to CP_3 since momenta are in general complex. The signature is (1,1) so that bi-spinors λ, $\tilde{\lambda}$ have real basis and twistor Fourier transform can be defined as ordinary Fourier transform. The reality of M^2 or induced spinors at string world sheets might allow to have SUSY without Majorana spinors.

   The reduction of external momenta to M^2 implies that real and imaginary parts are parallel and light-like. At classical level this poses strong conditions on preferred extremals. This does not require that color and electroweak quantum numbers are complex. The reason is that they emerge as labels of wave functions in twistor space $T(CP_2)$ representing wave functions in the moduli space of transversal E^2s with corresponding helicity identifiable as em charge.

   Localization of the light-like 8-momentum is possible to preferred M_0^2. Localization does not imply the disappearance of color wave function. The transversal E^2 momentum degrees of freedom however disappear. In the case of leptons and hadrons complete localization could be a good approximation but not in the case of quarks.

Elementary particles have fundamental fermions as building bricks

The assumption that the physics of elementary particles reduces at fundamental level to that of fundamental fermions has strong implications, when combined with the twistor Grassmann approach.

1. In TGD elementary particle would correspond to a pair of wormhole throats of wormhole connecting two space-time sheets with Minkowski signature. Wormhole itself would have Euclidian signature. Wormhole contacts would be connected by monopole flux tube with fermionic quantum numbers at the 4 wormhole throats defining the partonic 2-surfaces.

2. Fundamental vertices are associated with 2-surfaces at which light-like 3-surfaces carrying fermions and antifermions as string world sheet boundaries are glued together along their ends. Note that these surfaces are analogous to vertices of Feynman diagrams and singular as 4-surfaces but 3-surfaces are smooth unlike for stringy vertices.

3. Fermion lines correspond to the boundaries of string world sheets at the light-like orbits of partonic 2-surface at which the signature of the induced metric changes. At momentum space M^8 this picture should also make sense since space-time surfaces in M^8 and H would correspond to each other by M^8-H duality. At the level of M^8 the orbits of fermion lines could be seen as light-like geodesics along with twistor spheres move. At the edges of string world sheets they would intersect at single point and give rise to external massless particle.

4. The basic vertex is 4-fermion vertex in which fermions scatter classically and assignable to the 2-surface at which the ends of light-like 3-surfaces representing partonic orbits intersect. There would be no local 4-fermion vertex. Fermions would move as free particles in the background and the background would gives rise to the interaction between fermions at partonic vertices analogous to vertices of Feynman diagrams. This would automatically resolve possible problems caused by divergences and would be analogous to the vanishing of bosonic loops from WCW functional integration.

5. FFB couplings could be identified in terms of $FF(F\overline{F})$ couplings, where $F\overline{F}$ is associated with the same partonic orbit. These couplings would not be fundamental.

What could SUSY mean in TGD?

Extended super-conformal invariance is basic symmetry of TGD but it is not whether it possible to have SUSY (space-time supersymmetry) in TGD framework. Certainly the SUSY in question is not $\mathcal{N}=1$ SUSY since Majorana spinors are definitely excluded. $\mathcal{N}=2$ SUSY generated by right-handed neutrino and antineutrino can be however considered.

1. If one allows the boundaries of string world sheets carry fermion number bounded only by statistics (all spin-charge states for quarks and leptons would define maximal $\mathcal{N}$ for SUSY). This would allow local vertices for fermions and does not look like an attractive option unless SUSY manages to cancel the divergences.

2. SUSY could mean addition of fermions as separate lines to the orbits of wormhole throat. This SUSY would be broken and only approximately local. The question what the propagator for the many-fermion state at same string line is, is not quite obvious. SUSY would suggest propagator determined by the total spin of the state. I have also considered the possibility that the propagator is just the product of fermionic propagators acting on tensor power of single fermion spaces. The propagator behaves as $1/p^N$ for N fermion state and only for $N=1,2$ one would have the usual behavior. This option is not attractive.

3. SUSY could mean addition of right-handed neutrino or its antiparticle to the throat. The short range of weak interactions is explained by assuming that pair of right-handed neutrino and left-handed neutrino compensates the weak isospin at the second wormhole throat carrying quantum numbers of quark or lepton.

   Addition of right-handed neutrino or its antiparticle or both to a given boundary component could give rise to $\mathcal{N}=2$ SUSY. The breaking of SUSY could correspond to different p-adic length scales for spartners. Mass formula could be exactly the same and provided by p-adic thermodynamics. Why the p-adic mass scale would depend so much on the presence of covariantly constant ν_R having no color and ew interactions nor even gravitational interaction, remains to be understood. If the extensions of rationals are different for the members of SUSY multiplet, the corresponding preferred p-adic primes would be different and this could explain the widely different p-adic mass scales. One can of course ask the covariant constancy means that ν_R does not have any coupling to anything and its presence is undetectable.

5.5.3 Options for the construction of scattering amplitudes

There are several guidelines in the construction of scattering amplitudes.

1. SH in strongest form would mean that string word sheets and partonic 2-surfaces are all that is needed. In number theoretical vision also fixing the extension of rationals associated with the intersection of realities and p-adicities is needed and leads to a hierarchy of extensions which could realized discrete coupling constant evolution. SH would suggest that hybrids for analogs of string diagrams and Feynman diagrams code for the scattering amplitudes.

2. QCC suggests that the eigenvalues of the Cartan algebra generators of symmetries are equal to classical Noether charges. A weaker condition is that the eigenvalues of fermionic generators not affecting space-time surfaces are equal to the classical Noether charges. The generators have also bosonic parts acting in WCW.

   A prediction following from the condition that there is charge transfer between Euclidian and Minkowskian space-time regions is that the classical charges must be complex valued guaranteed if Kähler coupling strength as a spectrum of complex values. One proposal is that the spectrum of zeros of Riemann zeta determines if [L6]. This supports the twistorial view that momenta in the internal lines can be regarded as complex light-like on mass shell momenta.

3. QCC also suggests that scattering diagrams have space-time correlates. The lines of diagrams correspond to light-like orbits of partons at which the signature of induced metric changes. Vertices correspond to partonic 2-surfaces at which these 3-D lines meet. At fermion level fermion lines at partonic orbits correspond to boundaries of string world sheets.

This however leaves several alternative visions concerning the construction of scattering amplitudes.

What scattering diagrams are?

What does one mean with scattering diagrams is not at alle clear.

1. Are they counterparts of Feynman diagrams so that one would have a superposition of all space-time topologies corresponding to these diagrams? Probably not.

2. Or are they counterparts of twistor Grassmannian diagrams in which all particles are on mass shell but with possibly complex light-like quaternionic 8-momenta in $M^8 = M^4 \times E^4$ with $M^4 = M_0^2 \times E^2$. Why this option is interesting is that twistor Grassmann diagrams allow large number of moves reducing their number.

   This would translate to a conserved and massive longitudinal M^2-momentum; which for a special choice of M^2 is light-like, a wave function in the space of transversal E^2 momenta; color partial wave in the moduli space of E^2 planes for given M_0^2; and em charge describable as CP_2 helicity and allowing twistorialization.

   There is however a problem: the transverse E^6-momentum makes M^2 momentum massive and twistorialization fails. But what if the 8-momenta are real and in twistorial description M^2 momentum becomes complex but light-like. The square for the real part of M^2 momentum would be equal to the square of real E^6 momentum and twistor approach would apply! This map would be define the essence of M^2-twistorialization.

In ZEO one can interpret the construction of preferred extremals as a boundary value problem with ends of space-time surfaces at the boundaries of CD and the light-like orbits of partonic 2-surfaces defining a closed 3-surface and defining the scattering diagram as 3-D boundary. If so, it might be possible to construct rather large number of diagrams, even counterpartz of loop diagrams.

The situation would be analogous to the construction of soap films spanned by wires with wire network analogous to the network formed by the partonic orbits. Also an analogy with 4-D tensor network suggests strongly itself and scattering diagrams representing zero energy states would correxpond to the states of the tensor network.

The basic space-time vertex would be 3-vertex defined by partonic 2-surface.The basic fermionic vertex would be 4-fermion vertex in which fermions do not exchange gauge boson but interact classically at the 2-D vertex. All particles emerge as bound states of fundamental fermions at boundaries of string world sheets.

1. The basic view would be that M^2 momenta, and transversal momenta correspond to M^4-momenta. The moduli space for $M_0^2 \times E^2$ planes corresponds to CP_2 and color quantum numbers. M^2 helicities and electroweak quantum numbers would be coded to the weights twistor wave functions in twistor space if $M^2 \times CP_2$.

2. One approach to scattering amplitudes relies on symmetries. Twistor Grassmannian approach suggest strongly Yangian symmetry. The diagrams should be representations of multi-local Yangian algebra with basic algebra being that of the conformal group of M^4 restricted to M^2.

   This would give nicely real projective space RP^3 allowing to solve some problems of the standard twistor approach. In color degrees of freedom one would have color Yangian: hadrons could correspond to the multilocal generators created by multi-local Yangian generators. The E^2 degrees of freedom would correspond to states generated by Kac-Moody algebra and also now one could have Yangian algebra. The states for the representation of Yangian itself would be singlets.

Besides fermionic lines there are string world sheets. Infinite-D 2-D conformal group and Kac-Moody symmetries act as symmetries for string world sheets. The super-symplectic group would the isometry group of WCW and would give rise to conditions analogous to Super Virasoro conditions. These conditions would be satisfied by preferred extremals realizing number theoretic variant of SH. Also these symmetries would be extended to their Yangian versions naturally.

3. One can argue that classical field equations do not allow all possible diagrams. More precisely, for a given extension of rationals adelic physics allows only finite number diagrams and the extension induces a natural cutoff as minimal distance between points with coordinates in the extension representing intersection of reality and p-adicities [L21].

   The assumption that the end points of fermionic lines at partonic 2-surfaces at ends of CD and at the vertices carry fermions would give an immediate connection with the adelic physics. As the dimension of the extension increases, the number of the points in the intersection increases and more lines appear in the allowed diagrams. This would give rise to a discrete coupling constant evolution, hierarchy of Planck constants, and p-adic length scale hypothesis.

   Quantum criticality strongly suggests that coupling constant evolution is locally trivial and is discretized with discrete steps realized as phase transitions changing the extension. Galois group would be the fundamental number theoretic symmetry group acting on the intersection and its order would correspond to $h_{eff}/h = n$ allowing to realize the analogs of perturbative phases of gauge theories as perturbative phases.

4. The discreteness of coupling constant evolution demands that loop corrections vanish. This makes perfect sense for the functional integral over WCW. But what about fermionic degrees of freedom and topological counterparts of scattering diagrams, which very probably do not correspond to Feynman diagrams but could be analogous to twistor diagrams? For fermions there is actually no perturbation theory since effective 4-fermion vertices correspond to classical scattering of external fermions at partonic 2-surfaces defining the vertices. This is not a problem since thanks to h_{eff} guaranteeing the existence of perturbative expansion.

Three roads to follow

In ZEO construction of scattering amplitudes is basically a construction of zero energy states and one must be very cautious in applying QFT intuitions relying on positive energy ontology. One ends up to to a road fork.

Option I: Can one interpret the topological space-time diagrams as analogs of Feynman diagrams and assume that by quantum criticality the sum over the topological loops vanish? This option looks rather ad hoc.

Option II: Can one assume - with inspiration coming from adelic physics - that the number of these loops with fixed states at the boundaries of CD is finite and one just sums over these states with weights given by the exponential of the space-time action?

Here one encounters problems with number theoretical universality [L21]. One has superposition of vacuum exponentials over the diagrams and number theoretical universality demands that the ratio of given exponential to the sum is in the extension of rationals involved. This is very tough order - perhaps too tough.

Option III: Can one follow number theoretical vision suggesting that scattering diagrams correspond to computations in some sense [L11]. This leads to a new road fork.

1. Option IIIa): Could one generalize the old-fashioned string duality and require that there exist a huge symmetry allowing to transform the scattering diagrams using basic moves to tree diagrams? The basic moves would allow to shift the end of line past vertex and to remove self energy loop and hence the transformation to tree diagrams would become possible. Originally it was inspired by the idea that the vertices of the scattering diagram correspond to products and co-products in quantum algebra and that the condition involved can be interpreted as algebraic identities.

   Twistor Grassmannian diagrams indeed allow moves allowing surprising simplification allowing to show that all loop corrections with a given number of loops sum up to something proportional to a tree diagram [B43].

The assumption that the states moving in the internal lines have light-like quaternionic M^8 momenta gives very strong constraints on the moves and it might well be that the moves are not possible in the general case. Even if the move is possible, the value of the action exponential can change so that this option seems to demand mathematical miracles. The proposed manner to achieve number theoretical universality however eliminates action exponentials.

The mathematical miracle might be made possible by the possibility to find preferred M_0^2 in which the 2-momentum of fermion line is light-like. If M_0^2 is constant along entire fermion line, it seems to be possible perform the gliding operation past vertices as will be found. Note that ach fermion can wander around the network formed by the partonic orbits.

Note that the different space-time surface realizing equivalent computations would be cognitively non-equivalent since the cognitive representation defined by the points in extension of rationals would be different. Optimum computation would have smallest number of points and would correspond to tree diagram.

2. Option IIIb): Should one sum over the possible diagrams so that one would have quantum superposition of computations. This is done for loop diagrams in twistor Grassmann approach. Infinite sum is however awkward number theoretically. Adelic vision suggests that the number of loops is finite. The action exponentials would not disappear from the scattering amplitudes and are very problematic from the point of view of number theoretical universality.

3. Option IIIc): Could one regard the light-like partonic orbits as part of the dynamical system - this is what effectively is done if they form part of connected 3-surface defining the topological scattering diagram - and assume that each such diagram corresponds to a different physical situation analogous to a computation?

   One can argue that one must be also able to localize the zero energy state to single computation by state function reduction [L25]! State function reduction to single diagram should be possible. A rather classical picture about space-time would emerge: one would have just a superposition of space-time surfaces with the same topology and same action apart from quantum fluctuations around the point which is maximum with stationary phase. One would also have color wave functions and momentum wave functions in cm degrees of freedom of partonic 2-surfaces as WCW degrees of freedom.

   The action exponential, which is very problematic from the point of view of number theoretic vision, would be cancelled from the functional integral since it is normalized by the action exponential. The dependence on coupling parameters is however visible in the boundary conditions at boundaries of CD stating the vanishing of most supersymplectic charges and identifying the remaining super-symplectic charges and also isometry charge with the fermionic counterparts.

   This picture would be extremely simple and would be analogous to that of integrable quantum field theories in which the integral over small fluctuations gives Gaussian determinant and action exponential (now Gaussian determinant is cancelled by the metric determinant coming the Kähler metric of WCW) [K66].

   One can argue that the absence of loops makes it impossible to have non-perturbative effects. This is not true in adelic physics. Recall that the original motivation for $h_{eff} = n \times h$ was that this phase is generated with perturbation theory ceases to converge [K21, K22, K23, K24]. The large value of h_{eff} scales down the coupling strengths proportional to $1/h_{eff}$ and perturbation theory works again.

It must be admitted that one must accept all these options. Number theoretical universality of scattering amplitudes would select $IIIa$) and the need to realize given topological diagram using complex enough extension of rationals supports Option IIIc). I believe that the large number of the options reflects my limited mathematical understanding of the situation a careful analysis of the general implications of the options allows to pinpoint the most feasible one.

5.5.4 About problems related to the construction of twistor amplitudes

The dream is to construct twistorially fermionic scattering amplitudes and this requires the identification of fermionic 4-vertex. There are however several conceptual problems to be solved.

Could M^2 momenta be massive?

The naïve objection against massive particles is that one loses the twistorial description both in M^4 sense and M^2 sense. Real quaternionic M^8 momenta are massless but the transversal momentum in E^6 degrees of freedom makes M^2 momenta and M^4 momenta for arbitrary choice of M^4 are massive, and one cannot describe the M^2 and M^4 momenta using the helicity spinor pair $(\lambda, \tilde{lambda})$. The beautiful formalism seems to be lost.

1. The naïve argument is however wrong in TGD framework where particles are massless in M^8 sense. This means that mass does not correspond to $\overline{\Psi}\Psi$ in Dirac action but to comes from E^4 momentum (CP_2 "momentum"). 8-D chiral symmetry is unbroken as required by separate conservation of lepton and baryon numbers. In preferred M_0^2 one can indeed make M^2-momentum light-like.

2. Furthermore, 4-fermion twistor amplitudes are *holomorphic* functions of λ_i . There is no dependence of $\tilde{\lambda}$ and therefore no information about light-likeness! Why this amplitude could not describe the scattering of fermions only apparently massive in TGD Universe? Note that the momentum conserving delta function depends on the masses of the particles so that mass-dependence would be purely kinematical and analogous to the dependence on transverse momentum squared. Note that this argument makes sense also for M^4 twistorialization. If this view is correct then twistors are something more profound than momenta.

3. For M^2 twistorialization end would end up to effective (2,2) signature favored by twistorialization. (1,1) signature of real M^2 becomes (2,2) signature for complexified M^2 and real twistor space RP^3 is replaced with CP_3. This looks attractive description. If this picture is correct, all the nice results such as the possibility to assume reduction of amplitudes to positive Grassmannian remain unaffected.

Momentum conservation and mass shell conditions in 4-vertex

What is the exact meaning of the mass shell condition?

1. $H = M^4 \times CP_2$ harmonics would suggest that it mass squared in M^4 is eigenvalue of spinor d'Alembertian plus possible super-conformal contribution from Super Virasoro algebra, which is integer valued in suitable units. M^4-momentum decomposes to longitudinal M_0^2 momentum and transversal E^2 momentum. Super Virasoro algebra in transversal degrees of freedom suggests quantization of E^2 mass squared in integer multiples of a basic unit.

2. The CP_2 part of wave function in H corresponds in M^8 to a wave function in the moduli space of transversal planes E^2 assignable to M_0^2 and is involved only if the deformations of M^4 (or equivalently E^2) are present.

3. In the preferred frame M_0^4 the wave function would be strictly localized in single point of CP_2 and have maximally uncertain color quantum numbers. This kind of localization does look feasible physically. For instance, for color singlet CP_2 wave function of right-handed neutrino there is no localization. For sharp localization of 8-momentum to M_0^2 both color degrees and transvervsal E^2 degrees of freedom would effectively disappear.

4. The wave function in transversal E^2 momentum space with interpretation in terms of transversal momentum distribution - this at least in the case of hadrons.

5. The physically motivated assumption is that string world sheets at which the data determining the modes of induced spinor fields carry vanishing W fields and also vanishing generalized Kähler form $J(M^4) + J(CP_2)$. Em charge would be the only remaining electroweak degree of freedom. The identification as the helicity assignable to $T(CP_2)$ twistor sphere looks therefore natural. Note that the contribution to mass squared would be proportional to Q_{em}^2 so

that one would obtain the electroweak mass splitting automatically. This is true also for CP_2 spinor harmonics.

How plausible topological loops are?

Topological loops are associated with the networks formed from the orbits of partonic 2-surfaces meeting at their ends (this would define topological 3-vertex containing fermionic 4-vertex). The tree topologies would provide a nice space-time description of particle reactions but loops could be possible? The original vision about construction of WCW geometry indeed was that the space-time surfaces with fixed ends are unique.

In the original vision the non-determinism of Kähler action inspired the hypothesis that loops are possible but volume term removes to high extent this non-determinism. In the recent vision the fusion of 3-surfaces at the ends of CD with light-like parton orbits to single 3-surface as a boundary condition (analogous to a fixing of a frame for soap films) would define the scattering diagram classically. There is no reason why it could not contain topological loops. Option IIIa) assuming that one can transform the diagrams ot tree diagrams, is therefore attractive.

1. There are also conditions from space-time dynamics. Twistor graph topologies correlate with space-time topologies since fermion line are inside the parton orbits and at vertices the ends of the orbits meet. Topological vertices would be basically 3-vertices for partonic 2-surfaces. The fermion and anti-fermion lines associated with the effective boson exchange would be naturally associated with opposite throats of wormhole contact.

   By above argument one can in ZEO pose at space-time level conditions fixing the vertices and identify the graph topology as a topology of the network of light-like 3-surfaces defining the diagram as boundary of 3-surface defined by the union of the ends of space-time and by parton orbits forming a connected surface.

2. There is a further delicacy to be taken into account - measurement resolution coded by the extension of rationals involved. This might allow to interpret addition of loops as in quantum field theories: as a result of increased measurement resolution determined dynamically by the intersection of reality and p-adicities. Different computation yielding the same result would not be cognitively equivalent since these intersections would be different.

3. If this view is correct, one can obtain also loops but non-negativity of energy for a given arrow of time for quantum state would allow only loops resulting from the decay and re-fusion of partonic 2-surfaces. Tadpoles appearing in BCFW recursion formula are impossible if the energy is non-negative. One can of course ask whether the sign of energy could be also negative if complex four-momenta are allowed. If so, one could have also tadpoles classically.

Identification of the fundamental 4-fermion vertex

The fundamental 4-fermion vertex would not be local 4-fermion vertex but correspond to classical scattering at partonic 2-surface. This saves from the TGD counterparts of the problems of QFT approach produced by non-renormalizability.

What would be this 4-fermion vertex? Yangian invariance suggests that the classical interaction between fermions must be expressible in terms of fictive 3-vertex of SUSY theories describing classical interaction as exchange of a fictive boson. This leaves 3 options.

Option I: 4-fermion vertex could be fusion of two 3-vertices with complex massless 8-momenta in M^8 picture. For instance, the exchanged momentum could be complex massless momentum and external momenta real on-mass-shell momenta. This vertex does not have QFT counterpart as such.

Loops could be absent either in the strong sense twistorial loops are absent (Option Ia) or in the sense that corresponding Feynman diagrams contain no loops (Option Ib). In particular, formation of BCFW bridge would not be allowed for Option Ia). Given diagram would be twistorial tree diagram obtained by replacing the vertices of ordinary tree diagram with these 4-vertices with complex massless fermions in 8-D sense.

Option II: 4-fermion could be identified as BCFW bridge associated with a tree Feynman diagram describing an exchange of a fictive boson. This 4-vertex would be analogous to an exchange of ordinary boson and counterpart for a QFT tree diagram. One can even forget the presence of the fictive boson exchange and write the formula for the simplest Yangian invariant as a candidate for four-fermion vertex.

Option III: If one allows higher fermion numbers at the same line, it is also natural to allow branching of lines. This requires allowance of 3-vertex as branching of fermion line as analog of splitting of open string (now strings are actually closed if they continue to another space-time sheet through wormhole contact). The situation would resemble that in SUSY. One cannot completely exclude this possibility.

Consider now the construction of 4-fermion vertex in more detail.

1. The helicities of fermions are $h_i = \pm 1$ and the general conjecture for the 4-fermion twistorial scattering amplitude is the simplest possible holomorphic rational function in λ_i, which does not depend on $\tilde{\lambda}_i$, and satisfies the condition that the scaling $\lambda_i \to t\lambda_i$ introduces the scaling factor t^{-2}.

2. The rule is that fermions correspond to 2 positive powers of λ_i and antifermions to 2 negative powers in λ_i: schematically the $F_1F_2\bar{F}_3\bar{F}_4$ vertex is of form $\lambda_1^2\lambda_2^1/\lambda_3^2\lambda_4^2$ and constructible from $\langle\lambda_i, \lambda_j$. One can multiply any term in the expression of vertex by a rational function of for which the weights associated with λ_i vanish. Ratios $P_i(f)/P_j(f)$ of functions $P(f)$ obtained by via odd permutations P of the arguments λ_i of function

$$f(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = \langle\lambda_1, \lambda_2\rangle\langle\lambda_2, \lambda_3\rangle\langle\lambda_3, \lambda_4\rangle\langle\lambda_4, \lambda_1\rangle$$

3. invariant under 4 cyclic permutations. The number of these functions would be $4!/4 = 3! = 6$ corresponding to the 6 orbits of an odd permutation under the cyclic group Z_4. The simplest assumption is that these functions are not involved.

   The simplest guess for the 4-fermion scattering amplitude would be following:

$$T(F_1, F_2, \overline{F}_3, \overline{F}_4) = J \times \frac{\langle\lambda_1, \lambda_2\rangle^2}{\langle\lambda_3, \lambda_4\rangle^2} \quad . \tag{5.5.5}$$

   Charge conjugation would take the function to itse inverse. J is constant.

4. In 4-fermion vertex one has exchange of fictive boson and annihilation to fictive boson and the particles i, j in the vertex should contribute $\langle\lambda_i, \lambda_j\rangle$ to the scattering amplitudes.

Remarkably, this amplitude is holomorphic in λ_i and has no dependence on $\tilde{\lambda}_i$ and therefore carries no information about whether the momenta are light-like or not. It seems that one could allow massive fermions characterized by (λ_i, μ_i) and fermion masses would not be a problem! As already explained in TGD mass is not M^8-scalar and states are massless in 8-D sense: hence twistorialization should work!

One could construct more complex diagrams in very simple manner using these basic diagrams as building bricks just as in the twistor Grassmann approach. One could form product of diagrams A and B using merge operation [B43] identifying twistor variables Z_a and Z_b belonging to the two diagrams A and B to be fused.

For Option Ia) the diagram would represent repeated on mass shell 4-fermion scatterings but with of mass shell particles having complex momenta in 8-D sense. Real on mass shell particles would have massless but real 8-D momenta and physical polarizations.

The conservation of baryon and lepton numbers implies for all options that only $G(m, n = 2\times m)$ Grassmannians are needed. This simplifies considerably the twistor Grassmannian approach.

Why fermions as fundamental particles (to be distinguished from elementary particles in TGD) are so special?

1. The mass of the fundamental fermion is not visible in the holomorphic basic amplitude being visible only via momentum conserving delta function $\delta(\sum_i \lambda_i\tilde{\mu}_i)$. This property holds true also for more complex diagrams. Massivation does not require in TGD framework $\bar{\Psi}\Psi$ term

in Dirac action since M^4-massive fermions are M^8-massless and have only chiral couplings in 8-D sense. Scalar coupling would also break separate baryon and lepton conservation. Mass term correspond to a momentum in $E^4 \subset M^4 \times E^4 = M^8$ degrees of freedom. Massivation without losing 8-D light-likeness is consistent with conformal symmetry and with 8-D twistor approach.

2. Fermions are exceptional in the sense that the number of helicities is same for both massive and massless fermions. In particular, 4-fermion amplitude has $k = n/2$ and positive Grassmannian $G(n/2, n)$ with special symmetry property that one can take either negative or positive helicities in preferred role, could be important. For massless states with higher spin the number of helicities is 2 and maximal spin is $J_{max} = h_{max}/2$. For M^4 -massive states also the lower helicities $h_{max} - 2k$ are possible. The scattering amplitudes remain holomorphic.

3. For SUSY one would have all helicities $h(k) = h_{max} - k$ and the general form of amplitude could be written from the knowledge of $h(k)$. The number of fermions at the boundary of string world sheets could be maximal allowed by statistics. This would give SUSY in TGD sense but would require splitting of string boundaries: it is not clear whether this can be allowed. For light-like orbits of partonic 2-surface it has been assumed.

   Sparticles could correspond to states with higher fermion number at given partonic orbits. In this case one expects only approximate SUSY: the p-adic primes characterizing different SUSY states could be different. In adelic physics different p-adic prime could correspond to a different extension of rationals: one might say that the particles inside super-multiplets are at different levels in number theoretic evolution!

BCFW recursion formula as a consistency condition: BCFW homology

The basic consistency condition is that the boundary operation in the BCFW recursion formula gives zero so that the recursion formula can be solved without introducing sum over topological loops. The twistorial trees would have no boundaries but would not be boundaries and would be therefore closed in what might be called BCFW homology. Diagrams would correspond to closed forms.

Consider first the proposal assuming that all diagrams are equivalent with twistorial string diagrams with fermionic 4-vertex as the basic vertex. The boundary operation appearing in BCFW formula gives two terms [B20, B43, B18]. Recall that options I, II, and III correspond to twistorial diagrams without loops created by BCFW bridges, to twistor diagrams assignable to Feynman diagrams without loops, and to diagrams analogous to SUSY diagrams for which fermion lines carry also higher fermion number and can split.

1. The first term results as one BCFW bridge by contracting the three lines connecting the external particles to a larger diagram to a point in all possible ways. The non-vanishing of this term does not force loops in the sense of Feynman diagrams. For Option Ia) (no twistorial loops) there are no BCFW boxes to be reduced so that the outcome is zero.

   For option Ib) (no Feynman loops) a BCFW box diagram for which the two outward direct lines of the bridge are fictive, this operation makes sense and reduces the box to that describing the basic 4-fermion vertex. Same is true for the option II. For option III the operation would be essentially the same as in SUSY.

2. Second term corresponds to entangled removal of a fermion and anti-fermion and if it is non-vanishing, loops are unavoidable. This operation creates a closed fermionic loop to which several internal lines couple. By QCC the fermionic loop would be associated with a topological loop. One can argue that the topological tadpole loop must be closed time loop and that this is not possible since the sign of energy must change at the top and bottom of the loop, where the arrow of time changes: actually the energy should vanish. The same result would obtained if one requires that the energy identified as real part of complexified energy is non-negative for all on mass shell particles.

   Consider the 4-fermion vertex to which the fermionic tadpole loop is associated. Entangled removal gives for the members of a pair of external lines opposite momenta and helicities in twistor-diagrammatics. If so, there exist a vertex for which one fermion scatters in forward direction. Momentum conservation implies the same for the second fermion. One would obtain amplitude, which equals to unity rather than vanishing! Integration over four-momenta would give divergence. However, if the 4-momentum in the tadpole vanishes, the corresponding helicity spinor and also the amplitude vanishes. QCC indeed demands that fermionic loop corresponds to a time loop possible only only if the energy and by time-likeness also 3-momentum vanishes.

It seems that only the simplest option - Option Ia) - is consistent with the BCFW reduction formula. One can say that scattering diagrams are closed objects in the BCFW cohomology. Closedness condition might allow also topological loops, which are not tadpole loops: say decay of fermion to 3 fermions fusing back to the fermion.

Under what conditions fermionic self energy loop is removable?

Scattering diagram as a representation of computation demands that the fermionic "self energy" loop involving two external fermions gives free propagator. The situation in which the vertex contains only *light-like* complex momenta in M_0^2 can be considered as an example. In fact, one can always choose in M^8 the frame for given component of state in this manner.

1. The three fermion/antifermion internal lines in the loop would be light-like in complex 2-D sense as also external momentum. For external momenta $Re(p(M^2))$ would be light-like and orthogonal to light-like $Im(p(M^2))$: it is not clear whether $Im(p(M^2))$ vanishes.

   Light-likeness condition gives $Re(k)^2 - Im(k)^2 = 0$ and $Re(k) \cdot Im(k) = 0$, and $Re(k) = \pm Im(k)$ as a solution meaning that $Re(k)$ is proportional to a light-like vector $(1,1)$ or $(1-1)$. This applies to p, k_1,k_2, and $p - k_1 - k_2$. All these vectors are proportional to the same light-like vector in M^2.

   Apart from the degeneracy for sign factors the situation is equivalent with real 2-D case and one has from momentum conservation that the real parts of the virtual momenta are light-like and parallel and one has $Re(k_i) = \lambda_i p$ leaving two real parameters λ_i.

2. The only possible outcome from the integral is proportional to $D_F(p)$. The outcome is non-vanishing if the proportionality constant is proportional to $1/p^2$. This dependence should come from 4-fermion vertices. The integrand is proportional to the product $\lambda_1\lambda_2(1-\lambda_1-\lambda_2)$ and involves times the $D_F(p)$. Vertices give the inverses of these scaling factors. Since the outcome should be proportional to $1/D_F$ and lines are proportional to p^3, the 4- vertices should give a factor $1/p^2$ each.

   Assuming this one obtains integrand $1/(\lambda_1\lambda_2(1-(\lambda_1-\lambda_2)^2$. The integral over λ_i is of proportional to

   $$I = \int d\lambda_1 d\lambda_2/\lambda_1\lambda_2(1-\lambda_1-\lambda_2) \ .$$

   The ranges of integration are from $(-\infty,\infty)$.

   One can decompose the integral to four parts so that integration ranges are positive. The outcome is

   $$I = \int dlog(\lambda_1)dlog(\lambda 2)\left[\frac{1}{1-\lambda_1-\lambda_2}+\frac{1}{1+\lambda_1+\lambda_2}-\frac{1}{1+\lambda_1-\lambda_2}-\frac{1}{1-\lambda_1+\lambda_2}\right] \ .$$

   The change of variables $(u,v) = (\lambda_1+\lambda_2, \lambda_1-\lambda_2)$ transforms the integral to a product of integrals

   $$I = \int dudv\frac{1}{1-u^2}\int dv\frac{1}{1-v^2} \ .$$

   The interpretation as residue integral gives the outcome $I = (4\pi)^2$.

   Residue integration gives finite result for this integrals. One can worry about the singularity of the vertices for M_0^2 on mass shell momenta. The problem is that p is on mass shell so that the outcome from loop diverges. The outcome is $D_F$would be however finite.

Gliding conditions for 4-vertices

One can construct also loop diagrams with loops understood in twistorial sense. The interpretation of twistor diagram as computation requires that there exist moves reducing general loopy diagrams to tree diagrams. This requires that the vertices connected by a fermionic loop lines can be glided along fermion lines such that they become nearest neighbors and that these loops can be removed without affective the diagram.

If these diagrams are acceptable mathematically, moves reducing these loop diagrams to twistorial tree diagrams should exist. Could the basic rule be following?

1. One can glide the vertices past each other along fermion lines and reduce loops connecting points at different part of graph to the analogs of self-energy loops located at single fermion lines. These loops involve decay of fermion to 2 fermions and 1 anfifermion which then fuse to single fermion. All fermions are on mass shell in complex sense. The situation thus reduces to single fermion self energy loop if the gliding is possible always. Mass shell conditions could however prevent this.

2. To single fermion line one can assign D_F - the inverse of massless fermion propagator - having formal interpretation as a density matrix. The loop would not vanish but would give rise to a inverse of fermionic propagator so that the overall outcome should be just D_F. Is it possible to achieve this?

Under what conditions the gliding is possible?

1. Suppose that the 4-vertex V_1 is glided along fermion line past second 4-vertex V_1. V_1 corresponds to momenta $(P_{i,in}, P_{i1,in} - P, P_{i,1}, P_{i,2})$. The momentum $P_i = \sum_{k=1}^{2} P_{i,k}$ of 2 particles emanates from V_i so that the outgoing and incoming momenta are $P_{i,in} - P_i$, and $P_{i,in}$ $i = 1, 2$. Furthermore $P_{1,in} = P_{2,in} - P_2$. These complex momenta are on M^2 mass shell in the proposed sense.

2. Can one perform the gliding without changing the M_0^2-momenta $P_{i,1}$ and $P_{i,2}$? Gliding is possible if the on mass shell condition is satisfied also for $P_{2,in} - P_1 + P_2$ rather than only $P_{2,in} + P_2$. If the mass squared spectrum is integer valued in suitable units the condition reduces to the requirement that $2P_{2,in} \cdot P_1$ is real and integer valued.

   These conditions are independent of the conditions for $2P_{2,in} \cdot P_2$ coming from V_2, the conditions would correlate P_1 and P_2. The construction of the amplitude would involve non-local conditions on vertices rather than only momentum conservation and mass shell conditions at vertices as expected.

   M^2-momentum is however light-like for a special choice $M^2 = M_0^2$. If M_0^2 same along connected fermion lines, the gliding condition would make sense. M_0^2 is constant of motion along fermion line which can wander along the network formed by partonic orbits.

   In fact, M_0^2 must be same for all fermions in given vertex so that its is constant for all connected regions of fermionic part of the graph. Is there any hope of having non-trivial scattering amplitude or must all momenta be light-like and parallel in plane M_0^2? Tree diagrams certainly give rise to non-trivial scattering. One can also assign to all internal lines this kind of networks with M_0^2 that assignable to the internal line. It is quite possible that for general graphs allowing different M_0^2s in internal lines and loops, the reduction to tree graph is not possible.

3. The analogs of these conditions apply also to tree graphs. So that one must either sum over trees with different orderings of vertices or pose additional conditions on the M^2-momenta say the assumption that they are light-like and proportional to the same real momentum $(1, \pm 1)$ along the fermion line.

To conclude: if M_0^2 is constant of motion along the connected networks of fermion lines, the gliding conditions could be satisfied. Action exponentials do not produce trouble if one identifies the basis of zero energy states in such a way that every maximum of action gives its own separate amplitude (state) as also number theoretic universality demands. The most attractive option number theoretically is the option IIIa) assuming that localization of zero energy state to single computation is possible as quantum measurement: different localizations would have different intersections between reality and p-adicities and would correspond to different computation sequences as cognitive processes. The idea that twistor diagrams are closed forms in the sense that tadpole diagrams vanish is also very attractive and natural in this framework.

Permutation as basic data for a scattering diagram

In twistor Grassmannian approach to $\mathcal{N} = 4$ SUSY the data determining the Yangian invariants defining the basic building bricks of the amplitudes can be constructed using two 3-vertices. For the first (second) kind of vertex the helicity spinors λ_i ($\tilde{\lambda}_i$) are parallel that is $\lambda_1 \propto \lambda_2 \propto \lambda_3$ ($\overline{\lambda}_1 \propto \overline{\lambda}_2 \propto \overline{\lambda}_3$) and can be chosen to be identical by complex scaling invariant: momentum conservation reduces to that for $\tilde{\lambda}_i$ (λ_i). The graphical notation for the two vertices is as a small white *resp.* black disk [B43, B18] (see Fig. 3.3.35 `http://tinyurl.com/zbj9ad7`).

There are two basic moves leaving the amplitude unaffected (see Fig. 3.3.38 at `http://tinyurl.com/zbj9ad7`). Merging symmetry implies that 4-vertices satisfy a symmetry analogous to the duality of old-fashioned hadron physics: an internal line connecting black (white) vertices as exchange in s-channel can be transformed to an exchange in t-channel: $1+2 \rightarrow 3+4 \equiv 1+3 \rightarrow 2+4$. Merging symmetry allows to transform the diagram into a form in which neighboring vertices have opposite colors. Square move symmetry follows from the cyclic symmetry of the 4-particle amplitude and means black$\leftrightarrow$white replacement in 4-vertex.

These two moves do not affect the permutation defining the diagram. A given diagram is represented as a disk with external lines ordered cyclically along its boundary. The permutation of the n external particles associated with the diagram is constructed from the two 3-particle diagrams is defined by the following rule.

Start from k:th point at boundary end and go to the left in each white vertex and to the right in each black vertex (see Fig. 3.3.35 at `http://tinyurl.com/zbj9ad7`).

This leads to a particle $P(k)$ and the outcome is a permutation $P : k \rightarrow P(k)$ charactering the twistor diagram.

Moves do not affect the permutation associated with the diagram and leave the amplitude unaffected. BCFW bridge can be interpreted as a permutation of two neighboring external lines and allows to generate non-equivalent diagrams.

This permutation symmetry generalizes to 4-D SUSY the role of permutations in 1+1-D integrable field theories, where the scattering S-matrix induces only a phase shift of the wave functions of identical particles. The scattering diagram depends only on the permutation of particles induced by the scattering event. Yang-Baxter relation expresses this . Scattering corresponds to particles passing by each other and diagram is drawn in M^2 plane.

1. In 1+1-D integrable theory 3+3 scattering reduces to 2 particle scatterings. This can be illustrated using world lines in M^2 plane (see the illustration of `http://tinyurl.com/gogn75s`). The particle 2 can be taken to be at rest and 1 and 3 move with opposite velocities. There are three 2-particle scatterings of i and j as crossings of world-lines of i and j (pass-by spatially): denote the crossing by ij.

   For the diagram on the left hand side one has crossings 12, 13 and 23 with this time order. For the second case one has crossings 23, 13, and 12 in this time order. Graphically YB relation (see the illustration of `http://tinyurl.com/gogn75s`) says that the scattering amplitude fo 3+3 scattering does not depend on whether the position of the stationary particle 2 is to the left or right from the point at which the second scattering occurs: the time order of scatterings 12 and 23 does not matter.

2. Mathematically the two-particle scatterings are described by operators $R_{12}(u)$, $R_{13}(u+v)$, and $R_{23}(v)$ representing basic braiding operation $ij \rightarrow ji$. u, $u+v$, and v are parameters characterizing the Lorentz boosts determining the velocities of particles. YB equation reads as

$$R_{12}(u)R_{13}(u+v)R_{23}(v) = R_{23}(v)R_{13}(u+v)R_{12}(u) \ .$$

For a graphical illustration see http://tinyurl.com/gogn75s. The first and third R-matrices are permuted and the outcome is trivial. In pass-by interpretation YB equation states that the two ways to realize $123 \rightarrow 321$ give the same amplitude.

Instead of pass-by one could assume a reconnection of the world lines at the intersection: world lines are split and future pieces are permuted and connected to the past pieces again. With this interpretation one has $123 \rightarrow 123$ (the illustration of Wikipedia article corresponds to this interpretation).

3. At the static limit $u, v \rightarrow 0$ YB equation gives rise to an identity satisfied by braiding matrices. The pass-by at this limit can be interpreted as permutation lifted to braiding (braid groups is covering group of permutation group).

2+2 vertices are fundamental in integrable theories in M^2. Also in TGD 2+2 vertices for fundamental fermions are proposed to be fundamental, and the effective reduction to M^2 is crucial in many respects and reflects $M^8 - CP_2$ duality and 8-D quaternionic light-likeness implying that 2+2 fermion vertices reduce to vertices in M^2. TGD could be an integrable theory able to circumvent the limitations of integrable QFTs in M^2.

1. How could the 2+2-fermionic scattering matrix relate to the R-matrix? In TGD framework the scattering involves momentum transfer even in M_0^2 frame: the parallel light-like M^2 momenta are rescaled in momentum conserving manner. Could R matrix appear as additional factor in the scattering? The earlier picture indeed is that the fermion lines at partonic orbits can experience braiding described by R-matrix at the static limit (string world sheet boundaries would braid!).

2. In TGD the scattering of 2 fermions could occur in two ways by classical interactions at partonic 2-surface. The world lines either cross each other or not. In M^2 the first contribution is planar and second one non-planar. Both options should contribute to the 4-fermion amplitude but this is not be visible in the proposed form of the amplitude. Does the proposed 4-fermion scattering amplitude allow this interpretation?

   In $\mathcal{N} = 4$ SUSY the addition of BCFW bridge would permute the two external particles. In TGD the introduction of BCFW bridge would force to have bosonic lines in the BCFW bridge. This is not possible. The only manner to have BCFW diagram is to allow SUSY perhaps realized as and addition right-handed neutrinos to the fermion lines but this would force to allow splitting of fermion lines requiring splitting of strings.

3. Annihilations of fermion-antifermion pairs to bosons are not possible in 1+1-D QFTs but in TGD topological 3-vertices allow them. Boson would correspond to the final $B \equiv F\overline{F}$ pair at same parton orbit. There are two ways to achieve the annihilation. In s-channel $F\overline{F} \rightarrow vacuum \rightarrow F\overline{F} \equiv B$ is possible. Both F_1 coming from past and F_2 from future scatter classically backwards in time to give $\overline{F}_1$ travelling back to past and $\overline{F}_2$ travelling back to future. In t-channel one can have braiding ($F\overline{F} \rightarrow \overline{F}F \equiv B$.

About unitarity for scattering amplitudes

The first question is what one means with S-matrix in ZEO. I have considered several proposals for the counterparts of S-matrix [K49].In the original U-matrix, M-matrix and S-matrix were introduced but it seems that U-matrix is not needed.

1. The first question is whether the unitary matrix is between zero energy states or whether it characterizes zero energy states themselves as time-like entanglement coefficients between positive and negative energy parts of zero energy states associated with the ends of CD. One can argue that the first option is not sensible since positive and negative energy parts of zero energy states are strongly correlated rather than forming a tensor product: the S-matrix would in fact characterize this correlation partially.

   The latter option is simpler and is natural in the proposed identification of conscious entity - self - as a generalized Zeno effect, that is as a sequence of repeated state function reductions at either boundary of CD shifting also the boundary of CD farther away from the second boundary so that the temporal distance between the tips of CD increases. Each shift of this kind is a step in which superposition of states with different distances of upper boundary from lower boundary results followed by a localization fixing the active boundary and inducing unitary transformation for the states at the original boundary.

2. The proposal is that the the proper object of study for given CD is M-matrix. M-matrix is a product for a hermitian square root of diagonalized density matrix ρ with positive elements and unitary S-matrix $S: M = \sqrt{\rho}S$. Density matrix ρ could be interpreted in this approach as a non-trivial Hilbert space metric. Unitarity conditions are replaced with the conditions $MM^\dagger = \rho$ and $M^\dagger M = \rho$. For the single step in the sequence of reductions at active boundary of CD one has $M \rightarrow MS(\Delta T)$ so that one has $S \rightarrow SS(\Delta T)$. $S(\Delta T)$ depends on the time interval ΔT measured as the increase in the proper time distance between the tips of CD assignable to the step.

What does unitarity mean in the twistorial approach?

1. In accordance with the idea that scattering diagrams is a representation for a computation, suppose that the deformations of space-time surfaces defining a given topological diagram as a maximum of the exponent of Kähler function, are the basic objects. They would define different quantum phases of a larger quantum theory regarded as a square root of thermodynamics in ZEO and analogous to those appearing also in QFTs. Unitarity would hold true for each phase separately.

   The topological diagrams would not play the role of Feynman diagrams in unitarity conditions although their vertices would be analogous to those appearing in Feynman diagrams. This would reduce the unitarity conditions to those for fermionic states at partonic 2-surfaces at the ends of CDs, actually at the ends of fermionic lines assigned to the boundaries of string world sheets.

2. The unitarity conditions be interpreted stating the orthonormality of the basis of zero energy states assignable with given topological diagram. Since 3-surfaces as points of WCW appearing as argument of WCW spinor field are pairs consisting of 3-surfaces at the opposite boundaries of CD, unitarity condition would state the orthonormality of modes of WCW spinor field. If might be even that no mathematically well-defined inner product assignable to either boundary of CD exists since it does not conform with the view provided by WCW geometry. Perhaps this approach might help in identifying the correct form of S-matrix.

3. If only tree diagrams constructed using 4-fermion twistorial vertex are allowed, the unitarity relations would be analogous to those obtained using only tree diagrams. They should express the discontinuity for T in $S = 1 + iT$ along unitary cut as $Disc(T) = TT^\dagger$. T and $T^\dagger$ would be T-matrix and its time reversal.

4. The correlation between the structure of the fermionic scattering diagram and topological scattering diagrams poses very strong restrictions on allowed scattering reactions for given topological scattering diagram. One can of course have many-fermion states at partonic 2-surfaces and this would allow arbitrarily high fermion numbers but physical intuition suggests that for given partonic 2-surface (throat of wormhole contact) the fermion number is only 0, 1, or perhaps 2 in the case of supersymmetry possibly generated by right-handed neutrino.

   The number of fundamental fermions both in initial and final states would be finite for this option. In quantum field theory with only masive particles the total energy in the final state poses upper bound on the number of particles in the final state. When massless particles are allowed there is no upper bound. Now the complexity of partonic 2-surface poses an upper bound on fermions.

   This would dramatically simplify the unitarity conditions but might also make impossible to satisfy them. The finite number of conditions would be in spirit with the general philosophy behind the notion of hyper-finite factor. The larger the number of fundamental fermions associated with the state, the higher the complexity of the topological diagram. This would conform with the idea about QCC. One can make non-trivial conclusions about the total energy at which the phase transitions changing the topology of space-time surface defined by a topological diagram must take place.

5.5.5 Criticism

One can criticize the proposed vision.

What about loops of QFT?

The idea about cancellation of loop corrections in functional integral and moves allowing to transform scattering diagrams represented as networks of partonic orbits meeting at partonic 2-surfaces defining topological vertices is nice.

Loops are however unavoidable in QFT description and their importance is undeniable. Photon-photon (see http://tinyurl.com/lqhdujm) scattering is described by a loop diagram in which fermions appear in box like loop. Magnetic moment of muon see http://tinyurl.com/p7znfmd) involves a triangle loop. A further, interesting case is CP violation for mesons (see http://tinyurl.com/oop4apy) involving box-like loop diagrams.

Apart from divergence problems and problems with bound states, QFT works magically well and loops are important. How can one understand QFT loops if there are no fundamental loops? How could QFT emerge from TGD as an approximate description assuming lengths scale cutoff?

The key observation is that QFT basically replaces extended particles by point like particles. Maybe loop diagrams can be "unlooped" by introducing a better resolution revealing the non-point like character of the particles. What looks like loop for a particle line becomes in an improved resolution a tree diagram describing exchange of particle between sub-lines of line of the original diagram. In the optimal resolution one would have the scattering diagrams for fundamental fermions serving as building bricks of elementary particles.

To see the concrete meaning of the "unlooping" in TGD framework, it is necessary to recall the qualitative view about what elementary particles are in TGD framework.

1. The fundamental fermions are assigned to the boundaries of string world sheets at the light-like orbits of partonic 2-surfaces: both fermions and bosons are built from them. The classical scatterings of fundamental fermions at the 2-D partonic 2-surface defining the vertices of topological scattering diagrams give rise to scattering amplitudes at the level of fundamental fermions and twistor lift with 8-D light-likeness suggests essentially unique expressions for the 4-fermion vertex.

2. Elementary particle is modelled as a pair of wormhole contacts (Euclidian signature of metric) connecting two space-time sheets with throats at the two sheets connected by monopole flux tubes. All elementary particles are hadronlike systems but at recent energies the substructure is not visible. The fundamental fermions at the wormhole throats at given space-time sheet are connected by strings. There are altogether 4 wormhole throats per elementary particle in the simplest model.

   Elementary boson corresponds to fundamental fermion and antifermion at opposite wormhole throats with very small size (CP_2 size). Elementary fermion has only single fundamental fermion at either throat. There is $\nu_L \overline{\nu}_R$ pair or its CP conjugate at the other end of the flux tube to neutralize the weak isospin. The flux tube has length of order Compton length (or elementary particle or of weak boson) gigantic as compared to the size of the wormhole contact.

3. The vertices of topological diagram involve joining of the stringy diagrams associated with elementary particles at their ends defined by wormhole contacts. Wormhole contacts defining the ends of partonic orbits of say 3 interacting particles meet at the vertex - like lines in Feynman diagram - and fundamental fermion scattering redistributes fundamental fermions between the outgoing partonic orbits.

4. The important point is that there are $2 \times 2 = 4$ ways for the wormhole contacts at the ends of two elementary particle flux tubes to join together. This makes a possible a diagrams in which particle described by a string like object is emitted at either end and glued back at the other end of string like object. This is basically tree diagram at the level of wormhole contacts but if one looks it at a resolution reducing string to a point, it becomes a loop diagram.

5. Improvement of the resolution reveals particles inside particles, which can scatter by tree diagrams. This allows to "unloop" the QFT loops. By increasing resolution new space-time sheets with smaller size emerge and one obtains "unlooped" loops in shorter scales. The space-time sheets are characterized by p-adic length scale and primes near powers of 2 are favored. p-Adic coupling constant evolution corresponds to the gradual "unlooping" by going to shorter and shorter p-adic length scales revealing smaller and smaller space-time sheets.

The loop diagrams of QFTs could thus be seen as a direct evidence of the fractal many-sheeted space-time and quantum criticality and number theoretical universality (NTU) of TGD Universe. Quantum critical dynamics makes the dynamics universal and this explains the unreasonable success of QFT models as far as length scale dependence of couplings constants is considered. The weak point of QFT models is that they are not able to describe bound states: this indeed requires that the extended structure of particles as 3-surfaces is taken into account.

Can action exponentials really disappear?

The disappearance of the action exponentials from the scattering amplitudes can be criticized. In standard approach the action exponentials associated with extremals determine which configurations are important. In the recent case they should be the 3-surfaces for which Kähler action is maximum and has stationary phase. But what would select them if the action exponentials disappear in scattering amplitudes?

The first thing to notice is that one has functional integral around a maximum of vacuum functional and the disappearance of loops is assumed to follow from quantum criticality. This would produce exponential since Gaussian and metric determinants cancel, and exponentials would cancel for the proposal inspired by the interpretation of diagrams as computations. One could in fact *define* the functional integral in this manner so that a discretization making possible NTU would result.

Fermionic scattering amplitudes should depend on space-time surface somehow to reveal that space-time dynamics matters. In fact, QCC stating that classical Noether charges for bosonic action are equal to the eigenvalues of quantal charges for fermionic action in Cartan algebra would bring in the dependence of scattering amplitudes on space-time surface via the values of Noether charges. For four-momentum this dependence is obvious. The identification of $h_{eff}/h = n$ as the dimension of the extension dividing the order of its Galois group would mean that the basic unit for discrete charges depends on the extension characterizing the space-time surface. Also the cognitive representations defined by the set of points for which preferred embedding space coordinates are in this extension. Could the cognitive representations carry maximum amount of information for maxima? For instance, the number of the points in extension be maximal. Could the maximum configurations correspond to just those points of WCW, which have preferred coordinates in the extension of rationals defining the adele? These 3-surfaces would be in the intersection of reality and p-adicities and would define cognitive representation.

These ideas suggest that the usual quantitative criterion for the importance of configurations could be equivalent with a purely number theoretical criterion. p-Adic physics describing cognition and real physics describing matter would lead to the same result. Maximization for action would correspond to maximization for information.

Irrespective of these arguments, the intuitive feeling is that the exponent of the bosonic action must have physical meaning. It is number theoretically universal if action satisfies $S = q_1 + iq_2\pi$. This condition could actually be used to fix the dependence of the coupling parameters on the extension of rationals [L12]. By allowing sum over several maxima of vacuum functional these exponentials become important. Therefore the above ideas are interesting speculations but should be taken with a big grain of salt.

5.6 Appendix: Some background about twistors

In the following I try to summarize my view about how the ideas related to the twistor approach to scattering amplitudes evolved. A readable summary of specialist about twistor approach is given in the article *Scattering amplitudes* of Elvang and Huang [B18]. Also the thesis *Grassmannian Origin of Scattering Amplitudes* of Trnka [B43] gives a good summary about the work done in association with Nima Arkani-Hamed. I am not a specialist and have not been endowed with practical calculations so that my representation considers only the basic ideas and their relationship to TGD. In the following I summarize my very partial view about the development of ideas.

5.6.1 The pioneering works of Penrose and Witten

The pioneering work of Penrose discussed in *The Central Programme of Twistor Theory* [B41] on twistors initiated the twistor program, which had already had applications in Yang-Mills theories int he description of instantons. The key vision is that massless field equations reduce to holomorphy in twistor formulation.

Witten's *Perturbative Gauge Theory As a String Theory In Twistor Space* [B16] in 2003 initiated the progress leading to dramatic understanding of the planar scattering amplitudes of $\mathcal{N} = 4$ SUSY and eventually to the notion of amplituhedron. The abstract gives some idea about the key ideas.

Perturbative scattering amplitudes in Yang-Mills theory have many unexpected properties, such as holomorphy of the maximally helicity violating amplitudes. To interpret these results, we Fourier transform the scattering amplitudes from momentum space to twistor space, and argue that the transformed amplitudes are supported on certain holomorphic curves. This in turn is apparently a consequence of an equivalence between the perturbative expansion of $\mathcal{N} = 4$ super Yang-Mills theory and the D-instanton expansion of a certain string theory, namely the topological B model whose target space is the Calabi-Yau supermanifold $CP_{3|4}$.

Witten's observation was that the twistor Fourier transform of the scattering amplitudes of YM theories seem to be localized at 2-dimensional complex surfaces of twistor space and this led him to propose that twistor string theory in the twistor space CP_3 could allow to describe the scattering amplitudes. The basic problem of the twistor approach relates to space-time signature: all works nicely in signature (2,2), which suggests that something might be wrong in the basic assumptions.

5.6.2 BCFW recursion formula

BCFW recursion was first derived for tree amplitudes and later generalized to planar loop diagrams.

1. *Twistor diagram recursion for all gauge-theoretic tree amplitudes* by Hodges [B2] in 2005 and *Direct Proof of Tree-Level Recursion Relation in Yang- Mills Theory* by Britto, Cachazo, Feng, and Witten [B9] in 2005 proposed at tree level a recursion formula for the tree level MHV amplitudes of Yang-Mills theory in twistor space.

2. *Scattering Amplitudes and BCFW Recursion in Twistor Space* By Mason and Skinner [B9] discussed BCFW recursion relations for tree diagrams of YM theories.

3. *The S-Matrix in Twistor Space* by Arkani-Hamed, Cachazo, Cheung and Kaplan [B21] in 2009 discussed NkMHV amplitudes with more than two negative helicities (MHV amplitudes have 2 negative helicities are are extremely simple).

This work is carried out in metric signature (2,2), where the twistor transform reduces to ordinary Fourier transform. The other signatures are problematic. Only planar diagrams are considered. *On-Shell Structures of MHV Amplitudes Beyond the Planar Limit* [B25] in 2014 of Arkani-Hamed *et al* consider the problem posed by the non-planar diagrams.

5.6.3 Yangian symmetry and Grassmannian

The discovery of dual super-conformal invariance is one of the key steps of progress. This symmetry means extension of the conformal algebra from space-time level to the level of twistor space so that the dual superconformal invariance acts also on so called momentum twistors assigned with the twistor diagram. These dual conformal symmetries extend to a Yangian algebra containing besides local generators also multilocal generators. The dual conformal generators are bi-local generators and have weight $n = 1$. The Yangian symmetry is completely general and expected to generalize.

In the following I list the abstracts of some important articles.

1. *Magic identities for conformal four-point integrals* by Drummond, Henn, Smirnov, and Sokatchev [B27] in 2006 initiated the development of ideas. The interpretation is as dual conformal invariance generator by the weight 1 generators of Yangian.

   We propose an iterative procedure for constructing classes of off-shell four-point conformal integrals which are identical. The proof of the identity is based on the conformal properties of a sub-integral common for the whole class. The simplest example are the so-called "triple scalar box" and "tennis court" integrals. In this case we also give an independent proof using the method of Mellin-Barnes representation which can be applied in a similar way for general off-shell Feynman integrals.

2. *Yangian symmetry of scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory* [B15] by Drummond, Henn, and Plefka in 2009 continued this work and discussed Yangian algebra as as a symmetry having besides local generators also multilocal generators.

   Tree-level scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory have recently been shown to transform covariantly with respect to a "dual" superconformal symmetry algebra, thus extending the conventional superconformal symmetry algebra psu(2, 2|4) of the theory. In this paper we derive the action of the dual superconformal generators in on-shell superspace and extend the dual generators suitably to leave scattering amplitudes invariant. We then study the algebra of standard and dual symmetry generators and show that the inclusion of the dual superconformal generators lifts the psu(2,2|4) symmetry algebra to a Yangian. The non-local Yangian generators acting on amplitudes turn out to be cyclically invariant due to special properties of psu(2,2|4). The representation of the Yangian generators takes the same form as in the case of local operators, suggesting that the Yangian symmetry is an intrinsic property of planar $\mathcal{N} = 4$ super Yang-Mills, at least at tree level.

3. *Dual Superconformal Invariance, Momentum Twistors and Grassmannians* [B39] by Mason and Skinner introduces momentum twistors and Grassmannians.

   Dual superconformal invariance has recently emerged as a hidden symmetry of planar scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory. This symmetry can be made manifest by expressing amplitudes in terms of "momentum twistors", as opposed to the usual twistors that make the ordinary superconformal properties manifest. The relation between momentum twistors and on-shell momenta is algebraic, so the translation procedure does not rely on any choice of space-time signature. We show that tree amplitudes and box coefficients are succinctly generated by integration of holomorphic delta-functions in momentum twistors over cycles in a Grassmannian. This is analogous to, although distinct from, recent results obtained by Arkani-Hamed et al. in ordinary twistor space. We also make contact with Hodges' polyhedral representation of NMHV amplitudes in momentum twistor space.

4. *A Duality For The S Matrix* [B20] in 2009 by Arkani-Hamed *et al* discusses also Yangian invariance and introduces central ideas in algebraic geometry: Grassmannians, higher-dimensional residue theorems, intersection theory, and the Schubert calculus.

   We propose a dual formulation for the S Matrix of $\mathcal{N} = 4$ SYM. The dual provides a basis for the "leading singularities" of scattering amplitudes to all orders in perturbation theory, which are sharply defined, IR safe data that uniquely determine the full amplitudes at tree level and 1-loop, and are conjectured to do so at all loop orders. The scattering amplitude for n particles in the sector with k negative helicity gluons is associated with a simple integral over the space of k planes in n dimensions, with the action of parity and cyclic symmetries manifest. The residues of the integrand compute a basis for the leading singularities. A given leading singularity is associated with a particular choice of integration contour, which we explicitly identify at tree level and 1-loop for all NMHV amplitudes as well as the 8 particle N^2MHV amplitude. We also identify a number of 2-loop leading singularities for up to 8 particles. There are a large number of relations among residues which follow from the multi-variable generalization of Cauchy's theorem known as the "global residue theorem". These relations imply highly non-trivial identities guaranteeing the equivalence of many different representa- tions of the same amplitude. They also enforce the cancellation of non-local poles as well as consistent infrared structure at loop level. Our conjecture connects the physics *of scattering amplitudes to a particular subvariety in a Grassmannian; space-time locality is reflected in the topological properties of this space.*

5. *The All-Loop Integrand For Scattering Amplitudes in Planar $\mathcal{N} = 4$ SYM* [B22] by Arkani-Hamed *et al* in 2010.

   We give an explicit recursive formula for the all L-loop integrand for scattering amplitudes in $\mathcal{N} = 4$ SYM in the planar limit, manifesting the full Yangian symmetry of the theory. This generalizes the BCFW recursion relation for tree amplitudes to all loop orders, and extends the Grassmannian duality for leading singularities to the full amplitude. It also provides a new physical picture for the meaning of loops, associated with canonical operations for removing particles in a Yangian-invariant way. Loop amplitudes arise from the "entangled" removal of pairs of particles, and are naturally presented as an integral over lines in momentum-twistor space. As expected from manifest Yangian-invariance, the integrand is given as a sum over non-local terms, rather than the familiar decomposition in terms of local scalar integrals with rational coefficients. Knowing the integrands explicitly, it is straightforward to express them in local forms if desired; this turns out to be done most naturally using a novel basis of chiral, tensor integrals written in momentum-twistor space, each of which has unit leading singularities. As simple illustrative examples, we present a number of new multi-loop results written in local form, including the 6- and 7-point 2-loop NMHV amplitudes. Very concise expressions are presented for all 2-loop MHV amplitudes, as well as the 5-point 3-loop MHV amplitude. The structure of the loop integrand strongly suggests that the integrals yielding the physical amplitudes are "simple", and determined by IR-anomalies. We briefly comment on extending these ideas to more general planar theories.

5.6.4 Amplituhedron

The latest development in twistorial revolution was the notion of amplituhedron. Since I do not have intuitive understanding about amplituhedron and since amplituhedron does not have role in the twistorialization of TGD as I understand it now, I provide only abstracts about two articles to it.

1. *The Amplituhedron* [B6] by Arkani-Hamed and Trnka in 2013.

   Perturbative scattering amplitudes in gauge theories have remarkable simplicity and hidden infinite dimensional symmetries that are completely obscured in the conventional formulation of field theory using Feynman diagrams. This suggests the existence of a new understanding for scattering amplitudes where locality and unitarity do not play a central role but are derived consequences from a different starting point. In this note we provide such an understanding for $\mathcal{N} = 4$ SYM scattering amplitudes in the planar limit, which we identify as "the volume" of a new mathematical object–the Amplituhedron–generalizing the positive Grassmannian. Locality and unitarity emerge hand-in-hand from positive geometry.

2. *Positive Amplitudes in the Amplituhedron* [B5] by Arkani-Hamed *et al* in 2014.

   The all-loop integrand for scattering amplitudes in planar $\mathcal{N} = 4$ SYM is determined by an "amplitude form" with logarithmic singularities on the boundary of the amplituhedron. In this note we provide strong evidence for a new striking property of the superamplitude, which we conjecture to be true to all loop orders: the amplitude form is positive when evaluated inside the amplituhedron. The statement is sensibly formulated thanks to the natural "bosonization" of the superamplitude associated with the amplituhedron geometry. However this positivity is not manifest in any of the current approaches to scattering amplitudes, and in particular not in the cellulations of the amplituhedron related to on-shell diagrams and the positive Grassmannian. The surprising positivity of the form suggests the existence of a "dual amplituhedron" formulation where this feature would be made obvious. We also suggest that the positivity is associated with an extended picture of amplituhedron geometry, with the amplituhedron sitting inside a co-dimension one surface separating "legal" and "illegal" local singularities of the amplitude. We illustrate this in several simple examples, obtaining new expressions for amplitudes not associated with any triangulations, but following in a more invariant manner from a global view of the positive geometry.

Chapter 6

The Recent View about Twistorialization in TGD Framework

6.1 Introduction

The construction of scattering amplitudes is a dream that I have had since the birth of TGD for four decades ago. Various ideas have gradually emerged, some of them have turned out to be wrong, and some of them have survived. At this age I must admit that the dream about explicit algorithms that any graduate student could apply to construct the scattering amplitudes, would require a collective effort and probably will not be realized during my lifetime.

I have however identified a set of general powerful principles leading to a generalization of the recipes for constructing twistorial amplitudes and already now these principles suggest the possibility of rather concrete realizations. In the sequel several additional insights are developed in more detail. Some of them are discussed already earlier in the formulation of $M^8 - H$ duality [L20] in adelic framework [L23, L22] and in the chapters developing the TGD based generalization of twistor Grasmannian approach [L3, L11, L12, L24].

1. A proposal made already earlier [L24] is that scattering diagrams as analogs of twistor diagrams are constructible as tree diagrams for CDs connected by free particle lines. Loop contributions are not even well-defined in zero energy ontology (ZEO) and are in conflict with number theoretic vision. The coupling constant evolution would be discrete and associated with the scale of CDs (p-adic coupling constant evolution) and with the hierarchy of extensions of rationals defining the hierarchy of adelic physics.

2. Logarithms appear in the coupling constant evolution in QFTs. The identification of their number theoretic versions as rational number valued functions required by number-theoretical universality for both the integer characterizing the size scale of CD and for the hierarchy of Galois groups leads to an answer to a long-standing question what makes small primes and primes near powers of them physically special. The primes $p \in \{2, 3, 5\}$ indeed turn out to be special from the point of view of number theoretic logarithm.

3. The reduction of the scattering amplitudes to tree diagrams is in conflict with unitarity in 4-D situation. The imaginary part of the scattering amplitude would have discontinuity proportional to the scattering rate only for many-particle states with light-like total momenta. Scattering rates would vanish identically for the physical momenta for many-particle states.

   In TGD framework the states would be however massless in 8-D sense. Massless pole corresponds now to a continuum for M^4 mass squared and one would obtain the unitary cuts from a pole at $P^2 = 0$! Scattering rates would be non-vanishing only for many-particle states having light-like 8-momentum, which would pose a powerful condition on the construction of many-particle states. Single particle momenta cannot be however light-like for this kind of states unless they are parallel. They must be also complex as they indeed are already in classical TGD.

In fact, BCFW deformation $p_i \rightarrow p_i + zr_i$, $r_i \cdot r_j = 0$ creates at z-poles of the resulting amplitude pairs of zero energy states for which complex single particle momenta are not light-like but sum up to massless momentum. One can interpret these zero energy analogs of resonances, states inside CDs formed from massless external particles as they arrive to CD. This strong form of conformal symmetry has highly non-trivial implications concerning color confinement.

4. The key idea is number theoretical discretization [L23] in terms of "cognitive representations" as space-time time points with M^8-coordinates in an extension of rationals and therefore shared by both real and various p-adic sectors of the adele. Discretization realizes measurement resolution, which becomes an inherent aspect of physics rather than something forced by observed as outsider. This fixes the space-time surface completely as a zero locus of real or imaginary part of octonionic polynomial.

   This must imply the reduction of "world of classical worlds" (WCW) corresponding to a fixed number of points in the extension of rationals to a finite-dimensional discretized space with maximal symmetries and Kähler structure [K35, K19, K66].

   The simplest identification for the reduced WCW would be as complex Grassmannian - a more general identification would be as a flag manifold. More complex options can of course be considered. The Yangian symmetries of the twistor Grassmann approach known to act as diffeomorphisms respecting the positivity of Grassmannian and emerging also in its TGD variant would have an interpretation as general coordinate invariance for the reduced WCW. This would give a completely unexpected connection with supersymmetric gauge theories and TGD.

5. M^8 picture [L20] implies the analog of SUSY realized in terms of polynomials of super-octonions whereas H picture suggests that supersymmetry is broken in the sense that many-fermion states as analogs of components of super-field at partonic 2-surfaces are not local. This requires breaking of SUSY. At M^8 level the breaking could be due to the reduction of Galois group to its subgroup G/H, where H is normal subgroup leaving the point of cognitive representation defining space-time surface invariant. As a consequence, local many-fermion composite in M^8 would be mapped to a non-local one in H by $M^8 - H$ correspondence.

6.2 General view about the construction of scattering amplitudes in TGD framework

Before twistorial considerations a general vision about the basic principles of TGD and construction of scattering amplitudes in TGD framework is in order.

6.2.1 General principles behind S-matrix

Although explicit formulas for scattering amplitudes are probably too much to hope, one can try to develop a convincing general view about principles behind the S-matrix.

World of Classical Worlds

The first discovery was what I called the "world of classical worlds" (WCW) [K35, K19, K66] as a generalization of loop space allowing to replace path integral approach failing in TGD work. This led to a generalization of Einstein's geometrization program to an attempt to geometrize entire quantum physics. The geometry of WCW would be essentially unique from its mere existence since the existence of Riemann connection requires already in the case of loop spaces maximal isometries. Super-symplectic and super-conformal symmetries generalizing the 2-D conformal symmetries by replacing 2-D surfaces with light-like 3-surfaces (metrically 2-D!) would define the isometries.

Physical states would be classical spinor fields in the infinite-dimensional WCW and spinors at given point of WCW would be fermionic Fock states. Gamma matrices would be linear combinations of fermionic oscillator operators associated with the analog of massless Dirac equation at space-time surface determined by the variational principle whose preferred extremals the space-time surfaces are. Strong form of holography implied by strong form of general coordinate invariance would imply that it is enough to consider the restrictions of the induced spinor fields at string world sheets and partonic 2-surfaces (actually at discrete points at them defining the ends of boundaries of string world sheets) [K86, K66].

Zero Energy Ontology and generalization of quantum measurement theory to a theory of consciousness

The attempts to understand S-matrix led to the question about what does state function reduction really mean. This eventually led to the discovery of Zero Energy Ontology (ZEO) in which time=constant snapshot as a physical state is replaced with preferred extremal satisfying infinite number of additional gauge conditions [L25]. Temporal pattern becomes the fundamental entity: this conforms nicely with the view neuroscientists and computational scientists for whom behavior and program are basic notions. One can say that non-deterministic state function reduction replaces this kind time evolution with new one. One gets rid of the basic difficulty of ordinary quantum measurement theory.

Causal diamond (CD) is the basic geometric object of ZEO. The members of the state pair defining zero energy state - the analog of physical event characterized by initial and final states - have opposite total conserved quantum numbers and reside at the opposite light-like boundaries of CD being associated with 3-surfaces connected by a space-time surface, the preferred extremal. CDs form a fractal hierarchy ordered by their discrete size scale.

One ends up to a quite radical prediction: the arrow of time changes in “big” state function reduction changing the roles of active and passive boundaries of CD. The state function reductions occurring in elementary reactions represent an example of “big” state function reduction. The sequence of “small” state function reductions - analogs of so called weak measurements - defines self as a conscious entity having CD as embedding space correlate [L25].

In ZEO based view about WCW 3-surfaces X^3 are pairs of 3-surfaces at boundaries of CD connected by preferred extremals of the action principle. WCW spinors are pairs of fermionic Fock states at these 3-surfaces and WCW spinor fields are WCW spinors depending on X^3 . They satisfy the analog of massless Dirac equation which boils down to the analogs of Super Virasoro conditions including also gauge conditions for a sub-algebra of super-symplectic algebra. S-matrix describing time evolution followed by “small” state function reduction relates two WCW spinor fields of this kind.

Generalization of twistor Grassmannian approach to TGD framework

Twistorial approach generalizes from M^4 to $H = M^4 \times CP_2$. One possible motivation could be the fact that ordinary twistor approach describes only scattering of massless particles. In the proposed generalization particles are massless in 8-D sense and in general massive in 4-D sense [L3, L11, L12, L24].

1. The existence of twistor lift of Kähler action as 6-D analog of Kähler action fixes the choice of H uniquely: only M^4 and CP_2 allow twistor space with Kähler structure. The 12-D product of the twistor spaces of M^4 and CP_2 induces twistor structure for 6-D surface X^6 under additional conditions guaranteeing that the X^6 is twistor space of 4-D surface X^4 (S^2 bundle over X^4) - its twistor lift. The conjecture that 6-D Kähler action indeed gives rise to twistor spaces of X^4 as preferred extremals.

2. This conjecture is the analog for Penrose's original twistor representation of Maxwellian fields reducing dynamics of massless fields to homology. There is also an analogy with massless fields. Dimensional reduction of Kähler action occurs for 6-surfaces, which represent twistor spaces and the external particles entering CD would be minimal surfaces defining simultaneous preferred extremals of Kähler action satisfying infinite number of additional gauge conditions. Minimal surfaces indeed satisfy generalization of massless field equations.

In the interior of CD defining interaction region there is a coupling to Kähler 4-force and one has analog of massless particle coupling to Maxwellian field.

3. 6-D Kähler action would give the preferred extremals via the analog of dimensional reduction essential for the twistor space property requiring that one has S^2 bundle over space-time surface. I have considered the generalization of the standard twistorial construction of scattering amplitudes of $\mathcal{N} = 4$ SUSY to TGD context. In particular, the crucial Yangian invariance of the amplitudes holds true also now in both M^4 and CP_2 sectors.

4. Skeptic could argue that TGD generalization of twistors does not tell anything about the origin of the Yangian symmetry. During writing of this contribution I however realized that the hierarchy of Grassmannians realizing the Yangian symmetries could be seen as a hierarchy of reduced WCWs associated with the hierarchy of adeles defined by the hierarchy of extensions of rationals. The isometries of Grassmannian would emerge in the reduction of the isometry group of WCW to a finite-D isometry group of Grassmannian and would be caused by finite measurement resolution described number theoretically. Of course, one can consider also more general flag manifolds with Kähler property as candidates for the analogs of Grassmannians. I will represent the argument in more detail later.

   This could also relate to the postulated infinite hierarchy of hyper-finite factors of type II_1 (HFFs) [K85, K28] as a correlate for the finite measurement resolution with included subfactor inducing transformations which act trivially in the measurement resolution used.

Remark: There is an amusing connection with empiria. Topologist Barbara Shipman observed that honeybee dance allows a description in terms of flag manifold $F = SU(3)/U(1) \times U(1)$, which is the space for the choices of quantization axes of color quantum numbers and also the twistor space in CP_2 degrees of freedom [A7]. This suggest that QCD type physics might make sense in macroscopic length scales. p-Adic length scale hypothesis and the predicted long range classical color gauge fields suggest a hierarchy of QCD type physics. One can indeed construct a TGD based model of honeybee dance with aconcrete interpretation and representation for the points of F at space-time level [L28].

$M^8 - H$ duality

$M^8 - H$ duality provides two equivalent ways to see the dynamics with either M^8 or $H = M^4 \times CP_2$ as embedding space [L20]. One might speak of number theoretic compactification which is a completely non-dynamical analog for spontaneous compactification.

1. In M^8 picture the space-time corresponds to a zero locus for either imaginary part $IM(P)$ or real part $RE(P)$ of octonionic polynomial ($RE(o)$ and $IM(o)$ are defined by the decomposition $o = RE(o) + I_4 IM(o)$, where I_4 is octonion unit orthogonal to quaternionic subalgebra). The dynamics is purely algebraic and ultra-local.

2. At the level of H the dynamics is dictated by variational principle and partial differential equations. Space-time surfaces are preferred extremals of the twistor lift of Kähler action reduced to a sum of 4-D Kähler action and volume term analogous to cosmological term in GRT. The equivalence of these descriptions gives powerful constraints and should follow from the infinite number of gauge conditions at the level of H associated with a sub-algebra of supersymplectic algebra implying the required dramatic reduction of degrees of freedom [K19, K66]. One has a hierarchy of these sub-algebras, which presumably relates to the hierarchy of HFFs and hierarchy of extensions of rationals.

   H picture works very nicely in applications. For instance, the notions of field body and magnetic body are crucial in all applications.

The notion of quaternionicity, which is a central element of $M^8 - H$ duality has a deep connection with causality which I have not noticed earlier. At the level of momentum space quaternionicity means that 8-momenta -, which by $M^8 - H$-duality correspond to 4-momenta at level of M^4 and color quantum numbers at the level of CP_2 - are quaternionic. Quaternionicity means that the time component of 8-momentum, which is parallel to real octonion unit, is non-vanishing. The 8-momentum itself must be time-like, in fact light-like. In this case one can always regard the momentum as momentum in some quaternionic sub-space. Causality requires a fixed sign for the time component of the momentum.

It must be however noticed that 8-momentum can be complex: also the 4-momentum can be complex at the level of $M \times CP_2$ already classically. A possible interpretation is in terms of decay width as part of momentum as it indeed is in phenomenological description of unstable particles.

Could one require that the quaternionic momenta form a linear space with respect to octonionic sum? This is the case if the energy - that is the time-like part parallel to the real octonionic unit - has a fixed sign. The sum of the momenta is quaternionic in this case since the sum of light-like momenta is in general time-like and in special case light-like. If momenta with opposite signs of energy are allowed, the sum can become space-like and the sum of momenta is co-quaternionic.

This result is technically completely trivial as such but has a deep physical meaning. Quaternionicity at the level of 8-momenta implies standard view about causality: only time-like or at most light-like momenta and fixed sign of time-component of momentum.

Adelic physics

The adelization of ordinary physics fusing real number based physics and various p-adic variants of physics in order to describe cognition.

1. Adelic physics [L23, L22] gives powerful number theoretic constraints when combined with $M^8 - H$ duality and leads to the vision about evolutionary hierarchy defined by extensions of rationals. The higher the level in the hierarchy, the higher the dimension n of the extension identified in terms of Planck constant $h_{eff}/h = n$ labelling the levels of dark matter hierarchy.

2. Adelic hypothesis allows to sharpen the strong form of holography to a statement that discrete cognitive representations consisting of a finite number of points identified as points of space-time surface with M^8 coordinates in the extension of rationals fixes the space-time surface itself. This dramatic reduction would be basically due to finite measurement resolution realized as an inherent property of dynamics. Cognitive representation in fact gives the WCW coordinates of the space-time surface in WCW! WCW reduces to a number theoretic discretization of a finite-dimensional space with Kähler structure and presumably maximal isometries.

3. In ZEO space-time surface becomes analogous to a computer program determined in terms of finite net of numbers! Of course, at the QFT limit of TGD giving standard model and GRT space-time is locally much more complex since one approximates the many-sheeted space-time with single slightly curved region of M^4. This is the price paid for getting rid (or losing) the topological richness of the many-sheeted space-time crucial for the understanding living matter and even physics in galactic scales.

4. Skeptic can argue that this discretization of WCW leads to the loss of WCW geometry based on real numbers. One can however consider also continuous values for the points of cognitive representations and assigning metric to the points of cognitive representation. Metric could be defined as kind of induced metric. One slices CD by parallel CDs by shift the CD along the axis connecting its tips. This allows to see the point of cognitive representation as point at one particular CD. One shifts slightly the point along its CD. Embedding space metric allows to deduce the infinitesimal line element ds^2 and to deduce the metric components. This allows a definition of differential geometry so that the analog of WCW metric makes sense as a hierarchy of finite-dimensional metrics for space-time surfaces characterize by the cognitive representations.

   The interpretation in real context would be in terms of finite measurement resolution and the hierarchy would correspond to a hierarchy of hyper-finite factors (HFFs) [K85, K28], whose defining property is that they allow arbitrarily precise finite-dimensional approximations. What would be new is that the hierarchy of extensions of rationals would define a hierarchy of discretizations and hierarchy of HFFs.

Thabove list involves several unproven conjectures, which I can argue to be intuitively obvious with the experience of four decades: I cannot of course expect that a colleague reading for the first time about TGD would share these intuitions.

6.2.2 Classical TGD

Classical TGD is now rather well understood both in both $H = M^4 \times CP_2$ and M^8 pictures. Applications of classical TGD are in H picture and rather detailed phenomenology has emerged. M^8 picture has led to a rather precise vision about adelic physics and to understanding of finite measurement resolution.

Classical TGD in M^8 picture

Classical TGD in M^8 picture is discussed in [L20].

1. In M^8 picture one ends to an extremely simple number theoretic construction of space-time surfaces fixing only discrete or even finite number of space-time points to obtain space-time surface for a given extension of rationals. The reason is that space-time surfaces are zero loci for $RE(P)$ or $IM(P)$ of octonionic polynomials obtained by continuing real polynomial with coefficients in an extension of rationals to an octonionic polynomial.

   Needless to say, the hierarchy of algebraic extensions of rationals is what makes the dynamics at given level so simple. The coordinates of space-time surface as a point of WCW must be in the extension of rationals. As noticed, the points of space-time surface defining the cognitive representation determining the space-time surface serve as its natural WCW coordinates.

2. The highly non-trivial point is that no variational principle is involved with M^8 construction. Therefore it seems that neither WCW metric nor Kähler function is needed. If this is the case, the exponential of Kähler function definable as action exponential does not appear in scattering amplitudes and must disappear also at H-side from the scattering amplitudes.

3. Skeptic could argue that one loses general coordinate invariance in this approach. This is not true. Linear M^8 coordinates are the only possible option and forced already by symmetries. The choice octonionic and quaternionic structures fixes the linear M^8 coordinates almost uniquely since time direction is associated with real octonion unit and one spatial direction to special imaginary unit defining spin quantization axis. In algebraic approach identifying space-time surface as a zero locus of $RE(P)$ or $IM(P)$ these coordinates define space-time coordinates highly uniquely.

   Skeptic could also argue that number theoretic discretization implies reduction of the basic symmetry groups to their discrete sub-groups. This is true and one can argue that this loss of symmetry is due to the use of cognitive representations with finite resolution. Points with algebraic coordinates could be seen as a choices of representatives from a set of points, which are equivalent as far as measurement resolution is considered.

4. A physically important complication related to M^8 dynamics is the possibility of different octonionic and quaternionic structures. For instance, external particles arriving into CD correspond to different octonionic and quaternionic structures in general since Lorentz boost affects the octonionic structure changing the direction of time axis, which corresponds to the real octonionic unit. In color degrees of freedom one has wave function over different quaternionic structures: essentially color partial waves labelled by color quantum numbers [K42].

   One can apply Poincare transformations and color rotations (or transformation in sub-groups of these groups if one requires that the image points belong to the same extension) to the discrete cognitive representation defining space-time surface. The moduli spaces for these structures are essential for the understanding the standard Poincare and color quantum numbers and standard conservation laws in M^8 picture. Also the size scales of CDs define moduli as also Lorentz boosts leaving either boundary of CD unaffected.

Classical TGD in H picture

At the H side one action principle has partial differential equations and infinite number of gauge conditions associated with a sub-algebra of super-symplectic algebra selecting only extremely few preferred extremals of the action principle in terms of gauge conditions for a sub-algebra of super-symplectic algebra. This dynamics is conjectured to follow from the assumption that 6-D lift of space-time surface X^4 to a CP_1 bundle over X^4 is twistor space of X^4. This condition requires the analog of dimensional reduction since S^2 fiber is dynamically trivial.

For 6-D preferred extremals identifiable as twistor spaces of space-time surfaces the 6-D Kähler action in the product of twistor spaces of M^4 and CP_2 is assumed to dimensionally reduce to 4-D Kähler action plus volume term identifiable as the analog of cosmological constant term. This picture reproduces a description of scattering events highly analogous to that emerging in M^8. External particles correspond to minimal surfaces as analogs of free massless fields and all couplings disappear from the value of the action. The interior of CD corresponds to non-trivial coupling to Kähler 4-force which does not vanish. In M^8 picture one has associative and non-associative regions as counterparts of these regions.

What is remarkable is that the dynamics determined by partial differential equations plus gauge conditions would be equivalent with the number theoretic dynamics determined in terms of zero loci for real or imaginary parts of octonionic polynomials.

6.2.3 Scattering amplitudes in ZEO

The construction of scattering amplitudes even at the level of principle is far from well-understood. I have discussed rather concrete proposals for the twistorial construction but the feeling is that something is still missing [L3, L11, L12, L24]. This feeling might well reflect my quite too limited mathematical understanding of twistors and experience about practical construction of the scattering amplitudes. Later I will discuss possible identification of the missing piece of puzzle.

Consider first the general picture about the construction of scattering amplitudes suggested by ZEO inspired theory of quantum measurement theory defining also a theory of consciousness.

1. The portions of space-time surfaces outside CD correspond to external particles. They satisfy associativity conditions at M^8 side making possible to map them to minimal surfaces in $H = M^4 \times CP_2$ satisfying various infinite number of gauge conditions for a sub-algebra of super-symplectic algebra isomorphic with it.

   Remark: There is an additional condition requiring that associative tangent space or normal space contains fixed complex subspace of quaternions. It is not quite clear whether this condition can be generalized so that the distribution of these spaces is integrable.

   At both sides the dynamics of external particles is in a well-defined sense critical at both sides and does not depend at all on coupling constants.

2. Inside CDs associativity conditions break down in M^8 and one cannot map this spacetime region - call it X^4 - to H [L20]. It is however possible to construct counterpart of X^4 in H as a preferred extremal for the twistor lift of Kähler action by fixing the 3-surfaces at the boundaries of CD (boundary conditions). The dependence on couplings at the level of H would come from the vanishing conditions for classical Noether charges, which depend on coupling parameters.

3. If the two descriptions of the scattering amplitudes are equivalent, the dependence on coupling parameters in H should have a counterpart in M^8. Coupling constants making sense only at H side are expected to depend on the size scale of CD and on the extension of rationals defining the adele [L23, L22]. Coupling constants should be determined completely by the boundary values of Noether charges at the ends of space-time surface, and therefore by the 3-D ends of associative space-time regions representing external particles at M^8 side. This would suggest that coupling constants are functions of the coefficients of the polynomials and the points of cognitive representation.

Zero energy ontology and the life cycle of self

ZEO meant a decisive step in the understanding of quantum TGD since it solved the basic paradox of quantum measurement problem by forcing to realize that subjective and geometric time are not the same thing [L25].

1. Both the passive boundary of CD and the members of state pairs at it are unaffected during the sequence of state reductions analogous to weak measurements (see http://tinyurl.com/zt36hpb) defining self as a generalized Zeno effect. The members of state pairs associated with the active boundary change and the active boundary itself drifts farther away from the passive one in the sequence of "small" state function reductions.

   Also the space-time surfaces connecting passive and active boundaries change during the sequence of weak measurements. Only the 3-surfaces at the passive boundary are unaffected. Hence the geometric past relative to the active boundary changes during the life cycle of self. In positive energy ontology (PEO) this is not possible.

2. In "big" state function reduction the roles of passive and active boundary are changed and the arrow of time identifiable as the direction in which CD grows changes. In consciousness theory "big" state function reduction corresponds to the death of self and subsequent re-incarnations as a self with an opposite arrow of geometric time.

3. In ZEO the life cycle of self corresponds to a sequence of steps. Single step begins with a unitary time evolution in which a superposition of states associated with CDs larger than the original CD emerges. Then follows the analog of weak measurement leading to a localization to a CD in the moduli space of CDs so that it has a fixed and in general larger size. A measurement of geometric time occurs and gives rise to an experience about the flow of time.

   This option would allow to identify the total S-matrix as a product of the S-matrices associated with various steps in spirit with the interpretation as a generalized Zeno effect.

   Remark: In the usual description one fixes the time interval to which one assigns the S-matrix. There is no division to steps giving rise to the experience of time flow.

4. The measurement of geometric time would be a partial measurement reducing more general unitary time evolution to a unitary time evolution in the standard sense. Can one generalize the notion of partial measurement to other observables so that one would still have unitary time evolution albeit in more restricted sense? Or should one consider giving up the unitary time evolution?

   These observables should commute with the observables having the states at passive boundary as eigenstates: otherwise the state at passive boundary would change. If this picture makes sense, the "big" reduction to the opposite boundary meaning the death of self would necessarily occur when all observables commuting with the eigen observables at the passive boundary have been measured. It could of course occur already earlier.

   Should one allow measurements of all observables commuting with the eigen observables at the passive boundary. This would lead to partial de-coherence of the zero energy state. In TGD inspired quantum biology this could allow to understand aging as an unavoidable gradual loss of the quantum coherence.

More detailed interpretation of ZEO

There are several questions related to the detailed interpretation of ZEO. The intuitive picture is that inside CD representing self one has collection of sub-CDs representing sub-selves identified as mental images of self. On can loosely say, that sub-CDs represent mind. The sub-CDs are connected by on mass shell lines, which correspond to external particles - matter. Sub-CDs can also have sub-CDs and the hierarchy can have several levels.

The states at the boundaries of CD have opposite total quantum numbers. One can consider two interpretations.

1. In positive energy ontology (PEO) the notion of zero energy state could be seen only as an elegant manner to express conservation laws. This is done in QFT quite generally - also in twistor approach. Also the largest CD would have external particles emanating from its boundaries travelling to the geometric past and future. One would have however have only information about the interior of the CD possessed by conscious entity for which CD plus its sub-CDs (mental images) serve as correlates.

