HYPER-FINITE FACTORS, P-ADIC LENGTH SCALE HYPOTHESIS AND DARK MATTER HIERARCHY: PART I

Matti Pitkänen

Rinnekatu 2-4 A 8, Karkkila, 03620, Finland

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0.1 PREFACE

This book belongs to a series of online books summarizing the recent state Topological Geometrodynamics (TGD) and its applications. TGD can be regarded as a unified theory of fundamental interactions but is not the kind of unified theory as so called GUTs constructed by graduate students at seventies and eighties using detailed recipes for how to reduce everything to group theory. Nowadays this activity has been completely computerized and it probably takes only a few hours to print out the predictions of this kind of unified theory as an article in the desired format. TGD is something different and I am not ashamed to confess that I have devoted the last 37 years of my life to this enterprise and am still unable to write The Rules.

If I remember correctly, I got the basic idea of Topological Geometrodynamics (TGD) during autumn 1977, perhaps it was October. What I realized was that the representability of physical space-times as 4-dimensional surfaces of some higher-dimensional space-time obtained by replacing the points of Minkowski space with some very small compact internal space could resolve the conceptual difficulties of general relativity related to the definition of the notion of energy. This belief was too optimistic and only with the advent of what I call zero energy ontology the understanding of the notion of Poincare invariance has become satisfactory. This required also the understanding of the relationship to General Relativity.

It soon became clear that the approach leads to a generalization of the notion of space-time with particles being represented by space-time surfaces with finite size so that TGD could be also seen as a generalization of the string model. Much later it became clear that this generalization is consistent with conformal invariance only if space-time is 4-dimensional and the Minkowski space factor of imbedding space is 4-dimensional. During last year it became clear that 4-D Minkowski space and 4-D complex projective space CP_2 are completely unique in the sense that they allow twistor space with Kähler structure.

It took some time to discover that also the geometrization of also gauge interactions and elementary particle quantum numbers could be possible in this framework: it took two years to find the unique internal space (CP_2) providing this geometrization involving also the realization that family replication phenomenon for fermions has a natural topological explanation in TGD framework and that the symmetries of the standard model symmetries are much more profound than pragmatic TOE builders have believed them to be. If TGD is correct, main stream particle physics chose the wrong track leading to the recent deep crisis when people decided that quarks and leptons belong to same multiplet of the gauge group implying instability of proton.

There have been also longstanding problems.

- Gravitational energy is well-defined in cosmological models but is not conserved. Hence the conservation of the inertial energy does not seem to be consistent with the Equivalence Principle. Furthermore, the imbeddings of Robertson-Walker cosmologies turned out to be vacuum extremals with respect to the inertial energy. About 25 years was needed to realize that the sign of the inertial energy can be also negative and in cosmological scales the density of inertial energy vanishes: physically acceptable universes are creatable from vacuum. Eventually this led to the notion of zero energy ontology (ZEO) which deviates dramatically from the standard ontology being however consistent with the crossing symmetry of quantum field theories. In this framework the quantum numbers are assigned with zero energy states located at the boundaries of so called causal diamonds defined as intersections of future and past directed light-cones. The notion of energy-momentum becomes length scale dependent since one has a scale hierarchy for causal diamonds. This allows to understand the non-conservation of energy as apparent.

  Equivalence Principle as it is expressed by Einstein's equations follows from Poincare invariance once it is realized that GRT space-time is obtained from the many-sheeted space-time of TGD by lumping together the space-time sheets to a regionof Minkowski space and endowing it with an effective metric given as a sum of Minkowski metric and deviations of the metrices of space-time sheets from Minkowski metric. Similar description relates classical gauge potentials identified as components of induced spinor connection to Yang-Mills gauge potentials in GRT space-time. Various topological inhomogenities below resolution scale identified as particles are described using energy momentum tensor and gauge currents.

- From the beginning it was clear that the theory predicts the presence of long ranged classical electro-weak and color gauge fields and that these fields necessarily accompany classical electromagnetic fields.

  It took about 26 years to gain the maturity to admit the obvious: these fields are classical correlates for long range color and weak interactions assignable to dark matter. The only possible conclusion is that TGD physics is a fractal consisting of an entire hierarchy of fractal copies of standard model physics. Also the understanding of electro-weak massivation and screening of weak charges has been a long standing problem, and 32 years was needed to discover that what I call weak form of electric-magnetic duality gives a satisfactory solution of the problem and provides also surprisingly powerful insights to the mathematical structure of quantum TGD.

  The latest development was the realization that the well- definedness of electromagnetic charge as quantum number for the modes of the induced spinors field requires that the CP_2 projection of the region in which they are non-vanishing carries vanishing W boson field and is 2-D. This implies in the generic case their localization to 2-D surfaces: string world sheets and possibly also partonic 2-surfaces. This localization applies to all modes except covariantly constant right handed neutrino generating supersymmetry and mplies that string model in 4-D space-time is part of TGD. Localization is possible only for Kähler-Dirac assigned with Kähler action defining the dynamics of space-time surfaces. One must however leave open the question whether W field might vanish for the space-time of GRT if related to many-sheeted space-time in the proposed manner even when they do not vanish for space-time sheets.

I started the serious attempts to construct quantum TGD after my thesis around 1982. The original optimistic hope was that path integral formalism or canonical quantization might be enough to construct the quantum theory but the first discovery made already during first year of TGD was that these formalisms might be useless due to the extreme non-linearity and enormous vacuum degeneracy of the theory. This turned out to be the case.

- It took some years to discover that the only working approach is based on the generalization of Einstein's program. Quantum physics involves the geometrization of the infinite-dimensional "world of classical worlds" (WCW) identified as 3-dimensional surfaces. Still few years had to pass before I understood that general coordinate invariance leads to a more or less unique solution of the problem and in positive energyontology implies that space-time surfaces are analogous to Bohr orbits. This in positive energy ontology in which space-like 3-surface is basic object. It is not clear whether Bohr orbitology is necessary also in ZEO in which space-time surfaces connect space-like 3-surfaces at the light-like boundaries of causal diamond CD obtained as intersection of future and past directed light-cones (with CP_2 factor included). The reason is that the pair of 3-surfaces replaces the boundary conditions at single 3-surface involving also time derivatives. If one assumes Bohr orbitology then strong correlations between the 3-surfaces at the ends of CD follow. Still a couple of years and I discovered that quantum states of the Universe can be identified as classical spinor fields in WCW. Only quantum jump remains the genuinely quantal aspect of quantum physics.

- During these years TGD led to a rather profound generalization of the space-time concept. Quite general properties of the theory led to the notion of many-sheeted space-time with sheets representing physical subsystems of various sizes. At the beginning of 90s I became dimly aware of the importance of p-adic number fields and soon ended up with the idea that p-adic thermodynamics for a conformally invariant system allows to understand elementary particle massivation with amazingly few input assumptions. The attempts to understand p-adicity from basic principles led gradually to the vision about physics as a generalized number theory as an approach complementary to the physics as an infinite-dimensional spinor geometry of WCW approach. One of its elements was a generalization of the number concept obtained by fusing real numbers and various p-adic numbers along common rationals. The number theoretical trinity involves besides p-adic number fields also quaternions and octonions and the notion of infinite prime.

- TGD inspired theory of consciousness entered the scheme after 1995 as I started to write a book about consciousness. Gradually it became difficult to say where physics ends and consciousness theory begins since consciousness theory could be seen as a generalization of quantum measurement theory by identifying quantum jump as a moment of consciousness and by replacing the observer with the notion of self identified as a system which is conscious as long as it can avoid entanglement with environment. The somewhat cryptic statement "Everything is conscious and consciousness can be only lost" summarizes the basic philosophy neatly.

The idea about p-adic physics as physics of cognition and intentionality emerged also rather naturally and implies perhaps the most dramatic generalization of the space-time concept in which most points of p-adic space-time sheets are infinite in real sense and the projection to the real imbedding space consists of discrete set of points. One of the most fascinating outcomes was the observation that the entropy based on p-adic norm can be negative. This observation led to the vision that life can be regarded as something in the intersection of real and p-adic worlds. Negentropic entanglement has interpretation as a correlate for various positively colored aspects of conscious experience and means also the possibility of strongly correlated states stable under state function reduction and different from the conventional bound states and perhaps playing key role in the energy metabolism of living matter.

If one requires consistency of Negentropy Mazimization Pronciple with standard measurement theory, negentropic entanglement defined in terms of number theoretic negentropy is necessarily associated with a density matrix proportional to unit matrix and is maximal and is characterized by the dimension n of the unit matrix. Negentropy is positive and maximal for a p-adic unique prime dividing n.

- One of the latest threads in the evolution of ideas is not more than nine years old. Learning about the paper of Laurent Nottale about the possibility to identify planetary orbits as Bohr orbits with a gigantic value of gravitational Planck constant made once again possible to see the obvious. Dynamical quantized Planck constant is strongly suggested by quantum classical correspondence and the fact that space-time sheets identifiable as quantum coherence regions can have arbitrarily large sizes. Second motivation for the hierarchy of Planck constants comes from bio-electromagnetism suggesting that in living systems Planck constant could have large values making macroscopic quantum coherence possible. The interpretation of dark matter as a hierarchy of phases of ordinary matter characterized by the value of Planck constant is very natural.

  During summer 2010 several new insights about the mathematical structure and interpretation of TGD emerged. One of these insights was the realization that the postulated hierarchy of Planck constants might follow from the basic structure of quantum TGD. The point is that due to the extreme non-linearity of the classical action principle the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is one-to-many and the natural description of the situation is in terms of local singular covering spaces of the imbedding space. One could speak about effective value of Planck constant $h_{eff} = n \times h$ coming as a multiple of minimal value of Planck constant. Quite recently it became clear that the non-determinism of Kähler action is indeed the fundamental justification for the hierarchy: the integer n can be also interpreted as the integer characterizing the dimension of unit matrix characterizing negentropic entanglement made possible by the many-sheeted character of the space-time surface.

  Due to conformal invariance acting as gauge symmetry the n degenerate space-time sheets must be replaced with conformal equivalence classes of space-time sheets and conformal transformations correspond to quantum critical deformations leaving the ends of space-time surfaces invariant. Conformal invariance would be broken: only the sub-algebra for which conformal weights are divisible by n act as gauge symmetries. Thus deep connections between conformal invariance related to quantum criticality, hierarchy of Planck constants, negentropic entanglement, effective p-adic topology, and non-determinism of Kähler action perhaps reflecting p-adic non-determinism emerges.

  The implications of the hierarchy of Planck constants are extremely far reaching so that the significance of the reduction of this hierarchy to the basic mathematical structure distinguishing between TGD and competing theories cannot be under-estimated.

From the point of view of particle physics the ultimate goal is of course a practical construction recipe for the S-matrix of the theory. I have myself regarded this dream as quite too ambitious taking into account how far reaching re-structuring and generalization of the basic mathematical structure of quantum physics is required. It has indeed turned out that the dream about explicit formula is unrealistic before one has understood what happens in quantum jump. Symmetries and general physical principles have turned out to be the proper guide line here. To give some impressions about what is required some highlights are in order.

- With the emergence of ZEO the notion of S-matrix was replaced with M-matrix defined between positive and negative energy parts of zero energy states. M-matrix can be interpreted as a complex square root of density matrix representable as a diagonal and positive square root of density matrix and unitary S-matrix so that quantum theory in ZEO can be said to define a square root of thermodynamics at least formally. M-matrices in turn bombine to form the rows of unitary U-matrix defined between zero energy states.

- A decisive step was the strengthening of the General Coordinate Invariance to the requirement that the formulations of the theory in terms of light-like 3-surfaces identified as 3-surfaces at which the induced metric of space-time surfaces changes its signature and in terms of space-like 3-surfaces are equivalent. This means effective 2-dimensionality in the sense that partonic 2-surfaces defined as intersections of these two kinds of surfaces plus 4-D tangent space data at partonic 2-surfaces code for the physics. Quantum classical correspondence requires the coding of the quantum numbers characterizing quantum states assigned to the partonic 2-surfaces to the geometry of space-time surface. This is achieved by adding to the modified Dirac action a measurement interaction term assigned with light-like 3-surfaces.

- The replacement of strings with light-like 3-surfaces equivalent to space-like 3-surfaces means enormous generalization of the super conformal symmetries of string models. A further generalization of these symmetries to non-local Yangian symmetries generalizing the recently discovered Yangian symmetry of $\mathcal{N} = 4$ supersymmetric Yang-Mills theories is highly suggestive. Here the replacement of point like particles with partonic 2-surfaces means the replacement of conformal symmetry of Minkowski space with infinite-dimensional super-conformal algebras. Yangian symmetry provides also a further refinement to the notion of conserved quantum numbers allowing to define them for bound states using non-local energy conserved currents.

- A further attractive idea is that quantum TGD reduces to almost topological quantum field theory. This is possible if the Kähler action for the preferred extremals defining WCW Kähler function reduces to a 3-D boundary term. This takes place if the conserved currents are so called Beltrami fields with the defining property that the coordinates associated with flow lines extend to single global coordinate variable. This ansatz together with the weak form of electric-magnetic duality reduces the Kähler action to Chern-Simons term with the condition that the 3-surfaces are extremals of Chern-Simons action subject to the constraint force defined by the weak form of electric magnetic duality. It is the latter constraint which prevents the trivialization of the theory to a topological quantum field theory. Also the identification of the Kähler function of WCW as Dirac determinant finds support as well as the description of the scattering amplitudes in terms of braids with interpretation in terms of finite measurement resolution coded to the basic structure of the solutions of field equations.

- In standard QFT Feynman diagrams provide the description of scattering amplitudes. The beauty of Feynman diagrams is that they realize unitarity automatically via the so called Cutkosky rules. In contrast to Feynman's original beliefs, Feynman diagrams and virtual particles are taken only as a convenient mathematical tool in quantum field theories. QFT approach is however plagued by UV and IR divergences and one must keep mind open for the possibility that a genuine progress might mean opening of the black box of the virtual particle.

  In TGD framework this generalization of Feynman diagrams indeed emerges unavoidably. Light-like 3-surfaces replace the lines of Feynman diagrams and vertices are replaced by 2-D partonic 2-surfaces. The approximate localization of the nodes of induced spinor fields to 2-D string world sheets (and possibly also to partonic 2-surfaces) implies a stringy formulation of the theory analogous to stringy variant of twistor formalism with string world sheets having interpretation as 2-braids. In TGD framework fermionic variant of twistor Grassmann formalism leads to a stringy variant of twistor diagrammatics in which basic fermions can be said to be on mass-shell but carry non-physical helicities in the internal lines. This suggests the generalization of the Yangian symmetry to infinite-dimensional super-conformal algebras.

What I have said above is strongly biased view about the recent situation in quantum TGD. This vision is single man's view and doomed to contain unrealistic elements as I know from experience. My dream is that young critical readers could take this vision seriously enough to try to demonstrate that some of its basic premises are wrong or to develop an alternative based on these or better premises. I must be however honest and tell that 32 years of TGD is a really vast bundle of thoughts and quite a challenge for anyone who is not able to cheat himself by taking the attitude of a blind believer or a light-hearted debunker trusting on the power of easy rhetoric tricks.

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Matti Pitkänen

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Neither TGD nor these books would exist without the help and encouragement of many people. The friendship with Heikki and Raija Haila and their family and Kalevi and Ritva Tikkanen and their family have been kept me in contact with the everyday world and without this friendship I would not have survived through these lonely 45 lonely years most of which I have remained unemployed as a scientific dissident. I am happy that my children have understood my difficult position and like my friends have believed that what I am doing is something valuable although I have not received any official recognition for it.

During the last decade Tapio Tammi has helped me quite concretely by providing the necessary computer facilities and being one of the few persons in Finland with whom to discuss my work. Pertti Kärkkäinen is my old physicist friend and has provided continued economic support for a long time. I have also had stimulating discussions with Samuli Penttinen who has also helped to get through the economical situations in which there seemed to be no hope. The continual updating of fifteen online books means quite a heavy bureaucracy at the level of bits and without a systemization one ends up with endless copying and pasting and internal consistency is soon lost. Tommi Ullgren has provided both economic support and encouragement during years. Pekka Rapinoja has offered his help in this respect and I am especially grateful to him for my Python skills.

During the last five years I have had inspiring discussions with many people in Finland interested in TGD. We have had video discussions with Sini Kunnas and had podcast discussions with Marko Manninen related to the TGD based view of physics and consciousness. Marko has also helped in the practical issues related to computers and quite recently he has done a lot of testing of chatGPT helping me to get an overall view of what it is. The discussions in a Zoom group involving Marko Manninen, Tuomas Sorakivi and Rode Majakka have given me the valuable opportunity to clarify my thoughts.

The collaboration with Lian Sidorov was extremely fruitful and she also helped me to survive economically through the hardest years. The participation in CASYS conferences in Liege has been an important window to the academic world and I am grateful for Daniel Dubois and Peter Marcer for making this participation possible. The discussions and collaboration with Eduardo de Luna and Istvan Dienes stimulated the hope that the communication of new vision might not be a mission impossible after all. Also blog discussions have been very useful. During these years I have received innumerable email contacts from people around the world. I am grateful to Mark McWilliams, Paul Kirsch, Gary Ehlenberg, and Ulla Matfolk and many others for providing links to possibly interesting websites and articles. We have collaborated with Peter Gariaev and Reza Rastmanesh. These contacts have helped me to avoid the depressive feeling of being some kind of Don Quixote of Science and helped me to widen my views: I am grateful for all these people.

In the situation in which the conventional scientific communication channels are strictly closed it is important to have some loop hole through which the information about the work done can at least in principle leak to the public through the iron wall of academic censorship. Without any exaggeration I can say that without the world wide web I would not have survived as a scientist nor as an individual. Homepage and blog are however not enough since only the formally published result is a result in recent day science. Publishing is however impossible without direct support from power holders- even in archives like arXiv.org.

Situation changed as Andrew Adamatsky proposed the writing of a book about TGD when I had already gotten used to the thought that my work would not be published during my lifetime. The Prespacetime Journal and two other journals related to quantum biology and consciousness - all of them founded by Huping Hu - have provided this kind of loophole. In particular, Dainis Zeps, Phil Gibbs, and Arkadiusz Jadczyk deserve my gratitude for their kind help in the preparation of an article series about TGD catalyzing a considerable progress in the understanding of quantum TGD. Also the viXra archive founded by Phil Gibbs and its predecessor Archive Freedom have been of great help: Victor Christianto deserves special thanks for doing the hard work needed to run Archive Freedom. Also the Neuroquantology Journal founded by Sultan Tarlaci deserves a special mention for its publication policy.

And last but not least: there are people who experience as a fascinating intellectual challenge to spoil the practical working conditions of a person working with something which might be called unified theory: I am grateful for the people who have helped me to survive through the virus attacks, an activity which has taken roughly one month per year during the last half decade and given a strong hue of grey to my hair.

For a person approaching his 73th birthday it is somewhat easier to overcome the hard feelings due to the loss of academic human rights than for an inpatient youngster. Unfortunately the economic situation has become increasingly difficult during the twenty years after the economic depression in Finland which in practice meant that Finland ceased to be a constitutional state in the strong sense of the word. It became possible to depose people like me from society without fear about public reactions and the classification as dropout became a convenient tool of ridicule to circumvent the ethical issues. During the period when the right wing held political power this trend was steadily strengthening and the situation is the same as I am writing this. In this kind of situation the concrete help from individuals has been and will be of utmost importance. Against this background it becomes obvious that this kind of work is not possible without the support from outside and I apologize for not being able to mention all the people who have helped me during these years.

Karkkila, August 30, 2023, Finland

Matti Pitkänen

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Chapter 1

Introduction

1.1 Basic Ideas of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged towards the end of 1977 - would emerge now it would be seen as an attempt to solve the difficulties of these approaches to unification.

The basic physical picture behind the geometric vision of TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model. After 1995 number theoretic vision started to develop and was initiated by the success of mass calculations based on p-adic thermodynamics. Number theoretic vision involves all number fields and is complementary to the geometric vision: one can say that this duality is analogous to momentum-position duality of wave mechanics. TGD can be also regarded as topological quantum theory in a very general sense as already the attribute "Topological" in "TGD" makes clear. Space-time surfaces as minimal surfaces can be regarded as representatives of homology equivalence classes and p-adic topologies generalize the notion of local topology and apply to the description of correlates of cognition.

1.1.1 Geometric Vision Very Briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K3].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional embedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).

2. Induction procedure (a standard procedure in fiber bundle theory, see Appendix) allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections of CP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface. Spinor connection is also projected. The interpretation is that distances are measured in embedding space metric and parallel translation using spinor connection of embedding space.

Twistor lift of TGD means that one can lift space-time surfaces in H to 6-D surfaces a analogs of twistor space of space-time surface in the Cartesian product of the twistor spaces of M^4 and CP_2, which are the only 4-manifolds allowing twistor space with Kähler structure [A57]. The twistor structure would be induced in some sense, and should coincide with that associated with the induced metric. Clearly, the 2-spheres defining the fibers of twistor spaces of M^4 and CP_2 must allow identification: this 2-sphere defines the S^2 fiber of the twistor space of the space-time surface. This poses a constraint on the embedding of the twistor space of space-time surfaces as sub-manifold in the Cartesian product of twistor spaces. The existence of Kähler structure allows to lift 4-D Kähler action to its 6-D counterparts and the 6-D counterpart of twistor space is obtained by its dimensional reduction so that one obtains a sphere bundle. This makes possible twistorialization for all space-time surfaces: in general relativity the general metric does not allow this.

3. A geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from the standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

   M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of embedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field-like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

   Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particles in space-time can be identified as a topological inhomogeneities in background space-time surface which looks like the space-time of general relativity in long length scales.

   One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distances of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

   This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which the standard model and general relativity follow as a topological simplification, however forcing a dramatic increase of the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. The basic problem is that one has long ranged classical electroweak gauge fields. The resolution of the problem is that the quantum averages of induced weak and color gauge fields vanish due to the fact that color rotations affect both space-time surfaces and induced weak and color fields. Only the averages of

electromagnetic fields are nonvanishing. The correlations functions for weak fields are nonvanishing below Compton lengths of weak bosons. In living matter large values of effective Planck constant labelling phases of ordinary matter identified as dark matter make possible long ranged weak fields and color fields.

6. General coordinate invariance requires holography so that space-time surfaces are analogous to Bohr orbits for particles identified as 3-surfaces. Bohr orbit property would be naturally realized by a 4-D generalization of holomorphy of string world sheets and implies that the space-time surfaces are minimal surfaces apart from singularities. This holds true for any action as long as it is general coordinate invariant and constructible in terms of the induced geometry. String world sheets and light-like orbits of partonic 2-surfaces correspond to singularities at which the minimal surface property of the space-time surfaces realizing the preferred extremal property fails. Preferred extremals are not completely deterministic, which implies what I call zero energy ontology (ZEO) meaning that the Bohr orbits are the fundamental objects. This leads to a solution of the basic paradox of quantum measurement theory. Also the mathematically ill-defined path integral disappears and leaves only the well-defined functional integral over the Bohr orbits.

7. A string model-like picture emerges from TGD and one ends up with a rather concrete view about the topological counterpart of Feynman diagrammatics. The natural stringy action would be given by the string world sheet area, which is present only in the space-time regions with Minkowskian signature. Gravitational constant could be present as a fundamental constant in string action and the ratio $\hbar/G/R^2$ would be determined by quantum criticality conditions. The hierarchy of Planck constants $h_{eff}/h = n$ assigned to dark matter in TGD framework would allow to circumvent the objection that only objects of length of order Planck length are possible since string tension given by $T = 1/\hbar_{eff}G$ apart from numerical factor could be arbitrary small. This would make possible gravitational bound states as partonic 2-surfaces as structures connected by strings and solve the basic problem of superstring theories. This option allows the natural interpretation of M^4 type vacuum extremals with CP_2 projection, which is Lagrange manifold as good approximations for space-time sheets at macroscopic length scales. String area does not contribute to the Kähler function at all.

   Whether induced spinor fields associated with Kähler-Dirac action and de-localized inside the entire space-time surface should be allowed remains an open question: super-conformal symmetry strongly suggests their presence. A possible interpretation for the corresponding spinor modes could be in terms of dark matter, sparticles, and hierarchy of Planck constants.

It is perhaps useful to make clear what TGD is not and also what new TGD can give to physics.

1. TGD is *not* just General Relativity made concrete by using embeddings: the 4-surface property is absolutely essential for unifying standard model physics with gravitation and to circumvent the incurable conceptual problems of General Relativity. The many-sheeted space-time of TGD gives rise only at the macroscopic limit to GRT space-time as a slightly curved Minkowski space. TGD is *not* a Kaluza-Klein theory although color gauge potentials are analogous to gauge potentials in these theories.

   TGD space-time is 4-D and its dimension is due to completely unique conformal properties of light-cone boundary and 3-D light-like surfaces implying enormous extension of the ordinary conformal symmetries. Light-like 3-surfaces represent orbits of partonic 2-surfaces and carry fundamental fermions at 1-D boundaries of string world sheets. TGD is *not* obtained by performing Poincare gauging of space-time to introduce gravitation and is plagued by profound conceptual problems.

2. TGD is *not* a particular string model although string world sheets emerge in TGD very naturally as loci for spinor modes: their 2-dimensionality makes among other things possible quantum deformation of quantization known to be physically realized in condensed matter, and conjectured in TGD framework to be crucial for understanding the notion of finite measurement resolution. Hierarchy of objects of dimension up to 4 emerge from TGD: this obviously means analogy with branes of super-string models.

TGD is *not* one more item in the collection of string models of quantum gravitation relying on Planck length mystics. Dark matter becomes an essential element of quantum gravitation and quantum coherence in astrophysical scales is predicted just from the assumption that strings connecting partonic 2-surfaces are responsible for gravitational bound states.

TGD is *not* a particular string model although AdS/CFT duality of super-string models generalizes due to the huge extension of conformal symmetries and by the identification of WCW gamma matrices as Noether super-charges of super-symplectic algebra having a natural conformal structure.

3. TGD is *not* a gauge theory. In TGD framework the counterparts of also ordinary gauge symmetries are assigned to super-symplectic algebra (and its Yangian [A17] [B21, B15, B16]), which is a generalization of Kac-Moody algebras rather than gauge algebra and suffers a fractal hierarchy of symmetry breakings defining hierarchy of criticalities. TGD is *not* one more quantum field theory like structure based on path integral formalism: path integral is replaced with functional integral over 3-surfaces, and the notion of classical space-time becomes an exact part of the theory. Quantum theory becomes formally a purely classical theory of WCW spinor fields: only state function reduction is something genuinely quantal.

4. TGD view about spinor fields is *not* the standard one. Spinor fields appear at three levels. Spinor modes of the embedding space are analogs of spinor modes characterizing incoming and outgoing states in quantum field theories. Induced second quantized spinor fields at space-time level are analogs of stringy spinor fields. Their modes are localized by the well-definedness of electro-magnetic charge and by number theoretic arguments at string world sheets. Kähler-Dirac action is fixed by supersymmetry implying that ordinary gamma matrices are replaced by what I call Kähler-Dirac gamma matrices - this something new. WCW spinor fields, which are classical in the sense that they are not second quantized, serve as analogs of fields of string field theory and imply a geometrization of quantum theory.

5. TGD is in some sense an extremely conservative geometrization of entire quantum physics: *no* additional structures such as gauge fields as independent dynamical degrees of freedom are introduced: Kähler geometry and associated spinor structure are enough. "Topological" in TGD should not be understood as an attempt to reduce physics to torsion (see for instance [B14]) or something similar. Rather, TGD space-time is topologically non-trivial in all scales and even the visible structures of the everyday world represent non-trivial topology of space-time in the TGD Universe.

6. Twistor space - or rather, a generalization of twistor approach replacing masslessness in 4-D sense with masslessness in 8-D sense and thus allowing description of also massive particles - emerged originally as a technical tool, and its Kähler structure is possible only for $H = M^4 \times CP_2$. It however turned out that much more than a technical tool is in question. What is genuinely new is the infinite-dimensional character of the Kähler geometry making it highly unique, and its generalization to p-adic number fields to describe correlates of cognition. Also the hierarchy of Planck constants $h_{eff} = n \times h$ reduces to the quantum criticality of the TGD Universe and p-adic length scales and Zero Energy Ontology represent something genuinely new.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last 45 years to the realization of this dream and this has resulted in 26 online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

A collection of 30 online books is now (August 2023) under preparation. The goal is to minimize overlap between the topics of the books and make the focus of a given book sharper.

1.1.2 Two Visions About TGD as Geometrization of Physics and Their Fusion

As already mentioned, TGD as a geometrization of physics can be interpreted both as a modification of general relativity and generalization of string models.

TGD as a Poincare Invariant Theory of Gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_\times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A43, A56, A31, A50].

The identification of the space-time as a sub-manifold [A44, A80] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

TGD as a Generalization of the Hadronic String Model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Quite recently, it has turned out that fermionic strings inside space-time surfaces define an exact part of quantum TGD and that this is essential for understanding gravitation in long length scales. Also the analog of AdS/CFT duality emerges in that the Kähler metric can be defined either in terms of Kähler function identifiable as Kähler action assignable to Euclidian space-time regions or Kähler action + string action assignable to Minkowskian regions.

The recent view about construction of scattering amplitudes is very "stringy". By strong form of holography string world sheets and partonic 2-surfaces provide the data needed to construct scattering amplitudes. Space-time surfaces are however needed to realize quantum-classical correspondence necessary to understand the classical correlates of quantum measurement. There is a huge generalization of the duality symmetry of hadronic string models.

The proposal is that scattering amplitudes can be regarded as sequences of computational operations for the Yangian of super-symplectic algebra. Product and co-product define the basic vertices and realized geometrically as partonic 2-surfaces and algebraically as multiplication for the elements of Yangian identified as super-symplectic Noether charges assignable to strings. Any computational sequences connecting given collections of algebraic objects at the opposite boundaries of causal diamond (CD) produce identical scattering amplitudes.

Fusion of the Two Approaches via a Generalization of the Space-Time Concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there could be "vapor phase" that is a "gas" of particle like 3-surfaces and string like objects (counterpart of the "baby universes" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possible existence vapour phase.

. What one obtains is what I have christened as many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. ??** in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory the physical system does not possess this kind of field identity. The notion of the magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology. The existence of monopole flux tubes requiring no current as a source of the magnetic field makes it possible to understand the existence of magnetic fields in cosmological and astrophysical scales.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces and identifiable as analogs of Bohr orbits. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification of the lines of generalized Feynman diagrams is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian . Also the Euclidian 4-D regions can have a similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about a strong form of holography.

The understanding of the super symplectic invariance leads to the proposal that super symplectic algebra and other Kac-Moody type algebras labelled by non-negative multiples of basic conformal weights allow a hierarchy of symmetry breakings in which the analog of gauge symmetry breaks down to a genuine dynamical symmetry. This gives rise to fractal hierarchies of algebras and symmetry breakings. This breaking can occur also for ordinary conformal algebras if one restricts the conformal weights to be non-negative integers.

1.1.3 Basic Objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four embedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particles topologically condense to several space-time sheets simultaneously and experience the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the embeddability to 8-D embedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation makes it possible to understand the relationship to GRT space-time and how the Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of the space-time sheets from Minkowski metric. Poincare invariance strongly suggests classical EP for the GRT limit in long length scales at least. One can also consider other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with a very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of the standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Topological Field Quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other thins this leads to models for cell membrane, nerve pulse, and EEG.

1.1.4 Quantum TGD as Spinor Geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones.

World of Classical Worlds

The notion of WCW reduces the interacting quantum theory to a theory of free WCW spinor fields.

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude identified as WCW spinor in the configuration space CH ("world of classical worlds", WCW) consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included.

2. 4-D general coordinate invariance forces holography and replaces the ill-defined path integral over all space-time surfaces with a discrete sum over 4-D analogs of Bohr orbits for particles identified as 3-surfaces. Holography means that basic objects are these analogs of Bohr orbits. Since there is no quantization at the level of WCW, one has an analog of wave mechanics with point-like particles replaced with 4-D Bohr orbits.

3. One must geometrize WCW as the space of Bohr orbits. In an infinite-dimensional situation the existence of geometry requires maximal symmetries already in the case of loop spaces. Physics is unique from its mathematical existence.

   WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operators, appearing in the field equations of the theory [1]

Identification of Kähler function

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

It took long time to realize that there is no discretization in 4-D sense - this would lead to difficulties with basic symmetries. Rather, the discretization occurs for the parameters characterizing co-dimension 2 objects representing the information about space-time surface so that they belong to some algebraic extension of rationals. These 2-surfaces - string world sheets and partonic 2-surfaces - are genuine physical objects rather than a computational approximation. Physics itself approximates itself, one might say! This is of course nothing but strong form of holography.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factorc coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

   Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The way to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this way almost topological QFT results. But only "almost" since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

WCW spinor fields

Classical WCW spinor fields are analogous to Schrödinger amplitudes and the construction of WCW Kähler geometry reduces to the second quantization of free spinor fields of H.

[1] There are four kinds of Dirac operators in TGD. The geometrization of quantum theory requires Kähler metric definable either in terms of Kähler function identified as a the bosonic action for Euclidian space-time regions or as anti-commutators for WCW gamma matrices identified as conformal Noether super-charges associated with the second quantized modified Dirac action consisting of string world sheet term and possibly also modified Dirac action in Minkowskian space-time regions. These two possible definitions reflect a duality analogous to AdS/CFT duality.

1. The WCW metric is given by anticommutators of WCW gamma matrices which also have interpretation as supercharges assignable to the generators of WCW isometries and allowing expression as non-conserved Noether charges. Holography implies zero energy ontology (ZEO) meaning that zero energy states are superpositions of Bohr orbits connecting boundaries of causal diamond (CD). CDs form a fractal hierarchy and their space forming the spine of WCW is finite-dimensional and can be geometrized. The alternative interpretation is as a superposition of pairs of ordinary 3-D fermionic states assignable to the ends of the space-time surfaces.

2. There are several Dirac operators. WCW Dirac operator D_{WCW} appears in Super-symplectic gauge conditions analogous to Super Virasoro conditions. The algebraic variant of the H Dirac operator D_H appears in fermionic correlation functions: this is due to the fact that free fermions appearing as building bricks of WCW gamma matrices are modes of D_H. The modes of $_DH$ define the ground states of super-symplectic representations. There is also the modified Dirac operator D_{X^4} acting on the induced spinors at space-time surfaces and it is dictated by symmetry one the action fixing the space-time surfaces as Bohr orbits is fixed. D_H is needed since it determines the expressions of WCW gamma matrices as Noether charges assignable to 3-surfaces at the ends of WCW.

The role of modified Dirac action

1. By quantum classical correspondence, the construction of WCW spinor structure in sectors assignable to CDs reduces to the second quantization of the induced spinor fields of H. The basic action is so called modified Dirac action in which gamma matrices are replaced with the modified) gamma matrices defined as contractions of the canonical momentum currents of the bosonic action defining the space-time surfaces with the embedding space gamma matrices. In this way one achieves super-conformal symmetry and conservation of fermionic currents among other things and a consistent Dirac equation.

   Modified Dirac action is needed to define WCW gamma matrices as super charges assignable to WCW isometry generators identified as generators of symplectic transformations and by holography are needed only at the 3-surface at the boundaries of WCW. It is important to notice that the modified Dirac equation does not determine propagators since induced spinor fields are obtained from free second quantized spinor fields of H. This means enormous simplification and makes the theory calculable.

2. An important interpretational problem relates to the notion of the induced spinor connection. The presence of classical W boson fields is in conflict with the classical conservation of em charge since the coupling to classical W fields changes em charge.

   One way out of the problem is the fact that the quantum averages of weak and gluon fields vanish unlike the quantum average of the em field. This leads to a rather precise understanding of electroweak symmetry breaking as being due the fact that color symmetries rotate space-time surfaces and also affect the induced weak fields.

   One can also consider a stronger condition. If one requires that the spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrinos generating super-symmetries forms an exception. The vanishing of the Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that the string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

   The localization would simplify the mathematics enormously and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

   At the light-like 3-surfaces the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes. One can pose the condition that the algebraic analog of the massless Dirac equation is satisfied by the modes of the modified-Dirac action assignable to the Chern-Simons-Kähler action.

1.1.5 Construction of scattering amplitudes

Reduction of particle reactions to space-time topology

Particle reactions are identified as topology changes [A65, A86, A100]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \to B + C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

During years this naïve and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects un-expected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "configuration space". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

Construction of the counterparts of S-matrices

What does one mean with the counterpart of S-matrix in the TGD framework has been a long standing problem. The development of ZEO based quantum measurement theory has led to a rough overall view of the situation.

1. There are two kinds of state function reductions (SFRs). "Small" SFRs (SSFRs) following the TGD counterpart of a unitary time evolution defines a sequence of SFRs, which is analogous to a sequence of repeated quantum measurements associated with the Zeno effect. In wave mechanics nothing happens in these measurements. In quantum optics these measurements correspond to weak measurements. In TGD SSFR affects the zero energy state but leaves the 3-D state at the passive boundary of CD unaffected.

2. In TGD framework each SSFR is preceded by a counterpart of a unitary time evolution, which means dispersion in the space of CDs and unitary time evolution in fermionic degrees of freedom such that the passive boundary of CDs and 3-D states at it are unaffected but a superposition of CDs with varying active boundaries in the space of CDs is formed. In SSFR a localization in the space of CDs occurs such that the active is fixed. In a statistical sense the size of the CD increases and the increasing distance between the tips of the CD gives rise to the arrow of geometric time.

3. Also "big" SFRS (BSFRs) can occur and they correspond to ordinary SFRs. In BSFR the roles of the active and passive boundary are changed and this means that the arrow of time is changed. Big SFR occurs when the SSFR corresponds to a quantum measurement, which does not commute with the operators, which define the states at the passive boundary of CD as their eigenstates. This means a radical deviation from standard quantum measurement theory and has predictions in all scales.

4. One can assign the counterpart of S-matrix to the unitary time evolution between two subsequent SSFRs and also to the counterpart of S-matrix associated with BSFR. At least in the latter case the dimension of the state space can increase since at least BSFRs lead to the increase of the dimension of algebraic extension of rationals assignable to the space-time surface by $M^8 - H$ duality. Unitarity is therefore replaced with isometry.

5. I have also considered the possibility that unitary S-matrix could be replaced in the fermionic degrees of freedom with Kähler metric of the state space satisfying analogs of unitarity conditions but it seems that this is un-necessary and also too outlandish an idea.

The notion of M-matrix

1. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operators associated with WCWs associated with the spaces of CDs with fixed passive boundary: this would define an S-matrix assignable to SFR. Also the analog of S-matrix for the localizations of the states to the active boundary assignable to the BSFR changing the state at the passive boundary of CD is needed.

2. If one allows entanglement between positive and energy parts of the zero energy state but assumes that the states at the passive boundary are fixed, one must introduce the counterpart of the density matrix, or rather its square root. This classical free field theory would dictate what I have called M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. A biven M-matrix in turn would decompose to a product of a hermitian square root of density matrix and unitary S-matrix.

3. M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in a well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the M-matrices commuting with S-matrix means that they span infinite-dimensional Lie algebras acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in a well-defined sense.

4. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible and would correspond to a hierarchy of CDs with the temporal distances between tips coming as integer multiples of the CP_2 time.

   The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer n are naturally proportional to a representation matrix of scaling: $S(n) = S^n$, where S is unitary S-matrix associated with the minimal CD [K76]. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of S and represented as scaling with respect to the logarithm of the proper time distance between the tips of CD.

5. I have also considered the notion of U-matrix. U-matrix elements between M-matrices for various CDs are proportional to the inner products $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2} \lambda]$, where λ represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and H^i form an orthonormal basis of Hermitian square roots of density matrices. $\circ$ tells that S acts at the active boundary of CD only. I have proposed a general representation for the U-matrix, reducing its construction to that of the S-matrix.

1.1.6 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space ("world of classical worlds", WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name "TGD as a generalized number theory". It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of complexified counterparts of classical number fields, and the notion of infinite prime. Note that one can identify subrings such as hyper-quaternions and hyper-octonions as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product.

The Threads in the Development of Quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

2. The discussions with Tony Smith initiated a fourth thread which deserves the name "TGD as a generalized number theory". The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and rather fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the "physics as generalized number theory" thread.

3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called "physics as a generalized number theory". In the following I adopt this view. This reduces the number of threads to three corresponding to geometric, number theoretic and topological views of physics.

TGD forces the generalization of physics to a quantum theory of consciousness, and TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations.

Number theoretic vision very briefly

Number theoretic vision about quantum TGD involves notions like adelic physics, M^8-H duality and number theoretic universality. A short review of the basic ideas that have developed during years is in order.

1. The physical interpretation of M^8 is as an analog of momentum space and M^8-H duality is analogous to momentum-position duality of ordinary wave mechanics.

2. Adelic physics means that all classical number fields, all p-adic number fields and their extensions induced by extensions of rationals and defining adeles, and also finite number fields are basic mathematical building bricks of physics.

   The complexification of M^8, identified as complexified octonions, would provide a realization of this picture and M^8-H duality would map the algebraic physics in M^8 to the ordinary physics in $M^4\times CP_2$ described in terms of partial differential equations.

3. Negentropy Maximization Principle (NMP) states that the conscious information assignable with cognition representable measured in terms of p-adic negentropy increases in statistical sense.

   NMP is mathematically completely analogous to the second law of thermodynamics and number theoretic evolution as an unavoidable statistical increase of the dimension of the algebraic extension of rationals characterizing a given space-time region implies it. There is no paradox involved: the p-adic negentropy measures the conscious information assignable to the entanglement of two systems regarded as a conscious entity whereas ordinary entropy measures the lack of information about the quantums state of either entangled system.

4. Number theoretical universality requires that space-time surfaces or at least their M^8-H duals in M^8_c are defined for both reals and various p-adic number fields. This is true if they are defined by polynomials with integer coefficients as surfaces in M^8 obeying number theoretic holography realized as associativity of the normal space of 4-D surface using as holographic data 3-surfaces at mass shells identified in terms of roots of a polynomial. A physically motivated additional condition is that the coefficients of the polynomials are smaller than their degrees.

5. Galois confinement is a key piece of the number theoretic vision. It states that the momenta of physical states are algebraic integers in the extensions of rationals assignable to the space-time region considered. These numbers are in general complex and are not consistent with particle in box quantization. The proposal is that physical states satisfy Galois confinement being thus Galois singlets and having therefore total momenta, whose components are ordinary integers, when momentum unit defined by the scale of causal diamond (CD) is used.

6. The notion of p-adic prime was introduced in p-adic mass calculations that started the developments around 1995. p-Adic length scale hypothesis states that p-adic primes near powers of 2 have a special physical role (as possibly also the powers of other small primes such as $p=3$).

   The proposal is that p-adic primes correspond to ramified primes assignable to the extension and identified as divisors of the polynomial defined by the products of the root differences for the roots of the polynomial defining space-time space and having interpretation as values of, in general complex, virtual mass squared.

p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired "Universe as Computer" vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduces the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.
2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structure. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of embedding space and space-time concept and one can speak about real and p-adic space-time sheets. One can talk about adelic space-time, embedding space, and WCW.

The corresponds of real 4-surfaces with the p-adic ones is induced by number theoretical discretization using points of 4-surfaces $Y^4 \subset M^8_c$ identifiable as 8-momenta, whose components are assumed to be algebraic integers in an extension of rationals defined by the extension of rationals associated with a polynomial P with integer coefficients smaller than the degree of P. These points define a cognitive representation, which is universal in the sense that it exists also in the algebraic extensions of p-adic numbers. The points of the cognitive representations associated with the mass shells with mass squared values identified as roots of P are enough since $M^8 - H$ duality can be used at both M^8 and H sides and also in the p-adic context. The mass shells are special in that they allow for Minkowski coordinates very large cognitive representations unlike the interiors of the 4-surfaces determined by holography by using the data defined by the 3-surfaces at the mass shells. The higher the dimension of the algebraic extension associated with P, the better the accuracy of the cognitive representation.

Adelization providing number theoretical universality reduces to algebraic continuation for the amplitudes from this intersection of reality and various p-adicities - analogous to a back of a book - to various number fields. There are no problems with symmetries but canonical identification is needed: various group invariant of the amplitude are mapped by canonical identification to various p-adic number fields. This is nothing but a generalization of the mapping of the p-adic mass squared to its real counterpart in p-adic mass calculations.

This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book). One can also understand how preferred p-adic primes could emerge as so called ramified primes of algebraic extension of rationals in question and characterizing string world sheets and partonic 2-surfaces. Preferred p-adic primes would be ramified primes for extensions for which the number of p-adic continuations of two-surfaces to space-time surfaces (imaginations) allowing also real continuation (realization of imagination) would be especially large. These ramifications would be winners in the fight for number theoretical survival. Also a generalization of p-adic length scale hypothesis emerges from NMP [K72].

The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to “mind stuff”, the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of complexified quaternions and octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

The infinite primes at the first level of hierarchy, which represent analogs of bound states, can be mapped to irreducible polynomials, which in turn characterize the algebraic extensions of rationals defining a hierarchy of algebraic physics continuable to real and p-adic number fields. The products of infinite primes in turn define more general algebraic extensions of rationals. The interesting question concerns the physical interpretation of the higher levels in the hierarchy of infinite primes and integers mappable to polynomials of $n > 1$ variables.

1.1.7 An explicit formula for $M^8 - H$ duality

$M^8 - H$ duality is a generalization of momentum-position duality relating the number theoretic and geometric views of physics in TGD and, despite that it still involves poorly understood aspects, it has become a fundamental building block of TGD. One has 4-D surfaces $Y^4 \subset M^8_c$, where M^8_c is complexified M^8 having interpretation as an analog of complex momentum space and 4-D spacetime surfaces $X^4 \subset H = M^4 \times CP_2$. M^8_c, equivalently E^8_c, can be regarded as complexified octonions. M^8_c has a subspace M^4_c containing M^4.

Comment: One should be very cautious with the meaning of "complex". Complexified octonions involve a complex imaginary unit i commuting with the octonionic imaginary units I_k. i is assumed to also appear as an imaginary unit also in complex algebraic numbers defined by the roots of polynomials P defining holographic data in M^8_c.

In the following $M^8 - H$ duality and its twistor lift are discussed and an explicit formula for the dualities are deduced. Also possible variants of the duality are discussed.

Holography in H

$X^4 \subset H$ satisfies holography and is analogous to the Bohr orbit of a particle identified as a 3-surface. The proposal is that holography reduces to a 4-D generalization of holomorphy so that X^4 is a simultaneous zero of two functions of complex CP_2 coordinates and of what I have called Hamilton-Jacobi coordinates of M^4 with a generalized Kähler structure.

The simplest choice of the Hamilton-Jacobi coordinates is defined by the decomposition $M^4 = M^2 \times E^2$, where M^2 is endowed with hypercomplex structure defined by light-like coordinates (u, v), which are analogous to z and $\overline{z}$. Any analytic map $u \to f(u)$ defines a new set of light-like coordinates and corresponds to a solution of the massless d'Alembert equation in M^2. E^2 has some complex coordinates with imaginary unit defined by i.

The conjecture is that also more general Hamilton-Jacobi structures for which the tangent space decomposition is local are possible. Therefore one would have $M^4 = M^2(x) \times E^2(x)$. These would correspond to non-equivalent complex and Kähler structures of M^4 analogous to those possessed by 2-D Riemann surfaces and parametrized by moduli space.

Number theoretic holography in M^8_c

$Y^4 \subset M^8_c$ satisfies number theoretic holography defining dynamics, which should reduce to associativity in some sense. The Euclidian complexified normal space $N^4(y)$ at a given point y of Y^4 is required to be associative, i.e. quaternionic. Besides this, $N^4(i)$ contains a preferred complex Euclidian 2-D subspace $Y^2(y)$. Also the spaces $Y^2(x)$ define an integrable distribution. I have assumed that $Y^2(x)$ can depend on the point y of Y^4.

These assumptions imply that the normal space $N(y)$ of Y^4 can be parameterized by a point of $CP_2 = SU(3)/U(2)$. This distribution is always integrable unlike quaternionic tangent space distributions. $M^8 - H$ duality assigns to the normal space $N(y)$ a point of CP_2. M^4_c point y is mapped to a point $x \in M^4 \subset M^4 \times CP_2$ defined by the real part of its inversion (conformal transformation): this formula involves effective Planck constant for dimensional reasons.

The 3-D holographic data, which partially fixes 4-surfaces Y^4 is partially determined by a polynomial P with real integer coefficients smaller than the degree of P. The roots define mass squared values which are in general complex algebraic numbers and define complex analogs of mass shells in $M^4_c \subset M^8_c$, which are analogs of hyperbolic spaces H^3. The 3-surfaces at these mass shells define 3-D holographic data continued to a surface Y^4 by requiring that the normal space of Y^4 is associative, i.e. quaternionic. These 3-surfaces are not completely fixed but an interesting conjecture is that they correspond to fundamental domains of tessellations of H^3.

What does the complexity of the mass shells mean? The simplest interpretation is that the space-like M^4 coordinates (3-momentum components) are real whereas the time-like coordinate (energy) is complex and determined by the mass shell condition. One would have $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3 when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as a time coordinate. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to an equation of mass shell when $\sqrt{(Re(E)^2 - Im(E)^2)}$, expressed in terms of $Re(E)$, is taken as new energy coordinate $E_{eff} = \sqrt{(Re(E)^2 - Im(E)^2)}$. Is this deformation of H^3 in imaginary time direction equivalent with a region of the hyperbolic 3-space H^3?

One can look at the formula in more detail. Mass shell condition gives $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3, when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as an effective energy. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to a dispersion relation for $Re(E)^2$.

$$Re(E)^2 = \frac{1}{2}(Re(m^2) - Im(m^2) + p^2)(1 \pm \sqrt{1 + \frac{2Im(m^2)^2}{(Re(m^2) - Im(m^2) + p^2)^2}} . \quad (1.1.1)$$

Only the positive root gives a non-tachyonic result for $Re(m^2) - Im(m^2) > 0$. For real roots with $Im(m^2) = 0$ and at the high momentum limit the formula coincides with the standard formula. For $Re(m^2) = Im(m^2)$ one obtains $Re(E)^2 \to Im(m^2)/\sqrt{2}$ at the low momentum limit $p^2 \to 0$. Energy does not depend on momentum at all: the situation resembles that for plasma waves.

Can one find an explicit formula for $M^8 - H$ duality?

The dream is an explicit formula for the $M^8 - H$ duality mapping $Y^4 \subset M^8_c$ to $X^4 \subset H$. This formula should be consistent with the assumption that the generalized holomorphy holds true for X^4.

The following proposal is a more detailed variant of the earlier proposal for which Y^4 is determined by a map g of $M^4_c \to SU(3)_c \subset G_{2,c}$, where $G_{2,c}$ is the complexified automorphism group of octonions and $SU(3)_c$ is interpreted as a complexified color group.

This map defines a trivial $SU(3)_c$ gauge field. The real part of g however defines a non-trivial real color gauge field by the non-linearity of the non-abelian gauge field with respect to the gauge potential. The quadratic terms involving the imaginary part of the gauge potential give an additional condition to the real part in the complex situation and cancel it. If only the real part of g contributes, this contribution would be absent and the gauge field is non-vanishing.

How could the automorphism $g(x) \subset SU(3) \subset G_2$ give rise to $M^8 - H$ duality?

1. The interpretation is that $g(y)$ at given point y of Y^4 relates the normal space at y to a fixed quaternionic/associative normal space at point y_0, which corresponds is fixed by some subgroup $U(2)_0 \subset SU(3)$. The automorphism property of g guarantees that the normal space is quaternionic/associative at y. This simplifies the construction dramatically.
2. The quaternionic normal sub-space (which has Euclidian signature) contains a complex sub-space which corresponds to a point of sphere $S^2 = SO(3)/O(2)$, where $SO(3)$ is the quaternionic automorphism group. The interpretation could be in terms of a selection of spin quantization axes. The local choice of the preferred complex plane would not be unique and is analogous to the possibility of having non-trivial Hamilton Jacobi structures in M^4 characterized by the choice of $M^2(x)$ and equivalently its normal subspace $E^2(x)$.

   These two structures are independent apart from dependencies forced by the number theoretic dynamics. Hamilton-Jacobi structure means a selection of the quantization axis of spin and energy by fixing a distribution of light-like tangent vectors of M^4 and the choice of the quaternionic normal sub-space fixes a choice of preferred quaternionic imaginary unit defining a quantization axis of the weak isospin.
3. The real part $Re(g(y))$ defines a point of $SU(3)$ and the bundle projection $SU(3) \rightarrow CP_2$ in turn defines a point of $CP_2 = SU(3)/U(2)$. Hence one can assign to g a point of CP_2 as $M^8 - H$ duality requires and deduce an explicit formula for the point. This means a realization of the dream.
4. The construction requires a fixing of a quaternionic normal space N_0 at y_0 containing a preferred complex subspace at a single point of Y^4 plus a selection of the function g. If M^4 coordinates are possible for Y^4, the first guess is that g as a function of complexified M^4 coordinates obeys generalized holomorphy with respect to complexified M^4 coordinates in the same sense and in the case of X^4. This might guarantee that the $M^8 - H$ image of Y^4 satisfies the generalized holomorphy.
5. Also space-time surfaces X^4 with M^4 projection having a dimension smaller than 4 are allowed. I have proposed that they might correspond to singular cases for the above formula: a kind of blow-up would be involved. One can also consider a more general definition of Y^4 allowing it to have a M^4 projection with dimension smaller than 4 (say cosmic strings). Could one have implicit equations for the surface Y^4 in terms of the complex coordinates of $SU(3)_c$ and M^4? Could this give for instance cosmic strings with a 2-D M^4 projection and CP_2 type extremals with 4-D CP_2 projection and 1-D light-like M^4 projection?

What could the number theoretic holography mean physically?

What could be physical meaning of the number theoretic holography? The condition that has been assumed is that the CP_2 coordinates at the mass shells of $M^4_c \subset M^8_c$ mapped to mass shells H^3 of $M^4 \subset M^4 \times CP_2$ are constant at the H^3. This is true if the $g(y)$ defines the same CP_2 point for a given component X^3_i of the 3-surface at a given mass shell. g is therefore fixed apart from a local $U(2)$ transformation leaving the CP_2 point invariant. A stronger condition would be that the CP_2 point is the same for each component of X^3_i and even at each mass shell but this condition seems to be unnecessarily strong.

Comment: One can o criticize this condition as too strong and one can consider giving up this condition. The motivation for this condition is that the number of algebraic points at the 3-surfaces associated with H^3 explodes since the coordinates associated with normal directions vanish. Kind of cognitive explosion would be in question.

$SU(3)$ corresponds to a subgroup of G_2 and one can wonder what the fixing of this subgroup could mean physically. G_2 is 14-D and the coset space $G_2/SU(3)$ is 6-D and a good guess is that it is just the 6-D twistor space $SU(3)/U(1) \times U(1)$ of CP_2: at least the isometries are the same.

The fixing of the $SU(3)$ subgroup means fixing of a CP_2 twistor. Physically this means the fixing of the quantization axis of color isospin and hypercharge.

Twistor lift of the holography

What is interesting is that by replacing $SU(3)$ with G_2, one obtains an explicit formula form the generalization of M^8-H duality to that for the twistorial lift of TGD!

One can also consider a twistorial generalization of the above proposal for the number theoretic holography by allowing local G_2 automorphisms interpreted as local choices of the color quantization axis. G_2 elements would be fixed apart from a local $SU(3)$ transformation at the components of 3-surfaces at mass shells. The choice of the color quantization axes for a connected 3-surface at a given mass shell would be the same everywhere. This choice is indeed very natural physically since 3-surface corresponds to a particle.

Is this proposal consistent with the boundary condition of the number theoretical holography mean in the case of 4-surfaces in M^8_c and $M^4\times CP_2$?

1. The selection of $SU(3)\subset G_2$ for ordinary M^8-H duality means that the $G_{2,c}$ gauge field vanishes everywhere and the choice of color quantization axis is the same at all points of the 4-surface. The fixing of the CP_2 point to be constant at H^3 implies that the color gauge field at $H^3\subset M^8_c$ and its image $H^3\subset H$ vanish. One would have color confinement at the mass shells H^3_i, where the observations are made. Is this condition too strong?
2. The constancy of the G_2 element at mass shells makes sense physically and means a fixed color quantization axis. The selection of a fixed $SU(3)\subset G_2$ for entire space-time surface is in conflict with the non-constancy of G_2 element unless G_2 element differs at different points of 4-surface only by a multiplication of a local $SU(3)_0$ element, that is local $SU(3)$ transformation. This kind of variation of the G_2 element would mean a fixed color group but varying choice of color quantization axis.
3. Could one consider the possibility that the local $G_{2,c}$ element is free and defines the twistor lift of M^8-H duality as something more fundamental than the ordinary M^8-H duality based on $SU(3)_c$. This duality would make sense only at the mass shells so that only the spaces $H^3\times CP_2$ assignable to mass shells would make sense physically? In the interior CP_2 would be replaced with the twistor space $SU(3)/U(1)\times U(1)$. Color gauge fields would be non-vanishing at the mass shells but outside the mass shells one would have G_2 gauge fields.

   There is also a physical objection against the G_2 option. The 14-D Lie algebra representation of G_2 acts on the imaginary octonions which decompose with respect to the color group to $1\oplus 3\oplus \overline{3}$. The automorphism property requires that 1 can be transformed to 3 or $\overline{3}$ to themselves: this requires that the decomposition contains $3\oplus\overline{3}$. Furthermore, it must be possible to transform 3 and $\overline{3}$ to themselves, which requires the presence of 8. This leaves only the decomposition $8\oplus 3\oplus\overline{3}$. G_2 gluons would both color octet and triplets. In the TDG framework the only conceivable interpretation would be in terms of ordinary gluons and leptoquark-like gluons. This does not fit with the basic vision of TGD.

The choice of twistor as a selection of quantization axes should make sense also in the M^4 degrees of freedom. M^4 twistor corresponds to a choice of light-like direction at a given point of M^4. The spatial component of the light-like vector fixes the spin quantization axis. Its choice together with the light-likeness fixes the time direction and therefore the rest system and energy quantization axis. Light-like vector fixes also the choice of M^2 and of E^2 as its orthogonal complement. Therefore the fixing of M^4 twistor as a point of $SU(4)/SU(3)\times U(1)$ corresponds to a choice of the spin quantization axis and the time-like axis defining the rest system in which the energy is measured. This choice would naturally correspond to the Hamilton-Jacobi structure fixing the decompositions $M^2(x)\times E^2(x)$. At a given mass shell the choice of the quantization axis would be constant for a given X^3_i.

1.1.8 Hierarchy of Planck Constants and Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that

Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark Matter as Large $\hbar$ Phases

D. Da Rocha and Laurent Nottale [E13] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr}. Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K100].

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Certain experimental findings suggest the identification $h_{eff} = n\times = h_{gr}$. The large value of h_{gr} can be seen as a way to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description). The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n. Macroscopic quantum coherence in astrophysical scales is implied. If also Kähler-Dirac action is present, part of the interior degrees of freedom associated with the Kähler-Dirac part of conformal algebra become physical. A possible is that tfermionic oscillator operators generate super-symmetries and sparticles correspond almost by definition to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) of bunch of n low energy gravitons.

Hierarchy of Planck Constants from the Anomalies of Neuroscience and Biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K90, K91, K87]) support the view that dark matter might be a key player in living matter.

Dark Matter as a Source of Long Ranged Weak and Color Fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff}.

1.1.9 Twistors in TGD and connection with Veneziano duality

The twistorialization of TGD has two aspects. The attempt to generalize twistor Grassmannian approach emerged first. It was however followed by the realization that also the twistor lift of TGD at classical space-time level is needed. It turned out that the progress in the understanding of the classical twistor lift has been much faster - probably this is due to my rather limited technical QFT skills.

Twistor lift at space-time level

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [K114]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A57]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2. The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric.

In fact, it is enough to generalize the induction of spinor structure to that of twistor structure so that the induced twistor structure need not be identical with the ordinary twistor structure possibly assignable to the space-time surface. The induction procedure reduces to a dimensional reduction of 6-D Kähler action giving rise to 6-D surfaces having bundle structure with twistor sphere as fiber and space-time as base. The twistor sphere of this bundle is imbedded as sphere in the product of twistor spheres of twistor spaces of M^4 and CP_2.

This condition would define the dynamics, and the original conjecture was that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

It however turned out that twistor lift of TGD is much more than a mere technical tool. First of all, the dimensionally reduction of 6-D Kähler action contained besides 4-D Kähler action also a volume term having interpretation in terms of cosmological constant. This need not bring anything new, since all known extremals of Kähler action with non-vanishing induced Kähler form are minimal surfaces. There is however a large number of embeddings of twistor sphere of space-time surface to the product of twistor spheres. Cosmological constant has spectrum and depends on length scale, and the proposal is that coupling constant evolution reduces to that for cosmological constant playing the role of cutoff length. That cosmological constant could transform from a mere nuisance to a key element of fundamental physics was something totally new and unexpected.

1. The twistor lift of TGD at space-time level forces to replace 4-D Kähler action with 6-D dimensionally reduced Kähler action for 6-D surface in the 12-D Cartesian product of 6-D twistor spaces of M^4 and CP_2. The 6-D surface has bundle structure with twistor sphere as fiber and space-time surface as base.

   Twistor structure is obtained by inducing the twistor structure of 12-D twistor space using dimensional reduction. The dimensionally reduced 6-D Kähler action is sum of 4-D Kähler action and volume term having interpretation in terms of a dynamical cosmological constant depending on the size scale of space-time surface (or of causal diamond CD in zero energy ontology (ZEO)) and determined by the representation of twistor sphere of space-time surface in the Cartesian product of the twistor spheres of M^4 and CP_2.

2. The preferred extremal property as a representation of quantum criticality would naturally correspond to minimal surface property meaning that the space-time surface is separately an extremal of both Kähler action and volume term almost everywhere so that there is no coupling between them. This is the case for all known extremals of Kähler action with non-vanishing induced Kähler form.

   Minimal surface property could however fail at 2-D string world sheets, their boundaries and perhaps also at partonic 2-surfaces. The failure is realized in minimal sense if the 3-surface has 1-D edges/folds (strings) and 4-surface 2-D edges/folds (string world sheets) at which some partial derivatives of the embedding space coordinates are discontinuous but canonical momentum densities for the entire action are continuous.

   There would be no flow of canonical momentum between interior and string world sheet and minimal surface equations would be satisfied for the string world sheet, whose 4-D counterpart in twistor bundle is determined by the analog of 4-D Kähler action. These conditions allow the transfer of canonical momenta between Kähler- and volume degrees of freedom at string world sheets. These no-flow conditions could hold true at least asymptotically (near the boundaries of CD).

   $M^8 - H$ duality suggests that string world sheets (partonic 2-surfaces) correspond to images of complex 2-sub-manifolds of M^8 (having tangent (normal) space which is complex 2-plane of octonionic M^8).

3. Cosmological constant would depend on p-adic length scales and one ends up to a concrete model for the evolution of cosmological constant as a function of p-adic length scale and other number theoretic parameters (such as Planck constant as the order of Galois group): this conforms with the earlier picture.

   Inflation is replaced with its TGD counterpart in which the thickening of cosmic strings to flux tubes leads to a transformation of Kähler magnetic energy to ordinary and dark matter. Since the increase of volume increases volume energy, this leads rapidly to energy minimum at some flux tube thickness. The reduction of cosmological constant by a phase transition however leads to a new expansion phase. These jerks would replace smooth cosmic expansion of GRT. The discrete coupling constant evolution predicted by the number theoretical vision could be understood as being induced by that of cosmological constant taking the role of cutoff parameter in QFT picture [L73].

Twistor lift at the level of scattering amplitudes and connection with Veneziano duality

The classical part of twistor lift of TGD is rather well-understood. Concerning the twistorialization at the level of scattering amplitudes the situation is much more difficult conceptually - I already mentioned my limited QFT skills.

1. From the classical picture described above it is clear that one should construct the 8-D twistorial counterpart of theory involving space-time surfaces, string world sheets and their boundaries, plus partonic 2-surfaces and that this should lead to concrete expressions for the scattering amplitudes.

   The light-like boundaries of string world sheets as carriers of fermion numbers would correspond to twistors as they appear in twistor Grassmann approach and define the analog for the massless sector of string theories. The attempts to understand twistorialization have been restricted to this sector.

2. The beautiful basic prediction would be that particles massless in 8-D sense can be massive in 4-D sense. Also the infrared cutoff problematic in twistor approach emerges naturally and reduces basically to the dynamical cosmological constant provided by classical twistor lift.

   One can assign 4-momentum both to the spinor harmonics of the embedding space representing ground states of super-conformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this could be seen as a concretization of Equivalence Principle. Also a connection with string model emerges.

3. As far as symmetries are considered, the picture looks rather clear. Ordinary twistor Grassmannian approach boils down to the construction of scattering amplitudes in terms of Yangian invariants for conformal group of M^4. Therefore a generalization of super-symplectic symmetries to their Yangian counterpart seems necessary. These symmetries would be gigantic but how to deduce their implications?

4. The notion of positive Grassmannian is central in the twistor approach to the scattering amplitudes in $calN = 4$ SUSYs. TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings.

   Braid interpretation encourages the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

This is however not the only vision about a solution of non-planarity. Quantum criticality provides different view leading to a totally unexpected connection with string models, actually with the Veneziano duality, which was the starting point of dual resonance model in turn leading via dual resonance models to super string models.

1. Quantum criticality in TGD framework means that coupling constant evolution is discrete in the sense that coupling constants are piecewise constant functions of length scale replaced by dynamical cosmological constant. Loop corrections would vanish identically and the recursion formulas for the scattering amplitudes (allowing only planar diagrams) deduced in twistor Grassmann would involve no loop corrections. In particular, cuts would be replaced by sequences of poles mimicking them like sequences of point charge mimic line charges. In momentum discretization this picture follows automatically.

2. This would make sense in finite measurement resolution realized in number theoretical vision by number-theoretic discretization of the space-time surface (cognitive representation) as points with coordinates in the extension of rationals defining the adele [L49]. Similar discretization would take place for momenta. Loops would vanish at the level of discretization but what would happen at the possibly existing continuum limit: does the sequence of poles integrate to cuts? Or is representation as sum of resonances something much deeper?

3. Maybe it is! The basic idea of behind the original Veneziano amplitudes (see http://tinyurl.com/yyhwvbqb) was Veneziano duality. This 4-particle amplitude was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see http://tinyurl.com/yyvkx7as) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged. Later came superstring models and led to M-theory. Now it has become clear that something went wrong, and it seems that one must return to the roots. Could the return to the roots mean a careful reconsideration of the dual resonance model?

4. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

   The resonances had zero width, which was not consistent with unitarity. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of twistor Grassmann approach.

5. Veneziano duality is true only for s- and t- channels but not been s- and u-channel. Stringy description makes t-channel and s-channel pictures equivalent. Could it be that in fundamental description u-channels diagrams cannot be distinguished from s-channel diagrams or t-channel diagrams? Could the stringy representation of the scattering diagrams make u-channel twist somehow trivial if handles of string world sheet representing stringy loops in turn representing the analog of non-planarity of Feynman diagrams are absent? The permutation of external momenta for tree diagram in absence of loops in planar representation would be a twist of π in the representation of planar diagram as string world sheet and would not change the topology of the string world sheet and would not involve non-trivial world sheet topology.

   For string world sheets loops would correspond to handles. The presence of handle would give an edge with a loop at the level of 3-surface (self energy correction in QFT). Handles are not allowed if the induced metric for the string world sheet has Minkowskian signature. If the stringy counterparts of loops are absent, also the loops in scattering amplitudes should be absent.

   This argument applies only inside the Minkowskian space-time regions. If string world sheets are present also in Euclidian regions, they might have handles and loop corrections could emerge in this manner. In TGD framework strings (string world sheets) are identified to 1-D edges/folds of 3-surface at which minimal surface property and topological QFT property fails (minimal surfaces as calibrations). Could the interpretation of edge/fold as discontinuity of some partial derivatives exclude loopy edges: perhaps the branching points would be too singular?

A reduction to a sum over s-channel resonances is what the vanishing of loops would suggest. Could the presence of string world sheets make possible the vanishing of continuous cuts even at the continuum limit so that continuum cuts would emerge only in the approximation as the density of resonances is high enough?

The replacement of continuous cut with a sum of *infinitely* narrow resonances is certainly an approximation. Could it be that the stringy representation as a sum of resonances with *finite* width is an essential aspect of quantum physics allowing to get rid of infinities necessarily accompanying loops? Consider now the arguments against this idea.

1. How to get rid of the problems with unitarity caused by the zero width of resonances? Could *finite* resonance widths make unitarity possible? Ordinary twistor Grassmannian approach predicts that the virtual momenta are light-like but complex: obviously, the imaginary part of the energy in rest frame would have interpretation as resonance with.

   In TGD framework this generalizes for 8-D momenta. By quantum-classical correspondence (QCC) the classical Noether charges are equal to the eigenvalues of the fermionic charges in Cartan algebrable (maximal set of mutually commuting observables) and classical TGD indeed predicts complex momenta (Kähler coupling strength is naturally complex). QCC thus supports this proposal.

2. Sum over resonances/exchanges picture is in conflict with QFT picture about scattering of particles. Could *finite* resonance widths due to the complex momenta give rise to the QFT type scattering amplitudes as one develops the amplitudes in Taylor series with respect to the resonance width? Unitarity condition indeed gives the first estimate for the resonance width.

   QFT amplitudes should emerge in an approximation obtained by replacing the discrete set of finite width resonances with a cut as the distance between poles is shorter than the resolution for mass squared.

   In superstring models string tension has single very large value and one cannot obtain QFT type behavior at low energies (for instance, scattering amplitudes in hadronic string model are concentrated in forward direction). TGD however predicts an entire hierarchy of p-adic length scales with varying string tension. The hierarchy of mass scales corresponding roughly to the lengths and thickness of magnetic flux tubes as thickened cosmic strings and characterized by the value of cosmological constant predicted by twistor lift of TGD. Could this give rise to continuous QCT type cuts at the limit when measurement resolution cannot distinguish between resonances?

   The dominating term in the sum over sums of resonances in t-channel gives near forward direction approximately the lowest mass resonance for strings with the smallest string tension. This gives the behavior $1/(t - m_{min}^2)$, where m_{min} corresponds to the longest mass scale involved (the largest space-time sheet involved), approximating the $1/t$-behavior of massless theories. This also brings in IR cutoff, the lack of which is a problem of gauge theories. This should give rise to continuous QFT type cuts at the limit when measurement resolution cannot distinguish between resonances.

1.2 Bird's Eye of View about the "Topics of Hyper-finite Factors and Hierarchy of Planck Constants: Part I"

The book "Bird's Eye of View about the Topics of Hyper-finite Factors and Hierarchy of Planck Constants: Part I" is organized to 2 parts.

The first part of the part I is devoted to hyper-finite factors and hierarchy of Planck constants.

1. Configuration space spinors indeed define a canonical example about hyper-finite factor of type II_1. The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors of type II_1 could provide the mathematics needed to develop a more explicit view about the construction of M-matrix. This has turned out to be the case to the extent that a general master formula for M-matrix with interactions described as a deformation of ordinary tensor product to Connes tensor products emerges.

2. The idea about hierarchy of Planck constants emerged from anomalies of biology and the strange finding that planetary orbits could be regarded as Bohr orbits but with a gigantic value of Planck constant. This lead to the vision that dark matter corresponds to ordinary particles but with non-standard value of Planck constant and to a generalization of the 8-D imbedding space to a book like structure with pages partially characterized by the value of Planck constant. Using the intuition provided by the inclusions of hyper-finite factors of type II_1 one ends up to a prediction for the spectrum of Planck constants associated with M^4 and CP_2 degrees of freedom. This inspires the proposal that dark matter could be in quantum Hall like phase localized at light-like 3-surfaces with macroscopic size and behaving in many respects like black hole horizons.

In the 2nd part of part I some applications are discussed.

1. The first chapter is devoted to the applications of TGD view about quantum criticality and in various fields of physics and also in biology. There is also a chapter about the Nottale's formula for the gravitational Planck constant $h_{gr} = GMm/v_0 = n_{gr}h_0$. The are arguments suggesting how the value of the velocity parameter $v_0 < c$ is determined.

The possibility that Planck length l_P and CP_2 length R are identical is considered. The first guess $G = R^2/\hbar_{gr}$ holding true at gravitational flux tubes predicts too large variation of G. One can however interpret space-time surfaces as n_1-fold coverings of CP_2 and n_2-fold coverings of M^4 and one has $h_{gr} = n_1 n_2$. The value of n_2 is expected to be bounded by strong constraints. The formula $G = R^2/n_2\hbar_0$ gives a correct size scale for G for $n_2 \sim 10^{3.5}$ and one could also understand why the variation of G is relatively small but too large to be consistent with the existing views.

2. TGD based view about quasars as TGD analogs of white holes is discussed. They would be much more structured than ordinary blackholes. Quasars, galaxies, and even stars and planets would emerge as tangles to cosmic strings, and have topology resembling that of dipole magnetic field. They would possess magnetic fields and empty disk around the central region, and lack horizon. The inspiration came from the notion of MECO and one can ask whether even ordinary blackholes are replaced in TGD framework by these objects or their time reversals.

3. There is also a chapter devoted to the vision about 3-space as a network with nodes connected by flux tubes making possible quantum entanglement between nodes possible in arbitrary long length scales due to the hierarchy of Planck constants.

1.3 Sources

The eight online books about TGD [K122, K115, K94, K81, K29, K77, K56, K103] and nine online books about TGD inspired theory of consciousness and quantum biology [K111, K22, K86, K20, K53, K64, K67, K102, K110] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (`http://tinyurl.com/ybv8dt4n`) contains a lot of material about TGD. In particular, a TGD glossary at `http://tinyurl.com/yd6jf3o7`).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (`http://tinyurl.com/ycyrxj4o` founded by Lian Sidorov and in *Prespacetime Journal* (`http://tinyurl.com/ycvktjhn`), *Journal of Consciousness Research and Exploration* (`http://tinyurl.com/yba4f672`), and *DNA Decipher Journal* (`http://tinyurl.com/y9z52khg`), all of them founded by Huping Hu. One can find the list about the articles published at `http://tinyurl.com/ybv8dt4n`. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

1.4 The contents of the book

1.4.1 PART I: HYPER-FINITE FACTORS AND HIERARCHY OF PLANCK CONSTANTS

What von Neumann Right After All?

The work with TGD inspired model for topological quantum computation led to the realization that von Neumann algebras, in particular so called hyper-finite factors of type II_1, seem to provide the mathematics needed to develop a more explicit view about the construction of S-matrix. The original discussion has transformed during years from free speculation reflecting in many aspects my ignorance about the mathematics involved to a more realistic view about the role of these algebras in quantum TGD. The discussions of this chapter have been restricted to the basic notions are discussed and only short mention is made to TGD applications discussed in second chapter.

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1.

The definitions of adopted by von Neumann allow however more general algebras. Type I_n algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless being replaced by the notion of state which is generalization of the notion of thermodynamical state. The fascinating feature of this notion of state is that it defines a unique modular automorphism of the factor defined apart from unitary inner automorphism and the question is whether this notion or its generalization might be relevant for the construction of M-matrix in TGD. It however seems that in TGD framework based on Zero Energy Ontology identifiable as "square root" of thermodynamics a square root of thermodynamical state is needed.

The inclusions of hyper-finite factors define an excellent candidate for the description of finite measurement resolution with included factor representing the degrees of freedom below measurement resolution. The would also give connection to the notion of quantum group whose physical interpretation has remained unclear. This idea is central to the proposed applications to quantum TGD discussed in separate chapter.

Evolution of Ideas about Hyper-finite Factors in TGD

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors, could provide the mathematics needed to develop a more explicit view about the construction of M-matrix generalizing the notion of S-matrix in zero energy ontology (ZEO). In this chapter I will discuss various aspects of hyper-finite factors and their possible physical interpretation in TGD framework.

1. Hyper-finite factors in quantum TGD

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type III_1 appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

1. The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type II_1. Therefore also the Clifford algebra at a given point (light-like 3-surface) of world of classical worlds (WCW) is HFF of type II_1. If the fermionic Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type II_1. Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type II_∞ results.

2. WCW is a union of sub-WCWs associated with causal diamonds (CD) defined as intersections of future and past directed light-cones. One can allow also unions of CDs and the proposal is that CDs within CDs are possible. Whether CDs can intersect is not clear.

3. The assumption that the M^4 proper distance a between the tips of CD is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that a can have all possible values. Since $SO(3)$ is the isotropy group of CD, the CDs associated with a given value of a and with fixed lower tip are parameterized by the Lobatchevski space

$L(a) = SO(3,1)/SO(3)$. Therefore the CDs with a free position of lower tip are parameterized by $M^4 \times L(a)$. A possible interpretation is in terms of quantum cosmology with a identified as cosmic time. Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type III_1. If one allows all values of a, one ends up with $M^4 \times M^4_+$ as the space of moduli for WCW.

4. An interesting special aspect of 8-dimensional Clifford algebra with Minkowski signature is that it allows an octonionic representation of gamma matrices obtained as tensor products of unit matrix 1 and 7-D gamma matrices γ_k and Pauli sigma matrices by replacing 1 and γ_k by octonions. This inspires the idea that it might be possible to end up with quantum TGD from purely number theoretical arguments. One can start from a local octonionic Clifford algebra in M^8. Associativity (co-associativity) condition is satisfied if one restricts the octonionic algebra to a subalgebra associated with any hyper-quaternionic and thus 4-D sub-manifold of M^8. This means that the induced gamma matrices associated with the Kähler action span a complex quaternionic (complex co-quaternionic) sub-space at each point of the sub-manifold. This associative (co-associative) sub-algebra can be mapped a matrix algebra. Together with $M^8 - H$ duality this leads automatically to quantum TGD and therefore also to the notion of WCW and its Clifford algebra which is however only mappable to an associative (co-associative(algebra and thus to HFF of type II_1.

2. Hyper-finite factors and M-matrix

HFFs of type III_1 provide a general vision about M-matrix.

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.
2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.
3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology (ZEO): the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.
4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions meaning the analog of state function collapse in zero modes fixing the classical conserved charges equal to the quantal counterparts. Classical charges would be parameters characterizing zero modes.

A concrete construction of M-matrix motivated the recent rather precise view about basic variational principles is proposed. Fundamental fermions localized to string world sheets can be said to propagate as massless particles along their boundaries. The fundamental interaction vertices correspond to two fermion scattering for fermions at opposite throats of wormhole contact and the inverse of the conformal scaling generator L_0 would define the stringy propagator characterizing this interaction. Fundamental bosons correspond to pairs of fermion and antifermion at opposite throats of wormhole contact. Physical particles correspond to pairs of wormhole contacts with monopole Kähler magnetic flux flowing around a loop going through wormhole contacts.

3. Connes tensor product as a realization of finite measurement resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In ZEO $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.
2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.
3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$- "averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

4. Analogs of quantum matrix groups from finite measurement resolution?

The notion of quantum group replaces ordinary matrices with matrices with non-commutative elements. In TGD framework I have proposed that the notion should relate to the inclusions of von Neumann algebras allowing to describe mathematically the notion of finite measurement resolution.

In this article I will consider the notion of quantum matrix inspired by recent view about quantum TGD and it provides a concrete representation and physical interpretation of quantum groups in terms of finite measurement resolution. The basic idea is to replace complex matrix elements with operators expressible as products of non-negative hermitian operators and unitary operators analogous to the products of modulus and phase as a representation for complex numbers.

The condition that determinant and sub-determinants exist is crucial for the well-definedness of eigenvalue problem in the generalized sense. The weak definition of determinant meaning its development with respect to a fixed row or column does not pose additional conditions. Strong definition of determinant requires its invariance under permutations of rows and columns. The permutation of rows/columns turns out to have interpretation as braiding for the hermitian operators defined by the moduli of operator valued matrix elements. The commutativity of all sub-determinants is essential for the replacement of eigenvalues with eigenvalue spectra of hermitian operators and sub-determinants define mutually commuting set of operators.

The resulting quantum matrices define a more general structure than quantum group but provide a concrete representation and interpretation for quantum group in terms of finite measurement resolution if q is a root of unity. For $q = \pm 1$ (Bose-Einstein or Fermi-Dirac statistics) one obtains quantum matrices for which the determinant is apart from possible change by sign factor invariant under the permutations of both rows and columns. One could also understand the fractal structure of inclusion sequences of hyper-finite factors resulting by recursively replacing operators appearing as matrix elements with quantum matrices.

5. Quantum spinors and fuzzy quantum mechanics

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with "true" and "false". The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and decoherence is not a problem as long as it does not induce this transition.

Trying to fuse the basic mathematical ideas of quantum TGD to a single coherent whole

The theoretical framework behind TGD involves several different strands and the goal is to unify them to a single coherent whole. TGD involves number theoretic and geometric visions about physics and $M^8 - H$ duality, analogous to Langlands duality, is proposed to unify them. Also quantum classical correspondence (QCC) is a central aspect of TGD. One should understand both the $M^8 - H$ duality and QCC at the level of detail.

The following mathematical notions are expected to be of relevance for this goal.

1. Von Neumann algebras, call them M, in particular hyperfinite factors of type II_1 (HFFs), are in a central role. Both the geometric and number theoretic side, QCC could mathematically correspond to the relationship between M and its commutant M'.

   For instance, symplectic transformations leave induced Kähler form invariant and various fluxes of Kähler form are symplectic invariants and correspond to classical physics commuting with quantum physics coded by the super symplectic algebra (SSA). On the number theoretic side, the Galois invariants assignable to the polynomials determining space-time surfaces are analogous classical invariants.

2. The generalization of ordinary arithmetics to quantum arithmetics obtained by replacing $+$ and $\times$ with $\oplus$ and $\otimes$ allows us to replace the notions of finite and p-adic number fields with their quantum variants. The same applies to various algebras.

3. Number theoretic vision leads to adelic physics involving a fusion of various p-adic physics and real physics and to hierarchies of extensions of rationals involving hierarchies of Galois groups involving inclusions of normal subgroups. The notion of adele can be generalized by replacing various p-adic number fields with the p-adic representations of various algebras.

4. The physical interpretation of the notion of infinite prime has remained elusive although a formal interpretation in terms of a repeated quantization of a supersymmetric arithmetic QFT is highly suggestive. One can also generalize infinite primes to their quantum variants. The proposal is that the hierarchy of infinite primes generalizes the notion of adele.

The formulation of physics as Kähler geometry of the "world of classical worlds" (WCW) involves f 3 kinds of algebras A; supersymplectic isometries SSA acting on $\delta M^4_+ \times CP_2$, affine algebras Aff acting on light-like partonic orbits, and isometries I of light-cone boundary δM^4_+, allowing hierarchies A_n.

The braided Galois group algebras at the number theory side and algebras $\{A_n\}$ at the geometric side define excellent candidates for inclusion hierarchies of HFFs. $M^8 - H$ duality suggests that n corresponds to the degree $n$of the polynomial P defining space-time surface and that the n roots of P correspond to n braid strands at H side. Braided Galois group would act in A_n and hierarchies of Galois groups would induce hierarchies of inclusions of HFFs. The ramified primes of P would correspond to physically preferred p-adic primes in the adelic structure formed by p-adic variants of A_n with $+$ and $\times$ replaced with $\oplus$ and $\otimes$.

TGD view about Mckay Correspondence, ADE Hierarchy, Inclusions of Hyperfinite Factors, and Twistors

In this chapter 4 topics are discussed. McKay correspondence, SUSY, and twistors are discussed from TGD point of view, and new aspects of $M^8 - H$ duality are considered.

1. *McKay correspondence in TGD framework*

There are two mysterious looking correspondences involving ADE groups. McKay correspondence between McKay graphs characterizing tensor products for finite subgroups of $SU(2)$ and Dynkin diagrams of affine ADE groups is the first one. The correspondence between principal diagrams characterizing inclusions of hyper-finite factors of type II_1 (HFFs) with Dynkin diagrams for a subset of ADE groups and Dynkin diagrams for affine ADE groups is the second one.

These correspondences are discussed from number theoretic point of view suggested by TGD and based on the interpretation of discrete subgroups of $SU(2)$ as subgroups of the covering group of quaternionic automorphisms $SO(3)$ (analog of Galois group) and generalization of these groups to semi-direct products $Gal(K) \triangleleft SU(2)_K$ of Galois group for extension K of rationals with the discrete subgroup $SU(2)_K$ of $SU(2)$ with representation matrix elements in K. The identification of the inclusion hierarchy of HFFs with the hierarchy of extensions of rationals and their Galois groups is proposed.

A further mystery whether $Gal(K) \triangleleft SU(2)_K$ could give rise to quantum groups or affine algebras. In TGD framework the infinite-D group of isometries of "world of classical worlds" (WCW) is identified as an infinite-D symplectic group for which the discrete subgroups characterized by K have infinite-D representations so that hyper-finite factors are natural for their representations. Symplectic algebra SSA allows hierarchy of isomorphic sub-algebras SSA_n. The gauge conditions for SSA_n and $[SSA_n, SSA]$ would define measurement resolution giving rise to hierarchies of inclusions and ADE type Kac-Moody type algebras or quantum algebras representing symmetries modulo measurement resolution.

A concrete realization of ADE type Kac-Moody algebras is proposed. It relies on the group algebra of $Gal(K) \triangleleft SU(2)_K$ and free field representation of ADE type Kac-Moody algebra identifying the free scalar fields in Kac-Moody Cartan algebra as group algebra elements defined by the traces of representation matrices (characters).

2. New aspects of $M^8 - H$ duality

$M^8 - H$ duality is now a central part of TGD and leads to new findings. $M^8 - H$ duality can be formulated both at the level of space-time surfaces and light-like 8-momenta. Since the choice of M^4 in the decomposition of momentum space $M^8 = M^4 \times E^4$ is rather free, it is always possible to find a choice for which light-like 8-momentum reduces to light-like 4-momentum in M^4 - the notion of 4-D mass is relative. This leads to what might be called $SO(4) - SU(3)$ duality corresponding to the hadronic and partonic views about hadron physics. Particles, which are eigenstates of mass squared are massless in $M^4 \times CP_2$ picture and massive in M^8 picture. The massivation in this picture is a universal mechanism having nothing to do with dynamics and results in zero energy ontology automatically if the zero energy states are superpositions of states with different masses. p-Adic thermodynamics describes this massivation. Also a proposal for the realization of ADE hierarchy emerges.

4-D space-time surfaces correspond to roots of octonionic polynomials $P(o)$ with real coefficients corresponding to the vanishing of the real or imaginary part of $P(o)$. These polynomials however allow universal roots, which are not 4-D but analogs of 6-D branes and having topology of S^6. Their M^4 projections are time =constant snapshots $t = r_n, r_M \leq r_n$ 3-balls of M^4 light-cone (r_n is root of $P(x)$). At each point the ball there is a sphere S^3 shrinking to a point about boundaries of the 3-ball. These special values of M^4 time lead to a deeper understanding of ZEO based quantum measurement theory and consciousness theory.

3. Is the identification of twistor space of M^4 really correct?

The critical questions concerning the identification of twistor space of M^4 as $M^4 \times S^2$ led to consider a more conservative identification as CP_3 with hyperbolic signature (3,-3) and replacement of H with $H = cd_{conf} \times CP_2$, where cd_{conf} is CP_2 with hyperbolic signature (1,-3). This approach reproduces the nice results of the earlier picture but means that the hierarchy of CDs in M^8 is mapped to a hierarchy of spaces cd_{conf} with sizes of CDs. This conforms with Poincare symmetry from which everything started since Poincare group acts in the moduli space of octonionic structures of M^8. Note that also the original form of $M^8 - H$ duality continues to make sense and results from the modification by projection from $CP_{3,h}$ to M^4 rather than $CP_{2,h}$.

The outcome of octo-twistor approach applied at level of M^8 together with modified $M^8 - H$ duality leads to a nice picture view about twistorial description of massive states based on quaternionic generalization of twistor (super-)Grassmannian approach. A radically new view is that descriptions in terms of massive and massless states are alternative options, and correspond to two different alternative twistorial descriptions and leads to the interpretation of p-adic thermodynamics as completely universal massivation mechanism having nothing to do with dynamics. As a side product emerges a deeper understanding of ZEO based quantum measurement theory and consciousness theory relying on the universal roots of octonionic polynomials of M^8, which are not 4-D but analogs of 6-D branes. This part of article is not a mere side track since by $M^8 - H$ duality the finite sub-groups of $SU(2)$ of McKay correspondence appear quite concretely in the description of the measurement resolution of 8-momentum.

Does TGD predict spectrum of Planck constants?

The quantization of Planck constant has been the basic theme of TGD since 2005. The basic idea was stimulated by the suggestion of Nottale that planetary orbits could be seen as Bohr orbits with enormous value of Planck constant given by $\hbar_{gr} = GM_1M_2/v_0$, where the velocity parameter v_0 has the approximate value $v_0 \simeq 2^{-11}$ for the inner planets. This inspired the ideas that quantization is due to a condensation of ordinary matter around dark matter concentrated near Bohr orbits and that dark matter is in macroscopic quantum phase in astrophysical scales. The second crucial empirical input were the anomalies associated with living matter. The recent version of the chapter represents the evolution of ideas about quantization of Planck constants from a perspective given by seven years's work with the idea. A very concise summary about the situation is as follows.

1. Basic physical ideas

The basic phenomenological rules are simple.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Effective embedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies.

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

   The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of embedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

   In astrophysics and cosmology the implications are even more dramatic. The interpretation of $\hbar_{gr}$ introduced by Nottale in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in astronomical scales.

The gravitational Compton length $GM/v_0 = r_S/2v_0$ does not depend on m so that all particles around say Sun say same gravitational Compton length.

By the independence of gravitational acceleration and gravitational Compton length on particle mass, it is enough to assume that only microscopic particles couple to the dark gravitons propagating along flux tubes mediating gravitational interaction. Therefore $h_{gr} = h_{eff}$ could be true in microscopic scales and would predict that cyclotron energies have no dependence on the mass of the charged particle meaning that the spectrum ordinary photons resulting in the transformation of dark photons to ordinary photons is universal. An attractive identification of these photons would be as bio-photons with energies in visible and UV range and thus inducing molecular transitions making control of biochemistry by dark photons. This changes the view about gravitons and suggests that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation. The energy of the graviton is gigantic unless the emission is assume to take place from a microscopic systems with large but not gigantic h_{gr}.

3. Why Nature would like to have large - maybe even gigantic - value of effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

2. Space-time correlates for the hierarchy of Planck constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of embedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology (ZEO), it became clear that the notion of singular covering space of the embedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. In ZEO 3-surfaces are unions of space-like 3-surface at opposite boundaries of CD. The non-determinism of Kähler action due to the huge vacuum degeneracy would naturally explain the existence of several space-time sheets connecting the two 3-surfaces at the opposite boundaries of CD. Quantum criticality suggests strongly conformal invariance and the identification of n as the number of conformal equivalence classes of these space-time sheets. Also a connection with the notion of negentropic entanglement emerges.

Mathematical speculations inspired by the hierarchy of Planck constants

This chapter contains the purely mathematical speculations about the hierarchy of Planck constants (actually only effective hierarchy if the recent interpretation is correct) as separate from the material describing the physical ideas, key mathematical concepts, and the basic applications. These mathematical speculations emerged during the first stormy years in the evolution of the ideas about Planck constant and must be taken with a big grain of salt. I feel myself rather conservative as compared to the fellow who produced this stuff for 7 years ago. This all is of course very relative. Many readers might experience this recent me as a reckless speculator.

The first speculative question is about possible relationship between Jones inclusions of hyperfinite factors of type II_1 (hyper-finite factors are von Neuman algebras emerging naturally in TGD framework). The basic idea is that the discrete groups assignable to inclusions could correspond to discrete groups acting in the effective covering spaces of embedding space assignable to the hierarchy of Planck constants.

There are also speculations relating to the hierarchy of Planck constants, Mc-Kay correspondence, and Jones inclusions. Even Farey sequences, Riemann hypothesis and and N-tangles are discussed. Depending on reader these speculations might be experienced as irritating or entertaining. It would be interesting to go this stuff through in the light of recent understanding of the effective hierarchy of Planck constants to see what portion of its survives.

Negentropy Maximization Principle

In TGD Universe the moments of consciousness are associated with quantum jumps between quantum histories. The proposal is that the dynamics of consciousness is governed by Negentropy Maximization Principle (NMP), which states the information content of conscious experience is maximal. The formulation of NMP is the basic topic of this chapter.

NMP codes for the dynamics of standard state function reduction and states that the state function reduction process following U-process gives rise to a maximal reduction of entanglement entropy at each step. In the generic case this implies at each step a decomposition of the system to unique unentangled subsystems and the process repeats itself for these subsystems. The process stops when the resulting subsystem cannot be decomposed to a pair of free systems since energy conservation makes the reduction of entanglement kinematically impossible in the case of bound states. The natural assumption is that self loses consciousness when it entangles via bound state entanglement.

There is an important exception to this vision based on ordinary Shannon entropy. There exists an infinite hierarchy of number theoretical entropies making sense for rational or even algebraic entanglement probabilities. In this case the entanglement negentropy can be negative so that NMP favors the generation of negentropic entanglement (NE), which is not bound state entanglement in standard sense since the condition that state function reduction leads to an eigenstate of density matrix requires the final state density matrix to be a projection operator.

NE might serve as a correlate for emotions like love and experience of understanding. The reduction of ordinary entanglement entropy to random final state implies second law at the level of ensemble. For the generation of NE the outcome of the reduction is not random: the prediction is that second law is not a universal truth holding true in all scales. Since number theoretic entropies are natural in the intersection of real and p-adic worlds, this suggests that life resides in this intersection. The existence effectively bound states with no binding energy might have important implications for the understanding the stability of basic bio-polymers and the key aspects of metabolism. A natural assumption is that self experiences expansion of consciousness as it entangles in this manner. Quite generally, an infinite self hierarchy with the entire Universe at the top is predicted.

There are two options to consider. Strong form of NMP, which would demand maximal negentropy gain: this would not allow morally responsible free will if ethics is defined in terms of evolution as increase of NE resources. Weak form of NMP would allow self to choose also lower-dimensional sub-space of the projector defining the final state sub-space for strong form of NMP. Weak form turns out to have several highly desirable consequences: it favours dimensions of final state space coming as powers of prime, and in particular dimensions which are primes near powers of prime: as a special case, p-adic length scale hypothesis follows. Weak form of NMP allows also quantum computations, which halt unlike strong form of NMP.

Besides number theoretic negentropies there are also other new elements as compared to the earlier formulation of NMP.

1. ZEO modifies dramatically the formulation of NMP since U-matrix acts between zero energy states and can be regarded as a collection of orthonormal M-matrices, which generalize the ordinary S-matrix and define what might be called a complex square root of density matrix so that kind of a square root of thermodynamics at single particle level justifying also p-adic mass calculations based on p-adic thermodynamics is in question.
2. The hierarchy of Planck constants labelling a hierarchy of quantum criticalities is a further new element having important implications for conciousness and biology.
3. Hyper-finite factors of type II_1 represent an additional technical complication requiring separate treatment of NMP taking into account finite measurement resolution realized in terms of inclusions of these factors.

NMP has wide range of important implications.

1. In particular, one must give up the standard view about second law and replace it with NMP taking into account the hierarchy of CDs assigned with ZEO and dark matter hierarchy labelled by the values of Planck constants, as well as the effects due to NE. The breaking of second law in standard sense is expected to take place and be crucial for the understanding of evolution.
2. Self hierarchy having the hierarchy of CDs as embedding space correlate leads naturally to a description of the contents of consciousness analogous to thermodynamics except that the entropy is replaced with negentropy.
3. In the case of living matter NMP allows to understand the origin of metabolism. NMP demands that self generates somehow negentropy: otherwise a state function reduction to tjhe opposite boundary of CD takes place and means death and re-incarnation of self. Metabolism as gathering of nutrients, which by definition carry NE is the manner to avoid this fate. This leads to a vision about the role of NE in the generation of sensory qualia and a connection with metabolism. Metabolites would carry NE and each metabolite would correspond to a particular qualia (not only energy but also other quantum numbers would correspond to metabolites). That primary qualia would be associated with nutrient flow is not actually surprising!
4. NE leads to a vision about cognition. Negentropically entangled state consisting of a superposition of pairs can be interpreted as a conscious abstraction or rule: negentropically entangled Schrödinger cat knows that it is better to keep the bottle closed.
5. NMP implies continual generation of NE. One might refer to this ever expanding universal library as "Akaschic records". NE could be experienced directly during the repeated state function reductions to the passive boundary of CD - that is during the life cycle of sub-self defining the mental image. Another, less feasible option is that interaction free measurement is required to assign to NE conscious experience. As mentioned, qualia characterizing the metabolite carrying the NE could characterize this conscious experience.
6. A connection with fuzzy qubits and quantum groups with NE is highly suggestive. The implications are highly non-trivial also for quantum computation allowed by weak form of NMP since NE is by definition stable and lasts the lifetime of self in question.

Philosophy of Adelic Physics

The p-adic aspects of Topological Geometrodynamics (TGD) will be discussed. Introduction gives a short summary about classical and quantum TGD. This is needed since the p-adic ideas are inspired by TGD based view about physics.

p-Adic mass calculations relying on p-adic generalization of thermodynamics and super-symplectic and super-conformal symmetries are summarized. Number theoretical existence constrains lead to highly non-trivial and successful physical predictions. The notion of canonical identification mapping p-adic mass squared to real mass squared emerges, and is expected to be a key player of adelic physics allowing to map various invariants from p-adics to reals and vice versa.

A view about p-adicization and adelization of real number based physics is proposed. The proposal is a fusion of real physics and various p-adic physics to single coherent whole achieved by a generalization of number concept by fusing reals and extensions of p-adic numbers induced by given extension of rationals to a larger structure and having the extension of rationals as their intersection.

The existence of p-adic variants of definite integral, Fourier analysis, Hilbert space, and Riemann geometry is far from obvious and various constraints lead to the idea of number theoretic universality (NTU) and finite measurement resolution realized in terms of number theory. An attractive manner to overcome the problems in case of symmetric spaces relies on the replacement of angle variables and their hyperbolic analogs with their exponentials identified as roots of unity and roots of e existing in finite-dimensional algebraic extension of p-adic numbers. Only group invariants - typically squares of distances and norms - are mapped by canonical identification from p-adic to real realm and various phases are mapped to themselves as number theoretically universal entities.

Also the understanding of the correspondence between real and p-adic physics at various levels - space-time level, embedding space level, and level of "world of classical worlds" (WCW) - is a challenge. The gigantic isometry group of WCW and the maximal isometry group of embedding space give hopes about a resolution of the problems. Strong form of holography (SH) allows a non-local correspondence between real and p-adic space-time surfaces induced by algebraic continuation from common string world sheets and partonic 2-surfaces. Also local correspondence seems intuitively plausible and is based on number theoretic discretization as intersection of real and p-adic surfaces providing automatically finite "cognitive" resolution. he existence p-adic variants of Kähler geometry of WCW is a challenge, and NTU might allow to realize it.

I will also sum up the role of p-adic physics in TGD inspired theory of consciousness. Negentropic entanglement (NE) characterized by number theoretical entanglement negentropy (NEN) plays a key role. Negentropy Maximization Principle (NMP) forces the generation of NE. The interpretation is in terms of evolution as increase of negentropy resources.

The Recent View about SUSY in TGD Universe

The progress in understanding of $M^8 - H$ duality throws also light to the problem whether SUSY is realized in TGD and what SUSY breaking does mean. It is now rather clear that sparticles are predicted and SUSY remains exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them.

The question how to realize super-field formalism at the level of $H = M^4 \times CP_2$ led to a dramatic progress in the identification of elementary particles and SUSY dynamics. The most surprising outcome was the possibility to interpret leptons and corresponding neutrinos as local 3-quark composites with quantum numbers of anti-proton and anti-neutron. Leptons belong to the same super-multiplet as quarks and are antiparticles of neutron and proton as far quantum numbers are consided. One implication is the understanding of matter-antimatter asymmetry. Also bosons can be interpreted as local composites of quark and anti-quark.

Hadrons and hadronic gluons would still correspond to the analog of monopole phase in QFTs. Homology charge would appear as space-time correlate for color at space-time level and explain color confinement. Also color octet variants of weak bosons, Higgs, and Higgs like particle and the predicted new pseudo-scalar are predicted. They could explain the successes of conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC).

One ends up with the precise understanding of quantum criticality and understand the relation between its descriptions at M^8 level and H-level. Polynomials describing a hierarchy of dark matters describe also a hierarchy of criticalities and one can identify inclusion hierarchies as sub-hierarchies formed by functional composition of polynomials. The Wick contractions of quark-antiquark monomials appearing in the expansion of super-coordinate of H could define the analog of radiative corrections in discrete approach. $M^8 - H$ duality and number theoretic vision require that the number of non-vanishing Wick contractions is finite. The number of contractions is indeed bounded by the finite number of points in cognitive representation and increases with the degree of the octonionic polynomial and gives rise to a discrete coupling constant evolution parameterized by the extensions of rationals.

Quark oscillator operators in cognitive representation correspond to quark field q. Only terms with quark number 1 appear in q and leptons emerge in Kähler action as local 3-quark composites. Internal consistency requires that q must be the super-spinor field satisfying super Dirac equation. This leads to a self-referential condition $q_s = q$ identifying q and its super-counterpart q_s. Also super-coordinate h_s must satisfy analogous condition $(h_s)_s = h_s$, where $h_s \rightarrow (h_s)_s$ means replacement of h in the argument of h_s with h_s.

The conditions have an interpretation in terms of a fixed point of iteration and expression of quantum criticality. The coefficients of various terms in q_s and h_s are analogous to coupling constants can be fixed from this condition so that one obtains discrete number theoretical coupling constant evolution. The basic equations are quantum criticality condition $h_s = (h_s)_s$, $q = q_s$, $D_{\alpha,s}\Gamma^{\alpha}_s = 0$ coming from Kähler action, and the super-Dirac equation $D_s q = 0$.

One also ends up to the first completely concrete proposal for how to construct S-matrix directly from the solutions of super-Dirac equations and super-field equations for space-time super-surfaces. The idea inspired by WKB approximation is that the exponent of the super variant of Kähler function including also super-variant of Dirac action defines S-matrix elements as its matrix elements between the positive and negative energy parts of the zero energy states formed from the corresponding vacua at the two boundaries of CD annihilated by annihilation operators and *resp.* creation operators. The states would be created by the monomials appearing in the super-coordinates and super-spinor.

Super-Dirac action vanishes on-mass-shell. The proposed construction relying on ZEO allows however to get scattering amplitudes between all possible states using the exponential of super-Kähler action. Super-Dirac equation is however needed and makes possible to express the derivatives of the quark oscillator operators (values of quark field at points of cognitive representation) so that one can use only the points of cognitive representation without introducing lattice discretization. Discrete coupling constant evolution conforms with the fact that the contractions of oscillator operators occur at the boundary of CD and their number is limited by the finite number of points of cognitive representation.

1.4.2 PART II: SOME APPLICATIONS

Quantum criticality and dark matter: part I

Quantum criticality is one of the corner stone assumptions of TGD. The value of Kähler coupling strength fixes quantum TGD and is analogous to critical temperature. TGD Universe would be quantum critical. What does this mean is however far from obvious and I have pondered the notion repeatedly both from the point of view of mathematical description and phenomenology. Dark matter as a hierarchy of phases of ordinary matter labelled by the value of effective Planck constant h_{eff} following as prediction of adelic physics suggests a general approach to quantum criticality. In the first part of the chapter about quantum criticality general ideas about quantum criticality and phase transitions are discussed.

Quantum criticality and dark matter: part II

Quantum criticality is one of the corner stone assumptions of TGD. The value of Kähler coupling strength fixes quantum TGD and is analogous to critical temperature. TGD Universe would be quantum critical. What does this mean is however far from obvious and I have pondered the notion repeatedly both from the point of view of mathematical description and phenomenology. Dark matter as a hierarchy of phases of ordinary matter labelled by the value of effective Planck constant h_{eff} following as prediction of adelic physics suggests a general approach to quantum criticality. In the second part of the chapter about quantum criticality condensed matter applications are discussed.

Quantum criticality and dark matter: part III

Quantum criticality is one of the corner stone assumptions of TGD. The value of Kähler coupling strength fixes quantum TGD and is analogous to critical temperature. TGD Universe would be quantum critical. What does this mean is however far from obvious and I have pondered the notion repeatedly both from the point of view of mathematical description and phenomenology. Dark matter as a hierarchy of phases of ordinary matter labelled by the value of effective Planck constant h_{eff} following as prediction of adelic physics suggests a general approach to quantum criticality. In the third part of the chapter about quantum criticality biological applications are discussed.

Quantum criticality and dark matter: part IV

Quantum criticality is one of the corner stone assumptions of TGD. The value of Kähler coupling strength fixes quantum TGD and is analogous to critical temperature. TGD Universe would be quantum critical. What does this mean is however far from obvious and I have pondered the notion repeatedly both from the point of view of mathematical description and phenomenology. Dark matter as a hierarchy of phases of ordinary matter labelled by the value of effective Planck constant h_{eff} following as prediction of adelic physics suggests a general approach to quantum criticality. In the fourth part of the chapter about quantum criticality applications, which might be labelled as miscellaneous, are discussed.

About the Nottale's formula for h_{gr} and the possibility that Planck length l_P and CP_2 length R are identical giving $G = R^2/\hbar_{eff}$?

Nottale's formula for the gravitational Planck constant $\hbar_{gr} = GMm/v_0$ involves parameter v_0 with dimensions of velocity. I have worked with the quantum interpretation of the formula but the physical origin of v_0 - or equivalently the dimensionless parameter $\beta_0 = v_0/c$ (to be used in the sequel) appearing in the formula has remained open hitherto. In this chapter a possible interpretation based on many-sheeted space-time concept, many-sheeted cosmology, and zero energy ontology (ZEO) is discussed. In ZEO the non-changing parts of zero energy states are assigned to the passive boundary of CD and β_0 should be assigned to it.

There are two measures for the size of the system. The M^4 size L_{M^4} is identifiable as the maximum of the radial M^4 distance from the tip of CD associated with the center of mass of the system along the light-like geodesic at the boundary of CD. System has also size L_{ind} defined defined in terms of the induced metric of the space-time surface, which is space-like at the boundary of CD. One has $L_{ind} < L_H$. The identification $\beta_0 = L_{M^4}/L_H$ does not allow the identification of $L_H = L_{M^4}$. L_H would however naturally corresponds to the size of the magnetic body of the system in turn identifiable as the size of CD.

One can deduce an estimate for β_0 by approximating the space-time surface as Robertson-Walker cosmology expected to be a good approximation near the passive light-like boundary of CD. The resulting formula is tested for planetary system and Earth. The dark matter assignable to Earth can be identified as the innermost part of inner core with volume, which is .01 per cent of the volume of Earth. Also the consistency of the Bohr quantization for dark and ordinary matter is discussed and leads to a number theoretical condition on the ratio of the ordinary and dark masses.

$\beta_0/4\pi$ is analogous to gravitational fine structure constant for $h_{eff} = h_{gr}$. Could one see it as fundamental coupling parameter appearing also in other interactions at quantum criticality in which ordinary perturbation series diverges? Remarkably, the value of G does not appear at all in the perturbative expansion in this region! Could G have several values? This suggests the generalization $G = l_P^2/\hbar \rightarrow G = R^2/\hbar_{eff}$ so that G would indeed have a spectrum and that Planck length l_P would be equal to CP_2 radius R so that only one fundamental length would be associated with twistorialization. Ordinary Newton's constant would be given by $G = R^2/h_{eff}$ with h_{eff}/h_0 having value in the range $10^7 - 10^8$.

The second topic of the chapter relates to the the fact that the measurements of G give differing results with differences between measurements larger than the measurement accuracy. This suggests that there might be some new physics involved. In TGD framework the hierarchy of Planck constants $h_{eff} = nh_0$, $h = 6h_0$ together with the condition that theory contains CP_2 size scale R as only fundamental length scale, suggest the possibility that Newtons constant is given by $G = R^2/\hbar_{eff}$, where R replaces Planck length ($l_P = \sqrt{\hbar G} \rightarrow l_P = R$) and $\hbar_{eff}/h$ is in the range $10^6 - 10^7$. The spectrum of Newton' constant is consistent with Newton's equations if the scaling of $\hbar_{eff}$ inducing scaling G is accompanied by opposite scaling of M^4 coordinates in $M^4 \times CP_2$: dark matter hierarchy would correspond to discrete hierarchy of scales given by breaking of scale invariance. In the special case $h_{eff} = h_{gr} = GMm/v0$ quantum critical dynamics as gravitational fine structure constant $(v_0/c)/4\pi$ as coupling constant and it has no dependence of the value of G or masses M and m.

In this chapter I consider a possible interpretation for the finding of a Chinese research group measuring two different values of G differing by 47 ppm in terms of varying h_{eff}. Also a model for fountain effect of superfluidity as de-localization of wave function and increase of the maximal height of vertical orbit due to the change of the gravitational acceleration g at surface of Earth induced by a change of h_{eff} due to super-fluidity is discussed. Also Podkletnov effect is considered. TGD inspired theory of consciousness allows to speculate about levitation experiences possibly induced by the modification of G_{eff} at the flux tubes for some part of the magnetic body accompanying biological body in TGD based quantum biology.