In this picture the arrow of time is fixed since it must be same for all sub-CDs in order to void inconsistency with the basic idea about self as generalized Zeno effect realized as a sequence of weak measurements.

2. ZEO suggest a more radical interpretation. Zero energy state defines an event. There would be the largest CD defining self and sub-CDs would correspond to mental images. There would be no external particles emanating from the boundaries of the largest CD. In this framework it becomes possible to speak about the death of self as the first state function reduction to the opposite boundary changing the roles of active and passive boundaries of self.

This picture should be consistent with what we know about arrow of time and in TGD framework with the idea that the arrow of time can also change - in particular in living matter.

1. How would the standard arrow of time emerge in ZEO? One could see the emergence of the global arrow of geometric time as a process in which the size of the largest CD increases: the sub-CDs are forced to have the same arrow of time as the largest CD and cannot make state function reductions on opposite boundary (die) independently of it. During evolution the size of the networks with the same arrow of geometric time increases and fixed arrow of geometric time is established in longer scales.

2. This picture cannot be quite correct. The applications of TGD inspired consciousness require that the mental images of self can have arrow of geometric time opposite to that of self. For instance, motor actions could be sensory perceptions in non-standard arrow of time. Memory could be communications with brain of geometric past - seeing in time direction - involving signals to geometric past requiring temporary reversals of the arrow of time at some level of self-hierarchy. Hence space-time regions with different arrows of time but forming a connected space-time surface ought to be possible.

   Many-sheeted space-time means a hierarchy of space-time sheets connected by what I call wormhole contacts having Euclidian signature of the induced metric. Space-time sheets at different levels of the hierarchy are not causally connected in the sense that one cannot speak of signal propagation in the regions of Euclidian signature. This suggests that the space-time sheets connected by wormhole contacts can have different arrows of geometric time and are associated with their own CDs.

   In this manner one would avoid the paradox resulting when sub-self - mental image - dies so that its passive boundary becomes active and the particles emanating from it end up to the passive boundary of CD, where no changes are allowed during the life cycle of self. If the particles emanating from time-reversed sub-self and up to boundaries of parallel CD, the problem is circumvented.

3. Wormhole contacts induce an interaction between Minkowskian space-time sheets that they connect. The interaction is not mediated by classical signals but by boundary conditions at the boundaries between Minkowskian regions and Euclidian wormhole contact. These two boundaries are light-like orbits of opposite wormhole throats (partonic 2-surfaces).

   In number theoretic picture the presence of wormhole contact is reflected in the properties set of points in extension of rationals defining the cognitive representation in turn defining the space-time surface. In particular, the points associated with wormhole contact have space-like distance although they are at opposite boundaries of CD and have time-like distance in the metric of embedding space. This kind of point pairs associated with wormhole contacts serve serve as a tell-tale signature for them.

6.3 The counterpart of the twistor approach in TGD

The analogs of twistor diagrams could emerge in TGD [L11, L24] in the following manner in ZEO.

1. Portions of space-time surfaces inside CDs would appear as analogs of vertices and the spacetime surfaces connecting them as analogs of propagator lines. The "lines" connecting sub-CDs would carry massless on mass shell states but possibly with complex momenta

analogous to those appearing in twistor diagrams. This is true also classically at level of H: the coupling constants appearing in the action defining classical dynamics - at least Kähler coupling strength - are complex so that also conserved quantities have also imaginary parts.

Remark: At the level of M^8 one does not have action principle and cannot speak of Noether charges. Here the conserved charged are associated with the symmetries of the moduli spaces such as the moduli spaces for octonion and quaternion structures [L20]. The identification of the classical charges in Cartan algebra at H level with the quantum numbers labeling wave functions in moduli space at M^8 level could be seen as a realization of quantum classical correspondence.

2. At space-time level the vertices of twistor diagrams correspond to partonic 2-surfaces in the interior of given CD. In H description fermionic lines along the light-like orbits of partonic 2-surfaces scatter at partonic 2-surfaces. If each partonic 2-surface defining a vertex is surrounded by a sub-CD, these two views about TGD variants of twistor diagrams are unified. Sub-CD can of course contain more complex structures such as pair of wormhole contacts assignable to an elementary particle.

6.3.1 Could the classical number theoretical dynamics define the hard core of the scattering amplitudes?

The natural hope is that the simple picture about classical dynamics at the level of M^8 should have similar counterpart at the level of scattering amplitudes in M^8. The above arguments suggest that the scattering diagrams correspond to CDs connected by external particle lines representing on mass shell particles. These surfaces are associative at the level of M^8 and minimal surfaces at the level of H. This suggests that scattering amplitude for single CD serves as a building brick for scattering amplitudes: the rest would be “just kinematics” dictated by the enormous symmetries of WCW.

1. Everything in the construction should reduce to a hard core around which one would have integrations (or sums for number theoretic realization of finite measurement resolution) over various moduli characterizing the standard quantum numbers. Twistors for M^4 and CP_2 and the moduli for the choices of CDs should correspond to essentially kinematic contribution involving no genuine dynamics.

2. The scattering amplitudes should make sense in all sectors of adele. This poses powerful constraints on them. The exponential of Kähler function reducing to action exponential can in principle appear in the description at H-side but cannot be present at M^8 side. Therefore it should disappear also at the level of H.

   If the scattering amplitude at the level of H is sum over contributions with the same value of the action exponential, the exponentials indeed cancel and I have proposed that this condition holds true. In perturbative quantum field theory it holds practically always and in integrable theories is exact. This would mean enormous simplification since all information about the action principle in H would appear in the vanishing conditions for the Noether charges of the subalgebra of super-symplectic algebra at the ends of the space-time surface. These Noether changes indeed depend on the action principle and thus on coupling constants.

3. Could the hard core in the construction of the scattering amplitudes be just the choice of the cognitive representation as points if M^8 belonging to the algebraic extension defining the adele and determining space-time surface in terms of octonionic polynomial inside this CD defining the interaction region?

   The set of points of extension of rationals in the cognitive representation defines space-time surface and also its WCW coordinates. The restriction to a cognitive representation with given number of points in given extension of rationals would mean a reduction of WCW to a finite-dimensional sub-space.

   The first wild guess is that this space is Kähler manifold with maximal symmetries - just as WCW is. A further wild guess is that these reduced WCWs are Grassmannians and correspond to those appearing in the twistor Grassmannian approach. A more general conjecture is inspired by the vision that super-symplectic gauge conditions effectively reduce the super-symplectic algebra to a Kac-Moody algebra of a finite-dimensional Lie group - perhaps belonging to ADE hierarchy. The flag manifolds associated with these Lie groups define more general homogenous spaces as candidates for the reduced WCWs.

4. One must allow the action of Galois group and this gives several options for given set X of points in algebraic extension.

    (a) One can construct $X^4(X)$ in terms of octonionic polynomial and construct a representation of Galois group as superposition of space-time surfaces obtained from space-time surface by the action of Galois group on X giving rise to new sets $X_g = g(X)$.

    (b) One can also consider the action of Galois group on X and get larger set Y of points and construct single multi-sheeted surface $X^4(Y)$. This surface corresponds to Planck constant $h_{eff}/h = n$, where n is the dimension of algebraic extension.

    (c) One can also consider the actions of sub-groups of $H \subset Gal$ to X to get space-time surface with $h_{eff}/h = m$ dividing n. There are several options corresponding to representations for all sub-groups of Galois group. A hierarchy of symmetry breakings seems to be involved with unbroken symmetry associated with the largest value of h_{eff}/h.

5. In this picture the hard core would reduce to the classical number theoretical dynamics of space-time surface in M^8. The additional degrees of freedom would be due to the possibility of different octonionic and quaternionic structures and choices of size scales and Lorentz boosts and translations of CDs. The symmetries would dictate the S-matrix in the moduli degrees of freedom: the dream is that this part of the dynamics reduces to kinematics, so to say.

    The discrete coupling constant evolution would be determined by the hierarchy of extensions of rationals and by the hierarchy of p-adic length scales. The cancellation of radiative corrections in the sense of sub-CDs inside CDs could be achieved by replacing coupling constant evolution with its discrete counterpart.

If this dream has something to do with reality, the construction of scattering amplitudes would reduce to their construction in moduli degrees of freedom and here the generalization of twistorial approach relying on Yangian symmetry allowing to identify scattering amplitudes as Yangian invariants might "trivialize" the situation. It will be found that the Yangian symmetry could corresponds to general coordinate transformations for the reduced WCW forced by the restriction of the spacetime surfaces to those allowed by octonionic polynomials with coefficients in the extensiom of rationals.

6.3.2 Do loop contributions to the scattering amplitudes vanish in TGD framework?

In TGD scattering amplitudes interpreted as zero energy states would correspond at embedding space level to collections of space-time surfaces inside CDs analogous to vertices and connected by lines defined by the space-time surfaces representing on-mass-shell particles. One would have massless particles in 8-D sense. The quaternionicity of 8-momentum leads to $M^4 \times CP_2$ picture and CP_2 twistors should replace E^4 twistors of M^8 approach.

Why loop corrections should vanish?

There are several arguments suggesting that the loop contributions should vanish in TGD framework. This would give rise to a discrete coupling constant evolution analogous to a sequence of phase transitions between different critical coupling parameters. Amplitudes would be obtained as tree diagrams.

1. In ZEO it is far from clear what the basic operation defining the loop contribution could even mean. One would have zero energy state for which the members of added particle pair have opposite but momenta but the amplitude is superposition of states with varying momenta. Why should one allow zero energy states containing one particle which is not an eigenstate of momentum? This suggests that ZEO does not allow loop contributions at all: the distinction between PEO and ZEO would make itself visible in rather dramatic manner.

2. The restriction of the BCFW to tree diagrams is internally consistent since the loop term is identically vanishing in this case. The first term in the BCFW for diagram with l loops involves a factor with $l > 0$ loops which vanishes. In $l = 1$ case the second term is obtained from $(n + 2, l - 1 = 0)$ diagram by generating loop but this vanishes by assumption.

3. Number theoretic vision does not favor the decomposition of the amplitude to an infinite sum of amplitudes since this is expected to lead to the emergence of transcendental numbers and functions in the amplitude in conflict with the number theoretical universality.

   Loops indeed give logarithms and poly-logarithms of rational functions of external momenta in Grassmannian approach. This violates the number theoretical universality since the p-adic counterpart of logarithm exist only for the argument of form $x = 1 + O(p)$. This condition cannot hold true for all primes simultaneously.

Discrete coupling constant evolution suggests the vanishing of loops. One can imagine two alternative mechanisms for the vanishing of loop contributions. Either the loop contributions do not make sense at all in ZEO, or the sum of loop contributions for the critical values of coupling constants vanishes. The summing up of loop contributions to zero for critical values of couplings should happen for all values of external momenta and other quantum numbers: this does not look plausible.

General number theoretic ideas about coupling constant evolution

The discrete coupling constant evolution would be associated with the scale hierarchy for CDs and the hierarchy of extensions of rationals.

1. Discrete p-adic coupling constant evolution would naturally correspond to the dependence of coupling constants on the size of CD. For instance, I have considered a concrete but rather ad hoc proposal for the evolution of Kähler couplings strength based on the zeros of Riemann zeta [L6]. Number theoretical universality suggests that the size scale of CD identified as the temporal distance between the tips of CD using suitable multiple of CP_2 length scale as a length unit is integer, call it l. The prime factors of the integer could correspond to preferred p-adic primes for given CD.

2. I have also proposed that the so called ramified primes of the extension of rationals correspond to the physically preferred primes. Ramification is algebraically analogous to criticality in the sense that two roots understood in very general sense coincide at criticality. Could the primes appearing as factors of l be ramified primes of extension? This would give strong correlation between the algebraic extension and the size scale of CD.

In quantum field theories coupling constants depend in good approximation logarithmically on mass scale, which would be in the case of p-adic coupling constant evolution replaced with an integer n characterizing the size scale of CD or perhaps the collection of prime factors of n (note that one cannot exclude rational numbers as size scales). Coupling constant evolution could also depend on the size of extension of rationals characterized by its order and Galois group.

In both cases one expects approximate logarithmic dependence and the challenge is to define "number theoretic logarithm" as a rational number valued function making thus sense also for p-adic number fields as required by the number theoretical universality.

1. Coupling constant evolution with respect to CD size scale

Consider first the coupling constant as a function of the length scale $l_{CD}(n)/l_{CD}(1) = n$.

1. The number $\pi(n)$ of primes $p \leq n$ behaves approximately as $\pi(n) = n/log(n)$. This suggests the definition of what might be called "number theoretic logarithm" as $Log(n) \equiv n/\pi(n)$. Also iterated logarithms such $log(log(x))$ appearing in coupling constant evolution would have number theoretic generalization.

2. If the p-adic variant of $Log(n)$ is mapped to its real counterpart by canonical identification involving the replacement $p \rightarrow 1/p$, the behavior can very different from the ordinary logarithm. $Log(n)$ increases however very slowly so that in the generic case one can expect $Log(n) < p_{max}$, where p_{max} is the largest prime factor of n, so that there would be no dependence on p for p_{max} and the image under canonical identification would be number theoretically universal.

   For $n = p^k$, where p is small prime the situation changes since $Log(n)$ can be larger than small prime p. Primes p near primes powers of 2 and perhaps also primes near powers of 3 and 5 - at least - seem to be physically special. For instance, for Mersenne prime $M_k = 2^k - 1$ there would be dramatic change in the step $M_k \rightarrow M_k + 1 = 2^k$, which might relate to its special physical role.

3. One can consider also the analog of $Log(n)$ as

$$Log(n) = \sum_p k_p Log(p) \quad ,$$

   where p^{k_i} is a factor of n. $Log(n)$ would be sum of number theoretic analogs for primes factors and carry information about them.

   One can extend the definition of $Log(x)$ to the rational values $x = m/n$ of the argument. The logarithm $Log_b(n)$ in base $b = r/s$ can be defined as $Log_b(x) = Log(x)/Log(b)$.

4. For $p \in \{2, 3, 5\}$ one has $Log(p) > log(p)$, where for larger primes one has $Log(p) < log(p)$. One has $Log(2) = 2 > log(2) = .693...$, $Log(3) = 3k/2 > log(3) = 1.099$, $Log(5) = 5/3 = 1.666.. > log(5) = 1.609$. For $p = 7$ one has $Log(7) = 7/4 \simeq 1.75 < log(7) \simeq 1.946$. Hence these primes and CD size scales n involving large powers of $p \in \{2, 3, 5\}$ ought to be physically special as indeed conjectured on basis of p-adic calculations and some observations related to music and biological evolution [K52, K55, K64, K44].

   In particular, for Mersenne primes $M_k = 2^k - 1$ one would have $Log(M_k) \simeq klog(2)$ for large enough k. For $Log(2^k)$ one would have $k \times Log(2) = 2k > log(2^k) = klog(2)$: there would be sudden increase in the value of $Log(n)$ at $n = M_k$. This jump in p-adic length scale evolution might relate to the very special physical role of Mersenne primes strongly suggested by p-adic mass calculations [K42].

5. One can wonder whether one could replace the $log(p)$ appearing as a unit in p-adic negentropy [K46] with a rational unit $Log(p) = p/\pi(p)$ to gain number theoretical universality? One could therefore interpret the p-adic negentropy as real or p-adic number for some prime. Interestingly, $|Log(p)|_p = 1/p$ approaches zero for large primes p (eye cannot see itself!) whereas $|Log(p)|_q = 1/|\pi(p)|_q$ has large values for the prime power factors q^r of $\pi(p)$.

2. The dependence of $1/\alpha_K$ on the extension of rationals

Consider next the dependence on the extension of rationals. The natural algebraization of the problem is to consider the Galois group of the extension.

1. Consider first the counterparts of primes and prime factorization for groups. The counterparts of primes are simple groups, which do not have normal subgroups H satisfying $gH = Hg$ implying invariance under automorphisms of G. Simple groups have no decomposition to a product of sub-groups. If the group has normal subgroup H, it can be decomposed to a product $H \times G/H$ and any finite group can be decomposed to a product of simple groups.

   All simple finite groups have been classified (see `http://tinyurl.com/jn44bxe`). There are cyclic groups, alternating groups, 16 families of simple groups of Lie type, 26 sporadic groups. This includes 20 quotients G/H by a normal subgroup of monster group and 6 groups which for some reason are referred to as pariahs.

2. Suppose that finite groups can be ordered so that one can assign number $N(G)$ to group G. The roughest ordering criterion is based on $ord(G)$. For given order $ord(G)=n$ one has all groups, which are products of cyclic groups associated with prime factors of n plus products involving non-Abelian groups for which the order is not prime. $N(G) > ord(G)$ thus holds true. For groups with the same order one should have additional ordering criteria, which could relate to the complexity of the group. The number of simple factors would serve as an additional ordering criterion.

   If its possible to define $N(G)$ in a natural manner then for given G one can define the number $\pi_1(N(G))$ of simple groups (analogs of primes) not larger than G. The first guess is that that the number $\pi_1(N(G))$ varies slowly as a function of G. Since Z_i is simple group, one has $\pi_1(N(G)) \geq \pi(N(G))$.

3. One can consider two definitions of number theoretic logarithm, call it Log_1.

$$\begin{array}{ll} a) & Log_1(N(G)) = \frac{N(G)}{\pi_1(N(G))} \ , \\ b) & Log_1(G) = \sum_i k_i Log_1(N(G_i)) \ , \quad Log_1(N(G_i)) = \frac{N(G_i)}{\pi_1(N(G_i))} \ . \end{array} \tag{6.3.1}$$

   Option a) does not provide information about the decomposition of G to a product of simple factors. For Option b) one decomposes G to a product of simple groups G_i: $G = \prod_i G_i^{k_i}$ and defines the logarithm as Option b) so that it carries information about the simple factors of G.

4. One could organize the groups with the same order to same equivalence class. In this case the above definitions would give

$$\begin{array}{ll} a) & Log_1(ord(G)) = \frac{ord(G)}{\pi_1(ord(G))} < Log(ord(G)) \ , \\ b) & Log_1(ord(G)) = \sum_i k_i Log(ord(G_i)) \ , \quad Log_1(ord(G_i)) = \frac{ord(G_i)}{\pi_1(ord(G_i))} \ . \end{array} \tag{6.3.2}$$

   Besides groups with prime orders there are non-Abelian groups with non-prime orders. The occurrence of same order for two non-isomorphic finite simple groups is very rare (see `http://tinyurl.com/ydd6uomb`). This would suggests that one has $\pi_1(ord(G)) < ord(G)$ so that $Log_1(ord(G))/ord(G) < 1$ would be true.

5. For orders $n(G) \in \{2,3,5\}$ one has $Log_1(n(G)) = Log(n(G)) > log(n(G))$ so that the ordes $n(G)$ involving large factors of $p \in \{2,3,5\}$ would be special also for the extensions of rationals. S_3 with order 6 is the first non-abelian simple group. One has $\pi(S_3) = 4$ giving $Log(6) = 6/4 = 1.5 < log(6) = 1.79$ so that S_3 is different from the simple groups below it.

To sum up, number theoretic logarithm could provide answer to the long-standing question what makes Mersenne primes and also other small primes so special.

Considerations related to coupling constant evolution and Riemann zeta

I have made several number theoretic peculations related to the possible role of zeros of Riemann zeta in coupling constant evolution. The basic problem is that it is not even known whether the zeros of zeta are rationals, algebraic numbers or genuine transcendentals or belong to all these categories. Also the question whether number theoretic analogs of ζ defined for p-adic number fields could make sense in some sense is interesting.

1. Is number theoretic analog of ζ possible using $Log(p)$ instead of $log(p)$?

The definition of $Log(n)$ based on factorization $Log(n) \equiv \sum_p k_p Log(p)$ allows to define the number theoretic version of Riemann Zeta $\zeta(s) = \sum n^{-s}$ via the replacement $n^{-s} = exp(-log(n)s) \rightarrow exp(-Log(n)s)$.

1. In suitable region of plane number-theoretic Zeta would have the usual decomposition to factors via the replacement $1/(1-p^{-s}) \rightarrow 1/(1-exp(-Log(p)s)$. p-Adically this makes sense for $s=O(p)$ and thus only for a finite number of primes p for positive integer valued s: one obtains kind of cut-off zeta. Number theoretic zeta would be sensitive only to a finite number of prime factors of integer n.

2. This might relate to the strong physical indications that only a finite number of cognitive representations characterized by p-adic primes are present in given quantum state: the ramified primes for the extension are excellent candidates for these p-adic primes. The size scale n of CD could also have decomposition to a product of powers of ramified primes. The finiteness of cognition conforms with the cutoff: for given CD size n and extension of rationals the p-adic primes labelling cognitive representations would be fixed.

3. One can expand the regions of converge to larger p-adic norms by introducing an extension of p-adics containing e and some of its roots (e^p is automatically a p-adic number). By introducing roots of unity, one can define the phase factor $exp(-iLog(n)Im(s))$ for suitable values of $Im(s)$. Clearly, $exp(-ipIm(s))/\pi(p))$ must be in the extension used for all primes p involved. One must therefore introduce prime roots $exp(i/\pi(p))$ for primes appearing in cutoff. To define the number theoretic zeta for all p-adic integer values of $Re(s)$ and all integer values of $Im(s)$, one should allow all roots of unity $(ep(i2\pi/n))$ and all roots $e^{1/n}$: this requires infinite-dimensional extension.

4. One can thus define a hierarchy of cutoffs of zeta: for this the factorization of Zeta to a finite number of "prime factors" takes place in genuine sense, and the points $Im(s)=ik\pi(p)$ give rise to poles of the cutoff zeta as poles of prime factors. Cutoff zeta converges to zero for $Re(s)\rightarrow\infty$ and exists along angles corresponding to allowed roots of unity. Cutoff zeta diverges for $(Re(s)=0, Im(s)=ik\pi(p))$ for the primes p appearing in it.

Remark: One could modify also the definition of ζ for complex numbers by replacing $exp(log(n)s)$ with $exp(Log(n)s)$ with $Log(n)=\sum_p k_pLog(p)$ to get the prime factorization formula. I will refer to this variant of zeta as modified zeta ($\tilde{\zeta}$) below. $\tilde{\zeta}$ would carry explicit number theoretic information via the dependence of its "prime factors" $1/(1-exp(-Log(p)s))$.

2. Could the values of $1/\alpha_K$ *be given as zeros of* ζ *or of* $\tilde{\zeta}$

In [L6] I have discussed the possibility that the zeros $s=1/2+iy$ of Riemann zeta at critical line correspond to the values of complex valued Kähler coupling strength α_K: $s=i/\alpha_K$. The assumption that p^{iy} is root of unity for some combinations of p and y [$log(p)y=(r/s)2\pi$] was made. This does not allow s to be complex rational. If the exponent of Kähler action disappears from the scattering amplitudes as M^8-H duality requires, one could assume that s has rational values but also algebraic values are allowed.

1. If one combines the proposed idea about the Log-arithmic dependence of the coupling constants on the size of CD and algebraic extension with $s=i/\alpha_K$ hypothesis, one cannot avoid the conjecture that the zeros of zeta are complex rationals. It is not known whether this is the case or not. The rationality would not have any strong implications for number theory but the existence irrational roots would have (see `http://tinyurl.com/y8bbnhe3`). Interestingly, the rationality of the roots would have very powerful physical implications if TGD inspired number theoretical conjectures are accepted.

   The argument discussed below however shows that complex rational roots of zeta are not favored by the observations [A20] about the Fourier transform for the characteristic function for the zeros of zeta. Rather, the findings suggest that the imaginary parts [L5] should be rational multiples of 2π, which does not conform with the vision that $1/\alpha_K$ is algebraic number. The replacement of $log(p)$ with $Log(p)$ and of 2π with is natural p-adic approximation in an extension allowing roots of unity however allows $1/\alpha_K$ to be an algebraic number. Could the spectrum of $1/\alpha_K$ correspond to the roots of ζ or of $\tilde{\zeta}$?

2. A further conjecture discussed in [L6] was that there is 1-1 correspondence between primes $p\simeq 2^k$, k prime, and zeros of zeta so that there would be an order preserving map $k\rightarrow s_k$. The

support for the conjecture was the predicted rather reasonable coupling constant evolution for α_K. Primes near powers of 2 could be physically special because $Log(n)$ decomposes to sum of $Log(p)$:s and would increase dramatically at $n = 2^k$ slightly above them.

In an attempt to understand why just prime values of k are physically special, I have proposed that k-adic length scales correspond to the size scales of wormhole contacts whereas particle space-time sheets would correspond to $p \simeq 2^k$. Could the logarithmic relation between L_p and L_k correspond to logarithmic relation between p and $\pi(p)$ in case that $\pi(p)$ is prime and could this condition select the preferred p-adic primes p?

3. The argument of Dyson for the Fourier transform of the characteristic function for the set of zeros of ζ

Consider now the argument suggesting that the roots of zeta cannot be complex rationals. On basis of numerical evidence Dyson [A20] (`http://tinyurl.com/hjbfsuv`) has conjectured that the Fourier transform for the characteristic function for the critical zeros of zeta consists of multiples of logarithms $log(p)$ of primes so that one could regard zeros as one-dimensional quasi-crystal.

This hypothesis makes sense if the zeros of zeta decompose into disjoint sets such that each set corresponds to its own prime (and its powers) and one has $p^{iy} = U_{m/n} = exp(i2\pi m/n)$ (see the appendix of [L5]). This hypothesis is also motivated by number theoretical universality [K84, L23].

1. One can re-write the discrete Fourier transform over zeros of ζ at critical line as

$$f(x) = \sum_y exp(ixy)) \quad , \quad y = Im(s) \quad .$$

The alternative form reads as

$$f(u) = \sum_s u^{iy} \quad , \quad u = exp(x) \quad .$$

$f(u)$ is located at powers p^n of primes defining ideals in the set of integers.

For $y = p^n$ one would have $p^{iny} = exp(inlog(p)y)$. Note that $k = nlog(p)$ is analogous to a wave vector. If $exp(inlog(p)y)$ is root of unity as proposed earlier for some combinations of p and y, the Fourier transform becomes a sum over roots of unity for these combinations: this could make possible constructive interference for the roots of unity, which are same or at least have the same sign. For given p there should be several values of $y(p)$ with nearly the same value of $exp(inlog(p)y(p))$ whereas other values of y would interfere deconstructively.

For general values $y = x^n$ $x \neq p$ the sum would not be over roots of unity and constructive interference is not expected. Therefore the peaking at powers of p could take place. This picture does not support the hypothesis that zeros of zeta are complex rational numbers so that the values of $1/\alpha_K$ correspond to zeros of zeta and would be therefore complex rationals as the simplest view about coupling constant evolution would suggest.

Remark: Mumford has argued (`http://tinyurl.com/zemw27o`) that the Fourier transform should include also the trivial zeros at $s = -2, -4, -6...$ giving and exponentially small contributions and providing a slowly varying background to the Fourier transform.

2. What if one replaces $log(p)$ with $Log(p) = p/\pi(p)$, which is rational and thus ζ with $\tilde{\zeta}$? For large enough values of p $Log(p) \simeq log(p)$ finite computational accuracy does not allow distinguish $Log(p)$ from $log(p)$. For $Log(p)$ one could thus understand the finding in terms of constructive interference for the roots of unity if the roots of zeta are of form $s = 1/2 + i(m/n)2\pi$. The value of y cannot be rational number and $1/\alpha_K$ would have real part equal to y proportional to 2π which would require infinite-D extension of rationals. In p-adic sectors infinite-D extension does not conform with the finiteness of cognition.

Remark: It is possible to check by numerical calculations whether the locus of complex zeros of $\tilde{\zeta}$ is at line $Res(2) = 1/2$. If so, then Fourier transform would make sense. One can also check whether the peaks at $nlog(p)$ are shifted to $nLog(p)$: for $p = 2$ one would have $Log(2) = 2 > log(2)$. The positions of peaks should shift to the right for $p = 2, 3, 5$ and to the left for $p > 5$. This should be easy to check by numerical calculations.

3. Numerical calculations have however finite accuracy, and allow also the possibility that y is algebraic number approximating rational multiple of 2π in some natural manner. In p-adic sectors would obtain the spectrum of y and $1/\alpha_K$ as algebraic numbers by replacing 2π in the formula $is = \alpha_K = i/2 + q \times 2\pi$, $q = r/s$, with its approximate value:

$$2\pi \rightarrow sin(2\pi/n)n = i\frac{n}{2}(exp(i2\pi/n) - exp(-i2\pi/n))$$

for an extension of rationals containing n:th of unity. Maximum value of n would give the best approximation. This approximation performed by fundamental physics should appear in the number theoretic scattering amplitudes in the expressions for $1/\alpha_K$ to make it algebraic number.

y can be approximated in the same manner in p-adic sectors and a natural guess is that $n = p$ defines the maximal root of unity as $exp(i2\pi/p)$. The phase $exp(ilog(p)y)$ for $y = qsin(2\pi/n(y))$, $q = r/s$, is replaced with the approximation induced by $log(p) \rightarrow Log(p)$ and $2\pi \rightarrow sin(2\pi/n)n$ giving

$$exp(ilog(p)y) \rightarrow exp(iq(y)sin(2\pi/n(y))\frac{p}{\pi(p)}) \ .$$

If s in $q = r/s$ does not contain higher powers of p, the exponent exists p-adically for this extension and can can be expanded in positive powers of p as

$$\sum_n i^n q^n sin(2\pi/p)^n (p/\pi(p))^n \ .$$

This makes sense p-adically.

Also the actual complex roots of ζ could be algebraic numbers:

$$s = i/2 + q \times sin(\frac{2\pi}{n(y)})n(y) \ .$$

If the proposed correlation between p-adic primes $p \simeq 2^k$, k prime and zeros of zeta predicting a reasonable coupling constant evolution for $1/\alpha_K$ is true, one can have naturally, $n(y) = p(y)$, where p is the p-adic prime associated with y: the accuracy in angle measurement would increase with the size scale of CD. For given p there could be several roots y with same $p(y)$ but different $q(y)$ giving same phases or at least phases with same sign of real part.

Whether the roots of $tilde\zeta$ are algebraic numbers and at critical line $Re(s) = 1/2$ is an interesting question.

Remark: This picture allows many variants. For instance, if one assumes standard zeta, one could consider the possibility that the roots y_p associated with p and giving rise to constructive interference are of form $y = q \times (Log(p)/log(p)) \times sin(2\pi/p)p$, $q = r/s$.

4. Could functional equation and Riemann hypothesis generalize?

It is interesting to list the elementary properties of the $\tilde{\zeta}$ before trying to answer to the questions of the title.

1. The replacement $log(n) \rightarrow Log(n) \equiv sum_p k_p Log(p)$ implies that $\tilde{\zeta}$ codes explicitly number theoretic information. Note that $Log(n)$ satisfies the crucial identity $Log(mn) = Log(m) + Log(n)$. $\tilde{\zeta}$ is an analog of partition function with rational number valued $Log(n)$ taking the role of energy and $1/s$ that of a complex temperature. In ZEO this partition function like entity could be associated with zero energy state as a "square root" of thermodynamical partition function: in this case complex temperatures are possible. $|\tilde{\zeta}|^2$ would be the analog of ordinary partition function.

2. Reduction of $\tilde{\zeta}$ to a product of "prime factors" $1/[1 - exp(-Log(p)s)]$ holds true by $Log(n) \equiv sum_p k_p Log(p)$, $Log(p) = p/\pi(p)$.

3. $\tilde{\zeta}$ is a combination of exponentials $exp(-Log(n)s)$, which converge for $Re(s) > 0$. For ζ one has exponentials $exp(-log(n)s)$, which also converge for $Re(s) > 0$: the sum $\sum n^{-s}$ does not however converge in the region $Re(s) < 1$. Presumably $\tilde{\zeta}$ fails to converge for $Re(s) \leq 1$. The behavior of terms $exp(-Log(n)s)$ for large values of n is very similar to that in ζ.

4. One can express ζ o in terms of η function defined as

$$\eta(s) = \sum (-1)^n n^{-s} \quad .$$

The powers $(-1)^n$ guarantee that η converges (albeit not absolutely) inside the critical strip $0 < s < 1$.

By using a decomposition of integers to odd and even ones, one can express ζ in terms of η:

$$\zeta = \frac{\eta(s)}{(-1 + 2^{-s+1})} \quad .$$

This definition converges inside critical strip. Note the pole at $s = 1$ coming from the factor.

One can define also $\tilde{\eta}$:

$$\tilde{\eta}(s) = \sum (-1)^n e^{-Log(n)s} \quad .$$

The formula relating $\tilde{\zeta}$ and $\tilde{\eta}$ generalizes: 2^{-s} is replaced with $exp(-2s)$ $(Log(2) = 2)$:

$$\tilde{\zeta} = \frac{\tilde{\eta}(s)}{-1 + 2exp^{-2s}} \quad .$$

This definition $\tilde{\zeta}$ converges in the critical strip $Re(s) \in (0,1)$ and also for $Re(s) > 1$. $\tilde{\zeta}(1-s)$ converges for $Re(s) < 1$ so that in $\tilde{\eta}$ representation both converge.

Note however that the poles of ζ at $s = 1$ has shifted to that at $s = log(2)/2$ and is below $Re(s) = 1/2$ line. If a symmetrically posioned pole at $s = 1 - log(2)/2$ is not present in $\tilde{\eta}$, functional equation cannot be true.

5. $Log(n)$ approaches $log(n)$ for integers n not containing small prime factors p for which $\pi(n)$ differs strongly from $p/log(p)$. This suggests that allowing only terms $exp(-Log(n)s)$ in the sum defining $\tilde{\zeta}$ not divisible by primes $p < p_{max}$ might give a cutoff $\tilde{\zeta}^{cut,p_{max}}(s)$ behaving very much like ζ from which "prime factors" $1/(1 - exp(-Log(p)s)$, $p < p_{max}$ are dropped of. This is just division of $\tilde{\zeta}$ by these factors and at least formally, this does not affect the zeros of $\tilde{\zeta}$. Arbitrary number of factors can be droped. Could this mean that $\tilde{\zeta}^{cut}$ has same or very nearly same zeros as ζ at critical line? This sounds paradoxical and might reflect my sloppy thinking: maybe the lack of the absolute implies that the conclusion is incorrect.

The key questions are whether $\tilde{\zeta}$ allows a generalization of the functional equation $\xi(s) = \xi(1-s)$ with $\xi(s) = \frac{1}{2}s(s-1)\Gamma(s/2)\pi^{-s/2}\zeta(s)$ and whether Riemann hypothesis generalizes. The derivation of the functional equation is quite a tricky task and involves integral representation of ζ .

1. One can start from the integral representation of ζ true for $s > 0$.

$$\zeta(s) = \frac{1}{(1-2^{1-s})\Gamma(s)} \int_0^\infty \frac{t^{s-1}}{e^t + 1} dt \ , \quad Re(s) > 0 \quad .$$

deducible from the expression in terms of $\eta(s)$. The factor $1/(1 + e^t)$ can be expanded in geometric series $1/(1 + e^t) = \sum (-1)^n exp(nt)$ converning inside the critical strip. One formally performs the integrations by taking nt as an integration variable. The integral gives the result $\sum (-1)^n / n^z)\Gamma(s)$.

The generalization of this would be obtained by a generalization of geometric series:

$$1/(1+e^t) = \sum (-1)^n exp(nt) \rightarrow \sum (-1)^n e^{exp(Log(n))t}$$

in the integral representation. This would formally give $\tilde{\zeta}$: the only difference is that one takes $u = exp(Log(n))t$ as integration variable.

One could try to prove the functional equation by using this representation. One proof (see `http://tinyurl.com/yak93hyr`) starts from the alternative expression of ζ as

$$\zeta(s) = \frac{1}{\Gamma(s)} \int_1^\infty \frac{t^{s-1}}{e^t - 1} dt \ , \ Re(s) > 1 \ .$$

One modifies the integration contour to a contour C coming from $+\infty$ above positive real axis, circling the origin and returning back toc$+\infty$ below the real axes to get a modified representation of ζ:

$$\zeta(s) = \frac{1}{2isin(\pi s)\Gamma(s)} \int_1^\infty \frac{(-w)^{s-1}}{e^w - 1} dw \ , \ Re(s) > 1 \ .$$

One modifies the C further so that the origin is circled around a square with vertices at $\pm(2n+1)\pi$ and $\pm i(2n+1)\pi$.

One calculates the integral the integral along C as a residue integral. The poles of the integrand proportional to $1/(1-e^t)$ are at imaginary axis and correspond to $w = ir2\pi$, $r \in Z$. The residue integral gives the other side of the functional equation.

2. Could one generalize this representation to the recent case? One must generalize the geometric series defined by $1/(e^w - 1)$ to $-\sum e^{exp(Log(n))w}$. The problem is that one has only a generalization of the geometric series and not closed form for the counterpart of $1/(exp(w)-1)$ so that one does not know what the poles are. The naïve guess is that one could compute the residue integrals term by term in the sum over n. An equally naïve guess would be that for the poles the factors in the sum are equal to unity as they would be for Riemann zeta. This would give for the poles of n:th term the guess $w_{n,r} = r2\pi/exp(Log(n)$, $r \in Z$. This does not however allow to deduce the residue at poles.Note that the poles of $\tilde{\eta}$ at $s = log(2)/2$ suggests that functional equation is not true.

There is however no need for a functional equation if one is only interested in $F(s) \equiv \tilde{\zeta}(s) + \tilde{\zeta}(1-s)$ at the critical line! Also the analog of Riemann hypothesis follows naturally!

1. In the representation using $\tilde{\eta}$ $F(s)$ converges at critical striple and is *real*(!) at the critical line $Re(s) = 1/2$ as follows from the fact that $1 - s = \overline{s}$ for $Re(s) = 1/2$! Hence $F(s)$ is expected to have a large number of zeros at critical line. Presumably their number is infinite, since $F(s)^{cut,p_{max}}$ approaches $2\zeta^{cut,p_{max}}$ for large enough p_{max} at critical line.

2. One can define a different kind of cutoff of $\tilde{\zeta}$ for given n_{max}: $n < n_{max}$ in the sum over $e^{-Log(n)s}$. Call this cutoff $\tilde{\zeta}^{cut,n_{max}}$. This cutoff must be distinguished from the cutoff $\tilde{\zeta}^{cut,p_{max}}$ obtained by dropping the "prime factors" with $p < pmax$. The terms in the cutoff are of the form $u^{\sum k_p p/\pi(p)}$, $u = exp(-s)$. It is analogous to a polymomial but with fractional powers of u. It can be made a polynomial by a change of variable $u \rightarrow v = exp(-s/a)$, where a is the product of all $\pi(p)$:s associated with all the primes involved with the integers $n < n_{max}$.

   One could solve numerically the zeros of $\zeta(\tilde{s}) + \zeta(\tilde{s})$ using program modules calculating $\pi(p)$ for a given p and roots of a complex polynomial in given order. One can check whether also all zeros of $\zeta(\tilde{s}) + \zeta(\tilde{s})$ might reside at critical line.

3. One an define also $F(s)^{cut,n_{max}}$ to be distinguished from $F(s)^{cut,p_{max}}$. It reduces to a sum of terms $exp(-Log(n)/2)cos(-Log(n)y)$ at critical line, $n < n_{max}$. Cosines come from roots of unity. $F(s)$ function is not sum of rational powers of $exp(-iy)$ unlike $\zeta(\tilde{s})$. The existence of zero could be shown by showing that the sign of this function varies as function of y. The functions $cos(-Log(n)y)$ have period $\Delta y = 2\pi/Log(n)$. For small values of n the

exponential terms $exp(-Log(n)/2)$ are largest so that they dominate. For them the periods Δy are smallest so that one expected that the sign of both $F(s)$ and $F(s)^{cut,n_{max}}$ varies and forces the presence of zeros.

One could perhaps interpret the system as quantum critical system. The rather large rapidly varying oscillatory terms with $n < n_{max}$ with small $Log(n)$ give a periodic infinite set of approximate roots and the exponentially smaller slowly varying higher terms induce small perturbations of this periodic structure. The slowly varying terms with large $Log(n)$ become however large near the $Im(s) = 0$ so that here the there effect is large and destroys the period structure badly for small root of $\hat{\zeta}$.

Is the vanishing of the loop corrections consistent with unitarity?

Skeptic could argue that the vanishing of loop corrections is not consistent with unitarity. The following argument however shows that the fact that momenta in TGD framework are 8-D light-like momenta could save the situation. If not only single particle states but also *many-particle states* have light-like 8-momenta, the discontinuity of the amplitude at pole $P^2(M^8) = 0$ implies the discontinuity of the amplitude as function of $s \equiv P^2(M^4)$ along s-axis.

Minkowskian contribution to mass squared would essentially the sum of conformal (stringy) contribution from vibrational degrees of freedom and color contribution from CP_2 degrees of freedom. This suggests a weak form of color confinement: many-particle states could have vanishing color hyper charge and isospin but the eigenvalue value of color Casimir operator would be non-vanishing.

To get more concrete view about the situation the reader is encouraged to study the slides of Jaroslav Trnka explaining BCFW recursion formula [B35] (see `http://tinyurl.com/pqjzffj`) or the article [B18] of Elvang and Huang (see `http://tinyurl.com/y9rhbzhk`).

1. Unitarity condition $SS^\dagger = Id$ for S-matrix $S = 1 + iT$ gives $i(T - T^\dagger) = TT^\dagger$. For forward scattering the physical interpretation is that the discontinuity of $-2Im(T) = i(T - T^\dagger)$ in forward scattering as a function of total mass s above kinematical threshold along real axis is essentially the total scattering rate.

2. For a given tree amplitude, which is rational function, one replaces external momenta p_i with $\hat{p}_i = p_i + zr_i$. r_i real, light-like and orthogonal to each other and their sum vanishes. This gives on mass shell scattering amplitude with complex light-like momenta satisfying conservation conditions.

3. One can consider any non-trivial subset I of momenta and for this set one has $\hat{P}_I^2 = P_i^2 + 2zP \cdot R_I$, where one has $P_I = \sum_i p_i$ and $R_I = \sum_i r_i$. This gives

$$\hat{P}_I^2 = -P_I^2 \frac{(z - z_I)}{z_I} \quad , \quad z_I = \frac{P_I^2}{2P_I \cdot R_I} \quad .$$

The poles of the modified amplitude $\hat{A}_n(z)$ come from the propagators at $\hat{P}_I^2 = 0$ and correspond to the points $z = z_I$.

4. From the modified scattering amplitude $\hat{A_n}(z)$ one can obtain the original scattering amplitude by performing a residue integral for $\hat{A}_n(z)/z$ along a curve enclosing the poles z_I. This gives

$$A_n = \hat{A_n}(z = 0) + \sum_{z_I} Res_{z=z_I}(\frac{\hat{A}_n(z)}{z}) + B_n \quad .$$

B_n comes from the possible pole at $z = \infty$ and is often assumed to vanish. If so, the amplitude factorizes into a sum of products

$$Res_{z=z_I} \frac{\hat{A}_n(z)}{z} = \sum_I \hat{A}_L(z_I) \frac{1}{P_I^2} \hat{A}_R(z_I) \quad .$$

The amplitudes appearing in the product are for modified complex momenta.

The vanishing of loop corrections thus implies that the product terms $\hat{A}_L(1/P^2)\hat{A}_R$ in the BCFW formula give rational functions having no cuts just as the number theoretical vision demands. The discontinuities of the imaginary part of the amplitude are at poles and reduce to the products $\hat{A}_L\hat{A}_R$ with complex on-mass- shell light-like momenta as unitarity demands.

For forward scattering the discontinuity would be essentially positive definite total scattering rate. It would be however non-vanishing only at $P^2 = 0$ so that scattering rate could be non-vanishing only for $P^2 = 0$! This does not make sense in 4-D physics. Is it possible to overcome this difficulty in TGD framework?

1. The first thing to notice is that classical TGD predicts complex Noether charges since for instance Kähler coupling strength has imaginary part. This would suggest that the momenta of incoming particles could be complex. Could complex value of $P(M^4) \equiv P$ implying

$$P^2 = Re(P)^2 - Im(P)^2 + i2Re(P) \cdot Im(P) = 0$$

save the situation? The condition requires that $Re(P)$ and $Im(P)$ are light-like and parallel so that one would obtain only light-like four-momenta as total M^4 momenta.

2. However, in TGD light-likeness holds true in 8-D sense for single particle states: this led to the proposed generalization of twistor approach allowing particles to be massive in 4-D sense. $M^8 - H$ duality allows to speak about light-like M^8 momenta satisfying quaternionicity condition. The wave functions in CP_2 degrees of freedom emerge from momentum wave functions in M^8 degrees of freedom respecting quaternionicity. The condition $P^2(M^8) = 0$ implies that $Re[P(M^8)]$ and $Im[P(M^8)]$ are light-like and parallel. $Im[P(M^8)]$ can be arbitrarily small. One has also $Re[P(M^4)]^2 = Re[P(E^4)]^2$ and $Im[P(M^4)]^2 = Im[P(E^4)]^2$.

3. Could one pose the condition $P^2(M^8) = 0$ also on *many-particle states* or only to the many-particle states appearing as complex massless poles in the BCFW conditions? Kind of strong form of conformal invariance would be in question: not only single-particle states but also many-particle states would be massless in 8-D sense. Now $s = Re[P(M^4))]^2 = Re[P(E^4))]^2$ could have a continuum of values. The discontinuity along s-axis required by unitarity would would emerge from the discontinuity due to the pole at $P^2(M^8) = 0$! Hence 8-dimensional light-likeness in strong sense would be absolutely essential for having vanishing loop corrections together with non-vanishing scattering rates!

Here one must be however extremely careful.

1. In BCFW approach the expression of residue integral as sum of poles in the variable z associated with the amplitude obtained by the deformation $p_i \rightarrow p_i + zr_i$ of momenta ($\sum r_i = 0$, $r_i \cdot r_j = 0$) leads to a decomposition of the tree scattering amplitude to a sum of products of amplitudes in resonance channels with complex momenta at poles. The products involve $1/P^2$ factor giving pole and the analog of cut in unitary condition. Proof of tree level unitarity is achieved by using complexified momenta as a mere formal trick and complex momenta are an auxiliary notion. The complex massless poles are associated with groups I of particles whereas the momenta of particles inside I are complex and non-light-like.

2. Could BCFW deformation give a description of massless bound states massless particles so that the complexification of the momenta would describe the effect of bound state formation on the single particle states by making them non-light-like? This makes sense if one assumes that all 8-momenta - also external - are complex. The classical charges are indeed complex already classically since Kähler coupling strength is complex [L6]. A possible interpretation for the imaginary part is in terms of decay width characterizing the life-time of the particle and defining a length of four-vector.

3. The basic question in the construction of scattering amplitudes is what happens inside CD for the external particles with light-like momenta. The BCFW deformation leading to factorization suggests an answer to the question. The factorized channel pair corresponds to two CDs inside which analogs of M and $N - M$ particle bound states of external massless

particles would be formed by the deformation $p_i \rightarrow p_i + zr_i$ making particle momenta non-light-like. The allowed values of z would correspond to the physical poles. The factorization of BCFW scattering amplitude would correspond to a decomposition to products of bound state amplitudes for pairs of CDs. The analogs of bound states for zero energy states would be in question. BCFW factorization could be continued down to the lowest level below which no factorization is possible.

4. One can of course worry about the non-uniqueness of the BCFW deformation. For instance, the light-like momenta r_i must be parallel ($r_i = \lambda_i r$) but the direction of r is free. Also the choice of λ_i is free to a high extent. BCFW expression for the amplitude as a residue integral over z is however unique. What could this non-uniqueness mean?

   Suppose one accepts the number theoretic vision that scattering amplitudes are representations for sequences of algebraic manipulations. These representations are bound to be highly non-unique since very many sequences can connect the same initial and final expressions. The space-time surface associated with given representation of the scattering amplitude is not unique since each computation corresponds to different space-time surface. There however exists a representation with maximal simplicity.

   Could these two kinds of non-uniqueness relate?

It is indeed easy to see that many-particle states with light-like single particle momenta cannot have light-like momenta unless the single-particle momenta are parallel so that in non-parallel case one must give up light-likeness condition also in complex sense.

1. The condition of light-likeness in complex sense allows the vanishing of real and imaginary mass squared for individual particles

$$Im(p_i) = \lambda_i Re(p_i) \ , \quad (Re(p_i))^2 = (Im(p_i))^2 = 0 \ . \tag{6.3.3}$$

   Real and imaginary parts are parallel and light-like in 8-D sense. All λ_i have same sign and p_i has positive or negative time component depending on whether positive or negative energy part of zero energy state is in question.

2. The remaining two conditions come from the vanishing of the real and imaginary parts of the total mass squared:

$$\sum_{i\neq j} Re(p_i)\cdot Re(p_j) - Im(p_i)\cdot Im(p_j) = 0 \ , \quad \sum_{i\neq j} Re(p_i)\cdot Im(p_j) = 0 \ . \tag{6.3.4}$$

   By using proportionality of $Im(p_i)$ and $Re(p_i)$ one can express the conditions in terms of the real momenta

$$\sum_{i\neq j}(1-\lambda_i\lambda_j) Re(p_i)\cdot Re(p_j) = 0 \ , \quad \sum_{i\neq j} \lambda_j Re(p_i)\cdot Re(p_j) = 0 \ . \tag{6.3.5}$$

   For positive/negative energy part of zero energy state the sign of time component of momentum is fixed and therefore λ_i have fixed sign. Suppose that λ_i have fixed sign. Since the inner products $p_i \cdot p_j$ of time-like vectors with fixed sign of time compomemet are all positive or negative the second term can vanish only if one has $p_i \cdot p_j = 0$. If the sign of λ_i can vary, one can satisfy the condition linear in λ_i but not the first condition as is easy to see in 2-particle case.

3. States with light-like parallel 8-momenta are allowed and one can ask whether this kind of states might be realized inside magnetic flux tubes identified as carriers of dark matter in TGD sense. The parallel light-like momenta in 8-D sense would give rise to a state analogous to super-conductivity. Could this be true also for quarks inside hadrons assumed to move in parallel in QCD based model. This also brings in mind the earlier intuitive proposal that the momenta of fermions and antifermions associated with partonic 2-surfaces must be parallel so that the propagators for the states containing altogether n fermions and antifermions would behave like $1/(p^2)^{n/2}$ and would not correspond to ordinary particles.

These arguments are formulated in M^8 picture. What could this mean in $M^4 \times CP_2$ picture?

1. The intuitive expectation is that $Re[P(E^4)]^2$ corresponds to the eigenvalue Λ of CP_2 d'Alembertian so that the higher the momentum, the larger the value of Λ. CP_2 d'Alembertian would be essentially the M^4 mass squared of the state. This would allow vanishing color quantum numbers Y and I_3 but force symmetry breaking $SU(3) \rightarrow SU(2) \times U(1)$. This picture is not quite accurate: also the vibrational degrees of freedom contribute to the mass squared what might be called stringy contribution.

2. Could the geometry of CP_2 induce this symmetry breaking? For instance, Kähler gauge potential depends on the $U(2)$ invariant "radial" coordinate of CP_2 and is invariant only under $U(2)$ rotations and changes by gauge transformation in other color rotations. Could one assign the symmetry breaking to the choice of color quantization axes boiling down at the classical level to the fixing of CP_2 Kähler function would?

   One would have color confinement in weak sense: in QCD picture physical states correspond to color singlet representations. This is certainly very strong statement in a sharp conflict with the standard view about color confinement. It would make sense in TGD framework, where color as a spin like quantum number is replaced with angular momentum like quantum number. One could say that macroscopic systems perform macroscopic color rotation. The model for the honeybee dance [L28] conforms with this view and actually led to the proposal for a modification of cosmic string type extremals $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$ by putting Y^2 in 2-D rigid body color rotation along both time axis and spatial axis of the string world sheet X^2.

3. This picture raises again the old question about the relationship of color and electroweak quantum numbers in TGD framework. Could one regard electroweak quantum numbers as a spin related to color group $SU(3)$ just as one can relate ordinary spin with Lorentz transformations? Color quantum numbers of say quarks would be analogous to orbital angular momentum. The realization of the action of the electroweak $U(2)_{ew}$ on CP_2 spinors indeed involves also geometric color rotation affecting the gauge potentials in the general case and $U(2)_{ew}$ can be identified as holonomy group of CP_2 spinor connection and sugroup of $SU(3)$. One could also see electroweak symmetry breaking as a further symmetry breaking $U(2) \rightarrow U(1) \times U(1)$ assignable with the flag manifold $SU(3)/U(1) \times U(1)$ parameterizing different choices of color quantization axes and having interpretation as CP_2 twistor space.

Remark: Number theoretic vision means that the quaternionic M^8-momenta are discrete with components having values in the extension of rationals. $P^2(M^4)$ becomes discrete if one poses $P^2(M^8) = 0$ condition for all states. The values of discontinuity of $Im(T)$ correspond now to a discrete sequence of poles along s-axis approximating cut. At the continuum limit this discrete sequence of poles becomes cut. Continuum limit would correspond to a finite measurement resolution in which one cannot distinguish the poles from each other.

6.3.3 Grassmannian approach and TGD

Grassmannian approach has provided besides technical progress deeper views about twistorialization and also led to the understanding of the Yangian symmetry.

Grassmannian twistorialization - or what I understand about it

The twistorialization of the scattering amplitudes works for planar amplitudes in massless theories and involves the following ingredients.

1. All scattering amplitudes are expressible in terms of on-mass-shell scattering amplitudes with massless on-mass-shell particles in complex sense.

2. The scattering amplitude is sum over contributions with varying number of loops. BCFW recursion relation allows to construct scattering amplitudes from their singularities using 3-particle amplitudes as building brick amplitudes. There are two types of singularities.

   For the first type of singularity one has on-shell internal line and one obtains a sum over all possible decompositions of the scattering amplitude to a product of on-mass-shell scattering amplitudes multiplied by delta function for momentum squared of the internal line. Second type of singularity corresponds to the so called forward limit and is obtained from $(n+2, k)$ amplitude by contracting two added adjacent particles to form a loop so that their momenta are opposite and integrating over the momentum.

3. The singular term is algebraically analogous to an exterior derivative of the scattering amplitude and can be integrated explicitly: the integration adds BCFW bridge to the both terms such that the forward limit loop in the second term is under the bridge. The outcome is BCFW formula for l-loop amplitude with n external particles with k negative helicities consisting of these two terms.

Twistor Grassmannian approach expresses the on mass shell scattering amplitudes appearing as building bricks as residue integrals over Grassmannian $Gr(n, k)$, where n is the number of particles and k is the number of negative helicities. The Grassmannian approach is described in a concise form in the slides by Jaroslav Trnka [B35] (see `http://tinyurl.com/pqjzffj`).

1. The construction of the on-mass-shell scattering amplitudes appearing in BCFW formula as residue integrals in Grassmannians follows by expressing the momentum conserving delta functions in twistor description in terms of auxiliary variables serving as coordinates of Grassmannian $G(n, k, C)$ for the on mass shell tree amplitude with n external particles having k negative helicities. Grassmannian has dimension $d = (n-k)k$ and can be identified as the space of k-planes - or equivalently $n-k$-planes in C^N. Grasmannian has a representation as homogenous space $G(n, k, C) = U(n)/U(n-k) \times U(k)$ having $SU(n)$ as the group of isometries. For $k = 1$ one obtains projective space which is also symmetric space (allowing reflection along geodesic lines as isometries).

2. Grassmannians emerge as an auxiliary construct, and the multiple residue integral over Grassmannian gives sum of residues so that the introduction of Grassmannians might look like un-necessary complication. The selection of points of Grassmannian for given external quantum numbers by residue integral given at the same time the value of the amplitude might however have some deeper meaning.

   The construction involves standard mathematics, which is however new for physicists. For instance, notions such as Plücker coordinates, Schubert cells and cell decomposition appear. One can relate to each other various widely different looking expressions for the amplitudes as being associated with different cell decompositions of Grassmannian. The singularities of the integrand of the scattering amplitude defined as a multiple residue integral over $G(k, n)$ define a hierarchy of Schubert cells.

3. The so called positive Grassmannian [B19] defines a subset of singularities appearing in the scattering amplitudes of $\mathcal{N} = 4$ SUSY. The points of positive Grassmannian $Gr_+(k, n)$ are representable as $k \times n$ matrices with positive $k \times k$ determinants. The singularities correspond to the boundaries of $Gr_+(k, n)$ with some $k \times k$ determinants vanishing. For tree diagrams the singularities correspond to poles appearing in the factorized term of the BCFW decomposition of the scattering amplitude. The positivity conditions hold true also for the twistors representing external particles.

4. Positivity conditions guarantee the convexity of the integration region determined by the C-matrix as point of $Gr_+(k,n)$ appearing in the conditions dictating the integration region.

   To better understand the meaning of positivity one can first consider triangle call it T - as a representation of positive Grassmannian $Gr_+(1,3) = P^2_+$. Any interior points of T can be regarded as center of mass for suitable positive masses at the vertices of the triangle. These conditions generalizes to the case of general polygons, which must be convex. If the number of vertices of the polygon is larger than 3, convexity is not automatically satisfied, and requires additional conditions.

   This description generalizes to Grassmannians $Gr_+(k,n)$. Masses define the analog of C-matrix as element of $Gr_+(k,n)$ appearing in the twistor approach and the vertices of the triangle are analogous to the twistors associated with external particles combining to form a point of $Gr(4,n)$. Positivity condition is generalized to the condition that $k \times k$ minors of the $k \times n$ matrix are positive.

5. Also the twistors associated with the external particles must satisfy analogs of the positivity conditions. This involves the replacement of $Gr(4,n)$ associated with twistors of the external particles with $Gr_+(k+4,n)$. The additional k components of the twistors are Grassman numbers and determined by the superparts of the twistors (see the slides of Trnka at `http://tinyurl.com/pqjzffj`. I must admit that I did not understand this.

6. Residue integral can be defined in terms of what is called canonical form Ω - analog of volume form - having logarithmic singularities at the boundaries of the $Gr_+(k,n)$. Hence one can perform a reduction of the residue integral to a sum of integrals over $G(k,k+4)$ instead of $G(k,n)$ (actually not so surprising since the residue integrals give as outcome the residues at discrete points!).

   This leads to a reduction of the residue integral over $Gr_+(k,n)$ to a sum of lower dimensional residue integrals over triangulation defined by $Gr_+(k,k+4)$ represented as surfaces of $Gr_+(k,n)$ glued together along sides. The geometric analog would be decomposition of polygon to a union of triangles.

   This simplifies the situation dramatically [B43, B35, B19] and leads to the notion of amplituhedron [B6, B5]. What is so remarkable, is the simplicity of the expressions for all-loop amplitudes and the fact that positivity implies locality and unitarity for $\mathcal{N}=4$ SUSY.

7. It should be possible to construct Ω explicitly having the desired singularities which would be in TGD framework poles with $P^2(M^8) = P^2(M^4 \times CP_2) = 0$ if the proposed realization of unitary makes sense? Could one just assumes that Ω vanishes for that part of the boundary of $Gr_+(k,n)$, which gives loop singularities? Could these points $Gr_+(k,n)$ be transcendental and excluded for this reason?

If loop corrections are vanishing as ZEO strongly suggests, only tree amplitudes are needed. Therefore it is appropriate to summarize what I have managed to understand about the construction of the tree amplitudes with general value of k in the amplituhedron approach.

1. The notion of amplituhedron relies on the mapping of $G(k,n)$ to $G_+(k,k+m)$ $n \geq k+m$. Actually a map from $G(k,n) \times G(k+4,n) \rightarrow G_+(k,k+m)$ is in question. $m=4$ identifiable as the apparent dimension of twistor space without projective identification giving the actual dimension $d=3$. n is the number of external particles and k the number of negative helicities.

   The value of m is $m=4$ and follows from the conditions that amplitudes come out correctly. The constraint $Y = C \cdot Z$, where Y corresponds to point of $G_+(k,k+4)$ and Z to the point of $G(k+4,n)$ performs this mapping, which is clearly many-to one. One can decompose integral over $G_+(k,n)$ to integrals over positive regions $G_+(k,k+4)$ intersecting only along their common boundary portions. The decomposition of a convex polygon in plane to triangles represent the basic example of this kind of decomposition. Obviously there are several decompositions of this kind.

2. Each decomposition defines a sum of contributions to the scattering amplitude involving integration of a projectively invariant volume form over the positive region in question. The

form has a logarithmic singularity at the boundaries of the integration region but spurious singularities cancel so that only the contribution of the genuine boundary of $G_+(k,k+4)$ remains. There are additional delta function constraints fixing the integral completely in real case.

3. In complex case one has residue integral. The proposed generalization to the complex case is by analytic continuation. TGD inspired proposal is that the positivity condition in the real case is generalized to the condition that the positive coordinates are replaced by complex coordinates of hyperbolic space representable as upper half plane or equivalently as the unit disk obtained from upper half plane by exponential mapping $w = exp(iz)$. The measure $d\alpha/\alpha$ would correspond to $dz = dw/w$. If taken over boundary circle labelled by discrete phase factors $exp(i\phi)$ given by roots of unity the integral would be numerically a discrete Riemann sum making no sense p-adically but residue theorem could allow to avoid the discretizaton and to define the p-adic variant of the integral by analytic continuation. These conditions would be completely general conditions on various projectively invariant moduli involved.

4. One must extend the bosonic twistors Z_a of external particles by adding k coordinates. This extension looks very difficult to understand intuitively. Somewhat surprisingly, these coordinates are anti-commutative super-coordinates expressible as linear combinations of fermionic parts of super-twistor using coefficients, which are also Grassmann numbers. Integrating over these one ends up with the standard expression of the amplitude using canonical integration measure for the regions in the decomposition of amplituhedron. An interesting question is whether the addition of k-dimensional anti-commutative parts to Z_a expressible in terms of super-coordinates is only a trick or whether it could have some physical interpretation.

Grassmannians as reduced WCWs?

Grassmannians appear as auxiliary spaces in twistor approach. Could Grassmannians and the procedure assigning to external momenta and helicities discrete set of points of Grassmannian and scattering amplitude have some concrete interpretation in TGD framework?

1. The points of cognitive representation define WCW coordinates for space-time surface. For a fixed number of points in cognitive representation WCW is effectively replaced with a finite-dimensional reduced WCW. These points would naturally correspond to the points defining ends of fermionic lines at partonic 2-surfaces. WCW has Kähler metric with Euclidian signature. This could be true also for its reduction.

2. The experience with twistorialization suggests that these spaces could be simply Grassmannians $Gr(n,r,C)$ consisting or r-dimensional complex planes of n-dimensional complex space representable as coset spaces $U(n)/U(n-r)\times U(r)$ appearing as auxiliary spaces in the construction of twistor amplitudes.

   Note that the correlation between quantum states and geometry would be present since n corresponds to the number of external particles and r to those with negative helicity in ordinary twistor Grassmann approach. In TGD framework discretized variants of these spaces corresponding to the extension of rationals used would appear. Yangian symmetries could correspond to general coordinate transformations for the reduced WCW acting as gauge symmetry. These transformations act as diffeomorphisms for so called positive Grassmannians also in the standard twistorialization. If the reduced WCWs indeed correspond to twistor Grassmannians, one would have a completely unexpected connection with supersymmetric QFTs.

3. The reduction of WCW to a finite dimensional Kähler manifold suggests that also WCW spinors become ordinary spinors for Kähler manifold so that gamma matrices form a finite-D fermionic oscillator operator algebra. WCW has maximal symmetries and it would not be surprising if also the finite-D Kähler manifold would possess maximal symmetries. Note that WCW gamma matrices together with isometry generators of WCW give rise to a super-symplectic algebra involving a generalization of 2-D conformal invariance replacing 2-D surfaces with light-like 3-surfacs.

4. The interpretation of supersymmetry would be different from the standard one. Kähler structure implies that $\mathcal{N}$ is even and Majorana spinors are absent and both baryon and lepton number can be conserved separately. The ordinary fermionic oscillator algebra is a Clifford algebra and could be interpreted in terms of a broken supersymmetry.

Also more general flag manifolds than Grassmannians can be considered. If these spaces are homogenous spaces they have maximal isometries. They should have also Kähler structure. Compactness looks also a highly desirable property. The gauge conditions for the subalgebra of super-symplectic algebra state that the sub-algebra and its commutator with the entire algebra annihilate physical states and give rise to vanishing classical Noether charges. This would effectively reduce the super-symplectic algebra to a finite-D Lie group or Kac-Moody algebra of a finite-dimensional Lie group - perhaps belonging to the ADE hierarchy as the hierarchy of inclusions of HFFs as an alternative correlate for the realization of finite measurement resolution suggests. The flag manifolds associated with these Lie groups define more general homogenous spaces as candidates for the reduced WCWs.

Interpretation for Grassmannian residue integrations

The identification of Grassmannians (or possibly more general spaces) as reduced WCWs would give a genuine physical interpretation for the Grassmannian integrations as residue integrations over reduced WCW. What looks mysterious and maybe even frustrating is that the outcome of the entire process is sum over discrete residues: what does this mean?

1. The residue integration is only over a surface of reduced WCW with dimension equal to one half of that of WCW. One has integrand, which depends on the external quantum numbers coded in terms of twistors and on coordinates of reduced WCW. The residue integration is analogous to summation over amplitude associated with space-time surfaces coded by different cognitive representations.

2. One can argue that a continuous residue integral over Grassmannian is not consistent with the number theoretic discretization. The outcome is however discrete set of space-time surfaces labelled by cognitive representations as points of Grassmannian. Of the points in question are in the extension and if this is equivalent with the corresponding property for the coordinates of Grassmannian, there should be no problems. The restriction of external momenta to the extension of rationals might guarantee this.

3. The full multiple residue integral leaves only pole contributions, which correspond to a discrete collection of space-time surfaces (at least the set of space-time surfaces obtained by the action of Galois group), that is discrete set of points of reduced WCW. It seems that the entire residue integration is just a way to realize quantum classical correspondence by associating to the external quantum numbers space-time surfaces and corresponding cognitive representations - and of course, also the scattering amplitude.

4. One can also ask whether the positivity of Grassmannian might relate to the fact that p-adic numbers as ordinary integers are always non-negative (most of them infinite). The positivity might be necessary in order to have number theoretic universality. If the minors associated with the C-matrix serve as coordinates for $Gr_+(k,n)$ they could be interpreted also as p-adic numbers. If they are allowed to be negative, one encounters problems since p-adic numbers are not well-ordered and one cannot say whether p-adic number is negative or positive.

Posible description of SUSY and its breaking in TGD framework

Although twistor description make sense also in the absence of supersymmetry, super-symmetry is an essential part of the elegance of the Grassmannian approach. For the ordinary SUSY one has gluons and their superpartners characterized in terms of super-twistors. In TGD one has two pictures [L20, L24].

1. At the level H fermions as fundamental particles are described in terms of second quantized induced spinor fields, whose oscillator operators can be used to build gamma matrices for

WCW [K86, K66]. In TGD universe all known elementary particles would be composites of fundamental fermions represented as lines at the light-like orbits of partonic 2-surfaces (wormhole throats) and ordinary elementary particles involve a pair of wormhole contacts with throats containing these fermion lines. It is assumed that the fermions are at different points: this allows to avoid problems due to infinities.

In the proposed generalization of twistor approach $2 \to 2$ fermion scattering in the classical fields at partonic 2-surface would define the basic $2 \to 2$-vertex replacing 3-vertices of twistorial SUSY. Essentially one has only two-vertices describing the redistribution of fermions at partonic 2-surface between orbits of the partonic 2-surfaces meeting at it. This is different from $\mathcal{N} = 4$ SUSY [L11]. If one allows completely local multi-fermion states at the level of H one cannot avoid fermionic contact interactions.

The many-fermion states associated with partonic 2-surfaces would define the analogs of super-multiplets. One can wonder whether a SUSY type description could exist as a limit when the partonic 2-surface is approximated with single point so that also positions of fermions are approximated as single point. SUSY would be only approximate.

2. At the level of M^8 I have proposed the use of polynomials P of super-octonion serving as analogs of super-gluon fields to construct scattering amplitudes [L20]. This allows geometric description of all particles using super-multiplets. Each monomial of theta parameters would give rise to its own space-time surface by the condition that either $IM(P)$ or $RE(P)$ vanishes for the corresponding polynomial P. This condition would reduce the components of super-field to algebraic surfaces.

   There is however an important difference from H picture. The members of super-multiplet defined by P correspond to the coefficients of monomials of theta parameters having interpretation as analogs of oscillator operators. Super-partners would be in this sense point-like objects unlike in H approach, where this can hold true only approximately.

Could H- and M^8 pictures be equivalent and could one understand the breaking of SUSY in this framework?

1. $M^8 - H$ correspondence as a map of associative space-time regions from M^8 to minimal surfaces in H makes sense for the external particles and thus at boundaries of CDs. It assigns to a point of the partonic 2-surface $X^2 \subset X^4 \subset M^8$ the quaternionic tangent space of X^4 at it characterized by a point of CP_2. M^4 point is mapped to itself. There is additional condition requiring that quaternionic tangent space contains fixed complex sub-space but this is not relevant now.

2. Could this map be one-to-many so that super-field component describing purely many-fermion state would be mapped to several points at the image of X^2 in H describing multi-local many-fermion state? This is possible if the points in M^8 are singular in the sense that the action of a normal subgroup H of Galois group Gal leaves the point invariant so that Gal reduces to Gal/H: symmetry breaking takes place.

   The tangent spaces of the degenerate points are however different and are mapped to different points of CP_2 in $M^8 - H$ correspondence making sense at boundaries of CDs but not in their interiors. One would have several fermions with same M^4 coordinates but different CP_2 coordinates and the outcome would be many-fermion state. In the case of 2-fermion state the different values of CP_2 coordinates would be associated with the opposite throats of a wormhole contact whose orbit defines light-like 3-surface. Could light-likeness inducing the reduction of the metric dimension of the tangent space from 4 to 3 somehow induce also this degeneration?

3. Could symmetry breaking as a degeneration of Gal action to that for Gal/H take place for the conditions defining the 4-surfaces associated with the higher components of super-octonion and induce the breaking of SUSY at the level of M^8 manifesting as the non-locality of the fermion state at the level of H? This degeneration would be a typical manifestation of quantum criticality: criticality in general means coincidence of two roots.

6.3.4 Summary

Since the contribution means in well-defined sense a breakthrough in the understanding of TGD counterparts of scattering amplitudes, it is useful to summarize the basic results deduced above as a polished answer to a Facebook question.

There are two diagrammatics: Feynman diagrammatics and twistor diagrammatics.

1. Virtual state is an auxiliary mathematical notion related to Feynman diagrammatics coding for the perturbation theory. Virtual particles in Feynman diagrammatics are off-mass-shell.

2. In standard twistor diagrammatics one obtains counterparts of loop diagrams. Loops are replaced with diagrams in which particles in general have complex four-momenta, which however light-like: on-mass-shell in this sense. BCFW recursion formula provides a powerful tool to calculate the loop corrections recursively.

3. Grassmannian approach in which Grassmannians $Gr(k,n)$ consisting of k-planes in n-D space are in a central role, gives additional insights to the calculation and hints about the possible interpretation.

4. There are two problems. The twistor counterparts of non-planar diagrams are not yet understood and physical particles are not massless in 4-D sense.

In TGD framework twistor approach generalizes.

1. Massless particles in 8-D sense can be massive in 4-D sense so that one can describe also massive particles. If loop diagrams are not present, also the problems produced by non-planarity disappear.

2. There are no loop diagrams- radiative corrections vanish. ZEO does not allow to define them and they would spoil the number theoretical vision, which allows only scattering amplitudes, which are rational functions of data about external particles. Coupling constant evolution - something very real - is now discrete and dictated to a high degree by number theoretical constraints.

3. This is nice but in conflict with unitarity if momenta are 4-D. But momenta are 8-D in M^8 picture (and satisfy quaternionicity as an additional constraint) and the problem disappears! There is single pole at zero mass but in 8-D sense and *also many-particle states* have vanishing mass in 8-D sense: this gives all the cuts in 4-D mass squared for all many-particle state. For many-particle states not satisfying this condition scattering rates vanish: these states do not exist in any operational sense! This is certainly the most significant new discovery in the recent contribution.

   BCFW recursion formula for the calculation of amplitudes trivializes and one obtains only tree diagrams. No recursion is needed. A finite number of steps are needed for the calculation and these steps are well-understood at least in 4-D case - even I might be able to calculate them in Grassmannian approach!

4. To calculate the amplitudes one must be able to explicitly formulate the twistorialization in 8-D case for amplitudes. I have made explicit proposals but have no clear understanding yet. In fact, BCFW makes sense also in higher dimensions unlike Grassmannian approach and it might be that the one can calculate the tree diagrams in TGD framework using 8-D BCFW at M^8 level and then transform the results to $M^4 \times CP_2$.

What I said above does yet contain anything about Grassmannians.

1. The mysterious Grassmannians $Gr(k,n)$ might have a beautiful interpretation in TGD: they could correspond at M^8 level to reduced WCWs which is a highly natural notion at $M^4 \times CP_2$ level obtained by fixing the numbers of external particles in diagrams and performing number theoretical discretization for the space-time surface in terms of cognitive representation consisting of a finite number of space-time points.

Besides Grassmannians also other flag manifolds - having Kähler structure and maximal symmetries and thus having structure of homogenous space G/H - can be considered and might be associated with the dynamical symmetries as remnants of super-symplectic isometries of WCW.

2. Grassmannian residue integration is somewhat frustrating procedure: it gives the amplitude as a sum of contributions from a finite number of residues. Why this work when outcome is given by something at finite number of points of Grassmannian?!

   In M^8 picture in TGD cognitive representations at space-time level as finite sets of points of space-time determining it completely as zero locus of real or imaginary part of octonionic polynomial would actually give WCW coordinates of the space-time surface in finite resolution.

   The residue integrals in twistor diagrams would be the manner to realize quantum classical correspondence by associating a space-time surface to a given scattering amplitude by fixing the cognitive representation determining it. This would also give the scattering amplitude.

   Cognitive representation would be highly unique: perhaps modulo the action of Galois group of extension of rationals. Symmetry breaking for Galois representation would give rise to supersymmetry breaking. The interpretation of supersymmetry would be however different: many-fermion states created by fermionic oscillator operators at partonic 2-surface give rise to a representation of supersymmetry in TGD sense.

6.4 New insights about quantum criticality for twistor lift inspired by analogy with ordinary criticality

Quantum criticality (QC) is one of the basic ideas of TGD. Zero energy ontology (ZEO) is second key notion and leads to a theory of consciousness as a formulation of quantum measurement theory making observer part of the quantum system in terms of notion of self identified as a generalized Zeno effect or analog for a sequence of weak measurements, and solving the basic paradox of standard quantum measurement theory, which one usually tries to avoid by introducing some "interpretation".

ZEO allows to see quantum theory could be seen as "square root" of thermodynamics. It occurred to me that it would be interesting to apply this vision in the case of quantum criticality to perhaps gain additional insights about its meaning. We have a picture about criticality in the framework of thermodynamics: what would be the analogy in ZEO based interpretation of Quantum TGD? Could it help to understand more clearly the somewhat poorly understood views about the notion of self, which as a quantum physical counterpart of observer becomes in ZEO a key concept of fundamental physics?

The basic ingredients involved are discrete coupling constant evolution, zero energy ontology (ZEO) implying that quantum theory is analogous to "square root" of thermodynamics, self as generalized Zeno effect as counterpart of observer made part of the quantum physical system, $M^8 \leftrightarrow M^4 \times CP_2$ duality, and quantum criticality. A further idea is that vacuum functional is analogous to a thermodynamical partition function as exponent of energy $E = TS - PV$.

The correspondence rules are simple. The mixture of phases with different 3-volumes per particle in a critical region of thermodynamical system is replaced with a superposition of space-time surfaces of different 4-volumes assignable to causal diamonds (CDs) with different sizes. Energy E is replaced with action S for preferred extremals defining Kähler function in the "world of classical worlds" (WCW). S is sum of Kähler action and 4-volume term, and these terms correspond to entropy and volume in the generalization $E = TS - PV \rightarrow S$. P resp. T corresponds to the inverse of Kähler coupling strength α_K resp. cosmological constant Λ. Both have discrete spectrum of values determined by number theoretically determined discrete coupling constant evolution. Number theoretical constraints force the analog of micro-canonical ensemble so that S as the analog of E is constant for all 4-surfaces appearing in the quantum superposition. This implies quantization rules for Kähler action and volume, which are very strong since α_K is complex.

This kind of quantum critical zero energy state is created in unitary evolution created in single step in the process defining self as a generalized Zeno effect. This unitary process implying time de-localization is followed by a weak measurement reducing the state to a fixed CD so that the clock time identified as the distance between its tips is well-defined. The condition that the action is same for all space-time surfaces in the superposition poses strong quantization conditions between the value of Kähler action (Kähler coupling strength is complex) and volume term proportional to cosmological constant. The outcome is that after sufficiently large number of steps no space-time surfaces satisfying the conditions can be found, and the first reduction to the opposite boundary of CD must occur - self dies. This is the classical counterpart for the fact that eventually all state function reduction leaving the members of state pairs at the passive boundary of CD invariant are made and the first reduction to the opposite boundary remains the only option.

The generation of magnetic flux tubes provides a way to satisfy the constancy conditions for the action so that the existing phenomenology as well as TGD counterpart of cyclic cosmology as re-incarnations of cosmic self follows as a prediction. This picture allows to add details to the understanding of the twistor lift of TGD at classical level and allows an improved understanding of the p-adic length scale evolution of cosmological constant solving the standard problem caused by the huge value of Λ. The sign of Λ is predicted correctly.

This picture generalizes to the twistor lift of TGD and cosmology provides an interesting application. One ends up with a precise model for the p-adic coupling constant evolution of the cosmological constant Λ explaining the positive sign and smallness of Λ in long length scales as a cancellation effect for M^4 and CP_2 parts of the Kähler action for the sphere of twistor bundle in dimensional reduction, a prediction for the radius of the sphere of M^4 twistor bundle as Compton length associated with Planck mass (2π times Planck length), and a prediction for the p-adic coupling constant evolution for Λ and coupling strength of M^4 part of Kähler action giving also insights to the CP breaking and matter antimatter asymmetry. The observed two values of Λ could correspond to two different p-adic length scales differing by a factor of $\sqrt{2}$.

6.4.1 Some background

Some TGD background is needed to understand the ideas proposed in the sequel.

Discrete coupling constant evolution

The most obvious implication is discrete coupling constant evolution in which the set of values for coupling constants is discrete and analogous to the set of the critical values of temperature [L35] (see `http://tinyurl.com/y9hlt3rp`). Zeros of Riemann Zeta or its slight modification suggest themselves as the spectrum for the Kähler coupling strength. This discrete coupling constant evolution requires that loop corrections vanish. This vision is realized concretely in the generalization of the twistorial approach to the construction of scattering amplitudes [L35].

Non-manifest unitarity is the basic problem of the twistor Grassmann approach. A generalization of the BCFW formula without the loop corrections gives scattering amplitudes satisfying unitary constraints. The needed cuts are be replaced by sequences of massless poles in 8-D sense and cuts approximate these sequences (consider electrostatic analogy in which line charge approximates a discrete sequences of poles). The replacement cuts with sequences of poles is forced by the number theoretic discretization of momenta so that they belong to an extension of rationals defining the adele [L23] (see `http://tinyurl.com/ycbhse5c`).

Non-planar loop diagrams are a chronic problem of twistor approach since there is no general rule loop integrations allowing to combine them neatly. Also this problem disappears now.

M^8-H duality plays key role in the twistorial approach [L20] (see `http://tinyurl.com/yd43o2n2`). In the ordinary twistor approach all momenta are light-like so that it does not apply to massive particles. TGD solves this problem: at M^8 level one has quaternionic light-like 8-D momenta, which correspond to massive 4-D momenta in M^8 picture. In $H=M^4\times CP_2$ picture ground states of super-conformal representations are constructed in terms of spinor harmonics of in $M^4\times CP_2$, which are products plane-waves characterized by massive 4-momenta and color wave functions associated with massless Dirac equation in H. Also the analog of Dirac equation for the induced spinor fields at space-time surface is massless [K86] (see `http://tinyurl.com/yc2po5gf`).

ZEO and self as generalized Zeno effect

ZEO allows to see self as generalized Zeno effect [L25](see `http://tinyurl.com/ycxm2tpd`).

1. Generalized Zeno effect can be regarded as a sequence of "small" state function reductions analogous to weak measurements performed at active boundary of causal diamond (CD). In usual Zeno effect the state is unaffected under repeated measurements: now the same is true at passive boundary of CD whereas the members of state pairs at the active boundary change. The unitary evolutions followed by these evolutions leave thus passive boundary and states at it invariant whereas active boundary shifts farther away from the passive boundary and the members of state pairs at it are affected. This gives rise to the experienced flow of time.

   The change of states is characterized unitary S-matrix. Each unitary evolution involves delocalization in the space of CDs so that one has quantum superposition of CDs with sizes not smaller than the CD to which the state was localized at previous reduction. This gives rise to a steady increase of clock time defined as the distance between the tips of CD. Self dies and reincarnates as a self with opposite direction of clock time when the first unitary evolution at the passive boundary followed by a weak measurement at it takes place. Self dies when all observables leaving the states at passive boundary invariant are measured. There are no choices to be made anymore.

2. Quantum TGD as "square root " of thermodynamics means that the partition function of thermodynamics is replaced by its "square root" defined by the vacuum functional identified as exponent of Kähler function of "world of classical worlds" (WCW). Kähler function is analogous to energy $E = TS - PV$ in thermodynamics with T replaced with the inverse of complex Kähler coupling strength and P with cosmological constant, which have discrete spectrum of values.

   One has the analog of micro-canonical ensemble for which only states with given energy are possible. Now the action (Kähler function) is same for the space-time surfaces assignable to the zero energy states involved. This condition allows to get rid of the exponentials defining the vacuum functional otherwise appearing in the scattering amplitudes. This condition is strongly suggested by number theoretic universality for which these exponentials are extremely troublesome since both the exponent and exponential should belong to the extension of rationals used.

   This implies a huge simplification in the construction of the amplitudes [L20] (see `http://tinyurl.com/yd43o2n2`) because finite measurement resolution effectively replaces space-time surfaces with their cognitive representation defined by a discrete set of space-time points with embedding space coordinates in the extension of rationals defining the adele.This representation codes for the space-time surface if it corresponds to zero locus of real or imaginary part (in quaterionic sense) of an octonionic polynomial with real coefficients. WCW coordinates are given by the cognitive representation and are discrete. One is led to enumerative algebraic geometry.

$M^8 - H$ duality

$M^8 - H$ duality [L20] (see `http://tinyurl.com/yd43o2n2`) states that the purely algebraic dynamics determined by the vanishing of real or imaginary part for octonionic polynomial is dual to the dynamics dictated by partial differential equations for an action principle.

1. There are two options for how to identify M^8 counterparts of space-time surfaces in terms roots of four polynomials defining real or imaginary part of an octonionic polynomial obtained as a continuation of real polynomial.

   (a) One can allow all roots $x + iy$ and project them to M^4 or M^8 from M^8_c. One can decompose these surfaces to regions with associative (quaternionic) tangent space or normal space and they are analogous to external particles of a twistor diagram entering CD and to interaction regions in which associativity does not hold true and which correspond to interiors of CD. One can criticizes the projection as somewhat adhoc process.

(b) It became later clear that one can also consider space-time surface as Minkowskian real regions so that the projection to a sub-space $M^4 \subset M^8_c$ of complexified octonions is invariant under the conjugation $i \to -i, I_k \to -I_k$, where I_k are quaternionic units. M^4_c parts of space-time coordinates would be form $m = m^0 + iI_k m^k$, m^0, m^k real. This conditions need not or even cannot be posed on E^4_c coordinates since $M^8 - H$ duality assigns to the tangent space of space-time surface a CP_2 point irrespective of whether the point is in M^8_c or M^8.

2. At the level of H external particles correspond to minimal surfaces, which are also extremals of Kähler action and in accordance with the number theoretical universality and quantum criticality do not depend on the coupling parameters at all. They are obtained by a map taking the 4-surfaces in M^8 to those in H. These conditions should be equivalent with the condition that the 6-D surfaces X^6 in 12-D twistor space of H define twistor bundles of space-time surfaces X^4.

3. The space-time regions in the interiors of CDs are not minimal surfaces so that Kähler action and volume term couple dynamically and coupling parameters characterize the extremals. The analog is motion of point like particle in the Maxwell field defined by induced Kähler form: this is generalize to the motion of 3-D object with purely internal Kähler field and that associated with wormhole contacts and mediating interaction with larger and smaller space-time sheets.

   In these regions the map mediating $M^8 - H$ duality does not exist since one cannot label the tangent spaces of space-time surface by points of CP_2. The non-existence of this map is due to the failure of either associativity of tangent space or normal space at M^8 level. The initial values at boundaries of CD for the incoming preferred extremals however allows to fix the time evolution in the interior of CD. This is essentially due to the infinite number of gauge conditions for the super-symplectic algebra.

   It has later turned out [L35] that it might be possible to take the associativity conditions to extreme in the sense that they would hold everywhere apart from a set of discrete points and space-time surface would be minimal surfaces at all points except this finite set of points. There would be transfer of conserved quantities assignable to the volume term and the 4-D Kähler action (coming as dimensionally reduced 6-D Kähler action for the twistor lift of TGD) only at these points and elementary fermions would be naturally assignable to these points.

Quantum criticality

Quantum criticality is a further key notion of TGD and was originally motivated by the idea that Kähler coupling strength must be unique in order that the theory is unique.

1. The first implication of quantum criticality is quantization of various coupling strengths as analogs of critical temperature and of other critical parameters such as pressure. This quantization is required also by number theoretical universality in the adelic approach: coupling constant parameters must belong to the extension of rationals used.

2. Second implication of quantum criticality is a huge generalization of conformal symmetries to their 4-D analogs. The key observation is that 3-D light-like surfaces allow a generalization of conformal invariance to get the Kac-Moody algebra associated with the isometries of H (at least) as symmetries. In the case of boundary of CD this leads to what I call supersymplectic invariance: the symplectic transformations of the two components of $\delta CD \times CP_2$ act as isometries of WCW. This algebra allows a fractal hierarchy of sub-algebras isomorphic to the algebra itself and gauge conditions state that this kind of sub-algebra and its commutator with the entire algebra annihilate physical states and classical Noether charges for them vanish [L35] (see `http://tinyurl.com/y9hlt3rp`). By quantum classical correspondence (QCC) the eigenvalues of quantum charges are equal to the classical Noether charges in Cartan algebra of supersymplectic algebra.

3. The third implication is the understanding of preferred extremals in $H = M^4 \times CP_2$ and their counterparts at the level of M^8. Associativity condition at the level of M^8 satisfied by the spacetime surfaces representing external particles arriving into CD corresponds to quantum criticality posing conditions on the coefficients of octonionic polynomials. The space-time regions inside CD the space-time surfaces do not satisfy associativity conditions and are not critical.

4. TGD as "square root" of thermodynamics idea suggests a fourth application of quantum criticality. This analogy might allow a better understanding of self as Zeno effect. This application will be studied in the sequel.

6.4.2 Analogy of the vacuum functional with thermodynamical partition function

Consider first the thermodynamical view about criticality. I have discussed criticality from slightly different perspective in [L31] (see `http://tinyurl.com/ydhknc2c`).

1. Thermodynamical states in critical region, where phases with different densities - say liquid and gas - are present serves as a basic example. This situation is actually a problem of the approach relying on partition function as van der Waals equation predicting 3 different densities for the density of molecules as function of pressure and temperature. Cusp catastrophe gives a view about situation: number density n is behavior variable and P and T are the control variables.

2. The experimental fact is that the density is constant as function of volume V for fixed temperature T whereas van der Waals predicts dependence on V. The phase corresponding to the middle sheet of the cusp is not at all present and the portions of liquid and gas phases vary. Maxwell's rules (area rule and lever rule) allow to solve the problem plaguing actually all approaches based on partition function. Lever rule assumes that there are actually two kinds of "elements" present. Molecules are the first element but what the second element could be? TGD identification is as magnetic tubes [L31].

3. In the more general case in which the catastrophe is more general than cusp and has more sheets, two or more phases with different volumes are present and their volumes and possibly other behavior variables analogous to volume vary at criticality.

4. If one applies criticality in stronger sense by requiring that the function which has extremum as function of n at the surface represented by cusp catastrophe has same value at different sheets of the cusp, only the boundary line of the cusp having V-shaped projection in (p, T)-plane remains.

Generalization of thermodynamical criticality to TGD context

The generalization of this picture to TGD framework replaces the mixture of thermodynamical phases with different volumes with quantum superposition of space-time surfaces with different 4-volumes assignable to CDs with different quantized sizes (by number theoretical constraints).

1. Vacuum functional, which is exponent of Kähler function of WCW expressible as Kähler action for its preferred extremal, can be regarded as a complex "square root" of thermodynamical partition function Z meaning that its real valued modulus squared is analogous to partition function [L3, L11, L12, L24].

   Action S, whose value for preferred extremal defines Kähler function of WCW serves as the analog of energy assumed to have expression $E = PV - TS$, which is not generally true but implied by the condition that E is homogenous as function of conjugate variable pairs P, V and T, S. The analogs of P and T correspond to coupling constant parameters. Pressure p is replaced with the coefficient of volume term in action - essentially cosmological constant. T is replaced with the coefficient $1/\alpha_K$ of Kähler action representing entropy (or negentropy depending on situation).

Remark: Note that T corresponds now to $1/\alpha_K$ rather than α_K analogous to temperature when Kähler action S_K is regarded as analog of energy E rather than entropy S.

2. Quantum criticality in the sense of ZEO is the counterpart for the criticality in thermodynamics. The mixture of thermodynamical phases with different 3-volumes is replaced with quantum superposition of zero energy states with 4-surface having same action S but different 4-volumes assignable to different CDs. Critical system consists of several phases with same values of coupling parameters α_K and Λ but different 4-volume.

   There is also a number theoretic constraint identifiable as the counterpart of the constant energy condition defining micro-canonical ensemble. The exponent of action S must cancel from the scattering amplitudes to avoid serious existence problems in the p-adic sectors of adele associated with given extension of rationals. Criticality means thus that $exp(S)$ has same value for all preferred extremals involved. Real parts are same for all of them and imaginary parts of the action exponential are fixed modulo multiple of 2π. The analog in the case of van der Waals equation of state that the allowed states are associated with the boundary of the projection of the cusp catastrophe to (p,T) plane.

   Critical quantum states are superpositions of space-time surfaces with different 4-volumes associated with CDs with quantized size scales (distance between tips) and are generated by unitary evolution. The value of time as size of CD (distance between its tips) is not well-defined in these states.

   Remark: Quantum critical states are "timeless" as meditative practices would express it.

   This kind of superposition is created by unitary evolution operator at each step in the sequence of unitary evolutions followed by a state function reduction measuring clock time as the distance between the tips of CD. Localization to single CD is the outcome and only superposition with same time-scale and same S but possibly different 4-volumes.

3. The condition that action is same is very strong and applies to both real and imaginary parts of action (α_K is complex). The proposal [L6, L35] (see `http://tinyurl.com/yas6ofhv` and `http://tinyurl.com/y9hlt3rp`) is that the coupling constant evolution as p-adic length scale $p \simeq 2^k$, k prime corresponds to zero of Riemann ζ for $1/\alpha_K$ or is proportional to it by rational multiplier q. For $q = 1$ $Re(1/\alpha_K)$ analogous to the ordinary temperature would be equal to $Re(s) = 1/2$ for the zeros at the critical line and imaginary parts would correspond to the imaginary parts $Im(s)$ of the zeros. Constancy of the action S would boil down to the conditions

$$Re(S_K) + Re(S_{vol}) = constant \ \ , \ \ Im(S_K) + Im(S_{vol}) = constant \ \ mod \ \ 2\pi \ . \tag{6.4.1}$$

   Note that the condition for imaginary part is a typical quantization condition.

   4-volume can can have arbitrary large values but for S_K this is probably not the case - this already by the quantization conditions. Hence one expects that there is some maximal possible volume for preferred extremals and thus maximal distance between the tips of CDs involved.

   When the zero energy state is a superposition of only space-time surfaces with this maximal volume, further unitary evolutions are not possible and the first state function reduction to the opposite boundary of CD happens (death of self and reincarnation with opposite direction of clock time). Self has finite lifetime! This would be the classical correlate for the situation in which no quantum measurements leaving invariant the members of state pairs at the passive boundary of CD are possible.

The constancy of $Re(S)$

How the cancellation of real part of $\Delta(Re(S_K)) + \Delta(Re(S_{vol}))$ could take place?

1. The physical picture is that the time evolution giving rise to self starts from flux tube dominated phase obtained in the first state function reduction to the opposite boundary of CD and that also asymptotically one obtains flux tube dominated phase again but the flux tubes are scaled up. This is the TGD view about quantum cosmology as a sequences of selves and of their time reversals [K71] [L10] (see `http://tinyurl.com/y7fmaapa`). This picture suggests that the generation of magnetic flux tubes allows to satisfy the $\Delta Re(S_K) + \Delta Re(S_{vol}) = 0$ condition: in Minkowskian regions the change magnetic part of $\Delta Re(S_K)$ tends to cancel $\Delta Re(S_{vol})$ whereas the electric part is of the same sign. Therefore magnetic flux tubes are favored.

   If the sign of the volume term is negative the exponential defining the vacuum functional decreases with volume. If the relative sign of S_K and S_{vol} is negative, the magnetic part of the action is positive. The generation of flux tubes generates positive magnetic action ΔS_K helping to cancel the change ΔS_{vol}.

   The additional conditions coming from the imaginary parts are analogous to semiclassical quantization conditions.

2. The proposed picture can be realized by a proper choice of the relative signs of volume term and Kähler action term. The relative sign comes automatically correct for a positive value of cosmological constant Λ. For this choice the total action density is

$$L_{tot} = (L_K + \frac{\Lambda}{8\pi G})\sqrt{g_4} \quad . \tag{6.4.2}$$

   This choice gives positive vacuum energy density associated with the volume term.

3. The density of Kähler action associated with CP_2 degrees of freedom is

$$L_{K,CP_2} = -\frac{1}{4g^2} J^{\mu\nu} J_{\mu\nu} \quad . \tag{6.4.3}$$

   The action is proportional to $E^2 - B^2$ in Minkowskian regions and magnetic term has sign opposite to that of volume term so that these terms can compensate with the condition guaranteeing constant action. The overall sign of action in the exponent can be chosen so that the exponential vanishes for large volumes. This suggests that the volume term is negative in the vacuum functional (Kähler function as negative of the action for preferred extremal). Euclidian regions, where CP_2 part of Kähler action is of form $B^2 + E^2$ and tends to cancel the volume term.

4. There is also Kähler action in M^4 degrees of freedom. In twistor lift dimensional reduction occurs for 6-D Kähler action and M^4 part and CP_2 part contribute to Kähler action. The S^2 parts of these actions must give rise to a cosmological constant decreasing like the inverse of p-adic length scale squared. This is achieved if the Kähler contributions have opposite signs so that M^4 contribution has a non-standard sign. This is possible if M^4 Kahler form is proportional to imaginary unit and M^4 Kähler coupling strength contains additional scaling factor.

   The induced Kähler form must be sum of the M^4 parts and CP_2 parts and also the action must be sum of M^4 and CP_2 parts. This is achieved if the charge matrices of these two Kähler forms are orthogonal (the trace of their product vanishes). Since CP_2 part couples to both 1 and Γ_9 giving rise to Kähler charges proportional to 1 for quarks and 3 for leptons having opposite chiralities, the corresponding charges would be proportional to 3 for quarks and -1 for leptons.

   The imaginary unit multiplying M^4 Kähler form disappears in action and field equations and one obtains

$$L_K = -\frac{1}{4g_K^2}(\epsilon^2 J^2(M^4) + J^2(CP_2) \ , \qquad (6.4.4)$$

where ϵ is purely imaginary so that one has $\epsilon^2 < 0$. Since the fields are induced, negative sign for M^4 Kähler action is not expected to lead to difficulties if M^4 term is small.

Some examples are in order.

1. For cosmic string extremals Kähler action is multiple of volume action. The condition that the two actions cancel would give a constraint between Λ and α_K. Net string tension would be reduced from the value determined by CP_2 scale to a rather small value. This need not occur generally but might be true for very short p-adic length scales, where Λ is large as required by the large value of string tension associated with Kähler action. For thickened cosmic strings (magnetic flux tubes) the value of string tension assignable to Kähler action is reduced and the condition can be satisfied for smaller values of Λ.

2. For CP_2 type extremals assignable to wormhole contacts serving as basic building bricks of elementary particles the action would be finite for all size scales of CD. Both magnetic and electric contribution to the action are of same sign. For Euclidian regions with 4-D space-time projection with so strong electric field that it changes the signature of the induced metric the same is true.

3. One can ask whether blackhole interiors as Euclidian regions correspond to these Euclidian space-time sheets or to highly tangled magnetic flux tubes with length considerably longer than Schwartschild radius for which cancellation also can occur (see `http://tinyurl.com/ydhknc2c`). Both pictures are consistent in many-sheeted space-time: magnetic flux tube tangle could topologically condense to a space-time sheet with Euclidian signature. Cancellation cannot last for ever so that also blackholes are unstable against big state function reduction changing the arrow of time. Blackhole evaporation might relate to this instability.

The constancy of $Im(S)$ modulo 2π

If cosmological constant is real, the condition for the constancy of imaginary part of ΔS modulo 2π applies only to the case of S_K and implies that ΔS_K is fixed modulo 2π in the superposition of space-time surfaces. If zeros of ζ [L6] (see `http://tinyurl.com/yas6ofhv`) or its modification $Zeta$) [L35]) (see `http://tinyurl.com/y9hlt3rp`) give the spectrum of $1/\alpha_K$ the value of $\Delta S_{K,red} = \int Tr(J^2)dV$ is given as multiples of $2\pi n/y$, where y is imaginary part for a zero of zeta. The constancy of $Re(S)$ implies that the 4-volume ΔV is quantized as multiples of $2\pi n/\Lambda$. These conditions bring in mind semiclassical quantization of the action in multiples of $\hbar$.

It however turns out that twistor lift forces same phase for M^4 and CP_2 parts of the Kähler action so that the quantization condition for volume is lost. The reason is that $1/\alpha_K(M^4)$ and $1/\alpha_K(CP_2)$ are proportional to

$$\frac{1}{\alpha_{K,6}} = \frac{1}{\alpha_{K,4}R^2} \ , \qquad (6.4.5)$$

where R^2 has dimensions of length squared.

6.4.3 Is the proposed picture consistent with twistor lift of Kähler action?

Is it possible to realize the cancellation of real parts of ΔS_{vol} and ΔS_K (modulo 2π for imaginary part) for the twistor lift of Kähler action? Does the sign of the cosmological constant Λ come out correctly (wrong sign of Λ is the probably fatal problem of M-theory)? Can one understand the p-adic evolution of the cosmological constant Λ implying that Λ becomes small in long p-adic length scales and thus solving the key problem related to Λ?

Dimensional reduction of the twistor lift

The condition that the induction of the product of twistor bundles of M^4 and CP_2 to the space-time surface gives the twistor bundle of the space-time surface is conjectured to determine the dynamics of the space-time surfaces. A generalization of 4-D Kähler action to 6-D Kähler action is proposed to give this dynamics, and to dimensionally reduce to a sum of Kähler actions associated with M^4 and CP_2 Kähler forms plus cosmological term.

1. Twistor bundles are sphere bundles. For the extremals of 6-D Kähler action dimensional reduction takes place since 6-D extremals must be twistor bundle of corresponding space-time surface. Therefore S^2 degrees of freedom are frozen and become non-dynamical.

   One could say that the spheres appearing as fibers of twistor bundles of M^4 and CP_2 are identified in the embedding map. The simplest correspondence between $S^2(M^4)$ and $S^2(CP_2)$ identifies (θ_1,ϕ_1) for $S^2(M^4)$ with (θ_2,ϕ_2) for $S^2(CP_2)$. This means that $S^2(X^6)$ is mapped in the same manner to $S^2(M^4)$ and $S^2(CP_2)$.

   One can imagine also correspondence with n-fold winding based on the identification $(\theta_1,\phi_1) = (\theta_2,n\phi_2)$. The area of $S^2(M^4)$ are becomes n-fold and the S^2 part of the Kähler action using θ_2 as coordinate transforms as $S_K(S^2(M^4)n=1) \rightarrow S_K(S^2(M^4)n) = n^2 S_K(S^2(M^4))$. $n=1$ is the most plausible option physically.

2. What the proposed general vision implies for cosmological constant as a sum of $S^2(M^4)$ and $S^2(CP_2)$ parts of 6-D Kähler action giving in dimensional reduction 4-D volume term responsible for the cosmological constant and 4-D Kähler action. If the charge matrices of M^4 and CP_2 parts of Kähler form are orthogonal one can induce Kähler form. If the coupling to M^4 Kähler form is imaginary, M^4 and CP_2 contributions to the total Kähler action have opposite signs. M^4 and CP_2 parts have opposite signs of magnetic terms and the sign of CP_2 magnetic part is opposite to the volume term.

3. The dimensionally reduced action is obtained by integrating the 6-D Kähler action over S^2 fiber. The integration gives the area $A(S^2)$ of the S^2 fiber, which in the metric induced from the spheres of twistor space of X^4 is given by

   $$A(S^2) = (1+r^2)4\pi R^2(S^2(CP_2)) \ , \quad r = \frac{R(S^2(CP_2))}{R(S^2(M^4))} \ . \tag{6.4.6}$$

   The very natural but un-checked assumption is that the radius of $S^2(CP_2)$ equals to the radius $R(CP_2)$ of the geodesic sphere of CP_2:

   $$R(S^2(CP_2)) = R(CP_2) \ . \tag{6.4.7}$$

   One obtains

   $$L = -\frac{1}{16\pi\alpha_{K,6}}\left[J^2(CP_2)+\epsilon^2 J^2(M^4)+J^2(S^2(CP_2))+\epsilon^2 J^2(S^2(M^4))\right] A(S^2) \ . \tag{6.4.8}$$

   The immediate conclusion is that the phases of Kähler action and volume term are same so that the quantization condition for imaginary part of the action is not obtained.

4. The Kähler coupling strengths $\alpha_K(CP_2)$ and $\alpha_K(M^4)$ can be read from the first term

   $$\begin{aligned} \frac{1}{\alpha_K(CP_2)} &= \frac{1}{\alpha_{K,4}4\pi(1+r^2)}\frac{R^2(CP_2)}{R^2} \ , \\ \frac{1}{\alpha_K(M^4)} &= \frac{\epsilon^2}{\alpha_K(CP_2)} \ . \end{aligned} \tag{6.4.9}$$

One can choose the factor R^2 to be the area of S^2 by suitably renormalizing $1/\alpha_K$. This would give simpler expression

$$\begin{aligned} \frac{1}{\alpha_K(CP_2)} &= \frac{1}{\alpha_{K,4}} \ , \\ \frac{1}{\alpha_K(M^4)} &= \frac{\epsilon^2}{\alpha_K(CP_2)} \ . \end{aligned} \tag{6.4.10}$$

5. One can deduce constraints on the value of the ϵ^2 from the smallness of the contributions of the corresponding $U(1)$ gauge potential to the ordinary Coulomb potential affecting the energies of atoms by a coupling proportional to mass number A rather than Z as for Coulomb potential. This allows to distinguish between isotopes. This gives very stringent bounds on ϵ^2. I have earlier derived an upper bound treating this term as a perturbation and by considering the contribution to the Coulomb energy of hydrogen atom [L18] (see `http://tinyurl.com/y8xcem2d`). One obtains $\epsilon^2 \leq 10^{-10}$. The upper bound is also the size scale of CP breaking induced by M^4 part and characterizes also matter-antimatter asymmetry.

Cosmological constant

Consider next the prediction for the cosmological constant term.

1. The S^2 parts of the actions have constant values. The natural normalization of Kähler form of $J(S^2(X))$, $X = M^4, CP_2$ is as $J^2 = -2$. This a convention is the overall scale of normalization can be chosen freely by rescaling $1/\alpha_{K,4}$. Taking into account the fact that index raising is carried out by induced metric one finds that the cosmological term given the sum of M^4 and CP_2 contributions to S^2 part of Kähler action multiplied by $A(S^2)$

$$\Lambda = \frac{1}{16\pi\alpha_K}\frac{2}{(1+r^2)R^2(CP_2)}(1+\frac{\epsilon^2}{r^4}) \ . \tag{6.4.11}$$

If ϵ is imaginary one can achieve the cancellation giving rise to small cosmological constant.

2. The empirical condition on cosmological constant (see `https://en.wikipedia.org/wiki/Cosmological_constant`) can be expressed in terms of critical mass density corresponding to flat 3-space as

$$\begin{aligned} \Lambda &= 3\Omega_\Lambda H^2 \ , \qquad \Omega \simeq .691 \ , \\ H &= \frac{da}{dt}a \qquad \frac{da}{dt} = \frac{1}{\sqrt{g_{aa}}} \ . \end{aligned} \tag{6.4.12}$$

Here a corresponds to the proper time for the light-cone M^4_+ and t for the proper time for the space-time surface, which is Lorentz invariant under the Lorentz group leaving the boundary δM^4_+.

From this one obtains a condition for allowing to get idea about the discrete evolution of Λ with p-adic length scale occurring in jumps:

$$1+\frac{\epsilon^2}{r^4} = 24\pi\alpha_K(1+r^2)R^2(CP_2)\times\Omega_\Lambda H^2 \ . \tag{6.4.13}$$

In an excellent approximation one must have $\epsilon \simeq r^2$, $r = R(M^4)/(CP_2)$. One can consider two obvious guesses. One has either $R(M^4) = L_{Pl} = \sqrt{G}$ - that is Planck length - or one has the Compton length associated with Planck mass given by $R(M^4) = 2\pi l_{Pl}$. The first option gives in reasonable approximation $r = 2^{-11}$ and $\epsilon^2 = r^4 = 2^{-44} \sim .6 \times 10^{-13}$. The second option gives $\epsilon^2 \simeq .9 \times 10^{-10}$. This values corresponds roughly to the CP_2 breaking parameter and matter-antimatter asymmetry and M^4 part of the Kähler action indeed gives rise to CP_2 breaking. I have earlier derived an upper bound for ϵ by demanding that the Kähler U(1) forces does not give rise to observable effects in the energy levels of hydrogen atom. The upper bound is of the same magnitude as the estimate for ϵ^2 for the Compton scale option.

3. If one accepts p-adic length scale hypothesis $L_p \propto \sqrt{p}$, $p \simeq 2^k$ [K50], one expects $\Lambda(k) \propto 1/L(k)^2$ [L12] (see `http://tinyurl.com/ybrhguux`). How to achieve this? The only possibility is that the parameter ϵ^2 is subject to coupling constant evolution. One would have for the cosmological constant

$$\Lambda(k) \propto \frac{\epsilon^2}{r^4} - 1 \propto \frac{1}{L^2(k)} \propto 2^{-k} \quad . \tag{6.4.14}$$

This would suggest for the 2-adic coupling constant evolution of ϵ the expression

$$\epsilon^2 = -r^4(1-X) \quad , \quad X = 24\pi\alpha_K(1+r^2)R^2(CP_2) \times \Omega_\Lambda H^2 = q \times 2^{-k} \quad . \tag{6.4.15}$$

where q is rational number. Note that from p-adic length scale hypothesis one has $2^{-k} \propto 1/L^2(k)$. One can consider also p-adic primes near powers of small prime in which case one obtains different evolution.

4. For Ω_Λ constant this would predict quantization of Hubble constant as $\Omega_\Lambda H^2 \propto 1/L(k)^2$ determined by naïve scaling dimension. The ratio of Hubble constants for two subsequent scales would be $H(k)/H(k+1) = \sqrt{2}$ if Ω is constant. The observed - and poorly understood - variation of Hubble constant from cosmological studies and distance ladder studies is in the range $50 - 73.2$ km/s/Mpc. Cosmological studies correspond to longer scales so that the smaller value of H is consistent with the decrease of H. The ratio of these upper and lower bounds is $1.46 < \sqrt{2} \simeq 1.141$ (see `http://tinyurl.com/yd6m8sca` and `http://tinyurl.com/ycr4ffm4`).

   Remark: The uncertainty in the value of Hubble constant is reflected as uncertainty in the distances D deduced from cosmic redshift $z \simeq HD/c$. This is taken into account in the definition of cosmological distant unit $h^{-1}Mpc$, where h is in the range $.5-.75$ corresponding to a scale factor 1.5 rather near to $\sqrt{2}$.

5. Piecewise constant evolution means that acceleration parameter is positive since constant value of H gives

$$\frac{d^2a}{dt^2} = \frac{(da/dt)^2}{a} = aH^2 > 0 \quad . \tag{6.4.16}$$

If the phase transitions reducing H by factor 1/2 occur at $a(k) = 2^{k/2}a_0$, one has

$$\frac{d^2a}{dt^2} \propto 2^{-k/2} \quad . \tag{6.4.17}$$

Acceleration would be reduced gradually with rate determined by its naïve scaling dimension.

Solution of Hubble constant discrepancy from the length scale dependence of cosmological constant

One can critize this proposal. The recent best values of the Hubble constant are 67.0 km/s/Mpc and and 73.5 km/s/Mpc and their ratio is about 1.1 rather than $\sqrt{2}$. Therefore the hypothesis that H satisfies p-adic length scale hypothesis might be too strong. In the following a proposal in which the variation of H could be due to the variation of cosmological constant Λ satisfying p-adic length scale hypothesis is discussed.

The discrepancy of the two determinations of Hubble constant has led to a suggestion that new physics might be involved (see `http://tinyurl.com/yabszzeg`).

1. Planck observatory deduces Hubble constant H giving the expansion rate of the Universe from CMB data something like 360,000 y after Big Bang, that is from the properties of the cosmos in long length scales. Riess's team deduces H from data in short length scales by starting from galactic length scale and identifies standard candles (Cepheid variables), and uses these to deduce a distance ladder, and deduces the recent value of $H(t)$ from the redshifts.

2. The result from short length scales is 73.5 km/s/Mpc and from long scales 67.0 km/s/Mpc deduced from CMB data. In short length scales the Universe appears to expand faster. These results differ too much from each other. Note that the ratio of the values is about 1.1. There is only 10 percent discrepancy but this leads to conjecture about new physics: cosmology has become rather precise science!

TGD could provide this new physics. I have already earlier considered this problem but have not found really satisfactory understanding. The following represents a new attempt in this respect.

1. The notions of length scale are fractality are central in TGD inspired cosmology. Many-sheeted space-time forces to consider space-time always in some length scale and p-adic length scale defined the length scale hierarchy closely related to the hierarchy of Planck constants $h_{eff}/h_0 = n$ related to dark matter in TGD sense. The parameters such as Hubble constant depend on length scale and its value differ because the measurements are carried out in different length scales.

2. The new physics should relate to some deep problem of the recent day cosmology. Cosmological constant Λ certainly fits the bill. By theoretical arguments Λ should be huge making even impossible to speak about recent day cosmology. In the recent day cosmology Λ is incredibly small.

3. TGD predicts a hierarchy of space-time sheets characterized by p-adic length scales (Lk) so that cosmological constant Λ depends on p-adic length scale $L(k)$ as $\Lambda \propto 1/GL(k)^2$, where $p \simeq 2^k$ is p-adic prime characterizing the size scale of the space-time sheet defining the sub-cosmology. p-Adic length scale evolution of Universe involve as sequence of phase transitions increasing the value of $L(k)$. Long scales $L(k)$ correspond to much smaller value of Λ.

4. The vacuum energy contribution to mass density proportional to Λ goes like $1/L^2(k)$ being roughly $1/a^2$, where a is the light-cone proper time defining the "radius" $a = R(t)$ of the Universe in the Robertson-Walker metric $ds^2 = dt^2 - R^2(t)d\Omega^2$. As a consequence, at long length scales the contribution of Λ to the mass density decreases rather rapidly.

   Must however compare this contribution to the density ρ of ordinary matter. During radiation dominated phase it goes like $1/a^4$ from $T \propto 1/a$ and form small values of a radiation dominates over vacuum energy. During matter dominated phase one has $\rho \propto 1/a^3$ and also now matter dominates. During predicted cosmic string dominated asymptotic phase one has $\rho \propto 1/a^2$ and vacuum energy density gives a contribution which is due to Kähler magnetic energy and could be comparable and even larger than the dark energy due to the volume term in action.

5. The mass density is sum $\rho_m + \rho_d$ of the densities of matter and dark energy. One has $\rho_m \propto H^2$. $\Lambda \propto 1/L^2(k)$ implies that the contribution of dark energy in long length scales is considerably smaller than in the recent cosmology. In the Planck determination of H it is however assumed that cosmological constant is indeed constant. The value of H in long length scales is under-estimated so that also the standard model extrapolation from long to short length scales gives too low value of H. This is what the discrepancy of determinations of H performed in two different length scales indeed demonstrate.

A couple of remarks are in order.

1. The twistor lift of TGD [L3, L12] [L32] suggests an alternative parameterization of vacuum energy density as $\rho_{vac} = 1/L^4(k_1)$. k_1 is roughly square root of k. This gives rise to a pair of short and long p-adic length scales. The order of magnitude for $1/L(k_1)$ is roughly the same as that of CMB temperature T: $1/L(k_1) \sim T$. Clearly, the parameters $1/T$ and R correspond to a pair of p-adic length scales. The fraction of dark energy density becomes smaller during the cosmic evolution identified as length scale evolution with largest scales corresponding to earliest times. During matter dominated era the mass density going like $1/a^3$ would to dominate over dark energy for small enough values of a. The asymptotic cosmology should be cosmic string dominated predicting $1/GT^2(k)$. This does not lead to contradiction since Kähler magnetic contribution rather than that due to cosmological constant dominates.

2. There are two kinds of cosmic strings: for the other type only volume action is non-vanishing and for the second type both Kähler and volume action are non-vanishing but the contribution of the volume action decreases as function of the length scale.

6.5 Further comments about classical field equations in TGD framework

In the sequel some remarks about field equations defining space-time surfaces in TGD framework are made.

First three dualities at the level of field equations are discussed. These dualities are rather obvious but extremely important concerning the physical interpretation of TGD.

The earlier proposal that external particles correspond to minimal surfaces is strengthened. Also the interaction regions would correspond to minimal surfaces. The strongest condition would be that the minimal surface property break down at reaction vertices only associated with partonic 2-surfaces defining the 2-D counterparts of vertices: this would mean physical exchange of classical conserved charges between volume part of the action and Kähler action just at these points. This condition might be too strong.

The strongest condition could mean strengthening of the strong form of holography to $M^4 \times CP_2$ counterpart of the proposed number theoretic holography based on the notion of cognitive representation at the level of M^8 [L20] and also justification for the proposed construction of twistor Grassmannian variants of scattering amplitudes involving also data at a discrete set of points [L35].

6.5.1 Three dualities at the level of field equations

The basic field equations of TGD allow several dualities. There are 3 of them at the level of basic field equations (and several other dualities such as $M^8 - M^4 \times CP_2$ duality).

1. The first duality is the analog of particle-field duality. The spacetime surface describing the particle (3-surface of $H = M^4 \times CP_2$ instead of point-like particle) corresponds to the particle aspect whereas the fields inside it geometrized in terms of sub-manifold geometry correspond to the field aspect. Particle orbit serves as wave guide for field, one might say.

2. Second duality is particle-spacetime duality. Particle identified as 3-D surface means that particle orbit is space-time surface glued to a larger space-time surface by topological sum contacts. It depends on the scale used, whether it is more appropriate to talk about particle or of space-time.

3. The third duality is hydrodynamics- massless field theory duality. Hydrodynamical equations state local conservation of Noether currents. Field equations indeed reduce to local conservation conditions of Noether currents associated with the isometries of H. One the other hand, these equations have interpretation as non-linear geometrization of massless wave equation with coupling to Maxwell fields. This realizes the ultimate dream of theoretician: symmetries dictate the dynamics completely. This is expected to be realized also at the level of scattering amplitudes and the generalization of twistor Grassmannian amplitudes could realize this in terms of Yangian symmetry.

Hydrodynamics-wave equations duality generalizes to the fermionic sector and involves super-conformal symmetry.

1. What I call modified gamma matrices Γ^α are obtained as contractions of the partial derivatives of the action defining space-time surface with respect to the gradients of embedding space coordinate with embedding space gamma matrices [K86]. The divergence $D_\alpha\Gamma^\alpha$ vanishes by field equations for the space-time surface and this is necessary for the internal consistency the Dirac equation ($\overline{\Psi}$ satisfies essentially the same equation as Ψ). Γ^α reduce to ordinary ones if the space-time surface is M^4 and one obtains ordinary massless Dirac equation.

2. Modified Dirac equation [K86] expressess conservation of super current and actually infinite number of super currents obtained by contracting second quantized induced spinor field with the solutions of modified Dirac. This corresponds to the super-hydrodynamic aspect. On the other hand, modified Dirac equation corresponds to fermionic analog of massless wave equation.

6.5.2 Are space-time surfaces minimal surfaces everywhere except at 2-D interaction vertices?

If one starts from the analogy with complex analysis, the natural hypothesis would be that singular surfaces are co-dimension 2 surfaces - string world sheets and partonic 2-surfaces, which are at the ends of space-time surfaces and define topological reaction vertices. Light-like 3-surfaces as partonic orbits would be formally analogous to cuts of analytic function.

One can argue [L42] that the singular surface defines a sub-manifold giving a deltafunction like contribution to the action density and that one can assign conserved quantities to this surface. This requires that the singular contributions to the energy momentum tensor and canonical momentum currents as spacetime vectors are parallel to the singular surface. There must be one time-like or light-like direction and singular points do not satisfy this condition. There can be however an exchange of conserved charged between Kähler and volume degrees of freedom for the singular surfaces [L42]. One can also consider the possibility that the exchange is non-vanishing at singular points only. This option, which is perhaps non-realistic would be the strongest and will be discussed below.

String boundaries represent orbits of fundamental point-like fermions located at 3-D light-like surfaces which represent orbits of partonic 2-surfaces. String world sheets are minimal surfaces and correspond to stringy objects associated with say hadrons. There are also degrees of freedom associated with space-time interior. One have objects of various dimension which all are minimal surfaces. Modified Dirac equation extends the field equations to supersymmetric system and assigns fermionic degrees of freedom to these minimal surfaces of varying dimension.

From the physics point of view, the singular surfaces are analogous to carriers of currents acting as point- and string-like sources of massless field equations (more general option allows also string world sheets as carriers of currents).

The action S determining space-time surfaces as preferred extremals follows from twistor lift [L3, L24, L12, L35] and equals to the sum of volume term Vol multiplied by the TGD counterpart of cosmological constant and Kähler action S_K. The field equation is a geometric generalization of d'Alembert (Laplace) equation in Minkowskian (Eucidian) regions of space-time surface coupled with induced Kähler form analogous to Maxwell field. Generalization of equations of motion for particle by replacing it with 3-D surface is in question and the orbit of particle defines a region of space-time surface.

1. Zero energy ontology (ZEO) suggests that the external particles arriving to the boundaries of given causal diamond (CD) are like free massless particles and correspond to minimal surfaces as a generalization of light-like geodesic. This dynamic reduces to mere algebraic conditions and there is no dependence on the coupling parameters appearing in S. In contrast to this, in the interaction regions inside CDs there could be a coupling between Vol and S_K due to the non-vanishing divergences of energy momentum currents associated with the two terms in action cancelling each other.

2. Similar algebraic picture emerges from M^8-H duality [L20] at the level of M^8 and from what is known about preferred extremals of S assumed to satisfy infinite number of super-symplectic gauge conditions at the 3-surfaces defining the ends of space-time surface at the opposite boundaries of CD.

   At M^8 side of M^8-H duality associativity is realized as quaternionicity of either tangent or normal space of the space-time surface. The condition that there is 2-D integral distribution of sub-spaces of tangent spaces defining a distribution of complex planes as subspaces of octonionic tangent space implies the map of the space-time surface in M^8 to that of H. Given point m_8 of M^8 is mapped to a point of $M^4\times CP_2$ as a pair of points (m_4,s) formed by $M^4\subset M^8$ projection m_4 of m_8 point and by CP_2 point s parameterizing the tangent space or the normal space of $X^4\subset M^8$.

   Remark: The assumption about integrable distribution of $M^2(x)$ defining string world sheet in M^4 might be too general: $M^2x)$ could not depend on x.

   If associativity or even the condition about the existence of the integrable distribution of 2-planes fails, the map to $M^4\times CP_2$ is lost. One could cope with the situation since the gauge conditions at the boundaries of CD would allow to construct preferred extremal connecting the 3-surfaces at the boundaries of CD if this kind of surface exists at all. One can however wonder whether giving up the map $M^8\to H$ is necessary.

3. Number theoretic dynamics in M^8 involves no action principle and no coupling constants, just the associativity and the integrable distribution of complex planes $M^2(x)$ of complexified octonions. This suggests that also the dynamics at the level of H involves coupling constants only via boundary conditions. This is the case for the minimal surface solutions suggesting that M^8-H duality maps the surfaces satisfying the above mentioned conditions to minimal surfaces. The universal dynamics conforms also with quantum criticality.

4. One can argue that the dependence of field equations on coupling parameters of S leading to a perturbative series in coupling parameters in the interior of the space-time surface inside CD spoils the extremely beautiful purely algebraic picture about the construction of solutions of field equations using conformal invariance assignable to quantum criticality. Classical perturbation series is also in conflict with the vision that the TGD counterparts twistorial Grassmannian amplitudes do not involve any loop contributions coming as powers of coupling constant parameters [L35].

To sum up, both M^8-H duality, number theoretic vision, quantum criticality, twistor lift of TGD reducing dynamics to the condition about the existence of induced twistor structure, and the proposal for the construction of twistor scattering amplitudes suggest an extremely simple picture about the situation. The divergences of the energy momentum currents of Vol and S_K would be non-vanishing delta function type singularities only at discrete points at partonic 2-surfaces defining generalized vertices so that minimal surface equations would hold almost everywhere as the original proposal indeed stated.

1. The fact that all the known extremals of field equations for S are minimal surfaces conforms with the idea. This might be due to the fact that these extremals are especially easy to construct but could be also true quite generally apart from singular points. The divergences of the energy momentum currents associated with S_K and Vol vanish separately: this follows from the analog of holomorphy reducing the field equations to purely algebraic conditions.

   It is essential that Kähler current j_K vanishes or is light-like so that its contraction with the gradients of the embedding space coordinates vanishes. Second condition is that in transversal degrees of freedom energy momentum tensor is tensor of form (1,1) in the complex sense and second fundamental form consists of parts of type (1,1) and (-1-1). In longitudinal degrees of freedom the trace H^k of the second fundamental form $H^k_{\alpha\beta} = D_\beta \partial_\alpha h^k$ vanishes.

2. Minimal surface equations are a non-linear analog of massless field equation but one would like to have also the analog of massless particle. The 3-D light-like boundaries between Minkowskian and Euclidian space-time regions are indeed analogs of massless particles as are also the string like word sheets, whose exact identification is not yet fully understood. In any case, they are crucial for the construction of scattering amplitudes in TGD based generalization of twistor Grassmannian approach. At M^8 side these points could correspond to singularities at which Galois group of the extension of rationals has a subgroup leaving the point invariant. The points at which roots of polynomial as function of parameters coincide would serve as an analog.

   The intersections of string world sheets with the orbits of partonic 2-surface are 1-D light-like curves X^1_L defining fermion lines. The twistor Grassmannian proposal [L35] is that the ends of the fermion lines at partonic 2-surfaces defining vertices provide the information needed to construct scattering amplitudes so that information theoretically the construction of scattering amplitudes would reduce to an analog of quantum field theory for point-like particles.

3. Number theoretic vision discretizes coupling constant evolution: the values of coupling constants are labelled by parameters of extension of rationals and p-adic primes. This implies that twistor scattering amplitudes for given discrete values of coupling constants involve no radiative corrections [L35]: the construction of twistor Grassmannian amplitudes would be extremely simple. Note that infinite perturbation series would break the expression of scattering amplitudes as rational functions with coefficients int he extension of rationals defining the adele [L23, L22]. The cuts for the scattering amplitudes would be replaced by sequences of poles. This is unavoidable also because there is number theoretical discretization of momenta from the condition that their components belong to an extension of rationals defining the adele.

What could the reduction of cuts to poles for twistorial scattering amplitudes at the level of momentum space [L35] mean at space-time level?

1. Poles of an analytic function are co-dimension 2 objects. d'Alembert/Laplace equations holding true in Minkowskian/Euclidian signatures express the analogs of analyticity in 4-D case. Co-dimension 2 rule forces to ask whether partonic 2-surfaces defining the vertices and string world sheets could serve analogs of poles at space-time level? In fact, the light-like orbits X^3_L of partonic 2-surfaces allow a generalization of 2-D conformal invariance since they are metrically 2-D so that X^3_L and string world sheets could serve in the role of poles.

   X^3_L could be seen as analogs of orbits of bubbles in hydrodynamical flow in accordance with the hydrodynamical interpretations. Particle reactions would correspond to fusions and decays of these bubbles. Strings would connect these bubbles and give rise to tensor networks and serve as space-time correlates for entanglement. Reaction vertices would correspond to common ends for the incoming and outgoing bubbles. They would be analogous to the lines of Feynman diagram meeting at vertex: now vertex would be however 2-D partonic 2-surface.

2. What can one say about the singularities associated with the light-like orbits of partonic 2-surfaces? The divergence of the Kähler part T_K of energy momentum current T is proportional to a sum of contractions of Kähler current j_K with gradients ∇h^k of H coordinates. j_K need not be vanishing: it is enough that its contraction with ∇h^k vanishes and this is true if j_K is light-like. This is the case for so called massless extremals (MEs). For the other known extremals j_K vanishes.

   Could the Kähler current j_K be light-like and non-vanishing and singular at X^3_L and at string world sheets? This condition would provide the long sought-for precise physical identification of string world sheets. This would also induce to the modified Dirac action a 2-D contribution. Minimal surface equations would hold true also at these two kinds of surfaces apart from possible singular points. Even more: j_K could be non-vanishing and thus also singular only at the 1-D intersections X^1_L of string world sheets with X^3_L - I have called these curves fermionic lines.

What it means that j_K is singular - that is has 2-D delta function singularity at string world sheets? j_K is defined as divergence of the induced Kähler form J so that one can use the standard definition of derivative to define j_K at string world sheet as the limiting value $j^\alpha_K = (Div_{+-}J)^\alpha = lim_{\Delta x^n\rightarrow 0}(J^{\alpha n}_+ - J^{\alpha n}_-)/\Delta x^n$, where x^n is a coordinate normal to the string world sheet. If J is discontinuous, this gives rise to a singular current located at string world sheet. This current should be light like to guarantee that energy momentum currents are divergenceless. If J is not light-like, it gives rise to isometry currents with non-vanishing divergence at string world sheet. This is guaranteed if the isometry currents $T^{\alpha A}$ are continuous through the string world sheet.

3. If the light-like j_K at partonic orbits is localized at fermionic lines X^1_L, the divergences of isometry currents could be non-vanishing and singular only at the vertices defined at partonic 2-surfaces at which fermionic lines X^1_L meet. The divergences $DivT_K$ and $DivT_{Vol}$ would be non-vanishing only at these vertices. They should of course cancel each other: $DivT_K = -DivT_{Vol}$.

4. $DivT_K$ should be non-vanishing and singular only at the intersections of string world sheets and partonic 2-surfaces defining the vertices as the ends of fermion lines. How to translate this statement to a more precise mathematical form? How to precisely define the notions of divergence at the singularity?

   The physical picture is that there is a sharing of conserved isometry charges of the incoming partonic orbit $i = 1$ determined T_K between 2 outgoing partonic orbits labelled by $j = 2, 3$. This implies charge transfer from $i = 1$ to the partonic orbits $j = 2, 3$ such that the sum of transfers sum up to the total incoming charge. This must correspond to a non-vanishing divergence proportional to delta function. The transfer of the isometry charge for given pair i, j of partonic orbits that is $Div_{i\rightarrow j}T_K$ must be determined as the limiting value of the quantity $\Delta_{i\rightarrow j}T^{\alpha,A}_K/\Delta x^\alpha$ as Δx^α approaches zero. Here $\Delta_{i\rightarrow j}T^{\alpha,A}_K$ is the difference of the components of the isometry currents between partonic orbits i and j at the vertex. The outcome is proportional delta function.

5. Similar description applies also to the volume term. Now the trace of the second fundamental form would have delta function singularity coming from $Div_{i\rightarrow j}T_K$. The condition $Div_{i\rightarrow j}T_K = -Div_{i\rightarrow j}T_{Vol}$ would bring in the dependence of the boundary conditions on coupling parameters so that space-time surface would depend on the coupling constants in accordance with quantum-classical correspondence. The manner how the coupling constants make themselves visible in the properties of space-time surface would be extremely delicate.

This picture conforms with the vision about scattering amplitudes at both M^8 and H sides of $M^8 - H$ duality.

1. M^8 dynamics based on algebraic equations for space-time surfaces [L20] leads to the proposal that scattering amplitudes can be constructed using the data only at the points of space-time surface with M^8 coordinates in the extension of the rationals defining the adele [L22, L23]. I call this discrete set of points cognitive representation with motivations coming from TGD inspired theory of consciousness [K52].

2. At H side the information theoretic interpretation would be that all information needed to construct scattering amplitudes would come from points at which the divergences of the energy momentum tensors of S_K and Vol are non-vanishing and singular.

Both pictures would realize extremely strong form of holography, much stronger than the strong form of holography that stated that only partonic 2-surfaces and string world sheets are needed.

6.6 Still about twistor lift of TGD

Twistor lift of TGD led to a dramatic progress in the understanding of TGD but also created problems with previous interpretation. The new element was that Kähler action as analog of Maxwell action was replaced with dimensionally reduced 6-D Kähler action decomposing to 4-D Kähler action and volume term having interpretation in terms of cosmological constant.

One can of course ask whether the resulting induced twistor structure is acceptable. Certainly it is not equivalent with the standard twistor structure. In particular, the condition $J^2 = -g$ is lost. In the case of induced Kähler form at X^4 this condition is also lost. For spinor structure the induction guarantees the existence and uniqueness of the spinor structure, and the same applies also to the induced twistor structure being together with the unique properties of twistor spaces of M^4 and CP_2 the key motivation for the notion.

There are some potential problems related to the definition of Kähler function. The most natural identification is as 6-D dimensionally reduced Kähler action.

1. WCW metric must be Euclidian - that positive definite. Since it is defined in terms of second partial derivatives of the Kähler function with respect to complex WCW coordinates and their conjugates, the preferred extremals must be completely stable to guarantee that this quadratic form is positive definite. This condition excludes extremals for which this is not the case. There are also other identifications for the preferred extremal property and stability condition would is a obvious additional condition. Note that at quantum criticality the quadratic form would have some vanishing eigenvalues representing zero modes of the WCW metric.

2. Vacuum functional of WCW is exponent of Kähler function identified as negative of Kähler action for a preferred extremal. The potential problem is that Kähler action contains both electric and magnetic parts and electric part can be negative. For the negative sign of Kähler action the action must remain bounded, otherwise vacuum functional would have arbitrarily large values. This favours the presence of magnetic fields for the preferred extremals and magnetic flux tubes are indeed the basic entities of TGD based physics.

3. One can ask whether the sign of Kähler action for preferred extremals is same as the overall sign of the diagonalized Kähler metric: this would exclude extremals dominated by Kähler electric part of action or at least force the electric part be so small that WCW metric has the same overall signature everywhere.

If one accepts the proposal that the preferred extremals are minimal surfaces (the known extremals are), extremal property is satisfied for both 4-D Kähler action and volume term separately except at finite set of singular points at which there is transfer of conserved charges between the two degrees of freedom. In this principle this would allow the identification of Kähler function as either 4-D Kähler function or 4-D volume term (actually magnetic S^2 part of 6-D Kähler action). This option looks however rather ad hoc.

6.6.1 Is the cosmological constant really understood?

The interpretation of the coefficient of the volume term as cosmological constant has been a long-standing interpretational issue and caused many moments of despair during years. The intuitive picture has been that cosmological constant obeys p-adic length scale scale evolution meaning that Λ would behave like $1/L_p^2 = 1/p \simeq 1/2^k$ [L12].

This would solve the problems due to the huge value of Λ predicted in GRT approach: the smoothed out behavior of Λ would be $\Lambda \propto 1/a^2$, a light-cone proper time defining cosmic time, and the recent value of Λ - or rather, its value in length scale corresponding to the size scale of the observed Universe - would be extremely small. In the very early Universe - in very short length scales - Λ would be large.

A simple solution of the problem would be the p-adic length scale evolution of Λ as $\Lambda \propto 1/p$, $p \simeq 2^k$. The flux tubes would thicken until the string tension as energy density would reach stable minimum. After this a phase transition reducing the cosmological constant would allow further thickening of the flux tubes. Cosmological expansion would take place as this kind of phase transitions (for a mundane application of this picture see [K29]).

This would solve the basic problem of cosmology, which is understanding why cosmological constant manages to be so small at early times. Time evolution would be replaced with length scale evolution and cosmological constant would be indeed huge in very short scales but its recent value would be extremely small.

I have however not really understood how this evolution could be realized! Twistor lift seems to allow only a very slow (logarithmic) p-adic length scale evolution of Λ [L34]. Is there any cure to this problem?

1. The magnetic energy decreases with the area S of flux tube as $1/S \propto 1/p \simeq 1/2^k$, where $\sqrt{p}$ defines the transversal length scale of the flux tube. Volume energy (magnetic energy associated with the twistor sphere) is positive and increases like S. The sum of these has minimum for certain radius of flux tube determined by the value of Λ. Flux tubes with quantized flux would have thickness determined by the length scale defined by the density of dark energy: $L \sim \rho_{vac}^{-1/4}$, $\rho_{dark} = \Lambda/8\pi G$. $\rho_{vac} \sim 10^{-47}$ GeV^4 (see `http://tinyurl.com/k4bwlzu`) would give $L \sim 1$ mm, which would could be interpreted as a biological length scale (maybe even neuronal length scale).

2. But can Λ be very small? In the simplest picture based on dimensionally reduced 6-D Kähler action this term is not small in comparison with the Kähler action! If the twistor spheres of M^4 and CP_2 give the same contribution to the induced Kähler form at twistor sphere of X^4, this term has maximal possible value!

   The original discussions in [L3, L12] treated the volume term and Kähler term in the dimensionally reduced action as independent terms and Λ was chosen freely. This is however not the case since the coefficients of both terms are proportional to $(1/\alpha_K^2)S(S^2)$, where $S(S^2)$ is the area of the twistor sphere of 6-D induced twistor bundle having space-time surface as base space. This are is same for the twistor spaces of M^4 and CP_2 if CP_2 size defines the only fundamental length scale. I did not even recognize this mistake.

The proposed fast p-adic length scale evolution of the cosmological constant would have extremely beautiful consequences. Could the original intuitive picture be wrong, or could the desired p-adic length scale evolution for Λ be possible after all? Could non-trivial dynamics for dimensional reduction somehow give it? To see what can happen one must look in more detail the induction of twistor structure.

1. The induction of the twistor structure by dimensional reduction involves the identification of the twistor spheres S^2 of the geometric twistor spaces $T(M^4) = M^4 \times S^2(M^4)$ and of T_{CP_2} having $S^2(CP_2)$ as fiber space. What this means that one can take the coordinates of say $S^2(M^4)$ as coordinates and embedding map maps $S^2(M^4)$ to $S^2(CP_2)$. The twistor spheres $S^2(M^4)$ and $S^2(CP_2)$ have in the minimal scenario same radius $R(CP_2)$ (radius of the geodesic sphere of CP_2. The identification map is unique apart from $SO(3)$ rotation R of either twistor sphere possibly combined with reflection P. Could one consider the possibility that R is not trivial and that the induced Kähler forms could almost cancel each other?

2. The induced Kähler form is sum of the Kähler forms induced from $S^2(M^4)$ and $S^2(CP_2)$ and since Kähler forms are same apart from a rotation in the common S^2 coordinates, one has $J_{ind} = J + RP(J)$, where R denotes a rotation and P denotes reflection. Without reflection one cannot get arbitrary small induced Kähler form as sum of the two contributions. For mere reflection one has $J_{ind} = 0$.

   Remark: It seems that one can do with reflection if the Kähler forms of the twistor spheres are of opposite sign in standard spherical coordinates. This would mean that they have have opposite orientation.

   One can choose the rotation to act on (y,z)-plane as $(y,z) \to (cy + sz, -sz + cy)$, where s and c denote the cosines of the rotation angle. A small value of cosmological constant is obtained for small value of s. Reflection P can be chosen to correspond to $z \to -z$. Using coordinates $(u = cos(\Theta), \Phi)$ and their primed counterparts and by writing the reflection followed by rotation explicitly in coordinates (x,y,z) one finds $u' = -cu - s\sqrt{1-u^2}sin(\Phi)$, $\Phi' = arctan[(su/\sqrt{1-u^2}cos(\Phi) + ctan(\Phi)]$. In the lowest order in s one has $u' = -u - s\sqrt{1-u^2}sin(\Phi)$, $\Phi' = \Phi + scos(\Phi)(u/\sqrt{1-u^2})$.

3. Kähler form J^{tot} is sum of unrotated part $J = du \wedge d\Phi$ and $J' = du' \wedge d\Phi'$. J' equals to the determinant $\partial(u', \Phi')/\partial(u, \Phi)$. A suitable spectrum for s could reproduce the proposal $\Lambda \propto 2^{-k}$ for Λ. The S^2 part of 6-D Kähler action equals to $(J^{tot}_{\theta\phi})^2/\sqrt{g_2}$ and in the lowest order proportional to s^2. For small values of s the integral of Kähler action for S^2 over S^2 is proportional to s^2.

   One can write the S^2 part of the dimensionally reduced action as $S(S^2) = s^2F^2(s)$. Very near to the poles the integrand has $1/[sin(\Theta) + O(s)]$ singularity and this gives rise to a logarithmic dependence of F on s and one can write: $F = F(s, log(s))$. In the lowest order one has $s \simeq 2^{-k/2}$, and in improved approximation one obtains a recursion formula $s_n(S^2, k) = 2^{-k/2}/F(s_{n-1}, \log(s_{n-1})$ giving renormalization group evolution with k replaced by anomalous dimension $k_{n,a} = k + 2log[F(s_{n-1}, log(s_{n-1})]$ differing logarithmically from k.

4. The sum $J + RP(J)$ defining the induced Kähler form in $S^2(X^4)$ is covariantly constant since both terms are covariantly constant by the rotational covariance of J.

5. The embeddings of $S^2(X^4)$ as twistor sphere of space-time surface to both spheres are holomorphic since rotations are represented as holomorphic transformations. Also reflection as $z \rightarrow 1/z$ is holomorphic. This in turn implies that the second fundamental form in complex coordinates is a tensor having only components of type $(1, 1)$ and $(-1, -1)$ whereas metric and energy momentum tensor have only components of type $(1, -1)$ and $(-1, 1)$. Therefore all contractions appearing in field equations vanish identically and $S^2(X^4)$ is minimal surface and Kähler current in $S^2(X^4)$ vanishes since it involves components of the trace of second fundamental form. Field equations are indeed satisfied.

6. The solution of field equations becomes a family of space-time surfaces parameterized by the values of the cosmological constant Λ as function of S^2 coordinates satisfying $\Lambda/8\pi G = \rho_{vac} = J \wedge (*J)(S^2)$. In long length scales the variation range of Λ would become arbitrary small.

7. If the minimal surface equations solve separately field equations for the volume term and Kähler action everywhere apart from a discrete set of singular points, the cosmological constant affects the space-time dynamics only at these points. The physical interpretation of these points is as seats of fundamental fermions at partonic 2-surface at the ends of light-like 3-surfaces defining their orbits (induced metric changes signature at these 3-surfaces). Fermion orbits would be boundaries of fermionic string world sheets.

   One would have family of solutions of field equations but particular value of Λ would make itself visible only at the level of elementary fermions by affecting the values of coupling constants. p-Adic coupling constant evolution would be induced by the p-adic coupling constant evolution for the relative rotations R combined with reflection for the two twistor spheres. Therefore twistor lift would not be mere manner to reproduce cosmological term but determine the dynamics at the level of coupling constant evolution.

8. What is nice that also $\Lambda = 0$ option is possible. This would correspond to the variant of TGD involving only Kähler action regarded as TGD before the emergence of twistor lift. Therefore the nice results about cosmology [K71] obtained at this limit would not be lost.

6.6.2 Does p-adic coupling constant evolution reduce to that for cosmological constant?

One of the chronic problems if TGD has been the understanding of what coupling constant evolution could be defined in TGD.

1. The notion of quantum criticality is certainly central. The continuous coupling constant evolution having no counterpart in the p-adic sectors of adele would contain as a sub-evolution discrete p-adic coupling constant evolution such that the discrete values of coupling constants allowing interpretation also in p-adic number fields are fixed points of coupling constant evolution.

Quantum criticality is realized also in terms of zero modes, which by definition do not contribute to WCW metric. Zero modes are like control parameters of a potential function in catastrophe theory. Potential function is extremum with respect to behavior variables replaced now by WCW degrees of freedom. The graph for preferred extremals as surface in the space of zero modes is like the surface describing the catastrophe. For given zero modes there are several preferred extremals and the catastrophe corresponds to the regions of zero mode space, where some branches of coincide. The degeneration of roots of polynomials is a concrete realization for this.

Quantum criticality would also mean that coupling parameters effectively disappear from field equations. For minimal surfaces (generalization of massless field equation allowing conformal invariance characterizing criticality) this happens since they are separately extremals of Kähler action and of volume term.

Quantum criticality is accompanied by conformal invariance in the case of 2-D systems and in TGD this symmetry extends to its 4-D analogas isometries of WCW.

2. In the case of 4-D Kähler action the natural hypothesis was that coupling constant evolution should reduce to that of Kähler coupling strength $1/\alpha_K$ inducing the evolution of other coupling parameters. Also in the case of the twistor lift $1/\alpha_K$ could have similar role. One can however ask whether the value of the 6-D Kähler action for the twistor sphere $S^2(X^4)$ defining cosmological constant could define additional parameter replacing cutoff length scale as the evolution parameter of renormalization group.

3. The hierarchy of adeles should define a hierarchy of values of coupling strengths so that the discrete coupling constant evolution could reduce to the hierarchy of extensions of rationals and be expressible in terms of parameters characterizing them.

4. I have also considered number theoretical existence conditions as a possible manner to fix the values of coupling parameters. The condition that the exponent of Kähler function should exist also for the p-adic sectors of the adele is what comes in mind as a constraint but it seems that this condition is quite too strong.

   If the functional integral is given by perturbations around single maximum of Kähler function, the exponent vanishes from the expression for the scattering amplitudes due to the presence of normalization factor. There indeed should exist only single maximum by the Euclidian signature of the WCW Kähler metric for given values of zero modes (several extrema would mean extrema with non-trivial signature) and the parameters fixing the topology of 3-surfaces at the ends of preferred extremal inside CD. This formulation as counterpart also in terms of the analog of micro-canonical ensemble (allowing only states with the same energy) allowing only discrete sum over extremals with the same Kähler action [L33].

5. I have also considered more or less ad hoc guesses for the evolution of Kähler coupling strength such as reduction of the discrete values of $1/\alpha_K$ to the spectrum of zeros of Riemann zeta or actually of its fermionic counterpart [L6]. These proposals are however highly ad hoc.

As I started once again to consider coupling constant evolution I realized that the basic problem has been the lack of explicit formula defining what coupling constant evolution really is.

1. In quantum field theories (QFTs) the presence of infinities forces the introduction of momentum cutoff. The hypothesis that scattering amplitudes do not depend on momentum cutoff forces the evolution of coupling constants. TGD is not plagued by the divergence problems of QFTs. This is fine but implies that there has been no obvious manner to define what coupling constant evolution as a continuous process making sense in the real sector of adelic physics could mean!

2. Cosmological constant is usually experienced as a terrible head ache but it could provide the helping hand now. Could the cutoff length scale be replaced with the value of the length scale defined by the cosmological constant defined by the S^2 part of 6-D Kähler action? This parameter would depend on the details of the induced twistor structure. It was shown above that if the moduli space for induced twistor structures corresponds to rotations of S^2 possibly

combined with the reflection, the parameter for coupling constant restricted to that to $SO(2)$ subgroup of $SO(3)$ could be taken to be taken $s = sin(\epsilon)$.

3. RG invariance would state that the 6-D Kähler action is stationary with respect to variations with respect to s. The variation with respect to s would involve several contributions. Besides the variation of $1/\alpha_K(s)$ and the variation of the $S(^2)$ part of 6-D Kähler action defining the cosmological constant, there would be variation coming from the variations of 4-D Kähler action plus 4-D volume term . This variation vanishes by field equations. As matter of fact, the variations of 4-D Kähler action and volume term vanish separately except at discrete set of singular points at which there is energy transfer between these terms. This condition is one manner to state quantum criticality stating that field equations involved no coupling parameters.

   One obtains explicit RG equation for α_K and Λ having the standard form involving logarithmic derivatives. The form of the equation would be

$$\frac{dlog(\alpha_K)}{ds} = -\frac{S(S^2)}{S_K(X^4) + S(S^2)}\frac{dlog(S(S^2))}{ds} \quad . \tag{6.6.1}$$

   The equation contains the ratio $S(S^2)/(S_K(X^4) + S(S^2))$ of actions as a parameter. This does not conform with idea of micro-locality. One can however argue that this conforms with the generalization of point like particle to 3-D surface. For preferred extremal the action is indeed determined by the 3 surfaces at its ends at the boundaries of CD. This implies that the construction of quantum theory requires the solution of classical theory.

   In particular, the 4-D classical theory is necessary for the construction of scattering amplitudes. and one cannot reduce TGD to string theory although strong form of holography states that the data about quantum states can be assigned with 2-D surfaces. Even more: $M^8 - H$ correspondence implies that the data determining quantum states can be assigned with discrete set of points defining cognitive representations for given adel This set of points depends on the preferred extremal!

4. How to identify quantum critical values of α_K? At these points one should have $dlog(\alpha_K)/ds = 0$. This implies $dlog(S(S^2)/ds = 0$, which in turn implies $dlog(\alpha_K)/ds = 0$ unless one has $S_K(X^4) + S(S^2) = 0$. This condition would make exponent of 6-D Kähler action trivial and the continuation to the p-adic sectors of adele would be trivial. I have considered also this possibility [L34].

   The critical values of coupling constant evolution would correspond to the critical values of S and therefore of cosmological constant. The basic nuisance of theoretical physics would determine the coupling constant evolution completely! Critical values are in principle possible. Both the numerator $J^2_{u\Phi}$ and the numerator $1/\sqrt{det(g)}$ increase with ϵ. If the rate for the variation of these quantities with s vary it is possible to have a situation in which the one has

$$\frac{dlog(J^2_{u\Phi})}{ds} = -\frac{dlog(\sqrt{det(g)})}{ds} \quad . \tag{6.6.2}$$

5. One should demonstrate that the critical values of s are such that the continuation to p-adic sectors of the adele makes sense. For preferred extremals cosmological constant appears as a parameter in field equations but does not affect the field equations expect at the singular points. Singular points play the same role as the poles of analytic function or point charges in electrodynamics inducing long range correlations. Therefore the extremals depend on parameter s and the dependence should be such that the continuation to the p-adic sectors is possible.

A naïve guess is that the values of s are rational numbers. Above the proposal $s = 2^{-k/2}$ motivated by p-adic length scale hypothesis was considered but also $s = p^{-k/2}$ can be considered. These guesses might be however wrong, the most important point is that there is that one can indeed calculate $\alpha_K(s)$ and identify its critical values.

6. What about scattering amplitudes and evolution of various coupling parameters? If the exponent of action disappears from scattering amplitudes, the continuation of scattering amplitudes is simple. This seems to be the only reasonable option. In the adelic approach [L23] amplitudes are determined by data at a discrete set of points of space-time surface (defining what I call cognitive representation) for which the points have M^8 coordinates belong to the extension of rationals defining the adele.

   Each point of $S^2(X^4)$ corresponds to a slightly different X^4 so that the singular points depend on the parameter s, which induces dependence of scattering amplitudes on s. Since coupling constants are identified in terms of scattering amplitudes, this induces coupling constant evolution having discrete coupling constant evolution as sub-evolution.

The following argument suggests a connection between p-adic length scale hypothesis and evolution of cosmological constant but must be taken as an ad hoc guess: the above formula is enough to predict the evolution.

1. p-Adicization is possible only under very special conditions [L23], and suggests that anomalous dimension involving logarithms should vanish for $s = 2^{-k/2}$ corresponding to preferred p-adic length scales associated with $p \simeq 2^k$. Quantum criticality in turn requires that discrete p-adic coupling constant evolution allows the values of coupling parameters, which are fixed points of RG group so that radiative corrections should vanish for them. Also anomalous dimensions Δk should vanish.

2. Could one have $\Delta k_{n,a} = 0$ for $s = 2^{-k/2}$, perhaps for even values $k = 2k_1$? If so, the ratio c/s would satisfy $c/s = 2^{k_1} - 1$ at these points and Mersenne primes as values of c/s would be obtained as a special case. Could the preferred p-adic primes correspond to a prime near to but not larger than $c/s = 2^{k_1} - 1$ as p-adic length scale hypothesis states? This suggest that we are on correct track but the hypothesis could be too strong.

3. The condition $\Delta d = 0$ should correspond to the vanishing of dS/ds. Geometrically this would mean that $S(s)$ curve is above (below) $S(s) = xs^2$ and touches it at points $s = x2^{-k}$, which would be minima (maxima). Intermediate extrema above or below $S = xs^2$ would be maxima (minima).

6.6.3 Appendix: Explicit formulas for the evolution of cosmological constants

What is needed is induced Kähler form $J(S^2(X^4)) \equiv J$ at the twistor sphere $S^2(X^4) \equiv S^2$ associated with space-time surface. $J(S^2(X^4))$ is sum of Kähler forms induced from the twistor spheres $S^2(M^4)$ and $S^2(CP_2)$.

$$J(S^2(X^4) \equiv J = P[J(S^2(M^4)) + J(S^2(CP_2))] \ , \tag{6.6.3}$$

where P is projection taking tensor quantity T_{kl} in $S^2(M^4) \times S^2(CP_2)$ to its projection in $S^2(X^4)$. Using coordinates y^k for $S^2(M^4)$ or $S^(CP_2)$ and x^μ forS^2, P is defined as

$$P : T_{kl} \to T_{\mu\nu} = T_{kl} \frac{\partial y^k}{\partial x^\mu} \frac{\partial y^l}{\partial x^\nu} \ . \tag{6.6.4}$$

For the induced metric $g(S^2(X^4)) \equiv g$ one has completely analogous formula

$$g = P[g(J(S^2(M^4)) + g(S^2(CP_2))] \ . \tag{6.6.5}$$

The expression for the coefficient K of the volume part of the dimensionally reduced 6-D Kähler action density is proportional to

$$L(S^2) = J^{\mu\nu} J_{\mu\nu} \sqrt{det(g)} \ . \tag{6.6.6}$$

(Note that $J_{\mu\nu}$ refers to S^2 part 6-D Kähler action). This quantity reduces to

$$L(S^2) = (\epsilon^{\mu\nu} J_{\mu\nu})^2 \frac{1}{\sqrt{det(g)}} \ . \tag{6.6.7}$$

where $\epsilon^{\mu\nu}$ is antisymmetric tensor density with numerical values +,-1. The volume part of the action is obtained as an integral of K over S^2:

$$S(S^2) = \int_{S^2} L(S^2) = \int_{-1}^{1} du \int_0^{2\pi} d\Phi \frac{J_{u\Phi}^2}{\sqrt{det(g)}} \ . \tag{6.6.8}$$

$(u, \Phi) \equiv (cos(\Theta, \Phi)$ are standard spherical coordinates of $S^2)$ varying in the ranges $[-1, 1]$ and $[0, 2\pi]$.

This the quantity that one must estimate.

General form for the embedding of twistor sphere

The embedding of $S^2(X^4) \equiv S^2$ to $S^2(M^4) \times S^2(CP_2)$ must be known. Dimensional reduction requires that the embeddings to $S^2(M^4)$ and $S^2(CP_2)$ are isometries. They can differ by a rotation possibly accompanied by reflection

One has

$$(u(S^2(M^4)), \Phi(S^2(M^4)) = (u(S^2(X^4), \Phi(S^2(X^4)) \equiv (u, \Phi) \ ,$$

$$\left[u(S^2(CP_2)), \Phi(S^2(CP_2))\right] \equiv (v, \Psi) = RP(u, \Phi)$$

where RP denotes reflection P following by rotation R acting linearly on linear coordinates (x,y,z) of unit sphere S^2). Note that one uses same coordinates for $S^2(M^4)$ and $S^2(X^4)$. From this action one can calculate the action on coordinates u and Φ by using the definite of spherical coordinates.

The Kähler forms of $S^2(M^4)$ resp. $S^2(CP_2)$ in the coordinates $(u = cos(\Theta), \Phi)$ resp.(v, Ψ) are given by $J_{u\Phi} = \epsilon = \pm 1$ resp. $J_{v\Psi} = \epsilon = \pm 1$. The signs for $S^2(M^4)$ and $S^2(CP_2)$ are same or opposite. In order to obtain small cosmological constant one must assume either

1. $\epsilon = -1$ in which case the reflection P is absent from the above formula (RP $\rightarrow$ R).

2. $\epsilon = 1$ in which case P is present. P can be represented as reflection $(x, y, z) \rightarrow (x, y, -z)$ or equivalently $(u, \Phi) \rightarrow (-u, \Phi)$.

Rotation R can represented as a rotation in (y,z)-plane by angle ϕ which must be small to get small value of cosmological constant. When the rotation R is trivial, the sum of induced Kähler forms vanishes and cosmological constant is vanishing.

6.6.4 Induced Kähler form

One must calculate the component $J_{u\Phi}(S^2(X^4)) \equiv J_{u\Phi}$ of the induced Kähler form and the metric determinant $det(g))$ using the induction formula expressing them as sums of projections of M^4 and CP_2 contributions and the expressions of the components of $S^2(CP_2)$ contributions in the coordinates for $S^2(M^4)$. This amounts to the calculation of partial derivatives of the transformation R (or RP) relating the coordinates (u, Φ) of $S^2(M^4)$ and to the coordinates (v, Ψ) of $S^2(CP_2)$.

In coordinates (u, Φ) one has $J_{u\Phi}(M^4) = \pm 1$ and similar expression holds for $J(v\Psi) S^2(CP_2)$. One has

$$J_{u\Phi} = 1 + \frac{\partial(v,\Psi)}{\partial(u,\Phi)} \ . \tag{6.6.9}$$

where right-hand side contains the Jacobian determinant defined by the partial derivatives given by

$$\frac{\partial(v,\Psi)}{\partial(u,\Phi)} = \frac{\partial v}{\partial u}\frac{\partial \Psi}{\partial \Phi} - \frac{\partial v}{\partial \Phi}\frac{\partial \Psi}{\partial u} \ . \tag{6.6.10}$$

Induced metric

The components of the induced metric can be deduced from the line element

$$ds^2(S^2(X^4) \equiv ds^2 = P[ds^2(S^2(M^4)) + ds^2(S^2(CP_2))] \ .$$

where P denotes projection. One has

$$P(ds^2(S^2(M^4))) = ds^2(S^2(M^4)) = \frac{du^2}{1-u^2} + (1-u^2)d\Phi^2 \ .$$

and

$$P[ds^2(S^2(CP_2))] = P[\frac{(dv)^2}{1-v^2} + (1-v^2)d\Psi^2] \ ,$$

One can express the differentials $(dv, d\Psi)$ in terms of $(du, d\Phi)$ once the relative rotation is known and one obtains

$$P[ds^2(S^2(CP_2))] = \frac{1}{1-v^2}[\frac{\partial v}{\partial u}du + \frac{\partial v}{\partial \Phi}d\Phi]^2 + (1-v^2)[\frac{\partial \Psi}{\partial u}du + \frac{\partial \Psi}{\partial \Phi}d\Phi]^2 \ .$$

This gives

$$\begin{array}{l} P[ds^2(S^2(CP_2))] \\ = [(\frac{\partial v}{\partial u})^2\frac{1}{1-v^2} + (1-v^2)(\frac{\partial \Psi}{\partial u})^2]du^2 \\ +[(\frac{\partial v}{\partial \Phi})^2\frac{1}{1-v^2} + (\frac{\partial \Psi}{\partial \Phi})^2 1-v^2]d\Phi^2 \\ +2[\frac{\partial v}{\partial u}\frac{\partial v}{\partial \Phi}\frac{1}{(1-v^2)} + \frac{\partial \Psi}{\partial u}\frac{\partial \Psi}{\partial \Phi}(1-v^2)]dud\Phi \ . \end{array}$$

From these formulas one can pick up the components of the induced metric $g(S^2(X^4)) \equiv g$ as

$$\begin{array}{l} g_{uu} = \frac{1}{1-u^2} + (\frac{\partial v}{\partial u})^2\frac{1}{1-v^2} + (1-v^2)(\frac{\partial \Psi}{\partial u})^2] \ , \\ g_{\Phi\Phi} = 1-u^2 + (\frac{\partial v}{\partial \Phi})^2\frac{1}{1-v^2} + (\frac{\partial \Psi}{\partial \Phi})^2(1-v^2) \\ g_{u\Phi} = g_{\Phi u} = \frac{\partial v}{\partial u}\frac{\partial v}{\partial \Phi}\frac{1}{(1-v^2)} + \frac{\partial \Psi}{\partial u}\frac{\partial \Psi}{\partial \Phi}(1-v^2) \ . \end{array} \tag{6.6.11}$$

The metric determinant $det(g)$ appearing in the integral defining cosmological constant is given by

$$det(g) = g_{uu}g_{\Phi\Phi} - g_{u\Phi}^2 \ . \tag{6.6.12}$$

Coordinates (v,Ψ) in terms of (u,Φ)

To obtain the expression determining the value of cosmological constant one must calculate explicit formulas for (v,Ψ) as functions of (u,Φ) and for partial derivations of (v,Ψ) with respect to (u,Φ).

Let us restrict the consideration to the RP option.

1. P corresponds to $z \rightarrow -z$ and to

$$u \rightarrow -u \ . \tag{6.6.13}$$

2. The rotation R $(x,y,z) \rightarrow (x',y',z')$ corresponds to

$$x' = x, \quad y' = sz + cy = su + c\sqrt{1-u^2}sin(\Phi) \ , \quad z' = v = cu - s\sqrt{1-u^2}sin(\Phi) \tag{6.6.14}$$

   Here one has $(s,c) \equiv (sin(\epsilon), cos(\epsilon)$, where ϵ is rotation angle, which is extremely small for the value of cosmological constant in cosmological scales.

   From these formulas one can pick v and $\Psi = arctan(y'/x)$ as

$$v = cu - s\sqrt{1-u^2}sin(\Phi) \quad \Psi = arctan[\frac{su}{\sqrt{1-u^2}}cos(\Phi) + tan(\Phi)] \ . \tag{6.6.15}$$

3. RP corresponds to

$$v = -cu - s\sqrt{1-u^2}sin(\Phi) \quad \Psi = arctan[-\frac{su}{\sqrt{1-u^2}}cos(\Phi) + tan(\Phi)] \ . \tag{6.6.16}$$

Various partial derivatives

Various partial derivates are given by

$$\begin{aligned}
\frac{\partial v}{\partial u} &= -1 + s\frac{u}{\sqrt{1-u^2}}sin(\Phi) \ , \\
\frac{\partial v}{\partial \Phi} &= -s\frac{u}{\sqrt{1-u^2}}cos(\Phi) \ , \\
\frac{\partial \Psi}{\partial \Phi} &= (-s\frac{u}{\sqrt{1-u^2}}sin(\Phi) + c)\frac{1}{X} \ , \\
\frac{\partial \Psi}{\partial u} &= \frac{scos(\Phi)(1+u-u^2)}{(1-u^2)^{3/2}}\frac{1}{X} \ , \\
X &= cos^2(\Phi) + [-s\frac{u}{\sqrt{1-u^2}} + csin(\Phi)]^2 \ .
\end{aligned} \tag{6.6.17}$$

Using these expressions one can calculate the Kähler and metric and the expression for the integral giving average value of cosmological constant. Note that the field equations contain S^2 coordinates as external parameters so that each point of S^2 corresponds to a slightly different space-time surface.

Calculation of the evolution of cosmological constant

One must calculate numerically the dependence of the action integral S over S^2 as function of the parameter $s = sin(\epsilon))$. One should also find the extrema of S as function of s.

Especially interesting values are very small values of s since for the cosmological constant becomes small. For small values of s the integrand (see Eq. 6.6.8) becomes very large near poles having the behaviour $1/\sqrt{g} = 1/(sin(\Theta) + O(s))$ coming from $\sqrt{g}$ approaching that for the standard metric of S^2. The integrand remains finite for $s \neq 0$ but this behavior spoils the analytic dependence of integral on s so that one cannot do perturbation theory around $s = 0$. The expected outcome is a logarithmic dependence on s.

In the numerical calculation one must decompose the integral over S^2 to three parts.

1. There are parts coming from the small disks D^2 surrounding the poles: these give identical contributions by symmetry. One must have criterion for the radius of the disk and the natural assumption is that the disk radius is of order s.

2. Besides this one has a contribution from S^2 with disks removed and this is the regular part to which standard numerical procedures apply.

One must be careful with the expressions involving trigonometric functions which give rise to infinite if one applies the formulas in straightforward manner. These infinities are not real and cancel, when one casts the formulas in appropriate form inside the disks.

1. The limit $u \to \pm 1$ at poles involves this kind of dangerous quantities. The expression for the determinant appearing in $J_{u\Phi}$ remains however finite and $J_{u\phi}^2$ vanishes like s^2 at this limit. Also the metric determinant $1/\sqrt{g}$ remains finite expect at $s = 0$.

2. Also the expression for the quantity X in $\Psi = arctan(X)$ contains a term proportional to $1/cos(\Phi)$ approaching infinity for $\Phi \to \pi/2, 3\pi/2$. The value of $\Psi = arc(tan(X)$ remains however finite and equal to $\pm\Phi$ at this limit depending on on the sign of us.

Concerning practical calculation, the relevant formulas are given in Eqs. 6.6.7, 6.6.8, 6.6.9, 6.6.10, 6.6.11, 6.6.12, and 6.6.17.

The calculation would allow to test the conjectures already discussed.

1. There indeed exist extrema satisfying thus $dS/ds = 0$.

2. These extrema correspond to $s = 2{-}k$ or more generally $s = p^{-k}$. This conjecture is inspired by p-adic length scale hypothesis.

3. A further conjecture is that for certain integer values of integer k the integral $S(S^2)$ of Eq. 6.6.8 is of form $S(S^2) = xs^2$ for $s = 2^{-k}$, where x is a universal numerical constant.

   This would realize the idea that p-adic length scales realized as scales associated with cosmological constant correspond to fixed points of renormalization group evolution implying that radiative corrections otherwise present cancel. In particular, the deviation from $s = 2^{-d/2}$ would mean anomalous dimension replacing $s = 2^{-d/2}$ with $s^{-(d+\Delta d)/2}$ for $d = k$ the anomalies dimension Δd would vanish.

4. The condition $\Delta d = 0$ should be equivalent with the vanishing of the dS/ds. Geometrically this means that $S(s)$ curve is above (below) $S(s) = xs^2$ and touches it at points $s = x2^{-k}$, which would be minima (maxima). Intermediate extrema above or below $S = xs^2$ would be maxima (minima).

6.7 More about the construction of scattering amplitudes in TGD framework

The construction of scattering amplitudes in TGD framework has been a longstanding problem, and I have considered several proposals - perhaps the most realistic proposal relies on the generalization of twistor Grassmann approach to TGD context [L35]. These approaches have however suffered from their ad hoc character.

One reason for the slow progress might be the fact that I have not conditioned Feynman diagrams into my spine: I have intentionally avoided this in the fear that it would prevent genuine thinking. Second reason is that TGD is really different and my mathematical skills are rather limited. For instance, in TGD classical theory is an exact part of quantum theory and particles are replaced with 3-surfaces: there is no hope of starting from Lagrangian with simple non-linearities and writing Feynman rules and deducing beta functions.

There are several questions waiting for an answer. How to achieve unitarity? What it is to be a particle in classical sense? Can one identify TGD analogs of quantum fields? Could scattering amplitudes have interpretation as Fourier transforms of n-point functions for the analogs of quantum fields?

Unitarity is certainly the issue #1 and in the sequel almost trivial solution to unitarity problem is proposed. Also quantum classical correspondence is discussed.

6.7.1 Some background

Supersymplectic algebra

Let us collect what I think is known in TGD framework.

1. The "world of classical worlds" (WCW) [K66] geometry does not exist without maximal group of isometries and WCW is assumed to possess super-symplectic algebra (SSA) assignable to light-cone boundary (boundaries of causal diamonds (CDs)) as isometries. Also Kac-Moody algebras for isometries of embedding space realized at the light-like partonic orbits serving as boundaries between Euclidian and Minkowskian regions of space-time surface are expected to be of key importance (for p-adic mass calculations applying these symmetries see [K42].

   SSA has a fractal hierarchy of isomorphic sub-algebras and the proposal is that one has hierarchy of criticalities such that sub-SSA and its commutator with SSA annihilate the physical states so that SSA effectively reduces to a finite-D Lie-algebra generating the physical states. Sub-SSA takes the role of gauge algebra and one could say that it represents finite measurement resolution. This hierarchy would correspond to a hierarchies of inclusions of von Neumann algebras known as hyper-finite factors of type II_1 [K85, K28].

   It seems obvious to me that the scattering amplitudes should allow a formulation in terms of SSA effectively reducing to finite-D Lie-algebra of corresponding Kac-Moody algebra plus Kac-Moody algebras associated with embedding space isometries.

   Remark: Conformal weights of SSA associated with the radial light-like coordinate are non-negative so that one has analogy with Yangian algebra. The TGD variant of twistor Grassmann approach [L24] [L35] strongly suggests that SSA extends to Yangian having multi-local generators with locus corresponding to partonic 2-surface.

2. There are both classical and fermionic Noether charges associated with SSA and the Kac-Moody algebras [K19, K86, K66]. Quantum-classical correspondence (QCC) suggests that the eigenvalues for Cartan algebra Noether charges in the fermionic representation correspond to bosonic charges assignable to the dimensionally reduced Kähler action. One obtains also fermionic super-charges in 1-1 correspondence with the modes of the induced spinor field. Super-charges are very much like oscillator operators creating or annihilating fermions and there is a temptation to think that these fermionic SSA and Kac-Moody charges take the role of operators creating fermionic and bosonic states.

   One could think of constructing many-particle states at both boundaries of causal diamond (CD) by decomposing SSA to Cartan algebra and to parts acting like creation and annihilation operators. States would be created by the generators acting like oscillator operators.

   The time evolution dictated by preferred extremals and corresponding modified Dirac equation would transform initial states at boundary A of CD to final states at boundary B. This time evolution is determined by preferred extremal property and by modified Dirac equation [K86]. Time evolution is not obtained by exponentiating quantum Hamiltonian as in

QFT approach. The existence of infinite-D SSA of Noether changes should make it possible to prove unitarity.

General argument for unitarity

The argument for unitarity is very general and based on zero energy ontology (ZEO). Causal diamond (CD) containing space-time surfaces having ends at its opposite boundaries is central for ZEO. Zero energy states are quantum superpositions of space-time surfaces, which are preferred extremals of dimensionally reduced 6-D Kähler action decomposing to 4-D Kähler action and volume term. CD has two boundaries: the active boundary (B) and passive boundary (A) and space-time surfaces as preferred extremals have ends at these boundaries [L25].

In ZEO one has two kinds of state function reductions.

1. At the active boundary (B) one has "small" state function reductions as counterparts of weak measurements following unitary time evolutions shifting the active boundary B farther from passive boundary A in statistical sense. During each unitary time evolution there is a de-localization with respect to the distance between the tips of CD followed by localization serving also as time measurement. This would yield the correlation between experienced time as sequence of these weak measurements and geometric time identified as distance between the tips of CD.

   Also measurements of observables commuting with the observables, whose eigenstates the states at boundary A are, are possible. Passive boundary (A) and the members of zero energy states associated with it do not change, and this gives rise to what one might call generalized Zeno effect.

   S-matrix would correspond to the evolution between two weak measurements for the states at the active boundary of CD and expected to be unitary. At passive boundary of CD and states at it would not be affected. The time evolution in the fermionic sector would be induced by the modified Dirac equation. Now one can express the states at new active boundary in terms of those at old active boundary and one would obtain unitary S-matrix by expressing the final states in terms of the state basic for the original boundary.

2. In "big" state function reduction the roles of passive boundary A and active boundary B are changed. The states at B are superpositions of states in the state basis for SSA. Unitary S-matrix would be obtained by expressing these states in terms of SSA basis.

Unitarity does not seem to be a problem since the conservation of Cartan charges for SSA in the fermionic representation would not allow breaking of unitarity. The time evolution would be induced by the preferred extremal property and modified Dirac equation.

Scattering amplitudes would involve an integration over positions of particles meaning that instead of single 4-surface one would have large number of them contributing to single scattering amplitude. Different position would correspond to different values of zero modes not contributing to WCW metric. Number theoretical vision [L23, L22] demands that the exponent of action is same for all of these surfaces: with inspiration coming from the idea about quantum TGD as square root of thermodynamics, I have indeed proposed [L33] this quantum analog of micro-canonical ensemble (for which energy is constant) as a way to get rid of difficulties in the realization of number theoretical universality. The number theoretically cumbersome action exponents would cancel out from the scattering amplitudes.

6.7.2 Does 4-D action generate lower-dimensional terms dynamically?

The original proposal was that the action defining the preferred extremals is 4-D Kähler action. Later it became obvious that there must be also 2-D string world sheet term present and probably also 1-D term associated with string boundaries at partonic 2-surfaces. The question has been whether these lower-D terms in the action are primary of generated dynamically. By super-conformal symmetry the same question applies to the fermionic part of the action. The recent formulation based on the twistor lift of TGD contains also volume term but the question remains the same.

Quantum criticality would be realized as a minimal surface property realized by holomorphy in suitably generalized sense [L39, L34]. The reason is that the holomorphic solutions of minimal surface equations involve no coupling parameters as the universality of the dynamics at quantum criticality demands.

Minimal surface equation would be true apart from possible singular surfaces having dimension $D = 2, 1, 0$. $D = 2$ corresponds to string world sheets and partonic 2-surfaces. If there are 0-D singularities they would be associated with the ends of orbits of partonic 2-surfaces at boundaries of causal diamond (CD). Minimal surfaces are solutions of non-linear variant of massless d'Alembertian having as effective sources the singular surfaces at which d'Alembertian equation fails. The analogy with gauge theories is highly suggestive: singular surfaces would act as sources of massless field.

Strings world sheets seem to be necessary. The basic question is whether the singular surfaces are postulated from the beginning and there is action associated with them or whether they emerge dynamical from 4-D action. One can consider two extreme options.

Option I: There is an explicit assignment of action to the singular surfaces from the beginning. A transfer of Noether charges between space-time interior and string world sheets is possible. This kind of transfer process can take place also between string world sheets and their light-like boundaries and happens if the normal derivatives of embedding space coordinates are discontinuous at the singular surface.

Option II: No separate action is assigned with the singular surfaces. There could be a transfer of Noether charges between 4-D Kähler and volume degrees of freedom at the singular surfaces causing the failure of minimal surface property in 4-D sense. But could singular surfaces carry Noether currents as 2-D delta function like densities?

This is possible if the discontinuity of the normal derivatives generates a 2-D singular term to the action. Conservation laws require that at string world sheets energy momentum tensor should degenerate to a 2-D tensor parallel to and concentrated at string world sheet. Only 4-D action would be needed - this was actually the original proposal. Strings and particles would be essentially edges of space-time - this is not possible in GRT. Same could happen also at its boundaries giving rise to point like particles. Super-conformal symmetry would make this possible also in the fermionic sector.

For both options the singular surfaces would provide a concrete topological picture about the scattering process at the level of single space-time surface and telling what happens to the initial state. The question is whether Option I actually reduces to Option II. If the 2-D term is generated to 4-D action dynamically, there is no need to postulate primary 2-D action.

Can Option II generate separate 2-D action dynamically?

The following argument shows that Option II with 4-D primary action can generate dynamically 2-D term into the action so that no primary action need to be assigned with string world sheets.

1. Dimensional hierarchy of surfaces and strong form of holography

String world sheets having light-like boundaries at the light-like orbits of partonic 2-surfaces are certainly needed to realize strong form of holography [K86]. Partonic 2-surfaces emerge automatically as the ends of the orbits of wormhole contacts.

1. There could (but neet not) be a separate terms in the primary action corresponding to string world sheets and their boundaries. This hierarchy bringing in mind branes would correspond to the hierarchy of classical number fields formed by reals, complex numbers, quaternions (space-time surface), and octonions (embedding space in M^8-side of M^8 duality). The tangent - or normal spaces of these surfaces would inherit real, complex, and quaternionic structures as induced structure. The number theoretic interpretation would allow to see these surfaces as images of those surfaces in M^8 mapped to H by $M^8 - H$ duality. Therefore it would be natural to assign action to these surfaces.

2. This makes in principle possible the transfer of classical and quantum charges between space-time interior and string world sheets and between from string world sheets to their light-like boundaries. TGD variant of twistor Grassmannian approach [L24, L35] relies on the assumption that the boundaries of string world sheets at partonic orbits carry quantum numbers.

Quantum criticality realized in terms of minimal surface property realized holomorphically is central for TGD and one can ask whether it could play a role in the definition of S-matrix and identification of particles as geometric objects.

3. For preferred extremals string world sheets (partonic 2-surfaces) would be complex (co-complex) manifolds in octonionic sense. Minimal surface equations would hold true outside string world sheets. Conservation of various charges would require that the divergences of canonical momentum currents at string world sheet would be equal to the discontinuities of the normal components of the canonical momentum currents in interior. These discontinuities would correspond to discontinuities of normal derivatives of embedding space coordinates and are acceptable. Similar conditions would hold true at the light-like boundaries of string world sheets at light-like boundaries of parton orbits. String world sheets would not be minimal surfaces and minimal surface property for space-time surface would fail at these surfaces.

   Quantum criticality for string world sheets would also correspond to minimal surface property. If this is realized in terms of holomorphy, the field equations for Kähler and volume parts at string world sheets would be satisfied separately and the discontinuities of normal components for the canonical momentum currents in the interior would vanish at string world sheets.

4. The idea about asymptotic states as free particles would suggest that normal components of canonical momentum currents are continuous near the boundaries of CD as boundary conditions at least. The same must be true at the light-like boundaries of string world sheets. Minimal surface property would reduce to the property of being light-like geodesics at light-like partonic 2-surface. If this is not assumed, the orbit is space-like. Even if these conditions are realized, one can imagine the possibility that at string world sheets 4-D minimal surface equation fails and there is transfer of charges between Kähler and volume degrees of freedom (Option II) and therefore breaking of quantum criticality.

   If the exchange of Noether charges vanishes everywhere at string world sheets and boundaries, one could argue that they represent independent degrees of freedom and that TGD reduces to string model. The proposed equation for coupling constant evolution however contains a coefficients depending on the total action so that this would not be the case.

5. Assigning action to the lower-D objects requires additional coupling parameters. One should be able to express these parameters in terms of the parameters appearing in 4-D action (α_K and cosmological constant). For string sheets the action containing cosmological term is 4-D and Kähler action for $X^2 \times S^2$, where S^2 is non-dynamical twistor sphere is a good guess. Kähler action gets contributions from X^2 and S^2. If the 2-D action is generated dynamically as a singular term of 4-D action its coupling parameters are those of 4-D action.

6. There is a temptation to interpret this picture as a realization of strong form of holography (SH) in the sense that one can deduce the space-time surfaces by using data at string world sheets and partonic 2-surfaces and their light-like orbits. The vanishing of normal components of canonical momentum currents would fix the boundary conditions.

   If double holography $D = 4 \rightarrow D = 2 \rightarrow D = 1$ were satisfied it might be even possible to reduce the construction of S-matrix to the proposed variant of twistor Grassmann approach. This need not be the case: p-adic mass calculations rely on p-adic thermodynamics for the excitions of massless particles having CP_2 mass scale and it would seem that the double holography can makes sense for massless states only.

   In M^8-picture [L20] the information about space-time surface is coded by a polynomial defined at real line having coefficients in an extension of rationals. This real line for octonions corresponds to the time axis in the rest system rather than light-like orbit as light-like boundary of string world sheet.

2. Stringy quantum criticality?

The original intuition [L39] was that there are canonical momentum currents between Kähler and volume degrees of freedom at singular surfaces but no transfer of canonical momenta between interior and string world sheets nor string world sheets and their boundaries. Also string world sheets would be minimal surfaces as also the intuition from string models suggests. Could also the stringy quantum criticality be realized?

1. Some embedding space coordinates h^k must have discontinuous partial derivatives in directions normal to the string world sheet so that 3-surface has 1-D edge along fermionic string connecting light-like curves at partonic 2-surfaces in both Minkowskian and Euclidian regions. A closed highly flattened rectangle with long and short edges would be associated with closed monopole flux tube in the case of wormhole contact pairs assigned with elementary particles. 3-surfaces would be "edgy" entities and space-time surfaces would have 2-D and 1-D edges. In condensed matter physics these edges would be regarded as defects.

2. Quantum criticality demands that the dynamics of string world sheets and of interior effectively decouple. Same must take place for the dynamics of string world sheets and their boundaries. Decoupling allows also string world sheets to be minimal surfaces as analogs of complex surfaces whereas string world sheet boundaries would be light-like (their deformations are always space-like) so that one obtains both particles and string like objects.

3. By field equations the sums for the divergences of stringy canonical momentum currents and the corresponding singular parts of these currents in the interior must vanish. By quantum criticality in interior the divergencespf Kähler and volume terms vanish separately. Same must happen for the sums in case of string world sheets and their boundaries. The discontinuity of normal derivatives implies that the contribution from the normal directions to the divergence reduces to the sum of discontinuities in two normal directions multiplied by 2-D delta function. Thid contribution is in the general case equal to the divergence of corresponding stringy canonical momentum current but must vanish if one has quantum criticality also at string world sheets and their boundaries.

   The separate continuity of Kähler and volume parts of canonical momentum currents would guarantee this but very probably implies the continuity of the induced metric and Kähler form and therefore of normal derivatives so that there would be no singularity. However, the condition that total canonical momentum currents are continuous makes sense, and indeed implies a transfer of various conserved charges between Kähler action and volume degrees of freedom at string world sheets and their boundaries in normal directions as was conjectured in [L39].