TGD View about Quasars

The work of Rudolph Schild and his colleagues Darryl Letier and Stanley Robertson (among others) suggests that quasars are not supermassive blackholes but something else - MECOs, magnetic eternally collapsing objects having no horizon and possessing magnetic moment. Schild et al argue that the same applies to galactic blackhole candidates and active galactic nuclei, perhaps even to ordinary blackholes as Abhas Mitra, the developer of the notion of MECO proposes.

In the sequel TGD inspired view about quasars relying on the general model for how galaxies are generated as the energy of thickened cosmic strings decays to ordinary matter is proposed. Quasars would not be be blackhole like objects but would serve as an analog of the decay of inflaton field producing the galactic matter. The energy of the string like object would replace galactic dark matter and automatically predict a flat velocity spectrum.

TGD is assumed to have standard model and GRT as QFT limit in long length scales. Could MECOs provide this limit? It seems that the answer is negative: MECOs represent still collapsing objects. The energy of inflaton field is replaced with the sum of the magnetic energy of cosmic string and positive volume energy, which both decrease as the thickness of flux tube increases. The liberated energy transforms to ordinary particles and their dark variants in TGD sense. Time reversal of blackhole would be more appropriate interpretation. One can of course ask, whether the blackhole candidates in galactic nuclei are time reversals of quasars in TGD sense.

The writing of the article led also to a considerable understanding of two key aspects of TGD. The understanding of twistor lift and p-adic evolution of cosmological constant improved considerably. Also the understanding of gravitational Planck constant and the notion of space-time as a covering space became much more detailed in turn allowing much more refined view about the anatomy of magnetic body.

Solar Surprise

The detection of gamma rays from Sun has yielded a surprises. There are 5 times more gamma rays than expected and the spectrum has a deep and narrow dip around 30-50 GeV. Spectrum continues to much higher energies than expected, at least up to 100 GeV. One proposal is that there could be dark matter in the interior of Sun yielding the gamma rays but is unclear how they could get to the surface without experiencing the same fate as ordinary gammas from nuclear reactions.

The findings provide a test bench for TGD based view about magnetic fields and the first challenge is to understand the solar cycle. The model is follows from the model for the formation of galaxies, stars, and planets as tangles of long cosmic strings thickened to flux tube. Wormhole magnetic fields correspond to closed flux tubes with monopole flux returning along different sheet. If M^4 projections of the sheets co-incide and test particle touching them experiences no net magnetic force but the energy of flux tubes is dark making itself visible through gravitational fields. For disjoint projections sheets carry measurable magnetic fields.

Polarization reversal could be understood as a quantum analog of spontaneous magnetization generating first dipole loops of type II (I) taking measured B to zero. After this dipole loops of type I (II) would split by reconnection and decay to smaller loops and leave Sun. This defines first half-cycle and for second half-cycle the roles of loops are changed.

The model discussed explains qualitatively the findings in terms of cosmic rays entering to the flux tubes of dipole fields and accelerated in the electric field of the closed flux tube and making possibly several cycles before being detected. This predicts band structure of the spectrum.

The model suggests also inversion as a Z_2 symmetry changing the roles of the flux tube portions in the interior and exterior of the solar surface. Inversion symmetry is also a symmetry of Maxwell's equations. The notions of of monopole flux tube and associated approximate Z_2 symmetry acting either as reflection or inversion could be universal. Z_2 can be also represented as a subgroup of the group of Galois symmetries predicted by adelic physics.

This picture leads to highly non-trivial predictions. For instance, the "Axis of Evil" anomaly of CMB can be understood. For instance, quantum correlations in cosmological scales explain why the plane of planetary system makes itself visible in CMB. One can also add highly non-trivial detail to the TGD inspired view about quantum biology and consciousness.

Holography and Quantum Error Correcting Codes: TGD View

Preskill et all suggest a highly interesting representation of holography in terms of quantum error correction codes The idea is that time= constant section of AdS, which is hyperbolic space allowing tessellations, can define tensor networks. So called perfect tensors are building bricks of the tensor networks providing representation for holography and at the same time defining error correcting codes by mapping localized interior states (logical qubits) to highly entangled non-local boundary states (physical qubits).

There are three observations that put bells ringing and actually motivated this article.

1. Perfect tensors define entanglement which TGD framework corresponds negentropic entanglement playing key role in TGD inspired theory of consciousness and of living matter.
2. In TGD framework the hyperbolic tesselations are realized at hyperbolic spaces $H_3(a)$ defining light-cone proper time hyperboloids of M^4 light-cone.
3. TGD replaces AdS/CFT correspondence with strong form of holography.

A very attractive idea is that in living matter magnetic flux tube networks defining quantum computational networks provide a realization of tensor networks realizing also holographic error correction mechanism: negentropic entanglement - perfect tensors - would be the key element. As I have proposed, these flux tube networks would define kind of central nervous system make it possible for living matter to experience consciously its biological body using magnetic body.

These networks would also give rise to the counterpart of condensed matter physics of dark matter at the level of magnetic body: the replacement of lattices based on subgroups of translation group with infinite number of tesselations means that this analog of condensed matter physics describes quantum complexity.

Part I

HYPER-FINITE FACTORS AND HIERARCHY OF PLANCK CONSTANTS

Chapter 2

Was von Neumann Right After All?

2.1 Introduction

The work with TGD inspired model [K5] for topological quantum computation [B28] led to the realization that von Neumann algebras [A82, A99, A87, A63] , in particular so called hyper-finite factors of type II_1 [A48] , seem to provide the mathematics needed to develop a more explicit view about the construction of S-matrix. The lecture notes of R. Longo [A85] give a concise and readable summary about the basic definitions and results related to von Neumann algebras and I have used this material freely in this chapter.

The original discussion has transformed during years from a free speculation reflecting in many aspects my ignorance about the mathematics involved to a more realistic view about the role of these algebras in quantum TGD. In this chapter I will discuss various aspects of hyperfinite factors with only a brief digression to TGD inspired applications whose evolution discussed in separate chapter [K48].

2.1.1 Philosophical Ideas Behind Von Neumann Algebras

The goal of von Neumann was to generalize the algebra of quantum mechanical observables. The basic ideas behind the von Neumann algebra are dictated by physics. The algebra elements allow Hermitian conjugation * and observables correspond to Hermitian operators. Any measurable function $f(A)$ of operator A belongs to the algebra and one can say that non-commutative measure theory is in question.

The predictions of quantum theory are expressible in terms of traces of observables. Density matrix defining expectations of observables in ensemble is the basic example. The highly non-trivial requirement of von Neumann was that identical a priori probabilities for a detection of states of infinite state system must make sense. Since quantum mechanical expectation values are expressible in terms of operator traces, this requires that unit operator has unit trace: $tr(Id) = 1$.

In the finite-dimensional case it is easy to build observables out of minimal projections to 1-dimensional eigen spaces of observables. For infinite-dimensional case the probably of projection to 1-dimensional sub-space vanishes if each state is equally probable. The notion of observable must thus be modified by excluding 1-dimensional minimal projections, and allow only projections for which the trace would be infinite using the straightforward generalization of the matrix algebra trace as the dimension of the projection.

The non-trivial implication of the fact that traces of projections are never larger than one is that the eigen spaces of the density matrix must be infinite-dimensional for non-vanishing projection probabilities. Quantum measurements can lead with a finite probability only to mixed states with a density matrix which is projection operator to infinite-dimensional subspace. The simple von Neumann algebras for which unit operator has unit trace are known as factors of type II_1 [A48] .

The definitions of adopted by von Neumann allow however more general algebras. Type I_n

algebras correspond to finite-dimensional matrix algebras with finite traces whereas I_∞ associated with a separable infinite-dimensional Hilbert space does not allow bounded traces. For algebras of type III non-trivial traces are always infinite and the notion of trace becomes useless being replaced by the notion of state which is generalization of the notion of thermodynamical state. The fascinating feature of this notion of state is that it defines a unique modular automorphism of the factor defined apart from unitary inner automorphism and the question is whether this notion or its generalization might be relevant for the construction of M-matrix in TGD. It however seems that in TGD framework based on Zero Energy Ontology identifiable as "square root" of thermodynamics a square root of thermodynamical state is needed.

The inclusions of hyper-finite factors define an excellent candidate for the description of finite measurement resolution with included factor representing the degrees of freedom below measurement resolution. The would also give connection to the notion of quantum group whose physical interpretation has remained unclear. This idea is central to the proposed applications to quantum TGD discussed in separate chapter.

2.1.2 Von Neumann, Dirac, And Feynman

The association of algebras of type I with the standard quantum mechanics allowed to unify matrix mechanism with wave mechanics. Note however that the assumption about continuous momentum state basis is in conflict with separability but the particle-in-box idealization allows to circumvent this problem (the notion of space-time sheet brings the box in physics as something completely real).

Because of the finiteness of traces von Neumann regarded the factors of type II_1 as fundamental and factors of type III as pathological. The highly pragmatic and successful approach of Dirac [A83] based on the notion of delta function, plus the emergence of s [A89] , the possibility to formulate the notion of delta function rigorously in terms of distributions [A49, A72] , and the emergence of path integral approach [A88] meant that von Neumann approach was forgotten by particle physicists.

Algebras of type II_1 have emerged only much later in conformal and topological quantum field theories [A92, A41] allowing to deduce invariants of knots, links and 3-manifolds. Also algebraic structures known as bi-algebras, Hopf algebras, and ribbon algebras [A29] relate closely to type II_1 factors. In topological quantum computation [B28] based on braid groups [A101] modular S-matrices they play an especially important role.

In algebraic quantum field theory [B31] defined in Minkowski space the algebras of observables associated with bounded space-time regions correspond quite generally to the type III_1 hyper-finite factor [B6, B32].

I have restricted the considerations of this chapter mostly to the technical aspects and Appendix includes sections about inclusions of HFFs. The evolution of ideas about possible applications to quantum TGD is summarized in chapter, which was originally part of this chapter [K48].

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

2.2 Von Neumann Algebras

In this section basic facts about von Neumann algebras are summarized using as a background material the concise summary given in the lecture notes of Longo [A85] .

2.2.1 Basic Definitions

A formal definition of von Neumann algebra [A99, A87, A63] is as a *-subalgebra of the set of bounded operators $\mathcal{B}(\mathcal{H})$ on a Hilbert space $\mathcal{H}$ closed under weak operator topology, stable under the conjugation $J =^*: x \to x^*$, and containing identity operator Id. This definition allows also von Neumann algebras for which the trace of the unit operator is not finite.

Identity operator is the only operator commuting with a simple von Neumann algebra. A general von Neumann algebra allows a decomposition as a direct integral of simple algebras, which von Neumann called factors. Classification of von Neumann algebras reduces to that for factors.

$\mathcal{B}(\mathcal{H})$ has involution * and is thus a *-algebra. $\mathcal{B}(\mathcal{H})$ has order order structure $A \geq 0$: $(Ax, x) \geq 0$. This is equivalent to $A = BB^*$ so that order structure is determined by algebraic structure. $\mathcal{B}(\mathcal{H})$ has metric structure in the sense that norm defined as supremum of $||Ax||$, $||x|| \leq 1$ defines the notion of continuity. $||A||^2 = inf\{\lambda > 0 : AA^* \leq \lambda I\}$ so that algebraic structure determines metric structure.

There are also other topologies for $\mathcal{B}(\mathcal{H})$ besides norm topology.

1. $A_i \rightarrow A$ strongly if $||Ax - A_i x|| \rightarrow 0$ for all x. This topology defines the topology of C^* algebra. $\mathcal{B}(\mathcal{H})$ is a Banach algebra that is $||AB|| \leq ||A|| \times ||B||$ (inner product is not necessary) and also C^* algebra that is $||AA^*|| = ||A||^2$.
2. $A_i \rightarrow A$ weakly if $(A_i x, y) \rightarrow (Ax, y)$ for all pairs (x, y) (inner product is necessary). This topology defines the topology of von Neumann algebra as a sub-algebra of $\mathcal{B}(\mathcal{H})$.

Denote by M' the commutant of $\mathcal{M}$ which is also algebra. Von Neumann's bicommutant theorem says that $\mathcal{M}$ equals to its own bi-commutant. Depending on whether the identity operator has a finite trace or not, one distinguishes between algebras of type II_1 and type II_∞. II_1 factor allow trace with properties $tr(Id) = 1$, $tr(xy) = tr(yx)$, and $tr(x^*x) > 0$, for all $x \neq 0$. Let $L^2(\mathcal{M})$ be the Hilbert space obtained by completing $\mathcal{M}$ respect to the inner product defined $\langle x|y\rangle = tr(x^*y)$ defines inner product in $\mathcal{M}$ interpreted as Hilbert space. The normalized trace induces a trace in M', natural trace $Tr_{M'}$, which is however not necessarily normalized. JxJ defines an element of M': if $\mathcal{H} = L^2(\mathcal{M})$, the natural trace is given by $Tr_{M'}(JxJ) = tr_M(x)$ for all $x \in M$ and bounded.

2.2.2 Basic Classification Of Von Neumann Algebras

Consider first some definitions. First of all, Hermitian operators with positive trace expressible as products xx^* are of special interest since their sums with positive coefficients are also positive.

In quantum mechanics Hermitian operators can be expressed in terms of projectors to the eigen states. There is a natural partial order in the set of isomorphism classes of projectors by inclusion: $E < F$ if the image of $\mathcal{H}$ by E is contained to the image of $\mathcal{H}$ by a suitable isomorph of F. Projectors are said to be metrically equivalent if there exist a partial isometry which maps the images $\mathcal{H}$ by them to each other. In the finite-dimensional case metric equivalence means that isomorphism classes are identical $E = F$.

The algebras possessing a minimal projection E_0 satisfying $E_0 \leq F$ for any F are called type I algebras. Bounded operators of n-dimensional Hilbert space define algebras I_n whereas the bounded operators of infinite-dimensional separable Hilbert space define the algebra I_∞. I_n and I_∞ correspond to the operator algebras of quantum mechanics. The states of harmonic oscillator correspond to a factor of type I.

The projection F is said to be finite if $F < E$ and $F \equiv E$ implies $F = E$. Hence metric equivalence means identity. Simple von Neumann algebras possessing finite projections but no minimal projections so that any projection E can be further decomposed as $E = F + G$, are called factors of type II.

Hyper-finiteness means that any finite set of elements can be approximated arbitrary well with the elements of a finite-dimensional sub-algebra. The hyper-finite II_∞ algebra can be regarded as a tensor product of hyper-finite II_1 and I_∞ algebras. Hyper-finite II_1 algebra can be regarded as a Clifford algebra of an infinite-dimensional separable Hilbert space sub-algebra of I_∞.

Hyper-finite II_1 algebra can be constructed using Clifford algebras $C(2n)$ of $2n$-dimensional spaces and identifying the element x of $2^n \times 2^n$ dimensional $C(n)$ as the element $diag(x, x)/2$ of $2^{n+1} \times 2^{n+1}$-dimensional $C(n + 1)$. The union of algebras $C(n)$ is formed and completed in the weak operator topology to give a hyper-finite II_1 factor. This algebra defines the Clifford algebra of infinite-dimensional separable Hilbert space and is thus a sub-algebra of I_∞ so that hyper-finite II_1 algebra is more regular than I_∞.

von Neumann algebras possessing no finite projections (all traces are infinite or zero) are called algebras of type III. It was later shown by [A32] [A19] that these algebras are labeled by a parameter varying in the range $[0, 1]$, and referred to as algebras of type III_x. III_1 category contains a unique hyper-finite algebra. It has been found that the algebras of observables associated with bounded regions of 4-dimensional Minkowski space in quantum field theories correspond to hyper-finite factors of type III_1 [A85] . Also statistical systems at finite temperature correspond to factors of type III and temperature parameterizes one-parameter set of automorphisms of this algebra [B6] . Zero temperature limit correspond to I_∞ factor and infinite temperature limit to II_1 factor.

2.2.3 Non-Commutative Measure Theory And Non-Commutative Topologies And Geometries

von Neumann algebras and C^* algebras give rise to non-commutative generalizations of ordinary measure theory (integration), topology, and geometry. It must be emphasized that these structures are completely natural aspects of quantum theory. In particular, for the hyper-finite type II_1 factors quantum groups and Kac Moody algebras [B34] emerge quite naturally without any need for ad hoc modifications such as making space-time coordinates non-commutative. The effective 2-dimensionality of quantum TGD (partonic or stringy 2-surfaces code for states) means that these structures appear completely naturally in TGD framework.

Non-commutative measure theory

von Neumann algebras define what might be a non-commutative generalization of measure theory and probability theory [A85] .

1. Consider first the commutative case. Measure theory is something more general than topology since the existence of measure (integral) does not necessitate topology. Any measurable function f in the space $L^\infty(X,\mu)$ in measure space (X,μ) defines a bounded operator M_f in the space $\mathcal{B}(L^2(X,\mu))$ of bounded operators in the space $L^2(X,\mu)$ of square integrable functions with action of M_f defined as $M_f g = fg$.
2. Integral over $\mathcal{M}$ is very much like trace of an operator $f_{x,y} = f(x)\delta(x,y)$. Thus trace is a natural non-commutative generalization of integral (measure) to the non-commutative case and defined for von Neumann algebras. In particular, generalization of probability measure results if the case $tr(Id) = 1$ and algebras of type I_n and II_1 are thus very natural from the point of view of non-commutative probability theory.

The trace can be expressed in terms of a cyclic vector Ω or vacuum/ground state in physicist's terminology. Ω is said to be cyclic if the completion $\overline{M\Omega} = H$ and separating if $x\Omega$ vanishes only for $x = 0$. Ω is cyclic for $\mathcal{M}$ if and only if it is separating for M'. The expression for the trace given by

$$Tr(ab) = \left(\frac{(ab+ba)}{2},\Omega\right) \quad (2.2.1)$$

is symmetric and allows to defined also inner product as $(a,b) = Tr(a^*b)$ in $\mathcal{M}$. If Ω has unit norm $(\Omega,\Omega) = 1$, unit operator has unit norm and the algebra is of type II_1. Fermionic oscillator operator algebra with discrete index labeling the oscillators defines II_1 factor. Group algebra is second example of II_1 factor.

The notion of probability measure can be abstracted using the notion of state. State ω on a C^* algebra with unit is a positive linear functional on $\mathcal{U}$, $\omega(1) = 1$. By so called KMS construction [A85] any state ω in C^* algebra $\mathcal{U}$ can be expressed as $\omega(x) = (\pi(x)\Omega,\Omega)$ for some cyclic vector Ω and π is a homomorphism $\mathcal{U} \to \mathcal{B}(\mathcal{H})$.

Non-commutative topology and geometry

C^* algebras generalize in a well-defined sense ordinary topology to non-commutative topology.

1. In the Abelian case Gelfand Naimark theorem [A85] states that there exists a contravariant functor F from the category of unital abelian C^* algebras and category of compact topological spaces. The inverse of this functor assigns to space X the continuous functions f on X with norm defined by the maximum of f. The functor assigns to these functions having interpretation as eigen states of mutually commuting observables defined by the function algebra. These eigen states are delta functions localized at single point of X. The points

of X label the eigenfunctions and thus define the spectrum and obviously span X. The connection with topology comes from the fact that continuous map $Y \rightarrow X$ corresponds to homomorphism $C(X) \rightarrow C(Y)$.

2. In non-commutative topology the function algebra $C(X)$ is replaced with a general C^* algebra. Spectrum is identified as labels of simultaneous eigen states of the Cartan algebra of C^* and defines what can be observed about non-commutative space X.

3. Non-commutative geometry can be very roughly said to correspond to *-subalgebras of C^* algebras plus additional structure such as symmetries. The non-commutative geometry of Connes [A20] is a basic example here.

2.2.4 Modular Automorphisms

von Neumann algebras allow a canonical unitary evolution associated with any state ω fixed by the selection of the vacuum state Ω [A85] . This unitary evolution is an automorphism fixed apart form unitary automorphisms $A \rightarrow UAU^*$ related with the choice of Ω.

Let ω be a normal faithful state: $\omega(x^*x) > 0$ for any x. One can map $\mathcal{M}$ to $L^2(\mathcal{M})$ defined as a completion of $\mathcal{M}$ by $x \rightarrow x\Omega$. The conjugation * in $\mathcal{M}$ has image at Hilbert space level as a map $S_0 : x\Omega \rightarrow x^*\Omega$. The closure of S_0 is an anti-linear operator and has polar decomposition $S = J\Delta^{1/2}$, $\Delta = SS^*$. Δ is positive self-adjoint operator and J anti-unitary involution. The following conditions are satisfied

$$\begin{aligned} \Delta^{it}\mathcal{M}\Delta^{-it} &= \mathcal{M} \ , \\ J\mathcal{M}J &= \mathcal{M}' \ . \end{aligned} \tag{2.2.2}$$

Δ^{it} is obviously analogous to the time evolution induced by positive definite Hamiltonian and induces also the evolution of the expectation ω as $\pi \rightarrow \Delta^{it}\pi\Delta^{-it}$.

2.2.5 Joint Modular Structure And Sectors

Let $\mathcal{N} \subset \mathcal{M}$ be an inclusion. The unitary operator $\gamma = J_N J_M$ defines a canonical endomorphisms $M \rightarrow N$ in the sense that it depends only up to inner automorphism on $\mathcal{N}$, γ defines a sector of $\mathcal{M}$. The sectors of $\mathcal{M}$ are defined as $Sect(\mathcal{M}) = End(\mathcal{M})/Inn(\mathcal{M})$ and form a semi-ring with respected to direct sum and composition by the usual operator product. It allows also conjugation.

$L^2(\mathcal{M})$ is a normal bi-module in the sense that it allows commuting left and right multiplications. For $a, b \in M$ and $x \in L^2(\mathcal{M})$ these multiplications are defined as $axb = aJb^*Jx$ and it is easy to verify the commutativity using the factor $Jy^*J \in \mathcal{M}'$. [A32] [A20] has shown that all normal bi-modules arise in this way up to unitary equivalence so that representation concepts make sense. It is possible to assign to any endomorphism ρ index $Ind(\rho) \equiv M : \rho(\mathcal{M})$. This means that the sectors are in 1-1 correspondence with inclusions. For instance, in the case of hyper-finite II_1 they are labeled by Jones index. Furthermore, the objects with non-integral dimension $\sqrt{[\mathcal{M} : \rho(\mathcal{M})]}$ can be identified as quantum groups, loop groups, infinite-dimensional Lie algebras, etc...

2.2.6 Basic Facts About Hyper-Finite Factors Of Type III

Hyper-finite factors of type II_1, II_∞ and III_1, III_0, III_λ, $\lambda \in (0, 1)$, allow by definition hierarchy of finite approximations and are unique as von Neumann algebras. Also hyper-finite factors of type II_∞ and type III could be relevant for the formulation of TGD. HFFs of type II_∞ and III could appear at the level operator algebra but that at the level of quantum states one would obtain HFFs of type II_1. These extended factors inspire highly non-trivial conjectures about quantum TGD. The book of Connes [A20] provides a detailed view about von Neumann algebras in general.

Basic definitions and facts

A highly non-trivial result is that HFFs of type II_∞ are expressible as tensor products $II_\infty = II_1 \otimes I_\infty$, where II_1 is hyper-finite [A20] .

1. *The existence of one-parameter family of outer automorphisms*

The unique feature of factors of type III is the existence of one-parameter unitary group of outer automorphisms. The automorphism group originates in the following manner.

1. Introduce the notion of linear functional in the algebra as a map $\omega : \mathcal{M} \rightarrow C$. ω is said to be hermitian it respects conjugation in $\mathcal{M}$; positive if it is consistent with the notion of positivity for elements of $\mathcal{M}$ in which case it is called weight; state if it is positive and normalized meaning that $\omega(1) = 1$, faithful if $\omega(A) > 0$ for all positive A; a trace if $\omega(AB) = \omega(BA)$, a vector state if $\omega(A)$ is "vacuum expectation" $\omega_\Omega(A) = (\Omega, \omega(A)\Omega)$ for a non-degenerate representation $(\mathcal{H}, \pi)$ of $\mathcal{M}$ and some vector $\Omega \in \mathcal{H}$ with $||\Omega|| = 1$.
2. The existence of trace is essential for hyper-finite factors of type II_1. Trace does not exist for factors of type III and is replaced with the weaker notion of state. State defines inner product via the formula $(x, y) = \phi(y^*x)$ and * is isometry of the inner product. *-operator has property known as pre-closedness implying polar decomposition $S = J\Delta^{1/2}$ of its closure. Δ is positive definite unbounded operator and J is isometry which restores the symmetry between $\mathcal{M}$ and its commutant $\mathcal{M}'$ in the Hilbert space $\mathcal{H}_\phi$, where $\mathcal{M}$ acts via left multiplication: $\mathcal{M}' = J\mathcal{M}J$.
3. The basic result of Tomita-Takesaki theory is that Δ defines a one-parameter group $\sigma_\phi^t(x) = \Delta^{it}x\Delta^{-it}$ of automorphisms of $\mathcal{M}$ since one has $\Delta^{it}\mathcal{M}\Delta^{-it} = \mathcal{M}$. This unitary evolution is an automorphism fixed apart from unitary automorphism $A \rightarrow UAU^*$ related with the choice of ϕ. For factors of type I and II this automorphism reduces to inner automorphism so that the group of outer automorphisms is trivial as is also the outer automorphism associated with ω. For factors of type III the group of these automorphisms divided by inner automorphisms gives a one-parameter group of $Out(\mathcal{M})$ of outer automorphisms, which does not depend at all on the choice of the state ϕ.

More precisely, let ω be a normal faithful state: $\omega(x^*x) > 0$ for any x. One can map $\mathcal{M}$ to $L^2(\mathcal{M})$ defined as a completion of $\mathcal{M}$ by $x \rightarrow x\Omega$. The conjugation * in $\mathcal{M}$ has image at Hilbert space level as a map $S_0 : x\Omega \rightarrow x^*\Omega$. The closure of S_0 is an anti-linear operator and has polar decomposition $S = J\Delta^{1/2}$, $\Delta = SS^*$. Δ is positive self-adjoint operator and J anti-unitary involution. The following conditions are satisfied

$$\begin{aligned} \Delta^{it}\mathcal{M}\Delta^{-it} &= \mathcal{M} \ , \\ J\mathcal{M}J &= \mathcal{M}' \ . \end{aligned} \tag{2.2.3}$$

Δ^{it} is obviously analogous to the time evolution induced by positive definite Hamiltonian and induces also the evolution of the expectation ω as $\pi \rightarrow \Delta^{it}\pi\Delta^{-it}$. What makes this result thought provoking is that it might mean a universal quantum dynamics apart from inner automorphisms and thus a realization of general coordinate invariance and gauge invariance at the level of Hilbert space.

2. Classification of HFFs of type III

Connes achieved an almost complete classification of hyper-finite factors of type III completed later by others. He demonstrated that they are labeled by single parameter $0 \leq \lambda \leq 1]$ and that factors of type III_λ, $0 \leq \lambda < 1$ are unique. Haagerup showed the uniqueness for $\lambda = 1$. The idea was that the group has an invariant, the kernel $T(M)$ of the map from time like R to $Out(M)$, consisting of those values of the parameter t for which σ_ϕ^t reduces to an inner automorphism and to unity as outer automorphism. Connes also discovered also an invariant, which he called spectrum $S(\mathcal{M})$ of $\mathcal{M}$ identified as the intersection of spectra of $\Delta_\phi \backslash \{0\}$, which is closed multiplicative subgroup of R^+.

Connes showed that there are three cases according to whether $S(\mathcal{M})$ is

1. R^+, type III_1
2. $\{\lambda^n, n \in Z\}$, type III_λ.
3. $\{1\}$, type III_0.

   The value range of λ is this by convention. For the reversal of the automorphism it would be that associated with $1/\lambda$.

Connes constructed also an explicit representation of the factors $0 < \lambda < 1$ as crossed product II_∞ factor $\mathcal{N}$ and group Z represented as powers of automorphism of II_∞ factor inducing the scaling of trace by λ. The classification of HFFs of type III reduced thus to the classification of automorphisms of $\mathcal{N} \otimes \mathcal{B}(\mathcal{H}$. In this sense the theory of HFFs of type III was reduced to that for HFFs of type II_∞ or even II_1. The representation of Connes might be also physically interesting.

Probabilistic view about factors of type III

Second very concise representation of HFFs relies on thermodynamical thinking and realizes factors as infinite tensor product of finite-dimensional matrix algebras acting on state spaces of finite state systems with a varying and finite dimension n such that one assigns to each factor a density matrix characterized by its eigen values. Intuitively one can think the finite matrix factors as associated with n-state system characterized by its energies with density matrix ρ defining a thermodynamics. The logarithm of the ρ defines the single particle quantum Hamiltonian as $H = log(\rho)$ and $\Delta = \rho = exp(H)$ defines the automorphism σ_ϕ for each finite tensor factor as $exp(iHt)$. Obviously free field representation is in question.

Depending on the asymptotic behavior of the eigenvalue spectrum one obtains different factors [A20] .

1. Factor of type I corresponds to ordinary thermodynamics for which the density matrix as a function of matrix factor approaches sufficiently fast that for a system for which only ground state has non-vanishing Boltzmann weight.
2. Factor of type II_1 results if the density matrix approaches to identity matrix sufficiently fast. This means that the states are completely degenerate which for ordinary thermodynamics results only at the limit of infinite temperature. Spin glass could be a counterpart for this kind of situation.
3. Factor of type III results if one of the eigenvalues is above some lower bound for all tensor factors in such a way that neither factor of type I or II_1 results but thermodynamics for systems having infinite number of degrees of freedom could yield this kind of situation.

This construction demonstrates how varied representations factors can have, a fact which might look frustrating for a novice in the field. In particular, the infinite tensor power of $M(2, C)$ with state defined as an infinite tensor power of $M(2, C)$ state assigning to the matrix A the complex number $(\lambda^{1/2}A_{11} + \lambda^{-1/2}\ \phi(A) = A_{22})/(\lambda^{1/2} + \lambda^{-1/2})$ defines HFF III_λ [A20] , [C20] . Formally the same algebra which for $\lambda = 1$ gives ordinary trace and HFF of type II_1, gives III factor only by replacing trace with state. This simple model was discovered by Powers in 1967.

It is indeed the notion of state or thermodynamics is what distinguishes between factors. This looks somewhat weird unless one realizes that the Hilbert space inner product is defined by the "thermodynamical" state ϕ and thus probability distribution for operators and for their thermal expectation values. Inner product in turn defines the notion of norm and thus of continuity and it is this notion which differs dramatically for $\lambda = 1$ and $\lambda < 1$ so that the completions of the algebra differ dramatically.

In particular, there is no sign about I_∞ tensor factor or crossed product with Z represented as automorphisms inducing the scaling of trace by λ. By taking tensor product of I_∞ factor represented as tensor power with induces running from $-\infty$ to 0 and II_1 HFF with indices running from 1 to ∞ one can make explicit the representation of the automorphism of II_∞ factor inducing scaling of trace by λ and transforming matrix factors possessing trace given by square root of index $\mathcal{M} : \mathcal{N}$ to those with trace 2.

2.3 Braid Group, Von Neumann Algebras, Quantum TGD, And Formation Of Bound States

The article of Vaughan Jones in [A101] discusses the relation between knot theory, statistical physics, and von Neumann algebras. The intriguing results represented stimulate concrete ideas about how to understand the formation of bound states quantitatively using the notion of join along boundaries bond. All mathematical results represented in the following discussion can be found in [A101] and in the references cited therein so that I will not bother to refer repeatedly to this article in the sequel.

2.3.1 Factors Of Von Neumann Algebras

Von Neumann algebras M are algebras of bounded linear operators acting in Hilbert space. These algebras contain identity, are closed with respect to Hermitian conjugation, and are topologically complete. Finite-dimensional von Neuman algebras decompose into a direct sum of algebras M_n, which act essentially as matrix algebras in Hilbert spaces $\mathcal{H}_{nm}$, which are tensor products $C^n \otimes \mathcal{H}_m$. Here $\mathcal{H}_m$ is an m-dimensional Hilbert space in which M_n acts trivially. m is called the multiplicity of M_n.

A factor of von Neumann algebra is a von Neumann algebra whose center is just the scalar multiples of identity. The algebra of bounded operators in an infinite-dimensional Hilbert space is certainly a factor. This algebra decomposes into "atoms" represented by one-dimensional projection operators. This kind of von Neumann algebras are called type I factors.

The so called type II_1 factors and type III factors came as a surprise even for Murray and von Neumann. II_1 factors are infinite-dimensional and analogs of the matrix algebra factors M_n. They allow a trace making possible to define an inner product in the algebra. The trace defines a generalized dimension for any subspace as the trace of the corresponding projection operator. This dimension is however continuous and in the range $[0,1]$: the finite-dimensional analog would be the dimension of the sub-space divided by the dimension of $\mathcal{H}_n$ and having values $(0, 1/n, 2/n, ..., 1)$. II_1 factors are isomorphic and there exists a minimal "hyper-finite" II_1 factor is contained by every other II_1 factor.

Just as in the finite-dimensional case, one can to assign a multiplicity to the Hilbert spaces where II_1 factors act on. This multiplicity, call it $dim_M(\mathcal{H})$ is analogous to the dimension of the Hilbert space tensor factor $\mathcal{H}_m$, in which II_1 factor acts trivially. This multiplicity can have all positive real values. Quite generally, von Neumann factors of type I and II_1 are in many respects analogous to the coefficient field of a vector space.

2.3.2 Sub-Factors

Sub-factors $N \subset M$, where N and M are of type II_1 and have same identity, can be also defined. The observation that M is analogous to an algebraic extension of N motivates the introduction of index $|M : N|$, which is essentially the dimension of M with respect to N. This dimension is an analog for the complex dimension of CP_2 equal to 2 or for the algebraic dimension of the extension of p-adic numbers.

The following highly non-trivial results about the dimensions of the tensor factors hold true.

1. If $N \subset M$ are II_1 factors and $|M : N| < 4$, there is an integer $n \geq 3$ such $|M : N| = r = 4cos^2(\pi/n)$, $n \geq 3$.
2. For each number $r = 4cos^2(\pi/n)$ and for all $r \geq 4$ there is a sub-factor $R_r \subset R$ with $|R : R_r| = r$.

   One can say that M effectively decomposes to a tensor product of N with a space, whose dimension is quantized to a certain algebraic number r. The values of r corresponding to $n = 3, 4, 5, 6...$ are $r = 1, 2, 1 + \Phi \simeq 2.61, 3, ...$ and approach to the limiting value $r = 4$. For $r \geq 4$ the dimension becomes continuous.

An even more intriguing result is that by starting from $N \subset M$ with a projection e_N: $M \to N$ one can extend M to a larger II_1 algebra $\langle M, e_N \rangle$ such that one has

$$\begin{aligned} |\langle M, e_N \rangle : M| &= |M : N| \ , \\ tr(xe_N) &= |M : N|^{-1} tr(x) \ , \ \ x \in M \ . \end{aligned} \tag{2.3.1}$$

One can continue this process and the outcome is a tower of II_1 factors $M_i \subset M_{i+1}$ defined by $M_1 = N$, $M_2 = M$, $M_{i+1} = \langle M_i, e_{M_{i-1}} \rangle$. Furthermore, the projection operators $e_{M_i} \equiv e_i$ define a Temperley-Lieb representation of the braid algebra via the formulas

$$\begin{aligned} e_i^2 &= e_i \ , \\ e_i e_{i\pm 1} e_i &= \tau e_i \ , \ \tau = 1/|M:N| \\ e_i e_j &= e_j e_i \ , \ |i-j| \geq 2 \ . \end{aligned} \tag{2.3.2}$$

Temperley Lieb algebra will be discussed in more detail later. Obviously the addition of a tensor factor of dimension r is analogous with the addition of a strand to a braid.

The hyper-finite algebra R is generated by the set of braid generators $\{e_1, e_2, ...\}$ in the braid representation corresponding to r. Sub-factor R_1 is obtained simply by dropping the lowest generator e_1, R_2 by dropping e_1 and e_2, etc..

2.3.3 Ii$_1$ Factors And The Spinor Structure Of WCW

The following observations serve as very suggestive guidelines for how one could interpret the above described results in TGD framework.

1. The discrete spectrum of dimensions $1, 2, 1+\Phi, 3, ..$ below $r < 4$ brings in mind the discrete energy spectrum for bound states whereas the for $r \geq 4$ the spectrum of dimensions is analogous to a continuum of unbound states. The fact that r is an algebraic number for $r < 4$ conforms with the vision that bound state entanglement corresponds to entanglement probabilities in an extension of rationals defining a finite-dimensional extension of p-adic numbers for every prime p.
2. The discrete values of r correspond precisely to the angles ϕ allowed by the unitarity of Temperley-Lieb representations of the braid algebra with $d = -\sqrt{r}$. For $r \geq 4$ Temperley-Lieb representation is not unitary since $cos^2(\pi/n)$ becomes formally larger than one (n would become imaginary and continuous). This could mean that $r \geq 4$, which in the generic case is a transcendental number, represents unbound entanglement, which in TGD Universe is not stable against state preparation and state function reduction processes.
3. The formula $tr(xe_N) = |M:N|^{-1} tr(x)$ is completely analogous to the formula characterizing the normalization of the link invariant induced by the second Markov move in which a new strand is added to a braid such that it braids only with the leftmost strand and therefore does not change the knot resulting as a link closure. Hence the addition of a single strand seems to correspond to an introduction of an r-dimensional sub-factor to II$_1$ factor.

In TGD framework the generation of bound state has the formation of (possibly braided join along boundaries bonds as a space-time correlate and this encourages a rather concrete interpretation of these findings. Also the I$_1$ factors themselves have a nice interpretation in terms of the WCW spinor structure.

1. The interpretation of II$_1$ factors in terms of Clifford algebra of WCW

The Clifford algebra of an infinite-dimensional Hilbert space defines a II$_1$ factor. The counterparts for e_i would naturally correspond to the analogs of projection operators $(1+\sigma_i)/2$ and thus to operators of form $(1+\Sigma_{ij})/2$, defined by a subset of sigma matrices. The first guess is that the index pairs are $(i,j) = (1,2), (2,3), (3,4), ...$ The dimension of the Clifford algebra is 2^N for N-dimensional space so that $\Delta N = 1$ would correspond to $r = 2$ in the classical case and to one qubit. The problem with this interpretation is $r > 2$ has no physical interpretation: the formation of bound states is expected to reduce the value of r from its classical value rather than increase it.

One can however consider also the sequence $(i,j) = (1, 1+k), (1+k, 1+2k), (1+2k, 1+3k), ...$ For $k = 2$ the reduction of r from $r = 4$ would be due to the loss of degrees of freedom due to the formation of a bound state and $(r = 4, \Delta N = 2)$ would correspond to the classical limit resulting at the limit of weak binding. The effective elimination of the projection operators from the braid algebra would reflect this loss of degrees of freedom. This interpretation could at least be an appropriate starting point in TGD framework.

In TGD Universe physical states correspond to WCW spinor fields, whose gamma matrix algebra is constructed in terms of second quantized free induced spinor fields defined at space-time sheets. The original motivation was the idea that the quantum states of the Universe correspond to the modes of purely classical free spinor fields in the infinite-dimensional configuration space of 3-surfaces (the "world of classical worlds", WCW) possessing general coordinate invariant (in 4-dimensional sense!) Kähler geometry. Quantum information-theoretical motivation could have come from the requirement that these fields must be able to code information about the properties of the point (3-surface, and corresponding space-time sheet). Scalar fields would treat the 3-surfaces as points and are thus not enough. Induced spinor fields allow however an infinite number of modes: according to the naïve Fourier analyst's intuition these modes are in one-one correspondence with the points of the 3-surface. Second quantization gives much more. Also non-local information about the induced geometry and topology must be coded, and here quantum entanglement for states generated by the fermionic oscillator operators coding information about the geometry of 3-surface provides enormous information storage capacity.

In algebraic geometry also the algebra of the embedding space of algebraic variety divided by the ideal formed by functions vanishing on the surface codes information about the surface: for instance, the maximal ideals of this algebra code for the points of the surface (functions of embedding space vanishing at a particular point). The function algebra of the embedding space indeed plays a key role in the construction of WCW-geometry besides second quantized fermions.

The Clifford algebra generated by the WCW gamma matrices at a given point (3-surface) of WCW of 3-surfaces could be regarded as a II_1-factor associated with the local tangent space endowed with Hilbert space structure (WCW Kähler metric). The counterparts for e_i would naturally correspond to the analogs of projection operators $(1+\sigma_i)/2$ and thus operators of form $(G_{\overline{A}B} \times 1 + \Sigma_{\overline{A}B})$ formed as linear combinations of components of the Kähler metric and of the sigma matrices defined by gamma matrices and contracted with the generators of the isometries of WCW. The addition of single complex degree of freedom corresponds to $\Delta N = 2$ and $r = 4$ and the classical limit and would correspond to the addition of single braid. $(r < 4, \Delta N < 2)$ would be due to the binding effects.

$r = 1$ corresponds to $\Delta N = 0$. The first interpretation is in terms of strong binding so that the addition of particle does not increase the number of degrees of freedom. In TGD framework $r = 1$ might also correspond to the addition of zero modes which do not contribute to the WCW metric and spinor structure but have a deep physical significance. $(r = 2, \Delta N = 1)$ would correspond to strong binding reducing the spinor and space-time degrees of freedom by a factor of half. $r = \Phi^2$ $(n = 5)$ *resp.* $r = 3$ $(n = 6)$ corresponds to $\Delta N_r \simeq 1.3885$ *resp.* $\Delta N_r = 1.585$. Using the terminology of quantum field theories, one might say that in the infinite-dimensional context a given complex bound state degree of freedom possesses anomalous real dimension $r < 2$. $r \geq 4$ would correspond to a unbound entanglement and increasingly classical behavior.

2.3.4 About Possible Space-Time Correlates For The Hierarchy Of II_1 Sub-Factors

By quantum classical correspondence the infinite-dimensional physics at WCW level should have definite space-time correlates. In particular, the dimension r should have some fractal dimension as a space-time correlate.

1. Quantum classical correspondence

Join along boundaries bonds serve as correlates for bound state formation. The presence of join along boundaries bonds would lead to a generation of bound states just by reducing the degrees of freedom to those of connected 3-surface. The bonds would constrain the two 3-surfaces to single space-like section of embedding space.

This picture would allow to understand the difficulties related to Bethe-Salpeter equations for bound states based on the assumption that particles are points moving in M^4. The restriction of particles to time=constant section leads to a successful theory which is however non-relativistic. The basic binding energy would relate to the entanglement of the states associated with the bonded 3-surfaces. Since the classical energy associated with the bonds is positive, the binding energy tends to be reduced as r increases.

By spin glass degeneracy join along boundaries bonds have an infinite number of degrees of freedom in the ordinary sense. Since the system is infinite-dimensional and quantum critical, one expects that the number r of degrees freedom associated with a single join along boundaries bond is universal. Since join along boundaries bonds correspond to the strands of a braid and are correlates for the bound state formation, the natural guess is that $r = 4cos^2(\pi/n)$, $n = 3, 4, 5, ...$ holds true. $r < 4$ should characterize both binding energy and the dimension of the effective tensor factor introduced by a new join along boundaries bond.

The assignment of 2 "bare" and $\Delta N \leq 2$ renormalized real dimensions to single join along boundaries bond is consistent with the effective two-dimensionality of anyon systems and with the very notion of the braid group. The picture conforms also with the fact that the degrees of freedom in question are associated with metrically 2-dimensional light-like boundaries (of say magnetic flux tubes) acting as causal determinants. Also vibrational degrees of freedom described by Kac-Moody algebra are present and the effective 2-dimensionality means that these degrees of freedom are not excited and only topological degrees of freedom coded by the position of the puncture remain.

($r \geq 4, \Delta N \geq 2$), if possible at all, would mean that the tensor factor associated with the join along boundaries bond is effectively more than 4-dimensional due to the excitation of the vibrational Kac-Moody degrees of freedom. The finite value of r would mean that most of theme are eliminated also now but that their number is so large that bound state entanglement is not possible anymore.

The introduction of non-integer dimension could be seen as an effective description of an infinite-dimensional system as a finite-dimensional system in the spirit of renormalization group philosophy. The non-unitarity of $r \geq 4$ Temperley-Lieb representations could mean that they correspond to unbound entanglement unstable against state function reduction and preparation processes. Since this kind of entanglement does not survive in quantum jump it is not representable in terms of braid groups.

2. Does r define a fractal dimension of CP_2 projection of partonic 2-surface?

On basis of the quantum classical correspondence one expects that r should define some fractal dimension at the space-time level. Since r varies in the range $1, .., 4$ and corresponds to the fractal dimension of 2-D Clifford algebra the corresponding spinors would have dimension $d = \sqrt{r}$. There are two options.

1. $D = r/2$ is suggested on basis of the construction of quantum version of M^d.
2. $D = log_2(r)$ is natural on basis of the dimension $d = 2^{D/2}$ of spinors in D-dimensional space.

r can be assigned with CP_2 degrees of freedom in the model for the quantization of Planck constant based on the explicit identification of Josephson inclusions in terms of finite subgroups of $SU(2) \subset SU(3)$. Hence D should relate to the CP_2 projection of the partonic 2-surface and one could have $D = D(X^2)$, the latter being the average dimension of the CP_2 projection of the partonic 2-surface for the preferred extremals of Kähler action.

Since a strongly interacting non-perturbative phase should be in question, the dimension for the CP_2 projection of the space-time surface must be at least $D(X^4) = 2$ to guarantee that non-vacuum extremals are in question. This is true for $D(X^2) = r/2 \geq 1$. The logarithmic formula $D(X^2) = log_2(r) \geq 0$ gives $D(X^2) = 0$ for $n = 3$ meaning that partonic 2-surfaces are vacua: space-time surface can still be a non-vacuum extremal.

As n increases, the number of CP_2 points covering a given M^4 point and related by the finite subgroup of $G \subset SU(2) \subset SU(3)$ defining the inclusion increases so that the fractal dimension of the CP_2 projection is expected to increase also. $D(X^2) = 2$ would correspond to the space-time surfaces for which partons have topological magnetic charge forcing them to have a 2-dimensional CP_2 projection. There are reasons to believe that the projection must be homologically non-trivial geodesic sphere of CP_2.

2.3.5 Could Binding Energy Spectra Reflect The Hierarchy Of Effective Tensor Factor Dimensions?

If one takes completely seriously the idea that join along boundaries bonds are a correlate of binding then the spectrum of binding energies might reveal the hierarchy of the fractal dimensions $r(n)$. Hydrogen atom and harmonic oscillator have become symbols for bound state systems. Hence it is of interest to find whether the binding energy spectrum of these systems might be expressed in terms of the "binding dimension" $x(n) = 4 - r(n)$ characterizing the deviation of dimension from that at the limit of a vanishing binding energy. The binding energies of hydrogen atom are in a good approximation given by $E(n)/E(1) = 1/n^2$ whereas in the case of harmonic oscillator one has $E(n)/E_0 = 2n+1$. The constraint $n \geq 3$ implies that the principal quantum number must correspond $n-2$ in the case of hydrogen atom and to $n-3$ in the case of harmonic oscillator.

Before continuing one must face an obvious objection. By previous arguments different values of r correspond to different values of $\hbar$. The value of $\hbar$ cannot however differ for the states of hydrogen atom. This is certainly true. The objection however leaves open the possibility that the states of the light-like boundaries of join along boundaries bonds correspond to reflective level and represent some aspects of the physics of, say, hydrogen atom.

In the general case the energy spectrum satisfies the condition

$$\frac{E_B(n)}{E_B(3)} = \frac{f(4-r(n))}{f(3)} , \tag{2.3.3}$$

where f is some function. The simplest assumption is that the spectrum of binding energies $E_B(n) = E(n) - E(\infty)$ is a linear function of $r(n) - 4$:

$$\frac{E_B(n)}{E_B(3)} = \frac{4-r(n)}{3} = \frac{4}{3}sin^2(\frac{\pi}{n}) \rightarrow \frac{4\pi^2}{3} \times \frac{1}{n^2} . \tag{2.3.4}$$

In the linear approximation the ratio $E(n+1)/E(n)$ approaches $(n/n+1)^2$ as in the case of hydrogen atom but for small values the linear approximation fails badly. An exact correspondence results for

$$\frac{E(n)}{E(1)} = \frac{1}{n^2} ,$$

$$n = \frac{1}{\pi \arcsin\left(\sqrt{1-r(n+2)/4}\right)} - 2 .$$

Also the ionized states with $r \geq 4$ would correspond to bound states in the sense that two particle would be constrained to move in the same space-like section of space-time surface and should be distinguished from genuinely free states when particles correspond to disjoint space-time sheets.

For the harmonic oscillator one express $E(n) - E(0)$ instead of $E(n) - E(\infty)$ as a function of $x = 4 - r$ and one would have

$$\frac{E(n)}{E(0)} = 2n+1 ,$$

$$n = \frac{1}{\pi \arcsin\left(\sqrt{1-r(n+3)/4}\right)} - 3 .$$

In this case ionized states would not be possible due to the infinite depth of the harmonic oscillator potential well.

2.3.6 Four-Color Problem, II_1 Factors, And Anyons

The so called four-color problem can be phrased as a question whether it is possible to color the regions of a plane map using only four colors in such a way that no adjacent regions have the same color (for an enjoyable discussion of the problem see [A67]). One might call this kind of coloring complete. There is no loss of generality in assuming that the map can be represented as a graph with regions represented as triangle shaped faces of the graph. For the dual graph the coloring of faces becomes coloring of vertices and the question becomes whether the coloring is possible in such a way that no vertices at the ends of the same edge have same color. The problem can be generalized by replacing planar maps with maps defined on any two-dimensional surface with or without boundary and arbitrary topology. The four-color problem has been solved with an extensive use of computer [A25] but it would be nice to understand why the complete coloring with four colors is indeed possible.

There is a mysterious looking connection between four-color problem and the dimensions $r(n) = 4cos^2(\pi/n)$, which are in fact known as Beraha numbers in honor of the discoverer of this connection [A53] . Consider a more general problem of coloring two-dimensional map using m

colors. One can construct a polynomial $P(m)$, so called chromatic polynomial, which tells the number of colorings satisfying the condition that no neighboring vertices have the same color. The vanishing of the chromatic polynomial for an integer value of m tells that the complete coloring using m colors is not possible.

$P(m)$ has also other than integer valued real roots. The strange discovery due to Beraha is that the numbers $B(n)$ appear as approximate roots of the chromatic polynomial in many situations. For instance, the four non-integral real roots of the chromatic polynomial of the truncated icosahedron are very close to $B(5)$, $B(7)$, $B(8)$ and $B(9)$. These findings led Beraha to formulate the following conjecture. Let P_i be a sequence of chromatic polynomials for a graph for which the number of vertices approaches infinity. If r_i is a root of the polynomial approaching a well-defined value at the limit $i \rightarrow \infty$, then the limiting value of $r(i)$ is Beraha number.

A physicist's proof for Beraha's conjecture based on quantum groups and conformal theory has been proposed [A53] . It is interesting to look for the a possible physical interpretation of 4-color problem and Beraha's conjecture in TGD framework.

1. In TGD framework $B(n)$ corresponds to a renormalized dimension for a 2-spin system consisting of two qubits, which corresponds to 4 different colors. For $B(n) = 4$ two spin 1/2 fermions obeying Fermi statistics are in question. Since the system is 2-dimensional, the general case corresponds to two anyons with fractional spin $B(n)/4$ giving rise to $B(n) < 4$ colors and obeying fractional statistics instead of Fermi statistics. One can replace coloring problem with the problem whether an ideal antiferro-magnetic lattice using anyons with fractional spin $B(n)/4$ is possible energetically. In other words, does this system form a quantum mechanical bound state even at the limit when the lengths of the edges approach to zero.
2. The failure of coloring means that there are at least two neighboring vertices in the lattice with the property that the spins at the ends of the same edge are in the same direction. Lattice defect would be in question. At the limit of an infinitesimally short edge length the failure of coloring is certainly not an energetically favored option for fermionic spins ($m = 4$) but is allowed by anyonic statistics for $m = B(n) < 4$. Thus one has reasons to expect that when anyonic spin is $B(n)/4$ the formation of a purely 2-anyon bound states becomes possible and they form at the limit of an infinitesimal edge length a kind of topological macroscopic quantum phase with a non-vanishing binding energy. That $B(n)$ are roots of the chromatic polynomial at the continuum limit would have a clear physical interpretation.
3. Only $B(n) < 4$ defines a sub-factor of von Neumann algebra allowing unitary Temperley-Lieb representations. This is consistent with the fact that for $m = 4$ complete coloring must exists. The physical argument is that otherwise a macroscopic quantum phase with non-vanishing binding energy could result at the continuum limit and the upper bound for r from unitarity would be larger than 4. For $m = 4$ the completely anti-ferromagnetic state would represent the ground state and the absence of anyon-pair condensate would mean a vanishing binding energy.

2.4 Inclusions Of II_1 And III_1 Factors

Inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras have physical interpretation as a mathematical description for sub-system-system relation. For type I algebras the inclusions are trivial and tensor product description applies as such. For factors of II_1 and III the inclusions are highly non-trivial. The inclusion of type II_1 factors were understood by Vaughan Jones [A2] and those of factors of type III by Alain Connes [A19] .

Sub-factor $\mathcal{N}$ of $\mathcal{M}$ is defined as a closed *-stable C-subalgebra of $\mathcal{M}$. Let $\mathcal{N}$ be a sub-factor of type II_1 factor $\mathcal{M}$. Jones index $\mathcal{M} : \mathcal{N}$ for the inclusion $\mathcal{N} \subset \mathcal{M}$ can be defined as $\mathcal{M} : \mathcal{N} = dim_N(L^2(\mathcal{M})) = Tr_{N'}(id_{L^2(\mathcal{M})})$. One can say that the dimension of completion of $\mathcal{M}$ as $\mathcal{N}$ module is in question.

2.4.1 Basic Findings About Inclusions

What makes the inclusions non-trivial is that the position of $\mathcal{N}$ in $\mathcal{M}$ matters. This position is characterized in case of hyper-finite II_1 factors by index $\mathcal{M} : \mathcal{N}$ which can be said to the dimension of $\mathcal{M}$ as $\mathcal{N}$ module and also as the inverse of the dimension defined by the trace of the projector from $\mathcal{M}$ to $\mathcal{N}$. It is important to notice that $\mathcal{M} : \mathcal{N}$ does not characterize either $\mathcal{M}$ or $\mathcal{M}$, only the embedding.

The basic facts proved by Jones are following [A2] .

1. For pairs $\mathcal{N} \subset \mathcal{M}$ with a finite principal graph the values of $\mathcal{M} : \mathcal{N}$ are given by

$$\begin{aligned} &a)\ \ \mathcal{M} : \mathcal{N} = 4cos^2(\pi/h)\ \ ,\ \ \ h \geq 3\ \ , \\ &b)\ \ \mathcal{M} : \mathcal{N} \geq 4\ \ . \end{aligned} \tag{2.4.1}$$

   the numbers at right hand side are known as Beraha numbers [A53] . The comments below give a rough idea about what finiteness of principal graph means.

2. As explained in [B34] , for $\mathcal{M} : \mathcal{N} < 4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra g with h equal to the Coxeter number h of the Lie algebra given in terms of its dimension and dimension r of Cartan algebra r as $h = (dimg(g)-r)/r$. The Lie algebras of $SU(n)$, E_7 and D_{2n+1} are however not allowed. For $\mathcal{M} : \mathcal{N} = 4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of SU(2) and the interpretation proposed in [A96] is following. The ADE diagrams are associated with the $n = \infty$ case having $\mathcal{M} : \mathcal{N} \geq 4$. There are diagrams corresponding to infinite subgroups: SU(2) itself, circle group U(1), and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection. The diagrams corresponding to finite subgroups are extension of A_n for cyclic groups, of D_n dihedral groups, and of E_n with n=6,7,8 for tetrahedron, cube, dodecahedron. For $\mathcal{M} : \mathcal{N} < 4$ ordinary Dynkin graphs of D_{2n} and E_6, E_8 are allowed.

The interpretation of [A96] is that the subfactors correspond to inclusions $\mathcal{N} \subset \mathcal{M}$ defined in the following manner.

1. Let G be a finite subgroup of SU(2). Denote by R the infinite-dimensional Clifford algebras resulting from infinite-dimensional tensor power of $M_2(C)$ and by R_0 its subalgebra obtained by restricting $M_2(C)$ element of the first factor to be unit matrix. Let G act by automorphisms in each tensor factor. G leaves R_0 invariant. Denote by R_0^G and R^G the sub-algebras which remain element wise invariant under the action of G. The resulting Jones inclusions $R_0^G \subset R^G$ are consistent with the ADE correspondence.

2. The argument suggests the existence of quantum versions of subgroups of SU(2) for which representations are truncations of those for ordinary subgroups. The results have been generalized to other Lie groups.

3. Also $SL(2,C)$ acts as automorphisms of $M_2(C)$. An interesting question is what happens if one allows G to be any discrete subgroups of SL(2,C). Could this give inclusions with $\mathcal{M} : \mathcal{N} > 4$?. The strong analogy of the spectrum of indices with spectrum of energies with hydrogen atom would encourage this interpretation: the subgroup SL(2,C) not reducing to those of SU(2) would correspond to the possibility for the particle to move with respect to each other with constant velocity.

2.4.2 The Fundamental Construction And Temperley-Lieb Algebras

It was shown by Jones [A52] that for a given Jones inclusion with $\beta = \mathcal{M} : \mathcal{N} < \infty$ there exists a tower of finite II_1 factors $\mathcal{M}_k$ for $k = 0, 1, 2, ...$ such that

1. $\mathcal{M}_0 = \mathcal{N}$, $\mathcal{M}_1 = \mathcal{M}$,

2. $\mathcal{M}_{k+1} = End_{\mathcal{M}_{k-1}}\mathcal{M}_k$ is the von Neumann algebra of operators on $L^2(\mathcal{M}_k)$ generated by $\mathcal{M}_k$ and an orthogonal projection $e_k : L^2(\mathcal{M}_k) \rightarrow L^2(\mathcal{M}_{k-1})$ for $k \geq 1$, where $\mathcal{M}_k$ is regarded as a subalgebra of $\mathcal{M}_{k+1}$ under right multiplication.

It can be shown that $\mathcal{M}_{k+1}$ is a finite factor. The sequence of projections on $\mathcal{M}_\infty = \cup_{k\geq 0}\mathcal{M}_k$ satisfies the relations

$$\begin{array}{ll} e_i^2 = e_i \ , & e_i^{=} e_i \ , \\ e_i = \beta e_i e_j e_i & \text{for } \ |i-j| = 1 \ , \\ e_i e_j = e_j e_i & \text{for } \ |i-j| \geq 2 \ . \end{array} \qquad (2.4.2)$$

The construction of hyper-finite II_1 factor using Clifford algebra $C(2)$ represented by 2×2 matrices allows to understand the theorem in $\beta = 4$ case in a straightforward manner. In particular, the second formula involving β follows from the identification of x at $(k-1)^{th}$ level with $(1/\beta)diag(x,x)$ at k^{th} level.

By replacing 2×2 matrices with $\sqrt{\beta} \times \sqrt{\beta}$ matrices one can understand heuristically what is involved in the more general case. $\mathcal{M}_k$ is $\mathcal{M}_{k-1}$ module with dimension $\sqrt{\beta}$ and $\mathcal{M}_{k+1}$ is the space of $\sqrt{\beta} \times \sqrt{\beta}$ matrices $\mathcal{M}_{k-1}$ valued entries acting in $\mathcal{M}_k$. The transition from $\mathcal{M}_k$ to $\mathcal{M}_{k-1}$ linear maps of $\mathcal{M}_k$ happens in the transition to the next level. x at $(k-1)^{th}$ level is identified as $(x/\beta) \times Id_{\sqrt{\beta} \times \sqrt{\beta}}$ at the next level. The projection e_k picks up the projection of the matrix with $\mathcal{M}_{k-1}$ valued entries in the direction of the $Id_{\sqrt{\beta} \times \sqrt{\beta}}$.

The union of algebras $A_{\beta,k}$ generated by $1, e_1, ..., e_k$ defines Temperley-Lieb algebra A_β [A94] . This algebra is naturally associated with braids. Addition of one strand to a braid adds one generator to this algebra and the representations of the Temperley Lieb algebra provide link, knot, and 3-manifold invariants [A101] . There is also a connection with systems of statistical physics and with Yang-Baxter algebras [A28] .

A further interesting fact about the inclusion hierarchy is that the elements in $\mathcal{M}_i$ belonging to the commutator $\mathcal{N}'$ of $\mathcal{N}$ form finite-dimensional spaces. Presumably the dimension approaches infinity for $n \to \infty$.

2.4.3 Connection With Dynkin Diagrams

The possibility to assign Dynkin diagrams ($\beta < 4$) and extended Dynkin diagrams ($\beta = 4$ to Jones inclusions can be understood heuristically by considering a characterization of so called bipartite graphs [A98] , [B34] by the norm of the adjacency matrix of the graph.

Bipartite graphs Γ is a finite, connected graph with multiple edges and black and white vertices such that any edge connects white and black vertex and starts from a white one. Denote by $w(\Gamma)$ ($b(\Gamma)$) the number of white (black) vertices. Define the adjacency matrix $\Lambda = \Lambda(\Gamma)$ of size $b(\Gamma) \times w(\Gamma)$ by

$$w_{b,w} = \left\{ \begin{array}{ll} m(e) & \text{if there exists } e \text{ such that } \ \delta e = b - w \ , \\ 0 & \text{otherwise} \ . \end{array} \right. \qquad (2.4.3)$$

Here $m(e)$ is the multiplicity of the edge e.

Define norm $||\Gamma||$ as

$$\begin{array}{rcl} ||X|| & = & max\{||X||; \ ||x|| \leq 1\} \ , \\ ||\Gamma|| & = & ||\Lambda(\Gamma)|| = \left|\left| \begin{array}{ll} 0 & \Lambda(\Gamma) \\ \Lambda(\Gamma)^t & 0 \end{array} \right|\right| \ . \end{array} \qquad (2.4.4)$$

Note that the matrix appearing in the formula is $(m+n) \times (m+n)$ symmetric square matrix so that the norm is the eigenvalue with largest absolute value.

Suppose that Γ is a connected finite graph with multiple edges (sequences of edges are regarded as edges). Then

1. If $||\Gamma|| \leq 2$ and if Γ has a multiple edge, $||\Gamma|| = 2$ and $\Gamma = \tilde{A}_1$, the extended Dynkin diagram for $SU(2)$ Kac Moody algebra.
2. $||\Gamma|| < 2$ if and only Γ is one of the Dynkin diagrams of A,D,E. In this case $||\Gamma|| = 2cos(\pi/h)$, where h is the Coxeter number of Γ.
3. $||\Gamma|| = 2$ if and only if Γ is one of the extended Dynkin diagrams $\tilde{A}, \tilde{D}, \tilde{E}$.

This result suggests that one can indeed assign to the Jones inclusions Dynkin diagrams. To really understand how the inclusions can be characterized in terms bipartite diagrams would require a deeper understanding of von Neumann algebras. The following argument only demonstrates that bipartite graphs naturally describe inclusions of algebras.

1. Consider a bipartite graph. Assign to each white vertex linear space $W(w)$ and to each edge of a linear space $W(b, w)$. Assign to a given black vertex the vector space $\oplus_{\delta e=b-w} W(b, w) \otimes W(w)$ where (b, w) corresponds to an edge ending to b.
2. Define $\mathcal{N}$ as the direct sum of algebras $End(W(w))$ associated with white vertices and $\mathcal{M}$ as direct sum of algebras $\oplus_{\delta e=b-w} End(W(b, w)) \otimes End(W(w))$ associated with black vertices.
3. There is homomorphism $N \rightarrow M$ defined by embedding direct sum of white endomorphisms x to direct sum of tensor products x with the identity endomorphisms associated with the edges starting from x.

It is possible to show that Jones inclusions correspond to the Dynkin diagrams of A_n, D_{2n}, and E_6, E_8 and extended Dynkin diagrams of ADE type. In particular, the dual of the bi-partite graph associated with $\mathcal{M}_{n-1} \subset \mathcal{M}_n$ obtained by exchanging the roles of white and black vertices describes the inclusion $\mathcal{M}_n \subset \mathcal{M}_{n+1}$ so that two subsequent Jones inclusions might define something fundamental (the corresponding space-time dimension is $2 \times log_2(\mathcal{M} : \mathcal{N}) \leq 4$.

2.4.4 Indices For The Inclusions Of Type III_1 Factors

Type III_1 factors appear in relativistic quantum field theory defined in 4-dimensional Minkowski space [B6] . An overall summary of basic results discovered in algebraic quantum field theory is described in the lectures of Longo [A85] . In this case the inclusions for algebras of observables are induced by the inclusions for bounded regions of M^4 in axiomatic quantum field theory. Tomita's theory of modular Hilbert algebras [A81] , [B32] forms the mathematical corner stone of the theory.

The basic notion is Haag-Kastler net [A78] consisting of bounded regions of M^4. Double cone serves as a representative example. The von Neumann algebra $\mathcal{A}(O)$ is generated by observables localized in bounded region O. The net satisfies the conditions implied by local causality:

1. Isotony: $O_1 \subset O_2$ implies $\mathcal{A}(O_1) \subset \mathcal{A}(O_2)$.
2. Locality: $O_1 \subset O_2'$ implies $\mathcal{A}(O_1) \subset \mathcal{A}(O_2)'$ with O' defined as $\{x : \langle x, y\rangle < 0 \text{ for all } y \in O\}$.
3. Haag duality $\mathcal{A}(O')' = \mathcal{A}(O)$.

   Besides this Poincare covariance, positive energy condition, and the existence of vacuum state is assumed.

DHR (Doplicher-Haag-Roberts) [A36] theory allows to deduce the values of Jones index and they are squares of integers in dimensions $D > 2$ so that the situation is rather trivial. The 2-dimensional case is distinguished from higher dimensional situations in that braid group replaces permutation group since the paths representing the flows permuting identical particles can be linked in $X^2 \times T$ and anyonic statistics [D33, D36] becomes possible. In the case of 2-D Minkowski space M^2 Jones inclusions with $\mathcal{M} : \mathcal{N} < 4$ plus a set of discrete values of $\mathcal{M} : \mathcal{N}$ in the range $(4, 6)$ are possible. In [A85] some values are given ($\mathcal{M} : \mathcal{N} = 5, 5.5049..., 5.236...., 5.828...$).

At least intersections of future and past light cones seem to appear naturally in TGD framework such that the boundaries of future/past directed light cones serve as seats for incoming/outgoing states defined as intersections of space-time surface with these light cones. III_1 sectors cannot thus be excluded as factors in TGD framework. On the other hand, the construction of S-matrix at space-time level is reduced to II_1 case by effective 2-dimensionality.

2.5 TGD And Hyper-Finite Factors Of Type II_1

By effective 2-dimensionality of the construction of quantum states the hyper-finite factors of type II_1 fit naturally to TGD framework. In particular, infinite dimensional spinors define a canonical representations of this kind of factor. The basic question is whether only hyper-finite factors of type II_1 appear in TGD framework. Affirmative answer would allow to interpret physical M-matrix as time like entanglement coefficients.

2.5.1 What Kind Of Hyper-Finite Factors One Can Imagine In TGD?

The working hypothesis has been that only hyper-finite factors of type II_1 appear in TGD. The basic motivation has been that they allow a new view about M-matrix as an operator representable as time-like entanglement coefficients of zero energy states so that physical states would represent laws of physics in their structure. They allow also the introduction of the notion of measurement resolution directly to the definition of reaction probabilities by using Jones inclusion and the replacement of state space with a finite-dimensional state space defined by quantum spinors. This hypothesis is of course just an attractive working hypothesis and deserves to be challenged.

WCW spinors

For WCW spinor s the HFF II_1 property is very natural because of the properties of infinite-dimensional Clifford algebra and the inner product defined by the WCW geometry does not allow other factors than this. A good guess is that the values of conformal weights label the factors appearing in the tensor power defining WCW spinor s. Because of the non-degeneracy and super-symplectic symmetries the density matrix representing metric must be essentially unit matrix for each conformal weight which would be the defining characteristic of hyper-finite factor of type II_1.

Bosonic degrees of freedom

The bosonic part of the super-symplectic algebra consists of Hamiltonians of CH in one-one correspondence with those of $\delta M^4_\pm \times CP_2$. Also the Kac-Moody algebra acting leaving the light-likeness of the partonic 3-surfaces intact contributes to the bosonic degrees of freedom. The commutator of these algebras annihilates physical states and there are also Virasoro conditions associated with ordinary conformal symmetries of partonic 2-surface [K35] . The labels of Hamiltonians of WCW and spin indices contribute to bosonic degrees of freedom.

Hyper-finite factors of type II_1 result naturally if the system is an infinite tensor product finite-dimensional matrix algebra associated with finite dimensional systems [A20] . Unfortunately, neither Virasoro, symplectic nor Kac-Moody algebras do have decomposition into this kind of infinite tensor product. If bosonic degrees for super-symplectic and super-Kac Moody algebra indeed give I_∞ factor one has HFF if type II_∞. This looks the most natural option but threatens to spoil the beautiful idea about M-matrix as time-like entanglement coefficients between positive and negative energy parts of zero energy state.

The resolution of the problem is surprisingly simple and trivial after one has discovered it. The requirement that state is normalizable forces to project M-matrix to a finite-dimensional sub-space in bosonic degrees of freedom so that the reduction $I_\infty \rightarrow I_n$ occurs and one has the reduction $II_\infty \rightarrow II_1 \times I_n = II_1$ to the desired HFF.

One can consider also the possibility of taking the limit $n \rightarrow \infty$. One could indeed say that since I_∞ factor can be mapped to an infinite tensor power of $M(2, C)$ characterized by a state which is not trace, it is possible to map this representation to HFF by replacing state with trace [A20] . The question is whether the forcing the bosonic foot to fermionic shoe is physically natural. One could also regard the II_1 type notion of probability as fundamental and also argue that it is required by full super-symmetry realized also at the level of many-particle states rather than mere single particle states.

How the bosonic cutoff is realized?

Normalizability of state requires that projection to a finite-dimensional bosonic sub-space is carried out for the bosonic part of the M-matrix. This requires a cutoff in quantum numbers of super-conformal algebras. The cutoff for the values of conformal weight could be formulated by replacing integers with Z_n or with some finite field $G(p, 1)$. The cutoff for the labels associated with Hamiltonians defined as an upper bound for the dimension of the representation looks also natural.

Number theoretical braids which are discrete and finite structures would define space-time correlate for this cutoff. p-Adic length scale $p \simeq 2^k$ hypothesis could be interpreted as stating the fact that only powers of p up to p^k are significant in p-adic thermodynamics which would correspond to finite field $G(k, 1)$ if k is prime. This has no consequences for p-adic mass calculations since already the first two terms give practically exact results for the large primes associated with elementary particles [K77] .

Finite number of strands for the theoretical braids would serve as a correlate for the reduction of the representation of Galois group S_∞ of rationals to an infinite produce of diagonal copies of finite-dimensional Galois group so that same braid would repeat itself like a unit cell of lattice i condensed matter [K59] .

HFF of type III for field operators and HFF of type II_1 for states?

One could also argue that the Hamiltonians with fixed conformal weight are included in fermionic II_1 factor and bosonic factor I_∞ factor, and that the inclusion of conformal weights leads to a factor of type III. Conformal weight could relate to the integer appearing in the crossed product representation $III = Z \times_{cr} II_\infty$ of HFF of type III [A20] .

The value of conformal weight is non-negative for physical states which suggests that Z reduces to semigroup N so that a factor of type III would reduce to a factor of type II_∞ since trace would become finite. If unitary process corresponds to an automorphism for II_∞ factor, the action of automorphisms affecting scaling must be uni-directional. Also thermodynamical irreversibility suggests the same. The assumption that state function reduction for positive energy part of state implies unitary process for negative energy state and vice versa would only mean that the shifts for positive and negative energy parts of state are opposite so that $Z \to N$ reduction would still hold true.

HFF of type II_1 for the maxima of Kähler function?

Probabilistic interpretation allows to gain heuristic insights about whether and how hyper-finite factors of type type II_1 might be associated with WCW degrees of freedom. They can appear both in quantum fluctuating degrees of freedom associated with a given maximum of Kähler function and in the discrete space of maxima of Kähler function.

Spin glass degeneracy is the basic prediction of classical TGD and means that instead of a single maximum of Kähler function analogous to single free energy minimum of a thermodynamical system there is a fractal spin glass energy landscape with valleys inside valleys. The discretization of WCW in terms of the maxima of Kähler function crucial for the p-adicization problem, leads to the analog of spin glass energy landscape and hyper-finite factor of type II_1 might be the appropriate description of the situation.

The presence of the tensor product structure is a powerful additional constraint and something analogous to this should emerge in WCW degrees of freedom. Fractality of the many-sheeted space-time is a natural candidate here since the decomposition of the original geometric structure to parts and replacing them with the scaled down variant of original structure is the geometric analog of forming a tensor power of the original structure.

2.5.2 Direct Sum Of HFFs Of Type II_1 As A Minimal Option

HFF II_1 property for the Clifford algebra of WCW means a definite distinction from the ordinary Clifford algebra defined by the fermionic oscillator operators since the trace of the unit matrix of the Clifford algebra is normalized to one. This does not affect the anti-commutation relations at the basic level and delta functions can appear in them at space-time level. At the level of momentum space I_∞ property requires discrete basis and anti-commutators involve only Kronecker deltas. This conforms with the fact that HFF of type II_1 can be identified as the Clifford algebra associated with a separable Hilbert space.

II_∞ factor or direct sum of HFFs of type II_1?

The expectation is that super-symplectic algebra is a direct sum over HFFs of type II_1 labeled by the radial conformal weight. In the same manner the algebra defined by fermionic anti-commutation relations at partonic 2-surface would decompose to a direct sum of algebras labeled by the conformal weight associated with the light-like coordinate of X_l^3. Super-conformal symmetry suggests that also the configuration space degrees of freedom correspond to a direct sum of HFFs of type II_1.

One can of course ask why not $II_\infty = I_\infty \times II_1$ structures so that one would have single factor rather than a direct sum of factors.

1. The physical motivation is that the direct sum property allow to decompose M-matrix to direct summands associated with various sectors with weights whose moduli squared have an interpretation in terms of the density matrix. This is also consistent with p-adic thermodynamics where conformal weights take the place of energy eigen values.
2. II_∞ property would predict automorphisms scaling the trace by an arbitrary positive real number $\lambda \in R_+$. These automorphisms would require the scaling of the trace of the projectors of Clifford algebra having values in the range $[0,1]$ and it is difficult to imagine how these automorphisms could be realized geometrically.

How HFF property reflects itself in the construction of geometry of WCW?

The interesting question is what HFF property and finite measurement resolution realizing itself as the use of projection operators means concretely at the level of WCW geometry.

Super-Hamiltonians define the Clifford algebra of the configuration space. Super-conformal symmetry suggests that the unavoidable restriction to projection operators instead of complex rays is realized also WCW degrees of freedom. Of course, infinite precision in the determination of the shape of 3-surface would be physically a completely unrealistic idea.

In the fermionic situation the anti-commutators for the gamma matrices associated with WCW individual Hamiltonians in 3-D sense are replaced with anti-commutators where Hamiltonians are replaced with projectors to subspaces of the space spanned by Hamiltonians. This projection is realized by restricting the anti-commutator to partonic 2-surfaces so that the anti-commutator depends only the restriction of the Hamiltonian to those surfaces.

What is interesting that the measurement resolution has a concrete particle physical meaning since the parton content of the system characterizes the projection. The larger the number of partons, the better the resolution about WCW degrees of freedom is. The degeneracy of WCW metric would be interpreted in terms of finite measurement resolution inherent to HFFs of type II_1, which is not due to Jones inclusions but due to the fact that one can project only to infinite-D subspaces rather than complex rays.

Effective 2-dimensionality in the sense that WCW Hamiltonians reduce to functionals of the partonic 2-surfaces of X^3_l rather than functionals of X^3_l could be interpreted in this manner. For a wide class of Hamiltonians actually effective 1-dimensionality holds true in accordance with conformal invariance.

The generalization of WCW Hamiltonians and super-Hamiltonians by allowing integrals over the 2-D boundaries of the patches of X^3_l would be natural and is suggested by the requirement of discretized 3-dimensionality at the level of WCW.

By quantum classical correspondence the inclusions of HFFs related to the measurement resolution should also have a geometric description. Measurement resolution corresponds to braids in given time scale and as already explained there is a hierarchy of braids in time scales coming as negative powers of two corresponding to the addition of zero energy components to positive/negative energy state. Note however that particle reactions understood as decays and fusions of braid strands could also lead to a notion of measurement resolution.

2.5.3 Bott Periodicity, Its Generalization, And Dimension $D = 8$As An Inherent Property Of The Hyper-Finite II_1 Factor

Hyper-finite II_1 factor can be constructed as infinite-dimensional tensor power of the Clifford algebra $M_2(C) = C(2)$ in dimension $D = 2$. More precisely, one forms the union of the Clifford algebras $C(2n) = C(2)^{\otimes n}$ of $2n$-dimensional spaces by identifying the element $x \in C(2n)$ as block diagonal elements $diag(x,x)$ of $C(2(n+1))$. The union of these algebras is completed in weak operator topology and can be regarded as a Clifford algebra of real infinite-dimensional separable Hilbert space and thus as sub-algebra of I_∞. Also generalizations obtained by replacing complex numbers by quaternions and octions are possible.

1. The dimension 8 is an inherent property of the hyper-finite II_1 factor since Bott periodicity theorem states $C(n+8) = C_n(16)$. In other words, the Clifford algebra $C(n+8)$ is equivalent

with the algebra of 16×16 matrices with entries in $C(n)$. Or articulating it still differently: $C(n+8)$ can be regarded as 16×16 dimensional module with $C(n)$ valued coefficients. Hence the elements in the union defining the canonical representation of hyper-finite II_1 factor are $16^n \times 16^n$ matrices having $C(0)$, $C(2)$, $C(4)$ or $C(6)$ valued valued elements.

2. The idea about a local variant of the infinite-dimensional Clifford algebra defined by power series of space-time coordinate with Taylor coefficients which are Clifford algebra elements fixes the interpretation. The representation as a linear combination of the generators of Clifford algebra of the finite-dimensional space allows quantum generalization only in the case of Minkowski spaces. However, if Clifford algebra generators are representable as gamma *matrices*, the powers of coordinate can be absorbed to the Clifford algebra and the local algebra is lost. Only if the generators are represented as quantum versions of octonions allowing no matrix representation because of their non-associativity, the local algebra makes sense. From this it is easy to deduce both quantum and classical TGD.

2.5.4 The Interpretation Of Jones Inclusions In TGD Framework

By the basic self-referential property of von Neumann algebras one can consider several interpretations of Jones inclusions consistent with sub-system-system relationship, and it is better to start by considering the options that one can imagine.

How Jones inclusions relate to the new view about sub-system?

Jones inclusion characterizes the embedding of sub-system $\mathcal{N}$ to $\mathcal{M}$and $\mathcal{M}$ as a finite-dimensional $\mathcal{N}$-module is the counterpart for the tensor product in finite-dimensional context. The possibility to express $\mathcal{M}$ as $\mathcal{N}$ module $\mathcal{M}/\mathcal{N}$ states fractality and can be regarded as a kind of self-referential "Brahman=Atman identity" at the level of infinite-dimensional systems.

Also the mysterious looking almost identity $CH^2 = CH$ for the WCW would fit nicely with the identity $M \oplus M = M$. $M \otimes M \subset M$ in WCW Clifford algebra degrees of freedom is also implied and the construction of $\mathcal{M}$ as a union of tensor powers of $C(2)$ suggests that $M \otimes M$ allows $\mathcal{M} : \mathcal{N} = 4$ inclusion to $\mathcal{M}$. This paradoxical result conforms with the strange self-referential property of factors of II_1.

The notion of many-sheeted space-time forces a considerable generalization of the notion of sub-system and simple tensor product description is not enough. Topological picture based on the length scale resolution suggests even the possibility of entanglement between sub-systems of un-entangled sub-systems. The possibility that hyper-finite II_1-factors describe the physics of TGD also in bosonic degrees of freedom is suggested by WCW super-symmetry. On the other hand, bosonic degrees could naturally correspond to I_∞ factor so that hyper-finite II_∞ would be the net result.

The most general view is that Jones inclusion describes all kinds of sub-system-system inclusions. The possibility to assign conformal field theory to the inclusion gives hopes of rather detailed view about dynamics of inclusion.

1. The topological condensation of space-time sheet to a larger space-time sheet mediated by wormhole contacts could be regarded as Jones inclusion. $\mathcal{N}$ would correspond to the condensing space-time sheet, $\mathcal{M}$ to the system consisting of both space-time sheets, and $\sqrt{\mathcal{M}:\mathcal{N}}$ would characterize the number of quantum spinorial degrees of freedom associated with the interaction between space-time sheets. Note that by general results $\mathcal{M} : \mathcal{N}$ characterizes the fractal dimension of quantum group ($\mathcal{M} : \mathcal{N} < 4$) or Kac-Moody algebra ($\mathcal{M} : \mathcal{N} = 4$) [B34] .

2. The branchings of space-time sheets (space-time surface is thus homologically like branching like of Feynman diagram) correspond naturally to n-particle vertices in TGD framework. What is nice is that vertices are nice 2-dimensional surfaces rather than surfaces having typically pinch singularities. Jones inclusion would naturally appear as inclusion of operator spaces $\mathcal{N}_i$ (essentially Fock spaces for fermionic oscillator operators) creating states at various lines as sub-spaces $N_i \subset M$ of operators creating states in common von Neumann factor $\mathcal{M}$. This would allow to construct vertices and vertices in natural manner using quantum groups or Kac-Moody algebras.

The fundamental $\mathcal{N} \subset \mathcal{M} \subset \mathcal{M} \otimes_N \mathcal{M}$ inclusion suggests a concrete representation based on the identification $N_i = M$, where M is the universal Clifford algebra associated with incoming line and $\mathcal{N}$ is defined by the condition that $\mathcal{M}/\mathcal{N}$ is the quantum variant of Clifford algebra of H. N-particle vertices could be defined as traces of Connes products of the operators creating incoming and outgoing states. It will be found that this leads to a master formula for S-matrix if the generalization of the old-fashioned string model duality implying that all generalized Feynman diagrams reduce to diagrams involving only single vertex is accepted.

3. If 4-surfaces can branch as the construction of vertices requires, it is difficult to argue that 3-surfaces and partonic/stringy 2-surfaces could not do the same. As a matter fact, the master formula for S-matrix to be discussed later explains the branching of 4-surfaces as an apparent effect. Despite this one can consider the possibility that this kind of joins are possible so that a new kind of mechanism of topological condensation would become possible. 3-space-sheets and partonic 2-surfaces whose p-adic fractality is characterized by different p-adic primes could be connected by "joins" representing branchings of 2-surfaces. The structures formed by soap film foam provide a very concrete illustration about what would happen. In the TGD based model of hadrons [K79] it has been assumed that join along boundaries bonds (JABs) connect quark space-time space-time sheets to the hadronic space-time sheet. The problem is that, at least for identical primes, the formation of join along boundaries bond fuses two systems to single bound state. If JABs are replaced joins, this objection is circumvented.

4. The space-time correlate for the formation of bound states is the formation of JABs. Standard intuition tells that the number of degrees of freedom associated with the bound state is smaller than the number of degrees of freedom associated with the pair of free systems. Hence the inclusion of the bound state to the tensor product could be regarded as Jones inclusion. On the other hand, one could argue that the JABs carry additional vibrational degrees of freedom so that the idea about reduction of degrees of freedom might be wrong: free system could be regarded as sub-system of bound state by Jones inclusion. The self-referential holographic properties of von Neumann algebras allow both interpretations: any system can be regarded as sub-system of any system in accordance with the bootstrap idea.

5. Maximal deterministic regions inside given space-time sheet bounded by light-like causal determinants define also sub-systems in a natural manner and also their inclusions would naturally correspond to Jones inclusions.

6. The TGD inspired model for topological quantum computation involves the magnetic flux tubes defined by join along boundaries bonds connecting space-time sheets having light-like boundaries. These tubes condensed to background 3-space can become linked and knotted and code for quantum computations in this manner. In this case the addition of new strand to the system corresponds to Jones inclusion in the hierarchy associated with inclusion $\mathcal{N} \subset \mathcal{M}$. The anyon states associated with strands would be represented by a finite tensor product of quantum spinors assignable to $\mathcal{M}/\mathcal{N}$ and representing quantum counterpart of H-spinors.

One can regard $\mathcal{M} : \mathcal{N}$ degrees of freedom correspond to quantum group or Kac-Moody degrees of freedom. Quantum group degrees of freedom relate closely to the conformal and topological degrees of freedom as the connection of II_1 factors with topological quantum field theories and braid matrices suggests itself. For the canonical inclusion this factorization would correspond to factorization of quantum H-spinor from WCW spinor .

A more detailed study of canonical inclusions to be carried out later demonstrates what this factorization corresponds at the space-time level to a formation of space-time sheets which can be regarded as multiple coverings of M^4 and CP_2 with invariance group $G = G_a \times G_b \subset SL(2,C) \times SU(2)$, $SU(2) \subset SU(3)$. The unexpected outcome is that Planck constants assignable to M^4 and CP_2 degrees of freedom depend on the canonical inclusions. The existence of macroscopic quantum phases with arbitrarily large Planck constants is predicted.

It would seem possible to assign the $\mathcal{M} : \mathcal{N}$ degrees quantum spinorial degrees of freedom to the interface between subsystems represented by $\mathcal{N}$ and $\mathcal{M}$. The interface could correspond to the wormhole contacts, joins, JABs, or light-like causal determinants serving as boundary between maximal deterministic regions, etc... In terms of the bipartite diagrams representing the inclusions, joins (say) would correspond to the edges connecting white vertices representing sub-system (the entire system without the joins) to black vertices (entire system).

About the interpretation of $\mathcal{M}:\mathcal{N}$ degrees of freedom

The Clifford algebra $\mathcal{N}$ associated with a system formed by two space-time sheet can be regarded as $1 \leq \mathcal{M}:\mathcal{N} \leq 4$-dimensional module having $\mathcal{N}$ as its coefficients. It is possible to imagine several interpretations the degrees of freedom labeled by β.

1. The $\beta = \mathcal{M}:\mathcal{N}$ degrees of freedom could relate to the interaction of the space-time sheets. Beraha numbers appear in the construction of S-matrices of topological quantum field theories and an interpretation in terms of braids is possible. This would suggest that the interaction between space-time sheets can be described in terms of conformal quantum field theory and the S-matrices associated with braids describe this interaction. Jones inclusions would characterize the effective number of active conformal degrees of freedom. At $n = 3$ limit these degrees of freedom disappear completely since the conformal field theory defined by the Chern-Simons action describing this interaction would become trivial ($c = 0$ as will be found).
2. The interpretation in terms of embedding space Clifford algebra would suggest that β-dimensional Clifford algebra of $\sqrt{\beta}$-dimensional spinor space is in question. For $\beta = 4$ the algebra would be the Clifford algebra of 2-dimensional space. $\mathcal{M}/\mathcal{N}$ would have interpretation as complex quantum spinors with components satisfying $z_1z_2 = qz_2z_1$ and its conjugate and having fractal complex dimension $\sqrt{\beta}$. This would conform with the effective 2-dimensionality of TGD. For $\beta < 4$ the fractal dimension of partonic quantum spinors defining the basic conformal fields would be reduced and become $d = 1$ for $n = 3$: the interpretation is in terms of strong correlations caused by the non-commutativity of the components of quantum spinor. For number theoretical generalizations of infinite-dimensional Clifford algebras $Cl(C)$ obtained by replacing C with Abelian complexification of quaternions or octonions one would obtain higher-dimensional spinors.

2.5.5 WCW, Space-Time, Embedding Space AndHyper-Finite Type II_1 Factors

The preceding considerations have by-passed the question about the relationship of WCW tangent space to its Clifford algebra. Also the relationship between space-time and embedding space and their quantum variants could be better. In particular, one should understand how effective 2-dimensionality can be consistent with the 4-dimensionality of space-time.

Super-conformal symmetry and WCW Poisson algebra as hyper-finite type II_1 factor

It would be highly desirable to achieve also a description of the WCW degrees of freedom using von Neumann algebras. Super-conformal symmetry relating fermionic degrees of freedom and WCW degrees of freedom suggests that this might be the case. Super-symplectic algebra has as its generators configuration space Hamiltonians and their super-counterparts identifiable as CH gamma matrices. Super-symmetry requires that the Clifford algebra of CH and the Hamiltonian vector fields of CH with symplectic central extension both define hyper-finite II_1 factors. By super-symmetry Poisson bracket corresponds to an anti-commutator for gamma matrices. The ordinary quantized version of Poisson bracket is obtained as $\{P_i, Q_j\} \rightarrow [P_i, Q_j] = J_{ij}Id$. Finite trace version results by assuming that Id corresponds to the projector CH Clifford algebra having unit norm. The presence of zero modes means direct integral over these factors.

WCW gamma matrices anti-commuting to identity operator with unit norm corresponds to the tangent space $T(CH)$ of CH. Thus it would be not be surprising if $T(CH)$ could be imbedded in the sigma matrix algebra as a sub-space of operators defined by the gamma matrices generating this algebra. At least for $\beta = 4$ construction of hyper-finite II_1 factor this definitely makes sense.

The dimension of WCW defined as the trace of the projection operator to the sub-space spanned by gamma matrices is obviously zero. Thus WCW has in this sense the dimensionality of single space-time point. This sounds perhaps absurd but the generalization of the number concept implied by infinite primes indeed leads to the view that single space-time point is infinitely structured in the number theoretical sense although in the real sense all states of the point are equivalen. The reason is that there is infinitely many numbers expressible as ratios of infinite integers having unit real norm in the real sense but having different p-adic norms.

How to understand the dimensions of space-time and embedding space?

One should be able to understand the dimensions of 3-space, space-time and embedding space in a convincing matter in the proposed framework. There is also the question whether space-time and embedding space emerge uniquely from the mathematics of von Neumann algebras alone.

1. The dimensions of space-time and embedding space

Two sub-sequent inclusions dual to each other define a special kind of inclusion giving rise to a quantum counterpart of $D = 4$ naturally. This would mean that space-time is something which emerges at the level of cognitive states.

The special role of classical division algebras in the construction of quantum TGD [K107] , $D = 8$ Bott periodicity generalized to quantum context, plus self-referential property of type II_1 factors might explain why 8-dimensional embedding space is the only possibility.

State space has naturally quantum dimension $D \leq 8$ as the following simple argument shows. The space of quantum states has quark and lepton sectors which both are super-symmetric implying $D \leq 4$ for each. Since these sectors correspond to different Hamiltonian algebras (triality one for quarks and triality zero for leptonic sector), the state space has quantum dimension $D \leq 8$.

2. How the lacking two space-time dimensions emerge?

3-surface is the basic dynamical unit in TGD framework. This seems to be in conflict with the effective 2-dimensionality [K107] meaning that partonic 2-surface code for quantum states, and with the fact that hyper-finite II_1 factors have intrinsic quantum dimension 2.

A possible resolution of the problem is that the foliation of 3-surface by partonic two-surfaces defines a one-dimensional direct integral of isomorphic hyper-finite type II_1 factors, and the zero mode labeling the 2-surfaces in the foliation serves as the third spatial coordinate. For a given 3-surface the contribution to the WCW metric can come only from 2-D partonic surfaces defined as intersections of 3-D light-like CDs with $X^7_\pm$ [K36] . Hence the direct integral should somehow relate to the classical non-determinism of Kähler action.

1. The one-parameter family of intersections of light-like CD with $X^7_\pm$ inside $X^4 \cap X^7_\pm$ could indeed be basically due to the classical non-determinism of Kähler action. The contribution to the metric from the normal light-like direction to $X^3 = X^4 \cap X^7_\pm$ can cause the vanishing of the metric determinant $\sqrt{g_4}$ of the space-time metric at $X^2 \subset X^3$ under some conditions on X^2. This would mean that the space-time surface $X^4(X^3)$ is not uniquely determined by the minimization principle defining the value of the Kähler action, and the complete dynamical specification of X^3 requires the specification of partonic 2-surfaces X^2_i with $\sqrt{g_4} = 0$.

2. The known solutions of field equations [K18] define a double foliation of the space-time surface defined by Hamilton-Jacobi coordinates consisting of complex transversal coordinate and two light-like coordinates for M^4 (rather than space-time surface). Number theoretical considerations inspire the hypothesis that this foliation exists always [K107] . Hence a natural hypothesis is that the allowed partonic 2-surfaces correspond to the 2-surfaces in the restriction of the double foliation of the space-time surface by partonic 2-surfaces to X^3, and are thus locally parameterized by single parameter defining the third spatial coordinate.

3. There is however also a second light-like coordinate involved and one might ask whether both light-like coordinates appear in the direct sum decomposition of II_1 factors defining $T(CH)$. The presence of two kinds of light-like CDs would provide the lacking two space-time coordinates and quantum dimension $D = 4$ would emerge at the limit of full non-determinism. Note that the duality of space-like partonic and light-like stringy 2-surfaces conforms with this interpretation since it corresponds to a selection of partonic/stringy 2-surface inside given 3-D CD whereas the dual pairs correspond to different CDs.

4. That the quantum dimension would be $2D_q = \beta < 4$ above CP_2 length scale conforms with the fact that non-determinism is only partial and time direction is dynamically frozen to a high degree. For vacuum extremals there is strong non-determinism but in this case there is no real dynamics. For CP_2 type extremals, which are not vacuum extremals as far action and small perturbations are considered, and which correspond to $\beta = 4$ there is a complete non-determinism in time direction since the M^4 projection of the extremal is a light-like

random curve and there is full 4-D dynamics. Light-likeness gives rise to conformal symmetry consistent with the emergence of Kac Moody algebra [K18] .

3. Time and cognition

In a completely deterministic physics time dimension is strictly speaking redundant since the information about physical states is coded by the initial values at 3-dimensional slice of space-time. Hence the notion of time should emerge at the level of cognitive representations possible by to the non-determinism of the classical dynamics of TGD.

Since Jones inclusion means the emergence of cognitive representation, the space-time view about physics should correspond to cognitive representations provided by Feynman diagram states with zero energy with entanglement defined by a two-sided projection of the lowest level S-matrix. These states would represent the "laws of quantum physics" cognitively. Also space-time surface serves as a classical correlate for the evolution by quantum jumps with maximal deterministic regions serving as correlates of quantum states. Thus the classical non-determinism making possible cognitive representations would bring in time. The fact that quantum dimension of space-time is smaller than $D = 4$ would reflect the fact that the loss of determinism is not complete.

4. Do space-time and embedding space emerge from the theory of von Neumann algebras and number theor

The considerations above force to ask whether the notions of space-time and embedding space emerge from von Neumann algebras as predictions rather than input. The fact that it seems possible to formulate the S-matrix and its generalization in terms of inherent properties of infinite-dimensional Clifford algebras suggest that this might be the case.

Inner automorphisms as universal gauge symmetries?

The continuous outer automorphisms Δ^{it} of HFFs of type III are not completely unique and one can worry about the interpretation of the inner automorphisms. A possible resolution of the worries is that inner automorphisms act as universal gauge symmetries containing various super-conformal symmetries as a special case. For hyper-finite factors of type II_1 in the representation as an infinite tensor power of $M_2(C)$ this would mean that the transformations non-trivial in a finite number of tensor factors only act as analogs of local gauge symmetries. In the representation as a group algebra of S_∞ all unitary transformations acting on a finite number of braid strands act as gauge transformations whereas the infinite powers $P \times P \times ...$, $P \in S_n$, would act as counterparts of global gauge transformations. In particular, the Galois group of the closure of rationals would act as local gauge transformations but diagonally represented finite Galois groups would act like global gauge transformations and periodicity would make possible to have finite braids as space-time correlates without a loss of information.

Do unitary isomorphisms between tensor powers of II_1 define vertices?

What would be left would be the construction of unitary isomorphisms between the tensor products of the HFFs of type $II_1 \otimes I_n = II_1$ at the partonic 2-surfaces defining the vertices. This would be the only new element added to the construction of braiding M-matrices.

As a matter fact, this element is actually not completely new since it generalizes the fusion rules of conformal field theories, about which standard example is the fusion rule $\phi_i = {c_i}^{jk} \phi_j \phi_k$ for primary fields. These fusion rules would tell how a state of incoming HFF decomposes to the states of tensor product of two outgoing HFFs.

These rules indeed have interpretation in terms of Connes tensor products $\mathcal{M} \otimes_{\mathcal{N}} ... \otimes_{\mathcal{N}} \mathcal{M}$ for which the sub-factor $\mathcal{N}$ takes the role of complex numbers [A45] so that one has $\mathcal{M}$ becomes $\mathcal{N}$ bimodule and "quantum quantum states" have $\mathcal{N}$ as coefficients instead of complex numbers. In TGD framework this has interpretation as quantum measurement resolution characterized by $\mathcal{N}$ (the group G characterizing leaving the elements of $\mathcal{N}$ invariant defines the measured quantum numbers).

2.5.6 Quaternions, Octonions, And Hyper-Finite Type II_1Factors

Quaternions and octonions as well as their hyper counterparts obtained by multiplying imaginary units by commuting $\sqrt{-1}$ and forming a sub-space of complexified division algebra, are in in a central role in the number theoretical vision about quantum TGD [K107] . Therefore the question arises whether complexified quaternions and perhaps even octonions could be somehow inherent properties of von Neumann algebras. One can also wonder whether the quantum counterparts of quaternions and octonions could emerge naturally from von Neumann algebras. The following considerations allow to get grasp of the problem.

Quantum quaternions and quantum octonions

Quantum quaternions have been constructed as deformation of quaternions [A91] . The key observation that the Glebsch Gordan coefficients for the tensor product $3\otimes 3 = 5\oplus \oplus 3\oplus 1$ of spin 1 representation of $SU(2)$ with itself gives the anti-commutative part of quaternionic product as spin 1 part in the decomposition whereas the commutative part giving spin 0 representation is identifiable as the scalar product of the imaginary parts. By combining spin 0 and spin 1 representations, quaternionic product can be expressed in terms of Glebsh-Gordan coefficients. By replacing GGC:s by their quantum group versions for group $sl(2)_q$, one obtains quantum quaternions.

There are two different proposals for the construction of quantum octonions [A75, A1] . Also now the idea is to express quaternionic and octonionic multiplication in terms of Glebsch-Gordan coefficients and replace them with their quantum versions.

1. The first proposal [A75] relies on the observation that for the tensor product of $j = 3$ representations of $SU(2)$ the Glebsch-Gordan coefficients for $7\otimes 7 \rightarrow 7$ in $7\otimes 7 = 9\oplus 7\oplus 5\oplus 3\oplus 1$ defines a product, which is equivalent with the antisymmetric part of the product of octonionic imaginary units. As a matter fact, the antisymmetry defines 7-dimensional Malcev algebra defined by the anti-commutator of octonion units and satisfying b definition the identity

$$[[x,y,z]\,,x] = [x,y,[x,z]] \quad , \quad [x,y,z] \equiv [x,[y,z]] + [y,[z,x]] + [z,[x,y]] \quad . \tag{2.5.1}$$

   7-element Malcev algebra defining derivations of octonionic algebra is the only complex Malcev algebra not reducing to a Lie algebra. The $j = 0$ part of the product corresponds also now to scalar product for imaginary units. Octonions are constructed as sums of $j = 0$ and $j = 3$ parts and quantum Glebsch-Gordan coefficients define the octonionic product.

2. In the second proposal [A1] the quantum group associated with $SO(8)$ is used. This representation does not allow unit but produces a quantum version of octonionic triality assigning to three octonions a real number.

Quaternionic or octonionic quantum mechanics?

There have been numerous attempts to introduce quaternions and octonions to quantum theory. Quaternionic or octonionic quantum mechanics, which means the replacement of the complex numbers as coefficient field of Hilbert space with quaternions or octonions, is the most obvious approach (for example and references to the literature see for instance [A73] .

In both cases non-commutativity poses serious interpretational problems. In the octonionic case the non-associativity causes even more serious obstacles [B38, A73] , [B38] .

1. Assuming that an orthonormalized state basis with respect to an octonion valued inner product has been found, the multiplication of any basis with octonion spoils the orthonormality. The proposal to circumvent this difficulty discussed in [B38] , [B38] eliminates non-associativity by assuming that octonions multiply states one by one (rather than multiplying each other before multiplying the state). Effectively this means that octonions are replaced with 8×8-matrices.

2. The definition of the tensor product leads also to difficulties since associativity is lost (recall that Yang-Baxter equation codes for associativity in case of braid statistics [A29]).

3. The notion of hermitian conjugation is problematic and forces a selection of a preferred imaginary unit, which does not look nice. Note however that the local selection of a preferred imaginary unit is in a key role in the proposed construction of space-time surfaces as hyper-quaternionic or co-hyper-quaternionic surfaces and allows to interpret space-time surfaces either as surfaces in 8-D Minkowski space M^8 of hyper-octonions or in $M^4 \times CP_2$. This selection turns out to have quite different interpretation in the proposed framework.

Hyper-finite factor II_1 has a natural Hyper-Kähler structure

In the case of hyper-finite factors of type II_1 quaternions a more natural approach is based on the generalization of the Hyper-Kähler structure rather than quaternionic quantum mechanics. The reason is that also WCW tangent space should and is expected to have this structure [K36] . The Hilbert space remains a complex Hilbert space but the quaternionic units are represented as operators in Hilbert space. The selection of the preferred unit is necessary and natural. The identity operator representing quaternionic real unit has trace equal to one, is expected to give rise to the series of quantum quaternion algebras in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ having interpretation as N-modules.

The representation of the quaternion units is rather explicit in the structure of hyper-finite II_1 factor. The $\mathcal{M} : \mathcal{N} \equiv \beta = 4$ hierarchical construction can be regarded as Connes tensor product of infinite number of 4-D Clifford algebras of Euclidian plane with Euclidian signature of metric $(diag(-1,-1))$. This algebra is nothing but the quaternionic algebra in the representation of quaternionic imaginary units by Pauli spin matrices multiplied by i.

The imaginary unit of the underlying complex Hilbert space must be chosen and there is whole sphere S^2 of choices and in every point of WCW the choice can be made differently. The space-time correlate for this local choice of preferred hyper-octonionic unit [K107] . At the level of WCW geometry the quaternion structure of the tangent space means the existence of Hyper-Kähler structure guaranteeing that WCW has a vanishing Einstein tensor. It it would not vanish, curvature scalar would be infinite by symmetric space property (as in case of loop spaces) and induce a divergence in the functional integral over 3-surfaces from the expansion of $\sqrt{g}$ [K36] .

The quaternionic units for the II_1 factor, are simply limiting case for the direct sums of 2×2 units normalized to one. Generalizing from $\beta = 4$ to $\beta < 4$, the natural expectation is that the representation of the algebra as $\beta = \mathcal{M} : \mathcal{N}$-dimensional $\mathcal{N}$-module gives rise to quantum quaternions with quaternion units defined as infinite sums of $\sqrt{\beta} \times \sqrt{\beta}$ matrices.

At Hilbert space level one has an infinite Connes tensor product of 2-component spinor spaces on which quaternionic matrices have a natural action. The tensor product of Clifford algebras gives the algebra of 2×2 quaternionic matrices acting on 2-component quaternionic spinors (complex 4-component spinors). Thus double inclusion could correspond to (hyper-)quaternionic structure at space-time level. Note however that the correspondence is not complete since hyper-quaternions appear at space-time level and quaternions at Hilbert space level.

Von Neumann algebras and octonions

The octonionic generalization of the Hyper-Kähler manifold does not make sense as such since octonionic units are not representable as linear operators. The allowance of anti-linear operators inherently present in von Neumann algebras could however save the situation. Indeed, the Cayley-Dickson construction for the division algebras (for a nice explanation see [A67]), which allows to extend any * algebra, and thus also any von Neumann algebra, by adding an imaginary unit it and identified as *, comes in rescue.

The basic idea of the Cayley-Dickson construction is following. The * operator, call it J, representing a conjugation defines an *anti-linear* operator in the original algebra A. One can extend A by adding this operator as a new element to the algebra. The conditions satisfied by J are

$$a(Jb) = J(a^*b) \quad , \quad (aJ)b = (ab^*)J \quad , \quad (Ja)(bJ^{-1}) = (ab)^* \quad . \tag{2.5.2}$$

In the associative case the conditions are equivalent to the first condition.

It is intuitively clear that this addition extends the hyper-Kähler structure to an octonionic structure at the level of the operator algebra. The quantum version of the octonionic algebra is fixed by the quantum quaternion algebra uniquely and is consistent with the Cayley-Dickson construction. It is not clear whether the construction is equivalent with either of the earlier proposals [A75, A1] . It would however seem that the proposal is simpler.

Physical interpretation of quantum octonion structure

Without further restrictions the extension by J would mean that vertices contain operators, which are superpositions of linear and anti-linear operators. This would give superpositions of states and their time-reversals and mean that state could be a superposition of states with opposite values of say fermion numbers. The problem disappears if either the linear operators A or anti-linear operators JA can be used to construct physical states from vacuum. The fact, that space-time surfaces are either hyper-quaternionic or co-hyper-quaternionic, is a space-time correlate for this restriction.

The $HQ - coHQ$ duality discussed in [K107] states that the descriptions based on hyper-quaternionic and co-hyper-quaternionic surfaces are dual to each other. The duality can have two meanings.

1. The vacuum is invariant under J so that one can use either complexified quaternionic operators A or their co-counterparts of form JA to create physical states from vacuum.
2. The vacuum is not invariant under J. This could relate to the breaking of CP and T invariance known to occur in meson-antimeson systems. In TGD framework two kinds of vacua are predicted corresponding intuitively to vacua in which either the product of all positive or negative energy fermionic oscillator operators defines the vacuum state, and these two vacua could correspond to a vacuum and its J conjugate, and thus to positive and negative energy states. In this case the two state spaces would not be equivalent although the physics associated with them would be equivalent.

The considerations of [K107] related to the detailed dynamics of $HQ - coHQ$ duality demonstrate that the variational principles defining the dynamics of hyper-quaternionic and co-hyper-quaternionic space-time surfaces are antagonistic and correspond to world as seen by a conscientous book-keeper on one hand and an imaginative artist on the other hand. HQ case is conservative: differences measured by the magnitude of Kähler action tend to be minimized, the dynamics is highly predictive, and minimizes the classical energy of the initial state. $coHQ$ case is radical: differences are maximized (this is what the construction of sensory representations would require). The interpretation proposed in [K107] was that the two space-time dynamics are just different predictions for what would happen (has happened) if no quantum jumps would occur (had occurred). A stronger assumption is that these two views are associated with systems related by time reversal symmetry.

What comes in mind first is that this antagonism follows from the assumption that these dynamics are actually time-reversals of each other with respect to M^4 time (the rapid elimination of differences in the first dynamics would correspond to their rapid enhancement in the second dynamics). This is not the case so that T and CP symmetries are predicted to be broken in accordance with the CP breaking in meson-antimeson systems [K73] and cosmological matter-antimatter asymmetry [K101] .

2.5.7 Does The Hierarchy Of Infinite Primes Relate To The Hierarchy Of II_1 Factors?

The hierarchy of Feynman diagrams accompanying the hierarchy defined by Jones inclusions $\mathcal{M}_0 \subset \mathcal{M}_1 \subset ...$ gives a concrete representation for the hierarchy of cognitive dynamics providing a representation for the material world at the lowest level of the hierarchy. This hierarchy seems to relate directly to the hierarchy of space-time sheets.

Also the construction of infinite primes [K105] leads to an infinite hierarchy. Infinite primes at the lowest level correspond to polynomials of single variable x_1 with rational coefficients, next level to polynomials x_1 for which coefficients are rational functions of variable x_2, etc... so that a natural ordering of the variables is involved.

If the variables x_i are hyper-octonions (subs-space of complexified octonions for which elements are of form $x + \sqrt{-1}y$, where x is real number and y imaginary octonion and $\sqrt{-1}$ is commuting imaginary unit, this hierarchy of states could provide a realistic representation of physical states as far as quantum numbers related to embedding space degrees of freedom are considered in M^8 picture dual to $M^4 \times CP_2$ picture [K107] . Infinite primes are mapped to space-time surfaces in a way analogous to the mapping of polynomials to the loci of their zeros so that infinite primes, integers, and rationals become concrete geometrical objects.

Infinite primes are also obtained by a repeated second quantization of a super-symmetric arithmetic quantum field theory. Infinite rational numbers correspond in this description to pairs of positive energy and negative energy states of opposite energies having interpretation as pairs of initial and final states so that higher level states indeed represent transitions between the states. For these reasons this hierarchy has been interpreted as a correlate for a cognitive hierarchy coding information about quantum dynamics at lower levels. This hierarchy has also been assigned with the hierarchy of space-time sheets. Just as the hierarchy of generalized Feynman diagrams provides self representations of the lowest matter level and is coded by it, finite primes code the hierarchy of infinite primes.

Infinite primes, integers, and rationals have finite p-adic norms equal to 1, and one can wonder whether a Hilbert space like structure with dimension given by an infinite prime or integer makes sense, and whether it has anything to do with the Hilbert space for which dimension is infinite in the sense of the limiting value for a dimension of sub-space. The Hilbert spaces with dimension equal to infinite prime would define primes for the tensor product of these spaces. The dimension of this kind of space defined as any p-adic norm would be equal to one.