4. What about the situation in fermionic degrees of freedom? The action for string world sheet X^2 would be essentially of Kähler action for $X^2 \times S^2$, where S^2 is twistor sphere. Since the modified gamma matrices appearing in the modified Dirac equation are determined in terms of canonical momentum densities assignable to the modified Dirac action, there could be similar transfer of charges involved with the fermionic sector and the divergences of Noether charges and super-charges assignable to the volume action are non-vanishing at the singular surfaces. The above mechanism would force decoupling between interior spinors and string world sheets spinors also for the modified Dirac equation since modified gamma matrices are determined by the bosonic action.

   Remark: There is a delicacy involved with the definition of modified gamma matrices, which for volume term are proportional to the induced gamma matrices (projections of the embedding space gamma matrices to space-time surface). Modified gamma matrices are proportional to the contractions $T^{\alpha}_k\Gamma^k$ of canonical momentum densities $T^{\alpha_k} = \partial L/\partial(\partial_\alpha h^k)$ with embedding space gamma matrices Γ^k. To get dimension correctly in the case of volume action one must divide away the factor $\Lambda/8\pi G$. Therefore fermionic super-symplectic currents do not involve this factor as required.

It remains an open question whether the string quantum criticality is realized everywhere or only near the ends of string world sheets near boundaries of CD.

3. String world sheet singularities as infinitely sharp edges and dynamical generation of string world sheet action

The condition that the singularities are 2-D string world sheets forces 1-D edges of 3-surfaces to be infinitely sharp.

Consider an edge at 3-surface. The divergence from the discontinuity contains contributions from two normal coordinates proportional to a delta function for the normal coordinate and coming from the discontinuity. The discontinuity must be however localized to the string rather than 2-surface. There must be present also a delta function for the second normal coordinate. Hence the value of also discontinuity must be infinite. One would have infinitely sharp edge. A concrete example is provided by function $y = |x|^\alpha$ $\alpha < 1$. This kind of situation is encountered in Thom's catastrope theory for the projection of the catastrophe: in this case one has $\alpha = 1/2$. This argument generalizes to 3-D case but visualization is possible only as a motion of infinitely sharp edge of 3-surface.

Kähler form and metric are second degree monomials of partial derivatives so that an attractive assumption is that $g_{\alpha\beta}$, $J_{\alpha\beta}$ and therefore also the components of volume and Kähler energy momentum tensor are continuous. This would allow $\partial_{n_i} h^k$ to become infinite and change sign at the discontinuity as the idea about infinitely sharp edge suggests. This would reduce the continuity conditions for canonical momentum currents to rather simple form

$$T^{n_i n_j} \Delta \partial_{n_j} h^k = 0 \ . \tag{6.7.1}$$

which in turn would give

$$T^{n_i n_j} = 0 \tag{6.7.2}$$

stating that canonical momentum is conserved but transferred between Kähler and volume degrees of freedom. One would have a condition for a continuous quantity conforming with the intuitive view about boundary conditions due to conservation laws. The condition would state that energy momentum tensor reduces to that for string world sheet at the singularity so that the system becomes effectively 2-D. I have already earlier proposed this condition.

The reduction of 4-D locally to effectively 2-D system raises the question whether any separate action is needed for string world sheets (and their boundaries)? The generated 2-D action would be similar to the proposed 2-D action. By super-conformal symmetry similar generation of 2-D action would take place also in the fermionic degrees of freedom. I have proposed also this option already earlier. This would mean that Option II is enough.

The following gives a more explicit analysis of the singularities. The vanishing on the discontinuity for the sum of normal derivative gives terms with varying degree of divergence. Denote by n_i resp. t_i the coordinate indices in the normal resp. tangent space. Suppose that some derivative $\partial_{n_i} h^k$ become infinite at string. One can introduce degree n_D of divergence for a quantity appearing as part of canonical momentum current as the degree of the highest monomial consisting of the diverging derivatives $\partial_{n_i} h^k$ appearing in quantity in question. For the leading term in continuity conditions for canonical momentum currents of total action one should have $n_D = 2$ to give the required 2-D delta function singularity.

- $\partial_{n_i} h^k$ has $n_D \leq 1$. If it is also discontinuous - say changes sign - one has $n_D = 2$ for $\Delta \partial_{n_i} h^k$ in direction n_i.
- One has $n_D(g_{t_i t_j}) = 0$, $n_D(g_{t_i n_j}) = 1$, $n_D(g_{n_i n_i}) = 2$ and $n_D(g_{n_i n_j}) = 1$ or 2 for $i \neq j$. One has $n_D(g) = 4$ ($g = det(g_{\alpha\beta})$). For contravariant metric one gas $n_D(g^{t_i t_j}) = 0$ and $n_D(g^{n_i j}) = n_D(g^{n_i n_j}) = -2$ as is easy to see from the formula for $g^{\alpha\beta}$ in terms of cofactors.
- Both Kähler and volume terms in canonical momentum current are proportional to $\sqrt{g}$ with $n_D(\sqrt{g}) = 2$ having leading term proportional to 2-determinant $\sqrt{det(g_{n_i n_j})}$. In Kähler action the leading term comes from tangent space part J_{ij} and has $n_D = -1$ coming from the partial derivative. The remaining parts involving $J_{t_i n_j}$ or $J_{n_i n_j}$ have $n_D < 0$.
- Consider the behavior of the contribution of volume term to the canonical momentum currents. For $g^{n_i t_j} \partial_{t_j} h^k \sqrt{g}$ one has $n_D = 0$ so that this term is finite. For $g^{n_i n_j} \partial_{n_j} h^k \sqrt{g}$ one has $n_D \leq 1$ and this term can be infinite as also its discontinuity coming solely from the change of sign for $\partial_{n_j} h^k$. If $\partial_{n_j} h^k$ is infinite and changes sign, one can have $n_D = 2$ as required by 2-D delta function singularity.

The continuity condition for the canonical momentum current would state the vanishing of $n_D = 2$ discontinuity but would not imply separate vanishing of discontinuity for Kähler and volume parts of canonical momentum currents - this in accordance with the idea about canonical momentum transfer. If the sign of partial derivative only changes the coefficient of the partial derivative must vanish so that the condition reduces to the condition $T^{n_i n_j} = 0$ already given for the components of the total energy momentum tensor, which would be continuous by the above assumption.

4. A connection with Higgs vacuum expectation?

What about the physical interpretation of the singular divergences of the isometry currents J_A of the volume action located at string world sheet?

1. The divergences of J_A are proportional to the trace of the second fundamental form H formed by the covariant derivatives of gradients $\partial_\alpha h^k$ of H-coordinates in the interior and vanish. The singular contribution at string world sheets is determined by the discontinuity of the isometry current J_A and involves only the first derivatives $\partial_\alpha h^k$.

2. One of the first questions after ending up with TGD for 41 years ago was whether the trace of H in the case of CP_2 coordinates could serve as something analogous to Higgs vacuum expectation value. The length squared for the trace has dimensions of mass squared. The discontinuity of the isometry currents for $SU(3)$ parts in $h = u(2)$ and its complement t, whose complex coordinates define $u(2)$ doublet. $u(2)$ is in correspondence with electroweak algebra and t with complex Higgs doublet. Could an interpretation as Higgs or even its vacuum expectation make sense?

3. p-Adic thermodynamics explains fermion masses elegantly (understanding of boson masses is not in so good shape) in terms of thermal mixing with excitations having CP_2 mass scale and assignable to short string associated with wormhole contacts. There is also a contribution from long strings connecting wormhole contacts and this could be important for the understanding of weak gauge boson masses. Could the discontinuity of isometry currents in t determine this contribution to mass. Edges/folds would carry mass.

4. The non-singular part of the divergence multiplying 2-D delta function has dimension 1/length squared and the square of this vector in CP_2 metric has dimension of mass squared. Could the interpretation of the discontinuity as Higgs expectation make sense? If so, Higgs expectation would vanish in the space-time interior.

   Could the interior modes of the induced spinor field - or at least the interior mode of right-handed neutrino ν_R having no couplings to weak or color fields - be massless in 8-D or even 4-D sense? Could ν_R and $\overline{\nu}_R$ generate an unbroken $\mathcal{N} = 2$ SUSY in interior whereas inside string world sheets right-handed neutrino and antineutrino would be eaten in neutrino massivation and the generators of $\mathcal{N} = 2$ SUSY would be lost somewhat like charged components of Higgs!

   If so, particle physicists would be trying to find SUSY from wrong place. Space-time interior would be the correct place. Would the search of SUSY be condensed matter physics rather than particle physics?

Summarizing the recent view about elementary particles

It is interesting to see how elementary particles and their basic interaction vertices could be realized in this framework.

1. In TGD framework particle would correspond to pair of wormhole contact associated with closed magnetic flux tube carrying monopole flux. Strongly flattened rectangle with Minkowskian flux tubes as long edges with length given by weak scale and Euclidian wormhole contacts as short edges with CP_2 radius as lengths scale is a good visualization. 3-particle vertex corresponding to the replication of this kind of flux tube rectangle to two rectangles would replace 3-vertex of Feynman graph. There is analogy with DNA replication. Similar replication is expected to be possible also for the associated closed fermionic strings.

2. Denote the wormhole contacts by A and B and their opposite throats by A_i and B_i, $i = 1, 2$. For fermions A_1 can be assumed to carry the electroweak quantum numbers of fermion. For electroweak bosons A_1 and A_2 (for instance) could carry fermion and anti-fermion, whose quantum numbers sum up to those of ew gauge boson. These "corner fermions" can be called *active*.

   Also other distributions of quantum numbers must be considered. Fermion and anti-fermion could in principle reside at the same throat - say A_1. One can however assume that second wormhole contact, say A has quantum numbers of fermion or weak boson (or gluon) and second contact carries quantum numbers screening weak isospin.

3. The model assumes that the weak isospin is neutralized in length scales longer than the size of the flux tube structure given by electro-weak scale. The screening fermions can be called *passive*. If the weak isospin of $W^{\pm}$ boson is neutralized in the scale of flux tube, 2 $\nu_L\overline{\nu}_R$ pairs are needed (lepton number for these pairs must vanish) for W^-. For Z $\nu_L\overline{\nu}_R$ and $\overline{\nu}_L nu_R$ are needed. The pairs of passive fermions could reside in the interior of flux tube, at string world sheet or at its corners just like active fermions. The first extreme is that the neutralizing neutrino-antineutrino pairs reside in interior at the opposite long edges of the rectangular *flux tube*. Second extreme is that they are at the corners of rectangular *closed string*.

4. Rectangular closed string containing active fermion at wormhole A (say) and with members of isospin neutralizing neutrino-antineutrino pair at the throats of B serves as basic units. In scales shorter than string length the end A would behave like fermion with weak isospin. At longer scales physical fermion would be hadron like entity with vanishing isospin and one could speak of confinement of weak isospin.

   From these physical fermions one can build gauge bosons as bound states. Weak bosons and also gluons would be pairs of this kind of fermionic closed strings connecting wormhole contacts A and B. Gauge bosons (and also gravitons) could be seen as composites of string like physical fermions with vanishing net isospin rather than those of point like fundamental fermions.

5. The decay of weak boson to fermion-antifermion pair would be flux tube replication in which closed strings representing physical fermion and anti-fermion continue along different copies of flux tube structure. The decay of boson to two bosons - say $W \rightarrow WZ$ - by replication of flux tube would require creation of a pair of physical fermionic closed strings representing Z. This would correspond to a V-shaped vertex with the edge of V representing closed fermionic closed string turning backwards in time. In decays like $Z \rightarrow W^+W^-$ two closed fermion strings would be created in the replication of flux tube. Rectangular fermionic string would turns backwards in time in the replication vertex and the rectangular strings of Z would be shared between W^+ and W^-.

This mesonlike picture about weak bosons as bound states of fermions sounds complex as compared with standard model picture. On the other hand only the spinor fields assignable to single fermion family are present.

A couple of comments concerning this picture are in order.

1. M^8 duality provides a different perspective. In M^8 picture these vertices could correspond to analogs of local 3 particle vertices for octonionic superfield, which become nonlocal in the map taking $M^8 = M^4 \times CP_2$ surfaces to surfaces in $H = M^4 \times CP_2$. The reason is that M^4 point is mapped to M^4 point but the tangent space at E^4 point is mapped to a point of CP_2. If the point in M^8 corresponds to a self-intersection point the tangent space at the point is not unique and point is mapped to two distinct points. There local vertex in M^8 would correspond to non-local vertex in H and fermion lines could just begin. This would mean that at H-level fermion line at moment of replication and V-shaped fermion line pair beginning at different point of throat could correspond to 3-vertex at M^8 level.

2. The 3-vertex representing replication could have interpretation in terms of quantum criticality: in reversed direction of time two branches of solution of classical field equations would coincide.

Gravitation as a square of gauge interaction

I encountered in FB a link to an interesting popular article (see http://tinyurl.com/y5r4glgg) about theoretical physicist Henrik Johansson who has worked with supergravity in Wallenberg Academy. He has found strong mathematical evidence for a new duality. Various variants of super quantum gravity support the view that supersymmetric quantum theories of gravitation can be seen as a double copy of a gauge theory. One could say that spin 2 gravitons are gluons with color charge replaced with spin. Since the information about charges disappears, gluons can be understood very generally as gauge bosons for given gauge theory, not necessarily QCD.

The article of C. D. White [B12] (see https://arxiv.org/pdf/1708.07056.pdf) entitled "The double copy: gravity from gluons" explains in more detail the double copy duality and also shows that it relates in many cases also exact classical solutions of Einsteins equations and YM theories. One starts from L-loop scattering amplitude involving products of kinematical factors n_i and color factors c_i and replaces color factors with extra kinematical factors $\tilde{n}_i$. The outcome is an L-loop amplitude for gravitons.

As if gravitation could be regarded as a gauge theory with polarization and/or momenta identified giving rise to effective color charges. This is like taking gauge potential and giving it additional index to get metric tensor. This naïve analogy seems to hold true at the level of scattering amplitudes and also for many classical solutions of field equations. Could one think that gravitons as states correspond to gauge singlets formed from two gluons and having spin 2? Also spin 1 and spin 0 states would be obtained and double copies involve also them.

TGD view about elementary particles indeed predicts that gravitons be regarded in certain sense pairs of gauge bosons. Consider now gravitons and assume for simplicity that spartners of fundamental fermions - identifiable as local multi-fermion states allowed by statistics - are not involved: this does not change the situation much [L48]. Graviton's spin 2 requires 2 fermions and 2 anti-fermions: fermion or anti-fermion at each throat. For gauge bosons fermion and anti-fermion at two throats is enough. One could therefore formally see gravitons as pairs of two gauge bosons in accordance with the idea about graviton is a square of gauge boson.

The fermion contents of the monopole flux tube associated with elementary particle determines quantum numbers of the flux tube as particle and characterizes corresponding interaction. The interaction depends also on the charges at the ends of the flux tube. This leads to a possible interpretation for the formation of bound states in terms of flux tubes carrying quantum numbers of particles.

1. These long flux tubes can be arbitrarily long for large values of $\hbar_{eff} = n \times \hbar_0$ assigned to the flux tube. A plausible guess for for the expression of $\hbar$ in terms of $_0$ is as $\hbar = 6 \times \hbar_0$ [L13, L29]. The length of the flux tube scales like $\hbar_{eff}$.

2. Nottale [E1] proposed that it makes sense to speak about gravitational Planck constant h_{gr}. In TGD this idea is generalized and interpreted in framework of generalized quantum theory [K70, K57, K7]. For flux tubes assignable to gravitational bound states along which gravitons propagate, one would have $\hbar_{eff} = \hbar_{gr} = GMm/v_0$, where $v_0 < c$ is parameter with dimensions of velocity. One could write interaction strength as

$$GMm = v_0 \times \hbar_{gr} \ .$$

3. $\hbar_{gr}$ obtained from this formula must satisfy $\hbar_{gr} > \hbar$. This generalizes to other interactions. For instance, one has one would have

$$Z_1 Z_2 e^2 = \frac{v_0 \hbar_{em}}{}$$

for electromagnetic flux tubes in the case that ones $h_{em} > \hbar$. The interpretation of the velocity parameter v_0 is discussed in [K7].

One could even turn the situation around and say that the value of $\hbar_{eff}$ fixes the interaction strength. $\hbar_{eff}$ would depend on fermion content and thus of virtual particle and also on the masses or other charges at the ends of the flux tube. The longer the range of the interaction, the larger the typical value of $\hbar_{eff}$.

4. The interpretation could be in terms long length scale quantum fluctuations at quantum criticality. Particles generate U-shaped monopole flux tubes with varying length proportional to $\hbar_{gr}$. If these U-shaped flux tubes from two different particles find each other, they reconnect to flux tube pairs connecting particles and give rise to interaction. What comes in mind is tiny curious and social animals studying their environment.

5. I have indeed proposed this picture in biology: the U-shaped flux tubes would be tentacles with which bio-molecules (in particular) would be scanning their environment. This scanning would be the basic mechanism behind immune system. It would also make possible for bio-molecules to find each in molecular crowd and provide a mechanism of catalysis. Could this picture apply completely generally? Would even elementary particles be scanning their environment with these tentacles?

6. Could one interpret the flux tubes as analogs of virtual particles or could they replace virtual particles of quantum field theories? The objection is that flux tubes would have time-like momenta whereas virtual particle analogs would have space-like momenta. The interpretation makes sense only if the associated momenta are between space-like and time-like that is light-like so that flux tube would correspond to mass shell particle. But this is the case in twistor approach to gauge theories also in TGD [L48] (see `http://tinyurl.com/y62no62a`).

   Perhaps the following interpretation is more appropriate. Flux tubes are accompanied by strings and string world sheets can be interpreted as stringy description of gravitation and other interactions.

Kähler calibrations: an idea before its time?

While updating book introductions I was surprised to find that I had talked about so called calibrations of sub-manifolds as something potentially important for TGD and later forgotten the whole idea! A closer examination however demonstrated that I had ended up with the analog of this notion completely independently later as the idea that preferred extremals are minimal surfaces apart form 2-D singular surfaces, where there would be exchange of Noether charges between Kähler and volume degrees of freedom.

1. The original idea that I forgot too soon was that the notion of calibration (see `http://tinyurl.com/y3lyead3`) generalizes and could be relevant for TGD. A calibration in Riemann manifold M means the existence of a k-form ϕ in M such that for any orientable k-D sub-manifold the integral of ϕ over M equals to its k-volume in the induced metric. One can say that metric k-volume reduces to homological k-volume.

   Calibrated k-manifolds are minimal surfaces in their homology class, in other words their volume is minimal. Kähler calibration is induced by the k^{th} power of Kähler form and defines calibrated sub-manifold of real dimension $2k$. Calibrated sub-manifolds are in this case precisely the complex sub-manifolds. In the case of CP_2 they would be complex curves (2-surfaces) as has become clear.

2. By the Minkowskian signature of M^4 metric, the generalization of calibrated sub-manifold so that it would apply in $M^4 \times CP_2$ is non-trivial. Twistor lift of TGD however forces to introduce the generalization of Kähler form in M^4 (responsible for CP breaking and matter antimatter asymmetry) and calibrated manifolds in this case would be naturally analogs of string world sheets and partonic 2-surfaces as minimal surfaces. Cosmic strings are Cartesian products of string world sheets and complex curves of CP_2. Calibrated manifolds, which do not reduce to Cartesian products of string world sheets and complex surfaces of CP_2 should also exist and are minimal surfaces.

   One can also have 2-D calibrated surfaces and they could correspond to string world sheets and partonic 2-surfaces which also play key role in TGD. Even discrete points assignable to partonic 2-surfaces and representing fundamental fermions play a key role and would trivially correspond to calibrated surfaces.

3. Much later I ended up with the identification of preferred extremals as minimal surfaces by totally different route without realizing the possible connection with the generalized calibrations. Twistor lift and the notion of quantum criticality led to the proposal that preferred

extremals for the twistor lift of Kähler action containing also volume term are minimal surfaces. Preferred extremals would be separately minimal surfaces and extrema of Kähler action and generalization of complex structure to what I called Hamilton-Jacobi structure would be an essential element. Quantum criticality outside singular surfaces would be realized as decoupling of the two parts of the action. May be all preferred extremals be regarded as calibrated in generalized sense.

If so, the dynamics of preferred extremals would define a homology theory in the sense that each homology class would contain single preferred extremal. TGD would define a generalized topological quantum field theory with conserved Noether charges (in particular rest energy) serving as generalized topological invariants having extremum in the set of topologically equivalent 3-surfaces.

It is interesting to recall that the original proposal for the preferred extremals as absolute minima of Kähler action has transformed during years to a proposal that they are absolute minima of volume action within given homology class and having fixed ends at the boundaries of CD.

4. The experience with CP_2 would suggest that the Kähler structure of M^4 defining the counterpart of form ϕ is unique. There is however infinite number of different closed self-dual Kähler forms of M^4 defining what I have called Hamilton-Jacobi structures. These forms can have subgroups of Poincare group as symmetries. For instance, magnetic flux tubes correspond to given cylindrically symmetry Kähler form. The problem disappears as one realizes that Kähler structures characterize families of preferred extremals rather than M^4 itself.

If the notion of calibration indeed generalizes, one ends up with the same outcome - preferred extremals as minimal surfaces with 2-D string world sheets and partonic 2-surfaces as singularities - from many different directions.

1. Quantum criticality requires that dynamics does not depend on coupling parameters so that extremals must be separately extremals of both volume term and Kähler action and therefore minimal surfaces for which these degrees of freedom decouple except at singular 2-surfaces, where the necessary transfer of Noether charges between two degrees of freedom takes place at these. One ends up with string picture but strings alone are of course not enough. For instance, the dynamical string tension is determined by the dynamics for the twistor lift.

2. Almost topological QFT picture implies the same outcome: topological QFT property fails only at the string world sheets.

3. Discrete coupling constant evolution, vanishing of loop corrections, and number theoretical condition that scattering amplitudes make sense also in p-adic number fields, requires a representation of scattering amplitudes as sum over resonances realized in terms of string world sheets.

4. In the standard QFT picture about scattering incoming states are solutions of free massless field equations and interaction regions the fields have currents as sources. This picture is realized by the twistor lift of TGD in which the volume action corresponds to geodesic length and Kähler action to Maxwell action and coupling corresponds to a transfer of Noether charges between volume and Kähler degrees of freedom. Massless modes are represented by minimal surfaces arriving inside causal diamond (CD) and minimal surface property fails in the scattering region consisting of string world sheets.

5. Twistor lift forces M^4 to have generalize Kähler form and this in turn strongly suggests a generalization of the notion of calibration. At physics side the implication is the understanding of CP breaking and matter anti-matter asymmetry.

6. $M^8 - H$ duality requires that the dynamics of space-time surfaces in H is equivalent with the algebraic dynamics in M^8. The effective reduction to almost topological dynamics implied by the minimal surface property implies this. String world sheets (partonic 2-surfaces) in H would be images of complex (co-complex sub-manifolds) of $X^4 \subset M^8$ in H. This should allows to understand why the partial derivatives of embedding space coordinates can be

discontinuous at these edges/folds but there is no flow between interior and singular surface implying that string world sheets are minimal surfaces (so that one has conformal invariance).

The analogy with foams in 3-D space deserves to be noticed.

1. Foams can be modelled as 2-D minimal surfaces with edges meeting at vertices. TGD space-time could be seen as a dynamically generated foam in 4-D many-sheeted space-time consisting of 2-D minimal surfaces such that also the 4-D complement is a minimal surface. The counterparts for vertices would be light-like curves at light like orbits of partonic 2-surfaces from which several string world sheets can emanate.

2. Can one imagine something more analogous to the usual 3-D foam? Could the light-like orbits of partonic 2-surfaces define an analog of ordinary foam? Could also partonic 2-surfaces have edges consisting of 2-D minimal surfaces joined along edges representing strings connecting fermions inside partonic 2-surface?

   For years ago I proposed what I called as symplectic QFT (SQFT) as an analog of conformal QFT and as part of quantum TGD [K15]. SQFT would have symplectic transformations as symmetries, and provide a description for the symplectic dynamics of partonic 2-surfaces. SQFT involves an analog of triangulation at partonic 2-surfaces and Kähler magnetic fluxes associated with them serve as observables. The problem was how to fix this kind of network. Partonic foam could serve as a concrete physical realization for the symplectic network and have fundamental fermions at vertices. The edges at partonic 2-surfaces would be space-like geodesics. The outcome would be a calibration involving objects of all dimensions $0 \leq D \leq 4$ - a physical analog of homology theory.

6.7.3 Twistors in TGD and connection with Veneziano duality

The twistorialization of TGD has two aspects. The attempt to generalize twistor Grassmannian approach emerged first. It was however followed by the realization that also the twistor lift of TGD at classical space-time level is needed. It turned out that the progress in the understanding of the classical twistor lift has been much faster - probably this is due to my rather limited technical QFT skills.

Twistor lift at space-time level

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [L3]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A18]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2. The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure coincides with the twistor structure defined by the induced metric.

In fact, it is enough to generalize the induction of spinor structure to that of twistor structure so that the induced twistor structure need not be identical with the ordinary twistor structure possibly assignable to the space-time surface. The induction procedure reduces to a dimensional reduction of 6-D Kähler action giving rise to 6-D surfaces having bundle structure with twistor sphere as fiber and space-time as base. The twistor sphere of this bundle is imbedded as sphere in the product of twistor spheres of twistor spaces of M^4 and CP_2.

This condition would define the dynamics, and the original conjecture was that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

It however turned out that twistor lift of TGD is much more than a mere technical tool. First of all, the dimensionally reduction of 6-D Kähler action contained besides 4-D Kähler action also a volume term having interpretation in terms of cosmological constant. This need not bring anything new, since all known extremals of Kähler action with non-vanishing induced Kähler form are minimal surfaces. There is however a large number of embeddings of twistor sphere of space-time surface to the product of twistor spheres. Cosmological constant has spectrum and depends on length scale, and the proposal is that coupling constant evolution reduces to that for cosmological constant playing the role of cutoff length. That cosmological constant could transform from a mere nuisance to a key element of fundamental physics was something totally new and unexpected.

1. The twistor lift of TGD at space-time level forces to replace 4-D Kähler action with 6-D dimensionally reduced Kähler action for 6-D surface in the 12-D Cartesian product of 6-D twistor spaces of M^4 and CP_2. The 6-D surface has bundle structure with twistor sphere as fiber and space-time surface as base.

   Twistor structure is obtained by inducing the twistor structure of 12-D twistor space using dimensional reduction. The dimensionally reduced 6-D Kähler action is sum of 4-D Kähler action and volume term having interpretation in terms of a dynamical cosmological constant depending on the size scale of space-time surface (or of causal diamond CD in zero energy ontology (ZEO)) and determined by the representation of twistor sphere of space-time surface in the Cartesian product of the twistor spheres of M^4 and CP_2.

2. The preferred extremal property as a representation of quantum criticality would naturally correspond to minimal surface property meaning that the space-time surface is separately an extremal of both Kähler action and volume term almost everywhere so that there is no coupling between them. This is the case for all known extremals of Kähler action with non-vanishing induced Kähler form.

   Minimal surface property could however fail at 2-D string world sheets, their boundaries and perhaps also at partonic 2-surfaces. The failure is realized in minimal sense if the 3-surface has 1-D edges/folds (strings) and 4-surface 2-D edges/folds (string world sheets) at which some partial derivatives of the embedding space coordinates are discontinuous but canonical momentum densities for the entire action are continuous.

   There would be no flow of canonical momentum between interior and string world sheet and minimal surface equations would be satisfied for the string world sheet, whose 4-D counterpart in twistor bundle is determined by the analog of 4-D Kähler action. These conditions allow the transfer of canonical momenta between Kähler- and volume degrees of freedom at string world sheets. These no-flow conditions could hold true at least asymptotically (near the boundaries of CD).

   $M^8 - H$ duality suggests that string world sheets (partonic 2-surfaces) correspond to images of complex 2-sub-manifolds of M^8 (having tangent (normal) space which is complex 2-plane of octonionic M^8).

3. Cosmological constant would depend on p-adic length scales and one ends up to a concrete model for the evolution of cosmological constant as a function of p-adic length scale and other number theoretic parameters (such as Planck constant as the order of Galois group): this conforms with the earlier picture.

   Inflation is replaced with its TGD counterpart in which the thickening of cosmic strings to flux tubes leads to a transformation of Kähler magnetic energy to ordinary and dark matter. Since the increase of volume increases volume energy, this leads rapidly to energy minimum at some flux tube thickness. The reduction of cosmological constant by a phase transition however leads to a new expansion phase. These jerks would replace smooth cosmic expansion of GRT. The discrete coupling constant evolution predicted by the number theoretical vision could be understood as being induced by that of cosmological constant taking the role of cutoff parameter in QFT picture [L34].

Twistor lift at the level of scattering amplitudes and connection with Veneziano duality

The classical part of twistor lift of TGD is rather well-understood. Concerning the twistorialization at the level of scattering amplitudes the situation is much more difficult conceptually - I already mentioned my limited QFT skills.

1. From the classical picture described above it is clear that one should construct the 8-D twistorial counterpart of theory involving space-time surfaces, string world sheets and their

boundaries, plus partonic 2-surfaces and that this should lead to concrete expressions for the scattering amplitudes.

The light-like boundaries of string world sheets as carriers of fermion numbers would correspond to twistors as they appear in twistor Grassmann approach and define the analog for the massless sector of string theories. The attempts to understand twistorialization have been restricted to this sector.

2. The beautiful basic prediction would be that particles massless in 8-D sense can be massive in 4-D sense. Also the infrared cutoff problematic in twistor approach emerges naturally and reduces basically to the dynamical cosmological constant provided by classical twistor lift.

   One can assign 4-momentum both to the spinor harmonics of the embedding space representing ground states of super-conformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this could be seen as a concretization of Equivalence Principle. Also a connection with string model emerges.

3. As far as symmetries are considered, the picture looks rather clear. Ordinary twistor Grassmannian approach boils down to the construction of scattering amplitudes in terms of Yangian invariants for conformal group of M^4. Therefore a generalization of super-symplectic symmetries to their Yangian counterpart seems necessary. These symmetries would be gigantic but how to deduce their implications?

4. The notion of positive Grassmannian is central in the twistor approach to the scattering amplitudes in $calN = 4$ SUSYs. TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings.

   Braid interpretation encourages the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

This is however not the only vision about a solution of non-planarity. Quantum criticality provides different view leading to a totally unexpected connection with string models, actually with the Veneziano duality, which was the starting point of dual resonance model in turn leading via dual resonance models to super string models.

1. Quantum criticality in TGD framework means that coupling constant evolution is discrete in the sense that coupling constants are piecewise constant functions of length scale replaced by dynamical cosmological constant. Loop corrections would vanish identically and the recursion formulas for the scattering amplitudes (allowing only planar diagrams) deduced in twistor Grassmann would involve no loop corrections. In particular, cuts would be replaced by sequences of poles mimicking them like sequences of point charge mimic line charges. In momentum discretization this picture follows automatically.

2. This would make sense in finite measurement resolution realized in number theoretical vision by number-theoretic discretization of the space-time surface (cognitive representation) as points with coordinates in the extension of rationals defining the adele [L23]. Similar discretization would take place for momenta. Loops would vanish at the level of discretization but what would happen at the possibly existing continuum limit: does the sequence of poles integrate to cuts? Or is representation as sum of resonances something much deeper?

3. Maybe it is! The basic idea of behind the original Veneziano amplitudes (see `http://tinyurl.com/yyhwvbqb`) was Veneziano duality. This 4-particle amplitude was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see `http://tinyurl.com/yyvkx7as`) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged. Later came superstring models and led to M-theory. Now it has become clear that something went wrong, and it

seems that one must return to the roots. Could the return to the roots mean a careful reconsideration of the dual resonance model?

4. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

   The resonances had zero width, which was not consistent with unitarity. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of twistor Grassmann approach.

5. Veneziano duality is true only for s- and t- channels but not been s- and u-channel. Stringy description makes t-channel and s-channel pictures equivalent. Could it be that in fundamental description u-channels diagrams cannot be distinguished from s-channel diagrams or t-channel diagrams? Could the stringy representation of the scattering diagrams make u-channel twist somehow trivial if handles of string world sheet representing stringy loops in turn representing the analog of non-planarity of Feynman diagrams are absent? The permutation of external momenta for tree diagram in absence of loops in planar representation would be a twist of π in the representation of planar diagram as string world sheet and would not change the topology of the string world sheet and would not involve non-trivial world sheet topology.

   For string world sheets loops would correspond to handles. The presence of handle would give an edge with a loop at the level of 3-surface (self energy correction in QFT). Handles are not allowed if the induced metric for the string world sheet has Minkowskian signature. If the stringy counterparts of loops are absent, also the loops in scattering amplitudes should be absent.

   This argument applies only inside the Minkowskian space-time regions. If string world sheets are present also in Euclidian regions, they might have handles and loop corrections could emerge in this manner. In TGD framework strings (string world sheets) are identified to 1-D edges/folds of 3-surface at which minimal surface property and topological QFT property fails (minimal surfaces as calibrations). Could the interpretation of edge/fold as discontinuity of some partial derivatives exclude loopy edges: perhaps the branching points would be too singular?

A reduction to a sum over s-channel resonances is what the vanishing of loops would suggest. Could the presence of string world sheets make possible the vanishing of continuous cuts even at the continuum limit so that continuum cuts would emerge only in the approximation as the density of resonances is high enough?

The replacement of continuous cut with a sum of *infinitely* narrow resonances is certainly an approximation. Could it be that the stringy representation as a sum of resonances with *finite* width is an essential aspect of quantum physics allowing to get rid of infinities necessarily accompanying loops? Consider now the arguments against this idea.

1. How to get rid of the problems with unitarity caused by the zero width of resonances? Could *finite* resonance widths make unitarity possible? Ordinary twistor Grassmannian approach predicts that the virtual momenta are light-like but complex: obviously, the imaginary part of the energy in rest frame would have interpretation as resonance with.

   In TGD framework this generalizes for 8-D momenta. By quantum-classical correspondence (QCC) the classical Noether charges are equal to the eigenvalues of the fermionic charges in Cartan algebrable (maximal set of mutually commuting observables) and classical TGD indeed predicts complex momenta (Kähler coupling strength is naturally complex). QCC thus supports this proposal.

2. Sum over resonances/exchanges picture is in conflict with QFT picture about scattering of particles. Could *finite* resonance widths due to the complex momenta give rise to the QFT type scattering amplitudes as one develops the amplitudes in Taylor series with respect to the resonance width? Unitarity condition indeed gives the first estimate for the resonance width.

   QFT amplitudes should emerge in an approximation obtained by replacing the discrete set of finite width resonances with a cut as the distance between poles is shorter than the resolution for mass squared.

   In superstring models string tension has single very large value and one cannot obtain QFT type behavior at low energies (for instance, scattering amplitudes in hadronic string model are concentrated in forward direction). TGD however predicts an entire hierarchy of p-adic length scales with varying string tension. The hierarchy of mass scales corresponding roughly to the lengths and thickness of magnetic flux tubes as thickened cosmic strings and characterized by the value of cosmological constant predicted by twistor lift of TGD. Could this give rise to continuous QCT type cuts at the limit when measurement resolution cannot distinguish between resonances?

   The dominating term in the sum over sums of resonances in t-channel gives near forward direction approximately the lowest mass resonance for strings with the smallest string tension. This gives the behavior $1/(t-m_{min}^2)$, where m_{min} corresponds to the longest mass scale involved (the largest space-time sheet involved), approximating the $1/t$-behavior of massless theories. This also brings in IR cutoff, the lack of which is a problem of gauge theories. This should give rise to continuous QFT type cuts at the limit when measurement resolution cannot distinguish between resonances.

Number-theoretic approach to unitarity

Twistorialization leads to the proposal that cuts in the scattering amplitudes are replaced with sums over poles, and that also many-particle states have discrete momentum and mass squared spectrum having interpretation in terms of bound states. Gravitation would be the natural physical reason for the discreteness of the mass spectrum and in string models it indeed emerges as "stringy" mass spectrum. The situation is very similar to that in dual resonance models, which were precedessors of super string theories.

Number theoretical discretization based on the hierarchy of extensions of rationals defining extensions of p-adic number fields gives rise to cognitive representatations as discrete sets of space-time surface and discretization of 4-momenta and S-matrix with discrete momentum labels. In number theoretic discretization cuts reduce automatically to sequences of poles. Whether this discretization is an approximation reflecting finite cognitive resolution or whether finite cognitive representation is a property of physical states reflecting itself as a condition that various parameters characterizing them belong to the extension considered, remains an open question.

One can approach the unitarity conditions also number theoretically. In the discretization forced by the extension of rationals the amplitudes are defined between states having a discrete spectrum of 4-momenta. Unitarity condition reduces to a purely algebraic condition involving only sums. In these conditions the Dirac delta functions associated with the mass squared of the resonances are replaced with Kronecker deltas.

1. For given extension of rationals the unitary conditions are purely algebraic equations

$$i(T_{mn}+\overline{T}_{nm}) = \sum_r T_{mr}\overline{T}_{nr} = T_{mn}\overline{T}_{nn} + T_{mm}\overline{T}_{mn} + \sum_{r\neq m,n} T_{mr}\overline{T}_{nr} \quad .$$

   where T_{mn} belongs the extension. Complex imaginary unit i corresponds to that appearing in the extension of octonions in M^8-H duality [L20].

2. In the forward direction $m=n$ one obtains

$$2Im(T_{mm}) = Re(T_{mm})^2 + Im(T_{mm})^2 + P_m \quad , \quad P_m = \sum_{r\neq m} T_{mr}\overline{T}_{mr} \quad .$$

P_m represents total probability for non-forward scattering.

3. One can think of solving $Im(T_{mm})$ algebraically from this second order polynomial in the lowest order approximation in which $T_{mn} = 0$ for $m \neq n$. This gives

$$2Im(T_{mm}) = 1 + \sqrt{1 - P_m - Re(T_{mm})^2} \ .$$

Reality requires $1 - Re(T_{mm})^2 - P_m \geq 0$ giving

$$Re(T_{mm})^2 + P_m \leq 1 \ .$$

This condition is identically true by unitarity since probability for scattering cannot be larger than 1.

Besides this the real root must belong to the original extension of rationals. For instance, if the extension of rationals is trivial, the quantity $1 - P_m - Re(T_{mm})^2$ must be a square of rational y giving $1 - P_m = y^2 + Re(T_{mm})^2$. In the case of extension y is replaced with a number in the extension. I am not enough of number theorist to guess how powerful this kind of number theoretical conditions might be. In any case, the general ansatz for S is a unitary matrix in extension of rationals and this kind of matrices form a group so that there is no hope about unique solution.

4. One could think of iterative solution of the conditions by assuming in the zeroth order approximation $T_{mn} = 0$ for $m \neq n$ giving $Re(T_{mm})^2 + Im(T_{mm})^2 = 1$ reducing to $cos^2(\theta) + sin^2(\theta) = 1$. For trivial extension of rationals θ would correspond to Pythagorean triangle.

For non-diagonal elements of T_{mn} one would obtain at the next step the conditions

$$i(T_{mn} + \overline{T}_{nm}) = T_{mn}\overline{T}_{nn} + T_{mm}\overline{T}_{nm} \ .$$

This gives a 2 linear equations for T_{mn}.

5. These conditions are not enough to give unique solution. Time reversal invariance gives additional conditions and might help in this respect. T invariance is slightly broken but CPT symmetry could replace T symmetry in the general situation.

Time reversal operator T (to be not confused with T_{mn} above) is anti-unitary operator and one has $S^\dagger = T(S)$. In wave mechanics one can show that T-invariant S-matrix and thus also T-matrix is symmetric: $S = S^T$. The matrices of this kind do not form a group so that the conditions can be very powerful.

Combined with the above equations symmetry gives

$$2Im(T_{mn}) = T_{mn}\overline{T}_{nn} + T_{mm}\overline{T}_{mn} \ .$$

The two conditions for T_{mn} in principle fix it completely in this order.

One obtains from the real part of the equation

$$2Im(T_{mn}) = Re[T_{mn}\overline{T}_{nn} + T_{mm}\overline{T}_{mn}] \ .$$

The vanishing of the imaginary part gives

$$Im[T_{mn}\overline{T}_{nn} + T_{mm}\overline{T}_{mn}] = 0 \ .$$

giving a linear relation between the real and imaginary parts of T_{mn}. No new number theoretical conditions emerge. This relation requires that real and imaginary parts belong to the extension.

6. At higher orders one must feed the resulting ansatz to the unitarity conditions for the diagonal elements T_{nn}. One can hope that the lowest order ansatz leads to rather unique solution by iteration of the unitarity conditions. In higher order conditions the higher order corrections appear linearly so that no new number theoretic conditions emerge at higher orders.

   Physical picture suggests that the S-matrices could be obtained by an iterative procedure. Since infinitely long procedure very probably leads out of the extension, one can ask whether the procedure should stop after finite steps. This property would pose an additional conditions to the S-matrix.

7. Diagonal matrices are solutions to the conditions and for then the diagonal elements are roots of unity in the extension of rationals considered. The automorphisms $S_d \rightarrow US_dU^{-1}$ produce new S-matrices and if the unitary matrix U is orthogonal real matrix in algebraic extension satisfying therefore $UU^T = 1$, the condition $S = S^T$ is satisfied. There are therefore a large number of solutions.

   S-matrices diagonalizable in the extension are not the only solutions. The diagonalization of a unitary matrix $S = S^T$ in general gives a diagonal S-matrix, for which the roots of unity in general do not belong to the extension. Also the diagonalizating matrix fails to be in the extension. This non-diagonalizability might have deep physics content and explain why the physically natural state basis is not the one in which S-matrix is diagonal. In the case of density matrix it would guarantee stability of entanglement.

To sum up, number theoretic conditions could give rise to highly unique discrete S-matrices, when CPT symmetry can be formulated purely algebraically and be combined with unitarity. CPT symmetry might not however allow formulation in terms of automorphisms of diagonal unitary matrices analogous to orthogonal transformations.

6.7.4 Summary

It seems that unitarity of S-matrix reduces to the existence of maximal group of WCW isometries. The conservation of charges implies conservation of probability and unitarity.

Disjoint 3-surfaces and also those topologically condensed at larger space-time sheets would have interpretation as topological representations of particles in this approach. The special role of the partonic orbits suggests holography in the sense that these orbits have particle interpretation. Similar holography would make sense true for string world sheets and their boundaries. Action could therefore contain parts associated with $D = 2$ and $D = 1$ surfaces so that oscillator operators associated with these would be involved in the construction of states.

The transfer of quantum numbers from space-time interior to string world sheets could take place in interaction regions for Option I for which one assigns action to singular surfaces identified as surfaces having complex or real tangent space at M^8 level. The transfer would naturally vanish near the boundaries of CD. Same applies to the transfer from string world sheets to their boundaries. For Option II two the string world sheets would not carry Noether currents and only minimal surface property could fail at these surfaces: therefore this option is not realistic. Also for Option I there could be breaking of minimal surface property in this sense and the discontinuity of normal component for Noether currents would imply it automatically.

When this picture is combined with the twistor Grassmannian inspired view about scattering amplitudes using the constraints coming from quantum criticality, discreteness of the coupling constant evolution, and the existence of amplitudes as rational functions with coefficients in a extension of rationals allowing p-adic variants, one ends up to a picture in which amplitudes reduces to sums over resonances - this was just what was assumed in Veneziano model besides s-t duality.

This picture does not conform with QFT picture in superstring framework, where one has single large string tension so that poles cannot be approximated by cuts for low energies. In TGD framework this can be the case since string tension has spectrum reducing to that for cosmological constant. Since momenta are already classically predicted to be complex, resonance poles have finite width and one can in principle understand also unitarity. Therefore twistorialization in TGD framework leads to string models, and strings are indeed an essential part of twistorialization in TGD framework.

6.8 Scattering amplitudes and orbits of cognitive representations under subgroup of symplectic group respecting the extension of rationals

Number theorist Minhyong Kim has speculated about very interesting general connection between number theory and physics [A21, A24] (see http://tinyurl.com/y86bckmo). The reading of a popular article about Kim's work revealed that number theoretic vision about physics provided by TGD has led to a very similar ideas and suggests a concrete realization of Kim's ideas [L45]. The identification of points of algebraic surface with coordinates, which are rational or in extension of rationals, gives rise to what one can call identification problem. In TGD framework the embedding space coordinates for points of space-time surface belonging to the extension of rationals defining the adelic physics in question are common to reals and all extensions of p-adics induced by the extension. These points define what I call cognitive representation, whose construction means solving of the identification problem.

Cognitive representation defines discretized coordinates for a point of "world of classical worlds" (WCW) taking the role of the space of spaces in Kim's approach. The symmetries of this space are proposed by Kim to help to solve the identification problem. The maximal isometries of WCW necessary for the existence of its Kähler geometry provide symmetries identifiable as symplectic symmetries. The discrete subgroup respecting extension of rationals acts as symmetries of cognitive representations of space-time surfaces in WCW, and one can identify symplectic invariants characterizing the space-time surfaces at the orbits of the symplectic group.

This picture could be applied to the construction of scattering amplitudes with finite cognitive precision in terms of cognitive representations and their orbits under subgroup S_D of symplectic group respecting the extension of rationals defining the adele. One could pose to S_D the additional condition that it leaves the value of action invariant: call this group $S_{D,S}$: this would define what I have called micro-canonical ensemble (MCE).

The obvious question is whether the simplest zero energy states could correspond to single orbit of S_D or whether several orbits are required. For the more complex option zero energy states would be superposition of states corresponding to several orbits of S_D with coefficients constructed of symplectic invariants. The following arguments lead to the conclusion that MCE and single orbit orbit option are non-realistic, and raise the question whether the orbits of S_D could combine to an orbit of its Yangian analog. A generalization of the formula for scattering amplitudes in terms of n-point functions emerges and somewhat surprisingly one finds that the unitarity is an automatic consequence of state orthonormalization in zero energy ontology (ZEO).

6.8.1 Zero energy states

The degrees of freedom at WCW level can be divided to zero modes, which do not contribute to WCW metric and correspond to symplectic invariants and to dynamical degrees of freedom which correspond to the orbits of symplectic group of $\delta M^4_\pm \times CP_2$. The assumption is that symplectic group indeed acts as isometries. The general proposal for the state construction in continuum case should have a discrete analog. There are good reasons to hope that the zero energy states in the degrees of freedom corresponding to the orbits of the discrete variant S_D of the symplectic grop are analogous to spherical harmonics and are dictated completely by symmetry considerations.

Quantum superposition of space-time surfaces - preferred extremals - defines zero energy state. The natural question is whether zero energy state could correspond to single orbit of S_D or whether several of them are needed.

1. Preferred extremal is fixed more or less uniquely by its ends, which are 3-surfaces at the opposite light-like boundaries of CD. The interpretation is in terms of holography forced also by general coordinate invariance requiring that one must be able to assign to a given 3-surface a unique space-time surface at which general coordinate transformations act. In ZEO 3-surface means union of 3-surface at opposite ends of CD.

   The idea about preferred extremals as analogs of Bohr orbits suggests that the 3-surface at the either end determines the 3-surface at the opposite end highly uniquely. The proposal that preferred extremals are minimal surfaces apart from singular 2-surfaces identifiable as string world sheet, means that they are separately extremals of both Kähler action and volume term supports this expectation as also the condition that sub-algebra of symplectic group Lie algebra isomorphic to it gives rise to vanishing Noether charges and also the Noether charges associated with its commutator with the full algebra vanish.

The condition that the zero energy state at the active boundary of CD is superposition of many-particle states with different particle number in topological sense suggests that this is not the case.

Even stronger form of holography would be that the data at string world sheets and partonic 2-surfaces determines the preferred extremal completely. In number theoretic vision one can consider even stronger number theoretic holography: if octonionic polynomials code for the space-time surfaces as M^8-H holography suggests [L20], cognitive representation consisting of discrete set of points with M^8 coordinates in extension of rationals would determine the preferred extremals.

2. Also fermionic degrees of freedom at the ends are involved. Quantum classical correspondence (QCC) states that the classical charges in Cartan sub-algebra of symmetries are equal to the eigenvalues of quantal charges constructible in terms of fermionic oscillator operator algebra. Many-fermion states would correspond to preferred extremals and the fermionic statistics requires that one has superposition over corresponding 4-surfaces. The state at second end of CD is quantum entangled, and fermionic statistics suggests entanglement at both ends.

Symplectic isometries have subgroup with parameters in the extension of rationals defining the adele: call this subgroup S_D. Denote the subgroup of S_D leaving action invariant by $S_{D,S}$. The representations of S_D (or possibly $S_{D,S}$) are expected to be important concerning the construction of scattering amplitudes and on basic of zero energy state property one expects that the action of S_D ($S_{D,S}$) on the opposite ends of space-time surface compensate each other for zero energy states.

A reasonable looking question is whether simplest zero energy states could corresponds to single orbit of S_D. One expects that the number of points defining the cognitive representation is same for all preferred extremals at its orbit. There are several questions to be answered.

1. The existence of preferred extremals connecting given 3-surface with fixed topological particular number to 3-surface at the second end of CD having varying topological particle number looks rather plausible. Topological particle number can be identified either as number of disjoint 3-surfaces and number of disjoint partonic 2-surfaces carrying fermions.

   Can single orbit of S_D contain space-time surfaces with varying topological particle number at the other end of CD? If not , one must allow some minimal number of orbits of S_D in the definition of minimal zero energy state. This option looks the most realistic one.

2. What is the precise definition of cognitive representation?

3. Micro-canonical ensemble (MCE) hypothesis states that action is same for all space-time surfaces appearing in zero energy state. Can this hypothesis be consistent with the presence of many-particle states with different topological particle number? CP_2 type extremals represent particles and have non-vanishing actions. Also the action of symplectic group in general changes the Kähler action although the action is constant at co-dimension 1 surface of WCW so that the subgroup $S_{D,S}$ should act at this surface. It would seem that one must allow the variation of action and this is a challenge for number theoretic universality since the number theoretically non-universal part of action exponentials must be common to all space-time surfaces involves and must cancel in S-matrix.

What does one mean with cognitive representation? Is single orbit of S_D enough? Can one assume MCE? These are the key questions to be considered.

6.8.2 The action of symplectic isometries on cognitive representations

The action of S_D on cognitive representation defining the adele is straightforward. It is not however quite clear how to identify the cognitive representation.

1. Cognitive representation in question corresponds to a set of points of space-time surface with M^8 coordinates in extension of rationals defining the adele (a stronger condition is that also $M^4 \times CP_2$ coordinates satisfy the same condition).

2. Does cognitive representation contain only the points at the ends of CD, either end, or also interior points? Or does cognitive representation consists of singular points at which non-trivial subgroup of Galois group leaves the point invariant? The singular points could correspond to fundamental fermions at partonic 2-surfaces.

   Remark: If the fermionic lines are light-like geodesic they would correspond as cognitive representations exceptionally informative and easy ones containing infinite number of points of extensions essentially the number line defined by the extension. This raises the question whether the simplest string world sheets identifiable as planes M^2 could be the most interesting singularities of preferred extremals identified as singular minimal surfaces. Canonical embedding of M^4 is also cognitively easy.

   The condition that the actions of symplectic group at opposite boundaries of CD compensate each other makes sense only if one restricts the cognitive representations at either boundary of CD. This would exclude interior points.

   Could one allow also points in the interior of space-time surface by generalizing the view about symplectic invariance of zero energy state? For instance, could the partonic 2-surface defining vertices in the interior contain points of the cognitive representation. Does the allowance of the points of cognitive representation in interior mean giving up strict determinism and does the variational principle with volume term allow it (mere 4-D Kähler action allows huge vacuum degeneracy).

3. When does the point of cognitive representation correspond to a fundamental fermion? I have proposed [L20] that this is the case if the point is critical in number theoretical sense meaning that there is subgroup of Galois group leaving it invariant: the sheets corresponding to different elements of Galois sub-group would coincide at critical point. The number of singular points and thus number of fundamental fermions might vary.

4. Could the number of singular points vary for the 4-surfaces at the orbit so that the number of fundamental fermions would vary too? Could this allow to have superposition of many-particle states as active part of the zero energy state? This does not seem plausible since the number of points of cognitive representations must be S_D invariant. Several orbits of S_D seem to be required.

The role of Galois group of extension of rationals must be important.

1. Galois group act do not affect space-time surface but only inside the cognitive representation. Galois group can also have subgroup leaving invariant given point. A possible interpretation is as number theoretic correlate for fundamental fermion.

2. A natural hypothesis is that the sub-group of symplectic group leaving the cognitive representation invariant acts as Galois group. A goo analogous for Galois group is provide by the rotation group $SO(3)$ serving as isotropy group of time-like 4-momentum having vanishing 3-momentum in the rest system. For induced representations $SO(3)$ acts in spin degrees of freedom. In the recent case Galois group could act in number theoretic spin degrees of freedom. Could the action of Galois group be physically non-trivial. For instance, could the ordinary symmetries be represented as Galois transformations in fermionic degrees of freedom?

Symplectic invariants characterize the representation and Kähler fluxes for M^4 and CP_2 Kähler forms define this kind of invariants. Also higher fluxes are possible. The general state as superposition of states associated with the over orbits of S_D would have functions of these invariants as coefficients.

6.8.3 Zero energy states and generalization of micro-canonical ensemble

The space-time surfaces in micro-canonical ensemble (MCE) [L33] would have same action so that Kähler function would be constant. It is interesting to discuss this hypothesis in light of the idea that simplest zero energy state corresponds to a finite set of orbits of $S_{D,S}$.

Is micro-canonical ensemble consistent with zero energy state- S_D orbit correspondence?

The assumption that action is constant at the orbit is not problematic. Kähler function must vary in order to give rise to non-trivial Kähler metric. Kähler function is however constant at co-dimension 1 surfaces of WCW. For instance, the Kähler function of CP_2 is function of the radial coordinate invariant under subgroups invariant under $U(2)$ but not under $SU(3)$.

1. The simplest variant of MCE is that single space-time surface is involved. The action of $S_{D,S}$ would be essentially trivial - zero momentum would be more familiar Minkowski analogy. One would get rid of the action exponentials: this would solve the problems related to number theoretical universality caused by the fact that the exponential need not exist in various p-adic number fields.

2. A more realistic hypothesis is that $S_{D,S}$ has several 4-surfaces at its orbit. If the number of surfaces is N the sum of action exponentials is N-fold and the exponential disappears from the S-matrix elements in analogy with what happens in the full theory without discretization by cancellation of the exponential strong suggested by what happens in QFTs.

MCE has however problems.

1. It is not at all clear whether one can make restriction to a subgroup preserving the action. To gain some perspective, not that in the case of CP_2 this would mean restriction to $r = constant$ surface of CP_2 and this is not possible. In the case of rotation group this would mean restriction to sphere.

   Physically it is also obvious that one should allow in the zero energy state all 4-surfaces which are allowed by the conditions posed by preferred extremal property and there seems no good reason to prevent final states with varying particle topological particle number.

2. Also the standard view about S-matrix suggests at active boundary of CD a superposition of final states with different topological particle numbers having different number disjoint 3-surfaces or same number of disjoint 3-surfaces but varying number of partonic 2-surfaces. That the action of S_D changes the number of the disjoint 3-surfaces is in conflict with naïve intuitions but one must remember that number theoretic discretization loses information about connectedness.

3. If the zero energy state has at the active boundary 3-surfaces with a varying topological particle number identified as a number of CP_2 type extremals with unique maximal action, one expects that action exponential is not constant along the orbit of S_D. If the subgroup of S_D, call it $S_{D,S}$, preserves the value of the action, one must allow orbits of S_D with varying value of action. This would give superposition MCEs. Action preserving subgroup would be analogous to the little group of Poincare group preserving the momentum of particle. As notice, also several orbits of S_D must be allowed.

The conclusions seems to be that MCE is physically non-realistic.

Can one generalize micro-canonical ensemble to single orbit of S_D?

Suppose that the orbit of S_D contains many-particle states having in final state varying particle numbers measured as number disjoint 3-surfaces or partonic 2-surfaces. Is there any hope of understanding these many-particle states in terms of single representation of S_D?

1. The orbit of S_D must have 4-surfaces with varying value of action. This is possible if the action exponentials differ by a multiplicative rational number so that the number theoretically problematic part cancels out from the S-matrix since it appears in both denominator and numerator of the expression defining S-matrix element.

2. That cognitive representations at the orbit would have same number of points at all points of orbits is intuitively in conflict with varying topological particle number. If Galois group has a subgroup of order $m > 1$ acting trivially on points representing fundamental fermions, the number of points in the representation is effectively reduced since m points are replaced by 1 point. This could allows to have a varying particle numbers identified as the number of points of cognitive representation.

   If CP_2 type extremals in the final state serve as correlates for particles, one should understand how their addition is possible. Their addition to the state would require that some non-degenerate points of representation become degenerate. If the number N points is large, it is quite possible to have rather large number of fundamental fermions in the final state. The degeneration of these points would give rise to fermions. There is however an upper bound which also comes from infrared cutoff for energy.

3. It is not clear whether S_D can transform to each other points with different value of m. The problem is that idea that S_D maps some points to single point is in conflict with the idea that S_D action is bijective. It seems that this idea simply fails.

The conclusion seems to be that one must allow several orbits on basis of purely classical picture and QCC suggesting the possibility of finals states with varying topological particles number.

Could ZEO allow to understand the possibility of particle creation and annihilation?

The idea about quantum superposition of states with varying particle number in topological sense is natural if one believes in QFT based intuition. Just for fun one can ask whether ZEO could provide a loophole.

In ZEO "self" corresponds to a sequence of unitary time evolutions changing the state at active boundary. The active boundary itself becomes de-localized. "Small" state function reduction induces localization of the active boundary. This means measurement of clock time as temporal distance between CDs. The time increment ΔT between subsequent values of clock time varies, and one expects that particle number changes in each unitary evolution. The big state function reduction occurs at some time T, the lifetime of self, and one can assume that the value of T varies statistically.

Could one think that the particle number in topological is actually well-defined after each small reduction? The ensemble of detected particle reactions providing the data allowing to deduce the cross sections. Could the variation of intervals ΔT and the variation for the duration T gives rise to a variation of detected particle numbers in the final state. If this is the case the unitary time evolutions and "small" state function reductions would be very "classical". If so ZEO would simplify dramatically the structure of S-matrix.

To make this mechanism more detailed, one can add the existing wisdom about CP_2 type extremals as building bricks of particles.

1. The action is expected to depend on particle number and different numbers of CP_2 type extremals assignable to which fundamental fermions are assigned correspond to different values of actions. This is not a problem now since would not have have superposition over states with different number of CP_2 type extremals and even micro-canonical ensemble could make sense.

2. The addition of particle to the final state during the unitary evolution taking the active boundary farther away from the passive boundary would correspond to a creation of CP_2 type extremal. Simplest mechanism is 3-vertex defined by partonic 2-surface at which CP_2 type extremal replicates. The outgoing lines in the analogs of twistor diagrams would be unstable against replication. Replication is suggested to be universal process in TGD and the

replication of magnetic body (MB) would induce DNA replication in TGD inspired quantum biology.

3. A possible interpretation would be in terms of quantum criticality. CP_2 type extremals would be unstable against decay. One could also interpret the analog of twistor diagram as a sequence of algebraic operations.

In this framework the scattering rates would be determined by a hierarchy of S-matrices labelled by different values of total durations $T_n \sum_{k=1}^{n} \Delta T_k$ for a sequence of unitary evolution followed by time localization. In standard picture they would correspond to single infinitely long time evolution. It would not be surprising if this difference could exclude the proposal as unrealistic.

Could one regard zero energy state involving several orbits of S_D as an orbit of Yangian analog of S_D?

QCC suggest strongly that one must allow zero energy states, which correspond to several orbits of S_D. An interesting possibility is that these orbits could be integrated to a representation of a larger group. What suggests itself is the possibly existing Yangian variant of S_D in which the group action is not local anymore even at the level of WCW. The Yangian of projective transformations of M^4 indeed appears in twistor Grassmannian approach and gives rise to huge symmetries behind the success of twistor Grassmannian approach. I have proposed that super-symplectic variant of Grassmannian indeed exists [L3, L24, L12, L35].

6.8.4 How to construct scattering amplitudes?

Lubos Motl (see `http://tinyurl.com/y5lndpn3`) told about two new hep-th papers, by Pate, Raclariu, and Strominger (see `http://tinyurl.com/yxqx237b`) and by Nandan, Schreiber, Volovich, Zlotnikov (see `http://tinyurl.com/y642yspf`) related to a new approach to scattering amplitudes based on the replacement of the quantum numbers associated with Poincare group labelling particles appearing in the scattering amplitudes with quantum numbers associated with the representations of Lorentz group.

Why I got interested was that in zero energy ontology (ZEO) the key object is causal diamond (CD) defined as intersection of future and past directed M^4 light-cones with points replaced with CP_2. Space-time surfaces are inside CD and have ends at its light-like boundaries. The Lorentz symmetries associated with ithe boundaries of CD could be more natural than Poincare symmetry, which would emerge in the integration over the positions of CDs of external particles arriving to the opposite light-like boundaries of the big CD defining the scattering region where preferred extremal describing the scattering event resides.

I did my best to understand the articles and - of course relate these ideas to TGD, where the construction of scattering amplitudes is the basic challenge. My technical skills are too limited for to meet this challenge at the level of explicit formulas but I can try to understand the physics and mathematics brought in by TGD.

While playing with more or less crazy and short-lived ideas inspired by the reading of the articles I finally realized that there is perhaps no point in starting from quantum field theories. TGD is not quantum field theory and I must start from TGD itself.

In TGD framework the picture inspired by adelic physics [L22, L23] is roughly following.

1. Cognitive representations realizing number theoretic universality of adelic physics consist of points of embedding space with coordinates in the extension of rationals. The number of points is typically finite. Cognitive representation should contain as subset the points associated with n-point functions, which are essentially correlation functions.

   Fundamental fermions are building bricks of elementary particles, and a good guess is that fundamental fermions correspond to singular points for which the action of subgroup of Galois group of extension is trivial so that several points collapse together.

2. One must sum over the orbits of a subgroup S_D of symplectic group of light-cone boundary acting as isometries of both boundaries of CD. S_D consists of isometries with parameters in the extension of rationals defining the adele. All orbits needed to represent the pairs of

initial and final 3-surfaces at the boundaries of CD allowed by the action principle must be realized so that single orbit very probably is not enough.

3. Correlations code for the quantum dynamics. In quantum field theories quantum fluctuations of fields at distinct points of space-time correlate and give rise to n-point functions expressible in terms of propagators and vertices: massless fields and conformal fields define the basic example. Operator algebra or path integral describes them mathematically.

   In TGD correlations between embedding space points belonging to the space-time surface result from classical deterministic dynamics: the points of 3-surface at opposite boundaries of CD are not independent.

   This dynamics is non-linear geometric analog for the dynamics of massless fields: space-time sheets as preferred extremals are indeed minimal surfaces with string world sheets appearing as singularities. Minimal surface property is forced by the volume action implied by the twistor lift and having interpretation in terms of cosmological constant. The correlation between points at the same boundary of CD are expected to be independent since these 3-surfaces chosen rather freely as analogs of boundary values for fields.

   Fermionic dynamics governed by modified Dirac action is dictated completely by super-symplectic and super-conformal symmetries. Second quantization of fermions at space-time level is necessary to realized WCW spinor structure: WCW gamma matrices are linear combinations of fermionic oscillator operators.

4. This suggests that the attempts to guess the conformal field theory producing the correlation functions makes things much more complex than they actually are. It should be possible to understand how these correlations emerge from the classical dynamics of space-time surfaces.

As the first brave guess one could try to calculate directly the correlations of spinor harmonics of embedding space assigned with these points.

1. Sum over the symplectic orbits of cognitive representations must be involved as also vacuum expectation values in the fermionic sector for fermionic fields which must appear in vertices for external particles. At the level of cognitive representations anti-commutators for oscillator operators involve Kronecker deltas so that one has discretized variant of second quantization.

2. This could be achieved by expanding the restriction $\Psi^A_{|X^3}$ of the embedding space harmonic Ψ^A restricted to 3-surface at end of space-time surface as sum of modes Ψ_n of the induced spinor field. This would be counterpart for the induction procedure. One can assign to singular points bilinear of type $\overline{\Psi}^A_{|X^3} D^{\leftrightarrow}\Psi$, where Ψ is second quantized induced spinor field expressible as sum over its modes multiplied by oscillator operators. D is modified Dirac operator. This gives as vacuum expectations propagators connecting fermions vertices at the opposite ends of space-time surface.

3. A more concrete picture must rely on a concrete model for elementary particles. Elementary particles have as building bricks pair of wormhole contacts with fermion lines at the light-like orbits of the throats at which the signature of the metric changes from Minkowskian to Euclidian. Particle is necessarily a pair of two wormhole contacts and flux tube connects them at both space-time sheets and forms a closed flux tube carrying monopole flux.

   All particles consist of fundamental fermions and anti-fermions: for instance gauge bosons involve fermion and anti-fermion responsible for the quantum numbers at the opposite throats of second wormhole contact. Second wormhole contact involves neutrino pair neutralizing electroweak isospin in scales longer than the size of the flux tube structure.

4. The topological counterpart of 3-vertex appearing in Feynman diagram corresponds to a replication of this kind of 3-surface highly analogous to bio-replication. In replication vertex, there is no singularity of 3-surface analogous to that appearing in the vertices of stringy diagrams but space-time surface is singular just like 1-D manifold is singular for at vertex of Feynman diagram.

These singular replicating 3-surfaces and the partonic 2-surfaces give rise to the counterparts of interaction vertices. Fermionic 4-vertex is impossible and fermion lines can only be reshared between outgoing partonic orbits. This is however not enough as will be found. It will be found that also the creation of fermion pair as effective turning of fermion lines entering along "upper" wormhole throat and turning back at Euclidian wormhole throat and continuing along the orbit of "lower" wormhole throat must be possible.

To see how this conclusion emerges consider the following problem. One should obtain also emission of bosons identified as fermion pairs from fermion line. One has incoming fermion and outgoing fermion and fermion pair describing boson which represents gauge boson or graviton with vanishing B and L. Fermionic 4-vertex is not allowed since this would bring in divergences.

1. The appearance of a sub-CD assignable to the partonic 2-surface is possible but does not solve the problem considered. There would be incoming fermion line at lower boundary and 1 fermion line and fermion and anti-fermion line associated with the boson at the "upper" boundary. There would be non-locality in the scale of the partonic 2-surface and sub-CD meaning that the lines can end to vacuum. Now one would encounter the same difficulty but only in shorter scale.

2. Could one say that fermion line turns backwards in time? A line turning back could be described as an annihilation of fermion pair to vacuum carrying classical gauge field, which is standard process. In QFT picture this would be achieved if a bilinear $\overline{\Psi}D\Psi$ is allowed in the vertex where annihilation takes place. Not in TGD: fermionic action vanishes identically by field equations expressing essentially the conservation of fermion current and various super currents obtained as contractions fermion field with modes.

   Could fermion-anti-fermion pair creation occur at singular points associated with partonic surfaces representing the turning of fermion line backwards in time. This looks still too singular.

   Rather, the turning backwards in time should mean that a fermion line arriving from future along the orbit of "upper" throat (say) goes through Euclidian wormhole throat and continues along the orbit of "lower" throat back to future than making discontinuous turn-around. Euclidian regions of space-time surface representing one key distinction between GRT and TGD would thus be absolutely essential for the generalized scattering diagrams. An exchange of momentum with classical field would be Feynman diagrammatic manner to say this.

   New oscillator operator pairs emerge at the partonic vertices and would correspond to the above described turn-around for fermion line at wormhole contact. Fermion pairs present at the "lower" boundary of CD could also disappear.

3. The anti-commutation relations fermions are modified due to the presence of vacuum gauge fields so that the anti-commutator of fermionic creation operators $a_m^\dagger$ and anti-fermionic creation operators $b_n^\dagger$ is non-vanishing. A proper formulation of the fermionic anti-commutation relations at the ends of space-time surface is needed and in discretization provided by cognitive representation this should be relatively straightforward.

   One can imagine that although standard anti-commutation relations at the lower end of space-time surface hold true, the time evolution of Ψ in the presence of vacuum gauge potentials implies that the vacuum expectations $\langle vac|a_m^\dagger b_n^\dagger|vac\rangle$ are non-vanishing. This would require that for instance $b_n^\dagger$ and a_n are mixed.

There are still questions to be answered.

1. Is the first guess enough? It is not as becomes clear after a thought about the continuum limit. The WCW degrees of freedom are described at continuum limit in terms of super-symplectic algebra (SSA) acting on ground state are neglected. Embedding space spinor modes characterizee only the ground staes of these representations. These degrees of freedom are essential already in elementary particle physics [K42].

   Sub-algebra SSA_m of SSA with conformal weights coming as m-multiples of those of SSA and its commutator with SSA annihilate the physical states, and one obtains a hierarchy. How to describe these states in the discretization? The natural possibility are the representations of S_D such that $(S_D)_m$ and the subgroup generated by the commutator algebra are represented trivially. One has non-trivial $(S_D)_m$ representations at both ends of WCW such that the action of S_D on the tensor product acts trivially.

There are also fermionic degrees of freedom. The challenge is to identify among other things WCW gamma matrices as fermionic super charges and it would be nice if all charges were Noether charges. The simplest guess is that the algebra generated by fermionic Noether charges Q^A for symplectic transformations $h^k \to h^k + j^{Ak}$ assumed to induce isometries of WCW and Noether supercharges Q_n and their conjugates for the shifts $\Psi \to \Psi + \epsilon u_n$, where u_n is a solution of the modified Dirac equation, is enough.

The commutators $\Gamma_n^A = [Q^A, Q_n]$ are super-charges labelled by (A, n). One would like to identify them as gamma matrices of WCW. The problem is that they are labelled by (A, n) whereas isometry generators are labelled by A only. There should be one-one correspondence. Do all supercharges Γ_n^A except Γ_0^A corresponding to $u_0 = constant$ annihilate the physical states so that one would have 1-1 correspondence. This would be analogous to what happens quite generally in super-conformal algebras.

The generators of this fermionic algebra could be used to generate more general states. One should also construct the discretized versions of the generators as sums over points of the cognitive representation at the ends of space-time surface. Note that this requires tangent space data.

2. What about the conservation of four-momentum and other conservation laws? This can be handled by quantum classical correspondence (QCC). The momentum and color labels defined by fermionic quantum numbers in Cartan algebra can be assumed to be equal to the corresponding classical Noether charges for particle-like space-time surfaces entering to CD. The technical problem is that if one knows only the discretization - even with tangent space data - one does not know the values of these charges! It might be that $M^8 - H$ correspondence in which M^8 side fixes space-time surfaces as roots for real or imaginary parts of octonionic polynomials from the data at discrete set of points is needed.

3. ZEO means deviations from ordinary description. S_D invariance of zero energy state forces sum over the 4-surfaces of the orbit with identical coefficients. Symplectic invariance implies time-like entanglement. One can describe this in terms of hermitian square root Ψ of density matrix satisfying $\Psi^\dagger \Psi = \rho$. The coefficients of different orbits need not be same and allows description in terms of dynamical density matrix. If there is Yangian symmetry also this entanglement is analogous to the entanglement due to statistics.

   Surprisingly - and somewhat disappointingly after decades of attempts to understand unitarity in TGD - unitarity is trivial in ZEO since state basis is defined essentially by the rows of matrices and orthogonality conditions their orthogonality and therefore unitarity. More concretely, for single state at the passive end state function normalization to unity defined by inner product as sum over 3-surfaces at active end would give conservation of probability. Orthogonality of the state basis with inner product as sum over surfaces passive boundary gives orthogonality for the coefficients defining rows of a matrix and therefore unitarity. In the case that single orbit or even several of them defines the states one obtains the same result.

   What then guarantees the orthogonality of zero energy states? In ordinary quantum mechanics the property of being eigenstates of some hermitian operator guarantees orthogonality. In TGD zero energy states would be solutions of the analog of massless Dirac equation in WCW consisting of pairs of 3-surfaces with members at the ends of preferred extremals inside CD. This generalizes Super Virosoro conditions of superconformal theories and would provide the orthonormal state basis.

The outcome would be amazingly simple. There would be no propagators, no vertices, just spinor harmonics of embedding assigned with these $n = n_1 + n_2$ points at the boundaries of CD, and summation over the orbits of the symplectic group. All these mathematical objects would emerge from classical dynamics. The sum over the orbits for chosen spinor harmonics would produce n-point functions, vertices and propagators. It is difficult to imagine anything simpler and quantum classical correspondence would be complete.

6.9 Minimal surfaces: comparison of the perspectives of mathematician and physicist

The popular article "*Math Duo Maps the Infinite Terrain of Minimal Surfaces*" (see `http://tinyurl.com/yyetb7c7`) was an exceptional representative of its species. It did not irritate the reader with non-sense hype but gave very elegant and thought provoking representation of very abstract ideas in mathematics.

6.9.1 Progress in the understanding of closed minimal surfaces

The article tells about the work of mathematicians Fernando Coda Marques and Andre Neves based on a profound and - as they tell - extremely hard-to-understand work of Jon Pitts forgotten by mathematics community. It is comforting that at least in mathematics good work is eventually recognized.

The results of Marques and Neves are about minimal hyper-surfaces imbedded in various spaces with dimension varying between 3 and 7 and clearly extremely general. These spaces have varying topologies and are called "shapes" in the popular article.

Some examples of minimal surfaces

To begin it is good to have some examples about minimal surfaces.

1. For mathematician any lower-dimensional manifold in some embedding space is surface, even 1-D curve! Simplest minimal surfaces are indeed 1-D geodesic lines. In flat 3-space they are straight lines of infinite length but at the surface of sphere they are big circles.

2. Soap films are 2-D minimal surfaces spanned by frames and familiar for every-one. Frame is necessary for having minimal surface, which does not collapse to a point or extend to infinity and possibly self-intersect.

   Why minimal surfaces are not nice closed surfaces of finite size not intersecting themselves is due to the fact that the equations for minimal surface state the vanishing of the sum of external curvatures defined by the trace of so called second fundamental form defined by the covariant derivatives of tangent vectors of the minimal surface.

   One can say that for 2-D minimal surface the external curvatures in 2 orthogonal directions at given point of surface are of opposite sign. Surface looks locally like saddle rather than sphere. In n-dimensional case the sum of n principal curvatures - eigenvalues of second fundamental form as matrix- sum up to zero for each normal direction: more general saddle.

   In flat embedding space this implies the saddle property always but in curved space it might happen that the covariant derivatives replacing the ordinary derivatives in the definition of second fundamental form - having interpretation as generalized acceleration - can change the situation and the question is whether non-flat closed embedding space could contain closed minimal surfaces.

   Indeed, in compact spaces with non-flat metric minimal surface can be closed and there is a century old theorem by Birkhoff stating that sphere has always at least one closed geodesic independent of metric. In the case of ordinary sphere this geodesic is big circle, the equator. In complex projective space CP_2 there is infinite number of 2-D minimal surfaces which are closed: geodesic spheres are the simplest examples.

3. A good example about a non-closed 1-D surface is generic geodesic in torus with points labelled by two angles (ϕ_1, ϕ_2) in flat metric. The geodesic lines are of form $\phi_1 = \alpha\phi_2$). For non-rational value of α the curve winds the torus infinitely many times and has infinite length. For $\alpha = m/n$ the curve winds m times around second non-contractible circles and n times around the second one. Note that now the geodesic line is absolute minimum: this

is caused by the non-contractibility. It can be only shifted in both directions so that the minimum has 2-D degeneracy.

4. In spaces allowing Kähler structure - means that imaginary unit i satisfying $i^2 = -1$ has a representation as antisymmetric tensor - any complex algebraic surfaces representable as root for a set of polynomials, whose number is smaller than complex dimension of the space, is a minimal surface. This huge variety of minimal surfaces is due to the presence of complex structure.

What does minimal surface property mean?

Consider now what minimal surface property really means.

1. Strictly speaking, minimal surfaces are stationary with respect to the *local* variations of volume only. This is practically always true for physical variational principles defined by an action. For a great circle at sphere the minimality of length with respect to small variations is easy to understand by drawing to see what this variation means. With respect to non-local variations meaning shift toward North or East the area decreases so that one has maximum! This leads to the term Minimax principle used by Jon Pitts and his followers as a powerful guideline.

   In fact, minimal surfaces can be both minima and maxima for volume simultaneously. The general extremum as solution of equations defined by a general action principle is saddle. Minimum with respect to some variables and maximum with respect to others and minimal surfaces are this kind of objects in the general case.

2. There is a deep connection with Morse theory in topology (see `http://tinyurl.com/ych4chg9`). Morse function gives information about the topology of space. Morse function is a continuous monotonously increasing function from the space to real line and its extrema provide information about the topology of the space. Morse function can be seen as a kind of height function, a particular coordinate for the space.

   The height as z-coordinate for torus imbedded in 3-space gives a classical example of height function. As z varies on obtains 1-D intersections of torus. The minimum of z corresponds to a single point, above it one has circle, then circle decomposes to 2 circles at lower saddle, and circles fuse back to circle at upper saddle, which becomes a point at maximum. Therefore the extrema of heigh function tell about how the topology of the cross section of the torus varies with height: point-circle-2 circles-circle-point. The area of surface serves as a Morse function and minimal points are analogous to the points of the torus at which cross section changes its topology.

   A good guess is that the volume of the surface serves as a Morse function and thus gives information about the topology of rather abstract infinite-dimensional space: the space of surfaces. Minimal surfaces would be analogous to the critical points of height function at torus: points at which the cross section changes its topology.

3. Minimax property states the fact that minimal surfaces are in in generic situation saddle points in the space of surfaces. There would be a strange correspondence. The points of minimal surfaces are locally saddles in the finite-dimensional embedding space H and minimal surfaces represent saddle points in the finite-dimensional space of surfaces in H. This strange local-global correspondence bringing in mind holography might be behind a general principle: saddle property could have representations at two levels: points of the surface and points of the space of surfaces.

   Are minimal surfaces a rare exception or could it be that for a general action principle the extremals are saddles locally and that the space of all field configurations (not only extremals) contains the extremals as saddle points?

   Remark: Minimal surfaces might be very special and related to what corresponds in physics to criticality implying that the dynamics in certain sense universal. The space of surfaces corresponds in TGD as the space of 3-surfaces and is analogous to Wheeler's superspace consisting of 3-metrics. By holography forced by 4-D general coordinate invariance 3-surfaces in question must be in one-one correspondence with 4-D surfaces identified as space-time surfaces. I have christened this space world of classical worlds (WCW). Space-time surfaces are 4-D minimal surfaces in 8-D $H = M^4 \times CP_2$ but possessing lower dimensional singularities having interpretation as orbits of string like objects and point like particles. Minimal surface property would be a correlate for quantum criticality so that minimal surface would be very special.

The question and the answer

The question that Marquez and Neves posed to themselves was under which conditions compact space allows a closed minimal surface not intersecting itself or whether all candidates intersect themselves or have infinite volume. In fact, Marquez and Neves restricted the consideration to hyper-surfaces. A possible good reason for this is that there is only one field like dynamical degree of freedom for co-dimension 1 - the coordinate in the normal direction- and this is expected to simplify the situation considerably. From the tone of the article - "-hyper" has been dropped away - one has a temptation to guess that the results are much more general.

The basic result of Marques and Neves was rather astonishing. In almost all closed spaces with dimension between 3 and 7 there exists an infinite series of imbedded *closed* minimal hyper-surfaces (embedding means that there are no self-intersections). No frames needed! The irony was that they could not prove their result for spaces with roundest metrics (no bumps making metric positively curved, which in turn helps to have minimal surface property without local saddle property). Song however generalized this result to apply for arbitrary closed embedding spaces [A6] (see `http://tinyurl.com/yycbw4lx`).

What helped in the proof was a surprising result by Marques, Neves, and Liokumovich that the volume for these minimal hyper-surfaces depends on the volume of the compact embedding space only [A23] (see `http://tinyurl.com/y59pdawj`)!

This dependence suggests that these closed minimal hyper-surfaces manage to visit a dense set of points of the embedding space without intersecting themselves: in this manner they could "measure" the volume. Marques, Neves and Irie show that there is infinite set of imbedded minimal hyper-surfaces in spaces of dimension $3 \leq n \leq 7$ intersecting any given ball of the embedding space [A19] (see `http://tinyurl.com/y3u3bvnc`). Even more, these minimal surfaces tend to fill space in some sense evenly.

A natural guess inspired by Minimax Principle is that minimals surfaces correspond to saddle points for the volume as functional of surface defining Morse function. The volume is analogous to action in TGD framework.

Two remarks are in order.

1. As noticed, the popular article says that these results hold for minimal surfaces. The articles however restrict the consideration to minimal hyper-surfaces.

2. The theorem about the dependence of volume of hyper-surface on the volume of embedding space was inspired by a result proven by Weyl for the high frequencies of drum defined as a boundary of some space: these frequencies depend on the volume of the space, not on the shape of drum! One can understand this intuitively by the fact that high frequency vibrations correspond to short wave lengths and therefore depend only on the *local* properties of the space and not on the global topology. The dependence on volume comes from boundary conditions at the boundaries of the volume.

   In the case of minimal hyper-surfaces the analogy would suggests that the addition of details to the minimal hyper-surface corresponds to the increase of the frequency for drum. Boundary conditions for drum would be replaced by the compactness of the embedding space leading to the quantization of the volume analogous to that for frequency.

3. The infinite geodesic on flat torus described above is a rough analog for omni-presence although it is not closed. Also complex surfaces in CP_2 defined as zero loci of polynomials of complex coordinates (ξ^1, ξ^2) modified to contain irrational powers of ξ^i could define this kind of omni-present surfaces having however infinite area. There is however infinite number of minimal surfaces defined by complex polynomials, which are closed but not omni-present.

6.9.2 Minimal surfaces and TGD

In TGD framework surfaces satisfying minimal surface equations almost everywhere - play a central role.

Space-time surfaces as singular minimal surfaces

From the physics point this is not surprising since minimal surface equations are the geometric analog for massless field equations.

1. The boundary value problem in TGD is analogous to that defining soap films spanned by frames: space-time surface is thus like a 4-D soap film. Space-time surface has 3-D ends at the opposite boundaries of causal diamond of M^4 with points replaced with CP_2: I call this 8-D object just causal diamond (CD). Geometrically CD brings in mind big-bang followed by big crunch.

   These 3-D ends are like the frame of a soap film. This and the Minkowskian signature guarantees the existence of minimal surface extremals. Otherwise one would expect that the non-compactness does not allow minimal surfaces as non-self-intersecting surfaces.

2. Space-time is a 4-surface in 8-D $H = M^4 \times CP_2$ and is a minimal surface, which can have 2-D or 1-D singularities identifiable as string world sheets having 1-D singularities as light-like orbits - they could be geodesics of space-time surface.

   Remark: I considered in [L30] the possibility that the minimal surface property could fail only at the reaction vertices associated with partonic 2-surfaces defining the ends of string world sheet boundaries. This condition however seems to be too strong. It is essential that the singular surface defines a sub-manifold giving deltafunction like contribution to the action density and that one can assign conserved quantities to this surface. This requires that the singular contributions to energy momentum tensor and canonical momentum currents as spacetime vectors are parallel to the singular surface. Singular points do not satisfy this condition.

   String boundaries represent orbits of fundamental point-like fermions located at 3-D light-like surfaces which represent orbits of partonic 2-surfaces. String world sheets are minimal surfaces and correspond to stringy objects associated with say hadrons. There are also degrees of freedom associated with space-time interior. One have objects of various dimension which all are minimal surfaces. Modified Dirac equation extends the field equations to supersymmetric system and assigns fermionic degrees of freedom to these minimal surfaces of varying dimension.

   From the physics point of view, the singular surfaces are analogous to carriers of currents acting as point- and string-like sources of massless field equations.

3. Geometrically string world sheets are analogous to folds of paper sheet. Space-time surfaces are extremals of an action which is sum of volume term having interpretation in terms of cosmological constant and what I call Kähler action - analogous to Maxwell action. Outside singularities one has minimal surfaces stationary with respect to variations of both volume term and Kähler action - note the analogy with free massless field. At singularities there is an exchange of conserved quantities between volume and Kähler degrees of freedom analogous to the interaction of charged particle with electromagnetic field. One can see TGD as a generalization of a dynamics of point-like particle coupled to Maxwell field by making particle 3-D surface.

4. The condition that the exchange of conserved charges such as four-momentum is restricted to lower-D surfaces realizes preferred extremal property as a consequence of quantum criticality demanding a universal dynamics independent of coupling parameters [L39]. Indeed, outside the singularities the minimal surfaces dynamics has no explicit dependence on coupling constants provided local minimal surface property guarantees also the local stationarity of Kähler action.

   Preferred extremal property has also other formulations. What is essential is the generalization of super-conformal symmetry playing key role in super string models and in the theory of 2-D critical systems so that field equations reduce to purely algebraic conditions just like for analytic functions in 2-D space providing solutions of Laplace equations.

5. TGD provides a large number of specific examples about closed minimal surfaces [K5]. Cosmic strings are objects, which are Cartesian products of minimal surfaces (string world sheets) in M^4 and of complex algebraic curves (2-D surfaces). Both are minimal surfaces and extremize also Kähler action. These algebraic surfaces are non-contractible and characterized by homology charge having interpretation as Kähler magnetic charge. These surfaces are genuine minima just like the geodesics at torus.

   CP_2 contains two kinds of geodesic spheres, which are trivially minimal surfaces. The reason is that the second fundamental form defining as its trace the analogs of external curvatures in the normal space of the surfaces vanishes identically. The geodesic sphere of the first kind is non-contractible minimal surface and absolute minimum. Geodesic spheres of second kind is contractible and one has Minimax type situation.

   These geodesic spheres are analogous to 2-planes in flat 3-space with vanishing external curvatures. For a generic minimal surface in 3-space the principal curvatures are non-vanishing and sum up to zero. This implies that minimal surfaces look locally like saddles. For 2-plane the curvatures vanish identically so that saddle is not formed.

Kähler action as Morse function in the space of minimal surfaces

It was found that surface volume could define a Morse function in the space of surfaces. What about the situation in TGD, where volume is replaced with action which is sum of volume term and Kähler action [L35, L34, L39]?

Morse function interpretation could appear in two ways. The first possibility is that the action defines an analog of Morse function in the space of 4-surfaces connecting given 3-surfaces at the boundaries of CD. Could it be that there is large number of preferred extremals connecting given 3-surfaces at the boundaries of CD? This would serve as analogy for the existence of infinite number of closed surfaces in the case of compact embedding space. The fact that preferred extremals extremize almost everywhere two different actions suggests that this is not the case but one must consider also this option.

1. The simplest realization of general coordinate invariance would allow only single preferred extremal but I have considered also the option for which one has several preferred extremals. In this case one encounters problem with the definition of Kähler function which would become many-valued unless one is ready to replace 3-surfaces with its covering so that each preferred extremal associated with the given 3-surface gives rise to its own 3-surface in the covering space. Note that analogy with the definition of covering space of say circle by replacing points with the set of homologically equivalence classes of closed paths at given point (rotating arbitrary number of times around circle).

2. Number theoretic vision [K84, L11] suggests that these possibly existing different preferred extremals are analogous to same algebraic computation but performed in different ways or theorem proved in different ways. There is always the shortest manner to do the computation and an attractive idea is that the physical predictions of TGD do not depend on what preferred extremal is chosen.

3. An interesting question is whether the "drum theorem" could generalize to TGD framework. If there exists infinite series of preferred extremals which are singular minimal surfaces, the volume of space-time surface for surfaces in the series would depend only on the volume of the CD containing it. The analogy with the high frequencies and drum suggests that the surfaces in the series have more and more local details. In number theoretic vision this would correspond to emergence of more and more un-necessary pieces to the computation. One cannot exclude the possibility that these details are analogs for what is called loop corrections in quantum field theory.

4. If the action defines Morse action, the preferred extremals give information about its topology. Note however that the requirement that one has extremum of both volume term and

Kähler action almost everywhere is an extremely strong additional condition and corresponds physically to quantum criticality.

Remark: The original assumption was that the space-time surface decomposes to critical regions which are minimal surfaces locally and to non-critical regions inside which there is flow of canonical momentum currents between volume and Kähler degrees of freedom. The stronger hypothesis is that this flow occurs at 2-D and 1-D surfaces only.

Kähler function as Morse function the space of 3-surfaces

The notion of Morse function can make sense also in the space of 3-surfaces - the world of classical worlds which in zero energy ontology consists of pairs of 3-surfaces at opposite boundaries of CD connected by preferred extremal of Kähler action [K19, K66, L35, L34]. Kähler action for the preferred extremal is assumed to define Kähler function defining Kähler metric of WCW via its second derivates $\partial_K \partial_{\overline{L}} K$. Could Kähler function define a Morse function?

1. First of all, Morse function must be a genuine function. For general Kähler metric this is not the case. Rather, Kähler function K is a section in a $U(1)$ bundle consisting of patches transforming by real part of a complex gradient as one moves between the patches of the bundle. A good example is CP_2, which has non-trivial topology, and which decomposes to 3 coordinate patches such that Kähler functions in overlapping patches are related bythe analog of U(1) gauge transformation.

   Kähler action for preferred extremal associated with given 3-surface is however uniquely defined unless one includes Chern-Simons term which changes in U(1) gauge transformation for Kähler gauge potential of CP_2.

2. What could one conclude about the topology of WCW if the action for preferred extremal defines a Morse function as a functional of 3-surface? This function cannot have saddle points: in a region of WCW around saddle point the WCW metric depending on the second derivatives of Morse function would not be positive definite, and this is excluded by the positivity of Hilbert space inner product defined by the Kähler metric essential for the unitarity of the theory. This would suggest that the space of 3-surfaces has very simple topology if Kähler function.

   This is too hasty conclusion! WCW metric is expected to depend also on zero modes, which do not contribute to the WCW line element. What suggests itself is bundle structure. Zero modes define the base space and dynamical degrees of freedom contributing to WCW line element as fiber. The space of zero modes can be topologically complex.

There is a fascinating open problem related to the metric of WCW.

1. The conjecture is that WCW metric possess the symplectic symmetries of $\Delta M^4_+ \times CP_2$ as isometries. In infinite dimensional case the existence of Riemann/Kähler geometry is not at all obvious as the work of Dan Freed demonstrated in the case of loops spaces [A11], and the maximal group of isometries would guarantee the existence of WCW Kähler geometry. Geometry would be determined by symmetries alone and all points of the space would be metrically equivalent. WCW would be an infinite-dimensional analog of symmetric space.

2. Isometry group property does not require that symplectic symmetries leave Kähler action, and even less volume term for preferred extremal, invariant. Just the opposite: if the action would remain invariant, Kähler function and Kähler metric would be trivial!

3. The condition for the existence of symplectic isometries must fix the ratio of the coefficients of Kähler action and volume term highly uniquely. The physical interpretation is in terms of quantum criticality realized mathematically in terms of the symplectic symmetry serving as analog of ordinary conformal symmetry characterizing 2-D critical systems. Note that at classical level quantum criticality realized as minimal surface property says nothing outside singular surfaces since the field equations in this regions are algebraic. At singularities the situation changes. Note also that the minimal surface property is a geometric analog of masslessness which in turn is a correlate of criticality.

4. Twistor lift of TGD [?]eads to a proposal for the spectra of Kähler coupling strength and cosmological constant allowed by quantum criticality [L34]. What is surprising that cosmological constant identified as the coefficient of the volume term takes the role of cutoff mass in coupling constant evolution in TGD framework. Coupling constant evolution discretizes in accordance with quantum criticality which must give rise to infinite-D group of WCW isometries. There is also a connection with number theoretic vision in which coupling constant evolution has interpretation in terms of extensions of rationals [K84, L23, L20].

Can one apply the mathematical results about closed minimal surfaces to TGD?

The general mathematical thinking involved with the new results is applied also in TGD as should be clear from the above. But can one apply the new mathematical results described above to TGD? Unfortunately not as such. There are several reasons for this.

1. The dimension of $H = M^4 \times CP_2$ is $D = 8 > 7$. M^4 is non-compact and also the signature of M^4 metric is Minkowskian rather than Euclidian. Could one apply these results to special kinds of 4-surface such as stationary surfaces $M^1 \times X^3$, $X^3 \subset E^3 \times CP_2$. No: the problem is that E^3 is non-compact.

2. In TGD one does not consider closed space-time surfaces but analogs of soap films spanned by a frame defined by the 3-surfaces at the opposite ends of CD. Note that the singular surfaces of dimension $D = 2, 1$ are analogous to frames with boundaries at the ends of space-time surface.

3. In TGD framework preferred extremal property requires that space-time surface is both minimal surface and extremal of Kähler action outside singularities. This is known to be the case for all known extremals. This poses very strong conditions on extremals and seems to mean the existence of a generalization of Kähler structure and conformal invariance to 4-D situation. This drops a large number of minimal surface extremals from consideration

4. Minimal surfaces filling space evenly do not have any reasonable physical interpretation. Maybe this could be used to argue that one must have $D = 8$ and that signature must be Minkowskian in order to have soap films rather than closed minimal surfaces.

   What about E^4 with Euclidian signature instead of M^4 and closed space-time surfaces in analogy with Euclidian field theories? Would the projections of closed minimal 4-surfaces in $E^4 \times CP_2$ which are also extremals of Kähler action reduce to a point in E^4 and complex 2-surfaces in CP_2: Euclidianized TGD would degenerate to an Euclidian version of string model. Also in $H = S^4 \times CP_2$ the situation might be same since the property of being extremal of Kähler action is very powerful. It is however essential that also M^4 has analog of Kähler structure: S^4 does not have it although it allows twistor structure so that this options drops out.

5. Can one apply the results of Marques, Neves and others about hyper-surfaces to TGD? What comes in mind is a minimal 4-surface, which is a Cartesian product of geodesic line $M^1 \subset M^4$ and 3-D hyper-surface $X^3 \subset CP_2$ visiting all points of CP_2 and having a finite volume. If the action would contain only the volume term, this extremal would be possible. The action however contains Kähler action and this very probably excludes this extremal.

Chapter 7

About TGD counterparts of twistor amplitudes

7.1 Introduction

The twistor program was originally introduced by Penrose [B41]. The application of twistors to gauge theories, in particular $\mathcal{N} = 4$ SUSY, led to a dramatic progress in the mathematical understanding of these theories. For beginners like me (still), the article of Elvang and Huang [B18] is an extremely helpful introduction to twistor scattering amplitudes.

I am not a specialist in the field. Therefore the following list of works that have had effect in my attempts to understand how twistors might relate to TGD, must look rather random in the eyes of a professional. It however gives some idea about the timeline of ideas.

- Witten's work (2003) [B16] on perturbative string theory in twistor space.
- The proof of Britto, Cachazo, Feng and Witten (2005) [B9] for tree level recursion relation (BCFW recursion) in Yang-Mills theory.
- The work of Hodges (2005) [B2] about twistor diagram recursion for gauge-theory amplitudes.
- The works of Mason and Skinner (2009) on scattering amplitudes and BCFW recursion in twistor space [B38] and on dual superconformal invariance, momentum twistors and Grassmannians (2009) [B39]. There is also the work of Bullimore, Mason and Skinner (2009) on twistor strings, Grassmannians and leading singularities [B10].
- The work of Drummond, Henn and Plefka (2009) [B15] on Yangian symmetry of scattering amplitudes in $\mathcal{N} = 4$ SUSY.
- The work of Goncharov *et al* (2010) [B28] on classical polylogarithms for amplitudes and Wilson loops.
- Nima Arkani-Hamed and colleagues have made impressive contributions. There is a work by Arkani-Hamed *et al* on S-Matrix in twistor space (2009) [B21, B20]; a work about unification of residues and Grassmannian dualities (2010) [B23]; a proposal for all-loop integrand for scattering amplitudes for planar $\mathcal{N} = 4$ SUSY (2011) [?] a work on scattering amplitudes and positive Grassmannian (2012) [B19]; the proposal of amplituhedron (2013) [B6] and work about positive amplitudes in amplituhedron [B5] (2014); a proposal of MHV on-shell amplitudes beyond the planar limit (2014) [B25] ; the notion of associahedron (2017) [B4].

The TGD approach to twistors [L3, L24] [L35, L49] has developed gradually during the last decade. The evolution of ideas began with the attempt to geometrize twistors in the same way as standard model gauge fields are geometrized in TGD. Only quite recently, the number theoretic approach to twistors has started to evolve.

The twistor lift of TGD geometrizes the notion of twistor by replacing the twistor field configurations with 6-D surfaces assigning to space-time surfaces analog of its twistor space obtained by inducing the twistor structure of the product $T(M^4) \times T(CP_2)$ of the twistor spaces of M^4 and CP_2. The construction requires that these twistor spaces have a Kähler structure. M^4 and CP_2 are unique in that only their twistor spaces allow a Kähler structure [A18]. Therefore TGD is mathematically unique: the same conclusion is forced by standar model symmetries and $M^8 - H$ duality. This gives strong motivation for an attempt to construct the TGD counterparts of the twistor scattering amplitudes.

The number theoretic view about twistors based on $M^8 - H$ duality [L52, L53, L68] has developed during this year (2021) and this article tries to articulate this vision and leads to a proposal for how to construct twistor scattering amplitudes in the TGD framework.

7.1.1 Some background

In the following, the basic facts related to twistors are described. I cannot say anything about the technicalities of the twistorial computations and my basic aim is to clarify myself the contents of the notions involved and understand how the twistors diagrammatics might generalize to the TGD context.

Basic facts about twistors and bi-spinors

It is convenient to start by summarizing the basic facts about bi-spinors and their conjugates allowing to express massless momenta as $p^{aa'} = \lambda^a\tilde{\lambda}^{a'}$ with $\tilde{\lambda}$ defined as complex conjugate of λ and having opposite chirality (see `http://tinyurl.com/y6bnznyn`).

1. When λ is scaled by a complex number $\tilde{\lambda}$ suffers an opposite scaling. The bi-spinors allow the definition of various inner products

$$\begin{aligned} \langle\lambda,\mu\rangle &= \epsilon_{ab}\lambda^a\mu^b \ , \\ \left[\tilde{\lambda},\tilde{\mu}\right] &= \epsilon_{a'b'}\tilde{\lambda}^{a'}\tilde{\mu}^{b'} \ , \\ p\cdot q &= \langle\lambda,\mu\rangle\left[\tilde{\lambda},\tilde{\mu}\right] \ , \ (q_{aa'} = \mu_a\tilde{\mu}_{a'}) \ . \end{aligned} \tag{7.1.1}$$

2. Spinor indices are lowered and raised using antisymmetric tensors $\epsilon^{\alpha\beta}$ and $\epsilon_{\dot{\alpha}\dot{\beta}}$. If the particle has spin one can assign it a positive or negative helicity $h = \pm 1$. Positive helicity can be represented by introducing artitrary negative (positive) helicity bispinor μ_a ($\mu_{a'}$) not parallel to λ_a ($\mu_{a'}$) so that one can write for the polarization vector

$$\begin{aligned} \epsilon_{aa'} &= \frac{\mu_a\tilde{\lambda}_{a'}}{\langle\mu,\lambda\rangle} \ , \ \text{positive helicity} \ , \\ \epsilon_{aa'} &= \frac{\lambda_a\tilde{\mu}_{a'}}{\left[\tilde{\mu},\tilde{\lambda}\right]} \ , \ \text{negative helicity} \ . \end{aligned} \tag{7.1.2}$$

In the case of momentum twistors the μ part is determined by different criterion to be discussed later.

3. What makes 4-D twistors unique is the existence of the index raising and lifting operations using antisymmetric ϵ tensors. In higher dimensions they do not exist and this causes difficulties. For octonionic twistors with quaternionic components possibly only in $D = 8$ the situation changes.

Also massive momenta and any point of M^4 can be expressed in terms of helicity spinors but momenta different by a light-like momenta on some light-like geodesic give rise to the same twistor.

1. One has $p_{a\dot{b}} = \mu_a \tilde{\lambda}_{\dot{b}}$. The spinors μ and λ are determined only modulo opposite complex scalings. One can say that the twistor line (sphere CP_1) determines a point of M^4. A possible interpretation is that the points of CP_1 correspond to the choices of spin quantization axis for momentum p and the scaling changes its direction.

2. The incidence relation $\mu^a = p^{a\dot{b}}\lambda_{\dot{b}}$ is also true for $p^{a\dot{b}}+k\lambda^a\lambda\dot{b}$, for any k, so that the points of a light-like line in M^4 are mapped to a point of the twistor space and therefore would correspond to the same direction of spin quantization axis. Physically this could be interpreted by saying that this is the case because the points with a light-like separation are not causally independent.

Twistors allow an elegant formulation of the kinematics and the Mandelstam variable $s_{ij} = (p_i - p_j)^2 = m_i^2 + m_j^2 - 2p_i \cdot p_j$ can be expressed in terms of twistors by expressing p as

$$p = |\mu\rangle[\tilde{\lambda}| + |\tilde{\mu}]\langle\lambda|$$

Since the states are massive, the inner product $p_1 \cdot 2$ can be expressed as

$$p_1 \cdot p_2 = \langle\lambda_1\mu_2\rangle[\tilde{\lambda}_1\tilde{\mu}_2] \ ,$$

Since <> and [] are not complex conjugates of each other and can be regarded as independent complex variables. For massless case this is not case that the expression for $p_1 \cdot p_2$ reduces to modulus squared=

The notion of momentum twistor is nicely explained by Claude Durr in the slides of a talk "Momentum twistors, special functions and symbols" (`https://cutt.ly/AY7QYv3`). Momentum twistors are essential in the twistorial construction of the scattering amplitudes.

1. The notion makes sense for planar diagrams for which the momenta can be ordered. For non-planar diagrams this is not the case. Whether the embedding of non-planar diagrams to a surface with some minimal genus could allow the ordering (if two lines which cross in plane, the other line could go along the handle), is not clear to me.

2. One ends up with the momentum twistors Z_i, as opposed to ordinary twistors denoted by W_i, by performing a Fourier transform of a massless twistor amplitude, which is holomorphic in variables $\langle\lambda_i\lambda_j\rangle$ so that the relation of the helicity spinor μ to λ is essentially that of wave vector to a position vector. The helicity spinor pair $Z = (\omega, \lambda)$, where ω is essentially the complex conjugate of λ in massless case is replaced with (ω, μ). This transform makes sense also in the massive case.

   Momentum twistors correspond to what are called dual or area momenta. The ordinary momenta p_i can be expressed as their differences $p_i = x_{i+1} - x_i$ and area momenta in turn as as $x_i = \sum_{1\leq k \leq i} x_k$. The term area momentum comes from the observation that the planar diagrams divide the plane into disjoint regions and the area momenta can be assigned to these regions.

3. At the level of symmetries the possibility of momentum twistors means extension of the algebra of conformal symmetries of M^4 to a Yangian algebra whose generators are labeled by non-negative integers and which are poly-local so that the corresponding charges contain multilocal contributions (note that potential energy is bilocal and somewhat analogous notion). The generators generating conformal symmetries in the space of area momenta correspond to generators of conformal weight $h = 1$ and whereas ordinary conformal generators have conformal weight $h = 0$.

Remark: TGD suggests the interpretation of two kinds of twistors in terms of $M^8 - H$ duality. Area momenta and momentum twistors could correspond to M^8 level and ordinary momenta and twistors to H level. M^8 indeed has interpretation as analog of momentum space and $M^8 - H$ duality as the TGD counterpart of momentum-position duality having no generalization in quantum field theories where momentum and position are not dynamical variables.

MHV amplitudes as basic amplitudes

The following comments about MHV amplitudes sketch only the main points as I see them from my limited TGD perspective. One reason for this, besides my very limited practical experience with these amplitudes, is that it seems that The TGD approach in its recent form does not force their introduction.

The article of Elvang and Huang [B18] provides an excellent summary about the construction of twistor amplitudes explaining the important details (see also the slides by Claude Durr at `https://cutt.ly/AY7QYv3`). Maximally helicity violating (MHV) amplitudes with $k = 2$ negative helicity gluons are defined as tree amplitudes of say $\mathcal{N} = 4$ SUSY and involve gluons and their superpartners. It is convenient to drop the group theory factor $Tr(T_1T_2\cdots T_n)$ related to gluons.

NMHV amplitudes have $k > 2$ and can be classified by the number of loops as also $k = 2$ diagrams. NMHV diagrams are constructible in terms of MHV diagrams and the construction is known as BSFW construction which by recursion reduces these diagrams to $k = 2$ diagrams, about which 3-gluon vertices is the simplest example. To my amateurish understanding, it is not yet clear whether also the planar Feynman diagrams allow twistorialization. The basic problem is that the area moment x_i with $p_i = x_{i+1} - x_i$ must be ordered and this is not possible for non-planar diagrams.

The construction gives a recursion formula allowing to express the amplitudes in terms of MHV tree amplitudes. Rather remarkably, all loop amplitudes are proportional to the tree level MHV amplitudes so that the singularity structure of the amplitudes is completely determined by the MHV amplitudes. A holography at the level of momentum space is realized in the sense that the singularities dictate the amplitudes completely.

1. The starting point is the observation that tree amplitude with $k = 0$ or $k = 1$ vanishes. The simplest MHV amplitudes have exactly $k = n - 2$ gluons of same helicity- taken by a convention to be negative - have extremely simple form in terms of the spinors and reads as

$$A_n = \frac{\langle \lambda_x, \lambda_y \rangle^4}{\prod_{i=1}^n \langle \lambda_i, \lambda_{i+1} \rangle} \quad (7.1.3)$$

When the sign of the helicities is changed $\langle .. \rangle$ is replaced with $[..]$.

2. It is essential that the amplitudes are expressible in terms of the antisymmetric bi-linears $\langle \lambda_i, \lambda_j \rangle = \epsilon^{ab}\lambda_{i,a}\lambda_{j,b}$. This implies holomorphy and homogeneity with respect to $\langle \lambda_i, \lambda_j \rangle$ follows for massless field theories by the cancellation of the []:s or ¡¿:s of spinor inner products with [] or ¡¿ appearing in $p_i \cdot p_j$ appearing in the massless propagator.

3. $k = 2$ MHV amplitudes take the role of vertices in the construction of amplitudes with $k > 2$ negative helicity gluons. These amplitudes are connected together by off-shell propagator factors $1/P^2$. MHV diagrams allow to develop expressions for the planar on tree amplitudes and also of loop amplitudes using recursion.

4. The treatment of off-mass shell gluons forces to introduce an arbitrary fixed spinor η such that η is not a complex conjugate of λ. η is not the helicity spinor μ assignable uniquely to a massive particle (now a virtual particle). This assumption makes sense for momentum twistors assignable to internal lines of the MHV diagrams since area momenta are in general off-mass-shell.

Yangian symmetry, Grassmannians, positive Grassmannians, and amplituhedron

The work by Nima Arkani Hamed [B20, B24, B19, B1, B6, B43] and other pioneers has led to a very beautiful vision in which the twistorial scattering amplitudes $A_{k,n}$ for $\mathcal{N} = 4$ SUSY are expressible as residue integrals over Grassmannians $Gr_{k,n}$ of integrands which depend on the twistors characterizing the external only via delta functions forcing the integration to surfaces of $Gr(k,n)$. BCFW diagrams and therefore the Grassmannian integrals as their representations are Yangian invariants.

> The amplitudes are defined as residue integrals over $Gr(k,n)$ and contain data about momenta coded by twistors in the arguments of delta functions. The counterparts of the $< ij >$ or $ij]$ determining the integrand are the Plücker coordinates defined as the k-minors, that is determinants of the $k \times n$ matrices, characterizing the point of $Gr(k,n)$. The included minors are taken in cyclic order and contain subsequent columns [B18] (https://cutt.ly/yY7QzQg). One integrates over the k-planes, or equivalently, over $n-k$-planes, of C^n and the integral is residue integral. $Gr(n,k) = U(n)/U(k) \times U(n-k)$ has also an interpretation as a flag-manifold. The residues are located in the positive Grassmannian $Gr^{\geq 0}_{n,k}$. The integral reduces to a mere residue selecting a special k-plane of Grassmannian (note that a gauge fixing eliminating gauge degrees of freedom due to the $Gr(k)$ and $Gr(n)$ symmetries is performed). In the massless case, the delta function constraints state that the n-helicity spinors are orthogonal to $k-D$ and $n-k$-D planes of $GR_{k,n}$ and the conditions imply momentum conservation. In the massive case, the momentum conservation constraint states $\sum p_i = |\mu_i > [\tilde{\lambda}_i| + |\tilde{\mu}]_i < \lambda|_i = 0$. Also now, the interpretation as the inner product of n-helicity spinors is suggestive. A technically important detail is that the quadratic momentum delta function $\delta(\sum_i \lambda_i \tilde{lambda}_i)$ is forced by a product of linear delta function constraints associated with part of $Gr_(k,n)$ to two parts corresponding to k and $n-k$ gluons with opposite helicities. The gauge invariance of these parts with respect to $Gl(k)$ and $Gl(n-k)$ allows a coordinate choice in $Gr(k,n)$ simplifying the calculation drastically.

This work has led to the notions of positive Grassmannian $Gr^{\geq 0}_{k,n}$ [B18] (https://arxiv.org/abs/2110.10856) defined as a sub-space of Grassmannian in which all Plücker coordinates defined by the $k \times k$ minors appearing in the expression of the twistor amplitude are non-negative. Any $n \times (k+m)$, whose minors are positive induces a map from $Gr^{\geq 0}_{k,n}$ whose image is the amplituhedron $\mathcal{A}_{\backslash,\Vert,\Updownarrow}$ (https://arxiv.org/pdf/1912.06125.pdf and https://en.wikipedia.org/wiki/Amplituhedron) introduced by Arkani-Hamed and Trnka. For $m = 4$ the BSFW recurrence relations for the scattering amplitudes can be used to produce collections of $4k$-dimensional cells in $Gr^{\geq 0}_{k,n}$, whose images are conjectured to sub-divide the amplituhedron. $\mathcal{A}_{\backslash,\Vert,\Updownarrow}$ generalizes the positive Grassmannian.

Tree-level amplituhedron can be regarded as a generalization of convex hull of external data and the scattering amplitudes can be extracted from a unique differential form having poles at the boundaries of the amplituhedron.

7.1.2 How to generalize twistor amplitudes in the TGD framework?

Twistor approach works so beautifully in massless case such as $calN = 4$ SUSY because the scattering amplitudes for massless gluons can be written as holomorphic homogeneous functions of arguments constructed from the helicity spinors characterizing the momenta of the external massless particles.

It is always best to start from a problem and the basic problem of the twistor approach is that physical particles are not massless. In the massive QFT, one cannot write a simple twistorial expression of the amplitudes, which would be holomorphic homogenous polynomials in the twistor components and involve only the twistor bilinears $< ij >$ or $[ij]$. The reason is that the external and internal particles are massive. For massive particles, the Mandelstam variables $s_{ij} = (p_i - p_j)^2$ do not factorize as $s_{ij} =< ij > [ij]$.

> The intuitive TGD based proposal has been that since quark spinors are massless in 8-D sense in H, the masslessness in the 8-D sense could somehow solve the problems caused by the massivation in the construction of twistor scattering amplitudes. However, no obvious mechanism has been identified. One step in this direction was however the realization that in H quarks propagate with well-defined M^4 chiralities and only the $D^2(H)$ of Dirac operator annihilates the spinors. M^8 quark momenta are in general complex as algebraic integers. They are identifiable as the counterparts of the area momenta x_i of the momentum twistor

space whereas H momenta can be identified as ordinary momenta. The total momenta of Galois confined states have as components ordinary integers and the momentum spectra in H and M^8 are identical by $M^8 - H$ duality. The mass squared spectrum is quantized as integers for Galois confined states in accordance with supersymplectic invariance implying "stringy" mass spectrum. The natural first guess is that in H the free quarks satisfy the Dirac equation $D(H)\Psi = 0$. There are however excellent reasons to ask whether H spinors satisfy $D(M^4)\Psi = 0$. If so, the M^8 spinors as octonionic spinors would correspond to off-mass shell states with mass squared values given by the roots $m^2 = r_n$ of P, which in general are complex. This conforms with an idea that the super-symplectic conformal weights have an imaginary part and conformal confinement forces total conformal weights to be integers. This would give rise to twistor holomorphy.

The outcome is an extremely simple proposal for the scattering amplitudes.

1. Vertices correspond to trilinears of Galois confined many-quark states as states of super symplectic algebra acting as isometries of the "world of classical worlds" (WCW).

2. Both M^8 and H quarks are on-shell with H momentum p_i and M^8 momenta x_i, x_{i+1}, $p_i = x_{i+1} - x_i$. Dirac operator $x^k\gamma_k$ restricted to a fixed helicity L, R appears as a vertex factor and has an interpretation as a residue of a pole from an on-mass-shell propagator D so that a correspondence with twistorial construction becomes obvious. M^8 quarks are effectively massless but off-shell but the helicity spinors μ and λ are independent unlike for massless particles.

3. The solutions of the octonionic Dirac operator $D(X^4)$ is expressible in terms of helicity spinors of given chirality and this gives two independent holomorphic factors: in the case of massless theories they would be complex conjugates and the other one must cancel by a spinor contraction.

4. The scattering amplitudes would be rational functions in accordance with the number theoretic vision.

5. In the TGD framework the construction of the scattering amplitudes for a single space-time surface is not enough. One must also understand what the WCW integration could mean at the level of scattering amplitudes based on cognitive representations. WCW integration would be naturally replaced by a summation over polynomials such that the corresponding 4-surface correspond at the level of H maxima of the Kähler function. Monic polynomials are highly suggestive.

   A connection with the p-adicization emerges via the identification of the p-adic prime as one of the ramified primes of P. Only (monic) polynomials having a common ramified prime are allowed in the sum. The counterpart of the vacuum functional $exp(-K)$ is naturally identified as the discriminant D of the extension associated with P and p-adic coupling constant evolution emerges from the identification of $exp(-K)$ with D. This leads to the proposal that discriminant equals the exponent of Kähler function. This forces the identification of p-adic prime as ramified prime and fixes coupling constant evolution to a high degree.

7.1.3 Scattering as recombination of quarks to Galois singlets

The view about scattering event is as follows.

1. External particles are Galois singlets consisting of off-mass shell massless quarks with mass squared values coming as roots of the polynomial P characterizing the interaction region. External particles are characterized by polynomials P_i satisfying $P_i(0) = 0$. P is identified as the functional composite of P_i since it inherits the roots (mass squared values) of the incoming particles. The TGD view about cognitive state function reduction [L58] allows only cyclic permutations of P_i in the superposition.

2. The scattering event is essentially a re-combination of incoming Galois singlets to new Galois singlets and quarks propagate freely: hence OZI rule generalizes. Also a connection with the dual resonance models emerges. Finiteness is manifest since the integration of virtual moments is restricted to a summation over a finite number of mass shells.

7.1.4 Comparison with the gauge theory picture

There are several differences between the standard twistor approach applied in gauge theories and the TGD based vision.

1. Vertices involve external H line and two internal N^8 lines. If it indeed does not make sense to speak about internal on-mass-shell quark lines in H, the BCFW construction using MHV amplitudes as building bricks and utilizing now also internal H quark lines, is not needed. One can of course ask, whether the M^8 quark lines could be regarded as analogs of lines connecting different MHV diagrams replaced with Galois singlets. It seems that also Grassmannians, positive Grassmannians, and amplituhedron are unnecessary.

2. The identification of the twistor amplitudes as Yangian invariants is extremely attractive. The proposal has been that the super-symplectic algebra (SSA) and the extended half-Kac Moody algebra of isometries acting as symmetries of WCW extend to Yangians and that the higher charges of Grassmannisans with conformal weight $h > 0$ correspond to multiparticle contributions to conserved charges with potential energy as a very familiar 2-particle example.

   Hence the TGD based construction should produce the scattering amplitudes as Yangian invariants. One cannot of course exclude the possibility that the integration over the "world of classical worlds", which is not considered in this article, could produce analogs BCFW diagrams and their Grassmannian representations.

   Since ordinary particles correspond basically to massless Galois singlests with mass resulting from p-adic thermodynamics, it is very natural to expect that the QFT limit of TGD is a massless QFT. At this limit, the twistor Grassmannian approach would be very natural.

3. Another difference relates to the M^4 conformal invariance of the twistor approach. M^4 conformal invariance is not a symmetry of TGD and the fact that quarks in M^8 are massive in the M^4 sense, reflects this. Massivation forces to extend the twistor holomorphy to both bi-spinors defining the twistor for massive momenta. By the properties of M^8 mass, the masses do not appear explicitly in the amplitudes so effectively the M^8 quarks are massless off-mass shell states. The Yangians would be therefore associated with various super-symplectic algebras rather than with the M^4 conformal group.

4. In the TGD framework, the loop corrections are predicted to vanish and the scattering amplitudes for a given space-time surface would therefore be rational functions in accordance with the number theoretic vision. The absence of logarithmic radiative corrections is not a problem: the coupling constant evolution would be discrete and defined by the hierarchy of extensions of rationals. Also this supports the view that Grassmannians are not needed.

7.1.5 What about unitarity?

Unitary, locality, and the failure to find the twistorial counterparts of non-planar Feynman diagrams are the basic problems of the twistor Grassmannian approach. Also the non-existence of twistor spaces for most Riemannian manifolds is a problem in GRT framework but in TGD the existence of twistor spaces for M^4 and CP_2 solves this problem. In the TGD framework, the replacement of point-like particles with 3-surfaces leads to the loss of locality at the fundamental level. The analogs of non-planar diagrams are eliminated since only cyclic permutations of P_i are allowed.

This leaves only the problem with unitarity. Unitary is essentially a non-relativistic concept and unitary time evolution is a completely ad hoc notion. My feeling is that this problem reflects a lack of some deep principle. In the spirit of Einstein's program for the geometrization of physics, I have proposed in [L59] a geometrization of the state space. Replace the unitary S-matrix with the Kähler metric of Hilbert space. If this metric is non-trivial it is by infinite dimension highly unique. The unitarity conditions are replaced with the conditions $g^{A\overline{B}}g^{\overline{B}C} = \delta^A_C$. The twistorial scattering amplitudes as zero energy states define the Kähler metric $g_{A\overline{B}}$ of quark state space, which is non-vanishing between the 3-D state spaces associated with the opposite boundaries of CD. $g^{A\overline{B}}$ could be constructed as the inverse of this metric.

Scattering probabilities are identified as products of covariant and contravariant matrix elements of the metric and are complex but real and imaginary parts are separately conserved.

The interpretation in terms of Fisher information is possible. Due to the infinite-D character of the state space, the Kähler geometry exists only if it has a maximal group of isometries and is a unique constant curvature geometry. Also the interpretation of this approach in zero energy ontology is discussed.

7.1.6 Objections and critical questions

Objections and critical questions are the best way to make progress by making the picture more precise, and allowing us to see which assumptions might not be final. For instance, twistor holomorphy, M^4 conformal symmetry number theoretically, and many other arguments strongly suggest that free quark spinors do not satisfy $D(H)\Psi = 0$ but $D(M^4)\Psi = 0$ and are therefore massless. The propagation of any massive particle along a light-like geodesic is however effectively massless and CP_2 type extremals have light-like M^4 projection so that one must leave this issue open.

7.1.7 Number theoretical generalizations of scattering amplitudes

Last section discusses the number theoretical generalizations of the scattering amplitudes. For an iterate of fixed P (say large number of gravitons), the roots of the iterate of P defined virtual mass squared values, approach to the Julia set of P. The construction of scattering amplitudes thus leads to chaos theory at the limit of an infinite number of identical particles.

The construction generalizes also to the surfaces defined by real analytic functions and the fermionic variant of Riemann zeta and elliptic functions are discussed as examples.

7.2 TGD related considerations and ideas

The goal is to generalize twistorial construction of scattering amplitudes in the simplest possible manner to the TGD framework. One of the key challenges is the twistorial description of massivation. In this section I summarize briefly the ideas of TGD which seem to be relevant for the construction of the twistor amplitudes.

7.2.1 The basic view about ZEO and causal diamonds

In the following are listed the ideas and concepts behind ZEO [K89] that seem to be rather stable.

1. General Coordinate Invarince (GCI) plays a crucial role in the construction of the Kähler geometry of WCW and implies holography, Bohr orbitology and zero energy ontology (ZEO) [L47, L68] [K89].

2. X^3 is more or less equivalent with Bohr orbit/preferred extremal $X^4(X^3)$. A finite failure of determinism is however possible and is discussed in [L71]. Preferred extremals would be simultaneous extremals of both volume action and Kähler action outside singularities and thus minimal surfaces analogous to soap films spanned by frames. Zero energy states are superpositions of $X^4(X^3)$. Quantum jump is consistent with causality of field equations.

3. Causal diamond (CD=$cd \times CP_2$) defined as intersection of future and past directed light cones (cds) plays the role of quantization volume, and is not arbitrarily chosen. CD determines momentum scale and discretization unit for momentum (see **Fig. 9 Fig. 10**).

4. The opposite light-like boundaries of CD correspond for fermions dual vacuums (bra and ket) annihilated by fermion annihilation - *resp.* creation operators. These vacuums are also time reversals of each other.

   The first guess is that zero energy states in the fermionic degrees of freedom correspond to pairs of this kind of states located at the opposite boundaries of CD. This seems to be the correct view in H. At the M^8 level the natural identification is in terms of states localized at points inside light-cones with opposite time directions. The slicing would be by mass shells (hyperboloids) at the level of M^8 and by CDs with same center point at the level of H.

5. Zeno effect can be understood if the states at either cone of CD do not change in "small" state function reductions (SSFRs). SSFRs are analogs of weak measurements (`https://cutt.ly/nURW3QE`). One could call this half-cone call as a passive half-cone. I have also talked about passive boundary.

   The time evolutions between SSFRs induce a delocalization in the moduli space of CDs. Passive boundary/half-cone of CD does not change. The active boundary/half-cone of CD changes in SSFRs and also the states at it change. Sequences of SSFRs replace the CD with a quantum superposition of CDs in the moduli space of CDs. SSFR localizes CD in the moduli space and corresponds to time measurement since the distance between CD tips corresponds to a natural time coordinate identifiable as geometric time. The size of the CD is bound to increase in a statistical sense: this corresponds to the arrow of geometric time.

6. There is no reason to assume that the same boundary of CD is always the active boundary. In "big" SFRs (BSFRs) their roles would indeed change so that the arrow of time would change. The outcome of BSFR is a superposition of space-time surfaces leading to the 3-surface in the final state. BSFR looks like deterministic time evolution leading to the final state [L38] as observed by Minev *et al* [L38].

7. h_{eff} hierarchy [K21, K22, K23, K24] implied by the number theoretic vision [L52, L53] makes possible quantum coherence in arbitrarily long length scales at the magnetic bodies (MBs) carrying $h_{eff} > h$ phases of ordinary matter. ZEO forces the quantum world to look classical for an observer with an opposite arrow of time. Therefore the question about the scale in which the quantum world transforms to classical, becomes obsolete.

8. Change of the arrow of time changes also the thermodynamic arrow of time. A lot of evidence for this in biology. Provides also a mechanism of self-organization [L43]: dissipation with reversed arrow of time looks like self-organization [L94].

7.2.2 Galois confinement

The notion of Galois confinement emerged originally in TGD inspired quantum biology [L94, L56, L60, L64]. Galois group for the extension of rationals determined by the polynomial defining the space-time surface $X^4 \subset M^8$ acts as a number theoretical symmetry group and therefore also as a physical symmetry group.

1. The idea that physical states are Galois singlets transforming trivially under the Galois group emerged first in quantum biology. TGD suggests that ordinary genetic code is accompanied by dark realizations at the level of magnetic body (MB) realized in terms of dark proton triplets at flux tubes parallel to DNA strands and as dark photon triplets ideal for communication and control [L56, L64, L63]. Galois confinement is analogous to color confinement and would guarantee that dark codons and even genes, and gene pairs of the DNA double strand behave as quantum coherent units.

2. The idea generalizes also to nuclear physics and suggests an interpretation for the findings claimed by Eric Reiter [L69] in terms of dark N-gamma rays analogous to BECs and forming Galois singlets. They would be emitted by N-nuclei - also Galois singlets - quantum coherently [L69]. Note that the findings of Reiter are not taken seriously because he makes certain unrealistic claims concerning quantum theory.

It seems that Galois confinement might define a notion, which is much more general than thought originally. To understand what is involved, it is best to proceed by making questions.

1. Why not also hadrons could be Galois singlets so that the somewhat mysterious color confinement would reduce to Galois confinement? This would require the reduction of the color group to its discrete subgroup acting as Galois group in cognitive representations. Could also nuclei be regarded as Galois confined states? I have indeed proposed that the protons of dark proton triplets are connected by color bonds [L46, L55, L19].

2. Could all bound states be Galois singlets? The formation of bound states is a poorly understood phenomenon in QFTs. Could number theoretical physics provide a universal mechanism for the formation of bound states? The elegance of this notion is that it makes the notion of bound state number theoretically universal, making sense also in the p-adic sectors of the adele.

3. Which symmetry groups could/should reduce to their discrete counterparts? TGD differs from standard in that Poincare symmetries and color symmetries are isometries of H and their action inside the space-time surface is not well-defined. At the level of M^8 octonionic automorphism group G_2 containing as its subgroup $SU(3)$ and quaternionic automorphism group $SO(3)$ acts in this way. Also super-symplectic transformations of $\delta M^4_\pm \times CP_2$ act at the level of H. In contrast to this, weak gauge transformations acting as holonomies act in the tangent space of H.

   One can argue that the symmetries of H and even of WCW should/could have some kind of reduction to a discrete subgroup acting at the level of X^4. The natural guess is that the group in question is Galois group acting on cognitive representation consisting of points (momenta) of M^8_c with coordinates, which are algebraic integers for the extension.

   Momenta as points of M^8_c would provide the fundamental representation of the Galois group. Galois singlet property would state that the sum of (in general complex) momenta is a rational integer invariant under Galois group. If it is a more general rational number, one would have fractionation of momentum and more generally charge fractionation. Hadrons, nuclei, atoms, molecules, Cooper pairs, etc.. would consist of particles with momenta, whose components are algebraic, possibly complex, integers.

   Also other quantum numbers, in particular color, could correspond to representations of the Galois group. In the case of angular momentum, Galois confinement would allow algebraic fractional angular momenta summing up to the usual half-odd integer valued spin.

4. Why Galois confinement would be needed? For particles in a box of size L, the momenta are integer valued as multiples of the basic unit $p_0 = \hbar n \times 2\pi/L$. Group transformations for the Cartan group are typically represented as exponential phase factors, which must be roots of unity for discrete groups. For rational valued momenta this fixes the allowed values of group parameters. In the case of plane waves, momentum quantization is implied by periodic boundary conditions.

   For algebraic integers, the conditions satisfied by rational momenta in general fail. Galois confinement for the momenta would however guarantee that they are integer valued and boundary conditions can be satisfied for the bound states.

7.2.3 How could p-adicization and hyper-finite factors relate?

Factors of type I are von Neumann algebras acting in the ordinary Hilbert space allowing a discrete enumerable basis. Also hyperfinite factors of type II_1 (HFFs in the sequel) play a central role in quantum TGD [K85, K28]. HFFs replace the problematic factors of type III encountered in algebraic quantum field theories. Note that von Neumann himself regarded factors of type III pathological.

HFFs and p-adic physics as key notions of TGD

HFFs have rather unintuitive properties, which I have summarized in [K85, K28].

1. The Hilbert spaces associated with HFFs do not have a discrete basis and one could say that the dimension of Hilbert spaces associated with HFFs corresponds to the cardinality of reals. However, the dimension of the Hilbert space identified as a trace $Tr(Id)$ of the unit operator is finite and can be taken equal to 1.

2. HFFs have subfactors and the inclusion of sub-HFFs as analogs of tensor factors give rise to subfactors with dimension smaller than 1 defining a fractal dimension. For Jones inclusions

these dimensions are known and form a discrete set algebraic numbers. In the TGD framework, the included tensor factor allows an interpretation in terms of a finite measurement resolution. The inclusions give rise to quantum groups and their representations as analogs of coset spaces.

p-Adic numbers represent a second key notion of TGD.

1. p-Adic number fields emerged in p-adic mass calculations [K42, K47, K48] [L81]. Their properties led to a proposal that they serve as correlates of cognition. All p-adic number fields are possible and can be combined to form adele and the outcome is what could be called adelic physics [L23, L22].

2. Also the extensions of p-adic number fields induced by the extensions of rationals are involved and define a hierarchy of extensions of adeles. The ramified primes for a given polynomial define preferred p-adic primes. For a given space-time region the extension is assignable to the coefficients for a pair of polynomials or even Taylor coefficients for two analytic functions defining the space-time surface as their common root.

3. The inclusion hierarchies for the extensions of rationals accompanied by inclusion hierarchies of Galois groups for extensions of extensions of .... are analogous to the inclusion hierarchies of HFFs.

Holography = holomorphy vision

Before discussing how p-adic and real physics relate, one must summarize the recent formulation of TGD based on holography = holography correspondence.

1. The recent formulation of TGD allows to identify space-time surfaces in the imbedding space $H = M^4 \times CP_2$ as common roots for the pair (f_1, f_2) of generalized holomorphic functions defined in H. If the Taylor coefficients of f_i are in an extension of rationals, the conditions defining the space-time surfaces make sense also in an extension of p-adic number fields induced by this extension. As a special case this applies to the case when the functions f_i are polynomials. For the completely Taylor coefficients of generalized holomorphic functions f_i, the p-adicization is not possible. The Taylor series for f_i must also converge in the p-adic sense. For instance, this is the case for $exp(x)$ only if the p-adic norm of x is not smaller than 1.

2. The notion of Galois group can be generalized when the roots are not anymore points but 4-D surfaces [L88]. However, the notion of ramified prime becomes problematic.

   The notion of ramified primes makes sense if one allows 4 polynomials (P_1, P_2, P_3, P_4) instead of two. The roots of 3 polynomials (P_1, P_2, P_3) give rise to 2-surfaces as string world sheets and the simultaneous roots of (P_1, P_2, P_3, P_4) can be regarded as roots of the fourth polynomial and are identified as physical singularities identifiable as vertices [L90].

   Also the maps defined by analytic functions g in the space of function pairs (f_1, f_2) generate new space-time surfaces. One can assign Galois group and ramified primes to h if it is a polynomial P in an extension of rationals. The composition of polynomials P_i defines inclusion hierarchies with increasing algebraic complexity and as a special case one obtains iterations, an approach to chaos, and 4-D analogs of Mandelbrot fractals.

Canonical identification and cognitive representations

Consider now the relationship between real and p-adic physics.

1. The connection between real and p-adic physics is defined by common points of reals and p-adic numbers defining a discretization at the space-time level and therefore a finite measurement resolution. This correspondence generalizes to the level of the space-time surfaces

and defines a highly unique discretization depending only on the pinary cutoff for the algebraic integers involved. The discretization, I call it cognitive representation, is not completely unique since the choice of the generalized complex coordinates for H is not completely unique although the symmetries of H make it highly unique.

2. This picture leads to a vision in which reals and various p-adic number fields and their extensions induced by rationals form a gigantic book in which pages meet at the back of the book at the common points belonging to rationals and their extensions.

What it means to be a point "common" for reals and p-adics, is not quite clear. These common numbers belong to an algebraic extension of rationals inducing that of p-adic numbers. Since a discretization is in question, one can require that these common numbers have a *finite* pinary expansion in powers of p. For points with coordinates in an algebraic extension of rationals and having p-adic norm equal to 1, a direct identification is possible. In the general case, one can consider two options for the correspondence between p-adic discretization and its real counterpart.

1. The real number and the number in the extension have the same finite pinary expansions. This correspondence is however highly irregular and not continuous at the limit when an infinite number of powers of p are allowed.

2. The real number and its p-adic counterpart are related by canonical identification I. The coefficients of the units of the algebraic extension are finite real integers and mapped to p-adic numbers by $x_R = I(x_p) = \sum x_n p^{-n} \rightarrow x_p = \sum x_n p^n$. The inverse of I has the same form. This option is favored by the continuity of I as a map from p-adics to reals at the limit of an infinite number of pinary digits.

   Canonical identification has several variants. In particular, rationals m/n such that m and n have no common divisors and have finite pinary expansions can be mapped their p-adic counterparts and vice versa by using the map $m/n \rightarrow I(m)/I(n)$. This map generalizes to algebraic extensions of rationals.

The detailed properties of the canonical identification deserve a summary.

1. For finite integers I is a bijection. At the limit when an infinite number of pinary digits is allowed, I is a surjection from p-adics to reals but not a bijection. The reason is that the pinary expansion of a real number is not unique. In analogy with 1=.999...for decimal numbers, the pinary expansion $[(p-1)/p]\sum_{k\geq 0} p^{-k}$ is equal to the real unit 1. The inverse images of these numbers under canonical identification correspond to $x_p = 1$ and $y_p = (p-1)p\sum_{k\geq 0} p^k$. y_p has p-adic norm $1/p$ and an infinite pinary expansion.

   More generally, I maps real numbers $x = \sum_{n<N} x_n p^{-n} + x_N p^{-N}$ and $y = \sum_{n<N} x_n p^{-n} + (x_N - 1)p^{-N} + p^{-N-1}(p-1)\sum_{k\geq 0} p^{-k}$ to the same real number so that at the limit of infinite number of pinary digits, the inverse of I is two value for finite real integers if one allows the two representations. For rationals formed from finite integers there are 4 inverse images for $I(m/n) = I(m)/I(n)$.

2. One can consider 3 kinds of p-adic numbers. p-Adic integers correspond to finite ordinary integers with a finite pinary expansion. p-Adic rationals are ratios of finite integers and have a periodic pinary expansion. p-Adic transcendentals correspond to reals with non-periodic pinary expansion. For real transcendentals with infinite non-periodic pinary expansion the p-adic valued inverse image is unique since x_R does not have a largest pinary digit.

3. Negative reals are problematic from the point of view of canonical identification. The reason is that p-adic numbers are not well-ordered so that the notion of negative p-adic number is not well-defined unless one restricts the consideration to finite p-adic integers and the their negatives as $-n = (p-1)(1-p)n = (p-1)(1+p+p^2+...)n$. As far as discretizations are considered this restriction is very natural. The images of n and $-n$ under I would correspond to the same real integer but being represented differently. This does not make sense.

Should one modify I so that the p-adic $-n$ is mapped to real $-n$? This would work also for the rationals. The p-adic counterpart of a real with infinite and non-periodic pinary expansion and its negative would correspond to the same p-adic number. An analog of compactification of the real number to a p-adic circle would take place.

Analogies between p-adicization and HFFs

Both hyperfinite factors and p-adicization allow a description of a finite measurement resolution. Therefore a natural question is whether the strange properties of hyperfinite factors, in particular the fact that the dimension D of Hilbert space equals to the cardinality of reals on one hand and to a finite number ($D = 1$ in the convention used) on the other hand, could have a counterpart in the p-adic sector. What is the cardinality of p-adic numbers defined in terms of canonical identification? Could it be finite?

1. Consider real finite real integers $x = \sum_{n=0}^{N-1} x_n p^n$ but with $x = 0$ excluded. Each pinary digit has p values and the total cardinality of these numbers of this kind is $p^N - 1$. These real integers correspond to two kinds of p-adic integers in canonical identification so that the total number is $2p^N - 2$. One must also include zero so that the total cardinality is $M = 2p^N - 1$. Identify M as a p-adic integer. Its padic norm equals 1.

2. As a p-adic number, M corresponds to $M_p = 2p^N + (p-1)(1+p+p^2+...) = p^N + p^{N+1} + (p-1)(1+p+...-p^N))$. One can write $M_p = p^N + p^{N+2} + (p-1)(1+p+...-p^N - p^{N+1})$. One can continue in this way and obtains at the limit $N \to \infty$ $p^{N\to\infty}(1+p+...) + (p-1)(1+p+...+p^{N-1})$. The first term has a vanishing p-adic norm. The canonical image of this number equals p at the limit $N \to \infty$. The cardinality of p-adic numbers in this sense would be that of the corresponding finite field! Does this have some deep meaning or is it only number theoretic mysticism?

7.2.4 No loops in TGD

There are several arguments suggesting that there is no counterpart for loops of quantum field theories (QFTs) in TGD. Purely rational scattering amplitudes are required by number theoretic vision but the logarithmic corrections from loops would spoil the number theoretic beauty.

Loops however give rise to coupling constant evolution, which is a physical fact. What could be the TGD counterpart of coupling constant evolution?

1. The number theoretic and p-adic coupling constant evolutions, which are discrete rather than continuous, look natural. The effective coupling constant should be renormalized because the allowed momentum exchanges depend on the roots of a polynomial P or at least on their number. If the p-adic prime p corresponds to a ramified prime of extension, the dependence of the effective coupling parameters on the extension of rationals defined by P implies dependence on the prime p characterizing the p-adic length scale. The emerging picture will be described in more detail in the next section.

   In the scatterin amplitudes, a power of coupling g identifiable as Kähler coupling constant g_K appears. Also the factors from Galois singlets appear as well as the states, which correspond to the super-symplectic representations.

   It seems that for given external momenta a sum of several terms appear. If the number of momenta is small, a higher dimension of extension gives a larger number of diagrams and this could lead to number theoretic coupling constant evolution. If a given extension of rationals prefers some p-adic primes, mot naturally the ramified primes of the extension, number theoretic coupling constant evolution translates to a p-adic coupling constant evolution.

2. Does the integration over the WCW give Kähler coupling strength and various couplings or is Kähler coupling present at vertices from the beginning? The latter option would look natural. $M^8 - H$ duality strongly suggests that the exponent $exp(-K)$ of Kähler function K defining vacuum functional has a number theoretic counterpart. The unique counterpart would be the discriminant of the polynomial P and suggests that the value of $exp(-K)$ is equal to discriminant for maxima of K, which would naturally correspond to the space-time surface defining the cognitive representation.

7.2.5 Twistor lift of TGD

One could end up with the twistor lift of TGD from problems of the twistor Grassmannian approach originally due to Penrose [B41] and developed to a powerful computational tool in $\mathcal{N} = 4$ SYM [B17, B9, B25, B4]. For a very readable representation see [B18].

Twistor lift of TGD [L17, L50, L51] generalizes the ordinary twistor approach [L36, L37]. The 4-D masslessness implying problems in twistor approach is replaced with 8-D masslessness so that masses can be non-vanishing in 4-D sense. This gives hopes about massive twistorialization.

The basic recipe is simple: replace fields with surfaces. Twistors as field configurations are replaced with 6-D surfaces in the 12-D product $T(M^4) \times T(CP_2)$ of 6-D twistor spaces $T(M^4)$ and $T(CP_2)$ having the structure of S^2 bundle and analogous to twistor space $T(X^4)$. Bundle structure requires dimensional reduction. The induction of twistor structure allows to avoid the problems with the non-existence of twistor structure for arbitrary 4-geometry encountered in GRT.

he pleasant surprise was that the wistor space has the necessary Kähler structure only for M^4 and CP_2 [A18]: this had been discovered already when started to develop TGD! Since the Kähler structure is necessary for the twistor lift of TGD (the action principle is 6-D variant of Kähler action), TGD is unique. One outcome is length scale dependent cosmological constant Λ assignable to any system - even hadron - taking a central role in the theory [L24]. At long length scales Λ approaches zero and this solves the basic problem associated with it. At this limit action reduces to Kähler action, which for a long time was the proposal for the variational principle.

7.2.6 Yangian of supersymplectic algebra

The notion of Yangian for conformal symmetry group of Minkowski space plays a key role in the construction of scattering amplitudes in $\mathcal{N} = 4$ SUSY as Yangian invariants. There are excellent reasons to expect that also in TGD the scattering amplitudes are Yangian invariants.

Yangian symmetry

The notion equivalent to that of Yangian [A26] [B14, B15, B31] was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras.

The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [L3]. Besides ordinary product in the enveloping algebra there is co-product Δ, which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant and concrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [B14]. Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with the discrete index n being replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N} = 4$ SUSY). One of the conditions conditions is that the tensor product $R \otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n = 0$ and $n = 1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n = 1$ generators with themselves are however something different for a non-vanishing deformation parameter h.

Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n + 1$-local in the sense that they involve $n + 1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, there is not much to say. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

1. The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A2] and Virasoro algebras [A3] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras.

2. In the twistor approach conformal symmetries of M^4 are crucial. The isometries of H do not include scalings and inversions. The massless states of the super-symplectic representation would allow conformal invariance of M^4 as dynamical symmetries.

   There are however several alternatives.

   (a) The spectrum of the Dirac operator $D(H)$ contains only right-handed neutrino ν_R as a massless state and if M^4 Kähler structure is assumed it becomes tachyon.

   (b) The second option is that $D(M^4)$ annilates spinor modes. Dirac propagator would reduce to a delta function in CP_2 degrees of freedom. This option is favored by $M^8 - H$ duality and also by the associativity of the octonionic spinors implying that M^8 momenta reduce to M^4 momenta. This is actually achieved by a suitable choice of $M^4 \subset M^8$ always.

   (c) If $D(M^4)$ contains no coupling to M^4 Kähler gauge potential $A(M^4)$, on-mass-shell quarks are massless and realize M^4 conformal invariance. The appearance of roots polynomials as mass squared values in quark propagators would realize number theoretic breaking of M^4 conformal invariance at the level scattering amplitudes and allow twistor holomorphy.

   If $A(M^4)$ coupling is present, all quarks appear as spin doublets with positive and negative mass squared. M^4 conformal symmetry at the quark level is achieved only at long length scales when the spin term vanishes. The quark propagator in the scattering amplitudes would contain the coupling to $A(M^4)$ so that twistor holomorphy seems to be lost. M^4 gauge potential could explain small CP breaking, and one can imagine that the induced M^4 gauge potential appears only in the modified Dirac equation for the induced spinors.

3. The formal generalization looks surprisingly straightforward at the formal level. In zero energy ontology one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($cd \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones.

The polygon with light-like momenta would be naturally replaced with a polygon with more general momenta in zero energy ontology and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.

4. This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $cd \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely?

1. At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups.

   This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.

2. Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of the partonic 2-surface. This picture also justifies p-adic thermodynamics applied to either symplectic or isometry Super-Virasoro and giving thermal contribution to the vacuum conformal and thus to mass squared.

3. The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.

4. Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

How could the Yangian structure of the super-symplectic algebra emerge?

The isometries of WCW should generalize conformal symmetries of string models and supersymplectic transformations of the light-like boundary of CD are a highly natural candidate in this respect.

1. The crucial observation is that the 3-D light-cone boundary δM^4_+ has metric, which is effectively 2-D. Also the light-like 3-surfaces $X^3_L \subset X^4$ at which the Minkowskian signature of the induced metric changes to Euclidian are metrically 2-D. This gives an extended conformal invariance in both cases with complex coordinate z of the transversal cross section and radial light-coordinate r replacing z as coordinate of string world sheet. Dimensions $D = 4$ for X^4 and M^4 are therefore unique.

2. $\delta M^4_+ \times CP_2$ allows the group symplectic transformations of $S^2 \times CP_2$ made local with respect to the light-like radial coordinate r. The proposal is that the symplectic transformations define isometries of WCW [K19].

3. To the light-like partonic orbits one can assign Kac-Moody symmetries assignable to $M^4 \times CP_2$ isometries with additional light-like coordinate. They could correspond to Kac-Moody symmetries of string models assignable to elementary particles.

   The preferred extremal property raises the question whether the symplectic and generalized Kac-Moody symmetries are actually equivalent. The reason is that isometries are the only normal subgroup of symplectic transformations so that the remaining generators would naturally annihilate the physical states and act as gauge transformations. Classically the gauge conditions would state that the Noether charges vanish: this would be one manner to express preferred extremal property.

Consider next the general structure of the super-symplectic algebra (SSA).

1. The SSA and the TGD analogs of Kac-Moody algebras assignable to light-like partonic 3-surfaces have the property that the conformal weights assigned to the light-like coordinate r are non-negative integers. One can say that they are analogs of "half"-Kac-Moody algebras. Same holds true for the Yangian algebras, which suggests that these algebras could extend to Yangian algebras.

2. SCA (and also the Kac-Moody analogs) has fractal hierarchies of sub-algebras isomorphic to the algebra SSA itself at the lowest level. The conformal weights of sub-algebra SSA_n an n-multiplets of those of SSA: one obtains hierarchies of sub-algebras $SSA \supset SCA_{n_1} \supset SSA_{n_2 n_1}, ...$

3. This leads to the proposal that there is a hierarchy of analogs of "gauge symmetry" breakings. For the maximal "gauge symmetry", the entire SSA annihilates the states and classical Noether charges vanish. For SSA_n, only SSA_n and the commutator $[SSA_n, SSA]$ annihilate the physical states.

   One can ask whether these hierarchies could correspond to the hierarchies of extensions for rationals defined by the composition of polynomials defining 4-surfaces in M^8 and by $M^8 - H$ duality in H.

Cognitive representations play a key role and correspond to many quarks states.

1. Cognitive representations consist of the points of $X^4 \subset M^8$ with $M^4 \subset M^8$ coordinates belonging to an extension of rationals defined by a polynomial P defining X^4. It has become clear that here only the mass shells corresponding to the roots r_n of P need to be considered and that only algebraic integers defining the components of M^4 momenta need to be considered.

2. Cognitive representations consist of only those points which are "active", i.e. contain quark or antiquark. $M^8 - H$ duality maps the cognitive representations to H. The points of a given mass shell to the light-like boundary of CD. Momentum p as a point of $M^4 \subset M^8$

is mapped to a geodesic line starting from the center of CD and yields the image point as its intersection with the boundary of CD. The momenta at a given mass shell are actually mapped to the boundaries of all CDs forming a Russian doll hierarchy with common center points.

3. The cognitive representation codes for the physical states in quark degrees of freedom and should reflect themselves in the properties of the SSA state construction. The natural condition is that the Hamiltonians of SSA generate transformations leaving invariant the image points of cognitive representation at the boundaries of CD. This requires that the Hamiltonians vanish at the points of the cognitive representation. This is achieved if the Hamiltonians are obtained by multiplying the usual Hamiltonians, which can be chosen to define irreducible representations of $SU(2) \times SU(3)$, by a Hamiltonian H_{cogn}, which vanishes at the points of the cognitive representation.

   The condition that also the super-generators vanish at the points of cognitive representation implies that also the corresponding Hamiltonian vector field j vanishes so that at the points of cognitive representation all Hamiltonians vanish and are extrema. One would have a modification of the hierarchy of SSA_n but the gauge conditions would remain as such. These conditions could be regarded as a realization of quantum criticality.

4. The cognitive representation defined by the multi-quark states in M^8 would modify the SSA in H by multiplying its Hamiltonians with H_{cogn}. The level of WCW the role of the subalgebra SSA_{cogn} defined by cognitive representation would be similar to the algebra of isotropy group $SO(3)$ of particle momentum as a subgroup of $SO(3,1)$.

   This suggests that the induction procedure generating the irreducible representations for finite-dimensional Lie groups generalizes. The representations of $SO(3)$ have as an analog the representations of SSA_{cogn}. From these representations one would obtain by general symplectic transformations states analogous to the Lorentz boosts of a particle at rest. Note that for cognitive representations the Galois group acts non-trivially but one would have Galois singlet. One could have it in geometric sense so that the momenta would simply add up as vectors or in quantum sense as a many-quark state, with quarks at different points of the mass shell or at different mass shells.

How could one understand the generalization of the duality between momenta and area momenta?

1. The duality between ordinary momentum space and area momentum space means that dual conformal transformations act on area momenta x_i as symmetries of the scattering amplitudes. At the level of ordinary momenta this symmetry extends conformal symmetry algebra to a Yangian algebra.

2. Is this possible in the case of $M^8 - H$ duality? Does SSA realized at CD boundaries have a counterpart at the $M^4 \subset H$ mass shells? The counterparts of SSA transformations in M^8 must map the mass shells to itself and leave the points of the cognitive representation invariant. In the interior of $X^4 \subset M^8$ they would induce a deformation of X^4 consistent with the assumption that X^4 is obtained as a local element of $CP_2 = SU(3)/U(2)$, i.e. the deformation is induced by $SU(3)$ element $g(x)$ acting as octonionic automorphism such that $U(2) \subset SU(3)$ leaves the image point invariant. This would guarantee $M^8 - H$ duality.

   This deformation at the mass shell would induce in $X^4 \subset H$ an action having interpretation in terms of a local $SU(3)$ (CP_2) transformation, or possibly an symplectic transformation of CP_2 local with respect to light-cone. At the level of H one has group symplectic transformations of $S^2 \times CP_2$ expressible in terms of Hamiltonian in irreps of $SU(3)$.

3. Could the local $SU(3)/U(2) = CP_2$ transformations be representable as symplectic transformations as the duality would suggest? Does this somehow relate to the facts that both CP_2 and its twistor space $SU(3)/U(1) \times U(1)$ have Kähler structure [A18] and therefore also symplectic structure: this in fact makes CP_2 and M^4 completely unique.

4. What about the M^8 counterparts S^2 Hamiltonians. Could they somehow correspond to quaternionic automorphism group $SO(3)$. Could $SO(3)$ correspond to the allowed symplectic (contact) transformations for the mass shell itself whereas $SU(3)$ would act in the interior of $X^4 \subset M^8$?

The dual conformal transformations induce bilocal transformations in the ordinary Minkowski space and this leads to the notion of Yangian, which also implies higher multi-local actions. Why would be the physical origin of this multilocality?

1. Quantum group structure is involved and bi-local elements should correspond to tensor products $f_{abc}T^b \otimes T_c$ of Lie-algebra generators. This generalizes to higher multilocal states. Galois confinement is a multilocal phenomenon in M^8. $M^8 - H$ duality maps this multilocality to H. The simplest bi-local state is the quark-antiquark pair with total momentum which is an ordinary integer (necessarily non-tachyonic even if the roots r_n had negative real parts). Leptons would be tri-local states of quarks in CP_2 scale.

   The multilocality of the Galois confined many quark states in M^8 strongly suggests that the total charges include, besides the 1-local contributions, there are also multilocal contributions to Noether charges.

2. Galois confinement should force the multilocality of the symmetry generators. In particular, since the total momenta of quarks sum up to an ordinary integer, one cannot perform Lorentz transformations for them independently but one must transform several momenta simultaneously in order to guarantee that the total momentum changes in such a way that Galois confinement condition is satisfied.

   The Galois group acts also on spinors which can have number theoretic analogs of spinor space assignable to algebraic extensions as linear spaces and providing a finite-D number theoretic counterpart for WCW spinors. Therefore the generators of Lorentz transformations must contain bi-local and also n-local terms. Same applies to scalings and conformal transformations and in fact to all other symmetries.

3. In the case of energy, these multilocal contributions could have an interpretation as binding energy or potential energy depending on the distance between the image points of different momenta at the boundary of CD. The question is how these multilocal contributions would emerge in H for the super-symplectic algebra having a representation as classical Noether charges and fermionic Noether charges.

4. The notion of gravitational coupling constant suggests strongly that conserved quantities have besides the local contribution also bilocal contribution for which gravitional Planck constant defines unit of quantization. A possible identification is as a bilocal Yangian contribution.

In $\mathcal{N} = 4$ SUSY, scattering amplitudes are invariants of the Yangian defined by conformal transformations of M^4 and its dual acting in the space of area momenta. Since SSA is proposed to act as isometries of the "world of classical worlds" (WCW), also zero energy states having interpretation as scattering amplitudes should be Yangian invariants.

7.2.7 $M^8 - H$ duality and twistorialization of scattering amplitudes

The precise formulation of twistor amplitudes has remained a challenge although I have considered several proposals in this direction. The progress made in the understanding of the details of $M^8 - H$ duality [L68] motivate the attempts to find more explicit formulation for the scattering amplitudes. The following tries to give a brief overall vision.

1. In its recent form $M^8 - H$ duality predicts the twistor spaces of M^4 and CP_2 and their map to each other having interpretation in terms of 6-D twistor spaces of space-time surfaces as 6-surfaces in the product of the twistor spaces of M^4 and CP_2 replacing space-time surfaces with their twistor spaces in the twistor lift of TGD [L68].

2. Momentum twistors and space-time twistors are related by M^8-duality. M^8 momenta are identified as area momenta different from M^4-momenta in H. The notion of area momentum makes sense only for planar diagrams (it is not clear to me whether the embedding of diagrams genus g topology could allow a definition of area momentum).

3. In the usual twistor Grassmann approach to massless QFTs, the momenta of internal lines are massless and thus on-mass-shell but complex. The simplest option conforming is that both area momenta x_i and H-momenta p_i are on-mass-shell. Area momenta are indeed in general complex as algebraic integers. For a given polynomial P area mass squared spectrum of quarks is fixed as - in general complex - roots of polynomial P.

4. What looks first like a problem is that H momenta have naturally integer valued components (periodic boundary conditions) and mass squared is integer using a suitable unit determined by the p-adic length L_p for the CD. However, at the M^8 side the momenta have components which are algebraic integers in the extension determined by the polynomial P.

   A natural solution of the problem is provided by Galois confinement requiring that momentum components of confined states, which are Galois singlets, are integer valued rather than algebraic integers. This provides a universal mechanism for the formation of bound states. This allows also to have identical spectra for area momenta and ordinary momenta.

   In this picture, the particle would be a Galois singlet formed as a composite of quarks. This notion of a particle is extremely general as compared to the QFT view about elementary particles. The external lines of twistor diagrams carrying H quantum numbers would correspond to states in the representations of super-symplectic algebra (SSA) with Yangian structure.

5. The second quantization for quark fields of H means an enormous simplification. One avoids all problems related to quantization in a curved background. Here an essential role is played by the Kähler structure of M^4 forced by the twistor lift. The generators of supersymplectic algebra and generalized Kac-Moody algebras can be expressed in terms of quark oscillator operators.

6. For given H momenta, the momentum transfers are fixed by $p_i = x_{i-1} - x_i$. The twistor sphere S^2 characterizes the momentum directions. Momentum plus S^2 point s characterized by helicity spinor, defines a point in the twistor space and the geometric interpretation for s is that it characterizes the direction of spin quantization axis.

   The direction of quantization axies is defined only apart from a sign and for spin 1/2 particles the interpretation is as the sign of the spin projection. For massless states the spin axis is parallel to momentum.

7. Galois confinement is crucial. The conditions allow integer valued H momenta only if the area momenta correspond to Galois bound states of quarks. Entire composite of quarks at the same mass shell propagates as particle with total momentum which has integer components. By duality one can assign to the momentum p_i quantum numbers in supersymplectic representation.

   Clearly the notion of a particle as a Galois singlet is very general and corresponds to a multilocal state in both M^8 and H leading also to the notion of Yangian. In H, a particle is a state of a super-symplectic representation. At the level of M^8 it is a Galois confined state. These states correspond to each other.

The basic ideas related to the construction of scattering amplitudes are as follows.

1. $M^8 - H$ duality remains as such. $M^8 - H$ duality maps. Total area momenta X_i of Galois confined states to points at the boundary of corresponding CD with size determined by the total area momentum by $M^8 - H$ duality.

2. Basic vertices for Galois confined states involve many-quark Galois singlet in H with total momentum P_i and 2 many-quark Galois singlets in M^8 involving area momenta X_i and X_{i+1} satisfying $P_i = X_{i+1} - X_i$. The scattering amplitude reduces to quark level and one can say that quark lines connect different mass shells of $X^4 \subset M^8$.

3. 3-vertices are between two M^8 Galois singlets and super-symplectic Galois singlet in H at different M^8 mass shells and lines connecting them carrying momenta calculated at the level of H. Quarks in Galois singlets have collinear rational parts which are analogous to SUSY where monomials of theta parameters assignable to higher spin states are analogous to collinear many-fermion states.

7.3 Are holomorphic twistor amplitudes for massive particles possible in TGD?

Massive particles are believed to make twistorialization impossible. For instance, for a scalar field theory with Yukawa coupling to fermions, the part of scattering amplitude involving vertex with Yukawa coupling plus scalar propagator gives $g < 12 > \times 1/(p_1 - p_2)^2$. For massless particles, one has $(p_1 - p_2)^2 =< pq > [pq]$ and the expression reduces to $g/ < pq >$. This is essential for the holomorphy in twistor components in turn reflecting conformal invariance.

In MHV construction the MHV amplitudes with 2 negative helicities are used as building bricks of twistorial representations of more complex planar tree amplitudes and loop amplitudes connecting them with off-mass-shell lines involving propagators. The obvious question is whether this construction could be generalized.

The simplest MHV diagrams would be replaced with diagrams assignable to single CD and involving only on-mass-shell area momenta in M^8 and on-mass-shell area momenta in H as external particles. One would take several diagrams of this kind and connect them by a line carrying off-mass-shell M^8 momentum and quantum numbers of a state in SSA representation. In a given vertex involving this kind of virtual H-line, the on-mass-shell fermion momenta would be replaced by two 2 on-mass-shell area momenta and off-mass-shell momentum of the scalar particle would correspond to M^8 momentum.

The intuitive idea is that somehow 8-D massless at the level of H solves the problem but it is not at all clear whether it is possible to obtain twistor holomorphy somehow. One hint comes from the fact that twistors associated with massive particles involve two independent helicity spinors μ and λ? Could one have holomorphy with respect to both? A further hint comes from the observation that at the level of H tachyonic right-handed neutrino makes possible the construction of massless states. A further hint comes from Galois confinement: could the external particles be Galois confined states and could the propagating particles be quarks in M^8 having complex masses coming as roots of the polynomial P?

7.3.1 Is it possible to have twistor holomorphy for massive scalar and fermions?

Consider first the simple example of massive fermions and a massive scalar field. Assume that fermions are on-mass-shell with masses m_1 and m_2 and scalar off-mass-shell with mass m.

1. Assume Dirac spinors expressible in terms of left and right handed components. For massive scalar particle, the propagator factor reads as $(p_1 - p_2)^2 - m^2 = m_1^2 + m_2^2 - m^2 - 2(p_1 \cdot p_2)$.

2. The completeness relation for spinor modes reads in massive case as $p^k \gamma_k + m = O(p)$, $O(p) = |p\rangle [p| + |p[\langle p|$

   One can express $O(p)$ as $p^k \gamma_k = O(p) - m$. One obtains for Dirac spinor with left and right handed parts

$$2p_1 \cdot p_2 = \frac{1}{4} Tr[(O(p_1) - m)(O(p_2) - m)] = -m^2 - \frac{1}{4} Tr[O(p_1)O(p_2)] \ .$$

   For

$$m_1^2 + m_2^2 = 2m^2 \ ,$$

   the propagator factor reduces to $1/(Tr(O(p)O(q)) =< pq > [pq]$ as if the particles were massless. The part of the amplitude considered would reduce to $g < pq >$.

3. Could the masses for the generalized twistor diagram satisfy a generalization of the condition $m_1^2 + m_2^2 = 2m^2$ guaranteeing the holomorphy with respect to $< .. >$ or [..]? The prediction for spinors would be an effective prediction of massless QFT. Note that this result is also true when the masses are identical. This in turn might relate to SUSY. The additivity of mass squared values might in turn relate to 2-D conformal invariance in which mass squared operator is scaling generator and mass squared values are conformal weights. 2-D conformal invariance would generalize to its 4-D counterpart.

   Could this picture generalize to TGD in such a way that external on mass states correspond to states constructed in H area momenta are off-mass-shell? It is easy to see that this generalization does not work as such.

7.3.2 Scattering amplitudes in a picture based on $M^8 - H$ duality

The basic assumptions are inspired by $M^8 - H$ duality, ZEO, and geometric view about helicity spinors.

The first guess is that area momenta x_i are assignable to M^8 quarks and are at complex mass shells $m^2 = r_n$. x_i algebraic integers in the extension determined by a polynomial P. Galois confinement implies that the quark momenta associated with mass shells belong to quark composites forming Galois singlets and have a total momentum, which is integer valued with respect to the p-adic mass scale assignable to the mass shell. Also mass squared values would be integers. For general Galois singlets the momenta are assignable to several mass shells $m^2 = r_n$ and thus multi-local entities in M^8, which suggests possible origin of the Yangian symmetry. The mass shells are mapped to the boundaries of corresponding CD in H by $M^8 - H$ duality mapping p-adic mass scale m to its inverse defining p-adic length scale $L = \hbar_{eff}/m$ implying multi-locality in H. CDs form a Russian doll-like structure.Assume that the incoming momenta p_i are H assignable to supersymplectic representations constructed from spinor harmonics in H for a second quantized quark field. $M^8 - H$ duality suggests that the momentum and mass squared spectra are identical at M^8 and H sides. This conforms with Galois confinement at M^8 side. Particles would be Galois confined multi-quark states. Assume that twistors and momentum twistors have a geometric interpretation so that helicity spinors do not represent fermions but points in the CP_1 fiber of CP_3 as a bundle and the states with given spin correspond to wave functions in CP_2 having also half-integer spins. Twistor amplitudes would be constructed as contractions of these wave functions with the scattering amplitudes that the basic scattering amplitude would be independent of spin. In this framework, the many-quark states constructed by elements of Clifford algebra would be analogous to components of a super-field. By Galois confinement, the rational parts of quark momenta would be collinear, which conforms with the basic idea of SUSY that n-monomials of theta parameters are analogous to states of p collinear fermions. The spin of a given state would correspond to a product of spin 1/2 sherical harmonics in the space defined by the helicity spinor. A huge generalization of the notion of particle would be in question. Particle would correspond to an arbitrary Galois singlet assignable to single CD. This would conform with the WCW picture in which physical states of the Universe correspond to WCW spinor fields identified as zero energy states. Vertices would correspond to the states of Yangian supersymplectic representation identifiable as mode of WCW spinor field and representing general fermionic state analogous to a component of super field but without Majorana condition. In the standard model, all couplings except the coupling of Higgs to itself and to fermions respect helicity conservation. Assume that this is true also in TGD so that one can decompose quark spinors to left and right handed parts and that they can be described by spin wave functions in the fiber of twistor space corresponding to the momentum of the quark. Note however that the helicity twistors would be purely geometric quantities rather than representing spinor basis of a fermion. At the level of the twistor space of H, spin states would be described by partial waves at the twistor sphere. At the level of M^8 twistor space, a completely geometric description as a point of twistor space characterizing momentum and spin quantization axis and the sign of the spin 1/2 projection is possible. Helicity spinors μ and $\tilde{\lambda}$ would characterize the direction of the spin quantization axis as a point twistor sphere S^2. This conforms with the fact that for massive particles the direction of helicity spinor is not unique since the spin μ is determined only apart from a spinor proportional to λ. For massless particles the direction of the quantization axis is unique. Since only quarks with spin 1/2 are fundamental fermions, the twistor sphere with a fixed radius is enough. This interpretation is similar to the interpretation of the twistor sphere of $SU(3)/U(1) \times U(1)$ as a characterizer of the color quantization axes. For many-quark states a common quantization axis would force the spins to be parallel or antiparallel. The sum of spins associated with different momenta as different points of twistor space would be the sum of these spins.

The special twistorial role of quarks as spin 1/2 particles supports the idea that the construction of scattering amplitudes should be reduced to quark level although the physical states are Galois singlets. The situation would be very similar to that in QCD, where the challenge is to understand how the scattering amplitudes between hadrons are constructible in terms of scattering amplitudes for quarks and gluons. The basic problem in QCD is that a mechanism for the formation of bound states is missing: in TGD it is provided by Galois confinement.

The basic assumption is therefore that the quarks in M^8 are on-mass-shell states with $m^2 = r_n$. If Galois singlets were regarded as fundamental objects, one would encounter problems with the description of spin degrees of freedom. Situation is essentially the same as in hadron physics.

One can speak about Galois singlet states as a generalization of super-field but without Majorana conditions with oscillator operator monomials replacing the components of superfield: Galois singlets having quark momenta with parallel rational components would in this sense propagate linearly. Each quark Dirac operator $p^k\gamma_k$ is added to the vertex and is expressible in terms of a pair of holomorphic quantities $< .. >$ and [..] which are independent for massive quarks.

7.3.3 Twistor amplitudes using only mass shell M^8 momenta as internal lines

The simplest proposal for the twistor amplitudes assignable to single 4-surface assumes that the physical particles correspond to Galois singlets with integer valued momentum components p_i and integer valued mass squared spectrum. The components of quark momenta in M^8 would be algebraic integers.

$M^8 - H$ duality requires that physical states in M^8 and H correspond to each other and have the same mass and momentum spectrum. A stronger form of $M^8 - H$ duality would force the identification of the quark momenta in M^8 and H. Quark momenta would be virtual momenta. If the coupling to M^4 Kähler potential is not present, the twistor holomorphy is achieved if spinor modes satisfy $D(M^4)\Psi = 0$.

What could be the basic assumptions?

The following summarize the assumptions, which look plausible.

1. All quark states in both H and M^8 are on-mass-shell states with momenta which are algebraic integers in the extensions determined by polynomial P determining the quark mass shells $m^2 = r_n$ as its roots. Momenta for Galois singlets could also be rationals but periodic boundary conditions allow only integers.

   The physical states are Galois singlets with integer valued momenta in a given p-adic length scale. Mass squared values are integers and one obtains a stringy mass squared spectrum. By $M^8 - H$ duality the spectra at M^8 and H sides are identical.

2. The analog of the idea that the scattering amplitudes are poles of residue integral in momentum space is adopted. This means that in M^8 the purely algebraic 4-D quark Dirac operators $D(M^4)$, rather than propagators as in Feynman diagrams, act on the vertex defined by the trilinear of 3 Galois singlets (particles do not propagate in momentum space as they do in x-space!). The Galois singlets have an interpretation as representations of super-symplectic algebra.

The Galois singlet with total momentum $P_i = \sum p_{i,k}$ corresponds to H-state and the two other Galois singlets corresponds to states with area momenta X_i, X_{i+1} having similar decompositions $X_k =\sim x_{i,r}$ in terms of in general complex algebraic integer valued area momenta x_i. The complex on-mass-shell area momenta are analogous to the complex on-mass- shell light-like virtual momenta in the twistor Grassmann approach.

3. The total momentum of the vertex is conserved and gives a constraint on the quark momenta associated with the 3 states. In each vertex one has sum over all possible quark momenta consistent with the Galois singlet property and the structure of the state. Momentum conservation at vertex does not make sense at quark level since fermion number conservation would fail unless one introduces fundamental bosons.

   Momentum conservation constraints $P_i = X_{i+1} - X_i$, which completely fixes the momentum exchanges as $2X_i \cdot X_j = P_i^2 - X_{i+1}^2 - X_i^2 - 2(X_i - X_j)^2$. Momentum conservation implies in ZEO that one can see scattering diagrams as polygons having momenta at mass shells at the half-light-cones of M^8.

4. An essential constraint is that the rational parts of the area momenta x_i are parallel to each other. This gives rise to an analogy with supersymmetry in which one could regard the higher components of the super field as parallelly propagating Majorana fermions.

5. The propagator lines correspond in M^8 to vertex factors with the analog of $D = x_i^k \gamma_k$ acting on Galois singlet i. This would mean that one has a residue of the Feynman propagator. By adding a multiplicative factor m^2, one could equally well use Feynman propagator $1/D = D/m^2$, where $m^2 = r_n$ is quark mass squared. The number of diagrams is limited by the number of roots and only the number of Galois singlets poses a limit to the summation if one considers only amplitudes for a single surface X^4.

   In principle all pairs of Galois singlets in M^8 with a non-vanishing trilinear overlap with a given Galois singlet in H are allowed in the vertex. Note that same Galois singlets can contain quarks assignable to different quark mass shells $m^2 = r_n$.

6. The details of the algebraic extension are not visible in the properties of Galois singlets as analogs of hadrons. The details of algebraic extension are however visible in the details of quark propagators and give rise to a number theoretic coupling constant evolution as will be found. Also the increase of the dimension of extension with the degree of P implies that the number of contributing diagrams increases.

   In principle, also roots r_n with negative rational parts are possible and one cannot exclude tachyonic states. From tachyonic states one can form non-tachyonic ones by requiring that the 3-momenta sum up to zero.

7. The big difference with respect to standard massive QFTs is that although the states are massive, they propagate with well-defined helicities. There is therefore a doubling of helicity spinors appearing as L-R degeneration. The division to positive and negative helicities corresponds to the presence of quarks and antiquarks.

8. It seems that quarks and antiquarks can correspond to the same CD and to the same diagram of the proposed kind. For a single space-time surface BCFW construction does not make sense since it would require an off-mass-shell H particle. One must however notice that the quark propagators bring in mind the $1/P^2$ lines connecting BCFW sub-diagrams and Galois singlets bring in mind the MHV diagrams.

   Can one construct Galois singlets from both quarks and antiquarks? It would seem that in this case the scattering amplitudes involve products of holomorphic and antiholomorphic monomials of the twistor variables. This option looks intuitively more plausible.

A possible solution of the mass problem

The basic problem of the twistor approach is that physical particles are not massless. The intuitive TGD based proposal has been that since quark spinors are massless in H, the masslessness in the 8-D sense could somehow solve the problems caused by the massivation in the construction of twistor scattering amplitudes.

1. The first key observation stimulated by the recent findings about right-handed neutrino candidate [L65] was that although neutrinos are massive, their right-handed component has not been observed. This leads to a proposal that in H quarks should propagate with well-defined chiralities so that only the square of Dirac equation $D^2(H)\Psi = 0$ is satisfied.

2. At the level of M^8 the octonionic M^4 quark spinor reducing to a quaternionic spinor corresponds to H spinors. A spinor with a given chirality can be identified as a helicity spinor λ_{dota} and is annihilated by the operator $p^{a\dot{b}} = \mu^a \lambda^{\dot{a}}$. This makes sense by the fact that in the TGD Universe quarks are the only fundamental particles implying that all other particles, including elementary particles, emerge as their many particle states as Galois singlets.

   The M^8 counterpart of the 8-D massless condition in H is the restriction of the quark momenta to mass shells $m^2 = r_n$ determined as roots of P. The M^8 counterpart of Dirac equation in H is octonionic Dirac equation, which is algebraic. The solution is a helicity spinor $\tilde{\lambda}$ associated with the massive momentum p.

What about tachyons?

Polynomials P allow also roots r_n, which are negative and correspond to tachyonic mass shells. Should one restrict the roots inside the future light-cone? Should one require that the mass squared values of the masses of Galois singlets are non-negative integers? In principle, one can have integer valued momenta with tachyonic mass squared. The sum of this kind of momenta however gives always a non-tachyonic state if the energies are of the same sign as they are for a given half-light-cone.

1. M^4 Kähler structure implies that covariantly constant right- handed neutrino in CP_2 is a tachyon [L65]. This gives rise to the highly desired tachyon required by p-adic mass calculations [K42, K17]: with it the scale of mass spectrum would be huge and given by CP_2 mass. Tachyonic property is not consistent with the unitarity and ν_R cannot appear as a free particle.

2. Situation remains the same if the right-handed neutrino spinor mode is a good approximation for a Galois and color singlet of 3 quarks assignable to the same wormhole throat in H. ν_R as Galois singlet with tachyonic mass can be understood if tachyonic mass squared values are allowed for quarks.

   Could all quark masses could be tachyonic? Could this explain quark confinement? By generalizing slightly, also complex mass squared values for quarks could be seen as tachyonic so that Galois confinement would be essentially quark confinement.

3. A long-standing question has been whether ν_R could generate $N = 2$ SUSY. It seems that the tachyon property does not allow the analog of ordinary SUSY. States without ν_R would have huge masses of order CP_2 mass. One can also say that $calN = 2$ SUSY is broken in CP_2 scale.

Is the proposed picture consistent with coupling constant evolution?

Can one understand the discrete number theoretic coupling constant evolution in the proposed framework? As the number of roots of P increases, the number of scattering diagrams with N external particles with fixed momenta p_i increases since the number of Galois confined states characterized by mass shells $m_i^2 = n_i$ increases.

The number of diagrams contributing to the scattering increases and it becomes possible to speak about number theoretical coupling constant evolution. Otherwise the dependence on polynomials P is rather weak and brings in mind logarithmic coupling constant evolution replaced in TGD by discrete p-adic length scale evolution.

How does this relate to the p-adic coupling constant evolution and p-adic length scale hypothesis $p \simeq 2^k$, k some selected integer? For instance, could the p-adic primes preferred by a given extension correspond to the ramified primes of the extension dividing the product $\prod_i (r_i - r_j)$?

1. The dimensionless roots of $P(x)$ are of the form $r_n = R_n/M_p$, where R_n is the dimensional root of $P(M_p x)$. M_p would define the p-adic mass scale and the p-adic length scale of the corresponding CD. This would suggest that p-adic coupling constant evolution is not related to number theoretic coupling constant evolution.

2. On the other hand, the scattering amplitudes depend on the p-adic scale of the momenta. The reduction of scattering amplitudes to homogeneous functions of the factors $p_i \cdot p_j$ appearing in propagator denominators implies very simple dependence on momenta and the characteristic logarithmic dependence is absent. Does this mean that there should be a correlation between the p-adic length scale and algebraic extension? Why should a given extension prefer some p-adic primes, say ramified primes?

3. What about the vertices between Galois singlets, which involve a trilinear of an on-mass-shell state in H and two M^8 off-mass-shell states? How does the p-adic mass scale manifest itself in the properties of these Galois singlets? The conditions for Galois singlet property are scale invariant and the scale invariance is only broken by the condition that mass squared values are roots of polynomial P.

   M^8-H duality suggests the identification of the discriminant D of the polynomial as an exponent $exp(-K)$ of Kähler function defining vacuum functional and the identification of p-adic prime as a ramified prime dividing D. The real mass squared value would be determined by the canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$ for ramified prime and depend on P.

4. p-Adic physics depends on the value of p-adic prime p. Could this bring in the p-adic coupling constant evolution and preferred p-adic primes number theoretically? The dimension of extensions of p-adics induced by a given extension of rationals depends on p since some roots exist as ordinary p-adic numbers. If p-adic physics as physics of cognition is essential also for real physics as p-adic mass calculations [K42, K17] suggest, it could force the natural selection of preferred p-adic primes and p-adic length scale evolution.

5. Only the identification of the preferred p-adic primes as ramified primes of extension comes into mind. What could make them so special? The p-adic variant of the polynomial P has a double root in order $O(p) = 0$ for a ramified prime. Double root is the mathematical counterpart of criticality and quantum criticality indeed is the basic dynamical principle of TGD. Could something which is of order $O(p^0)$ become order $O(p)$ for a ramified prime? The roots of P correspond to mass squared values: one would have $m_1^2 - m_2^2 = r_1 - r_2 = O(p)$ p-adically.

   For instance, could it be a generic mass squared scale defined by the difference $m_1^2 - m_2^2$ reduces from $M^2(CP_2)$ to $M(CP_2)^2/p$ for ramified primes or p-adic mass scale $M_p = M^2(CP)/p$ reduces to secondary $\mathrm{p}_a dicmassscale \mathrm{M}_{p,2} = M^2(CP)/p^2$. Could the interpretation be in terms of emergence of a massless excitation as counterpart of quantum criticality. Kind of number theoretic analog of Goldstone boson.

   There is some support for this idea. In the living matter, the 10 Hz biorhythm is fundamental. It corresponds to the secondary p-adic length scale of the electron characterized by Mersenne prime $M_{127} = 2^{127} - 1$ [K42]. 10 Hz biorhythm could correspond to a kind of Goldstone boson. This argument still leaves open the question why ramified primes near powers of 2 (or of a small integer such as 3 [**?**, **?**]) should be so special?

6. One can even speculate with the possibility that a kind of natural selection takes place already at this level. A high number of zero energy states could be possible for Galois singlet states associated with very special polynomials. In the functional composition $P_1 \circ P_2$ of polynomials conservation of roots takes place if the condition $P_i(0) = 0$ is satisfied. This could make possible evolutionary hierarchies in which conserved roots would be analogous to conserved genes.

An open challenge is to formulate a precise criterion fixing what diagrams are allowed. The intuitive picture is that the lines of the diagrams connecting mass shells $m_i^2 = n_i$ diagrams define convex polygons.

7.3.4 How can one include the WCW degrees of freedom?

The above consideration has been restricted to a single cognitive representation defined by a polynomial P. Already the inclusion of color degrees of freedom requires color partial waves in H and the superposition over space-time surfaces related by color rotation and therefore WCW spinor fields.

"Objective" and "subjective" representations of physics

The usual understanding of Uncertainty Principle (UP) requires that one has a WCW spinor field providing for instance the analogs of the plane waves in the center of mass degrees of freedom for 3-surface. This representation at the level of WCW might be called "objective" representation since one looks at the system from the H or WCW perspective. The localization of particles to the space-time surface violates UP in this "objective" sense.

Discrete cognitive representations define in ZEO what might be called a "subjective" representation of the Poincare and color group since one looks at the system from the perspective of a single space-time surface.

1. The "subjective" representations of isometries would be realized as flows inside X^4 rather than in H. The flows would be defined by the projections of Killing vectors on the space-time surface [L68].

2. The "subjective" representation is actually highly analogous to quantum group representation. For instance, for many-sheeted space-time surface, rotation by 2π would not bring the particle to a different space-time sheet and one would obtain charge fractionalization closely related to the hierarchy of many-sheeted structure corresponding to $h_{eff}/h_0 = n$ hierarchy were n is the dimension of the extension of rationals determined by the polynomial P. This representation could be restricted to Cartan algebra and does not require a 2-D system since the Cartan algebra effectively replaces the 2-D system.

3. The notion of "subjective" representation allows to generalize the gravitational and inertial mass to all conserved charges. Inertial charges would relate to the action in H and gravitational charges to the quantum group charges for flows restricted to $X^4 \subset H$. $M^8 - H$ duality indeed maps the momenta at mass shells associated with $X^4 \subset M^8$ to positions at the boundaries of CD and the action of Lorentz symmetries keeps the image points at the boundaries of CD.

Is WCW needed at the level of M^8?

The inclusion of WCW degrees of freedom is necessary for several reasons. WCW provides the "objective" perspective extending the "subjective" perspective provided by scattering amplitudes at a single space-time surface. Also the understanding of classical physics as an exact correlate of quantum physics requires WCW.

WCW has been introduced at the level of H and the question whether the notion of WCW makes sense also at the level of M^8, has remained open for a long time.

It is now clear that the polynomials P alone determine only the mass shells as their roots [L68]. Could the adelization and p-adization alone serve as the counterpart of WCW for M^8?

On the other hand, the interiors of 4-surfaces in M^8 involve the local CP_2 element and at the mass shells one has a local $S^2 = SO(3)/SO(2)$ element. Hence WCW might be realized at both sides as $M^8 - H$ duality suggests. An interesting conjecture is that by $M^8 - H$ duality, the two WCWs are one and the same thing. Therefore it would seem that adelization does not provide the counterpart of WCW in M^8.

Summation over polynomials as M^8 analog for the WCW integration

What could be the "cognitive" M^8 analog of WCW and integration over WCW?

1. The preferred extremal property of space-time surface $X^4 \subset H$ means that it is defined by its intersections with the boundary of CD. $M^8 - H$ duality requires that this is the case also

in M^8. This would mean that the polynomial P determines, not only the 3-D mass shells of selected M^4 as its roots contained in $X^4 \subset M_c^8$, but also the 4-surface as an $SU(3)/U(2)$ local deformation of M^4 containing them and mapped to H by $M^8 - H$ duality.