One cannot exclude the possibility that infinite primes could express the infinite dimensions of hyper-finite III_1 factors, which cannot be excluded and correspond to that part of quantum TGD which relates to the embedding space rather than space-time surface. Indeed, infinite primes code naturally for the quantum numbers associated with the embedding space. Secondly, the appearance of 7-D light-like causal determinants $X^7_\pm = M^4_\pm \times CP_2$ forming nested structures in the construction of S-matrix brings in mind similar nested structures of algebraic quantum field theory [B6] . If this is were the case, the hierarchy of Beraha numbers possibly associated with the phase resolution could correspond to hyper-finite factors of type II_1, and the decomposition of space-time surface to regions labeled by p-adic primes and characterized by infinite primes could correspond to hyper-finite factors of type III_1 and represent embedding space degrees of freedom.

The state space would in this picture correspond to the tensor products of hyper-finite factors of type II_1 and III_1 (of course, also factors I_n and I_∞ are also possible). III_1 factors could be assigned to the sub-WCWs defined by 3-surfaces in regions of M^4 expressible in terms of unions and intersections of $X^7_\pm = M^4_\pm \times CP_2$. By conservation of four-momentum, bounded regions of this kind are possible only for the states of zero net energy appearing at the higher levels of hierarchy. These sub-WCWs would be characterized by the positions of the tips of light cones $M^4_\pm \subset M^4$ involved. This indeed brings in continuous spectrum of four-momenta forcing to introduce non-separable Hilbert spaces for momentum eigen states and necessitating III_1 factors. Infinities would be avoided since the dynamics proper would occur at the level of space-time surfaces and involve only II_1 factors.

2.6 HFFs Of Type III And TGD

One can imagine several ways for how HFFs of type III could emerge in TGD although the proposed view about M-matrix in zero energy ontology suggests that HFFs of type III_1 should be only an auxiliary tool at best. Same is suggested with interpretational problems associated with them. Both TGD inspired quantum measurement theory, the idea about a variant of HFF of type II_1 analogous to a local gauge algebra, and some other arguments, suggest that HFFs of type III could be seen as a useful idealization allowing to make non-trivial conjectures both about quantum TGD and about HFFs of type III. Quantum fields would correspond to HFFs of type III and II_∞ whereas physical states (M-matrix) would correspond to HFF of type II_1. I have summarized first the problems of III_1 factors so that reader can decide whether the further reading is worth of it.

2.6.1 Problems Associated With The Physical Interpretation Of III_1 Factors

Algebraic quantum field theory approach [B31, B6] has led to a considerable understanding of relativistic quantum field theories in terms of hyper-finite III_1 factors. There are however several reasons to suspect that the resulting picture is in conflict with physical intuition. Also the infinities of non-trivial relativistic QFTs suggest that something goes wrong.

Are the infinities of quantum field theories due the wrong type of von Neumann algebra?

The infinities of quantum field theories involve basically infinite traces and it is now known that the algebras of observables for relativistic quantum field theories for bounded regions of Minkowski space correspond to hyper-finite III_1 algebras, for which non-trivial traces are always infinite. This might be the basic cause of the divergence problems of relativistic quantum field theory.

On basis of this observations there is some temptation to think that the finite traces of hyper-finite II_1 algebras might provide a resolution to the problems but not necessarily in QFT context. One can play with the thought that the subtraction of infinities might be actually a process in which III_1 algebra is transformed to II_1 algebra. A more plausible idea suggested by dimensional regularization is that the elimination of infinities actually gives rise to II_1 inclusion at the limit $\mathcal{M} : \mathcal{N} \rightarrow 4$. It is indeed known that the dimensional regularization procedure of quantum field theories can be formulated in terms of bi-algebras assignable to Feynman diagrams and [A21] and the emergence of bi-algebras suggests that a connection with II_1 factors and critical role of dimension $D = 4$ might exist.

Continuum of inequivalent representations of commutation relations

There is also a second difficulty related to type III algebras. There is a continuum of inequivalent representations for canonical commutation relations [A84] . In thermodynamics this is blessing since temperature parameterizes these representations. In quantum field theory context situation is however different and this problem has been usually put under the rug.

Entanglement and von Neumann algebras

In quantum field theories where 4-D regions of space-time are assigned to observables. In this case hyper-finite type III_1 von Neumann factors appear. Also now inclusions make sense and has been studiedin fact, the parameters characterizing Jones inclusions appear also now and this due to the very general properties of the inclusions.

The algebras of type III_1 have rather counter-intuitive properties from the point of view of entanglement. For instance, product states between systems having space-like separation are not possible at all so that one can speak of intrinsic entanglement [A58] . What looks worse is that the decomposition of entangled state to product states is highly non-unique.

Mimicking the steps of von Neumann one could ask what the notion of observables could mean in TGD framework. Effective 2-dimensionality states that quantum states can be constructed using the data given at partonic or stringy 2-surfaces. This data includes also information about normal derivatives so that 3-dimensionality actually lurks in. In any case this would mean that observables are assignable to 2-D surfaces. This would suggest that hyper-finite II_1 factors appear in quantum TGD at least as the contribution of single space-time surface to S-matrix is considered. The contributions for WCW degrees of freedom meaning functional (not path-) integral over 3-surfaces could of course change the situation.

Also in case of II_1 factors, entanglement shows completely new features which need not however be in conflict with TGD inspired view about entanglement. The eigen values of density matrices are infinitely degenerate and quantum measurement can remove this degeneracy only partially. TGD inspired theory of consciousness has led to the identification of rational (more generally algebraic entanglement) as bound state entanglement stable in state function reduction. When an infinite number of states are entangled, the entanglement would correspond to rational (algebraic number) valued traces for the projections to the eigen states of the density matrix. The symplectic transformations of CP_2 are almost $U(1)$ gauge symmetries broken only by classical gravitation.

They imply a gigantic spin glass degeneracy which could be behind the infinite degeneracies of eigen states of density matrices in case of II_1 factors.

2.6.2 Quantum Measurement Theory And HFFs Of Type III

The attempt to interpret the HFFs of type III in terms of quantum measurement theory based on Jones inclusions leads to highly non-trivial conjectures about these factors.

Could the scalings of trace relate to quantum measurements?

What should be understood is the physical meaning of the automorphism inducing the scaling of trace. In the representation based of factors based on infinite tensor powers the action of g should transform single $n \times n$ matrix factor with density matrix Id/n to a density matrix e_{11} of a pure state.

Obviously the number of degrees of freedom is affected and this can be interpreted in terms of appearance or disappearance of correlations. Quantization and emergence of non-commutativity indeed implies the emergence of correlations and effective reduction of degrees of freedom. In particular, the fundamental quantum Clifford algebra has reduced dimension $\mathcal{M} : \mathcal{N} = r \leq 4$ instead of $r = 4$ since the replacement of complex valued matrix elements with $\mathcal{N}$ valued ones implies non-commutativity and correlations.

The transformation would be induced by the shift of finite-dimensional state to right or left so that the number of matrix factors overlapping with I_∞ part increases or is reduced. Could it have interpretation in terms of quantum measurement for a quantum Clifford factor? Could quantum measurement for $\mathcal{M}/\mathcal{N}$ degrees of freedom reducing the state in these degrees of freedom to a pure state be interpreted as a transformation of single finite-dimensional matrix factor to a type I factor inducing the scaling of the trace and could the scalings associated with automorphisms of HFFs of type III also be interpreted in terms of quantum measurement?

This interpretation does not as such say anything about HFF factors of type III since only a decomposition of II_1 factor to I_2^k factor and II_1 factor with a reduced trace of projector to the latter. However, one can ask whether the scaling of trace for HFFs of type III could correspond to a situation in which infinite number of finite-dimensional factors have been quantum measured. This would correspond to the inclusion $\mathcal{N} \subset \mathcal{M}_\infty = \cup_n \mathcal{M}_n$ where $\mathcal{N} \subset \mathcal{M} \subset ...\mathcal{M}_n...$ defines the canonical inclusion sequence. Physicist can of course ask whether the presence of infinite number of I_2-, or more generally, I_n-factors is at all relevant to quantum measurement and it has already become clear that situation at the level of M-matrix reduces to I_n.

Could the theory of HHFs of type III relate to the theory of Jones inclusions?

The idea about a connection of HFFs of type III and quantum measurement theory seems to be consistent with the basic facts about inclusions and HFFs of type III_1.

1. Quantum measurement would scale the trace by a factor $2^k/\sqrt{\mathcal{M} : \mathcal{N}}$ since the trace would become a product for the trace of the projector to the newly born $M(2,C)^{\otimes k}$ factor and the trace for the projection to $\mathcal{N}$ given by $1/\sqrt{\mathcal{M} : \mathcal{N}}$. The continuous range of values $\mathcal{M} : \mathcal{N} \geq 4$ gives good hopes that all values of λ are realized. The prediction would be that $2^k\sqrt{\mathcal{M} : \mathcal{N}} \geq 1$ holds always true.
2. The values $\mathcal{M} : \mathcal{N} \in \{r_n = 4cos^2(\pi/n)\}$ for which the single $M(2,C)$ factor emerges in state function reduction would define preferred values of the inverse of $\lambda = \sqrt{\mathcal{M} : \mathcal{N}/4}$ parameterizing factors III_λ. These preferred values vary in the range $[1/2, 1]$.
3. $\lambda = 1$ at the end of continuum would correspond to HFF III_1 and to Jones inclusions defined by infinite cyclic subgroups dense in $U(1) \subset SU(2)$ and this group combined with reflection. These groups correspond to the Dynkin diagrams A_∞ and D_∞. Also the classical values of $\mathcal{M} : \mathcal{N} = n^2$ characterizing the dimension of the quantum Clifford $\mathcal{M} : \mathcal{N}$ are possible. In this case the scaling of trace would be trivial since the factor n to the trace would be compensated by the factor $1/n$ due to the disappearance of $\mathcal{M}/\mathcal{N}$ factor III_1 factor.
4. Inclusions with $\mathcal{M} : \mathcal{N} = \infty$ are also possible and they would correspond to $\lambda = 0$ so that also III_0 factor would also have a natural identification in this framework. These factors

correspond to ergodic systems and one might perhaps argue that quantum measurement in this case would give infinite amount of information.

5. This picture makes sense also physically. p-Adic thermodynamics for the representations of super-conformal algebra could be formulated in terms of factors of type I_∞ and in excellent approximation using factors I_n. The generation of arbitrary number of type II_1 factors in quantum measurement allow this possibility.

The end points of spectrum of preferred values of λ are physically special

The fact that the end points of the spectrum of preferred values of λ are physically special, supports the hopes that this picture might have something to do with reality.

1. The Jones inclusion with $q = exp(i\pi/n)$, $n = 3$ (with principal diagram reducing to a Dynkin diagram of group SU(3)) corresponds to $\lambda = 1/2$, which corresponds to HFF III_1 differing in essential manner from factors III_λ, $\lambda < 1$. On the other hand, $SU(3)$ corresponds to color group which appears as an isometry group and important subgroup of automorphisms of octonions thus differs physically from the ADE gauge groups predicted to be realized dynamically by the TGD based view about McKay correspondence [K59] .
2. For $r = 4$ SU(2) inclusion parameterized by extended ADE diagrams $M(2, C)^{\otimes 2}$ would be created in the state function reduction and also this would give $\lambda = 1/2$ and scaling by a factor of 2. Hence the end points of the range of discrete spectrum would correspond to the same scaling factor and same HFF of type III. SU(2) could be interpreted either as electro-weak gauge group, group of rotations of th geodesic sphere of $\delta M^4_\pm$, or a subgroup of SU(3). In TGD interpretation for McKay correspondence a phase transition replacing gauge symmetry with Kac-Moody symmetry.
3. The scalings of trace by factor 2 seem to be preferred physically which should be contrasted with the fact that primes near prime powers of 2 and with the fact that quantum phases $q = exp(i\pi/n)$ with n equal to Fermat integer proportional to power of 2 and product of the Fermat primes (the known ones are 5, 17, 257, and $2^{16} + 1$) are in a special role in TGD Universe.

2.6.3 What Could One Say About II_1 Automorphism Associated With The II_∞ Automorphism Defining Factor Of Type III?

An interesting question relates to the interpretation of the automorphisms of II_∞ factor inducing the scaling of trace.

1. If the automorphism for Jones inclusion involves the generator of cyclic automorphism subgroup Z_n of II_1 factor then it would seem that for other values of λ this group cannot be cyclic. SU(2) has discrete subgroups generated by arbitrary phase q and these are dense in $U(1) \subset SU(2)$ sub-group. If the interpretation in terms of Jones inclusion makes sense then the identification $\lambda = \sqrt{\mathcal{M} : \mathcal{N}}/2^k$ makes sense.
2. If HFF of type II_1 is realized as group algebra of infinite symmetric group [K59] , the outer automorphism induced by the diagonally imbedded finite Galois groups can induce only integer values of n and Z_n would correspond to cyclic subgroups. This interpretation conforms with the fact that the automorphisms in the completion of inner automorphisms of HFF of type II_1 induce trivial scalings. Therefore only automorphisms which do not belong to this completion can define HFFs of type III.

2.6.4 What Could Be The Physical Interpretation Of Two Kinds Of Invariants Associated With HFFs Type III?

TGD predicts two kinds of counterparts for S-matrix: M-matrix and U-matrix. Both are expected to be more or less universal.

There are also *two* kinds of invariants and automorphisms associated with HFFs of type III.

1. The first invariant corresponds to the scaling $\lambda \in]0,1[$ of the trace associated with the automorphism of factor of II_∞. Also the end points of the interval make sense. The inverse of this scaling accompanies the inverse of this automorphism.
2. Second invariant corresponds to the time scales $t = T_0$ for which the outer automorphism σ_t reduces to inner automorphism. It turns out that T_0 and λ are related by the formula $\lambda^{iT_0} = 1$, which gives the allowed values of T_0 as $T_0 = n2\pi/log(\lambda)$ [A20] . This formula can be understood intuitively by realizing that λ corresponds to the eigenvalue of the density matrix $\Delta = e^H$ in the simplest possible realization of the state ϕ.

The presence of two automorphisms and invariants brings in mind U matrix characterizing the unitary process occurring in quantum jump and M-matrix characterizing time like entanglement.

1. If one accepts the vision based on quantum measurement theory then λ corresponds to the scaling of the trace resulting when quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ reduces to a tensor power of $M(2, C)$ factor in the state function reduction. The proposed interpretation for U process would be as the inverse of state function reduction transforming this factor back to $\mathcal{M}/\mathcal{N}$. Thus U process and state function reduction would correspond naturally to the scaling and its inverse. This picture might apply not only in single particle case but also for zero energy states which can be seen as states associated the a tensor power of HFFs of type II_1 associated with partons.
2. The implication is that U process can occur only in the direction in which trace is reduced. This would suggest that the full III_1 factor is not a physical notion and that one must restrict the group Z in the crossed product $Z \times_{cr} II_\infty$ to the group N of non-negative integers. In this kind of situation the trace is well defined since the traces for the terms in the crossed product comes as powers λ^{-n} so that the net result is finite. This would mean a reduction to II_∞ factor.
3. Since time t is a natural parameter in elementary particle physics experiment, one could argue that σ_t could define naturally M-matrix. Time parameter would most naturally correspond to a parameter of scaling affecting all $M^4_\pm$ coordinates rather than linear time. This conforms also with the fundamental role of conformal transformations and scalings in TGD framework.

The identification of the full M-matrix in terms of σ does not seem to make sense generally. It would however make sense for incoming and outgoing number theoretic braids so that σ could define universal braiding M-matrices. Inner automorphisms would bring in the dependence on experimental situation. The reduction of the braiding matrix to an inner automorphism for critical values of t which could be interpreted in terms of scaling by power of p. This trivialization would be a counterpart for the elimination of propagator legs from M-matrix element. Vertex itself could be interpreted as unitary isomorphism between tensor product of incoming and outgoing HFFs of type II_1 would code all what is relevant about the particle reaction.

2.6.5 Does The Time Parameter T Represent Time Translation Or Scaling?

The connection $T_n = n2\pi/log(\lambda)$ would give a relationship between the scaling of trace and value of time parameter for which the outer automorphism represented by σ reduces to inner automorphism. It must be emphasized that the time parameter t appearing in σ need not have anything to do with time translation. The alternative interpretation is in terms of $M^4_\pm$ scaling (implying also time scaling) but one cannot exclude even preferred Lorentz boosts in the direction of quantization axis of angular momentum.

Could the time parameter correspond to scaling?

The central role of conformal invariance in quantum TGD suggests that t parameterizes scaling rather than translation. In this case scalings would correspond to powers of $(K\lambda)^n$. The numerical factor K which cannot be excluded a priori, seems to reduce to $K = 1$.

1. The scalings by powers of p have a simple realization in terms of the representation of HFF of type II_∞ as infinite tensor power of $M(p,C)$ with suitably chosen densities matrices in factors to get product of I_∞ and II_1 factor. These matrix algebras have the remarkable property of defining prime tensor power factors of finite matrix algebras. Thus p-adic fractality would reflect directly basic properties of matrix algebras as suggested already earlier. That scalings by powers of p would correspond to automorphism reducing to inner automorphisms would conform with p-adic fractality.
2. Also scalings by powers $[\sqrt{\mathcal{M}:\mathcal{N}}/2^k]^n$ would be physically preferred if one takes previous arguments about Jones inclusions seriously and if also in this case scalings are involved. For $q = exp(i\pi/n)$, $n = 5$ the minimal value of n allowing universal topological quantum computation would correspond to a scaling by Golden Mean and these fractal scalings indeed play a key role in living matter. In particular, Golden Mean makes it visible in the geometry of DNA.

Could the time parameter correspond to time translation?

One can consider also the interpretation of σ_t as time translation. TGD predicts a hierarchy of Planck constants parameterized by rational numbers such that integer multiples are favored. In particular, integers defining ruler and compass polygons are predicted to be in a very special role physically. Since the geometric time span associated with zero energy state should scale as Planck constant one expects that preferred values of time t associated with σ are quantized as rational multiples of some fundamental time scales, say the basic time scale defined by CP_2 length or p-adic time scales.

1. For $\lambda = 1/p$, p prime, the time scale would be $T_n = nT_1$, $T_1 = T_0 = 2\pi/log(p)$ which is not what p-adic length scale hypothesis would suggest.
2. For Jones inclusions one would have $T_n/T_0 = n2\pi/log(2^{2k}/\mathcal{M}:\mathcal{N})$. In the limit when λ becomes very small (the number k of reduced $M(2,C)$ factors is large one obtains $T_n = (n/k)t_1$, $T_1 = T_0\pi/log(2)$. Approximate rational multiples of the basic length scale would be obtained as also predicted by the general quantization of Planck constant.

p-Adic thermodynamics from first principles

Quantum field theory at non-zero temperature can be formulated in the functional integral formalism by replacing the time parameter associated with the unitary time evolution operator $U(t)$ with a complexified time containing as imaginary part the inverse of the temperature: $t \rightarrow t + i\hbar/T$. In the framework of standard quantum field theory this is a mere computational trick but the time parameter associated with the automorphisms σ_t of HFF of type III is a temperature like parameter from the beginning, and its complexification would naturally lead to the analog of thermal QFT.

Thus thermal equilibrium state would be a genuine quantum state rather than fictive but useful auxiliary notion. Thermal equilibrium is defined separately for each incoming parton braid and perhaps even braid (partons can have arbitrarily large size). At elementary particle level p-adic thermodynamics could be in question so that particle massivation would have first principle description. p-Adic thermodynamics is under relatively mild conditions equivalent with its real counterpart obtained by the replacement of p^{L_0} interpreted as a p-adic number with p^{-L_0} interpreted as a real number.

2.6.6 HFFs Of Type III And The Dynamics In $M^4_\pm$ Degrees Of Freedom?

HFFs of type III could be also assigned with the poorly understood dynamics in $M^4_\pm$ degrees of freedom which should have a lot of to do with four-dimensional quantum field theory. Hyper-finite factors of type III_1 might emerge when one extends II_1 to a local algebra by multiplying it with hyper-octonions replaced as analog of matrix factor and considers hyper-quaternionic subalgebra. The resulting algebra would be the analog of local gauge algebra and the elements of algebra would be analogous to conformal fields with complex argument replaced with hyper-octonionic, -quaternionic, or -complex one. Since quantum field theory in M^4 gives rise to hyper-finite III_1 factors one might guess that the hyper-quaternionic restriction indeed gives these factors.

The expansion of the local HFF II_∞ element as $O(m) = \sum_n m^n O_n$, where M^4 coordinate m is interpreted as hyper-quaternion, could have interpretation as expansion in which O_n belongs to $\mathcal{N} g^n$ in the crossed product $\mathcal{N} \times_{cr} \{g^n, n \in Z\}$. The analogy with conformal fields suggests that the power g^n inducing λ^n fold scaling of trace increases the conformal weight by n.

One can ask whether the scaling of trace by powers of λ defines an inclusion hierarchy of sub-algebras of conformal sub-algebras as suggested by previous arguments. One such hierarchy would be the hierarchy of sub-algebras containing only the generators O_m with conformal weight $m \geq n$, $n \in Z$.

It has been suggested that the automorphism Δ could correspond to scaling inside light-cone. This interpretation would fit nicely with Lorentz invariance and TGD in general. The factors III_λ with λ generating semi-subgroups of integers (in particular powers of primes) could be of special physical importance in TGD framework. The values of t for which automorphism reduces to inner automorphism should be of special physical importance in TGD framework. These automorphisms correspond to scalings identifiable in terms of powers of p-adic prime p so that p-adic fractality would find an explanation at the fundamental level.

If the above mentioned expansion in powers of m^n of $M^4_\pm$ coordinate makes sense then the action of σ^t representing a scaling by p^n would leave the elements O invariant or induce a mere inner automorphism. Conformal weight n corresponds naturally to n-ary p-adic length scale by uncertainty principle in p-adic mass calculations.

The basic question is the physical interpretation of the automorphism inducing the scaling of trace by λ and its detailed action in HFF. This scaling could relate to a scaling in M^4 and to the appearance in the trace of an integral over M^4 or subspace of it defining the trace. Fractal structures suggests itself strongly here. At the level of construction of physical states one always selects some minimum non-positive conformal weight defining the tachyonic ground state and physical states have non-negative conformal weights. The interpretation would be as a reduction to HHF of type II_∞ or even II_1.

2.6.7 Could The Continuation Of Braidings To Homotopies Involve Δ^{It} Automorphisms

The representation of braidings as special case of homotopies might lead from discrete automorphisms for HFFs type II_1 to continuous outer automorphisms for HFFs of type III_1. The question is whether the periodic automorphism of II_1 represented as a discrete sub-group of $U(1)$ would be continued to $U(1)$ in the transition.

The automorphism of II_∞ HFF associated with a given value of the scaling factor λ is unique. If Jones inclusions defined by the preferred values of λ as $\lambda = \sqrt{\mathcal{M} : \mathcal{N}}/2^k$ (see the previous considerations), then this automorphism could involve a periodic automorphism of II_1 factor defined by the generator of cyclic subgroup Z_n for $\mathcal{M} : \mathcal{N} < 4$ besides additional shift transforming II_1 factor to I_∞ factor and inducing the scaling.

2.6.8 HFFs Of Type III As Super-Structures Providing Additional Uniqueness?

If the braiding M-matrices are as such highly unique. One could however consider the possibility that they are induced from the automorphisms σ_t for the HFFs of type III restricted to HFFs of type II_∞. If a reduction to inner automorphism in HFF of type III implies same with respect to HFF of type II_∞ and even II_1, they could be trivial for special values of time scaling t assignable to the partons and identifiable as a power of prime p characterizing the parton. This would allow to eliminate incoming and outgoing legs. This elimination would be the counterpart of the division of propagator legs in quantum field theories. Particle masses would however play no role in this process now although the power of padic prime would fix the mass scale of the particle.

2.7 Appendix: Inclusions Of Hyper-Finite Factors Of Type II_1

Many names have been assigned to inclusions: Jones, Wenzl, Ocneacnu, Pimsner-Popa, Wasserman [A76] . It would seem to me that the notion Jones inclusion includes them all so that various names would correspond to different concrete realizations of the inclusions conjugate under outer automorphisms.

1. According to [A76] for inclusions with $\mathcal{M} : \mathcal{N} \leq 4$ (with $A_1^{(1)}$ excluded) there exists a countable infinity of sub-factors with are pairwise non inner conjugate but conjugate to $\mathcal{N}$.
2. Also for any finite group G and its outer action there exists uncountably many sub-factors which are pairwise non inner conjugate but conjugate to the fixed point algebra of G [A76] . For any amenable group G the inclusion is also unique apart from outer automorphism [A45] .

Thus it seems that not only Jones inclusions but also more general inclusions are unique apart from outer automorphism.

Any *-endomorphism σ, which is unit preserving, faithful, and weakly continuous, defines a sub-factor of type II_1 factor [A76] . The construction of Jones leads to a standard inclusion sequence $\mathcal{N} \subset \mathcal{M} \subset \mathcal{M}^1 \subset ....$ This sequence means addition of projectors e_i, $i < 0$, having visualization as an addition of braid strand in braid picture. This hierarchy exists for all factors of type II. At the limit $\mathcal{M}^\infty = \cup_i \mathcal{M}^i$ the braid sequence extends from $-\infty$ to ∞. Inclusion hierarchy can be understood as a hierarchy of Connes tensor powers $\mathcal{M} \otimes_\mathcal{N} \mathcal{M} .... \otimes_\mathcal{N} \mathcal{M}$. Also the ordinary tensor powers of hyper-finite factors of type II_1 (HFF) as well as their tensor products with finite-dimensional matrix algebras are isomorphic to the original HFF so that these objects share the magic of fractals.

Under certain assumptions the hierarchy can be continued also in opposite direction. For a finite index an infinite inclusion hierarchy of factors results with the same value of index. σ is said to be basic if it can be extended to *-endomorphisms from $\mathcal{M}^1$ to $\mathcal{M}$. This means that the hierarchy of inclusions can be continued in the opposite direction: this means elimination of strands in the braid picture. For finite factors (as opposed to hyper-finite ones) there are no basic *-endomorphisms of $\mathcal{M}$ having fixed point algebra of non-abelian G as a sub-factor [A76] .

2.7.1 Jones Inclusions

For hyper-finite factors of type II_1 Jones inclusions allow basic *-endomorphism. They exist for all values of $\mathcal{M} : \mathcal{N} = r$ with $r \in \{4cos^2(\pi/n) | n \geq 3\} \cap [4, \infty)$ [A76] . They are defined for an algebra defined by projectors e_i, $i \geq 1$. All but nearest neighbor projectors commute. $\lambda = 1/r$ appears in the relations for the generators of the algebra given by $e_i e_j e_i = \lambda e_i$, $|i-j| = 1$. $\mathcal{N} \subset \mathcal{M}$ is identified as the double commutator of algebra generated by e_i, $i \geq 2$.

This means that principal graph and its dual are equivalent and the braid defined by projectors can be continued not only to $-\infty$ but that also the dropping of arbitrary number of strands is possible [A76] . It would seem that ADE property of the principal graph meaning single root length codes for the duality in the case of $r \leq 4$ inclusions.

Irreducibility holds true for $r < 4$ in the sense that the intersection of $Q' \cap P = P' \cap P = C$. For $r \geq 4$ one has $dim(Q' \cap P) = 2$. The operators commuting with Q contain besides identify operator of Q also the identify operator of P. Q would contain a single finite-dimensional matrix factor less than P in this case. Basic *-endomorphisms with $\sigma(P) = Q$ is $\sigma(e_i) = e_{i+1}$. The difference between genuine symmetries of quantum TGD and symmetries which can be mimicked by TGD could relate to the irreducibility for $r < 4$ and raise these inclusions in a unique position. This difference could partially justify the hypothesis [K47] that only the groups $G_a \times G_b \subset SU(2) \times SU(2) \subset SL(2, C) \times SU(3)$ define orbifold coverings of $H_\pm = M^4_\pm \times CP_2 \to H_\pm / G_a \times G_b$.

2.7.2 Wassermann's Inclusion

Wasserman's construction of $r = 4$ factors clarifies the role of the subgroup of $G \subset SU(2)$ for these inclusions. Also now $r = 4$ inclusion is characterized by a discrete subgroup $G \subset SU(2)$ and is given by $(1 \otimes \mathcal{M})^G \subset (M_2(C) \times \mathcal{M})^G$. According to [A76] Jones inclusions are irreducible also for $r = 4$. The definition of Wasserman inclusion for $r = 4$ seems however to imply that the identity matrices of both $\mathcal{M}^G$ and $(M(2,C) \otimes \mathcal{M})^G$ commute with $\mathcal{M}^G$ so that the inclusion should be reducible for $r = 4$.

Note that G leaves both the elements of $\mathcal{N}$ and $\mathcal{M}$ invariant whereas $SU(2)$ leaves the elements of $\mathcal{N}$ invariant. $M(2,C)$ is effectively replaced with the orbifold $M(2,C)/G$, with G acting as automorphisms. The space of these orbits has complex dimension $d = 4$ for finite G.

For $r < 4$ inclusion is defined as $M^G \subset M$. The representation of G as outer automorphism must change step by step in the inclusion sequence $... \subset \mathcal{N} \subset \mathcal{M} \subset ...$ since otherwise G would act trivially as one proceeds in the inclusion sequence. This is true since each step brings in additional finite-dimensional tensor factor in which G acts as automorphisms so that although $\mathcal{M}$ can be invariant under $G_{\mathcal{M}}$ it is not invariant under $G_{\mathcal{N}}$.

These two inclusions might accompany each other in TGD based physics. One could consider $r < 4$ inclusion $\mathcal{N} = \mathcal{M}^G \subset \mathcal{M}$ with G acting non-trivially in $\mathcal{M}/\mathcal{N}$ quantum Clifford algebra. $\mathcal{N}$ would decompose by $r = 4$ inclusion to $\mathcal{N}_1 \subset \mathcal{N}$ with $SU(2)$ taking the role of G. $\mathcal{N}/\mathcal{N}_1$ quantum Clifford algebra would transform non-trivially under $SU(2)$ but would be G singlet.

In TGD framework the G-invariance for $SU(2)$ representations means a reduction of S^2 to the orbifold S^2/G. The coverings $H_\pm \to H_\pm/G_a \times G_b$ should relate to these double inclusions and $SU(2)$ inclusion could mean Kac-Moody type gauge symmetry for $\mathcal{N}$. Note that the presence of the factor containing only unit matrix should relate directly to the generator d in the generator set of affine algebra in the McKay construction [K59] . The physical interpretation of the fact that almost all ADE type extended diagrams ($D_n^{(1)}$ must have $n \geq 4$) are allowed for $r = 4$ inclusions whereas D_{2n+1} and E_6 are not allowed for $r < 4$, remains open.

2.7.3 Generalization From $Su(2)$ To Arbitrary Compact Group

The inclusions with index $\mathcal{M} : \mathcal{N} < 4$ have one-dimensional relative commutant $\mathcal{N}' \cup \mathcal{M}$. The most obvious conjecture that $\mathcal{M} : \mathcal{N} \geq 4$ corresponds to a non-trivial relative commutant is wrong. The index for Jones inclusion is identifiable as the square of quantum dimension of the fundamental representation of $SU(2)$. This identification generalizes to an arbitrary representation of arbitrary compact Lie group.

In his thesis Wenzl [A55] studied the representations of Hecke algebras $H_n(q)$ of type A_n obtained from the defining relations of symmetric group by the replacement $e_i^2 = (q-1)e_i + q$. H_n is isomorphic to complex group algebra of S_n if q is not a root of unity and for $q = 1$ the irreducible representations of $H_n(q)$ reduce trivially to Young's representations of symmetric groups. For primitive roots of unity $q = exp(i2\pi/l)$, $l = 4, 5...$, the representations of $H_n(\infty)$ give rise to inclusions for which index corresponds to a quantum dimension of any irreducible representation of $SU(k)$, $k \geq 2$. For $SU(2)$ also the value $l = 3$ is allowed for spin 1/2 representation.

The inclusions are obtained by dropping the first m generators e_k from $H_\infty(q)$ and taking double commutant of both H_∞ and the resulting algebra. The relative commutant corresponds to $H_m(q)$. By reducing by the minimal projection to relative commutant one obtains an inclusion with a trivial relative commutant. These inclusions are analogous to a discrete states superposed in continuum. Thus the results of Jones generalize from the fundamental representation of $SU(2)$ to all representations of all groups $SU(k)$, and in fact to those of general compact groups as it turns out.

The generalization of the formula for index to square of quantum dimension of an irreducible representation of $SU(k)$ reads as

$$\mathcal{M} : \mathcal{N} = \prod_{1 \leq r < s \leq k} \frac{sin^2\left((\lambda_r - \lambda_s + s - r)\pi/l\right)}{sin^2\left((s-r)n/l\right)} . \tag{2.7.1}$$

Here λ_r is the number of boxes in the r^{th} row of the Yang diagram with n boxes characterizing the representations and the condition $1 \leq k \leq l - 1$ holds true. Only Young diagrams satisfying the condition $l - k = \lambda_1 - \lambda_{r_{max}}$ are allowed.

The result would allow to restrict the generalization of the embedding space in such a way that only cyclic group Z_n appears in the covering of $M^4 \to M^4/G_a$ or $CP_2 \to CP_2/G_b$

factor. Be as it may, it seems that quantum representations of any compact Lie group can be realized using the generalization of the embedding space. In the case of SU(2) the interpretation of higher-dimensional quantum representations in terms of Connes tensor products of 2-dimensional fundamental representations is highly suggestive.

The groups $SO(3,1) \times SU(3)$ and $SL(2,C) \times U(2)_{ew}$ have a distinguished position both in physics and quantum TGD and the vision about physics as a generalized number theory implies them. Also the general pattern for inclusions selects these groups, and one can say that the condition that all possible statistics are realized is guaranteed by the choice $M^4 \times CP_2$.

1. $n > 2$ for the quantum counterparts of the fundamental representation of $SU(2)$ means that braid statistics for Jones inclusions cannot give the usual fermionic statistics. That Fermi statistics cannot "emerge" conforms with the role of infinite-D Clifford algebra as a canonical representation of HFF of type II_1. $SO(3,1)$ as isometries of H gives Z_2 statistics via the action on spinors of M^4 and $U(2)$ holonomies for CP_2 realize Z_2 statistics in CP_2 degrees of freedom.

2. $n > 3$ for more general inclusions in turn excludes Z_3 statistics as braid statistics in the general case. $SU(3)$ as isometries induces a non-trivial Z_3 action on quark spinors but trivial action at the embedding space level so that Z_3 statistics would be in question.

Chapter 3

Evolution of Ideas about Hyper-finite Factors in TGD

3.1 Introduction

This chapter has emerged from a splitting of a chapter devote to the possible role of von Neumann algebras known as hyper-finite factors in quantum TGD. Second chapter emerging from the splitting is a representation of basic notions to chapter "Was von Neumann right after all?" [K125] representing only very briefly ideas about application to quantum TGD only briefly.

In the sequel the ideas about TGD applications are reviewed more or less chronologically. A summary about evolution of ideas is in question, not a coherent final structure, and as always the first speculations - in this case roughly for a decade ago - might look rather weird. The vision has however gradually become more realistic looking as deeper physical understanding of factors has evolved slowly.

The mathematics involved is extremely difficult for a physicist like me, and to really learn it at the level of proofs one should reincarnate as a mathematician. Therefore the only practical approach relies on the use of physical intuition to see whether HFFs might the correct structure and what HFFs do mean. What is needed is a concretization of the extremely abstract mathematics involved: mathematics represents only the bones to which physics should add flesh.

3.1.1 Hyper-Finite Factors In Quantum TGD

The following argument suggests that von Neumann algebras known as hyper-finite factors (HFFs) of type III_1 appearing in relativistic quantum field theories provide also the proper mathematical framework for quantum TGD.

1. The Clifford algebra of the infinite-dimensional Hilbert space is a von Neumann algebra known as HFF of type II_1. There also the Clifford algebra at a given point (light-like 3-surface) of world of classical worlds (WCW) is therefore HFF of type II_1. If the fermionic Fock algebra defined by the fermionic oscillator operators assignable to the induced spinor fields (this is actually not obvious!) is infinite-dimensional it defines a representation for HFF of type II_1. Super-conformal symmetry suggests that the extension of the Clifford algebra defining the fermionic part of a super-conformal algebra by adding bosonic super-generators representing symmetries of WCW respects the HFF property. It could however occur that HFF of type II_∞ results.
2. WCW is a union of sub-WCWs associated with causal diamonds (CD) defined as intersections of future and past directed light-cones. One can allow also unions of CDs and the proposal is that CDs within CDs are possible. Whether CDs can intersect is not clear.
3. The assumption that the M^4 proper distance a between the tips of CD is quantized in powers of 2 reproduces p-adic length scale hypothesis but one must also consider the possibility that a can have all possible values. Since $SO(3)$ is the isotropy group of CD, the CDs associated with a given value of a and with fixed lower tip are parameterized by the Lobatchevski space

$L(a) = SO(3,1)/SO(3)$. Therefore the CDs with a free position of lower tip are parameterized by $M^4 \times L(a)$. A possible interpretation is in terms of quantum cosmology with a identified as cosmic time [K101] . Since Lorentz boosts define a non-compact group, the generalization of so called crossed product construction strongly suggests that the local Clifford algebra of WCW is HFF of type III_1. If one allows all values of a, one ends up with $M^4 \times M^4_+$ as the space of moduli for WCW.

4. An interesting special aspect of 8-dimensional Clifford algebra with Minkowski signature is that it allows an octonionic representation of gamma matrices obtained as tensor products of unit matrix 1 and 7-D gamma matrices γ_k and Pauli sigma matrices by replacing 1 and γ_k by octonions. This inspires the idea that it might be possible to end up with quantum TGD from purely number theoretical arguments. This seems to be the case. One can start from a local octonionic Clifford algebra in M^8. Associativity condition is satisfied if one restricts the octonionic algebra to a subalgebra associated with any hyper-quaternionic and thus 4-D sub-manifold of M^8. This means that the Kähler-Dirac gamma matrices associated with the Kähler action span a complex quaternionic sub-space at each point of the sub-manifold. This associative sub-algebra can be mapped a matrix algebra. Together with $M^8 - H$ duality [K126, K35] this leads automatically to quantum TGD and therefore also to the notion of WCW and its Clifford algebra which is however only mappable to an associative algebra and thus to HFF of type II_1.

3.1.2 Hyper-Finite Factors And M-Matrix

HFFs of type III_1 provide a general vision about M-matrix.

1. The factors of type III allow unique modular automorphism Δ^{it} (fixed apart from unitary inner automorphism). This raises the question whether the modular automorphism could be used to define the M-matrix of quantum TGD. This is not the case as is obvious already from the fact that unitary time evolution is not a sensible concept in zero energy ontology.

2. Concerning the identification of M-matrix the notion of state as it is used in theory of factors is a more appropriate starting point than the notion modular automorphism but as a generalization of thermodynamical state is certainly not enough for the purposes of quantum TGD and quantum field theories (algebraic quantum field theorists might disagree!). Zero energy ontology requires that the notion of thermodynamical state should be replaced with its "complex square root" abstracting the idea about M-matrix as a product of positive square root of a diagonal density matrix and a unitary S-matrix. This generalization of thermodynamical state -if it exists- would provide a firm mathematical basis for the notion of M-matrix and for the fuzzy notion of path integral.

3. The existence of the modular automorphisms relies on Tomita-Takesaki theorem, which assumes that the Hilbert space in which HFF acts allows cyclic and separable vector serving as ground state for both HFF and its commutant. The translation to the language of physicists states that the vacuum is a tensor product of two vacua annihilated by annihilation oscillator type algebra elements of HFF and creation operator type algebra elements of its commutant isomorphic to it. Note however that these algebras commute so that the two algebras are not hermitian conjugates of each other. This kind of situation is exactly what emerges in zero energy ontology: the two vacua can be assigned with the positive and negative energy parts of the zero energy states entangled by M-matrix.

4. There exists infinite number of thermodynamical states related by modular automorphisms. This must be true also for their possibly existing "complex square roots". Physically they would correspond to different measurement interactions giving rise to Kähler functions of WCW differing only by a real part of holomorphic function of complex coordinates of WCW and arbitrary function of zero mode coordinates and giving rise to the same Kähler metric of WCW.

3.1.3 Connes Tensor Product As A Realization Of Finite Measurement Resolution

The inclusions $\mathcal{N} \subset \mathcal{M}$ of factors allow an attractive mathematical description of finite measurement resolution in terms of Connes tensor product but do not fix M-matrix as was the original optimistic belief.

1. In zero energy ontology $\mathcal{N}$ would create states experimentally indistinguishable from the original one. Therefore $\mathcal{N}$ takes the role of complex numbers in non-commutative quantum theory. The space $\mathcal{M}/\mathcal{N}$ would correspond to the operators creating physical states modulo measurement resolution and has typically fractal dimension given as the index of the inclusion. The corresponding spinor spaces have an identification as quantum spaces with non-commutative $\mathcal{N}$-valued coordinates.
2. This leads to an elegant description of finite measurement resolution. Suppose that a universal M-matrix describing the situation for an ideal measurement resolution exists as the idea about square root of state encourages to think. Finite measurement resolution forces to replace the probabilities defined by the M-matrix with their $\mathcal{N}$ "averaged" counterparts. The "averaging" would be in terms of the complex square root of $\mathcal{N}$-state and a direct analog of functionally or path integral over the degrees of freedom below measurement resolution defined by (say) length scale cutoff.
3. One can construct also directly M-matrices satisfying the measurement resolution constraint. The condition that $\mathcal{N}$ acts like complex numbers on M-matrix elements as far as $\mathcal{N}$ "averaged" probabilities are considered is satisfied if M-matrix is a tensor product of M-matrix in $\mathcal{M}(\mathcal{N}$ interpreted as finite-dimensional space with a projection operator to $\mathcal{N}$. The condition that $\mathcal{N}$ averaging in terms of a complex square root of $\mathcal{N}$ state produces this kind of M-matrix poses a very strong constraint on M-matrix if it is assumed to be universal (apart from variants corresponding to different measurement interactions).

3.1.4 Concrete Realization Of The Inclusion Hierarchies

A concrete construction of M-matrix motivated by the recent rather precise view about basic variational principles of TGD allows to identify rather concretely the inclusions of HFFs in TGD framework and relate them to the hierarchies of broken conformal symmetries accompanying quantum criticalities.

1. Fundamental fermions localized to string world sheets can be said to propagate as massless particles along their boundaries. The fundamental interaction vertices correspond to two fermion scattering for fermions at opposite throats of wormhole contact and the inverse of the conformal scaling generator L_0 would define the stringy propagator characterizing this interaction. Fundamental bosons correspond to pairs of fermion and antifermion at opposite throats of wormhole contact. Physical particles correspond to pairs of wormhole contacts with monopole Kähler magnetic flux flowing around a loop going through wormhole contacts.
2. The formulation of scattering amplitudes in terms of Yangian of the super-symplectic algebra leads to a rather detailed view about scattering amplitudes [K114]. In this formulation scattering amplitudes are representations for sequences of algebraic operations connecting collections of elements of Yangian and sequences produce the same result. A huge generalization of the duality symmetry of the hadronic string models is in question.
3. The reduction of the hierarchy of Planck constants $h_{eff}/h = n$ to a hierarchy of quantum criticalities accompanied by a hierarchy of sub-algebras of super-symplectic algebra acting as conformal gauge symmetries leads to the identification of inclusions of HFFs as inclusions of WCW Clifford algebras characterizing by $n(i)$ and $n(i+1) = m(i) \times n(i)$ so that hierarchies of von Neuman algebras, of Planck constants, and of quantum criticalities would be very closely related. In the transition $n(i) \rightarrow n(i+1) = m(i) \times n(i)$ the measurement accuracy indeed increases since some conformal gauge degrees of freedom are transformed to physical ones. An open question is whether one could interpret $m(i)$ as the integer characterizing inclusion: the problem is that also $m(i) = 2$ with $\mathcal{M} : \mathcal{N} = 4$ seems to be allowed whereas Jones inclusions allow only $m \geq 3$.

Even more, number theoretic universality and strong form of holography leads to a detailed vision about the construction of scattering amplitudes suggesting that the hierarchy of algebraic extensions of rationals relates to the above mentioned hierarchies.

3.1.5 Analogs of quantum matrix groups from finite measurement resolution?

The notion of quantum group replaces ordinary matrices with matrices with non-commutative elements. In TGD framework I have proposed that the notion should relate to the inclusions of von Neumann algebras allowing to describe mathematically the notion of finite measurement resolution.

In this article I will consider the notion of quantum matrix inspired by recent view about quantum TGD and it provides a concrete representation and physical interpretation of quantum groups in terms of finite measurement resolution. The basic idea is to replace complex matrix elements with operators expressible as products of non-negative hermitian operators and unitary operators analogous to the products of modulus and phase as a representation for complex numbers.

The condition that determinant and sub-determinants exist is crucial for the well-definedness of eigenvalue problem in the generalized sense. The weak definition of determinant meaning its development with respect to a fixed row or column does not pose additional conditions. Strong definition of determinant requires its invariance under permutations of rows and columns. The permutation of rows/columns turns out to have interpretation as braiding for the hermitian operators defined by the moduli of operator valued matrix elements. The commutativity of all sub-determinants is essential for the replacement of eigenvalues with eigenvalue spectra of hermitian operators and sub-determinants define mutually commuting set of operators.

The resulting quantum matrices define a more general structure than quantum group but provide a concrete representation and interpretation for quantum group in terms of finite measurement resolution if q is a root of unity. For $q = \pm 1$ (Bose-Einstein or Fermi-Dirac statistics) one obtains quantum matrices for which the determinant is apart from possible change by sign factor invariant under the permutations of both rows and columns. One could also understand the fractal structure of inclusion sequences of hyper-finite factors resulting by recursively replacing operators appearing as matrix elements with quantum matrices.

3.1.6 Quantum Spinors And Fuzzy Quantum Mechanics

The notion of quantum spinor leads to a quantum mechanical description of fuzzy probabilities. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with "true" and "false". The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and de-coherence is not a problem as long as it does not induce this transition.

This chapter represents a summary about the development of the ideas with last sections representing the recent latest about the realization and role of HFFs in TGD. I have saved the reader from those speculations that have turned out to reflect my own ignorance or are inconsistent with what I regarded established parts of quantum TGD.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

3.2 A Vision About The Role Of HFFs In TGD

It is clear that at least the hyper-finite factors of type II_1 assignable to WCW spinors must have a profound role in TGD. Whether also HFFs of type III_1 appearing also in relativistic quantum field theories emerge when WCW spinors are replaced with spinor fields is not completely clear. I have proposed several ideas about the role of hyper-finite factors in TGD framework. In particular, Connes tensor product is an excellent candidate for defining the notion of measurement resolution.

In the following this topic is discussed from the perspective made possible by ZEO and the recent advances in the understanding of M-matrix using the notion of bosonic emergence. The conclusion is that the notion of state as it appears in the theory of factors is not enough for the purposes of quantum TGD. The reason is that state in this sense is essentially the counterpart of thermodynamical state. The construction of M-matrix might be understood in the framework of factors if one replaces state with its "complex square root" natural if quantum theory is regarded as a "complex square root" of thermodynamics. It is also found that the idea that Connes tensor product could fix M-matrix is too optimistic but an elegant formulation in terms of partial trace for the notion of M-matrix modulo measurement resolution exists and Connes tensor product allows interpretation as entanglement between sub-spaces consisting of states not distinguishable in the measurement resolution used. The partial trace also gives rise to non-pure states naturally.

The newest element in the vision is the proposal that quantum criticality of TGD Universe is realized as hierarchies of inclusions of super-conformal algebras with conformal weights coming as multiples of integer n, where n varies. If n_1 divides n_2 then various super-conformal algebras C_{n_2} are contained in C_{n_1}. This would define naturally the inclusion.

3.2.1 Basic facts about factors

In this section basic facts about factors are discussed. My hope that the discussion is more mature than or at least complementary to the summary that I could afford when I started the work with factors for more than half decade ago. I of course admit that this just a humble attempt of a physicist to express physical vision in terms of only superficially understood mathematical notions.

Basic notions

First some standard notations. Let $\mathcal{B}(\mathcal{H})$ denote the algebra of linear operators of Hilbert space $\mathcal{H}$ bounded in the norm topology with norm defined by the supremum for the length of the image of a point of unit sphere $\mathcal{H}$. This algebra has a lot of common with complex numbers in that the counterparts of complex conjugation, order structure and metric structure determined by the algebraic structure exist. This means the existence involution -that is *- algebra property. The order structure determined by algebraic structure means following: $A \geq 0$ defined as the condition $(A\xi, \xi) \geq 0$ is equivalent with $A = B^*B$. The algebra has also metric structure $||AB|| \leq ||A||||B|$ (Banach algebra property) determined by the algebraic structure. The algebra is also C^* algebra: $||A^*A|| = ||A||^2$ meaning that the norm is algebraically like that for complex numbers.

A von Neumann algebra $\mathcal{M}$ [A16] is defined as a weakly closed non-degenerate *-subalgebra of $\mathcal{B}(\mathcal{H})$ and has therefore all the above mentioned properties. From the point of view of physicist it is important that a sub-algebra is in question.

In order to define factors one must introduce additional structure.

1. Let $\mathcal{M}$ be subalgebra of $\mathcal{B}(\mathcal{H})$ and denote by $\mathcal{M}'$ its commutant $(\mathcal{H})$ commuting with it and allowing to express $\mathcal{B}(\mathcal{H})$ as $\mathcal{B}(\mathcal{H}) = \mathcal{M} \vee \mathcal{M}'$.

2. A factor is defined as a von Neumann algebra satisfying $\mathcal{M}'' = \mathcal{M}$ $\mathcal{M}$ is called factor. The equality of double commutant with the original algebra is thus the defining condition so that also the commutant is a factor. An equivalent definition for factor is as the condition that the intersection of the algebra and its commutant reduces to a complex line spanned by a unit operator. The condition that the only operator commuting with all operators of the factor is unit operator corresponds to irreducibility in representation theory.

3. Some further basic definitions are needed. $\Omega \in \mathcal{H}$ is cyclic if the closure of $\mathcal{M}\Omega$ is $\mathcal{H}$ and separating if the only element of $\mathcal{M}$ annihilating Ω is zero. Ω is cyclic for $\mathcal{M}$ if and only if it is separating for its commutant. In so called standard representation Ω is both cyclic and separating.

4. For hyperfinite factors an inclusion hierarchy of finite-dimensional algebras whose union is dense in the factor exists. This roughly means that one can approximate the algebra in arbitrary accuracy with a finite-dimensional sub-algebra.

The definition of the factor might look somewhat artificial unless one is aware of the underlying physical motivations. The motivating question is what the decomposition of a physical system to non-interacting sub-systems could mean. The decomposition of $\mathcal{B}(\mathcal{H})$ to $\vee$ product realizes this decomposition.

1. Tensor product $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is the decomposition according to the standard quantum measurement theory and means the decomposition of operators in $\mathcal{B}(\mathcal{H})$ to tensor products of mutually commuting operators in $\mathcal{M} = \mathcal{B}(\mathcal{H}_1)$ and $\mathcal{M}' = \mathcal{B}(\mathcal{H}_2)$. The information about $\mathcal{M}$ can be coded in terms of projection operators. In this case projection operators projecting to a complex ray of Hilbert space exist and arbitrary compact operator can be expressed as a sum of these projectors. For factors of type I minimal projectors exist. Factors of type I_n correspond to sub-algebras of $\mathcal{B}(\mathcal{H})$ associated with infinite-dimensional Hilbert space and I_∞ to $\mathcal{B}(\mathcal{H})$ itself. These factors appear in the standard quantum measurement theory where state function reduction can lead to a ray of Hilbert space.
2. For factors of type II no minimal projectors exists whereas finite projectors exist. For factors of type II_1 all projectors have trace not larger than one and the trace varies in the range $(0,1]$. In this case cyclic vectors Ω exist. State function reduction can lead only to an infinite-dimensional subspace characterized by a projector with trace smaller than 1 but larger than zero. The natural interpretation would be in terms of finite measurement resolution. The tensor product of II_1 factor and I_∞ is II_∞ factor for which the trace for a projector can have arbitrarily large values. II_1 factor has a unique finite tracial state and the set of traces of projections spans unit interval. There is uncountable number of factors of type II but hyper-finite factors of type II_1 are the exceptional ones and physically most interesting.
3. Factors of type III correspond to an extreme situation. In this case the projection operators E spanning the factor have either infinite or vanishing trace and there exists an isometry mapping $E\mathcal{H}$ to $\mathcal{H}$ meaning that the projection operator spans almost all of $\mathcal{H}$. All projectors are also related to each other by isometry. Factors of type III are smallest if the factors are regarded as sub-algebras of a fixed $\mathcal{B}(\mathcal{H})$ where $\mathcal{H}$ corresponds to isomorphism class of Hilbert spaces. Situation changes when one speaks about concrete representations. Also now hyper-finite factors are exceptional.
4. Von Neumann algebras define a non-commutative measure theory. Commutative von Neumann algebras indeed reduce to $L^\infty(X)$ for some measure space (X,μ) and vice versa.

Weights, states and traces

The notions of weight, state, and trace are standard notions in the theory of von Neumann algebras.

1. A weight of von Neumann algebra is a linear map from the set of positive elements (those of form a^*a) to non-negative reals.
2. A positive linear functional is weight with $\omega(1)$ finite.
3. A state is a weight with $\omega(1) = 1$.
4. A trace is a weight with $\omega(aa^*) = \omega(a^*a)$ for all a.
5. A tracial state is a weight with $\omega(1) = 1$.

A factor has a trace such that the trace of a non-zero projector is non-zero and the trace of projection is infinite only if the projection is infinite. The trace is unique up to a rescaling. For factors that are separable or finite, two projections are equivalent if and only if they have the same trace. Factors of type I_n the values of trace are equal to multiples of $1/n$. For a factor of type I_∞ the value of trace are $0,1,2,....$ For factors of type II_1 the values span the range $[0,1]$ and for factors of type II_∞ n the range $[0,\infty)$. For factors of type III the values of the trace are 0, and ∞.

Tomita-Takesaki theory

Tomita-Takesaki theory is a vital part of the theory of factors. First some definitions.

1. Let $\omega(x)$ be a faithful state of von Neumann algebra so that one has $\omega(xx^*) > 0$ for $x > 0$. Assume by Riesz lemma the representation of ω as a vacuum expectation value: $\omega = (\cdot\Omega, \Omega)$, where Ω is cyclic and separating state.

2. Let

$$L^\infty(\mathcal{M}) \equiv \mathcal{M} \ , \quad L^2(\mathcal{M}) = \mathcal{H} \ , \qquad L^1(\mathcal{M}) = \mathcal{M}_* \ , \tag{3.2.1}$$

where $\mathcal{M}_*$ is the pre-dual of $\mathcal{M}$ defined by linear functionals in $\mathcal{M}$. One has ${\mathcal{M}_*}^* = \mathcal{M}$.

3. The conjugation $x \to x^*$ is isometric in $\mathcal{M}$ and defines a map $\mathcal{M} \to L^2(\mathcal{M})$ via $x \to x\Omega$. The map $S_0; x\Omega \to x^*\Omega$ is however non-isometric.
4. Denote by S the closure of the anti-linear operator S_0 and by $S = J\Delta^{1/2}$ its polar decomposition analogous that for complex number and generalizing polar decomposition of linear operators by replacing (almost) unitary operator with anti-unitary J. Therefore $\Delta = S^*S > 0$ is positive self-adjoint and J an anti-unitary involution. The non-triviality of Δ reflects the fact that the state is not trace so that hermitian conjugation represented by S in the state space brings in additional factor $\Delta^{1/2}$.
5. What x can be is puzzling to physicists. The restriction fermionic Fock space and thus to creation operators would imply that Δ would act non-trivially only vacuum state so that $\Delta > 0$ condition would not hold true. The resolution of puzzle is the allowance of tensor product of Fock spaces for which vacua are conjugates: only this gives cyclic and separating state. This is natural in ZEO.

The basic results of Tomita-Takesaki theory are following.

1. The basic result can be summarized through the following formulas

$$\Delta^{it} M \Delta^{-it} = \mathcal{M} \ , J\mathcal{M}J = \mathcal{M}' \ .$$

2. The latter formula implies that $\mathcal{M}$ and $\mathcal{M}'$ are isomorphic algebras. The first formula implies that a one parameter group of modular automorphisms characterizes partially the factor. The physical meaning of modular automorphisms is discussed in [A34, A66] Δ is Hermitian and positive definite so that the eigenvalues of $log(\Delta)$ are real but can be negative. Δ^{it} is however not unitary for factors of type II and III. Physically the non-unitarity must relate to the fact that the flow is contracting so that hermiticity as a local condition is not enough to guarantee unitarity.
3. $\omega \to \sigma_t^\omega = Ad\Delta^{it}$ defines a canonical evolution -modular automorphism- associated with ω and depending on it. The Δ:s associated with different ω:s are related by a unitary inner automorphism so that their equivalence classes define an invariant of the factor.

Tomita-Takesaki theory gives rise to a non-commutative measure theory which is highly non-trivial. In particular the spectrum of Δ can be used to classify the factors of type II and III.

Modular automorphisms

Modular automorphisms of factors are central for their classification.

1. One can divide the automorphisms to inner and outer ones. Inner automorphisms correspond to unitary operators obtained by exponentiating Hermitian Hamiltonian belonging to the factor and connected to identity by a flow. Outer automorphisms do not allow a representation as a unitary transformations although $log(\Delta)$ is formally a Hermitian operator.
2. The fundamental group of the type II_1 factor defined as fundamental group group of corresponding II_∞ factor characterizes partially a factor of type II_1. This group consists real numbers λ such that there is an automorphism scaling the trace by λ. Fundamental group typically contains all reals but it can be also discrete and even trivial.
3. Factors of type III allow a one-parameter group of modular automorphisms, which can be used to achieve a partial classification of these factors. These automorphisms define a flow in the center of the factor known as flow of weights. The set of parameter values λ for which ω is mapped to itself and the center of the factor defined by the identity operator (projector to the factor as a sub-algebra of $\mathcal{B}(\mathcal{H})$) is mapped to itself in the modular automorphism defines

the Connes spectrum of the factor. For factors of type III_λ this set consists of powers of $\lambda < 1$. For factors of type III_0 this set contains only identity automorphism so that there is no periodicity. For factors of type III_1 Connes spectrum contains all real numbers so that the automorphisms do not affect the identity operator of the factor at all.

The modules over a factor correspond to separable Hilbert spaces that the factor acts on. These modules can be characterized by M-dimension. The idea is roughly that complex rays are replaced by the sub-spaces defined by the action of $\mathcal{M}$ as basic units. M-dimension is not integer valued in general. The so called standard module has a cyclic separating vector and each factor has a standard representation possessing antilinear involution J such that $\mathcal{M}' = J\mathcal{M}J$ holds true (note that J changes the order of the operators in conjugation). The inclusions of factors define modules having interpretation in terms of a finite measurement resolution defined by $\mathcal{M}$.

Crossed product as a way to construct factors of type III

By using so called crossed product crossedproduct for a group G acting in algebra A one can obtain new von Neumann algebras. One ends up with crossed product by a two-step generalization by starting from the semidirect product $G \triangleleft H$ for groups defined as $(g_1, h_1)(g_2, h_2) = (g_1 h_1(g_2), h_1 h_2)$ (note that Poincare group has interpretation as a semidirect product $M^4 \triangleleft SO(3,1)$ of Lorentz and translation groups). At the first step one replaces the group H with its group algebra. At the second step the the group algebra is replaced with a more general algebra. What is formed is the semidirect product $A \triangleleft G$ which is sum of algebras Ag. The product is given by $(a_1, g_1)(a_2, g_2) = (a_1 g_1(a_2), g_1 g_2)$. This construction works for both locally compact groups and quantum groups. A not too highly educated guess is that the construction in the case of quantum groups gives the factor $\mathcal{M}$ as a crossed product of the included factor $\mathcal{N}$ and quantum group defined by the factor space $\mathcal{M}/\mathcal{N}$.

The construction allows to express factors of type III as crossed products of factors of type II_∞ and the 1-parameter group G of modular automorphisms assignable to any vector which is cyclic for both factor and its commutant. The ergodic flow θ_λ scales the trace of projector in II_∞ factor by $\lambda > 0$. The dual flow defined by G restricted to the center of II_∞ factor does not depend on the choice of cyclic vector.

The Connes spectrum - a closed subgroup of positive reals - is obtained as the exponent of the kernel of the dual flow defined as set of values of flow parameter λ for which the flow in the center is trivial. Kernel equals to $\{0\}$ for III_0, contains numbers of form $log(\lambda)Z$ for factors of type III_λ and contains all real numbers for factors of type III_1 meaning that the flow does not affect the center.

Inclusions and Connes tensor product

Inclusions $\mathcal{N} \subset \mathcal{M}$ of von Neumann algebras have physical interpretation as a mathematical description for sub-system-system relation. In [K125] there is more extensive TGD colored description of inclusions and their role in TGD. Here only basic facts are listed and the Connes tensor product is explained.

For type I algebras the inclusions are trivial and tensor product description applies as such. For factors of II_1 and III the inclusions are highly non-trivial. The inclusion of type II_1 factors were understood by Vaughan Jones [A2] and those of factors of type III by Alain Connes [A19] .

Formally sub-factor $\mathcal{N}$ of $\mathcal{M}$ is defined as a closed *-stable C-subalgebra of $\mathcal{M}$. Let $\mathcal{N}$ be a sub-factor of type II_1 factor $\mathcal{M}$. Jones index $\mathcal{M} : \mathcal{N}$ for the inclusion $\mathcal{N} \subset \mathcal{M}$ can be defined as $\mathcal{M} : \mathcal{N} = dim_N(L^2(\mathcal{M})) = Tr_{N'}(id_{L^2(\mathcal{M})})$. One can say that the dimension of completion of $\mathcal{M}$ as $\mathcal{N}$ module is in question.

Basic findings about inclusions

What makes the inclusions non-trivial is that the position of $\mathcal{N}$ in $\mathcal{M}$ matters. This position is characterized in case of hyper-finite II_1 factors by index $\mathcal{M} : \mathcal{N}$ which can be said to the dimension of $\mathcal{M}$ as $\mathcal{N}$ module and also as the inverse of the dimension defined by the trace of the projector from $\mathcal{M}$ to $\mathcal{N}$. It is important to notice that $\mathcal{M} : \mathcal{N}$ does not characterize either $\mathcal{M}$ or $\mathcal{M}$, only the embedding.

The basic facts proved by Jones are following [A2] .

1. For pairs $\mathcal{N} \subset \mathcal{M}$ with a finite principal graph the values of $\mathcal{M} : \mathcal{N}$ are given by

$$a) \ \mathcal{M} : \mathcal{N} = 4cos^2(\pi/h) \ \ , \quad h \geq 3 \ \ ,$$
$$b) \ \mathcal{M} : \mathcal{N} \geq 4 \ \ . \tag{3.2.2}$$

   the numbers at right hand side are known as Beraha numbers [A53] . The comments below give a rough idea about what finiteness of principal graph means.

2. As explained in [B34] , for $\mathcal{M} : \mathcal{N} < 4$ one can assign to the inclusion Dynkin graph of ADE type Lie-algebra g with h equal to the Coxeter number h of the Lie algebra given in terms of its dimension and dimension r of Cartan algebra r as $h = (dimg(g) - r)/r$. For $\mathcal{M} : \mathcal{N} < 4$ ordinary Dynkin graphs of D_{2n} and E_6, E_8 are allowed. The Dynkin graphs of Lie algebras of $SU(n)$, E_7 and D_{2n+1} are however not allowed. $E_6, E_7, andE_8$ correspond to symmetry groups of tetrahedron, octahedron/cube, and icosahedron/dodecahedron. The group for octahedron/cube is missing: what could this mean?

   For $\mathcal{M} : \mathcal{N} = 4$ one can assign to the inclusion an extended Dynkin graph of type ADE characterizing Kac Moody algebra. Extended ADE diagrams characterize also the subgroups of $SU(2)$ and the interpretation proposed in [A96] is following-

   The ADE diagrams are associated with the $n = \infty$ case having $\mathcal{M} : \mathcal{N} \geq 4$. There are diagrams corresponding to infinite subgroups: A_∞ corresponding to $SU(2)$ itself, $A_{-\infty,\infty}$ corresponding to circle group $U(1)$, and infinite dihedral groups (generated by a rotation by a non-rational angle and reflection.

   One can construct also inclusions for which the diagrams corresponding to finite subgroups $G \subset SU(2)$ are extension of A_n for cyclic groups, of D_n dihedral groups, and of E_n with $n = 6, 7, 8$ for tetrahedron, cube, dodecahedron. These extensions correspond to ADE type Kac-Moody algebras.

   The extension is constructed by constructing first factor R as infinite tensor power of $M_2(C)$ (complexified quaternions). Sub-factor R_0 consists elements of of R of form $Id \otimes x$. $SU(2)$ preserves R_0 and for any subgroup G of $SU(2)$ one can identify the inclusion $N \subset M$ in terms of $N = R_0^G$ and $M = R^G$, where $N = R_0^G$ and $M = R^G$ consists of fixed points of R_0 and R under the action of G. The principal graph for $N \subset M$ is the extended Coxeter-Dynk graph for the subgroup G.

   Physicist might try to interpret this by saying that one considers only sub-algebras R_0^G and R^G of observables invariant under G and obtains extended Dynkin diagram of G defining an ADE type Kac-Moody algebra. Could the condition that Kac-Moody algebra elements with non-vanishing conformal weight annihilate the physical states state that the state is invariant under R_0 defining measurement resolution. Besides this the states are also invariant under finite group G? Could R_0^G and R^G correspond just to states which are also invariant under finite group G.

Connes tensor product

The basic idea of Connes tensor product is that a sub-space generated sub-factor $\mathcal{N}$ takes the role of the complex ray of Hilbert space. The physical interpretation is in terms of finite measurement resolution: it is not possible to distinguish between states obtained by applying elements of $\mathcal{N}$.

Intuitively it is clear that it should be possible to decompose $\mathcal{M}$ to a tensor product of factor space $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$:

$$\mathcal{M} \ = \ \mathcal{M}/\mathcal{N} \otimes \mathcal{N} \ . \tag{3.2.3}$$

One could regard the factor space $\mathcal{M}/\mathcal{N}$ as a non-commutative space in which each point corresponds to a particular representative in the equivalence class of points defined by $\mathcal{N}$. The connections between quantum groups and Jones inclusions suggest that this space closely relates to quantum groups. An alternative interpretation is as an ordinary linear space obtained by mapping $\mathcal{N}$ rays to ordinary complex rays. These spaces appear in the representations of quantum groups. Similar procedure makes sense also for the Hilbert spaces in which $\mathcal{M}$ acts.

Connes tensor product can be defined in the space $\mathcal{M}\otimes\mathcal{M}$ as entanglement which effectively reduces to entanglement between $\mathcal{N}$ sub-spaces. This is achieved if $\mathcal{N}$ multiplication from right is equivalent with $\mathcal{N}$ multiplication from left so that $\mathcal{N}$ acts like complex numbers on states. One can imagine variants of the Connes tensor product and in TGD framework one particular variant appears naturally as will be found.

In the finite-dimensional case Connes tensor product of Hilbert spaces has a rather simple representation. If the matrix algebra N of $n\times n$ matrices acts on V from right, V can be regarded as a space formed by $m\times n$ matrices for some value of m. If N acts from left on W, W can be regarded as space of $n\times r$ matrices.

1. In the first representation the Connes tensor product of spaces V and W consists of $m\times r$ matrices and Connes tensor product is represented as the product VW of matrices as $(VW)_{mr}e^{mr}$. In this representation the information about N disappears completely as the interpretation in terms of measurement resolution suggests. The sum over intermediate states defined by N brings in mind path integral.
2. An alternative and more physical representation is as a state

$$\sum_n V_{mn}W_{nr}e^{mn}\otimes e^{nr}$$

in the tensor product $V\otimes W$.
3. One can also consider two spaces V and W in which N acts from right and define Connes tensor product for $A^\dagger\otimes_N B$ or its tensor product counterpart. This case corresponds to the modification of the Connes tensor product of positive and negative energy states. Since Hermitian conjugation is involved, matrix product does not define the Connes tensor product now. For $m=r$ case entanglement coefficients should define a unitary matrix commuting with the action of the Hermitian matrices of N and interpretation would be in terms of symmetry. HFF property would encourage to think that this representation has an analog in the case of HFFs of type II_1.
4. Also type I_n factors are possible and for them Connes tensor product makes sense if one can assign the inclusion of finite-D matrix algebras to a measurement resolution.

Factors in quantum field theory and thermodynamics

Factors arise in thermodynamics and in quantum field theories [A85, A34, A66] . There are good arguments showing that in HFFs of III_1 appear are relativistic quantum field theories. In non-relativistic QFTs the factors of type I appear so that the non-compactness of Lorentz group is essential. Factors of type III_1 and III_λ appear also in relativistic thermodynamics.

The geometric picture about factors is based on open subsets of Minkowski space. The basic intuitive view is that for two subsets of M^4, which cannot be connected by a classical signal moving with at most light velocity, the von Neumann algebras commute with each other so that $\vee$ product should make sense.

Some basic mathematical results of algebraic quantum field theory [A66] deserve to be listed since they are suggestive also from the point of view of TGD.

1. Let $\mathcal{O}$ be a bounded region of R^4 and define the region of M^4 as a union $\cup_{|x|<\epsilon}(\mathcal{O}+x)$ where $(\mathcal{O}+x)$ is the translate of O and $|x|$ denotes Minkowski norm. Then every projection $E\in\mathcal{M}(\mathcal{O})$ can be written as WW^* with $W\in\mathcal{M}(\mathcal{O}_\epsilon)$ and $W^*W=1$. Note that the union is not a bounded set of M^4. This almost establishes the type III property.
2. Both the complement of light-cone and double light-cone define HFF of type III_1. Lorentz boosts induce modular automorphisms.
3. The so called split property suggested by the description of two systems of this kind as a tensor product in relativistic QFTs is believed to hold true. This means that the HFFs of

type III_1 associated with causally disjoint regions are sub-factors of factor of type I_∞. This means

$$\mathcal{M}_1 \subset \mathcal{B}(\mathcal{H}_1) \times 1 \ , \ \mathcal{M}_2 \subset 1 \otimes \mathcal{B}(\mathcal{H}_2) \ .$$

An infinite hierarchy of inclusions of HFFs of type III_1s is induced by set theoretic inclusions.

3.2.2 TGD and factors

The following vision about TGD and factors relies heavily on zero energy ontology, TGD inspired quantum measurement theory, basic vision about quantum TGD, and bosonic emergence.

The problems

Concerning the role of factors in TGD framework there are several problems of both conceptual and technical character.

1. Conceptual problems

It is safest to start from the conceptual problems and take a role of skeptic.

1. Under what conditions the assumptions of Tomita-Takesaki formula stating the existence of modular automorphism and isomorphy of the factor and its commutant hold true? What is the physical interpretation of the formula $\mathcal{M}' = J\mathcal{M}J$ relating factor and its commutant in TGD framework?
2. Is the identification $M = \Delta^{it}$ sensible is quantum TGD and ZEO, where M-matrix is "complex square root" of exponent of Hamiltonian defining thermodynamical state and the notion of unitary time evolution is given up? The notion of state ω leading to Δ is essentially thermodynamical and one can wonder whether one should take also a "complex square root" of ω to get M-matrix giving rise to a genuine quantum theory.
3. TGD based quantum measurement theory involves both quantum fluctuating degrees of freedom assignable to light-like 3-surfaces and zero modes identifiable as classical degrees of freedom assignable to interior of the space-time sheet. Zero modes have also fermionic counterparts. State preparation should generate entanglement between the quantal and classical states. What this means at the level of von Neumann algebras?
4. What is the TGD counterpart for causal disjointness. At space-time level different space-time sheets could correspond to such regions whereas at embedding space level causally disjoint CDs would represent such regions.

2. Technical problems

There are also more technical questions.

1. What is the von Neumann algebra needed in TGD framework? Does one have a a direct integral over factors? Which factors appear in it? Can one construct the factor as a crossed product of some group G with direct physical interpretation and of naturally appearing factor A? Is A a HFF of type II_∞? assignable to a fixed CD? What is the natural Hilbert space $\mathcal{H}$ in which A acts?
2. What are the geometric transformations inducing modular automorphisms of II_∞ inducing the scaling down of the trace? Is the action of G induced by the boosts in Lorentz group. Could also translations and scalings induce the action? What is the factor associated with the union of Poincare transforms of CD? $log(\Delta)$ is Hermitian algebraically: what does the non-unitarity of $exp(log(\Delta)it)$ mean physically?
3. Could Ω correspond to a vacuum which in conformal degrees of freedom depends on the choice of the sphere S^2 defining the radial coordinate playing the role of complex variable in the case of the radial conformal algebra. Does *-operation in $\mathcal{M}$ correspond to Hermitian conjugation for fermionic oscillator operators and change of sign of super conformal weights?

The exponent of the Kähler-Dirac action gives rise to the exponent of Kähler function as Dirac determinant and fermionic inner product defined by fermionic Feynman rules. It is implausible that this exponent could as such correspond to ω or Δ^{it} having conceptual roots in thermodynamics rather than QFT. If one assumes that the exponent of the Kähler-Dirac action defines a "complex square root" of ω the situation changes. This raises technical questions relating to the notion of square root of ω.

1. Does the complex square root of ω have a polar decomposition to a product of positive definite matrix (square root of the density matrix) and unitary matrix and does $\omega^{1/2}$ correspond to the modulus in the decomposition? Does the square root of Δ have similar decomposition with modulus equal equal to $\Delta^{1/2}$ in standard picture so that modular automorphism, which is inherent property of von Neumann algebra, would not be affected?
2. Δ^{it} or rather its generalization is defined modulo a unitary operator defined by some Hamiltonian and is therefore highly non-unique as such. This non-uniqueness applies also to $|\Delta|$. Could this non-uniqueness correspond to the thermodynamical degrees of freedom?

ZEO and factors

The first question concerns the identification of the Hilbert space associated with the factors in ZEO. As the positive or negative energy part of the zero energy state space or as the entire space of zero energy states? The latter option would look more natural physically and is forced by the condition that the vacuum state is cyclic and separating.

1. The commutant of HFF given as $\mathcal{M}' = J\mathcal{M}J$, where J is involution transforming fermionic oscillator operators and bosonic vector fields to their Hermitian conjugates. Also conformal weights would change sign in the map which conforms with the view that the light-like boundaries of CD are analogous to upper and lower hemispheres of S^2 in conformal field theory. The presence of J representing essentially Hermitian conjugation would suggest that positive and zero energy parts of zero energy states are related by this formula so that state space decomposes to a tensor product of positive and negative energy states and M-matrix can be regarded as a map between these two sub-spaces.
2. The fact that HFF of type II_1 has the algebra of fermionic oscillator operators as a canonical representation makes the situation puzzling for a novice. The assumption that the vacuum is cyclic and separating means that neither creation nor annihilation operators can annihilate it. Therefore Fermionic Fock space cannot appear as the Hilbert space in the Tomita-Takesaki theorem. The paradox is circumvented if the action of * transforms creation operators acting on the positive energy part of the state to annihilation operators acting on negative energy part of the state. If J permutes the two Fock vacuums in their tensor product, the action of S indeed maps permutes the tensor factors associated with $\mathcal{M}$ and $\mathcal{M}'$.

It is far from obvious whether the identification $M = \Delta^{it}$ makes sense in ZEO.

1. In ZEO M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the state. M-matrix is essentially "complex square root" of the density matrix and quantum theory similar square root of thermodynamics. The notion of state as it appears in the theory of HFFs is however essentially thermodynamical. Therefore it is good to ask whether the "complex square root of state" could make sense in the theory of factors.
2. Quantum field theory suggests an obvious proposal concerning the meaning of the square root: one replaces exponent of Hamiltonian with imaginary exponential of action at $T \to 0$ limit. In quantum TGD the exponent of Kähler-Dirac action giving exponent of Kähler function as real exponent could be the manner to take this complex square root. Kähler-Dirac action can therefore be regarded as a "square root" of Kähler action.
3. The identification $M = \Delta^{it}$ relies on the idea of unitary time evolution which is given up in ZEO based on CDs? Is the reduction of the quantum dynamics to a flow a realistic idea? As will be found this automorphism could correspond to a time translation or scaling for either upper or lower light-cone defining CD and can ask whether Δ^{it} corresponds to the exponent of scaling operator L_0 defining single particle propagator as one integrates over t. Its complex square root would correspond to fermionic propagator.

4. In this framework $J\Delta^{it}$ would map the positive energy and negative energy sectors to each other. If the positive and negative energy state spaces can identified by isometry then $M = J\Delta^{it}$ identification can be considered but seems unrealistic. $S = J\Delta^{1/2}$ maps positive and negative energy states to each other: could S or its generalization appear in M-matrix as a part which gives thermodynamics? The exponent of the Kähler-Dirac action does not seem to provide thermodynamical aspect and p-adic thermodynamics suggests strongly the presence exponent of $exp(-L_0/T_p)$ with T_p chose in such manner that consistency with p-adic thermodynamics is obtained. Could the generalization of $J\Delta^{n/2}$ with Δ replaced with its "square root" give rise to padic thermodynamics and also ordinary thermodynamics at the level of density matrix? The minimal option would be that power of Δ^{it} which imaginary value of t is responsible for thermodynamical degrees of freedom whereas everything else is dictated by the unitary S-matrix appearing as phase of the "square root" of ω.

Zero modes and factors

The presence of zero modes justifies quantum measurement theory in TGD framework and the relationship between zero modes and HFFs involves further conceptual problems.

1. The presence of zero modes means that one has a direct integral over HFFs labeled by zero modes which by definition do not contribute to WCW line element. The realization of quantum criticality in terms of Kähler-Dirac action [K126] suggests that also fermionic zero mode degrees of freedom are present and correspond to conserved charges assignable to the critical deformations of the pace-time sheets. Induced Kähler form characterizes the values of zero modes for a given space-time sheet and the symplectic group of light-cone boundary characterizes the quantum fluctuating degrees of freedom. The entanglement between zero modes and quantum fluctuating degrees of freedom is essential for quantum measurement theory. One should understand this entanglement.
2. Physical intuition suggests that classical observables should correspond to longer length scale than quantal ones. Hence it would seem that the interior degrees of freedom outside CD should correspond to classical degrees of freedom correlating with quantum fluctuating degrees of freedom of CD.
3. Quantum criticality means that Kähler-Dirac action allows an infinite number of conserved charges which correspond to deformations leaving metric invariant and therefore act on zero modes. Does this super-conformal algebra commute with the super-conformal algebra associated with quantum fluctuating degrees of freedom? Could the restriction of elements of quantum fluctuating currents to 3-D light-like 3-surfaces actually imply this commutativity. Quantum holography would suggest a duality between these algebras. Quantum measurement theory suggests even 1-1 correspondence between the elements of the two super-conformal algebras. The entanglement between classical and quantum degrees of freedom would mean that prepared quantum states are created by operators for which the operators in the two algebras are entangled in diagonal manner.
4. The notion of finite measurement resolution has become key element of quantum TGD and one should understand how finite measurement resolution is realized in terms of inclusions of hyper-finite factors for which sub-factor defines the resolution in the sense that its action creates states not distinguishable from each other in the resolution used. The notion of finite measurement resolution suggests that one should speak about entanglement between sub-factors and corresponding sub-spaces rather than between states. Connes tensor product would code for the idea that the action of sub-factors is analogous to that of complex numbers and tracing over sub-factor realizes this idea.
5. Just for fun one can ask whether the duality between zero modes and quantum fluctuating degrees of freedom representing quantum holography could correspond to $\mathcal{M}' = J\mathcal{M}J$? This interpretation must be consistent with the interpretation forced by zero energy ontology. If this crazy guess is correct (very probably not!), both positive and negative energy states would be observed in quantum measurement but in totally different manner. Since this identity would simplify enormously the structure of the theory, it deserves therefore to be shown wrong.

Crossed product construction in TGD framework

The identification of the von Neumann algebra by crossed product construction is the basic challenge. Consider first the question how HFFs of type II_∞ emerge, how modular automorphisms act on them, and how one can understand the non-unitary character of the Δ^{it} in an apparent conflict with the hermiticity and positivity of Δ.

1. The Clifford algebra at a given point of WCW(CD) (light-like 3-surfaces with ends at the boundaries of CD) defines HFF of type II_1 or possibly a direct integral of them. For a given CD having compact isotropy group $SO(3)$ leaving the rest frame defined by the tips of CD invariant the factor defined by Clifford algebra valued fields in WCW(CD) is most naturally HFF of type II_∞. The Hilbert space in which this Clifford algebra acts, consists of spinor fields in WCW(CD). Also the symplectic transformations of light-cone boundary leaving light-like 3-surfaces inside CD can be included to G. In fact all conformal algebras leaving CD invariant could be included in CD.

2. The downwards scalings of the radial coordinate r_M of the light-cone boundary applied to the basis of WCW (CD) spinor fields could induce modular automorphism. These scalings reduce the size of the portion of light-cone in which the WCW spinor fields are non-vanishing and effectively scale down the size of CD. $exp(iL_0)$ as algebraic operator acts as a phase multiplication on eigen states of conformal weight and therefore as apparently unitary operator. The geometric flow however contracts the CD so that the interpretation of $exp(itL_0)$ as a unitary modular automorphism is not possible. The scaling down of CD reduces the value of the trace if it involves integral over the boundary of CD. A similar reduction is implied by the downward shift of the upper boundary of CD so that also time translations would induce modular automorphism. These shifts seem to be necessary to define rest energies of positive and negative energy parts of the zero energy state.

3. The non-triviality of the modular automorphisms of II_∞ factor reflects different choices of ω. The degeneracy of ω could be due to the non-uniqueness of conformal vacuum which is part of the definition of ω. The radial Virasoro algebra of light-cone boundary is generated by $L_n = L^*_{-n}$, $n \neq 0$ and $L_0 = L_0^*$ and negative and positive frequencies are in asymmetric position. The conformal gauge is fixed by the choice of $SO(3)$ subgroup of Lorentz group defining the slicing of light-cone boundary by spheres and the tips of CD fix $SO(3)$ uniquely. One can however consider also alternative choices of $SO(3)$ and each corresponds to a slicing of the light-cone boundary by spheres but in general the sphere defining the intersection of the two light-cone does not belong to the slicing. Hence the action of Lorentz transformation inducing different choice of $SO(3)$ can lead out from the preferred state space so that its representation must be non-unitary unless Virasoro generators annihilate the physical states. The non-vanishing of the conformal central charge c and vacuum weight h seems to be necessary and indeed can take place for super-symplectic algebra and Super Kac-Moody algebra since only the differences of the algebra elements are assumed to annihilate physical states.

Modular automorphism of HFFs type III_1 can be induced by several geometric transformations for HFFs of type III_1 obtained using the crossed product construction from II_∞ factor by extending CD to a union of its Lorentz transforms.

1. The crossed product would correspond to an extension of II_∞ by allowing a union of some geometric transforms of CD. If one assumes that only CDs for which the distance between tips is quantized in powers of 2, then scalings of either upper or lower boundary of CD cannot correspond to these transformations. Same applies to time translations acting on either boundary but not to ordinary translations. As found, the modular automorphisms reducing the size of CD could act in HFF of type II_∞.

2. The geometric counterparts of the modular transformations would most naturally correspond to any non-compact one parameter sub-group of Lorentz group as also QFT suggests. The Lorentz boosts would replace the radial coordinate r_M of the light-cone boundary associated with the radial Virasoro algebra with a new one so that the slicing of light-cone boundary with spheres would be affected and one could speak of a new conformal gauge. The temporal distance between tips of CD in the rest frame would not be affected. The effect would seem

to be however unitary because the transformation does not only modify the states but also transforms CD.

3. Since Lorentz boosts affect the isotropy group $SO(3)$ of CD and thus also the conformal gauge defining the radial coordinate of the light-cone boundary, they affect also the definition of the conformal vacuum so that also ω is affected so that the interpretation as a modular automorphism makes sense. The simplistic intuition of the novice suggests that if one allows wave functions in the space of Lorentz transforms of CD, unitarity of Δ^{it} is possible. Note that the hierarchy of Planck constants assigns to CD preferred M^2 and thus direction of quantization axes of angular momentum and boosts in this direction would be in preferred role.

4. One can also consider the HFF of type III_λ if the radial scalings by negative powers of 2 correspond to the automorphism group of II_∞ factor as the vision about allowed CDs suggests. $\lambda = 1/2$ would naturally hold true for the factor obtained by allowing only the radial scalings. Lorentz boosts would expand the factor to HFF of type III_1. Why scalings by powers of 2 would give rise to periodicity should be understood.

The identification of M-matrix as modular automorphism Δ^{it}, where t is complex number having as its real part the temporal distance between tips of CD quantized as 2^n and temperature as imaginary part, looks at first highly attractive, since it would mean that M-matrix indeed exists mathematically. The proposed interpretations of modular automorphisms do not support the idea that they could define the S-matrix of the theory. In any case, the identification as modular automorphism would not lead to a magic universal formula since arbitrary unitary transformation is involved.

Quantum criticality and inclusions of factors

Quantum criticality fixes the value of Kähler coupling strength but is expected to have also an interpretation in terms of a hierarchies of broken conformal gauge symmetries suggesting hierarchies of inclusions.

1. In ZEO 3-surfaces are unions of space-like 3-surfaces at the ends of causal diamond (CD). Space-time surfaces connect 3-surfaces at the boundaries of CD. The non-determinism of Kähler action allows the possibility of having several space-time sheets connecting the ends of space-time surface but the conditions that classical charges are same for them reduces this number so that it could be finite. Quantum criticality in this sense implies non-determinism analogous to that of critical systems since preferred extremals can co-incide and suffer this kind of bifurcation in the interior of CD. This quantum criticality can be assigned to the hierarchy of Planck constants and the integer n in $h_{eff} = n \times h$ [K47] corresponds to the number of degenerate space-time sheets with same Kähler action and conserved classical charges.

2. Also now one expects a hierarchy of criticalities and since criticality and conformal invariance are closely related, a natural conjecture is that the fractal hierarchy of sub-algebras of conformal algebra isomorphic to conformal algebra itself and having conformal weights coming as multiples of n corresponds to the hierarchy of Planck constants. This hierarchy would define a hierarchy of symmetry breakings in the sense that only the sub-algebra would act as gauge symmetries.

3. The assignment of this hierarchy with super-symplectic algebra having conformal structure with respect to the light-like radial coordinate of light-cone boundary looks very attractive. An interesting question is what is the role of the super-conformal algebra associated with the isometries of light-cone boundary $R_+ \times S^2$ which are conformal transformations of sphere S^2 with a scaling of radial coordinate compensating the scaling induced by the conformal transformation. Does it act as dynamical or gauge symmetries?

4. The natural proposal is that the inclusions of various superconformal algebras in the hierarchy define inclusions of hyper-finite factors which would be thus labelled by integers. Any sequences of integers for which n_i divides n_{i+1} would define a hierarchy of inclusions proceeding in reverse direction. Physically inclusion hierarchy would correspond to an infinite hierarchy of criticalities within criticalities.

3.2.3 Can one identify M-matrix from physical arguments?

Consider next the identification of M-matrix from physical arguments from the point of view of factors.

A proposal for M-matrix

The proposed general picture reduces the core of U-matrix to the construction of S-matrix possibly having the real square roots of density matrices as symmetry algebra. This structure can be taken as a template as one tries to to imagine how the construction of M-matrix could proceed in quantum TGD proper.

1. At the bosonic sector one would have converging functional integral over WCW . This is analogous to the path integral over bosonic fields in QFTs. The presence of Kähler function would make this integral well-defined and would not encounter the difficulties met in the case of path integrals.

2. In fermionic sector 1-D Dirac action and its bosonic counterpart imply that spinors modes localized at string world sheets are eigenstates of induced Dirac operator with generalized eigenvalue $p^k\gamma_k$ defining light-like 8-D momentum so that one would obtain fermionic propagators massless in 8-D sense at light-light geodesics of embedding space. The 8-D generalization of twistor Grassmann approach is suggestive and would mean that the residue integral over fermionic virtual momenta gives only integral over massless momenta and virtual fermions differ from real fermions only in that they have non-physical polarizations so that massless Dirac operator replacing the propagator does not annihilate the spinors at the other end of the line.

3. Fundamental bosons (not elementary particles) correspond to wormhole contacts having fermion and antifermion at opposite throats and bosonic propagators are composite of massless fermion propagators. The directions of virtual momenta are obviously strongly correlated so that the approximation as a gauge theory with gauge symmetry breaking in almost massless sector is natural. Massivation follows necessary from the fact that also elementary particles are bound states of two wormhole contacts.

4. Physical fermions and bosons correspond to pairs of wormhole contacts with throats carrying Kähler magnetic charge equal to Kähler electric charge (dyon). The absence of Dirac monopoles (as opposed to homological magnetic monopoles due to CP_2 topology) implies that wormhole contacts must appear as pairs (also large numbers of them are possible and 3 valence quarks inside baryons could form Kähler magnetic tripole). Hence elementary particles would correspond to pairs of monopoles and are accompanied by Kähler magnetic flux loop running along the two space-time sheets involved as well as fermionic strings connecting the monopole throats.

   There seems to be no specific need to assign string to the wormhole contact and if is a piece of deformed CP_2 type vacuum extremal this might not be even possible: the Kähler-Dirac gamma matrices would not span 2-D space in this case since the CP_2 projection is 4-D. Hence massless fermion propagators would be assigned only with the boundaries of string world sheets at Minkowskian regions of space-time surface. One could say that physical particles are bound states of massless fundamental fermions and the non-collinearity of their four-momenta can make them massive. Therefore the breaking of conformal invariance would be due to the bound state formation and this would also resolve the infrared divergence problems plaguing Grassmann twistor approach by introducing natural length scale assignable to the size of particles defined by the string like flux tube connecting the wormhole contacts. This point is discussed in more detail in [K114].

   The bound states would form representations of super-conformal algebras so that stringy mass formula would emerge naturally. p-Adic mass calculations indeed assume conformal invariance in $CP2$ length scale assignable to wormhole contacts. Also the long flux tube strings contribute to the particle masses and would explain gauge boson masses.

5. The interaction vertices would correspond topologically to decays of 3-surface by splitting in complete analogy with ordinary Feynman diagrams. At the level of orbits of partonic 2-surface

the vertices would be represented by partonic 2-surfaces. In [K114] the interpretation of scattering ampiltudes as sequences of algebraic operations for the Yangian of super-symplectic algebra is proposed: product and co-product would define time 3-vertex and its time reversal. At the level of fermions the diagrams reduce to braid diagrams since fermions are "free". At vertices fermions can however reflect in time direction so that fermion-antifermion annihilations in classical fields can be said to appear in the vertices.

The Yangian is generated by super-symplectic fermionic Noether charges assignable to the strings connecting partonic 2-surfaces. The interpretation of vertices as algebraic operations implies that all sequences of operations connecting given collections of elements of Yangian at the opposite boundaries of CD give rise to the same amplitude. This means a huge generalization of the duality symmetry of hadronic string models that I have proposed already earlier: the chapter [K16] is a remnant of an "idea that came too early". The propagators are associated with the fermionic lines identifiable as boundaries of string world sheets. These lines are light-like geodesics of H and fermion lines correspond topartial wave in the space S^3 of light like 8-momenta with fixed M^4 momentum. For external lines M^8 momentum corresponds to the $M^4 \times CP_2$ quantum numbers of a spinor harmonic.

The amplitudes can be formulated using only partonic 2-surfaces and string world sheets and the algebraic continuation to achieve number theoretic Universality should be rather straightforward: the parameters characterizing 2-surfaces - by conformal invariance various conformal moduli - in the algebraic extension of rationals are replaced with real and various p-adic numbers.

6. Wormhole contacts represent fundamental interaction vertex pairs and propagators between them and one has stringy super-conformal invariance. Therefore there are excellent reasons to expect that the perturbation theory is free of divergences. Without stringy contributions for massive conformal excitations of wormhole contacts one would obtain the usual logarithmic UV divergences of massless gauge theories. The fact that physical particles are bound states of massless particles, gives good hopes of avoiding IR divergences of massless theories.

The figures ??, ?? (`http://tgdtheory.fi/appfigures/elparticletgd.jpg` `http://tgdtheory.fi/appfigures/tgdgrpahs.jpg`) in the appendix of this book illustrate the relationship between TGD diagrammatics, QFT diagrammatics and stringy diagrammatics. In [K114] a more detailed construction based on the generalization of twistor approach and the idea that scattering amplitudes represent sequences of algebraic operation in the Yangian of super-symplectic algebra, is considered.

Quantum TGD as square root of thermodynamics

ZEO (ZEO) suggests strongly that quantum TGD corresponds to what might be called square root of thermodynamics. Since fermionic sector of TGD corresponds naturally to a hyper-finite factor of type II_1, and super-conformal sector relates fermionic and bosonic sectors (WCW degrees of freedom), there is a temptation to suggest that the mathematics of von Neumann algebras generalizes: in other worlds it is possible to speak about the complex square root of ω defining a state of von Neumann algebra [A85] [K125]. This square root would bring in also the fermionic sector and realized super-conformal symmetry. The reduction of determinant with WCW vacuum functional would be one manifestation of this supersymmetry.

The exponent of Kähler function identified as real part of Kähler action for preferred extremals coming from Euclidian space-time regions defines the modulus of the bosonic vacuum functional appearing in the functional integral over WCW. The imaginary part of Kähler action coming from the Minkowskian regions is analogous to action of quantum field theories and would give rise to interference effects distinguishing thermodynamics from quantum theory. This would be something new from the point of view of the canonical theory of von Neumann algebra. The saddle points of the imaginary part appear in stationary phase approximation and the imaginary part serves the role of Morse function for WCW.

The exponent of Kähler function depends on the real part of t identified as Minkowski distance between the tips of CD. This dependence is not consistent with the dependence of the canonical unitary automorphism Δ^{it} of von Neumann algebra on t [A85], [K125] and the natural interpretation is that the vacuum functional can be included in the definition of the inner product for spinors fields of WCW. More formally, the exponent of Kähler function would define ω in bosonic degrees of freedom.

Note that the imaginary exponent is more natural for the imaginary part of Kähler action coming from Minkowskian region. In any case, one has combination of thermodynamics and QFT and the presence of thermodynamics makes the functional integral mathematically well-defined.

Number theoretic vision requiring number theoretical universality suggests that the value of CD size scales as defined by the distance between the tips is expected to come as integer multiples of CP_2 length scale - at least in the intersection of real and p-adic worlds. If this is the case the continuous faimily of modular automorphisms would be replaced with a discretize family.

Quantum criticality and hierarchy of inclusions

Quantum criticality and related fractal hierarchies of breakings of conformal symmetry could allow to understand the inclusion hierarchies for hyper-finite factors. Quantum criticality - implied by the condition that the Kähler-Dirac action gives rise to conserved currents assignable to the deformations of the space-time surface - means the vanishing of the second variation of Kähler action for these deformations. Preferred extremals correspond to these 4-surfaces and $M^8-M^4\times CP_2$ duality would allow to identify them also as associative (co-associative) space-time surfaces.

Quantum criticality is basically due to the failure of strict determinism for Kähler action and leads to the hierarchy of dark matter phases labelled by the effective value of Planck constant $h_{eff}=n\times h$. These phases correspond to space-time surfaces connecting 3-surfaces at the ends of CD which are multi-sheeted having n conformal equivalence classes.

Conformal invariance indeed relates naturally to quantum criticality. This brings in n discrete degrees of freedom and one can technically describe the situation by using n-fold singular covering of the embedding space [K47]. One can say that there is hierarchy of broken conformal symmetries in the sense that for $h_{eff}=n\times h$ the sub-algebra of conformal algebras with conformal weights coming as multiples of n act as gauge symmetries. This implies that classical symplectic Noether charges vanish for this sub-algebra. The quantal conformal charges associated with induced spinor fields annihilate the physical states. Therefore it seems that the measured quantities are the symplectic charges and there is not need to introduce any measurement interaction term and the formalism simplifies dramatically.

The resolution increases with $h_{eff}/h=n$. Also the number of of strings connecting partonic 2-surfaces (in practice elementary particles and their dark counterparts plus bound states generated by connecting dark strings) characterizes physically the finite measurement resolution. Their presence is also visible in the geometry of the space-time surfaces through the conditions that induced W fields vanish at them (well-definedness of em charge), and by the condition that the canonical momentum currents for Kähler action define an integrable distribution of planes parallel to the string world sheet. In spirit with holography, preferred extremal is constructed by fixing string world sheets and partonic 2-surfaces and possibly also their light-like orbits (should one fix wormhole contacts is not quite clear). If the analog of AdS/CFT correspondence holds true, the value of Kähler function is expressible as the energy of string defined by area in the effective metric defined by the anti-commutators of K-D gamma matrices.

Super-symplectic algebra, whose charges are represented by Noether charges associated with strings connecting partonic 2-surfaces extends to a Yangian algebra with multi-stringy generators [K114]. The better the measurement resolution, the larger the maximal number of strings associated with the multilocal generator.

Kac-Moody type transformations preserving light-likeness of partonic orbits and possibly also the light-like character of the boundaries of string world sheets carrying modes of induced spinor field underlie the conformal gauge symmetry. The minimal option is that only the light-likeness of the string end world line is preserved by the conformal symmetries. In fact, conformal symmetries was originally deduced from the light-likeness condition for the M^4 projection of CP_2 type vacuum extremals.

The inclusions of super-symplectic Yangians form a hierarchy and would naturally correspond to inclusions of hyperfinite factors of type II_1. Conformal symmetries acting as gauge transformations would naturally correspond to degrees of freedom below measurement resolution and would correspond to included subalgebra. As h_{eff} increases, infinite number of these gauge degrees of freedom become dynamical and measurement resolution is increased. This picture is definitely in conflict with the original view but the reduction of criticality in the increase of h_{eff} forces it.

Summary

On basis of above considerations it seems that the idea about "complex square root" of the state ω of von Neumann algebras might make sense in quantum TGD. Also the discretized versions of modular automorphism assignable to the hierarchy of CDs would make sense and because of its non-uniqueness the generator Δ of the canonical automorphism could bring in the flexibility needed one wants thermodynamics. Stringy picture forces to ask whether Δ could in some situation be proportional $exp(L_0)$, where L_0 represents as the infinitesimal scaling generator of either super-symplectic algebra or super Kac-Moody algebra (the choice does not matter since the differences of the generators annihilate physical states in coset construction). This would allow to reproduce real thermodynamics consistent with p-adic thermodynamics. Note that also p-adic thermodynamics would be replaced by its square root in ZEO.

3.2.4 Finite measurement resolution and HFFs

The finite resolution of quantum measurement leads in TGD framework naturally to the notion of quantum M-matrix for which elements have values in sub-factor $\mathcal{N}$ of HFF rather than being complex numbers. M-matrix in the factor space $\mathcal{M}/\mathcal{N}$ is obtained by tracing over $\mathcal{N}$. The condition that $\mathcal{N}$ acts like complex numbers in the tracing implies that M-matrix elements are proportional to maximal projectors to $\mathcal{N}$ so that M-matrix is effectively a matrix in $\mathcal{M}/\mathcal{N}$ and situation becomes finite-dimensional. It is still possible to satisfy generalized unitarity conditions but in general case tracing gives a weighted sum of unitary M-matrices defining what can be regarded as a square root of density matrix.

About the notion of observable in ZEO

Some clarifications concerning the notion of observable in zero energy ontology are in order.

1. As in standard quantum theory observables correspond to hermitian operators acting on either positive or negative energy part of the state. One can indeed define hermitian conjugation for positive and negative energy parts of the states in standard manner.
2. Also the conjugation $A \rightarrow JAJ$ is analogous to hermitian conjugation. It exchanges the positive and negative energy parts of the states also maps the light-like 3-surfaces at the upper boundary of CD to the lower boundary and vice versa. The map is induced by time reflection in the rest frame of CD with respect to the origin at the center of CD and has a well defined action on light-like 3-surfaces and space-time surfaces. This operation cannot correspond to the sought for hermitian conjugation since JAJ and A commute.
3. In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. Chern-Simons Dirac terms to which Kähler action reduces could be responsible for the breaking of CP and T symmetries as they appear in CKM matrix.
4. ZEO gives Cartan sub-algebra of the Lie algebra of symmetries a special status. Only Cartan algebra acting on either positive or negative states respects the zero energy property but this is enough to define quantum numbers of the state. In absence of symmetry breaking positive and negative energy parts of the state combine to form a state in a singlet representation of group. Since only the net quantum numbers must vanish ZEO allows a symmetry breaking respecting a chosen Cartan algebra.
5. In order to speak about four-momenta for positive and negative energy parts of the states one must be able to define how the translations act on CDs. The most natural action is a shift of the upper (lower) tip of CD. In the scale of entire CD this transformation induced

Lorentz boost fixing the other tip. The value of mass squared is identified as proportional to the average of conformal weight in p-adic thermodynamics for the scaling generator L_0 for either super-symplectic or Super Kac-Moody algebra.

Inclusion of HFFs as characterizer of finite measurement resolution at the level of S-matrix

The inclusion $\mathcal{N} \subset \mathcal{M}$ of factors characterizes naturally finite measurement resolution. This means following things.

1. Complex rays of state space resulting usually in an ideal state function reduction are replaced by $\mathcal{N}$-rays since $\mathcal{N}$ defines the measurement resolution and takes the role of complex numbers in ordinary quantum theory so that non-commutative quantum theory results. Non-commutativity corresponds to a finite measurement resolution rather than something exotic occurring in Planck length scales. The quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ creates physical states modulo resolution. The fact that $\mathcal{N}$ takes the role of gauge algebra suggests that it might be necessary to fix a gauge by assigning to each element of $\mathcal{M}/\mathcal{N}$ a unique element of $\mathcal{M}$. Quantum Clifford algebra with fractal dimension $\beta = \mathcal{M} : \mathcal{N}$ creates physical states having interpretation as quantum spinors of fractal dimension $d = \sqrt{\beta}$. Hence direct connection with quantum groups emerges.
2. The notions of unitarity, hermiticity, and eigenvalue generalize. The elements of unitary and hermitian matrices and $\mathcal{N}$-valued. Eigenvalues are Hermitian elements of $\mathcal{N}$ and thus correspond entire spectra of Hermitian operators. The mutual non-commutativity of eigenvalues guarantees that it is possible to speak about state function reduction for quantum spinors. In the simplest case of a 2-component quantum spinor this means that second component of quantum spinor vanishes in the sense that second component of spinor annihilates physical state and second acts as element of $\mathcal{N}$ on it. The non-commutativity of spinor components implies correlations between then and thus fractal dimension is smaller than 2.
3. The intuition about ordinary tensor products suggests that one can decompose Tr in $\mathcal{M}$ as

$$Tr_{\mathcal{M}}(X) = Tr_{\mathcal{M}/\mathcal{N}} \times Tr_{\mathcal{N}}(X) \ . \tag{3.2.4}$$

Suppose one has fixed gauge by selecting basis $|r_k\rangle$ for $\mathcal{M}/\mathcal{N}$. In this case one expects that operator in $\mathcal{M}$ defines an operator in $\mathcal{M}/\mathcal{N}$ by a projection to the preferred elements of $\mathcal{M}$.

$$\langle r_1|X|r_2\rangle = \langle r_1|Tr_{\mathcal{N}}(X)|r_2\rangle \ . \tag{3.2.5}$$

4. Scattering probabilities in the resolution defined by $\mathcal{N}$ are obtained in the following manner. The scattering probability between states $|r_1\rangle$ and $|r_2\rangle$ is obtained by summing over the final states obtained by the action of $\mathcal{N}$ from $|r_2\rangle$ and taking the analog of spin average over the states created in the similar from $|r_1\rangle$. $\mathcal{N}$ average requires a division by $Tr(P_{\mathcal{N}}) = 1/\mathcal{M} : \mathcal{N}$ defining fractal dimension of $\mathcal{N}$. This gives

$$p(r_1 \to r_2) = \mathcal{M} : \mathcal{N} \times \langle r_1|Tr_{\mathcal{N}}(SP_{\mathcal{N}}S^{\dagger})|r_2\rangle \ . \tag{3.2.6}$$

This formula is consistent with probability conservation since one has

$$\sum_{r_2} p(r_1 \to r_2) = \mathcal{M} : \mathcal{N} \times Tr_N(SS^{\dagger}) = \mathcal{M} : \mathcal{N} \times Tr(P_N) = 1 \ . \tag{3.2.7}$$

5. Unitarity at the level of $\mathcal{M}/\mathcal{N}$ can be achieved if the unit operator Id for $\mathcal{M}$ can be decomposed into an analog of tensor product for the unit operators of $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$ and M decomposes to a tensor product of unitary M-matrices in $\mathcal{M}/\mathcal{N}$ and $\mathcal{N}$. For HFFs of type II projection operators of $\mathcal{N}$ with varying traces are present and one expects a weighted sum of unitary M-matrices to result from the tracing having interpretation in terms of square root of thermodynamics.

6. This argument assumes that $\mathcal{N}$ is HFF of type II_1 with finite trace. For HFFs of type III_1 this assumption must be given up. This might be possible if one compensates the trace over $\mathcal{N}$ by dividing with the trace of the infinite trace of the projection operator to $\mathcal{N}$. This probably requires a limiting procedure which indeed makes sense for HFFs.

Quantum M-matrix

The description of finite measurement resolution in terms of inclusion $\mathcal{N} \subset \mathcal{M}$ seems to boil down to a simple rule. Replace ordinary quantum mechanics in complex number field C with that in $\mathcal{N}$. This means that the notions of unitarity, hermiticity, Hilbert space ray, etc.. are replaced with their $\mathcal{N}$ counterparts.

The full M-matrix in $\mathcal{M}$ should be reducible to a finite-dimensional quantum M-matrix in the state space generated by quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ which can be regarded as a finite-dimensional matrix algebra with non-commuting $\mathcal{N}$-valued matrix elements. This suggests that full M-matrix can be expressed as M-matrix with $\mathcal{N}$-valued elements satisfying $\mathcal{N}$-unitarity conditions.

Physical intuition also suggests that the transition probabilities defined by quantum S-matrix must be commuting hermitian $\mathcal{N}$-valued operators inside every row and column. The traces of these operators give $\mathcal{N}$-averaged transition probabilities. The eigenvalue spectrum of these Hermitian matrices gives more detailed information about details below experimental resolution. $\mathcal{N}$-hermicity and commutativity pose powerful additional restrictions on the M-matrix.

Quantum M-matrix defines $\mathcal{N}$-valued entanglement coefficients between quantum states with $\mathcal{N}$-valued coefficients. How this affects the situation? The non-commutativity of quantum spinors has a natural interpretation in terms of fuzzy state function reduction meaning that quantum spinor corresponds effectively to a statistical ensemble which cannot correspond to pure state. Does this mean that predictions for transition probabilities must be averaged over the ensemble defined by "quantum quantum states"?

Quantum fluctuations and inclusions

Inclusions $\mathcal{N} \subset \mathcal{M}$ of factors provide also a first principle description of quantum fluctuations since quantum fluctuations are by definition quantum dynamics below the measurement resolution. This gives hopes for articulating precisely what the important phrase "long range quantum fluctuations around quantum criticality" really means mathematically.

1. Phase transitions involve a change of symmetry. One might hope that the change of the symmetry group $G_a \times G_b$ could universally code this aspect of phase transitions. This need not always mean a change of Planck constant but it means always a leakage between sectors of embedding space. At quantum criticality 3-surfaces would have regions belonging to at least two sectors of H.

2. The long range of quantum fluctuations would naturally relate to a partial or total leakage of the 3-surface to a sector of embedding space with larger Planck constant meaning zooming up of various quantal lengths.

3. For M-matrix in $\mathcal{M}/\mathcal{N}$ regarded as $calN$ module quantum criticality would mean a special kind of eigen state for the transition probability operator defined by the M-matrix. The properties of the number theoretic braids contributing to the M-matrix should characterize this state. The strands of the critical braids would correspond to fixed points for $G_a \times G_b$ or its subgroup.

M-matrix in finite measurement resolution

The following arguments relying on the proposed identification of the space of zero energy states give a precise formulation for M-matrix in finite measurement resolution and the Connes tensor product involved. The original expectation that Connes tensor product could lead to a unique M-matrix is wrong. The replacement of ω with its complex square root could lead to a unique hierarchy of M-matrices with finite measurement resolution and allow completely finite theory despite the fact that projectors have infinite trace for HFFs of type III_1.

1. In ZEO the counterpart of Hermitian conjugation for operator is replaced with $\mathcal{M} \rightarrow J\mathcal{M}J$ permuting the factors. Therefore $N \in \mathcal{N}$ acting to positive (negative) energy part of state corresponds to $N \rightarrow N' = JNJ$ acting on negative (positive) energy part of the state.
2. The allowed elements of N much be such that zero energy state remains zero energy state. The superposition of zero energy states involved can however change. Hence one must have that the counterparts of complex numbers are of form $N = JN_1J \vee N_2$, where N_1 and N_2 have same quantum numbers. A superposition of terms of this kind with varying quantum numbers for positive energy part of the state is possible.
3. The condition that N_{1i} and N_{2i} act like complex numbers in $\mathcal{N}$-trace means that the effect of $JN_{1i}J \vee N_{2i}$ and $JN_{2i}Ji \vee N_{1i}$ to the trace are identical and correspond to a multiplication by a constant. If $\mathcal{N}$ is HFF of type II_1 this follows from the decomposition $\mathcal{M} = \mathcal{M}/\mathcal{N} \otimes \mathcal{N}$ and from $Tr(AB) = Tr(BA)$ assuming that M is of form $M = M_{\mathcal{M}/\mathcal{N}} \times P_{\mathcal{N}}$. Contrary to the original hopes that Connes tensor product could fix the M-matrix there are no conditions on $M_{\mathcal{M}/\mathcal{N}}$ which would give rise to a finite-dimensional M-matrix for Jones inclusions. One can replaced the projector $P_{\mathcal{N}}$ with a more general state if one takes this into account in * operation.
4. In the case of HFFs of type III_1 the trace is infinite so that the replacement of Tr_N with a state ω_N in the sense of factors looks more natural. This means that the counterpart of * operation exchanging N_1 and N_2 represented as $SA\Omega = A^*\Omega$ involves Δ via $S = J\Delta^{1/2}$. The exchange of N_1 and N_2 gives altogether Δ. In this case the KMS condition $\omega_{\mathcal{N}}(AB) = \omega_{\mathcal{N}}\Delta A)$ guarantees the effective complex number property [A8] .
5. Quantum TGD more or less requires the replacement of ω with its "complex square root" so that also a unitary matrix U multiplying Δ is expected to appear in the formula for S and guarantee the symmetry. One could speak of a square root of KMS condition [A8] in this case. The QFT counterpart would be a cutoff involving path integral over the degrees of freedom below the measurement resolution. In TGD framework it would mean a cutoff in the functional integral over WCW and for the modes of the second quantized induced spinor fields and also cutoff in sizes of causal diamonds. Discretization in terms of braids replacing light-like 3-surfaces should be the counterpart for the cutoff.
6. If one has M-matrix in $\mathcal{M}$ expressible as a sum of M-matrices of form $M_{\mathcal{M}/\mathcal{N}} \times M_{\mathcal{N}}$ with coefficients which correspond to the square roots of probabilities defining density matrix the tracing operation gives rise to square root of density matrix in M.

Is universal M-matrix possible?

The realization of the finite measurement resolution could apply only to transition probabilities in which $\mathcal{N}$-trace or its generalization in terms of state ω_N is needed. One might however dream of something more.

1. Maybe there exists a universal M-matrix in the sense that the same M-matrix gives the M-matrices in finite measurement resolution for all inclusions $\mathcal{N} \subset \mathcal{M}$. This would mean that one can write

$$M = M_{\mathcal{M}/\mathcal{N}} \otimes M_{\mathcal{N}} \tag{3.2.8}$$

   for any physically reasonable choice of $\mathcal{N}$. This would formally express the idea that M is as near as possible to M-matrix of free theory. Also fractality suggests itself in the sense that $M_{\mathcal{N}}$ is essentially the same as $M_{\mathcal{M}}$ in the same sense as $\mathcal{N}$ is same as $\mathcal{M}$. It might be that the trivial solution $M = 1$ is the only possible solution to the condition.
2. $M_{\mathcal{M}/\mathcal{N}}$ would be obtained by the analog of $Tr_{\mathcal{N}}$ or ω_N operation involving the "complex square root" of the state ω in case of HFFs of type III_1. The QFT counterpart would be path integration over the degrees of freedom below cutoff to get effective action.
3. Universality probably requires assumptions about the thermodynamical part of the universal M-matrix. A possible alternative form of the condition is that it holds true only for canonical choice of "complex square root" of ω or for the S-matrix part of M:

$$S = S_{\mathcal{M}/\mathcal{N}} \otimes S_{\mathcal{N}} \tag{3.2.9}$$

for any physically reasonable choice $\mathcal{N}$.

4. In TGD framework the condition would say that the M-matrix defined by the Kähler-Dirac action gives M-matrices in finite measurement resolution via the counterpart of integration over the degrees of freedom below the measurement resolution.

An obvious counter argument against the universality is that if the M-matrix is "complex square root of state" cannot be unique and there are infinitely many choices related by a unitary transformation induced by the flows representing modular automorphism giving rise to new choices. This would actually be a well-come result and make possible quantum measurement theory.

In the section "Handful of problems with a common resolution" it was found that one can add to both Kähler action and Kähler-Dirac action a measurement interaction term characterizing the values of measured observables. The measurement interaction term in Kähler action is Lagrange multiplier term at the space-like ends of space-time surface fixing the value of classical charges for the space-time sheets in the quantum superposition to be equal with corresponding quantum charges. The term in Kähler-Dirac action is obtained from this by assigning to this term canonical momentum densities and contracting them with gamma matrices to obtain Kähler-Dirac gamma matrices appearing in 3-D analog of Dirac action. The constraint terms would leave Kähler function and Kähler metric invariant but would restrict the vacuum functional to the subset of 3-surfaces with fixed classical conserved charges (in Cartan algebra) equal to their quantum counterparts.

Connes tensor product and space-like entanglement

Ordinary linear Connes tensor product makes sense also in positive/negative energy sector and also now it makes sense to speak about measurement resolution. Hence one can ask whether Connes tensor product should be posed as a constraint on space-like entanglement. The interpretation could be in terms of the formation of bound states. The reducibility of HFFs and inclusions means that the tensor product is not uniquely fixed and ordinary entanglement could correspond to this kind of entanglement.

Also the counterpart of p-adic coupling constant evolution would makes sense. The interpretation of Connes tensor product would be as the variance of the states with respect to some subgroup of $U(n)$ associated with the measurement resolution: the analog of color confinement would be in question.

2-vector spaces and entanglement modulo measurement resolution

John Baez and collaborators [A61] are playing with very formal looking formal structures obtained by replacing vectors with vector spaces. Direct sum and tensor product serve as the basic arithmetic operations for the vector spaces and one can define category of n-tuples of vectors spaces with morphisms defined by linear maps between vectors spaces of the tuple. n-tuples allow also element-wise product and sum. They obtain results which make them happy. For instance, the category of linear representations of a given group forms 2-vector spaces since direct sums and tensor products of representations as well as n-tuples make sense. The 2-vector space however looks more or less trivial from the point of physics.

The situation could become more interesting in quantum measurement theory with finite measurement resolution described in terms of inclusions of hyper-finite factors of type II_1. The reason is that Connes tensor product replaces ordinary tensor product and brings in interactions via irreducible entanglement as a representation of finite measurement resolution. The category in question could give Connes tensor products of quantum state spaces and describing interactions. For instance, one could multiply M-matrices via Connes tensor product to obtain category of M-matrices having also the structure of 2-operator algebra.

1. The included algebra represents measurement resolution and this means that the infinite-D sub-Hilbert spaces obtained by the action of this algebra replace the rays. Sub-factor takes the role of complex numbers in generalized QM so that one obtains non-commutative quantum mechanics. For instance, quantum entanglement for two systems of this kind would not be

between rays but between infinite-D subspaces corresponding to sub-factors. One could build a generalization of QM by replacing rays with sub-spaces and it would seem that quantum group concept does more or less this: the states in representations of quantum groups could be seen as infinite-dimensional Hilbert spaces.

2. One could speak about both operator algebras and corresponding state spaces modulo finite measurement resolution as quantum operator algebras and quantum state spaces with fractal dimension defined as factor space like entities obtained from HFF by dividing with the action of included HFF. Possible values of the fractal dimension are fixed completely for Jones inclusions. Maybe these quantum state spaces could define the notions of quantum 2-Hilbert space and 2-operator algebra via direct sum and tensor production operations. Fractal dimensions would make the situation interesting both mathematically and physically.

Suppose one takes the fractal factor spaces as the basic structures and keeps the information about inclusion.

1. Direct sums for quantum vectors spaces would be just ordinary direct sums with HFF containing included algebras replaced with direct sum of included HFFs.
2. The tensor products for quantum state spaces and quantum operator algebras are not anymore trivial. The condition that measurement algebras act effectively like complex numbers would require Connes tensor product involving irreducible entanglement between elements belonging to the two HFFs. This would have direct physical relevance since this entanglement cannot be reduced in state function reduction. The category would defined interactions in terms of Connes tensor product and finite measurement resolution.
3. The sequences of super-conformal symmetry breakings identifiable in terms of inclusions of super-conformal algebras and corresponding HFFs could have a natural description using the 2-Hilbert spaces and quantum 2-operator algebras.

3.2.5 Questions about quantum measurement theory in Zero Energy Ontology

The following summary about quantum measurement theory in ZEO is somewhat out-of-date and somewhat sketchy. For more detailed view see [K72, K121, K9].

Fractal hierarchy of state function reductions

In accordance with fractality, the conditions for the Connes tensor product at a given time scale imply the conditions at shorter time scales. On the other hand, in shorter time scales the inclusion would be deeper and would give rise to a larger reducibility of the representation of $\mathcal{N}$ in $\mathcal{M}$. Formally, as $\mathcal{N}$ approaches to a trivial algebra, one would have a square root of density matrix and trivial S-matrix in accordance with the idea about asymptotic freedom.

M-matrix would give rise to a matrix of probabilities via the expression $P(P_+ \rightarrow P_-) = Tr[P_+ M^\dagger P_- M]$, where P_+ and P_- are projectors to positive and negative energy energy $\mathcal{N}$-rays. The projectors give rise to the averaging over the initial and final states inside $\mathcal{N}$ ray. The reduction could continue step by step to shorter length scales so that one would obtain a sequence of inclusions. If the U-process of the next quantum jump can return the M-matrix associated with $\mathcal{M}$ or some larger HFF, U process would be kind of reversal for state function reduction.

Analytic thinking proceeding from vision to details; human life cycle proceeding from dreams and wild actions to the age when most decisions relate to the routine daily activities; the progress of science from macroscopic to microscopic scales; even biological decay processes: all these have an intriguing resemblance to the fractal state function reduction process proceeding to shorter and shorter time scales. Since this means increasing thermality of M-matrix, U process as a reversal of state function reduction might break the second law of thermodynamics.

The conservative option would be that only the transformation of intentions to action by U process giving rise to new zero energy states can bring in something new and is responsible for evolution. The non-conservative option is that the biological death is the U-process of the next quantum jump leading to a new life cycle. Breathing would become a universal metaphor for what happens in quantum Universe. The 4-D body would be lived again and again.

quantum classical correspondence is realized at parton level?

Quantum classical correspondence must assign to a given quantum state the most probable space-time sheet depending on its quantum numbers. The space-time sheet $X^4(X^3)$ defined by the Kähler function depends however only on the partonic 3-surface X^3, and one must be able to assign to a given quantum state the most probable X^3 - call it X^3_{max} - depending on its quantum numbers.

$X^4(X^3_{max})$ should carry the gauge fields created by classical gauge charges associated with the Cartan algebra of the gauge group (color isospin and hypercharge and electromagnetic and Z^0 charge) as well as classical gravitational fields created by the partons. This picture is very similar to that of quantum field theories relying on path integral except that the path integral is restricted to 3-surfaces X^3 with exponent of Kähler function bringing in genuine convergence and that 4-D dynamics is deterministic apart from the delicacies due to the 4-D spin glass type vacuum degeneracy of Kähler action.

Stationary phase approximation selects X^3_{max} if the quantum state contains a phase factor depending not only on X^3 but also on the quantum numbers of the state. A good guess is that the needed phase factor corresponds to either Chern-Simons type action or an action describing the interaction of the induced gauge field with the charges associated with the braid strand. This action would be defined for the induced gauge fields. YM action seems to be excluded since it is singular for light-like 3-surfaces associated with the light-like wormhole throats (not only $\sqrt{det(g_3)}$ but also $\sqrt{det(g_4)}$ vanishes).

The challenge is to show that this is enough to guarantee that $X^4(X^3_{max})$ carries correct gauge charges. Kind of electric-magnetic duality should relate the normal components F_{ni} of the gauge fields in $X^4(X^3_{max})$ to the gauge fields F_{ij} induced at X^3. An alternative interpretation is in terms of quantum gravitational holography.

One is forced to introduce gauge couplings and also electro-weak symmetry breaking via the phase factor. This is in apparent conflict with the idea that all couplings are predictable. The essential uniqueness of M-matrix in the case of HFFs of type II_1 (at least) however means that their values as a function of measurement resolution time scale are fixed by internal consistency. Also quantum criticality leads to the same conclusion. Obviously a kind of bootstrap approach suggests itself.

Quantum measurements in ZEO

ZEO based quantum measurement theory leads directly to a theory of conscious entities. The basic idea is that state function reduction localizes the second boundary of CD so that it becomes a piece of light-cone boundary (more precisely $\delta M^4_\pm \times CP_2$).

Repeated reductions are possible as in standard quantum measurement theory and leave the passive boundary of CD. Repeated reduction begins with U process generating a superposition of CDs with the active boundary of CD being de-localized in the moduli space of CDs, and is followed by a localization in this moduli space so that single CD is the outcome. This process tends to increase the distance between the ends of the CD and has interpretation as a space-time correlate for the flow of subjective time.

Self as a conscious entity corresponds to this sequence of repeated reductions on passive boundary of CD. The first reduction at opposite boundary means death of self and its re-incarnation at the opposite boundary of CD. Also the increase of Planck constant and generation of negentropic entanglement is expected to be associated with this state function reduction.

Weak form of NMP is the most plausible variational principle to characterize the state function reduction. It does not require maximal negentropy gain for state function reductions but allows it. In other words, the outcome of reduction is n-dimensional eigen space of density matrix space but this space need not have maximum possible dimension and even 1-D ray is possible in which case the entanglement negentropy vanishes for the final state and system becomes isolated from the rest of the world. Weak form of NMP brings in free will and can allow also larger negentropy gain than the strong form if n is a product of primes. The beauty of this option is that one can understand how the generalization of p-adic length scale hypothesis emerges.

Hyper-finite factors of type II_1 and quantum measurement theory with a finite measurement resolution

The realization that the von Neumann algebra known as hyper-finite factor of type II_1 is tailor made for quantum TGD has led to a considerable progress in the understanding of the mathematical structure of the theory and these algebras provide a justification for several ideas introduced earlier on basis of physical intuition.

Hyper-finite factor of type II_1 has a canonical realization as an infinite-dimensional Clifford algebra and the obvious guess is that it corresponds to the algebra spanned by the gamma matrices of WCW. Also the local Clifford algebra of the embedding space $H = M^4 \times CP_2$ in octonionic representation of gamma matrices of H is important and the entire quantum TGD emerges from the associativity or co-associativity conditions for the sub-algebras of this algebra which are local algebras localized to maximal associative or co-associate sub-manifolds of the embedding space identifiable as space-time surfaces.

The notion of inclusion for hyper-finite factors provides an elegant description for the notion of measurement resolution absent from the standard quantum measurement theory.

1. The included sub-factor creates in ZEO states not distinguishable from the original one and the formally the coset space of factors defining quantum spinor space defines the space of physical states modulo finite measurement resolution.

2. The quantum measurement theory for hyperfinite factors differs from that for factors of type I since it is not possible to localize the state into single ray of state space. Rather, the ray is replaced with the sub-space obtained by the action of the included algebra defining the measurement resolution. The role of complex numbers in standard quantum measurement theory is taken by the non-commutative included algebra so that a non-commutative quantum theory is the outcome.

3. This leads also to the notion of quantum group. For instance, the finite measurement resolution means that the components of spinor do not commute anymore and it is not possible to reduce the state to a precise eigenstate of spin. It is however perform a reduction to an eigenstate of an observable which corresponds to the probability for either spin state.

4. For HFFs the dimension of infinite-dimensional state space is finite and 1 by convention. For included HFF $\mathcal{N} \subset \mathcal{M}$ the dimension of the tensor factor space containing only the degrees of freedom which are above measurement resolution is given by the index of inclusion $d = \mathcal{M} : \mathcal{N}$. One can say that the dimension associated with degrees of freedom below measurement resolution is $D = 1/d$. This number is never large than 1 for the inclusions and contains a set of discrete values $d = 4cos^2(2\pi/n)$, $n \geq 3$, plus the continuum above it. The fractal generalization of the formula for entanglement entropy gives $S = -log(1/D) = -log(d) \leq 0$ so that one can say that the entanglement negentropy assignable to the projection operators to the sub-factor is positive except for $n = 3$ for which it vanishes. The non-measured degrees of freedom carry information rather than entropy.

5. Clearly both HFFs of type I and II allow entanglement negentropy and allow to assign it with finite measurement resolution. In the case of factors its is not clear whether the weak form of NMP allows makes sense.

As already explained, the topology of the many-sheeted space-time encourages the generalization of the notion of quantum entanglement in such a way that unentangled systems can possess entangled sub-systems. One can say that the entanglement between sub-selves is not visible in the resolution characterizing selves. This makes possible sharing and fusion of mental images central for TGD inspired theory of consciousness. These concepts find a deeper justification from the quantum measurement theory for hyper-finite factors of type II_1 for which the finite measurement resolution is basic notion.

Hierarchies of conformal symmetry breakings, Planck constants, and inclusions of HFFs

The basic almost prediction of TGD is a fractal hierarchy of breakings of symplectic symmetry as a gauge symmetry.

It is good to briefly summarize the basic facts about the symplectic algebra assigned with $\delta M^4_\pm \times CP_2$ first.

1. Symplectic algebra has the structure of Virasoro algebra with respect to the light-like radial coordinate r_M of the light-cone boundary taking the role of complex coordinate for ordinary conformal symmetry. The Hamiltonians generating symplectic symmetries can be chosen to be proportional to functions $f_n(r_M)$. What is the natural choice for $f_n(r_M)$ is not quite clear. Ordinary conformal invariance would suggests $f_n(r_M) = r_M^n$. A more adventurous possibility is that the algebra is generated by Hamiltonians with $f_n(r_M) = r^{-s}$, where s is a root of Riemann Zeta so that one has either $s = 1/2 + iy$ (roots at critical line) or $s = -2n$, $n > 0$ (roots at negative real axis).
2. The set of conformal weights would be linear space spanned by combinations of all roots with integer coefficients $s = n - iy$, $s = \sum n_i y_i$, $n > -n_0$, where $-n_0 \geq 0$ is negative conformal weight. Mass squared is proportional to the total conformal weight and must be real demanding $y = \sum y_i = 0$ for physical states: I call this conformal confinement analogous to color confinement. One could even consider introducing the analog of binding energy as "binding conformal weight".

   Mass squared must be also non-negative (no tachyons) giving $n_0 \geq 0$. The generating conformal weights however have negative real part -1/2 and are thus tachyonic. Rather remarkably, p-adic mass calculations force to assume negative half-integer valued ground state conformal weight. This plus the fact that the zeros of Riemann Zeta has been indeed assigned with critical systems forces to take the Riemannian variant of conformal weight spectrum with seriousness. The algebra allows also now infinite hierarchy of conformal sub-algebras with weights coming as n-ples of the conformal weights of the entire algebra.
3. The outcome would be an infinite number of hierarchies of symplectic conformal symmetry breakings. Only the generators of the sub-algebra of the symplectic algebra with radial conformal weight proportional to n would act as gauge symmetries at given level of the hierarchy. In the hierarchy n_i divides n_{i+1}. In the symmetry breaking $n_i \rightarrow n_{i+1}$ the conformal charges, which vanished earlier, would become non-vanishing. Gauge degrees of freedom would transform to physical degrees of freedom.
4. What about the conformal Kac-Moody algebras associated with spinor modes. It seems that in this case one can assume that the conformal gauge symmetry is exact just as in string models.

The natural interpretation of the conformal hierarchies $n_i \rightarrow n_{i+1}$ would be in terms of increasing measurement resolution.

1. Conformal degrees of freedom below measurement resolution would be gauge degrees of freedom and correspond to generators with conformal weight proportional to n_i. Conformal hierarchies and associated hierarchies of Planck constants and n-fold coverings of space-time surface connecting the 3-surfaces at the ends of causal diamond would give a concrete realization of the inclusion hierarchies for hyper-finite factors of type II_1 [K125].

   n_i could correspond to the integer labelling Jones inclusions and associating with them the quantum group phase factor $U_n = exp(i2\pi/n)$, $n \geq 3$ and the index of inclusion given by $|M : N| = 4cos^2(2\pi/n)$ defining the fractal dimension assignable to the degrees of freedom above the measurement resolution. The sub-algebra with weights coming as n-multiples of the basic conformal weights would act as gauge symmetries realizing the idea that these degrees of freedom are below measurement resolution.
2. If $h_{eff} = n \times h$ defines the conformal gauge sub-algebra, the improvement of the resolution would scale up the Compton scales and would quite concretely correspond to a zoom analogous to that done for Mandelbrot fractal to get new details visible. From the point of view of cognition the improving resolution would fit nicely with the recent view about h_{eff}/h as a kind of intelligence quotient.

   This interpretation might make sense for the symplectic algebra of $\delta M^4_\pm \times CP_2$ for which the light-like radial coordinate r_M of light-cone boundary takes the role of complex coordinate. The reason is that symplectic algebra acts as isometries.

3. If Kähler action has vanishing total variation under deformations defined by the broken conformal symmetries, the corresponding conformal charges are conserved. The components of WCW Kähler metric expressible in terms of second derivatives of Kähler function can be however non-vanishing and have also components, which correspond to WCW coordinates associated with different partonic 2-surfaces. This conforms with the idea that conformal algebras extend to Yangian algebras generalizing the Yangian symmetry of $\mathcal{N} = 4$ symmetric gauge theories. The deformations defined by symplectic transformations acting gauge symmetries the second variation vanishes and there is not contribution to WCW Kähler metric.

4. One can interpret the situation also in terms of consciousness theory. The larger the value of h_{eff}, the lower the criticality, the more sensitive the measurement instrument since new degrees of freedom become physical, the better the resolution. In p-adic context large n means better resolution in angle degrees of freedom by introducing the phase $exp(i2\pi/n)$ to the algebraic extension and better cognitive resolution. Also the emergence of negentropic entanglement characterized by $n \times n$ unitary matrix with density matrix proportional to unit matrix means higher level conceptualization with more abstract concepts.

The extension of the super-conformal algebra to a larger Yangian algebra is highly suggestive and gives and additional aspect to the notion of measurement resolution.

1. Yangian would be generated from the algebra of super-conformal charges assigned with the points pairs belonging to two partonic 2-surfaces as stringy Noether charges assignable to strings connecting them. For super-conformal algebra associated with pair of partonic surface only single string associated with the partonic 2-surface. This measurement resolution is the almost the poorest possible (no strings at all would be no measurement resolution at all!).

2. Situation improves if one has a collection of strings connecting set of points of partonic 2-surface to other partonic 2-surface(s). This requires generalization of the super-conformal algebra in order to get the appropriate mathematics. Tensor powers of single string super-conformal charges spaces are obviously involved and the extended super-conformal generators must be multi-local and carry multi-stringy information about physics.

3. The generalization at the first step is simple and based on the idea that co-product is the "time inverse" of product assigning to single generator sum of tensor products of generators giving via commutator rise to the generator. The outcome would be expressible using the structure constants of the super-conformal algebra schematically a $Q^1_A = f_A^{BC} Q_B \otimes Q_C$. Here Q_B and Q_C are super-conformal charges associated with separate strings so that 2-local generators are obtained. One can iterate this construction and get a hierarchy of n-local generators involving products of n stringy super-conformal charges. The larger the value of n, the better the resolution, the more information is coded to the fermionic state about the partonic 2-surface and 3-surface. This affects the space-time surface and hence WCW metric but not the 3-surface so that the interpretation in terms of improved measurement resolution makes sense. This super-symplectic Yangian would be behind the quantum groups and Jones inclusions in TGD Universe.

4. n gives also the number of space-time sheets in the singular covering. One possible interpretation is in terms measurement resolution for counting the number of space-time sheets. Our recent quantum physics would only see single space-time sheet representing visible manner and dark matter would become visible only for $n > 1$.

It is not an accident that quantum phases are assignable to Yangian algebras, to quantum groups, and to inclusions of HFFs. The new deep notion added to this existing complex of high level mathematical concepts are hierarchy of Planck constants, dark matter hierarchy, hierarchy of criticalities, and negentropic entanglement representing physical notions. All these aspects represent new physics.

3.2.6 Planar Algebras And Generalized Feynman Diagrams

Planar algebras [A11] are a very general notion due to Vaughan Jones and a special class of them is known to characterize inclusion sequences of hyper-finite factors of type II_1 [A35] . In the following an argument is developed that planar algebras might have interpretation in terms of planar projections of generalized Feynman diagrams (these structures are metrically 2-D by presence of one light-like direction so that 2-D representation is especially natural). In [K27] the role of planar algebras and their generalizations is also discussed.

Planar algebra very briefly

First a brief definition of planar algebra.

1. One starts from planar k-tangles obtained by putting disks inside a big disk. Inner disks are empty. Big disk contains $2k$ braid strands starting from its boundary and returning back or ending to the boundaries of small empty disks in the interior containing also even number of incoming lines. It is possible to have also loops. Disk boundaries and braid strands connecting them are different objects. A black-white coloring of the disjoint regions of k-tangle is assumed and there are two possible options (photo and its negative). Equivalence of planar tangles under diffeomorphisms is assumed.
2. One can define a product of k-tangles by identifying k-tangle along its outer boundary with some inner disk of another k-tangle. Obviously the product is not unique when the number of inner disks is larger than one. In the product one deletes the inner disk boundary but if one interprets this disk as a vertex-parton, it would be better to keep the boundary.
3. One assigns to the planar k-tangle a vector space V_k and a linear map from the tensor product of spaces V_{k_i} associated with the inner disks such that this map is consistent with the decomposition k-tangles. Under certain additional conditions the resulting algebra gives rise to an algebra characterizing multi-step inclusion of HFFs of type II_1.
4. It is possible to bring in additional structure and in TGD framework it seems necessary to assign to each line of tangle an arrow telling whether it corresponds to a strand of a braid associated with positive or negative energy parton. One can also wonder whether disks could be replaced with closed 2-D surfaces characterized by genus if braids are defined on partonic surfaces of genus g. In this case there is no topological distinction between big disk and small disks. One can also ask why not allow the strands to get linked (as suggested by the interpretation as planar projections of generalized Feynman diagrams) in which case one would not have a planar tangle anymore.

General arguments favoring the assignment of a planar algebra to a generalized Feynman diagram

There are some general arguments in favor of the assignment of planar algebra to generalized Feynman diagrams.

1. Planar diagrams describe sequences of inclusions of HFF:s and assign to them a multi-parameter algebra corresponding indices of inclusions. They describe also Connes tensor powers in the simplest situation corresponding to Jones inclusion sequence. Suppose that also general Connes tensor product has a description in terms of planar diagrams. This might be trivial.
2. Generalized vertices identified geometrically as partonic 2-surfaces indeed contain Connes tensor products. The smallest sub-factor N would play the role of complex numbers meaning that due to a finite measurement resolution one can speak only about N-rays of state space and the situation becomes effectively finite-dimensional but non-commutative.
3. The product of planar diagrams could be seen as a projection of 3-D Feynman diagram to plane or to one of the partonic vertices. It would contain a set of 2-D partonic 2-surfaces. Some of them would correspond vertices and the rest to partonic 2-surfaces at future and past directed light-cones corresponding to the incoming and outgoing particles.
4. The question is how to distinguish between vertex-partons and incoming and outgoing partons. If one does not delete the disk boundary of inner disk in the product, the fact that lines arrive at it from both sides could distinguish it as a vertex-parton whereas outgoing partons would correspond to empty disks. The direction of the arrows associated with the lines of planar diagram would allow to distinguish between positive and negative energy partons (note however line returning back).

5. One could worry about preferred role of the big disk identifiable as incoming or outgoing parton but this role is only apparent since by compactifying to say S^2 the big disk exterior becomes an interior of a small disk.

A more detailed view

The basic fact about planar algebras is that in the product of planar diagrams one glues two disks with identical boundary data together. One should understand the counterpart of this in more detail.

1. The boundaries of disks would correspond to 1-D closed space-like stringy curves at partonic 2-surfaces along which fermionic anti-commutators vanish.
2. The lines connecting the boundaries of disks to each other would correspond to the strands of number theoretic braids and thus to braidy time evolutions. The intersection points of lines with disk boundaries would correspond to the intersection points of strands of number theoretic braids meeting at the generalized vertex.

   [Number theoretic braid belongs to an algebraic intersection of a real parton 3-surface and its p-adic counterpart obeying same algebraic equations: of course, in time direction algebraicity allows only a sequence of snapshots about braid evolution].
3. Planar diagrams contain lines, which begin and return to the same disk boundary. Also "vacuum bubbles" are possible. Braid strands would disappear or appear in pairwise manner since they correspond to zeros of a polynomial and can transform from complex to real and vice versa under rather stringent algebraic conditions.
4. Planar diagrams contain also lines connecting any pair of disk boundaries. Stringy decay of partonic 2-surfaces with some strands of braid taken by the first and some strands by the second parton might bring in the lines connecting boundaries of any given pair of disks (if really possible!).
5. There is also something to worry about. The number of lines associated with disks is even in the case of k-tangles. In TGD framework incoming and outgoing tangles could have odd number of strands whereas partonic vertices would contain even number of k-tangles from fermion number conservation. One can wonder whether the replacement of boson lines with fermion lines could imply naturally the notion of half-k-tangle or whether one could assign half-k-tangles to the spinors of WCW ("world of classical worlds") whereas corresponding Clifford algebra defining HFF of type II_1 would correspond to k-tangles.

3.2.7 Miscellaneous

The following considerations are somewhat out-of-date: hence the title "Miscellaneous".

Connes tensor product and fusion rules

One should demonstrate that Connes tensor product indeed produces an M-matrix with physically acceptable properties.

The reduction of the construction of vertices to that for n-point functions of a conformal field theory suggest that Connes tensor product is essentially equivalent with the fusion rules for conformal fields defined by the Clifford algebra elements of $CH(CD)$ (4-surfaces associated with 3-surfaces at the boundary of causal diamond CD in M^4), extended to local fields in M^4 with gamma matrices acting on WCW spinor s assignable to the partonic boundary components.

Jones speculates that the fusion rules of conformal field theories can be understood in terms of Connes tensor product [A96] and refers to the work of Wassermann about the fusion of loop group representations as a demonstration of the possibility to formula the fusion rules in terms of Connes tensor product [A23] .

Fusion rules are indeed something more intricate that the naïve product of free fields expanded using oscillator operators. By its very definition Connes tensor product means a dramatic reduction of degrees of freedom and this indeed happens also in conformal field theories.

1. For non-vanishing n-point functions the tensor product of representations of Kac Moody group associated with the conformal fields must give singlet representation.

2. The ordinary tensor product of Kac Moody representations characterized by given value of central extension parameter k is not possible since k would be additive.

3. A much stronger restriction comes from the fact that the allowed representations must define integrable representations of Kac-Moody group [A30] . For instance, in case of $SU(2)_k$ Kac Moody algebra only spins $j \leq k/2$ are allowed. In this case the quantum phase corresponds to $n = k + 2$. $SU(2)$ is indeed very natural in TGD framework since it corresponds to both electro-weak $SU(2)_L$ and isotropy group of particle at rest.

Fusion rules for localized Clifford algebra elements representing operators creating physical states would replace naïve tensor product with something more intricate. The naïvest approach would start from M^4 local variants of gamma matrices since gamma matrices generate the Clifford algebra Cl associated with $CH(CD)$. This is certainly too naïve an approach. The next step would be the localization of more general products of Clifford algebra elements elements of Kac Moody algebras creating physical states and defining free on mass shell quantum fields. In standard quantum field theory the next step would be the introduction of purely local interaction vertices leading to divergence difficulties. In the recent case one transfers the partonic states assignable to the light-cone boundaries $\delta M^4_\pm(m_i) \times CP_2$ to the common partonic 2-surfaces X^2_V along $X^3_{L,i}$ so that the products of field operators at the same space-time point do not appear and one avoids infinities.

The remaining problem would be the construction an explicit realization of Connes tensor product. The formal definition states that left and right $\mathcal{N}$ actions in the Connes tensor product $\mathcal{M} \otimes_\mathcal{N} \mathcal{M}$ are identical so that the elements $nm_1 \otimes m_2$ and $m_1 \otimes m_2 n$ are identified. This implies a reduction of degrees of freedom so that free tensor product is not in question. One might hope that at least in the simplest choices for $\mathcal{N}$ characterizing the limitations of quantum measurement this reduction is equivalent with the reduction of degrees of freedom caused by the integrability constraints for Kac-Moody representations and dropping away of higher spins from the ordinary tensor product for the representations of quantum groups. If fusion rules are equivalent with Connes tensor product, each type of quantum measurement would be characterized by its own conformal field theory.

In practice it seems safest to utilize as much as possible the physical intuition provided by quantum field theories. In [K34] a rather precise vision about generalized Feynman diagrams is developed and the challenge is to relate this vision to Connes tensor product.

Connection with topological quantum field theories defined by Chern-Simons action

There is also connection with topological quantum field theories (TQFTs) defined by Chern- Simons action [A41] .

1. The light-like 3-surfaces X^3_l defining propagators can contain unitary matrix characterizing the braiding of the lines connecting fermions at the ends of the propagator line. Therefore the modular S-matrix representing the braiding would become part of propagator line. Also incoming particle lines can contain similar S-matrices but they should not be visible in the M-matrix. Also entanglement between different partonic boundary components of a given incoming 3-surface by a modular S-matrix is possible.

2. Besides CP_2 type extremals MEs with light-like momenta can appear as brehmstrahlung like exchanges always accompanied by exchanges of CP_2 type extremals making possible momentum conservation. Also light-like boundaries of magnetic flux tubes having macroscopic size could carry light-like momenta and represent similar brehmstrahlung like exchanges. In this case the modular S-matrix could make possible topological quantum computations in $q \neq 1$ phase [K5] . Notice the somewhat counter intuitive implication that magnetic flux tubes of macroscopic size would represent change in quantum jump rather than quantum state. These quantum jumps can have an arbitrary long geometric duration in macroscopic quantum phases with large Planck constant [K44] .

There is also a connection with topological QFT defined by Chern-Simons action allowing to assign topological invariants to the 3-manifolds [A41] . If the light-like CDs $X^3_{L,i}$ are boundary components, the 3-surfaces associated with particles are glued together somewhat like they are glued in the process allowing to construct 3-manifold by gluing them together along boundaries. All 3-manifold topologies can be constructed by using only torus like boundary components.

This would suggest a connection with 2+1-dimensional topological quantum field theory defined by Chern-Simons action allowing to define invariants for knots, links, and braids and 3-manifolds using surgery along links in terms of Wilson lines. In these theories one consider gluing of two 3-manifolds, say 3-spheres S^3 along a link to obtain a topologically non-trivial 3-manifold. The replacement of link with Wilson lines in $S^3\#S^3 = S^3$ reduces the calculation of link invariants defined in this manner to Chern-Simons theory in S^3.

In the recent situation more general structures are possible since arbitrary number of 3-manifolds are glued together along link so that a singular 3-manifolds with a book like structure are possible. The allowance of CDs which are not boundaries, typically 3-D light-like throats of wormhole contacts at which induced metric transforms from Minkowskian to Euclidian, brings in additional richness of structure. If the scaling factor of CP_2 metric can be arbitrary large as the quantization of Planck constant predicts, this kind of structure could be macroscopic and could be also linked and knotted. In fact, topological condensation could be seen as a process in which two 4-manifolds are glued together by drilling light-like CDs and connected by a piece of CP_2 type extremal.

3.3 Fresh View About Hyper-Finite Factors In TGD Framework

In the following I will discuss the basic ideas about the role of hyper-finite factors in TGD with the background given by a work of more than half decade. First I summarize the input ideas which I combine with the TGD inspired intuitive wisdom about HFFs of type II_1 and their inclusions allowing to represent finite measurement resolution and leading to notion of quantum spaces with algebraic number valued dimension defined by the index of the inclusion.

Also an argument suggesting that the inclusions define "skewed" inclusions of lattices to larger lattices giving rise to quasicrystals is proposed. The core of the argument is that the included HFF of type II_1 algebra is a projection of the including algebra to a subspace with dimension $D \leq 1$. The projection operator defines the analog of a projection of a bigger lattice to the included lattice. Also the fact that the dimension of the tensor product is product of dimensions of factors just like the number of elements in finite group is product of numbers of elements of coset space and subgroup, supports this interpretation.

One also ends up with a detailed identification of the hyper-finite factors in orbital degrees of freedom in terms of symplectic group associated with $\delta M^4_\pm \times CP_2$ and the group algebras of their discrete subgroups define what could be called "orbital degrees of freedom" for WCW spinor fields. By very general argument this group algebra is HFF of type II, maybe even II_1.

3.3.1 Crystals, Quasicrystals, Non-Commutativity And Inclusions Of Hyperfinite Factors Of Type II_1

I list first the basic ideas about non-commutative geometries and give simple argument suggesting that inclusions of HFFs correspond to "skewed" inclusions of lattices as quasicrystals.

1. Quasicrystals (see `http://tinyurl.com/67kz3qo`) (say Penrose tilings) [A13] can be regarded as subsets of real crystals and one can speak about "skewed" inclusion of real lattice to larger lattice as quasicrystal. What this means that included lattice is obtained by projecting the larger lattice to some lower-dimensional subspace of lattice.

2. The argument of Connes concerning definition of non-commutative geometry can be found in the book of Michel Lapidus at page 200. Quantum space is identified as a space of equivalence classes. One assigns to pairs of elements inside equivalence class matrix elements having the element pair as indices (one assumes numerable equivalence class). One considers irreducible representations of the algebra defined by the matrices and identifies the equivalent irreducible representations. If I have understood correctly, the equivalence classes of irreps define a discrete point set representing the equivalence class and it can often happen that there is just

single point as one might expect. This I do not quite understand since it requires that all irreps are equivalent.

3. It seems that in the case of linear spaces - von Neumann algebras and accompanying Hilbert spaces - one obtains a connection with the inclusions of HFFs and corresponding quantum factor spaces that should exist as analogs of quantum plane. One replaces matrices with elements labelled by element pairs with linear operators in HFF of type II_1. Index pairs correspond to pairs in linear basis for the HFF or corresponding Hilbert space.

4. Discrete infinite enumerable basis for these operators as a linear space generates a lattice in summation. Inclusion $N \subset M$ defines inclusion of the lattice/crystal for N to the corresponding lattice of M. Physical intuition suggests that if this inclusion is "skewed" one obtains quasicrystal. The fact the index of the inclusion is algebraic number suggests that the coset space M/N is indeed analogous to quasicrystal.

   More precisely, the index of inclusion is defined for hyper-finite factors of type II_1 using the fact that quantum trace of unit matrix equals to unity $Tr(Id(M)) = 1$, and from the tensor product composition $M = (M/N) \times N$ given $Tr(Id(M)) = 1 = Ind(M/N)Tr(P(M \rightarrow N))$, where $P(M \rightarrow N$ is projection operator from M to N. Clearly, $Ind(M/N) = 1/Tr(P(M \rightarrow N))$ defines index as a dimension of quantum space M/N.

   For Jones inclusions characterized by quantum phases $q = exp(i2\pi/n)$, $n = 3, 4, ...$ the values of index are given by $Ind(M/N) = 4cos^2(\pi/n)$, $n = 3, 4, ...$ There is also another range inclusions $Ind(M/N) \geq 4$: note that $Tr(P(M \rightarrow N))$ defining the dimension of N as included sub-space is never larger than one for HFFs of type II_1. The projection operator $P(M \rightarrow N)$ is obviously the counterpart of the projector projecting lattice to some lower-dimensional sub-space of the lattice.

5. Jones inclusions are between linear spaces but there is a strong analogy with non-linear coset spaces since for the tensor product the dimension is product of dimensions and for discrete coset spaces G/H one has also the product formula $n(G) = n(H) \times n(G/H)$ for the numbers of elements. Noticing that space of quantum amplitudes in discrete space has dimension equal to the number of elements of the space, one could say that Jones inclusion represents quantized variant for classical inclusion raised from the level of discrete space to the level of space of quantum states with the number of elements of set replaced by dimension. In fact, group algebras of infinite and enumerable groups defined HFFs of type II under rather general conditions (see below).

   Could one generalize Jones inclusions so that they would apply to non-linear coset spaces analogs of the linear spaces involved ? For instance, could one think of infinite-dimensional groups G and H for which Lie-algebras defining their tangent spaces can be regarded as HFFs of type II_1? The dimension of the tangent space is dimension of the non-linear manifold: could this mean that the non-linear infinite-dimensional inclusions reduce to tangent space level and thus to the inclusions for Lie-algebras regarded hyper-finite factors of type II_1 or more generally, type II? This would would rise to quantum spaces which have finite but algebraic valued quantum dimension and in TGD framework take into account the finite measurement resolution.

6. To concretize this analogy one can check what is the number of points map from 5-D space containing aperiodic lattice as a projection to a 2-D irrational plane containing only origin as common point with the 5-D lattice. It is easy to get convinced that the projection is 1-to-1 so that the number of points projected to a given point is 1. By the analogy with Jones inclusions this would mean that the included space has same von Neumann dimension 1 - just like the including one. In this case quantum phase equals $q = exp(i2\pi/n)$, $n = 3$ - the lowest possible value of n. Could one imagine the analogs of $n > 3$ inclusions for which the number of points projected to a given point would be larger than 1? In 1-D case the rational lines $y = (k/l)x$ define 1-D rational analogs of quasi crystals. The points $(x, y) = (m, n)$, $m \ mod \ l = 0$ are projected to the same point. The number of points is now infinite and the ratio of points of 2-D lattice and 1-D crystal like structure equals to l and serves as the analog for the quantum dimension $d_q = 4cos^2(\pi/n)$.

To sum up, this this is just physicist's intuition: it could be wrong or something totally trivial from the point of view of mathematician. The main message is that the inclusions of HFFs might define also inclusions of lattices as quasicrystals.

3.3.2 HFFs And Their Inclusions In TGD Framework

In TGD framework the inclusions of HFFs have interpretation in terms of finite measurement resolution. If the inclusions define quasicrystals then finite measurement resolution would lead to quasicrystals.

1. The automorphic action of N in $M \supset N$ and in associated Hilbert space H_M where N acts generates physical operators and accompanying stas (operator rays and rays) not distinguishable from the original one. States in finite measurement resolution correspond to N-rays rather than complex rays. It might be natural to restrict to unitary elements of N.

   This leads to the need to construct the counterpart of coset space M/N and corresponding linear space H_M/H_N. Physical intuition tells that the indices of inclusions defining the "dimension" of M/N are algebraic numbers given by Jones index formula.

2. Here the above argument would assign to the inclusions also inclusions of lattices as quasicrystals.

Degrees of freedom for WCW spinor field

Consider first the identification of various kinds of degrees of freedom in TGD Universe.

1. Very roughly, WCW ("world of classical worlds") spinor is a state generated by fermionic creation operators from vacuum at given 3-surface. WCW spinor field assigns this kind of spinor to each 3-surface. WCW spinor fields decompose to tensor product of spin part (Fock state) and orbital part ("wave" in WCW) just as ordinary spinor fields.
2. The conjecture motivated by super-symmetry has been that both WCW spinors and their orbital parts (analogs of scalar field) define HFFs of type II_1 in quantum fluctuating degrees of freedom.
3. Besides these there are zero modes, which by definition do not contribute to WCW Kähler metric.
   (a) If the zero zero modes are symplectic invariants, they appear only in conformal factor of WCW metric. Symplectically invariant zero modes represent purely classical degrees of freedom - direction of a pointer of measurement apparatus in quantum measurement - and in given experimental arrangement they entangle with quantum fluctuating degrees of freedom in one-one manner so that state function reduction assigns to the outcome of state function reduction position of pointer. I forget symplectically invariant zero modes and other analogous variables in the following and concentrate to the degrees of freedom contributing WCW line-element.
   (b) There are also zero modes which are not symplectic invariants and are analogous to degrees of freedom generated by the generators of Kac-Moody algebra having vanishing conformal weight. They represent "center of mass degrees of freedom" and this part of symmetric algebra creates the representations representing the ground states of Kac-Moody representations. Restriction to these degrees of freedom gives QFT limit in string theory. In the following I will speak about "cm degrees of freedom".

The general vision about symplectic degrees of freedom (the analog of "orbital degrees of freedom" for ordinary spinor field) is following.

1. WCW (assignable to given CD) is a union over the sub-WCWs labeled by zero modes and each sub-WCW representing quantum fluctuating degrees of freedom and "cm degrees of freedom" is infinite-D symmetric space. If symplectic group assignable to $\delta M^4_+ \times CP_2$ acts as as isometries of WCW then "orbital degrees of freedom" are parametrized by the symplectic group or its coset space (note that light-cone boundary is 3-D but radial dimension is light-like so that symplectic - or rather contact structure - exists).

   Let S^2 be $r_M = constant$ sphere at light-cone boundary (r_M is the radial light-like coordinate fixed apart from Lorentz transformation). The full symplectic group would act as isometries of WCW but does not - nor cannot do so - act as symmetries of Kähler action except in the huge vacuum sector of the theory correspond to vacuum extremals.

2. WCW Hamiltonians can be deduced as "fluxes" of the Hamiltonians of $\delta M^4_+ \times CP_2$ taken over partonic 2-surfaces. These Hamiltonians expressed as products of Hamiltonians of S^2 and CP_2 multiplied by powers r_M^n. Note that r_M plays the role of the complex coordinate z for Kac-Moody algebras and the group G defining KM is replaced with symplectic group of $S^2 \times CP_2$. Hamiltonians can be assumed to have well-defined spin (SO(3)) and color (SU(3)) quantum numbers.
3. The generators with vanishing radial conformal weight ($n = 0$) correspond to the symplectic group of $S^2 \times CP_2$. They are not symplectic invariants but are zero modes. They would correspond to "cm degrees of freedom" characterizing the ground states of representations of the full symplectic group.

Discretization at the level of WCW

The general vision about finite measurement resolution implies discretization at the level of WCW.

1. Finite measurement resolution at the level of WCW means discretization. Therefore the symplectic groups of $\delta M^4_+ \times CP_2$ *resp.* $S^2 \times CP_2$ are replaced by an enumerable discrete subgroup. WCW is discretized in both quantum fluctuating degrees of freedom and "center of mass" degrees of freedom.
2. The elements of the group algebras of these discrete groups define the "orbitals parts" of WCW spinor fields in discretization. I will later develop an argument stating that they are HFFs of type II - maybe even II_1. Note that also function spaces associated with the coset spaces of these discrete subgroups could be considered.
3. Discretization applies also in the spin degrees of freedom. Since fermionic Fock basis generates quantum counterpart of Boolean algebra the interpretation in terms of the physical correlates of Boolean cognition is motivated (fermion number 1/0 and various spins in decomposition to a tensor product of lower-dimensional spinors represent bits). Note that in ZEO fermion number conservation does not pose problems and zero states actually define what might be regarded as quantum counterparts of Boolean rules $A \rightarrow B$.
4. Note that 3-surfaces correspond by the strong form of GCI/holography to collections of partonic 2-surfaces and string world sheets of space-time surface intersecting at discrete set of points carrying fermionic quantum numbers. WCW spinors are constructed from second quantized induced spinor fields and fermionic Fock algebra generates HFF of type II_1.

Does WCW spinor field decompose to a tensor product of two HFFs of type II_1?

The group algebras associated with infinite discrete subgroups of the symplectic group define the discretized analogs of waves in WCW having quantum fluctuating part and cm part. The proposal is that these group algebras are HFFs of type II_1. The spinorial degrees of freedom correspond to fermionic Fock space and this is known to be HFF. Therefore WCW spinor fields would defined tensor product of HFFs of type II_1. The interpretation would be in terms of supersymmetry at the level of WCW. Super-conformal symmetry is indeed the basic symmetry of TGD so that this result is a physical "must". The argument goes as follows.

1. In non-zero modes WCW is symplectic group of $\delta M^4_+ \times CP_2$ (call this group just *Sympl*) reduces to the analog of Kac-Moody group associated with $S^2 \times CP_2$, where S^2 is $r_M =$ constant sphere of light-cone boundary and z is replaced with radial coordinate. The Hamiltonians, which do not depend on r_M would correspond to zero modes and one could not assign metric to them although symplectic structure is possible. In "cm degrees of freedom" one has symplectic group associated with $S^2 \times CP_2$.
2. Finite measurement resolution, which seems to be coded already in the structure of the preferred extremals and of the solutions of the Kähler-Dirac equation, suggests strongly that this symplectic group is replaced by its discrete subgroup or symmetric coset space. What this group is, depends on measurement resolution defined by the cutoffs inherent to the solutions. These subgroups and coset spaces would define the analogs of Platonic solids in WCW!

3. Why the discrete infinite subgroups of *Sympl* would lead naturally to HFFs of type II? There is a very general result stating that group algebra of an enumerable discrete group, which has infinite conjugacy classes, and is amenable so that its regular representation in group algebra decomposes to all unitary irreducibles is HFF of type II. See for examples about HFFs of type II listed in Wikipedia article (see `http://tinyurl.com/y8445w8q`) [A7].
4. Suppose that the group algebras associated the discrete subgroups *Sympl* are indeed HFFs of type II or even type II_1. Their inclusions would define finite measurement resolution the orbital degrees of freedom for WCW spinor fields. Included algebra would create rays of state space not distinguishable experimentally. The inclusion would be characterized by the inclusion of the lattice defined by the generators of included algebra by linearity. One would have inclusion of this lattice to a lattice associated with a larger discrete group. Inclusions of lattices are however known to give rise to quasicrystals (Penrose tilings are basic example), which define basic non-commutative structures. This is indeed what one expects since the dimension of the coset space defined by inclusion is algebraic number rather than integer.
5. Also in fermionic degrees of freedom finite measurement resolution would be realized in terms of inclusions of HFFs- now certainly of type II_1. Therefore one could obtain hierarchies of lattices included as quasicrystals.

What about zero modes which are symplectic invariants and define classical variables? They are certainly discretized too. One might hope that one-one correlation between zero modes (classical variables) and quantum fluctuating degrees of freedom suggested by quantum measurement theory allows to effectively eliminate them. Besides zero modes there are also modular degrees of freedom associated with partonic 2-surfaces defining together with their 4-D tangent space data basis objects by strong form of holography. Also these degrees of freedom are automatically discretized. But could one consider finite measurement resolution also in these degrees of freedom. If the symplectic group of $S^2 \times CP_2$ defines zero modes then one could apply similar argument also in these degrees of freedom to discrete subgroups of $S^2 \times CP_2$.

3.3.3 Little Appendix: Comparison Of WCW Spinor Fields With Ordinary Second Quantized Spinor Fields

In TGD one identifies states of Hilbert space as WCW spinor fields. The analogy with ordinary spinor field helps to understand what they are. I try to explain by comparison with QFT.

Ordinary second quantized spinor fields

Consider first ordinary fermionic QFT in fixed space-time. Ordinary spinor is attached to an space-time point and there is $2^{D/2}$ dimensional space of spin degrees of freedom. Spinor field attaches spinor to every point of space-time in a continuous/smooth manner. Spinor fields satisfying Dirac equation define in Euclidian metric a Hilbert space with a unitary inner product. In Minkowskian case this does not work and one must introduce second quantization and Fock space to get a unitary inner product. This brings in what is essentially a basic realization of HFF of type II_1 as allowed operators acting in this Fock space. It is operator algebra rather than state space which is HFF of type II_1 but they are of course closely related.

Classical WCW spinor fields as quantum states

What happens TGD where one has quantum superpositions of 4-surface/3-surfaces by GCI/partonic 2-surfaces with 4-D tangent space data by strong form of GCI.

1. First guess: space-time point is replaced with 3-surface. Point like particle becomes 3-surface representing particle. WCW spinors are fermionic Fock states at this surface. WCW spinor fields are Fock state as a functional of 3-surface. Inner product decomposes to Fock space inner product plus functional integral over 3-surfaces (no path integral!). One could speak of quantum multiverse. Not single space-time but quantum superposition of them. This quantum multiverse character is something new as compared to QFT.
2. Second guess: forced by ZEO, by geometrization of Feynman diagrams, etc.

(a) 3-surfaces are actually not connected 3-surfaces. They are collections of components at both ends of CD and connected to single connected structure by 4-surface. Components of 3-surface are like incoming and outgoing particles in connected Feynman diagrams. Lines are identified as regions of Euclidian signature or equivalently as the 3-D light-like boundaries between Minkowskian and Euclidian signature of the induced metric.

(b) Spinors(!!) are defined now by the fermionic Fock space of second quantized induced spinor fields at these 3-surfaced and by holography at 4-surface. This fermionic Fock space is assigned to all multicomponent 3-surfaces defined in this manner and WCW spinor fields are defined as in the first guess. This brings integration over WCW to the inner product.

3. Third, even more improved guess: motivated by the solution ansatz for preferred extremals and for Kähler-Dirac equation [K126] giving a connection with string models.

   The general solution ansatz restricts all spinor components but right-handed neutrino to string world sheets and partonic 2-surfaces: this means effective 2-dimensionality. String world sheets and partonic 2-surfaces intersect at the common ends of light-like and space-like braids at ends of CD and at along wormhole throat orbits so that effectively discretization occurs. This fermionic Fock space replaces the Fock space of ordinary second quantization.

3.4 The idea of Connes about inherent time evolution of certain algebraic structures from TGD point of view

Jonathan Disckau asked me about what I think about the proposal of Connes represented in the summary of progress of noncommutative geometry in "Noncommutative Geometry Year 2000" [A22] (see `https://arxiv.org/abs/math/0011193`) that certain mathematical structures have inherent time evolution coded into their structure.

I have written years ago about Connes's proposal. At that time I was trying to figure out how to understand the construction of scattering amplitudes in the TGD framework and the proposal of Connes looked attractive. Later I had to give up this idea. However, the basic idea is beautiful. One should only replace the notion of time evolution from a one-parameter group of automorphisms to something more interesting. Also time evolution as increasing algebraic complexity is a more attractive interpretation.

The inclusion hierarchies of hyperfinite factors (HFFs) - closely related to the work of Connes - are a key element of TGD and crucial for understanding evolutionary hierarchies in TGD. Is it possible that mathematical structure evolves in time in some sense? The TGD based answer is that quantum jump as a fundamental evolutionary step - moment of subjective time evolution - is a necessary new element. The sequence of moments of consciousness as quantum jumps would have an interpretation as hopping around in the space of mathematical structures leading to increasingly complex structures.

The generalization of the idea of Connes is discussed in this framework. In particular, the inclusion hierarchies of hyper-finite factors, the extension hierarchies of rationals, and fractal inclusion hierarchies of subalgebras of supersymplectic algebra isomorphic with the entire algebra are proposed to be more or less one and the same thing in TGD framework.

The time evolution operator of Connes could corresponds to super-symplectic algebra (SSA) to the time evolution generated by $exp(iL_0\tau)$ so that the operator Δ of Connes would be identified as $\Delta = exp(L_0)$. This identification allows number theoretical universality if τ is quantized. Furthermore, one ends up with a model for the subjective time evolution by small state function reductions (SSFRs) for SSA with SSA_n gauge conditions: the unitary time evolution for given SSFR would be generated by a linear combination of Virasoro generators not annihilating the states. This model would generalize the model for harmonic oscillator in external force allowing exact S-matrix.

3.4.1 Connes proposal and TGD

In this section I develop in more detail the analog of Connes proposal in TGD framework.

What does Connes suggest?

One must first make clear what the automorphism of HFFs discovered by Connes is.

1. Tomita-Takesaki theory

Tomita-Takesaki theory is a vital part of the theory of factors. I have described the theory earlier [K76, K48].
First some definitions.

1. Let $\omega(x)$ be a faithful state of von Neumann algebra so that one has $\omega(xx^*) > 0$ for $x > 0$. Assume by Riesz lemma the representation of ω as a vacuum expectation value: $\omega = (\cdot\Omega, \Omega)$, where Ω is cyclic and separating state.
2. Let

$$L^{\infty}(\mathcal{M}) \equiv \mathcal{M} \ , \quad L^2(\mathcal{M}) = \mathcal{H} \ , \quad L^1(\mathcal{M}) = \mathcal{M}_* \ , \qquad (3.4.1)$$

where $\mathcal{M}_*$ is the pre-dual of $\mathcal{M}$ defined by linear functionals in $\mathcal{M}$. One has $\mathcal{M}_*^{\ *} = \mathcal{M}$.
3. The conjugation $x \to x^*$ is isometric in $\mathcal{M}$ and defines a map $\mathcal{M} \to L^2(\mathcal{M})$ via $x \to x\Omega$. The map $S_0; x\Omega \to x^*\Omega$ is however non-isometric.
4. Denote by S the closure of the anti-linear operator S_0 and by $S = J\Delta^{1/2}$ its polar decomposition analogous that for complex number and generalizing polar decomposition of linear operators by replacing (almost) unitary operator with anti-unitary J. Therefore $\Delta = S^*S > 0$ is positive self-adjoint and J an anti-unitary involution. The non-triviality of Δ reflects the fact that the state is not trace so that hermitian conjugation represented by S in the state space brings in additional factor $\Delta^{1/2}$.
5. What x can be is puzzling to physicists. The restriction fermionic Fock space and thus to creation operators would imply that Δ would act non-trivially only vacuum state so that $\Delta > 0$ condition would not hold true. The resolution of puzzle is the allowance of tensor product of Fock spaces for which vacua are conjugates: only this gives cyclic and separating state. This is natural in ZEO.

The basic results of Tomita-Takesaki theory are following.

1. The basic result can be summarized through the following formulas

$$\Delta^{it} M \Delta^{-it} = \mathcal{M} \ , J\mathcal{M}J = \mathcal{M}' \ .$$

2. The latter formula implies that $\mathcal{M}$ and $\mathcal{M}'$ are isomorphic algebras. The first formula implies that a one parameter group of modular automorphisms characterizes partially the factor. The physical meaning of modular automorphisms is discussed in [A34, A66] Δ is Hermitian and positive definite so that the eigenvalues of $log(\Delta)$ are real but can be negative. Δ^{it} is however not unitary for factors of type II and III. Physically the non-unitarity must relate to the fact that the flow is contracting so that hermiticity as a local condition is not enough to guarantee unitarity.
3. $\omega \to \sigma_t^{\omega} = Ad\Delta^{it}$ defines a canonical evolution -modular automorphism- associated with ω and depending on it. The Δ:s associated with different ω:s are related by a unitary inner automorphism so that their equivalence classes define an invariant of the factor.

Tomita-Takesaki theory gives rise to a non-commutative measure theory which is highly non-trivial. In particular the spectrum of Δ can be used to classify the factors of type II and III.
The definition of Δ^{it} reduces in eigenstate basis of Δ to the definition of complex function d^{it}. Note that is positive so that the logarithm of d is real.
In TGD framework number theoretic universality poses additional conditions. In diagonal basis $e^{log(d)it}$ must exist. A simply manner to solve the conditions is $e = exp(m/r)$ existing p-adically for an extension of rational allowing r:th root of e. This requires also quantization of as a root of unity so that the exponent reduces to a root of unity.

2. Modular automorphisms

Modular automorphisms of factors are central for their classification.

1. One can divide the automorphisms to inner and outer ones. Inner automorphisms correspond to unitary operators obtained by exponentiating Hermitian Hamiltonian belonging to the factor and connected to identity by a flow. Outer automorphisms do not allow a representation as a unitary transformations although $log(\Delta)$ is formally a Hermitian operator.
2. The fundamental group of the type II_1 factor defined as fundamental group group of corresponding II_∞ factor characterizes partially a factor of type II_1. This group consists of real numbers λ such that there is an automorphism scaling the trace by λ. Fundamental group typically contains all reals but it can be also discrete and even trivial.
3. Factors of type III allow a one-parameter group of modular automorphisms, which can be used to achieve a partial classification of these factors. These automorphisms define a flow in the center of the factor known as flow of weights. The set of parameter values λ for which ω is mapped to itself and the center of the factor defined by the identity operator (projector to the factor as a sub-algebra of $\mathcal{B}(\mathcal{H})$) is mapped to itself in the modular automorphism defines the Connes spectrum of the factor. For factors of type III_λ this set consists of powers of $\lambda < 1$. For factors of type III_0 this set contains only identity automorphism so that there is no periodicity. For factors of type III_1 Connes spectrum contains all real numbers so that the automorphisms do not affect the identity operator of the factor at all.

The modules over a factor correspond to separable Hilbert spaces that the factor acts on. These modules can be characterized by M-dimension. The idea is roughly that complex rays are replaced by the sub-spaces defined by the action of $\mathcal{M}$ as basic units. M-dimension is not integer valued in general. The so called standard module has a cyclic separating vector and each factor has a standard representation possessing antilinear involution J such that $\mathcal{M}' = J\mathcal{M}J$ holds true (note that J changes the order of the operators in conjugation). The inclusions of factors define modules having interpretation in terms of a finite measurement resolution defined by $\mathcal{M}$.

3. Objections against the idea of Connes

One can represent objections against this idea.

1. Ordinary time evolution in wave mechanics is a unitary automorphism, so that in this framework they would not have physical meaning but act as gauge transformations. If outer automorphisms define time evolutions, they must act as gauge transformations. One would have an analog of gauge field theory in HFF. This would be of course highly interesting: when I gave up the idea of Connes, I did not consider this possibility. Super-symplectic algebras having fractal structure are however extremely natural candidate for defining HFF and there is infinite number of gauge conditions.
2. An automorphism is indeed in question so that the algebraic system would not be actually affected. Therefore one cannot say that HFF has inherent time evolution and time. However, one can represent in HFF dynamical systems obeying this inherent time evolution. This possibility is highly interesting as a kind of universal gauge theory.

   On the other hand, outer automorphisms affect the trace of the projector defining the identity matrix for a given factor. Does the scaling factor Λ represent some kind of renormalization operation? Could it relate to the action of scalings in the TGD framework where scalings replace time translations at the fundamental level? What the number theoretic vision of TGD could mean? Could this quantize the continuous spectrum of the scalings Λ for HFFs so that they belong to the extension? Could one have a spectrum of Λ for each extension of rationals? Are different extensions related by inclusions of HFFs?
3. The notion of time evolution itself is an essentially Newtonian concept: selecting a preferred time coordinate breaks Lorentz invariance. In TGD however time coordinate is replace by scaling parameter and the situation changes.
4. The proposal of Connes is not general enough if evolution is interpreted as an increase of complexity.

For these reasons I gave up the automorphism proposed by Connes as a candidate for defining time evolution giving rise to scattering amplitudes in TGD framework.

Two views about TGD

The two dual views about what TGD is described briefly in [L110].

1. Physics as geometry of the world of "world of classical worlds" (WCW) identified as the space of space-time surfaces in $M^4 \times CP_2$ [K95]. Twistor lift of TGD [K98] implies that the space-time surfaces are minimal surfaces which can be also regarded as extermals of the Kähler action. This implies holography required by the general coordinate invariance in TGD framework.
2. TGD as generalized number theory forcing to generalize physics to adelic physics [L49] fusing real physics as correlate of sensory experience and various p-adic physics as correlates of cognition. Now space-times are naturally co-associative surfaces in complexified M^8 (complexified octonions) defined as "roots" of octonionic polynomials determined by polynomials with rational coefficients [L100, L101, L117]. Now holography extends dramatically: finite number of rational numbers/roots of rational polynomial/points of space-time region dictate it.

$M^8 - H$ duality relates these two views and is actually a generalization of Fourier transform and realizes generalization of momentum-position duality.

The notion of time evolution in TGD

Concerning various time evolutions in TGD, the general situation is now rather well understood.

There are two quantal time evolutions: geometric one assignable to single CD and and subjective time evolution which reflects the generalization of point-like particle to a 3-surface and the introduction of CD as 4-D perceptive field of particle in ZEO [L91].

1. Geometric time evolution corresponds to the standard scattering amplitudes for which I have a general formula now in terms of zero energy ontology (ZEO) [L112, L100, L101, L117]. The analog of S-matrix corresponds to entanglement coefficients between members of zero energy state at opposite boundaries of causal diamond (CD).
2. Subjective time evolution of conscious entity corresponds to a sequence of "small" state function reductions (SSFRs) as moments of consciousness: each SSFR is preceded by an analog of unitary time evolution, call it U. SSFRs are the TGD counterparts of "weak" measurements. $U(t)$ is generated by the scaling generator L_0 scaling light-like radial coordinate of light-cone boundary and is a generalization of corresponding operator in superconformal and string theories and defined for super-symplectic algebras acting as isometries of the world of classical worlds (WCW) [L117]. $U(t)$ is not the exponential of energy as a generator of time translation as in QFTs but an exponential of the mass squared operator and corresponds to the scaling of radial light-like coordinate r of the light-like boundary of CD: r is analogous to the complex coordinate z in conformal field theories.

   Also "big" SFRs (BSFRs) are possible and correspond to "ordinary" SFRs and in TGD framework mean death of self in the universal sense and followed by reincarnation as time reversed subjective time evolution [L77].
3. There is also classical time evolution at the level of space-time surfaces. Here the assumption that X^4 belongs to $H = M^4 \times CP_2$ defines Minkowski coordinates of M^4 as almost unique space-time coordinates of X^4 is the M^4 projection of X^4 is 4-D. This generalizes also to the case of M^8. Symmetries make it possible to identify an essentially a unique time coordinate.

   This means enormous simplification. General coordinate invariance is a marvellous symmetry but it leads to the problem of specifying space-time coordinates that is finding preferred coordinates. This seems impossible since 3-metric is dynamical. M^4 provides a fixed reference system and the problem disappears. M^4 is dynamical by its Minkowskian signature and one can speak about classical signals.
4. There is also classical time evolution for the induced spinor fields. At the level of H the spinor field is a superposition of modes of the massless Dirac operator (massless in 8-D sense). This spinor field is free and second quantized. Second quantization of induced spinor trivializes and this is absolutely crucial for obtaining scattering amplitudes for fermions and avoiding the usual problems for quantization of fermions in curved background.

The induced spinor field is a restriction of this spinor field to the space-time surface and satisfies modified Dirac equation automatically. There is no need for second quantization at the level of space-time surface and propagators etc.... are directly calculable. This is an enormous simplification.

There are therefore as many as 4 time evolutions and subjective time evolution by BSFRs and possibly also by SSFRs is a natural candidate for time evolution as genuine evolution as emergence of more complex algebraic structures.

Could the inherent time evolution of HFF have a physical meaning in TGD after all?

The idea about inherent time evolution defined by HFF itself as one parameter group of outer automorphisms is very attractive by its universality: physics would become part of mathematics.

1. Thermodynamic interpretation, with inverse temperature identified as an analog of time coordinate, comes first in mind but need not be the correct interpretation.
2. Outer automorphisms should act at a very fundamental level analogous to the state space of topological field theories. Fundamental group is after all in question! The assignment of the S-matrix of particle physics to the outer automorphism does not look reasonable since the time evolution would be with respect to the linear Minkowski coordinate, which is not Lorentz invariant.

For these reasons I gave up the idea of Connes when considering it for the first time. However, TGD inspired theory of consciousness as a generalization of quantum measurement theory has evolved since then and the situation is different now.

The sequence of SSFRs defines subjective time evolution having no counterpart in QFTs. Each SSFR is preceded by a unitary time evolution, which however corresponds to the scaling of the light-like radial coordinate of the light-cone boundary [L117] rather than time translation. Hamiltonian is replaced with the scaling generator L_0 acting as Lorentz invariant mass squared operator so that Lorentz invariance is not lost.

Could the time evolution assignable to L_0 correspond to the outer automorphism of Connes when one poses an infinite number of gauge conditions making inner automorphisms gauge transformations? The connection of Connes proposal with conformal field theories and with TGD is indeed suggestive.

1. Conformally invariant systems obey infinite number of gauge conditions stating that the conformal generators L_n, $n > 0$, annihilate physical states and carry vanishing Noether charges.

   These gauge conditions bring in mind the condition that infinitesimal inner automorphisms do not change the system physically. Does this mean that Connes outer automorphism generates the time evolution and inner automorphisms act as gauge symmetries? One would have an analog of gauge field theory in HFF.
2. In TGD framework one has an infinite hierarchy of systems satisfying conditions analogous to the conformal gauge conditions. The generators of the super-symplectic algebra (SCA) acting as isometries of the "world of classical worlds" (WCW) are labelled by non-negative conformal weight n and it has infinite hierarchy of algebras SCA$_k$ isomorphic to it with conformal weights given by k-multiple of those of the entire algebra, $k = 1, 2, ...$

   Gauge conditions state for SCA_k that the generators of SCA_k and its commutator with SCA annihilate physical states. The interpretation is in terms of a hierarchy of improving measurement resolutions with degrees of freedom below measurement resolution acting like gauge transformations.

   The Connes automorphism would "see" only the time evolution in the degrees of freedom above measurement resolution and as k increases, their number would increase.

In the case of hyperfinite factors of type II_1 (HFFs) the fundamental group of corresponding factor II_∞ consists of all reals: I hope I am right here.

1. The hyperfinite factors of type II_1 and corresponding factors II_∞ are natural in the TGD context. Therefore the spectrum would consist of reals unless one poses additional conditions.

2. Could the automorphisms correspond to the scalings of the lightcone proper time, which replace time translations as fundamental dynamics. Also in string models scalings take the role of time translations.
3. In zero energy ontology (ZEO) the scalings would act in the moduli space of causal diamonds which is finite-dimensional. This moduli space defines the backbone of the "world of classical worlds". WCW itself consists of a union of sub-WCs as bundle structures over CDs [L136]. The fiber consists of space-time surfaces inside a given CD analogous to Bohr orbits and satisfying holography reducing to generalized holomorphy. The scalings as automorphisms scale the causal diamonds. The space of CDs is a finite-dimensional coset space and has also other symmetry transformations.
4. The number theoretic vision suggests a quantization of the spectrum of Λ so that for a given extension of rationals the spectrum would belong to the extension. HFFs would be labelled at least partially by the extensions of rationals. The recent view of M^8-H duality [L138] is dramatically simpler than the earlier view [L100, L101, ?] and predicts that the space-time regions are determined by a pair of analytic functions with rational coefficients forced by number theoretical universality meaning that the space-time surfaces have interpretation also as p-adic surfaces.

   The simplest analytic functions are polynomials with integer coefficients and if one requires that the coefficients are smaller than the degree of the polynomial, the number of polynomials is finite for a given degree. This would give very precise meaning for the concept of number theoretic evolution.

   There would be an evolutionary hierarchy of pairs of polynomials characterized by increasing complexity and one can assign to these polynomials extension of rationals characterized by ramified primes depending on the polynomials. The ramified primes would have interpretation as p-adic primes characterizing the space-time region considered. Extensions of rationals and ramified primes could also characterize HFFs. This is a rather dramatic conjecture at the level of pure mathematics.
5. Scalings define renormalization group in standard physics. Now they scale the size of the CD. Could the scalings as automorphisms of HFFs correspond to discrete renormalization operations?

Three views about finite measurement resolution

Evolution could be seen physically as improving finite measurement resolution: this applies to both sensory experience and cognition. There are 3 views about finite measurement resolution (FMR) in TGD.

1. Hyper finite factors (HFFs) and FMR

HFFs are an essential part of Connes's work and I encountered them about 15 years ago or so [K125, K48].

The inclusions of hyper-finite factors HFFs provide one of the three - as it seems equivalent - ways to describe finite measurement resolution (FMR) in TGD framework: the included factor defines an analog for gauge degrees of freedom which correspond to those below measurement resolution.

2. Cognitive representations and FMR

Another description for FMR in the framework of adelic physics would be in terms of cognitive representations [L83]. First some background about M^8-H duality.

1. There are number theoretic and geometric views about dynamics. In algebraic dynamics at the level of M^8, the space-time surfaces are roots of polynomials. There are no partial differential equations like in the geometric dynamics at the level of H.
2. The algebraic "dynamics" of space-time surfaces in M^8 is dictated by co-associativity, which means that the normal space of the space-time surface is associative and thus quaternionic. That normal space rather than tangent space must be associative became clear last year [L100, L101].

3. M^8-H duality maps these algebraic surfaces in M^8 to $H=M^4\times CP_2$ and the one obtains the usual dynamics based on variational principle giving minimal surfaces which are non-linear analogs for the solutions of massless field equations. Instead of polynomials the natural functions at the level of H are periodic functions used in Fourier analysis [L117].

At level of complexified M^8 cognitive representation would consist of points of co-associative space-time surface X^4 in complexified M^8 (complexified octonions), whose coordinates belong to extension of rationals and therefore make sense also p-adically for extension of p-adic numbers induced by extension of rationals. M^8-H duality maps the cognitive representations to H.

Cognitive representations form a hierarchy: the larger the extension of rationals, the larger the number of points in the extension and in the unique discretization of space-time surface. Therefore also the measurement resolution improves.

The surprise was that the cognitive representations which are typically finite, are for the "roots" of octonionic polynomials infinite [L100, L101]. Also in this case the density of points of cognitive representation increases as the dimension of extensions increases.

The understanding of the physical interpretation of M^8-H duality increased dramatically during the last half year.

1. X^4 in M^8 is highly analogous to momentum space (4-D analog of Fermi ball one might say) and H to position space. Physical states correspond to discrete sets of points - 4-momenta - in X^4. This is just the description used in particle physics for physical states. Time and space in this description are replaced by energy and 4-momentum. At the level of H one space-time and classical fields and one talks about frequencies and wavelengths instead of momenta.
2. M^8-H duality is a generalization of Fourier transform. Hitherto I have assumed that the space-time surface in M^8 is mapped to H. The momentum space interpretation at the level of M^8 however requires that the image must be a superposition of translates of the image in plane wave with some momentum: only the translates inside some bigger CD are allowed - this means infrared cutoff.

   The total momentum as sum of momenta for two half-cones of CD in M^8 is indeed well-defined. One has a generalization of a plane wave over translational degrees of freedom of CD and restricted to a bigger CD.

   At the limit of infinitely large size for bigger CD, the result is non-vanishing only when the sum of the momenta for two half-cones of CD vanishes: this corresponds to conservation of 4-momentum as a consequence of Poincare invariance rather than assumption as in the earlier approach [L117].

   This generalizes the position-momentum duality of wave mechanics lost in quantum field theory. Point-like particle becomes a quantum superposition of space-time surfaces inside the causal diamond (CD). Plane wave is a plane wave for the superposition of space-time surfaces inside CD having the cm coordinates of CD as argument.

3. Inclusion hierarchy of supersymplectic algebras and FMR

The third inclusion hierarchy allowing to describe finite measurement resolution is defined by supersymplectic algebras acting as the isometries of the "world of classical worlds" (WCW) consisting of space-time surfaces are preferred extremals ("roots" of polynomials in M^8 and minimal surfaces satisfying infinite-D set of additional "gauge conditions" in H).

At a given level of hierarchy generators with conformal weight larger than n act like gauge generators as also their commutators with generators with conformal weight smaller than n correspond to vanishing Noether charges. This defines "gauge conditions".

To sum up, there are therefore 3 hierarchies allowing to describe finite measurement resolution and they must be essentially equivalent in TGD framework.

Three evolutionary hierarchies

There are three evolutionary hierarchies: hierarchies of extensions of extensions of... ofrationals...; inclusions of inclusions of .... of HFFs, and inclusions of isomorphic super symplectic algebras.

1. Extensions of rationals

The extensions of rationals become algebraically increasingly complex as their dimension increases. The co-associative space-time surfaces in M^8 are "roots" of real polynomials with rational coefficients to guarantee number theoretical universality and this means space-time surfaces are characterized by extension of rationals.

Each extension of rationals defines extensions for p-adic number fields and entire adele. The interpretation is as a cognitive leap: the system's intelligence/algebraic complexity increases when the extension is extended further.

The extensions of extensions of .... define hierarchies with Galois groups in certain sense products of extensions involved. Exceptional extensions are those which do not allow this decomposition. In this case Galois group is a simple group. Simple groups are primes of finite groups and correspond to elementary particles of cognition. Kind of fundamental, non-decomposable ideas. Mystic might speak of pure states of consciousnesswith no thoughts.

In the evolution by quantum jumps the dimension of extension increases in statistical sense and evolution is unavoidable. This evolution is due to subjective time evolution by quantum jumps, something which is in spirit with Connes proposal but replaces time evolution by a sequence of evolutionary leaps.

2. Inclusions of HFFs as a hierarchy

HFFs are fractals. They have infinite inclusion hierarchies in which sub-HFF isomorphicto entire HFFs is included to HFF.

Also the hierarchies of inclusions define evolutionary hierarchies: HFF which is isomorphic with original becomes larger and in some sense more complex than the included factor. Also now one has sequences of inclusions of inclusions of.... These sequences would correspond to sequences for extensions of extensions... of rationals. Note that the inclusion hierarchy would be the basic object: not only single HFF in the hierarchy.

3. Inclusions of supersymplectic algebras as an evolutionary hierarchy

The third hierarchy is defined by the fractal hierarchy of sub-algebras of supersymplectic algebra isomorphic to the algebra itself. At a given level of hierarchy generators with conformal weight larger than n correspond to gauge degrees of freedom. As n increases the number of physical degrees of freedom above measurement resolution increases which means evolution. This hierarchy should correspond rather concretely to that for the extensions of rationals. These hierarchies would be essentially one and the same thing in the TGD Universe.

TGD based model for subjective time development

The understanding of subjective time development as sequences of SSFRs preceded by unitary "time" evolution has improved quite considerably recently [L117]. The idea is that the subjective time development as a sequence of scalings at the light-cone boundary generated by the vibrational part $\hat{L}_0$ of the scaling generator $L_0 = p^2 - \hat{L}_0$ (L_0 annihilates the physical states). Also p-adic mass calculations use $\hat{L}_0$.

For more than 10 years ago [K76, K48], I considered the possibility that Connes time evolution operator that he assigned with thermo-dynamical time could have a significant role in the definition of S-matrix in standard sense but had to give up the idea.

It however seems that for super-symplectic algebra $\hat{L}_0$ generates an outer automorphism since the algebra has only generators with conformal with $n > 0$ and its extension to included also generators with $n \leq 0$ is required to introduce L_0: since L_0 contains annihilation operators, it indeed generates outer automorphism in SCA. The two views could be equivalent! Whereas Connes considered thermo-dynamical time evolution, in TGD framework the time evolution would be subjective time evolution by SSFRs.

1. The guess would be that the exponential of the scaling operator L_0 gives the time evolution. The problem is that L_0 annihilates the physical states. The solution of the problem would be the same as in p-adic thermodynamics. L_0 decomposes as $L_0 = p^2 - \hat{L}_0$ and the vibrational part $\hat{L}_0$ this gives mass spectrum as eigenvalues of p^2. The thermo-dynamical state in p-adic thermodynamics is $p^{\hat{L}_0\beta}$. This operator exists p-adically in the p-adic number field defined by prime p.

2. Could unitary subjective time development involve the operator $exp(i2\pi L_0\tau)$ $\tau = log(T/T_0)$? This requires $T/T_0 = exp(n/m)$ guaranteeing that exponential is a root of unity for an eigenstate of L_0. The scalings are discretized and scalings come as powers of $e^{1/m}$. This is possible using extensions of rationals generated by a root of e. The unique feature of p-adics is that e^p is ordinary p-adic number. This alone would give periodic time evolution for eigenstates of L_0 with integer eigenvalues n.

SSA and SSA_n

Supersymplectic algebra SSA has fractal hierarchies of subalgebras SSA_n. The integers in a given hierarchy are of forn $n_1, n_1n_2, n_1n_2n_3, ...$ and correspond naturally to hierarchies of inclusions of HFFs. Conformal weights are positive: $n > 0$. For ordinary conformal algebras also negative weights are allowed. Yangians have only non-negative weights. This is of utmost importance.

SSA_n with generators have radial light-like conformal weights coming as multiples of n. SSA_n annihilates physical states and $[SSA_n, SSA]$ does the same. Hence the generators with conformal weight larger than n annihilate the physical states.

What about generators with conformal weights smaller than n? At least a subset of them need not annihilate the physical states. Since L_n are superpositions of creation operators, the idea that analogs of coherent states could be in question.

It would be nice to have a situation in which L_n, $n < m$ commute. $[L_k, L_l] = 0$ effectively for $k + l \geq m$.

The simplest way to obtain a set of effectively commuting operators is to take the generators L_k, $[m/2] < k < m$, where $[m/2]$ is nearest integer larger than $m/2$.

This raises interesting questions.

1. Could the Virasoro generators $O(\{c_k\}) = \sum_{k\in[m/2],m]} c_kL_k$ as linear combinations of creation operators generate a set of coherent states as eigenstates of their Hermitian conjugates.
2. Some facts about coherent states are in order.
   (a) When one adds to quantum harmonic oscillator Hamiltonian oscillator a time dependent perturbation which lasts for a finite the vacuum state evolves to an oscillator vacuum whose position is displacemented. The displacement is complex and is a Fourier component of the external force $f(t)$ corresponding to the harmonic oscillator frequency ω. Time evolution picks up only this component.
   (b) Coherent state property means that the state is eigenstate of the annihilation creation operator with eivengeu $\alpha = -ig(\omega)$ where $g(omega) = \int f(u)exp(-i\omega u)du$ is Fourier transform of $f(t)$.
   (c) Coherent states are not orthogonal and form an overcomplete set. The overlaps of coherent states are proportional to a Gaussian depending on the complex parameters characterizing them. One can however develop any state in terms of coherent states as a unique expansion since one can represent unitary in terms of coherent states.
   (d) Coherent state obtained from the vacuum state by time evolution in presence of $f(t)$ by a unitary displacement operator $D(\alpha) = exp(\alpha a^\dagger - \overline{\alpha}a)$. (`https://en.wikipedia.org/wiki/Displacement_operator`).
   The displacement operator is a unitary operator and in the general case the displacement is complex. The product of two displacement operators would be apart from a phase factor a displacement operator associated with the sum of displacements.
   (e) Harmonic oscillator coherent states are indeed maximally classical since wave packets have minimal width in both q and p space. Furthermore, the classical expectation values for q and p obey classical equations of motion.

These observations raise interesting questions about how the evolution by SSFRs could be modelled.

1. Instead of harmonic oscillator in q-space, one would have time evolution in the space of scalings of causal diamond parameterized by the scaling parameter $\tau = log(T/T_0)$, where T can be identified as the radial light-like coordinate of light-cone boundary.
   The analogs of harmonic oscillator states would be defined in this space and would be essentially wave packets with ground state minimizing the width of the wave packet.

2. The role of harmonic oscillator Hamiltonian in absence of external force would be taken by the generator $\hat{L}_0$ ($L_0 = p^2 - \hat{L}_0$ acts trivially) and gives rise to mass squared quantization. The situation would be highly analogous to that in p-adic thermodynamics. The role of ω would be taken by the minimal conformal weight h_{min} such that the eigenvalues of L_0 are its multiples. It seems that this weight must be equal to $h_{min} = 1$.

   The commutations of $\hbar L_0$ with L_k, $k > 0$ would be as for L_0 so what the replacement should not affect the situation.

3. The scaling parameter τ is analogous to the spatial coordinate q for the harmonic oscillator. Can one identify the analog of the external force $f(t)$ acting during unitary evolution between two SSFRs? Or is it enough to use only the analog of $g(\omega \to h_{min} = 1)$ - that is the coefficients C_k.

   To identify $f(t)$, one needs a time coordinate t. This was already identified as τ. This one would have $q = t$, which looks strange. The space in which time evolution is the space of scalings and the time evolutions are scalings and thus time evolution means translation in this space. The analog for this would be Hamiltonian $H = i\hbar d/dq$.

   Number theoretical universality allows only the values of $\tau = r/s$ whose exponents give roots of unity. Also $exp(n\tau)$ makes sense p-adically for these values. This would mean that the Fourier transform defining g would become discrete and be sum over the values $f(\tau = r/s)$.

4. What happens if one replaces $\hat{L}_0$ with L_0. In this case one would have the replacement of ω with $h_{vac} = 0$. Also the analog of Fourier transform with zero frequency makes sense. $\hat{L_0} = p^2 - L_0$ is the most natural choice for the Hamiltonian defining the time evolution operator but is trivial. Could $\Delta^{i\tau}$ describe the inherent time evolution. It would be outer automorphism since it is not defined solely in terms of SCA. So: could one have $\Delta = exp(\hat{L_0})$ so that $\Delta^{i\tau}$ coincide with $exp(i\hat{L_0}\tau)$? This would mean the identification

$$\Delta = exp(\hat{L_0}) \ ,$$

   which is a positive definite operator. The exponents coming from $exp(iL_0\tau)$ can be number theoretically universal if $\tau = log(T/T_0)$ is a rational number implying $T/T_0 = exp(r/s)$, which is possible number theoretically) and the extension of rationals contains some roots of e.

5. Could one have $\Delta = L_0$? Also now that positivity condition would be satisfied if SSA conformal weights satisfy $n > 0$.

   The problem with this operation is that it is not number theoretically universal since the exponents $exp(ilog(n)\tau)$ do not exist p-adically without introducing infinite-D extension of p-adic number making $log(n)$ well-defined.

   What is however intriguing is that the "time" evolution operator $\Delta^{i\tau}$ in the eigenstate basis would have trace equal to $Tr(\Delta^{i\tau}) \sum d(n) n^{i\tau}$, where $d(n)$ is the degeneracy of the state. This is a typical zeta function: for Riemann Zeta one has $d(n) = 1$.

   For $\Delta = exp(L_0)$ option $Tr(\Delta^{i\tau}) = \sum d(n) exp(in\tau)$ exists for $\tau = r/s$ if r:th root of e belongs to the extension of p-adics.

To sum up, one would have Gaussian wave packet as harmonic oscillator vacuum in the space of scaled variants of CD. The unitary time evolution associated with SSFR would displace the peak of the wave packet to a larger scalings. The Gaussian wave function in the space of scaled CDs has been proposed earlier.

Could this time evolution make sense and be even realistic?

1. The analogs of harmonic oscillator states are defined in the space of scalings as Gaussians and states obtained from them using oscillator operators. There would be a wave function in the moduli space of CDs analogous to a state of harmonic oscillator.

2. SSFR following the time evolutions would project to an eigenstate of harmonic oscillator having in general displaced argument. The unitary displacement operator D should commute with the operators having the members of zero energy states at the passive boundary of CD as eigenstates. This poses strong conditions. At least number theoretic measurements could satisfy these conditions.

3. SSFRs are identified as weak measurements as near as possible to classical measurements. Time evolution by the displacement would be indeed highly analogous to classical time evolution for theeharmonic oscillator.
4. The unitary displacement operator corresponds to the arbitrary external force on the harmonic oscillator and it seems that it would be selected in SSFR for the unitary evolution after SSFR. This means fixing the coefficients C_k in the operator $\sum C_k L_k$.

What is the subjective "time" evolution operator when in the case of SSA_n?

1. The scaling analog of the unitary displacement operator D as $D = \sum exp(\sum C_k L_k - \overline{C}_k L_{-k})$ is highly suggestive and would take the oscillator vacuum to a coherent state. Coefficients C_k would be proportional to τ. There would be a large number of choices for the unitary displacement operator. One can also consider complex values of τ since one has complexified M^8.
2. There should be a normalization for the coefficients: without this it is not possible to talk about a special value of τ does not make sense. For instance, the sum of their moduli squared could be equal to 1. This would give interpretation as a quantum state in the degrees of freedom considered. The width of the Gaussian would increase slowly during the unitary time evolution and be proportional to $log(T/T_0)$.

   The width of the Gaussian would increase slowly as a function of T during the unitary time evolution and be proportional to $log(T/T_0)$. The condition that c_k are proportional the same complex number times τ is too strong.
3. The arbitrariness in the choice of C_k would bring in a kind of non-determinism as a selection of this superposition. The ability to engineer physical systems is in conflict with the determinism of classical physics and also difficult to understand in standard quantum physics. Could one interpret this choice as an analog for engineering a Hamiltonian as in say quantum computation or build-up of an electric circuit for some purpose? Could goal directed action correspond to this choice?

   If so engineerable degrees of freedom would correspond to intermediate degrees of freedom associated with L_k, $[m/2] \leq k \leq m$. They would be totally absent for $k = 1$ and this would correspond to a situation analogous to the standard physics without any intentional action.

3.5 MIP*= RE: What could this mean physically?

I received a very interesting link to a popular article (`https://cutt.ly/sfd5UQF`) explaining a recently discovered deep result in mathematics having implications also in physics. The article [A70] (`https://cutt.ly/rffiYdc`) by Zhengfeng Ji, Anand Natarajan, Thomas Vidick, John Wright, and Henry Yuen has a rather concise title "**MIP*=RE**". In the following I try to express the impressions of a (non-mainstream) physicist about the result.

The following is the result expressed using the concepts of computer science about which I know very little at the hard technical level. The results are however told to state something highly non-trivial about physics.

1. RE (recursively enumerable languages) denotes all problems solvable by computer. P denotes the problems solvable in a polynomial time. NP does not refer to a non-polynomial time but to "non-deterministic polynomial acceptable problems" - I hope this helps the reader- I am a little bit confused! It is not known whether P = NP is true.
2. IP problems (P is now for "prover" that can be solved by a collaboration of an interrogator and prover who tries to convince the interrogator that her proof is convincing with high enough probability. MIP involves multiple l provers treated as criminals trying to prove that they are innocent and being not allowed to communicate. MIP* is the class of solvable problems in which the provers are allowed to entangle.

The finding, which is characterized as shocking, is that *all* problems solvable by a Turing computer belong to this class: MIP*=RE. All problems solvable by computer would reduce to problems in the class MIP*! Quantum computation would indeed add something genuinely new to the classical computation.

Quantum entanglement would play an essential role in quantum computation. Also the implications for physics are highly non-trivial.

1. Connes embedding problem asking whether all infinite-D matrices can always be approximated by finite-D matrices has a negative solution.Therefore MIP*= RE does not hold true for hyperfinite factors of type II_1 (HFFs) central in quantum TGD. Also the Tirelson problem finds a solution. The measurements of commuting observers performed by two observers are equivalent to the measurements of tensor products of observables only in finite-D case and for HFFs. That quantum entanglement would not have any role in HFFs is in conflict with intuition.

2. In the TGD framework finite measurement resolution is realized in terms of HFFs at Hilbert space level and in terms of cognitive representations at space-time level defined purely number-theoretically. This leads to a hierarchy of adeles defined by extensions of rationals and the Hilbert spaces must have algebraic extensions of rationals as a coefficient field. Therefore one cannot in general case find a unitary transformation mapping the entangled situation to an unentangled one, and quantum entanglement plays a key role. It seems that computationalism formulated in terms of recursive functions of natural numbers must be formulated for the hierarchy of extensions of rationals in terms of algebraic integers.
3. In TGD inspired theory of consciousness entanglement between observers could be seen as a kind of telepathy helping to perform conscious quantum computations. Zero energy ontology also suggests a modification of the views about quantum computation. TGD can be formulated also for real and p-adic continua identified as correlates of sensory experience and cognition, and it seems that computational paradigm need not apply in the continuum cases.

3.5.1 Two physically interesting applications

There are two physically interesting applications of the theorem interesting also from the TGD point of view and force to make explicit the assumptions involved.

About the quantum physical interpretation of MP*

To proceed one must clarify the quantum physical interpretation of MIP*.

Quantum measurement requires entanglement of the observer O with the measured system M. What is basically measured is the density matrix of M (or equivalently that of O). State function reduction gives as an outcome a state, which corresponds to an eigenvalue of the density matrix. Note that this state can be an entangled state if the density matrix has degenerate eigenvalues. Quantum measurement can be regarded as a question to the measured system: "What are the values of given commuting observables?". The final state of quantum measurement provides an eigenstate of the observables as the answer to this question. M would be in the role of the prover and O_i would serve as interrogators.

In the first case multiple interrogators measurements would entangle M with unentangled states of the tensor product $H_1 \otimes H_2$ for O followed by a state function reduction splitting the state of M to un-entangled state in the tensor product $M_1 \otimes M_2$.

In the second case the entire M would be interrogated using entanglement of M with entangled states of $H_1 \otimes H_2$ using measurements of several commuting observables. The theorem would state that interrogation in this manner is more efficient in infinite-D case unless HFFs are involved.

3. This interpretation differs from the interpretation in terms of computational problem solving in which one would have several provers and one interrogator. Could these interpretations be dual as the complete symmetry of the quantum measurement with respect to O and M suggests? In the case of multiple provers (analogous to accused criminals) it is advantageous to isolate them. In the case of multiple interrogators the best result is obtained if the interrogator does not effectively split itself into several ones.

Connes embedding problem and the notion of finite measurement/cognitive resolution

Alain Connes formulated what has become known as Connes embedding problem. The question is whether infinite matrices forming factor of type II_1 can be *always* approximated by finite-D matrices that is imbedded in a *hyperfinite* factor of type II_1 (HFF). Factors of type II and their HFFs are special classes of von Neumann algebras possibly relevant for quantum theory.

This result means that if one has measured of a complete set of for a product of commuting observables acting in the full space, one can find in the finite-dimensional case a unitary transformation transforming the observables to tensor products of observables associated with the factors of a tensor product. In the infinite-D case this is not true.

What seems to put alarms ringing is that in TGD there are excellent arguments suggesting that the state space has HFFs as building bricks. Does the result mean that entanglement cannot help in quantum computation in TGD Universe? I do not want to live in this kind of Universe!

Tsirelson problem

Tsirelson problem (see this) is another problem mentioned in the popular article as a physically interesting application. The problem relates to the mathematical description of quantum measurement.

Three systems are considered. There are two systems O_1 and O_2 representing observers and the third representing the measured system M. The measurement reducing the entanglement between M and O_1 or O_2 can regarded as producing correspondence between state of M and O_1 or O_2, and one can think that O_1 or O_2 measures only obserservables in its own state space as a kind of image of M. There are two ways to see the situation. The provers correspond now to the observers and the two situations correspond to provers without and with entanglement.

Consider first a situation in which one has single Hilbert space H and single observer O. This situation is analogous to IP.

1. The state of the system is described statistically by a density matrix - not necessarily pure state -, whose diagonal elements have interpretation as reduction probabilities of states in this bases. The measurement situation fixes the state basis used. Assume an ensemble of identical copies of the system in this state. Assume that one has a complete set of commuting observables.
2. By measuring all observables for the members of the ensemble one obtains the probabilities as diagonal elements of the density matrix. If the observable is the density matrix having no- degenerate eigenvalues, the situation is simplified dramatically. It is enough to use the density matrix as an observable. TGD based quantum measurement theory assumes that the density matrix describing the entanglement between two subsystems is in a universal observable measure in state function reductions reducing their entanglement.
3. Can one deduce also the state of M as a superposition of states in the basic chosen by the observer? This basis need not be the same as the basis defined by - say density matrix if the system has interacted with some system and this ineracton has led to an eigenstate of the density matrix. Assume that one can prepare the latter basis by a physical process such as this kind of interaction.

   The coefficients of the state form a set of N^2 complex numbers defining a unitary $N \times N$ matrix. Unitarity conditions give N conditions telling that the complex rows and unit vectors: these numbers are given by the measurement of all observables. There are also $N(N-1)$ conditions telling that the rows are orthogonal. Together these $N + N(N-1) = N^2$ numbers fix the elements of the unitary matrix and therefore the complex coefficients of the state basis of the system can be deduced from a complete set of measurements for all elements of the basis.

Consider now the analog of the MIS* involving more than one observer. For simplicity consider two observers.

1. Assume that the state space H of M decomposes to a tensor product $H = H_1 \otimes H_2$ of state spaces H_1 and H_2 such that O_1 measures observables X_1 in H_1 and O_2 measuresobservables X_2 in H_2. The observables X_1 and X_2 commute since they act in different tensor factors.

The absence of interaction between the factors corresponds to the inability of the provers to communicate. As in the previous case, one can deduce the probabilities for the various outcomes of the joint measurements interpreted as measurements of a complete set of observables $X_1 \otimes X_2$.

2. One can also think that the two systems form a single system O so that O_1 and O_2 can entangle. This corresponds to a situation in which entanglement between the provers is allowed. Now X_1 and X_2 are not in general independent but also now they must commute. One can deduce the probabilities for various outcomes as eigenstates of observables X_1X_2 and deduce the diagonal elements of the density matrix as probabilities.

Are these ways to see the situation equivalent? Tsirelson demonstrated that this is the case for finite-dimensional Hilbert spaces, which can indeed be decomposed to a tensor product of factors associated with O_1 and O_2. This means that one finds a unitary transformation transforming the entangled situation to an unentangled one and to tensor product observables.

For the infinite-dimensional case the situation remained open. According to the article, the new result implies that this is not the case. For hyperfinite factors the situation can be approximated with a finite-D Hilbert space so that the situations are equivalent in arbitrary precise approximation.

3.5.2 The connection with TGD

The result looks at first a bad news from the TGD point of view, where HFFs are highly suggestive. One must be however very careful with the basic definitions.

Measurement resolution

Measurement resolution is the basic notion.

1. There are intuitive physicist's arguments demonstrating that in TGD the operator algebras involved with TGD are HFFs provides a description of finite measurement resolution. The inclusion of HFFs defines the notion of resolution: included factor represents the degrees of freedom not seen in the resolution used [K125, K48] (`http://tgdtheoryd.fi/pfpool/vNeumann.pdf`) and `http://tgdtheoryd.fi/pfpool/vNeumannnew.pdf`).

   Hyperfinite factors involve new structures like quantum groups and quantum algebras reflecting the presence of additional symmetries: actually the "world of classical worlds" (WCW) as the space of space-time surfaces as maximal group of isometries and this group has a fractal hierarchy of isomorphic groups imply inclusion hierarchies of HFFs. By the analogs of gauge conditions this infinite-D group reduces to a hierarchy of effectively finite-D groups. For quantum groups the infinite number of irreps of the corresponding compact group effectively reduces to a finite number of them, which conforms with the notion of hyper-finiteness.

   It looks that the reduction of the most general quantum theory to TGD-like theory relying on HFFs is not possible. This would not be surprising taking into account gigantic symmetries responsible for the cancellation of infinities in TGD framework and the very existence of WCW geometry.

2. Second TGD based approach to finite resolution is purely number theoretic [L50] and involves adelic physics as a fusion of the real physics with various p-adic physics as correlates of cognition. Cognitive representations are purely number theoretic and unique discretizations of space-time surfaces defined by a given extension of rationals forming an evolutionary hierarchy: the coordinates for the points of space-time as a 4-surface of the embedding space $H = M^4 \times CP_2$ or of its dual M^8 are in the extension of rationals defining the adele. In the case of M^8 the preferred coordinates are unique apart from time translation. These two views would define descriptions of the finite resolution at the level of space-time and Hilbert space. In particular, the hierarchies of extensions of rationals should define hierarchies of inclusions of HFFs.

For hyperfinite factors the analog of MIP*=RE cannot hold true. Doesn't the TGD Universe allow a solution of all the problems solvable by Turing Computer? There is a loophole in this argument.

1. The point is that for the hierarchy of extensions of rationals also Hilbert spaces have as a coefficient field the extension of rationals! Unitary transformations are restricted to matrices with elements in the extension. In general it is not possible to realize the unitary transformation mapping the entangled situation to an un-entangled one! The weakening of the theorem would hold true for the hierarchy of adeles and entanglement would give something genuinely new for quantum computation!
2. A second deep implication is that the density matrix characterizing the entanglement between two systems cannot in general be diagonalized such that all diagonal elements identifiable as probabilities would be in the extension considered. One would have stable or partially stable entanglement (could the projection make sense for the states or subspaces with entanglement probability in the extension). For these bound states the binding mechanism is purely number theoretical. For a given extension of p-adic numbers one can assign to algebraic entanglement also information measure as a generalization of Shannon entropy as a p-adic entanglement entropy (real valued). This entropy can be negative and the possible interpretation is that the entanglement carries conscious information.

What about transcendental extensions?

During the writing of this article an interesting question popped up.

1. Also transcendental extensions of rationals are possible, and one can consider the generalization of the computationalism by also allowing functions in transcendental extensions. Could the hierarchy of algebraic extensions could continue with transcendental extensions? Could one even play with the idea that the discovery of transcendentals meant a quantum leap leading to an extension involving for instance e and π as basic transcendentals? Could one generalize the notion of polynomial root to a root of a function allowing Taylor expansion $f(x) = \sum q_n x^n$ with rational coefficients so that the roots of $f(x) = 0$ could be used define transcendental extensions of rationals?

2. Powers of e or its root define and infinite-D extensions having the special property that they are finite-D for p-adic number fields because e^p is ordinary p-adic number. In the p-adic context e can be defined as a root of the equation $x^p - \sum p^n/n! = 0$ making sense also for rationals. The numbers $log(p_i)$ such that p_i appears a factor for integers smaller than p define infinite-D extension of both rationals and p-adic numbers. They are obtained as roots of $e^x - p_i = 0$.
3. The numbers $(2n+1)\pi$ $(2n\pi)$ can be defined as roots of $sin(x) = 0$ $(cos(x) = 0$. The extension by π is infinite-dimensional and the conditions defining it would serve as consistency conditions when the extension contains roots of unity and effectively replaces them.
4. What about other transcendentals appearing in mathematical physics? The values $\zeta(n)$ of Riemann Zeta appearing in scattering amplitudes are for even values of n given by $\zeta(2n) = (-1)^{n+1}B_{2n}(2\pi)^{2n}/2(2n+1)!$. This follows from the functional identity for Riemann zeta and from the expression $\zeta(-n) = (-1)^n B_{n+1}/(n+1)$ ($(B(-1/2) = -1/2)$ (https://cutt.ly/dfgtgmw). The Bernoulli numbers B_n are rational and vanish for odd values of n. An open question is whether also the odd values are proportional to π^n with a rational coefficient or whether they represent "new" transcendentals.

What about the situation for the continuum version of TGD?

At least the cognitively finitely representable physics would have the HFF property with coefficient field of Hilbert spaces replaced by an extension of rationals. Number theoretical universality would suggest that HFF property characterizes also the physics of continuum TGD. This leads to a series of questions.

1. Does the new theorem imply that in the continuum version of TGD all quantum computations allowed by the Turing paradigm for real coefficients field for quantum states are not possible: $MIP* \subset RE$? The hierarchy of extensions of rationals allows utilization of entanglement, and one can even wonder whether one could have $MIP* = RE$ at the limit of algebraic numbers.

2. Could the number theoretic vision force change also the view about quantum computation? What does RE actually mean in this framework? Can one really assume complex entanglement coefficients in computation. Does the computational paradigm makes sense at all in the continuum picture?

   Are both real and p-adic continuum theories unreachable by computation giving rise to cognitive representations in the algebraic intersubsection of the sensory and cognitive worlds? I have indeed identified real continuum physics as a correlate for sensory experience and various p-adic physics as correlates of cognition in TGD: They would represent the computionally unreachable parts of existence.

   Continuum physics involves transcendentals and in mathematics this brings in analytic formulas and partial differential equations. At least at the level of mathematical consciousness the emergence of the notion of continuum means a gigantic step. Also this suggests that transcendentality is something very real and that computation cannot catch all of it.

3. Adelic theorem allows to express the norm of a rational number as a product of inverses of its p-adic norms. Very probably this representation holds true also for the analogs of rationals formed from algebraic integeres. Reals can be approximated by rationals. Could extensions of all p-adic numbers fields restricted to the extension of rationals say about real physics only what can be expressed using language?

Also fermions are highly interesting in the recent context. In TGD spinor structure can be seen as a square root of Kähler geometry, in particular for the "world of classical worlds" (WCW). Fermions are identified as correlates of Boolean cognition. The continuum case for fermions does not follow as a naïve limit of algebraic picture.

1. The quantization of the induced spinors in TGD looks different in discrete and continuum cases. Discrete case is very simple since equal-time anticommutators give discrete Kronecker deltas. In the continuum case one has delta functions possibly causing infinite vacuum energy like divergences in conserved Noether charges (Dirac sea).

2. In [L106] (https://cutt.ly/zfftoK6) I have proposed how these problems could be avoided by avoiding anticommutators giving delta-function. The proposed solution is based on zero energy ontology and TGD based view about space-time. One also obtains a long-sought-for concrete realization for the idea that second quantized induce spinor fields are obtained as restrictions of second quantized free spinor fields in $H = M^4 \times CP_2$ to space-time surface. The fermionic variant of $M^8 - H$-duality [L107] provides further insights and gives a very concrete picture about the dynamics of fermions in TGD.

These considerations relate in an interesting manner to consciousness. Quantum entanglement makes in the TGD framework possible telepathic sharing of mental images represented by sub-selves of self. For the series of discretizations of physics by HFFs and cognitive representations associated with extensions of rationals, the result indeed means something new.

What does one mean with quantum computation in TGD Universe?

The TGD approach raises some questions about computation.

1. The ordinary computational paradigm is formulated for Turing machines manipulating natural numbers by recursive algorithms. Programs would essentially represent a recursive function $n \rightarrow f(n)$. What happens to this paradigm when extensions of rationals define cognitive representations as unique space-time discretizations with algebraic numbers as the limit giving rise to a dense in the set of reals.

   The usual picture would be that since reals can be approximated by rationals, the situation is not changed. TGD however suggests that one should replace at least the quantum version of the Turing paradigm by considering functions mapping algebraic integers (algebraic rational) to algebraic integers.

   Quite concretely, one can manipulate algebraic numbers without approximation as a rational and only at the end perform this approximation and computations would construct recursive functions in this manner. This would raise entanglement to an active role even if one has

HFFs and even if classical computations could still look very much like ordinary computation using integers.

This suggests that computationalism usually formulated in terms of recursive functions of natural or rational numbers could be replaced with a hierarchy of computationalisms for the hierarchy of extensions of rationals. One would have recursively definable functions defined and having values in the extensions of rationals. These functions would be analogs of analytic functions (or polynomials) with the complex variable replaced with an integer or a rational of the extension. This poses very powerful constraints and there are good reasons to expect an increase of computational effectiveness. One can hope that at the limit of algebraic numbers of these functions allow arbitrarily precise approximations to real functions. If the real world phenomena can be indeed approximated by cognitive representations in the TGD sense, one can imagine a highly interesting approach to AI.

2. ZEO brings in also time reversal occurring in "big" (ordinary) quantum jumps and this modifies the views about quantum computation. In ZEO based conscious quantum computation halting means "death" and "reincarnation" of conscious entity, self? How the processes involving series of haltings in this sense differs from ordinary quantum computation: could one shorten the computation time by going forth and back in time.

There are many interesting questions to be considered. $M^8 - H$ duality gives justifications for the vision about algebraic physics. TGD leads also to the notion of infinite prime and I have considered the possibility that infinite primes could give a precise meaning for the dimension of infinite-D Hilbert space. Could the number-theoretic view about infinite be considerably richer than the idea about infinity as limit would suggest [K105].

The construction of infinite primes is analogous to a repeated second quantization of arithmetic supersymmetric quantum field theory allowing also bound states at each level and a concrete correspondence with the hierarchy of space-time sheets is suggestive. For the infinite primes at the lowest level of the hierarchy single particle states correspond to rationals and bound states to polynomials and therefore to the sets of their roots. This strongly suggests a connection with M^8 picture.

Could the number field of computable reals (p-adics) be enough for physics?

For some reason I have managed to not encounter the notion of computable number (see `https://cutt.ly/pTeSSfR`) as opposed to that of non-computable number (see `https://cutt.ly/gTeD9vF`). The reason is perhaps that I have been too lazy to take computationalism seriously enough.

Computable real number is a number, which can be produced to an arbitrary accuracy by a Turing computer, which by definition has a finite number of internal states, has input which is natural number and produces output which is natural numbers. Turing computer computes values of a function from natural numbers to itself by applying a recursive algorithm.

The following three formal definitions of the notion are equivalent.

1. The real number a is computable, if it can be expressed in terms of a computable function $n \rightarrow f(n)$ from natural numbers to natural numbers characterized by the property

$$\frac{f(n)-1}{n} \leq a \leq \ (\frac{f(n)+1}{n} .$$

For rational $a = q$, $f(n) = nq$ satisfies the conditions. Note that this definition does not work for p-adic numbers since they are not well-ordered.

2. The number a is computable if for an arbitrarily small rational number ϵ there exists a computable function producing a rational number r satisfying $|r - x \leq \epsilon$. This definition works also for p-adic numbers since it involves only the p-adic norm which has values which are powers of p and is therefore real valued.

3. a is computable if there exists a computable sequence of rational numbers r_i converging to a such that $|a - r_i| \leq 2^{-i}$ holds true. This definition works also for 2-adic numbers and its variant obtained by replacing 2 with the p-adic prime p makes sense for p-adic numbers.

The set R_c of computable real numbers and the p-adic counterparts $Q_{p,c}$ of R_c, have highly interesting properties.

1. R_c is enumerable and therefore can be mapped to a subset of rationals: even the ordering can be preserved. Also $Q_{p,c}$ is enumerable but now one cannot speak of ordering. As a consequence, most real (p-adic) numbers are non-computable. Note that the pinary expansion of a rational is periodic after some pinary digit. For a p-adic transcendental this is not the case.
2. Algebraic numbers are computable so that one can regard R_c as a kind of completion of algebraic numbers obtained by adding computable reals. For instance, π and e are computable. 2π can be computed by replacing the unit circle with a regular polygon with n sides and estimating the length as nL_n. L_n the length of the side. e can be computed from the standard formula. Interestingly, e^p is an ordinary p-adic number. An interesting question is whether there are other similar numbers. Certainly many algebraic numbers correspond to ordinary p-adic numbers.
3. R_c ($Q_{p,c}$) is a number field since the arithmetic binary operations $+, -\times, /$ are computable. Also differential and integral calculus can be constructed. The calculation of a derivative as a limit can be carried out by restricting the consideration to computable reals and there is always a computable real between two computable reals. Also Riemann sum can be evaluated as a limit involving only computable reals.
4. An interesting distinction between real and p-adic numbers is that in the sum of real numbers the sum of arbitrarily high digits can affect even all lower digits so that it requires computational work to predict the outcome. For p-adic numbers memory digits affect only the higher digits. This is why p-adic numbers are tailor made for computational purposes.Canonical identification $\sum x_n p^n \rightarrow \sum x_n p^{-n}$ used in p-adic mass calculations to map p-adic mass squared to its real counterpart [K66] maps p-adics to reals in a continuous manner. For integers this corresponds is 2-to-1 due to the fact that the p-adic numbers $-1 = (p-1)/(1-p)$ and $1/p$ are mapped to p.
5. For computable numbers, one cannot define the relation =. One can only define equality in some resolution ϵ. The category theoretical view about equality is also effective and conforms with the physical view.