2. In the full theory, one has integration over WCW spinor fields. Number theoretical approach means number theoretically unique discretization using cognitive representation rather than its "active" points (containing quark) defining a representation of the Galois group.

   The natural proposal is that WCW integration reduces to a summation over some subset of polynomials and amplitudes associated with the corresponding cognitive representations for which the area momenta for quarks are algebraic integers. External momenta would be ordinary integers for a given p-adic prime p. Therefore the summation over polynomials of varying degree makes sense for amplitudes with fixed external momenta if one uses extension of rationals containing all extensions defined by the polynomials.

3. The rational coefficients of polynomials would serve as WCW coordinates for the polynomials. The assumption that they are rational, however, creates a problem since the summation over rationals defining the coefficients understood as real numbers does not define an analog of integration measure.

One can imagine two number theoretical solutions of the problem: both are inspired by p-adic thermodynamics [K50, K34].

1. One manner to overcome the problem would be a restriction of the coefficients of P to integers. This is natural if the polynomials are monic polynomials of the form $x^n + an - 1x^{n-1} + ...$ This would mean a loss of scaling invariance since $P(kx)$ is not a monic polynomial. The good news is that this might select preferred p-adic primes and explain even the p-adic length scale hypothesis.

2. For a monic polynomial of degree n, the summation would reduce to a summation over $n-1$ integers. The roots would be powers of a single generating root r_0 giving rise to a basis for algebraic integers, and one would have fractility since the quark mass shells correspond to the powers for the modulus of the generating root. The moduli for the differences of roots would be proportional to the power of the modulus of the root and it would be natural to assign p-adic prime to the root with the smallest modulus. This option is highly attractive both physically and mathematically.

3. One expects a rapid p-adic convergence in the sense that polynomials with coefficients, which differ by a large power of p give to scattering amplitudes p-adically very similar contributions. The sum over these contributions should converge rapidly.

   It would seem that the exponent of Kähler function must enter into the picture and give rise to something resembling p-adic thermodynamics with the Boltzmann weight $exp(-E/T)$ being replaced with p-adic number p^{S/T_p}, where the p-adic temperature T_p is inverse integer and S is integer valued. p-Adic number p^{S/T_p} would correspond to the exponent $exp(-K)$ of Kähler function for the H imasage of the surface associated with P. Canonical identification would map p^{S/T_p} to its p-adic norm p^{-S/T_p} identified with $exp(-K)$.

4. The values of S/T_p correspond to the maxima of the Kähler function K for preferred extremals. These exponents exist p-adically only if the value of Kähler coupling strength α_K as an analog of inverse of a critical temperature satisfies strong number theoretic conditions reducing the exponent to an integer power of p (unless one assumes that also the roots of p can appear in the extension considered). These conditions would give rise to a p-adic coupling constant evolution for α_K and also to a coupling constant evolution as a function of algebraic extension.

5. One expects that these conditions can be satisfied only in a very restricted subset of preferred extremals so that one should assume a localization of WCW spinor field to a subset of maxima of the Kähler function. TGD is analogous to a complex square root of thermodynamics and this kind of localization takes place quite generally (spontaneous magnetization) in thermodynamics and also in quantum field theories (Higgs mechanism).

For spin glass discussed from the TGD point of view in [L67], this kind of localization occurs also and in the ultrametric topology of the spin glass energy landscape emerges naturally. p-Adic topologies represent basic examples about ultrametric topologies. The TGD inspired proposal indeed is that p-adic thermodynamics [K50, K42] allows the formulation of spin glass thermodynamics free of ad hoc assumptions.

TGD is Universe is indeed highly analogous to a spin glass in long scales, where the action approaches Kähler action having a huge vacuum degeneracy involving classical non-determinism as the length scale dependent cosmological constant Λ predicted by the twistor lift [L24, L35] approaches zero. An attractive proposal is that this kind of localization has a purely number theoretic origin making p-adic thermodynamics for a suitably chosen value of α_K possible [L67].

6. Also the summation over amplitudes associated with different polynomials of various degrees is in principle possible and could correspond to the summation appearing in perturbation theory and to the summation appearing in p-adic thermodynamics.

One cannot exclude a more general option in which there is a summation over all polynomials with rational coefficients analogous to the summation over the valleys of the energy landscape for spin glass phase.

1. For general rational polynomials, one would have a scaling invariance $P(x) \rightarrow P(kx)$. There would be a summation over scaled roots of P and rationally scaled mass shells. For monic polynomials the scaling invariance is lost and this seems the only realistic possibility.

2. One might hope that the summations over rationals assigned to the coefficients of P with fixed degree reduce to a p-adic integration and that a p-adic integration measure for this integral exists and reduces essentially to summation over p-adic integers with a given norm p^k plus to a summation over the norms p^k at the limit when the norm approaches infinity (`https://cutt.ly/UUbit6f`). Here the problem is that there is no natural lower bound on the p-adic norm of the coefficients as for monic polynomials and the integral need not converge.

The restriction to monic polynomials looks highly attractive. Another possible restriction is that polynomials are proportional to x so that the roots of P are also the roots of the functional composite $P \circ Q$. This restriction might be also an outcome of a number theoretical evolution.

M^8 analog of vacuum functional

The vacuum functional as an exponent of the Kähler function determines the physics at WCW level. $M^8 - H$ duality suggests that it should have a counterpart at the level of M^8 and appear as a weight function in the summation. Adelic physics requires that weight function is a power of p-adic prime and ramified primes of the extension are the natural candidates in this respect.

1. The discriminant D of the algebraic extension defined by a polynomial P with rational coefficients (`https://en.wikipedia.org/wiki/Discriminant`) is expressible as a square for the product of the non-vanishing differences $r_i - r_j$ of the roots of P. For a polynomial P with rational coefficients, D is a rational number as one can see for polynomial $P = ax^2 + bx + c$ from its expression $D = b^2 - 4ac$. For monic polynomials of form $x^n + a_{n-1}x^{m-1} + ...$ with integer coefficients, D is an integer. In both cases, one can talk about ramified primes as prime divisors of D.

   If the p-adic prime $p$is identified as a ramified prime, D is a good candidate for the weight function since it would be indeed proportional to a power of p and have p-adic norm proportional to negative power of p. Hence the p-adic interpretation of the sum over scattering amplitudes for polynomials P is possible if p corresponds to a ramified prime for the polynomials allowed in the amplitude.

   p-Adic thermodynamics [K42] suggest that p-adic valued scattering amplitudes are mapped to real numbers by applying to the Lorentz invariants appearing in the amplitude the canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$ mapping p-adics to reals in a continuous manner

2. For monic polynomials, the roots are powers of a generating root, which means that D is proportional to a power of the generating root, which should give rise to some power of p. When the degree of the monic polynomial increases, the overall power of p increases so that the contributions of higher polynomials approach zero very rapidly in the p-adic topology. For the p-adic prime $p = M_{127} = 2^{127} - 1 \sim 10^{38}$ characterizing electrons, the convergence is extremely rapid.

   Polynomials of lowest degree should give the dominating contribution and the scattering amplitudes should be characterized by the degree of the lowest order polynomial appearing in it. For polynomials with a low degree n the number of particles in the scattering amplitude could be very small since the number n of roots is small. The sum $x_i + p_i$ cannot belong to the same mass shell for timelike p_i so that the minimal number of roots r_n increases with the number of external particles.

3. $M^8 - H$ duality requires that the sum over polynomials corresponds to a WCW integration at H-side. Therefore the exponent of Kähler function at its maximum associated to a given polynomial should be apart from a constant numerical factor equal to the discriminant D in canonical identification.

   The condition that the exponent of Kähler function as a sum of the Kähler action and the volume term for the preferred extremal $X^4 \subset H$ equals to power of D apart from a proportionality factor, should fix the discrete number theoretical and p-adic coupling constant evolutions of Kähler coupling strength and length scale dependent cosmological constant proportional to inverse of a p-adic length scale squared. For Kähler action alone, the evolution is logarithmic in prime p since the function reduces to the logarithm of D.

$M^8 - H$ duality suggests that the exponent $exp(-K)$ of Kähler function has an M^8 counterpart with a purely number theoretic interpretation. The discriminant D of the polynomial P is the natural guess. For monic polynomials D is integer having ramified primes as factors.

There are two options for the correspondence between $exp(-K)$ at its maximum and D assuming that P is monic polynomial.

1. In the real topology, one would naturally have $exp(-K) = 1/D$. For monic polynomials with high degree, D becomes large so that $exp(-K)$ is large.

2. In a p-adic topology defined by p-adic prime p identified as a ramified prime of D, one would have naturally $exp(-K) = I(D)$, where one has $I(x) = \sum x_n p^n = \sum x_n p^{-n}$.

   If p is the largest ramified prime associated with D, this option gives the same result as the real option, which suggests a unique identification of the p-adic prime p for a given polynomial P. P would correspond to a unique p-adic length scale L_p and a given L_p would correspond to all polynomials P for which the largest ramified prime is p.

   This might provide some understanding concerning the p-adic length scale hypothesis stating that p-adic primes tend to be near powers of integer. In particular, understanding about why Mersenne primes are favored might emerge. For instance, Mersennes could correspond to primes for which the number of polynomials having them as the largest ramified prime is especially large. The quantization condition $exp(-K) = D(p)$ could define which p-adic primes are the fittest ones.

The condition that $exp(-K)$ at its maximum equals to D via canonical identification gives a powerful number theoretic quantization condition. Is this condition realized for preferred extremals as extremals of both Kähler action and volume term, or should one regard these conditions as additional conditions?

1. P fixes only the mass-shells as its roots r_n. The real parts of these roots belong to the same M^4. $M^8 - H$ duality is realized by assuming that the mass shells are connected by a 4-surface X^4, which is a deformation of M^4 by a local $SU(3)$ element $g(x)$ such that the subgroup $U(2)$ leaves the points of deformation invariant: this condition gives rise to an explicit form of $M^8 - H$ duality.

   P itself poses no conditions on the local CP_2 element. Could the condition $exp(-K) = I(D)$ for the image of $X^4 \subset M^8$ in H fix the $g(x)$ and thus $X^4 \subset H$?

2. The twistor lift should determine the surface $X^4 \subset H$. The counterpart of twistor lift is defined also at the level of M^8. It maps 6-D surface connecting 5-D mass shells of M^8 as roots of P identified as a local $SU(3)$ deformation of M^6 remaining invariant under $U(1) \times U(1)$ at each point. Hence a point of CP_2 twistor space is assigned to M^6 identified locally as a point of M^4 twistor space.

   One can assign to the twistor space of X^4 as 6-surface $X^6 \subset T(M^4) \times T(CP_2)$ 6-D Kähler action reducing to 4-D Kähler action plus volume term by a dimensional reduction required by the bundle property. One can define the twistorial variant of WCW with the Kähler function K_6 defined by the 6-D Kähler action for X^6. The vacuum functional $exp(-K_6)$ would be the same as for WCW.

   Since S^2 degrees are non-dynamical, the two WCWs are more or less one and the same thing apart from delicacies of non-trivial windings numbers for the maps from the fiber S^2 of $T(X^4)$ to the fibers of $T(M^4)$ and $T(CP_2)$.

3. The $U(2)$ *resp.* $U(1) \times U(1)$ invariant points of the deformation of M^4 *resp.* M^6 would define X^4 *resp.* its twistor space $T(X^4)$. The condition that the image of the deformed M^6 is a preferred extremal of 6-D Kähler action, should determine $g(x)$. $I(D) = exp(-K)$ fixes the 6-D Kähler action action.

4. The formulation of the variational problem in H as a variational problem in $M^4 \subset M^8$ might provide some insight. The 6-D Kähler action for $X^6 \subset H$ naturally assigns an action to the deformed $M^6 \subset M^8$. At the level of M^8, the quantization condition $exp(-K) = I(D)$ plus the boundary conditions defined by the roots of P would select $X^6 \subset M^8$ as a preferred extremal of 6-D Kähler action. This condition could also induce a natural selection of p-adic primes explaining p-adic length scale hypothesis.

The evolution of α_K and of cosmological constant from number theory?

I have considered earlier the evolution of cosmological constant [L3, L24, L35] but it is interesting to look at it in a more detail from the number theoretic perspective.

1. There are three parameters involved: Kähler coupling strength α_K and the winding numbers n_1 and n_2 for the maps of the twistor sphere $T(X^4)$ of $X^4 \subset H$ to the twistor spheres $S^2(M^4)$ and $S^2(CP_2)$ associated with the twistor spaces $T(M^4)$ and $T(CP_2)$: these maps essentially identify the latter twistor spheres.

2. The 6-D Kähler action for $X^6 = T(X^4) \subset T(M^4) \times T(CP_2)$ is proportional to Kähler coupling strength and the scale factor $1/R^2$, which is equal to CP_2 radius squared. The recent interpretation is that CP_2 radius corresponds to the Planck length L_{Pl} scaled up by h_{eff}/h_0. So that for $h_{eff} = h_0$, the CP_2 radius would reduce to Planck length apart from a numerical constant.

3. Dimensional reduction is necessary in order that X^6 has the structure of the induced twistor bundle with $X^4 \subset H$ as a base-space. This requires maps of the twistor sphere S^2 of the twistor space $T(X^4)$ of $X^4 \subset H$ to the twistor spheres $S^2(M^4)$ and $S^2(CP_2)$: this map identifies these twistor spheres locally.

4. Dimensional reduction gives rise to the usual 4-D Kähler action and a volume term with a cosmological constant Λ determined by the Kähler action for the S^2 part of 6-D Kähler action. The induced Kähler form in S^2 is the sum of the contributions from $S^2(M^4)$ and $S^2(CP_2)$.

   Unless the winding numbers of the maps differ from unit, the induced Kähler form is zero or twice the Kähler form of $S^2(CP_2)$ depending on the relative sign of the Kähler forms, whose normalization is fixed by the condition that the magnetic flux is quantized to unity. The form of the maps in spherical coordinates (θ, ϕ) for $S^2(X^4)$ is given by $\theta(M^4) = \theta(CP_2) = \theta$ and $\phi(M^4) = n_1\phi$ and $\phi(CP_2) = n_2\phi$.

5. If the winding numbers n_i are different and of opposite sign (assuming the same sign for Kähler forms), the induced Kähler form is given by $J = (n_2 - n_1)J(S^2(CP_2))$, where n_i are positive.

   The induced line element is $ds^2 = d\theta^2 + sin^2(\theta)(n_1^2 + n_2^2)\phi^2$. The determinant $\sqrt{g}$ of the induced metric of S^2 is $\sqrt{g} = \sqrt{n_1^2 + n_2^2}\sqrt{g(CP_2)}$. The contravariant induced Kähler form is given by

$$J^{\theta\phi} = \frac{g^{\theta\theta}g^{\phi\phi}}{J}_{\theta\phi} = (n_1 - n_2)/n_1^2 + n_2^2 J^{\theta\phi}(CP_2) \ . \tag{7.3.1}$$

   The Kähler action for S^2 is given by

$$J^{\theta\phi}J_{\theta\phi}\sqrt{g} = \frac{n_1 - n_2}{\sqrt{n_1^2 + n_2^2}} J^{\theta\phi}(CP_2)J_{\theta\phi}(J(CP_2))\sqrt{g(CP_2)} \ .$$

   For small values of $n_1 - n_2$ and large values of $n_1 \sim n_2$ the contribution to action behaves like $\Delta n/n_1$ and can become arbitrarily small. This would predict that cosmological constant approaches to zero in long p-adic length scales.

   This poses a condition on the integers n_i depending on the p-adic prime p identified as a ramified prime: $\Delta n/n_1$ should behave like the inverse of the p-adic length scale scale. The p-adic length scale evolution of both α_K and integers n_i should follow from the condition that the total action equals to the discriminant D (also a polynomial of discriminant can in principle be considered but this seems artificial). The best one can hope is that $M^8 - H$ duality completely fixes both coupling constant evolutions.

6. For the cosmological constant Λ in cosmological scales, the dark energy density is parameterized as $\rho_{vac} = 1/L^4$, $L \sim L_{neuron}$, where $L_{neuron} \simeq 10^{-4}$ m corresponds to the size scale of neuron.

   This rough estimate follows from the identification $\Lambda/8\pi G = 1/L^4$ giving $L(8\pi G/\Lambda)^{1/4}$. Λ itself would correspond to an inverse of p-adic length square, which is of order of the horizon size (naturally the size of cosmological CD).

Do Grassmannians emerge at the QFT limit of TGD?

There is no obvious use for Grassmannians and related concepts in the construction of twistor amplitudes for a space-time surface associated with a given polynomial P.

However, a given scattering amplitude is a sum of contributions associated with monic polynomials P with an increasing number of roots such that a given p-adic prime p appears as their ramified prime. The discriminant D is assumed to play the role of the vacuum functional $exp(-K)$. This picture is highly analogous to a perturbation theory in a given p-adic length scale.

This suggests that QFT with massless particles is a reasonable approximation of TGD at the QFT limit and that the basic twistorial structures could appear at this limit.

Apart from masses given by p-adic thermodynamics [K42, K17], elementary particles, to be distinguished from fundamental quarks, correspond to massless states so that massless QFT is a good guess for the QFT limit.

The emergence of the massless states requires M^4 Kähler structure forced by the twistor lift [L65]. This breaks the Lorentz symmetry to that of $M^2 \times E2$ and the transversal degrees of freedom correspond to harmonic oscillator type degrees of freedom just as in string model and are characterized by two conformal weights. This spontaneous breaking of Lorentz symmetry characterizes massless particles and hadronic quarks. It makes possible the required tachyonic ν_R making it possible to construct massless ground states in p-adic mass calculations.

1. In M^8, the mass shells in general correspond to complex roots. It is possible to have tachyonic Galois confined states. Covariantly constant right-handed neutrino ν_R would be such a state and needed to construct massless Galois confined physical states in H.

2. In H, only the ν_R constructed from quarks is tachyonic in the approximation that H-spinor mode with Kähler charge $Q_K = 3$ describes leptons as 3-quark Galois singlets. $M^8 - H$ duality suggests that there are no other tachyonic quark states and that all Galois confined states are non-tachyonic so that the momenta belong to the interior of the light-cone in M^8.

3. If the amplitudes in the massless sector are indeed Yangian invariants, Grassmannians would emerge naturally at the QFT limit.

The following series of questions is an attempt to crystallize my ignorance.

1. Could a QFT based on twistorial amplitudes for massless Galois confined external particles in H provide a QFT limit of TGD?

2. Could the sum over amplitudes for different polynomials having a given p-adic prime p as a ramified prime correspond to structure resembling that produced in BCFW recursion?

3. Or could MHV structure emerge at the level of a single polynomial P: this is the case if the quark propagators connecting Galois singlets in the amplitudes be regarded as analogs of the propagators $1/P^2$ connecting parts of MHV amplitudes?

4. How the coupling constant evolution emerges at the QFT limit. Number theoretic approach does not allow logarithmic contributions coming from loops but it would not be surprising if the discrete p-adic coupling constant evolution would allow a logarithmic coupling evolution as a reasonable approximation.

   This is also suggested by the fact that the expression of α_K in terms of discriminant D involves logarithm of the p-adic length scale (, that is p). If $exp(-K)$ equals to the image $I(D)$ under canonical identification, one has $\alpha_K = S/log(I(D))$, where $S = K\alpha_K$ is the total action without the proportionality factor $1/\alpha_K$. For ramified primes α_K is proportional to $1/log(p)$.

7.3.5 What about the twistorialization in CP_2 degrees of freedom?

The proposed picture does not use CP_2 twistor space at all. One should understand why this is the case.

The treatment of color degrees of freedom involves several new aspects. First of all, color is not a spin-like quantum number in the TGD framework.

1. One can identify colored states as color partial waves in WCW degrees of freedom associated with the center of mass degrees of freedom of 3-surface. H spinor modes can be indeed regarded as color partial waves in H.

   It would seem that one cannot speak of color for a single space-time surface. This is indeed true for an "objective" view about the isometries of H. One can however define the "subjective" representations of the isometries by replacing them with flows defined by the projections of Killing vectors to the space-time surfaces [L68].

   For cognitive representations the "subjective" representations could in some situations be reduced to those for the discrete Galois group. One can wonder whether color confinement could reduce to Galois confinement.

2. "Subjective" representations are analogous to quantum group representations [L68]. Objective-subjective dichotomy coud also generalize the inertial-gravitational dichotomy. Note that one can also assign Noether charges to the projected flows. This applies also to supersymplectic symmetries.

The treatment of CP_2 degrees of freedom for the twistor amplitudes remains a challenge and in the following I can only try to clarify my thoughts.

1. Twistor lift strongly suggests that $M^8 - H$ duality defines a map of the twistor spaces of H and M^8 to each other. The M^8 counterparts of 6-D twistor space as a surface $X^6 \subset T(M^4) \times T(CP_2)$ would be 6-D surface with a commutative normal space defined by a deformation of complexified Minkowski space M^6 by a local $SU(3)$ element, which is left-invariant under $U(1) \subset U(1)$. This would give a 6-surface Y^6 as a counterpart of the 6-surface. It would seem that M^6 should correspond to the twistor $T(M^4)$, perhaps via the identification with a projective space of M^8 by 2-D projective scalings (perhaps by hypercomplex numbers).

2. This map would preserve S^2 bundle structure so that the twistor spheres of $T(M^4)$ and $T(CP_2)$ would be mapped to each other. This looks strange at first but conforms with the general picture.

   At the level of $T(H)$ twistor wave functions at the twistor spheres S^2 of $T(M^4)$ and $T(CP_2)$, which have been identified, describe spin and color or electroweak quantum numbers (the holonomy group of the spinor connection of CP_2 defining weak gauge group can be identified as $U(2) \subset SU(3)$). This implies a correlation for spin and electroweak spin doublets defined quarks apart from the sign factors.

   In the algebraic picture a single point of M^8 does not define only the momentum of quark momentum: rather quark momentum and spin corresponds to a single point of $X^6 \subset M^8$. Fermi statistics boil down to the condition that each point of X^6 can contain only a single quark. Also now directions of the quantization axis characterize the sign of spin and electroweak spin.

3. Spin-isospin correspondence makes sense only because quarks are both spin and weak isospin doublets. The fact that spin value $\pm 1/2$ corresponds to the two directions of the quantization axis allows all possible pairings of spin and electroweak (or color) isospin.

   This map between $T(M^4)$ and $T(CP_2)$ can be understood at M^8 level and generalizes the mapping of M^4 to CP_2 for a space-time surface with 4-D M^4 projection. There are 4-suraces X^4 for which the dimensions of the projections M^4 or CP_2 projection are not maximal. These 4-surfaces correspond to singularities in which normal space at the points of the singularity is not unique [L71].

   It is enough that the twistor spheres of $T(M^4)$ and $T(CP_2)$ are mapped to each other by locally 1-to-1 projection to the twistor sphere of $T(X^4)$: the base space of the twistor space X^6 need not have 4-D projection to M^4 or CP_2.

4. CP_2 twistors can be regarded as functions of M^4 twistors for a given space-time surface with 4-D M^4 projection. The implications for the construction of scattering amplitudes remain to be understood.

How color degrees of freedom are described at M^8 level? There are two equivalent ways to understand the emergence of CP_2 in $M^8 - H$ duality.

1. The normal spaces of $X^4 \subset M^8$ define an integrable distribution. Normal space of X^4 is regarded as a CP_2 point characterizing the deformation of fixed M^4 [L68, L52, L53] so that one obtains $M^8 - H$ duality.

   This distribution contains an integrable distribution of commutative 2-surfaces in turn defining a 6-D surface X^6, which is a good candidate for the counterpart of twistor space. The assignment of the normal space defines a point of the twistor space $SU(3)/U(1) \times U(1)$.

2. Second view [L68, L52, L53], which emerged only quite recently from the detailed study of the surfaces determined by polynomials P, is that the element of local $SU(3)$ naturally defines a deformation of X^4, which is invariant under left or right action by $U(2) \subset SU(3)$ so that local element of $CP(2)$ is in question. This means that color $SU(3)$ corresponds to a subgroup of the automorphism group G_2 of octonions. P as such does not etermines the local CP_2 element. What determines P, will be discussed later.

   The counterpart for the distribution of commutative normal spaces of X^6 is a deformation of M^6, or its variant with some signature of metric, defined by a local element of $SU(3)$ such that the imagine point remains invariant by $U(1) \times U(1) \subset SU(3)$ so that it assigns a point of the twistor space $SU(3)/U(1)U(1)$ to each point of X^6.

3. The equivalence of these views is not rigorously proven. Note that the polynomial P itself defines only 3-D complex mass shells as its roots and the 4-surface connecting them is determined from the condition that $M^8 - H$ duality makes sense.

There is an objection against CP_2 type extremal as a blow-up of 1-D singularity of $X^4 \subset M^8$. Is it really possible to describe CP_2 type extremal as 1-D singularity of $X^{\subset} M^8$ using the $U(2)$ invariant map $M^4 \to CP_2$?

1. The line singularity can be identified as an 1-D intersection of 2 Minkowskian space-time sheets as roots of P. At H level, this leads to a generation of wormhole contact with an Euclidean signature of metric, CP_2 type extremal, connecting the space-time sheets. The Minkowskian space-time becomes Euclidean at the wormhole throats.

2. At each point of 1-D curve L the singularity should be 3-D surface in CP_2. This requires that the normal space is non-unique and the normal spaces at a point x of L form a 3-D surface in CP_2. If one however thinks about how this could be achieved, one ends up with a problem. One can think that the images of an arbitrarily small sphere S^2 around the point of L is a sphere of CP_2. At the limit one would obtain 2-D rather than 3-D surface of CP_2.

3. The $U(2)$ invariant local $SU(3)$ transformation as a deformation of M^4 defining a local CP_2 transformation is not quite enough to describe the situation. The solution is to consider its inverse as a map from CP_2 to M^4 having a singularity at which a 4-D region of CP_2 is mapped to a line of M^4.

7.4 What about unitarity?

Unitarity is a poorly understood problem of the twistor approach and also of TGD.

7.4.1 What do we mean with time evolution?

The first questions relate to the identification of the TGD counterpart of S-matrix.

1. Zero energy states correspond to superpositions of pairs of ordinary 3-D states assignable to the opposite boundaries of CD. The simplest assumption corresponds to the idea about state preparation is that the states are unentangled. Unitarity would mean that the 3-D zero energy states at the active boundary of CD are orthogonal if the 3-D states at the passive boundary of CD are orthogonal. The scattering amplitudes considered in this article would naturally correspond to zero energy states. Is there any reason for zero energy states to satisfy this kind of orthogonality?

2. The time evolutions between "small" state function reductions (SSFRs) are assumed to increase the size of CD in a statistical sense at least and affect the states at the active boundary of CD but leave the "visible" part of the state at the passive boundary unaffected. These time evolutions are proposed to correspond to the scalings of CD rather than time translations. In this case unitarity would look a reasonable property.

   The sequence of (ordinary) "big" SFRs (BSFRs) could allow approximate description as being associated with unitary time evolutions with time translations rather than scalings and followed by BSFR changing the arrow of time. The characteristic features of these time evolutions would be polynomial and exponential decay and the relaxation of spin glass would be a key example about time evolution by SSFRs [L67].

7.4.2 What really occurs in BSFR?

It has been assumed hitherto that a time reversal occurs in BSFR. The assumption that SSFRs correspond to a sequence of time evolutions identified as scalings, forces to challenge this assumption. Could BSFR involve a time reflection T natural for time translations or inversion $I : T \to 1/T$ natural for the scalings or their combination TI?

I would change the scalings increasing the size of CD to scalings reducing it. Could any of these options: time reversal T, inversion I, or their combination TI take place in BSFRs whereas arrow would remain as such in SSFRs? T (TI) would mean that the active boundary of CD is frozen and CD starts to increase/decrease in size.

There is considerable evidence for T in BSFRs identified as counterparts of ordinary SFRs but could it be accompanied by I?

1. Mere I in BSFR would mean that CD starts to decrease but the arrow of time is not changed and passive boundary remains passive boundary. What comes to mind is blackhole collapse.

   I have asked whether the decrease in size could take place in BSFR and make it possible for the self to get rid of negative subjective memories from the last moments of life, start from scratch and live a "childhood". Could this somewhat ad hoc looking reduction of size actually take place by a sequence of SSFRs? This brings into mind the big bang and big crunch. Could this period be followed by a BSFR involving inversion giving rise to increase of the size of CD as in the picture considered hitherto?

2. If BSFR involves TI, the CD would shift towards a fixed time direction like a worm, and one would have a fixed arrow of time from the point of view of the outsider although the arrow of time would change for sub-CD. This modified option might be consistent with the recent picture, in particular with the findings made in the experiments of Minev *et al* [L38] [L38].

   This kind of shifting must be assumed in the TGD inspired theory of consciousness. For instance, after images as a sequence of time reversed lives of sub-self, do not remain in the geometric past but follow the self in travel through time and appear periodically (when their arrow of time is the same as of self). The same applies to sleep: it could be a period with a reversed arrow of time but the self would shift towards the geometric future during this period: this could be interpreted as a shift of attention towards the geometric future. Also this option makes it possible for the self to have a "childhood"".

3. However, the idea about a single arrow of time does not look attractive. Perhaps the following observation is of relevance. If the arrow of time for sub-CD correlates with that of sub-CD, the change of the arrow of time for CD, would induce its change for sub-CDs and now the sub-CDs would increase in the opposite direction of time rather than decrease.

7.4.3 Should unitarity be replaced with the Kähler-like geometry of the fermionic state space?

After these preliminaries we can state the key question. Is unitarity possible at all and should it be replaced with some deeper principle? I have considered these questions several times and in [L59] a rather radical solution was proposed.

Assigning an S-matrix to a unitary time evolution works in non-relativistic theory but fails already in the generic QFT and correlation functions replace S-matrix.

1. Einstein's great vision was to geometrize gravitation by reducing it to the curvature of space-time. Could the same recipe work for quantum theory? Could the replacement of the flat Kähler metric of Hilbert space with a non-flat one allow the identification of the analog of unitary S-matrix as a geometric property of Hilbert space? Kähler metric is required to geometrize hermitian conjugation. It turns out that the Kähler metric of a Hilbert bundle determined by the Kähler metric of its base space could replace the unitary S-matrix.

2. An amazingly simple argument demonstrates that one can construct scattering probabilities from the matrix elements of Kähler metric and assign to the Kähler metric a unitary S-matrix assuming that some additional conditions guaranteeing that the probabilities are real and non-negative are satisfied. If the probabilities correspond to the real part of the complex analogs of probabilities, it is enough to require that they are non-negative: complex analogs of probabilities would define the analog of the Teichmüller matrix.

   Teichmüller space parameterizes the complex structures of Riemann surface: could the allowed WCW Kähler metrics - or rather the associated complex probability matrices - correspond to complex structures for some space? By the strong form of holography (SH), the most natural candidate would be Cartesian product of Teichmüller spaces of partonic 2 surfaces with punctures and string world sheets.

3. Under some additional conditions one can assign to Kähler metric a unitary S-matrix but this does not seem necessary. The experience with loop spaces suggests that for infinite-D Hilbert spaces the existence of non-flat Kähler metric requires a maximal group of isometries. Hence one expects that the counterpart of S-matrix is highly unique.

4. In the TGD framework the "world of classical worlds" (WCW) has Kähler geometry allowing spinor structure. WCW spinors correspond to Fock states for second quantized spinors at space-time surface and induced from second quantized spinors of the embedding space. Scattering amplitudes would correspond to the Kähler metric for the Hilbert space bundle of WCW spinor fields realized in zero energy ontology and satisfying Teichmüller condition guaranteeing non-negative probabilities.

5. Equivalence Principle generalizes to the level of WCW and its spinor bundle. In ZEO one can assign also to the Kähler space of zero energy states spinor structure and this strongly suggests an infinite hierarchy of second quantizations starting from space-time level, continuing at the level of WCW, and continuing further at the level of the space of zero energy states. This would give an interpretation for an old idea about infinite primes as an infinite hierarchy of second quantizations of an arithmetic quantum field theory.

6. There is also an objection. The transition probabilities would be given by $P(A,B) = g^{A,\overline{B}}g_{\overline{B},A}$ and the analogs for unitarity conditions would be satisfied by $g^{A,\overline{B}}g_{\overline{B},C} = \delta^A_C$. The problem is that $P(A,B)$ is not real without further conditions. Can one imagine any physical interpretation for the imaginary part of $Im(P(A,B))$?

In this framework, the twistorial scattering amplitudes as zero energy states define the covariant Kähler metric $g_{A\overline{B}}$, which is non-vanishing between the 3-D state spaces associated with the opposite boundaries of CD. $g^{A\overline{B}}$ could be constructed as the inverse of this metric. The problem with the unitarity would disappear.

Explicit expressions for scattering probabilities

The proposed identification of scattering probabilities as $P(A\to B) = g^{A\overline{B}}g_{\overline{B}A}$ in terms of components of the Kähler metric of the fermionic state space.

Contravariant component $g^{A\overline{B}}$ of the metric is obtained as a geometric series $\sum_{n\geq 0} T^n$ from from the deviation $T_{A\overline{B}} = g_{A\overline{B}} - \delta_{A\overline{B}}$ of the covariant metric $g_{A\overline{B}}$ from $\delta A\overline{B}$.

g this is not a diagonal matrix. It is convenient to introduce the notation Z^A, $A\in\{1,...,n\}$ $Z^{\overline{A}} = Z^{n+k}$, $k = n+1,...,2n$. So that the $g_{\overline{B}C}$ corresponds to $g_{k+n,l} = \delta_{k,l} + T_{k,l}$. and one has $g^{A\overline{B}}$ to $g^{k,l+n} = \delta_{k,l} + T^1_{k,l}$.

The condition $g^{A\overline{B}}g_{\overline{B}C} = \delta^A_C$ gives

$$g^{k,l+n}g_{l+n,m} = \delta^k_m \quad . \tag{7.4.1}$$

giving

$$\sum_l(\delta_{k,l} + T^1_{k,l})(\delta_{l,m} + T_{l,m}) = \delta_{k,m} + (T^1 + T + T^1T)_{km} = \delta_{k,m} \quad . \quad . \tag{7.4.2}$$

which resembles the corresponding condition guaranteeing unitarity. The condition gives

$$T_1 = -\frac{T}{1+T} = -\sum_{n>1}((-1)^n T^n . \quad . \tag{7.4.3}$$

The expression for $P(A\to B)$ reads as

$$P(A \to B) = g^{A\overline{B}} g_{\overline{B}A} \\ = [1 - \frac{T}{1+T} + T^\dagger - (\frac{T}{1+T})_{AB} T^\dagger]_{AB} \quad . \tag{7.4.4}$$

It is instructive to compare the situation with unitary S-matrix $S = 1 + T$. Unitarity condition $SS^\dagger = 1$ gives

$$T^\dagger = -\frac{T}{1+T} \quad ,$$

and

$$P(A \to B) = \delta_{AB} + T_{AB} + T^\dagger_{AB} + T^\dagger_{AB} T_{AB} = [\delta_{AB} - (\frac{T}{1+T})_{AB} + T_{AB} - (\frac{T}{1+T})_{AB} T_{AB} \quad .$$

The formula is the same as in the case of Kähler metric.

7.4.4 Critical questions

One can pose several critical questions helping to further develop the proposed number theoretic picture.

Is mere recombinatorics enough as fundamental dynamics?

Fundamental dynamics as mere re-combination of free quarks to Galois singlets is attractive in its simplicity but might be an over-simplification. Can quarks really continue with the same momenta in each SSFR and even BSFR?

1. For a given polynomial P, there are several Galois singlets with the same incoming integer-valued total momentum p_i. Also quantum superpositions of different Galois singlets with the same incoming momenta p_i but fixed quark and antiquark numbers are in principle possible. One must also remember Galois singlet property in spin degrees of freedom.

2. WCW integration corresponds to a summation over polynomials P with a common ramified prime (RP) defining the p-adic prime. For each P of the Galois singlets have different decomposition to quark momenta.

   One can even consider the possibility that only the total quark number as the difference of quark and antiquark numbers is fixed so that polynomials P in the superposition could correspond to different numbers of quark-antiquark pairs.

3. One can also consider a generalization of Galois confinement by replacing classical Galois singlet property with a Galois-singlet wave function in the product of quark momentum spaces allowing classical Galois non-singlets in the superposition.

   Hydrogen atom serves as an illustration: electron at origin would correspond to classical ground state and s-wave correspond to a state invariant under rotations such that the position of electron is not anymore invariant under rotations. The proposal for transition amplitudes remains as such otherwise.

Note however that the basic dynamics at the level of a single polynomial would be recombinatorics for all these options.

General number theoretic picture of scattering

Only the interaction region has been considered hitherto. One must however understand how the interaction region is determined by the 4-surfaces and polynomials associated with incoming Galois singlets. Also the details of the map of p-adic scatting amplitude to a real one must be understood.

The intuitive picture about scattering is as follows.

1. The incoming particle "i" is characterized by p-adic prime p_i, which is RP for the corresponding 4-surface in M^8. Also the "adelic" option that all RPs characterize the particle, is considered below.

2. The interaction region corresponds to a polynomial P. The integration over WCW corresponds to a sum over several P:s with at least one common RP allowing to map the superposition of amplitudes to real amplitude by canonical identification I: $\sum x_n p^n \rightarrow \sum x_n p^{-n}$.

   If one gives up the assumption about a shared RP, the real amplitude is obtained by applying I to the amplitudes in the superposition such that RP varies. Mathematically, this is an ugly option.

3. If there are several shared RPs, in the superposition over polynomials P, one can consider an adelic picture. The amplitude would be mapped by I to a product of the real amplitudes associated with the shared RP:s. This brings in mind the adelic theorem stating that rational number is expressible as a product of the inverses of its p-adic norms. The map I indeed generalizes the p-adic norm as a map of p-adics to reals. Could one say that the real scattering amplitude is a product of canonical images of the p-adic amplitudes for the shared RP:s? Witten has proposed this kind of adelic representation of real string vacuum amplitude.

   Whether the adelization of the scattering amplitudes in this manner makes sense physically is far from clear. In p-adic thermodynamics one must choose a single p-adic prime p as RP. Sum over ramified primes for mass squared values would give CP_2 mass scale if there are small p-adic primes present.

The incoming polynomials P_i should determine a unique polynomial P assignable to the interaction regions as CD to which particles arrive. How?

1. The natural requirement would be that P possess the RPs associated with P_i:s. This can be realized if the condition $P_i = 0$ is satisfied and P is a functional composite of polynomials P_i. All permutations π of $1, ..., n$ are allowed: $P = P_{i_1} \circ P_{i_2} \circ ....P_{i_n}$ with $(i_1, ...i_n) = (\pi(1), ..., \pi(n))$. P possesses the roots of P_i.

   Different permutations π could correspond to different permutations of the incoming particles in the proposal for scattering amplitudes so that the formation of area momenta $x_{i+1} = \sum_{k=1}^{i} p_k$ in various orders would corresponds to different orders of functional compositions.

2. Number theoretically, interaction would mean composition of polynomials. I have proposed that so-called cognitive measurements as a model for analysis could be assigned with this kind of interaction [L58, L60]. The preferred extremal property realized as a simultaneous extremal property for both Kähler action and volume action suggests that the classical non-determinism due to singularities as analogs of frames for soap films serves as a classical correlate for quantum non-determinism [L71].

3. If each incoming state "i" corresponds to a superposition of P_i:s with some common RPs, only the RP:s shared by all compositions P from these would appear in the adelic image. If all polynomials P_i are unique (no integration over WCW for incoming particles), the canonical image of the amplitude could be the product over images associated with common RPs.

   The simplest option is that a complete localization in WCW occurs for each external state, perhaps as a result of cognitive state preparation and reduction, so that P has the RP:s of P_is as RP:s and adelization could be maximal.

Do the notions of virtual state, singularity and resonance have counterparts?

Is the proposal physically acceptable? Does the approach allow to formulate the notions of virtual state, singularity and resonance, which are central for the standard approach?

1. The notion of virtual state plays a key role in the standard approach. On-mass-shell internal lines correspond to singularities of S-matrix and in a twistor approach for $\mathcal{N} = 4$ SUSY, they seem to be enough to generate the full scattering amplitudes.

   If only off-mass-shell scattering amplitudes between on- mass-shell states are allowed, one can argue that only the singularities are allowed, which is not enough.

2. Resonance should correspond to the factorization of S-matrix at resonance, when the intermediate virtual state reduces to an on-mass-shell state. Can the approach based on Kähler metric allow this kind of factorization if the building brick of the scattering amplitudes as the deviation of the covariant Kähler metric from the unit matrix $\delta_{A\overline{B}}$ is the basic building bricks and defined between on mass shell states?

   Note that in the dual resonance model, the scattering amplitude is some over contribution of resonances and I have proposed that a proper generalization of this picture could make sense in the TGD framework.

The basic question concerns the number theoretical identification of on-mass-shell and off-mass-shell states.

1. Galois singlets with integer valued momentum components are the natural identification for on-mass-shell states. Galois non-singlet would be off-mass-shell state naturally having complex quark masses and momentum components as algebraic integers.

   Virtual states could be arbitrary states without any restriction to the components of quark momentum except that they are in the extension of rationals and the condition coming from momentum conservation, which forces intermediate states to be Galois singlets or products of them.

   Therefore momentum conservation allows virtual states as on mass shell states, that is intermediate states, which are Galois singlets but consist of Galois non-singlets identified as off-mass-shell lines. The construction of bound states formed from Galois non-singlets would indeed take place in this way.

2. The expansion of the contravariant part of the scattering matrix $T_1 = T/(1+T)$ appearing in the probability

$$\begin{aligned} P(A \to B) &= g^{A\overline{B}} g_{\overline{B}A} \\ &= [1 - \tfrac{T}{1+T} + T^{\dagger} - (\tfrac{T}{1+T})_{AB} T^{\dagger}]_{AB} \quad . \end{aligned}$$

   would give a series of analogs of diagrams in which Galois singlets of intermediate states are deformed to non-singlets states.

3. Singularities and resonances would correspond to the reduction of an intermediate state to a product of Galois singlets.

What about the planarity condition in TGD?

The simplest proposal inspired by the experience with the twistor amplitudes is that only planar polygon diagrams are possible since otherwise the area momenta are not well-defined. In the TGD framework, there is no obvious reason for not allowing diagrams involving permutations of external momenta with positive energies *resp.* negative energies since the area momenta $x_{i+1} = \sum_{k=1}^{i} p_k$ are well-defined irrespective of the order. The only manner to uniquely order the Galois singlets as incoming states is with respect to their mass squared values given by integers.

Generalized OZI rule

In TGD, only quarks are fundamental particles and all elementary particles and actually all physical states in the fermionic sector are composites of them. This implies that quark and antiquark numbers are separately conserved in the scattering diagrams and the particle reaction only means the-arrangement of the quarks to a new set of Galois singlets.

At the level of quarks, the scattering would be completely trivial, which looks strange. One would obtain a product of quark propagators connecting the points at mass shells with opposite energies plus entanglement coefficients arranging them at positive and negative energy light-cones to groups which are Galois singlets.

This is completely analogous to the OZI role. In QCD it is of course violated by generation of gluons decaying to quark pairs. In TGD, gauge bosons are also quark pairs so that there is no problem of principle.

There is an objection against this picture.

1. If particle reactions are mere recombinations of Galois singlets with Galois singlets, the quark and antiquark numbers N_q and $N_{\overline{q}}$ of quark and antiquark numbers are separately conserved (as also their difference $N_q - N_{\overline{q}}$). This forbids many reactions, for instance those in which a gauge boson is emitted unless one assumes that many quark states are superpositions of states with a varying total quark number N. This would mean that the extremely simple re-combinatorics picture is lost.

2. Crossing symmetry, which is a symmetry of standard QFTs, suggests a solution to the problem. Crossing symmetry would mean that one can transfer quarks between initial and final states by changing the sign of the quark four-momentum so that momentum conservation is not violated. Crossing means analytic continuation of the scattering amplitude by replacing incoming (outgoing) momentum p with outgoing (incoming) momentum $-p$. The scattering amplitudes for reactions for which the quark number is conserved can be constructed using mere recombinatorics, and the remaining amplitudes would be obtained by crossing.

3. Crossing must respect the Galois singlet property. For instance, the crossing of a single quark destroys Galois singlet. Unless one allows destruction and recombination of Galois singlets, the crossing can apply only to Galois singlets. These rules bring to mind the vanishing of twistor amplitudes when one gluon has negative helicity and the remaining gluons have positive helicity.

7.4.5 Western and Eastern ontologies of physics

This picture forces us to ask whether something deeper might lurk behind the usual ideas about particle physics in which scattering rates encode the information. Could the imaginary part of $P(A, B)$ have a well-defined physical meaning in some more general framework?

1. In ZEO, single classical time evolution and zero energy state as a pair of initial and final states becomes the basic entity. One can even ask whether it might make sense to speak about probability density for different zero energy states as time evolutions, events.

   Could the "western" view about existing reality evolving in time be replaced with an ontology in which events in both classical sense (zero energy states) and quantum transitions would be what really exists.

   In the "eastern" view, the relevant probabilities would not be for transitions $A \rightarrow B$ for a given state A but for the occurrence of these transitions $A \rightarrow B$ in given state, whatever its definition might be, and one would measure the relative rates for occurrence for the various transitions $A \rightarrow B$.

   The ensemble would not consist of entities A but transitions $A \rightarrow B$. In biology and neuroscience, the states are indeed replaced with behaviors. IKn computer science the program, rather than the state of the computer, is the basic notion.

2. In order to develop this picture at the level of scattering amplitudes, one could start from the QFT description for the n-point correlation functions used to construct S-matrix. One adds to the exponent of action a term, which is a combination of small current terms assignable to external particles and calculates functional Taylor series with respect to the small parameters. The Taylor coefficients are identified as n-point functions.

   In QFTs this is regarded as a mere calculational trick and the "state" defined by the exponential as an analog of that in statistical physics is defined by the exponential of action when the values of the parameters vanish.

   One can of course ask what it would mean if these parameters do not vanish. In perturbation theory one actually has this situation. These deformed states look formally like coherent states. Could the physical states at a deeper level correspond to these analogs of coherent states as analogs of thermo-dynamical states?

3. TGD can be formally regarded as a complex square root of thermodynamics, which suggests a generalization of the formulation of quantum theory as algebraic QFT promoted for instance by Connes [A10], and this is what this new interpretation would mean also physically.

4. In the TGD framework, one would add to the exponent of $exp(-K)$ a superposition of oscillator operator monomials of quark oscillator operators creating positive and negative energy parts of the zero energy states with complex coefficients Z_i as parameters and essentially defining coordinates for the Hilbert space. Z_i would be analogous to the complex numbers defining coherent states.

   The exponential can be expanded and fermionic vacuum expectation forces conservation of quark number and the combination of the positive and negative energy parts to give a non-vanishing result. At the limit of infinitely large CD conservation of 4-momentum is obtained.

5. The ordinary transition amplitudes are obtained by performing the limit $Z_i \rightarrow 0$, and calculating Taylor coefficients as transition amplitudes. The analog of $G_{A,\overline{B}}$ would be obtained for the analogs 2-point functions having as arguments the parts of zero energy states and $P(A,B) = Re(G_{A,\overline{B}}G_{\overline{B},A})$ would give transition probabilities. For Kähler geometry the analog of probability conservation and unitarity would hold true.

6. That these amplitudes are obtained as second derivatives with respect to the fermionic Hilbert space complex coordinates Z_i and $\overline{Z}_j$ conforms with the interpretation of the exponential containing the additional terms as a generalization of an exponential of Kähler function associated with the fermionic degrees of freedom. Kähler metric indeed corresponds to $\partial_{Z_I}\partial_{\overline{Z}_J}K$, where K is the Kähler function.

7. Could the expressions of higher n-point functions in fermionic degrees of freedom boil down to the curvature tensor and its covariant derivatives so that quantum theory would be geometrized? If one has a constant curvature space, as strongly suggested by the mere existence of infinite-D Kähler metric, then only $G_{A,\overline{B}}$ would be needed so that it is enough to measure only the scattering probabilities (rates at infinite-volume limit for CD).

   Could the parameters Z_i be non-vanishing and define a square root of a thermodynamic state as an analog of a coherent state? If a constant curvature metric is in question, the scattering rates for non-vanishing Z_i could be expressed in terms of those for $Z_i = 0$. Could different phases of quantum theory correlate with the value ranges of the parameters Z_i?

7.4.6 Connection with the notion of Fisher information

The notion of Fisher information (`https://cutt.ly/GUPvF37`) relates in an interesting manner to the proposed Kähler geometrization of quantum theory.

1. Fisher information matrix F is associated with a probability density function $f(X,Z)$ for random variables X_i depending on the parameters Z_i (Z_i are denoted by θ_i in the Wikipedia article at `https://cutt.ly/GUPvF37`). Matrix F gives information about the $f(X,\theta)$, which must be deduced from the measurements of X. The matrix element F_{ij} is essentially integral over X for the quantity $\langle \partial_{\theta_i}\partial_{\theta_j}log(f)\rangle$, where $\langle..\rangle$ denotes the expectation obtained by integrating over X. F_{ij} determines a statistical metric and for complex parameters Z_i one obtains a Kähler metric.

2. In TGD, X would correspond to WCW coordinates and f would be analogous to the vacuum functional $exp(-K)$ but containing also a parameter dependent part defined by the combination of positive and negative energy parts of the fermionic zero energy states. The complex coefficients Z_i *resp.* $\overline{Z}_i$ of monomials of creation *resp.* annihilation operators would define the parameters. Fermionic Kähler metric would have an interpretation as Fisher information, which can be also complex valued.

3. Also the higher derivatives with respect to coefficients of zero energy states would provide information about the vacuum functional. One would have n-point functions for zero energy states possibly reducing to covariant derivatives of the analog curvature tensor. If the space of fermionic zero energy states is analog of a constant curvature space, the scattering amplitudes at the limit $Z_i = 0$ would give all the needed information needed to calculate the scattering amplitudes for $Z_i \neq 0$. $P(A,B)$ would be complex as components of the Fisher information matrix.

4. Basically, the information provided by the scattering amplitudes would be about the generalization of the vacuum functional of WCW including also the fermionic part. Scattering amplitudes would give information Kähler function of the WCW metric and about parameters Z_i.

   The scattering amplitudes indeed correlate strongly with the properties of space-time surfaces determined by polynomials. The p-adic prime p, crucial for the real scattering amplitudes as canonical images of p-adic amplitudes, corresponds to a ramified prime for P and this means localization of the vacuum functional to polynomials having a ramified prime equal to p. The number of Galois singlets in the scattering amplitude means lower bound for the degree of P.

7.4.7 About the relationship of Kähler approach to the standard picture

The replacement of the notion of unitary S-matrix with Kähler metric of fermionic state space generalizes the notion of unitarity. The rows of the matrix defined by the contravariant metric are orthogonal to the columns of the covariant metric in the inner product $(T\circ U)_{A\overline{B}} = T_{A\overline{C}}\eta^{\overline{C}D}U_{D\overline{B}}$, where $\eta^{\overline{C}D}$ is flat contravariant Kähler metric of state space. Although the probabilities are complex, their real parts sum up to 1 and the sum of the imaginary parts vanishes.

The counterpart of the optical theorem in TGD framework

The Optical Theorem generalizes. In the standard form of the optical theorem $i(T-T^\dagger)_{mm} = 2Im(T) = TT^\dagger_{m,m}$ states that the imaginary part of the forward scattering amplitude is proportional to the total scattering rate. Both quantities are physical observables.

In the TGD framework the corresponding statement

$$T^{A\overline{B}}\eta_{\overline{B}C} + \eta^{A\overline{B}}T_{\overline{B}C} + T^{A\overline{B}}T_{\overline{B}C} = 0 \ . \tag{7.4.5}$$

Note that here one has $G = \eta + T$, where G and T are hermitian matrices. The correspondence with the standard situation would require the definition $G = \eta + iU$. The replacement $T \to T = iU$, where U is antihermitian matrix, gives

One has

$$i[U^{A\overline{B}}\eta_{\overline{B}C} + \eta^{A\overline{B}}I_{\overline{B}C}] = U^{A\overline{B}}U_{\overline{B}C} \ . \tag{7.4.6}$$

This statement does not reduce to single condition but gives two separate conditions.

1. The first condition is analogous to Optical Theorem:

$$Im[\eta^{A\overline{B}}U_{C\overline{B}} + U^{A\overline{B}}\eta_{\overline{B}C}] = -Re[U^{A\overline{B}}U_{\overline{B}C}] = Re[U^{A\overline{B}}U_{C\overline{B}}] \ . \tag{7.4.7}$$

2. Second condition is new and reflects the fact that the probabilities are complex. It is necessary to guarantee that the sum of the probabilities reduces to the sum of their real parts.

$$Re[\eta^{A\overline{B}}U_{C\overline{B}} + U^{A\overline{B}}\eta_{\overline{B}C}] = -Im[U^{A\overline{B}}U_{C\overline{B}}] \ . \tag{7.4.8}$$

   The challenge would be to find a physical meaning for the imaginary parts of scattering probabilities. This is discussed in [L59]. The idea is that the imaginary parts could make themselves visible in a Markov process involving a power of the complex probability matrix.

In the applications of the optical theorem, the analytic properties of the scattering matrix T make it possible to construct the amplitude as a function of mass shell momenta using its discontinuity at the real axis. Indeed, $2Im(T)$ for the forward scattering amplitude can be identified as the discontinuity $Disc(T)$. In the recent case, this identification would suggests the generalization

$$Disc[T^{A\overline{B}}\eta_{\overline{B}C}] = T^{A\overline{B}}\eta_{\overline{B}C} + \eta^{A\overline{B}}T_{C\overline{B}} \ . \tag{7.4.9}$$

Therefore covariant and contravariant Kähler metric could be limits of the same analytic function from different sides of the real axis. One assigns the hermitian conjugate of S-matrix to the time reflection. Are covariant and contravariant forms of Kähler metric related by time reversal? Does this mean that T symmetry is violated for a non-flat Kähler metric.

The emergence of QFT type scattering amplitudes at long length scale limit

The basic objection against the proposal for the scattering amplitudes is that they are non-vanishing only at mass shells with $m^2 = n$. A detailed analysis of this objection improves the understanding about how the QFT limit of TGD emerges.

1. The restriction to the mass shells replaces cuts of QFT approach with a discrete set of masses. The TGD counterpart of unitarity and optical theorem holds true at the discrete mass shells.

2. The p-adic mass scale for the reaction region is determined by the largest ramified prime RP for the functional composite of polynomials characterizing the Galois singlets participating in the reaction. For large values of ramified prime RP for the reaction region, the p-adic mass scale increases and therefore the momentum resolution improves.

3. For large enough RP below measurement resolution, one cannot distinguish the discrete sequence of poles from a continuum, and it is a good approximation to replace the discrete set of mass shells with a cut. The physical analogy for the discrete set of masses along the real axis is as a set of discrete charges forming a linear structure. When their density becomes high enough, the description as a line charge is appropriate and in 2-D electrostatistics this replaces the discrete set of poles with a cut.

This picture suggests that the QFT type description emerges at the limit when RP becomes very large. This kind of limit is discussed in the article considering the question whether a notion of a polynomial of infinite degree as an iterate of a polynomial makes sense [L62]. It was found that the number of the roots is expected to become dense in some region of the real line so that effectively the QFT limit is approached.

1. If the polynomial characterizing the scattering region corresponds to a composite of polynomials participating in the reaction, its degree increases with the number of external particles. At the limit of an infinite number of incoming particles, the polynomial approaches a polynomial of infinite degree. This limit also means an approach to a chaos as a functional iteration of the polynomial defining the space-time surface [L54]. In the recent picture, the iteration would correspond to an addition of particles of a given type characterized by a fixed polynomial. Could the characteristic features for the approach of chaos by iteration, say period doubling, be visible in scattering in some situations. Could p-adic length scale hypothesis stating that p-adic primes near powers of two are favored, relate to this.

2. For a large number of identical external particles, the functional composite defining RG involves iteration of polynomials associated with particles of a particular kind, if their number is very large. For instance, the radiation of IR photons and IR gravitons in the reaction increases the degree of RP by adding to P very high iterates of a photonic or gravitonic polynomial.

   Gravitons could have a large value of ramified prime as the approximately infinite range of gravitational interaction and the notion of gravitational Planck constant [L27, L70] originally proposed by Nottale [E1] suggest. If this is the case, graviton corresponds to a polynomial of very high degree, which increases the p-adic length scale of the reaction region and improves the momentum resolution. If the number of gravitons is large, this large RP appears at very many steps of the SFR cascade.

A connection with dual resonance models

There is an intriguing connection with the dual resonances models discussed already in [L35].

1. The basic idea behind the original Veneziano amplitudes (see `http://tinyurl.com/yyhwvbqb`) was Veneziano duality. The 4-particle amplitude of Veneziano was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see `http://tinyurl.com/yyvkx7as`) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged.

2. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have a representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

3. The resonances have zero width and the imaginary part of the amplitude has a discontinuity only at the resonance poles, which is not consistent with unitarity so that one must force unitarity by hand by an iterative procedure. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of the twistor Grassmann approach.

It is interesting to compare this picture with the twistor Grassman approach and TGD picture.

1. In the TGD framework, one just picks up the residue of what would be analogous to stringy scattering amplitude at mass shells. In the dual resonance models, one keeps the entire amplitude and encounters problems with the unitarity outside the poles. In the twistor Grassmann approach, one assumes that the amplitudes are completely determined by the singularities whereas in TGD they *are* the residues at singularities. At the limit of an infinite-fold iterate the amplitudes approach analogs of QFT amplitudes.

2. In the dual resonance model, the sums over s- and t-channel resonances are the same. This guarantees crossing symmetry. An open question is whether this can be the case also in the TGD framework. If this is the case, the continuum limit of the scattering amplitudes should have a close resemblance with string model scattering amplitudes as the $M^4 \times CP_2$ picture having magnetic flux tubes in a crucial role indeed suggests.

3. In dual resonance models, only the cyclic permutations of the external particles are allowed. As found, the same applies in TGD if the scattering event is a cognitive measurement [L58], only the cyclic permutations of the factors of a fixed functional composite are allowed. Non-cyclic permutations would produce the counterparts of non-planar diagrams and the cascade of cognitive state function reductions could not make sense for all polynomials in the superposition simultaneously. Remarkably, in the twistor Grassmann approach just the non-planar diagrams are the problem.

7.5 Some useful objections

The details of the proposed construction of the scattering amplitudes starting from twistors are still unclear and the best way to proceed is to invent objections and critical questions.

7.5.1 How the quark momenta in M^8 and H relate to each other?

The relationship between quark momenta in M^8 and H is not clear. There are four options to consider corresponding to the Dirac propagators in H and M^4 with or without coupling to $A(M^4)$. I assign to these options attributea $D(H,A)$, $D(H)$, $D(M^4,A)$ and $D(M^4)$. For all options something seems to go wrong.

Consider fits the list of criteria that the correct option should satisfy.

1. M^8-H duality suggests the same momentum and mass spectrum for quarks in M^8 and H.

   (a) However, the mass spectrum of color partial waves for quark spinors for $D(H)$ and $D(H,A)$ is very simple and characterized by 2 integers labeling triality $t=1$ representations of $SU(3)$ [L1]. Neither $D(H)$ or $D(H,A)$ allows a mass spectrum as algebraic roots of polynomials and seems to be excluded.

   (b) If M^8-H duality holds true in a strong sense so that these spectra are identical, the only possible conclusion seems to be that the propagator in both M^8 and H is just the M^4 Dirac propagator $D(M^4)$ and that the roots of the polynomial P give the spectrum of off-mass-shell masses. Also tachyonic mass squared values are allowed as roots of P. The real on-shell masses would be associated with Galois singlets.

2. Twistor holomorphy and associativity leave only the $D(M^4)$ option. The couplings to $A(M^4)$ and presence of $D(CP_2)$ spoil these properties. $D(M^4)$ option has very nice features. The integration over the momentum space reduces to a finite summation over virtual mass shells defined by the roots of P and one avoids divergences. This tightens the connection with QFTs. For $D(M^4()$ this nice property is lost. Massless quarks are also consistent with the QCD picture about quarks.

3. The predictions of p-adic mass calculations [K42, K17] were sensitive to the negative ground state conformal weight h_{vac} depending on the electroweak isospin and gave rise to electroweak symmetry breaking. h_{vac} could be generated by conformal generators with weights h coming as algebraic integers determined by P. This favors $D(H)$ and $D(H,A)$. $D(H,A)$ predicts tachyonic ν_R, which was necessary for the calculation. Only $D(H,A)$ survives.

4. For some years ago, I found that the space-time propagators for points of H connected by a light-like geodesic behave like massless propagators irrespective of mass. CP_2 type extermals have a light-like geodesic as an M^4 projection. This would suggest that quarks associated with CP_2 type extremals effectively propagate as massless particles even if one assumes that they correspond to modes of the full H Dirac operator. This allows us to consider $D(H)$ as an alternative. For this option most quarks in the interior of the space-time surface would be extremely massive and practically absent.

5. Suppose that one takes seriously the idea that the situation can be described also by using massless M^8 momenta. This implies that for some choices of $M^4\subset M^8$ the momentum is parallel to M^4 and therefore massless in 4-D sense. Only the quarks associated with the same M^4 can interact. Hence M^4 can be always chosen so that the on mass-shell 4-momenta are light-like. $D(H,A)$ option could be correct but $D(M^4)$ option would appear as an effective option obtained by a suitable choice of $M^4\subset M^8$.

6. The consideration of problems related to right-handed neutrino [L65] led to the question whether the quark spinor modes in H are annihilated only by the H d'Alembertian $D^2(H,A(M^4))$ but not by the H Dirac operator [L65]. The assumption that on mass shell H-spinors are annihilated by $D(M^4,A)$ leads to the same outcome.

   D^2 options allow different M^4 chiralities to propagate separately and solves problems related to the notion of right-handed neutrino ν_R (assumed to be 3-antiquark state and modellable using leptonic spinors in H. This also conforms with the right and left-handed character of the standard model couplings. However, the mixing of M^4 chiralities serves as a signature for the massivation and is lost.

   If leptons are allowed as fundamental fermions, $D(H)$ allows ν_R as a spinor mode, which is covariantly constant in CP_2. If leptons are not allowed, one can argue that ν_R as a 3-quark state can be modeled as a mode of H spinor with Kähler coupling yielding correct leptonic charges.

   The M^4 Kähler structure favored by the twistor lift of TGD [L35] implies that ν_R with negative mass squared appears as a mode of $D(H)$. This mode allows the construction of tachyonic ground states. This is lost for $D(M^4)$ with coupling to $A(M^4)$.

For $D(M^4, A)$, one obtains for all spinor modes states with both positive and negative mass squared from the $J_{kl}\Sigma^{kl}$ term. Physical on-mass- shell states with negative mass squared cannot be allowed. These would however allow to construct tachyonic ground states needed in the p-adic mass calculations. Now the problem is that $D(M^4, A)$ as propagator spoils twistor holomorphy.

7. Since the color group acts as symmetries, one can assume that spinor modes correspond to color partial waves as eigen states of CP_2 spinor d'Alembertian $D^2(CP_2)$. This predicts that different M^4 chiralities propagate independently. $D(M^4)$ and $D(M^4, A)$ options make the same prediction. For the $D(H)$ and $D(H, A)$ option one obtains a mixing of M^4 chiralities having interpretation in terms of massivation.

   For all options the correlation between color and electroweak quantum numbers is "wrong". This is however not a problem for off-mass-shell fundamental quarks since the physical states are obtained as SSA representations.

To sum up, $D(H, A)$ is strongly favored by the p-adic thermodynamics, by the possibility to build the physical quarks using SSA, by the fact that propagators over-light-like distances do not depend on mass, and also by the freedom to choose $M^4 \subset M^8$ in such a way that on mass shell spinor mode is massless. $D(M^4)$ is strongly favoured by $M^8 - H$ duality (associativity) and by twistor analyticity. Both options seem to be both right and wrong. This suggests that something is wrong with the interpretation of the notion of the Dirac propagator.

1. From the view point of H, M^8 quarks are off-mass-shell whereas from the M^8 point of view they are on-mass-shell. Suppose that off-mass shell quarks in the sense of $D(H, A)$ differ from on-mass-shell quarks only in that they have M^4 momentum $p_{off} = p_{on} + \Delta p$ differing by Δp from the on-mass shell momentum p_{on} with integer components and satisfying mass shell condition for $D(H)$. In CP_2 these states are on-mass-shell. Suppose that p_{off} is on M^8 mass shell determined as a root of P.

   With these assumptions, one can write Dirac operator as $D(H, A, off) = D(H, A, on) + \Delta p^k gamma_k$, whose action to incoming Galois singlets reduces to $D(H, A, off) = \Delta p^k gamma_k = D(M^4)$. This is just the free massless propagator.

2. The propagating entities would be basically solutions of $D(H, A)$ with an off-mass-shell M^4-momentum with Δp having mass. In particular, they are superpositions of components with left- and right-handed M^4 chiralities having opposite CP_2 chiralities and the mixing of M^4 chiralities can be seen as a signature of massivation. On the other hand, $D(M^4)$ does not depend on M^4 chirality. Maybe this option could avoid all objections!

7.5.2 Can one allow "wrong" correlation between color and electroweak quantum numbers for fundamental quarks?

For CP_2 harmonics, the correlation between color and electroweak quantum numbers is wrong [K42]. Therefore the physical quarks cannot correspond to the solutions of $D^2(H)\Psi = 0$. The same applies also to the solutions of $D(M^4)\Psi = 0$ if one assumes that they belong to irreducible representations of the color group as eigenstates of $D(CP_2)$.

How to construct quark states, which are physical in the sense that they are massless and color-electroweak correlation is correct?

1. The reduction of quark masses to zero requires a tachyonic ground state in p-adic mass calculations [K42]. The assumption that physical states are constructed using quarks, which are on-mass-shell in the M^8 sense but off-mass-shell in the H sense.

   Colored operators with non-vanishing conformal weight are required to make all quark states massless color triplets. This is possible only if the ground state is tachyonic, which gives strong support for M^4 Kähler structure.

2. This is achieved by the identification of physical quarks as states of super-symplectic representations. Also the generalized Kac-Moody algebra assignable to the light-like partonic

orbits or both of these representations can be considered. These representations could correspond to inertial and gravitational representations realized at "objective" embedding space level and "subjective" space-time level.

Supersymplectic generators are characterized by a conformal weight h completely analogous to mass squared. The conformal weights naturally correspond to algebraic integers associated with P. The mass squared values for the Galois singlets are ordinary integers.

3. It is plausible that also massless color triplet states of quarks can be constructed as color singlets. From these one can construct hadrons and leptons as color singlets for a larger extension of rationals. This conforms with the earlier picture about conformal confinement. These physical quarks constructed as states of super-symplectic representation, as opposed to modes of the H spinor field, would correspond to the quarks of QCD.

   One can argue that Galois confinement allows to construct physical quarks as color triplets for some polynomial Q and also color singlets bound states of these with extended Galois group for a higher polynomial $P \circ Q$ and with larger Galois group as representation of group $Gal(P)/Gal(Q)$ allowing representations of a discrete subgroup of color group.

7.5.3 Can one allow complex quark masses?

One objection relates to unitarity. Complex energies and mass squared values are not allowed in the standard picture based on unitary time evolution.

1. Here several new concepts lend a hand. Galois confinement could solve the problems if one considers only Galois singlets as physical particles. ZEO replaces quantum states with entangled pairs of positive and negative energy states at the boundaries of CD and entanglement coefficients define transition amplitudes.

   The notion of the unitary time evolution is replaced with the Kähler metric in quark degrees of freedom and its components correspond to transition amplitudes. The analog of the time evolution operator assignable to SSFRs corresponds naturally to a scaling rather than time translation and mass squared operator corresponds to an infinitesimal scaling.

2. The complex eigenvalues of mass squared as roots of P be allowed when unitarity at quark level is not required to achieve probability conservation. For complex mass squared values, the entanglement coefficients for quarks would be proportional to mass squared exponents $exp(im^2\lambda)$, λ the scaling parameter analogous to the duration of time evolution. For Galois singlets these exponentials would sum up to imaginary ones so that probability conservation would hold true.

7.5.4 Are M^8 spinors as octonionic spinors equivalent with H-spinors?

At the level of M^8 octonionic spinors are natural. M^8-H duality requires that they are equivalent with H-spinors. The most natural identification of octionic spinors is as bi-spinors, which have octonionic components. Associativity is satisfied if the components are complexified quaternionic so that they have the same number of components as quark spinors in H. The H spinors can be induced to $X^4 \subset M^8$ by using $M^8 - H$ duality. Therefore the M^8 and H pictures fuse together.

The quaternionicity condition for the octonionic spinors is essential. Octonionic spinor can be expressed as a complexified octonion, which can be identified as momentum p. It is not an on-mass shell spinor. The momenta allowed in scattering amplitudes belong to mass shells defined by the polynomial P. That octonionic spinor has only quaternionic components conforms with the quaternionicity of $X^4 \subset M^8$ eliminating the remaining momentum components and also with the use of $D(M^4)$.

7.5.5 Two objections against p-adic thermodynamics and their resolution

Unlike the Higgs mechanism, p-adic thermodynamics provides a universal description of massivation involving no other assumptions about dynamics except super-conformal symmetry, which guarantees the existence of p-adic Boltzmann weights.

There are two basic objections against p-adic thermodynamics. The mass calculations require the presence of states with negative conformal weights giving rise to tachyons. Furthermore, by conformal invariance L_0 should annihilate physical states so that all states should have vanishing mass squared! In this article a resolution of these objections, based on the very definition of thermodynamics and on number theoretic vision predicting quark states with discretized tachyonic mass, which are counterparts for virtual states in QFTs, is discussed.

Physical states for the entire Universe would be indeed massless but for subsystems such as elementary particles the thermal expectation of the mass squared is non-vanishing. This conforms with the formula of blackhole entropy stating that it is proportional to the mass square of the blackhole and vanishes for vanishing mass: this would indeed correspond to a pure state.

p-Adic thermodynamics

Number theoretic physics involves the combination of real and various p-adic physics to adelic physics [L22, L23], and classical number fields [K74]. p-Adic mass calculations is a rather successful application of p-adic thermodynamics for the mass squared operator identified as conformal scaling generator L_0. p-Adic thermodynamics can be also understood as a constraint on a real thermodynamics for the mass squared from the condition that it can be also regarded as a p-adic thermodynamics.

The motivation for p-adicization came from p-adic mass calculations [K42, K17].

1. p-Adic thermodynamics for mass squared operator M^2 proportional to scaling generator L_0 of Virasoro algebra. Mass squared thermal mass from the mixing of massless states with states with mass of order CP_2 mass.

2. $exp(-E/T) \rightarrow p^{L_0/T_p}$, $T_p = 1/n$. Partition function p^{L_0/T_p}. p-Adic valued mass squared mapped to a real number by canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$. Eigenvalues of L_0 must be integers for the Boltzmann weights to exist. Conformal invariance guarantees this.

3. p-adic length scale $L_p \propto \sqrt{p}$ from Uncertainty Principle ($M \propto 1/\sqrt{p}$). p-Adic length scale hypothesis states that p-adic primes characterizing particles are near to a power of 2: $p \simeq 2^k$. For instance, for an electron one has $p = M^{127} - 1$, Mersenne prime. This is the largest not completely super-astrophysical length scale.

   Also Gaussian Mersenne primes $M_{G,n} = (1 + i)^n - 1$ seem to be realized (nuclear length scale, and 4 biological length scales in the biologically important range 10 nm,2.5 μm).

4. p-Adic physics [K50] is interpreted as a correlate for cognition. Motivation comes from the observation that piecewise constant functions depending on a finite number of pinary digits have a vanishing derivative. Therefore they appear as integration constants in p-adic differential equations. This could provide a classical correlate for the non-determinism of imagination.

Objections and their resolution

The number theoretic picture leads to a deeper understanding of a long standing objection against p-adic thermodynamics [K42] as a thermodynamics for the scaling generator L_0 of Super Virasoro algebra.

If one requires super-Virasoro symmetry and identifies mass squared with a scaling generator L_0, one can argue that only massless states are possible since L_0 must annihilate these states! All states of the theory would be massless, not only those of fundamental particles as in conformally invariant theories to which twistor approach applies! This looks extremely beautiful mathematically but seems to be in conflict with reality already at single particle level!

The resolution of the objection is that *thermodynamics* is indeed in question.

1. Thermodynamics replaces the state of the entire system with the density matrix for the subsystem and describes approximately the interaction with the environment inducing the entanglement of the particle with it. To be precise, actually a "square root" of p-adic thermodynamics could be in question, with probabilities being replaced with their square roots having also phase factors. The excited states of the entire system indeed are massless [L77].

2. The entangling interaction gives rise to a superposition of products of single particle massive states with the states of environment and the entire mass squared would remain vanishing. The massless ground state configuration dominates and the probabilities of the thermal excitations are of order $O(1/p)$ and extremely small. For instance, for the electron one has $p = M_{127} = 2^{127} - 1 \sim 10^{38}$.

3. In the p-adic mass calculations [K42, K17], the effective environment for quarks and leptons would in a good approximation consist of a wormhole contact (wormhole contacts for gauge bosons and Higgs and hadrons). The many-quark state many-quark state associated with the wormhole throat (single quark state for quarks and 3-quark-state for leptons [L61].

4. In M^8 picture [L52, L53], tachyonicity is unavoidable since the real part of the mass squared as a root of a polynomial P can be negative. Also tachyonic real but algebraic mass squared values are possible. At the H level, tachyonicity corresponds to the Euclidean signature of the induced metric for a wormhole contact.

   Tachyonicity is also necessary: otherwise one does not obtain massless states. The super-symplectic states of quarks would entangle with the tachyonic states of the wormhole contacts by Galois confinement.

5. The massless ground state for a particle corresponds to a state constructed from a massive single state of a single particle super-symplectic representation (CP_2 mass characterizes the mass scale) obtained by adding tachyons to guarantee masslessness. Galois confinement is satisfied. The tachyonic mass squared is assigned with wormhole contacts with the Euclidean signature of the induced metric, whose throats in turn carry the fermions so that the wormhole contact would form the nearby environment.

   The entangled state is in a good approximation a superposition of pairs of massive single-particle states with the wormhole contact(s). The lowest state remains massless and massive single particle states receive a compensating negative mass squared from the wormhole contact. Thermal mass squared corresponds to a single particle mass squared and does not take into account the contribution of wormhole contacts except for the ground state.

6. There is a further delicate number theoretic element involved [L65, L71]. The choice of $M^4 \subset M^8$ for the system is not unique. Since M^4 momentum is an M^4 projection of a massless M^8 momentum, it is massless by a suitable choice of $M^4 \subset M^8$. This choice must be made for the environment so that both the state of the environment and the single particle ground state are massless. For the excited states, the choice of M^4 must remain the same, which forces the massivation of the single particle excitations and p-adic massivation.

All physical states are massless!

These arguments strongly suggest that pure states, in particular the state of the entire Universe, are massless. Mass would reflect the statistical description of entanglement using a density matrix. The proportionality between p-adic thermal mass squared (mappable to real mass squared by canonical identification) and the entropy for the entanglement of the subsystem-environment pair is therefore natural.

This proportionality conforms with the formula for the blackhole entropy, which states that the blackhole entropy is proportional to mass squared. Also p-adic mass calculations inspired the notion of blackhole-elementary particle analogy [K53] but without a deeper understanding of its origin.

One implication is that virtual particles are much more real in the TGD framework than in QFTs since they would be building bricks of physical states. A virtual particle with algebraic value of mass squared would have a discrete mass squared spectrum given by the roots of a rational, possibly monic, polynomial and $M^8 - H$ duality suggests an association to an Euclidean wormhole contact as the "inner" world of an elementary particle. Galois confinement, universally responsible for the formation of bound states, analogous to color confinement and possibly explaining it, would make these virtual states invisible [L72, L73].

Relationship with Higgs mechanism

Polynomials P have two kinds of solutions depending on whether their roots determine either mass or energy shells. For the energy option a space-time region corresponds by $M^8 - H$ duality to a solution spectrum in which the roots correspond to energies rather than mass squared values and light-cone proper time is replaced with linear Minkoski time [L52, L53]. The physical interpretation of the energy shell option has remained unclear.

The energy shell option gives rise to a p-adic variant of the ordinary thermodynamics and requires integer quantization of energy. This option is natural for massless states since scalings leave the mass shell invariant in this case. Scaling invariance and conformal invariance are not violated.

One can wonder what the role of these massless virtual quark states in TQC could be. A good guess is that the two options correspond to phases with broken *resp.* unbroken conformal symmetry. In gauge theories they correspond to phases with broken and unbroken gauge symmetries. The breaking of gauge symmetry indeed induces breaking of conformal symmetry and this breaking is more fundamental.

1. Particle massivation corresponds in gauge theories to symmetry breaking caused by the generation of the Higgs vacuum expectation value. Gauge symmetry breaking induces a breaking of conformal symmetry and particle massivation. In the TGD framework, the generation of entanglement between members of state pairs such that members having opposite values of mass squared determined as roots of polynomial P in the most general case, leads to a breaking of conformal symmetry for each tensor factor and the description in terms of p-adic thermodynamics gives thermal mass squared.

2. What about the situation when energy, instead of mass squared, comes as a root of P. Also now one can construct physical states from massless virtual quarks with energies coming as algebraic integers. Total energies would be ordinary integers. This gives massless entangled states, if the rational integer parts of 4-momenta are parallel. This brings in mind a standard twistor approach with parallel light-like momenta for on-mass shell states. Now however the virtual states can have transversal momentum components which are algebraic numbers (possibly complex) but sum up to zero.

   Quantum entangled states would be superpositions over state pairs with parallel massless momenta. Massless extremals (topological light rays) are natural classical space-time correlates for them. This phase would correspond to the phase with unbroken conformal symmetry.

3. One can also assign a symmetry breaking to the thermodynamic massivation. For the energy option, the entire Galois group appears as symmetry of the mass shell whereas for the mass squared option only the isotropy group does so. Therefore there is a symmetry breaking of the full Galois symmetry to the symmetry defined by the isotropy group. In a loose sense, the real valued argument of P serves as a counterpart of the Higgs field.

   If the symmetry breaking in the model of electroweak interaction corresponds to this kind of symmetry breaking, the isotropy group, which presumably involves also a discrete subgroup of quaternionic automorphisms as an analog of the Galois group. Quaternionic group could act as a discrete subgroup of $SU(2) \subset SU(2)_L \times U(1)$. The hierarchy of discrete subgroups associated with the hierarchy of Jones inclusions assigned with measurement resolution suggests itself. It has the isometry groups of Platonic solids as the groups with genuinely 3-D action. $U(1)$ factor could correspond to Z_n as the isotropy group of the Galois group. In the QCD picture about strong interactions there is no gauge symmetry breaking so that a description based on the energy option is natural. Hadronic picture would correspond to mass squared option and symmetry breaking to the isotropy group of the root.

To sum up, in the maximally symmetric scenario, conformal symmetry breaking would be only apparent, and due to the necessity to restrict to non-tachyonic subsystems using p-adic thermodynamics. Gauge symmetry breaking would be replaced with the replacement of the Galois group with the isotropy group of the root representing mass squared value. The argument of the polynomial defining space-time region would be the analog of the Higgs field.

7.5.6 Some further comments about the notion of mass

In the sequel some further comments related to the notion of mass are represented.

M^8-H duality and tachyonic momenta

Tachyonic momenta are mapped to space-like geodesics in H or possibly to the geodesics of X^4 [L52, L53, L68]. This description could allow to describe pair creation as turning of fermion backwards in time [L73]. Tachyonic momenta correspond to points of de Sitter space and are therefore outside CD and would go outside the space-time surface, which is inside CD. Could one avoid this?

1. Since the points of the twistor spaces $T(M^4)$ and $T(CP_2)$ are in 1-1 correspondence, one can use either $T(M^4)$ or $T(CP_2)$ so that the projection to M^4 or CP_2 would serve as the base space of $T(X^4)$. One could use CP_2 coordinates or M^4 coordinates as space-time coordinates if the dimension of the projection is 4 to either of these spaces. In the generic case, both dimensions are 4 but one must be very cautious with genericity arguments which fail at the level of M^8.

2. There are exceptional situations in which genericity fails at the level of H. String-like objects of the form $X^2\times Y^2\subset M^4\subset CP_2$ is one example of this. In this case, X^6 would not define 1-1 correspondence between $T(M^4)$ or $T(CP_2)$.

   Could one use partial projections to M^2 and S^2 in this case? Could $T(X^4)$ be divided locally into a Cartesian product of 3-D M^4 part projecting to $M^2\subset M^4$ and of 3-D CP_2 part projected to $Y^2\subset CP_2$.

3. One can also consider the possibility of defining the twistor space $T(M^2\times S^2)$. Its fiber at a given point would consist of light-like geodesics of $M^2\times S^2$. The fiber consists of direction vectors of light-like geodesics. S^2 projection would correspond to a geodesic circle $S^1\subset S^2$ going through a given point of S^2 and its points are parametrized by azimuthal angle Φ. Hyperbolic tangent $tanh(\eta)$ with range $[-1,1]$ would characterize the direction of a time like geodesic in M^2. At the limit of $\eta\rightarrow\pm\infty$ the S^2 contribution to the S^2 tangent vector to length squared of the tangent vector vanishes so that all angles in the range $(0,2\pi)$ correspond to the same point. Therefore the fiber space has a topology of S^2.

   There are also other special situations such as $M^1\times S^3$, $M^3\times S^1$ for which one must introduce specific twistor space and which can be treated in the same way.

During the writing of this article I realized that the twistor space of H defined geometrically as a bundle, which has as H as base space and fiber as the space of light-like geodesic starting from a given point of H need not be equal to $T(M^4)\times T(CP_2)$, where $T(CP_2)$ is identified as $SU(3)/U(1)\times U(1)$ characterizing the choices of color quantization axes.

1. The definition of $T(CP_2)$ as the space of light-like geodesics from a given point of CP_2 is not possible. One could also define the fiber space of $T(CP_2)$ geometrically as the space of geodesics emating from origin at $r=0$ in the Eguchi-Hanson coordinates [L2] and connecting it to the homologically non-trivial geodesic sphere S^2_G $r=\infty$. This relation is symmetric.

   In fact, all geodesics from $r=0$ end up to S^2. This is due to the compactness and symmetries of CP_2. In the same way, the geodesics from the North Pole of S^2 end up to the South Pole. If only the endpoint of the geodesic of CP_2 matters, one can always regard it as a point S^2_G.

   The two homologically non-trivial geodesic spheres associated with distinct points of CP_2 always intersect at a single point, which means that their twistor fibers contain a common geodesic line of this kind. Also the twistor spheres of $T(M^4)$ associated with distinct points of M^4 with a light-like distance intersect at a common point identifiable as a light-like geodesic connecting them.

2. Geometrically, a light-like geodesic of H is defined by a 3-D momentum vector in M^4 and 3-D color momentum along CP_2 geodesic. The scale of the 8-D tangent vector does not matter and the 8-D light-likeness condition holds true. This leaves 4 parameters so that $T(H)$ identified in this way is 12-dimensional.

   The M^4 momenta correspond to a mass shell H^3. Only the momentum direction matters so that also in the M^4 sector the fiber reduces to S^2 . If this argument is correct, the space of light-like geodesics at point of H has the topology of $S^2 \times S^2$ and $T(H)$ would reduce to $T(M^4) \times T(CP_2)$ as indeed looks natural.

Conformal confinement at the level of H

The proposal of [L81], inspired by p-adic thermodynamics, is that all states are massless in the sense that the sum of mass squared values vanishes. Conformal weight, as essentially mass squared value, is naturally additive and conformal confinement as a realization of conformal invariance would mean that the sum of mass squared values vanishes. Since complex mass squared values with a negative real part are allowed as roots of polynomials, the condition is highly non-trivial.

$M^8 - H$ duality [L52, L53] would make it natural to assign tachyonic masses with CP_2 type extremals and with the Euclidean regions of the space-time surface. Time-like masses would be assigned with time-like space-time regions. In [L79] it was found that, contrary to the beliefs held hitherto, it is possible to satisfy boundary conditions for the action action consisting of the Kähler action, volume term and Chern-Simons term, at boundaries (genuine or between Minkowskian and Euclidean space-time regions) if they are light-like surfaces satisfying also $detg_4 = 0$. Masslessness, at least in the classical sense, would be naturally associated with light-like boundaries (genuine or between Minkowskian and Euclidean regions).

About the analogs of Fermi torus and Fermi surface in H^3

Fermi torus (cube with opposite faces identified) emerges as a coset space of E^3/T^3, which defines a lattice in the group E^3. Here T^3 is a discrete translation group T^3 corresponding to periodic boundary conditions in a lattice.

In a realistic situation, Fermi torus is replaced with a much more complex object having Fermi surface as boundary with non-trivial topology. Could one find an elegant description of the situation?

1. Hyperbolic manifolds as analogies for Fermi torus?

The hyperbolic manifold assignable to a tessellation of H^3 defines a natural relativistic generalization of Fermi torus and Fermi surface as its boundary. To understand why this is the case, consider first the notion of cognitive representation.

1. Momenta for the cognitive representations [L80] define a unique discretization of 4-surface in M^4 and, by $M^8 - H$ duality, for the space-time surfaces in H and are realized at mass shells $H^3 \subset M^4 \subset M^8$ defined as roots of polynomials P. Momentum components are assumed to be algebraic integers in the extension of rationals defined by P and are in general complex.

   If the Minkowskian norm instead of its continuation to a Hermitian norm is used, the mass squared is in general complex. One could also use Hermitian inner product but Minkowskian complex bilinear form is the only number-theoretically acceptable possibility. Tachyonicity would mean in this case that the real part of mass squared, invariant under $SO(1,3)$ and even its complexification $SO_c(1,3)$, is negative.

2. The active points of the cognitive representation contain fermion. Complexification of H^3 occurs if one allows algebraic integers. Galois confinement [L80, L76] states that physical states correspond to points of H^3 with integer valued momentum components in the scale defined by CD.

   Cognitive representations are in general finite inside regions of 4-surface of M^8 but at H^3 they explode and involve all algebraic numbers consistent with H^3 and belonging to the extension of rationals defined by P. If the components of momenta are algebraic integers, Galois confinement allows only states with momenta with integer components favored by periodic boundary conditions.

Could hyperbolic manifolds as coset spaces $SO(1,3)/\Gamma$, where Γ is an infinite discrete subgroup $SO(1,3)$, which acts completely discontinuously from left or right, replace the Fermi torus? Discrete translations in E^3 would thus be replaced with an infinite discrete subgroup Γ. For a given P, the matrix coefficients for the elements of the matrix belonging to Γ would belong to an extension of rationals defined by P.

1. The division of $SO(1,3)$ by a discrete subgroup Γ gives rise to a hyperbolic manifold with a finite volume. Hyperbolic space is an infinite covering of the hyperbolic manifold as a fundamental region of tessellation. There is an infinite number of the counterparts of Fermi torus [L64]. The invariance respect to Γ would define the counterpart for the periodic boundary conditions.

   Note that one can start from $SO(1,3)/\Gamma$ and divide by $SO(3)$ since Γ and $SO(3)$ act from right and left and therefore commute so that hyperbolic manifold is $SO(3) \setminus SO(1,3)/\Gamma$.

2. There is a deep connection between the topology and geometry of the Fermi manifold as a hyperbolic manifold. Hyperbolic volume is a topological invariant, which would become a basic concept of relativistic topological physics (`https://cutt.ly/RVsdNl3`).

   The hyperbolic volume of the knot complement serves as a knot invariant for knots in S^3. Could this have physical interpretation in the TGD framework, where knots and links, assignable to flux tubes and strings at the level of H, are central. Could one regard the effective hyperbolic manifold in H^3 as a representation of a knot complement in S^3?

   Could these fundamental regions be physically preferred 3-surfaces at H^3 determining the holography and $M^8 - H$ duality in terms of associativity [L52, L53]. Boundary conditions at the boundary of the unit cell of the tessellation should give rise to effective identifications just as in the case of Fermi torus obtained from the cube in this way.

2 .De Sitter manifolds as tachyonic analogs of Fermi torus do not exist

Can one define the analogy of Fermi torus for the real 4-momenta having negative, tachyonic mass squared? Mass shells with negative mass squared correspond to De-Sitter space $SO(1,3)/SO(1,2)$ having a Minkowskian signature. It does not have analogies of the tessellations of H^3 defined by discrete subgroups of $SO(1,3)$.

The reason is that there are no closed de-Sitter manifolds of finite size since no infinite group of isometries acts discontinuously on de Sitter space: therefore these is no group replacing the Γ in H^3/Γ. (`https://cutt.ly/XVsdLwY`).

3.Do complexified hyperbolic manifolds as analogs of Fermi torus exist?

The momenta for virtual fermions defined by the roots defining mass squared values can also be complex. Tachyon property and complexity of mass squared values are not of course not the same thing.

1. Complexification of H^3 would be involved and it is not clear what this could mean. For instance, does the notion of complexified hyperbolic manifold with complex mass squared make sense.

2. $SO(1,3)$ and its infinite discrete groups Γ act in the complexification. Do they also act discontinuously? p^2 remains invariant if $SO(1,3)$ acts in the same way on the real and imaginary parts of the momentum leaves invariant both imaginary and complex mass squared as well as the inner product between the real and imaginary parts of the momenta. So that the orbit is 5-dimensional. Same is true for the infinite discrete subgroup Γ so that the construction of the coset space could make sense. If Γ remains the same, the additional 2 dimensions can make the volume of the coset space infinite. Indeed, the constancy of $p_1 \cdot p_2$ eliminates one of the two infinitely large dimensions and leaves one.