   Also the relations $\leq$ and $\geq$ fail to have computable counterparts since only the absolute value $|x-y|$ can appear in the definition and one loses the information about the well-ordered nature of reals. For p-adic numbers there is no well-ordering so that nothing is lost. A restriction to non-equal pairs however makes order relation computable. For p-adic numbers the same is true.
6. Computable number is obviously definable but there are also definanable numbers, which are not computable. Examples are Gödel numbers in a given coding scheme for statements, which are true but not provable. More generally, the Gödel numbers coding for undecidable problems such as the halting problem are uncomputable natural numbers in a given coding scheme. Chaitin's constant, which gives the probability that random Turing computation halts, is a non-computable but definable real number.
7. Computable numbers are arithmetic numbers, which are numbers definable in terms of first order logic using Peano's axioms. First order logic does not allow statements about statements and one has an entire hierarchy of statements about... about statements. The hierarchy of infinite primes defines an analogous hierarchy in the TGD framework and is formally similar to a hierarchy of second quantizations [K105].

3.6 Analogs Of Quantum Matrix Groups From Finite Measurement Resolution?

The notion of quantum group [?]eplaces ordinary matrices with matrices with non-commutative elements. This notion is physically very interesting, and in TGD framework I have proposed that it should relate to the inclusions of von Neumann algebras allowing to describe mathematically the notion of finite measurement resolution [?] These ideas have developed slowly through various side tracks.

In the sequel I will consider the notion of quantum matrix inspired by the recent view about quantum TGD relying on the notion of finite measurement resolution and show that under some additional conditions it provides a concrete representation and physical interpretation of quantum groups in terms of finite measurement resolution.

1. The basic idea is to replace complex matrix elements with operators, which are products of non-negative hermitian operators and unitary operators analogous to the products of modulus and phase as a representation for complex numbers. Modulus and phase would be non-commuting and have commutation relation analogous to that between momentum and plane-wave in accordance with the idea about quantization of complex numbers.
2. The condition that determinant and sub-determinants exist is crucial for the well-definedness of eigenvalue problem in the generalized sense. Strong/weak permutation symmetry of determinant requires its invariance apart from sign change under permutations of rows and/or columns. Weak permutation symmetry means development of determinant with respect to a fixed row or column and does not pose additional conditions. For weak permutation symmetry the permutation of rows/columns would however have a natural interpretation as braiding for the hermitian operators defined by the moduli of operator valued matrix elements and here quantum group structure emerges.
3. The commutativity of all sub-determinants is essential for the replacement of eigenvalues with eigenvalue spectra of hermitian operators and sub-determinants define mutually commuting set of operators.

Quantum matrices define a more general structure than quantum group but provide a concrete representation for them in terms of finite measurement resolution, in particular when q is a root of unity. For $q = \pm 1$ (Bose-Einstein or Fermi-Dirac statistics) one obtains quantum matrices for which the determinant is apart from possible change by a sign factor invariant under the permutations of both rows and columns. One can also understand the recursive fractal structure of inclusion sequences of hyper-finite factors resulting by replacing operators appearing as matrix elements with quantum matrices and a concrete connection with quantum groups emerges.

In Zero Energy Ontology (ZEO) M-matrix serving as the basic building brick of unitary U-matrix and identified as a hermitian square root of density matrix provides a possible application for this vision. Especially fascinating is the possibility of hierarchies of measurement resolutions represented as inclusion sequences realized as recursive construction of M-matrices. Quantization would emerge already at the level of complex numbers appearing as M-matrix elements.

This approach might allow to unify various ideas behind TGD. For instance, Yangian algebras emerging naturally in twistor approach are examples of quantum algebras. The hierarchy of Planck constants should have close relationship with inclusions and fractal hierarchy of sub-algebras of super-symplectic and other conformal algebras.

3.6.1 Well-definedness Of The Eigenvalue Problem As A Constraint To Quantum Matrices

Intuition suggests that the presence of degrees of freedom below measurement resolution implies that one must use density matrix description obtained by taking trace over the unobserved degrees of freedom. One could argue that in state function reduction with finite measurement resolution the outcome is not a pure state, or not even negentropically entangled state (possible in TGD framework) but a state described by a density matrix. The challenge is to describe the situation mathematically in an elegant manner.

1. There is present an infinite number of degrees of freedom below measurement resolution with which measured degrees of freedom entangle so that their presence affects the situation. One has a system with finite number degrees of freedom such as two-state system described by a quantum spinor. In this case observables as hermitian operators described by 2×2 matrices would be replaced by quantum matrices with elements, which in general do not commute.

   An attractive generalization of complex numbers appearing as elements of matrices is obtained by replacing them with products $H_{ij} = h_{ij} u_{ij}$ of hermitian operators h_{ij} with non-negative spectrum (modulus of complex number) and unitary operators u_{ij} (phase of complex number) suggests itself. The commutativity of h_{ij} and u_{ij} would look nice but is not necessary and is in conflict with the idea that modulus and phase of an amplitudes do not commute in quantum mechanics.

Very probably this generalization is trivial for mathematician. One could indeed interpret the generalization in terms of a tensor product of finite-dimensional matrices with possibly infinite-dimensional space of operators of Hilbert space. For the physicist the situation might be different as the following proposal for what hermitian quantum matrices could be suggests.

2. The modulus of complex number is replaced with a hermitian operator having non-negative eigenvalues. The representation as $h = AA^\dagger + A^\dagger A$ is would guarantee this. The phase of complex number would be replaced by a unitary operator U possibly allowing the representation $U = exp(iT)$, T hermitian. The commutativity condition

$$[h_{ij}, u_{ij}] = 0 \quad (3.6.1)$$

for a given matrix element is also suggestive but as already noticed, Uncertainty Principle suggests that modulus and phase do not commute as operators. The commutator of modulus and phase would naturally be equal to that between momentum operator and plane wave:

$$[h_{ij}, u_{ij}] = i\hbar \times u_{ij} \ , \quad (3.6.2)$$

Here $\hbar = h/2\pi$ can be chosen to be unity in standard quantum theory. In TGD it can be generalized to a hermitian operator H_{eff}/h with an integer valued spectrum of eigenvalues given by $h_{eff}/h = n$ so that ordinary and dark matter sectors would be unified to single structure mathematically.

3. The notions of eigenvalues and eigenvectors for a hermitian operator should generalize. Now hermitian operator H would be a matrix with formally the same structure as $N \times N$ hermitian matrix in commutative number field - say complex numbers - possibly satisfying additional conditions.

Hermitian matrix can be written as

$$H_{ij} = h_{ij}u_{ij} \quad \text{for i>j} \quad H_{ij} = u_{ij}h_{ij} \quad \text{for i<j} \ , \quad H_{ii} = h_i \ . \quad (3.6.3)$$

Hermiticity conditions $H_{ij} = H^\dagger_{ji}$ give

$$h_{ij} = h_{ji} \ , \quad u_{ij} = u^\dagger_{ji} \ . \quad (3.6.4)$$

Here it has been assumed that one has quantum SU(2). For quantum U(2) one would have $U_{11} = U^\dagger_{22} = h_a u_a$ with u_a commuting with other operators. The form of the conditions is same as for ordinary hermitian matrices and it is not necessary to assume commutativity $[h_{ij}, u_{ij}] = 0$. Generalization of Pauli spin matrices provides a simple illustration.

4. The well-definedness of eigenvalue problem gives a strong constraint on the notion of hermitian quantum matrix. Eigenvalues of hermitian operator are determined by the vanishing of determinant $det(H - \lambda I)$. Its expression involves sub-determinants and one must decide whether to demand that the definition of determinant is independent of which column or row one chooses to develop the determinant.

For ordinary matrix the determinant is expressible as sum of symmetric functions:

$$det(H - \lambda I) = \sum \lambda^n S_n(H) \ . \quad (3.6.5)$$

Elementary symmetric functions S_n - n-functions in following - have the property that they are sums of contributions from to n-element paths along the matrix with the property that path contains no vertical or horizontal steps. One has a discrete analog of path integral in which time increases in each step by unit. The analogy with fermionic path integral is also obvious. In the non-commutative case non-commutativity poses problems since different orderings of rows (or columns) along the same n-path give different results.

(a) For the first option one gives up the condition that determinant can be developed with respect to any row or column and defines determinant by developing it with respect to say first row or first column. If one developing with respect to the column (row) the permutations of rows (columns) do not affect the value of determinant or sub-determinants but permutations of columns (rows) do so unless one poses additional conditions stating that the permutations do not affect given contribution to the determinant or sub-determinant. It turns out that this option must be applied in the case of ordinary quantum group. For quantum phase $q = \pm 1$ the determinant is invariant under permutations of both rows and columns.

(b) Second manner to get rid of difficulty would be that n-path does not depend on the ordering of the rows (columns) differ only by the usual sign factor. For 2×2 case this would give

$$ad - bc = da - cb \ , \quad \text{(Option 2)} \tag{3.6.6}$$

These conditions state the invariance of the n-path under permutation group S_n permuting rows or columns.

(c) For the third option the elements along n-paths commute: paths could be said to be "classical". The invariance of N-path in this sense guarantees the invariance of all n-paths. In 2-D case this gives

$$[a, d] = 0 \ , \quad [b, c] = 0 \ . \quad \text{(Option 3)} \tag{3.6.7}$$

5. One should have a well-defined eigenvalue problem. If the n-functions commute, one can diagonalize the corresponding operators simultaneously and the eigenvalues problem reduces to possibly infinite number of ordinary eigenvalue problems corresponding to restrictions to given set of eigenvalues associated with $N-1$ symmetric functions. This gives an additional constraint on quantum matrices.

   In 2-dimensional case one would have the condition

$$[ad - bc, a + d] = 0 \ . \tag{3.6.8}$$

   Depending on how strong S_2 invariance one requires, one obtains 0, 1, 2 nontrivial conditions for 2×2 quantum matrices and 1 condition from the commutativity of n-functions besides hermiticity conditions.

   For $N \times N$-matrices one would have $N! - 1$ non-trivial conditions from the strong form of permutation invariance guaranteeing the permutation symmetry of n-functions and $N(N-1)/2$ conditions from the commutativity of n-functions.

6. The eigenvectors of the density matrix are obtained in the usual manner for each eigenvalue contributing to quantum eigenvalue. Also the diagonalization can be carried out by a unitary transformation for each eigenvalue separately. Hence the standard approach seems to generalize almost trivially.

   What makes the proposal non-trivial and possibly physically interesting is that the hermitian operators are not assumed to be just tensor products of $N \times N$ hermitian matrices with hermitian operators in Hilbert space.

The notion of unitary quantum matrix should also make sense. The naïve guess is that the exponentiation of a linear combination of ordinary hermitian matrices with coefficients, which are hermitian matrices gives quantum unitary matrices. In the case of U(1) the replacement of exponentiation parameter t in $exp(itX)$ with a hermitian operator gives standard expression for the exponent and it is trivial to see that unitary conditions are satisfied also in this case. Also in the case of SU(2) it is easy to verify that the guess is correct. One must also check that one indeed obtains a group: it could also happen that only semi-group is obtained.

In any case, one could speak of quantum matrix groups with coordinates replaced by hermitian matrices. These quantum matrix group need not be identical with quantum groups in the standard sense of the word. Maybe this could provide one possible meaning for quantization in the case of groups and perhaps also in the case of coset spaces G/H.

3.6.2 The Relationship To Quantum Groups And Quantum Lie Algebras

It is interesting to find out whether quantum matrices give rise to quantum groups under suitable additional conditions. The child's guess for these conditions is that the permutation of rows and columns correspond to braiding for the hermitian moduli h_{ij} defined by unitary operators U_{ij}.

Quantum groups and quantum matrices

The conditions for hermiticity and unitary do not involve quantum parameter q, which suggests that the naïve generalization of the notion of unitary matrix gives unitary group obtained by replacing complex number field with operator algebra gives group with coordinates defined by hermitian operators rather than standard quantum group. This turns out to be the case and it seems that quantum matrices provide a concrete representation for quantum group. The notion of braiding as that for operators h_{ij} can be said to emerge from the notion of quantum matrix.

1. Exponential of quantum hermitian matrix is excellent candidate for quantum unitary matrix. One should check the exponentiation indeed gives rise to a quantum unitary matrix. For $q = \pm 1$ this seems obvious but one should check this separately for other roots of unity. Instead of considering the general case, we consider explicit ansatz for unitary U(2) quantum matrix as $U = [a, b; -b^\dagger, a^\dagger]$. The conditions for unitary quantum group in the proposed sense would state the orthonormality and unit norm property of rows/columns.

   The explicit form of the conditions reads as

$$\begin{array}{ll} ab - ba = 0 \ , & ab^\dagger = b^\dagger a \ , \\ aa^\dagger + bb^\dagger = 1 \ , & a^\dagger a + b^\dagger b = 1 \ . \end{array} \tag{3.6.9}$$

   The orthogonality conditions are unique and reduce to the vanishing of commutators.

   Normalization conditions involve a choice of ordering. One possible manner to avoid the problem is to assume that both orderings give same unit length for row or column (as done above). If only the other option is assumed then only third or fourth equations is needed. The invariance of determinant under permutation of rows would imply $[a, a^\dagger] = [b, b^\dagger] = 0$ and the ordering problem would disappear.

2. One can look what conditions the explicit representation $U_{ij} = h_{ij}u_{ij}$ or equivalently $[h_au_a, h_bu_b; -u_b^\dagger h_b, u_a^\dagger h_a]$ gives. The intuitive expectation is that U(2) matrix decomposes to a product of commutating SU(2) matrix and U(1) matrices. This implies that u_a commutes with the other matrices involved. One obtains the conditions

$$h_ah_b = h_b(u_bh_au_b^\dagger) \ , \quad h_bh_a = (u_bh_au_b^\dagger)h_b \ . \tag{3.6.10}$$

   These conditions state that the permutation of h_a and h_b analogous to braiding operation is a unitary operation.

For the purposes of comparison consider now the corresponding conditions for $SU(2)_q$ matrix.

1. The $SU(2)_q$ matrix $[a, b; b^\dagger, a^\dagger]$ with *real* value of q (see `http://tinyurl.com/yb8tycag`) satisfies the conditions

$$\begin{array}{lll} ba = qab \ , & b^\dagger a = qab^\dagger , & bb^\dagger = b^\dagger b \ , \\ a^\dagger a + q^2 b^\dagger b = 1 \ , & aa^\dagger + bb^\dagger = 1 \ . & \end{array} \tag{3.6.11}$$

This gives $[a^\dagger, a] = (1-q^2)b^\dagger b$. The above conditions would correspond to $q = \pm 1$ but with complex numbers replaced with operator algebra. q-commutativity obviously replaces ordinary commutativity in the conditions and one can speak of q-orthonormality.

For complex values of q - in particular roots of unity - the condition $a^\dagger a + q^2 b^\dagger b = 1$ is in general not self-consistent since hermitian conjugation transforms q^2 to its complex conjugate. Hence this condition must be dropped for complex roots of unity.

2. Only for $q = \pm 1$ corresponding to Bose-Einstein and Fermi-Dirac statistics the conditions are consistent with the invariance of n-functions (determinant) under permutations of both rows and columns. Indeed, if 2×2 q-determinant is developed with respect to column, the permutation of rows does not affect its value. This is trivially true also in $N \times N$ dimensional case since the permutation of rows does not affect the n-paths at all.

   If the symmetry under permutations is weakened, nothing prevents from posing quantum orthogonality conditions also now and the decomposition to a product of positive and hermitian matrices give a concrete meaning to the notion of quantum group.

   Do various $n-$functions commute with each other for $SU(2)_q$? The only commutator of this kind is that for the trace and determinant and should vanish:

$$\left[b + b^\dagger, aa^\dagger + bb^\dagger\right] = 0 \ . \tag{3.6.12}$$

   Since $a^\dagger a$ and $aa^\dagger$ are linear combinations of $b^\dagger b = b^\dagger b$, they vanish. Hence it seems that TGD based view about quantum groups is consistent with the standard view.

3. One can look these conditions in TGD framework by restricting the consideration to the case of SU(2) ($u_a = 1$) and using the ansatz $U = [h_a, h_b u_b; -u_b^\dagger h_b, h_a]$. Orthogonality conditions read as

$$h_a h_b = q h_b (u_b h_a u_b^\dagger) \ , \quad h_b h_a = q (u_b h_a u_b^\dagger) h_b \ .$$

   If q is root of unity, these conditions state that the permutation of h_a and h_b analogous to a unitary braiding operation apart from a multiplication with quantum phase q. For $q = \pm 1$ the sign-factor is that in standard statistics. Braiding picture could help guess the commutators of h_{ij} in the case of $N \times N$ quantum matrices. The permutations of rows and columns would have interpretation as braidings and one could say that braided commutators of matrix elements vanish.

   The conditions from the normalization give

$$h_a^2 + h_b^2 = 1 \ , \quad h_a^2 + q^2 (u_b^\dagger h_b^2 u_b) = 1 \ . \tag{3.6.13}$$

   For complex q the latter condition does not make sense since $h_a^2 - 1$ and $u_b^\dagger h_b^2 u_b$ are hermitian matrices with real eigenvalues. Also for real values of $q \neq \pm 1$ one obtains contradicion since the spectra of unitarily related hermitian operators would differ by scaling factor q^2. Hence one must give up the condition involving q^2 unless one has $q = \pm 1$. Note that the term proportional to q^2 does not allow interpretation in terms of braiding.

4. Roots of unity are natural number theoretically as values of q but number theoretical universality allows the generic value of q would be a complex number existing simultaneously in all p-adic number properly extended. This would suggest the spectrum of q to come as

$$q(m,n) = e^{1/m} exp(\frac{i2\pi}{n}) \ . \tag{3.6.14}$$

   The motivation comes from the fact that e^p is ordinary p-adic number for all p-adic number fields so e and also any root of e defines a finite-dimensional extension of p-adic numbers [K124] [L10]. The roots of unity would be associated to the discretization of the ordinary angles in case of compact matrix groups. Roots of e would be associated with the discretization of hyperbolic angles needed in the case of non-compact matrix groups such as SL(2,C).

Also now unification of various values of q to single single operator Q, which is product of *commuting* hermitian and unitary operators and commuting with the hermitian operator H representing the spectrum of Planck constant would code the spectrum. Skeptic can of course wonder, whether the modulus and phase of Q can be assumed to commute. The relationship between integers associated with H and Q is interesting.

Quantum Lie algebras and quantum matrices

What about quantum Lie algebras? There are many notions of quantum Lie algebra and quantum group. General formulas for the commutation relations are well-known for Drinfeld-Jimbo type quantum groups (see `http://tinyurl.com/yb8tycag`). The simplest guess is that one just poses the defining conditions for quantum group, replaces complex numbers as coefficient module with operator algebra, and poses the above described conditions making possible to speak about eigenvalues and eigen vectors. One might however hope that this representation allows to realize the non-commutativity of matrix elements of quantum Lie algebra in a concrete manner.

1. For SU(2) the commutation relations for the elements X_+, X_-, h read as

$$[h, X_\pm] = \pm X_\pm \ \ , \ \ [X_+, X_-] = h \ . \tag{3.6.15}$$

   Here one can use the 2×2 matrix representations for the ladder operators $X^\pm$ and diagonal angular momentum generator h.

2. For $SU(2)_q$ one has

$$[h, X_\pm] = \pm X_\pm \ \ , \ \ [X_+, X_-] = \frac{q^h - q^{-h}}{q - q^{-1}} \ . \tag{3.6.16}$$

3. Using the ansatz for the generators but allowing hermitian operator coefficients in non-diagonal generators $X_\pm$, one obtains the condition
   For $SU(2)_q$ one would have

$$[X_+, X_-] = h_+^2 = h_-^2 = \frac{q^h - q^{-h}}{q - q^{-1}} \ . \tag{3.6.17}$$

   Clearly, the proposal might make possible to have concrete representations for the quantum Lie algebras making the decomposition to measurable and directly non-measurable degrees of freedom explicit.

The conclusion is that finite measurement resolution does not lead automatically to standard quantum groups although the proposed realization is consistent with them. Also the quantum phases $q = \pm 1$ $n = 1, 2$ are realized and correspond to strong permutation symmetry and Bose-Einstein and Fermi statistics.

3.6.3 About Possible Applications

The realization for the notion of finite measurement resolution is certainly the basic application but one can imagine also other applications where hermitian and unitary matrices appear.

Density matrix description of degrees of freedom below measurement resolution

Density matrix ρ obtained by tracing over non-observable degrees of freedom is a fundamental example about a hermitian matrix satisfying the additional condition $Tr(\rho) = 1$.

1. A state function reduction with a finite measurement resolution would lead to a non-pure state. This state would be describable using $N \times N$-dimensional quantum hermitian quantum density matrix satisfying the condition $Tr(\rho) = 1$ (or more generally $Tr_q(\rho) = 1$), and satisfying the additional conditions allowing to reduce its diagonalization to that for a collection of ordinary density matrices so that the eigenvalues of ordinary density matrix would be replaced by N quantum eigenvalues defined by infinite-dimensional diagonalized density matrices.

2. One would have N quantum eigenvalues - quantum probabilities - each decomposing to possibly infinite set of ordinary probabilities assignable to the degrees of freedom below measurement resolution and defining density matrix for non-pure states resulting in state function reduction.

Some questions

Some further questions pop up naturally.

1. One might hope that the quantum counterparts of hermitian operators are in some sense universal, at least in TGD framework (by quantum criticality). Could the condition that the commutator of hermitian generators is proportional to $i\hbar$ times hermitian generator pose additional constraints? In 2-D case this condition is satisfied for quantum SU(2) generators and very probably the same is true also in the general case. The possible problems result from the non-commutativity but $(XY)^\dagger = Y^\dagger X^\dagger$ identity takes care that there are no problems.
2. One can also raise physics related questions. What one can say about most general quantum Hamiltonians and their energy spectra, say quantum hydrogen atom? What about quantum angular momentum? If the proposed construction is only a concretization of abstract quantum group construction, then nothing new is expected at the level of representations of quantum groups.
3. Could the spectrum of h_{eff} define a quantum h as a hermitian positive definite operator? Could this allow a description for the presence of dark matter, which is not directly observable? Same question applies to the quantum parameter q.
4. M-matrices are basic building bricks of scattering amplitudes in ZEO. M-matrix is produce of hermitian "complex" square root H of density matrix satisfying $H^2 = \rho$ and unitary S-matrix S. It has been proposed that these matrices commute. The previous consideration relying on basic quantum thinking suggests that they relate like translation generator in radial direction and phase defined by angle and thus satisfy $[H, S] = i(H_{eff}/h) \times S$. This would give enormously powerful additional condition to S-matrix. One can also ask whether M-matrices in presence of degrees of freedom below measurement resolution is quantum version of M-matrix in the proposed sense.
5. Fractality is of of the key notions of TGD and characterizes also hyperfinite factors. I have proposed some realizations of fractality such as infinite primes and finite-dimensional Hilbert spaces taking the role of natural numbers and ordinary sum and product replaced with direct sum and tensor product. One could also imagine a fractal hierarchy of quantum matrices obtained by replacing the operators appearing as matrix elements of quantum matrix element by quantum matrices. This hierarchy could relate to the sequence of inclusions of HFFs.

3.7 Jones Inclusions And Cognitive Consciousness

WCW spinors have a natural interpretation in terms of a quantum version of Boolean algebra. Beliefs of various kinds are the basic element of cognition and obviously involve a representation of the external world or part of it as states of the system defining the believer. Jones inclusions mediating unitary mappings between the spaces of WCWs spinors of two systems are excellent candidates for these maps, and it is interesting to find what one kind of model for beliefs this picture leads to.

The resulting quantum model for beliefs provides a cognitive interpretation for quantum groups and predicts a universal spectrum for the probabilities that a given belief is true. This spectrum depends only on the integer n characterizing the quantum phase $q = exp(i2\pi/n)$ characterizing the Jones inclusion. For $n \neq \infty$ the logic is inherently fuzzy so that absolute knowledge is impossible. $q = 1$ gives ordinary quantum logic with qbits having precise truth values after state function reduction.

3.7.1 Does One Have A Hierarchy Of U- And M-Matrices?

U-matrix describes scattering of zero energy states and since zero energy states can be illustrated in terms of Feynman diagrams one can say that scattering of Feynman diagrams is in question.

The initial and final states of the scattering are superpositions of Feynman diagrams characterizing the corresponding M-matrices which contain also the positive square root of density matrix as a factor.

The hypothesis that U-matrix is the tensor product of S-matrix part of M-matrix and its Hermitian conjugate would make U-matrix an object deducible by physical measurements. One cannot of course exclude that something totally new emerges. For instance, the description of quantum jumps creating zero energy state from vacuum might require that U-matrix does not reduce in this manner. One can assign to the U-matrix a square like structure with S-matrix and its Hermitian conjugate assigned with the opposite sides of a square.

One can imagine of constructing higher level physical states as composites of zero energy states by replacing the S-matrix with M-matrix in the square like structure. These states would provide a physical representation of U-matrix. One could define U-matrix for these states in a similar manner. This kind of hierarchy could be continued indefinitely and the hierarchy of higher level U and M-matrices would be labeled by a hierarchy of n-cubes, $n = 1, 2$,... TGD inspired theory of consciousness suggests that this hierarchy can be interpreted as a hierarchy of abstractions represented in terms of physical states. This hierarchy brings strongly in mind also the hierarchies of n-algebras and n-groups and this forces to consider the possibility that something genuinely new emerges at each step of the hierarchy. A connection with the hierarchies of infinite primes [K105] and Jones inclusions are suggestive.

3.7.2 Feynman Diagrams As Higher Level Particles And Their Scattering As Dynamics Of Self Consciousness

The hierarchy of inclusions of hyper-finite factors of II_1 as counterpart for many-sheeted space-time lead inevitably to the idea that this hierarchy corresponds to a hierarchy of generalized Feynman diagrams for which Feynman diagrams at a given level become particles at the next level. Accepting this idea, one is led to ask what kind of quantum states these Feynman diagrams correspond, how one could describe interactions of these higher level particles, what is the interpretation for these higher level states, and whether they can be detected.

Jones inclusions as analogs of space-time surfaces

The idea about space-time as a 4-surface replicates itself at the level of operator algebra and state space in the sense that Jones inclusion can be seen as a representation of the operator algebra $\mathcal{N}$ as infinite-dimensional linear sub-space (surface) of the operator algebra $\mathcal{M}$. This encourages to think that generalized Feynman diagrams could correspond to image surfaces in II_1 factor having identification as kind of quantum space-time surfaces.

Suppose that the modular S-matrices are representable as the inner automorphisms $\Delta(\mathcal{M}_k^{it}$ assigned to the external lines of Feynman diagrams. This would mean that $\mathcal{N} \subset \mathcal{M}_k$ moves inside $calM_k$ along a geodesic line determined by the inner automorphism. At the vertex the factors $calM_k$ to fuse along $\mathcal{N}$ to form a Connes tensor product. Hence the copies of $\mathcal{N}$ move inside $\mathcal{M}_k$ like incoming 3-surfaces in H and fuse together at the vertex. Since all $\mathcal{M}_k$ are isomorphic to a universal factor $\mathcal{M}$, many-sheeted space-time would have a kind of quantum image inside II_1 factor consisting of pieces which are $d = \mathcal{M} : \mathcal{N}/2$-dimensional quantum spaces according to the identification of the quantum space as subspace of quantum group to be discussed later. In the case of partonic Clifford algebras the dimension would be indeed $d \leq 2$.

The hierarchy of Jones inclusions defines a hierarchy of S-matrices

It is possible to assign to a given Jones inclusion $\mathcal{N} \subset \mathcal{M}$ an entire hierarchy of Jones inclusions $\mathcal{M}_0 \subset \mathcal{M}_1 \subset \mathcal{M}_2$..., $\mathcal{M}_0 = N$, $\mathcal{M}_1 = M$. A possible interpretation for these inclusions would be as a sequence of topological condensations.

This sequence also defines a hierarchy of Feynman diagrams inside Feynman diagrams. The factor $\mathcal{M}$ containing the Feynman diagram having as its lines the unitary orbits of $\mathcal{N}$ under $\Delta_{\mathcal{M}}$ becomes a parton in $\mathcal{M}_1$ and its unitary orbits under $\Delta_{\mathcal{M}_1}$ define lines of Feynman diagrams in M_1. The concrete representation for M-matrix or projection of it to some subspace as entanglement coefficients of partons at the ends of a braid assignable to the space-like 3-surface representing a vertex of a higher level Feynman diagram. In this manner quantum dynamics would be coded and simulated by quantum states.

The outcome can be said to be a hierarchy of Feynman diagrams within Feynman diagrams, a fractal structure for which many particle scattering events at a given level become particles at the next level. The particles at the next level represent dynamics at the lower level: they have the property of "being about" representing perhaps the most crucial element of conscious experience. Since net conserved quantum numbers can vanish for a system in TGD Universe, this kind of hierarchy indeed allows a realization as zero energy states. Crossing symmetry can be understood in terms of this picture and has been applied to construct a model for M-matrix at high energy limit [K34] .

One might perhaps say that quantum space-time corresponds to a double inclusion and that further inclusions bring in N-parameter families of space-time surfaces.

Higher level Feynman diagrams

The lines of Feynman diagram in $\mathcal{M}_{n+1}$ are geodesic lines representing orbits of $\mathcal{M}_n$ and this kind of lines meet at vertex and scatter. The evolution along lines is determined by $\Delta_{\mathcal{M}_{n+1}}$. These lines contain within themselves $\mathcal{M}_n$ Feynman diagrams with similar structure and the hierarchy continues down to the lowest level at which ordinary elementary particles are encountered.

For instance, the generalized Feynman diagrams at the second level are ribbon diagrams obtained by thickening the ordinary diagrams in the new time direction. The interpretation as ribbon diagrams crucial for topological quantum computation and suggested to be realizable in terms of zero energy states in [K5] is natural. At each level a new time parameter is introduced so that the dimension of the diagram can be arbitrarily high. The dynamics is not that of ordinary surfaces but the dynamics induced by the $\Delta_{\mathcal{M}_n}$.

Quantum states defined by higher level Feynman diagrams

The intuitive picture is that higher level quantum states corresponds to the self reflective aspect of existence and must provide representations for the quantum dynamics of lower levels in their own structure. This dynamics is characterized by M-matrix whose elements have representation in terms of Feynman diagrams.

1. These states correspond to zero energy states in which initial states have "positive energies" and final states have "negative energies". The net conserved quantum numbers of initial and final state partons compensate each other. Gravitational energies, and more generally gravitational quantum numbers defined as absolute values of the net quantum numbers of initial and final states do not vanish. One can say that thoughts have gravitational mass but no inertial mass.

2. States in sub-spaces of positive and negative energy states are entangled with entanglement coefficients given by M-matrix at the level below.

To make this more concrete, consider first the simplest non-trivial case. In this case the particles can be characterized as ordinary Feynman diagrams, or more precisely as scattering events so that the state is characterized by $\hat{S} = P_{in}SP_{out}$, where S is S-matrix and P_{in} *resp.* P_{out} is the projection to a subspace of initial *resp.* final states. An entangled state with the projection of S-matrix giving the entanglement coefficients is in question.

The larger the domains of projectors P_{in} and P_{out}, the higher the representative capacity of the state. The norm of the non-normalized state $\hat{S}$ is $Tr(\hat{S}\hat{S}^{\dagger}) \leq 1$ for II_1 factors, and at the limit $\hat{S} = S$ the norm equals to 1. Hence, by II_1 property, the state always entangles infinite number of states, and can in principle code the entire S-matrix to entanglement coefficients.

The states in which positive and negative energy states are entangled by a projection of S-matrix might define only a particular instance of states for which conserved quantum numbers vanish. The model for the interaction of Feynman diagrams discussed below applies also to these more general states.

The interaction of $\mathcal{M}_n$ Feynman diagrams at the second level of hierarchy

What constraints can one pose to the higher level reactions? How Feynman diagrams interact? Consider first the scattering at the second level of hierarchy ($\mathcal{M}_1$), the first level $\mathcal{M}_0$ being assigned to the interactions of the ordinary matter.

1. Conservation laws pose constraints on the scattering at level $\mathcal{M}_1$. The Feynman diagrams can transform to new Feynman diagrams only in such a way that the net quantum numbers are conserved separately for the initial positive energy states and final negative energy states of the diagram. The simplest assumption is that positive energy matter and negative energy matter know nothing about each other and effectively live in separate worlds. The scattering matrix form Feynman diagram like states would thus be simply the tensor product $S \otimes S^\dagger$, where S is the S-matrix characterizing the lowest level interactions and identifiable as unitary factor of M-matrix for zero energy states. Reductionism would be realized in the sense that, apart from the new elements brought in by $\Delta_{\mathcal{M}_n}$ defining single particle free dynamics, the lowest level would determine in principle everything occurring at the higher level providing representations about representations about... for what occurs at the basic level. The lowest level would represent the physical world and higher levels the theory about it.
2. The description of hadronic reactions in terms of partons serves as a guide line when one tries to understand higher level Feynman diagrams. The fusion of hadronic space-time sheets corresponds to the vertices $\mathcal{M}_1$. In the vertex the analog of parton plasma is formed by a process known as parton fragmentation. This means that the partonic Feynman diagrams belonging to disjoint copies of $\mathcal{M}_0$ find themselves inside the same copy of $\mathcal{M}_0$. The standard description would apply to the scattering of the initial *resp.* final state partons.
3. After the scattering of partons hadronization takes place. The analog of hadronization in the recent case is the organization of the initial and final state partons to groups I_i and F_i such that the net conserved quantum numbers are same for I_i and F_i. These conditions can be satisfied if the interactions in the plasma phase occur only between particles belonging to the clusters labeled by the index i. Otherwise only single particle states in $\mathcal{M}_1$ would be produced in the reactions in the generic case. The cluster decomposition of S-matrix to a direct sum of terms corresponding to partitions of the initial state particles to clusters which do not interact with each other obviously corresponds to the "hadronization". Therefore no new dynamics need to be introduced.
4. One cannot avoid the question whether the parton picture about hadrons indeed corresponds to a higher level physics of this kind. This would require that hadronic space-time sheets carry the net quantum numbers of hadrons. The net quantum numbers associated with the initial state partons would be naturally identical with the net quantum numbers of hadron. Partons and they negative energy conjugates would provide in this picture a representation of hadron about hadron. This kind of interpretation of partons would make understandable why they cannot be observed directly. A possible objection is that the net gravitational mass of hadron would be three times the gravitational mass deduced from the inertial mass of hadron if partons feed their gravitational fluxes to the space-time sheet carrying Earth's gravitational field.
5. This picture could also relate to the suggested duality between string and parton pictures [K107] . In parton picture hadron is formed from partons represented by space-like 2-surfaces X_i^2 connected by join along boundaries bonds. In string picture partonic 2-surfaces are replaced with string orbits. If one puts positive and negative energy particles at the ends of string diagram one indeed obtains a higher level representation of hadron. If these pictures are dual then also in parton picture positive and negative energies should compensate each other. Interestingly, light-like 3-D causal determinants identified as orbits of partons could be interpreted as orbits of light like string word sheets with "time" coordinate varying in space-like direction.

Scattering of Feynman diagrams at the higher levels of hierarchy

This picture generalizes to the description of higher level Feynman diagrams.

1. Assume that higher level vertices have recursive structure allowing to reduce the Feynman diagrams to ordinary Feynman diagrams by a procedure consisting of finite steps.
2. The lines of diagrams are classified as incoming or outgoing lines according to whether the time orientation of the line is positive or negative. The time orientation is associated with the time parameter t_n characterizing the automorphism $\Delta_{\mathcal{M}_\backslash}^{it_n}$. The incoming and outgoing net quantum numbers compensate each other. These quantum numbers are basically the quantum numbers of the state at the lowest level of the hierarchy.
3. In the vertices the $\mathcal{M}_{n+1}$ particles fuse and $\mathcal{M}_n$ particles form the analog of quark gluon plasma. The initial and final state particles of $\mathcal{M}_n$ Feynman diagram scatter independently and the S-matrix S_{n+1} describing the process is tensor product $S_n \otimes S_n^\dagger$. By the clustering property of S-matrix, this scattering occurs only for groups formed by partons formed by the incoming and outgoing particles $\mathcal{M}_n$ particles and each outgoing $\mathcal{M}_{n+1}$ line contains and irreducible $\mathcal{M}_n$ diagram. By continuing the recursion one finally ends down with ordinary Feynman diagrams.

3.7.3 Logic, Beliefs, And Spinor Fields In The World Of Classical Worlds

Beliefs can be characterized as Boolean value maps $\beta_i(p)$ telling whether i believes in proposition p or not. Additional structure is brought in by introducing the map $\lambda_i(p)$ telling whether p is true or not in the environment of i. The task is to find quantum counterpart for this model.

The spectrum of probabilities for outcomes in state function reduction with finite measurement resolution is universal

Consider quantum two-spinor as a model of a system with finite measurement resolution implying that state function reduction do not anymore lead to a spin state with a precise value but that one can only predict the probability distribution for the outcome of measurement. These probabilities can be also interpreted as truth values of a belief in finite cognitive resolution.

It is actually possible to calculate the spectrum of the probabilities of truth values with rather mild additional assumptions.

1. Since the Hermitian operators $X_1 = (z^1\overline{z^1} + \overline{z^1}z^1)/2$ and $X_2 = (z^2\overline{z^2} + \overline{z^2}z^2)/2$ commute, physical states can be chosen to be eigen states of these operators and it is possible to assign to the truth values probabilities given by $p_1 = X_1/R^2$ and $p_2 = X_2/R^2$, $R^2 = X_1 + X_2$.
2. By introducing the analog of the harmonic oscillator vacuum as a state $|0\rangle$ satisfying $z^1|0\rangle = z^2|0\rangle = 0$, one obtains eigen states of X_1 and X_2 as states $|n_1, n_2\rangle = \overline{z^1}^{n_1}\overline{z^2}^{n_2}|0\rangle$, $n_1 \geq 0, n_2 \geq 0$. The eigenvalues of X_1 and X_2 are given by a modified harmonic oscillator spectrum as

$$X_1 = (1/2 + n_1 q^{n_2})r \ \ , \ \ X_2 = (1/2 + n_2 q^{n_1})r \ \ .$$

   The reality of eigenvalues (hermiticity) is guaranteed if one has $n_1 = N_1 n$ and $n_1 = N_2 n$ and implies that the spectrum of eigen states gets increasingly thinner for $n \to \infty$. This must somehow reflect the fractal dimension. The fact that large values of oscillator quantum numbers n_1 and n_2 correspond to the classical limit suggests that modulo condition guarantees approximate classicality of the logic for $n \to \infty$.
3. The probabilities p_1 and p_2 for the truth values given by $(p_1, p_2) = (1/2+N_1 n, 1/2+N_2 n)/[1+(N_1 + N_2)n]$ are rational and allow an interpretation as both real and p-adic numbers. This also conforms with the frequency interpretation for probabilities. All states are are inherently fuzzy and only at the limits $N_1 \gg N_2$ and $N_2 \gg N_1$ non-fuzzy states result. As noticed, $n = \infty$ must be treated separately and corresponds to an ordinary non-fuzzy qbit logic. At $n \to \infty$ limit one has $(p_1, p_2) = (N_1, N_2)/(N_1, N_2)$: at this limit $N_1 = 0$ or $N_2 = 0$ states are non-fuzzy.
4. A possible interpretation for the fuzziness is in terms of finite measurement resolution. The quantized probabilities could be assigned with diagonalized density matrix regarded as matrix with elements which are commuting hermitian operators. The generalized eigenvalues would be eigenvalues spectra. States would not be pure expect at the limits$N_1 \gg N_2$ and $N_2 \gg$

N_1. The non-purity of the state could be understood in terms of entanglement with the degrees of freedom below measurement resolution describable in terms of inclusion of von Neumann algebras. One could perhaps say that in finite measurement resolution the outcome of state function reduction is always non-pure state characterized by a universal density matrix obtained by tracing over non-visible degrees of freedom.

WCW spinors as logic statements

In TGD framework the infinite-dimensional WCW (CH) spinor fields defined in CH, the "world of classical worlds", describe quantum states of the Universe [K126] . WCW spinor field can be regarded as a state in infinite-dimensional Fock space and are labeled by a collection of various two valued indices like spin and weak isospin. The interpretation is as a collection of truth values of logic statements one for each fermionic oscillator operator in the state. For instance, spin up and down would correspond to two possible truth values of a proposition characterized by other quantum numbers of the mode.

The hierarchy of space-time sheet could define a physical correlate for the hierarchy of higher order logics (statements about statements about...). The space-time sheet containing N fermions topologically condensed at a larger space-time sheet behaves as a fermion or boson depending on whether N is odd or even. This hierarchy has also a number theoretic counterpart: the construction of infinite primes [K105] corresponds to a repeated second quantization of a super-symmetric quantum field theory.

Quantal description of beliefs

The question is whether TGD inspired theory of consciousness allows a fundamental description of beliefs.

1. Beliefs define a model about some subsystem of universe constructed by the believer. This model can be understood as some kind of representation of real word in the state space representing the beliefs.
2. One can wonder what is the difference between real and p-adic variants of WCW spinor fields and whether they could represent reality and beliefs about reality. WCW spinors (as opposed to spinor fields) are constructible in terms of fermionic oscillator operators and seem to be universal in the sense that one cannot speak about p-adic and real WCW spinors as different objects. Real/p-adic spinor fields however have real/p-adic space-time sheets as arguments. This would suggest that there is no fundamental difference between the logic statements represented by p-adic and real WCW spinors.
3. This vision is realized if the intersection of reality and various p-adicities corresponds to an algebraically universal set of consisting of partonic 2-surfaces and string world sheets for which defining parameters are WCW coordinates in an algebraic extension of rationals defining that for p-adic number fields. Induced spinor fields would be localized at string world sheets and their intersections with partonic 2-surfaces and would be number theoretically universal. If second quantized induced spinor fields are correlates of Boolean cognition, which is behind the entire mathematics, their number theoretical universality is indeed a highly natural condition. Also fermionic anticommutation relations are number theoretically universal. By conformal invariance the conformal moduli of string world sheets and partonic 2-surface would be the natural WCW coordinates for the 2-surfaces in question and I proposed their p-adicization already in p-adic mass calculations for two decades ago.

   This picture would provide an elegant realization for the p-adicization. There would be ne need to map real space-time surfaces directly to p-adic ones and vice versa and one would avoid problems related to general coordinate invariance (GCI) completely. Strong form of holography would assign to partonic surfaces the real and p-adic variants. Already p-adic mass calculations support the presence of cognition in all length scales.

These observations suggest a more concrete view about how beliefs emerge physically.

The idea that p-adic WCW spinor fields could serve as representations of beliefs and real WCW spinor fields as representations of reality looks very nice and conforms with the adelic vision that space-time is adele - a book-like structure contains space-time sheets in various number fields as pages glued together along back for which the parameters characterizing space-time surface are numbers in an algebraic extension of rationals. Real space-time surfaces would be correlates for sensory experience and p-adic space-time sheets for cognition.

3.7.4 Jones Inclusions For Hyperfinite Factors Of Type II_1 As A Model For Symbolic And Cognitive Representations

Consider next a more detailed model for how cognitive representations and beliefs are realized at quantum level. This model generalizes trivially to symbolic representations.

The Clifford algebra of gamma matrices associated with WCW spinor fields corresponds to a von Neumann algebra known as hyper-finite factor of type II_1. The mathematics of these algebras is extremely beautiful and reproduces basic mathematical structures of modern physics (conformal field theories, quantum groups, knot and braid groups,....) from the mere assumption that the world of classical worlds possesses infinite-dimensional Kähler geometry and allows spinor structure.

The almost defining feature is that the infinite-dimensional unit matrix of the Clifford algebra in question has by definition unit trace. Type II_1 factors allow also what are known as Jones inclusions of Clifford algebras $\mathcal{N} \subset \mathcal{M}$. What is special to II_1 factors is that the induced unitary mappings between spinor spaces are genuine inclusions rather than 1-1 maps.

The S-matrix associated with the real-to-p-adic quantum transition inducing belief from reality would naturally define Jones inclusion of CH Clifford algebra $\mathcal{N}$ associated with the real space-time sheet to the Clifford algebra $\mathcal{M}$ associated with the p-adic space-time sheet. The moduli squared of S-matrix elements would define probabilities for pairs or real and belief states.

In Jones inclusion $\mathcal{N} \subset \mathcal{M}$ the factor $\mathcal{N}$ is included in factor $\mathcal{M}$ such that $\mathcal{M}$ can be expressed as $\mathcal{N}$-module over quantum space $\mathcal{M}/\mathcal{N}$ which has fractal dimension given by Jones index $\mathcal{M} : \mathcal{N} = 4cos^2(\pi/n) \leq 4$, $n = 3, 4, ...$ varying in the range $[1, 4]$. The interpretation is as the fractal dimension corresponding to a dimension of Clifford algebra acting in $d = \sqrt{\mathcal{M} : \mathcal{N}}$-dimensional spinor space: d varies in the range $[1, 2]$. The interpretation in terms of a quantal variant of logic is natural.

Probabilistic beliefs

For $\mathcal{M} : \mathcal{N} = 4$ ($n = \infty$) the dimension of spinor space is $d = 2$ and one can speak about ordinary 2-component spinors with $\mathcal{N}$-valued coefficients representing generalizations of qubits. Hence the inclusion of a given $\mathcal{N}$-spinor as M-spinor can be regarded as a belief on the proposition and for the decomposition to a spinor in N-module $\mathcal{M}/\mathcal{N}$ involves for each index a choice $\mathcal{M}/\mathcal{N}$ spinor component selecting super-position of up and down spins. Hence one has a superposition of truth values in general and one can speak only about probabilistic beliefs. It is not clear whether one can choose the basis in such a way that $\mathcal{M}/\mathcal{N}$ spinor corresponds always to truth value 1. Since WCW spinor field is in question and even if this choice might be possible for a single 3-surface, it need not be possible for deformations of it so that at quantum level one can only speak about probabilistic beliefs.

Fractal probabilistic beliefs

For $d < 2$ the spinor space associated with $\mathcal{M}/\mathcal{N}$ can be regarded as quantum plane having complex quantum dimension d with two non-commuting complex coordinates z^1 and z^2 satisfying $z^1z^2 = qz^2z^1$ and $\overline{z^1z^2} = \overline{q}\overline{z^2z^1}$. These relations are consistent with hermiticity of the real and imaginary parts of z^1 and z^2 which define ordinary quantum planes. Hermiticity also implies that one can identify the complex conjugates of z^i as Hermitian conjugates.

The further commutation relations $[z^1, \overline{z^2}] = [z^2, \overline{z^1}] = 0$ and $[z^1, \overline{z^1}] = [z^2, \overline{z^2}] = r$ give a closed algebra satisfying Jacobi identities. One could argue that $r \geq 0$ should be a function $r(n)$ of the quantum phase $q = exp(i2\pi/n)$ vanishing at the limit $n \to \infty$ to guarantee that the algebra becomes commutative at this limit and truth values can be chosen to be non-fuzzy. $r = sin(\pi/n)$ would be the simplest choice. As will be found, the choice of $r(n)$ does not however affect at all the spectrum for the probabilities of the truth values. $n = \infty$ case corresponding to non-fuzzy quantum logic is also possible and must be treated separately: it corresponds to Kac Moody algebra instead of quantum groups.

The non-commutativity of complex spinor components means that z^1 and z^2 are not independent coordinates: this explains the reduction of the number of the effective number of truth values to $d<2$. The maximal reduction occurs to $d=1$ for $n=3$ so that there is effectively only single truth value and one could perhaps speak about taboo or dogma or complete disappearance of the notions of truth and false (this brings in mind reports about meditative states: in fact $n=3$ corresponds to a phase in which Planck constant becomes infinite so that the system is maximally quantal).

As non-commuting operators the components of d-spinor are not simultaneously measurable for $d<2$. It is however possible to measure simultaneously the operators describing the probabilities $z^1\overline{z^1}$ and $z^2\overline{z^2}$ for truth values since these operators commute. An inherently fuzzy Boolean logic would be in question with the additional feature that the spinorial counterparts of statement and its negation cannot be regarded as independent observables although the corresponding probabilities satisfy the defining conditions for commuting observables.

If one can speak of a measurement of probabilities for $d<2$, it differs from the ordinary quantum measurement in the sense that it cannot involve a state function reduction to a pure qubit meaning irreducible quantal fuzziness. One could speak of fuzzy qbits or fqbits (or quantum qbits) instead of qbits. This picture would provide the long sought interpretation for quantum groups.

The previous picture applies to all representations $M_1 \subset M_2$, where M_1 and M_2 denote either real or p-adic Clifford algebras for some prime p. For instance, real-real Jones inclusion could be interpreted as symbolic representations assignable to a unitary mapping of the states of a subsystem M_1 of the external world to the state space M_2 of another real subsystem. $p_1 \rightarrow p_2$ unitary inclusions would in turn map cognitive representations to cognitive representations. There is a strong temptation to assume that these Jones inclusions define unitary maps realizing universe as a universal quantum computer mimicking itself at all levels utilizing cognitive and symbolic representations. Subsystem-system inclusion would naturally define one example of Jones inclusion.

The spectrum of probabilities of truth values is universal

It is actually possible to calculate the spectrum of the probabilities of truth values with rather mild additional assumptions.

1. Since the Hermitian operators $X_1=(z^1\overline{z^1}+\overline{z^1}z^1)/2$ and $X_2=(z^2\overline{z^2}+\overline{z^2}z^2)/2$ commute, physical states can be chosen to be eigen states of these operators and it is possible to assign to the truth values probabilities given by $p_1=X_1/R^2$ and $p_2=X_2/R^2$, $R^2=X_1+X_2$.

2. By introducing the analog of the harmonic oscillator vacuum as a state $|0\rangle$ satisfying $z^1|0\rangle = z^2|0\rangle=0$, one obtains eigen states of X_1 and X_2 as states $|n_1,n_2\rangle=\overline{z^1}^{n_1}\overline{z^2}^{n_2}|0\rangle$, $n_1\geq 0, n_2\geq 0$. The eigenvalues of X_1 and X_2 are given by a modified harmonic oscillator spectrum as

$$X_1=(1/2+n_1q^{n_2})r \ \ , \ \ X_2=(1/2+n_2q^{n_1})r \ \ .$$

   The reality of eigenvalues (hermiticity) is guaranteed if one has $n_1=N_1n$ and $n_1=N_2n$ and implies that the spectrum of eigen states gets increasingly thinner for $n\rightarrow\infty$. This must somehow reflect the fractal dimension. The fact that large values of oscillator quantum numbers n_1 and n_2 correspond to the classical limit suggests that modulo condition guarantees approximate classicality of the logic for $n\rightarrow\infty$.

3. The probabilities p_1 and p_2 for the truth values given by $(p_1,p_2)=(1/2+N_1n,1/2+N_2n)/[1+(N_1+N_2)n]$ are rational and allow an interpretation as both real and p-adic numbers. This also conforms with the frequency interpretation for probabilities. All states are are inherently fuzzy and only at the limits $N_1\gg N_2$ and $N_2\gg N_1$ non-fuzzy states result. As noticed, $n=\infty$ must be treated separately and corresponds to an ordinary non-fuzzy qbit logic. At $n\rightarrow\infty$ limit one has $(p_1,p_2)=(N_1,N_2)/(N_1,N_2)$: at this limit $N_1=0$ or $N_2=0$ states are non-fuzzy.

4. A possible interpretation for the fuzziness is in terms of finite measurement resolution. The quantized probabilities could be assigned with diagonalized density matrix regarded as matrix

with elements which are commuting hermitian operators. The generalized eigenvalues would be eigenvalues spectra. States would not be pure expect at the limits$N_1 \gg N_2$ and $N_2 \gg N_1$. The non-purity of the state could be understood in terms of entanglement with the degrees of freedom below measurement resolution describable in terms of inclusion of von Neumann algebras. One could perhaps say that in finite measurement resolution the outcome of state function reduction is always non-pure state characterized by a universal density matrix obtained by tracing over non-visible degrees of freedom.

How to define variants of belief quantum mechanically?

Probabilities of true and false for Jones inclusion characterize the plausibility of the belief and one can ask whether this description is enough to characterize states such as knowledge, misbelief, doubt, delusion, and ignorance. The truth value of $\beta_i(p)$ is determined by the measurement of probability assignable to Jones inclusion on the p-adic side. The truth value of $\lambda_i(p)$ is determined by a similar measurement on the real side. β and λ appear completely symmetrically and one can consider all kinds of triplets $\mathcal{M}_1 \subset \mathcal{M}_2 \subset \mathcal{M}_3$ assuming that there exist unitary S-matrix like maps mediating a sequence $\mathcal{M}_1 \subset \mathcal{M}_2 \subset \mathcal{M}_3$ of Jones inclusions. Interestingly, the hierarchies of Jones inclusions are a key concept in the theory of hyper-finite factors of type II_1 and pair of inclusions plays a fundamental role.

Let us restrict the consideration to the situation when $\mathcal{M}_1$ corresponds to a real subsystem of the external world, $\mathcal{M}_2$ its real representation by a real subsystem, and $\mathcal{M}_3$ to p-adic cognitive representation of M_3. Assume that both real and p-adic sides involve a preferred state basis for qubits representing truth and false.

Assume first that both $\mathcal{M}_1 \subset \mathcal{M}_2$ and $\mathcal{M}_2 \subset \mathcal{M}_3$ correspond to $d = 2$ case for which ordinary quantum measurement or truth value is possible giving outcome true or false. Assume further that the truth values have been measured in both M_2 and M_3.

1. Knowledge corresponds to the proposition $\beta_i(p) \wedge \lambda_i(p)$.
2. Misbelief to the proposition $\beta_i(p) \wedge \neq \lambda_i(p)$.
   Knowledge and misbelief would involve both the measurement of real and p-adic probabilities
   .
3. Assume next that one has $d < 2$ form $\mathcal{M}_2 \subset \mathcal{M}_3$. Doubt can be regarded neither belief or disbelief: $\beta_i(p) \wedge \neq \beta_i(\neq p)$: belief is inherently fuzzy although proposition can be non-fuzzy.
   Assume next that truth values in $\mathcal{M}_1 \subset \mathcal{M}_2$ inclusion corresponds to $d < 2$ so that the basic propositions are inherently fuzzy.
4. Delusion is a belief which cannot be justified: $\beta_i(p) \wedge \lambda_i(p) \wedge \neq \lambda(\neq p))$. This case is possible if $d = 2$ holds true for $\mathcal{M}_2 \subset \mathcal{M}_3$. Note that also misbelief that cannot be shown wrong is possible.
   In this case truth values cannot be quantum measured for $\mathcal{M}_1 \subset \mathcal{M}_2$ but can be measured for $\mathcal{M}_2 \subset \mathcal{M}_3$. Hence the states are products of pure $\mathcal{M}_3$ states with fuzzy $\mathcal{M}_2$ states.
5. Ignorance corresponds to the proposition $\beta_i(p) \wedge \neq \beta_i(\neq p) \wedge \lambda_i(p) \wedge \neq \lambda(\neq p))$. Both real representational states and belief states are inherently fuzzy.

Quite generally, only for $d_1 = d_2 = 2$ ideal knowledge and ideal misbelief are possible. Fuzzy beliefs and logics approach to ordinary one at the limit $n \to \infty$, which according to the proposal of [K100] corresponds to the ordinary value of Planck constant. For other cases these notions are only approximate and quantal approach allows to characterize the goodness of the approximation. A new kind of inherent quantum uncertainty of knowledge is in question and one could speak about a Uncertainty Principle for cognition and symbolic representations. Also the unification of symbolic and various kinds of cognitive representations deserves to be mentioned.

3.7.5 Intentional Comparison Of Beliefs By Topological Quantum Computation?

Intentional comparison would mean that for a given initial state also the final state of the quantum jump is fixed. This requires the ability to engineer S-matrix so that it leads from a given state to single state only. Any S-matrix representing permutation of the initial states fulfills these conditions. This condition is perhaps unnecessarily strong.

Quantum computation is basically the engineering of S-matrix so that it represents a superposition of parallel computations. In TGD framework topological quantum computation based on the braiding of magnetic flux tubes would be represented as an evolution characterized by braid [K5] . The dynamical evolution would be associated with light-like boundaries of braids. This evolution has dual interpretations either as a limit of time evolution of quantum state (program running) or a quantum state satisfying conformal invariance constraints (program code).

The dual interpretation would mean that conformally invariant states are equivalent with engineered time evolutions and topological computation realized as braiding connecting the quantum states to be compared (beliefs represented as many-fermion states at the boundaries of magnetic flux tubes) could give rise to conscious computational comparison of beliefs. The complexity of braiding would give a measure for how much the states to be compared differ.

Note that quantum computation is defined by a unitary map which could also be interpreted as symbolic representation of states of system M_1 as states of system M_2 mediated by the braid of join along boundaries bonds connecting the two space-time sheets in question and having light-like boundaries. These considerations suggest that the idea about S-matrix of the Universe should be generalized so that the dynamics of the Universe is dynamics of mimicry described by an infinite collection of fermionic S-matrices representable in terms of Jones inclusions.

3.7.6 The Stability Of Fuzzy Qbits And Quantum Computation

The stability of fqbits against state function reduction might have deep implications for quantum computation since quantum spinors would be stable against state function reduction induced by the perturbations inducing de-coherence in the normal situation. If this is really true, and if the only dangerous perturbations are those inducing the phase transition to qbits, the implications for quantum computation could be dramatic. Of course, the rigidity of qbits could be just another way to say that topological quantum computations are stable against thermal perturbations not destroying anyons [K5] .

The stability of fqbits could also be another manner to state the stability of rational, or more generally algebraic, bound state entanglement against state function reduction, which is one of the basic hypothesis of TGD inspired theory of consciousness [K68] . For sequences of Jones inclusions or equivalently, for multiple Connes tensor products, one would obtain tensor products of quantum spinors making possible arbitrary complex configurations of fqbits. Anyonic braids in topological quantum computation would have interpretation as representations for this kind of tensor products.

3.7.7 Fuzzy Quantum Logic And Possible Anomalies In The Experimental Data For The EPR-Bohm Experiment

The experimental data for EPR-Bohm experiment [J7] excluding hidden variable interpretations of quantum theory. What is less known that the experimental data indicates about possibility of an anomaly challenging quantum mechanics [J2] . The obvious question is whether this anomaly might provide a test for the notion of fuzzy quantum logic inspired by the TGD based quantum measurement theory with finite measurement resolution.

The anomaly

The experimental situation involves emission of two photons from spin zero system so that photons have opposite spins. What is measured are polarizations of the two photons with respect to polarization axes which differ from standard choice of this axis by rotations around the axis of photon momentum characterized by angles α and β. The probabilities for observing polarizations (i,j), where i,j is taken Z_2 valued variable for a convenience of notation are $P_{ij}(\alpha,\beta)$, are predicted to be $P_{00} = P_{11} = cos^2(\alpha-\beta)/2$ and $P_{01} = P_{10} = sin^2(\alpha-\beta)/2$.

Consider now the discrepancies.

1. One has four identities $P_{i,i} + P_{i,i+1} = P_{ii} + P_{i+1,i} = 1/2$ having interpretation in terms of probability conservation. Experimental data of [J7] are not consistent with this prediction [J3] and this is identified as the anomaly.
2. The QM prediction $E(\alpha,\beta) = \sum_i (P_{i,i} - P_{i,i+1}) = cos(2(\alpha-\beta)$ is not satisfied neither: the maxima for the magnitude of E are scaled down by a factor $\simeq .9$. This deviation is not discussed in [J3] .

Both these findings raise the possibility that QM might not be consistent with the data. It turns out that fuzzy quantum logic predicted by TGD and implying that the predictions for the probabilities and correlation must be replaced by ensemble averages, can explain anomaly b) but not anomaly a). A "mundane" explanation for anomaly a) is proposed.

Predictions of fuzzy quantum logic for the probabilities and correlations

1. The description of fuzzy quantum logic in terms statistical ensemble

The fuzzy quantum logic implies that the predictions $P_{i,j}$ for the probabilities should be replaced with ensemble averages over the ensembles defined by fuzzy quantum logic. In practice this means that following replacements should be carried out:

$$\begin{aligned} P_{i,j} \quad &\rightarrow \quad P^2 P_{i,j} + (1-P)^2 P_{i+1,j+1} \\ &+ \quad P(1-P)\left[P_{i,j+1} + P_{i+1,j}\right] \quad . \end{aligned} \tag{3.7.1}$$

Here P is one of the state dependent universal probabilities/fuzzy truth values for some value of n characterizing the measurement situation. The concrete predictions would be following

$$\begin{aligned} P_{0,0} = P_{1,1} \quad &\rightarrow \quad A\frac{cos^2(\alpha-\beta)}{2} + B\frac{sin^2(\alpha-\beta)}{2} \\ &= \quad (A-B)\frac{cos^2(\alpha-\beta)}{2} + \frac{B}{2} \ , \\ P_{0,1} = P_{1,0} \quad &\rightarrow \quad A\frac{sin^2(\alpha-\beta)}{2} + B\frac{cos^2(\alpha-\beta)}{2} \\ &= \quad (A-B)\frac{sin^2(\alpha-\beta)}{2} + \frac{B}{2} \ , \\ A \quad &= \quad P^2 + (1-P)^2 \ \ , \ \ B = 2P(1-P) \ \ . \end{aligned} \tag{3.7.2}$$

The prediction is that the graphs of probabilities as a function as function of the angle $\alpha - \beta$ are scaled by a factor $1 - 4P(1-P)$ and shifted upwards by $P(1-P)$. The value of P, and one might hope even the value of n labeling Jones inclusion and the integer m labeling the quantum state might be deducible from the experimental data as the upward shift. The basic prediction is that the maxima of curves measuring probabilities $P_(i,j)$ have minimum at $B/2 = P(1-P)$ and maximum is scaled down to $(A-B)/2 = 1/2 - 2P(1-P)$.

If the P is same for all pairs i,j, the correlation $E = \sum_i (P_{ii} - P_{i,i+1})$ transforms as

$$E(\alpha,\beta) \quad \rightarrow \quad \left[1 - 4P(1-P)\right] E(\alpha,\beta) \ \ . \tag{3.7.3}$$

Only the normalization of $E(\alpha,\beta)$ as a function of $\alpha - \beta$ reducing the magnitude of E occurs. In particular the maximum/minimum of E are scaled down from $E = \pm 1$ to $E = \pm(1 - 4P(1-P))$.

From the figure 1b) of [J3] the scaling down indeed occurs for magnitudes of E with same amount for minimum and maximum. Writing $P = 1 - \epsilon$ one has $A - B \simeq 1 - 4\epsilon$ and $B \simeq 2\epsilon$ so that the maximum is in the first approximation predicted to be at $1 - 4\epsilon$. The graph would give $1 - P \simeq \epsilon \simeq .025$. Thus the model explains the reduction of the magnitude for the maximum and minimum of E which was not however considered to be an anomaly in [J2, J3] .

A further prediction is that the identities $P(i,i) + P(i+1,i) = 1/2$ should still hold true since one has $P_{i,i} + P_{i,i+1} = (A-B)/2 + B = 1$. This is implied also by probability conservation.

The four curves corresponding to these identities do not however co-incide as the figure 6 of [J3] demonstrates. This is regarded as the basic anomaly in [J2, J3] . From the same figure it is also clear that below $\alpha - \beta < 10$ degrees $P_{++} = P_{--}$ $\Delta P_{+-} = -\Delta P_{-+}$ holds true in a reasonable approximation. After that one has also non-vanishing ΔP_{ii} satisfying $\Delta P_{++} = -\Delta P_{--}$. This kind of splittings guarantee the identity $\sum_{ij} P_{ij} = 1$. These splittings are not visible in E.

Since probability conservation requires $P_{ii} + P_{ii+1} = 1$, a mundane explanation for the discrepancy could be that the failure of the conditions $P_{i,i}+P_{ii+1} = 1$ means that the measurement efficiency is too low for P_{+-} and yields too low values of $P_{+-}+P_{--}$ and $P_{+-}+P_{++}$. The constraint $\sum_{ij} P_{ij} = 1$ would then yield too high value for P_{-+}. Similar reduction of measurement efficiency for P_{++} could explain the splitting for $\alpha - \beta > 10$ degrees.

Clearly asymmetry with respect to exchange of photons or of detectors is in question.

1. The asymmetry of two photon state with respect to the exchange of photons could be considered as a source of asymmetry. This would mean that the photons are not maximally entangled. This could be seen as an alternative "mundane" explanation.
2. The assumption that the parameter P is different for the detectors does not change the situation as is easy to check.
3. One manner to achieve splittings which resemble observed splittings is to assume that the value of the probability parameter P depends on the *polarization pair*: $P = P(i,j)$ so that one has $(P(-,+), P(+,-)) = (P+\Delta, P-\Delta)$ and $(P(-,-), P(+,+)) = (P+\Delta_1, P-\Delta_1)$. $\Delta \simeq .025$ and $\Delta_1 \simeq \Delta/2$ could produce the observed splittings qualitatively. One would however always have $P(i,i) + P(i,i+1) \geq 1/2$. Only if the procedure extracting the correlations uses the constraint $\sum_{i,j} P_{ij} = 1$ effectively inducing a constant shift of P_{ij} downwards an asymmetry of observed kind can result. A further objection is that there are no special reason for the values of $P(i,j)$ to satisfy the constraints.

2. Is it possible to say anything about the value of n in the case of EPR-Bohm experiment?

To explain the reduction of the maximum magnitudes of the correlation E from 1 to $\sim .9$ in the experiment discussed above one should have $p_1 \simeq .9$. It is interesting to look whether this allows to deduce any information about the valued of n. At the limit of large values of $N_i n$ one would have $(N_1 - N_2)/(N_1 + N_2) \simeq .4$ so that one cannot say anything about n in this case. $(N_1, N_2) = (3, 1)$ satisfies the condition exactly. For $n = 3$, the smallest possible value of n, this would give $p_1 \simeq .88$ and for $n = 4$ $p_1 = .41$. With high enough precision it might be possible to select between $n = 3$ and $n = 4$ options if small values of N_i are accepted.

3.7.8 Category Theoretic Formulation For Quantum Measurement Theory With Finite Measurement Resolution?

I have been trying to understand whether category theory might provide some deeper understanding about quantum TGD, not just as a powerful organizer of fuzzy thoughts but also as a tool providing genuine physical insights. Marni Dee Sheppeard (or Kea in her blog Arcadian Functor at `http://tinyurl.com/yb3lsbjq`) is also interested in categories but in much more technical sense. Her dream is to find a category theoretical formulation of M-theory as something, which is not the 11-D something making me rather unhappy as a physicist with second foot still deep in the muds of low energy phenomenology.

Locales, frames, Sierpinski topologies and Sierpinski space

The ideas below popped up when Kea mentioned in M-theory lesson 51 the notions of locale and frame [A5] . In Wikipedia I learned that complete Heyting algebras, which are fundamental to category theory, are objects of three categories with differing arrows. CHey, Loc and its opposite category Frm (arrows reversed). Complete Heyting algebras are partially ordered sets which are complete lattices. Besides the basic logical operations there is also algebra multiplication (I have considered the possible role of categories and Heyting algebras in TGD in [K28]). From Wikipedia I also learned that locales and the dual notion of frames form the foundation of pointless topology [A12] . These topologies are important in topos theory which does not assume axiom of choice.

The so called particular point topology [A10] assumes a selection of single point but I have the physicist's feeling that it is otherwise rather near to pointless topology. Sierpinski topology [A14] is this kind of topology. Sierpinski topology is defined in a simple manner: the set is open only if it contains a given preferred point p. The dual of this topology defined in the obvious sense exists also. Sierpinski space consisting of just two points 0 and 1 is the universal building block of these topologies in the sense that a map of an arbitrary space to Sierpinski space provides it with Sierpinski topology as the induced topology. In category theoretical terms Sierpinski space is the initial object in the category of frames and terminal object in the dual category of locales. This category theoretic reductionism looks highly attractive.

Particular point topologies, their generalization, and number theoretical braids

Pointless, or rather particular point topologies might be very interesting from physicist's point of view. After all, every classical physical measurement has a finite space-time resolution. In TGD framework discretization by number theoretic braids replaces partonic 2-surface with a discrete set consisting of algebraic points in some extension of rationals: this brings in mind something which might be called a topology with a set of particular algebraic points. Could this preferred set belongs to any open set in the particular point topology appropriate in this situation?

Perhaps the physical variant for the axiom of choice could be restricted so that only sets of algebraic points in some extension of rationals can be chosen freely and the choices is defined by the intersection of p-adic and real partonic 2-surfaces and in the framework of TGD inspired theory of consciousness would thus involve the interaction of cognition with the material world. The extension would depend on the position of the physical system in the algebraic evolutionary hierarchy defining also a cognitive hierarchy. Certainly this would fit very nicely to the formulation of quantum TGD unifying real and p-adic physics by gluing real and p-adic number fields to single super-structure via common algebraic points.

Analogs of particular point topologies at the level of state space: finite measurement resolution

There is also a finite measurement resolution in Hilbert space sense not taken into account in the standard quantum measurement theory based on factors of type I. In TGD framework one indeed introduces quantum measurement theory with a finite measurement resolution so that complex rays become included hyper-finite factors of type II_1 (HFFs).

1. Could topology with particular algebraic points have a generalization allowing a category theoretic formulation of the quantum measurement theory without states identified as complex rays?
2. How to achieve this? In the transition of ordinary Boolean logic to quantum logic in the old fashioned sense (von Neuman again!) the set of subsets is replaced with the set of subspaces of Hilbert space. Perhaps this transition has a counterpart as a transition from Sierpinski topology to a structure in which sub-spaces of Hilbert space are quantum sub-spaces with complex rays replaced with the orbits of subalgebra defining the measurement resolution. Sierpinski space {0,1} would in this generalization be replaced with the quantum counterpart of the space of 2-spinors. Perhaps one should also introduce q-category theory with Heyting algebra being replaced with q-quantum logic.

Fuzzy quantum logic as counterpart for Sierpinksi space

The program formulated above might indeed make sense. The lucky association induced by Kea's blog was to the ideas about fuzzy quantum logic realized in terms of quantum 2-spinor that I had developed a couple of years ago. Fuzzy quantum logic would reflect the finite measurement resolution. I just list the pieces of the argument.

Spinors and qbits: Spinors define a quantal variant of Boolean statements, qbits. One can however go further and define the notion of quantum qbit, qqbit. I indeed did this for couple of years ago (the last section of this chapter).

Q-spinors and qqbits: For q-spinors the two components a and b are not commuting numbers but non-Hermitian operators: $ab = qba$, q a root of unity. This means that one cannot measure both a and b simultaneously, only either of them. $aa^\dagger$ and $bb^\dagger$ however commute so that probabilities for bits 1 and 0 can be measured simultaneously. State function reduction is not possible to a state in which a or b gives zero. The interpretation is that one has q-logic is inherently fuzzy: there are no absolute truths or falsehoods. One can actually predict the spectrum of eigenvalues of probabilities for say 1. Obviously quantum spinors would be state space counterparts of Sierpinski space and for $q \neq 1$ the choice of preferred spinor component is very natural. Perhaps this fuzzy quantum logic replaces the logic defined by the Heyting algebra.

Q-locale: Could one think of generalizing the notion of locale to quantum locale by using the idea that sets are replaced by sub-spaces of Hilbert space in the conventional quantum logic. Q-openness would be defined by identifying quantum spinors as the initial object, q-Sierpinski space. a (resp. b for the dual category) would define q-open set in this space. Q-open sets for other quantum spaces would be defined as inverse images of a (resp. b) for morphisms to this space. Only for q=1 one could have the q-counterpart of rather uninteresting topology in which all sets are open and every map is continuous.

Q-locale and HFFs: The q-Sierpinski character of q-spinors would conform with the very special role of Clifford algebra in the theory of HFFs, in particular, the special role of Jones inclusions to which one can assign spinor representations of $SU(2)$. The Clifford algebra and spinors of the world of classical worlds identifiable as Fock space of quark and lepton spinors is the fundamental example in which 2-spinors and corresponding Clifford algebra serves as basic building brick although tensor powers of any matrix algebra provides a representation of HFF.

Q-measurement theory: Finite measurement resolution (q-quantum measurement theory) means that complex rays are replaced by sub-algebra rays. This would force the Jones inclusions associated with $SU(2)$ spinor representation and would be characterized by quantum phase q and bring in the q-topology and q-spinors. Fuzzyness of qqbits of course correlates with the finite measurement resolution.

Q-n-logos: For other q-representations of SU(2) and for representations of compact groups (Appendix) one would obtain something which might have something to do with quantum n-logos, quantum generalization of n-valued logic. All of these would be however less fundamental and induced by q-morphisms to the fundamental representation in terms of spinors of the world of classical worlds. What would be however very nice that if these q-morphisms are constructible explicitly it would become possible to build up q-representations of various groups using the fundamental physical realization - and as I have conjectured [K94] - McKay correspondence and huge variety of its generalizations would emerge in this manner.

The analogs of Sierpinski spaces: The discrete subgroups of $SU(2)$, and quite generally, the groups Z_n associated with Jones inclusions and leaving the choice of quantization axes invariant, bring in mind the n-point analogs of Sierpinski space with unit element defining the particular point. Note however that $n \geq 3$ holds true always so that one does not obtain Sierpinski space itself. If all these n preferred points belong to any open set it would not be possible to decompose this preferred set to two subsets belonging to disjoint open sets. Recall that the generalized embedding space related to the quantization of Planck constant is obtained by gluing together coverings $M^4 \times CP_2 \rightarrow M^4 \times CP_2/G_a \times G_b$ along their common points of base spaces. The topology in question would mean that if some point in the covering belongs to an open set, all of them do so. The interpretation would be that the points of fiber form a single inseparable quantal unit.

Number theoretical braids identified as as subsets of the intersection of real and p-adic variants of algebraic partonic 2-surface define a second candidate for the generalized Sierpinski space with a set of preferred points.

Chapter 4

Trying to fuse the basic mathematical ideas of quantum TGD to a single coherent whole

4.1 Introduction

I have had a very interesting discussions with Baba Ilya Iyo Azza about von Neumann algebras [A64]. I have a background of physicist and have suffered a lot of frustration in trying to understand hyperfinite factors of type II_1 (HFFs, `https://cutt.ly/OX8uP32`) by trying to read mathematicians' articles.

I cannot understand without a physical interpretation and associations to my own big vision TGD. Again I stared at the basic definitions, ideas and concepts trying to build a physical interpretation. This is not my first attempt to understand the possible role of HFFs in TGD: I have written already earlier of the possible role of von Neumann algebras in the TGD framework [K125, K48]. In the sequel I try to summarize what I have possibly understood with my meager technical background.

In the first section I will redescribe the basic notions and ideas related to von Neumann algebras as I see them now, in particular HFFs, which seem to be especially relevant for TGD because of their "hyperfiniteness" property implying that they are effectively finite-D matrix algebras.

There are also more general factors of type II_1, in particular those related to the notion of free probability (`hhttps://cutt.ly/SX2ftyx`), which is a notion related to a theory of non-commutative random variables. The free group generated by a finite number of generators is basic notion and the group algebras associated with free groups are factors of type II_1. The isomorphism problem asks whether these algebras are isomorphic for different numbers of generators. These algebras are not hyperfinite and from the physics point of view this is not a good news.

4.1.1 Basic notions of HFFs from TGD perspective

In this section I will describe my recent, still rather primitive physicist's understanding of HFFs. Factor M and its commutant M' are central notions in the theory of von Neumann algebras. An important question, not discussed earlier, concerns the physical counterparts of M and M'. I will not discuss technical details: I have made at least a noble attempt to do this earlier [K125, K48].

1. In the TGD framework, one can distinguish between quantum degrees of freedom and classical ones, and classical physics can be said to be an exact part of quantum physics.
2. The formulation of physics as Kähler geometry of the "world of classical worlds" (WCW) is briefly summarized in the Appendix. The formulation involves hierarchies A_n of 3 kinds of algebras; supersymplectic algebras SSA_n acting on $\delta M^4_+ \times CP_2$ and assumed to induce isometries of WCW, affine algebras Aff_n associated with isometries and holonomies of $H = M^4 \times CP_2$ acting on light-like partonic orbits, and isometries I_n of the light-cone boundary δM^4_+.

At the H-side, quantum degrees of freedom are assignable to A_n, which would correspond to M.

In zero energy ontology (ZEO) [K130] states are quantum superpositions of preferred extremals. Preferred extremals depend on zero modes, which are symplectic invariants and do not appear in the line element of WCW. Zero modes serve as classical variables, which commute with super symplectic transformations and could correspond to M' for SSA_n at H-side. Similar identification of analogs of zero modes should be possible for Aff_n and I_n.

3. In the number theoretic sector at the M^8-side, braided group algebras would correspond to quantum degrees of freedom, that is M. M' would correspond to some number theoretic invariants of polynomials P determining the space-time surface in H by $M^8 - H$ duality [L100, L101]. The set of roots of P and ramified primes dividing the discriminant of P are such invariants.

4.1.2 Bird's eye view of HFFs in TGD

A rough bird's eye view of HFFs is discussed with an emphasis on their physical interpretation. There are two visions of TGD: the number theoretic view [L50, L49] and the geometric view [K57, K36, K126, K95] and $M^8 - H$ duality relates these views [L122, L100, L101].

1. At the M^8 side, the p-adic representations of braided group algebras of Galois groups associated with hierarchies of extensions of rationals define natural candidates for the inclusion hierarchies of HFFs.

   Braid groups represent basically permutations of tensor factors and the same applies to the braided Galois groups with S_n restricted to the Galois group.

   A good guess is that braid strands correspond to the roots of a polynomial labelling mass shells H^3 in $M^4 \subset M^8$.

2. The 3-D mass shells define a 4-surface in M^8 by holography based on associativity, which makes possible holography.

   The condition that the normal space N of the 4-surface X^4 in M^8 is associative and contains a 2-D commutative sub-space X^2, guarantees both holography and $M^8 - H$ duality mapping this 4-surface $X^4 \subset M^8$ to a space-time surface $Y^4 \subset H$.

   The 2-D commutative space $X^2 \subset N$ can be regarded as a normal space of the 6-D counterpart of twistor space $T(M^4)$. $T(M^4)$ is mapped by $M^8 - H$ duality to a point of the twistor space $T(CP_2) = SU(3)/U(1) \times U(1)$ of CP_2. This map is assumed to define the twistor space $T(Y^4) \subset T(M^4) \times T(CP_2)$ as a preferred extremal [L126, L127].

3. The physical picture strongly suggests that also string world sheets and partonic 2-surfaces in $Y^4 \subset H$ are needed. They are are assumed to correspond to singularities for the map to H. A natural conjecture is that the 2-D subspace $X^2 \subset N$ is mapped to a 2-D subspace $Y^2 \subset T$ of the tangent space T of X^4 by a multiplication with a preferred octonionic imaginary unit in T.

   How could this preferred octonionic unit be determined?

   (a) Complexified octonionic units in the tangent space of M^8_c decomposes under $SU(3) \subset G_2$, having interpretation as color group, to representations $1_1 \oplus 1_2 \oplus 3 \oplus \overline{3}$. 1_1 and 1_2 correspond to the real unit I_0 and imaginary unit I_1 and 3 and $\overline{3}$ correspond to color triplets analogous to quarks and antiquarks.

   (b) Complexified quaternionic sub-space defining N corresponds to color singlets I_0, I_1, and quarks I_2, I_3 with $(Y = -1/3, I_3 = 1/2)$ and $Y = 2/3, I_3 = 0)$. The complement T corresponds to quark I_4 $(Y = -1/3, I_3 = 1/2)$ and 3 antiquarks (I_5, I_6, I_7). The octonionic multiplication of the units of quaternionic subspace by quark I_4 gives T as the orthogonal complement of the quaternionic sub-space N.

   (c) This multiplication would assign to $X^2 \subset N$ 2-D subspace of T and also its orthogonal complement Y^2 in T. If the distributions of X^2 and Y^2 are integrable, they define the slicing of X^4 by partonic 2-surfaces and string world sheets. The tangent spaces for them would correspond to the local choice of I_0, I_1 and I_2, I_3. X^2 and Y^2 at different points would differ by a local SU(3) transformation. In fact, the 4-surface in M^8 would correspond to a complex color gauge transformation [L100, L101].

This choice could correspond to what I have called Hamilton-Jacobi (H-J) structure [K10] in X^4 defining a slicing of X^4 defined by an integrable distribution of pairs of orthonormal 2-surfaces analogous to the choice of massless wave vector and orthogonal polarization plane depending on the point of X^4 or equivalently on the point of M^4 as its projection. H-J structure would also define the analog of Kähler structure in M^4 strongly suggested by twistor lift.

The original proposal was that the H-J structure is associated with M^4, and one cannot completely exclude the possibility that the projection of the proposed slicing to M^4 defines H-J. The idea about single H-J structure is not physical. Dynamical H-J structure does not conform with the idea that M^4 is completely non-dynamical. However, if the H-J structure is determined by the choice $X^2 \subset N$ and defines H-J structure in X^4, this objection can be circumvented.

At the H side there are 3 algebras.

1. The subalgebras SSA_n of super-symplectic algebra (SSA) are assumed to induce isometries of WCW. Since SSA and also other algebras have non-negative conformal weights, it has a hierarchy of subalgebras SSA_n with conformal weights coming as n-multiples of those for SSA.
2. There are also affine algebras Aff associated with H isometries acting on light-like orbits of partonic 2-surfaces and having similar hierarchy of Aff_n. Both isometries and holonomies of H are involved.
3. Light-cone boundary allows infinite dimensional isometry group I consisting of generalized conformal transformation combined with a local scaling allowing similar hierarchy I_n.

One should understand how the number theoretic and geometric hierarchies relate to each other and a good guess is that braided group algebras act on braids assignable to SSA_n with n interpreted as the number of braid strands and thus the degree n of P.

Also the interpretational problems related to quantum measurement theory and probability interpretation are discussed from the TGD point of view, in which zero energy ontology (ZEO) allows us to solve the basic problem of quantum measurement theory.

4.1.3 M^8-H duality and HFFS

M^8-H duality [L100, L101] suggests that the hierarchies of extensions of rationals at the number theoretic side and hierarchies of HFFs at the geometric side are closely related.

The key idea is that the braided Galois groups at M^8-side interact on algebras $A_n \in \{SSA_n, Aff_n, I_n\}$ at H level as number theoretic braid groups permuting the tensor factors assignable to the braid strands, which correspond to the roots of the polynomial P.

The basic notions associated with a polynomial P with rational coefficients having degree n are its n roots, ramified primes as factors of the discriminant defined by the difference of its roots, and Galois group plus a set of Galois invariants such as symmetric polynomials of roots. The Galois group is the same for a very large number of polynomials P. The question concerns the counterparts of these notions at the level of H?

An educated guess is that the n roots of P label the strands of an n-braid in H assignable to A_n, ramified primes correspond to physically preferred p-adic primes in the adelic structure formed by various p-adic representations $A_{n,p}$ of the algebras A_n and the Galois group algebra associated with the polynomial P with degree n.

This picture suggests a generalization of arithmetics to quantum arithmetics based on the replacement of $+$ and $\times$ with $\oplus$ and $\otimes$ and replacement of numbers with representations of groups or algebras [L130]. This implies a generalization of adele by replacing p-adic numbers with the p-adic quantum counterparts of algebras A_n.

The mysterious McKay correspondence [A79] has inspired several articles during years [L38, L95, L94, L130] but it is fair to say that I do not really understand it. Hence I could not avoid the temptation to attack this mystery also in this article.

4.1.4 Infinite primes

The notion of infinite primes [K105, K71] is one of the ideas inspired by TGD, which has waited for a long time for its application. Their construction is analogous to a quantization of supersymmetric arithmetic quantum field theory.

1. The analog of Dirac sea X is defined by the product of finite primes and one "kicks" from sea a subset of primes defining a square free integer n_F to get the sum $X/n_F + n_F$. One can also add bosons to X/n_F *resp.* n_F multiplying it with integer n_{B_1} *resp.* n_{B_2}, which is divisible only by primes dividing Z/n_F *resp.* n_F.
2. This construction generalizes and one can form polynomials of X to get infinite primes analogous to bound states. One can consider instead of $P(X)$ a polynomial $P(X,Y)$, where Y is the product of all primes at the first level thus involving the product of all infinite primes already constructed, and repeat the procedure. One can repeat the procedure indefinitely and the formal interpretation is as a repeated quantization. The interpretation could be in terms of many-sheeted space-time or abstraction process involving formation of logical statements about statements about ...
3. The polynomials Q could also be interpreted as ordinary polynomials. If $Q(X) = P(X)$, where $P(X)$ is the polynomial defining a 4-surface in M^8, the space-time surface X^4 in H would correspond to infinite prime. This would give a "quantization" of P defining the space-time surface.

   The polynomial P defining 4-surface in H would fix various quantum algebras associated with it. The polynomials $P(X_1, X_2, ...X_n)$ could be interpreted as $n-1$-parameter families defining surfaces in the "world of classical worlds" (WCW) [L122] (for the development of the notion see [K57, K36, K126, K95]).
4. X is analogous to adele and infinite primes could be perhaps seen as a generalization of the notion of adele. One could assign p-adic variants of various HFFs to the primes defining the adele and $+$ and $\times$ could be replaced with $\oplus$ and $\otimes$. The physical interpretation of ramified primes of P is highly interesting.

In the last section, I try to guess how the fusion of these building blocks by using the ideas introduced in the previous sections could give rise to what might be called quantum TGD. It must be made clear that the twistor lift of TGD [L126, L127] is not considered in this work.

4.2 Basic notions related to hyperfinite factors of type II_1 from TGD point of view

In this section, the basic notions of hyperfinite factors (HFFs) as a physicists from the TGD point of view will be discussed. I have considered HFFs earlier several times [K125, K48] and will not discuss here the technical details of various notions.

4.2.1 Basic concepts related to von Neumann algebras

John von Neumann proposed that the algebras, which now carry his name are central for quantum theory [A64]. Von Neumann algebra decomposes to a direct integral of factors appearing and there are 3 types of factors corresponding to types I, II, and III.

Inclusion/embedding as a basic aspect of physics

Inclusion (`https://cutt.ly/NX8eWwa`, `https://cutt.ly/cX8eUuf`, `https://cutt.ly/4X8ePn6`)) is a central notion in the theory of factors. Inclusion/embedding involving induction of various geometric structures is a key element of classical and quantum TGD.

One starts from the algebra $B(H)$ of bounded operators in Hilbert space. This algebra has naturally hermitian conjugation $*$ as an antiunitary operation and therefore one talks of C_* algebras. von Neumann algebra is a subalgebra of $B(H)$. Already here an analog of inclusion is involved (`https://cutt.ly/3XkPO2s`). There are also inclusions between von Neumann algebras, in particular HFFs.

What could the inclusion of von Neumann algebra to $B(H)$ as subalgebra mean physically? In the TGD framework, one can identify several analogies.

1. Space-time is a 4-surface in $H = M^4 \times CP_2$: analog of inclusion reducing degrees of freedom.
2. Space-time is not only an extremal of an action [K10] [L125] but a preferred extremal (PE), which satisfies holography so that it is almost uniquely defined by a 3-surface. This guarantees general coordinate invariance at the level of H without path integral. I talk about preferred extremals (PEs) analogous to Bohr orbits. Space-time surface as PE is a 4-D minimal surface with singularities [L125]: there is an analogy with a soap film spanned by frames. This implies a small failure of determinism localizable at the analogs of frames so that holography is not completely unique.

   Holography means that very few extremals are physically possible. This Bohr orbit property conforms with the Uncertainty Principle. Also HFFs correspond to small sub-spaces of $B(H)$. Quantum classical correspondence suggests that this analogy is not accidental.

The notion of commutant and its physical interpretation in the TGD framework

The notion of the commutant M' of M, which also defines HFF, is also essential. What could be the physical interpretation of M'? TGD suggests 3 important hierarchies of HFFs as algebras A_n. A_n could correspond to super-symplectic algebras SSA_n acting at $\delta M^4_- \times CP_2$; to an affine algebras Aff_n acting at the light-like partonic orbits; or to an isometry algebra I_n acting at δM^4_+. All these HFF candidates have commutants and would have interpretation in terms of quantum-classical correspondence.

One can consider SSA as an example.

1. In TGD, one has indeed an excellent candidate for the commutant. Supersymplectic symmetry algebra (SSA) of $\delta M^4_+ \times CP_2$ (δM^4_+ denotes the boundary of a future directed light-cone) is proposed to act as isometries of the "world of classical worlds" (WCW) consisting of space-time surfaces as PEs (very, very roughly).

   Symplectic symmetries would be generated by Hamiltonians, which are products of Hamiltonians associated with δM^4_+ (metrically sphere S^2) and CP_2. Symplectic symmetries are conjectured to act as isometries of WCW and gamma matrices of WCW extend symplectic symmetries to super-symplectic ones.

   Hamiltonians and their super-counterparts generate the super-symplectic algebra (SSA) and quantum states are created by using them. SSA is a candidate for HFF. Call it M. What about M?
2. The symplectic symmetries leave invariant the induced Kähler forms of CP_2 and contact form of δM^4_+ (assignable to the analog of Kähler structure in M^4).
3. The wave functions in WCW depending of magnetic fluxes defined by these Kähler forms over 2-surfaces are physically observables which commute SSA and with M. These fluxes are in a central role in the classical view about TGD and define what might perhaps be regarded as a dual description necessary to interpret quantum measurements.

   Could M' correspond or at least include the WCW wave functions (actually the scalar parts multiplying WCW spinor fields with WCW spinor for a given 4-surface a fermionic Fock state) depending on these fluxes only? I have previously talked of these degrees of freedom as zero modes commuting with quantum degrees of freedom and of quantum classical correspondence.
4. There is $M - M'$ correspondence also for number theoretic degrees of freedom, which naturally appear from the number theoretic M^8 description mapped to H-description. Polynomials P associated with a given Galois group are analogous to symplectic degrees of freedom with given fluxes as symplectic invariants. Galois groups and Galois invariants are "classical" invariants at the M^8 side and should have counterparts on the H side. For instance, the degree n of polynomial P could correspond to the number of braid stran

More algebraic notions

There are further algebraic notions involved. The article of John Baez (`https://cutt.ly/VXlQyqD`) describes these notions nicely.

1. The condition $M'' = M$ is a defining algebraic condition for von Neumann algebras. What does this mean? Or what could its failure mean? Could M'' be larger than M? It would seem that this condition is achieved by replacing M with M''.

   $M'' = M$ codes algebraically the notion of weak continuity, which is motivated by the idea that functions of operators obtained by replacing classical observable by its quantum counterpart are also observables. This requires the notion of continuity. Every sequence of operators must approach an operator belonging to the von Neumann algebra and this can be required in a weak sense, that is for matrix elements of the operators.

   Does $M'' = M$ mean that the classical descriptions and quantum descriptions are somehow equivalent? At first, this looks nonsensical but when one notices that the scalar parts of WCW spinor fields correspond to wave functions in the zero mode of WCW which do not appear in the line element of WCW, this idea starts to look more sensible. In quantum measurements the outcome is indeed expressed in terms of classical variables. Zero modes and quantum fluctuating modes would provide dual descriptions of physics.

2. There is also the notion of hermitian conjugation defined by an antiunitary operator J: $a^\dagger = JAJ$. This operator is absolutely essential in quantum theory and in the TGD framework it is geometrized in terms of the Kähler form of WCW. The idea is that entire quantum theory, rather than only gravitation or gravitation and gauge interactions should be geometrized. Left multiplication by JaJ corresponds to right multiplication by a.

Connes tensor product and category theoretic notions

Connes tensor product (Connes fusion) [A20] appears in the construction of the hierarchy of inclusions of HFFs. For instance, matrix multiplication has an interpretation as Connes tensor product reduct tensor product of matrices to a matrix product. The number of degrees of freedom is reduced. The tensor product $A \otimes_R B$ depends on the coefficient ring R acting as right multiplication in A and left multiplication in B. If the dimension of R increases, the dimension of A (B) as a left/right R module is reduced. For instance, A as an A-module is 1-dimensional.

Also category theory related algebraic notions appear. I still do not have an intuitive grasp about category theory. In any case, one would have a so-called 2-category (`https://cutt.ly/3XkPO2s`). M and N correspond to 0-morphisms (objects). One can multiply arguments of functions in $L^2(M)$ and $L^2(N)$ by M or N.

Bimodule (`https://cutt.ly/EX885WA`) is a key notion. For instance the set of $R_{m,n}$ of $m \times n$ matrices is a bimodule, which is a left (right) module with respect to $m \times m$ ($n \times n$) matrices. One can replace matrices with algebras. The bimodule ${}_M M_M$ *resp.* ${}_N N_N$ is analogous to $m \times m$ *resp.* $n \times n$ matrices. They correspond to 1-morphisms, which behave like units. The bimodule ${}_M N_N$ *resp.* ${}_N N_M$ is analogous to $m \times n$ *resp.* $n \times m$ matrices. These two bimodules correspond to a generating 1-morphisms mapping N to M *resp.* M to N. Bimodule map corresponds to 2-morphisms. Connes tensor product defines what category theorists call a tensor functory.

The notions of factor and trace

The notion of factor as a building block of more complex structures is central and analogous to the notion of simple group or prime. Factor is a von Neumann algebra, which is simple in the sense that it has a trivial center consisting of multiples of unit operators. The algebra is direct sum or integral over different factors.

The notion of trace is fundamental and highly counter intuitive. For the factors of type I, it is just the ordinary trace and the trace $Tr(I)$ of the unit operator is equal to the dimension n of the Hilbert space. This notion is natural when direct sum is the key notion. For the other factors, the situation is different.

Factors can be classified into three types: $I, II, and III$.

1. For factors of type I associated with three bosons, the trace equals n in the n-D case and ∞ in the infinite-D case.

2. A highly non-intuitive and non-trivial axiom relating to HFFs as hyperfinite factors of type II_1 is that the trace of the unit operator satisfies $Tr(Id) = 1$: for factors of type II (see the

article of Popa at https://cutt.ly/KX8y0Fs). This definition is natural in the sense that being a subsystem means being a tensor factor rather than subspace.

The intuitive idea is that the density matrix for an infinite-D system identified as a unit operator gives as its trace total probability equal to one. These factors emerge naturally for free fermions. "Hyperfinite" expresses the fact that the approximation of a factor with its finite-D cutoff is an excellent approximation.

HFFs are extremely flexible and can look like arbitrarily high-dimensional factor I_n. For instance, one can extract any matrix algebra $M^n(C)$ as a tensor factor so that one has $M = M^n(C) \otimes M^{1/n}$ by the multiplicativity of dimensions in the tensor product. Should one interpret this by saying that measurement can separate from a factor an $n-D$ Hilbert space and that $M^{1/n}$ is something that remains inaccessible to the measurements considered? If one introduces the notion of measurement resolution in this manner, the description of measurement could be based on factors of tyoe I_n.

3. The factors of type II_∞ are tensor products of infinite-D factors of type I and HFFs and could describe free bosons and fermions.

4. In quantum field theory (QFT), factors of type III appear and in this case the notion of trace becomes useless. These factors are pathological and in QFT they lead to divergence difficulties. The physical reason is the idea about point-like particles, which is too strong an idealization.

   In the TGD framework, the generalization of a point-like particle to 3-surface saves from these difficulties and leads to factors of type I and HFFs. In TGD, finite measurement resolution is realized in terms of a unique number theoretic discretization, which further simplifies the situation in the TGD framework.

4.2.2 Standard construction for the hierarchy of HFFs

Consider now the standard construction leading to a hierarchy of HFFs and their inclusions.

1. One starts from an inclusion $M \subset N$ of HFFs. I will later consider what these algebras could be in the TGD framework.

2. One introduces the spaces $L^2(M)$ *resp.* $L^2(N)$ of square integrable functions in M *resp.* N.

   From the physics point of view, bringing in "L^2" is something extremely non-trivial. Space is replaced with wave functions in space: this corresponds to what is done in wave mechanics, that is quantization! One quantizes in M, particles as points of M are replaced by wave functions in M, one might say.

3. At the next step one introduces the projection operator e as a projection from $L^2(N)$ to $L^2(M)$: this is like projecting wave functions in N to wave functions in M. I wish I could really understand the physical meaning of this. The induction procedure for second quantized spinor fields in H to the space-time surface by restriction is completely analogous to this procedure.

   After that one generates a HFF as an algebra generated by e and $L^2(N)$: call it $\langle L^2(N), e\rangle$. One has now 3 HFFs and their inclusions: $M_0 \equiv M$, $M_1 \equiv N$, and $\langle L^2(N), e\rangle \equiv M_2$.

   An interesting question is whether this process could generalize to the level of induced spinor fields?

4. Even this is not enough! One constructs $L^2(M_2) \equiv M_3$ including M_2. One can continue this indefinitely. Physically this means a repeated quantization.

   One could ask whether one could build a hierarchy M_0, $L^2(M_0)$,..., $L^2(L^2...(M_0))..)$: why is this not done?

   The hierarchy of projectors e_i to M_i defines what is called Temperley-Lieb algebra [A94] involving quantum phase $q = exp(i\pi/n)$ as a parameter. This algebra resembles that of S_∞ but differs from it in that one has projectors instead of group elements. For the braid group $e_i^2 = 1$ is replaced with a sum of terms proportional to e_i and unit matrix: mixture of projector and permutation is in question.

5. There is a fascinating connection in TGD and theory of consciousness. The construction of what I call infinite primes [K105, K71] is structurally like a repeated second quantization of a supersymmetric arithmetic quantum field theory involving fermions and bosons labelled by the primes of a given level I conjectured that it corresponds physically to quantum theory in the many-sheeted space-time.

   Also an interpretation in terms of a hierarchy of statements about statements about .... bringing in mind hierarchy of logics comes to mind. Cognition involves generation of reflective levels and this could have the quantization in the proposed sense as a quantum physical correlate.

4.2.3 Classification of inclusions of HFFs using extended ADE diagrams

Extended ADE Dynkin diagrams for ADE Lie groups, which correspond to finite subgroups of $SU(2)$ by McKay correspondence [A79, A77, A59], discussed from the TGD point of view in [L130], characterize inclusions of HFFs.

For a subset of ADE groups not containing E_7 and D_{2n+1}, there are inclusions, which correspond to Dynkin diagrams corresponding quantum groups. What is interesting that E_6 (tetrahedron) and E_8 (icosahedron/dodecahedron) appear in the TGD based model of bioharmony and genetic code but not E_7 (cube and octahedron) [L116].

1. Why finite subgroups of $SU(2)$ (or $SU(2)_q$) should characterize the inclusions in the tunnel hierarchies with the same value of the quantum dimension $M_{n+1} : M_n$ of quantum group?

   In the TGD interpretation M_{n+1} reduces to a tensor product of M_n and quantum group, when M_n represents reduced measurement resolution and quantum group the added degrees of freedom. Quantum groups would represent the reduced degrees of freedom. This has a number theoretical counterpart in terms of finite measurement resolution obtained when an extension of ... of rationals is reduced to a smaller extension. The braided relative Galois group would represent the new degrees of freedom.

2. One can algebraically identify HFF as a "tunnel" obtained by iterated standard construction as an infinite tensor power of $GL(2, c)$ or $GL(n, C)$. The analog of the McKay graph for the irreps of a closed subgroup of $GL(2, C)$ defines an invariant characterizing the fusion rules involved with the reduction of the Connes tensor products. This invariant reduces to the McKay graph for the tensor products of the canonical 2-D representation with the irreps of a *finite* rather than only closed subgroups of $SU(2)$. This must take place also for $GL(n, C)$. Why?

   The reduction of degrees of freedom implied by the Connes tensor product seems to imply a discretization at the level of $SU(2)$ and replace closed subgroups of $SU(2)$ with finite subgroups. This conforms with the similarity of the representation theories of discrete and closed groups. In the case of quantum group representations only a finite number of ordinary finite-D group representations survive.

   All this conforms with the TGD view about the equivalence of number-theoretic discretization and inclusions as descriptions of finite measurement resolution.

In the TGD framework, $SU(2)$ could correspond to a covering group of quaternionic automorphisms and number theoretic discretization (cognitive representations) would naturally lead to discrete and finite subgroups of $SU(2)$.

4.3 TGD and hyperfinite factors of type II_1: a bird's eye of view

In this section, a tentative identification of hyperfinite factors of type II_1 (HFFS) in the TGD framework [K125, K48] is discussed. Also some general related to the interpretation of HFFs and their possible resolution in the TGD framework are considered.

4.3.1 Identification of HFFs in the TGD framework

Inclusion hierarchies of extensions of rationals and of HFFs

I have enjoyed discussions with Baba Ilya Iyo Azza about von Neumann algebras. Hyperfinite factors of type II_1 (HFF) (https://cutt.ly/lXp6MDB) are the most interesting von Neumann algebras from the TGD point of view. One of the conjectures motivated by TGD based physics, is that the inclusion sequences of extensions of rationals defined by compositions of polynomials define inclusion sequences of hyperfinite factors. It seems that this conjecture might hold true!

Already von Neumann demonstrated that group algebras of groups G satisfying certain additional constraints give rise to von Neuman algebras. For finite groups they correspond to factors of type I in finite-D Hilbert spaces.

The group G must have an infinite number of elements and satisfy some additional conditions to give a HFF. First of all, all its conjugacy classes must have an infinite number of elements. Secondly, G must be amenable. This condition is not anymore algebraic. Braid groups define HFFs.

To see what is involved, let us start from the group algebra of a finite group G. It gives a finite-D Hilbert space, factor of type I.

1. Consider next the braid groups B_n, which are coverings of S_n. One can check from Wikipedia that the relations for the braid group B_n are obtained as a covering group of S_n by giving up the condition that the permutations σ_i of nearby elements e_i, e_{i+1} are idempotent. Could the corresponding braid group algebra define HFF?

   It is. The number of conjugacy classes $g_i\sigma_i g_i^{-1}$, $g_i == \sigma_{i+1}$ is infinite. If one poses the additional condition $\sigma_i^2 = U \times 1$, U a root of unity, the number is finite. Amenability is too technical a property for me but from Wikipedia one learns that all group algebras, also those of the braid group, are hyperfinite factors of type II_1 (HFFs).

2. Any finite group is a subgroup G of some S_n. Could one obtain the braid group of G and corresponding group algebra as a sub-algebra of group algebra of B_n, which is HFF. This looks plausible.

3. Could the inclusion for HFFs correspond to an inclusion for braid variants of corresponding finite group algebras? Or should some additional conditions be satisfied? What the conditions could be?

Here the number theoretic view of TGD could comes to the rescue.

1. In the TGD framework, I am primarily interested in Galois groups. The vision/conjecture is that the inclusion hierarchies of extensions of rationals correspond to the inclusion hierarchies for hyperfinite factors. The hierarchies of extensions of rationals defined by the hierarchies of composite polynomials $P_n \circ ... \circ P_1$ have Galois groups, which define a hierarchy of relative Galois groups such that the Galois group G_k is a normal subgroup of G_{k+1}. One can say that the Galois group G is a semidirect product of the relative Galois groups.

2. One can decompose any finite subgroup to a maximal number of normal subgroups, which are simple and therefore do not have a further decomposition. They are primes in the category of groups.

3. Could the prime HFFs correspond to the braid group algebras of simple finite groups acting as Galois groups? Therefore prime groups would map to prime HFFs and the inclusion hierarchies of Galois groups induced by composite polynomials would define inclusion hierarchies of HFFs just as speculated.

   One would have a deep connection between number theory and HFFs.

How could HFFs emerge in TGD?

What could HFFs correspond to in the TGD framework? Consider first the situation at the level of M^8.

1. Braid group $B(G)$ of group (say Galois group as subgroup of S_n) and its group algebra would correspond to $B(G)$ and $L^2(B(G)$). Braided Galois group and its group algebra could correspond to $B(G)$ and $L^2(B(G))$.

The inclusion of Galois group algebra of extension to its extension could naturally define a Connes tensor product. The additional degrees of freedom brought in by extension of extension would be below measurement resolution.

2. Composite polynomials $P_n \circ .... \circ P_1$ are used instead of a product of polynomials naturally characterizing free n-particle states. Composition would describe interaction physically: the degree is the product of degrees of factors for a composite polynomial and sum for the product of polynomials.

   The multiplication rule for the dimensions holds also for the tensor product so that functional composition could be also seen as a number theoretic correlate for the formation of interacting many particle states.

3. Compositeness implies correlations and formation of bound states so that the number of degrees of freedom is reduced. The interpretation as bound state entanglement is suggestive. This hierarchical entanglement could be assigned with directed attention in the TGD inspired theory of consciousness [L111].

   An alternative interpretation is in terms of braids of braids of ... of braids with braid strands at a given level characterized by the roots of P_i. These interpretations could be actually consistent with each other.

4. Composite polynomials define hierarchies of Galois groups such that the included Galois group is a normal subgroup. This kind of hierarchy could define an increasing sequence of inclusions of braided Galois groups.

Consider the situation at the H level.

1. At the level of H, elements of the algebras $A \in \{SSA, Aff, I\}$a associated with super-symplectic symmetries acting at δM^4_+, affine isometries acting at light-like partonic orbites, isometries of δM^4_+, are labelled by conformal weights coming as non-negative integers. Also algebraic integers can be considered but for physical states conformal confinement requires integer valued conformal weights.

2. One can construct a hierarchy of representations of A such that subalgebras A_n with conformal weights $h \geq 0$ coming as multiples of n and the commutator $[A_n, A]$ annihilate the physical states. These representations are analogous to quantum groups and one can say that A_n defines a finite measurement resolution in A. A_{nk}, $k \geq 1$ is included by A_n for and one has a reversed sequence of inclusions.

   One can construct inclusion hierarchies defined by the sequences $1 \div n_1 \div n_2 \div ....$ "$n_{-1} = 1$" corresponds to SSA. The factor spaces $A_{n_k}/A_{n_{k+1}}$ are analogs of quantum group-like objects associated with Jones inclusions and the interpretation is in terms of finite measurement resolution defined by $A_{n_{k+1}}$.

   The factor spaces A/A_{n_k} define inclusion hierarchies with an increasing measurement resolution.

4.3.2 Could the notion of free probability be relevant in TGD?

In discussions with Baba Ilya Iyo Azza, I learned about the notion of free probability (`https://cutt.ly/LCY51sy`) assignable to von Neumann algebras. This algebra is II_1 factor. Originally, the notion was discovered by Voiolescu [A93] in order to attack some operator algebra problems, in particular free group factor isomorphism problem and Voiolescu demonstrated there is an infinity of von Neumann free group factors, which can be isomorphic. One can ask whether the free probability could have physical applications. In particular, whether the HFFs emerging naturally in TGD are consistent with this notion.

I try first to describe the notion of free probability as I understand it, on basis of what I learned in the discussions.

1. Free probability theory and classical probability theory differ because the latter is commutative and the former is highly noncommutative, and the notion of independence differs for them.

   In the classical theory, the expectations of the variables $X, Y, ...$ are commutative whereas in free probability theory they become observables represented by operators, which in general are non-commutative. The expectations for independent variables satisfy $E(XY) = E(X)E(Y)$ and more generally $E(X^mY^n) = E(X^m)E(Y^n)$. The expectations for powers are called moments.

The free probability theory generalizes independent variables to free, in general non-commutative, operators $a, b, ...$ of von Neumann algebra M. The mean value $E(X)$ is replaced with a vacuum expectation value $\tau(a)$ of a, as physicists would call it. The expectations define what mathematicians call a normal state. Here τ, defining the vacuum expectation, denotes a linear functional in M.

The random variable a can act on the argument of square integrable functions $F(m)$ defined in non-commutative von Neumann algebra M defining a commutative algebra $L^2(M)$. The action of a is non-commutative right or left multiplication of the argument of $F(m)$. One can speak of non-commutative probability space.

Could group algebras and braid group algebra represent free algebras? Unfornatunately not. It is known that HFF probability is not consistent with free algebra property.

2. In the classical theory of independent random variables, one has $E(XY) = E(X)E(Y)$ and it is possible to express all expectations of monomials of $X_1, X_2, ...$ of polynomials of variables $X_1, X_2, ..$ in terms of moments $E(X_i^n)$.

   For free probability theory an analogous situation prevails although the formulas are not identical. Consider factor M of type II_1, which in the case ff free algebras cannot be hyperfinite. A linear functional $\tau(a)$ corresponds to vacuum expectation value, using the language of physicists. One has $\tau(1) = 1$. This corresponds to the condition $Tr(Id) = 1$. One has pointless space in the sense that the projector to a ray of Hilbert state defined by M has a vanishing trace. This corresponds to a finite measurement resolution requiring that the trace of the projector characterizing quantum measurement is a nonvanishing number.

   $\tau(ab) = \tau(a)\tau(b)$ is true for the generators of the free algebra and states that there is no correlation between a and b. This is however not true in general.

   [Note that an analogous condition holds true for the correlators of free quantum field fields at the level of momentum space: the n-point correlation functions reduce to products of momentum space propagators.]

   For instance, one would have

$$\tau(abab) = \tau(a^2)\tau(b)^2 + \tau(a)^2\tau(b^2) - 2\tau(a)^2\tau(b)^2$$

   instead of $E(XYXY) = E(X^2)E(Y^2)$ for classical independent variables. Also now however only powers of a and b appear in the formula. This reduction of the expectations to the momenta $\tau(a^n)$ would hold quite generally.

3. A more precise definition is as follows (`https://cutt.ly/LCY51sy`). Unital subalgebras $A_1, ..., A_m$ are said to be freely independent if the expectation of the product $a_1...a_n$ is zero whenever each a_j has zero expectation, lies in an A_k, no *adjacent* a_j's come from the same subalgebra A_k, and n is nonzero. Random variables are freely independent if they generate freely independent unital subalgebras.

4. The lattice of non-crossing partitions (`https://cutt.ly/jCY6jNe`) for a finite set ordered cyclically, distinguishes free probability theory from lattice of all partition in the theory if independent random variables. Two partitions ab and xy are non-crossing if their elements do not correspond to order $axby$. The subsets of a non-crossing partition consist of elements, which are adjacent in this ordering and form connected subsets with k_i elements in which a cyclic subgroup $Z_{k_i} \subset Z_n$ acts. The expression of an element of S_n as a product of elements of cyclic subgroups Z_{n_k} of S_n corresponds to this kind of partition.

   Interestingly, in the construction of the non-planar parts of the twistor amplitudes similar cyclic ordering plays an important role. The problem of the twistor constructed are non-planar amplitudes which do not allow cyclic ordering. Could it be that the non-planar parts of the amplitudes do not have counterparts in a deeper theory utilizing HFFs? If so, free probability could code a very profound aspect of quantum theory.

Free random variables could correspond to the generators of von Neumann algebra of type II_1. My un-educated guess was that also HFFs realize free probability. I was wrong: thanks for

Baba Ilya Iyo Azza for noticing this. It however seems that something highly reminiscent of free probability emerges for the algebras involved with TGD.

1. In the TGD framework, the generators typically generate an algebra of observables having interpretation as algebra of symmetries, such as affine algebra or super symplectic algebra.
2. ab would correspond to the product of say affine algebra generators a and b labelled by quantum numbers which are additive in the product. $\tau(a)$ would vanish as a vacuum expectation value of a generator with non-vanishing quantum numbers so that for generators one should have $\tau(ab) = \tau(a)\tau(b) = 0$.

   ab is expressible in terms of commutator and anticommutator as the sum $[a, b]/2 + \{a, b\}/2$. Both terms vanish if the quantum numbers of ab are non-vanishing. Only when the quantum numbers of a and b are opposite, the vanishing need not take place. If a and its Hermitian conjugate $a^\dagger$ with opposite quantum numbers belong to the set of generators of the free algebra, one has $\tau(aa^\dagger) > 0$ and is different from $\tau(a)\tau(a^\dagger) = 0$.

   Therefore the hermitian conjugates of generators cannot belong to the generators of the algebra creating the physical states. This algebra is highly reminiscent of free algebra since all vacuum expectations for the products vanish.
3. For affine and conformal algebras this condition corresponds to the requirement that physical states are created using only the generators with non-negative conformal weight $n \geq 0$ analogous to the algebra of creation operators. Also the generators of this algebra, whose number is finite, satisfy this condition. One could speak of half-algebra.
4. In TGD half-algebras appear for a different reason. The TGD Universe is fractal in several senses of the word. Also the algebra A of observables is fractal in the sense that it contains an infinite hierarchy of sub-algebras A_n for which the conformal weights are n-multiples of those for A. The finite measurement resolution is realized by the conditions that A_n and $[A_n, A]$ annihilate physical states and that also the corresponding classical Noether charges vanish, which gives strong conditions on space-time surfaces. These sub-algebras define hierarchies of measurement resolutions related to inclusions of HFFs.

   If the generators of super symplectic algebra and extensions of affine algebras indeed define free algebras, the rules of free probability theory could bring in dramatic computational simplifications if the scattering amplitudes correspond to expectations for the polynomials of the free-algebra generators.
5. In ZEO, zero energy states are generated by this kind of half-algebra and its hermitian conjugate as superpositions of state pairs assigned to the opposite boundaries of causal diamond (CD= $cd \times CP_2$, where cd is the intersection of future and past directed light-ones).

   The members of the state pair are created by the half-algebra *resp.* its hermitian conjugate and are assigned with opposite boundaries of CD (intersection of light-ones with opposite time directions). The corresponding vacua are analogous to Dirac sea of negative energy fermions and its hermitian conjugate consisting of positive energy fermions. The zero energy states are analogous to pairs formed by Dirac's bras and kets. This allows to code the scattering matrix elements [L126, L127] as zero energy states.

4.3.3 Some objections against HFFs

One cannot avoid philosophical considerations related to the notion of probability and to the interpretations of quantum measurement theory (`https://cutt.ly/YXxSLS1`).

Standard measurement theory and HFFs

The standard interpretations of quantum measurement theory are known to lead to problems in the case of HFFs.

1. An important aspect related to the probabilistic interpretation is that physical states are characterized by a density matrix so that quantum theory reduces to a purely statistical theory. Therefore the phenomenon of interference central in the wave mechanics does not have a direct description.

Another problem is that for HFFs, pure states do not exist as so-called normal states, which are such that it is possible to assign a density operator to them. This is easy to understand intuitively since by the $Tr(Id) = 1$ property of the unit matrix, there is no minimal projection. Selection of a ray would correspond to an infinite precision and delta function type density operator. The axiom of choices in mathematics is quite a precise analogy.

One can of course argue that even if pure states as normal states are possible, in practice the studied system is entangled with the environment and that this forces the description in terms of a density matrix even when pure states are realized at the fundamental level.

2. In the purely statistical approach, the notion of quantum measurement must be formulated in terms of what occurs for the density matrix in quantum measurement. The expectation value of any observable A for the new density matrix generated in the measurement of observable O with a discrete spectrum must be a weighted sum for the expectations for the eigenstates of the observable with weights given by the state function reduction probabilities.

   Problems are however encountered when the spectrum contains discrete parts. In the TGD framework, the number theoretic discretization would make it possible to avoid these problems.

Should density matrix be replaced with a more quantal object?

These problems force us to ask whether there could be something deeply wrong with the notion of density matrix? The TGD inspired view of HFFs [K125, K48] suggests a generalization of the state as a density matrix to a "complex square root" of the density matrix. At the level of WCW as vacuum functional, it would be proportional to exponent of a real valued Kähler function of WCW identified as Kähler action for the space-time region as a preferred extremal and a phase factor defined by the analog of of action exponential. Zero energy state would be proportional to an exponent of Kähler function of WCW identified as Kähler action for space-time surface as a preferred extrema.

Problems with the interpretations of quantum theory

HFFs based probability concept has also problems with the interpretations of quantum theory, which actually strongly suggest that something is badly wrong with the standard ontology.

1. In TGD, this requires a generalization of quantum measurement theory [L91] [K130] based on zero energy ontology (ZEO) and Negentropy Maximization Principle (NMP) [K72] [L35], which is consistent with the second law [L118]. What is essential is that physics is extended to what I call adelic physics [L50, L49] to describe also the correlates of cognition. This brings in a measure for conscious information based on a p-adic generalization of Shannon entropy.

2. ZEO [K130] is forced by an almost exact holography in turn implied by general coordinate invariance for space-time as 4-surface. States in ZEO are superpositions of classical time evolutions and and is replaced by a new one in a state function reduction (SFR) [L91, L128]. The determinism of the unitary time evolution is consistent with the non-determinism of SFR. The basic problem of quantum measurement theory disappears since there are two times and two causalities. Causality of field equations and geometric time of physicists can be assigned to the classical time evolutions. The causality of free will and flow of experienced time can be assigned to a sequence of SFRs. The findings of Minev et al [L77] provide support for ZEO [L77].

   Quantum measurement as a reduction of entanglement can in principle occur for any entangled system pair if NMP favors it. There is no need to assume mysterious decoherence as a separate postulate. By NMP, entanglement negentropy can also increase by the formation of entangled states. Since entanglement negentropy is the sum of positive p-adic contribution and negative contribution from real entanglement and is positive, the increase of negentropy is consistent with the increase of real entanglement entropy.

   However, since classical determinism is slightly broken [L125] (there is analogy with the non-uniqueness of the minimal surfaces spanned by frames), the holography is not quite exact. This has important implications for the understanding of the space-time correlates of cognition and intentionality in the TGD framework.

The notion of finite measurement resolution and probabilistic interpretation

One can also ask whether something could go wrong with the quantum measurement theory itself. This notion of quantum measurement does not take into account the fact that the measurement resolution is finite.

The notion of finite measurement resolution realized in terms of inclusion, replacing Hilbert space ray with the included factor and reducing state space to quantum group like object, could allow us to overcome the problems due to the absence of minimal projectors for HFFs implying that the notion of Hilbert space ray does not make sense.

Quantum group like object would represent the degrees of freedom modulo finite measurement resolution described by the included factor. The quantum group representations form a finite subset of corresponding group representations and the state function reductions could occur to quantum group representations and the standard quantum measurement theory for factors of type I would generalize.

Connes tensor product and finite measurement resolution

In the TGD framework Connes tensor product could provide a description of finite measurement resolution in terms of inclusion.

1. In the TGD framework, inclusion of HFFs are interpreted in terms of measurement resolution. The included factor $M \subset N$ would represent the degrees of freedom below measurement resolution. N as M module would mean that M degrees of freedom are absorbed to the coefficient ring and are not visible in the physical states. Complex numbers as a coefficient ring of the Hilbert space are effectively replaced with M. In the number theoretic description of the measurement resolution, the extension of extension is replaced with the extension. The quantum group, N as M, quantum group with quantum dimension $N : M$ would characterize the observable degrees of freedom.

   This fits with the hierarchy of SSA_n:s. SSA_{n+1} would take the role of M and SSA_n that of N. This conforms with the physical intuition. Since n corresponds to conformal weight, the large values of n would naturally correspond to degrees of freedom below UV cutoff.

   Could also IR cutoff have a description in the super symplectic hierarchy of SSA_n:s. It should correspond to a minimal value for conformal weight. The finite size of CD defining a momentum unit gives a natural IR cutoff. The proposal is that the total momentum assignable to the either half-cone of CD defines by $M^8 - H$ duality the size scale L as $L = h_{eff}/M$ [L100, L101].

2. For the hierarchies of extensions of rationals the upper levels of the extension hierarchy would not be observed. The larger the value of $n = h_{eff}/h_0$, n a dimension of extension of rationals associated with polynomial P defining the space-time region by $M^8 - H$ duality, the longer the quantum coherence scale.

   In this case large values for the dimension of extension would correspond to IR cutoff. Therefore UV and IR cutoffs would correspond to number theoretic and geometric cutoffs. This conforms with the view that $M^8 - H$ duality as an analog of Langlands duality is between number theoretic and geometric descriptions.

3. Duality suggests that also UV cutoff should have a number theoretic description. In the number theoretic situation, Galois confinement for these levels might imply that they are indeed unobservable, just like color-confined quarks. In fact, the hypothesis $n = h_{eff}/h_0$, n a dimension of extension of rationals associated with polynomial P defining the space-time region by $M^8 - H$ duality, for the effective Planck constant leads to estimate for ordinary Planck constant as $h = n_0 h_0$ where n_0 corresponds to the order of permutation group S_7.

   Could the interpretation be that these degrees of freedom are Galois confined and unobservable in the scales at which measurements are performed. Smaller values of h_{eff} would appear only in length scales much below the electroweak scale and at the limit of CP_2 scale?

How finite measurement resolution could be realized using inclusions of HFFs?

The basic ideas are that finite measurement resolution corresponds to inclusions of HFFs on one hand, and to number theoretic discretizations defined by extensions of rationals. In both cases one has inclusion hierarchies.

One can consider realizations at the level of WCW (geometry) and at the level of number theory in terms of adelic structures assignable to the extensions of rationals. Space-time surfaces can be discretized and this induces discretization of WCW. Even more, WCW should be in some natural manner effectively discrete.

In [K57, K36, K95] the construction WCW Kähler metric is considered and the mere existence of the K "ahler metric is expected to require infinite-D isometry group and imply constant curvature property. The Kähler function K is defined in terms of action consisting of the Kähler action and volume part for a preferred extremal (PE). There are however zero modes present and the metric depends on the zero modes. Twistor lift fixes the choices of H uniquely [L126, L127].

How to define WCW functional integral and how to discretize it? I have proposed that the Gaussian approximation to WCW integration is exact and allows to define a discretization in terms of the maxima (maybe also other extrema) of Kähler function. The proposal is that the exponential of Kähler function should correspond to a number theoretic invariant, perhaps the discriminant of the polynomial P defining PE by $M^8 - H$ duality.

Consider first the standard realization of the restriction $P : N \to M$ reducing the measurement resolution.

1. The definition of a unitary S-matrix for HFFs is non-trivial. Usually one considers only density matrices expressible in terms of projection operators P to subspaces of HFF.

   I have earlier proposed the notion of a complex square root of the density matrix as a generalization of the density matrix. In a direct sum representation of S over projections, in which S-matrix is diagonal, and the projection operators would be multiplied by phase factors. This definition looks sensible at the level of WCW but perhaps as a generalization of the density matrix rather than the S-matrix.

   The exponent of Kähler function could have a modulus multiplied by a phase factor. Also an additional state dependent phase factor can be considered. The mathematical existence of the WCW integral fixes the modulus essentially uniquely to an exponent of Kähler function K multiplied by the metric volume element. K could also have an imaginary part.

2. The projected S-matrix PSP is unitary if the projection operator P must commute with S. S-matrix is realized at the level of HFFs so that the matrix representation does not make sense in a strict sense since the notion of ray is not sensical.

3. Projection $N \to M$ respects unitarity only if P commutes with S and $S^\dagger$. The S-matrix does not have matrix elements between M and N. This is a very tough condition.

How the finite measurement resolution could be realized in the TGD framework?

1. In WCW spin degrees of freedom plus algebras A_n. Number theoretic degrees of freedom are discrete and correspond to various p-adic degrees of freedom. Continuous WCW is associated with the real part of the adelic structure. Its number theoretic parts correspond to the p-adic degrees of freedom, which are discrete.

2. Discretization could be a natural and necessary part of the definition of WCW. Could discrete WCW degrees of freedom be identified in terms of symplectic and number theoretic invariants? They would represent for WCW spinor fields scalar degrees scalar degrees of freedom separable from spin degrees of freedom representable in terms of algebras A_n. These two kinds of degrees of freedom correspond to M and M' if the proposed general picture is correct.

Measurement resolution would be realized in terms of braid group algebras and algebras A_n defining the measurement resolution. What does this mean at the level of WCW?

1. Bosonic generators of SSA_n and possible other algebras A_n define tangent space basis for WCW. The gauge conditions stating that A_n and $[A_n, A]$ annihilate WCW spinor fields define a finite measurement resolution selecting only a subset of tangent space-generators and their super counterparts.

2. Consider first ideal measurement resolution in a function space. There is a complete basis of scalar functions Φ_m in a given space. The sum $\overline{\Phi}_m(x)\Phi_m(y) = \delta(x, y)$ would hold true for an infinite measurement resolution.

In a finite measurement resolution one uses only a finite subset of the scalar function basis, and completeness relation becomes non-local and is smoothed out: $\delta(x,y) \rightarrow D(x,y)$, which is non-vanishing for different point pairs x,y.

3. The condition of finite measurement resolution should define a partition of WCW to disjoint sets. In real topology, the condition $|x-y|^2$ would define a natural measurement resolution but would not define a partition.

   In p-adic topology, the situation is different: the p-adic distance function $d(x-y)$ has values p^{-n} and the sets $d(x-y) < d$ are either disjoint or identical. One would have the desired partition. Therefore it seems that p-adicization is essential and the p-adic variants of WCW, or rather regions of WCW, obtained by discretization could allow partitions corresponding to various p-adic number fields forming the adele. Different p-adic representations of algebras A_n would define measurement resolutions.

   There is a connection with spin glasses where spin energy landscape consisting of free energy minima allows ultrametric topology: p-adic topologies are indeed ultrametric. The TGD view of spin glasses is discussed in [L120]. One expects the decomposition of WCW to different p-adic topologies with ramified primes of polynomial P defining the p-adic sectors to which a given space-time surface can belong.

4. The consistency condition is that the transition probabilities $P(m \rightarrow n)$ between the states satisfying the gauge conditions representing finite measurement resolution, predicted by S-matrix or its TGD counterpat, should be constant should be constant in the subsets of WCW for which the completeness relation gives a non-vanishing $D(x,y)$ for the point pairs (x,y).

5. Does WCW have hierarchies of partitions such that the constancy of $P(m \rightarrow n)$ holds true within each partition?

   Do these partitions correspond to hierarchies of inclusions of HFFs defining increasing resolution? M^8-H duality does not allow all kinds of hierarchies. The hierarchies should be induced by the hierarchies of extensions of rationals. As the measurement precision increases, the partition involves an increasing number of sets and at the limit of ideal measurement resolution, the partition consists of algebraic points of WCW and of space-time surfaces.

6. $P=Q$ condition implying that space-time surfaces correspond to infinite prime, could appear as a consistency condition for allowed hierarchies. Preferred extremals and preferred polynomials would correspond to each other. Note that $P=Q$ conditions fixes the scaling of P.

In the TGD framework, one can challenge the idea, originally due to Wheeler, that transition probabilities are given by a unitary S-matrix.

1. The TGD based proposal is that in spin degrees of freedom, that is for many-fermion states for a given space-time surface, the counterpart of S-matrix could be be given by the analog of Kähler metric in the fermionic Hilbert space [L113]. This would mean a geometrization of quantum theory, at least in fermionic degrees of freedom.

   The transition probabilities would be given by $P(m \rightarrow n) = K_{\overline{m}n}K^{\overline{n}m}$ and the properties of Kähler metric K give analogs of unitary conditions and probability conservation plus some prediction distinguishing the proposal from the standard view.

2. In the infinite-D situation, the existence of Hilbert space Kähler metric in the fermionic sector is an extremely powerful condition and one expects that the Kähler metric is a unique constant curvature metric allowing a maximal group of isometries. This, together with p-adization, would help to satisfy the constancy conditions for $P(m \rightarrow n)$ inside the sets for which $D(x,y)$ is non-vanishing. In fact, one expects that since super-generators are proportional to isometry generators contracted with WCW gamma matrices the metric in the fermionic degrees of freedom is induced by Kähler metric in the basis of isometry generators.

3. This condition could allow a generalization to include the states obtained by application of the bosonic generations of A_n the to ground state. This would mean that in bosonic degrees of freedom Kähler metric of WCW in the isometry basis defines the transition probabilities. Tangent vectors of WCW correspond to the isometry generators. An arbitrary number of

isometry generators is involved in the definition of the state. However, the Kähler metric of WCW induces a Kähler metric in the algebra generated by the isometry generators, which is analogous to the algebra of tensors.

4.4 $M^8 - H$ duality and HFFs

$M^8 - H$ duality [L100, L101] gives strong constraints on the interpretation of HFFs at the number theoretic M^8 side and the geometric H side of the duality. One must also understand the relation between $M^8 - H$ duality and $M - M'$ duality, identifiable as quantum-classical correspondence (QCC).

Although McKay correspondence [A79, A77, A59, A47, A46] is not quite at the core of $M^8 - H$ duality, it is difficult to avoid its discussion. I have considered McKay correspondence also before [L38, L94, L95, L130].

4.4.1 Number theoretical level: M^8 picture

Braided Galois group algebras

For n-braids the permutation group has extension to a braid group B_n defining an infinite covering of S_n for which permutation corresponds to a geometric operation exchanging the two strands of a braid. There are also hierarchies of finite coverings.

S_n is replaced with the Galois group which is a subgroup of S_n and the property of being a subgroup of S_n allows to identify a braided Galois group as a braided Galois subgroup of braided S_n. In the same way one can identify the braided Galois group algebra defining HFF as a sub-algebra of HFF associated with braid group algebra defined by S_n. One can ask whether the property of being a number theoretic braid could be interpreted as a kind of symmetry breaking to S_n to the Galois group of P.

$M^8 - H$ duality [L100, L101] suggests that the roots correspond to braid strands of geometric braids in H. If so, the braided Galois group would be both topological and number theoretic: topology, natural at the level of H, and number theory, natural at the level of M^8, would meet by $M^8 - H$ duality.

This picture looks nice but one can make critical questions.

1. Can the n roots really correspond to n braid strands at the level of H? The n roots correspond to, in general complex, algebraic numbers associated with the extension of rationals. The real projections correspond to mass shells with different mass values mapped to light-cone proper time surfaces in H by $M^8 - H$ duality. Therefore the action of the Galois group changes mass squared values and does not commute with Lorentz transformations. This suggests a violation of causality.

   Should one restrict the Galois group to the isotropy group of a given root? This would mean number theoretic symmetry breaking and could relate to massivation. This restriction would however trivialize the braid.

2. Zero energy ontology (ZEO) could come to the rescue here. In fact, ZEO implies space-time surfaces are the basic objects rather than 3-surfaces so that quantum states are superpositions of space-time surfaces as preferred extremals (PEs). This is forced by the slight violation of determinism of field equations implying also a slight violation of ideal holography.

   Space-time surfaces are minimal surfaces [L125] analogous to soap films spanned by frames and there can be a slight violation of the strict determinism localized to frames as already 2-D case suggests. This could be also seen as violation of classical causality. At the level of consciousness theory it would be a classical correlate for the non-determinism of intentional free will.

   In particular, time-like braids for which the braiding is time-like and corresponds to a dynamical dance pattern, make sense. For these braids one can in principle select the mass squared value mapped to a value of light-cone proper time a to belong to the braid. The values of a need not be the same.

Also Galois confinement, which is a key aspect of the number theoretic vision, is involved.

1. Galois confinement states that physical states transform trivially under the Galois group of extension. This condition for physical states follows as a consequence of periodic boundary conditions for causal diamond (CD), which takes the role of box for a particle in a box.

   A weaker condition would be that singlet property holds only for the isotropy group of a given root of the polynomial P characterizing the space-time region and corresponding to mass squared value and at the level of H to a value of the light-cone proper time a.

2. In M^8, the momenta of particles are points at the mass shells of $M^4 \subset M^8$ identifiable as hyperbolic spaces $H^3 \subset M^4$ defined with mass squared values defined as the roots of P. The momenta correspond to algebraic integers (the momentum unit is defined by CD) for the extension defined by P, and in general they are complex. The interpretation is as virtual particles which form physical particles as composites. The physical states must have total momenta, which are ordinary integers. This gives the simplest form of Galois confinement.

3. Commutativity with the Lorentz group would favor the isotropy group instead of the full Galois group. One must be however very cautious since in zero energy ontology (ZEO) physical states correspond to a superposition of space-time surfaces and time-like braids are natural. There is a small violation of strict determinism at the level of preferred extremas. The labelling of braid strands based on the images of roots as mass squared values at level of H is quite natural and is not in conflict with causality.

The Galois group for a polynomial $P_n \circ ... \circ P_1$ has a decomposition to normal subgroups GA_i acting as Galois groups for the i:th sub-extension.

1. The number of roots is a product of the numbers of roots for P_i. Therefore the natural identification is that number theoretic braid groups allow a natural interpretation in terms of braids of braids ... of braids.

2. This hierarchy defines an inclusion hierarchy for the braided HFFs assignable to the polynomials $P_k \circ ... \circ P_1$, $k = 1, ..., n$. It is not quite clear to me whether these inclusions reduce to Jones inclusions and whether one can characterize the inclusions in the sequence by the same invariants as in the case of Jones inclusions.

3. In this picture the Connes tensor product would correspond to formation of composite polynomials $P \circ Q$. The reduction in the number of degrees of freedom from that for the ordinary tensor product of braided Galois group algebras would be due to interactions described in terms of polynomial decomposition. Various braids in the hierarchy could correspond to braids at different sheets of the many-sheeted space-time.

4. Any normal subgroup Gal_i of Galois group Gal defining a sequence of inclusions of normal sub-groups Gal_i can be trivially represented. By normal subgroup property, the elements of Gal can be represented as semidirect products of elements of the factor groups $G_i = Gal_i/Gal_{i-1}$. Any representation of Gal can be decomposed to a direct sum of tensor products of representations of G_i.

   From this decomposition it is clear that any group G_i in the decomposition can be trivially represented so that one obtains a rich structure of representation in which some G_i:s are trivially represented.

A possible interpretation is that in TGD, rational polynomials give discrete cognitive representations as approximations for physics. Cognitive representations are in the intersection of p-adicities and reality defined by the intersection of reals and extension of p-adics defined by the algebraic extension of the polynomial P defining a given space-time surface. Continuum theory would represent real numbers as a factor of the adele.

One can ask whether the various zeta functions consistent with the integer spectrum for the conformal weights and possibly also with conformal confinement, appear at the continuum limit and provide representations for the space-time surfaces at this limit? In this framework, it would be natural for the roots of zeta to be algebraic numbers [K96]. Also in the case of ζ, the virtual momenta of fermions would be algebraic integers for virtual fermions and integers for the physical states. This makes sense if the notions of Galois group and Galois confinement are sensible for ζ.

As noticed, the notion of ζ generalizes. The so-called global L-functions (`https://cutt.ly/3VNPYmp`) are formally similar to ζ and the extended Riemann Hypothesis could be true for them. The physical motivation for RH would be that it would allow fermion with any conformal weight to appear in a state which is conformal singlet. Algebraic integers for a finite extension of rationals replace integers in the ordinary ζ and one has an entire hierarchy of L-functions. Could one think that the global L-functions could define preferred extremals at the continuum limit?

How could the degrees of prime polynomials associated with simple Galois groups and ramified primes relate to the symmetry algebras acting in H?

The goal is to relate various parameters characterizing polynomials P for which braided Galois group algebras define HFFs to the parameters labelling the symmetry algebras defining hierarchies of HFFs at the level H. There are good reasons to believe that polynomial composition defines inclusion of HFFs and that this inclusion induces the inclusions for the symmetry algebras A_n at the level of H.

One can identify simple Galois groups as prime groups having no normal subgroups. The polynomial P associated with a simple Galois group cannot have no non-trivial functional decomposition $P_n \circ ... \circ P_1$ if one stays in the field of rationals (say). This leads to the notion of prime polynomials. Note that this notion of primeness does not correspond to the irreducibility stating that polynomials with coefficients in a given number field do not allow decomposition to lower degree polynomials.

A polynomial P is also partially characterized by ramified primes and discriminant defines a Galois invariant for the polynomial as also the symmetric polynomials formed from the roots.

How do these two notions of primeness relate to the p-adic prime decomposition of adelic structures defined by the algebras A_n, which act at the level of H and decomposed adelically to a tensor product of all $A_{n,p}$:s?

Simple Galois groups correspond to prime polynomials. This notion looks fundamental concerning the understanding of the situation at the level of H.

1. Polynomials can be factorized into composites of prime polynomials [A24, A69] (`https://cutt.ly/HXAKDzT` and `https://cutt.ly/5XAKCe2`). A polynomial, which does not have a functional composition to lower degree polynomials, is called a prime polynomial. It is not possible to assign to prime polynomials prime degrees except in special cases. Simple Galois groups with no normal subgroups must correspond to prime polynomials.

2. For a non-prime polynomial, the number N of the factors P_i, their degrees n_i are fixed and only their order can vary so that n_i and $n = \prod n_i$ is an invariant of a prime polynomial and of simple Galois group [A24, A69]. Note that this composition need not exist for monic polynomials even if the Galois group is not simple so that polynomial primes in the monic sense need not correspond to simple Galois groups.

3. The number of the roots of P_i is given by its order n_i, and since Galois group and its braided variant permute the roots as subgroup of S_{n_i}, it is natural to assume that the roots define an n_i-braid. The composite polynomial would define braid of braids of ... of braids. At the level of H the braid strands would correspond to flux tubes and braiding would have a geometric interpretation.

4. The integer n characterizing the algebra A_n acting in H would naturally correspond to the degree of n of P and the decomposition of P to polynomial primes would naturally correspond to an inclusion hierarchy A_{n_i} $A_{n_1} \subset A_{n_1 n_2} \subset ... \subset A_n$ with improving resolution allowing to see braids and braids of braids.

   The corresponding factor spaces realizing the notion of finite measurement resolution, would be analogous to quantum groups obtained when some number of the highest levels in the hierarchy of braids in the braid of braids of ... braids are neglected and the entire algebra is replaced with a quantum group-like structure. This means cutting off some number of the highest levels in the tree-like hierarchy. The trunk is described by a quantum group-like object.

5. This hierarchy corresponds to the hierarchy of Galois groups as normal subgroups assignable to braids in the decomposition and the hierarchy of corresponding braided Galois group algebras defining inclusions of HFFs. Galois group algebras would act as braid groups inc corresponding algebras A_n. Therefore number theoretic and geometric views would fuse together.

6. Connes tensor product is a central notion in the theory of HFFs and it could be naturally associated with the inclusions of brided Galois group algebras. The counterpart for the quantum group as factor space N/M of the factors would correspond to the inclusion $Gal_{i-1} \subset Gal_i$ as a normal subgroup. The inclusion defines group $G_i = Gal_i/Gl_{i-1}$. Also its braided variant is defined. The factor space of braided group algebras would be the counterpart of the quantum group G_i.

   Note that these quantum group-like objects could be much more general than the quantum groups defined by subgroups of $SU(2)$ appearing in Jones inclusions.

What about the interpretation of the ramified primes, which are Galois invariants as also the root spectrum (but not the roots themselves) and depends on the polynomial.

In accordance with the proposed physical interpretation of the ramified primes as preferred p-adic primes labelling particles in p-adic thermodynamics, ramified primes p_i would define preferred p-adic primes for the p-adic variants of the algebras A_n in the adelic generalization of A_n as tensor product of p-adic representations of $A_{n,p}$ of A_n. A_{n,p_i} would be physically and also mathematically special.

Both the degree n as the number of braids of P and the ramified primes of P would dictate the physically especially relevant algebras A_{n,p_i}. For instance, un-ramified primes could be such that corresponding p-adic degrees of freedom are not excited.

4.4.2 Geometric level: H picture

The hierarchies of algebras SSA_n, Aff_n and I_n

The algebras $A_n \in \{SSA_n, Aff_n, I_n\}$ for $n = p$ acting at the level of WCW seem to have special properties since the values of the conformal weights for the factor algebras defined by the conditions that A_n and $[A_n, A]$ annihilate physical states, allow the structure of finite field $G(p)$ or even its extension $G(p, k)$ for conformal weights in extension of rationals. The representations would be finite-D. Also the values $n = p^k$ seem special and the finite field representations of SSA_p could be extended to p-adic representations.

This raises the question, whether one could regard n as a p-adic number? The interpretation of n as the number of braid strands assignable to roots of the polynomial P with degree n defining the space-time surface, looks more approriate since it allows braid group algebra of P to act in SSA_n, This identification does not favor this interpretation.

A more plausible interpretation is that the p-adic primes, identifiable as ramified primes of P, characterize the p-adic representations of SSA_n. This also conforms with the interpretation of preferred p-adic primes characterizing elementary particles as ramified primes.

The polynomials with prime degree could be however physically special. The algebras SSA_p, with p defining the degree of polynomial p allow finite field representations, which extend to p-adic representations and one can ask whether the prime decomposition of n could allow some kind of inclusion hierarchy of representations.

This would also give a possible content for the p-adic length scale hypothesis $p \simeq 2^k$, k prime, or its generalization involving primes near powers of prime $q = 2, 3, 5, ...$ A more general form of p-adic length scale hypothesis would be $p \simeq q^n$, n the degree of P.

Commutants for algebras A_n and braid group algebras

For the super $A \in \{SSA, Aff, I\}$, the inclusion An_{nk} to SSA_n should define a Connes tensor product. One would obtain inclusion hierarchies labelled by divisibility hierarchies $n_1 \div n_2 \div .....$ For braid group algebras one obtains similar hierarchies realized in terms of composite polynomials.

What about the already mentioned "classical" degrees of freedom associated with the fluxes of the induced Kähler form? They should be included to M' at the level of H. The hierarchies of flux tubes within ... within flux tubes correspond to the hierarchies assignable to M' at the level of H.

The number theoretic degrees of freedom identifiable as invariants of Galois groups should be included to M' at the number theoretical level. The hierarchies of roots assignable to composite polynomials $P_n \circ ... \circ P_1$ with roots assigned to the strands of time like braid strands could correspond to these hierarchies at the level of M^8.

4.4.3 Wild speculations about McKay correspondence

McKay correspondence is loosely related to the HFFs in TGD framework [L38, L95, L94, L130] and I cannot avoid the temptation to try to understand it in TGD framework.

1. The origin of the McKay graphs for inclusions is intuitively understood. Representations of finite subgroups of $SU(2)$ are assignable to 2-D factors. These representations could correspond to closed subgroups of quaternionic $SU(2)$ on the basis of the reduction to $M_2(C)\otimes M_2(C)\otimes$..... A reduction of degrees of freedom happens for HFFs since they are subalgebras of $B(H)$ and this could reduce the closed subgroup to a finite subgroup.

   Also the interpretation N as tensor product of M and quantum group $SU(2)$ suggests the same since quantum groups have a finite number of irreps, when q equal is a root of unity. The analog of McKay graph coding fusion rules for the quantum group tensor products would reduce to McKay graphs.

2. Why would the McKay graphs for finite subgroups of $U(2)$ correspond to those for affine or ordinary Lie algebras? Could these Lie-algebras emerge from the inclusions. This is a mystery, at least to me.

3. In the TGD framework one can ask why there should be Weyl group of extended ADE Dynkin diagram assignable to SSA_n? SSA_n defines a representation of SSA with SSA_n and $[SSA_n, SSA]$ acting trivially. Could this representation correspond to an affine or ordinary ADE algebra? Similar question makes sense for all algebras $A_n \in \{SSA_n, Aff_n, I_n\}$. A_nn would define a cutoff of the SSA so that all generators with conformal weight larger than n would be represented trivially.

   Note that for $n = p$, the conformal weights of A_n would define a finite field and if algebraic integers also its extension. This case could correspond to polynomials defining cyclic extension of order p with roots coming as roots of unity.

4. The Weyl groups assignable to the "factor algebra" of SSA_n defined by the gauge conditions for A_n and $[A_n, A]$ and proposed to reduce to ADE type affine or ordinary Lie algebra should relate to Galois groups for polynomial P with degree n as number of braid strands.

   (a) Could the braid strands correspond to the roots of ADE algebra so that roots in the number theoretic sense would correspond to the roots in the group theoretic sense? This would conform with Langlands correspondence [K59, A40, A39] discussed from the TGD perspective in [K59] [L2, L28].

   (b) Could the Weyl groups allow identification as subgroups of corresponding Galois groups?

   Note that simple Galois groups correspond to so-called prime polynomials [A24, A69] allowing no decomposition to polynomials of lower degree so that the preferred values of n would correspond to prime polynomials.

5. Affine electroweak and color algebras an their M^4 counterparts would be special since they wuuld not emerge a dynamical symmetries of SSA_n but define algebras Aff_n and I_n related to the light-like partonic orbits. They would also correspond to symmetries both at the level of M^8 and H.

This inspires the following questions, which of course look very naive from the point of view of a professional mathematician. My only excuse is the strong conviction that the proposed picture is on the right track. I might be wrong.

1. The Jones inclusion of HFFs [A48, A96, A97] involves an extended or ordinary ADE Dynkin diagram assignable also to finite subgroups of $SU(2)$ by McKay correspondence [A79].

   Could the Weyl group of an extended ADE diagram really correspond to an affine algebra or quantum group assignable to A_n? If so, one would have dynamical symmetries and should relate to the "factor" space SSA/SSA_n in which SSA_n defines a measurement resolution.

2. HFF can be regarded algebraically as an infinite tensor power of $M_2(C)$. Does the representation as a 2×2 matrix imply the emergence of representations of a closed subgroup of $SU(2)$ or its quantum counterpart. Could the reduction of degrees of freedom due to the finite measurement resolution imply that the closed subgroup reduces to a finite subgroup?

3. The algebraic decomposition of HFF to an infinite tensor power of $M^2(C)$ would suggest that the including factor N with dimension 1 is equal to $M^{d_q} \otimes M^{1/d_q}$, where d_q is the quantum dimension characterizing eith M or N. Could these two objects correspond to an ADE type affine algebra and quantum group with inverse quantum dimensions? Or could either of them correspond to ADE type affine algebra or quantum group?
4. Could one think that the analog of McKay graph for the quantum group-like object assignable to affine group by a finite measurement resolution reduces to the McKay graph for a finite subgroup of $SU(2)$ because only a finite number of representations survives?
5. Could the finite subgroups of $SU(2)$ correspond to finite subgroups for the covering group of quaternion automorphisms acting naturally in M^8? Could these finite subgroups correspond to finite subgroups of the rotation group $SU(2)$ at H side?

Could only the n_C (dimension of Cartan algebra) roots appearing in the Dynkin diagram be represented as roots of a polynomial P in extension of rationals or its quantum variant? This option fails since the Dynkin diagram does not allow a symmetry group identifiable as the Galois group. The so called Steinberg symmetry groups (`https://cutt.ly/GXMb8Si`) act as automorphisms of Dynkin diagrams of ADE type groups and seem quite too small and fail to be transitive as action of the Galois group of an irreducible polynomial is.

$M^8 - H$ duality inspires the question whether a subgroup of Galois group could act as the Weyl group of ADE type affine or ordinary Lie algebra at H side.

1. The Galois group acts as a braid group and permutes the roots of P represented as braid strands. Weyl group permutes the roots of Lie algebra

   The crazy question is whether the roots of P and roots of the ADE type Lie-algebra could correspond to each other. Could the roots of P in $N \rightarrow$ 1-correspondence with the non-vanishing roots of the representation of Lie algebra or of its affine counterpart containing an additional root corresponding to the central extension?

   If the roots appearing in the Dynkin diagram correspond to a subset of roots of polynomial P, the Weyl group could correspond to a minimal subgroup of the Galois group generated by reflections and generating all non-vanishing roots of the Lie algebra.

2. The action of the Weyl group should give all roots for the representation of G. Could the Weyl group, which is generated by reflections, correspond to a minimal subgroup of Gal giving all roots as roots of P when applied to the McKay graph?

   The obvious objection is that the order of the Weyl group increases rapidly with the order of the Cartan group so that also the Gal and also the order of corresponding polynomials P would increase very rapidly. Gal is a subgroup of S_n having order $n!$ for a polynomial of degree n so that the degree of P need not be large and this is what matters.

   If the m braid strands labelled by the m roots correspond to the roots of the affine algebra, it would be natural to assign affine algebra generators to these roots with the braid strands. The condition $n = Nm$ implies that m divides n. For $Gal = S_n$ with order $n!$ this condition is very mild. $Gal = Z_p$ fixes the Lie algebra to A_p.

   The root space of the dynamical symmetry group would have dimension m, which is a factor of n. For Lie algebras A_n and D_{2n} (with $n \geq 4$) appear besides E_6 and E_8. For affine Lie algebras $\hat{A}_n$ or $hatD_n$ (with $n \geq 3$) and $\hat{E}_6$, $\hat{E}_7$ and $\hat{E}_8$ appear. For large values of n, there are two alternatives for even values of n.

3. One can also consider quantum arithmetics based on $\oplus$ and $\otimes$ and replace P with its quantum counterpart and solve it in the space of irreps of the finite subgroup G of $U(2)$ defining a quantum analog for an extension of rationals. The roots of the quantum variant of P would be direct sums of irreps of G.

   These quantum roots define nodes of a diagram. This diagram should include as nodes the roots of the Dynkin diagram defined by positive roots, whose number is the dimension n_C of Cartan algebra.

   Could the missing edges correspond to the edges of the Mac-Kay graph in the tensor product with a 2-D representation of $SU(2)$ restricted to a subgroup? The action of 2-D representation would generate the (extended) Dynkin diagram ADE type.

One can look this option in more detail.

1. Assume that adjoint representation *Adj* of an affine or ordinary ADE Lie group L emerges in the tensor product $M^2(C) \otimes ... \otimes M^2(C)$ allowing embedding of $SU(2)$ as diagonal embedding. One can imbed the finite subgroup $G \subset SU(2)$ as a diagonal group $G \times G \times ... \times G$ to $M^2(C) \otimes ... \otimes M^2(C)$.

   Also a given representation of G can be embedded as a direct sum of the copies of the representation, each acting in one factor of $M^2(C) \otimes ... \otimes M^2(C)$. The $2-D$ canonical representation of $G \subset SU(2)$ has a natural action in the $G \times G \times ... \times G$ to $M^2(C) \otimes ... \otimes M^2(C)$ and would generate a McKay graph.

   One can also embed G to L as $G \subset SU(2) \subset L$. *Adj* can be decomposed to irreps G. Therefore the tensor product action of various irreps of G, in particular the canonical 2-D representation, in *Adj* is well-defined. The tensor action of the 2-D canonical representation of G gives a McKay graph such that the nodes have weights telling how many times a given irrep appears in the decomposition of *Adj* to irreps of G. The weighted sum of the dimensions of irreps of G is equal to the dimension of *Adj*.

2. This construction is possible for any Lie group and some consistency conditions should be satisfied. That McKay graph is the same as the generalized Dynkin diagram would be such a consistency condition and leave only simply laced Lie groups.

3. What can one say about the weights of the weighted McKay graph? Could the weights be the number of the images of the positive root under the action of the Weyl group W of L.

   The McKay graph would correspond only to the n_C (dimension of the Cartan algebra) positive roots appearing in the Dynkin diagram of *Adj*. How to continue the Dynkin dynkin to a root diagram of *Adj*?

4. Could the n_C roots in the Dynkin diagram correspond to the roots of a polynomial P in a quantum extension of rationals with roots as irreps of G appearing in the McKay graph. The multiple of a given root would correspond to its orbit under W. The action of W as reflections in the quantum extension of rationals, spanned by the roots of *Adj*, as vectors with integer components would generate all roots of *Adj* as quantum algebraic integers in the quantum extension of rationals.

5. As proposed, one could interpret the Dynkin diagram as a subdiagram of the root diagram of *Adj* and identify its nodes as roots of *Gal* for a suitable polynomial P. The Weyl group could be the minimal transitive subgroup of *Gal*.

6. The Galois group of extension of ... of rationals is a semidirect of Galois groups which can be chosen to be simple so that the polynomials considered are prime polynomials unless one poses additional restrictions. What does this restriction mean for the ADE type Weyl group of assignable to the exrtension

4.5 About the selection of the action defining the Kähler function of the "world of classical worlds" (WCW)

The proposal is that space-time surfaces correspond to preferred extremals of some action principle, being analogous to Bohr orbits, so that they are almost deterministic. The action for the preferred extremal would define the Kähler function of WCW [K57, K95].

How unique is the choice of the action defining WCW Kähler metric? The problem is that twistor lift strongly suggests the identification of the preferred extremals as 4-D surfaces having 4-D generalization of complex structure and that a large number of general coordinate invariant actions constructible in terms of the induced geometry have the same preferred extremals.

4.5.1 Could twistor lift fix the choice of the action uniquely?

The twistor lift of TGD [K98] [L122, L126, L127] generalizes the notion of induction to the level of twistor fields and leads to a proposal that the action is obtained by dimensional reduction of the action having as its preferred extremals the counterpart of twistor space of the space-time surface identified as 6-D surface in the product $T(M^4) \times T(CP_2)$ twistor spaces of $T(M^4)$ and $T(CP_2)$ of M^4 and CP_2. Only M^4 and CP_2 allow a twistor space with Kähler structure [A57] so that TGD would be unique. Dimensional reduction is forced by the condition that the 6-surface has S^2-bundle structure characterizing twistor spaces and the base space would be the space-time surface.

1. Dimensional reduction of 6-D Kähler action implies that at the space-time level the fundamental action can be identified as the sum of Kähler action and volume term (cosmological constant). Other choices of the action do not look natural in this picture although they would have the same preferred extremals.

2. Preferred extremals are proposed to correspond to minimal surfaces with singularities such that they are also extremals of 4-D Kähler action outside the singularities. The physical analogue are soap films spanned by frames and one can localize the violation of the strict determinism and of strict holography to the frames.

3. The preferred extremal property is realized as the holomorphicity characterizing string world sheets, which generalizes to the 4-D situation. This in turn implies that the preferred extremals are the same for any general coordinate invariant action defined on the induced gauge fields and induced metric apart from possible extremals with vanishing CP_2 Kähler action.

   For instance, 4-D Kähler action and Weyl action as the sum of the tensor squares of the components of the Weyl tensor of CP_2 representing quaternionic imaginary units constructed from the Weyl tensor of CP_2 as an analog of gauge field would have the same preferred extremals and only the definition of Kähler function and therefore Kähler metric of WCW would change. One can even consider the possibility that the volume term in the 4-D action could be assigned to the tensor square of the induced metric representing a quaternionic or octonionic real unit.

Action principle does not seem to be unique. On the other hand, the WCW Kähler form and metric should be unique since its existence requires maximal isometries.

Unique action is not the only way to achieve this. One cannot exclude the possibility that the Kähler gauge potential of WCW in the complex coordinates of WCW differs only by a complex gradient of a holomorphic function for different actions so that they would give the same Kähler form for WCW. This gradient is induced by a symplectic transformation of WCW inducing a U(1) gauge transformation. The Kähler metric is the same if the symplectic transformation is an isometry.

Symplectic transformations of WCW could give rise to inequivalent representations of the theory in terms of action at space-time level. Maybe the length scale dependent coupling parameters of an effective action could be interpreted in terms of a choice of WCW Kähler function, which maximally simplifies the computations at a given scale.

1. The 6-D analogues of electroweak action and color action reducing to Kähler action in 4-D case exist. The 6-D analog of Weyl action based on the tensor representation of quaternionic imaginary units does not however exist. One could however consider the possibility that only the base space of twistor space $T(M^4)$ and $T(CP_2)$ have quaternionic structure.

2. Kähler action has a huge vacuum degeneracy, which clearly distinguishes it from other actions. The presence of the volume term removes this degeneracy. However, for minimal surfaces having CP_2 projections, which are Lagrangian manifolds and therefore have a vanishing induced Kähler form, would be preferred extremals according to the proposed definition. For these 4-surfaces, the existence of the generalized complex structure is dubious.

   For the electroweak action, the terms corresponding to charged weak bosons eliminate these extremals and one could argue that electroweak action or its sum with the analogue of color action, also proportional Kähler action, defines the more plausible choice. Interestingly, also the neutral part of electroweak action is proportional to Kähler action.

Twistor lift strongly suggests that also M^4 has the analog of Kähler structure. M^8 must be complexified by adding a commuting imaginary unit i. In the E^8 subspace, the Kähler structure of E^4 is defined in the standard sense and it is proposed that this generalizes to M^4 allowing also generalization of the quaternionic structure. M^4 Kähler structure violates Lorentz invariance but could be realized at the level of moduli space of these structures.

The minimal possibility is that the M^4 Kähler form vanishes: one can have a different representation of the Kähler gauge potential for it obtained as generalization of symplectic transformations acting non-trivially in M^4. The recent picture about the second quantization of spinors of $M^4 \times CP_2$ assumes however non-trivial Kähler structure in M^4.

4.5.2 Two paradoxes

TGD view leads to two apparent paradoxes.

1. If the preferred extremals satisfy 4-D generalization of holomorphicity, a very large set of actions gives rise to the same preferred extremals unless there are some additional conditions restricting the number of preferred extremals for a given action.
2. WCW metric has an infinite number of zero modes, which appear as parameters of the metric but do not contribute to the line element. The induced Kähler form depends on these degrees of freedom. The existence of the Kähler metric requires maximal isometries, which suggests that the Kähler metric is uniquely fixed apart from a conformal scaling factor Ω depending on zero modes. This cannot be true: galaxy and elementary particle cannot correspond to the same Kähler metric.

Number theoretical vision and the hierarchy of inclusions of HFFs associated with supersymplectic algebra actings as isometries of WcW provide equivalent realizations of the measurement resolution. This solves these paradoxes and predicts that WCW decomposes into sectors for which Kähler metrics of WCW differ in a natural way.

The hierarchy subalgebras of supersymplectic algebra implies the decomposition of WCW into sectors with different actions

Supersymplectic algebra of $\delta M^4_+ \times CP_2$ is assumed to act as isometries of WCW [L133]. There are also other important algebras but these will not be discussed now.

1. The symplectic algebra A of $\delta M^4_+ \times CP_2$ has the structure of a conformal algebra in the sense that the radial conformal weights with non-negative real part, which is half integer, label the elements of the algebra have an interpretation as conformal weights.

   The super symplectic algebra A has an infinite hierarchy of sub-algebras [L133] such that the conformal weights of sub-algebras $A_{n(SS)}$ are integer multiples of the conformal weights of the entire algebra. The superconformal gauge conditions are weakened. Only the subalgebra $A_{n(SS)}$ and the commutator $[A_{n(SS)}, A]$ annihilate the physical states. Also the corresponding classical Noether charges vanish for allowed space-time surfaces.

   This weakening makes sense also for ordinary superconformal algebras and associated Kac-Moody algebras. This hierarchy can be interpreted as a hierarchy symmetry breakings, meaning that sub-algebra $A_{n(SS)}$ acts as genuine dynamical symmetries rather than mere gauge symmetries. It is natural to assume that the super-symplectic algebra A does not affect the coupling parameters of the action.
2. The generators of A correspond to the dynamical quantum degrees of freedom and leave the induced Kähler form invariant. They affect the induced space-time metric but this effect is gravitational and very small for Einsteinian space-time surfaces with 4-D M^4 projection.

   The number of dynamical degrees of freedom increases with $n(SS)$. Therefore WCW decomposes into sectors labelled by $n(SS)$ with different numbers of dynamical degrees of freedom so that their Kähler metrics cannot be equivalent and cannot be related by a symplectic isometry. They can correspond to different actions.

Number theoretic vision implies the decomposition of WCW into sectors with different actions

The number theoretical vision leads to the same conclusion as the hierarchy of HFFs. The number theoretic vision of TGD based on $M^8 - H$ duality [L133] predicts a hierarchy with levels labelled by the degrees $n(P)$ of rational polynomials P and corresponding extensions of rationals characterized by Galois groups and by ramified primes defining p-adic length scales.

These sequences allow us to imagine several discrete coupling constant evolutions realized at the level H in terms of action whose coupling parameters depend on the number theoretic parameters.

1. Coupling constant evolution with respect to $n(P)$

The first coupling constant evolution would be with respect to $n(P)$.

1. The coupling constants characterizing action could depend on the degree $n(P)$ of the polynomial defining the space-time region by $M^8 - H$ duality. The complexity of the space-time surface would increase with $n(P)$ and new degrees of freedom would emerge as the number of the rational coefficients of P.
2. This coupling constant evolution could naturally correspond to that assignable to the inclusion hierarchy of hyperfinite factors of type II_1 (HFFs). I have indeed proposed [L133] that the degree $n(P)$ equals to the number $n(braid)$ of braids assignable to HFF for which super symplectic algebra subalgebra $A_{n(SS)}$ with radial conformal weights coming as $n(SS)$-multiples of those of entire algebra A. One would have $n(P) = n(braid) = n(SS)$. The number of dynamical degrees of freedom increases with n which just as it increases with $n(P)$ and $n(SS)$.
3. The actions related to different values of $n(P) = n(braid) = n(SS)$ cannot define the same Kähler metric since the number of allowed space-time surfaces depends on $n(SS)$.

   WCW could decompose to sub-WCWs corresponding to different actions, a kind of theory space. These theories would not be equivalent. A possible interpretation would be as a hierarchy of effective field theories.
4. Hierarchies of composite polynomials define sequences of polynomials with increasing values of $n(P)$ such that the order of a polynomial at a given level is divided by those at the lower levels. The proposal is that the inclusion sequences of extensions are realized at quantum level as inclusion hierarchies of hyperfinite factors of type II_1.

   A given inclusion hierarchy corresponds to a sequence $n(SS)_i$ such that $n(SS)_i$ divides $n(SS)_{i+1}$. Therefore the degree of the composite polynomials increases very rapidly. The values of $n(SS)_i$ can be chosen to be primes and these primes correspond to the degrees of so called prime polynomials [L129] so that the decompositions correspond to prime factorizations of integers. The "densest" sequence of this kind would come in powers of 2 as $n(SS)_i = 2^i$. The corresponding p-adic length scales (assignable to maximal ramified primes for given $n(SS)_i$) are expected to increase roughly exponentially, say as 2^{r2^i}. $r = 1/2$ would give a subset of scales $2^{r/2}$ allowed by the p-adic length scale hypothesis. These transitions would be very rare.

   A theory corresponding to a given composite polynomial would contain as sub-theories the theories corresponding to lower polynomial composites. The evolution with respect to $n(SS)$ would correspond to a sequence of phase transitions in which the action genuinely changes. For instance, color confinement could be seen as an example of this phase transition.
5. A subset of p-adic primes allowed by the p-adic length scale hypothesis $p \simeq 2^k$ defining the proposed p-adic length scale hierarchy could relate to n_S changing phase transition. TGD suggests a hierarchy of hadron physics corresponding to a scale hierarchy defined by Mersenne primes and their Gaussian counterparts [K73, K74]). Each of them would be characterized by a confinement phase transition in which n_S and therefore also the action changes.

2. Coupling constant evolutions with respect to ramified primes for a given value of $n(P)$

For a given value of $n(P)$, one could have coupling constant sub-evolutions with respect to the set of ramified primes of P and dimensions $n = h_{eff}/h_0$ of algebraic extensions. The action would only change by U(1) gauge transformation induced by a symplectic isometry of WCW. Coupling parameters could change but the actions would be equivalent.

The choice of the action in an optimal manner in a given scale could be seen as a choice of the most appropriate effective field theory in which radiative corrections would be taken into account. One can interpret the possibility to use a single choice of coupling parameters in terms of quantum criticality.

The range of the p-adic length scales labelled by ramified primes and effective Planck constants h_{eff}/h_0 is finite for a given value of $n(SS)$.

The first coupling constant evolution of this kind corresponds to ramified primes defining p-adic length scales for given $n(SS)$.

1. Ramified primes are factors of the discriminant $D(P)$ of P, which is expressible as a product of non-vanishing root differents and reduces to a polynomial of the n coefficients of P. Ramified primes define p-adic length scales assignable to the particles in the amplitudes scattering amplitudes defined by zero energy states.

   P would represent the space-time surface defining an interaction region in $N-$-particle scattering. The N ramified primes dividing $D(P)$ would characterize the p-adic length scales assignable to these particles. If $D(P)$ reduces to a single ramified prime, one has elementary particle [L129], and the forward scattering amplitude corresponds to the propagator.

   This would give rise to a multi-scale p-adic length scale evolution of the amplitudes analogous to the ordinary continuous coupling constant evolution of n-point scattering amplitudes with respect to momentum scales of the particles. This kind of evolutions extend also to evolutions with respect to $n(SS)$.

2. According to [L129], physical constraints require that $n(P)$ and the maximum size of the ramified prime of P correlate.

   A given rational polynomial of degree $n(P)$ can be always transformed to a polynomial with integer coefficients. If the integer coefficients are smaller than $n(P)$, there is an upper bound for the ramified primes. This assumption also implies that finite fields become fundamental number fields in number theoretical vision [L129].

3. p-Adic length scale hypothesis [L134] in its basic form states that there exist preferred primes $p \simeq 2^k$ near some powers of 2. A more general hypothesis states that also primes near some powers of 3 possibly also other small primes are preferred physically. The challenge is to understand the origin of these preferred scales.

   For polynomials P with a given degree $n(P)$ for which discriminant $D(P)$ is prime, there exists a maximal ramified prime. Numerical calculations suggest that the upper bound depends exponentially on $n(P)$.

   Could these maximal ramified primes satisfy the p-adic length scale hypothesis or its generalization? The maximal prime defines a fixed point of coupling constant evolution in accordance with the earlier proposal. For instance, could one think that one has $p \simeq 2^k$, $k = n(SS)$? Each p-adic prime would correspond to a p-adic coupling constant sub-evolution representable in terms of symplectic isometries.

Also the dimension n of the algebraic extension associated with P, which is identified in terms of effective Planck constant $h_{eff}/h_0 = n$ labelling different phases of the ordinary matter behaving like dark matter, could give rise to coupling constant evolution for given $n(SS)$. The range of allowed values of n is finite. Note however that several polynomials of a given degree can correspond to the same dimension of extension.

Number theoretic discretization of WCW and maxima of WCW Kähler function

Number theoretic approach involves a unique discretization of space-time surface and also of WCW. The question is how the points of the discretized WCW correspond to the preferred extremals.

1. The exponents of Kähler function for the maxima of Kähler function, which correspond to the universal preferred extremals, appear in the scattering amplitudes. The number theoretical approach involves a unique discretization of space-time surfaces defining the WCW coordinates of the space-time surface regarded as a point of WCW.

   In [L133] it is assumed that these WCW points appearing in the number theoretical discretization correspond to the maxima of the Kähler function. The maxima would depend on the action and would differ for ghd maxima associated with different actions unless they are not related by symplectic WCW isometry.

2. The symplectic transformations of WCW acting as isometries are assumed to be induced by the symplectic transformations of $\delta M^4_+ \times CP_2$ [K57, K36]. As isometries they would naturally permute the maxima with each other.

4.6 About the TGD based notions of mass, of twistors and hyperbolic counterpart of Fermi torus

The notion of mass in the TGD framework is discussed from the perspective of $M^8 - H$ duality [L100, L101, L134, L127].

1. In TGD, space-time regions are characterized by polynomials P with rational coefficients [L100, L101]. Galois confinement defines a universal mechanism for the formation of bound states. Momenta for virtual fermions have components, which are algebraic integers in an extension of rationals defined by a polynomial P characterizing a space-time region. For the physical many fermion states, the total momentum as the sum of fermion momenta has components, which are integers using the unit defined by the size of the causal diamond (CD) [L91, L119, L128].

2. This defines a universal number theoretical mechanism for the formation of bound states as Galois singlets. The condition is very strong but for rational coefficients it can be satisfied since the sum of all roots is always a rational number as the coefficient of the first order term.

3. Galois confinement implies that the sum of the mass squared values, which are in general complex algebraic numbers in E, is also an integer. Since the mass squared values correspond to conformal weights as also in string models, one has conformal confinement: states are conformal singlets. This condition replaces the masslessness condition of gauge theories [L134].

Also the TGD based notion of twistor space is considered at concrete geometric level.

1. Twistor lift of TGD means that space-time surfaces X^4 is $H = M^4 \times CP_2$ are replaced with 6-surfaces in the twistor space with induced twistor structure of $T(H) = T(M^4) \times T(CP_2)$ identified as twistor space $T(X^4)$. This proposal requires that T(H) has Kähler structure and this selects $M^4 \times CP_2$ as a unique candidate [A57] so that TGD is unique.

2. One ends up to a more precise understanding of the fiber of the twistor space of CP_2 as a space of "light-like" geodesics emanating from a given point. Also a more precise view of the induced twistor spaces for preferred extremals with varying dimensions of M^4 and CP_2 projections emerges. Also the identification of the twistor space of the space-time surface as the space of light-like geodesics itself is considered.

3. Twistor lift leads to a concrete proposal for the construction of scattering amplitudes. Scattering can be seen as a mere re-organization of the physical many-fermion states as Galois singlets to new Galois singlets. There are no primary gauge fields and both fermions and bosons are bound states of fundamental fermions. 4-fermion vertices are not needed so that there are no divergences.

4. There is however a technical problem: fermion and antifermion numbers are separately conserved in the simplest picture, in which momenta in $M^4 \subset M^8$ are mapped to geodesics of $M^4 \subset H$.The led to a proposal for the modification of $M^8 - H$ duality [L100, L101]. The modification would map the 4-momenta to geodesics of X^4. Since X^4 allows both Minkowskian and Euclidean regions, one can have geodesics, whose M^4 projection turns backwards in time. The emission of a boson as a fermion-antifermion pair would correspond to a fermion turning backwards in time. A more precise formulation of the modification shows that it indeed works

The third topic of this article is the hyperbolic generalization of the Fermi torus to hyperbolic 3-manifold H^3/Γ. Here $H^3 = SO(1,3)/SO(3)$ identifiable the mass shell $M^4 \subset M^8$ or its $M^8 - H$ dual in $H = M^4 \times CP_2$. Γ denotes an infinite subgroup of $SO(1,3)$ acting completely discontinuously in H^3. For virtual fermions also complexified mass shells are required and the question is whether the generalization of H^3/Γ, defining besides hyperbolic 3-manifold also tessellation of H^3 analogous to a cubic lattice of E^3.

4.6.1 Conformal confinement

The notion of mass distinguishes TGD from QFT. As in string models, mass squared corresponds to a conformal weight in TGD. However, in the TGD framework tachyonic states are not a curse but an essential part of the physical picture and conformal confinement, generalizing masslessness condition, states that the sum of conformal weights for physical states vanishes. This view conforms with the fact that Euclidean space-time regions are unavoidable at the level of H. Positive *resp.* negative *resp.* vanishing conformal weights can be assigned with Minkowskian *resp.* Euclidean space-time regions *resp.* light-like boundaries associated with them.

Mass squared as conformal weight, conformal confinement and its breaking

At the level of M^8, the momentum components for momenta as points of $H_c^3 \subset M_c^4 \subset M_c^8$ are (in general complex) algebraic integers in an extension of rationals defined by the polynomial P defining the space-time region. For physical states the momentum components for the sum of the momenta are ordinary integers when the momentum unit is defined by the size scales of causal diamond (CD). This scale corresponds to a p-adic length scale for p-adic prime, which is a ramified prime of the extension of rationals defined by the polynomial P.

For virtual many-fermion states the mass squared is an algebraic integer but an ordinary integer for the physical states [L134]. The question is whether the mass squared for the physical states can be negative so that one would have tachyons. The p-adic mass calculations require the presence of tachyonic mass squared values and the proposal is conformal confinement in the sense that the sum of mass squared values for the particles present in state and identifiable as conformal weights sum up to zero. Conformal confinement would generalize the masslessness condition of gauge field theories.

The observed mass squared values would correspond to the Minkowskian non-tachyonic parts of the mass squared values assignable to states, which in general are entangled states formed from tachyonic and non-tachyonic states. p-Adic thermodynamics would describe the entanglement in terms of the density matrix and observed mass squared would be thermal average. p-Adic thermodynamics leads to a breaking of the generalized conformal invariance and explains why different values of the Virasoro scaling generator L_0 are involved. Since complex mass squared values with a negative real part are allowed as roots of polynomials, the condition is highly non-trivial.

Association of mass squared values to space-time regions

$M^8 - H$ duality [L100, L101] would make it natural to assign tachyonic masses with CP_2 type extremals and with the Euclidean regions of the space-time surface. Time-like masses would be assigned with time-like space-time regions. In [L132] it was found that, contrary to the beliefs held hitherto, it is possible to satisfy boundary conditions for the action action consisting of the Kähler action, volume term and Chern-Simons term, at boundaries (genuine or between Minkowskian and Euclidean space-time regions) if they are light-like surfaces satisfying also $detg_4 = 0$. Masslessness, at least in the classical sense, would be naturally associated with light-like boundaries (genuine or between Minkowskian and Euclidean regions).

Riemann zeta, quantum criticality, and conformal confinement

The assumption that the space-time surface corresponds to rational polynomials in TGD is not necessary. One can also consider real analytic functions f [L127]. The condition that momenta of physical states have integer valued momentum components implies integer valued conformal weights poses extremely strong conditions on this kind of functions since the sum of the real parts of the roots of f must be an integer as a conformal weight identified as the sum of in general complex virtual mass squared values.

There are strong indications Riemann zeta (`https://cutt.ly/iVTV1kqs`) has a deep role in physics, in particular in the physics of critical systems. TGD Universe is quantum critical. What quantum criticality would mean at the space-time level is discussed in [L132]. This raises the question whether Riemann zeta could have a deep role in TGD.

First some background relating to the number theoretic view of TGD.

1. In TGD, space-time regions are characterized by polynomials P with rational coefficients [L100, L101]. Galois confinement defines a universal mechanism for the formation of bound states. Momenta for virtual fermions have components, which are algebraic integers in an extension of rationals defined by a polynomial P characterizing space-time region. For the physical many fermion states, the total momentum as the sum of fermion momenta has components, which are integers using the unit defined by the size of the causal diamond (CD).

   This defines a universal number theoretical mechanism for the formation of bound states. The condition is very strong but for rational coefficients it can be satisfied since the sum of all roots is always a rational number as the coefficient of the first order term.

2. Galois confinement implies that the sum of the mass squared values, which are in general complex algebraic numbers in E, is also an integer. Since the mass squared values correspond to conformal weights as also in string models, one can have conformal confinement: states would be conformal singlets. This condition replaces the masslessness condition of gauge theories [L134].

Riemann zeta [A42] (`https://cutt.ly/oVNSltD`)is not a polynomial but has infinite number of roots. How could one end up with Riemann zeta in TGD? One can also consider the replacement of the rational polynomials with analytic functions with rational coefficients or even more general functions [L127].

1. For real analytic functions roots come as pairs but building many-fermion states for which the sum of roots would be a real integer, is very difficult and in general impossible.

2. Riemann zeta and the hierarchy of its generalizations to extensions of rationals (Dedekind zeta functions, and L-functions in general) is however a complete exception! If the roots are at the critical line as the generalization of Riemann Hypothesis (RH) assumes, the sum of the root and its conjugate is equal to 1 and it is easy to construct many fermion states as 2N fermion states, such that they have integer value conformal weight.

   Since zeta has also trivial zeros for even negative integers interpretable in terms of tachyonic states, also conformal confinement with vanishing net conformal weight for physical states is possible. The trivial zeros would be associated with Euclidean space-time regions and non-trivial ones to Minkowskian ones.

   One can wonder whether one could see Riemann zeta as an analog of a polynomial such that the roots as zeros are algebraic numbers. This is however not necessary. Could zeta and its analogies allow it to build a very large number of Galois singlets and they would form a hierarchy corresponding to extensions of rationals. Could they represent a kind of second abstraction level after rational polynomials?

A possible interpretation is that in TGD, rational polynomials give discrete cognitive representations as approximations for physics. Cognitive representations are in the intersection of p-adicities and reality defined by the intersection of reals and extension of p-adics defined by the algebraic extension of the polynomial P defining a given space-time surface. Continuum theory would represent real numbers as a factor of the adele.

One can ask whether the various zeta functions consistent with the integer spectrum for the conformal weights and possibly also with conformal confinement, appear at the continuum limit and provide representations for the space-time surfaces at this limit? In this framework, it would be natural for the roots of zeta to be algebraic numbers [K96]. Also in the case of ζ, the virtual momenta of fermions would be algebraic integers for virtual fermions and integers for the physical states. This makes sense if the notions of Galois group and Galois confinement are sensible for ζ.

As noticed, the notion of ζ generalizes. The so-called global L-functions (`https://cutt.ly/3VNPYmp`) are formally similar to ζ and the extended Riemann Hypothesis (RH) could be true for them. The physical motivation for RH would be that it would allow a fermion with any conformal weight to appear in a state which is conformal singlet. Algebraic integers for a finite extension of rationals replace integers in the ordinary ζ and one has an entire hierarchy of L-functions. Could one think that the global L-functions could define preferred extremals at the continuum limit?

4.6.2 About the notion of twistor space

For the twistor lift of TGD, twistor space $T(X^4)$ of the space-time surface X^4 is identified an S^2 bundle over X^4 obtained by the induction of the twistor bundle $T(H) = T(M^4) \times T(CP_2)$. The definition of the $T(X^4)$ as 6-surface in $T(H)$ identifies the twistor spheres of $T(M^4)$ and $T(CP_2)$ and identifies it as a twistor sphere of $T(X^4)$.

The notion of twistor space for different different types of preferred extremals

I have not previously considered the notion of the induced twistor space for the different types of preferred extremals. Here some technical complications emerge.

1. Since the points of the twistor spaces $T(M^4)$ and $T(CP_2)$ are in 1-1 correspondence, one can use either $T(M^4)$ or $T(CP_2)$ so that the projection to M^4 or CP_2 would serve as the base space of $T(X^4)$. One could use either CP_2 coordinates or M^4 coordinates as space-time coordinates if the dimension of the projection is 4 to either of these spaces. In the generic case, both dimensions are 4 but one must be very cautious with genericity arguments, which turned out to fail at the level of M^8 [L100, L101].
2. There are exceptional situations in which genericity fails at the level of H. String-like objects of the form $X^2 \times Y^2 \subset M^4 \subset CP_2$ is one example of this. In this case, X^6 would not define 1-1 correspondence between $T(M^4)$ or $T(CP_2)$.

   Could one use partial projections to M^2 and S^2 in this case? Could $T(X^4)$ be divided locally into a Cartesian product of 3-D M^4 part projecting to $M^2 \subset M^4$ and of 3-D CP_2 part projected to $Y^2 \subset CP_2$?
3. One can also consider the possibility of defining the twistor space $T(M^2 \times S^2)$. Its fiber at a given point would consist of light-like geodesics of $M^2 \times S^2$. The fiber consists of direction vectors of light-like geodesics. S^2 projection would correspond to a geodesic circle $S^1 \subset S^2$ going through a given point of S^2 and its points are parametrized by azimuthal angle Φ. Hyperbolic tangent $tanh(\eta)$ with range $[-1, 1]$ would characterize the direction of a time like geodesic in M^2. At the limit of $\eta \rightarrow \pm\infty$ the S^2 contribution to the S^2 tangent vector to length squared of the tangent vector vanishes so that all angles in the range $(0, 2\pi)$ correspond to the same point. Therefore the fiber space has a topology of S^2.

   There are also other special situations such as $M^1 \times S^3$, $M^3 \times S^1$ for which one must introduce specific twistor space and which can be treated in the same way.

To deal with these special cases in which the dimensions of both M^4 and CP_2 are not equal to 4, one must allow also 6-surfaces X^6 which can have dimension of M^4 and CP_2 projections which are different from the canonical value 4. For CP_2 type extremals the dimension of CP_2 projection would be 6 and the dimension of M^4 projection would be 1. For cosmic strings the dimensions of M^4 projection and CP_2 projection would be 2.

The concrete definition of the twistor space of H as the space of light-like geodesics

During the writing of this article I realized that the twistor space of H defined geometrically as a bundle, which has as H as base space and fiber as the space of light-like geodesic starting from a given point of H, need not be equal to $T(M^4) \times T(CP_2)$, where $T(CP_2)$ is identified as $SU(3)/U(1) \times U(1)$ characterizing the choices of color quantization axes. Is this really the case?

1. The definition of $T(CP_2)$ as the space of light-like geodesics from a given point of CP_2 is not possible. One could also define the fiber space of $T(CP_2)$ geometrically as the space of geodesics emating from origin at $r = 0$ in the Eguchi-Hanson coordinates [K19] and connecting it to the homologically non-trivial geodesic sphere S^2_G $r = \infty$. This relation is symmetric.

   In fact, all geodesics from $r = 0$ end up to S^2. This is due to the compactness and symmetries of CP_2. In the same way, the geodesics from the North Pole of S^2 end up to the South Pole. If only the endpoint of the geodesic of CP_2 matters, one can always regard it as a point S^2_G.

The two homologically non-trivial geodesic spheres associated with distinct points of CP_2 always intersect at a single point, which means that their twistor fibers contain a common geodesic line of this kind. Also the twistor spheres of $T(M^4)$ associated with distinct points of M^4 with a light-like distance intersect at a common point identifiable as a light-like geodesic connecting them.

2. Geometrically, a light-like geodesic of H is defined by a 3-D momentum vector in M^4 and 3-D color momentum along CP_2 geodesic. The scale of the 8-D tangent vector does not matter and the 8-D light-likeness condition holds true. This leaves 4 parameters so that $T(H)$ identified in this way is 12-dimensional.

   The M^4 momenta corresponds to a mass shell H^3. Only the momentum direction matters so that also in the M^4 sector the fiber reduces to S^2. If this argument is correct, the space of light-like geodesics at point of H has the topology of $S^2 \times S^2$ and $T(H)$ would reduce to $T(M^4) \times T(CP_2)$ as indeed looks natural.

The twistor space of the space-time surface

The twistor lift of TGD allows to identify the twistor space of the space-time surface X^4 as the base space of the S^2 bundle induced from the 12-D twistor space $T(8) = T(M^4) \times CP(2)$ to the 6-surface $X^6 \subset T(H)$ by a local dimensional reduction to $X^4 \times S^2$ occurring for the preferred extremals of 6-D Kähler action existing only in case of $H = M^4 \times CP_2$.

Could the geometric definition of $T(X^4)$ as the space of light-like geodesics make sense in the Minkowskian regions of X^4?

1. By their definition, stating that the length of the tangent vector of the geodesic is conserved, the geodesic equations conserve the value of the velocity squared so that light-likeness can be forced via the initial values. This allows the assignment of a twistor sphere to a given point of a Minkowskian space-time region. Whether this assignment can be made global is not at all trivial and the difficulties related to the definition of twistor space in general relativity probably reflects this problem. If this is the case, then the direct geometric definition might not make sense unless the very special properties of the PEs come to rescue.

2. The twistor lift of TGD is proposed to modify the definition of the twistor space so that one can assign twistor structure to the space-time surface by inducing the twistor structure of H just as one can assign spinor structure with the space-time surface by inducing the spinor structure of H.

   Could the generalized holomorphic structure, implying that PEs are extremals of both volume and of 4-D Kähler action, make possible the existence of light-like geodesics and even allow to assign to a given point of the space-time surface sphere parametrizing light-like geodesics?

3. The light-like 3-surfaces X^3 representing partonic orbits carry fermionic lines as light-like geodesics and are therefore especially interesting. They are metrically 2-D and boundary conditions for the field equations force the vanishing of the determinant $det(g_4)$ of the induced metric at them so that the dimension of the tangent space is effectively reduced. Light-like 3-surfaces allow a generalization of isometries such that conformal symmetries accompanied by scaling of the light-like radial coordinate depending on transversal complex coordinates is isometry.

   It seems that to a given point of the space-like intersection, only a single light-like geodesic can be assigned so that the twistor space at a given point would consist of a single light-like geodesic. This would be caused by the light-likeness of X^3.

The geometric definition of the twistor space for CP_2

In the case of the Euclidean regions, the notion of a light-like geodesic does not make sense. The closed geodesics and the presence of pairs of points analogous to North pole-South pole pairs, where diverging geodesics meet, would be required. This condition is very strong and the minimal requirement is that the space has a positive curvature so that the geodesics do not diverge. Also symmetries seem to be necessary. Clearly, something new is required.

1. The addition of Kähler coupling term equal to an odd multiple of the induced Kähler gauge potential A to the spinor connection is an essential element in the definition of a generalized spinor structure of CP_2.
2. Should one replace the light-like geodesics with orbits of Kähler charged particles for which CP_2 has been replaced with $p - q_K A$. For the counterparts of light-like geodesics $p - q_K A$ would vanish and the analog of mass squared would vanish but one would have a line. For a geodesic p would be constant.

   Is it possible to have $A = constant$ along a closed geodesic? In the case of sphere, the Kähler gauge potential in the spherical coordinates is $(A_\theta = A_\phi = kcos(\theta)$ and is constant along the geodesics going through South and North Poles. Something like this could happen in the case of CP_2 but it seems that a special pair of homological non-trivial spheres S^2 invariant under $U(2) \subset SU(3)$ is selected. One might perhaps speak of symmetry breaking.

   To obtain entire S^2 of light-like geodesics in this sense, the geodesics must emanate from a coordinate singularity, the origin of Eguchi-Hanson coordinates at $r = 0$, where the values of the coordinates (θ, ϕ, ψ) correspond to the same point. The space for the light-like geodesics must be 2-D rather than 3-D. This must be forced by the $p - A = 0$ condition. For the homologically trivial geodesic sphere $r = \infty$, Ψ coordinate is redundant so that the conserved value of A_ψ must vanish for the light-like geodesics and the associated velocities cannot have component in the direction of Ψ.
3. Note that this definition could apply also in Minkowskian regions of space-time surface.

The description of particle reactions without vertices

In standard field theory, particles are point-like and particle reactions are described using vertices assignable to non-linear interaction terms in the action.

1. In the TGD framework, particles are replaced with 3-surfaces and elementary particles are assigned to partonic 2-surface whose orbits correspond to light-like 3-surfaces identifiables as the boundary regions between Minkowskian and Euclidean space-time regions and modelled as wormhole contacts between two space-time sheets with a Minkowskian signature. Vertices are replaced with topological vertices at which incoming partonic 2-surfaces, whose orbits are light-like 3-surfaces, meet at partonic 2-surfaces.
2. In TGD, all particles are composites of fundamental fermions assignable to the wormhole throats identified as partonic orbits. In particular, bosons consist of fermions and antifermions assignable to the throats of wormholes. Since wormhole contact contains homologically trivial 2-surface of CP_2, there is a monopole flux throwing out of the throat and one must have at least two wormhole contacts so that one obtains a closed monopole flux flowing between the sheets and forming a closed flux tube.
3. The light-like orbits of the partonic 2-surfaces contain fermionic lines defined at the ends of string world sheets connecting different partonic orbits. In QFT description, this would require a 4-fermion vertex as a fundamental vertex involving dimensional coupling constant and leading to a non-renormalizable QFT. Therefore there can be no vertices at the level of fermion lines.

In the number theoretic vision based on Galois confinement [L126, L127], the interactions correspond at the level of M^8 to re-arrangements of virtual fermions, having virtual momentum components in the extension of rationals defined by P, to new combinations required to be Galois singlets and therefore having momentum components, which are ordinary integers. Note that P fixes by holography the 4-surface in M^8 in turn defining the space-time surface in H by $M^8 - H$ duality based on associativity.

There is however a problem. If the particle reactions are mere re-arrangements of fundamental fermions and antifermions, moving along light-like geodesic lines in fixed time direction, the total numbers of fermions and antifermions are separately conserved. How can one overcome this problem without introducing the disastrous 4-fermion vertex?

Consider FFB vertex describing boson emission by fermion as a concrete example.

1. B is described as a pair of partonic surfaces containing at least one fermion-antifermion pair, which must be created in the vertex. Incoming particles for the topolocal FFB 3-vertex correspond to partonic orbits for incoming F and outgoing F, each containing one fermion line and possibly a pair of fermion and antifermion.

2. The idea is that boson emission as a pair creation could be described geometrically as a turning of fermion backwards in time. This forces us to reconsider the definition of M^8-H duality. The simplest view of M^8-H duality is that momenta of $M^4 \subset M^8$ are mapped to the geodesic lines of M^4. Tachyonic momenta in $M^4 \subset M^8$ would be mapped to space-like geodesics in H emanating from the center of CD which is a sub-CD of a larger CD in general. It seems that this definition does not allow us to understand boson emission by fermion in the way proposed in [L127].

3. This led to a proposal that the images of momenta could be geodesics of the space-time surface X^4, rather than H. Since X^4 allows also Euclidean regions and the interiors of the deformed CP_2 type extremals are Euclidean, one ends up with the idea that the geodesics lines of X^4 can have M^4 projections, which turn backwards in the time direction [L100, L101, L122].

   This would allow us to interpret the emission of a boson as a fermion-antifermion pair as the turning of a fermionic line backwards in time. Fermions lines would be identified as the boundaries of string world sheets. Sub-manifold gravitation would play a key role in the elimination of 4-fermion vertex and thus of QFT type divergences.

4. But is it possible to have a light-like geodesic arriving at the partonic 2-surface and continuing as a light-like geodesic in the Euclidean wormhole contact and returning back? The problem is that in Euclidean regions, ordinary light-like geodesics degenerate to points. The generalization of the light-like geodesics satisfying $p=qA$ implying $(p-qA)^2=0$ is possible. At the space-time level, these conditions could be true quite generally and give as a special case light-like geodesics with $p^2=0$ in the Minkowskian regions.

4.6.3 About the analogies of Fermi torus and Fermi surface in H^3

Fermi torus (cube with opposite faces identified) emerges as a coset space of E^3/T^3, which defines a lattice in the group E^3. Here T^3 is a discrete translation group T^3 corresponding to periodic boundary conditions in a lattice.

In a realistic situation, Fermi torus is replaced with a much more complex object having Fermi surface as boundary with non-trivial topology. Could one find an elegant description of the situation?

Hyperbolic manifolds as analogies for Fermi torus?

The hyperbolic manifold assignable to a tessellation of H^3 defines a natural relativistic generalization of Fermi torus and Fermi surface as its boundary. To understand why this is the case, consider first the notion of cognitive representation.

1. Momenta for the cognitive representations [L133] define a unique discretization of 4-surface in M^4 and, by M^8-H duality, for the space-time surfaces in H and are realized at mass shells $H^3 \subset M^4 \subset M^8$ defined as roots of polynomials P. Momentum components are assumed to be algebraic integers in the extension of rationals defined by P and are in general complex.

   If the Minkowskian norm instead of its continuation to a Hermitian norm is used, the mass squared is in general complex. One could also use Hermitian inner product but Minkowskian complex bilinear form is the only number-theoretically acceptable possibility. Tachyonicity would mean in this case that the real part of mass squared, invariant under $SO(1,3)$ and even its complexification $SO_c(1,3)$, is negative.

2. The active points of the cognitive representation contain fermion. Complexification of H^3 occurs if one allows algebraic integers. Galois confinement [L133, L130] states that physical states correspond to points of H^3 with integer valued momentum components in the scale defined by CD.

   Cognitive representations are in general finite inside regions of 4-surface of M^8 but at H^3 they explode and involve all algebraic numbers consistent with H^3 and belonging to the extension of rationals defined by P. If the components of momenta are algebraic integers, Galois confinement allows only states with momenta with integer components favored by periodic boundary conditions.

Could hyperbolic manifolds as coset spaces $SO(1,3)/\Gamma$, where Γ is an infinite discrete subgroup $SO(1,3)$, which acts completely discontinuously from left or right, replace the Fermi torus? Discrete translations in E^3 would thus be replaced with an infinite discrete subgroup Γ. For a given P, the matrix coefficients for the elements of the matrix belonging to Γ would belong to an extension of rationals defined by P.

1. The division of $SO(1,3)$ by a discrete subgroup Γ gives rise to a hyperbolic manifold with a finite volume. Hyperbolic space is an infinite covering of the hyperbolic manifold as a fundamental region of tessellation. There is an infinite number of the counterparts of Fermi torus [L116]. The invariance respect to Γ would define the counterpart for the periodic boundary conditions.

   Note that one can start from $SO(1,3)/\Gamma$ and divide by $SO(3)$ since Γ and $SO(3)$ act from right and left and therefore commute so that hyperbolic manifold is $SO(3)\setminus SO(1,3)/\Gamma$.

2. There is a deep connection between the topology and geometry of the Fermi manifold as a hyperbolic manifold. Hyperbolic volume is a topological invariant, which would become a basic concept of relativistic topological physics (`https://cutt.ly/RVsdNl3`).

   The hyperbolic volume of the knot complement serves as a knot invariant for knots in S^3. Could this have physical interpretation in the TGD framework, where knots and links, assignable to flux tubes and strings at the level of H, are central. Could one regard the effective hyperbolic manifold in H^3 as a representation of a knot complement in S^3?

   Could these fundamental regions be physically preferred 3-surfaces at H^3 determining the holography and M^8-H duality in terms of associativity [L100, L101]. Boundary conditions at the boundary of the unit cell of the tessellation should give rise to effective identifications just as in the case of Fermi torus obtained from the cube in this way.

De Sitter manifolds as tachyonic analogies of Fermi torus do not exist

Can one define the analogy of Fermi torus for the real 4-momenta having negative, tachyonic mass squared? Mass shells with negative mass squared correspond to De-Sitter space $SO(1,3)/SO(1,2)$ having a Minkowskian signature. It does not have analogies of the tessellations of H^3 defined by discrete subgroups of $SO(1,3)$.

The reason is that there are no closed de-Sitter manifolds of finite size since no infinite group of isometries acts completely discontinuously on de Sitter space: therefore these is no group replacing the Γ in H^3/Γ. (`https://cutt.ly/XVsdLwY`).

Do complexified hyperbolic manifolds as analogies of Fermi torus exist?

The momenta for virtual fermions defined by the roots defining mass squared values can also be complex. Tachyon property and complexity of mass squared values are not of course not the same thing.

1. Complexification of H^3 would be involved and it is not clear what this could mean. For instance, does the notion of complexified hyperbolic manifold with complex mass squared make sense.

2. $SO(1,3)$ and its infinite discrete groups Γ act in the complexification. Do they also act completely discontinuously? p^2 remains invariant if $SO(1,3)$ acts in the same way on the real and imaginary parts of the momentum leaves invariant both imaginary and complex mass squared as well as the inner product between the real and imaginary parts of the momenta. So that the orbit is 5-dimensional. Same is true for the infinite discrete subgroup Γ so that the construction of the coset space could make sense. If Γ remains the same, the additional 2 dimensions can make the volume of the coset space infinite. Indeed, the constancy of $p_1 \cdot p_2$ eliminates one of the two infinitely large dimensions and leaves one.

   Could one allow a complexification of $SO(1,3)$, $SO(3)$ and $SO(1,3)_c/SO(3)_c$? Complexified $SO(1,3)$ and corresponding subgroups Γ satisfy $OO^T=1$. Γ_c would be much larger and contain the real Γ as a subgroup. Could this give rise to a complexified hyperbolic manifold H_c^3 with a finite volume?

3. A good guess is that the real part of the complexified bilinear form $p \cdot p$ determines what tachyonicity means. Since it is given by $Re(p)^2 - Im(p)^2$ and is invariant under $SO_c(1,3)$ as also $Re(p) \cdot Im(p)$, one can define the notions of time-likeness, light-likeness, and space-likeness using the sign of $Re(p)^2 - Im(p^2)$ as a criterion. Note that $Re(p)^2$ and $Im(p)^2$ are separately invariant under $SO(1,3)$.

   The physicist's naive guess is that the complexified analogies of infinite discrete and discontinuous groups and complexified hyperbolic manifolds as analogies of Fermi torus exist for $Re(P^2) - Im(p^2) > 0$ but not for $Re(P^2) - Im(p^2) < 0$ so that complexified dS manifolds do not exist.

4. The bilinear form in H_c^3 would be complex valued and would not define a real valued Riemannian metric. As a manifold, complexified hyperbolic manifold is the same as the complex hyperbolic manifold with a hermitian metric (see `https://cutt.ly/qVsdS7Y` and `https://cutt.ly/kVsd3Q2`) but has different symmetries. The symmetry group of the complexified bilinear form of H_c^3 is $SO_c(1,3)$ and the symmetry group of the Hermitian metric is $U(1,3)$ containing $SO(1,3)$ as a real subgroup. The infinite discrete subgroups Γ for $U(1,3)$ contain those for $SO(1,3)$. Since one has complex mass squared, one cannot replace the bilinear form with hermitian one. The complex H^3 is not a constant curvature space with curvature -1 whereas H_c^3 could be such in a complexified sense.

4.7 The notion of generalized integer

This chapter was inspired by the article "Space Element Reduction Duplication (SERD) model produces photon-like information packets and light-like cosmological horizons" by Thomas L. Wood, published in Metodologia IV B: Journal of International and Finnish Methodology, expresses the basic assumptions of the SERD approach very coherently and in a systematic way so that it easy to criticize them and compare with other views, in my case the TGD view.

My criticism, summarized below, is based on a different interpretations of the discreteness. In TGD framework would be assignable to cognitive representations based on p-adic numbers fields involving extensions of rationals rather than being a feature of space-time. The introduction of continuous number fields (reals, complex numbers, quaternions, octonions) besides p-adic number fields brings in real space-time as sensory representation and one ends up to a generalization of the standard model proving a number theoretic interpretation for its symmetries.

The approach of Wood looks is essentially topological: for instance, the information propagating in the hypergraph is assumed to be topological and characterize the graph. In TGD, discrete structures analogs define cognitive representations of the continuous sensory world and are basically number theoretic. The description of the sensory world involves both topology and geometry.

4.7.1 The first reactions to the abstract

The abstract gives a very concise summary of the approach and I have added below my reactions to it. The following commentary is my attempt to understand the basic ideas of SERD. I have also used the third section of the article to clarify my views. I must admit that I didn't quite get the two basic principles in the beginning of the third section. I have slightly re-organized the abstract and hope that I have not done any damage.

[TW] This document describes a correspondence between photons and propagating information packets (PIPs) that are emergent out of the Space Element Reduction Duplication (SERD) model introduced in a rudimentary form in [1, 2]. The SERD model is a discrete background independent microscopic space-time description.

[MP] The assumption of discreteness at the fundamental space-time level raises several challenges. 4-D space-time with Minkowskian signature should somehow emerge. The mere hypergraph might possess under additional assumptions a local dimension defined homologically/combinatorially but would vary. Note that in standard homology theory an embedding to some space is required and would give a metric. Now the distance and other geometric notions look problematic to me.

One can also ask what kind of dynamics for hypergraphs could select the 4-D space-time? Should one have a variational principle of some kind?

The notion of symmetries is central in physics. Lorentz invariance or even Poincare invariance should emerge as approximate symmetries at least. Only discrete subgroups of these groups can emerge in the hypergraph approach. Lorentz invariance poses very, perhaps too, powerful constraints on the hypergraphs. The notion of discretized time is introduced. It should be Lorentz invariant and here the light-cone proper time a serves as an analog. a=constant sections would be analogs of hyperbolic 3-space H^3.

[TW] By observation of physically comparable behaviour emerging from this system, through analysis and computer simulation, we draw conclusions of what the form and dynamics of the true underlying space-time may be.

By treating elements of the system as fundamental observers, mathematical and empirical evidence is obtained of the existence of fully emergent light-like cosmological horizons, implying the existence of causally separated 'pocket universes'.

[MP] The emergence of the analogy with expanding cosmology presumably reflects the underlying dynamics implying the increase of the size of the hypergraph. The emergence of light-like causal horizons is natural if the dynamics involves maximal velocity of propagation for the signals. This is probably due to the locality of the basic dynamics involving only local changes of the hypergraph topology. Locality and classicality raise challenges if one wants to describe phenomena like quantum entanglement.

[TW] The SERD model is a hypergraph of connected hyperedges called Point Particles (PP) which represent the fundamental constituents of all matter and particles (and therefore observers) separated by strings of consecutive and fundamental elements or edges called Space Elements (SE).

[MP] I had to clarify myself what a hypergraph is. Hypergraph is a generalization of graphs. Also it contains the set of vertices/nodes. The notion of edge connecting a pair of vertices is however generalized to a hyperedge (PP) as a pair of subsets of vertices. PPs correspond to hyperedges as fundamental constituents of matter and formed by pairs of subsets of the set of nodes.

One could interpret this as a combinatorial counterpart for a length scale hierarchy of TGD in which a set can be approximated as a point. One might also interpret subsets of vertices as analogs of bound states of fundamental particles. In the TGD framework, many-sheeted space-time and various other hierarchies serve as its analogs.

Space elements (SEs) would bring in basic aspects of 3-space. It is said that they are infinitesimal or maximally small. SEs would be like edges (not hyperedges) of the hypergraph. Consecutive SEs in turn form interaction edges (IEs) connecting PPs. IEs store and transmit information relating to the structure space. What comes to mind is that functionally PPs are like neutrons and neuron groups and IEs are like axons.

[TW] All elements are separated by nodes called Information Gaps (IGs), that store propagating topological information of the hypergraph. Information gaps (IGs) are between PPs and SEs, between SEs and between PPs themselves.

[MP] What distinguishes the SERD model from physical theories, is that information takes the role of matter. Information is treated as some kind of substance. The basic objection is that conscious information is always about something, whereas matter just is.

IGs have the role of interfaces somewhat analogous to black-hole horizon assumed to store information in the holographic picture. One could see PPs as the nodes and IEs as the edges or SEs as the edges and IGs as the nodes. IGs could have synaptic contacts as analogs.

[TW] In time step (TS), SE can duplicate and reduce (disappear) while the PPs split and merge through discrete time. These processes create space or destroy it and increase or reduce the effective distance between PPs. Splitting generates an SE between the resulting PPs. These are known as the actions of the elements and create a highly dynamic multi-way system.

[MP] Time step (TS) is a further basic notion and corresponds to an elementary event as nearest neighbor interaction taking during the time chronon. The propagation rate for information is CS/TS and is analogous to maximal signal velocity. The counterpart of the space-time metric is thus brought in by the introduction of TS and CS.

SEs emerge or disappear so that the effective distances of the nearby points change: this would the counterpart for the dynamics of space-time metric in General Relativity. I understood that duplication and reduction effectively corresponds to the duplication or halving of the distance assignable to SE.

[TW] Elements have an 'awareness' of the information around them and communicate with their nearest neighbours through time.

[MP] The treatment of elements as fundamental observers is an interesting idea but can be criticized. Why not PPs? One could also argue that the SEs become conscious observers only under some additional assumptions. For instance, one can imagine that they represent matter and become fundamental conscious observers if fermions or fermion pairs can be assigned to them.

The abstract says nothing about quantum theory. To my view it is very difficult to imagine how quantum theory could emerge from an approach based on classical probability and some kind of quantum approach would be required to understand entanglement and state function reduction.

4.7.2 Fundamental discretization as a cognitive representation?

In the sequel TGD view of the discretization interpreted as cognitive representation is described. The surprise was the discovery of what I call generalized integers and rationals as a union of various p-adic number fields with different p-adic number fields glued together along numbers which belong to both p-adic number fields. I do not know whether mathematicians have played with this thought. This space has an ultrametric topology and could have application to the description of spin glass type systems [L120]. In TGD it could have application in the mathematical description of processes in which the p-adic prime associated with the particle changes.

Something is discrete but what it is?

Something is discrete at the fundamental level: is it space-time or only a discrete cognitive representation, a discretization of a continuous space-time? The essential assumption of SERD is that it is space-time, which is fundamentally discrete and realized as hypergraph. The basic problem is that it is not clear whether the notions of space-time dimensions, distance, angle, and curvature can emerge in a purely combinatorial approach in which only distance between nearby nodes is a metric notion. These notions also have a formal generalization to gauge theories.

The alternative approach would be based on the observation that cognition is discrete and finite. Cognition provides representations of the physical world. Could one assume that the physical world has continuous geometry and that only cognition is discrete?

Could the cognitive Universe consist of generalized integers?

Integers (and rationals) are the simplest discrete but infinite systems. Integers/rationals are usually assumed to have real topology. One can however imagine an infinite number of p-adic topologies, which are ultrametric and are defined by a p-adic norm having values coming as powers of prime p. p-Adic primes typically have an infinite expansion in powers of p and large powers of p have small p-adic norm in contrast to the real norm.

p-Adic integer/rational has expansion in powers of p and the inverse of the smallest power in the expansion determines the norm so that the notion of size is completely different for p-adic and real integers. Note that also the p-adic expansion of rationals involves an infinite number of powers of p but is periodic. p-Adic transcendentals do not have this property. Note also that p-adic integers modulo p define a finite field $G(p)$.

p-Adic integers are only weakly ordered. Only if two p-adic integers/ rationals have different p-adic norms, can one tell which is the larger one. One can however construct continuous maps from p-adics to reals to approximately preserve the norm. p-Adic norm is ultrametric and this property is essential in the thermodynamic models of spin glass energy landscape [L120].

One could, at least as the first guess, imagine that the Universe of cognition consists of integers/rationals or a finite subset of them and that one also allows integers/rationals, which are infinite as real integers but finite as p-adic integers for some prime p.

One can decompose generalized integers to subsets with different p-adic topologies.

1. Regions corresponding to two different p-adic topologies p_1 and p_2 have as an interface as the set integers, which have an expansion in powers of $n_{12} = p_1p_2$. Therefore the cognitive world decomposes into p-adic regions having interfaces, which consist of power series of $n_{12..k} = p_1^{k_1} ... \times p_k^{n_k}$. Ordinary integer n with a decomposition to primes belongs to the interface of the p-adic worlds corresponding to the prime factors.

How does this decomposition relate to adeles [L50, L48], which can be regarded as a Cartesian product of p-adic number fields defining and of reals [L50, L48]? Adeles correspond to a Cartesian product but now one has a union so that these concepts seems to be different. I do not know whether mathematicians have encountered the notion of generalized integers and rationals.

2. Each p-adic region decomposes into shells, kinds of analogs of mass shells, consisting of p-adic integers with p-adic norm given by a power of p.

3. The distance between the points of the cognitive sub-landscape corresponding to p would be defined by the p-adic norm. The points with the same p-adic norm would have a distance defined as the p-adic norm of their difference. This distance is the same for several point pairs so that p-adic topology is much rougher than the real topology. For instance the p-adic norm of numbers $1, ..., p-1$ is the same.

4. One could define a distance between points associated with p-adic topologies p_1 and p_2 as the shortest distance between them identified as the sum of the distances to the interface between these regions.

In this framework, the analog of a hypergraph would be simply a subset of generalized integers decomposing to p-adic integers labeled by some subset of primes.

1. The simplest dynamical operation, having now an interpretation as a cognitive operation, would be addition or removal of a p-adic integer corresponding to some value of p-adic prime or several of them. The addition would have an interpretation or worsening or improving the cognitive representation for some prime p.

2. Arithmetic operations for the points inside a region corresponding to a given p are possible. Arithmetic operations of finite integers are basic elements of at least human cognition and their sum and product would correspond to "particle reactions" in which two points fuse together to form a sum or product. If infinite integers can be expressed as power series of integers n_1 and n_2, they can be regarded as p-adic integers for the factors of n_1 and n_2 and both sum and product make sense for common prime factors. Note that the operations are well-defined also for generalized rationals.

3. What happens in the arithmetic operations information theoretically? In the product operation, the outcome is in the interface region associated with n_1 and n_2 and the information about factors is not lost since a measurement revealing prime factors can be done repeatedly.

   The projection operator applied to a quantum superposition of integers would project to a subspace of integers, which are divisible by a given prime p. This operation could be repeated for different primes and eventually give the prime number decomposition for some integer n in the superposition.

   One strange fact about idiot savants described by Oliver Sacks (this is discussed from the TGD point of view in [K97]) is that they can decompose integers into prime factors and obviously see the emergence of the prime factors. Could this kind of cognitive measurement be in question?

   Sum does not in general belong to the interface region of either integer and information is lost since many number pairs give rise to the same sum. Therefore sum and product are information-theoretically very different operations.

Could there be a quantum physical realization for the arithmetic operations? Could they relate to our conscious arithmetic thinking?

1. Consider first the sum operation. Quantum numbers, such as momenta, represented as integers or even algebraic integers are conserved in the physical reaction vertices. The conserved quantum numbers for the final state for a fusion reaction are sums of integers so that these reactions have an arithmetic interpretation.

2. In the case of a product, the fusion reaction should give a product of integers n_1 and n_2 or a representation of it? One should have conserved multiplicative quantum numbers in the vertex.

Phase factors as eigenvalues of unitary operators are such. They should form a multiplicative group as representation of integers or even rationals. Integer scalings define such a group. One can also consider eigenvalues $n^{i\phi}$, ϕ some fixed phase angle. The operator would therefore be a scaling represented unitarily by these phase factors.

Initial state would be a product of eigenstates of the scaling operator with eigenphases $n_1^{i\phi}$ and $n_2^{i\phi}$ and the final state would be a single particle state with the eigenvalue $n_1^{i\phi}n_2^{i\phi} = (n_1n_2)^{i\phi}$. One can say that n_1 acts on n_2 by scaling or vice versa. Interestingly, at the fundamental level scalings replace time translations in the TGD framework (and also in superstring theory), and this is especially so for spin glass phase [L120].

Interestingly, sum appears at the level of Lie algebras and product at the level of Lie groups.

In quantum groups also the reverse operations, co-product and co-sum, having pair creation as analog, are possible. For the co-sum the information increases for the product. These operations would be time reversals of each other. In the zero energy ontology (ZEO) of TGD time reversal occurs in "big" (ordinary) state function reductions (BSFRs) [L91, L128] [K130]. What comes to mind is that the idiot savants described by Sacks might perform a time reversal decomposing product to prime factors. The cognitive measurement would correspond to BSFR.

Note that ZEO also predicts "small" state function reductions (SSFRs), which do not change the arrow of time and give rise to the flow of consciousness whereas BSFR corresponds to a universal counterpart of death or of falling asleep. It is the TGD counterpart of repeated measurements in the Zeno effect and of weak measurements of quantum optics.

This cognitive world would in TGD correspond physically to the most general spin glass energy landscape having an ultrametric topology [L120].

The algebraic extensions of p-adic number fields are discrete

The proposed structure does not have any natural notion of dimension. We are however able to cognize higher dimensional spaces using formulas.

1. p-Adic number fields indeed allow infinite hierarchies of algebraic extensions obtained by adding to them roots of polynomials, which are algebraic numbers. These induce extensions of p-adic number fields as finite fields $G(p,k)$ having algebraic dimension, which is at most the dimension of the corresponding extensions of rationals.

2. It is natural to assume that cognitive representations are always finite. This suggests that the set of "populated" points of the cognitive space is discrete and even finite. Being "populated" could mean that a fermion, having an interpretation as a generator of Boolean algebra, is labelled by the algebraic number defining the point. In a more general formulation bringing in quaternions and octonions as number fields: algebraic complexified quaternions would define the momentum components of fermions.

   What has been said above, generalizes almost as such and one obtains a hierarchy of generalized integers as algebraic extensions of generalized integers at the lowest level. This could generalize the rational number based computationalism (Turing paradigm) to an entire hierarchy of cognitive computationalisms. The hierarchy of algebraic extensions suggests the same.

3. The algebraic complexity of generalized integers increases with the dimension of extension and in the TGD framework it corresponds to an evolutionary hierarchy. The dimension of extension defines what is identified in terms of an effective Planck constant.

But what about the real world?

A hierarchy of p-adicities and hierarchies of the algebraic extensions of p-adicities have been obtained. The 4-D world of sensory perceptions with its fundamental symmetries is however still missing. Could number theory come to rescue also here? This is indeed the case.

1. The fundamental continuous number fields consist of reals, complex numbers, quaternions and octonions with dimensions 1,2,4, 8 [L100, L101, L135]. Quaternions cannot as such correspond

to 4-D space-time since the number theoretic purely algebraic norm defines the Euclidean metric.

2. This norm can be however algebraically continued to the complexification of quaternions obtained by adding a commuting imaginary unit i commuting with quaternionic and octonionic imaginary units. This algebraic norm squared does not involve complex conjugation as the Hilbert space norm and is in general complex but real for the subspaces corresponding to various metric signatures (a given component of quaternion are either real or imaginary). One obtains therefore Minkowski space and even more: its variants with various metric signatures.

3. One can imagine a generalization of the notion of generalized integer so that one would have hierarchies of generalized complex numbers, quaternions and octonions and their complexifications for various extensions of rationals.

   A possible problem relates to the p-adic variants of quaternions, octonions and complex numbers. Consider the inverse $z^{-1} = (x-iy)/(x^2+y^2)$ of p-adic complex numbers $z = x+iy$. The problem is that $x^2 + y^2$ can vanish since there is no notion of sign of the number. For $p \ mod\ 4 = 1$, $\sqrt{-1}$ is an ordinary p-adic number, albeit with an infinite pinary expansion so that for $y = \sqrt{-1}x$, one has this problem.

   Could the finiteness of cognition solve the problem? If only finite p-adic integers and rationals can define momentum components of fermions (finite cognitive and measurement resolution), the problem disappears.

   Could one give up the field property for the p-adic variants of classical number fields? Already the complexification by i forces to give up the field property but has physical meaning since it makes Minkowski signature possible.

This would give Minkowski space M^4 as a special case. This is however not enough. One wants curved 4-D space-times. The basic structure is complexified octonions.

1. One should obtain 4-D surfaces of M^8 generalizing empty Minkowski space M^4. Octonions fail to be associative and at the level of M^8_c the natural proposal is that there is number theoretic dynamics based on associativity. The 4-D surfaces must be associative in some sense. The geometric vision predicts holography and this holography should have a number theoretic counterpart based on associativity.

2. The first guess is that the tangent space of 4-surface is associative and thus quaternionic. This gives only M^4 and is therefore trivial [L100, L101, L135].

   The requirement that the normal space of the 4-surface Y^4 in M^8_c is associative/quaternionic however works. If one requires that the normal subspace contains also a commutative (complex) subspace, one ends up to $M^8 - H$-duality ($H = M^4 \times CP_2$ mapping the associative 4-D surfaces Y^4 of M^8_c to space-time surfaces X^4 in H determined by holography forced by generalized coordinate invariance. The symmetries of H include Poincare symmetries and standard model symmetries.

3. At the level of M^8, associativity of the normal space allows also 6-D surfaces with 2-D commutative normal space and they can be interpreted in terms of analogs of 6-D twistor spaces of 4-D surfaces Y^4. They can be mapped to to the twistor space of H by $M^8 - H$ duality and define 6-D twistor spaces of space-time surfaces X^4 of H. What is beautiful is that the Kähler structure for the twistor space of H exists only for the choice $H = M^4 \times CP_2$, which is also forced by the associative dynamics [A57]! TGD is unique!

4. The dynamics would rely on holography but how to get the algebraic extensions? The roots of a polynomial P with rational or even integer coefficients satisfying some additional conditions would define the needed extension of rationals. The roots would in the general case define complex mass shells H^3_c as complex variants of hyperbolic 3-spaces H^3 in $M^4_c \subset M^8_c$ having interpretation as a momentum space. $M^8 - H$ duality serves as a generalization of momentum position duality. The 3-D surfaces as subsets of these H^3:s define the data of the associative holography and are contained by the 4-surface Y^4.

5. Cognitive representation would be defined as a unique number theoretic discretization of the 4-surface Y^4 of M^8_c consisting of points, whose number theoretically preferred linear Minkowski coordinates are algebraic integers in an extension defining the 4-surface in question.

This discretization induces discretization of the space-time surface via $M^8 - H$ duality. The cognitive representations are number-theoretically universal and belong to the intersections of realities and p-adicities.

6. The mass shells H_c^3 are very special since in the preferred Minkowski coordinates a cognitive explosion takes place. All algebraic rationals, in particular integers, are points of H_c^3. Algebraic integers are physically favored and define components of four-momenta. Galois confinement [L121] states that the total momenta have components which are ordinary integers when a suitable momentum unit is used.

4.8 Infinite primes as a basic mathematical building block

Infinite primes [K105, K58, K71] are one of the key ideas of TGD. Their precise physical interpretation and the role in the mathematical structure of TGD has however remained unclear.

3 new ideas are be discussed. Infinite primes could define a generalization of the notion of adele; quantum arithmetics could replace $+$ and $\times$ with $\oplus$ and $\otimes$ and ordinary primes with p-adic representations of say HFFs; the polynomial Q defining an infinite prime could be identified with the polynomial P defining the space-time surface: $P = Q$.

4.8.1 Construction of infinite primes

Consider first the construction of infinite primes [K105].

1. At the lowest level of hierachy, infinite primes (in real sense, p-adically they have unit norm) can be defined by polynomials of the product X of all primes as an analog of Dirac vacuum.

   The decomposition of the simplest infinite primes at the lowest level are of form $aX+b$, where the terms have no common prime divisors. More concretely $a = m_1/n_F$ $b = m_0 n_F$, where n_F is square free integer analogous and the integer m_1 and n_F have no common prime divisors divisors. The divisors of m_2 are divisors of n_F and m_i has interpretation as n-boson state. Power p^k corresponds to k-boson state with momenta p. $n_F = \prod p_i$ has interpretation as many-fermion state satisfying Fermi-Dirac statistics.

   The decomposition of lowest level infinite primes to infinite and finite part has a physical analogy as kicking of fermions from Dirac sea to form the finite part of infinite prime. These states have interpretation as analogs of free states of supersymmetric arithmetic quantum field theory (QFT) There is a temptation to interpret the sum $X/n_F + n_F$ as an analog of quantum superposition. Fermion number is well-defined if one assigns the number of factors of n_F to both n_F and X/n_F.

2. More general infinite primes correspond to polynomials $Q(X) = \sum_n q_n X^n$ required to define infinite integers which are not divisible by finite primes. Each summand $q_n X^n$ must be a infinite integer. This requires that q_n is given by $q_n = m_{B,n}/\prod_{i_1}^n n_{F,i}$ of square free integers $n_{F,i}$ having no common divisors.

   The coefficients $m_{B,n}$ representing bosonic states have no common primes with $\prod n_{F,i}$ and there exists no prime dividing all coefficients $m_{B,n}$: there is no boson with momentum p present in all states in the sum.

   These states have a formal interpretation as bound states of arithmetic supersymmetric QFT. The degree k of Q determines the number of particles in the bound states.

   The products of infinite primes at given level are infinite primes with respect to the primes at the lower levels but infinite integers at their own level. Sums of infinite primes are not in general infinite primes. For instance the sum and difference of $X/n_F + n_F$ and $X/n_F - n_F$ are not infinite primes.

3. At the next step one can form the product of all finite primes and infinite primes constructed in this manner and repeat the process as an analog to second quantization. This procedure can be repeated indefinitely. This repeated quantization a hierarchy of infinite primes, which could correspond to the hierarchy of space-time sheets.