   Could one allow a complexification of $SO(1,3)$, $SO(3)$ and $SO(1,3)_c/SO(3)_c$? Complexified $SO(1,3)$ and corresponding subgroups Γ satisfy $OO^T = 1$. Γ_c would be much larger and contain the real Γ as a subgroup. Could this give rise to a complexified hyperbolic manifold H^3_c with a finite volume?

3. A good guess is that the real part of the complexified bilinear form $p \cdot p$ determines what tachyonicity means. Since it is given by $Re(p)^2 - Im(p)^2$ and is invariant under $SO_c(1,3)$ as also $Re(p) \cdot Im(p)$, one can define the notions of time-likeness, light-likeness, and space-likeness using the sign of $Re(p)^2 - Im(p^2)$ as a criterion. Note that $Re(p)^2$ and $Im(p)^2$ are separately invariant under $SO(1,3)$.

   The physicist's naive guess is that the complexified analogs of infinite discrete and discontinuous groups and complexified hyperbolic manifolds as analogs of Fermi torus exist for $Re(P^2) - Im(p^2) > 0$ but not for $Re(P^2) - Im(p^2) < 0$ so that complexified dS manifolds do not exist.

4. The bilinear form in H_c^3 would be complex valued and would not define a real valued Riemannian metric. As a manifold, complexified hyperbolic manifold is the same as the complex hyperbolic manifold with a hermitian metric (see `https://cutt.ly/qVsdS7Y` and `https://cutt.ly/kVsd3Q2`) but has different symmetries. The symmetry group of the complexified bilinear form of H_c^3 is $SO_c(1,3)$ and the symmetry group of the Hermitian metric is $U(1,3)$ containing $SO(1,3)$ as a real subgroup. The infinite discrete subgroups Γ for $U(1,3)$ contain those for $SO(1,3)$. Since one has complex mass squared, one cannot replace the bilinear form with hermitian one. The complex H^3 is not a constant curvature space with curvature -1 whereas H_c^3 could be such in a complexified sense.

7.5.7 Is pair creation really understood in the twistorial picture?

Twistorialization leads to a beautiful picture about scattering amplitudes at the level of M^8 [L72, L73]. In the simplest picture, scattering would be just a re-organization of Galois singlets to new Galois singlets. Fundamental fermions would move as free particles.

The components of the 4-momentum of virtual fundamental fermion with mass m would be algebraic integers and therefore complex. The real projection of 4-momentum would be mapped by $M^8 - H$ duality to a geodesic of M^4 starting from either vertex of the causal diamond (CD) . Uncertainty Principle at classical level requires inversion so that one has $a = \hbar_{eff}/m$, where ab denotes light-cone proper time assignable to either half-cone of CD and m is the mass assignable to the point of the mass shell $H^3 \subset M^4 \subset M^8$.

The geodesic would intersect the $a = \hbar_{eff}/m$ 3-surface and also other mass shells and the opposite light-cone boundaries of CDs involved. The mass shells and CDs containing them would have a common center but Uncertainty Principle at quantum level requires that for each CD and its contents there is an analog of plane wave in CD cm degrees of freedom.

One can however criticize this framework. Does it really allow us to understand pair creation at the level of the space-time surfaces $X^4 \subset H$?

1. All elementary particles consist of fundamental fermions in the proposed picture. Conservation of fermion number allows pair creation occurring for instance in the emission of a boson as fermion-antifermion pair in $f \rightarrow f + b$ vertex.

2. The problem is that if only non-space-like geodesics of H are allowed, both fermion and antifermion numbers are conserved separately so that pair creation does not look possible. Pair creation is both the central idea and source of divergence problems in QFTs.

3. One can identify also a second problem: what are the anticommutation relations for the fermionic oscillator operators labelled by tachyonic and complex valued momenta? Is it possible to analytically continue the anticommutators to complexified $M^4 \subset H$ and $M^4 \subset M^8$? Only the first problem will be considered in the following.

Is it possible to understand pair creation in the proposed picture based on twistor scattering amplitudes or should one somehow bring the bff 3-vertex or actually $ff\overline{ff}$ vertex to the theory at the level of quark lines? This vertex leads to a non-renormalizable theory and is out of consideration.

One can first try to identify the key ingredients of the possible solution of the problem.

1. Crossing symmetry is fundamental in QFTs and also in TGD. For non-trivial scattering amplitudes, crossing moves particles between initial and final states. How should one define the crossing at the space-time level in the TGD framework? What could the transfer of the end of a geodesic line at the boundary of CDs to the opposite boundary mean geometrically?

2. At the level of H, particles have CP_2 type extremals -wormhole contacts - as building bricks. They have an Euclidean signature (of the induced metric) and connect two space-time sheets with a Minkowskian signature.

   The opposite throats of the wormhole contacts correspond to the boundaries between Euclidean and Minkowskian regions and their orbits are light-like. Their light-like boundaries, orbits of partonic 2-surfaces, are assumed to carry fundamental fermions. Partonic orbits allow light-like geodesics as possible representation of massless fundamental fermions.

   Elementary particles consist of at least two wormhole contacts. This is necessary because the wormhole contacts behave like Kähler magnetic charges and one must have closed magnetic field lines. At both space-time sheets, the particle could look like a monopole pair.

3. The generalization of the particle concept allows a geometric realization of vertices. At a given space-time sheet a diagram involving a topological 3-vertex would correspond to 3 light-like partonic orbits meeting at the partonic 2-surface located in the interior of X^4. Could the topological 3-vertex be enough to avoid the introduction of the 4-fermion vertex?

Could one modify the definition of the particle line as a geodesic of H to a geodesic of the space-time surface X^4 so that the classical interactions at the space-time surface would make it possible to describe pair creation without introducing a 4-fermion vertex? Could the creation of a fermion pair mean that a virtual fundamental fermion moving along a space-like geodesics of a wormhole throat turns backwards in time at the partonic 3-vertex. If this is the case, it would correspond to a tachyon. Indeed, in M^8 picture tachyons are building bricks of physical particles identified as Galois singlets.

1. If fundamental fermion lines are geodesics at the light-like partonic orbits, they can be light-like but are space-like if there is motion in transversal degrees of freedom.

2. Consider a geodesic carrying a fundamental fermion, starting from a partonic 2-surface at either light-like boundary of CD. As a free fermion, it would propagate to the opposite boundary of CD along the wormhole throat.

   What happens if the fermion goes through a topological 3-vertex? Could it turn backwards in time at the vertex by transforming first to a space-like geodesic inside the wormhole contact leading to the opposite throat and return back to the original boundary of CD? It could return along the opposite throat or the throat of a second wormhole contact emerging from the 3-vertex. Could this kind of process be regarded as a bifurcation so that it would correspond to a classical non-determinism as a correlate of quantum non-determinism?

3. It is not clear whether one can assign a conserved space-like M^4 momentum to the geodesics at the partonic orbits. It is not possible to assign to the partonic 2-orbit a 3-momentum, which would be well-defined in the Noether sense but the component of momentum in the light-like direction would be well-defined and non-vanishing.

   By Lorentz invariance, the definition of conserved mass squared as an eigenvalue of d'Alembertian could be possible. For light-like 3-surfaces the d'Alembertian reduces to the d'Alembertian for the Euclidean partonic 2-surface having only non-positive eigenvalues. If this process is possible and conserves M^4 mass squared, the geodesic must be space-like and therefore tachyonic.

   The non-conservation of M^4 momentum at single particle level (but not classically at n-particle level) would be due to the interaction with the classical fields.

4. In the M^8 picture, tachyons are unavoidable since there is no reason why the roots of the polynomials with integer coefficients could not correspond to negative and even complex mass squared values. Could the tachyonic real parts of mass squared values at M^8 level, correspond to tachyonic geodesics along wormhole throats possibly returning backwards along the another wormhole throat?

How does this picture relate to p-adic thermodynamics [L81] as a description of particle massivations?

1. The description of massivation in terms of p-adic thermodynamics [L81] suggests that at the fundamental level massive particles involve non-observable tachyonic contribution to the mass squared assignable to the wormhole contact, which cancels the non-tachyonic contribution.

   All articles, and for the most general option all quantum states could be massless in this sense, and the massivation would be due the restriction of the consideration to the non-tachyonic part of the mass squared assignable to the Minkowskian regions of X^4.

2. p-Adic thermodynamics would describe the tachyonic part of the state as "environment" in terms of the density matrix dictated to a high degree by conformal invariance, which this description would break. A generalization of the blackhole entropy applying to any system emerges and the interpretation for the fact that blackhole entropy is proportional to mass squared. Also gauge bosons and Higgs as fermion-antifermion pairs would involve the tachyonic contribution and would be massless in the fundamental description.

3. This could solve a possible and old problem related to massless spin 1 bosons. If they consist of a collinear fermion and antifermion, which are massless, they have a vanishing helicity and would be scalars, because the fermion and antifermion with parallel momenta have opposite helicities. If the fermion and antifermion are antiparallel, the boson has correct helicity but is massive.

   Massivation could solve the problem and p-adic thermodynamics indeed predicts that even photons have a very small thermal mass. Massless gauge bosons (and particles in general) would be possible in the sense that the positive mass squared is compensated by equally small tachyonic contribution.

4. It should be noted however that the roots of the polynomials in M^8 can also correspond to energies of massless states. This phase would be analogous to the Higgs=0 phase. In this phase, Galois symmetries would not be broken: for the massive phase Galois group permutes different mass shells (and different $a = constant$ hyperboloids) and one must restrict Galois symmetries to the isotropy group of a given root. In the massless phase ,Galois symmetries permute different massless momenta and no symmetry breaking takes place.

7.6 Antipodal duality and TGD

I learned of a new particle physics duality from the popular article "Particle Physicists Puzzle Over a New Duality" published in Quanta Magazine (`https://cutt.ly/jZOaDhd`). The article describes the findings of Dixon et al reported in the article "Folding Amplitudes into Form Factors: An Antipodal Duality" [B26] (`https://cutt.ly/EZOsfGl`) This work relies on the calculations of Goncharov et al published in the article "Classical Polylogarithms for Amplitudes and Wilson Loops" [B34] (`https://cutt.ly/sZOsuu6`).

The calculations of Goncharov et al lead to an explicit formula for the loop contributions to the 6-gluon scattering amplitude in $\mathcal{N} = 4$ SUSY. The new duality is called antipodal duality and relates 6-gluon amplitude for the forward scattering to a 3-gluon form factor of stress tensor analogous to a quantum field describing a scalar particle. This amplitude can be identified as a contribution to the scattering amplitude $h + g \rightarrow g + g$. The result is somewhat mysterious since in the standard model strong and electroweak interactions are completely separate.

7.6.1 Findings of Dixon et al

Consider first the findings of Dixon et al [B26].

1. One considers [B34] twistor amplitudes in $\mathcal{N} = \triangle$ SUSY. Only the maximally helicity violating amplitudes (MHV) are considered and one restricts the consideration to planar diagrams (to my best understanding, non-planar diagrams are still poorly understood). The contribution of the loop corrections is studied and the number of loops is rather high in the computations checking the claimed result.

6-gluon forward scattering amplitude and 3-gluon form factor of stress energy tensor regarded as a quantum field are discussed. Conformal invariance fixes the Lorentz invariants appearing in the 6-gluon forward amplitude and in the 3-gluon form factor of stress tensor to be 3 conformally invariant cross ratios formed from the 3 gluon momenta.

The claimed antipodal duality is found to hold true for each number of loops separately at the limit when one of conformal invariants approaches zero: the interpretation is that momentum exchange between 2 gluons vanishes at this limit. For 6-gluon forward amplitudes, this limit corresponds to in the 3-D space of conformal invariants to the edges of a tetrahedron.

2. $3g \rightarrow 3g$ scattering amplitude is studied at the limit when the scattering is in forward direction. One has effectively 3 gluons but not 3-gluon scattering since there is no momentum conservation constraining the total momentum of 3 gluons except effectively for the forward scattering of the stress tensor.

   As far as total quantum numbers are considered, the stress tensor can give rise to a quantum field behaving like Higgs as far as QCD is considered. The surprising finding is that the so-called antipodal duality applied to the 6-gluon amplitude gives a 3-gluon form factor of the stress tensor, which is scalar having no spin and vanishing color quantum numbers.

3. The antipodal transformation is carried for the 6-gluon amplitude in forward direction so that only 3 gluon momenta are involved. One starts from the 6-gluon amplitude constructed using the standard rules, which require that the amplitude involves only cyclic permutations of the gluons (elements of S_6 of the gluons.

   One considers permutation group $S_3 \subset S_6$ acting in the same way on the first 3 first and 3 remaining gluons, and constructs an S_3 singlet as a sum of the amplitudes obtained by applying S_3 transformations. S_3 operations are not allowed in the twistor diagrammatics since only planar amplitudes are considered usually (the construction of twistor counterparts of non-planar amplitudes is not well-understood).

4. One also constructs the 3-gluon form factor of stress energy tensor by using the twistor rules and considers the so-called soft limit at which the sum of the 3 gluon momenta vanishes so that the effectite particle assignable to the stress tensor scatters in the forward direction. It comes as a surprise that this amplitude is related to the amplitude obtained from the forward 6-gluon amplitude by the antipodal transformation.

5. The duality also involves a simple transformation of the 3 conformal invariants formed from the gluon momenta involved to the 3-gluon form factor of the energy momentum tensor. The antipodal duality holds true at the edges of the 2-D tetrahedron surface defined by the image of the 3-gluon form factor in the space of 3 conformal invariants characterizing the 6-gluon forward amplitude.

   The term antipodal derives from the fact that the 6-gluon amplitude can be expressed as a "word" formed from 6 "letters" and the above described transformation reverses the order of the letters.

6. It is conjectured that this result generalizes to large values of n so that antipodal images of $2n$-gluon scattering amplitude in forward direction could correspond to $n-$-gluon form factor for stress tensor energy and this in turn would be associated with scattering of Higgs and n gluons.

7.6.2 Questions

Since the stress tensor is a scalar, it is not totally surprising that a term proportional to this amplitude contributes to the scattering amplitude $h + g \rightarrow g + g$, where h denotes Higgs particle. What looks somewhat mysterious is that Higgs is an electro-weakly interacting particle and has no direct color interactions. The description of the scattering in the standard model involves electroweak interactions and involves at least one decay of a gluon to a quark pair in turn interacting with the Higgs.

This inspires several questions.

1. Can one consider more general subgroups $S_m \subset S_{2n}$ and by forming S_m singlets construct amplitudes with a physical interpretation?

2. Can one imagine a deep duality between color and electroweak interactions such that $\mathcal{N} = 4$ SUSY would reflect this duality? Could one even think that the strong and electroweak interactions are in some sense dual?

In TGD color interactions and electroweak interactions are related to the isometries and holonomies of $CP2$ and there indeed exists quite a number of pieces of evidence for this kind of duality. However, the possibility that electroweak or color interactions alone could provide a full description of scattering amplitudes looks unrealistic: both electroweak and color quantum numbers are needed. The number-theoretical view of TGD [L68, L23, L72, L73] could however come into rescue.

7.6.3 In what sense could electroweak and color interactions be dual?

Some kind of duality of electroweak and color interactions is suggested by the antipode duality having an interpretation in terms of Hopf algebras (`https://en.wikipedia.org/wiki/Hopf_algebra`): antipode generalizes the notion of inverse for an element of algebra.

TGD contains several mysterious looking and not-well understood features suggesting some kind of duality between electroweak and color interactions. What could make this duality possible in the TGD framework, would be the presence of Galois symmetry, which would allow us to describe electroweak or color particle multiplets number-theoretically using representations of the Galois group.

1. The electric-magnetic duality or Montonen-Olive duality (`https://en.wikipedia.org/wiki/Montonen\OT1\textendashOlive_duality`) is inspired by the homology of CP_2 in TGD [?]. The generalization of this duality in gauge theories relates the perturbative description of gauge interactions for gauge group G to a non-perturbative description in terms of magnetic monopoles associated with the dual gauge group G_L. Langlands duality [A13, A12], discussed from the TGD perspective in [K37, K38], relates the representations of Galois groups and those of Lie groups, and involves Lie group and its Langlands dual. Therefore gauge groups, magnetic monopoles and the corresponding dual gauge group, and number theory seem to be mathematically related, and TGD suggests a physical realization of this view.

2. The dual groups G and G_L should be very similar but electroweak gauge group $U(2)$ and color group $SU(3)$, albeit naturally related as holonomy and isometry groups of CP_2, do not satisfy this condition. Here the Galois group could come into rescue and provide the missing quantum numbers.

3. Depending on the situation, Galois confinement could relate to color confinement or electroweak confinement. In the context of electric-magnetic duality [K35, K5, K48], I have discussed electroweak confinement and as a possible dual description for the electroweak massivation, involving summation of electroweak $SU(2)$ quantum numbers to zero in the scale of monopole flux tubes assignable to elementary particles. The screening of weak isospin would take place by a pair of neutrino and right-handed neutrino in the Compton scale of weak boson or fermion: $h_{eff} > h$ allows longer scales.

4. Also magnetic charge or flux assignable to the flux tubes could make possible a topological description of color hypercharge topologically whereas color isospin could might have description in terms of weak iosospin. I considered this idea already in my thesis. As a matter of fact, already before the discovery of CP_2 around 1980, I proposed that magnetic (homology-) charges 2,-1,-1 for cP_2 could correspond to em charges 2/3,-1/3,-1/3 of quarks and that quark confinement could be a topological phenomenon. Maybe these almost forgotten ideas might find a place in TGD after all.

Consider now the possible duality between electroweak and color interactions.

H level

At the level of H spinors do not couple classically to gluons and color is not spin-like quantum number.

1. The proposal is that the zero energy states are singlets either with respect to the Galois group or the isotropy group of a given root. Z_3 as a subgroup or possibly normal subgroup of the Galois group would act on the space of fermion momenta for which components are algebraic integers belonging to the extension of rationals defined by P.

2. Color confinement could correspond to Galois confinement. Alternatively, the confinement of color isospin could correspond to Galois confinement whereas the confinement of color hypercharge would have a description in terms of the already mentioned monopole confinement. Both number theoretic and topological color would be invisible.

   Could antipodal duality be understood number-theoretically?

1. The antipodal duality produces an S_3 singlet from a twistor amplitude. Could color singlets correspond to Z_3 Galois-singlets and electroweak singlets above Compton scale to Z_2 singlets.

2. Could Z_2 be realized as an exchange of two gluons ordered cyclically in the amplitude? Could one think that S^6 acts as a Galois group or its isotropy group?

   The stress tensor as a Higgs like state is not a doublet. Could one obtain Higgs as a Z_2 doublet by allowing the entire orbit of S_3 but requiring only that Z_3 singlet property holds true?

3. Could all isotropy groups or even all subgroups of S^3 be allowed. Could S_n quite generally have a representation as a Galois group? This picture applies also to $2n$-gluon amplitudes but also more general conditions for Galois singlet property can be imagined.

M^8 level

The roles of color and electroweak quantum numbers are changed in $M^8 - H$ duality [L52, L53].

1. At the level of M^8, complexified octonionic 2-spinors [L48, L52, L53] decompose to the representations of the subgroup $SU(3) \subset G_2$ of octonionic automorphisms as $1 + \overline{1} + 3 + \overline{3}$. One obtains leptons and quarks with spin but electroweak quantum numbers do not appear as spin-like quantum numbers. This would suggest that one should assume both lepton and quark spinors at the level of H although the idea about leptons as 3-quark composites in CP_2 scale is attractive [L61].

   One can however construct octonionic spinor fields $M^4 \times E^4$ with the spinor partial waves belonging to the representations of $SO(4) = SU(2) \times SU(2)$ decomposing to representation of $U(2)$ with quantum numbers having interpretation as orbital angular momentum like electroweak quantum numbers.

2. At the level of 4-surfaces of M^8, weak isospin doublet could correspond to Galois doublet associated with a Z_2 factor of the Galois group.

Twistor space level

Also at the level of twistor spaces, the roles of electroweak and color numbers are changed in $M^8 - H$ duality.

1. At the level of H, $M^4 \times CP_2$ is replaced by the product of the twistor spaces $T(M^4)$ and $T(CP_2) = SU(3)/U(1) \times U(1)$. Since spinors are not involved anymore, electroweak quantum numbers disappear. Number theoretic description should apply. Here Galois subgroup Z_2 could help.

   This suggests that $U(2) \subset SU(3)$ must be interpreted in terms of electroweak quantum numbers. There indeed exists a natural embedding of the holonomy group of CP_2 to its isometry group. At the level of space-time, surface color hyper-charge and isopin could correspond to electroweak hyper-charge and isospin. This works if, for given electroweak quantum numbers, the choice of the quantization axes of color quantum numbers depends on the state so that the regions of space-time surface assignable to a fermion depends on its color quantum numbers in H. This would give a correlation between space-time geometry and quantum numbers.

2. At the level of M^8 the twistor space $T(E^4)$ contains information about weak quantum numbers but no information of color quantum numbers since octonionic spinors are given up. Z_6 as a subgroup of the Galois group could help now.

Also the induced twistor structure at the level of space-time surface in H and at the level of 4-surface in M^8 gives strong consistency conditions.

1. The induced twistor structure for the surface $T(X^4) \subset T(H)$ has S^2 bundle structure characterizing twistor space. This structure is obtained by dimensional reduction to $X^6 = X^4 \times S^2$ locally such that S^2 corresponds to the twistor sphere of both $T(M^4)$ and $T(CP_2)$.

2. For cognitive representations as unique number theoretic discretizations of the space-time surface, the twistor spheres S^2 of $T(M^4)$ *resp.* $T(CP_2)$ must correspond to each other. The point of S^2 represents the direction of the quantization axis and the value $\pm 1/2$ of spin *resp.* color isospin or appropriately normalized color hypercharge respectively.

   For quark triplets this kind of correlation can make sense between spin and color hypercharge only and only at the level of the space-time surface. Since the quantization directions of color isospin are not fixed, only the correlation between representations, rather states, is required and makes sense for quarks. This suggests that color isospin at the space-time level must correspond to Galois quantum number.

3. What about leptons? For leptons color hypercharge vanishes. However, both leptonic and quark-like induced spinors have anomalous hypercharge proportional to electromagnetic charge so that also leptonic spinors would form doublets with respect to anomalous color [L29].

The induced twistor structure for 4-surfaces in M^8 does not correspond to dimensional reduction but one expects an analogous correlation between spin and electroweak quantum numbers induced by the mapping of the twistor spheres S^2 to each other.

1. This correlation spin H-spinors correspond to tensor products of spin and electroweak doublets and all elementary particles are constructed from these.

2. Something seems to be however missing: also M^4 spinors should have a $U(1)$ charge to make the picture completely symmetric. The spinor lift strongly suggests that also M^4 has the analog of Kähler structure [L65] and this would give rise to $U(1)$ charge for M^4 spinors [L24] [K5]. This coupling could give rise to small CP breaking effects at the level of fundamental spinors [L65].

The experimental picture about strong and electroweak interactions suggests that the description of standard model interactions as either color interactions or electroweak interactions combined with a number theoretic/topological description of the missing quantum numbers is enough.

1. In hadron physics, only electroweak quantum numbers are visible. Color could be described using number-theory and topology and also these descriptions might be dual. In the QCD picture at high energies only color quantum numbers are visible and electroweak quantum numbers could be described number-theoretically. For a given particle, electroweak confinement would work above its Compton scale of weak scale.

2. In the old fashioned hadron physics conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC) relate hadron physics and electroweak physics in

a way which is not fully understood since also quark confinement is still poorly understood. PCAC reflects the massivation of hadrons and can be also seen as caused by the massivation of quarks and leptons and makes successful predictions. In the TGD framework PCAC is applied to the model of so-called lepto-hadrons [K80].

One can say that hadronic description uses $SO(4) = SU(2)_L \times U(2)_R$ or rather, $U_{ew}(2)$ as a symmetry group whereas QCD uses $SU(3)$ in accordance with the duality between color and electroweak interactions. This conforms with the $M^8 - H$ duality.

3. What about CP_2 type extremals (wormhole contacts), which have Euclidean metric. Could electroweak spin be described as the spin of an octo-spinor and could M^4 spin be described number-theoretically?

What about leptons? For leptons color hypercharge vanishes. However, both leptonic and quark-like induced spinors have anomalous hypercharge proportional to electromagnetic charge so that also leptonic spinors would form doublets with respect to anomalous color.

7.6.4 Antipodal duality and TGD: two years later

Avi Shrikumar asked about the antipodal duality (see this), which has been discovered in QCD but whose origin is not well-understood.

Antipodal duality implies connections between strong and electroweak interactions, which look mysterious since in the standard model these interactions are apparently independent. This kind of connections were discovered long before QCD and expressed in terms of the conserved vector current hypothesis (CVC) and partially conserved axial current PCAC hypothesis for the current algebra.

I looked at the antipodal duality as I learned [K6] of it but did not find any obvious explanation in TGD at that time. After that I however managed to develop a rather detailed understanding of how the scattering amplitudes emerge in the TGD framework. The basic ideas about the construction of vertices [L82, L91] are very helpful in the sequel.

1. In TGD, classical gravitational fields, color fields, electroweak fields are very closely related, being expressed in terms of CP_2 coordinates and their gradients, which define the basic field like variables when space-time surface 4-D M^4 projection. TGD predicts that also M^4 possesses Kähler structure and gives rise to the electroweak U(1) gauge field. It might give an additional contribution to the electroweak U(1) field or define an independent U(1) field.

   There is also a Higgs emission vertex and the CP_2 part for the trace of the second fundamental fundamental form behaves like the Higgs group theoretically. This trace can be regarded as a generalized acceleration and satisfies the analog of Newton' s equation and Einstein's equations. M^4 part as generalized M^4 acceeration would naturally define graviton emission vertex and CP_2 part Higgs emission vertex.

   This picture is bound to imply very strong connections between strong and weak interactions and also gravitation.

2. The construction of the vertices led to the outcome that all gauge theory vertices reduce to the electroweak vertices. Only the emission vertex corresponding to Kähler gauge potential and photon are vectorial and can contribute to gluon emission vertices so that strong interactions might involve only the Kähler gauge potentials of CP_2 and M^4 (something new).

3. The vertices involving gluons can involve only electroweak parity conserving vertices since color is not a spin-like quantum number in TGD but corresponds to partial waves in CP_2. This implies very strong connections between electroweak vertices and vertices involving gluon emission. One might perhaps say that one starts the U(1) electroweak vertex and its M^4 counterpart and assigns to the final state particles as a center of mass motion in CP_2.

If this view is correct, then the standard model would reflect the underlying much deeper connection between electroweak, color and gravitational interactions implied by the geometrization of the standard model fields and gravitational fields. This connection would remain hidden in the gauge theory approximation.

7.7 How could Julia sets and zeta functions relate to Galois confinement?

In this section the limit of large particle number of identical particles for the scattering is considered. It is found that the mass spectrum belongs to the Julia set of an infinitely iterated polynomial defining the many-particle state. Also a generalization replacing polynomials with real analytic functions is discussed and it is found that zeta functions and elliptic functions are especially interesting concerning conformal confinement as analog of Galois confinement.

7.7.1 The mass spectrum for an iterate of polynomial and chaos theory

Suppose that the number theoretic interaction in the scattering corresponds to a functional composition of the polynomials characterizing the external particles. If the number of the external particles is large, the composite can involve a rather high iterate of a single polynomial. This motivates the study of the scattering of identical particles described by the same polynomial P at the limit of a large particle number. These particles could correspond to elementary particles, in particular IR photons and gravitons. This situation leads to an iteration of a complex polynomial.

If the polynomials satisfy $P(0) = 0$ requiring $P(x) = xP_1(x)$, the roots of P are inherited. In this case fixed points correspond to the points with $P(x) = 1$. Assume also that the coefficients are rational. Monic polynomials are an especially interesting option.

For a k:th iterate of P, the mass squared spectrum is obtained as a union of spectra obtained as images of the spectrum of P under iterates P^{-r}, $r \leq k$, for the inverse of P, which is an n-valued algebraic function if P has degree n. This set is a subset of Fatou set (`https://cutt.ly/hOgq6Yy`)and for polynomials a subset of filled Julia set.

At the limit of large k, the limiting contributions to the spectrum approach a subset of Julia set defined as a P-invariant set which for polynomials is the boundary of the set for which the iteration divergences. The iteration of all roots except $x = 0$ (massless particles) leads to the Julia set asymptotically.

All inverse iterates of the roots of P are algebraic numbers. The Julia set itself is expected to contain transcendental complex numbers. It is not clear whether the inverse iterates at the limit are algebraic numbers or transcendentals. For instance, one can ask whether they could consist of n-cycles for various values of n consisting of algebraic points and forming a dense subset of the Julia set. The fact that the number of roots is infinite at this limit, suggests that a dense subset is in question.

The basic properties of Julia set deserve to be listed.

1. At the real axis , the fixed points satisfying $P(x) = x$ with $|dP/dx| > 1$ are repellers and belong to the Julia set. In the complex plane, the definition of points of the Julia set is $|P(w) - P(z)| \geq |w - z|$ for point w near to z.

2. Julia set is the complement of the Fatou set consisting of domains. Each Fatou domain contains at least one critical point with $dP/dz = 0$. At the real axis, this means that P has maximum or minimum. The iteration of P inside Fatou domain leads to a fixed point inside the Fatou set and inverse iteration to its boundary. The boundaries of Fatou domains combine to form the Julia set. In the case of polynomials, Fatou domains are labeled by the $n - 1$ solutions of $dP/dz = P_1 + zdP_1/dz = 0$.

3. Julia set is a closure of infinitely many periodic repelling orbits. The limit of inverse iteration leads towards these orbits. These points are fixed points for powers P^n of P.

4. For rational functions Julia set is the boundary of a set consisting of points whose iteration diverges to infinity. For polynomials Julia set is the boundary of the so-called filled Julia set consisting of points for which the iterate remains finite.

Chaos theory also studies the dependence of Julia set on the parameters of the polynomials. Mandelbrot fractal is associated to the polynomial $Q(z) = a + z^2$ for which origin is an stable critical point and corresponds to the boundary of the region in a-plane containing origin such that outside the boundary the iteration leads to infinity and in the interior to origin.

The critical points of P with $dP/dz = 0$ for $z = z_{cr}$ located inside Fatou domains are analogous to point $z = 0$ for $Q(z)$ associated with Fatou domains and quadratic polynomial $a + b(z - z_{cr})^2$, $b > 0$, would serve as an approximation. The variation of a is determined by the variation of the coefficients of P required to leave z_{cr} invariant.

Feigenbaum studied iteration of a polynomial $a - x^2$ for which origin is unstable critical point and found that the variation of a leads to a period doubling sequence in which a sequence of 2^n-cycles is generated (`https://cutt.ly/pOgwuqj`). Origin would correspond to an unstable critical point $dP(z)/dz = 0$ belonging to a Julia set.

The physical implications of this picture are highly interesting.

1. For a large number of interacting quarks, the mass squared spectrum of quarks as roots of the iterate of P in the interaction region would approach the Julia set as infinite inverse iterates of the roots of P. This conforms with the idea that the complexity increases with the particle number.

   Galois confinement forces the mass squared spectrum to be integer valued when one uses as a unit the p-adic mass scale defined by the larger ramified prime for the iterate. The complexity manifests itself only as the increase of the microscopic states in interaction regions.

2. Julia set contains a dense set consisting of repulsive n-cycles, which are fixed points of P and the natural expectation is that the mass spectrum decomposes into n-multiplets. Whether all values of n are allowed, is not clear to me. The limit of a large quark number would also mean an approach to (quantum) criticality.

To sum up, it would seem that chaos (or rather complexity-) theory could be an essential part of the fundamental physics of many-quark systems rather than a mere source of pleasures of mathematical aesthetics.

7.7.2 A possible generalization of number theoretic approach to analytic functions

$M^8 - H$ duality also allows the possibility that space-time surfaces in M^8 are defined as roots of real analytic functions. This option will be considered in this subsection.

Are polynomials 4-surfaces only an approximation

One of the open problems of the number-theoretic vision is whether the space-time surfaces associated with rational or even monic polynomials are an approximation or not.

1. One could argue that the cognitive representations are only a universal discretization obtained by approximating the 4-surface in M^8 by a polynomial. This discretization relies on an extension of rationals and more general than rational discretizations, which however appear via Galois confinement for the momenta of Galois singlets.

   One objection against space-time surfaces as being determined by polynomials in M^8 was that the resulting 4-surfaces in M^8 would bre algebraic surfaces. There seems to be no hope about Fourier analysis. The problem disappeared with the realization that polynomials determine only the 3-surfaces as mass-shells of M^4 and that $M^8 - H$ duality is realized by an explicit formula subject to $I(D) = exp{-}K$ condition.

2. Galois confinement provides a universal mechanism for the formation of bound states. Could evolution be a development of real states for which cognitive representations in terms of quarks become increasingly precise.

   That the quarks defining the active points of the representation are at 3-D mass shells would correspond to holography at the level of M^8. At the level of H they would be at the boundaries of CD. This would explain why we experience the world as 3-dimensional.

Also the 4-surfaces containing quark mass shells defined in terms of roots of arbitrary real analytic functions are possible.

1. Analytic functions could be defined in terms of Taylor or Laurent series. In fact, any representation can be considered. Also now one can consider representation involving only integers, rationals, algebraic numbers, and even reals as parameters playing a role of Taylor coefficients.

2. Does the notion of algebraic integers generalize? The roots of the holomorphic functions defining the meromorphic functions as their ratios define an extension of rationals, which is in the general transcendental. It is plausible that the notion of algebraic integers generalizes and one can assume that quarks have momentum components, which are transcendental integers. One can also define the transcendental analog of Galois confinement.

3. One can form functional composites to construct scattering amplitudes and this would make possible particle reactions between particles characterized by analytic functions. Iteration of analytic functions and approach to chaos would emerge as the functions involved appear very many times as one expects in case of IR photons and gravitons.

What about p-adicization requiring the definition discriminant D and identification of the ramified primes and maximal ramified prime? Under what conditions do these notions generalize?

1. One can start from rational functions. In the case of rational functions R, one can generalize the notion of discriminant and define it as a ratio $D = D_1/D_2$ of discriminants D_1 and D_2 for the polynomials appearing as a numerator and denominator of R. The value of D is finite irrespective of the values of the degrees of polynomials.

2. Analytic functions define function fields. Could a generalization of discriminant exist. If the analytic function is holomorphic, it has no poles so that D could be defined as the product of squares of root differences.

   If the roots appear as complex conjugate pairs, D is real. This is guaranteed if one has $f(\overline{z}) = \overline{f(z)}$. The real analyticity of f guarantees this and is necessary in the case of polynomials. A stronger condition would be that the parameters such as Taylor coefficients are rational.

   If the roots are rationals, the discriminant is a rational number and one can identify ramified primes and p-adic prime if the number of zeros is finite.

3. Meromorphic functions are ratios of two holomorphic functions. If the numbers of zeros are finite, the ratio of the discriminants associated with the numerator and denominator is finite and rational under the same assumptions as for holomorphic functions.

4. $M^8 - H$ duality leads to the proposal that the discriminant interpreted as a p-adic number for p-adic prime defined by the largest ramified prime, is equal to the exponent of $exp(-K)$ of Kähler function for the space-time surface in H.

   If one can assign ramified primes to D, it is possible to interpret D as a p-adic number having a finite real counterpart in canonical identification. For instance, if the roots of zeta are rationals, this could make sense.

Questions related to the emergence of mathematical consciousness

These considerations inspire further questions about the emergence of mathematical consciousness.

1. Could some mathematical entities such as analytic functions have a direct realization in terms of space-time surfaces? Could cognitive processes be identified as a formation of functional composites of analytic functions? They would be analogs of particle reactions in which the incoming particles consist of quarks, which are associated with mass-shells defined by the roots of analytic function.

   These composites would decay to products of polynomials in cognitive measurements involving a cascade of SSFRs reducing the entanglement between a relative Galois group and corresponding normal group acting as Galois group of rationals [L58].

2. Could the basic restriction to cognition come from the Galois confinement: momenta of composite states must be integers using p-adic mass scale as a unit.

   Or could one think that the normal sub-group hierarchies formed by Galois groups actually give rise to hierarchies of states, which are Galois confined for an extension of the Galois group.

   Could these higher levels relate to the emergence of consciousness about algebraic numbers. Could one extend computationalism allow also extensions of rationals and algebraic integers as discussed in [L57].

   Galois confinement for an extension of rationals would be analogous to the replacement of a description in terms of hadrons with that in terms of quarks and mean increase of cognitive resolution. Also Galois confinement could be generalized to its quantum version. One could have many quark states for which wave function in the space of total momenta is Galois singlet whereas total momenta are algebraic integers. S-wave states of a hydrogen atom define an obvious analog.

3. During the last centuries the evolution of mathematical consciousness has made huge steps due to the discovery of various mathematical concepts. Essentially a transformation of rational arithmetics with real analysis and calculus has taken place since the times of Newton. Could these evolutionary explosions correspond to the emergence of space-time surfaces defined by analytic functions or is it that only a conscious awareness about their existence has emerged?

Space-time surfaces defined by zeta functions and elliptic functions

Several physical interpretations of Riemann zeta have been proposed. Zeta has been associated with chaotic systems, and the interpretation of the imaginary parts of the roots of zeta as energies has been considered. Also an interpretation as a formal analog of a partition function has been considered. The interpretation as a scattering amplitude was considered by Grant Remmen [B33] (`https://cutt.ly/TID1kjU`).

1.Conformal confinement as Galois confinement for polynomials?

TGD suggests a totally different kind of approach in the attempts to understand Riemann Zeta. The basic notion is conformal confinement [K49].

1. The proposal is that the zeros of zeta correspond to complex conformal weights $s_n = 1/2+iy_n$. Physical states should be conformally confined meaning that the total conformal weight as the sum of conformal weights for a composite particle is real so that the state would have integer value conformal weight n, which is indeed natural. Also the trivial roots of zeta with $s = -2n$, $n > 0$, could be considered.

2. In $M^8 - H$ duality, the 4-surfaces $X^4 \subset M^8$ correspond to roots of polynomials P. M^8 has an interpretation as an analog of momentum space. The 4-surface involves mass shells $m^2 = r_n$, where r_n is the root of the polynomial P, algebraic complex number in general.

   The 4-surface goes through these 3-D mass-shells having $M^4 \subset M^8$ as a common real projection. The 4-surface is fixed from the condition that it defines $M^8 - H$ duality mapping it to $M^4 \times CP_2$. One can think X^4 as a deformation of M^4 by a local $SU(3)$ element such that the image points are $U(2)$ invariant and therefore define a point of CP_2. $SU(3)$ has an interpretation as octonionic automorphism.

3. Galois confinement states that physical states as many-quark states with quark momenta as algebraic integers in the extension defined by the polynomial have integer valued momentum components in the scale defined by the causal diamond also fixed by the p-adic prime identified as the largest ramified prime associated with the discriminant D of P.

   Mass squared in the stringy picture corresponds to conformal weight so that the mass squared values for quarks are analogous to conformal weights and the total conformal weight is integer by Galois confinement.

2.Conformal confinement for zeta functions

At least formally, TGD also allows a generalization of real polynomials to analytic functions. For a generic analytic function it is not possible to find superpositions of roots that would be integers and this could select Riemann Zeta and possible other analytic functions are those with infinite number of roots since they might allow a large number of bound states and be therefore winners in the number theoretic selection.

Riemann zeta is a highly interesting analytic function in this respect.

1. Actually an infinite hierarchy of zeta functions, one for any extension of rationals and conjectured to have zeros at the critical line, can be considered. Could one regard these zetas as analogous to polynomials with an infinite degree so that the allowed mass squared values for quarks would correspond to the roots of zeta?

2. Conformal confinement [K49] requires integer valued momentum components and total conformal weights as mass squared values. The fact that the roots of zetas appear as complex conjugates allows for a very large number of states with real conformal weights. This is however not enough. The fact that the roots are of the form $z_n = 1/2 + iy_n$ or $z = -2n$ implies that the conformal weights of Galois/conformal singlets are integer-valued and the spectrum is the same as in conformal field theories.

3. Riemann zeta has only a single pole at $s = 1$. Discriminant would be the product $\prod_{m\neq n}(y_m - y_n^2)\prod_{m\neq n} 4(m-n)^2 \prod_{m,n}(4m^2 + y_n^2)$ since the pole gives $D = 1$. D would be infinite.

4. Fermionic zeta $\zeta_F(s) = \zeta(s)/\zeta(2s)$ is analogous to the partition function for fermionic statistics and looks more appropriate in the case of quarks. In this case, the zeros are z_n *resp.* $z_n/2$ and the ratio of determinants would reduce to an infinite power of 2. The ramified prime would be the smallest possible: $p = 2$!

   $D = D_1/D_2$ would be infinite power of 2 and 2-adically zero so that $exp(-K)$ should vanish and Kähler function would diverge. 3-adically it would be infinite power of -1. If one can say that the number of roots is even, one has $D = 1$ 3-adically. Kähler function would be equal to zero, which is in principle possible.

   For Mersenne primes $M_n = 2^n - 1$, 2^n would be equal to $1 + M_n = 1 \ \ modM_n$ and one would obtain an infinite power $1 + M_n$, which is equal to 1 mod M_n. Could this relate to the special role of Mersenne primes?

5. What about Riemann Hypothesis? By $\zeta(\overline{s}) = \overline{zeta}(s)$, the zeros of zeta appear in complex conjugate pairs. By functional equation, also s and $1-s$ are zeros. Suppose that there is a zero $s_+ = s_0 + iy_n$ with $s_0 \neq 1/2$ in the interval $(0,1)$. This is accompanied by zeros $\overline{s}_+$, $1 - s_+$, $s_- = 1 - \overline{s}_+$. The sum of these four zeros is equal to $s = 2$. Therefore Galois singlet property does not allow us to say anything about the Riemann hypothesis.

3.Conformal confinement for elliptic functions

Elliptic functions (`https://cutt.ly/dINxAeQ`) provide examples of analytic functions with infinite number of roots forming a doubly periodic lattice and are therefore candidates for analogs of polynomials with infinite degree.

1. Weierstrass $\mathcal{P}(z)$-function $\mathcal{P}(z) = \sum_\lambda 1/(z-\lambda)^2$, where the summation is over the lattice defined by a complex modular parameter τ, is the fundamental elliptic function. The basic objection is that $\mathcal{P}(z)$ is not real analytic. Despite this it is interesting to look at its properties so that conformal weights do not appear in complex conjugate pairs. Therefore it is not clear whether conformal confinement is possible. One can also ask whether the notion of integer could be replaced with that of "modular" integers $m + n\tau$.

2. Elliptic functions are doubly periodic and characterized by the ratio τ of complex periods ω_1 and ω_2. One can assume the convention $\omega_1 = 1$ giving $\omega_2 = \tau$. The roots of the elliptic function for an infinite lattice and complex rational roots are of obvious interest concerning the generalization of Galois/conformal confinement.

3. The fundamental set of zeros is associated with a cell of this lattice. The finite number of zeros (with zero with multiplicity m counted as m zeros) in the cell is the same as the number poles and characterizes partially the elliptic function besides τ.

4. Weierstrass $\mathcal{P}$-function and its derivative $d\mathcal{P}/\lceil\ddagger$ are the building blocks of elliptic functions. A general elliptic function is a rational function of $\mathcal{P}$ and $d\mathcal{P}/\lceil\ddagger$. In even elliptic functions only the even funktion $\mathcal{P}$ appears.

5. The roots of Weierstrass $\mathcal{P}$-function $\mathcal{P}(z) = \sum_\lambda 1/(z-\lambda)^2$ appear in pairs $\pm z$ whereas the double poles at at the points of the modular lattice: see the article "The zeros of the Weierstrass $\mathcal{P}$-function and hypergeometric series" of Duke and Imamoglu [A31] (`https://cutt.ly/uIZSK4T`).

   The roots are given by Eichler-Zagier formula $z_\pm(m,n) = 1/2+m+n\tau\pm z_1$, where z_1 contains an imaginary transcendental part $log(5+2\sqrt{6})/2\pi)$ plus second part, which depends on τ (see formula 6) of `https://cutt.ly/uIZSK4T`).

6. Conformally confined states with conformal weights $h = 1+(m_1+m_2)+(n_1+n_2)\tau$ can be realized as pairs with conformal weights $(z_+(m_1,n_1), z_-(m_2,n_2)$. The condition $n_1 = -n_2$ guarantees integer-valued conformal weights and conformal confinement for a general value of τ.

7. A possible problem is that the total conformal weights can be also negative, which means tachyonicity. This is not a problem also in the case of Riemann zeta if trivial zeros are included.

   As a matter of fact, already at the level of M^8, M^4 Kähler structure implies that right-handed neutrino ν_R is a tachyon [L65]. However, ν_R provides the tachyon needed to construct massless super-symplectic ground states and also allows us to understand why neutrinos can be massive although right-handed neutrinos are not detected. The point is that only the square of Dirac equation in H is satisfied so that different M^4 chiralities can propagate independently.

   In M^8-H duality, non-tachyonicity makes it possible to map the momenta at mass shell to the boundary of CD in H. Hence the natural condition would be that the total conformal weight of a physical state is non-negative.

What about the notion of discriminant and ramified prime? One can assign to the algebraic extensions primes as prime ideals for algebraic integers and this suggests that the generalization of p-adicity and p-adic prime is possible. If this is the case also for transcendental extensions, it would be possible to define transcendental p-adicity.

One can however ask whether the discriminant is rational under some conditions. D could also allow factorization to the primes of the transcendental extension.

1. Elliptic functions are meromorphic and have the same number of poles and zeros in the basic cell so that there are some hopes that the ratio of discriminants is finite and even rational or integer for a suitable choice of the modular parameter τ as the ratio of the periods and the other parameters. Discriminant D as the ratio D_1/D_2 of the discriminants defined by the products of differences of roots and poles could be finite although they diverge separately.

2. For the Weierstrass $\mathcal{P}$-function, the zeros appear as pairs $\pm z_0$ and also as complex conjugate pairs. Complex pairs are required by real analyticity essential for the number theoretical vision. It might be possible to define the notion of ramified prime under some assumptions.

   For $z_+(m,n)$ or $z_-(m,n)$, the defining D_1 in D_1/D_2 would reduce to a product $\prod_{m,n}\Delta_{m,n})^2(\Delta_{m,n}+2z_1)(\Delta_{m,n}-2z_1)$, $\Delta_{m,n} = \Delta m + \Delta n\tau$, which is a complex integer valued if τ has integer components. D_1 would be a product of Gaussian integers.

3. The number of poles and zeros for the basic cell is the same so that D_2 as a product of the pole differences would have an identical general form. For large values of m,n, the factors in the product approach $\Delta_{m,n}$ for both zeros and poles so that the corresponding factors combine to a factor approaching unity.

The double poles of $\mathcal{P}(z) = \sum_\lambda 1/(z-\lambda)^2$ are at points of the lattice. One has $D_2 = \prod_{m,n} \Delta_{m,n})^4$. This gives

$$D = \frac{D_1}{D_2} = \prod_{m,n}(1+\frac{2z_0}{\Delta_{m,n}})(1-\frac{2z_0}{\Delta_{m,n}}) = \prod_{m,n}(1-4(\frac{2z_0}{\Delta_{m,n}})^2) \ .$$

Therefore D is finite and in general complex and transcendental so that the notion of ramified prime does not make sense as an ordinary prime. z_0 contains a transcendental constant term plus a term depending on τ (`https://cutt.ly/uIZSK4T`). Whether values of τ for which D is rational, might exist, is not clear.

In the number theoretic vision, the construction of many-particle states corresponds to the formation of functional composites of polynomials P. If the condition $P(0) = 0$ is satisfied, the $n-fold$ composite inherits the roots of $n-1$-fold composites and the roots are like conserved genes. If one multiplies zeta functions and elliptic functions by z, one obtains similar families and the formation of composites gives rise to iteration sequences and approach to chaos [L54].

Riemann zeta, quantum criticality, and conformal confinement

There are strong indications Riemann zeta (`https://cutt.ly/iVTV1kqs`) has a deep role in physics, in particular in the physics of critical systems. TGD Universe is quantum critical. What quantum criticality would mean at the space-time level is discussed in [L79]. This raises the question whether Riemann zeta could have a deep role in TGD.

First some background relating to the number theoretic view of TGD.

1. In TGD, space-time regions are characterized by polynomials P with rational coefficients [L52, L53]. Galois confinement defines a universal mechanism for the formation of bound states. Momenta for virtual fermions have components, which are algebraic integers in an extension of rationals defined by a polynomial P characterizing space-time region. For the physical many fermion states, the total momentum as the sum of fermion momenta has components, which are integers using the unit defined by the size of the causal diamond (CD).

   This defines a universal number theoretical mechanism for the formation of bound states. The condition is very strong but for rational coefficients it can be satisfied since the sum of all roots is always a rational number as the coefficient of the first order term.

2. Galois confinement implies that the sum of the mass squared values, which are in general complex algebraic numbers in E, is also an integer. Since the mass squared values correspond to conformal weights as also in string models, one has conformal confinement: states are conformal singlets. This condition replaces the masslessness condition of gauge theories [L81].

Riemann zeta is not a polynomial but has infinite number of root. How could one end up with Riemann zeta in TGD? One can also consider the replacement of the rational polynomials with analytic functions with rational coefficients or even more general functions [L73].

1. For real analytic functions roots come as pairs but building many-fermion states for which the sum of roots would be a real integer, is very difficult and in general impossible.

2. Riemann zeta and the hierarchy of its generalizations to extensions of rationals (Dedekind zeta functions) is however a complete exception! If the roots are at the critical line as the generalization of Riemann hypothesis assumes, the sum of the root and its conjugate is equal to 1 and it is easy to construct many fermion states as 2N fermion states, such that they have integer value conformal weight.

   One can wonder whether one could see Riemann zeta as an analog of a polynomial such that the roots as zeros are algebraic numbers. This is however not necessary. Could zeta and its analogies allow it to build a very large number of Galois singlets and they would form a hierarchy corresponding to extensions of rationals. Could they represent a kind of second abstraction level after rational polynomials?

Chapter 8

Twistors and holography= holomorphy vision

8.1 Introduction

Twistor lift of TGD [L3, L24, K6] relies on the replacement of space-time surfaces in $H = M^4 \times CP_2$ with the analogs of their 6-D twistor spaces X^6 as sphere bundles as surfaces in the twistor space $T(H) = T(M^4) \times T(CP_2)$ of H identified as the product of twistor spaces $T(M^4)$ and $T(CP_2)$.

Dimensional reduction for the extremals of 6-D Kähler action and the identification of the fiber spheres CP_1 of $T(M^4)$ and $T(CP_2)$ is needed to product to produce the X^6 as a sphere bundle over X^4. The dimensionally reduced action is 4-D Kähler action and volume term as in terms of an analog of dynamical, length scale dependent cosmological constant. Holography= holomorphy (H-H) vision [L88, L92] allows us to solve the field equations for the 4-D action exactly.

The structural analogies of the H-H based solutions with the twistor lift led to ask whether the twistor spheres of $T(M^4)$ and $T(CP_2)$ could be represented as surfaces inside space-time surfaces and whether the twistorialization of TGD could be carried out without the introduction of $T(H)$. As a matter of fact, this kind structural analogies should exist since the notion of twistor space is basically deduce from the geometry of M^4 and CP_2 rather than vice versa.

8.1.1 What twistors are?

The twistor space of M^4 can be defined purely geometrically. Twistor would describe fixing a coordinate frame with origin at a given M^4 point and a fixed quantization axis of spin defined by a direction of light-like momentum characterized by a point of CP_2. The light-like vector also defines a 2-D orthogonal plane. In massless field theories this corresponds to a choice of momentum vector and polarization vector. Light-like geodesics at a given point define the fiber at this point. Fiber is a 2-sphere. The bundle structure is non-trivial. The twistor spheres at points with-like separation have a common point. Not that the twistor sphere would be represented in M^4.

In the twistor Grassmannian approach [B18, B38, B23, B6, B43, B5, B25], the twistor space of M^4 is identified as $CP_3 = SU(4)/SU(3) \times U(1)$. One can end up with this identification in the following way.

1. Single bi-spinor represents a light-like momentum via the correspondence $p^k \rightarrow p^k\sigma_k$, where σ_k are Pauli spin matrices acting on complex bi-spinors. Light-likeness implies that the determinant of this 2×2 matrix vanishes. Determinant is a bilinear function or rows and columns so that the representation so that complex scalings of the bispinor do not affect the condition $p^2 = 0$.

   Twistors thus correspond to pairs of dotted (χ) and undotted (ψ) bi-spinors as conjugate representations of the Lorentz group defining the matrix $p^k\sigma_k$. Dotted and undotted bispinors are related by co-incidence relation $\chi = p^k\sigma_k\psi$: this does not fix ϕ uniquely since $\psi \rightarrow \psi + p^r k\sigma_k\phi$ leaves χ unaffects. χ and ψ span C^2 each so that one has C^4. The invariance of p^k under opposite complex scalings of the bi-spinors suggests that C^4 must be replaced with the projective space $CP_3 = SU(4)/SU(3) \times U(1))$. The problem is that the geometrically identified twistor space is non-compact whereas CP_3 is compact.

2. CP_3 should correspond to S^2 bundle over M^4 with S^2 consisting of light-like geodesics with common origin. Compact CP_3 should correspond to bundles over M^4. This cannot be true since M^4 is not compact. This leads to the proposal that compactification of M^4 is involved. This looks to me questionable.

3. The Minkowskian signature of M^4 leads to ask whether a more appropriate identification of the twistor space could be based on group theory and would be as a non-compact space $CP_{2,1} = SU(3,1)/SU(3) \times U(1)$. It should have one real time-like dimension and 5 space-like real dimensions: one complex coordinate should be hypercomplex and 2 coordinates should be complex. This would fit nicely with the H-H vision in which M^4 has one hypercomplex coordinate and one complex coordinate and a twistor sphere adds one complex coordinate. Note that now the scaling of hypercomplex coordinates with a complex number does not make sense so that the group theoretic view is necessary.

   One key problem of the twistor Grassmannian approach is that the natural signature of the Minkowski space would be (2,2) rather than (1,3). Could one think that for the signature (1,3) the two real time-like coordinates defining complex coordinates are transformed to a hypercomplex coordinate pair $(u = t + z, v = t - z)$. CP_3 naturally associated with the signature $(2,2)$ would be transformed to $CP_{2,1} \equiv SU(3,1)/SU(3) \times U(1)$ associated with the signature (1,3).

4. CP_3 ($CP_{2,1}$) is obtained by adding to E^6 the CP_2 ($CP_{1,1} = SU(2,1)/SU(2) \times U(1)$) at infinity. The set of geodesics directed from the origin of E^6 to infinity is indeed 4-D. CP_3 and $CP_{2,1}$ should allow an interpretation as a bundle with fiber CP_1.

   How could one understand this geometrically? Does the M^4 correspond to the 4-D space of homologically non-trivial 2-spheres in $CP_{2,1}$ as counterparts of twistor spheres? Is this a non-singular manifold? Note that when the points of M^4 as 2-spheres are connected by light-like geodesics, the corresponding 2-spheres must have an intersection point.

The twistor bundle of CP_2 is something completely new from the point of view of field theories. The definition of the twistor space $T(CP_2) = SU(3)/U(1) \times U(1)$ is as the space of choices of quantization axes of color isospin and hypercharge. The fiber is sphere as the set of geodesics directed rom the origin to the infinity, which corresponds to a homologically non-trivial 2-sphere added to E^4.

8.1.2 About the problems of twistorialization

Twistorialization is also plagued by other difficulties than those already mentioned. Besides the problems associated with the interpretation of CP_3 as twistor space, favoring the (2,2) signature of Minkowski space, there is a problem that the description of massive particles fails in the twistor approach. A heuristic guess is that light-likeness in the 8-D sense holding true for the modes of the second quantized induced spinor fields might help. The classical picture supports this too: for the light-like geodesics in $M^4 \times S^1 \subset M^4 \times CP_2$ M^4 projection of 8-momentum is indeed massive.

There is also the so-called googly problem and the problem that in general relativity only conformally flat space-times allow twistor structure.

Googly problem

Twistorialization means a geometrization of spin in the twistor Grassmannian approach [B18, B38, B23, B6, B43, B5, B25], which indeed allows a very elegant descirption of scattering amplitudes of spinning particles in $\mathcal{N} = 4$ SUSY. This requires massless fields. Spin corresponds to a partial wave in the twistor sphere and there is no need to introduce it as a separate internal degree of freedom. Holomorphy is an essential ingredient and analogous to holomorphy= holography hypothesis of TGD but realized at the level of surfaces rather than fields.

Googly problem means that anti-self-dual massless fields do not allow this geometrization. Only self-dual field configurations allow twistorialization in terms of holomorphic fields in twistor space. Could the fields with opposite chiralities correspond to holomorphic and antiholomorphic fields? Or does anti-holomorphy correspond to antiparticles? Why are both of them not allowed? How would one describe their interaction?

There are several notions involved: the notions of chirality/handedness, helicity and orientability, which is a property of space-time. Reflection P in M^4 changes chirality/helicity whereas charge conjugation changes the helicity. P is not a symmetry in the standard model.

A possible solution of the googly problem in terms of pin structures (see this) has been proposed. Also the reflections in M^4 would be symmetries unlike for spin structure. The two chiralities would be related by a symmetry transforming a left handed glove to a right-handed one if this symmetry is realized geometrically. Spatial reflection P and time reflection T change the orientation of M^4 but PT preserves it. P and T are not representable as transformations generated from identity and this seems to be the case also for PT. Could one somehow extend the Lie-group symmetries (Poincare group) so that PT is generated from identity. To me these proposals look artificial to me.

Conformal flatness is required in GRT

The existence of the twistor structure requires conformal invariance and massless fields in twistor space are indeed holomorphic and self-dual fields. Twistor structure is allowed only by conformally flat space-times. This condition is very strong and implies that the so-called Weyl tensor (see this) vanishes. The vanishing of the Weyl tensor implies that tidal forces describable in terms of geodesic deviation vanish. Also the trace of the energy momentum tensor must vanish as it indeed does for the Yang-Mills action. This condition is violated for typical solutions of Einstein's equations.

8.2 Twistorialization in TGD

There one can consider two, not mutually exclusive, approaches to twistorialization in TGD [L3, L24, K6]).

1. Twistor lift is based on the twistor space of $T(H)$ identified as the product $T(H) = T(M^4) \times T(CP_2)$ twistor spaces of M^4 and CP_2 is the first approach. It involves the identification of the twistor spheres of $T(M^4) \times T(CP_2)$ and dimensionally reduces the 6-D Kähler action of $T(H)$ to the sum of 4-D Kähler action and a volume term.

2. H-H principle [L88, L92]solves the field equations space-time surfaces and does not exclude $T(H)$ but can be extended to the level of $T(H)$. There are some indications that H-H alone could describe the twistorialization. The twistor spheres indeed have natural representation in $X^4 \subset H$. Since the notion of twistor is realized in terms of the geometry of the space-time, it would be natural that space-time surfaces provide representation of the twistor lift. If not, something might be wrong.

8.2.1 Some background

The existence of both left and right fermion chiralities are the source of googly problem. Reflection transforming right and left chiralities to each other is therefore closely related to the problem.

In TGD, parity violation is understood. The embedding space $H = M^4 \times CP_2$ is 8-dimensional. Both M^4 chiralities are predicted and parity violation and the strange-looking coupling structure of the Standard Model finds explanation. Spinor connection and second quantized free spinor field from H is induced to the spacetime surface are induced. The baryon and fermion numbers correspond to the two H-chiralities and the couplings to quarks and leptons are obtained correctly. For quarks/leptons, right- and left-handed M^4 chirality correspond to different CP_2 chiralities and the massivation requiring the mixing of M^4 chiralities automatically follows from the mixing of the chiralities for the massless Dirac equation in H.

The counterpart of Googly problem could be however encountered at the H-level if the twistorialization also now requires that only a single H-chirality is allowed. Only quarks or leptons would be fundamental fermions: I have considered both options. The idea about leptons as a bound state of quarks is discussed in [L61] and I have also considered the idea that quarks could be quarks, which have suffered charge fractionation. Now I have become skeptical about both options.

8.2.2 Twistor lift of TGD

Twistor Grassmannian approach [B18, B38, B23, B6, B43, B5, B25] provides an extremely economical description of scattering amplitudes in $\mathcal{N} = 4$ SUSY and even for more gauge theories. Therefore one can ask whether TGD could have a twistor lift and what would this mean?

1. Around the same time that I started developing TGD, it had been discovered that M^4 (or E^4 or S^4) and CP_2 are in a completely special position with respect to twistorization. Only they allow a twistor space, which has a Kähler structure [A18]. The Kähler structure indeed plays a key ontological role in TGD [L89, L90]. TGD is unique by the requirement that twistor lift exists and would correspond to replacing $M^4 \times CP_2$ with the product $T(H) = T(M^4) \times T(CP_2)$ of the twistor spaces $T(M^4)$ and $T(CP_2)$. This led to the proposal that M^4 has the analog of Kähler structure.

2. The induction procedure for gauge potentials would generalize to the twistor level [L3, L24]. The twistor space for the spacetime surface would be a 6-surface X^6 in $T(H) = T(M^4) \times T(CP_2)$ and therefore an S^2 bundle over the spacetime surface and a twistor structure would be induced on the 6-surface.

About the details of the twistor lift

Consider now this in more detail.

1. The requirement that X^6 is an S^2 bundle, requires a dimensional reduction of the 6-D Kähler action, which reduces it to the sum of the 4-D Kähler action and a volume term that can be interpreted as the emergence of the cosmological constant Λ [L3, L24, K6].

   The cosmological constant Λ is determined as the coefficient of the volume term of 4-D action. It is determined by the sum of Kähler action for the twistor sphere S^2 of X^6, which would depend on the induced metric and Kähler form. Λ would be dynamic and have a spectrum. The Kähler forms of both H and $T(H)$ have both M^4 and CP_2 parts. Their destructive interference can make the induced Kähler and therefore also cosmological constant very small. A natural guess is that it is inversely proportional to the square of the p-adic length scale and approaches zero in long length scales.

2. The induced metric and Kähler form of $T(X^6)$ are obtained in the usual way and in $S^2(X^4)$ the metric and Kähler form are the sum of contributions from $S^2(T(M^4)$ and $S^2(T(CP_2)$. This gives rise to a dynamical cosmological constant determined by the part of the 6-D Kähler action coming from $S^2(X^4)$.

In TGD, only fermions are fundamental particles and all particles, especially bosons, are built from these. Therefore twistor geometrization of fermion spin and isospin might make sense but is not necessary since second quantized free fermion fields in H gives fermionic propagators elegantly.

Color symmetry and rotations represent two exact symmetries realized as isometries and it might be possible to twistorialize the corresponding quantum numbers (spin and color hypercharge and - isospin). Electroweak symmetries are not exact symmetries and are not realized as isometries. Could twistorialization apply also to weak spin and hypercharge?

Is the twistorialization of the fermion spin, electroweak spin, and color quantum numbers possible? The correspondence between twistor spheres of $T(M^4)$ and $T(CP_2)$ poses very strong constraint. One can argue that if fermion spin and color charges can be twistorialized as points of these 2 twistor spheres, spin and color isospin and hypercharge would closely relate to each other. Can this make sense?

1. The first objection is that single quark spin sould correspond to 3 possible color charges. This could be understood in terms of the 3-valued character of the map $S^2(T(M^4))$ to $S^2(CP_2)$. It could allow to assign to a given spin of quark 3 different values of color charges as different space-time surfaces. This would be an analog for the representation of color as partial waves in CP_2 rather than as spin-like quantum number for fermions as in QCD?

2. The second objection is that leptons have no color quantum numbers. Could this correspond to the fact that for leptonic space-time surfaces $S^2(T(X^4)$ correspond a single point of $T(CP_2)$ so that only the twistor sphere of $T(M^4)$ is "activated"?

To sum up, in TGD only fermions are fundamental particles and all particles, especially bosons, are built from these. Therefore twistor geometrization of fermion spin and isospin might make sense but is not necessary since second quantized free fermion fields in H gives fermionic propagators elegantly.

How does twistor lift relate to H-J structure?

How does the induced twistor structure relate to the Hamilton-Jacobi (H-J) structure [L83] required by the existence of H-H structure?

1. H-J structure means the existence of 4 coordinates combining to form hypercomplex coordinate u and its conjugate v and complex coordinate w and its conjugate $\overline{w}$. The coordinate lines of u and v have light-like tangent vectors, which by integrability are proportional to gradients.

2. A point in twistor space corresponds to the choice of an M^4 point and of light-like direction at each point. Making this choice at each point of M^4 defines a section of $T(M^4)$. Could H-J structure correspond to a section of $T(M^4)$.

3. Or could it correspond to a section of $T(H)$. If the light-like coordinate curves for u and v at the space-time surface have CP_2 projections as geodesic lines, twistorialization for massive particles might be possible since M^4 projections could correspond to massive geodesics or more general curves which constant M^4 momentum squared. Induction of the twistor structure of H would generate correlations between twistor structures of M^4 and CP_2 at the space-time surface.

4. Under what conditions two H-J structures are equivalent? Physical intuition suggests that two H-J structures, which are related by a conformal diffeomorphism of H, generating new identical space-time surfaces in H-H vision, are equivalent. Conformal diffeomorphism would reduce to a transformation of hypercomplex coordinate u independent of complex coordinates and an analytic transformation of the 3 complex coordinates with coefficients depending on hypercomplex coordinates. It also looks natural that the conformal diffeomorphisms reduce to products of transformations acting in M^4 and CP_2 degrees of freedom.

5. How many H-J structures do exist? The obstacles come from topology, complex structure and integrability. A physics inspired guess is that they could correspond to self-dual or anti-self-dual solutions of the massless spin 1 field in M^4. One would have a polarization vector and light-like vector at each point. Massless extremals define such sections of $T(M^4)$ in H [K5].

8.2.3 Could holography= holomorphy vision make possible twistorialization without twistor lift?

H-H vision encourages to ask whether the homologically non-trivial twistor spheres of $T(M^4)$ and $T(CP_2)$ have representations as homologically non-trivial 2-surfaces inside space-time surfaces: this could mean that the introduction $T(H)$ might not be necessary. In particular, cosmological constant would have inherent representation in terms of the solution of field equations according to H-H vision. This does not seem to conform with the idea that $T(H)$ level determines the cosmological constant. On the other hand, number theoretic vision suggests that the couplings appearing in the classical action, including cosmological constant, are determined by the number theoretic expression for the action.

The possible representations of twistor spheres as 2surfaces inside $X^4 \subset H$, should be homologically non-trivial in X^4. One can indeed represent the twistor spheres of M^4 and CP_2 in a natural way at the space-time level.

1. The space-time surface X^4 must contain a homologically non-trivial geodesic sphere $S^2(CP_2)$ in order to allow the representation of CP_2 twistor sphere. Cosmic strings and monopole flux tubes do so but massless extremals do not.

2. The homological non-triviality of a sphere $S^2(M^4)$ embeddable inside the space-time surface X^4 is enough and is possible to realize dynamically. If the space-time surface is analogous to a magnetic monopole in the sense that that $S^2(M^4)$ is mapped to $S^2(CP_2)$, $S^2(M^4)$ cannot be contracted to a point inside X^4.

   For instance, the condition $f_2 = \xi_1 = w = 0$ mapping to each other the twistor spheres $S^2(M^4)$ and $S^2(CP_2)$ identified as homologically non-trivial spheres in X^4 defines also a section of $T(CP_2)$ as the analog of H-J structure of CP_2 [L88, L92].

Note that the homological non-triviality at the level of M^4 for the M^4 sphere $S^2(M^4)$ is not necessary but could be realized inside the CD if the CD has a hole, i.e. does not contain the line connecting its tips. This looks artificial.

H-H vision in more detail

The simplest variant of H-H vision is as follows.

1. A general solution to the field equations is obtained by requiring that the spacetime surfaces correspond to the roots for the pairs $(f_1, f_2) : H \to C^2$. f_1 and f_2 are analytic functions of the hypercomplex coordinate u and 3-complex coordinates (w, ξ_1, ξ_2) of H. The equations reduce to purely algebraic conditions and the solution is universal and valid for any action that is general coordinate-invariant and based on induced geometry [L92].

2. The surface $(f_1, f_2) = (0, 0)$ would define the intersection of 2 6-D surfaces $f_1 = 0$ and $f_2 = 0$. The functions f_i are analytic functions of a hypercomplex coordinate u and complex coordinate w of M^4 and of complex coordinates ξ_1, ξ_2 for CP_2. The choices of these coordinates correspond to different Hamilton-Jacobi (H-J) structures [L83] which could be identified as sections of the twistor space of M^4.

3. In the simplest situation u and v correspond to light-like coordinates $t + z$ and $t - z$. $w = x + iy$ as a planar coordinate could serve as a local complex coordinate of for the second hemisphere of a homologically non-trivial sphere CP_1 of causal diamond (CD) [L84]. Homological non-triviality means that the time-like axis connecting the vertices of the causal diamond CD defines a hole in the CD. CD with light-like boundaries could be sliced by light-cone boundaries parallel to its second boundary. CP_1 would parametrize the light-like geodesic emanating from the points at the axis of the light-cone boundary.

4. For instance, $f_2 = \xi_2 - w = 0$ gives rise to a 6-surface, which can be interpreted as a bundle-like structure in 2 ways. For the first interpretation, $(u, (v), \xi_1)$ serve as coordinates of the 4-D base space X^4 and w is the local coordinate for the twistor sphere realized as the homologically non-trivial sphere CP_1 of causal diamond (CD) acting as a fiber.

   The second interpretation is that $(u, (v), w)$ serve as coordinates of the 4-D base space and ξ_1 is the coordinate of the homologically non-trivial geodesic sphere CP_1 of CP_2 acting as a fiber. The condition $f_1 = 0$ fixes ξ_1 as a function of w and identifies the two fibers and determines X^4.

This picture is the simplest one and perhaps too simple.

1. The physical picture suggests that there is a dimensional hierarchy of surfaces with dimensions 4, 2, 0 [L92]. The introduction of f_3 would allow us to identify 2-D string world sheets or monopole flux tubes as roots of (f_1, f_2, f_3). The introduction of f_4 would make it possible to identify points of string world sheets as roots of (f_1, f_2, f_3, f_4) having interpretation as fermionic vertices. The analytic maps $g : C^2 \to C^2$ act as dynamical symmetries for $f = (f_1, f_2) : H \to C^2$.

   In the ase of $f = (f_1, f_2, f_3) : H \to C^3$ the analytic local diffeomorphisms of the space-time surfaces for 2-D roots $f = (f_1, f_2, f_3) = 0$ would act as dynamical symmetries.

2. A prediction, made already earlier [L86] is the breaking of extended conformal invariance as a gauge symmetry in the following sense. Various conformal algebras have non-negative conformal weights and have an infinite hierarchy of isomorphic algebras as sub-algebras. The conformal symmetries as gauge symmetries would transform into dynamical symmetries for finite dimensional subalgebra and this conforms with the p-adic mass calculations [L81].

3. One could assign to these sets of these 2-surfaces and points discriminants in the way as to the maps $g : C^2 \to C^2$ [L92]. This makes sense also for $f = (f_1, f_2, f_3, f_4)$: $H \to C^4$. The condition that the classical action exponential reduces to the product of exponents of all these 3 discriminants would determine the coupling constant evolution. This would correspond to the assignment of separate action exponentials to these surfaces of the dimensional hierarchy and also this would conform with the physical picture. Note that C^4 defines extended twistor space. Presumably this is a mere accident.

Objections against H-H without $T(H)$

Consider now the objections against H-H without $T(H)$.

1. $T(H)$ option for TGD based view of twistor space of H is very elegant and a rigorous proof that the equivalence with H-H option is lacking.

2. If f_2, appearing in a simple mode, is assumed to be surjective in either direction, all space-time surfaces involved would contain homologically non-trivial 2-spheres of both CD and CP_2. This would exclude for instance massless extremals and cosmic string type extremals [K5]. The problem disappears if the f_2 as a map between the homology spheres can also have winding number 0 in either direction or both directions. This could allow massless extremals, CP_2 type extemals, and cosmic strings and their deformations. Could the value of cosmological constant determined by f_2 vanish for MEs) and have its maximal value for CP_2 type extremals.

   Note that these extremals are possible also for the $T(H)$ option since the twistor sphere for X^6 can be identified with the twistor sphere of only $T(M^4)$ or $T(CP_2)$.

To sum up, HH vision is consistent with the $T(H)$ option. H-H without $T(h)$ would provide twistorialization without twistor lift.

8.2.4 Is the Googly problem an illusion in the TGD framework?

In the twistor Grassmann approach [B18, B38, B23, B6, B43, B5, B25] twistors are interpreted in terms of Majorana fermions of Weyl fermions of fixed chirality (this in fact is a problem of $\mathcal{N} = 1$ SUSYs). The TGD variant of twistor Grassmannian approach [L24, L35] relies on the assumption that the boundaries of string world sheets at partonic orbits carry quantum numbers.

It must be however emphasized that twistor description of fermions is not necessary in the TGD approach: the propagators for free fermion fields in H play a central role and vertices emerge from exotic smooth structures [L44] possible only in 4-dimensions and allowing the description of fermion pair creation for free fermion fields as a fundamental vertex [L91, L82].

Twistor space $T(H)$ as the space of choices for the quantization axes

It is possible to start from the geometrization of the space of light-like geodesics, where spinors emerge naturally. There is however no need to assume that the twistors correspond to ordinary spinors and have anything to do with fermions. Could the googly problem be an outcome of a wrong interpretation of the twistor space?

1. To get perspective it is better to start from the twistor space $T(CP_2) = SU(3)/U(1) \times U(1)$, which has a natural interpretation as a space for choices of color and isospin quantization axes. The first application was rather unexpected: the honeybee dance. Topologist Barbara Shipman [A7] had discovered that this space appears in the model of the honeybee dance. The idea that quarks could have something to with honeybee dance is of course total nonsense in the framework of standard particle physics but TGD predicts the possibility of quantum in all scales, in particular in biological length scales and this led to a TGD based model [K31] for the finding.

   The wave function in $T(CP_2)$ would correspond to the wave function in the space of choices of quantization axes. The choice of the quantization axes is an essential part of a quantum measurement. It would be very nice if it could correspond to a state function at a higher level, the level of an experimenter. This view would be consistent with the fact that fundamental fermions appear as basic building bricks of all elementary particles in TGD.

2. Could the S^2 part of the M^4 twistor also be interpreted as a choice of the point of M^4 as the origin of the rest frame and the choice of spin quantization axis as a point of S^2. In the case of a massless particle, the spin quantization axis is a direction that is the same as the direction of motion.

Could also the description of elementary particle quantum numbers using twistor wave functions make sense?

The idea about description of elementary particles with spin using wave functions in a twistor sphere is however extremely elegant. Could this make sense? There are two cases to be discussed.

Option I: twistor lift

Consider first the situation for the twistor lift involving $T(H)$. Classically this would mean that a point of a twistor sphere defines not only the direction of quantization axes but also the value of spin. In TGD this could make sense since all fundamental particles are fermions.

1. In M^8, proposed to relate to H by M^8-H duality as analog of momentum-position duality [L87, L92], momenta as discrete point of M^8 correspond to planewaves in H. This could apply also at the level of twistor space: at the level of H wave functions in twistor sphere would describe fermions spin. At the level of M^8, the point of M^4 twistor sphere would fix the direction of the spin quantization axis and also the spin value. Since the radius of S^2 is fixed, it would fix its magnitude to $s = 1/2$, i.e. the value of the Casimir operator I have built interpretations for twistors based on this observation and the $M^8 - H$ duality.

2. What about CP_2? The point in twistor space $T(CP_2) = SU(3)/U(1) \times U(1)$ would fix the directions of the color isospin and hypercharge quantization axes. Is only quark chirality allowed. Or could it be that for leptons the $S^2(T(X^4))$ is mapped at the level of $T(H)$ to a point of $S^2(T(CP_2))$?

The above arguments suggest abandoning the twistorization of fermion spin. This would not fit well with the twistor-Grassmann approach. On the other hand, despite the undeniable successes of the twistor Grassmann approach, Nima-Arkani Hamed and its other proponents seem to have given it up and the problems for this are obvious.

1. Quantum measurement theory suggests an interpretation of the section of the twistor bundle in terms of choices of quantization axes at a given point of H. The choices of the quantization axes and the spin measurement would mean a localization in the twistor space, in particular S^2. In fact, it would localize in both twistor spheres S^2 since twistor spheres are identified. This conforms with the fact that fermions correspond to isospin and spin doublets.

2. One can of course ask whether the wave functions in the space of choices of quantization axes could also have an interpretation as spin states of fermions.

Option II: H-H principle

One can consider the situation in the framework provided by H-H principle alone, which suggests that the space-time surface, in the case that f_2 is a surjection between the homology spheres, can be seen as a sphere bundle in which the sphere corresponds to the twistor sphere of CP_2 or to the twistor sphere of CD. The wave function in the twistor sphere would correspond to a wave function in either M^4 or CP_2.

But doesn't this allow only the representation of integer spin states and color particles waves with triality $t = 0$? Here the situation is not so simple. The space-time surfaces can have multiple coverings of M^4 and CP_2 and this can lead to the fractionation of quantum numbers for wave functions defined at space-time surfaces but not for this in H. This would be the basic mechanism leading to charge fractionalization and braid statistics, even non-Abelian, in the TGD Universe.

One can construct 4-surfaces for which charge fractionation happens. In M^4, simplest analogs of the 2-sheeted Riemann surface carrying geometric spin 1/2 are associated with $z^{1/2}$. A rotation of 2π along the surface would not lead to the starting point. For more general fractional powers $z^{1/n}$, onehas the spin fractionation occurring for the representations of quantum groups. The same argument applies to various other Cartan charges.

Could the correct conclusion be the following? The space-time wave functions assignable to the twistor spheres of the many-sheeted coverings of M^4 and CP_2, in turn closely related to the notion of effective Planck constants, allow the description of charge fractionation. The fundamental description using spinor modes of H does not however reduce to this kind of description.

The effect of discrete symmetries on H-J structure

Reflection and other discrete symmetries affect the H-J structure defining what hypercomplex and complex coordinates and what the analyticity of (f_1, f_2) means. This would represent these discrete symmetries as geometric transformations of the spacetime surface.

1. The reflection changing left- and right-handed fermions to each other, should also affect the elementary particle-like space-time surfaces associated with them and for the simples H-J structure $(u = t - x, v = t + x, w = x + iy)$ mean the transformation $(u, w \rightarrow (v, -w)$ for the arguments of f_i. Left- and right-handed fermions would live at different elementary particle-like space-time surfaces representing elementary particles and the googly problem, if it is a problem in TGD, could disappear. The nature of holomorphy would not be a fixed but dynamic property and characterize the solution of field equations.

2. Geometrically PT means $(u, w \rightarrow (-u, -w)$. If the functions f_i are odd or even under PT the space-time surface is not affected. If C corresponds to complex conjugation if CP_2 and CPT corresponds to identity, T should induce complex conjugation in CP_2.

3. Baryon and lepton number conservation requiring fixed and opposite H-chiralities for quarks and leptons does not allow independent reflections in M^4 and CP_2 but must be carried out simultaneously. But what is the counterpart of reflection in CP_2? Charge conjugation C is a good guess but does not change orientation: C looks like an analog of PT.

   CPT invariance in geometric sense would require that T is accompanied by complex conjugation in CP_2. CPT would act as $(u, w) \rightarrow (-u, -w)$ and trivially in CP_2. Odd/even property of f with respect to u and w would guarantee the invariance of the space-time surface. Here one must be cautious since in ZEO "big" state function reduction changes the arrow of the geometric time by mapping fermionic vacuum to its dual. This could explain why complex conjugation in CP_2 must be involved. T in this sense is not mere geometric time reflection. This would correspond to a realization at the Hilbert space level as an anti-unitary transformation involving hermitian conjugation analogous to complex conjugation in CP_2.

4. What is interesting is that CP acts as $(u,w) \rightarrow (v,-w)$ in M^4 and as complex conjugation in CP_2. This affects the space-time surfaces so that exact symmetry is not in question. This would conform with the small CP breaking and also with matter antimatter asymmetry, which could be understood if matter and antimatter correspond to different H-J structures so that they must live at different space-time surfaces.

Pin structure and TGD

Ordinary spin structure and also conformal structure require orientable manifolds. Pin structure extending SO(1,3) to O(1,3) containing also P, PT and PT has been discussed as one possible cure of the googly problem. Pin structure is also possible for non-orientable manifolds. P transforms M^4 chiralities of spinors to each other. In the electro-weak gauge transformations respect M^4 chirality but the Dirac equation in H and also the induced Dirac equation couples opposite M^4 chiralities. These couplings are analogous to mass or Higgs couplings.

In TGD, H is orientable so the pin group is not relevant in TGD. In TGD, the 8-D pin structure would mean that there are continuous symmetries that convert quarks into leptons in TGD. This is not possible due to different charges and color quantum numbers as well conservation laws of baryon and lepton number.

How does the possible non-orientability of the space-time surface affect the situation? Certainly non-orientable surfaces are possible but the holography= holomorphism hypothesis does not allow them since complex structure requires orientability.

8.3 Twistorialization at the level of M^8

M^8-H duality as analog of momentum-position duality for 3-surfaces as particles [L52, L53, L87, L92] is central part of TGD. I have already earlier considered several variants of what the twistor lift at the level of M^8 could mean. There are several questions to be answered.

8.3.1 Identification of the twistor spaces

What are the twistor spaces of $T(M^8)$ and $T(Y^4)$ for the M^8-H dual Y^4 of the space-time surfaces $X^4 \subset H$?

1. The 12-D space of light-like geodesics in $M(1,7)$ would be the naive guess for the twistor space of M^8. Now however the Minkowski metric of M^8 is number theoretic and given by real part of octonionic product and 14-D G_2, is the number theoretic symmetry group so that the 12-D $G_2/U(1) \times U(1)$ is the natural candidate for the octonionic twistor space of M^8. $U(1) \times U(1)$ has an interpretation as color Cartan algebra.

2. Without further conditions, the twistor sphere defined by light-like rays at a given point of M^8 is a 6-D and the space $S^6 = G_2/SU(3)$ is the natural identification for it. With this identification, the dimension of the total twistor space $T(M^8)$ would be $8+6=14$, the dimension of G_2. This does not conform with the identification as $T(M^8)=G_2$. It is also an open question whether S^6 possesses the twistorially highly desirable Kähler structure.

3. How could one reduce the dimension of the space of light-like rays of M^8 from 6 to 4? Could the condition that the light-rays are associated with a point of M^8-H dual $Y^4 \subset M^8$ are quaternionic, allow to achieve this. M^8-H duality in its recent form indeed requires that the normal space for a given point of $Y^4 \subset M^8$ as M^8-H dual of $X^4 \subset H$ is quaternionic and Minkowskian in number theoretic sense [L92]. This suggests a direct connection between twistorialization and M^8-H duality.

   (a) Could one require that the light-like 8-momentum has vanishing tangential component to Y^4 and is therefore quaternionic? This would replace the twistor sphere with a union of twistor spheres associated with Minkowskian mass shells $p^2=m^2$. The space of light rays would be 3- rather than 4-D and the wistor space of M^8 would be 11-D rather than 12-D. One dimension is missing.

(b) The physical intuition suggests that the light rays do not have a momentum component in the direction of the tangent space of Y^3 defining the 3-D holographic data but that they have a component tangential to Y^4 in a direction normal to Y^3. This would conform with non-point-likeness: by general coordinate invariance, the momentum component tangential to Y^3 would not correspond to anything physical.

The additional condition would be that these light-like vectors are quaternionic. The space of allowed 8-D light-like vectors would be 4-D and the twistor space could be G_2. The associativity of the dynamics at the level of M^8 requires that the normal space is quaternionic and thus Minkowskian and also contains a commutative subspace. Can these two quaternionicity conditions be consistent with each other? If so, 8-D associative light-likeness respecting the 3-dimensality of holographic data implies the desired 4-dimensionality of the analog of the twistor sphere.

4. The section of the twistor bundle assigned to Y^4 assigns to each point of Y^4 a light-like vector. If also quaternionic units are chosen in an integrable way, this would define the M^8 counterpart of the $H-J$ structure which, when mapped to H by M^8-H duality, would provide the H-J structure of H.

   If the selected light-like vectors have a vanishing tangential component in Y^4, the light-like vectors in H are in M^4. If this is not the case, the light-like vectors in M^4 have also CP_2 component. For instance, light-like geodesics in $M^4\times S^1$, $S^1\subset CP_2$ are possible. It therefore seems that the TGD view of twistorialization indeed makes possible the twistor description of massive particles.

The precise identification of the twistor spaces of M^8 is not obvious. The twistor space of M^8 should have 4-D fiber.

1. The condition that the twistor space allows Kähler structure and has $S^2\times S^2$ as a fiber might leave only the product $T(M^8)=T(M^4)\times T(E^4)$, which is consistent with $M^8=M^4\times E^4$. Whether one can identify $T(E^4)$ as CP_3 is quite not clear.

   In this case, the dimensional reduction of 6-D Kähler action to 4-D action involves the identification of the 2 twistor spheres $S^2(T(M^4))$ and $S^2(T(E^4))$. As in the case of $T(H)$, this identification need not and cannot always be 1-1.

   $Y^6=T(Y^4)$ decomposes locally to a Cartesian tensor product $Y^4\times S^2(T(Y^4))$, $Y^4\subset M^4\times E^4$: Y^4 need not correspond to a map $M^4\rightarrow E^4$ or vice versa.

   The twistor spheres of $S^2(T(M^4))$ and $S^2(T(CP_2))$ are mapped to each other. The consistency between the purely geometric and spinorial view of twistorialization requires that $S^2(T(M^4))$ and $S^2(T(CP_2))$ correspond to homologically non-trivial spheres in Y^4, which are therefore mapped to each other. Cosmological constant depends on the winding number of the identification.

2. Very naively, in the spinorial approach the extended twistor space C^4 is replaced with C^8. Division with 2-complex-dimensional planes CP_2 would give Grassmannian $Gr_c(2,8)$ with dimension $2\times(8-2)=12$, which is a complex manifold having the representation $U(8)/U(2)\times U(6)$. Intuition suggests that the fiber is CP_2. Minkowskian signature would suggest that $U(6)$ is replaced with $U(5,1)$ and $U(8)$ with $U(7,1)$.

   The existence of an analog of the previous dimensional reduction of 6-D Kähler action to 4-D action does not seem plausible. CP_2 fiber does not allow $S^2\times S^2$ as a sub-manifold.

3. The number theoretic $G_2/U(1)\times U(1)$ is the third possible identification but it is not clear whether it is consistent with the number theoretic M^4 signature and CP_2 fiber. It is far from clear whether the 4-D fibration exists and whether the fiber is $S^2\times S^2$.

8.3.2 About the spinorial aspects of M^8 twistorialization

What about the spinorial aspects of M^8 twistorialization? One should generalize a) the map of the points of sphere S^2 to the 2×2 matrices defined by a bi-spinor and its dual, b) the masslessness condition as vanishing of a determinant of the analog of the quaternionic matrix and c) the coincidence relation. One should also understand how the counterparts of the electroweak couplings are represented and solve the Dirac equation in M^8.

1. In the case of M^4, the light-like momenta are mapped to the bispinors providing a matrix representation of quaternions in terms of Pauli sigma matrices. A possible way to achieve this is to introduce octonionic spinor structure [K84, K74, L87] in which massless 8-D momenta correspond to octonions, which should be associative and therefore quaternionic. This would conform with the above identification of light rays.

2. Octonionic spinors, presumably complexified octonionic spinors with $i = \sqrt{-1}$ commuting with the octonionic units, should be also defined. The map of quaternionic massless 8-momenta to the octonionic counterparts of the Pauli spin matrices representing quaternionic basis would define octonionic spinors satisfying the quaternionicity condition. Massless Dirac equation can be solved in the standard way.

3. The matrices defined by bi-spinor pairs associated with M^4 twistors can be regarded as quaternions. The quaternionicity condition means that the octonionic spinor pairs actually reduce to M^4 bi-spinor pairs on a suitable basis, which however depends on the point of Y^4?

   If commutative i is introduced and quaternions are not replaced with their 2×2 matrix representations involving commuting imaginary units, a doubling of degrees of freedom takes place. Does this mean that both M^4 chiralities are obtained? Could this solve the googly problem in M^4?

Also in the case of octonionic spinors complexification would double the degrees of freedom. Does one obtain in this way both spin and electroweak spin?

1. What happens to M^8 spinors as tensor products of Minkowski spinors and electroweak spinors when the octonionic Dirac operator acts on a quaternionic subspace. The electroweak degrees of freedom do not disappear but become passive. One has 8-D complex spinors, which are enough to represent a single H-chirality if the octonionic gamma matrices, which are quaternionic at Y^4, are not represented in terms of Pauli sigma matrices and i is introduced.

2. The electroweak gauge potentials as induced spinor connection represent the geometric view of physics realized at the level of H. Number theoretical vision suggests that the M^8 spinor connection cannot involve sigma matrices, which would be defined as commutators of octonionic units and be octonionic units themselves. Kähler coupling is however possible.

   What could the form of the Kähler gauge potential be? The Kähler form should be apart from a multiplicative imaginary unit i equal to the theoretical flat metric of M^8 so that the Kähler function would represent harmonic oscillator potential.

   The octonionic Dirac equation would have a unique coupling to the Kähler gauge potential with Kähler coupling constant absorbed to it. This would guarantee that the solutions of the modified Dirac equation in M^8 have a finite norm. The solutions can be found by generalizing the procedure to solve Dirac equation in harmonic oscillator potential.