   At the n:th hierarchy level the polynomials are polynomials of n variables X_i. A possible interpretation would be that one has families of infinite primes at the first level labelled by n_1 parameters. If the polynomials $P(x)$ at the first level define space-time surfaces, the interpretation at the level of WCW could be that one has an $n-1$-D surface in WCW parametrized by $n-1$ parameters with rational values and defining a kind of sub-WCW. The WCW spinor fields would be restricted to this surface of WCW.

The Dirac vacuum X brings in mind adele, which is roughly a product of p-adic number fields. The primes of infinite prime could be interpreted as labels for p-adic number fields. Even more generally, they could serve as labels for p-adic representations of various algebras and one could even consider replacing the arithmetic operations with $\oplus$ and $\otimes$ to get the quantum variants of various number fields and of adeles.

The quantum counterparts of nfinite primes at the lowest and also at the higher levels of hierarchy could be seen as a generalization of adeles to quantum adeles.

4.8.2 Questions about infinite primes

One can ask several questions about infinite primes.

1. Could $\oplus$ and $\otimes$ replace $+$ and $-$ also for infinite primes. This would allow us to interpret the primes p as labels for algebras realized p-adically. This would give rise to quantal counterparts of infinite primes.
2. What could $+ \rightarrow \oplus$ for infinite primes mean physically? Could it make sense in adelic context? Infinite part has finite p-adic norms. The interpretation as direct sum conforms with the fermionic interpretation if the product of all finite primes is interpreted as Dirac sea. In this case, the finite and infinite parts of infinite prime would have the same fermion number.
3. Could adelization relate to the notion of infinite primes? Could one generalize quantum adeles based on $\oplus$ and $\otimes$ so that they would have parts with various degrees of infinity?

4.8.3 $P = Q$ hypothesis

One cannot avoid the idea that that polynomial, call it $Q(X)$, defining an infinite prime at the first level of the hierarchy, is nothing but the polynomial P defining a 4-surface in M^4 and therefore also a space-time surface. $P = Q$ would be a condition analogous to the variational principle defining preferred extremals (PEs) at the level of H.

There is however an objection.

1. $P = Q$ gives very powerful constraints on Q since it must define an infinite integer. The prime polynomials P are expected to be highly non-unique and an entire class of polynomials of fixed degree characterized by the Galois group as an invariant is in question. The same applies to polynomials Q as is easy to see: the only condition is that powers of $a_k X^k$ defining infinite integers have no common prime factors.
2. It seems that a composite polynomial $P_n \circ ... \circ P_1$ satisfying $P_i = Q_i$ cannot define an infinite prime or even infinite integer. Even infinite integer property requires very special conditions.
3. There is however no need to assume $P_i = Q_i$ conditions. It is enough to require that there exists a composite $P_n \circ ... \circ P_1$ of prime polynomials satisfying $P_n \circ ... \circ P_1 = Q$ defining an infinite prime.

   The physical interpretation would be that the interaction spoils the infinite prime property of the composites and they become analogs of off-mass-shell particles. Exactly this occurs for bound many-particle states of particles represented by P_i represented composite polynomials $P_1 \circ ... P_n$. The roots of the composite polynomials are indeed affected for the composite. Note that also products of Q_i are infinite primes and the interpretation is as a free many-particle state formed by bound states Q_i.

There is also a second objection against $P = Q$ property.

1. The proposed physical interpretation is that the ramified primes associated with $P = Q$ correspond to the p-adic primes characterizing particles. This would mean that the ramimied primes appearing in the infinite primes at the first level of the hierarchy should be physically special.

2. The first naive guess is that for the simplest infinite primes $Q(X) = (m_1/n_F)X + m_2n_F$ at the first level, the finite part m_2n_F has an identification as the discriminant D of the polynomial $P(X)$ defining the space-time surface. This guess has no obvious generalization to higher degree polynomials $Q(X)$ and the following argument shows that it does not make sense.

   Since Q is a rational polynomial of degree 1 there is only a single rational root and discriminant defined by the differences of distinct roots is ill-defined that $Q = P$ condition would not allow the simplest infinite primes.

   Therefore one must give either of these conjectures and since $P = Q$ conjecture dictates the algebraic structure of the quantum theory for a given space-time surface, it is much more attractive.

The following argument gives $P = Q$. One can assign to polynomial P invariants as symmetric functions of the roots. They are invariants under permutation group S_n of roots containing Galois group and therefore also Galois invariants (for polynomials of second order correspond to sum and product of roots appearing as coefficients of the polynomial in the representation $x^2 + bx + cx$). The polynomial Q having as coefficients these invariants is the original polynomial. This interpretation gives $P = Q$.

4.9 Summary of the proposed big picture

In the previous sections the plausible looking building blocks of the bigger picture of the TGD were discussed. Here I try to summarize a guess for the big picture.

4.9.1 The relation between $M^8 - H$ and $M - M'$ dualities

The first question is whether $M^8 - H$ duality between number theoretical and geometric physics, very probably relating to Langlands duality, corresponds to a duality between M and its commutant M'. Physical intuition suggests that these dualities are independent. M' would more naturally correspond to classical description as dual to quantum description using M. One would assign classical and quantum views to both number theoretic (M^8) and geometric (H) descriptions.

1. At the geometric side M would be realized in terms of HFFs associated with SSA_n, Aff_n and I acting in H. At the number theoretic side, braided Galois group algebras would define the HFFs and have natural action in SSA_n, A_n and I .
2. The descriptions in terms of preferred extremals in H and of polynomials P defining 4-surfaces in M^8 would correspond to classical descriptions. $P = Q$ condition would define preferred polynomials and infinite primes.
3. At the geometric side, M' would correspond to scalar factors of WCW wave functions symplectic invariants identifiable as Kähler magnetic fluxes at both M^4 and CP_2 sectors. They are zero modes and therefore do not contribute to the WCW line element.
4. At the number theoretic side, the wave functions would depend on Galois invariants. Discriminant D, set of roots to which braid strands can be assigned to define n-braid, and ramified primes dividing it in the case of polynomials with rational/integer coefficients are Galois invariants analogous to Kähler fluxes. They code information about the spectrum of virtual mass squared values as roots of P. The strands of braid as Galois invariant correspond to (possibly) monopole flux tubes and one assign them quantized magnetic fluxes as integer valued symplectic invariants.

4.9.2 Basic mathematical building blocks

The basic mathematical building blocks of quantum aspects of TGD involve at least the following ones.

1. The generalization of arithmetics and even number theory by replacing sum and product by direct sum and tensor product for various algebras and associated representations is a mathematical notion expected to be important and a straightforward generalization of adeles and infinite primes to their quantum counterparts is highly suggestive.

2. Quantum version of adelic physics obtained by replacing ordinary arithmetic operations with direct sum and tensor product relates closely to the fusion of real and various p-adic physics at quantum level.
3. The hierarchy of infinite primes suggested by the many-sheeted space-time suggests a profound generalization of the notion of adelic physics. Infinite primes are defined by polynomials of several variables the basic equation in the general form would be $Q(X_1, ..., X_n) = P(X_1, ..., X_n)$.

4.9.3 Basic algebraic structures at number theoretic side

Number theoretic side involves several key notions that must have counterparts at the geometric side.

1. Number theoretic side involves Galois groups as counterparts of symplectic symmetries and can be regarded as number theoretic variants of permutation symmetries and lead to the notion of braided Galois group, whose group algebra defines HFF.
2. Galois groups can be decomposed to a hierarchy of normal subgroups, which are simple and therefore primes in group theoretic sense. Simple Galois groups correspond to polynomial primes with respect to functional composition, and one can assign to a given Galois group a set of polynomials with fixed degrees although the polynomials and their order of polynomials in composition are not unique.
3. There is a large class of polynomials giving rise to a given Galois group and they bring in additional degrees of freedom. The variation of the polynomial coefficients corresponding to the same Galois group is analogous to symplectic transformations leaving the induced Kähler form invariant.

   The roots of polynomials define analogs for the strands of n-braid, discriminant D, and ramified primes dividing the discriminant. They are central Galois invariants analogous to Kähler magnetic fluxes at the geometry side.
4. Ramified primes characterize polynomials P but are not fixed by the Galois group, are analogous to the zero modes at the level of H. Magnetic fluxes are their counterparts at the level of H. I have proposed the interpretation of ramified primes p as p-adic primes characterizing elementary particles in the model of particle masses based on p-adic thermodynamics. These primes are rather large: for instance, $M_{127} = 2^{127} - 1$ would characterize electrons. It would however seem that the prime k in SSA_k corresponds to the prime characterizing simple Galois group.

   Also affine algebras Aff_n assignable to the light-like partonic orbits and isometries of H are present and also they appear in p-adic mass calculations based on p-adic thermodynamics. Could the adelic hierarchy p-adic variants of algebras SSA, Aff and I have adelic factors labelled by ramified primes p form also an adelic structure with respect to $\oplus$ and $\otimes$?

4.9.4 Basic algebraic structures at the geometric side

The symmetry algebras at the level of H define the key quantal structures.

1. The symmetries at the geometric side involve hierarchies A_n of algebras $A_n \in SSA_n, A_n, I_n$ defining hierarchies of factor algebras. The condition that subalgebras A_n and $[A_n, A]$ annihilate physical states gives rise to hierarchies of algebras, which would correspond to those for Galois groups for multiple extensions of rationals. The braided Galois groups for polynomials of degree n n roots/braids would act naturally in A_n so that it would have number theoretic braiding.
2. The decomposition of the Galois group to simple normal subgroups would correspond to a functional composite of prime polynomials, which corresponds to the inclusion hierarchy of HFFs associated with A_n with n identified as the degree of polynomial.

   The polynomials $Q(X)$ defining infinite prime have decomposition to polynomial primes but the polynomial primes in the decomposition cannot define infinite primes.

   Kähler magnetic fluxes for CP_2 and M^4 Kähler forms are symplectic invariants and represent zero modes. At the number theoretic side the discriminant and root spectrum (mass squared spectrum) are classical Galois invariants. States as Galois singlets are Galois invariants at quantum level.

The key equation, not encountered before in the TGD framework, is $P = Q$ motivated by the notion of infinite prime. It would assign to polynomial P unique algebraic structures defining what might be called its quantization. Without this structure one should give up the notion of infinite prime and lose the notion of preferred P as analog of preferred extremal.

4.10 Appendix: The reduction of quantum TGD to WCW geometry and spinor structure

The first attempts to build quantum TGD were based on the standard method used to quantize quantum field theories. The path integral over all possible space-time surfaces connecting initial and final 3-surfaces for an action exponential using for instance Kähler action, would have given the scattering amplitudes.

4.10.1 The problems

The first problem is that the integrand is a phase factor $exp(iS)$, where S could be the Kähler action. Phase factor has modulus 1 and the integral does not converge even formally. One would need a real exponent to have any hopes of convergence. This problem can be circumvented in free quantum field theory by algebraic tricks.

The second problem is that all conceivable actions are extremely nonlinear and new kinds of divergences appear in each order of perturbation theory. This is essentially due to the locality of the action principle involving interaction vertices with arbitrarily high numbers of particles. Also ordinary QFTs meet the same problem and for renormalizable theories the addition of counterterms with suitably infinite coefficients can cancel the divergences without the addition of an infinite number of counter terms. It became clear that there are no hopes of getting rid of the divergences in TGD by addition of counterterms. The situation is the same in general relativity although heroic and ingenious attempts to calculate scattering amplitudes have been made.

Only $\mathcal{N} = 4$ SUSY is a QFT that is hoped to be free of divergences without renormalization but here the problem is caused by the non-planar Feynman diagrams, to which the twistor approach does not apply.

4.10.2 3-D surfaces or 4-surfaces associated to them by holography replace point-like particles

The key idea of TGD is that point-like particles are replaced with 3-surfaces. This idea does not favour path integral approach.

1. In TGD, point-like particles are replaced with 3-surfaces. Local interaction vertices are smoothed out to non-local ones so that there should be no local divergences. Perhaps the path integral, derived originally as a representation of Schrödinger equation, is not only unnecessary but also a wrong way to compute anything in TGD. In superstring models, the replacement of a point-like particle with string indeed allows elimination of the local divergences.

   3-D surface should be the basic dynamical object. One should therefore have a functional integral over 3-surfaces, which is analogous to the Gaussian integral and converges.

2. This problem led to the idea of the "world of classical worlds" (WCW). 4-D General Coordinate Invariance implies that to a given 3-surface X^3 one must be able to assign a 4-surface $X^4(X^3)$ at which the 4-D general coordinate transformations act.

   Either 3-surfaces X^3 or almost unique 4-surfaces $X^4(X^3)$ are the fundamental objects so that holography holds true. At that time I did not talk about holography, which was introduced by Susskind much later, around 1995. Therefore the introduction of the path integral is not necessary.

   Later it became clear that the exact determinism of the classical dynamics can be lost, at least for Kähler action having huge spin glass degeneracy. Later 4-D Käehler action replaced in twistor lift of TGD by its sum with a volume term, and for this action the non-determinism is analogous to that for soap films spanned by frames, that is finite, and has physical interpretation.

4.10.3 WCW Kähler geometry as s geometrization of the entire quantum physics

This argument led to the vision about quantum TGD as WCW geometry, which generalizes Einstein's vision of geometrization of gravitational interaction to geometrization of all classical interactions and then to the geometrization of the entire quantum theory.

1. WCW is the space of all 3-surfaces or almost equivalently the space of 4-surfaces. Physical states correspond to WCW spinor fields.
2. WCW must have Kähler geometry since Kähler structure allows to geometrize the hermitian conjugation which is fundamental for quantum theory. Imaginary unit is represented geometrically by the Kähler form and the real unit by the Kähler metric. The tensor square Kähler form as an imaginary unit is equal to the negative of the real unit, that is the negative of the metric.
3. The construction of loop space geometries by Dan Freed [A37] led to a unique geometry of loop space. The mere existence of Riemann connection requires that the metric has maximal isometries and is unique apart from scaling. When basic objects are 3-D this condition is even more stringent. The Kähler geometry of WCW and thus physics could be unique from its mere mathematical existence!

   Why $H = M^4 \times CP_2$? The existence of the twistor lift fixes H uniquely since only M^4 (E^4) and CP_2 allow a twistor space with Kähler structure [A57]. The necessarily dimensionally reduced Kähler action at the twistor space level adds to the 4-D Kähler action a volume term removing the non-determinism and explaining cosmological constant and its smallness in long scales.
4. How is the Kähler geometry of WCW determined? The definition of the Kähler metric of WCW must assign to a 3-surface X^3 a more or less unique space-time surface $X^4(X^3)$ in order to have a general coordinate invariance. One must also have a connection with classical physics: classical physics must be an exact part of quantum physics and thus the definition of WCW Kähler geometry involves a classical action principle.

   The Kähler metric is defined by the Kähler function K. The idea is that K is the value of Kähler action S_K or of a more general action for a more or less unique space-time surface $X^4(X^3)$ containing a given 3-surface X^3.
5. It is convenient to speak of preferred extremal (PE) and there are several characterizations of what PE is. $M^8 - H$ duality gives the most concrete one. Twistor lift gives the second one and the gauge conditions associated with the WCW Dirac equation provide the third characterization.

4.10.4 Quantum physics as physics of free, classical spinor fields in WCW

How to develop quantum physics in WCW? The idea is that free, classical WCW spinor fields define all possible quantum states of the Universe and interactions reduce to topology. There would be no quantization at the level of WCW and the only genuinely quantal element of quantum theory would be state function reduction giving rise to conscious experience.

1. In order to have spinor fields in WCW, one must have the notion of spinor structure. Spinor structure is almost uniquely fixed by the metric and involves in an essential manner gamma matrices, which anticommute to metric.
2. The second quantization of H spinor fields assigns to the modes of H spinor fields fermionic oscillator operators. Why not build the conplexified gamma matrices of WCW (their hermitian conjugates) as linear combinations of the creation (annihilation) operators?! Second

quantization for the *free* H spinor field, is completely unique and straightforward and avoids all problems of quantization in curved space-time.

One could interpret the second quantization of free fermions and fermionic statistics in terms of WCW geometry, which is something completely new.

3. WCW spinors (for given 4-surface as point of WCW) would be fermionic Fock states created using fermionic oscillator operators and depend on the space-time surface $X^4(X^3)$ as a 4-surface almost uniquely determined by 3-surface X^3.

   The fermionic Fock state basis can be interpreted as a representation of Boolean logic so that Boolean logic could be seen as a "square root" of Kähler geometry.

   The WCW spinor field would correspond to a superposition of preferred extremals X^4 with a WCW spinor assigned with each X^4.

4.10.5 Dirac equation for WCW spinor fields

Free Dirac equation is the key equation for classical spinor fields.

1. In string models it corresponds to the analogs of super-Virasoro and super-Kac-Moody conditions stating conformal invariance and Kac-Moody invariance analogous but not quite equivalent with gauge symmetry.

2. In TGD, these conditions as a counterpart of the WCW Dirac equation generalize. Super symplectic algebra associated with $\delta M^4_+ \times CP_2$ (δM^4_+ denotes light-cone boundary) SSA, the infinite-D algebras of conformal symmetries ($Conf$) and isometries (I) of δM^4_+ (unique to the 4-D Minkowski space), and the affine algebras Aff associated with the light-like orbits of partonic 2-surfaces would be the basic algebras.

3. To each of these algebras, one can assign a generalization of the gauge conditions of conformal field theories. What is new is that one obtains a hierarchy of gauge conditions. The algebra in question, call it A, and sub-algebra A_n, $n \geq 0$, with conformal weights coming as n-multiples of weights for A, and the commutator $[A_n, A]$ annihilate the physical states. Also the corresponding classical Noether charges vanish, which gives strong conditions on space-time surfaces and decomposes WCW to sectors characterized by n.

4. In superstring models one has only $n = 0$. In the number theoretic vision, the hierarchy of values of n would actually correspond to the hierarchy of extensions of rationals. If $M^8 - H$ duality holds true, n corresponds to the degree of polynomial P defining the space-time surface and polynomials P would decompose WCW to sectors.

4.10.6 $M^8 - H$ duality at the level of WCW

WCW emerges in the geometric view of quantum TGD. $M^8 - H$ duality should lso work for WCW. What is the number theoretic counterpart of WCW? What is the geometric counterpart of the discretization characteristic to the number theoretic approach?

In the number theoretic vision in which WCW is discretized by replacing space-time surfaces with their number theoretical discretizations determined by the points of $X^4 \subset M^8$ having the octonionic coordinates of M^8 in an extension of rationals and therefore making sense in all p-adic number fields? How could an effective discretization of the real WCW at the geometric H level, making computations easy in contrast to all expectations, take place?

1. The key observation is that any functional or path integral with integrand defined as exponent of action, can be *formally* calculated as an analog of Gaussian integral over the extrema of the action exponential $exp(S)$. The configuration space of fields would be effectively discretized. Unfortunately, this holds true only for the so called integrable quantum field theories and there are very few of them and they have huge symmetries. But could this happen for WCW integration thanks to the maximal symmetries of the WCW metric?

2. For the Kähler function K, its maxima (or maybe extrema) would define a natural effective discretization of the sector of WCW corresponding to a given polynomial P defining an extension of rationals.

The discretization of the WCW defined by polynomials P defining the space-time surfaces should be equivalent with the number theoretical discretization induced by the number theoretical discretization of the corresponding space-time surfaces. Various p-adic physics and corresponding discretizations should emerge naturally from the real physics in WCW.

3. The physical interpretation is clear. The TGD Universe is analogous to the spin glass phase [L120]. The discretized WCW corresponds to the energy landscape of spin glass having an ultrametric topology. Ultrametric topology of WCW means that discretized WCW decomposes to p-adic sectors labelled by polynomials P. The ramified primes of P label various p-adic topologies associated with P.

Chapter 5

TGD view about McKay Correspondence, ADE Hierarchy, Inclusions of Hyperfinite Factors, $M^8 - H$ Duality, SUSY, and Twistors

5.1 Introduction

There are two mysterious looking correspondences involving ADE groups. McKay correspondence between McKay graphs characterizing tensor products for finite subgroups of $SU(2)$ and Dynkin diagrams of affine ADE groups is the first one. The correspondence between principal diagrams characterizing inclusions of hyper-finite factors of type II_1 (HFFs) with Dynkin diagrams for a subset of ADE groups and Dynkin diagrams for affine ADE groups is the second one.

I have considered the interpretation of McKay correspondence in TGD framework already earlier [K125, K48] but the decision to look it again led to a discovery of a bundle of new ideas allowing to answer several key questions of TGD.

1. Asking questions about $M^8 - H$ duality at the level of 8-D momentum space [L40] led to a realization that the notion of mass is relative as already the existence of alternative QFT descriptions in terms of massless and massive fields suggests (electric-magnetic duality). Depending on choice $M^4 \subset M^8$, one can describe particles as massless states in $M^4 \times CP_2$ picture (the choice is M^4_L depending on state) and as massive states (the choice is fixed M^4_T) in M^8 picture. p-Adic thermal massivation of massless states in M^4_L picture can be seen as a universal dynamics independent mechanism implied by ZEO. Also a revised view about zero energy ontology (ZEO) based quantum measurement theory as theory of consciousness suggests itself.
2. Hyperfinite factors of type II_1 (HFFs) [K125, K48] and number theoretic discretization in terms of what I call cognitive representations [L73] provide two alternative approaches to the notion of finite measurement resolution in TGD framework. One obtains rather concrete view about how these descriptions relate to each other at the level of 8-D space of light-like momenta. Also ADE hierarchy can be understood concretely.
3. The description of 8-D twistors at momentum space-level is also a challenge of TGD. 8-D twistorializations in terms of octo-twistors (M^4_T description) and $M^4 \times CP_2$ twistors (M^4_L description) emerge at embedding space level. Quantum twistors could serve as a twistor description at the level of space-time surfaces.

5.1.1 McKay correspondence in TGD framework

Consider first McKay correspondence in more detail.

1. McKay correspondence states that the McKay graphs characterizing the tensor product decomposition rules for representations of discrete and finite sub-groups of $SU(2)$ are Dynkin diagrams for the affine ADE groups obtained by adding one node to the Dynkin diagram of ADE group. Could this correspondence make sense for any finite group G rather than only discrete subgroups of $SU(2)$? In TGD Galois group of extensions K of rationals can be any finite group G. Could Galois group take the role of G?

2. Why the subgroups of $SU(2)$ should be in so special role? In TGD framework quaternions and octonions play a fundamental role at M^8 side of M^8-H duality [L40]. Complexified M^8 represents complexified octonions and space-time surfaces X^4 have quaternionic tangent or normal spaces. $SO(3)$ is the automorphism group of quaternions and for number theoretical discretizations induced by extension K of rationals it reduces to its discrete subgroup $SO(3)_K$ having $SU(2)_K$ as a covering. In certain special cases corresponding to McKay correspondence this group is finite discrete group acting as symmetries of Platonic solids. Could this make the Platonic groups so special? Could the semi-direct products $Gal(K) \triangleleft SU(2)_K$ take the role of discrete subgroups of $SU(2)$?

5.1.2 HFFs and TGD

The notion of measurement resolution is definable in terms of inclusions of HFFs and using number theoretic discretization of X^4. These definitions should be closely related.

1. The inclusions $\mathcal{N} \subset \mathcal{M}$ of HFFs with index $\mathcal{M} : \mathcal{N} < 4$ are characterized by Dynkin diagrams for a subset of ADE groups. The TGD inspired conjecture is that the inclusion hierarchies of extensions of rationals and of corresponding Galois groups could correspond to the hierarchies for the inclusions of HFFs. The natural realization would be in terms of HFFs with coefficient field of Hilbert space in extension K of rationals involved.

   Could the physical triviality of the action of unitary operators $\mathcal{N}$ define measurement resolution? If so, quantum groups assignable to the inclusion would act in quantum spaces associated with the coset spaces $\mathcal{M}/\mathcal{N}$ of operators with quantum dimension $d = \mathcal{M} : \mathcal{N}$. The degrees of freedom below measurement resolution would correspond to gauge symmetries assignable to $\mathcal{N}$.

2. Adelic approach [L49] provides an alternative approach to the notion of finite measurement resolution. The cognitive representation identified as a discretization of X^4 defined by the set of points with points having H (or at least M^8 coordinates) in K would be common to all number fields (reals and extensions of various p-adic number fields induced by K). This approach should be equivalent with that based on inclusions. Therefore the Galois groups of extensions should play a key role in the understanding of the inclusions.

How HFFs could emerge from TGD?

1. The huge symmetries of "world of classical words" (WCW) could explain why the ADE diagrams appearing as McKay graphs and principal diagrams of inclusions correspond to affine ADE algebras or quantum groups. WCW consists of space-time surfaces X^4, which are preferred extremals of the action principle of the theory defining classical TGD connecting the 3-surfaces at the opposite light-like boundaries of causal diamond $CD = cd \times CP_2$, where cd is the intersection of future and past directed light-cones of M^4 and contain part of $\delta M^4_\pm \times CP_2$. The symplectic transformations of $\delta M^4_+ \times CP_2$ are assumed to act as isometries of WCW. A natural guess is that physical states correspond to the representations of the super-symplectic algebra SSA.

2. The sub-algebras SSA_n of SSA isomorphic to SSA form a fractal hierarchy with conformal weights in sub-algebra being n-multiples of those in SSA. SSA_n and the commutator $[SSA_n, SSA]$ would act as gauge transformations. Therefore the classical Noether charges for these sub-algebras would vanish. Also the action of these two sub-algebras would annihilate the quantum states. Could the inclusion hierarchies labelled by integers $.. < n_1 < n_2 < n_3 ....$

with n_{i+1} divisible by n_i would correspond hierarchies of HFFs and to the hierarchies of extensions of rationals and corresponding Galois groups? Could n correspond to the dimension of Galois group of K.

3. Finite measurement resolution defined in terms of cognitive representations suggests a reduction of the symplectic group SG to a discrete subgroup SG_K, whose linear action is characterized by matrix elements in the extension K of rationals defining the extension. The representations of discrete subgroup are infinite-D and the infinite value of the trace of unit operator is problematic concerning the definition of characters in terms of traces. One can however replace normal trace with quantum trace equal to one for unit operator. This implies HFFs and the hierarchies of inclusions of HFFs [K125, K48]. Could inclusion hierarchies for extensions of rationals correspond to inclusion hierarchies of HFFs and of isomorphic sub-algebras of SSA?

Quantum spinors are central for HFFs. A possible alternative interpretation of quantum spinors is in terms of quantum measurement theory with finite measurement resolution in which precise eigenstates as measurement outcomes are replaced with universal probability distributions defined by quantum group. This has also application in TGD inspired theory of consciousness [K48]: the idea is that the truth value of Boolean statement is fuzzy. At the level of quantum measurement theory this would mean that the outcome of quantum measurement is not anymore precise eigenstate but that one obtains only probabilities for the appearance of different eigenstate. One might say that probability of eigenstates becomes a fundamental observable and measures the strength of belief.

5.1.3 New aspects of $M^8 - H$ duality

M^8-H duality ($H = M^4\times CP_2$) [L40] has become one of central elements of TGD. M^8-H duality implies two descriptons for the states.

1. $M^8 - H$ duality assumes that space-time surfaces in M^8 have associative tangent- or normal space M^4 and that these spaces share a common sub-space $M^2 \subset M^4$, which corresponds to complex subspace of octonions (also integrable distribution of $M^2(x)$ can be considered). This makes possible the mapping of space-time surfaces $X^4 \subset M^8$ to $X^4 \subset H = M^4 \times CP_2$) giving rise to $M^8 - H$ duality.

2. $M^8 - H$ duality makes sense also at the level of 8-D momentum space in one-one correspondence with light-like octonions. In $M^8 = M^4 \times E^4$ picture light-like 8-momenta are projected to a fixed quaternionic $M^4_T \subset M^8$. The projections to $M^4_T \supset M^2$ momenta are in general massive. The group of symmetries is for E^4 parts of momenta is $Spin(SO(4)) = SU(2)_L \times SU(2)_R$ and identified as the symmetries of low energy hadron physics.

   $M^4 \supset M^2$ can be also chosen so that the light-like 8-momentum is parallel to $M^4_L \subset M^8$. Now CP_2 codes for the E^4 parts of 8-momenta and the choice of M^4_L and color group $SU(3)$ as a subgroup of automorphism group of octonions acts as symmetries. This correspond to the usual description of quarks and other elementary particles. This leads to an improved understanding of $SO(4) - SU(3)$ duality. A weaker form of this duality $S^3 - CP_2$ duality: the 3-spheres S^3 with various radii parameterizing the E^4 parts of 8-momenta with various lengths correspond to discrete set of 3-spheres S^3 of CP_2 having discrete subgroup of $U(2)$ isometries.

3. The key challenge is to understand why the MacKay graphs in McKay correspondence and principal diagrams for the inclusions of HFFs correspond to ADE Lie groups or their affine variants. It turns out that a possible concrete interpretation for the hierarchy of finite subgroups of $SU(2)$ appears as discretizations of 3-sphere S^3 appearing naturally at M^8 side of $M^8 - H$ duality. Second interpretation is as covering of quaternionic Galois group. Also the coordinate patches of CP_2 can be regarded as piles of 3-spheres and finite measurement resolution. The discrete groups of $SU(2)$ define in a natural way a hierarchy of measurement resolutions realized as the set of light-like M^8 momenta. Also a concrete interpretation for Jones inclusions as inclusions for these discretizations emerges.

4. A radically new view is that descriptions in terms of massive and massless states are alternative options leads to the interpretation of p-adic thermodynamics as a completely universal

massivation mechanism having nothing to do with dynamics. The problem is the paradoxical looking fact that particles are massive in H picture although they should be massless by definition. The massivation is unavoidable if zero energy states are superposition of massive states with varying masses. The M^4_L in this case most naturally corresponds to that associated with the dominating part of the state so that higher mass contributions can be described by using p-adic thermodynamics and mass squared can be regarded as thermal mass squared calculable by p-adic thermodynamics.

5. As a side product emerges a deeper understanding of ZEO based quantum measurement theory and consciousness theory. 4-D space-time surfaces correspond to roots of octonionic polynomials $P(o)$ with real coefficients corresponding to the vanishing of the real or imaginary part of $P(o)$.

   These polynomials however allow universal roots, which are not 4-D but analogs of 6-D branes and having topology of S^6. Their M^4 projections are time =constant snapshots $t = r_n, r_M \leq r_n$ 3-balls of M^4 light-cone (r_n is root of $P(x)$). At each point the ball there is a sphere S^3 shrinking to a point about boundaries of the 3-ball.

   What suggests itself is following "braney" picture. 4-D space-time surfaces intersect the 6-spheres at 2-D surfaces identifiable as partonic 2-surfaces serving as generalized vertices at which 4-D space-time surfaces representing particle orbits meet along their ends. Partonic 2-surfacew would define the space-time regions at which one can pose analogs of boundary values fixing the space-time surface by preferred extremal property. This would realize strong form of holography (SH): 3-D holography is implied already by ZEO.

   This picture forces to consider a modification of the recent view about ZEO based theory of consciousness. Should one replace causal diamond (CD) with light-cone, which can be however either future or past directed. "Big" state function reductions (BSR) meaning the death and re-incarnation of self with opposite arrow of time could be still present. An attractive interpretation for the moments $t = r_n$ would be as moments assignable to "small" state function reductions (SSR) identifiable as "weak" measurements giving rise to sensory input of conscious entity in ZEO based theory of consciousness. One might say that conscious entity becomes gradually conscious about its roots in increasing order. The famous question "What it feels to be a bat" would reduce to "What it feels to be a polynomial?"! One must be however very cautious here.

5.1.4 What twistors are in TGD framework?

The basic problem of the ordinary twistor approach is that the states must be massless in 4-D sense. In TGD framework particles would be massless in 8-D sense. The meaning of 8-D twistorialization at space-time level is relatively well understood but at the level of momentum space the situation is not at all so clear.

1. In TGD particles are massless in 8-D sense. For M^4_L description particles are massless in 4-D sense and the description at momentum space level would be in terms of products of ordinary M^4 twistors and CP_2 twistors. For M^4_T description particles are massive in 4-D sense. How to generalize the twistor description to 8-D case?

   The incidence relation for twistors and the need to have index raising and lowering operation in 8-D situation suggest the replacement of the ordinary l twistors with either with octo-twistors or non-commutative quantum twistors.

2. I have assumed that what I call geometric twistor space of M^4 is simply $M^4 \times S^2$. It however turned out that one can consider standard twistor space CP_3 with metric signature (3,-3) as an alternative. This option reproduces the nice results of the earlier approach but the philosophy is different: there is no fundamental length scale but the hierarchy of causal diamonds (CDs) predicted by zero energy ontology (ZEO) gives rise to the breaking of the exact scaling invariance of M^8 picture. This forces to modify $M^8 - H$ correspondence so that it involves map from M^4 to CP_3 followed by a projection to hyperbolic variant $CP_{2,h}$ of CP_2. Note that also the original form of $M^8 - H$ duality continues to make sense and results from the modification by projection from $CP_{3,h}$ to M^4 rather than $CP_{2,h}$.

   M^4 in H would not be be replaced with conformally compactified M^4 (M^4_{conf}) but conformally compactified cd (cd_{conf}) for which a natural identification is as CP_2 with second complex coordinate replaced with hypercomplex coordinate. The sizes of twistor spaces of cd_{conf} using CP_2 size as unit would reflect the hierarchy of size scales for CDs. The consideration on the twistor space of M^8 in similar picture leads to the identification of corresponding twistor space as HP_3 - quaternionic variant of CP_3: the counterpart of CD_8 would be HP_2.

3. Octotwistors can be expressed as pairs of quaternionic twistors. Octotwistor approach suggests a generalization of twistor Grassmannian approach obtained by replacing the bi-spinors with complexified quaternions and complex Grassmannians with their quaternionic counterparts. Although TGD is not a quantum field theory, this proposal makes sense for cognitive representations identified as discrete sets of spacetime points with coordinates in the extension of rationals defining the adele [L49] implying effective reduction of particles to point-like particles.

4. The outcome of octo-twistor approach together with $M^8 - H$ duality leads to a nice picture view about twistorial description of massive states based on quaternionic generalization of twistor Grassmannian approach. A radically new view is that descriptions in terms of massive and massless states are alternative options, and correspond to two different alternative twistorial descriptions and leads to the interpretation of p-adic thermodynamics as completely universal massivation mechanism having nothing to do with dynamics. As a side product emerges a deeper understanding of ZEO based quantum measurement theory and consciousness theory relying on the universal roots of octonionic polynomials of M^8, which are not 4-D but analogs of 6-D branes. By $M^8 - H$ duality the finite sub-groups of $SU(2)$ of McKay correspondence appear quite concretely in the description of the measurement resolution of 8-momentum.

What about super-twistors in TGD framework?

1. The parallel progress in the understanding SUSY in TGD framework [L92] in turn led to the identification of the super-counterparts of M^8, H and of twistor spaces modifying dramatically the physical interpretation of SUSY. Super-spinors in twistor space would provide the description of quantum states. Super-Grassmannians would be involved with the construction of scattering amplitudes. Quaternionic super Grassmannians would be involved with M^8 description.

2. The great surprise from physics point of view is that in fermionic sector only quarks are allowed by $SO(1,7)$ triality and that anti-leptons are local 3-quark composites identifiable as spartners of quarks. Gauge bosons, Higgs and graviton would be also spartners and assignable to super-coordinates of embedding space expressible as super-polynomials of quark oscillator operators. Super-symmetrization means also quantization of fermions allowing local many-quark states.

3. SUSY breaking would be caused by the same universal mechanism as ordinary massivation of massless states. The mass formulas would be supersymmetric but the choice of p-adic prime identifiable as ramified prime of extension of rationals would depend on the state of super-multiplet. ZEO would make possible symmetry breaking without symmetry breaking as Wheeler might put it.

What about the interpretation of quantum twistors? They could make sense as 4-D spacetime description analogous to description at space-time level. Now one can consider generalization of the twistor Grassmannian approach in terms of quantum Grassmannians.

5.2 McKay correspondence

Consider first McKay correspondence from TGD point of view.

5.2.1 McKay graphs

McKay graps are defined in the following manner. Consider group G which is discrete but not necessarily finite. If the group is finite there is a finite number of irreducible representations χ_I. Select preferred representation V - usually V is taken to be the fundamental representation of G and form tensor products $\chi_I \otimes V$. Suppose irrep χ_J appears n_{ij} times in the tensor product

$\chi_I \otimes \chi_0$. Assign to each representation χ_I dot and connect the dots of χ_I and χ_J by n_{ij} arrows. This gives rise to MacKay graph.

Consider now the situation for finite-D groups of $SU(2)$. 2-D $SU(2)$ spinor representation as a fundamental representation is essential for obtaining the identification of McKay graphs as Dynkin diagrams of simply laced affine algebras having only single line connecting the roots (the angle between positive roots is 120 degrees) (see `http://tinyurl.com/z48d92t`).

1. For $SU(2)$ representations one has the basic rule $j_1 - 1/2 \leq j \leq j_1 + 1/2$ for the tensor product $j_1 \otimes 1/2$. This rule must be broken for finite subgroups of $SU(2)$ since the number of representations if finite so that branching point appears in McKay graph. In branching point the decomposition of $j_1 \otimes 1/2$ decomposes to 3 lower-dimensional representations of the finite subgroup takes place.
2. Simply lacedness means that given representation appears only once in $chi_I \otimes V$, when V is 2-D representation as it can be for a subgroup of $SU(2)$. Additional exceptional properties is the absence of loops ($n_{ii} = 0$) and connectedness of McKay graph.
3. One can consider binary icosahedral group (double covering of icosahedral group A_5 with order 60) as an example (for the McKay graph see `http://tinyurl.com/y2h55jwp`). The representations of A_5 are $1_A, 3_A, 3'_B, 4_A, 5_A$, where integer tells the dimension. Note that $SO(3)$ does not allow 4-D representation. For binary icosahedral group one obtains also the representations $2_A, 2'_B, 4_B, 6_A$. The McKay graph of E_8 contains one branching point in which one has the tensor product of 6-D and 2-D representations 6_A and 2_A giving rise to $5_A \oplus 3_C \oplus 4_B$.

McKay graphs can be defined for any finite group and they are not even unions of simply laced diagrams unless one has subgroups of $SU(2)$. Still one can wonder whether McKay correspondence generalizes from subgroups of $SU(2)$ to all finite groups. At first glance this does not seem possible but there might be some clever manner to bring in all finite groups.

The proposal has been that this McKay correspondence has a deeper meaning. Could simply laced affine ADE algebras, ADE type quantum algebras, and/or corresponding finite groups act as symmetry algebras in TGD framework?

5.2.2 Number theoretic view about McKay correspondence

Could the physical picture provided by TGD help to answer the above posed questions?

1. Could one identify discrete subgroups of $SU(2)$ with those of the covering group $SU(2)$ of $SO(3)$ of quaternionic automorphisms defining the continuous analog of Galois group and reducing to a discrete subgroup for a finite resolution characterized by extension K of rationals. The tensor products of 2-D spinor representation of these discrete subgroups $SU(2)_K$ would give rise to irreps appearing in the McKay graph.
2. In adelic physics [L49] extensions K of rationals define an evolutionary hierarchy with effective Planck constant $h_{eff}/h_0 = n$ identified as the dimension of K. The action of discrete and finite subgroups of various symmetry groups can be represented as Galois group action making sense at the level of X^4 [L40] for what I have called cognitive representations. By $M^8 - H$ duality also the Galois group of quaternions and its discrete subgroups appear naturally.

   This suggests a possible generalization of McKay correspondence so that it would apply to all finite groups G. Any finite group G can appear as Galois group. The Galois group $Gal(K)$ characterizing the extension of rationals induces in turn extensions of p-adic number fields appearing in the adele. Could the representation of G as Galois group of extension of rationals allow to generalize McKay correspondence?

Could the following argument inspired by these observations make sense?

1. $SU(2)$ is identified as spin covering of the quaternionic automorphism group. One can define the subgroups in matrix representation of $SU(2)$ based on complex numbers. One can replace complex numbers with the extension of rationals and speak of group $SU(2)_K$ identified as a discrete subgroup of $SU(2)$ having in general infinite order.

   The discrete finite subgroups $G \subset SU(2)$ appearing in the standard McKay correspondence correspond to extensions K of rationals for which one has $G = SU(2)_K$. These special extensions can be identified by studying the matrix elements of the representation of G and include the discrete groups Z_n acting as rotation symmetries of the Platonic solids. For instance, for icosahedral group Z_2,Z_3 and Z_5 are involved and correspond to roots of unity.

2. The semi-direct product $Gal \triangleleft SU(2)_K$ with group action

$$(gal_1, g_1)(gal_2, g_2) = (gal_1 \circ gal_2, g_1(gal_1 g_2))$$

makes sense. The action of $Gal \triangleleft SU(2)_K$ in the representation is therefore well-defined. Since all finite groups G can appear as Galois groups, it seems that one obtains extension of the McKay correspondence to semi-direct products involving all finite groups G representable as Galois groups.

3. A good guess is that the number of representations of $SU(2)_K$ involved is infinite if $SU(2)_K$ has infinite order. For $\tilde{A}_n$ and $\tilde{D}_n$ the branching occurs only at the ends of the McKay graph. For E_k, $k = 6, 7, 8$ the branching occurs in middle of the graph (see `http://tinyurl.com/y2h55jwp`). What happens for arbitrary G. Does the branching occur at all? One could argue that if the discrete subgroup has infinite order, the representation can be completed to a representation of $SU(2)$ in terms of real numbers so that the McKay graphs must be identical.

4. A concrete realization of ADE type Kac-Moody algebras is proposed. It relies on the group algebra of $Gal(K) \triangleleft SU(2)_K$ and free field representation of ADE type Kac-Moody algebra identifying the free scalar fields in Kac-Moody Cartan algebra as group algebra elements defined by the traces of representation matrices (characters).

5. A possible interpretation of quantum spinors is in terms of quantum measurement theory with finite measurement resolution in which precise eigenstates as measurement outcomes are replaced with universal probability distributions defined by quantum group [K48]. TGD inspired theory of consciousness is a possible application.

   Also the notion of quantum twistor [L97] can be considered. In TGD particles are massless in 8-D sense and in general massive in 4-D sense but 4-D twistors are needed also now so that a modification of twistor approach is needed. The incidence relation for twistors suggests the replacement of the usual twistors with non-commutative quantum twistors.

5.3 ADE diagrams and principal graphs of inclusions of hyperfinite factors of type II$_1$

Dynkin diagrams for ADE groups and corresponding affine groups characterize also the inclusions of hyperfinite factors of type II$_1$ (HFFs) [K48].

5.3.1 Principal graphs and Dynkin diagrams for ADE groups

1. If the index $\beta = \mathcal{M} : \mathcal{N}$ of the Jones inclusion satisfies $\beta < 4$, the affine Dynkin diagrams of $SU(n)$ (discrete symmetry groups of n-polygons) and E_7 (symmetry group of octahedron and cube) and $D(2n + 1)$ (symmetries of double 2n+1-polygons) are not allowed.

2. Vaughan Jones [A96] (see `http://tinyurl.com/y8jzvogn`) has speculated that these finite subgroups could correspond to quantum groups as kind of degenerations of Kac-Moody groups. Modulo arithmetics defined by the integer n defining the quantum phase suggests itself strongly. For $\beta = 4$ one can construct inclusions characterized by extended Dynkin diagram and any finite sub-group of $SU(2)$. In this case affine ADE hierarchy appear as principal graphs characterizing the inclusions. For $\beta < 4$ the finite measurement resolution could reduce affine algebra to quantum algebra.

3. The rule is that for odd values of n defining the quantum phase the Dynkin diagram does not appear. If Dynkin diagrams correspond to quantum groups, one can ask whether they allow only quantum group representations with quantum phase $q = exp(i\pi/n)$ equal to even root of unity.

5.3.2 Number theoretic view about inclusions of HFFs and preferred role of $SU(2)$

Consider next the TGD inspired interpretation.

1. TGD suggests the interpretation in terms of representations of $Gal(K(G)) \triangleleft G$ for finite subgroups G of $SU(2)$, where $K(G)$ would be an extension associated with G. This would generalize to subgroups of $SU(2)$ with infinite order in the case of arbitrary Galois group. Quantum groups have finite number of representations in 1-1-correspondence with terms of finite-D representations of G. Could the representations of $Gal(K(G)) \triangleleft G$ correspond to the representations of quantum group defined by G?

   This would conform with the vision that there are two ways to realize finite measurement resolution. The first one would be in terms of inclusions of hyper-finite factors. Second would be in terms cognitive representations defining a number theoretic discretization of X^4 with embedding space coordinates in the extension of rationals in which Galois group acts.

   In fact, also the discrete subgroup of infinite-D group of symplectic transformations of $\Delta M^4_+ \times CP_2$ would act in the cognitive representations and this suggests a far reaching implications concerning the understanding of the cognitive representations, which pose a formidable looking challenge of finding the set of points of X^4 in given extension of rationals [L87].

2. This brings in mind also the model for bio-harmony in which genetic code is defined in terms of Hamiltonian cycles associated with icosahedral and tetrahedral geometries [L5, L76]. One can wonder why the Hamiltonian cycles for cubic/octahedral geometry do not appear. On the other hand, according to Vaughan the Dynkin diagram for E_7 is missing from the hierarchy of so principal graphs characterizing the inclusions of HFFs for $\beta < 4$ (a fact that I failed to understand). Could the genetic code directly reflect the properties of the inclusion hierarchy?

How would the hierarchies of inclusions of HFFs and extensions of rationals relate to each other?

1. I have proposed that the inclusion hierarchies of extensions K of rationals accompanied by similar hierarchies of Galois groups $Gal(K)$ could correspond to a fractal hierarchy of sub-algebras of hyperfinite factor of type II_1. Quantum group representations in ADE hierarchy would somehow correspond to these inclusions. The analogs of coset spaces for two algebras in the hierarchy define would quantum group representations with quantum dimension characterizing the inclusion.

2. The quantum group in question would correspond to a quantum analog of finite-D group of $SU(2)$ which would be in completely unique role mathematically and physically. The infinite-D group in question could be the symplectic group of $\delta M^4_+ \times CP_2$ assumed to act as isometries of WCW. In the hierarchy of Galois groups the quantum group of finite group $G \subset SU(2)$ would correspond to Galois group of an extension K.

3. The trace of unit matrix defining the character associated with unit element is infinite for these representations for factors of type I. Therefore it is natural to assume that hyper-finite factor of type II_1 for which the trace of unit matrix can be normalized to 1. Sub-factors would have trace of projector with trace smaller than 1.

4. Do the ADE diagrams for groups $Gal(K(G)) \triangleleft G$ indeed correspond to quantum groups and affine algebras? Why the finite groups should give rise to affine/Kac-Moody algebras? In number theoretic vision a possible answer would be that depending on the value of the index β of inclusion the symplectic algebra reduces in the number theoretic discretization by gauge conditions specifying the inclusion either to Kac-Moody group ($\beta = 4$) or to quantum group ($\beta < 4$).

What about subgroups of groups other than $SU(2)$? According to Vaughan there has been work about inclusion hierarchies of $SU(3)$ and other groups and it seems that the results generalize and finite subgroups of say $SU(3)$ appear. In this case the tensor products of finite sub-groups McKay graphs do not however correspond to the principal graphs for inclusions. Could the number theoretic vision come in rescue with the replacement of discrete subgroup with Galois group and the identification of $SU(2)$ as the covering for the Galois group of quaternions?

5.3.3 How could ADE type quantum groups and affine algebras be concretely realized?

The questions discussed are following. How to understand the correspondence between the McKay graph for finite group $G \subset SU(2)$ and ADE (affine) group Dynkin diagram for $\beta < 4$ ($\beta = 4$)? How the nodes of McKay grap representing the irreps of finite group can correspond to the positive roots of a Dynkin diagram, which are essentially vectors defined by eigenvalues of Cartan algebra generators for complexified Lie-algebra basis.

The first thing that comes in mind is the construction of representation of Kac-Moody algebra using scalar fields labelled by Cartan algebra generators (see `http://tinyurl.com/y9lkeelk`): these representations are discussed by Edward Frenkel [A38]. The charged generators of Kac-Moody algebra in the complement of Cartan algebra are obtained by exponentiating the contractions of the vector formed by these scalar fields with roots to get $\alpha \cdot \Phi = \alpha_i \Phi^i$. The charged field is represented as a normal ordered product $: exp(i\alpha \cdot \Phi) :$. If one can assign to each irrep of G a scalar field in a natural manner one could achieve this.

Neglect first the presence of the group algebra of $Gal(K(G)) \triangleleft G$. The standard rule for the dimensions of the representations of finite groups reads as $\sum_i d_I^2 = n(G)$. For double covering of G one obtains this rule separately for integer spin representations and half-odd integers spin representations. An interesting possibility is that this could be interpreted in terms of supersymmetry at the level of group algebra in which representation of dimension d_I appears d_I times.

The group algebra of G and its covering provide a universal manner to realize these representations in terms of a basis for complex valued functions in group (for extensions of rationals also the values of the functions must belong to the extension).

1. Representation with dimension d_I occurs d_I times and corresponds to $d_I \times d_I$ representation matrices D^I_{mn} of representation χ_I, whose columns and rows provide representations for left- and right-sided action of G. The tensor product rules for the representations χ_I can be formulated as double tensor products. For basis states $|J,n\rangle$ in χ_I and $|J,n\rangle$ in χ_J one has

$$|I,m\rangle_{\otimes}|J,n\rangle = c^{K,p}_{I,m|J,n}|K,p\rangle \quad ,$$

where $c^{K,p}_{J,n|J,n}$ are Glebch-Gordan coefficients.

2. For the tensor product of matrices D^I_{mn} and D^J_{mn} one must apply this rule to both indices. The orthogonality properties of Glebsch-Gordan coefficients guarantee that the tensor product contains only terms in which one has same representation at left- and right-hand side. The orthogonality rule is

$$\sum_{m,n} c^{K,p}_{I,m|J,n} c^{K,q}_{I,r|J,s} \propto \delta_{K,L} \quad .$$

3. The number of states is $n(G)$ whereas the number $I(G)$ of irreps corresponds to the dimension of Cartan algebra of Kac-Moody algebra or of quantum group is smaller. One should be able to pick only one state from each representation D^I.

   The condition that the state X of group algebra is invariant under automorphism gXg^{-1} implies that the allowed states as function in group algebra are traces $Tr(D^I)(g)$ of the representation matrices. The traces of representation matrices indeed play fundamental role as automorphism invariants. This suggests that the scalar fields Φ_I in Kac-Moody algebra correspond to Hilbert space coefficients of $Tr(D^I)(g)$ as elements of group algebra labelled by the representation. The exponentiation of $\alpha \cdot \Phi$ would give the charged Kac-Moody algebra generators as free field representation.

4. For infinite sub-groups $G \subset SU(2)$ $d(G)$ is infinite. The traces are finite also in this case if the dimensions of the representations involved are finite. If one interprets the unit matrix as a function having value 1 in entire group $Tr(Id)$ diverges. Unit dimension for HFFs provide a more natural notion of dimension $d = n(G)$ of group algebra $n(G)$ as $d = n(G) = 1$. Therefore HFFs would emerge naturally.

It is easy to take into account $Gal(K(G))$. One can represent the elements of semi-direct product $Gal(K(G)) \triangleleft G$ as functions in $Gal(K(G)) \times G$ and the proposed construction brings in also the tensor products in the group algebra of $Gal(K(G))$. It is however essential that group algebra elements have values in K. This brings in tensor products of representations Gal and G and the number of representations is $n(Gal) \times n(G)$. The number of fields Φ_I as also the number of Cartan algebra generators of ADE Lie algebra increases from $I(G)$ to $I(Gal) \times I(G)$. The reduction of the extension of coefficient field for the Kac-Moody algebra from complex numbers to K splits the Hilbert space to sectors with smaller number of states.

5.4 $M^8 - H$ duality

The generalization of the standard twistor Grassmannian approach to TGD remains a challenge for TGD and one can imagine several approaches. $M^8 - H$ duality is essential for these approaches and will be discussed in the sequel.

The original form of $M^8 - H$ duality assumed $H = M^4 \times CP_2$ but quite recently it turned out that one could replace the twistor space of M^4 identified as $M^4 \times S^2$ with $CP_{3,h}$, which is hyperbolic variant of CP_3. This option forces to replace H with $H = CP_{2,h} \times CP_2$. $M^8 - H$ duality would consist of a map of M^4 point to corresponding twistor sphere in $CP_{3,h}$ and its projection to $CP_{2,h}$. This option will be discussed in the section about twistor lift of TGD.

5.4.1 $M^8 - H$ duality at the level of space-time surfaces

$M^8 - H$ duality [L40] relates two views about space-time surfaces X^4: as algebraic surfaces in complexified octonionic M^8 and as minimal surfaces with singularities in $H = M^4 \times CP_2$.

1. Octonion structure at the level of M^8 means a selection of a suitable decomposition $M^8 = M^4 \times E^4$ in turn determining $H = M^4 \times CP_2$. Choices of M^4 share a preferred 2-plane $M^2 \subset M^4$ belonging to the tangent space of allowed space-time surfaces $X^4 \subset M^8$ at various points. One can parameterize the tangent space of $X^4 \subset M^8$ with this property by a point of CP_2. Therefore X^4 can be mapped to a surface in $H = M^4 \times CP_2$: one M^8-duality. One can consider also the possibility that the choice of M^2 is local but that the distribution of $M^2(x)$ is integrable and defines string world sheet in M^4. In this case $M^2(x)$ is mapped to same $M^2 \subset H$.
2. Since 8-momenta p_8 are light-like one can always find a choice of $M^4_L \subset M^8$ such that p_8 belongs to M^4_L and is thus light-like. The momentum has in the general case a component orthogonal to M^2 so that M^4_L is unique by quaternionicity: quaternionic cross product for tangent space quaternions gives the third imaginary quaternionic unit. For a fixed M^4, call it M^4_T, the M^4 projections of momenta are time-like. When momentum belongs to M^2 the choices is non-unique and any $M^4 \subset M^2$ is allowed.
3. Space-time surfaces $X^4 \subset M^8$ have either quaternionic tangent- or normal spaces. Quantum classical correspondence (QCC) requires that charges in Cartan algebra co-incide with their classical counters parts determined as Noether charges by the action principle determining X^4 as preferred extremal. Parallelity of 8-momentum currents with tangent space of X^4 would conform with the naïve view about QCC. It does not however hold true for the contributions to four-momentum coming from string world sheet singularities (string world sheet boundaries are identified as carriers of quantum numbers), where minimal surface property fails.

An important aspect of $M^8 - H$ duality is the description of space-time surfaces $X^4_c \subset M^8_c$ as roots for the "real" or "imaginary" part in quaternionic sense of complexified-octonionic polynomial with real coefficients: these options correspond to complexified-quaternionic tangent - or normal spaces. The real space-time surfaces would be naturally obtained as "real" parts with respect to i of their complexified counterparts by projection from M^8_c to M^4_c. One could drop the subscripts "$_c$" but in the sequel they are kept.

Remark:O_c,O_c,C_c,R_c will be used in the sequel for complexifications of octonions, quaternions, etc.. number fields using commuting imaginary unit i appearing naturally via the roots of real polynomials.

M^8-H duality makes sense under 2 additional assumptions to be considered in the following more explicitly than in earlier discussions.

1. Space-time surface is identified as a 4-D root for a H_c-valued "imaginary" or "real" part of O_c valued polynomial obtained as an O_c continuation of a real polynomial P with rational coefficients, which can be chosen to be integers. For $P(x) = x^n+..$ ordinary roots are algebraic integers. The 4-D space-time surface is projection of this surface from M^8_c to M^8.

   The tangent space of space-time surface and thus space-time surface itself contains a preferred $M^2_c \subset M^4_c$ or more generally, an integrable distribution of tangent spaces $M^2_c(x)$. The string world sheet like entity defined by this distribution is 2-D surface $X^2_c \subset X^4_c$ in R_c sense.

   X^2c can be fixed by posing to the non-vanishing Q_c-valued part of octonionic polynomial condition that the C_c valued "real" or "imaginary" part in C_c sense for this polynomial vanishes. M^2_c would be the simplest solution but also more general complex sub-manifolds $X^2_c \subset M^4_c$ are possible. In general one would obtain book like structures as collections of several string world sheets having real axis as back.

   By assuming that R_c-valued "real" or "imaginary" part of the polynomial at this 2-surface vanishes. one obtains preferred M^1_c or E^1_c containing octonionic real and preferred imaginary unit or distribution of the imaginary unit having interpretation as complexified string. Together these kind 1-D surfaces in R_c sense would define local quantization axis of energy and spin. The outcome would be a realization of the hierarchy $R_{\rightarrow}C_c \rightarrow H_c \rightarrow O_c$ realized as surfaces.

   Remark: Also M^4_c appears as a special solution for any polynomial P. M^4_c seems to be like a universal reference solution with which to compare other solutions. M^4_c would intersect all other solutions along string world sheets X^2_c. Also this would give rise to a book like structures with 2-D string world sheet representing the back of given book. The physical interpretation of these book like structures remains open in both cases.

   I have proposed that string world sheets as singularities correspond to 2-D folds of space-time surfaces at which the dimension of the quaternionic tangent space degenerates from 4 to 2 [L84] [K10]. This interpretation is consistent with the identification as a book like structure with 2-pages. Also 1-D real and imaginary manifols could be interpreted as folds or equivalently books with 2 pages.

2. Associativity condition for tangent-/normal space is second essential condition and means that tangent - or normal space is quaternionic. The conjecture is that the identification in terms of roots of polynomials guarantees this and one can formulate this as rather convincing argument [L41, L42, L43].

One cannot exclude rational functions and or even real analytic functions in the sense that Taylor coefficients are octonionically real (propotional to octonionic real unit). Number theoretical vision - adelic physics [L49], suggests that polynomial coefficients are rational or perhaps in extensions of rationals. The real coefficients could in principle be replaced with complex numbers $a + ib$, where i commutes with the octonionic units and defines complexifiation of octonions. i appears also in the roots defining complex extensions of rationals.

1. In general the zero loci for imaginary or real part are 4-D but the 7-D light-cone δM^8_+ of M^8 with tip at the origin of coordinates is an exception [L40]. At δM^8_+ the octonionic coordinate o is light-like and one can write $o = re$, where 8-D time coordinate and radial coordinate are related by $t = r$ and one has $e = (1 + e_r)/\sqrt{2}$ such that one as $e^2 = e$.

   Polynomial $P(o)$ can be written at δM^8_+ as $P(o) = P(r)e$ and its roots correspond to 6-spheres S^6 represented as surfaces $t_M = t = r_N$, $r_M = \sqrt{r_N^2 - r_E^2} \leq r_N$, $r_E \leq r_N$, where the value of Minkowski time $t = r = r_N$ is a root of $P(r)$ and r_M denotes radial Minkowski coordinate. The points with distance r_M from origin of $t = r_N$ ball of M^4 has as fiber 3-sphere with radius $r = \sqrt{r_N^2 - r_E^2}$. At the boundary of S^3 contracts to a point.

2. These 6-spheres are analogous to 6-D branes in that the 4-D solutions would intersect them in the generic case along 2-D surfaces X^2. The boundaries $r_M = r_N$ of balls belong to the boundary of M^4 light-cone. In this case the intersection would be that of 4-D and 3-D surface, and empty in the generic case (it is however quite not clear whether topological notion of "genericity" applies to octonionic polynomials with very special symmetry properties).

3. The 6-spheres $t_M = r_N$ would be very special. At these 6-spheres the 4-D space-time surfaces X^4 as usual roots of $P(o)$ could meet. Brane picture suggests that the 4-D solutions connect the 6-D branes with different values of r_n.

   The basic assumption has been that particle vertices are 2-D partonic 2-surfaces and light-like 3-D surfaces - partonic orbits identified as boundaries between Minkowskian and Euclidian regions of space-time surface in the induced metric (at least at H level) - meet along their 2-D ends X^2 at these partonic 2-surfaces. This would generalize the vertices of ordinary Feynman diagrams. Obviously this would make the definition of the generalized vertices mathematically elegant and simple.

   Note that this does not require that space-time surfaces X^4 meet along 3-D surfaces at S^6. The interpretation of the times t_n as moments of phase transition like phenomena is suggestive. ZEO based theory of consciousness suggests interpretation as moments for state function reductions analogous to weak measurements ad giving rise to the flow of experienced time.

4. One could perhaps interpret the free selection of 2-D partonic surfaces at the 6-D roots as initial data fixing the 4-D roots of polynomials. This would give precise content to strong form of holography (SH), which is one of the central ideas of TGD and strengthens the 3-D holography coded by ZEO alone in the sense that pairs of 3-surfaces at boundaries of CD define unique preferred extremals. The reduction to 2-D holography would be due to preferred extremal property realizing the huge symplectic symmetries and making $M^8 - H$ duality possible as also classical twistor lift.

   I have also considered the possibility that 2-D string world sheets in M^8 could correspond to intersections $X^4 \cap S^6$? This is not possible since time coordinate t_M constant at the roots and varies at string world sheets.

   Note that the compexification of M^8 (or equivalently octonionic E^8) allows to consider also different variants for the signature of the 6-D roots and hyperbolic spaces would appear for $(\epsilon_1, \epsilon_i, .., \epsilon_8)$, $epsilon_i = \pm 1$ signatures. Their physical interpretation - if any - remains open at this moment.

5. The universal 6-D brane-like solutions S^6_c have also lower-D counterparts. The condition determining X^2 states that the C_c-valued "real" or "imaginary" for the non-vanishing Q_c-valued "real" or "imaginary" for P vanishes. This condition allows universal brane-like solution as a restriction of O_c to M^4_c (that is CD_c) and corresponds to the complexified time=constant hyperplanes defined by the roots $t = r_n$ of P defining "special moments in the life of self" assignable to CD. The condition for reality in R_c sense in turn gives roots of $t = r_n$ a hyper-surfaces in M^2_c.

5.4.2 $M^8 - H$ duality at the level of momentum space

$M^8 - H$ duality occurs also at the level of momentum space and has different meaning now.

1. At M^8 level 8-momenta are quaternionic and light-like. The choices of $M^4_L \supset M^2$ for which 8-momentum in M^4_L, are parameterized by CP_2 parameterizing also the choices of tangent or normal spaces of $X^4 \subset M^8$ at space-time level. This maps M^8 light-like momenta to light-like M^4_L momenta and to CP_2 point characterizing the M^4 and depending on 8-momentum. One can introduce CP_2 wave functions expressible in terms of spinor harmonics and generators of of a tensor product of Super-Virasoro algebras.

2. For a fixed choice M^4_T momenta in general time-like and the E^4 component of 8-momentum has value equal to mass squared. E^4 momenta are points of 3-sphere so that $SO(3)$ harmonics with $SO(4)$ symmetry could parametrize the states. The quantum numbers are $M^4_T \supset M^2$ momenta with fixed mass and the two angular momenta with identical values for S^3 harmonics, which correspond to the quantum states of a spherical quantum mechanical rigid body, and are given by the matrix elements $D^j_{m,n}$ $SU(2)$ group elements ($SO(4)$ decomposes to $SU(2)_L) \times SU(2)_R$ acting from left and right).

   This picture suggests what one might call $SO(4) - SU(3)$ duality at the level of momentum space. There would be two descriptions of states: as massless states with $SU(3)$ symmetry and massive states with $SO(4)$ symmetry.

3. What about the space formed by the choices of the space of the light-like 8-momenta? This space is the space for the choices of preferred M^2 and parameterized by the 6-D (symmetric space $G_2/SU(3)$, where $SU(3) \subset G_2$ leaving complex plane M^2 invariant is subgroup of quaternionic automorphic group $G(2)$ leaving octonionic real unit defining the rest system invariant. This space is moduli space for octonionic structures each of which defines family of space-time surfaces. 8-D Lorent transformations produce even more general octonionic structures. The space for the choices of color quantization axes is $SU(3)/U(1) \times U(1)$, the twistor space of CP_2.

Do M^4_L and M^4_T have analogs at the space-time level?

As found, the solutions of octonionic polynomials consisting of 4-D roots and special 6-D roots coming as 6-sphere S^6 s at 7-D light-cone of M^8. The roots at $t = r$ light-cone boundary are given by the roots $r = r_N$ of the polynomial $P(t)$ and correspond to M^4 slices $t_M = r_N, r_M \leq r_N$. At point r_M S^3 fiber as radius $r(S^3) = \sqrt{r_N^2 - r_M^2}$ and contracts to a point at its boundaries.

Could M^4_L and M_T have analogies at the space-time level?

1. The sphere S^3 associated M^4_T could have counterpart at the level of space-time description. The momenta in M^4_T would naturally be mapped to momenta in the section $t = r_n$ in this case the S^3:s of different mass squared values would naturally correspond to S^3:s assignable to the points of the balls $t = r_n$ and code for mass squared value.

   The counterpart of M^4_L should correspond to light-cone boundary but what does CP_2 correspond? Could the pile of S^3 associated with $t = r_n$ correspond also to CP_2. Could this be the case if there is wormhole contact carrying monopole flux at the origin so that the analog for the replacement of 3-sphere at $r_{CP_2} = \infty$ with homologically non-trivial 2-sphere would be realized?

2. Does the 6-sphere as a root polynomial have counterpart in H? The image should be consistent with $M^8 - H$ duality and correspond to a fixed structure depending on the root r_n only. Since S^3 associated with the E^4 momenta reduces to a point for M^4_L, the natural guess is that S^6 reduces to $t = r_n, 0 \leq r_M \leq r_n$ surface in H.

$S^3 - CP_2$ duality

$S^3 - CP_2$ duality at the level of quantum numbers suggest strongly itself. What does this require? One can approach the problem from two different perspectives.

1. The first approach would be that the representations of $SU(3)$ and $SO(4)$ groups somehow correspond to each other: one could speak of $SU(3) - SO(4)$ duality [K107, K124]. The original form of this duality was this. The color symmetries of quark physics at high energies would be dual to the $SO(4) = SU(2)_L \times SU(2)_R$ symmetries of the low energy hadron physics. Since the physical objects are partons and hadrons formed from the one cannot expect the duality to hold true at the level of details for the representations, and the comparison of the representations makes this clear.

2. The second approach relies on the notion of cognitive representation meaning discretization of CP_2 and S^3 and counting of points of cognitive representations providing discretization in terms of M^8 or H points belonging to the extension of rationals considered. In this case it is more natural to talk about $S^3 - CP_2$ duality.

   The basic observation is that the open region $0 \leq r < \infty$ of CP_2 in Eguchi-Hanson coordinates with r labeling 3-spheres $S^3(r)$ with finite radius can be regarded as pile of $S^3(r)$. In discretization one would have discrete pile of these 3-spheres with finite number of points in the extension of rationals. They points of given S^3 could be related by isometries in special cases.

How $S^3 - CP_2$ duality at the level of light-like M^8 momenta could emerge?

1. Consider first the situation in which one chooses $M^4 \supset M^2$ sub-spaces so that momentum projection to it is light-like. For cognitive representation the choices of $M^4 \supset M^2$ correspond to ad discrete set of points of CP_2 and thus points in the pile of S^3 with discrete radii since all

E^4 parts of momenta with fixed length squared to zero in this choice and each E^4 momentum with fixed lengthand thus identifiable as discrete point of S^3 would correspond to one choice. All these choices would give rise to a pile of S^3:s identifiable as set $0 \leq r < \infty$ of CP_2. The number of CP_2 points would be same as total number of points in the pile of discrete S^3s. This is what $S^3 - CP_2$ duality would say.

Remark: The volumes of CP_2 and S^3 with unit radius are $8\pi^2$ and $2\pi^2$ so that ration is rational number.

2. Consider now the situation for M^4_T so that one has non-vanishing M^4 mass squared equal to E^4 mass squared, having discretized values. The E^4 would momenta correspond to points for a pile of discretized S^3 and thus to the points of CP_2 by above argument. One would have $S^3 - CP_2$ correspondence also now as one indeed expects since the two ways to see the situation should be equivalent.
3. In the space of light-like M^8 momenta E^8 momenta could naturally organize into representations of finite discrete subgroups of $SU(2)$ appearing in McKay correspondence with ADE groups: the groups are cyclic groups, dihedral groups, and the isometry groups associated with tetrahedron, octahedron (cube) and icosahedron (dodecahedron) (see `http://tinyurl.com/yyyn9p95`).
4. Could a concrete connection with the inclusion hierarchy of HFFs be based on increasing momentum resolution realized in terms of these groups at sphere S^3. Notice however that for cyclic and dihedral groups the orbits are circles and pairs of circles for dihedral groups so that the discretization looks too simple and is rotationally asymmetric. Discretization should improve as n increases.

   One can of course ask why C_n and D_n with single direction of rotation axes would appear? Could it be that the directions of rotation axis correspond to the directions defined by the vertices of the 5 Platonic solids. Or could the orbits of fixed axis under the 5 Platonic orbits be allowed. Also this looks still too simple.

   Could the discretization labelled by n_{max} be determined by the product of the groups up to n_{max} and define a group with infinite order. One can consider also groups defined by subsets $\{n_1, n_2...n_3\}$ and these a pair of sequences with larger sequence containing the smaller one could perhaps define an inclusion. The groups C_n and D_n allow prime decomposition in obvious manner and it seems enough to include to the product only the groups C_p and D_p, where p is prime as generators so that one would have set $\{p_1, ...p_n\}$ of primes labelling these groups besides the Platonic groups. The extension of rationals used poses a cutoff on the number of groups involved and on the group elements representable since since too high roots of unity resulting in the multiplication of C_{p_i} and D_{p_j} do not belong to the extension.

   At the level of momentum space the hierarchy of finite discrete groups of $SU(2)$ would define the notion measurement resolution. The discrete orbits of $SU(2) \times U(1)$ at S^3 would be analogous to tessellations of sphere S^2 known as Platonic solids at sphere S^2 and appearing in the ADE correspondence assignable to Jones inclusions as description of measurement resolution. This would also explain also why Z_2 coverings of the subgroups of $SO(3)$ appear in ADE sequence.

   This picture is probably not enough for the needs of adelic physics [L49] allowing all extensions of rationals. Besides roots of unity only the roots of small integers $2, 3, 5$ associated with the geometry of Platonic solids would be included in these discretizations. One could interpret these discretizations in terms of subgroups of discrete automorphism groups of quaternions. Also the extensions of rationals are probably needed.

Could $S^3 - CP_2$ duality make sense at space-time level? Consider the space-time analog for the projection of M^8 momenta to fixed M^4_T.

1. Suppose that the 3-surfaces determining the space-time surfaces as algebraic surfaces in $X^4 \subset M^8$ are given at the surfaces $t = r_N, r_M \leq r_N$ and have a 3-D fiber which should be surface in CP_2. On can assign to each point of this ball $S^3(r_M)$ with radius going to zero at $r_M = r_N$. One has pile of $S^3(r_M)$ which corresponds to the region $0 \leq r < \infty$ of CP_2. This set is discretized. Suppose that the discretization is like momentum discretization. If so, the points would correspond to points of CP_2. It is not however clear why the discretization should be so symmetric as in momentum discretization.

2. The initial values could be chosen by choosing discrete set of points in this pile of S^3:s and this would give rise to a discrete set of points of CP_2 fixing tangent or normal plane of X^4 at these points. One should show that the selection of a point of S^6 at each point indeed determines quaternionic tangent or normal plane plane for a given polynomial $P(o)$ in M^8.

   It would seem that this correspondence need not hold true.

5.4.3 $M^8 - H$ duality and the two ways to describe particles

The isometry groups for $M^4 \times CP_2$ is $P \times SU(3)$ (P for Poincare group). The isometry group for $M^8 = M^4 \times E^4$ with a fixed choice of M^4 breaks down to $P \times SO(4)$. A further breaking by selection $M^4 \subset M^2$ of preferred octonionic complex plane M^2 necessary in the algebraic approach to space-time surfaces $X^4 \subset M^8$ brings in preferred rest system and reduces the Poincare symmetry further. At the space-time level the assumption that the tangent space of X^4 contains fixed M^2 or at least integral distribution of $M^2(x) \subset M^4$ is necessary for $M^8 - H$ duality [L40].

The representations $SO(4)$ and $SU(3)$ could provide alternative description of physics so that one would have what I have called $SO(4) - SU(3)$ duality [K107]. This duality could manifest in the description of strong interaction physics in terms of hadrons and quarks respectively (conserved vector current hypothesis and partially conserved axial current hypothesis based on $Spin(SO(4)) = SU(2) \times SU(2)_R$. The challenge is to understand in more detail this duality. This could allow also to understand better how the two twistor descriptions might relate.

$SO(4) - SU(3)$ duality implies two descriptions for the states and scattering amplitudes.

Option I: One uses projection of 8-momenta to a fixed $M^4_T \supset M^2$.

Option II: One assumes that $M^4_L \supset M^2$ is defines the frame in which quaternionic octonion momentum is parallel to M^4_L: this M^4_L depends on particle state and describes this dependence in terms of wave function in CP_2.

Option I: fixed $M^4_T \supset M^2$

For Option I the description would be in terms of a *fixed* $M^4_T \subset M^8 = M^4_T \times E^4$ and $M^2 \subset M^4_T$ fixed for both options. For given quaternionic light-like M^8 momentum one would have projection to M^4_T, which is in general massive. E^4 momentum would have same the length squared by light-likeness.

De-localization M^4_T mass squared equal to $p^2(M^4_T) = m^2$ in E^4 can be described in terms of $SO(4)$ harmonics at sphere having $p^2(E^4) = m^2$. This would be the description applied to hadrons and leptons and particles treated as massive particles. Particle mass would be due to the fixed choice of M^4_T. What dictates this choice is an interesting question. An interesting question is how these descriptions relate to QFT Higgs mechanism as (in principle) alternative descriptions: the choice of fixed M^4_T could be seen as analog for the generation of vacuum expectation of Higgs selecting preferred direction in the space of Higgs fields.

Option II: varying $M^4_L \supset M^2$

For Option II the description would use $M^4_L \supset M^2$, which is *not fixed* but chosen so that it contains light-like M^8 momentum. This would give light-like momentum in M^4_L identifiable as quaternionic sub-space of complexified octonions.

1. One could assign to the state wave function function for the choices of M^4 and by quaternionicity of 8-momenta this would correspond to a state in super-conformal representation with vanishing M^4_L mass: CP_2 point would code the information about E^4 component light-like 8-momentum. This description would apply to the partonic description of hadrons in terms of massless quarks and gluons.

2. For this option one could use the product of ordinary M^4 twistors and CP_2 twistors. One challenge would be the generalization of the twistor description to the case of CP_2 twistors.

p-Adic particle massivation and ZEO

The two pictures about description of light-like M^8 momenta do not seem to be quite consistent with the recent view about TGD in which H-harmonics describe massivation of massless particles. What looks like a problem is following.

1. The resulting particles are massive in M^4. But they should be massless in $M^4 \times CP_2$ description. The non-vanishing mass would suggest the correct description in terms of Option I for which the description in terms of E^4 momenta with length equal to mass and thus identifiable as points of S^3. Momentum space wave functions at S^3 - essentially rigid body wave functions given by representation matrices of $SU(2)$ could characterize the states rather than CP_2 harmonic.
2. The description based on CP_2 color partial waves however works and this would favor Option II with vanishing M^4 mass. What goes wrong?

To understand what might be involved, consider p-adic mass calculations.

1. The massivation of physical fermion states includes also the action of super-conformal generators changing the mass. The particles are originally massless and p-adic mass squared is generated thermally and mapped to real mass squared by canonical identification map.

   For CP_2 spinor harmonics mass squared is of order CP_2 mass squared and thus gigantic. Therefore the mass squared is assumed to contain negative tachyonic ground state contribution due to the negative half-odd integer valued conformal weight $h_{vac} < 0$ of vacuum. The origin of this contribution has remained a mystery in p-adic thermodynamics but it makes possible to construct massless states. h_{vac} cancels the spinorial contributions and the contribution from positive conformal weights of super-conformal generators so that the particle states are massless before thermalization. This would conform with the idea of using varying M^4_L and thus CP_2 description.
2. What does the choice of M^4 mean in terms of super-conformal representations? Could the mysterious vacuum conformal weight h_{vac} provide a description for the effect of the needed $SU(3)$ rotation of M^4 from standard orientation on super-conformal representation. The effect would be very simple and in certain sense reversal to the effect of Higgs vacuum expectation value in that it would cancel mass rather than generate it.

   An important prediction would be that heavy states should be absent from the spectrum in the sense that mass squared would be p-adically of order $O(p)$ or $O(p^2)$ (in real sense of order $O(1/p)$ or $O(1/p^2)$). The trick would be that the generation of h_0 as a representation of SU(3) rotation of M^4 makes always the dominating contribution to the mass of the state vanishing.

   Remark: If the canonical identification I mapping the p-adic mass integers to their real numbers is of the simplest form $m = \sum_n x_n p^n \rightarrow I(m) = \sum_n x_n p^{-n}$, it can happen that the image of rational m/n with p-adic norm not larger than 1 represented as p-adic integer by expanding it in powers of p, can be near to the maximal value of p and the mass of the state can be of order CP_2 mass - about 10^{-4} Planck masses. If the canonical identification is defined as $m/n \rightarrow I/(m)/I(n)$ the image of the mass is small for small values of m and n.
3. Unfortunately, it is easy to get convinced that this explanation of h_{vac} is not physically attractive. Identical mass spectra at the level of M^8 and H looks like a natural implication of $M^8 - H$-duality. $SU(3)$ rotation of M^4 in M^8 cannot however preserve the general form of $M^4 \times CP_2$ mass squared spectrum for the M^4 projections of M^8 momenta at level of M^8.

   Remark: For $H = M^4 \times CP_2$ the mass squared in given representation of Super-conformal symmetries is given as a sum of CP_2 mass squared for the spinor harmonic determining the ground state and of a Virasoro contribution proportional to a non-negative integer. The masses are required to independent of electroweak quantum numbers.

One can imagine two further identifications for the origin of h_{vac}.

1. Take seriously the possibility of complex momenta allowed by the complexification of M^8 by commuting imagine unit i and also suggested by the generalization of the twistorialization. The real and imaginary parts of light-like complex 8-momenta $p_8 = p_{8,Re} + ip_{8,Im}$ are orthogonal to each other: $p_{8,Re} \cdot p_{8,Im} = 0$ so that both real and imaginary parts of p_8 are

light-like: $p_{8,Re}^2 = p_{8,Im}^2 = 0$. The M^4 mass squared can be written has $m^2 = m_{Re}^2 - m_{Im}^2$ with $h_{vac} \propto -m_{Im}^2$. The representations of Super-conformal algebra would be labelled by $h_{vac} \propto m_{Im}^2$.

Could the needed wrong sign contribution to CP_2 mass squared mass make sense? CP_2 type extremals having light-like geodesic as M^4 projection are locally identical with CP_2 but because of light-like projection they can be regarded as CP_2 with a hole and thus non-compact. Boundary conditions allow analogs of CP_2 harmonics for which spinor d'Alembertian would have complex eigenvalues.

Does quantum-classical correspondence allow complex momenta: can the classical four-momenta assignable to 6-D Kähler action be complex? The value of Kähler coupling strength allows the action to have complex phase but parts with different phases are not allowed. Could the imaginary part to 4-momentum could come from the CP_2 type extremal with Euclidian signature of metric?

2. Second - not necessarily independent - idea relies on the observation that in TGD one has besides the usual conformal algebra acting on complex coordinate z also its analog acting on the light-like radial coordinate r of light-cone boundary. At light-cone boundary one has both super-symplectic symmetries of $\Delta M_+^4 \times CP_2$ and extension of super-conformal symmetries of sphere S^2 to analogs of conformal symmetries depending on z and r and it seems that one must chose between these two options. Also the extension of ordinary Kac-Moody algebra acts at the light-like orbits of partonic 2-surfaces.

   There are two scaling generators: the usual $L_0 = zd/dz$ and the second generator $L_{0,1} = ird/dr$. For $L_{0,1}$ at light-cone boundary powers of z^n are replaced with $(r/r_0)^{ik} = exp(iku)$, $u = log(r/r_0))$. Could it be that mass squared operator is proportional to $L_0 + L_{0,1}$ having eigenvalues $h = n - k$? The absence of tachyons requires $h \geq 0$. Could k characterize given Super-Virasoro representation? Could $k \geq 0$ serve as an analog of positive energy condition allowing to analytically continue $exp(iku)$ to upper u-plane? How to interpret this continuation?

   The 3-D generalization of super-symplectic symmetries at light-cone boundary and extended Ka-Moody symmetries at partonic 2-surfaces should be possible in some sense. Could the continuation to the upper u-plane correspond to the continuation of the extended conformal symmetries from light-cone boundary to future light-one and from light-partonic 2-surfaces to space-time interior?

Why p-adic massivation should occur at all? Here ZEO comes in rescue.

1. In ZEO one can have superposition of states with different 4-momenta, mass values and also other charges: this does not break conservation laws. How to fix M^4 in this case? One cannot do it separately for the states in superposition since they have different masses. The most natural choices is as the M^4 associated with the dominating contribution to the zero energy state. The outcome would be thermal massivation described excellently by p-adic thermodynamics [K66]. Recently a considerable increase in the understanding of hadron and weak boson masses took place [L98].

2. In ZEO quantum theory is square root of thermodynamics in a well-defined formal sense, and one can indeed assign to p-adic partition function a complex square root as a genuine zero energy state. Since mass varies, one must describe the presence of higher mass excitations in zero energy state as particles in M^4 assigned with the dominating part of the state so that the observed particle mass squared is essentially p-adic thermal expectation value over thermal excitations. p-Adic thermodynamics would thus describe the fact that the choice of M_L^4 cannot not ideal in ZEO and massivation would be possible only in ZEO.

3. Current quarks and constituent quarks are basic notions of hadron physics. Constituent quarks with rather large masses appear in the low energy description of hadrons and current quarks in high energy description of hadronic reactions. That both notions work looks rather paradoxical. Could massive quarks correspond to M_T picture and current quarks to M_L^4 picture but with p-adic thermodynamics forced by the superposition of mass eigenstates with different masses.

The massivation of ordinary massless fermion involves mixing of fermion chiralities. This means that the $SU(3)$ rotation determined by the dominating component in zero energy state must induce this mixing. This should be understood.

5.4.4 $M^8 - H$ duality and consciousness

$M^8 - H$ duality is one of the key ideas of TGD and one can ask whether it has implications for TGD inspired theory of consciousness and it indeed forces to challenge the recent ZEO based view about consciousness [L52] .

Objections against ZEO based theory of consciousness

Consider first objections against ZEO based view about consciousness.

1. ZEO (zero energy ontology) based view about conscious entity can be regarded as a sequence of "small" state function reductions (SSRs) identifiable as analogs of so called weak measurements at the active boundary of causal diamond (CD) receding reduction by reduction farther away from the passive boundary, which is unchanged as also the members of state pairs at it. One can say that weak measurements commute with the observables, whose eigenstates the states at passive boundary are. This asymmetry assigns arrow of time to the self having CD as embedding space correlate. "Big" state function reductions (BSRs) would change the roles of boundaries of CD and the arrow of time. The interpretation is as death and re-incarnation of the conscious entity with opposite arrow of time.

   The question is whether quantum classical correspondence (QCC) could allow to say something about the time intervals between subsequent values of temporal distance between weak state function reductions.

2. The questionable aspect of this view is that $t_M = constant$ sections look intuitively more natural as seats of quantum states than light-cone boundaries forming part of CD boundaries. The boundaries of CD are however favoured by the huge symplectic symmetries assignable to the boundary of M^4 light-cone with points replaced with CP_2 at level of H. These symmetries are crucial or the existence of the geometry of WCW ("world of classical worlds").

3. Second objection is that the size of CD increases steadily: this nice from the point of view of cosmology but the idea that CD as correlate for a conscious entity increases from CP_2 size to cosmological scales looks rather weird. For instance, the average energy of the state assignable to either boundary of CD would increase. Since zero energy state is a superposition of states with different energies classical conservation law for energy does not prevent this [L93]: essentially quantal effect due to the fact that the zero energy states are not exact eigenstates of energy could be in question. In BSRs the energy would gradually increase. Admittedly this looks strange and one must be keen for finding more conventional options.

4. Third objection is that re-incarnated self would not have any "childhood" since CD would increase all the time.

One can ask whether $M^8 - H$ duality and this braney picture has implications for ZEO based theory of consciousness. Certain aspects of $M^8 - H$ duality indeed challenge the recent view about consciousness based on ZEO (zero energy ontology) and ZEO itself.

1. The moments $t = r_n$ defining the 6-branes correspond classically to special moments for which phase transition like phenomena occur. Could $t = r_n$ have a special role in consciousness theory?

   (a) For some SSRs the increase of the size of CD reveals new $t = r_n$ plane inside CD. One can argue that these SSRS define very special events in the life of self. This would not modify the original ZEO considerably but could give a classical signature for how many ver special moments of consciousness have occurred: the number of the roots of P would be a measure for the lifetime of self and there would be the largest root after which BSR would occur.

   (b) Second possibility is more radical. One could one think of replacing CD with single truncated future- or past-directed light-cone containing the 6-D universal roots of P up to some r_n defining the upper boundary of the truncated cone? Could $t = r_n$ define a sequence of moments of consciousness? To me it looks more natural to assume that they are associated with very special moments of consciousness.

2. For both options SSRs increase the number of roots r_n inside CD/truncated light-one gradually and thus its size? When all roots of $P(o)$ would have been measured - meaning that the largest value r_{max} of r_n is reached -, BSR would be unavoidable.

   BSR could replace $P(o)$ with $P_1(r_1 - o)$: r_1 must be real and one should have $r_1 > r_{max}$. The new CD/truncated light-cone would be in opposite direction and time evolution would be reversed. Note that the new CD could have much smaller size size if it contains only the smallest root r_0. One important modification of ZEO becomes indeed possible. The size of CD after BSR could be much smaller than before it. This would mean that the re-incarnated self would have "childhood" rather than beginning its life at the age of previous self - kind of fresh start wiping the slate clean.

   One can consider also a less radical BSR preserving the arrow of time and replacing the polynomial with a new one, say a polynomial having higher degree (certainly in statistical sense so that algebraic complexity would increase).

Could one give up the notion of CD?

A possible alternative view could be that one the boundaries of CD are replaced by a pair of two $t = r_N$ snapshots $t = r_0$ and $t = r_N$. Or at least that these surfaces somehow serve as correlates for mental images. The theory might allow reformulation also in this case, and I have actually used this formulation in popular lectures since it is easier to understand by laymen.

1. Single truncated light-cone, whose size would increase in each SSR would be present now since the spheres correspond to balls of radius r_n at times r_n. If $r_0 = 0$, which is the case for $P(o) \propto o$, the tip of the light-cone boundary is one root. One cannot avoid association with big bang cosmology. For $P(0) \neq r_0$ the first conscious moment of the cosmology corresponds to $t = r_0$. One can wonder whether the emergence of consciousness in various scales could be described in terms of the varying value of the smallest root r_0 of $P(o)$.

   If one allows BSR:s this picture differs from the earlier one in that CDs are replaced with alternation of light-cones with opposite directions and their intersections would define CD.

2. For this option the preferred values of t for SSRs would naturally correspond to the roots of the polynomial defining $X^4 \subset M^8$. Moments of consciousness as state function reductions would be due to collisions of 4-D space-time surfaces X^4 with 6-D branes! They would replace the sequence of scaled CD sizes. CD could be replaced with light-one and with the increasing sequence $(r_0, ...r_n)$ of roots defining the ticks of clock and having positive and negative energy states at the boundaries r_0 and r_n.

3. What could be the interpretation for BSRs representing death of a conscious entity in the new variant of ZEO? Why the arrow of time would change? Could it be because there are no further roots of $P(o)$? The number of roots of $P(o)$ would give the number of small state function reductions?

   What would happen to $P(o)$ in BSR? The vision about algebraic evolution as increase of the dimension for the extension of rationals would suggest that the degree of $P(o)$ increases as also the number of roots if all complex roots are allowed. Could the evolution continue in the same direction or would it start to shift the part of boundary corresponding to the lowest root in opposite direction of time. Now one would have more roots and more algebraic complexity so that evolutionary step would occur.

   In the time reversal one would have naturally $t_{max} \geq r_{n_{max}}$ for the new polynomial $P(t - t_{max})$ having $r_{n_{max}}$ as its smallest root. The light-cone in M^8 with tip at $t = t_{max}$ would be in opposite direction now and also the slices $t - t_{max} = r'_n$ would increase in opposite direction! One would have two light-cones with opposite directions and the $t = r_n$ sections would replace boundaries of CDs. The reborn conscious entity would start from the lowest root so that also it would experience childhood.

This option could solve the argued problems of the previous scenario and give concrete connection with the classical physics in accordance with QCC. On the other hand, a minimal modification of original scenario combined with $M^8 - H$ duality with moments $t = r_n$ as special moments in the life of conscious entity allows also to solve these problems if the active boundary of CD is interpreted as boundary beyond which classical signals cannot contribute to perceptions.

What could be the minimal modification of ZEO based view about consciousness?

What would be the minimal modification of the earlier picture? Could one *assume* that CDs serve as embedding space correlates for the perceptive field?

1. Zero energy states would be defined as before that is in terms of 3-surfaces at boundaries of CD: this would allow a realization of huge symmetries of WCW and the active boundary A of CD would define the boundary of the region from which self can receive classical information about environment. The passive boundary P of CD would define the boundary of the region providing classical information about the state of self. Also now BSR would mean death and reincarnation with an opposite arrow of time. Now however CD would shrink in BSR before starting to grow in opposite time direction. Conscious entity would have "childhood".
2. If the geometry of CD were fixed, the size scale of the $t = r_n$ balls of M^4 would first increase and then start to decrease and contract to a point eventually at the tip of CD. One must however remember that the size of $t = r_n$ planes increases all the time as also the size of CD in the sequences of SSRs. Moments $t = r_n$ could represent special moments in the life of conscious entity taking place in SSRs in which $t = r_n$ hyperplane emerges inside CD with increased size. The recent surprising findings challenging the Bohrian view about quantum jumps [L77] can be understood in this picture [L77].
3. $t = r_n$ planes could also serve as correlates for memories. As CD increases at active boundary new events as $t = r_n$ planes would take place and give rise to memories. The states at $t = r_n$ planes are analogous to seats of boundary conditions in strong holography and the states at these planes might change in state function reductions - this would conform with the observations that our memories are not absolute.

To sum up, the original view about ZEO seems to be essentially correct. The introduction of moments $t = r_n$ as special moments in the life of self looks highly attractive as also the possibility of wiping the slate clear by reduction of the size of CD in BSR.

5.5 Could standard view about twistors work at space-time level after all?

While asking what super-twistors in TGD might be, I became critical about the recent view concerning what I have called geometric twistor space of M^4 identified as $M^4 \times S^2$ rather than CP_3 with hyperbolic metric. The basic motivations for the identification come from M^8 picture in which there is number theoretical breaking of Poincare and Lorentz symmetries. Second motivation was that M^4_{conf} - the conformally compactified M^4 - identified as group $U(2)$ [B2] (see `http://tinyurl.com/y35k5wwo`) assigned as base space to the standard twistor space CP_3 of M^4, and having metric signature (3,-3) is compact and is stated to have metric defined only modulo conformal equivalence class.

As found in the previous section, TGD strongly suggests that M^4 in $H = M^4 \times CP_2$ should be replaced with hyperbolic variant of CP_2, and it seems to me that these spaces are not identical. Amusingly, $U(2)$ and CP_2 are fiber and base in the representation of $SU(3)$ as fiber space so that the their identification does not seem plausible.

On can however ask whether the selection of a representative metric from the conformal equivalence class could be seen as breaking of the scaling invariance implied also by ZEO introducing the hierarchy of CDs in M^8. Could it be enough to have M^4 only at the level of M^8 and conformally compactified M^4 at the level of H? Should one have $H = cd_{conf} \times CP_2$? What cd_{conf} would be: is it hyperbolic variant of CP_2?

5.5.1 Getting critical

The only way to make progress is to become very critical now and then. These moments of almost despair usually give rise to a progress. At this time I got very critical about the TGD inspired identification of twistor spaces of M^4 and CP_2 and their properties.

Getting critical about geometric twistor space of M^4

Let us first discuss the recent picture and how to modify it so that it is consistent with the hierarchy of CDs. The key idea is that the twistor space and its base space represents CD so that one obtains scale hierarchy of twistor spaces as a realization of broken scale invariance giving rise to the p-adic length scale hierarchy.

1. I have identified the twistor space of M^4 simply as $T(M^4) = M^4 \times S^2$. The interpretation would be at the level of octonions as a product of M^4 and choices of M^2 as preferred complex sub-space of octonions with S^2 parameterizing the directions of spin quantization axes. Real octonion axis would correspond to time coordinate. One could talk about the space of of light-like directions. Light-like vector indeed defines M^2. This view could be defended by the breaking of both translation and Lorentz invariance in the octonionic approach due to the choice of M^2 and by the fact that it seems to work.

   Remark: $M^8 = M^4 \times E^4$ is complexified to M^8_c by adding a commuting imaginary unit i appearing in the extensions of rationals and ordinary M^8 represents its particular sub-space. Also in twistor approach one uses often complexified M^4.

2. The objection is that it is ordinary twistor space identifiable as CP_3 with (3,-3) signature of metric is what works in the construction of twistorial amplitudes. CP_3 has metric as compact space and coset space. Could this choice of twistor space make sense after all as geometric twistor space?

   Here one must pause and recall that the original key idea was that Poincare invariance is symmetry of TGD for $X^4 \subset M^4 \times CP_2$. Now Poincare symmetry has been transformed to a symmetry acting at the level of M^8 in the moduli space of octonion structures defined by the choice of the direction of octonionic real axis reducing Poincare group to $T \times SO(3)$ consisting of time translations and rotations. Fixing of M^2 reducrs the group to $T \times SO(2)$ and twistor space can be seen as the space for selections of quantization axis of energy and spin.

3. But what about the space H? The first guess is $H = M^4_{conf} \times CP_2$. According to [B2] (see `http://tinyurl.com/y35k5wwo`) one has $M^4_{conf} = U(2)$ such that $U(1)$ factor is time- like and $SU(2)$ factor is space-like. One could understand $M^4_{conf} = U(2)$ as resulting by addition and identification of metrically 2-D light-cone boundaries at $t = \pm\infty$. This is topologically like compactifying E^3 to S^3 and gluing the ends of cylinder $S^3 \times D^1$ together to the $S^3 \times S^1$.

   The conformally compactified Minkowski space M^4_{conf} should be analogous to base space of CP_3 regarded as bundle with fiber S^2. The problem is that one cannot imagine an analog of fiber bundle structure in CP_3 having $U(2)$ as base. The identification $H = M^4_{conf} \times CP_2$ does not make sense.

4. In ZEO based breaking of scaling symmetry it is CD that should be mapped to the analog of M^4_{conf} - call it cd_{conf}. The only candidate is $cd_{conf} = CP_2$ with one hypercomplex coordinate. To understand why one can start from the following picture. The light-like boundaries of CD are metrically equivalent to spheres. The light-like boundaries at $t = \pm\infty$ are identified as in the case of M^4_{conf}. In the case of CP_2 one has 3 homologically trivial spheres defining coordinate patches. This suggests that cd_{conf} is simply CP_2 with second complex coordinate made hypercomplex. M^4 and E^4 differ only by the signature and so would do cd_{conf} and CP_2.

   The twistor spheres of CP_3 associated with points of M^4 intersect at point if the points differ by light-like vector so that one has and singular bundle structure. This structure should have analog for the compactification of CD. CP_3 has also bundle structure $CP_3 \rightarrow CP_2$. The S^2 fibers and base are homologically non-trivial and complex analogs of mutually orthogonal line and plane and intersect at single point. This defines the desired singular bundle structure via the assignment of S^2 to each point of CP_2.

The M^4 points must belong to the interior of cd and this poses constraints on the distance of M^4 points from the tips of cd. One expects similar hierarchy of cds at the level of momentum space.

5. In this picture $M^4_{conf} = U(2)$ could be interpreted as a base space for the space of CDs with fixed direction of time axis identified as direction of octonionic real axis associated with various points of M^4 and therefore of M^4_{conf}. For Euclidian signature one would have base and fiber of the automorphism sub-group $SU(3)$ regarded as $U(2)$ bundle over CP_2: now one would have CP_2 bundle over $U(2)$. This is perhaps not an accident, and one can ask whether these spaces could be interpreted as representing local trivialization of $SU(3)$ as $U(2) \times CP_2$. This would give to metric cross terms between $U(2)$ and CP_2.

6. The proposed identification can be tested by looking whether it generalizes. What the twistor space for entire M^8 would be? $cd = CD_4$ is replaced with CD_8 and the discussion of the preceding chapter demonstrated that the only possible identification of the twistor space is now is as the 12-D hyperbolic variant of HP_3 whereas $CD_{8,conf}$ would correspond to 8-D hyperbolic variant of HP_2 analogous to hyperbolic variant of CP_2.

The outcome of these considerations is surprising.

1. One would have $T(H) = CP_3 \times F$ and $H = CP_{2,H} \times CP_2$ where $CP_{2,H}$ has hyperbolic metric with metric signature $(1, -3)$ having M^4 as tangent space so that the earlier picture can be understood as an approximation. This would reduce the construction of preferred extremals of 6-D Kähler action in $T(H)$ to a construction of polynomial holomorphic surfaces and also the minimal surfaces with singularities at string world sheets should result as bundle projection. Since $M^8 - H$ duality must respect algebraic dynamics the maximal degree of the polynomials involved must be same as the degree of the octonionic polynomial in M^8.

2. The hyperbolic variant Kähler form and also spinor connection of hyperbolic CP_2 brings in new physics beyond standard model. This Kähler form would serve as the analog of Kähler form assigned to M^4 earlier, and suggested to explain the observed CP breaking effects and matter antimatter asymmetry for which there are two explanations [L92].

Some comments about the Minkowskian signature of the hyperbolic counterparts of CP_3 and CP_2 are in order.

1. Why the metric of CP_3 could not be Euclidian just as the metric of F? The basic objection is that propagation of fields is not possible in Euclidian signature and one completely loses the earlier picture provided by $M^4 \times CP_2$. The algebraic dynamics in M^8 picture can hardly replace it.

2. The map assigning to the point M^4 a point of CP_3 involves Minkowskian sigma matrices but it seems that the Minkowskian metric of CP_3 is not explicitly involved in the construction of scattering amplitudes. Note however that the antisymmetric bi-spinor metric for the spin 1/2 representation of Lorentz group and its conjugate bring in the signature. $U(2,2)$ as representation of conformal symmetries suggests $(2,2)$ signature for 8-D complex twistor space with 2+2 complex coordinates representing twistors.

   The signature of CP_3 metric is not explicitly visible in the construction of twistor amplitudes but analytic continuations are carried out routinely. One has also complexified M^4 and M^8 and one could argue that the problems disappear. In the geometric situation the signatures of the subspaces differ dramatically. As already found, analytic continuation could allow to define the variants of twistor spaces elegantly by replacing a complex coordinate with a hyperbolic one.

   Remark: For E^4 CP_3 is Euclidian and if one has $E^4_{conf} = U(2)$, one could think of replacing the Cartesian product of twistor spaces with $SU(3)$ group having $M^4_{conf} = U(2)$ as fiber and CP_2 as base. The metric of $SU(3)$ appearing as subgroup of quaternionic automorphisms leaving $M^4 \subset M^8$ invariant would decompose to a sum of M^4_{conf} metric and CP_2 metric plus cross terms representing correlations between the metrics of M^4_{conf} and CP_2. This is probably mere accident.

$M^8 - H$ duality and twistor space counterparts of space-time surfaces

It seems that by identifying $CP_{3,h}$ as the twistor space of M^4, one could develop $M^8 - H$ duality to a surprisingly detailed level from the conditions that the dimensional reduction guaranteed by the identification of the twistor spheres takes place and the extensions of rationals associated with the polynomials defining the space-time surfaces at M^8- and twistor space sides are the same. The reason is that minimal surface conditions reduce to holomorphy meaning algebraic conditions involving first partial derivatives in analogy with algebraic conditions at M^8 side but involving no derivatives.

1. The simplest identification of twistor spheres is by $z_1 = z_2$ for the complex coordinates of the spheres. One can consider replacing z_i by its Möbius transform but by a coordinate change the condition reduces to $z_1 = z_2$.
2. At M^8 side one has either $RE(P) = 0$ or $IM(P) = 0$ for octonionic polynomial obtained as continuation of a real polynomial P with rational coefficients giving 4 conditions (RE/IM denotes real/imaginary part in quaternionic sense). The condition guarantees that tangent/normal space is associative.

   Since quaternion can be decomposed to a sum of two complex numbers: $q = z_1 + Jz_2$ $RE(P) = 0$ correspond to the conditions $Re(RE(P)) = 0$ and $Im(RE(P)) = 0$. $IM(P) = 0$ in turn reduces to the conditions $Re(IM(P)) = 0$ and $Im(IM(P)) = 0$.
3. The extensions of rationals defined by these polynomial conditions must be the same as at the octonionic side. Also algebraic points must be mapped to algebraic points so that cognitive representations are mapped to cognitive representations. The counterparts of both $RE(P) = 0$ and $IM(P) = 0$ should be satisfied for the polynomials at twistor side defining the same extension of rationals.
4. $M^8 - H$ duality must map the complex coordinates $z_{11} = Re(RE)$ and $z_{12} = Im(RE)$ ($z_{21} = Re(IM)$ and $z_{22} = Im(IM)$) at M^8 side to complex coordinates u_{i1} and u_{i2} with $u_{i1}(0) = 0$ and $u_{i2}(0) = 0$ for $i = 1$ or $i = 2$, at twistor side.

   Roots must be mapped to roots in the same extension of rationals, and no new roots are allowed at the twistor side. Hence the map must be linear: $u_{i1} = a_i z_{i1} + b_i z_{i2}$ and $u_{i2} = c_i z_{i1} + d_i z_{i2}$ so that the map for given value of i is characterized by SL(2,Q) matrix $(a_i, b_i; c_i, d_i)$.
5. These conditions do not yet specify the choices of the coordinates (u_{i1}, u_{i2}) at twistor side. At CP_2 side the complex coordinates would naturally correspond to Eguchi-Hanson complex coordinates (w_1, w_2) determined apart from color $SU(3)$ rotation as a counterpart of $SU(3)$ as sub-group of automorphisms of octonions.

   If the base space of the twistor space $CP_{3,h}$ of M^4 is identified as $CP_{2,h}$, the hyper-complex counterpart of CP_2, the analogs of complex coordinates would be (w_3, w_4) with w_3 hypercomplex and w_4 complex. A priori one could select the pair (u_{i1}, u_{i2}) as any pair $(w_{k(i)}, w_{l(i)})$, $k(i) \neq l(i)$. These choices should give different kinds of extremals: such as CP_2 type extremals, string like objects, massless extremals, and their deformations.

String world sheet singularitees and world-line singularities as their light-like boundaries at the light-like orbits of partonic 2-surfaces are conjectured to characterize preferred extremals as surfaces of H at which there is a transfer of canonical momentum currents between Kähler and volume degrees of freedom so that the extremal is not simultaneously an extremal of both Kähler action and volume term as elsewhere. What could be the counteparts of these surfaces in M^8?

1. The interpretation of the pre-images of these singularities in M^8 should be number theoretic and related to the identification of quaternionic imaginary units. One must specify two non-parallel octonionic imaginary units e^1 and e^2 to determine the third one as their cross product $e^3 = e^1 \times e^2$. If e^1 and e^2 are parallel at a point of octonionic surface, the cross product vanishes and the dimension of the quaternionic tangent/normal space reduces from $D = 4$ to $D = 2$.
2. Could string world sheets/partonic 2-surfaces be images of 2-D surfaces in M^8 at which this takes place? The parallelity of the tangent/normal vectors defining imaginary units e_i, $i = 1, 2$ states that the component of e_2 orthogonal to e_1 vanishes. This indeed gives 2 conditions in the space of quaternionic units. Effectively the 4-D space-time surface would degenerate into

2-D at string world sheets and partonic 2-surfacesa as their duals. Note that this condition makes sense in both Euclidian and Minkowskian regions.

3. Partonic orbits in turn would correspond surfaces at which the dimension reduces to D=3 by light-likeness - this condition involves signature in an essential manner - and string world sheets would have 1-D boundaries at partonic orbits.

Getting critical about implicit assumptions related to the twistor space of CP_2

One can also criticize the earlier picture about implicit assumptions related the twistor spaces of CP_2.

1. The possibly singular decomposition of F to a product of S^2 and CP_2 would has a description similar to that for CP_3. One could assign to each point of CP_2 base homologically non-trivial sphere intersecting it orthogonally.

2. I have assumed that the twistor space $T(CP_2) = F = SU(3)/U(1) \times U(1)$ allows Kaluza-Klein type metric meaning that the metric decomposes to a sum of the metrics assignable to the base CP_2 and fiber S^2 plus cross terms representing interaction between these degrees of freedom. It is easy to check that this assumption holds true for Hopf fibration $S^3 \to S^2$ having circle $U(1)$ as fiber (see `http://tinyurl.com/qbvktsx`). If Kaluza-Klein picture holds true, the metric of F would decompose to a sum of CP_2 metric and S^2 metric plus cross terms representing correlations between the metrics of CP_2 and S^2.

3. One should demonstrate that $F = SU(3)/U(1) \times U(1)$ has metric with the expected Kaluza-Klein property. One can represent $SU(3)$ matrices as products XYZ of 3 matrices. X represents a point of base space CP_2 as matrix, Y represents the point of the fiber $S^2 = U(2)/U(1) \times U(1)$ of F in similar manner as $U(2)$ matrix, and the Z represents $U(1) \times U(1)$ element as diagonal matrix [B2](see `http://tinyurl.com/y6c3pp2g`).

   By dropping $U(1) \times U(1)$ matrix one obtains a coordinatization of F. To get the line element of F in these coordinates one could put the coordinate differentials of $U(1) \times U(1)$ to zero in an expression of $SU(3)$ line element. This should leave sum of the metrics of CP_2 and S^2 with constant scales plus cross terms. One might guess that the left- and righ-invariance of the $SU(3)$ metric under $SU(3)$ implies KK property.

Also CP_3 should have the KK structure if one wants to realize the breaking of scaling invariance as a selection of the scale of the conformally compactified M^4. In absence of KK structure the space-time surface would depend parametrically on the point of the twistor sphere S^2.

5.5.2 The nice results of the earlier approach to M^4 twistorialization

The basic nice results of the earlier picture should survive in the new picture.

1. Central for the entire approach is twistor lift of TGD replacing space-time surfaces with 6-D surfaces in 12-D $T(M^4) \times T(CP_2)$ having space-time surfaces as base and twistor sphere S^2 as fiber. Dimensional reduction identifying twistor spheres of $T(M^4)$ an $T(CP_2)$ and makes these degrees of freedom non-dynamical.

2. Dimensionally reduced action 6-D Kähler action is sum of 4-D Kähler action and a volume term coming from S^2 contribution to the induced Kähler form. On interpretation is as a generalization of Maxwell action for point like charge by making particle a 3-surface.

   The interpretation of volume term is in terms of cosmological constant. I have proposed that a hierarchy of length scale dependent cosmological constants emerges. The hierarchy of cosmological constants would define the running length scale in coupling constant evolution and would correspond to a hierarchy of preferred p-aic length scales with preferred p-adic primes identified as ramified primes of extension of rationals.

3. The twistor spheres associated $M^4 \times S^2$ and F were assumed to have same radii and most naturally same Euclidian signature: this looks very nice since there would be only single fundamental length equal to CP_2 radius determining the radius of its twistor sphere. The vision to be discussed would be different. There would be no fundamental scale and length

scales would emerge through the length scale hierarchy assignable to CDs in M^8 and mapped to length scales for twistor spaces.

The identification of twistor spheres with same radius would give only single value of cosmological constant and the problem of understanding the huge discrepancy between empirical value and its naïve estimate would remain. I have argued that the Kähler forms and metrics of the two twistor spheres can be rotated with respect to each other so that the induced metric and Kähler form are rotated with respect to each other, and the magnetic energy density assignable to the sum of the induced Kähler forms is not maximal.

The definition of Kähler forms involving preferred coordinate frame would gives rise to symmetry breaking. The essential element is interference of real Kähler forms. If the signatures of twistor spheres were opposite, the Kähler forms differ by imaginary unit and the interference would not be possible.

Interference could give rise to a hierarchy of values of cosmological constant emerging as coefficient of the Kähler magnetic action assignable to $S^2(X^4)$ and predict length scale dependent value of cosmological constant and resolve the basic problem related to the extremely small value of cosmological constant.

4. One could criticize the allowance of relative rotation as adhoc: note that the resulting cosmological constant becomes a function depending on S^2 point. For instance, does the rotation really produce preferred extremals as minimal surfaces extremizing also Kähler action except at string world sheets? Each point of S^2 would correspond to space-time surface X^4 with different value of cosmological constant appearing as a parameter. Moreover, non-trivial relative rotation spoils the covariant constancy and $J^2(S^2) = -g(S^2)$ property for the S^2 part of Kähler form, and that this does not conform with the very idea of twistor space.

5. One nice implication would be that space-time surfaces would be minimal surfaces apart from 2-D string world sheet singularities at which there is a transfer of canonical momentum currents between Kähler and volume degrees of freedom. One can also consider the possibility that the minimal surfaces correspond to surfaces give as roots of 3 polynomials of hypercomplex coordinate of M^2 and remaining complex coordinates.

   Minimal surface property would be direct translation of masslessness and conform with the twistor view. Singular surfaces would represent analogs of Abelian currents. The universal dynamics for minimal surfaces would be a counterpart for the quantum criticality. At M^8 level the preferred complex plane M^2 of complexified octonions would represent the singular string world sheets and would be forced by number theory.