3. The octonionic Dirac operator, which reduces to the quaternionic M^4 Dirac operator and for the local quaternionic M^4 identified as a normal space, the fermions are massless. How to solve this problem? As found, the non-vanishing M^4 mass requires that the light-like M^8 momentum has a component in the direction of Y^4 having a natural interpretation as the analog of the square root of the Higgs field.

4. Complexified octonionic spinors form a complex 8-D space, which corresponds to a single fermion chirality. Do different H chiralities emerge from the mere octonionic picture or must one introduce them in the same way in the case of H? The couplings of quarks and leptons to the induced Kähler form are different and this should be true also at the level of M^8: it seems that both quarks and leptons should be introduced unless on is read consider either leptons or quarks as fundamental fermions.

5. Color $SU(3)$ acts as a number theoretic symmetry of octonions. $SU(3)$ as an automorphism group transforms to each other different quaternionic normal spaces represented as points of CP_2. This representation is realized at the level of H in terms of spinor harmonics. The idea that the low energy and higher energy models for hadron in terms of $SO(4)$ and $SU(3)$ symmetries are dual suggest that fermionic $SO(4)$ harmonics in M^8 could be analogous to the representation of color as spinor harmonics in CP_2.

This picture suggests that 6-D Kähler action as the Kähler function of the twistor space of M^8 could determine surfaces Y^4 as its preferred extremals and that holography= holomorphy principle determines the extremals also now. The 12-D twistor bundle with 4-D fiber should have Kähler structure. This gives very strong condition. One possibility is that it is just the Cartesian product of twistor spaces for M^4 and E^4.

Chapter i

Appendix

A-1 Introduction

Originally this appendix was meant to be a purely technical summary of basic facts but in its recent form it tries to briefly summarize those basic visions about TGD which I dare to regarded stabilized. I have added illustrations making it easier to build mental images about what is involved and represented briefly the key arguments. This chapter is hoped to help the reader to get fast grasp about the concepts of TGD.

The basic properties of embedding space and related spaces are discussed and the relationship of CP_2 to the standard model is summarized. The basic vision is simple: the geometry of the embedding space $H = M^4 \times CP_2$ geometrizes standard model symmetries and quantum numbers. The assumption that space-time surfaces are basic objects, brings in dynamics as dynamics of 3-D surfaces based on the induced geometry. Second quantization of free spinor fields of H induces quantization at the level of H, which means a dramatic simplification.

The notions of induction of metric and spinor connection, and of spinor structure are discussed. Many-sheeted space-time and related notions such as topological field quantization and the relationship many-sheeted space-time to that of GRT space-time are discussed as well as the recent view about induced spinor fields and the emergence of fermionic strings. Also the relationship to string models is discussed briefly.

Various topics related to p-adic numbers are summarized with a brief definition of p-adic manifold and the idea about generalization of the number concept by gluing real and p-adic number fields to a larger book like structure analogous to adele [L22, L23]. In the recent view of quantum TGD [L80], both notions reduce to physics as number theory vision, which relies on $M^8 - H$ duality [L52, L53] and is complementary to the physics as geometry vision.

Zero energy ontology (ZEO) [L47] [K89] has become a central part of quantum TGD and leads to a TGD inspired theory of consciousness as a generalization of quantum measurement theory having quantum biology as an application. Also these aspects of TGD are briefly discussed.

A-2 Embedding space $M^4 \times CP_2$

Space-times are regarded as 4-surfaces in $H = M^4 \times CP_2$ the Cartesian product of empty Minkowski space - the space-time of special relativity - and compact 4-D space CP_2 with size scale of order 10^4 Planck lengths. One can say that embedding space is obtained by replacing each point m of empty Minkowski space with 4-D tiny CP_2. The space-time of general relativity is replaced by a 4-D surface in H which has very complex topology. The notion of many-sheeted space-time gives an idea about what is involved.

Fig. 1. Embedding space $H = M^4 \times CP_2$ as Cartesian product of Minkowski space M^4 and complex projective space CP_2. `http://tgdtheory.fi/appfigures/Hoo.jpg`

Denote by M^4_+ and M^4_- the future and past directed lightcones of M^4. Denote their intersection, which is not unique, by CD. In zero energy ontology (ZEO) [L47, L66] [K89] causal diamond

(CD) is defined as cartesian product $CD \times CP_2$. Often I use CD to refer just to $CD \times CP_2$ since CP_2 factor is relevant from the point of view of ZEO.

Fig. 2. Future and past light-cones M^4_+ and M^4_-. Causal diamonds (CD) are defined as their intersections. `http://tgdtheory.fi/appfigures/futurepast.jpg`

Fig. 3. Causal diamond (CD) is highly analogous to Penrose diagram but simpler. `http://tgdtheory.fi/appfigures/penrose.jpg`

A rather recent discovery was that CP_2 is the only compact 4-manifold with Euclidian signature of metric allowing twistor space with Kähler structure. M^4 is in turn is the only 4-D space with Minkowskian signature of metric allowing twistor space with Kähler structure [A18] so that $H = M^4 \times CP_2$ is twistorially unique.

One can loosely say that quantum states in a given sector of "world of classical worlds" (WCW) are superpositions of space-time surfaces inside CDs and that positive and negative energy parts of zero energy states are localized and past and future boundaries of CDs. CDs form a hierarchy. One can have CDs within CDs and CDs can also overlap. The size of CD is characterized by the proper time distance between its two tips. One can perform both translations and also Lorentz boosts of CD leaving either boundary invariant. Therefore one can assign to CDs a moduli space and speak about wave function in this moduli space.

In number theoretic approach it is natural to restrict the allowed Lorentz boosts to some discrete subgroup of Lorentz group and also the distances between the tips of CDs to multiples of CP_2 radius defined by the length of its geodesic. Therefore the moduli space of CDs discretizes. The quantization of cosmic recession velocities for which there are indications, could relate to this quantization.

A-2.1 Basic facts about CP_2

CP_2 as a four-manifold is very special. The following arguments demonstrate that it codes for the symmetries of standard models via its isometries and holonomies.

CP_2 as a manifold

CP_2, the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space C^3 under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) \ . \tag{A-2.1}$$

Here λ is any non-zero complex number. Note that CP_2 can be also regarded as the coset space $SU(3)/U(2)$. The pair z^i/z^j for fixed j and $z^i \neq 0$ defines a complex coordinate chart for CP_2. As j runs from 1 to 3 one obtains an atlas of three coordinate charts covering CP_2, the charts being holomorphically related to each other (e.g. CP_2 is a complex manifold). The points $z^3 \neq 0$ form a subset of CP_2 homoeomorphic to R^4 and the points with $z^3 = 0$ a set homeomorphic to S^2. Therefore CP_2 is obtained by "adding the 2-sphere at infinity to R^4".

Besides the standard complex coordinates $\xi^i = z^i/z^3$, $i = 1, 2$ the coordinates of Eguchi and Freund [A14] will be used and their relation to the complex coordinates is given by

$$\begin{aligned} \xi^1 &= z + it \ , \\ \xi^2 &= x + iy \ . \end{aligned} \tag{A-2.2}$$

These are related to the "spherical coordinates" via the equations

$$\begin{aligned} \xi^1 &= rexp(i\frac{(\Psi+\Phi)}{2})cos(\frac{\Theta}{2}) \ , \\ \xi^2 &= rexp(i\frac{(\Psi-\Phi)}{2})sin(\frac{\Theta}{2}) \ . \end{aligned} \tag{A-2.3}$$

The ranges of the variables r, Θ, Φ, Ψ are $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively.

Considered as a real four-manifold CP_2 is compact and simply connected, with Euler number Euler number 3, Pontryagin number 3 and second $b = 1$.

Fig. 4. CP_2 as manifold. `http://tgdtheory.fi/appfigures/cp2.jpg`

Metric and Kähler structure of CP_2

In order to obtain a natural metric for CP_2, observe that CP_2 can be thought of as a set of the orbits of the isometries $z^i \to exp(i\alpha)z^i$ on the sphere S^5: $\sum z^i \bar{z}^i = R^2$. The metric of CP_2 is obtained by projecting the metric of S^5 orthogonally to the orbits of the isometries. Therefore the distance between the points of CP_2 is that between the representative orbits on S^5.

The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}} d\xi^a d\bar{\xi}^b \ , \tag{A-2.4}$$

where the Hermitian, in fact Kähler metric $g_{a\bar{b}}$ is defined by

$$g_{a\bar{b}} = R^2 \partial_a \partial_{\bar{b}} K \ , \tag{A-2.5}$$

where the function K, Kähler function, is defined as

$$\begin{aligned} K &= log(F) \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-2.6}$$

The Kähler function for S^2 has the same form. It gives the S^2 metric $dzd\bar{z}/(1+r^2)^2$ related to its standard form in spherical coordinates by the coordinate transformation $(r, \phi) = (tan(\theta/2), \phi)$.

The representation of the CP_2 metric is deducible from S^5 metric is obtained by putting the angle coordinate of a geodesic sphere constant in it and is given

$$\frac{ds^2}{R^2} = \frac{(dr^2 + r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2 + \sigma_2^2)}{F} \ , \tag{A-2.7}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{A-2.8}$$

R denotes the radius of the geodesic circle of CP_2. The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2 \sum_A e_k^A e_l^A \ , \tag{A-2.9}$$

are given by

$$\begin{aligned} e^0 &= \frac{dr}{F} \ , \quad e^1 = \frac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 &= \frac{r\sigma_2}{\sqrt{F}} \ , \quad e^3 = \frac{r\sigma_3}{F} \ . \end{aligned} \tag{A-2.10}$$

The explicit representations of vierbein vectors are given by

$$\begin{array}{llllll} e^0 & = & \frac{dr}{F} \ , & e^1 & = & \frac{r(sin\Theta cos\Psi d\Phi + sin\Psi d\Theta)}{2\sqrt{F}} \ , \\ e^2 & = & \frac{r(sin\Theta sin\Psi d\Phi - cos\Psi d\Theta)}{2\sqrt{F}} \ , & e^3 & = & \frac{r(d\Psi + cos\Theta d\Phi)}{2F} \ . \end{array} \tag{A-2.11}$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2) \ . \tag{A-2.12}$$

From this expression one finds that at coordinate infinity $r = \infty$ line element reduces to $\frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2)$ of S^2 meaning that 3-sphere degenerates metrically to 2-sphere and one can say that CP_2 is obtained by adding to R^4 a 2-sphere at infinity.

The vierbein connection satisfying the defining relation

$$de^A = -V^A_B \wedge e^B \ , \tag{A-2.13}$$

is given by

$$\begin{array}{llllll} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r - \frac{1}{r})e^3 \ , & V_{12} & = & (2r + \frac{1}{r})e^3 \ . \end{array} \tag{A-2.14}$$

The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{llllll} R_{01} & = & e^0 \wedge e^1 - e^2 \wedge e^3 \ , & R_{23} & = & e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ R_{02} & = & e^0 \wedge e^2 - e^3 \wedge e^1 \ , & R_{31} & = & -e^0 \wedge e^2 + e^3 \wedge e^1 \ , \\ R_{03} & = & 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ , & R_{12} & = & 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ . \end{array} \tag{A-2.15}$$

Metric defines a real, covariantly constant, and therefore closed 2-form J

$$J = -is_{a\bar{b}} d\xi^a d\bar{\xi}^b \ , \tag{A-2.16}$$

the so called Kähler form. Kähler form J defines in CP_2 a symplectic structure because it satisfies the condition

$$J^k_{\ r} J^{rl} = -s^{kl} \ . \tag{A-2.17}$$

The condition states that J and g give representations of real unit and imaginary units related by the formula $i^2 = -1$.

Kähler form is expressible locally in terms of Kähler gauge potential

$$J = dB \ , \tag{A-2.18}$$

where B is the so called Kähler potential, which is not defined globally since J describes homological magnetic monopole.

$dJ = ddB = 0$ gives the topological half of Maxwell equations (vanishing of magnetic charges and Faraday's induction law) and self-duality $*J = J$ reduces the remaining equations to $dJ = 0$. Hence the Kähler form can be regarded as a curvature form of a $U(1)$ gauge potential B carrying a magnetic charge of unit $1/2g$ (g denotes the gauge coupling).

The magnetic flux of J through a 2-surface in CP_2 is proportional to its homology equivalence class, which is integer valued. The explicit representations of J and B are given by

$$\begin{aligned} B &= 2re^3 \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) = \frac{r}{F^2}dr \wedge (d\Psi + cos\Theta d\Phi) + \frac{r^2}{2F}sin\Theta d\Theta \wedge d\Phi \ . \end{aligned} \tag{A-2.19}$$

The vierbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type (1, 1).

Useful coordinates for CP_2 are the so called canonical (or symplectic or Darboux) coordinates in which the Kähler potential and Kähler form have very simple expressions

$$\begin{aligned} B &= \sum_{k=1,2} P_k dQ_k \ , \\ J &= \sum_{k=1,2} dP_k \wedge dQ_k \ . \end{aligned} \tag{A-2.20}$$

The relationship of the canonical coordinates to the "spherical" coordinates is given by the equations

$$\begin{aligned} P_1 &= -\frac{1}{1+r^2} \ , \\ P_2 &= -\frac{r^2 cos\Theta}{2(1+r^2)} \ , \\ Q_1 &= \Psi \ , \\ Q_2 &= \Phi \ . \end{aligned} \tag{A-2.21}$$

Spinors In CP_2

CP_2 doesn't allow spinor structure in the conventional sense [A9]. However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of CP_2 play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space M. The parallel propagation around a closed curve with a base point x leads to a rotated vierbein at x: $e^A = R^A_B e^B$ and one can associate to each closed path an element of $SO(4)$.

Consider now a one-parameter family of closed curves $\gamma(v) : v \in (0,1)$ with the same base point x and $\gamma(0)$ and $\gamma(1)$ trivial paths. Clearly these paths define a sphere S^2 in M and the element $R^A_B(v)$ defines a closed path in $SO(4)$. When the sphere S^2 is contractible to a point e.g., homologically trivial, the path in $SO(4)$ is also contractible to a point and therefore represents a trivial element of the homotopy group $\Pi_1(SO(4)) = Z_2$.

For a homologically nontrivial 2-surface S^2 the associated path in $SO(4)$ can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group Spin(4) (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallel propagate also spinors and by the above construction associate a closed path of Spin(4) to the surface S^2. Now, however this path corresponds to a lift of the corresponding $SO(4)$ path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the non-allowed -1-factor associated with the parallel transport of the spinor around the sphere S^2 by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating -1-factor. For a $U(1)$ gauge potential this factor is given by the exponential

$exp(i2\Phi)$, where Φ is the magnetic flux through the surface. This factor has the value -1 provided the $U(1)$ potential carries half odd multiple of Dirac charge $1/2g$. In case of CP_2 the required gauge potential is half odd multiple of the Kähler potential B defined previously. In the case of $M^4 \times CP_2$ one can in addition couple the spinor components with different chiralities independently to an odd multiple of $B/2$.

Geodesic sub-manifolds of CP_2

Geodesic sub-manifolds are defined as sub-manifolds having common geodesic lines with the embedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors h^k_α (understood as vectors of H) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to H and X^4.

In [A28] a general characterization of the geodesic sub-manifolds for an arbitrary symmetric space G/H is given. Geodesic sub-manifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra g of the group G. The Lie triple system t is defined as a subspace of g characterized by the closedness property with respect to double commutation

$$[X,[Y,Z]] \in t \ \text{ for } \ X,Y,Z \in t \ . \tag{A-2.22}$$

$SU(3)$ allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that $SU(3)$ allows two nonequivalent $SU(2)$ algebras corresponding to subgroups $SO(3)$ (orthogonal 3×3 matrices) and the usual isospin group $SU(2)$. By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic sub-manifold of CP_2.

Standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S^2_I \ : \ \xi^1 = \bar{\xi}^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S^2_{II} \ : \ \xi^1 = \xi^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Phi = 0) \ .$$

The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for S^2_I. S^2_{II} is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

A-2.2 CP_2 geometry and Standard Model symmetries

Identification of the electro-weak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B36] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \tag{A-2.23}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \otimes \gamma_5$, $1 \otimes \gamma_5$ and $\gamma_5 \otimes 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4)$ having as its covering group $SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{A-2.24}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{aligned} V_{01} &= -\frac{e^1}{r} \ , & V_{23} &= \frac{e^1}{r} \ , \\ V_{02} &= -\frac{e^2}{r} \ , & V_{31} &= \frac{e^2}{r} \ , \\ V_{03} &= (r - \frac{1}{r})e^3 \ , & V_{12} &= (2r + \frac{1}{r})e^3 \ , \end{aligned} \tag{A-2.25}$$

and

$$B = 2re^3 \ , \tag{A-2.26}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \tag{A-2.27}$$

where one have defined

$$\begin{aligned} I_L^1 &= \frac{(\Sigma_{01} - \Sigma_{23})}{2} \ , \\ I_L^2 &= \frac{(\Sigma_{02} - \Sigma_{13})}{2} \ . \end{aligned} \tag{A-2.28}$$

A_{ch} is clearly left handed so that one can perform the identification of the gauge potential as

$$W^\pm = \frac{2(e^1 \pm ie^2)}{r} \ , \tag{A-2.29}$$

where $W^\pm$ denotes the charged intermediate vector boson.

The covariantly constant curvature tensor is given by

$$\begin{aligned} R_{01} &= -R_{23} = e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ R_{02} &= -R_{31} = e^0 \wedge e^2 - e^3 \wedge e^1 \ , \\ R_{03} &= 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ , \\ R_{12} &= 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ . \end{aligned} \tag{A-2.30}$$

The charged part of the curvature tensor is left handed.

This is to be compared with the Weyl tensor, which defines a representation of quaternionic imaginary units.

$$\begin{aligned} W_{03} = W_{12} &\equiv 2I_3 = 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ W_{01} = W_{23} &\equiv I_1 = -e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ W_{02} = W_{31} &\equiv I_2 = -e^0 \wedge e^2 - e^3 \wedge e^1 \ . \end{aligned} \tag{A-2.31}$$

The charged part of the Weyl tensor is right-handed and that the relative sign of the two terms in the curvature tensor and Weyl tensor are opposite.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$\begin{aligned} X &= re^3 \ , \\ Y &= \frac{e^3}{r} \ , \end{aligned} \tag{A-2.32}$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$\begin{aligned} \bar{\gamma} &= aX + bY \ , \\ \bar{Z}^0 &= cX + dY \ , \end{aligned} \tag{A-2.33}$$

where the normalization condition

$$ad - bc = 1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$\begin{aligned} A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\ &+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ . \end{aligned} \tag{A-2.34}$$

Identifying Σ_{12} and $\Sigma_{03} = 1 \times \gamma_5\Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \ . \tag{A-2.35}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-2.36}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$\begin{aligned} Q_{em} &= \Sigma^{12} + \frac{(n_+1_+ + n_-1_-)}{6} \ , \\ I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ . \end{aligned} \tag{A-2.37}$$

The fields γ and Z^0 are defined via the relations

$$\begin{aligned} \gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ , \\ Z^0 &= 4(a+b)\bar{Z}^0 = 4(X - Y) \ . \end{aligned} \tag{A-2.38}$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} , \quad \text{(A-2.39)}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of the Weinberg angle is a dynamical problem. The original approach was based on the assumption that it makes sense to talk about electroweak action defined at fundamental level and introduce a symmetry breaking by adding an additional term proportional to Kähler action. The recent view is that Kähler action plus volume term defines the fundamental action.

The Weinberg angle is completely fixed if one requires that the electroweak action contains no cross term of type γZ^0. This leads to a definite value for the Weinberg angle.

One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle. As a matter fact, color gauge action identifying color gauge field as proportional to $H^A J_{\alpha\beta}$ is proportional to Kähler action. A possible interpretation would be as a sum of electroweak and color gauge interactions.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+1_+ + n_-1_-) , \quad \text{(A-2.40)}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) , \end{aligned} \quad \text{(A-2.41)}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) . \quad \text{(A-2.42)}$$

Evaluating the expressions above, one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{12} , \\ Z^0 &= 2R_{03} . \end{aligned} \quad \text{(A-2.43)}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) . \quad \text{(A-2.44)}$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned} L_{ew} &= L_{sym} + fJ^{\alpha\beta}J_{\alpha\beta} , \\ L_{sym} &= \frac{1}{4g^2}Tr(F^{\alpha\beta}F_{\alpha\beta}) , \end{aligned} \quad \text{(A-2.45)}$$

where the trace is taken in spinor representation, in terms of γ and Z^0 one obtains for the coefficient X of the γZ^0 cross term (this coefficient must vanish) the expression

$$\begin{aligned} X &= -\frac{K}{2g^2} + \frac{fp}{18} \ , \\ K &= Tr\left[Q_{em}(I_L^3 - sin^2\theta_W Q_{em})\right] \ , \end{aligned} \tag{A-2.46}$$

This parameter can be calculated by substituting the values of quark and lepton charges and weak isospins.

In the general case the value of the coefficient K is given by

$$K = \sum_i \left[-\frac{(18+2n_i^2)sin^2\theta_W}{9}\right] \ , \tag{A-2.47}$$

where the sum is over the spinor chiralities, which appear as elementary fermions and n_i is the integer describing the coupling of the spinor field to the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9\sum_i 1}{(fg^2 + 2\sum_i(18+n_i^2))} \ . \tag{A-2.48}$$

In the scenario where both leptons and quarks are elementary fermions the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9}{(\frac{fg^2}{2} + 28)} \ . \tag{A-2.49}$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is not far from the typical value 9/24 of GUTs at high energies [B3]. The experimental value at the scale length scale of the electron can be deduced from the ratio of W and Z boson masses as $sin^2\theta_W = 1-(m_W/m_Z)^2 \simeq .22290$. This ratio and also the weak boson masses depend on the length scale.

If one interprets the additional term proportional to J as color action, one could perhaps interpret the value of Weinberg angle as expressing a connection between strong and weak coupling constant evolution. The limit $f \rightarrow 0$ should correspond to an infinite value of color coupling strength and at this limit one would have $sin^2\theta_W = \frac{9}{28}$ for $f/g^2 \rightarrow 0$. This does not make sense since the Weinberg angle is in the standard model much smaller in QCD scale Λ corresponding roughly to pion mass scale. The Weinberg angle is in principle predicted by the p-adic coupling constant evolution fixed by the number theoretical vision of TGD.

One could however have a sum of electroweak action, correction terms changing the value of Weinberg angle, and color action and coupling constant evolution could be understood in terms of the coupling parameters involved.

Electroweak symmetry breaking

One of the hardest challenges in the development of the TGD based view of weak symmetry breaking was the fact that classical field equations allow space-time surfaces with finite but arbitrarily large size. For a fixed space-time surface, the induced gauge fields, including classical weak fields, are long ranged. On the other hand, the large mass for weak bosons would require a short correlation length. How can one understand this together with the fact that a photon has a long correlation length?

In zero energy ontology quantum states are superpositions of space-time surfaces as analogs of almost unique Bohr orbits of particles identified as 3-D surfaces. For some reason the superposition should be such that the quantum averages of weak gauge boson fields vanish below the weak scale whereas the quantum average of electromagnetic fields is non-vanishing.

This is indeed the case.

1. The supersymplectic symmetries form isometries of the world of classical worlds (WCW) and they act in CP_2 degrees of freedom as symplectic transformations leaving the CP_2 symplectic form J invariant and therefore also its contribution to the electromagnetic field since this part is the same for all space-time surfaces in the superposition of space-time surfaces as a representation of supersymplectic isometry group (as a special case a representation of color group).

2. In TGD, color and electroweak symmetries acting as holonomies are not independent and for the $SU(2)_L$ part of induced spinor connection the symplectic transformations induces $SU(2)_L \times U(1)_R$ gauge transformation. This suggests that the quantum expectations of the induced weak fields over the space-time surfaces vanish above the quantum coherence scale. The averages of W and of the left handed part of Z^0 should therefore vanish.

3. $\langle Z^0 \rangle$ should vanish. For $U(1)_R$ part of Z^0, the action of gauge transformation is trivial in gauge theory. Now however the space-time surface changes under symplectic transformations and this could make the average of the right-handed part of Z^0 vanishing. The vanishing of the average of the axial part of the Z^0 is suggested by the partially conserved axial current hypothesis.

One can formulate this picture quantitatively.

1. The electromagnetic field [L85] contains, besides the induced Kähler form, also the induced curvature form R_{12}, which couples vectorially. Conserved vector current hypothesis suggests that the average of R_{12} is non-vanishing. One can express the neutral part of the induced gauge field in terms of induced spinor curvature and Kähler form J as

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) = J + 2e^0 \wedge e^3 \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) = 3J - 2e^0 \wedge e^3 \ , \end{aligned} \tag{A-2.50}$$

2. The induced fields γ and Z^0 (photon and Z- boson) can be expressed as

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{12} \ , \\ Z^0 &= 2R_{03} = 2(J + 2e^0 \wedge e^3) && \text{(A-2.51)} \\ per. & && \text{(A-2.52)} \end{aligned}$$

The condition $\langle Z^0 \rangle = 0$ gives $2\langle e^0 \wedge e^3 \rangle = -2J$ and this in turn gives $\langle R_{12} \rangle = 4J$. The average over γ would be

$$\langle \gamma \rangle = (3 - 4sin^2\theta_W)J \ .$$

For $sin^2\theta_W = 3/4\ langle \gamma \rangle$ would vanish.

The quantum averages of classical weak fields quite generally vanish. What about correlation functions?

1. One expects that the correlators of classical weak fields as color invariants, and perhaps even symplectic invariants, are non-vanishing below the Compton length since in this kind of situation the points in the correlation function belong to the same 3-surface representing particle, such as hadron.

2. The intuitive picture is that in longer length scales one has disjoint 3-surfaces with a size scale of Compton length. If the states associated with two disjoint 3-surfaces are separately color invariant there are no correlations in color degrees of freedom and correlators reduce to the products of expectations of classical weak fields and vanish. This could also hold when the 3-surfaces are connected by flux tube bonds.

   Below the Compton length weak bosons would thus behave as correlated massless fields. The Compton lengths of weak bosons are proportional to the value of effective Planck constant h_{eff} and in living systems the Compton lengths are proposed to be even of the order of cell size. This would explain the mysterious chiral selection in living systems requiring large parity violation.

3. What about the averages and correlators of color gauge fields? Classical color gauge fields are proportional to the products of Hamiltonians of color isometries induced Kähler form and the expectations of color Hamiltonians give vanishing average above Compton length and therefore vanishing average. Correlators are non-vanishing below the hadron scale. Gluons do not propagate in long scales for the same reason as weak bosons. This is implied by color confinement, which has also classical description in the sense that 3-surfaces have necessarily a finite size.

   A large value of h_{eff} allows colored states even in biological scales below the Compton length since in this kind of situation the points in the correlation function belong to the same 3-surface representing particle, such as dark hadron.

Discrete symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:

1. Symmetries must be realized as purely geometric transformations.

2. Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [B8] .

The action of the reflection P on spinors of is given by

$$\Psi \rightarrow P\Psi = \gamma^0 \otimes \gamma^0 \Psi \ . \tag{A-2.53}$$

in the representation of the gamma matrices for which γ^0 is diagonal. It should be noticed that W and Z^0 bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of P.

The guess that a complex conjugation in CP_2 is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$\begin{aligned} m^k &\rightarrow T(M^k) \ , \\ \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \gamma^1\gamma^3 \otimes 1\Psi \ . \end{aligned} \tag{A-2.54}$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in CP_2:

$$\begin{aligned} \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \Psi^\dagger\gamma^2\gamma^0 \otimes 1 \ . \end{aligned} \tag{A-2.55}$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

A-3 Induction procedure and many-sheeted space-time

Since the classical gauge fields are closely related in TGD framework, it is not possible to have space-time sheets carrying only single kind of gauge field. For instance, em fields are accompanied by Z^0 fields for extremals of Kähler action.

Classical em fields are always accompanied by Z^0 field and some components of color gauge field. For extremals having homologically non-trivial sphere as a CP_2 projection em and Z^0 fields are the only non-vanishing electroweak gauge fields. For homologically trivial sphere only W fields are non-vanishing. Color rotations does not affect the situation.

For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has U(1) holonomy by 2-dimensionality of the CP_2 projection. Color gauge field has $U(1)$ holonomy for all space-time surfaces and quantum classical correspondence suggest a weak form of color confinement meaning that physical states correspond to color neutral members of color multiplets.

A-3.1 Induction procedure for gauge fields and spinor connection

Induction procedure for gauge potentials and spinor structure is a standard procedure of bundle theory. If one has embedding of some manifold to the base space of a bundle, the bundle structure can be induced so that it has as a base space the imbedded manifold, whose points have as fiber the fiber if embedding space at their image points. In the recent case the embedding of space-time surface to embedding space defines the induction procedure. The induced gauge potentials and gauge fields are projections of the spinor connection of the embedding space to the space-time surface (see `http://tgdtheory.fi/appfigures/induct.jpg`).

Induction procedure makes sense also for the spinor fields of embedding space and one obtains geometrization of both electroweak gauge potentials and of spinors. The new element is induction of gamma matrices which gives their projections at space-time surface.

As a matter fact, the induced gamma matrices cannot appear in the counterpart of massless Dirac equation. To achieve super-symmetry, Dirac action must be replaced with Kähler-Dirac action for which gamma matrices are contractions of the canonical momentum currents of Kähler action with embedding space gamma matrices. Induced gamma matrices in Dirac action would correspond to 4-volume as action.

Fig. 9. Induction of spinor connection and metric as projection to the space-time surface. `http://tgdtheory.fi/appfigures/induct.jpg`.

A-3.2 Induced gauge fields for space-times for which CP$_2$ projection is a geodesic sphere

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional CP$_2$ projection, only vacuum extremals and space-time surfaces for which CP$_2$ projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing W fields and homologically non-trivial sphere to non-vanishing W fields but vanishing γ and Z^0. This can be verified by explicit examples.

$r = \infty$ surface gives rise to a homologically non-trivial geodesic sphere for which e_0 and e_3 vanish imply the vanishing of W field. For space-time sheets for which CP$_2$ projection is $r = \infty$ homologically non-trivial geodesic sphere of CP_2 one has

$$\gamma = (\frac{3}{4} - \frac{sin^2(\theta_W)}{2})Z^0 \simeq \frac{5Z^0}{8} \quad .$$

The induced W fields vanish in this case and they vanish also for all geodesic sphere obtained by $SU(3)$ rotation.

$Im(\xi^1) = Im(\xi^2) = 0$ corresponds to homologically trivial geodesic sphere. A more general representative is obtained by using for the phase angles of standard complex CP_2 coordinates constant values. In this case e^1 and e^3 vanish so that the induced em, Z^0, and Kähler fields vanish but induced W fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D CP$_2$ projection color rotations and weak symmetries commute.

A-3.3 Many-sheeted space-time

TGD space-time is many-sheeted: in other words, there are in general several space-sheets which have projection to the same M^4 region. Second manner to say this is that CP_2 coordinates are many-valued functions of M^4 coordinates. The original physical interpretation of many-sheeted space-time time was not correct: it was assumed that single sheet corresponds to GRT space-time and this obviously leads to difficulties since the induced gauge fields are expressible in terms of only four embedding space coordinates.

Fig. 10. Illustration of many-sheeted space-time of TGD. `http://tgdtheory.fi/appfigures/manysheeted.jpg`

Superposition of effects instead of superposition of fields

The first objection against TGD is that superposition is not possible for induced gauge fields and induced metric. The resolution of the problem is that it is effects which need to superpose, not the fields.

Test particle topologically condenses simultaneously to all space-time sheets having a projection to same region of M^4 (that is touches them). The superposition of effects of fields at various space-time sheets replaces the superposition of fields.This is crucial for the understanding also how GRT space-time relates to TGD space-time, which is also in the appendix of this book).

Wormhole contacts

Wormhole contacts are key element of many-sheeted space-time. One does not expect them to be stable unless there is non-trivial Kähler magnetic flux flowing through then so that the throats look like Kähler magnetic monopoles.

Fig. 11. Wormhole contact. `http://tgdtheory.fi/appfigures/wormholecontact.jpg`

Since the flow lines of Kähler magnetic field must be closed this requires the presence of another wormhole contact so that one obtains closed monopole flux tube decomposing to two Minkowskian pieces at the two space-time sheets involved and two wormhole contacts with Euclidian signature of the induced metric. These objects are identified as space-time correlates of elementary particles and are clearly analogous to string like objects.

The relationship between the many-sheeted space-time of TGD and of GRT space-time

The space-time of general relativity is single-sheeted and there is no need to regard it as surface in H although the assumption about representability as vacuum extremal gives very powerful constraints in cosmology and astrophysics and might make sense in simple situations.

The space-time of GRT can be regarded as a long length scale approximation obtained by lumping together the sheets of the many-sheeted space-time to a region of M^4 and providing it with an effective metric obtained as sum of M^4 metric and deviations of the induced metrics of various space-time sheets from M^4 metric. Also induced gauge potentials sum up in the similar manner so that also the gauge fields of gauge theories would not be fundamental fields.

Fig. 12. The superposition of fields is replaced with the superposition of their effects in many-sheeted space-time. `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg`

Space-time surfaces of TGD are considerably simpler objects that the space-times of general relativity and relate to GRT space-time like elementary particles to systems of condensed matter physics. Same can be said about fields since all fields are expressible in terms of embedding space coordinates and their gradients, and general coordinate invariance means that the number of bosonic field degrees is reduced locally to 4. TGD space-time can be said to be a microscopic description whereas GRT space-time a macroscopic description. In TGD complexity of space-time topology replaces the complexity due to large number of fields in quantum field theory.

Topological field quantization and the notion of magnetic body

Topological field quantization also TGD from Maxwell's theory. TGD predicts topological light rays ("massless extremals (MEs)") as space-time sheets carrying waves or arbitrary shape propagating with maximal signal velocity in single direction only and analogous to laser beams and carrying light-like gauge currents in the generi case. There are also magnetic flux quanta and electric flux quanta. The deformations of cosmic strings with 2-D string orbit as M^4 projection gives rise to magnetic flux tubes carrying monopole flux made possible by CP_2 topology allowing homological Kähler magnetic monopoles.

Fig. 13. Topological quantization for magnetic fields replaces magnetic fields with bundles of them defining flux tubes as topological field quanta. `http://tgdtheory.fi/appfigures/field.jpg`

The imbeddability condition for say magnetic field means that the region containing constant magnetic field splits into flux quanta, say tubes and sheets carrying constant magnetic field. Unless one assumes a separate boundary term in Kähler action, boundaries in the usual sense are forbidden except as ends of space-time surfaces at the boundaries of causal diamonds. One obtains typically pairs of sheets glued together along their boundaries giving rise to flux tubes with closed cross section possibly carrying monopole flux.

These kind of flux tubes might make possible magnetic fields in cosmic scales already during primordial period of cosmology since no currents are needed to generate these magnetic fields: cosmic string would be indeed this kind of objects and would dominated during the primordial period. Even superconductors and maybe even ferromagnets could involve this kind of monopole flux tubes.

A-3.4 Embedding space spinors and induced spinors

One can geometrize also fermionic degrees of freedom by inducing the spinor structure of $M^4 \times CP_2$.

CP_2 does not allow spinor structure in the ordinary sense but one can couple the opposite H-chiralities of H-spinors to an $n = 1$ ($n = 3$) integer multiple of Kähler gauge potential to obtain a respectable modified spinor structure. The em charges of resulting spinors are fractional (integer valued) and the interpretation as quarks (leptons) makes sense since the couplings to the induced spinor connection having interpretation in terms electro-weak gauge potential are identical to those assumed in standard model.

The notion of quark color differs from that of standard model.

1. Spinors do not couple to color gauge potential although the identification of color gauge potential as projection of $SU(3)$ Killing vector fields is possible. This coupling must emerge only at the effective gauge theory limit of TGD.

2. Spinor harmonics of embedding space correspond to triality $t = 1$ ($t = 0$) partial waves. The detailed correspondence between color and electroweak quantum numbers is however not correct as such and the interpretation of spinor harmonics of embedding space is as representations for ground states of super-conformal representations. The wormhole pairs associated with physical quarks and leptons must carry also neutrino pair to neutralize weak quantum numbers above the length scale of flux tube (weak scale or Compton length). The total color quantum numbers or these states must be those of standard model. For instance, the color quantum numbers of fundamental left-hand neutrino and lepton can compensate each other for the physical lepton. For fundamental quark-lepton pair they could sum up to those of physical quark.

The well-definedness of em charge is crucial condition.

1. Although the embedding space spinor connection carries W gauge potentials one can say that the embedding space spinor modes have well-defined em charge. One expects that this is true for induced spinor fields inside wormhole contacts with 4-D CP_2 projection and Euclidian signature of the induced metric.

2. The situation is not the same for the modes of induced spinor fields inside Minkowskian region and one must require that the CP_2 projection of the regions carrying induced spinor field is such that the induced W fields and above weak scale also the induced Z^0 fields vanish in order to avoid large parity breaking effects. This condition forces the CP_2 projection to be 2-dimensional. For a generic Minkowskian space-time region this is achieved only if the

spinor modes are localized at 2-D surfaces of space-time surface - string world sheets and possibly also partonic 2-surfaces.

3. Also the Kähler-Dirac gamma matrices appearing in the modified Dirac equation must vanish in the directions normal to the 2-D surface in order that Kähler-Dirac equation can be satisfied. This does not seem plausible for space-time regions with 4-D CP_2 projection.

4. One can thus say that strings emerge from TGD in Minkowskian space-time regions. In particular, elementary particles are accompanied by a pair of fermionic strings at the opposite space-time sheets and connecting wormhole contacts. Quite generally, fundamental fermions would propagate at the boundaries of string world sheets as massless particles and wormhole contacts would define the stringy vertices of generalized Feynman diagrams. One obtains geometrized diagrammatics, which brings looks like a combination of stringy and Feynman diagrammatics.

5. This is what happens in the the generic situation. Cosmic strings could serve as examples about surfaces with 2-D CP_2 projection and carrying only em fields and allowing delocalization of spinor modes to the entire space-time surfaces.

A-3.5 About induced gauge fields

In the following the induced gauge fields are studied for general space-time surface without assuming the preferred extremal property (Bohr orbit property). Therefore the following arguments are somewhat obsolete in their generality.

Space-times with vanishing em, Z^0, or Kähler fields

The following considerations apply to a more general situation in which the homologically trivial geodesic sphere and extremal property are not assumed. It must be emphasized that this case is possible in TGD framework only for a vanishing Kähler field.

Using spherical coordinates (r, Θ, Ψ, Φ) for CP_2, the expression of Kähler form reads as

$$\begin{aligned} J &= \frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta)d\Phi) + \frac{r^2}{2F} sin(\Theta) d\Theta \wedge d\Phi \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-3.1}$$

The general expression of electromagnetic field reads as

$$\begin{aligned} F_{em} &= (3+2p)\frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta)d\Phi) + (3+p)\frac{r^2}{2F} sin(\Theta) d\Theta \wedge d\Phi \ , \\ p &= sin^2(\Theta_W) \ , \end{aligned} \tag{A-3.2}$$

where Θ_W denotes Weinberg angle.

1. The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$\begin{aligned} \Psi &= k\Phi \ , \\ (3+2p)\frac{1}{r^2 F}(d(r^2)/d\Theta)(k + cos(\Theta)) &+ (3+p)sin(\Theta) = 0 \ , \end{aligned} \tag{A-3.3}$$

hold true. The conditions imply that CP_2 projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} \ , \\ X &= D\left[|\frac{k+u}{C}|\right]^{\epsilon} \ , \\ u &\equiv cos(\Theta) \ , \ C = k + cos(\Theta_0) \ , \ D = \frac{r_0^2}{1+r_0^2} \ , \ \epsilon = \frac{3+p}{3+2p} \ , \end{aligned} \tag{A-3.4}$$

where C and D are integration constants. $0 \leq X \leq 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \leq 1$ and $r = \infty$ to $X = 1$ giving $|u + k| = [(1 + r_0^2)/r_0^2)]^{(3+2p)/(3+p)}$ achieved only for

$$sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\frac{3+2p}{3+p}} \leq k+1 \ ,$$

where $sign(x)$ denotes the sign of x.

The expressions for Kähler form and Z^0 field are given by

$$\begin{aligned} J &= -\frac{p}{3+2p} X du \wedge d\Phi \ , \\ Z^0 &= -\frac{6}{p} J \ . \end{aligned} \tag{A-3.5}$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range Z^0 vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

2. The vanishing of Z^0 fields is achieved by the replacement of the parameter ϵ with $\epsilon = 1/2$ as becomes clear by considering the condition stating that Z^0 field vanishes identically. Also the relationship $F_{em} = 3J = -\frac{3}{4}\frac{r^2}{F} du \wedge d\Phi$ is useful.

3. The vanishing Kähler field corresponds to $\epsilon = 1, p = 0$ in the formula for em neutral space-times. In this case classical em and Z^0 fields are proportional to each other:

$$\begin{aligned} Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u} du \wedge d\Phi = (k+u) du \wedge d\Phi \ , \\ r &= \sqrt{\frac{X}{1-X}} \ , \ X = D|k+u| \ , \\ \gamma &= -\frac{p}{2} Z^0 \ . \end{aligned} \tag{A-3.6}$$

For a vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible.

The effective form of CP_2 metric for surfaces with 2-dimensional CP_2 projection

The effective form of the CP_2 metric for a space-time having vanishing em,Z^0, or Kähler field is of practical value in the case of vacuum extremals and is given by

$$\begin{aligned} ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta}d\Theta^2 + s^{eff}_{\Phi\Phi}d\Phi^2] \ , \\ s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\ s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ , \end{aligned} \quad \text{(A-3.7)}$$

and is useful in the construction of vacuum embedding of, say Schwartchild metric.

Topological quantum numbers

Space-times for which either em, Z^0, or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers (ω_1 and ω_2) are frequency type parameters, two (k_1 and k_2) are wave vector like quantum numbers, two of the quantum numbers (n_1 and n_2) are integers. The parameters ω_i and n_i will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of CP_2 coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$\begin{aligned} \Psi &= \omega_2 m^0 + k_2 m^3 + n_2\phi + \text{Fourier expansion} \ , \\ \Phi &= \omega_1 m^0 + k_1 m^3 + n_1\phi + \text{Fourier expansion} \ . \end{aligned} \quad \text{(A-3.8)}$$

m^0,m^3 and ϕ denote the coordinate variables of the cylindrical M^4 coordinates) so that one has $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$. The regions of the space-time surface with given values of the vacuum parameters ω_i,k_i and n_i and m and C are bounded by the surfaces at which space-time surface becomes ill-defined, say by $r > 0$ or $r < \infty$ surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters r_0 and Θ_0. At $r = \infty$ surfaces n_2,ω_2 and m can change since all values of Ψ correspond to the same point of CP_2: at $r = 0$ surfaces also n_1 and ω_1 can change since all values of Φ correspond to same point of CP_2, too. If $r = 0$ or $r = \infty$ is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global embedding for, say a constant magnetic field. Although global embedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate u in general possesses discontinuous derivative at $r = 0$ and $r = \infty$ surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteeing the absence of edges are satisfied.

For instance, the vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 \ , \quad \text{(A-3.9)}$$

is satisfied. In particular, the ratio ω_2/ω_1 is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter n_1 and n_2 (ω_1 and ω_2) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

A-4 The relationship of TGD to QFT and string models

The recent view of the relationship of TGD to QFT and string models has developed slowly during years and it seems that in a certain sense TGD means a return to roots: instead of QFT like description involving path integral one would have wave mechanics for 3-surfaces.

A-4.1 TGD as a generalization of wave mechanism obtained by replacing point-like particles with 3-surfaces

The first vision of TGD was as a generalization of quantum field theory (string models) obtained by replacing pointlike particles (strings) as fundamental objects with 3-surfaces.

The later work has revealed that TGD could be seen as a generalization of the wave mechanism based on the replacement of a point-like particle with 3-D surface. This is due to holography implied by general coordinate invariance. The definition of the metric of the "world of classical worlds" (WCW) must assign a unique or at least almost unique space-time surface to a given 3-surface. This 4-surface is analogous to Bohr orbit so that also Bohr orbitology becomes an exact part of quantum physics. The failure of strict determinism forces to replace 3-surfaces with 4-surfaces and this leads to zero energy ontology (ZEO) in which quantum states are superpositions of space-time surfaces [K35, K19, K66] [L68, L80].

Fig. 5. TGD replaces point-like particles with 3-surfaces. `http://tgdtheory.fi/appfigures/particletgd.jpg`

A-4.2 Extension of superconformal invariance

The fact that light-like 3-surfaces are effectively metrically 2-dimensional and thus possess generalization of 2-dimensional conformal symmetries with light-like radial coordinate defining the analog of second complex coordinate suggests that this generalization could work and extend the super-conformal symmetries to their 4-D analogs.

The boundary $\delta M^4_+ = S^2 \times R_+$- of 4-D light-cone M^4_+ is also metrically 2-dimensional and allows extended conformal invariance. Also the group of isometries of light-cone boundary and of light-like 3-surfaces is infinite-dimensional since the conformal scalings of S^2 can be compensated by S^2-local scaling of the light-like radial coordinate of R_+. These simple facts mean that 4-dimensional Minkowski space and 4-dimensional space-time surfaces are in a completely unique position as far as symmetries are considered.

In fact, this leads to a generalization of the Kac-Moody type symmetries of string models. $\delta M^4_+ \times CP_2$ allows huge supersymplectic symmetries for which the radial light-like coordinate of δM^4_+ plays the role of complex string coordinate in string models. These symmetries are assumed to act as isometries of WCW.

A-4.3 String-like objects and strings

String like objects obtained as deformations of cosmic strings $X^2 \times Y^2$, where X^2 is minimal surface in M^4 and Y^2 a holomorphic surface of CP_2 are fundamental extremals of Kähler action having string world sheet as M^4 projections. Cosmic strings dominate the primordial cosmology of the TGD Universe and the inflationary period corresponds to the transition to radiation dominated cosmology for which space-time sheets with 4-D M^4 projection dominate.

Also genuine string-like objects emerge from TGD. The conditions that the em charge of modes of induces spinor fields is well-defined requires in the generic case the localization of the modes at 2-D surfaces -string world sheets and possibly also partonic 2-surfaces. This in Minkowskian space-time regions.

Fig. 6. Well-definedness of em charge forces the localization of induced spinor modes to 2-D surfaces in generic situations in Minkowskian regions of space-time surface. `http://tgdtheory.fi/appfigures/fermistring.jpg`

A-4.4 TGD view of elementary particles

The TGD based view about elementary particles has two key aspects.

1. The space-time correlates of elementary particles are identified as pairs of wormhole contacts with Euclidean signature of metric and having 4-D CP_2 projection. Their throats behave effectively as Kähler magnetic monopoles so that wormhole throats must be connected by Kähler magnetic flux tubes with monopole flux so that closed flux tubes are obtained.

2. At the level of H Fermion number is carried by the modes of the induced spinor field. In space-time regions with Minkowski signature the modes are localized at string world sheets connecting the wormhole contacts.

Fig. 7. TGD view about elementary particles. a) Particle orbit corresponds to a 4-D generalization of a world line or b) with its light-like 3-D boundary (holography). c) Particle world lines have Euclidean signature of the induced metric. d) They can be identified as wormhole contacts. e) The throats of wormhole contacts carry effective Kähler magnetic charges so that wormhole contacts must appear as pairs in order to obtain closed flux tubes. f) Wormhole contacts are accompanied by fermionic strings connecting the throats at the same sheet: the strings do not extend inside the wormhole contacts. `http://tgdtheory.fi/appfigures/elparticletgd.jpg`

Particle interactions involve both stringy and QFT aspects.

1. The boundaries of string world sheets correspond to fundamental fermions. This gives rise to massless propagator lines in generalized Feynman diagrammatics. One can speak of "long" string connecting wormhole contacts and having a hadronic string as a physical counterpart. Long strings should be distinguished from wormhole contacts which due to their superconformal invariance behave like "short" strings with length scale given by CP_2 size, which is 10^4 times longer than Planck scale characterizing strings in string models.

2. Wormhole contact defines basic stringy interaction vertex for fermion-fermion scattering. The propagator is essentially the inverse of the superconformal scaling generator L_0. Wormhole contacts containing fermion and antifermion at its opposite throats behave like virtual bosons so that one has BFF type vertices typically.

3. In topological sense one has 3-vertices serving as generalizations of 3-vertices of Feynman diagrams. In these vertices 4-D "lines" of generalized Feynman diagrams meet along their 3-D ends. One obtains also the analogs of stringy diagrams but stringy vertices do not have the usual interpretation in terms of particle decays but in terms of propagation of particles along two different routes.

Fig. 8. a) TGD analogs of Feynman and string diagrammatics at the level of space-time topology. b) The 4-D analogs of both string diagrams and QFT diagrams appear but the interpretation of the analogs stringy diagrams is different. `http://tgdtheory.fi/appfigures/tgdgraphs.jpg`

A-5 About the selection of the action defining the Kähler function of the "world of classical worlds" (WCW)

The proposal is that space-time surfaces correspond to preferred extremals of some action principle, being analogous to Bohr orbits, so that they are almost deterministic. The action for the preferred extremal would define the Kähler function of WCW [K35, K66].

How unique is the choice of the action defining WCW Kähler metric? The problem is that twistor lift strongly suggests the identification of the preferred extremals as 4-D surfaces having 4-D generalization of complex structure and that a large number of general coordinate invariant actions constructible in terms of the induced geometry have the same preferred extremals.

A-5.1 Could twistor lift fix the choice of the action uniquely?

The twistor lift of TGD [L24] [L68, L72, L73] generalizes the notion of induction to the level of twistor fields and leads to a proposal that the action is obtained by dimensional reduction of the action having as its preferred extremals the counterpart of twistor space of the space-time surface identified as 6-D surface in the product $T(M^4) \times T(CP_2)$ twistor spaces of $T(M^4)$ and $T(CP_2)$

of M^4 and CP_2. Only M^4 and CP_2 allow a twistor space with Kähler structure [A18] so that TGD would be unique. Dimensional reduction is forced by the condition that the 6-surface has S^2-bundle structure characterizing twistor spaces and the base space would be the space-time surface.

1. Dimensional reduction of 6-D Kähler action implies that at the space-time level the fundamental action can be identified as the sum of Kähler action and volume term (cosmological constant). Other choices of the action do not look natural in this picture although they would have the same preferred extremals.

2. Preferred extremals are proposed to correspond to minimal surfaces with singularities such that they are also extremals of 4-D Kähler action outside the singularities. The physical analogue are soap films spanned by frames and one can localize the violation of the strict determinism and of strict holography to the frames.

3. The preferred extremal property is realized as the holomorphicity characterizing string world sheets, which generalizes to the 4-D situation. This in turn implies that the preferred extremals are the same for any general coordinate invariant action defined on the induced gauge fields and induced metric apart from possible extremals with vanishing CP_2 Kähler action.

   For instance, 4-D Kähler action and Weyl action as the sum of the tensor squares of the components of the Weyl tensor of CP_2 representing quaternionic imaginary units constructed from the Weyl tensor of CP_2 as an analog of gauge field would have the same preferred extremals and only the definition of Kähler function and therefore Kähler metric of WCW would change. One can even consider the possibility that the volume term in the 4-D action could be assigned to the tensor square of the induced metric representing a quaternionic or octonionic real unit.

Action principle does not seem to be unique. On the other hand, the WCW Kähler form and metric should be unique since its existence requires maximal isometries.

Unique action is not the only way to achieve this. One cannot exclude the possibility that the Kähler gauge potential of WCW in the complex coordinates of WCW differs only by a complex gradient of a holomorphic function for different actions so that they would give the same Kähler form for WCW. This gradient is induced by a symplectic transformation of WCW inducing a U(1) gauge transformation. The Kähler metric is the same if the symplectic transformation is an isometry.

Symplectic transformations of WCW could give rise to inequivalent representations of the theory in terms of action at space-time level. Maybe the length scale dependent coupling parameters of an effective action could be interpreted in terms of a choice of WCW Kähler function, which maximally simplifies the computations at a given scale.

1. The 6-D analogues of electroweak action and color action reducing to Kähler action in 4-D case exist. The 6-D analog of Weyl action based on the tensor representation of quaternionic imaginary units does not however exist. One could however consider the possibility that only the base space of twistor space $T(M^4)$ and $T(CP_2)$ have quaternionic structure.

2. Kähler action has a huge vacuum degeneracy, which clearly distinguishes it from other actions. The presence of the volume term removes this degeneracy. However, for minimal surfaces having CP_2 projections, which are Lagrangian manifolds and therefore have a vanishing induced Kähler form, would be preferred extremals according to the proposed definition. For these 4-surfaces, the existence of the generalized complex structure is dubious.

   For the electroweak action, the terms corresponding to charged weak bosons eliminate these extremals and one could argue that electroweak action or its sum with the analogue of color action, also proportional Kähler action, defines the more plausible choice. Interestingly, also the neutral part of electroweak action is proportional to Kähler action.

Twistor lift strongly suggests that also M^4 has the analog of Kähler structure. M^8 must be complexified by adding a commuting imaginary unit i. In the E^8 subspace, the Kähler structure of E^4 is defined in the standard sense and it is proposed that this generalizes to M^4 allowing also generalization of the quaternionic structure. M^4 Kähler structure violates Lorentz invariance but could be realized at the level of moduli space of these structures.

The minimal possibility is that the M^4 Kähler form vanishes: one can have a different representation of the Kähler gauge potential for it obtained as generalization of symplectic transformations acting non-trivially in M^4. The recent picture about the second quantization of spinors of $M^4 \times CP_2$ assumes however non-trivial Kähler structure in M^4.

A-5.2 Two paradoxes

TGD view leads to two apparent paradoxes.

1. If the preferred extremals satisfy 4-D generalization of holomorphicity, a very large set of actions gives rise to the same preferred extremals unless there are some additional conditions restricting the number of preferred extremals for a given action.

2. WCW metric has an infinite number of zero modes, which appear as parameters of the metric but do not contribute to the line element. The induced Kähler form depends on these degrees of freedom. The existence of the Kähler metric requires maximal isometries, which suggests that the Kähler metric is uniquely fixed apart from a conformal scaling factor Ω depending on zero modes. This cannot be true: galaxy and elementary particle cannot correspond to the same Kähler metric.

Number theoretical vision and the hierarchy of inclusions of HFFs associated with supersymplectic algebra actings as isometries of WcW provide equivalent realizations of the measurement resolution. This solves these paradoxes and predicts that WCW decomposes into sectors for which Kähler metrics of WCW differ in a natural way.

The hierarchy subalgebras of supersymplectic algebra implies the decomposition of WCW into sectors with different actions

Supersymplectic algebra of $\delta M^4_+ \times CP_2$ is assumed to act as isometries of WCW [L80]. There are also other important algebras but these will not be discussed now.

1. The symplectic algebra A of $\delta M^4_+ \times CP_2$ has the structure of a conformal algebra in the sense that the radial conformal weights with non-negative real part, which is half integer, label the elements of the algebra have an interpretation as conformal weights.

   The super symplectic algebra A has an infinite hierarchy of sub-algebras [L80] such that the conformal weights of sub-algebras $A_{n(SS)}$ are integer multiples of the conformal weights of the entire algebra. The superconformal gauge conditions are weakened. Only the subalgebra $A_{n(SS)}$ and the commutator $[A_{n(SS)}, A]$ annihilate the physical states. Also the corresponding classical Noether charges vanish for allowed space-time surfaces.

   This weakening makes sense also for ordinary superconformal algebras and associated Kac-Moody algebras. This hierarchy can be interpreted as a hierarchy symmetry breakings, meaning that sub-algebra $A_{n(SS)}$ acts as genuine dynamical symmetries rather than mere gauge symmetries. It is natural to assume that the super-symplectic algebra A does not affect the coupling parameters of the action.

2. The generators of A correspond to the dynamical quantum degrees of freedom and leave the induced Kähler form invariant. They affect the induced space-time metric but this effect is gravitational and very small for Einsteinian space-time surfaces with 4-D M^4 projection.

   The number of dynamical degrees of freedom increases with $n(SS)$. Therefore WCW decomposes into sectors labelled by $n(SS)$ with different numbers of dynamical degrees of freedom so that their Kähler metrics cannot be equivalent and cannot be related by a symplectic isometry. They can correspond to different actions.

Number theoretic vision implies the decomposition of WCW into sectors with different actions

The number theoretical vision leads to the same conclusion as the hierarchy of HFFs. The number theoretic vision of TGD based on M^8-H duality [L80] predicts a hierarchy with levels labelled by the degrees $n(P)$ of rational polynomials P and corresponding extensions of rationals characterized by Galois groups and by ramified primes defining p-adic length scales.

These sequences allow us to imagine several discrete coupling constant evolutions realized at the level H in terms of action whose coupling parameters depend on the number theoretic parameters.

1. Coupling constant evolution with respect to $n(P)$

The first coupling constant evolution would be with respect to $n(P)$.

1. The coupling constants characterizing action could depend on the degree $n(P)$ of the polynomial defining the space-time region by M^8-H duality. The complexity of the space-time surface would increase with $n(P)$ and new degrees of freedom would emerge as the number of the rational coefficients of P.

2. This coupling constant evolution could naturally correspond to that assignable to the inclusion hierarchy of hyperfinite factors of type II_1 (HFFs). I have indeed proposed [L80] that the degree $n(P)$ equals to the number $n(braid)$ of braids assignable to HFF for which super symplectic algebra subalgebra $A_{n(SS)}$ with radial conformal weights coming as $n(SS)$-multiples of those of entire algebra A. One would have $n(P)=n(braid)=n(SS)$. The number of dynamical degrees of freedom increases with n which just as it increases with $n(P)$ and $n(SS)$.

3. The actions related to different values of $n(P)=n(braid)=n(SS)$ cannot define the same Kähler metric since the number of allowed space-time surfaces depends on $n(SS)$.

   WCW could decompose to sub-WCWs corresponding to different actions, a kind of theory space. These theories would not be equivalent. A possible interpretation would be as a hierarchy of effective field theories.

4. Hierarchies of composite polynomials define sequences of polynomials with increasing values of $n(P)$ such that the order of a polynomial at a given level is divided by those at the lower levels. The proposal is that the inclusion sequences of extensions are realized at quantum level as inclusion hierarchies of hyperfinite factors of type II_1.

   A given inclusion hierarchy corresponds to a sequence $n(SS)_i$ such that $n(SS)_i$ divides $n(SS)_{i+1}$. Therefore the degree of the composite polynomials increases very rapidly. The values of $n(SS)_i$ can be chosen to be primes and these primes correspond to the degrees of so called prime polynomials [L74] so that the decompositions correspond to prime factorizations of integers. The "densest" sequence of this kind would come in powers of 2 as $n(SS)_i=2^i$. The corresponding p-adic length scales (assignable to maximal ramified primes for given $n(SS)_i$) are expected to increase roughly exponentially, say as 2^{r2^i}. $r=1/2$ would give a subset of scales $2^{r/2}$ allowed by the p-adic length scale hypothesis. These transitions would be very rare.

   A theory corresponding to a given composite polynomial would contain as sub-theories the theories corresponding to lower polynomial composites. The evolution with respect to $n(SS)$ would correspond to a sequence of phase transitions in which the action genuinely changes. For instance, color confinement could be seen as an example of this phase transition.

5. A subset of p-adic primes allowed by the p-adic length scale hypothesis $p\simeq 2^k$ defining the proposed p-adic length scale hierarchy could relate to n_S changing phase transition. TGD suggests a hierarchy of hadron physics corresponding to a scale hierarchy defined by Mersenne primes and their Gaussian counterparts [K47, K48]). Each of them would be characterized by a confinement phase transition in which n_S and therefore also the action changes.

2. Coupling constant evolutions with respect to ramified primes for a given value of $n(P)$

For a given value of $n(P)$, one could have coupling constant sub-evolutions with respect to the set of ramified primes of P and dimensions $n = h_{eff}/h_0$ of algebraic extensions. The action would only change by U(1) gauge transformation induced by a symplectic isometry of WCW. Coupling parameters could change but the actions would be equivalent.

The choice of the action in an optimal manner in a given scale could be seen as a choice of the most appropriate effective field theory in which radiative corrections would be taken into account. One can interpret the possibility to use a single choice of coupling parameters in terms of quantum criticality.

The range of the p-adic length scales labelled by ramified primes and effective Planck constants h_{eff}/h_0 is finite for a given value of $n(SS)$.

The first coupling constant evolution of this kind corresponds to ramified primes defining p-adic length scales for given $n(SS)$.

1. Ramified primes are factors of the discriminant $D(P)$ of P, which is expressible as a product of non-vanishing root differents and reduces to a polynomial of the n coefficients of P. Ramified primes define p-adic length scales assignable to the particles in the amplitudes scattering amplitudes defined by zero energy states.

   P would represent the space-time surface defining an interaction region in $N-$-particle scattering. The N ramified primes dividing $D(P)$ would characterize the p-adic length scales assignable to these particles. If $D(P)$ reduces to a single ramified prime, one has elementary particle [L74], and the forward scattering amplitude corresponds to the propagator.

   This would give rise to a multi-scale p-adic length scale evolution of the amplitudes analogous to the ordinary continuous coupling constant evolution of n-point scattering amplitudes with respect to momentum scales of the particles. This kind of evolutions extend also to evolutions with respect to $n(SS)$.

2. According to [L74], physical constraints require that $n(P)$ and the maximum size of the ramified prime of P correlate.

   A given rational polynomial of degree $n(P)$ can be always transformed to a polynomial with integer coefficients. If the integer coefficients are smaller than $n(P)$, there is an upper bound for the ramified primes. This assumption also implies that finite fields become fundamental number fields in number theoretical vision [L74].

3. p-Adic length scale hypothesis [L81] in its basic form states that there exist preferred primes $p \simeq 2^k$ near some powers of 2. A more general hypothesis states that also primes near some powers of 3 possibly also other small primes are preferred physically. The challenge is to understand the origin of these preferred scales.

   For polynomials P with a given degree $n(P)$ for which discriminant $D(P)$ is prime, there exists a maximal ramified prime. Numerical calculations suggest that the upper bound depends exponentially on $n(P)$.

   Could these maximal ramified primes satisfy the p-adic length scale hypothesis or its generalization? The maximal prime defines a fixed point of coupling constant evolution in accordance with the earlier proposal. For instance, could one think that one has $p \simeq 2^k$, $k = n(SS)$? Each p-adic prime would correspond to a p-adic coupling constant sub-evolution representable in terms of symplectic isometries.

Also the dimension n of the algebraic extension associated with P, which is identified in terms of effective Planck constant $h_{eff}/h_0 = n$ labelling different phases of the ordinary matter behaving like dark matter, could give rise to coupling constant evolution for given $n(SS)$. The range of allowed values of n is finite. Note however that several polynomials of a given degree can correspond to the same dimension of extension.

Number theoretic discretization of WCW and maxima of WCW Kähler function

Number theoretic approach involves a unique discretization of space-time surface and also of WCW. The question is how the points of the discretized WCW correspond to the preferred extremals.

1. The exponents of Kähler function for the maxima of Kähler function, which correspond to the universal preferred extremals, appear in the scattering amplitudes. The number theoretical approach involves a unique discretization of space-time surfaces defining the WCW coordinates of the space-time surface regarded as a point of WCW.

   In [L80] it is assumed that these WCW points appearing in the number theoretical discretization correspond to the maxima of the Kähler function. The maxima would depend on the action and would differ for ghd maxima associated with different actions unless they are not related by symplectic WCW isometry.

2. The symplectic transformations of WCW acting as isometries are assumed to be induced by the symplectic transformations of $\delta M^4_+ \times CP_2$ [K35, K19]. As isometries they would naturally permute the maxima with each other.

A-6 Number theoretic vision of TGD

Physics as number theory vision is complementary to the physics as geometry vision and has developed gradually since 1993. Langlands program is the counterpart of this vision in mathematics [L78].

The notion of p-adic number fields emerged with the motivation coming from the observation that elementary particle mass scales and mass ratios could be understood in terms of the so-called p-adic length scale hypothesis [K50, K42, K17]. The fusion of the various p-adic physics leads to what I call adelic physics [L22, L23]. Later the hypothesis about hierarchy of Planck constants labelling phases of ordinary matter behaving like dark matter emerged [K21, K22, K23, K24].

Eventually this led to that the values of effective Planck constant could be identified as the dimension of an algebraic extension of rationals assignable to polynomials with rational coefficients. This led to the number theoretic vision in which so-called $M^8 - H$ duality [L52, L53] plays a key role. M^8 (actually a complexification of real M^8) is analogous to momentum space so that the duality generalizes momentum position duality for point-like particles. M^8 has an interpretation as complexified octonions.

The dynamics of 4-surfaces in M^8 is coded by polynomials with rational coefficients, whose roots define mass shells H^3 of $M^4 \subset M^8$. It has turned out that the polynomials satisfy stringent additional conditions and one can speak of number theoretic holography [L74, L78]. Also the ordinary $3 \rightarrow 4$ holography is needed to assign 4-surfaces with these 3-D mass shells. The number theoretic dynamics is based on the condition that the normal space of the 4-surface in M^8 is associative (quaternionic) and contains a commutative complex sub-space. This makes it possible to assign to this surface space-time surface in $H = M^4 \times CP_2$.

At the level of H the space-time surfaces are by holography preferred extremals and are assumed to be determined by the twistor lift of TGD [L24] giving rise to an action which is sum of the Kähler action and volume term. The preferred extremals would be minimal surfaces analogous to soap films spanned by frames. Outside frames they would be simultaneous extremals of the Kähler action, which requires a generalization of the holomorphy characterizing string world sheets.

In the following only p-adic numbers and hierarchy of Planck constants will be discussed.

A-6.1 p-Adic numbers and TGD

p-Adic number fields

p-Adic numbers (p is prime: 2, 3, 5, ...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [A8]. p-Adic numbers are representable as power expansion of the prime number p of form

$$x = \sum_{k \geq k_0} x(k) p^k , \; x(k) = 0, ..., p-1 \; . \tag{A-6.1}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} \ . \tag{A-6.2}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) \ , \tag{A-6.3}$$

where $\varepsilon(x) = k + ...$ with $0 < k < p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

The distance function $d(x,y) = |x-y|_p$ defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x,z) \leq max\{d(x,y), d(y,z)\} \ . \tag{A-6.4}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x,y) \leq D \ . \tag{A-6.5}$$

This division of the metric space into classes has following properties:

1. Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.

2. Distances of points x and y inside single class are smaller than distances between different classes.

3. Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [B32]. The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

Canonical correspondence between p-adic and real numbers

The basic challenge encountered by p-adic physicist is how to map the predictions of the p-adic physics to real numbers. p-Adic probabilities provide a basic example in this respect. Identification via common rationals and canonical identification and its variants have turned out to play a key role in this respect.

1. Basic form of the canonical identification

There exists a natural continuous map $I : R_p \rightarrow R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \rightarrow x = \sum_{k<N} y_k p^{-k} \ , \\ y_k &\in \{0,1,..,p-1\} \ . \end{aligned} \tag{A-6.6}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also decimal expansion is not unique (1 = 0.999...) for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^{N} x_k p^{-k} \ , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \ . \end{aligned} \quad \text{(A-6.7)}$$

The p-adic images associated with these expansions are different

$$\begin{aligned} y_1 &= \sum_{k=N_0}^{N} x_k p^{k} \ , \\ y_2 &= \sum_{k=N_0}^{N-1} x_k p^{k} + (x_N - 1)p^{N} + (p-1)p^{N+1} \sum_{k=0,..} p^{k} \\ &= y_1 + (x_N - 1)p^N - p^{N+1} \ , \end{aligned} \quad \text{(A-6.8)}$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite pinary digits. The finite pinary digit expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

2. The topology induced by canonical identification

The topology induced by the canonical identification in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see **Fig. A-6.1**) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

Fig. 14. The real norm induced by canonical identification from 2-adic norm. `http://tgdtheory.fi/appfigures/norm.png`

The linear structure of the p-adic numbers induces a corresponding structure in the set of the non-negative real numbers and p-adic linearity in general differs from the ordinary concept of linearity. For example, p-adic sum is equal to real sum only provided the summands have no common pinary digits. Furthermore, the condition $x +_p y < max\{x, y\}$ holds in general for the p-adic sum of the real numbers. p-Adic multiplication is equivalent with the ordinary multiplication only provided that either of the members of the product is power of p. Moreover one has $x \times_p y < x \times y$ in general. The p-Adic negative -1_p associated with p-adic unit 1 is given by $(-1)_p = \sum_k (p-1)p^k$ and defines p-adic negative for each real number x. An interesting possibility is that p-adic linearity might replace the ordinary linearity in some strongly nonlinear systems so these systems would look simple in the p-adic topology.

These results suggest that canonical identification is involved with some deeper mathematical structure. The following inequalities hold true:

$$(x+y)_R \leq x_R + y_R \ ,$$
$$|x|_p |y|_R \leq (xy)_R \leq x_R y_R \ , \quad \text{(A-6.9)}$$

where $|x|_p$ denotes p-adic norm. These inequalities can be generalized to the case of $(R_p)^n$ (a linear vector space over the p-adic numbers).

$$(x+y)_R \leq x_R + y_R \ ,$$
$$|\lambda|_p |y|_R \leq (\lambda y)_R \leq \lambda_R y_R \ , \quad \text{(A-6.10)}$$

where the norm of the vector $x \in T_p^n$ is defined in some manner. The case of Euclidian space suggests the definition

$$(x_R)^2 = (\sum_n x_n^2)_R \ . \quad \text{(A-6.11)}$$

These inequalities resemble those satisfied by the vector norm. The only difference is the failure of linearity in the sense that the norm of a scaled vector is not obtained by scaling the norm of the original vector. Ordinary situation prevails only if the scaling corresponds to a power of p.

These observations suggests that the concept of a normed space or Banach space might have a generalization and physically the generalization might apply to the description of some non-linear systems. The nonlinearity would be concentrated in the nonlinear behavior of the norm under scaling.

3. Modified form of the canonical identification

The original form of the canonical identification is continuous but does not respect symmetries even approximately. This led to a search of variants which would do better in this respect. The modification of the canonical identification applying to rationals only and given by

$$I_Q(q = p^k \times \frac{r}{s}) = p^k \times \frac{I(r)}{I(s)} \quad \text{(A-6.12)}$$

is uniquely defined for rationals, maps rationals to rationals, has also a symmetry under exchange of target and domain. This map reduces to a direct identification of rationals for $0 \leq r < p$ and $0 \leq s < p$. It has turned out that it is this map which most naturally appears in the applications. The map is obviously continuous locally since p-adically small modifications of r and s mean small modifications of the real counterparts.

Canonical identification is in a key role in the successful predictions of the elementary particle masses. The predictions for the light elementary particle masses are within extreme accuracy same for I and I_Q but I_Q is theoretically preferred since the real probabilities obtained from p-adic ones by I_Q sum up to one in p-adic thermodynamics.

4. Generalization of number concept and notion of embedding space

TGD forces an extension of number concept: roughly a fusion of reals and various p-adic number fields along common rationals is in question. This induces a similar fusion of real and p-adic embedding spaces. Since finite p-adic numbers correspond always to non-negative reals n-dimensional space R^n must be covered by 2^n copies of the p-adic variant R_p^n of R^n each of which projects to a copy of R_+^n (four quadrants in the case of plane). The common points of p-adic and real embedding spaces are rational points and most p-adic points are at real infinity.

Real numbers and various algebraic extensions of p-adic number fields are thus glued together along common rationals and also numbers in algebraic extension of rationals whose number belong to the algebraic extension of p-adic numbers. This gives rise to a book like structure with rationals and various algebraic extensions of rationals taking the role of the back of the book. Note that Neper number is exceptional in the sense that it is algebraic number in p-adic number field Q_p satisfying $e^p \ mod \ p = 1$.

Fig. 15. Various number fields combine to form a book like structure. `http://tgdtheory.fi/appfigures/book.jpg`

For a given p-adic space-time sheet most points are literally infinite as real points and the projection to the real embedding space consists of a discrete set of rational points: the interpretation in terms of the unavoidable discreteness of the physical representations of cognition is natural. Purely local p-adic physics implies real p-adic fractality and thus long range correlations for the real space-time surfaces having enough common points with this projection.

p-Adic fractality means that M^4 projections for the rational points of space-time surface X^4 are related by a direct identification whereas CP_2 coordinates of X^4 at these points are related by I, I_Q or some of its variants implying long range correlates for CP_2 coordinates. Since only a discrete set of points are related in this manner, both real and p-adic field equations can be satisfied and there are no problems with symmetries. p-Adic effective topology is expected to be a good approximation only within some length scale range which means infrared and UV cutoffs. Also multi-p-fractality is possible.

The notion of p-adic manifold

The notion of p-adic manifold is needed in order to fuse real physics and various p-adic physics to a larger structure which suggests that real and p-adic number fields should be glued together along common rationals bringing in mind adeles. The notion is problematic because p-adic topology is totally disconnected implying that p-adic balls are either disjoint or nested so that ordinary definition of manifold using p-adic chart maps fails. A cure is suggested to be based on chart maps from p-adics to reals rather than to p-adics (see the appendix of the book)

The chart maps are interpreted as cognitive maps, "thought bubbles".

Fig. 16. The basic idea between p-adic manifold. `http://tgdtheory.fi/appfigures/padmanifold.jpg`

There are some problems.

1. Canonical identification does not respect symmetries since it does not commute with second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map arithmetic operations which requires pinary cutoff below which chart map takes rationals to rationals so that commutativity with arithmetics and symmetries is achieved in finite resolution: above the cutoff canonical identification is used

2. Canonical identification is continuous but does not map smooth p-adic surfaces to smooth real surfaces requiring second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map requiring completion of the image to smooth preferred extremal of Kähler action so that chart map is not unique in accordance with finite measurement resolution

3. Canonical identification violates general coordinate invariance of chart map: (cognition-induced symmetry breaking) minimized if p-adic manifold structure is induced from that for p-adic embedding space with chart maps to real embedding space and assuming preferred coordinates made possible by isometries of embedding space: one however obtains several inequivalent p-adic manifold structures depending on the choice of coordinates: these cognitive representations are not equivalent.

A-6.2 Hierarchy of Planck constants and dark matter hierarchy

Hierarchy of Planck constants was motivated by the "impossible" quantal effects of ELF em fields on vertebrate cyclotron energies $E = hf = \hbar \times eB/m$ are above thermal energy is possible only if $\hbar$ has value much larger than its standard value. Also Nottale's finding that planetary orbits migh be understood as Bohr orbits for a gigantic gravitational Planck constant.

Hierachy of Planck constant would mean that the values of Planck constant come as integer multiples of ordinary Planck constant: $h_{eff} = n \times h$. The particles at magnetic flux tubes characterized by h_{eff} would correspond to dark matter which would be invisible in the sense that only particle with same value of h_{eff} appear in the same vertex of Feynman diagram.

Hierarchy of Planck constants would be due to the non-determism of the Kähler action predicting huge vacuum degeneracy allowing all space-time surfaces which are sub-manfolds of any $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2. For agiven Y^2 one obtains new manifolds Y^2 by applying symplectic transformations of CP_2.

Non-determinism would mean that the 3-surface at the ends of causal diamond (CD) can be connected by several space-time surfaces carrying same conserved Kähler charges and having same values of Kähler action. Conformal symmetries defined by Kac-Moody algebra associated with the embedding space isometries could act as gauge transformations and respect the light-likeness property of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian (Minkowskianb space-time region transforms to wormhole contact say). The number of conformal equivalence classes of these surfaces could be finite number n and define discrete physical degree of freedom and one would have $h_{eff} = n \times h$. This degeneracy would mean "second quantization" for the sheets of n-furcation: not only one but several sheets can be realized.

This relates also to quantum criticality postulated to be the basic characteristics of the dynamics of quantum TGD. Quantum criticalities would correspond to an infinite fractal hierarchy of broken conformal symmetries defined by sub-algebras of conformal algebra with conformal weights coming as integer multiples of n. This leads also to connections with quantum criticality and hierarchy of broken conformal symmetries, p-adicity, and negentropic entanglement which by consistency with standard quantum measurement theory would be described in terms of density matrix proportional $n \times n$ identity matrix and being due to unitary entanglement coefficients (typical for quantum computing systems).

Formally the situation could be described by regarding space-time surfaces as surfaces in singular n-fold singular coverings of embedding space. A stronger assumption would be that they are expressible as as products of n_1 -fold covering of M^4 and n_2-fold covering of CP_2 meaning analogy with multi-sheeted Riemann surfaces and that M^4 coordinates are n_1-valued functions and CP_2 coordinates n_2 -valued functions of space-time coordinates for $n = n_1 \times n_2$. These singular coverings of embedding space form a book like structure with singularities of the coverings localizable at the boundaries of causal diamonds defining the back of the book like structure.

Fig. 17. Hierarchy of Planck constants. `http://tgdtheory.fi/appfigures/planckhierarchy.jpg`

A-6.3 $M^8 - H$ duality as it is towards the end of 2021

The view of $M^8 - H$ duality (see Appendix **??**) has changed considerably towards the end 2021 [L68] after the realization that this duality is the TGD counterpart of momentum position duality of wave mechanics, which is lost in QFTs. Therefore M^8 and also space-time surface is analogous to momentum space. This forced us to give up the original simple identification of the points $M^4 \subset M^4 \times E^4 = M^8$ and of $M^4 \times CP_2$ so that it respects Uncertainty Principle (UP).

The first improved guess for the duality map was the replacement with the inversion $p^k \rightarrow m^k = \hbar_{eff} p^k/p^2$ conforming in spirit with UP but turned out to be too naive.

The improved form [L68] of the $M^8 - H$ duality map takes mass shells $p^2 = m^2$ of $M^4 \subset M^8$ to cds with size $L(m) = \hbar_{eff}/m$ with a common center. The slicing by mass shells is mapped to a Russian doll like slicing by cds. Therefore would be no CDs in M^8 contrary to what I believed first.

Quantum classical correspondence (QCC) inspires the proposal that the point $p^k \in M^8$ is mapped to a geodesic line corresponding to momentum p^k starting from the common center of cds. Its intersection with the opposite boundary of cd with size $L(m)$ defines the image point. This is not yet quite enough to satisfy UP but the additional details [L68] are not needed in the sequel.

The 6-D brane-like special solutions in M^8 are of special interest in the TGD inspired theory of consciousness. They have an M^4 projection which is $E = E_n$ 3-ball. Here E_n is a root of the real polynomial P defining $X^4 \subset M^8_c$ (M^8 is complexified to M^8_c) as a "root" of its octonionic continuation [L52, L53]. E_n has an interpretation as energy, which can be complex. The original interpretation was as moment of time. For this interpretation, $M^8 - H$ duality would be a linear identification and these hyper planes would be mapped to hyperplanes in $M^4 \subset H$.

This motivated the term "very special moment in the life of self" for the image of the $E = E_n$ section of $X^4 \subset M^8$ [L41]. This notion does not make sense at the level M^8 anymore.

The modified $M^8 - H$ duality forces us to modify the original interpretation [L68]. The point $(E_n, p = 0)$ is mapped $(t_n = \hbar_{eff}/E_n, 0)$. The momenta (E_n, p) in $E = E_n$ plane are mapped to the boundary of cd and correspond to a continuous time interval at the boundary of CD: "very special moment" becomes a "very special time interval".

The quantum state however corresponds to a set of points corresponding to quark momenta, which belong to a cognitive representation and are therefore algebraic integers in the extension determined by the polynomial. These active points in E_n are mapped to a discrete set at the boundary of cd(m). A "very special moment" is replaced with a sequence of "very special moments".

So called Galois confinement [L60] forces the total momenta for bound states of quarks and antiquarks to be rational integers invariant under Galois group of extension of rationals determined by the polynomial P [L68]. These states correspond to states at boundaries of sub-CDs so that one obtains a hierarchy. Galois confinement provides a universal number theoretic mechanism for the formation of bound states.

A-7 Zero energy ontology (ZEO)

ZEO is implied by the holography forced in the TGD framework by general coordinate invariance.

A-7.1 Basic motivations and ideas of ZEO

The following gives a brief summary of ZEO [L47] [K89].

1. In ZEO quantum states are not 3-dimensional but superpositions of 4-dimensional deterministic time evolutions connecting ordinary initial 3-dimensional states. By holography they are equivalent to pairs of ordinary 3-D states identified as initial and final states of time evolution. One can say that in the TGD framework general coordinate invariance implies holography and the slight failure of its determinism in turn forces ZEO.

   Quantum jumps replace this state with a new one: a superposition of deterministic time evolutions is replaced with a new superposition. Classical determinism of individual time evolution is not violated and this solves the basic paradox of quantum measurement theory. There are two kinds of quantum jumps: ordinary ("big") state function reductions (BSFRs) changing the arrow of time and "small" state function reductions (SSFRs) (weak measurements) preserving it and giving rise to the analog of Zeno effect [L47].

2. To avoid getting totally confused it is good to emphasize some aspects of ZEO.

   (a) ZEO does not mean that physical states in the usual 3-D sense as snapshots of time evolution would have zero energy state pairs defining zero energy states as initial and final states have same conserved quantities such as energy. Conservation implies that one can adopt the conventions that the values of conserved quantities are opposite for these states so that their sum vanishes: one can think that incoming and outgoing particles come from geometric past and future is the picture used in quantum field theories.

   (b) ZEO means two times: subjective time as sequence of quantum jumps and geometric time as space-time coordinate. These times are identifiable but are strongly correlated.

3. In BSFRs the arrow of time is changed and the time evolution in the final state occurs backwards with respect to the time of the external observer. BSFRs can occur in all scales since TGD predicts a hierarchy of effective Planck constants with arbitrarily large values. There is empirical support for BSFRs.

   (a) The findings of Minev et al [L38] in atomic scale can be explained by the same mechanism [L38]. In BSFR a final zero energy state as a superposition of classical deterministic time evolutions emerges and for an observer with a standard arrow of time looks like a superposition of deterministic smooth time evolutions leading to the final state. Interestingly, once this evolution has started, it cannot be stopped unless one changes the stimulus signal inducing the evolution in which case the process does not lead to anywhere: the interpretation would be that BSFR back to the initial state occurs!

(b) Libets' experiments about active aspects of consciousness [J1] can be understood. Subject person raises his finger and neural activity starts before the conscious decision to do so. In the physicalistic framework it is thought to lead to raising of the finger. The problem with the explanation is that the activity beginning .5 seconds earlier seems to be dissipation with a reversed arrow of time: from chaotic and disordered to ordered at around .15 seconds. ZEO explanation is that macroscopic quantum jump occurred and generated a signal proceeding backwards in time and generated neural activity and dissipated to randomness.

(c) Earthquakes involve a strange anomaly: they are preceded by ELF radiation. One would expect that they generate ELF radiation. The identification as BSFR would explain the anomaly [L40]. In biology the reversal of the arrow of time would occur routinely and be a central element of biological self-organization, in particular self-organized quantum criticality (see [L43, L93].

A-7.2 Some implications of ZEO

ZEO has profound implications for understanding self-organization and self-organized quantum criticality in terms of dissipation with non-standard arrow of time looking like generation of structures [L43, L93]. ZEO could also allow understanding of what planned actions - like realizing the experiment under consideration - could be.

1. Second law in the standard sense does not favor - perhaps even not allow - realization of planned actions. ZEO forces a generalization of thermodynamics: dissipation with a non-standard arrow of time for a subsystem would look like self-organization and planned action and its realization.

   Could most if not all planned action be like this - induced by BSFR in the geometric future and only apparently planned? There would be however the experience of planning and realizing induced by the signals from geometric future by a higher level in the hierarchy of conscious entities predicted by TGD! In long time scales we would be realizing our fates or wishes of higher level conscious entities rather than agents with completely free will.

2. The notion of magnetic body (MB) serving as a boss of ordinary matter would be central. MB carries dark matter as $h_{eff} = nh_0$ phases of ordinary matter with n serving as a measure for algebraic complexity of extension of rationals as its dimension and defining a kind of universal IQ. There is a hierarchy of these phases and MBs labelled by extension of rationals and the value of n.

   MBs would form a hierarchy of bosses - a realization for master slave hierarchy. Ordinary matter would be at the bottom and its coherent behavior would be induced from quantum coherence at higher levels. BSFR for higher level MB would give rise to what looks like planned actions and experienced as planned action at the lower levels of hierarchy. One could speak of planned actions inducing a cascade of planned actions in shorter time scales and eventually proceeding to atomic level.

A-8 Some notions relevant to TGD inspired consciousness and quantum biology

Below some notions relevant to TGD inspired theory of consciousness and quantum biology.

A-8.1 The notion of magnetic body

Topological field quantization inspires the notion of field body about which magnetic body is especially important example and plays key role in TGD inspired quantum biology and consciousness theory. This is a crucial departure fromt the Maxwellian view. Magnetic body brings in third level to the description of living system as a system interacting strongly with environment. Magnetic body would serve as an intentional agent using biological body as a motor instrument and sensory receptor. EEG would communicated the information from biological body to magnetic body and Libet's findings from time delays of consciousness support this view.

The following pictures illustrate the notion of magnetic body and its dynamics relevant for quantum biology in TGD Universe.

Fig. 18. Magnetic body associated with dipole field. `http://tgdtheory.fi/appfigures/fluxquant.jpg`

Fig. 19. Illustration of the reconnection by magnetic flux loops. `http://tgdtheory.fi/appfigures/reconnect1.jpg`

Fig. 20. Illustration of the reconnection by flux tubes connecting pairs of molecules. `http://tgdtheory.fi/appfigures/reconect2.jpg`

Fig. 21. Flux tube dynamics. a) Reconnection making possible magnetic body to "recognize" the presence of another magnetic body, b) braiding, knotting and linking of flux tubes making possible topological quantum computation, c) contraction of flux tube in phase transition reducing the value of h_{eff} allowing two molecules to find each other in dense molecular soup. `http://tgdtheory.fi/appfigures/fluxtubedynamics.jpg`

A-8.2 Number theoretic entropy and negentropic entanglement

TGD inspired theory of consciousness relies heavily p-Adic norm allows an to define the notion of Shannon entropy for rational probabilities (and even those in algebraic extension of rationals) by replacing the argument of logarithm of probability with its p-adic norm. The resulting entropy can be negative and the interpretation is that number theoretic entanglement entropy defined by this formula for the p-adic prime minimizing its value serves as a measure for conscious information. This negentropy characterizes two-particle system and has nothing to do with the formal negative negentropy assignable to thermodynamic entropy characterizing single particle. Negentropy Maximization Principle (NMP) implies that number theoretic negentropy increases during evolution by quantum jumps. The condition that NMP is consistent with the standard quantum measurement theory requires that negentropic entanglement has a density matrix proportional to unit matrix so that in 2-particle case the entanglement matrix is unitary.

Fig. 22. Schrödinger cat is neither dead or alive. For negentropic entanglement this state would be stable. `http://tgdtheory.fi/appfigures/cat.jpg`

A-8.3 Life as something residing in the intersection of reality and p-adicities

In TGD inspired theory of consciousness p-adic space-time sheets correspond to space-time correlates for thoughts and intentions. The intersections of real and p-adic preferred extremals consist of points whose coordinates are rational or belong to some extension of rational numbers in preferred embedding space coordinates. They would correspond to the intersection of reality and various p-adicities representing the "mind stuff" of Descartes. There is temptation to assign life to the intersection of realities and p-adicities. The discretization of the chart map assigning to real space-time surface its p-adic counterpart would reflect finite cognitive resolution.

At the level of "world of classical worlds" (WCW) the intersection of reality and various p-adicities would correspond to space-time surfaces (or possibly partonic 2-surfaces) representable in terms of rational functions with polynomial coefficients with are rational or belong to algebraic extension of rationals.

The quantum jump replacing real space-time sheet with p-adic one (vice versa) would correspond to a buildup of cognitive representation (realization of intentional action).

Fig. 23. The quantum jump replacing real space-time surface with corresponding p-adic manifold can be interpreted as formation of though, cognitive representation. Its reversal would correspond to a transformation of intention to action. `http://tgdtheory.fi/appfigures/padictoreal.jpg`

A-8.4 Sharing of mental images

The 3-surfaces serving as correlates for sub-selves can topologically condense to disjoint large space-time sheets representing selves. These 3-surfaces can also have flux tube connections and this makes possible entanglement of sub-selves, which unentangled in the resolution defined by the size of sub-selves. The interpretation for this negentropic entanglement would be in terms of sharing of mental images. This would mean that contents of consciousness are not completely private as assumed in neuroscience.

Fig. 24. Sharing of mental images by entanglement of subselves made possible by flux tube connections between topologically condensed space-time sheets associated with mental images. `http://tgdtheory.fi/appfigures/sharing.jpg`

A-8.5 Time mirror mechanism

Zero energy ontology (ZEO) is crucial part of both TGD and TGD inspired consciousness and leads to the understanding of the relationship between geometric time and experience time and how the arrow of psychological time emerges. One of the basic predictions is the possibiity of negative energy signals propagating backwards in geometric time and having the property that entropy basically associated with subjective time grows in reversed direction of geometric time. Negative energy signals inspire time mirror mechanism (see **Fig.** `http://tgdtheory.fi/appfigures/timemirror.jpg` or **Fig.** 24 in the appendix of this book) providing mechanisms of both memory recall, realization of intentational action initiating action already in geometric past, and remote metabolism. What happens that negative energy signal travels to past and is reflected as positive energy signal and returns to the sender. This process works also in the reverse time direction.

Fig. 25. Zero energy ontology allows time mirror mechanism as a mechanism of memory recall. Essentially "seeing" in time direction is in question. `http://tgdtheory.fi/appfigures/timemirror.jpg`

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