   Masslessness would be realized as generalized holomorphy (allowing hyper-complexity in M^2 plane) as proposed in the original twistor approach but replacing holomorphic fields in twistor space with 6-D twistor spaces realized as holomorphic 6-surfaces.

5.5.3 ZEO and twistorialization as ways to introduce scales in M^8 physics

M^8 physics as such has no scales. One motivation for ZEO is that it brings in the scales as sizes of causal diamonds (CDs).

ZEO generates scales in M^8 physics

Scales are certainly present in physics and must be present also in TGD Universe.

1. In TGD Universe CP_2 scale plays the role of fundamental length scale, there is also the length scale defined by cosmological constant and the geometric mean of these two length scales defining a scale of order 10^{-4} meters emerging in the earlier picture and suggesting a biological interpretation.

   The fact that conformal inversion $m^k \to R^2 m^k/a^2$, $a^2 = m^k m_k$ is a conformal transformation mapping hyperboloids with $a \geq R$ and $a \leq R$ to each other, suggests that one can relate CP_2 scale and cosmological scale defined by Λ by inversion so that cell length scale would define one possible radius of cd_{conf}.

2. In fact, if one has $R(cd_{conf}) = x \times R(CP_2)$ one obtains by repeated inversions a hierarchy $R(k) = x^k R$ and for $x = \sqrt{p}$ one obtains p-adic length scale hierarchy coming as powers of $\sqrt{p}$,

which can be also negative. This suggests a connection with p-adic length scale hypothesis and connections between long length scale and short length scale physics: they could be related by inversion. One could perhaps see Universe as a kind of Leibnizian monadic system in which monads reflect each other with respect to hyperbolic surfaces $a = constant$. This would conform with the holography.

3. Without additional assumptions there is a complete scaling invariance at the level of M^8. The scales could come from the choice of 8-D causal diamond CD_8 as intersection of 8-D future and past directed light-cones inducing choice of cd in M^4. CD serves as a correlate for the perceptive field of a conscious entity in TGD inspired theory of consciousness and is crucial element of zero energy ontology (ZEO) allowing to solve the basic problem of quantum measurement theory.

Twistorial description of CDs

Could the map of the surfaces of 4-surfaces of M^8 to $cd_{conf} \times CP_2$ by a modification of $M^8 - H$ correspondence allow to describe these scales? If so, compactification via twistorialization and $M^8 - H$ correspondence would be the manner to describe these scales as something emergent rather than fundamental.

1. The simplest option is that the scale of cd_{conf} corresponds to that of CD_8 and CD_4. One should also understand what CP_2 scale corresponds. The simplest option is that CP_2 scale defines just length unit since it is difficult to imagine how this scale could appear at M^8 level. cd_{conf} scale squared would be multiple or CP_2 scale squared, say prime multiple of it, and assignable to ramified primes of extension of rationals. Inversions would produce further scales. Inversion would allow kind of hologram like representation of physics in long length scales in arbitrary short length scales and vice versa.
2. The compactness of cd_{conf} corresponds to periodic time assignable to over-critical cosmologies starting with big bang and ending with big crunch. Also CD brings in mind over-critical cosmology, and one can argue that the dynamics at the level of cd_{conf} reflects the dynamics of ZEO at the level of M^8.

Modification of H and $M^8 - H$ correspondence

It is often said that the metric of M^4_{conf} is defined only modulo conformal scaling factor. This would reflect projectivity. One can however endow projective space CP_3 with a metric with isometry group $SU(2,2)$ and the fixing of the metric is like gauge choice by choosing representative in the projective equivalence class. Thus CP_3 with signature (3,-3) might perhaps define geometric twistor space with base cd_{conf} rather than M^4_{conf} very much like the twistor space $T(CP_2) = F = SU(3)/U(1) \times U(1)$ at the level. Second projection would be to M^4 and map twistor sphere to a point of M^4. The latter bundle structure would be singular since for points of M^4 with light-like separation the twistor spheres have a common point: this is an essential feature in the construction of twistor amplitudes.

New picture requires a modification of the view about H and about $M^8 - H$ correspondence.

1. H would be replaced with $cd_{conf} \times CP_2$ and the corresponding twistor space with $CP_3 \times F$. $M^8 - H$ duality involves the decomposition $M^2 \subset M^4 \subset M^8 = M^4 \times CP_2$, where M^4 is quaternionic sub-space containing preferred place M^2. The tangent or normal space of X^4 would be characterized by a point of CP_2 and would be mapped to a point of CP_2 and the point of CP_2 - or rather point plus the space S^2 or light-like vectors characterizing the choices of M^2 - would mapped to the twistor sphere S^2 of CP_3 by the standard formulas.

   $S^2(cd_{conf})$ would correspond to the choices of the direction of preferred octonionic imaginary unit fixing M^2 as quantization axis of spin and $S^2(CP_2)$ would correspond to the choice of isospin quantization axis: the quantization axis for color hyperspin would be fixed by the choice of quaternionic $M^4 \subset M^8$. Hence one would have a nice information theoretic interpretation.
2. The M^4 point mapped to twistor sphere $S^2(CP_3)$ would be projected to a point of cd_{conf} and define $M^8 - H$ correspondence at the level of M^4. This would define compactification

and associate two scales with it. Only the ratio $R(cd_{conf})/R(CP_2)$ matters by the scaling invariance at M^8 level and one can just fixe the scale assignable to $T(CP_2)$ and call it CP_2 length scale.

One should have a concrete construction for the hyperbolic variants of CP_n.

1. One can represent Minkowski space and its variants with varying signatures as sub-spaces of complexified quaternions, and it would seem that the structure of sub-space must be lifted to the level of the twistor space. One could imagine variants of projective spaces CP_n, $n = 2, 3$ as and HP_n, $n = 2, 3$. They would be obtained by multiplying imaginary quaternionic unit I_k with the imaginary unit i commuting with quaternionic units. If the quaternions λ involved with the projectivization $(q_1, ..., q_n) \equiv \lambda(q_1, ..., q_n)$ are ordinary quaternions, the multiplication respects the signature of the subspace. By non-commutativity of quaternions one can talk about left- and right projective spaces.

2. One would have extremely close correspondence between M^4 and CP_2 degrees of freedom reflecting the $M^8 - H$ correspondence. The projection $CP_3 \to CP_2$ for E^4 would be replaced with the projection for the hyperbolic analogs of these spaces in the case of M^4. The twistor space of M^4 identified as hyperbolic variant of CP_3 would give hyperbolic variant of CP_2 as conformally compactified cd. The flag manifold $F = SU(3)/U(1) \times U(1)$ as twistor space of CP_2 would also give CP_2 as base space.

The general solution of field equations at the level of $T(H)$ would correspond to holomorphy in general sense for the 6-surfaces defined by 3 vanishing conditions for holomorphic functions - 6 real conditions. Effectively this would mean the knowledge of the exact solutions of field equations also at the level of H: TGD would be an integrable theory. Scattering amplitudes would in turn constructible from these solutions using ordinary partial differential equations [L92].

1. The first condition would identify the complex coordinates of $S^2(cd_{conf})$ and $S^2(CP_2)$: here one cannot exclude relative rotation represented as a holomorphic transformation but for $R(cd_{conf}) \gg R(CP_2)$ the effect of the rotation is small.

2. Besides this there would be vanishing conditions for 2 holomorphic polynomials. The coordinate pairs corresponding to $M^2 \subset M^4$ would correspond to hypercomplex behavior with hyper complex coordinate $u = \pm t - z$. t and z could be assigned with $U(1)$ fibers of Hopf fibrations $SU(2) \to S^2$.

3. The octonionic polynomial $P(o)$ of degree $n = h_{eff}/h_0$ with rational coefficients fixes the extension of rationals and since the algebraic extension should be same at both sides, the polynomials in twistor space should have same degree. This would give enormous boos concerning the understanding of the proposed cancellation of fermionic Wick contractions in SUSY scattering amplitudes forced by number theoretic vision [L92].

Possible problems related to the signatures

The different signatures for the metrics of the twistor spheres of cd_{conf} and CP_2 can pose technical problems.

1. Twistor lift would replace X^4 with 6-D twistor space X^6 represented as a 6-surface in $T(M^4) \times T(CP_2)$. X^6 is defined by dimensional reduction in which the twistor spheres $S^2(cd_{conf})$ and $S^2(CP_2)$ are identified and define the twistor sphere $S^2(X^4)$ of X^6 serving as a fiber whereas space-time surface X^4 serves as a base. The simplest identification is as $(\theta, \phi)_{S^2(M^4)} = (\theta, \phi)_{S^2(CP_2)}$: the same can be done for the complex coordinates $z_{S^2(M^4_{conf})} = z_{S^2(CP_2))}$). An open question is whether a Möbius transformation could relate the complex coordinates. The metrics of the spheres are of opposite sign and differ only by the scaling factors $R^2(cd_{conf})$ and $R^2(CP_2)$.

2. For cd_{conf} option the signatures of the 2 twistor spheres would be opposite (time-like for cd_{conf}). For $R(cd_{conf})/R(CP_2) = 1$. $J^2 = -g$ is the only consistent option unless the signature of space is not totally positive or negative and implies that the Kähler forms of the two twistor spheres differ by i. The magnetic contribution from $S^2(X^4)$ would give rise to an infinite value of cosmological constant proportional to $1/\sqrt{g_2}$, which would diverge

$R(cd_{conf})/R(CP_2) = 1$. There is however no need to assume this condition as in the original approach.

5.5.4 Hierarchy of length scale dependent cosmological constants in twistorial description

At the level of M^8 the hierarchy of CDs defines a hierarchy of length scales and must correspond to a hierarchy of length scale dependent cosmological constants. Even fundamental scales would emerge.

1. If one has $R(cd_{conf})/R(CP_2) >> 1$ as the idea about macroscopic cd_{conf} would suggest, the contribution of $S^2(cd_{conf})$ to the cosmological constant dominates and the relative rotation of metrics and Kähler form cannot affect the outcome considerably. Therefore different mechanism producing the hierarchy of cosmological constants is needed and the freedom to choose rather freely the ratio $R(cd_{conf})/R(CP_2)$ would provide the mechanism. What looked like a weakness would become a strength.
2. $S^2(cd_{conf}$ would have time-like metric and could have large scale. Is this really acceptable? Dimensional reduction essential for the twistor induction however makes $S^2(cd_{conf})$ non-dynamical so that time-likeness would not be visible even for large radii of $S^2(cd_{conf})$ expected if the size of cd_{conf} can be even macroscopic. The corresponding contribution to the action as cosmological constant has the sign of magnetic action and also Kähler magnetic energy is positive. If the scales are identical so that twistor spheres have same radius, the contributions to the induced metric cancel each other and the twistor space becomes metrically 4-D.
3. At the limit $R(cd_{conf}) \rightarrow RCP_2)$ cosmological constant coming from magnetic energy density diverges for $J^2 = -G$ option since it is proportional to $1/\sqrt{g_2}$. Hence the scaling factors must be different. The interpretation is that cosmological constant has arbitrarily large values near CP_2 length scale. Note however that time dependence is replaced with scale dependence and space-time sheets with different scales have only wormhole contacts.

It would seem that this approach could produce the nice results of the earlier approach. The view about how the hierarchy of cosmological constants emerges would change but the idea about reducing coupling constant evolution to that for cosmological constant would survive. The interpretation would be in terms of the breaking of scale invariance manifesting as the scales of CDs defining the scales for the twistor spaces involved. New insights about p-adic coupling constant evolution emerge and one finds a new "must" for ZEO. $H = M^4 \times CP_2$ picture would emerge as an approximation when cd_{conf} is replaced with its tangent space M^4. The consideration of the quaternionic generalization of twistor space suggests natural identification of the conformally compactified twistor space as being obtained from CP_2 by making second complex coordinate hyperbolic. This need not conform with the identification as $U(2)$.

5.6 How to generalize twistor Grassmannian approach in TGD framework?

One should be able to generalize twistor Grassmannian approach in TGD framework. The basic modification is replacement of 4-D light-like momenta with their 8-D counterparts. The octonionic interpretation encourages the idea that twistor approach could generalize to 8-D context. Higher-dimensional generalizations of twistors have been proposed but the basic problem is that the index raising and lifting operations for twistors do not generalize (see `http://tinyurl.com/y24lkwce`).

1. For octonionic twistors as pairs of quaternionic twistors index raising would not be lost working for M_T option and light-like M^8 momenta can be regarded sums of M^4_T and E^4 parts as also twistors. Quaternionic twistor components do not commute and this is essential for incidence relation requiring also the possibility to raise or lower the indices of twistors. Ordinary complex twistor Grassmannians would be replaced with their quaternionic countparts. The twistor space as a generalization of CP_3 would be 3-D quaternionic projective space $T(M^8) = HP_3$ with Minkowskian signature (6,6) of metric and having real dimension 12 as one might expect.

Another option realizing non-commutativity could be based on the notion of quantum twistor to be also discussed.

2. Second approach would rely on the identification of $M^4 \times CP_2$ twistor space as a Cartesian product of twistor spaces of M^4 and CP_2. For this symmetries are not broken, $M^4_L \subset M^8$ depends on the state and is chosen so that the projection of M^8 momentum is light-like so that ordinary twistors and CP_2 twistors should be enough. $M^8 - H$ relates varying M^4_L based and M^4_T based descriptions.

3. The identification of the twistor space of M^4 as $T(M^4) = M^4 \times S^2$ can be motivated by octonionic considerations but might be criticized as non-standard one. The fact that quaternionic twistor space HP_3 looks natural for M^8 forces to ask whether $T(M^4) = CP_3$ endowed with metric having signature (3,3) could work in the case of M^4. In the sequel also a vision based on the identification $T(M^4) = CP_3$ endowed with metric having signature (3,3) will be discussed.

5.6.1 Twistor lift of TGD at classical level

In TGD framework twistor structure is generalized [K114, K98, K14, L75]. The inspiration for TGD approach to twistorialization has come from the work of Nima Arkani-Hamed and colleagues [B21, B15, B16, B18, B45, B22, B5]. The new element is the formulation of twistor lift also at the level of classical dynamics rather than for the scattering amplitudes only [K114, K14, K98, L75].

1. The 4-D light-like momenta in ordinary twistor approach are replaced by 8-D light-like momenta so that massive particles in 4-D sense become possible. Twistor lift of TGD takes places also at the space-time level and is geometric counterpart for the Penrose's replacement of massless fields with twistors. Roughly, space-time surfaces are replaced with their 6-D twistor spaces represented as 6-surfaces. Space-time surfaces as preferred extremals are minimal surfaces with 2-D string world sheets as singularities. This is the geometric manner to express masslessness. X^4 is simultaneously also extremal of 4-D Kähler action outside singularities: this realizes preferred extremal property.

2. One can say that twistor structure of X^4 is induced from the twistor structure of $H = M^4 \times CP_2$, whose twistor space $T(H)$ is the Cartesian product of geometric twistor space $T(M^4) = M^4 \times CP_1$ and $T(CP_2) = SU(3)/U(1) \times U(1)$. The twistor space of M^4 assigned to momenta is usually taken as a variant of CP_3 with metric having Minkowski signature and both CP_1 fibrations appear in the more precise definition of $T(M^4)$. Double fibration [B43] (see `http://tinyurl.com/yb4bt74l`) means that one has fibration from $M^4 \times CP_1$ - the trivial CP_1 bundle defining the geometric twistor space to the twistors space identified as complex projective space defining conformal compactification of M^4. Double fibration is essential in the twistorialization of TGD [K50].

3. The basic objects in the twistor lift of classical TGD are 6-D surfaces in $T(H)$ having the structure of twistor space in the sense that they are CP_1 bundles having X^4 as base space. Dimensional reduction to CP_1 bundle effectively eliminates the dynamics in CP_1 degrees of freedom and its only remnant is the value of cosmological constant appearing as coefficient of volume term of the dimensionally reduced action containing also 4-D Kähler action. Cosmological term depends on p-adic length scales and has a discrete spectrum [L75, L73].

CP_1 has also an interpretation as a projective space constructed from 2-D complex spinors. Could the replacement of these 2-spinors with their quantum counterparts defining in turn quantum CP_1 realize finite quantum measurement resolution in M^4 degrees of freedom? Projective invariance for the complex 2-spinors would mean that one indeed has effectively CP_1.

5.6.2 Octonionic twistors or quantum twistors as twistor description of massive particles

For M^4_T option the particles are massive and the one encounters the problem whether and how to generalize the ordinary twistor description.

5.6.3 Basic facts about twistors and bi-spinors

It is convenient to start by summarizing the basic facts about bi-spinors and their conjugates allowing to express massless momenta as $p^{aa'} = \lambda^a\tilde{\lambda}^{a'}$ with $\tilde{\lambda}$ defined as complex conjugate of λ and having opposite chirality (see http://tinyurl.com/y6bnznyn).

1. When λ is scaled by a complex number $\tilde{\lambda}$ suffers an opposite scaling. The bi-spinors allow the definition of various inner products

$$\begin{aligned} \langle\lambda,\mu\rangle &= \epsilon_{ab}\lambda^a\mu^b \ , \\ \left[\tilde{\lambda},\tilde{\mu}\right] &= \epsilon_{a'b'}\tilde{\lambda}^{a'}\tilde{\mu}^{b'} \ , \\ p\cdot q &= \langle\lambda,\mu\rangle\left[\tilde{\lambda},\tilde{\mu}\right] \ , \ (q_{aa'} = \mu_a\tilde{\mu}_{a'}) \ . \end{aligned} \tag{5.6.1}$$

2. Spinor indices are lowered and raised using antisymmetric tensors $\epsilon^{\alpha\beta}$ and $\epsilon_{\dot{\alpha}\dot{\beta}}$. If the particle has spin one can assign it a positive or negative helicity $h = \pm 1$. Positive helicity can be represented by introducing artitrary negative (positive) helicity bispinor μ_a ($\mu_{a'}$) not parallel to λ_a ($\mu_{a'}$) so that one can write for the polarization vector

$$\begin{aligned} \epsilon_{aa'} &= \frac{\mu_a\tilde{\lambda}_{a'}}{\langle\mu,\lambda\rangle} \ , \ \text{positive helicity} \ , \\ \epsilon_{aa'} &= \frac{\lambda_a\tilde{\mu}_{a'}}{\left[\tilde{\mu},\tilde{\lambda}\right]} \ , \ \text{negative helicity} \ . \end{aligned} \tag{5.6.2}$$

In the case of momentum twistors the μ part is determined by different criterion to be discussed later.

3. What makes 4-D twistors unique is the existence of the index raising and lifting operations using ϵ tensors. In higher dimensions they do not exist and this causes difficulties. For octonionic twistors with quaternionic components possibly only in $D = 8$ the situation changes.

To get a very rough idea about twistor Grassmannian approach idea, consider tree amplitudes of $\mathcal{N} = 4$ SUSY as example and it is convenient to drop the group theory factor $Tr(T_1T_2\cdots T_n)$. The starting point is the observation that tree amplitude for which more than $n-2$ gluons have the same helicity vanish. MHV amplitudes have exactly $n-2$ gluons of same helicity- taken by a convention to be negative- have extremely simple form in terms of the spinors and reads as

$$A_n = \frac{\langle\lambda_x,\lambda_y\rangle^4}{\prod_{i=1}^{n}\langle\lambda_i,\lambda_{i+1}\rangle} \tag{5.6.3}$$

When the sign of the helicities is changed $\langle..\rangle$ is replaced with [..].

An essential point in what follows is that the amplitudes are expressible in terms of the antisymmetric bi-linears $\langle\lambda_i,\lambda_j\rangle$ making sense also for octotwistors and identifiable as quaternions rather than octonions.

$M^8 - H$ duality and two alternative twistorializations of TGD

$M^8 - H$ duality suggests two alternative twistorializations of TGD.

1. The first approach would be in terms of M^8 twistors suggested by quaternionic light-lineness of 8-momenta. M^8 twistors would be Cartesian products of M^4 and E^4 twistors. One can imagine a straightforward generalization of twistor scattering amplitudes in terms of generalized Grassmannian approach replacing complex Grassmannian with quaternionic Grassmannian, which is a mathematically well-defined notion.
2. Second approach would rely on $M^4 \times CP_2$ twistors, which are products of M^4 twistors and CP_2 twistors: this description works nicely at classical space-time level but at the level of momentum space the problem is how to describe massivation of M^4 momenta using twistors.

Why the components of twistors must be non-commutative?

How to modify the 4-D twistor description of light-like 4-momenta so that it applies to massive 4-momenta?

1. Twistor consists of a pair $(\mu_{\dot{\alpha}}, \lambda^{\alpha})$ of bi-spinors in conjugate representations of $SU(2)$. One can start from the 4-D incidence relations for twistors

$$\mu_{\dot{\alpha}} = p_{\alpha\dot{\alpha}}\lambda^{\alpha} \quad .$$

   Here $p_{\alpha\dot{\alpha}}$ denotes the representation of four-momentum $p^k\sigma_k$. The antisymmetric permutation symbols $\epsilon^{\alpha\beta}$ and its dotted version define antisymmetric "inner product" in twistor space. By taking the inner product of μ with itself, one obtains the commutation relation $\mu_1\mu_2 - \mu_2\mu_1 = 0$, which is consistent with right-hand side for massless particles with $p_kp^k = 0$.

2. In TGD framework particles are massless only in 8-D sense so that the right hand side in the contraction is in general non-vanishing. In massive case one can replace four-momentum with unit vector. This requires

$$\langle \mu 1, \mu_2 \rangle = \mu_1\mu_2 - \mu_2\mu_1 \neq 0 \quad .$$

   The components of 2-spinor become non-commutative.

This raises two questions.

1. Could the replacement of complex twistors by quaternionic twistors make them non-commutative and allow massive states?
2. Could non-commutative quantum twistors solve the problem caused by the light-likeness of momenta allowing 4-D twistor description?

Octotwistors or quantum twistors?

One should be able to generalize twistor amplitudes and twistor Grassmannian approach to TGD framework, where particles are massless in 8-D sense and massive in 4-D sense. Could twistors be replaced by octonionic or quantum twistors.

1. One can express mass squared as a product of commutators of components of the twistors λ and $\tilde{\lambda}$, which is essentially the conjugate of λ:

$$p \cdot p \quad = \quad \langle \lambda, \lambda \rangle \left[\tilde{\lambda}, \tilde{\lambda} \right] \quad . \tag{5.6.4}$$

   This operator should be non-vanishing for non-vanishing mass squared. Both terms in the product vanish unless commutativity fails so that mass vanishes. The commutators should have the quantum state as its eigenstate.

2. Also 4-momentum components should have well-defined values. Four-momentum has expression $p^{aa'} = \lambda^a\tilde{\lambda}^{a'}$ in massless case. This expression should generalized to massive case as such. Eigenvalue condition and reality of the momentum components requires that the components $p^{aa'}$ are commuting Hermitian operators.

   In twistor Grassmannian approach complex but light-like momenta are possible as analogs of virtual momenta. Also in TGD framework the complexity of Kähler coupling strength allows to consider complex momenta. For twistor lift they however differ from real momenta only by a phase factor associated with the $1/\alpha_K$ associated with 6-D Kähler action.

   Remark: I have considered also the possibility that states are eigenstates only for the longitudinal M^2 projection of 4-momentum with quark model of hadrons serving as a motivation.

   (a) Could this equation be obtained in massive case by regarding λ^a and $\tilde{\lambda}^{a'}$ as commuting octo-spinors and their complex conjugates? Octotwistors would naturally emerge in the description at embedding space level. I have already earlier considered the notion of octotwistor [K106] [L40]).

(b) Or could it be obtained for quantum bi-spinors having same states as eigenstates. Could quantum twistors as generalization of the ordinary twistors correspond to the reduction of the description from the level of M^8 or H to at space-time level so that one would have 4-D twistors and massive particles with 4-momentum identifiable as Noether charge for the action principle determining preferred extremals? I have considered also the notion of quantum spinor earlier [K48, K76, K68, K2, K123].

3. In the case of quantum twistors the generalization of the product of the quantities $\langle\lambda_i, \lambda_{i+1}\rangle$ appearing in the formula should give rise to c-number in the case of quantum spinors. Can one require that the quantities $\langle\lambda_i, \lambda_{i+1}\rangle$ or even $\langle\lambda_i, \lambda_j\rangle$ are c-numbers simultaneously? This would also require that $\langle\lambda, \lambda\rangle$ is non-vanishing c-number in massive case: also incidence relation suggest this condition. Could one think λ as an operator such that $\langle\lambda, \lambda\rangle$ has eigenvalue spectrum corresponding to the quantities $\langle\lambda_i, \lambda_{i+1}\rangle$ appearing in the scattering amplitude?

5.6.4 The description for M^4_T option using octo-twistors?

For option I with massive M^4_T projection of 8-momentum one could imagine twistorial description by using M^8 twistors as products of M^4_T and E^4 twistors, and a rather straightforward generalization of standard twistor Grassmann approach can be considered.

Could twistor Grassmannians be replaced with their quaternionic variants?

The first guess would simply replace $Gr(k, n)$ with $Gr(2k, 2n)$ 4-D twistors 8-D twistors. From twistor amplitudes with quaternionic M^8-momenta one could construct physical amplitudes by going from 8-momentum basis to the 4-momentum- basis with wave functions in irreps of $SO(3)$. Life is however not so simple.

1. The notion of ordinary twistor involves in an essential manner Pauli matrices σ_i satisfying the well-known anti-commutation relations. They should be generalized. In fact, σ_0 and $\sqrt{-1}\sigma_i$ can be regarded as a matrix representation for quaternionic units. They should have analogs in 8-D case.

   Octonionic units ie_i indeed provide this analog of sigma matrices. Octonionic units for the complexification of octonions allow to define incidence relation and representation of 8-momenta in terms of octo-spinors. They do not however allow matrix representation whereas time-like octonions allow interpretation as quaternion in suitable bases and thus matrix representation. Index raising operation is essential for twistors and makes dimension $D = 4$ very special. For naïve generalizations of twistors to higher dimensions this operation is lost (see `http://tinyurl.com/y24lkwce`).

2. Could one avoid multiplication of more than two octo-twistors in Grassmann amplitudes leading to difficulties with associativity. An important observation is that in the expressions for the twistorial scattering amplitudes only products $\langle\lambda_i, \lambda_j\rangle$ or $[\tilde{\lambda}_i, \tilde{\lambda}_{i+1}]$ but not both occur. These products are associative even if the spinors are replaced by quaternionic spinors.

   These operations are antisymmetric in the arguments, which suggests cross product for quaternions giving rise to imaginary quaternion so that the product of objects would give rise to a product of imaginary quaternions. This might be a problem since a large number of terms in the product would approach to zero for random imaginary quaternions.

   An ad hoc guess would be that scattering probability is proportional to the product of amplitude as product $\langle\lambda_i, \lambda_j\rangle$ and its "hermitian conjugate" with the conjugates $[\tilde{\lambda}_i, \tilde{\lambda}_{i+1}]$ in the reverse order (this does not affect the outcome) so that the result would be real. Scattering amplitude would be more like quaternion valued operator. Could one have a formulation of quantum theory or at least TGD view about quantum theory allowing this?

3. If ordinary massless 4-momenta correspond to quaternionic sigma matrices, twistors can be regarded as pairs of 2-spinors in matrix representation. Octonionic 8-momenta should correspond to pairs of 4-spinors. As already noticed, octonions do not however allow matrix representation! Octonions for a fixed decomposition $M^8 = M^4 \times E^4$ can be however decomposed to linear combination of two quaternions just like complex numbers to a combination of real numbers. These quaternions would have matrix representation and quaternionic analogs

of twistor pair $(\mu, \tilde{\lambda})$. One could perhaps formulate the generalization of twistor Grassmann amplitudes using these pairs. This would suggest replacement of complex bi-spinors with complexified quaternions in the ordinary formalism. This might allow to solve problems with associativity if only $\langle \lambda_i, \lambda_j \rangle$ or $[\tilde{\lambda}_i, \tilde{\lambda}_{i+1}]$ appear in the amplitudes.

4. The argument in the momentum conserving delta function $\delta(\lambda_i \tilde{\lambda}_i)$ should be real so that the conjugation with respect to i would not change the argument and non-commutativity would not be problem. In twistor Grassmann amplitudes the argument $C \cdot Z$ of delta momentum conserving function is linear in the components of complex twistor Z. If complex twistor is replaced with quaternionic twistor, the Grassmannian coordinates C in delta functions $\delta(C \cdot Z)$ must be replaced with quaternionic one.

The replacement of complex Grassmannians $Gr_C(k, n)$ with quaternionic Grassmannians $Gr_H(k, n)$ is therefore highly suggestive. Quaternionic Grassmannians (see `http://tinyurl.com/y23jsffn`) are quotients of symplectic Lie groups $Gr_H(k, n) = U_n(H)/(U_r(H) \times U_{n-r}(H))$ and thus well-defined. In the description using $Gl_H(k, n)$ matrices the matrix elements would be quaternions and $k \times k$ minors would be quaternionic determinants.

Remark: Higher-D projective spaces of octonions do not exist so that in this sense dimension $D = 8$ for embedding space would be maximal.

Twistor space of M^8 as quaternionic projective space HP_3?

The simplest Grassmannian corresponds to twistor space and one can look what one obtains in this case. One can also try to understand how to cope with the problems caused by Minkowskian signature.

1. In previous section it was found that the modification of H to $H = cd_{conf} \times CP_2$ with $cd_{conf} = CP_{2,h}$ identifiable as CP_2 with Minkowskian signature of metric is strongly suggestive.
2. For E^8 quaternionic twistor space as analog of CP_3 would be its quaternionic variant HP_3 with expected dimension $D = 16 - 4 = 12$. Twistor sphere would be replaced with its quaternionic counterpart $SU(2)_H/U(1)_H$ having dimension 4 as expected. $CD_{8,conf}$ as conformally compactified CD_8 must be 8-D. The space HP_2 has dimension 8 and is analog of CP_2 appearing as analog of base space of CP_3 identified as conformally compactified 4-D causal diamond cd_{conf}. The quaternionic analogy of $M^4_{conf} = U(2)$ identified as conformally compactified M^4 would be $U(2)_H$ having dimension $D = 10$ rather than 8.

   HP_3 and HP_2 might work for E^8 but it seems that the 4-D analog of twistor sphere should have signature (2,-2) whereas base space should have signature (1,-7). Some kind of hyperbolic analogs of these spaces obtained by replacing quaternions with their hypercomplex variant seem to be needed. The same receipe in the twistorialization of M^4 would give cd_{conf} as analog of CP_2 with second complex coordinate made hyperbolic. I have already considered the construction of hyperbolic analogs of CP_2 and CP_3 as projective spaces. These results apply to HP_2 and HP_3.
3. What about octonions? Could one define octonionic projective plane OP_2 and its hyperbolic variants corresponding to various sub-spaces of M^8? Euclidian OP_2 known as Cayley plane exists as discovered by Ruth Moufang in 1933. Octonionic higher-D projective spaces and Grassmannians do not however exist so that one cannot assign OP_3 as twistor spaces.

Can one obtain scattering amplitudes as quaternionic analogs of residue integrals?

Can one obtain complex valued scattering amplitudes (i commuting with octonionic units) in this framework?

1. The residue integral over quaternionic C-coordinates should make sense, and pick up the poles as vanishing points of minors. The outcome of repeated residue integrations should give a sum over poles with complex residues.
2. Residue calculus requires analyticity. The problem is that quaternion analyticity based on a generalization of Cauchy-Riemann equations allows only linear functions. One could define quaternion (and octonion) analyticity in restricted sense using powers series with real coefficients (or in extension involving i commuting with octonion units). The quaternion/octonion

analytic functions with real coefficients are closed with respect to sum and product. I have used this definition in the proposed construction of algebraic dynamics for in $X^4 \subset M^8$ [L40].

3. Could one define the residue integral purely algebraically? Could complexity of the coefficients (i) force complex outcome: if pole q_0 is not quaternionically real the function would not allow decompose to $f(q)/(q - q_0)$ with f allowing similar Taylor series at pole. If so, then the formulas of Grassmannian formalism could generalize more or less as such at M^8 level and one could map the predictions to predictions of $M^4 \times CP_2$ approach by analog of Fourier transform transforming these quantum state basis to each other.

This option looks rather interesting and involves the key number theoretic aspects of TGD in a crucial manner.

5.6.5 Do super-twistors make sense at the level of M^8?

By $M^8 - H$ duality [L40] there are two levels involved: M^8 and H. These levels are encountered both at the space-time level and momentum space level. Do super-octonions and super-twistors make sense at M^8 level?

1. At the level of M^8 the high uniqueness and linearity of octonion coordinates makes the notion of super-octonion natural. By $SO(8)$ triality octonionic coordinates (bosonic octet 8_0), octonionic spinors (fermionic octet 8_1), and their conjugates (anti-fermionic octet 8_{-1}) would for triplet related by triality. A possible problem is caused by the presence of separately conserved B and L. Together with fermion number conservation this would require $\mathcal{N} = 4$ or even $\mathcal{N} = 4$ SUSY, which is indeed the simplest and most beautiful SUSY.
2. At the level of the 8-D momentum space octonionic twistors would be pairs of two quaternionic spinors as a generalization of ordinary twistors. Super octo-twistors would be obtained as generalization of these.

The progress in the understanding of the TGD version of SUSY [L92] led to a dramatic progress in the understanding of super-twistors.

1. In non-twistorial description using space-time surfaces and Dirac spinors in H, embedding space coordinates are replaced with super-coordinates and spinors with super-spinors. Theta parameters are replaced with quark creation and annihilation operators. Super-coordinate is a super-polynomial consisting of monomials with vanishing total quark number and appearing in pairs of monomial and its conjugate to guarantee hermiticity.

   Dirac spinor is a polynomial consisting of powers of quark creation operators multiplied by monomials similar to those appearing in the super-coordinate. Anti-leptons are identified as spartners ofquarks identified as local 3-quark states. The multi-spinors appearing in the expansions describe as such local many-quark-antiquark states so that super-symmetrization means also second quantization. Fermionic and bosonic states assignable to H-geometry interact since super-Dirac action contains induced metric and couplings to induced gauge potentials.
2. The same recipe works at the level of twistor space. One introduces twistor super-coordinates analogous to super-coordinates of H and M^8. The super YM field of $\mathcal{N} = 4$ SUSY is replaced with super-Dirac spinor in twistor space. The spin degrees of freedom associated with twistor spheres S^2 would bring in 2 additional spin-like degrees of freedom.

   The most plausible option is that the new spin degrees are frozen just like the geometric S^2 degrees of freedom. The freezing of bosonic degrees of freedom is implied by the construction of twistor space of X^4 by dimensional reduction as a 6-D surface in the product of twistor spaces of M^4 and CP_2. Chirality conditions would allow only single spin state for both spheres.
3. Number theoretical vision implies that the number of Wick contractions of quarks and anti-quarks cannot be larger than the degree of the octonionic polynomial, which in turn should be same as that of the polynomials of twistor space giving rise to the twistor space of space-time surface as 6-surface. The resulting conditions correspond to conserved currents identifiable as Noether currents assignable to symmetries.

Also Grassmannian is replaced with super-Grassmannian and super-coordinates as matrix elements of super matrices are introduced.

1. The integrand of the Grassmannian integral defining the amplitude can be expanded in Taylor series with respect to theta parameters associated with the super coordinates C as rows of super $G(k,n)$ matrix.

2. The delta function $\delta(C,Z)$ factorizing into a product of delta functions is also expanded in Taylor series to get derivatives of delta function in which only coordinates appear. By partial integration the derivatives acting on delta function are transformed to derivatives acting on integrand already expanded in Taylor series in theta parameters. The integration over the theta parameters using the standard rules gives the amplitudes associated with different powers of theta parameters associated with Z and from this expression one can pick up the scattering amplitudes for various helicities of external particles.

The super-Grassmannian formalism is extremely beautiful but one must remember that one is dealing with quantum field theory. It is not at all clear whether this kind of formalism generalizes to TGD framework, where particle are 3-surfaces [L40]. The notion of cognitive representation effectively reducing 3-surfaces to a set of point-like particles strongly suggests that the generalization exists.

The progress in understanding of $M^8 - H$ duality throws also light to the problem whether SUSY is realized in TGD and what SUSY breaking does mean. It seems now clear that sparticles are predicted and SUSY remains in the simplest scenario exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them.

The increased undestanding of what twistorialization leads to an improved understanding of what twistor space in TGD could be. It turns out that the hyperbolic variant $CP_{3,h}$ of the standard twistor space CP_3 is a more natural identification than the earlier $M^4 \times S^2$ also in TGD framework but with a scale corresponding to the scale of CD at the level of M^8 so that one obtains a scale hierarchy of twistor spaces. Twistor space has besides the projection to M^4 also a bundle projection to the hyperbolic variant $CP_{2,h}$ of CP_2 so that a remarkable analogy between M^4 and CP_2 emerges. One can formulate super-twistor approach to TGD using the same formalism as will be discussed in this article for the formulation at the level of H. This requires introducing besides 6-D Kähler action and its super-variant also spinors and their super-variants in super-twistor space. The two formulations are equivalent apart from the hierarchy of scales for the twistor space. Also M^8 allows analog of twistor space as quaternionic Grassmannian HP_3 with signature (6,6). What about super- variant of twistor lift of TGD? consider first the situation before the twistorialization.

1. The parallel progress in the understanding SUSY in TGD framework [L92] leads to the identification of the super-counterparts of M^8, H and of twistor spaces modifying dramatically the physical interpretation of SUSY. Super-spinors in twistor space would provide the description of quantum states. Super-Grassmannians would be involved with the construction of scattering amplitudes. Quaternionic super Grassmannians would be involved with M^8 description.

2. In fermionic sector only quarks are allowed by $SO(1,7)$ triality and that anti-leptons are local 3-quark composites identifiable as spartners of quarks. Gauge bosons, Higgs and graviton would be also spartners and assignable to super-coordinates of embedding space expressible as super-polynomials of quark oscillator operators. Super-symmetrization means also quantization of fermions allowing local many-quark states.

3. SUSY breaking would be caused by the same universal mechanism as ordinary massivation of massless states. The mass formulas would be supersymmetric but the choice of p-adic prime identifiable as ramified prime of extension of rationals would depend on the state of super-multiplet. ZEO would make possible symmetry breaking without symmetry breaking as Wheeler might put it.

Super-counterpart of twistor lift using the proposed formalism

The construction of super-coordinates and super-spinors [L92] suggests a straightforward formulation of the super variant of twistor lift . One should only replace the super-embedding space and super-spinors with super-twistor space and corresponding super-spinors and formulate the theory using 6-D super-Kähler action and super-Dirac equation and the same general prescription for constructing S-matrix. Dimensional reduction should give essentially the 4-D theory apart from the variation of the radius of the twistor space predicting variation of cosmological constant. The size scale of CD would correspond to the size scale of the twistor space for M^4 and for CP_2 the size scale would serve as unit and would not vary.

The first step is the construction of ordinary variant of Kähler action and modified Dirac action for 6-D surfaces in 12-D twistor space.

1. Replace the spinors of H with the spinors of 12-D twistor space and assume only quark chirality. By the bundle property of the twistor space one can express the spinors as tensor products of spinors of the twistor spaces $T(M^4)$ and $T(CP_2)$. One can express the spinors of $T(M^4)$ tensor products of spinors of M^4 - and S^2 spinors locally and spinors of $T(CP_2)$ as tensor products of CP_2 - and S^2 spinors locally. Chirality conditions should reduce the number of 2 spin components for both $T(M^4)$ and $T(CP_2)$ to one so that there are no additional spin degrees of freedom.

   The dimensional reduction can be generalized by identifying the two S^2 fibers for the preferred extremals so that one obtains induced twistor structure. In spinorial sector the dimensional reduction must identify spinorial degrees of freedom of the two S^2s by the proposed chirality conditions also make them non-dynamical. The S^2 spinors covariantly constant in S^2 degrees of freedom.

2. Define the spinor structure of 12-D twistor space, define induced spinor structure at 6-D surfaces defining the twistor space of space-time surface. Define the twistor counterpart of the analog of modified Dirac action using same general formulas as in case of H.

Construct next the super-variant of this structure.

1. Introduce second quark oscillator operators labelled by the points of cognitive representation in 12-D twistor space effectively replacing 6-D surface with its discretization and having quantized quark field q as its continuum counterpart. Replace the coordinates of the 12-D twistor space with super coordinates h_s expressed in terms of quark and anti-quark oscillator operators labelled by points of cognitive representation, and having interpretation as quantized quark field q restricted to the points of representation.

2. Express 6-D Kähler action and Dirac action density in terms of super-coordinates h_s. The local monomials of q appear in h_s and therefore also in the expansion of super-variants of modified gamma matrices defined by 6-D ähler action as contractions of canonical momentum currents of the action density L_K with the gamma matrices of 12-D twistor space. In super-Kähler action also the local composites of q giving rise to currents formed from the local composites of 3-quarks and antiquarks and having interpretation as leptons and anti-leptons occur - leptons would be therefore spartners of squarks.

3. Perform super-expansion also for the induced spinor field q_s in terms of monomials of q. $q_s(q)$ obeys super-Dirac equation non-linear in q. But also q should satisfy super-Dirac action as an analog of quantized quark field and non-linearity indeed forces also q to have has super-expansion. Thus both quark field q and super-quark field q_s both satisfy super-Dirac equation.

   The only possibility is $q_s = q$ stating fixed point property under $q \rightarrow q_s$ having interpretation in terms of quantum criticality fixing the values of the coefficients of various terms in q_s and in the super-coordinate h_s having interpretation as coupling constants. One has quantum criticality and discrete coupling constant evolution with respect to extension of rationals characterizing adelic physics.

4. Super-Dirac action vanishes for its solutions and the exponent of super-action reduces to exponent of super-Kähler action, whose matrix elements between positive and negative energy parts of zero energy states give S-matrix elements.

   Super-Dirac action has however an important function: the derivatives of quark currents appearing in the super-Kähler action can be transformed to a linear strictly local action of super spinor connection ($\partial_\alpha \to A_{\alpha,s}$ effectively). Without this lattice discretization would be needed and cognitive representation would not be enough.

To sum up, the super variants of modified gamma matrices of the 6-surface would satisfy the condition $D_{\alpha,s}\Gamma_s^\alpha = 0$ expressing preferred extremal property and guaranteeing super-hermicity of D_s. q_s would obey super-Dirac equation $D_s q_s = 0$. The self-referential identification $q = q_s$ would express quantum criticality of TGD.

5.7 Could one describe massive particles using 4-D quantum twistors?

The quaternionic generalization of twistors looks almost must. But before this I considered also the possibility that ordinary twistors could be generalized to quantum twistors to describe particle massivation. Quantum twistors could provide space-time level description, which requires 4-D twistors, which cannot be ordinary M^4 twistors. Also the classical 4-momenta, which by QCC would be equal to M^8 momenta, are in general massive so that the ordinary twistor approach cannot work. One cannot of course exclude the possibility that octo-twistors are enough or that M^8_L description is equivalent with space-time description using quantum twistors.

5.7.1 How to define quantum Grassmannian?

The approach to twistor amplitude relies on twistor Grassmann approach [B17, B13, B10, B20, B21, B5] (see `http://tinyurl.com/yxllwcsn`). This approach should be replaced by replacing Grassmannian $GR(K,N) = Gl(n,C)/Gl(n-m,C) \times Gl(m,C)$ with quantum Grassmannian.

naïve approach to the definition of quantum Grassmannian

Quantum Grassmannian is a notion studied in mathematics and the approach of [A68] (see `http://tinyurl.com/y5q6kv6b`) looks reasonably comprehensible even for physicist. I have already earlier tried to understand quantum algebras and their possible role in TGD [K16]. It is however better to start as ignorant physicist and proceed by trial and error and find whether mathematicians have ended up with something similar.

1. Twistor Grassmannian scattering amplitudes involving k negative helicity gluons involve product of $k \times k$ minors of an $k \times n$ matrix C taken in cyclic order. C defines $k \times n$ coordinates for Grassmannian $Gr(k,n)$ of which part is redundant by the analogs of gauge symmetries $Gl(n-m,C) \times Gl(m,C)$. Here n is the number of external gluons and k the number of negative helicity gluons. The $k \times k$ determinants taken in cyclic order appear in the integrand over Grassmannian. Also the quantum variants of these determinants and integral over quantum Grassmannian should be well-defined and residue calculus gives hopes for achieving this.
2. One should define quantum Grassmannian as algebra according to my physicist's understanding algebra can be defined by starting from a free algebra generated by a set of elements - now the matrix elements of quantum matrix. One poses on these elements relations to get the algebra considered. What could these conditions be in the recent case.
3. A natural condition is that the definition allows induction in the sense that its restriction to quantum sub-matrices is consistent with the general definition of $k \times n$ quantum matrices. In particular, one can identify the columns and rows of quantum matrices as instances of quantum vectors.
4. How to generalize from 2×2 case to $k \times n$ case? The commutation relations for neighboring elements of rows and columns are fixed by induction. In 4×4 corresponding to M^4 twistors one would obtain for $(a_1, ..., a_4)$. $a_i a_{i+1} = q a_{i+1} a_i$ cyclically ($k = 1$ follows $k = 4$).

   What about commutations of a_i and a_{i+k}, $k > 1$. Is there need to say anything about these commutators? In twistor Grassmann approach only connected $k \times k$ minors in cyclic order appear. Without additional relations the algebra might be too large. One could argue that the simplest option is that one has $a_i a_{i+k} = q a_{i+k} a_i$ for k odd $a_i a_{i+k} = q^{-1} a_{i+k} a_i$ for k even. This is required from the consistency with cyclicity. These conditions would allow to define also sub-determinants, which do not correspond to connected $k \times k$ squares by moving the elements to a a connected patch by permutations of rows and columns.

5. What about elements along diagonal? The induction from 2×2 would require the commutativity of elements along right-left diagonals. Only commutativity of the elements along left-right diagonal be modified. Or is the commutativity lost only along directions parallel to left-right diagonal? The problem is that the left-right and right-left directions are transformed to each other in odd permutations. This would suggest that only even permutations are allowed in the definition of determinant
6. Could one proceed inductively and require that one obtains the algebra for 2×2 matrices for all 2×2 minors? Does this apply to all 2×2 minors or only to connected 2×2 minors with cyclic ordering of rows and columns so that top and bottom row are nearest neighbors as also right and left column. Also in the definition of 3×3 determinant only the connected developed along the top row or left column only 2×2 determinants involving nearest neighbor matrix elements appear. This generalizes to $k \times k$ case.

It is time to check how wrong the naïve intuition has been. Consider 2×2 matrices as simple example. In this case this gives only 1 condition ($ad - bc = -da + cb$) corresponding to the permutation of rows or columns. Stronger condition suggested by higher-D case would be $ad = da$ and $bc = cb$. The definition of 2×2 in [A68] however gives for quantum 2-matrices $(a, b; c, d)$ the conditions

$$\begin{array}{ll} ac = qca \ , & bd = qda \ , \\ ab = qba \ , cd = qdc \ , & \\ bc = cb \ , & ad - da = (q - q^{-1})bc \ . \end{array} \tag{5.7.1}$$

The commutativity along left-right diagonal is however lost for $q \neq 1$ so that quantum determinant depends on what row or column is used to expand it. The modification of the commutation relations along rows and columns is what one might expect and wants in order to achieve non-commutativity of twistor components making possible massivation in M^4 sense.

The limit $q \rightarrow 1$ corresponds to non-trivial algebra in general and would correspond to $\beta = 4$ for inclusions of HFFs expected to give representations of Kac-Moody algebras. At this limit only massless particles in 4-D sense are allowed. This suggests that the reduction of Kac-Moody algebras to quantum groups corresponds to symmetry breaking associated with massivation in 4-D sense.

Mathematical definition of quantum Grassmannian

It would seem that the proposed approach is reasonable. The article [A95] (see `http://tinyurl.com/yycflgrd`) proposing a definition of quantum determinant explains also the basic interpretation of what the non-commutativity of elements of quantum matrices does mean.

1. The first observation is that the commutation of the elements of quantum matrix corresponds to braiding rather than permutation and this operation is represented by R-matrix. The formula for the action of braiding is

$$R^{ab}_{cd} t^c_e t^d_f = t^a_d t^b_c R^{cd}_{ef} \ . \tag{5.7.2}$$

Here R-matrix is a solution of Yang-Baxter equaion and characterizes completely the commutation relations between the elements of quantum matrix. The action of braiding is obtained by applying the inverse of R-matrix from left to the equation. By iterating the braidings of nearest neighbors one can deduce what happens in the braiding exchanging quantum matrix elements which are not nearest neighbors. What is nice that the R-matrix would fix the quantum algebra, in particular quantum Grassmannian completely.
2. In the article the notion of quantum determinant is discussed and usually the definition of quantum determinant involves also the introduction of metric g^{ab} allowing the raising of the indices of the permutation symbol. One obtains formulas relating metric and R-matrix and restricting the choice of the metric. Note however that if ordinary permutation symbol is used there is no need to introduce the metric.

The definition quantum Grassmannian proposed does not involve hermitian conjugates of the matrices involved. One can define the elements of Grassmannian and Grassmannian residue integrals without reference to complex conjugation: could one do without hermitian conjugates? On the other hand, Grassmannians have complex structure and Kähler structure: could this require hermitian conjugates and commutation relations for these?

5.7.2 Two views about quantum determinant

If one wants to define quantum matrices in $Gr(k,n)$ so that quantal twistor-Grassmann amplitudes make sense, the first challenge is to generalize the notion of $k \times k$ determinant.

One can consider two approaches concerning the definition of quantum determinant.

1. The first guess is that determinant should not depend on the ordering of rows or columns apart from the standard sign factor. This option fails unless one modifies the definition of permutation symbol.
2. The alternative view is that permutation symbol is ordinary and there is dependence on the row or column with respect to which one develops. This dependence would however disappear in the scattering amplitudes. If the poles and corresponding residues associated with the $k \times k$-minors of the twistor amplitude remain invariant under the permutation, this is not a problem. In other words, the scattering amplitudes are invariant under braid group. This is what twistor Grassmann approach implies and also TGD predict.

For the first option quantum determinant would be braiding invariant. The standard definition of quantum determinant is discussed in detail in [A95] (see `http://tinyurl.com/yycflgrd`).

1. The commutation of the elements of quantum matrix corresponds to braiding rather than permutation and as found, this operation is represented by R-matrix.
2. Quantum determinant would change only by sign under the braidings of neighboring rows and columns. The braiding for the elements of quantum matrix would compensate the braiding for quantum permutation symbol. Permutation symbol is assumed to be q-antisymmetric under braiding of any adjacent indices. This requires that permutation $i_k \leftrightarrow i_{k+1}$ regarded as braiding gives a contraction of quantum permutation symbol $\epsilon_{i_1,\ldots 1_k}$ with $R^{ij}_{i_k i_{k+1}}$ plus scaling by some normalization factor λ besides the change of sign.

$$\epsilon_{a_1 \ldots a_k a_{k+1} \ldots a_n} = -\lambda \epsilon_{a_1 \ldots ij \ldots a_n} R^{ji}_{a_k a_{k+1}} \quad . \tag{5.7.3}$$

   The value of λ can be calculated.

3. The calculation however leads to the result that quantum determinant $\mathcal{D}$ satisfies $\mathcal{D}^2 = 1$! If the result generalizes for sub-determinants defined by $k \times k$-minors (, which need not be the case) would have determinants satisfying $\mathcal{D}^2 = 1$, and the idea about vanishing of $k \times k$-minor essential for getting non-trivial twistor scattering amplitude as residue would not make sense.

It seems that the braiding invariant definition of quantum determinant, which of course involves technical assumptions) is too restrictive. Does this mean that the usual definition requiring development with respect to preferred row is the physically acceptable option? This makes sense if only the integral but not integrand is invariant under braidings. Braiding symmetry would be analogous to gauge invariance.

5.7.3 How to understand the Grassmannian integrals defining the scattering amplitudes?

The beauty of the twistor Grassmannian approach is that the residue integrals over quantum $Gr(k,n)$ would reduce to sum over poles (or possibly integrals over higher-D poles). Could residue calculus provide a manner to integrate q-number valued functions of q-numbers? What would be the minimal assumptions allowing to obtain scattering amplitudes as c-numbers?

Consider first what the integrand to be replaced with its quantum version looks like.

1. Twistor scattering amplitudes involve also momentum conserving delta function expressible as $\delta(\lambda_a\tilde{\lambda}^a)$. This sum and - as it seems - also the summands should be c-numbers - in other words one has eigenstates of the operators defining the summands.
2. By introducing Grassmannian space $Gr(k,n)$ with coordinates $C_{\alpha,i}$ (see `http://tinyurl.com/yxllwcsn`), one can linearize $\delta(\lambda_a\tilde{\lambda}^a)$ to a product of delta functions $\delta(C\cdot Z) = \delta(C\cdot\tilde{\lambda})\times\delta(C^{\perp}\cdot\lambda)$ (I have not written the delta function is Grassmann parameters related to super coordinates). Z is the n-vector formed by the twistors associated with incoming particles.

   The $4\times k$ components of $C_{\alpha,k}Z^k$ should be c-numbers at least when they vanish. One should define quantum twistors and quantum Grassmannian and pose the constraints on the poles.

How to achieve the goal? Before proceeding it is good to recall the notion of non-commutative geometry (see `http://tinyurl.com/yxrcr8xv`). Ordinary Riemann geometry can be obtained from exterior algebra bundle, call it E. The Hilbert space of square integrable sections in E carries a representation of the space of continuous functions $C(M)$ by multiplication operators. Besides this there is unbounded differential operator D, which so called signature operator and defined in terms of exterior derivative and its dual: $D = d + d^*$. This spectral triple of algebra, Hilbert space, and operator D allows to deduce the Riemann geometry.

The dream is that one could assign to non-commutative algebras non-commutative spaces using this spectral triple. The standard q-p quantization is example of this: one obtains now Lagrange manifolds as ordinary commutative manifolds.

Consider now the situation in the case of quantum Grassmannian.

1. In the recent case the points defining the poles of the function - it might be that the eventual poles are not a set of discrete points but a higher-dimensional object - would form the commutative part of non-commutative quantum space. In this space the product of quantum minors would become ordinary number as also the argument $C\cdot Z$ of the delta function. This commutative sub-space would correspond to a space in which maximum number of minors vanish and residues reduce to c-numbers.

   Thus poles of the integrand of twistor amplitude would correspond to eigenstates for some $k\times k$ minors of Grassmannian with a vanishing eigenvalue. The residue at the pole at given step in the recursion pole by pole need not be c-number but the further residue integrals should eventually lead to a c-number or c-number valued integrand.
2. The most general option would be that the conditions hold true only in the sense that some $k\times k$ minors for $k\geq 2$ are c-numbers and have a vanishing eigenvalue but that smaller minors need not have this property. Also $C_{\alpha,k}Z^k$ should be c-numbers and vanish. Residue calculus would give rise to lower-D integrals in step-wise manner.

   The simplest and most general option is that one can speak only about eigenvalues of $k\times k$ minors. At pole it is enough to have one minor for which eigenvalue vanishes whereas other minors could remain quantal. In the final reduction the product of all non-vanishing $k\times k$ minors appearing in cyclic order in the integrand should have a well-defined c-number as eigenvalue. Does this allow the appearance of only cyclic minors.

   A stronger condition would be that all non-vanishing minors reduce to their eigenvalues. Could it be that only the n cyclic minors can commute simultaneously and serve as analogs of q-coordinates in phase space? The complex dimension of $G_C(n,k)$ is $d=(n-k)k$. If the space spaced by minors corresponds to Lagrangian manifold with real dimension not larger than d, one has $k\leq d=(n-k)k$. This gives $k\leq n/2(1+\sqrt{1-2/n})$ For $k=2$ this gives $k\leq n/2$. For $n\to\infty$ one has $k\leq n/2+1$. For $k>n/2$ one can change the roles of positive and negative helicities. It has been found that in certain sense the Grassmannian contributing to the twistor amplitude is positive.

   The notion of positivity found to characterize the part of Grassmannian contributing to the residue integral and also the minors and the argument of delta function [B19](see `http://tinyurl.com/yd9tf2ya`) would suggest that it is also real sub-space in some sense and this finding supports this picture.

   The delta function constraint forcing $C\cdot Z$ to zero must also make sense. $C\cdot Z$ defines $k\times 6$ matrix and also now one must consider eigenvalues of $C\cdot Z$. Positivity suggest reality also now. Z adds $4\times n$ degrees of freedom and the number $6\times k$ of additional conditions is smaller than $4\times n$. $6k\leq 4\times n$ combined with $k\leq n/2$ gives $k\leq n/2$ so that the conditions seems to be consistent.

3. The c-number property for the cyclic minors could define the analog of Lagrangian manifold for the phase space or Kähler manifold. One can of course ask, whether Kähler structure of $Gr(k,n)$ could generalize to quantum context and give the integration region as a sub-manifold of Lagrangian manifold of $Gr(k,n)$ and whether the twistor amplitudes could reduce to integral over sub-manifold of Lagrangian manifold of ordinary $Gr(k,n)$.

To sum up, I have hitherto thought that TGD allows to get rid of the idea of quantization of coordinates. Now I have encountered this idea from totally unexpected perspective in an attempt to understand how 8-D masslessness and its twistor description could relate to 4-D one. Grassmannians are however very simple and symmetric objects and have natural coordinates as $k\times n$ matrices interpretable as quantum matrices. The notion of quantum group could find very concrete application as a solution to the basic problem of the standard twistor approach. Therefore one can consider the possibility that they have quantum counterparts and at least the residue integrals reducing to c-numbers make sense for quantum Grassmannians in algebraic sense.

Chapter 6

Does TGD Predict Spectrum of Planck Constants?

6.1 Introduction

The quantization of Planck constant has been the basic them of TGD since 2005 and the perspective in the earlier version of this chapter reflected the situation for about year and one half after the basic idea stimulated by the finding of Nottale [E13] that planetary orbits could be seen as Bohr orbits with enormous value of Planck constant given by $\hbar_{gr} = GM_1M_2/v_0$, $v_0 \simeq 2^{-11}$ for the inner planets. The general form of $\hbar_{gr}$ is dictated by Equivalence Principle. This inspired the ideas that quantization is due to a condensation of ordinary matter around dark matter concentrated near Bohr orbits and that dark matter is in macroscopic quantum phase in astrophysical scales.

The second crucial empirical input were the anomalies associated with living matter. Mention only the effects of ELF radiation at EEG frequencies on vertebrate brain and anomalous behavior of the ionic currents through cell membrane. If the value of Planck constant is large, the energy of EEG photons is above thermal energy and one can understand the effects on both physiology and behavior. If ionic currents through cell membrane have large Planck constant the scale of quantum coherence is large and one can understand the observed low dissipation in terms of quantum coherence. This approach led to the formula $h_{eff} = n \times h$. Rather recently (2014) it became clear that for microscopic systems the identification $h_{eff} = h_{gr}$ makes sense and predicts universal energy spectrum for cyclotron energies of dark photons identifiable as energy spectrum of bio-photons in TGD inspired quantum biology.

6.1.1 Evolution Of Mathematical Ideas

The original formulation for the hierarchy of Planck constants was in terms of $h_{eff}/h = n$-fold singular coverings of the embedding space $H = M^4 \times CP_2$. Later it turned out that there is no need to postulate these covering spaces although they are a nice auxiliary tool allowing to understand why the phase of matter with different values of n behave like dark matter relative to each other: they are simply at different pages of the book-like structure formed by the covering spaces.

Few years ago it became clear that the hierarchy of Planck constants could be only effective but have the same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $h_{eff} = n \times h$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. This reduces the understanding of the effective hierarchy of Planck constants to quantum variant of multi-furcations for the dynamics of preferred extremals of Kähler action. The number of branches of multi-furcation defines the integer n in $\hbar_{eff} = n\hbar$.

One of the latest steps in the progress was the realization that the hierarchy of Planck constants can be understood in terms of quantum criticality of TGD Universe postulated from the beginning as a way to obtain a unique theory. In accordance with what is known about 2-D critical systems, quantum criticality should correspond to a generalization of conformal invariance. TGD indeed predicts several analogs of super-conformal algebras: so called super-symplectic algebra acting in $\delta M^4_\pm \times CP_2$ should act as isometries of WCW and its generators are labeled by conformal weights. Light-cone boundary $\delta M^4_\pm$ has an extension of conformal symmetries as conformal symmetries and an algebra isomorphic to the ordinary conformal algebra acts as its isometries. The light-like orbits of partonic 2-surfaces allow similar algebra of conformal symmetries and string world sheets and partonic 2-surfaces allow conformal symmetries.

The proposal is that super-symplectic algebra (at least it) defines a hierarchy of broken super-conformal gauge symmetries in the sense that the sub-algebra for which the conformal weights are n-ples of those for the entire algebra acts as gauge conformal symmetries. $n = h_{eff}/h$ giving a connection to the hierarchy of Planck constants would hold true. These sub-algebras are isomorphic to the full algebra and thus form a fractal hierarchy. One has infinite number of hierarchies of broken conformal symmetries defined by the sequences $n(i+1) = m_i \times n(i)$. In the phase transition increasing n conformal gauge symmetry is reduced and some gauge degrees of freedom transform to physical ones and criticality is reduced so that the transition takes place spontaneously. TGD Universe is like a ball at the top of hill at the top of hill at....

This view has far reaching implication for the understanding of living matter and leads to deep connections between different key ideas of TGD. The hierarchy has also a purely number theoretical interpretation in terms of hierarchy of algebraic extensions of rationals appearing naturally in the adelic formulation of quantum TGD. $n = h_{eff}/h$ would naturally correspond to an integer, which is product of so called ramified primes (rational primes for which the decomposition to primes of extension contains higher powers of these primes).

In this framework it becomes obvious that - instead of coverings of embedding space postulated in the original formulation - one has space-time surfaces representable as singular n-fold coverings. The non-determinism of Kähler action - key element of criticality - would be the basic reason for the appearance of singular coverings: two 3-surfaces at the opposite boundaries of CD are connected by n-sheeted space-time surfaces for which the sheets co-incide at the boundaries. Criticality must be accompanied by 4-D variant of conformal gauge invariance already described so that these space-time surfaces are replaced by conformal gauge equivalence classes.

These coverings are highly analogous to the covering space associated with the analytic function $w(z) = z^{1/n}$. If one uses w as a variable, the ordinary conformal symmetries generated by functions of z indeed correspond to the algebra generated by w^n and the sheets of covering correspond to conformal gauge equivalence classes not transformed to each other by conformal transformations.

6.1.2 The Evolution Of Physical Ideas

The evolution of physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

1. The basic idea was that ordinary matter condenses around dark matter which is a phase of matter characterized by non-standard value of Planck constant.
2. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase [K85] . If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E13] can be understood. Dark matter would resemble to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with minimum size of order Schwarstchild radius r_S of order scaled up Planck length: $r_S \sim \sqrt{\hbar G}$. Black hole entropy being inversely proportional to $\hbar$ is predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

3. Darkness is a relative concept and due to the fact that particles at different pages of book cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to different page of book are however possible and change Planck constant so that particle exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. This allows to conclude that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K116] , [I18] .

4. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially shocking outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L1, K116] , [L1] .

6.1.3 Basic Physical Picture As It Is Now

The basic phenomenological rules are simple and remained roughly the same during years.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Embedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K117].

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

   The hierarchy of Planck constants suggests also a new interpretation for FQHE (fractional quantum Hall effect) [K85] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of embedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

   In astrophysics and cosmology the implications are even more dramatic. It was [E13] who first introduced the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This changes the view about gravitons and suggests that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

3. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

4. The interpretation of the hierarchy of Planck constants as labels for quantum critical systems is especially powerful in TGD inspired quantum biology and consciousness theory. The increase of Planck constant by integer factor occurs spontaneously and means an increase of complexity and sensory and cognitive resolution - in other words evolution. Living matter is however fighting to stay at the existing level of criticality. The reason is that the changes involves state function reduction at the opposite boundary of CD and means death of self followed by re-incarnation.

   Negentropy Maximization Principle [K72] saves the system from this fate if it is able to generate negentropic entanglement by some other means. Metabolic energy suggested already earlier to be a carrier of negentropic entanglement makes this possible. Also other metabolites can carry negentropy. Therefore living systems are eating each other to satisfy the demands of NMP! Why this non-sensical looking Karma's cycle? The sub-systems of self defining sub-selves (mental images) are dying and re-incarnating and generating negentropy: self is a gardener and sub-selves are the fruit trees and the longer self lives, the more fruits are produced. Hence this process, which Buddhist would call attachment to ego is the ways to generate what I have called "Akashic records". Everything has its purpose.

In this chapter I try to summarize the evolution of the ideas related to Planck constant. I have worked hardly to achieve internal consistency but the old theory layers are there and might cause confusion.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

6.2 Experimental Input

In this section basic experimental inputs suggesting a hierarchy of Planck constants and the identification of dark matter as phases with non-standard value of Planck constant are discussed.

6.2.1 Hints For The Existence Of Large $\hbar$ Phases

Quantum classical correspondence suggests the identification of space-time sheets identifiable as quantum coherence regions. Since they can have arbitrarily large sizes, phases with arbitrarily large quantum coherence lengths and arbitrarily long de-coherence times seem to be possible in TGD Universe. In standard physics context this seems highly implausible. If Planck constant can have arbitrarily large values, the situation changes since Compton lengths and other quantum scales are proportional to $\hbar$. Dark matter is excellent candidate for large $\hbar$ phases.

The expression for $\hbar_{gr}$ in the model explaining the Bohr orbits for planets is of form $\hbar_{gr} = GM_1M_2/v_0$ [K100]. This suggests that the interaction is associated with some kind of interface between the systems, perhaps join along boundaries bonds/flux tubes connecting the space-time sheets associated with systems possessing gravitational masses M_1 and M_2. Also a large space-time sheet carrying the mutual classical gravitational field could be in question. This argument generalizes to the case $\hbar/\hbar_0 = Q_1Q_2\alpha/v_0$ in case of generic phase transition to a strongly interacting phase with α describing gauge coupling strength.

There exist indeed some experimental indications for the existence of phases with a large $\hbar$.

1. With inspiration coming from the finding of Nottale [E13] I have proposed an explanation of dark matter as a macroscopic quantum phase with a large value of $\hbar$ [K100]. Any interaction, if sufficiently strong, can lead to this kind of phase. The increase of $\hbar$ would make the fine structure constant α in question small and guarantee the convergence of perturbation series.
2. Living matter could represent a basic example of large $\hbar$ phase [K43, K12]. Even ordinary condensed matter could be "partially dark" in many-sheeted space-time [K45]. In fact, the realization of hierarchy of Planck constants leads to a considerably weaker notion of darkness stating that only the interaction vertices involving particles with different values of Planck constant are impossible and that the notion of darkness is relative notion. For instance, classical interactions and photon exchanges involving a phase transition changing the value of $\hbar$ of photon are possible in this framework.
3. There is claim about a detection in RHIC (Relativistic Heavy Ion Collider in Brookhaven) of states behaving in some respects like mini black holes [C17]. These states could have explanation as color flux tubes at Hagedorn temperature forming a highly tangled state and identifiable as stringy black holes of strong gravitation. The strings would carry a quantum coherent color glass condensate, and would be characterized by a large value of $\hbar$ naturally resulting in confinement phase with a large value of α_s [K101] . The progress in hadronic mass calculations led to a concrete model of color glass condensate of single hadron as many-particle state of super-symplectic gluons [K79, K73] - something completely new from the point of QCD - responsible for non-perturbative aspects of hadron physics. In RHIC events these color glass condensate would fuse to single large condensate. This condensate would be present also in ordinary black-holes and the blackness of black-hole would be darkness.
4. I have also discussed a model for cold fusion based on the assumption that nucleons can be in large $\hbar$ phase. In this case the relevant strong interaction strength is $Q_1Q_2\alpha_{em}$ for two nucleon clusters inside nucleus which can increase $\hbar$ so large that the Compton length of protons becomes of order atomic size and nuclear protons form a macroscopic quantum phase [K45, K43].

6.2.2 Quantum Coherent Dark Matter And $\hbar$

The argument based on gigantic value of $\hbar_{gr}$ explaining darkness of dark mater is attractive but one should be very cautious.

Consider first ordinary QED$e = \sqrt{\alpha 4\pi\hbar}$ appears in vertices so that perturbation expansion in powers of $\sqrt{\hbar}$ basically. This would suggest that large $\hbar$ leads to large effects. All predictions are however in powers of alpha and large $\hbar$ means small higher order corrections. What happens can be understood on basis of dimensional analysis. For instance, cross sections are proportional to $(\hbar/m)^2$, where m is the relevant mass and the remaining factor depends on $\alpha = e^2/(4\pi\hbar)$ only. In the more general case tree amplitudes with n vertices are proportional to e^n and thus to $\hbar^{n/2}$ and loop corrections give only powers of α which get smaller when $\hbar$ increases. This must relate to the powers of $1/\hbar$ from the integration measure associated with the momentum loop integrals affected by the change of α.

Consider now the effects of the scaling of $\hbar$. The scaling of Compton lengths and other quantum kinematical parameters is the most obvious effect. An obvious effect is due to the change of $\hbar$ in the commutation relations and in the change of unit of various quantum numbers. In particular, the right hand side of oscillator operator commutation and anti-commutation relations is scaled. A further effect is due to the scaling of the eigenvalues of the Kähler-Dirac operator $\hbar\Gamma^\alpha D_\alpha$.

The exponent $exp(K)$ of Kähler function K defining perturbation series in WCW degrees of freedom is proportional to $1/g_K^2$ and does not depend on $\hbar$ at all if there is only single Planck constant. The propagator is proportional to g_K^2. This can be achieved also in QED by absorbing e from vertices to e^2 in photon propagator. Hence it would seem that the dependence on α_K (and $\hbar$) must come from vertices which indeed involve Jones inclusions of the II_1 factors of the incoming and outgoing lines.

This however suggests that the dependence of the scattering amplitudes on $\hbar$ is purely kinematical so that all higher radiative corrections would be absent. This seems to leave only one option: the scale factors of covariant CD and CP_2 metrics can vary and might have discrete spectrum of values.

1. The invariance of Kähler action with respect to overall scaling of metric however allows to keep CP_2 metric fixed and consider only a spectrum for the scale factors of M^4 metric.
2. The first guess motivated by Schrödinger equation is that the scaling factor of covariant CD metric corresponds the ratio $r^2 = (\hbar/\hbar_0)^2$. This would mean that the value of Kähler action depends on r^2. The scaling of M^4 coordinate by r the metric reduces to the standard form but if causal diamonds with quantized temporal distance between their tips are the basic building blocks of WCW geometry as zero energy ontology requires, this scaling of $\hbar$ scales the size of CD by r so that genuine effect results since M^4 scalings are not symmetries of Kähler action.
3. In this picture r would code for radiative corrections to Kähler function and thus space-time physics. Even in the case that the radiative corrections to WCW functional integral vanish, as suggested by quantum criticality, they would be actually taken into account.

This kind of dynamics is not consistent with the original view about embedding space and forces to generalize the notion of embedding spaces since it is clear that particles with different Planck constants cannot appear in the same vertex of Feynman diagram. Somehow different values of Planck constant must be analogous to different pages of book having almost copies of embedding space as pages. A possible resolution of the problem cames from the realization that the fundamental structure might be the inclusion hierarchy of number theoretical Clifford algebras from which entire TGD could emerge including generalization of the embedding space concept.

6.2.3 The Phase Transition Changing The Value Of Planck Constant As A Transition To Non-Perturbative Phase

A phase transition increasing $\hbar$ as a transition guaranteeing the convergence of perturbation theory

The general vision is that a phase transition increasing $\hbar$ occurs when perturbation theory ceases to converge. Very roughly, this would occur when the parameter $x = Q_1Q_2\alpha$ becomes larger than one. The net quantum numbers for "spontaneously magnetized" regions provide new natural units for quantum numbers. The assumption that standard quantization rules prevail poses very strong restrictions on allowed physical states and selects a subspace of the original configuration space. One can of course, consider the possibility of giving up these rules at least partially in which case a spectrum of fractionally charged anyon like states would result with confinement guaranteed by the fractionization of charges.

The necessity of large $\hbar$ phases has been actually highly suggestive since the first days of quantum mechanics. The classical looking behavior of macroscopic quantum systems remains still a poorly understood problem and large $\hbar$ phases provide a natural solution of the problem.

In TGD framework quantum coherence regions correspond to space-time sheets. Since their sizes are arbitrarily large the conclusion is that macroscopic and macro-temporal quantum coherence are possible in all scales. Standard quantum theory definitely fails to predict this and the conclusion is that large $\hbar$ phases for which quantum length and time scales are proportional to $\hbar$ and long are needed.

Somewhat paradoxically, large $\hbar$ phases explain the effective classical behavior in long length and time scales. Quantum perturbation theory is an expansion in terms of gauge coupling strengths inversely proportional to $\hbar$ and thus at the limit of large $\hbar$ classical approximation becomes exact. Also the Coulomb contribution to the binding energies of atoms vanishes at this limit. The fact that we experience world as a classical only tells that large $\hbar$ phase is essential for our sensory perception. Of course, this is not the whole story and the full explanation requires a detailed anatomy of quantum jump.

The criterion for the occurrence of the phase transition increasing the value of $\hbar$

In the case of planetary orbits the large value of $\hbar_{gr} = 2GM/v_0$ makes possible to apply Bohr quantization to planetary orbits. This leads to a more general idea that the phase transition increasing $\hbar$ occurs when the system consisting of interacting units with charges Q_i becomes non-perturbative in the sense that the perturbation series in the coupling strength $\alpha Q_i Q_j$, where α is the appropriate coupling strength and $Q_i Q_j$ represents the maximum value for products of gauge charges, ceases to converge. Thus Mother Nature would resolve the problems of theoretician. A primitive formulation for this criterion is the condition $\alpha Q_i Q_j \geq 1$.

The first working hypothesis was the existence of dark matter hierarchies with $\hbar = \lambda^k \hbar_0$, $k = 0, 1, ...$, $\lambda = n/v_0$ or $\lambda = 1/nv_0$, $v_0 \simeq 2^{-11}$. This rule turned out to be quite too specific. The mathematically plausible formulation predicts that in principle any rational value for $r = \hbar(M^4)/\hbar(CP_2)$ is possible but there are certain number theoretically preferred values of r such as those coming powers of 2.

6.3 A Generalization of the Notion of Embedding Space as a Realization of the Hierarchy of Planck Constants

In the following the basic ideas concerning the realization of the hierarchy of Planck constants are summarized and after that a summary about generalization of the embedding space is given. In [K85] the important delicacies associated with the Kähler structure of generalized embedding space are discussed. The background for the recent vision is quite different from that for half decade ago. Zero energy ontology and the notion of causal diamond, number theoretic compactification leading to the precise identification of number theoretic braids, the realization of number theoretic universality, and the understanding of the quantum dynamics at the level of Kähler-Dirac action fix to a high degree the vision about generalized embedding space.

6.3.1 Basic Ideas

The first key idea in the geometric realization of the hierarchy of Planck constants emerges from the study of Schrödinger equation and states that Planck constant appears a scaling factor of M^4 metric. Second key idea is the connection with Jones inclusions inspiring an explicit formula for Planck constants. For a long time this idea remained heuristic must-be-true feeling but the recent view about quantum TGD provide a justification for it.

Scaling of Planck constant and scalings of CD and CP_2 metrics

The key property of Schrödinger equation is that kinetic energy term depends on $\hbar$ whereas the potential energy term has no dependence on it. This makes the scaling of $\hbar$ a non-trivial transformation. If the contravariant metric scales as $r = \hbar/\hbar_0$ the effect of scaling of Planck constant is realized at the level of embedding space geometry provided it is such that it is possible to compare the regions of generalized embedding space having different value of Planck constant.

In the case of Dirac equation same conclusion applies and corresponds to the minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$. Consider next the situation in TGD framework.

1. The minimal substitution $p - eA \rightarrow i\hbar\nabla - eA$ does not make sense in the case of CP_2 Dirac operator since, by the non-triviality of spinor connection, one cannot choose the value of $\hbar$ freely. In fact, spinor connection of CP_2 is defined in such a way that spinor connection corresponds to the quantity $\hbar eQA$, where denotes A gauge potential, and there is no natural manner to separate $\hbar e$ from it.

2. The contravariant CD metric scales like $\hbar^2$. In the case of Dirac operator in $M^4 \times CP_2$ one can assign separate Planck constants to Poincare and color algebras and the scalings of CD and CP_2 metrics induce scalings of corresponding values of $\hbar^2$. As far as Kähler action is considered, CP_2 metric could be always thought of being scaled to its standard form.

3. Dirac equation gives the eigenvalues of wave vector squared $k^2 = k^i k_i$ rather than four-momentum squared $p^2 = p^i p_i$ in CD degrees of freedom and its analog in CP_2 degrees of

freedom. The values of k^2 are proportional to $1/r^2$ so that p^2 does not depend on it for $p^i = \hbar k^i$: analogous conclusion applies in CP_2 degrees of freedom. This gives rise to the invariance of mass squared and the desired scaling of wave vector when $\hbar$ changes.

This consideration generalizes to the case of the induced gamma matrices and induced metric in X^4, Kähler-Dirac operator, and Kähler action which carry dynamical information about the ratio $r = \hbar_{eff}/\hbar_0$.

Kähler function codes for a perturbative expansion in powers of $\hbar(CD)/\hbar(CP_2)$

Suppose that one accepts that the spectrum of CD *resp.* CP_2 Planck constants is accompanied by a hierarchy of overall scalings of covariant CD (causal diamond) metric by $(\hbar(M^4)/\hbar_0)^2$ and CP_2 metric by $(\hbar(CP_2)/\hbar_0)^2$ followed by overall scaling by $r^2 = (\hbar_0/\hbar(CP_2))^2$ so that CP_2 metric suffers no scaling and difficulties with isometric gluing procedure of sectors are avoided.

The first implication of this picture is that the Kähler-Dirac operator determined by the induced metric and spinor structure depends on r in a highly nonlinear manner but there is no dependence on the overall scaling of the H metric. This in turn implies that the fermionic oscillator algebra used to define WCW spinor structure and metric depends on the value of r. Same is true also for Kähler action and configuration space Kähler function. Hence Kähler function is analogous to an effective action expressible as infinite series in powers of r.

This interpretation allows to overcome the paradox caused by the hypothesis that loop corrections to the functional integral over WCW defined by the exponent of Kähler function serving as vacuum functional vanish so that tree approximation is exact. This would imply that all higher order corrections usually interpreted in terms of perturbative series in powers of $1/\hbar$ vanish. The paradox would result from the fact that scattering amplitudes would not receive higher order corrections and classical approximation would be exact.

The dependence of both states created by Super Kac-Moody algebra and the Kähler function and corresponding propagator identifiable as contravariant WCW metric would mean that the expressions for scattering amplitudes indeed allow an expression in powers of r. What is so remarkable is that the TGD approach would be non-perturbative from the beginning and "semi-classical" approximation, which might be actually exact, automatically would give a full expansion in powers of r. This is in a sharp contrast to the usual quantization approach.

Jones inclusions and hierarchy of Planck constants

From the beginning it was clear that Jones inclusions of hyper-finite factors of type II_1 are somehow related to the hierarchy of Planck constants. The basic motivation for this belief has been that WCW Clifford algebra provides a canonical example of hyper-finite factor of type II_1 and that Jones inclusion of these Clifford algebras is excellent candidate for a first principle description of finite measurement resolution.

Consider the inclusion $\mathcal{N} \subset \mathcal{M}$ of hyper-finite factors of type II_1 [K125]. A deep result is that one can express $\mathcal{M}$ as $\mathcal{N} : \mathcal{M}$-dimensional module over N with fractal dimension $\mathcal{N} : \mathcal{M} = B_n$. $\sqrt{b_n}$ represents the dimension of a space of spinor space renormalized from the value 2 corresponding to $n = \infty$ down to $\sqrt{b_n} = 2cos(\pi/n)$ varying thus in the range $[1, 2]$. B_n in turn would represent the dimension of the corresponding Clifford algebra. The interpretation is that finite measurement resolution introduces correlations between components of quantum spinor implying effective reduction of the dimension of quantum spinors providing a description of the factor space $\mathcal{N}/\mathcal{M}$.

This would suggest that somehow the hierarchy of Planck constants must represent finite measurement resolution and since phase factors coming as roots of unity are naturally associated with Jones inclusions the natural guess was that angular resolution and coupling constant evolution associated with it is in question. This picture would suggest that the realization of the hierarchy of Planck constant in terms of a book like structure of generalized embedding space provides also a geometric realization for a hierarchy of Jones inclusions.

The notion of number theoretic braid and realization that the modified Dirac operator has only finite number of generalized eigenmodes -thanks to the vacuum degeneracy of Kähler action-finally led to the understanding how the notion of finite measurement resolution is coded to the Kähler action and the realized in practice by second quantization of induced spinor fields and how these spinor fields endowed with q-anti-commutation relations give rise to a representations of finite-quantum dimensional factor spaces $\mathcal{N}/\mathcal{M}$ associated with the hierarchy of Jones inclusions having generalized embedding space as space-time correlate. This means enormous simplification since infinite-dimensional spinor fields in infinite-dimensional world of classical worlds are replaced with finite-quantum-dimensional spinor fields in discrete points sets provided by number theoretic braids.

The study of a concrete model for Jones inclusions in terms of finite subgroups G of $SU(2)$ defining sub-algebras of infinite-dimensional Clifford algebra as fixed point sub-algebras leads to what looks like a correct track concerning the understanding of quantization of Planck constants.

The ADE diagrams of A_n and D_{2n} characterize cyclic and dihedral groups whereas those of E_6 and E_8 characterize tetrahedral and icosahedral groups. This approach leads to the hypothesis that the scaling factor of Planck constant assignable to Poincare (color) algebra corresponds to the order of the maximal cyclic subgroup of $G_b \subset SU(2)$ ($G_a \subset SL(2,C)$) acting as symmetry of space-time sheet in CP_2 (CD) degrees of freedom. It predicts arbitrarily large CD and CP_2 Planck constants in the case of A_n and D_{2n} under rather general assumptions.

There are two ways for how G_a and G_b can act as symmetries corresponding to G_i coverings and factors spaces. These coverings and factor spaces are singular and associated with spaces $\hat{CD}\backslash M^2$ and $CP_2\backslash S^2_I$, where S^2_I is homologically trivial geodesic sphere of CP_2. The physical interpretation is that M^2 and S^2_I fix preferred quantization axes for energy and angular moment and color quantum numbers so that also a connection with quantum measurement theory emerges.

6.3.2 The Vision

A brief summary of the basic vision behind the generalization of the embedding space concept needed to realize the hierarchy of Planck constants is in order before going to the detailed representation.

1. The hierarchy of Planck constants cannot be realized without generalizing the notions of embedding space and space-time because particles with different values of Planck constant cannot appear in the same interaction vertex. Some kind of book like structure for the generalized embedding space forced also by p-adicization but in different sense is suggestive. Both M^4 and CP_2 factors would have the book like structure so that a Cartesian product of books would be in question.
2. The study of Schrödinger equation suggests that Planck constant corresponds to a scaling factor of CD metric whose value labels different pages of the book. The scaling of M^4 coordinate so that original metric results in CD factor is possible so that the interpretation for scaled up value of $\hbar$ is as scaling of the size of causal diamond CD.
3. The light-like 3-surfaces having their 2-D and light-boundaries of CD are in a key role in the realization of zero energy states, and the infinite-D spaces of light-like 3-surfaces inside scaled variants of CD define the fundamental building brick of WCW (world of classical worlds). Since the scaling of CD does not simply scale space-time surfaces the effect of scaling on classical and quantum dynamics is non-trivial and a coupling constant evolution results and the coding of radiative corrections to the geometry of space-time sheets becomes possible. The basic geometry of CD suggests that the allowed sizes of CD come in the basic sector $\hbar = \hbar_0$ as powers of two. This predicts p-adic length scale hypothesis and lead to number theoretically universal discretized p-adic coupling constant evolution. Since the scaling is accompanied by a formation of singular coverings and factor spaces, different scales are distinguished at the level of topology. p-Adic length scale hierarchy affords similar characterization of length scales in terms of effective topology.
4. The idea that TGD Universe is quantum critical in some sense is one of the key postulates of quantum TGD. The basic ensuing prediction is that Kähler coupling strength is analogous to critical temperature. Quantum criticality in principle fixes the p-adic evolution of various coupling constants also the value of gravitational constant. The exact realization of quantum criticality would be in terms of critical sub-manifolds of M^4 and CP_2 common to all sectors of the generalized embedding space. Quantum criticality of TGD Universe means that the two kinds of number theoretic braids assignable to M^4 and CP_2 projections of the partonic 2-surface belong by the very definition of number theoretic braids to these critical sub-manifolds.

At the boundaries of CD associated with positive and negative energy parts of zero energy state in a given time scale partonic two-surfaces belong to a fixed page of the Big Book whereas light-like 3-surface decomposes to regions corresponding to different values of Planck constant much like matter decomposes to several phases at criticality.

The connection with Jones inclusions was originally a purely heuristic guess, and it took half decade to really understand why and how they are involved. The notion of measurement resolution is the key concept.

1. The key observation is that Jones inclusions are characterized by a finite subgroup $G \subset SU(2)$ and the this group also characterizes the singular covering or factor spaces associated with CD or CP_2 so that the pages of generalized embedding space could indeed serve as correlates for Jones inclusions.
2. The dynamics of Kähler action realizes finite measurement resolution in terms of finite number of modes of the induced spinor field automatically implying cutoffs to the representations of various super-conformal algebras typical for the representations of quantum groups associated with Jones inclusions. The interpretation of the Clifford algebra spanned by the fermionic oscillator operators is as a realization for the concept of the factor space $\mathcal{N}/\mathcal{M}$ of hyper-finite factors of type II_1 identified as the infinite-dimensional Clifford algebra $\mathcal{N}$ of the configuration space and included algebra $\mathcal{M}$ determining the finite measurement resolution for angle measurement in the sense that the action of this algebra on zero energy state has no detectable physical effects. $\mathcal{M}$ takes the role of complex numbers in quantum theory and makes physics non-commutative. The resulting quantum Clifford algebra has anti-commutation relations dictated by the fractionization of fermion number so that unit becomes $r = \hbar/\hbar_0$. $SU(2)$ Lie algebra transforms to its quantum variant corresponding to the quantum phase $q = exp(i2\pi/r)$.
3. G invariance for the elements of the included algebra can be interpreted in terms of finite measurement resolution in the sense that action by G invariant Clifford algebra element has no detectable effects. Quantum groups realize this view about measurement resolution for angle measurement. The G-invariance of the physical states created by fermionic oscillator operators which by definition are not G invariant guarantees that quantum states as a whole have non-fractional quantum numbers so that the leakage between different pages is possible in principle. This hypothesis is consistent with the TGD inspired model of quantum Hall effect [K85].
4. Concerning the formula for Planck constant in terms of the integers n_a and n_b characterizing orders of the maximal cyclic subgroups of groups G_a and G_b defining coverings and factor spaces associated with CD and CP_2 the basic constraint is that the overall scaling of H metric has no effect on physics. What matters is the ratio of Planck constants $r = \hbar(M^4)/\hbar(CP_2)$ appearing as a scaling factor of M^4 metric. This leaves two options if one requires that the Planck constant defines a homomorphism. The model for dark gravitons suggests a unique choice between these two options but one must keep still mind open for the alternative.
5. Jones inclusions appear as two variants corresponding to $\mathcal{N} : \mathcal{M} < 4$ and $\mathcal{N} : \mathcal{M} = 4$. The tentative interpretation is in terms of singular G-factor spaces and G-coverings of M^4 and CP_2 in some sense. The alternative interpretation assigning the inclusions to the two different geodesic spheres of CP_2 would mean asymmetry between M^4 and CP_2 degrees of freedom and is therefore not convincing.
6. The natural question is why the hierarchy of Planck constants is needed. Is it really necessary? Number theoretic Universality suggests that this is the case. One must be able to define the notion of angle -or at least the notion of phase and of trigonometric functions- also in the p-adic context. All that one can achieve naturally is the notion of phase defined as a root of unity and introduced by allowing algebraic extension of p-adic number field by introducing the phase. In the framework of TGD inspired theory of consciousness this inspires a vision about cognitive evolution as the gradual emergence of increasingly complex algebraic extensions of p-adic numbers and involving also the emergence of improved angle resolution expressible in terms of phases $exp(i2\pi/n)$ up to some maximum value of n. The coverings and factor spaces would realize these phases purely geometrically and quantum phases q assignable to

Jones inclusions would realize them algebraically. Besides p-adic coupling constant evolution based on the hierarchy of p-adic length scales there would be coupling constant evolution with respect to $\hbar$ and associated with angular resolution.

6.3.3 Hierarchy Of Planck Constants And The Generalization Of The Notion Of Embedding Space

In the following the recent view about structure of embedding space forced by the quantization of Planck constant is summarized. The question is whether it might be possible in some sense to replace H or its Cartesian factors by their necessarily singular multiple coverings and factor spaces. One can consider two options: either M^4 or the causal diamond CD. The latter one is the more plausible option from the point of view of WCW geometry.

The evolution of physical ideas about hierarchy of Planck constants

The evolution of the physical ideas related to the hierarchy of Planck constants and dark matter as a hierarchy of phases of matter with non-standard value of Planck constants was much faster than the evolution of mathematical ideas and quite a number of applications have been developed during last five years.

1. The starting point was the proposal of Nottale [E13] that the orbits of the 4 inner planets correspond to Bohr orbits with Planck constant $\hbar_{gr} = GMm/v_0$ and outer planets with Planck constant $\hbar_{gr} = 5GMm/v_0$, $v_0/c \simeq 2^{-11}$. The basic proposal [K100] was that ordinary matter condenses around dark matter which is a phase of matter characterized by a non-standard value of Planck constant whose value is gigantic for the space-time sheets mediating gravitational interaction. The interpretation of these space-time sheets could be as magnetic flux quanta or as massless extremals assignable to gravitons.
2. Ordinary particles possibly residing at these space-time sheet have enormous value of Compton length meaning that the density of matter at these space-time sheets must be very slowly varying. The string tension of string like objects implies effective negative pressure characterizing dark energy so that the interpretation in terms of dark energy might make sense [K101]. TGD predicted a one-parameter family of Robertson-Walker cosmologies with critical or overcritical mass density and the "pressure" associated with these cosmologies is negative.
3. The quantization of Planck constant does not make sense unless one modifies the view about standard space-time is. Particles with different Planck constant must belong to different worlds in the sense local interactions of particles with different values of $\hbar$ are not possible. This inspires the idea about the book like structure of the embedding space obtained by gluing almost copies of H together along common "back" and partially labeled by different values of Planck constant.
4. Darkness is a relative notion in this framework and due to the fact that particles at different pages of the book like structure cannot appear in the same vertex of the generalized Feynman diagram. The phase transitions in which partonic 2-surface X^2 during its travel along X_l^3 leaks to another page of book are however possible and change Planck constant. Particle (say photon -) exchanges of this kind allow particles at different pages to interact. The interactions are strongly constrained by charge fractionization and are essentially phase transitions involving many particles. Classical interactions are also possible. It might be that we are actually observing dark matter via classical fields all the time and perhaps have even photographed it [K116].
5. The realization that non-standard values of Planck constant give rise to charge and spin fractionization and anyonization led to the precise identification of the prerequisites of anyonic phase [K85]. If the partonic 2-surface, which can have even astrophysical size, surrounds the tip of CD, the matter at the surface is anyonic and particles are confined at this surface. Dark matter could be confined inside this kind of light-like 3-surfaces around which ordinary matter condenses. If the radii of the basic pieces of these nearly spherical anyonic surfaces - glued to a connected structure by flux tubes mediating gravitational interaction - are given by Bohr rules, the findings of Nottale [E13] can be understood. Dark matter would resemble

to a high degree matter in black holes replaced in TGD framework by light-like partonic 2-surfaces with a minimum size of order Schwartschild radius r_S of order scaled up Planck length $l_{Pl} = \sqrt{\hbar_{gr}G} = GM$. Black hole entropy is inversely proportional to $\hbar$ and predicted to be of order unity so that dramatic modification of the picture about black holes is implied.

6. Perhaps the most fascinating applications are in biology. The anomalous behavior ionic currents through cell membrane (low dissipation, quantal character, no change when the membrane is replaced with artificial one) has a natural explanation in terms of dark supra currents. This leads to a vision about how dark matter and phase transitions changing the value of Planck constant could relate to the basic functions of cell, functioning of DNA and amino-acids, and to the mysteries of bio-catalysis. This leads also a model for EEG interpreted as a communication and control tool of magnetic body containing dark matter and using biological body as motor instrument and sensory receptor. One especially amazing outcome is the emergence of genetic code of vertebrates from the model of dark nuclei as nuclear strings [L1, K116], [L1].

The most general option for the generalized embedding space

Simple physical arguments pose constraints on the choice of the most general form of the embedding space.

1. The fundamental group of the space for which one constructs a non-singular covering space or factor space should be non-trivial. This is certainly not possible for M^4, CD, CP_2, or H. One can however construct singular covering spaces. The fixing of the quantization axes implies a selection of the sub-space $H_4 = M^2 \times S^2 \subset M^4 \times CP_2$, where S^2 is geodesic sphere of CP_2. $\hat{M}^4 = M^4 \backslash M^2$ and $\hat{CP}_2 = CP_2 \backslash S^2$ have fundamental group Z since the codimension of the excluded sub-manifold is equal to two and homotopically the situation is like that for a punctured plane. The exclusion of these sub-manifolds defined by the choice of quantization axes could naturally give rise to the desired situation.

2. CP_2 allows two geodesic spheres which left invariant by $U(2$ *resp.* $SO(3)$. The first one is homologically non-trivial. For homologically non-trivial geodesic sphere $H_4 = M^2 \times S^2$ represents a straight cosmic string which is non-vacuum extremal of Kähler action (not necessarily preferred extremal). One can argue that the many-valuedness of $\hbar$ is un-acceptable for non-vacuum extremals so that only homologically trivial geodesic sphere S^2 would be acceptable. One could go even further. If the extremals in $M^2 \times CP_2$ can be preferred non-vacuum extremals, the singular coverings of M^4 are not possible. Therefore only the singular coverings and factor spaces of CP_2 over the homologically trivial geodesic sphere S^2 would be possible. This however looks a non-physical outcome.

   (a) The situation changes if the extremals of type $M^2 \times Y^2$, Y^2 a holomorphic surface of CP_3, fail to be hyperquaternionic. The tangent space M^2 represents hypercomplex sub-space and the product of the Kähler-Dirac gamma matrices associated with the tangent spaces of Y^2 should belong to M^2 algebra. This need not be the case in general.

   (b) The situation changes also if one reinterprets the gluing procedure by introducing scaled up coordinates for M^4 so that metric is continuous at $M^2 \times CP_2$ but CDs with different size have different sizes differing by the ratio of Planck constants and would thus have only piece of lower or upper boundary in common.

3. For the more general option one would have four different options corresponding to the Cartesian products of singular coverings and factor spaces. These options can be denoted by $C-C$, $C-F$, $F-C$, and $F-F$, where C (F) signifies for covering (factor space) and first (second) letter signifies for CD (CP_2) and correspond to the spaces $(\hat{CD}\hat{\times}G_a) \times (\hat{CP}_2\hat{\times}G_b)$, $(\hat{CD}\hat{\times}G_a) \times \hat{CP}_2/G_b$, $\hat{CD}/G_a \times (\hat{CP}_2\hat{\times}G_b)$, and $\hat{CD}/G_a \times \hat{CP}_2/G_b$.

4. The groups G_i could correspond to cyclic groups Z_n. One can also consider an extension by replacing M^2 and S^2 with its orbit under more general group G (say tetrahedral, octahedral, or icosahedral group). One expects that the discrete subgroups of $SU(2)$ emerge naturally in this framework if one allows the action of these groups on the singular sub-manifolds M^2 or S^2. This would replace the singular manifold with a set of its rotated copies in the case that the subgroups have genuinely 3-dimensional action (the subgroups which corresponds

to exceptional groups in the ADE correspondence). For instance, in the case of M^2 the quantization axes for angular momentum would be replaced by the set of quantization axes going through the vertices of tetrahedron, octahedron, or icosahedron. This would bring non-commutative homotopy groups into the picture in a natural manner.

About the phase transitions changing Planck constant

There are several non-trivial questions related to the details of the gluing procedure and phase transition as motion of partonic 2-surface from one sector of the embedding space to another one.

1. How the gluing of copies of embedding space at $M^2 \times CP_2$ takes place? It would seem that the covariant metric of CD factor proportional to $\hbar^2$ must be discontinuous at the singular manifold since only in this manner the idea about different scaling factor of CD metric can make sense. On the other hand, one can always scale the M^4 coordinates so that the metric is continuous but the sizes of CDs with different Planck constants differ by the ratio of the Planck constants.
2. One might worry whether the phase transition changing Planck constant means an instantaneous change of the size of partonic 2-surface in M^4 degrees of freedom. This is not the case. Light-likeness in $M^2 \times S^2$ makes sense only for surfaces $X^1 \times D^2 \subset M^2 \times S^2$, where X^1 is light-like geodesic. The requirement that the partonic 2-surface X^2 moving from one sector of H to another one is light-like at $M^2 \times S^2$ irrespective of the value of Planck constant requires that X^2 has single point of M^2 as M^2 projection. Hence no sudden change of the size X^2 occurs.
3. A natural question is whether the phase transition changing the value of Planck constant can occur purely classically or whether it is analogous to quantum tunnelling. Classical non-vacuum extremals of Chern-Simons action have two-dimensional CP_2 projection to homologically non-trivial geodesic sphere S^2_I. The deformation of the entire S^2_I to homologically trivial geodesic sphere S^2_{II} is not possible so that only combinations of partonic 2-surfaces with vanishing total homology charge (Kähler magnetic charge) can in principle move from sector to another one, and this process involves fusion of these 2-surfaces such that CP_2 projection becomes single homologically trivial 2-surface. A piece of a non-trivial geodesic sphere S^2_I of CP_2 can be deformed to that of S^2_{II} using 2-dimensional homotopy flattening the piece of S^2 to curve. If this homotopy cannot be chosen to be light-like, the phase transitions changing Planck constant take place only via quantum tunnelling. Obviously the notions of light-like homotopies (cobordisms) are very relevant for the understanding of phase transitions changing Planck constant.

How one could fix the spectrum of Planck constants?

The question how the observed Planck constant relates to the integers n_a and n_b defining the covering and factors spaces, is far from trivial and I have considered several options. The basic physical inputs are the condition that scaling of Planck constant must correspond to the scaling of the metric of CD (that is Compton lengths) on one hand and the scaling of the gauge coupling strength $g^2/4\pi\hbar$ on the other hand.

1. One can assign to Planck constant to both CD and CP_2 by assuming that it appears in the commutation relations of corresponding symmetry algebras. Algebraist would argue that Planck constants $\hbar(CD)$ and $\hbar(CP_2)$ must define a homomorphism respecting multiplication and division (when possible) by G_i. This requires $r(X) = \hbar(X)\hbar_0 = n$ for covering and $r(X) = 1/n$ for factor space or vice versa.
2. If one assumes that $\hbar^2(X)$, $X = M^4$, CP_2 corresponds to the scaling of the covariant metric tensor g_{ij} and performs an over-all scaling of H-metric allowed by the Weyl invariance of Kähler action by dividing metric with $\hbar^2(CP_2)$, one obtains the scaling of M^4 covariant metric by $r^2 \equiv \hbar^2/\hbar_0^2 = \hbar^2(M^4)/\hbar^2(CP_2)$ whereas CP_2 metric is not scaled at all.
3. The condition that $\hbar$ scales as n_a is guaranteed if one has $\hbar(CD) = n_a\hbar_0$. This does not fix the dependence of $\hbar(CP_2)$ on n_b and one could have $\hbar(CP_2) = n_b\hbar_0$ or $\hbar(CP_2) = \hbar_0/n_b$. The intuitive picture is that n_b- fold covering gives in good approximation rise to n_an_b sheets and

multiplies YM action action by $n_a n_b$ which is equivalent with the $\hbar = n_a n_b \hbar_0$ if one effectively compresses the covering to $CD \times CP_2$. One would have $\hbar(CP_2) = \hbar_0/n_b$ and $\hbar = n_a n_b \hbar_0$. Note that the descriptions using ordinary Planck constant and coverings and scaled Planck constant but contracting the covering would be alternative descriptions.

This gives the following formulas $r \equiv \hbar/\hbar_0 = r(M^4)/r(CP_2)$ in various cases.

	$C-C$	$F-C$	$C-F$	$F-F$
r	$n_a n_b$	$\frac{n_a}{n_b}$	$\frac{n_b}{n_a}$	$\frac{1}{n_a n_b}$

Preferred values of Planck constants

Number theoretic considerations favor the hypothesis that the integers corresponding to Fermat polygons constructible using only ruler and compass and given as products $n_F = 2^k \prod_s F_s$, where $F_s = 2^{2^s} + 1$ are distinct Fermat primes, are favored. The reason would be that quantum phase $q = exp(i\pi/n)$ is in this case expressible using only iterated square root operation by starting from rationals. The known Fermat primes correspond to $s = 0, 1, 2, 3, 4$ so that the hypothesis is very strong and predicts that p-adic length scales have satellite length scales given as multiples of n_F of fundamental p-adic length scale. $n_F = 2^{11}$ corresponds in TGD framework to a fundamental constant expressible as a combination of Kähler coupling strength, CP_2 radius and Planck length appearing in the expression for the tension of cosmic strings, and the powers of 2^{11} was proposed to define favored as values of n_a in living matter [K44].

The hypothesis that Mersenne primes $M_k = 2^k - 1$, $k \in \{89, 107, 127\}$, and Gaussian Mersennes $M_{G,k} = (1 + i)k - 1$, $k \in \{113, 151, 157, 163, 167, 239, 241..\}$ (the number theoretical miracle is that all the four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$ with $k \in \{151, 157, 163, 167\}$ are in the biologically highly interesting range 10 nm-2.5 μm) define scaled up copies of electro-weak and QCD type physics with ordinary value of $\hbar$ and that these physics are induced by dark variants of corresponding lower level physics leads to a prediction for the preferred values of $r = 2^{k_d}$, $k_d = k_i - k_j$, and the resulting picture finds support from the ensuing models for biological evolution and for EEG [K44]. This hypothesis - to be referred to as Mersenne hypothesis - replaces the rather ad hoc proposal $r = \hbar/\hbar_0 = 2^{11k}$ for the preferred values of Planck constant.

How Planck constants are visible in Kähler action?

$\hbar(M^4)$ and $\hbar(CP_2)$ appear in the commutation and anti-commutation relations of various superconformal algebras. Only the ratio of M^4 and CP_2 Planck constants appears in Kähler action and is due to the fact that the M^4 and CP_2 metrics of the embedding space sector with given values of Planck constants are proportional to the corresponding Planck. This implies that Kähler function codes for radiative corrections to the classical action, which makes possible to consider the possibility that higher order radiative corrections to functional integral vanish as one might expect at quantum criticality. For a given p-adic length scale space-time sheets with all allowed values of Planck constants are possible. Hence the spectrum of quantum critical fluctuations could in the ideal case correspond to the spectrum of $\hbar$ coding for the scaled up values of Compton lengths and other quantal lengths and times. If so, large $\hbar$ phases could be crucial for understanding of quantum critical superconductors, in particular high T_c superconductors.

6.4 Updated View About The Hierarchy Of Planck Constants

During last years the work with TGD proper has transformed from the discovery of brave visions to the work of clock smith. The challenge is to fill in the details, to define various notions more precisely, and to eliminate the numerous inconsistencies.

Few years has passed from the latest formulation for the hierarchy of Planck constant. The original hypothesis was that the hierarchy is real. In this formulation the embedding space was replaced with its covering space assumed to decompose to a Cartesian product of singular finite-sheeted coverings of M^4 and CP_2.

Few years ago came the realization that it could be only effective but have same practical implications. The basic observation was that the effective hierarchy need not be postulated separately but follows as a prediction from the vacuum degeneracy of Kähler action. In this formulation Planck constant at fundamental level has its standard value and its effective values come as its integer multiples so that one should write $\hbar_{eff} = n\hbar$ rather than $\hbar = n\hbar_0$ as I have done. For most practical purposes the states in question would behave as if Planck constant were an integer multiple of the ordinary one. It was no more necessary to assume that the covering reduces to a Cartesian product of singular coverings of M^4 and CP_2 but for some reason I kept this assumption.

It seems that the time is ripe for checking whether some polishing of this formulation might be needed. In particular, the work with TGD inspired quantum biology suggests a close connection between the hierarchy of Planck constants and negentropic entanglement. Also the connection with anyons and charge fractionalization (see `http://tinyurl.com/y89xp4bu`) has remained somewhat fuzzy [K85]. In particular, it seems that the formulation based on multi-furcations of space-time surfaces to N branches is not general enough: the N branches are very much analogous to single particle states and second quantization allowing all $0 < n \leq N$-particle states for given N rather than only N-particle states looks very natural: as a matter fact, this interpretation was the original one and led to the very speculative and fuzzy notion of N-atom, which I later more or less gave up. Quantum multi-furcation could be the root concept implying the effective hierarchy of Planck constants, anyons and fractional charges, and related notions- even the notions of N-nuclei, N-atoms, and N-molecules.

6.4.1 Basic Physical Ideas

The basic phenomenological rules are simple and there is no need to modify them.

1. The phases with non-standard values of effective Planck constant are identified as dark matter. The motivation comes from the natural assumption that only the particles with the same value of effective Planck can appear in the same vertex. One can illustrate the situation in terms of the book metaphor. Embedding spaces with different values of Planck constant form a book like structure and matter can be transferred between different pages only through the back of the book where the pages are glued together. One important implication is that light exotic charged particles lighter than weak bosons are possible if they have non-standard value of Planck constant. The standard argument excluding them is based on decay widths of weak bosons and has led to a neglect of large number of particle physics anomalies [K117].

2. Large effective or real value of Planck constant scales up Compton length - or at least de Broglie wave length - and its geometric correlate at space-time level identified as size scale of the space-time sheet assignable to the particle. This could correspond to the Kähler magnetic flux tube for the particle forming consisting of two flux tubes at parallel space-time sheets and short flux tubes at ends with length of order CP_2 size.

   This rule has far reaching implications in quantum biology and neuroscience since macroscopic quantum phases become possible as the basic criterion stating that macroscopic quantum phase becomes possible if the density of particles is so high that particles as Compton length sized objects overlap. Dark matter therefore forms macroscopic quantum phases. One implication is the explanation of mysterious looking quantal effects of ELF radiation in EEG frequency range on vertebrate brain: $E = hf$ implies that the energies for the ordinary value of Planck constant are much below the thermal threshold but large value of Planck constant changes the situation. Also the phase transitions modifying the value of Planck constant and changing the lengths of flux tubes (by quantum classical correspondence) are crucial as also reconnections of the flux tubes.

   The hierarchy of Planck constants suggests also a new interpretation for FQHE (see `http://tinyurl.com/y89xp4bu`) (fractional quantum Hall effect) [K85] in terms of anyonic phases with non-standard value of effective Planck constant realized in terms of the effective multi-sheeted covering of embedding space: multi-sheeted space-time is to be distinguished from many-sheeted space-time.

3. In astrophysics and cosmology the implications are even more dramatic if one believes that also $\hbar_{gr}$ corresponds to effective Planck constant interpreted as number of sheets of multi-furcation. It was Nottale (see `http://tinyurl.com/ya6f3s4l`) [E13] who first introduced

the notion of gravitational Planck constant as $\hbar_{gr} = GMm/v_0$, $v_0 < 1$ has interpretation as velocity light parameter in units $c = 1$. This would be true for $GMm/v_0 \geq 1$. The interpretation of $\hbar_{gr}$ in TGD framework is as an effective Planck constant associated with space-time sheets mediating gravitational interaction between masses M and m. The huge value of $\hbar_{gr}$ means that the integer $\hbar_{gr}/\hbar_0$ interpreted as the number of sheets of covering is gigantic and that Universe possesses gravitational quantum coherence in super-astronomical scales for masses which are large. This would suggest that gravitational radiation is emitted as dark gravitons which decay to pulses of ordinary gravitons replacing continuous flow of gravitational radiation.

It must be however emphasized that the interpretation of $\hbar_{gr}$ could be different, and it will be found that one can develop an argument demonstrating how $\hbar_{gr}$ with a correct order of magnitude emerges from the effective space-time metric defined by the anti-commutators appearing in the Kähler-Dirac equation.

4. Why Nature would like to have large effective value of Planck constant? A possible answer relies on the observation that in perturbation theory the expansion takes in powers of gauge couplings strengths $\alpha = g^2/4\pi\hbar$. If the effective value of $\hbar$ replaces its real value as one might expect to happen for multi-sheeted particles behaving like single particle, α is scaled down and perturbative expansion converges for the new particles. One could say that Mother Nature loves theoreticians and comes in rescue in their attempts to calculate. In quantum gravitation the problem is especially acute since the dimensionless parameter $GMm/\hbar$ has gigantic value. Replacing $\hbar$ with $\hbar_{gr} = GMm/v_0$ the coupling strength becomes $v_0 < 1$.

6.4.2 Space-Time Correlates For The Hierarchy Of Planck Constants

The hierarchy of Planck constants was introduced to TGD originally as an additional postulate and formulated as the existence of a hierarchy of embedding spaces defined as Cartesian products of singular coverings of M^4 and CP_2 with numbers of sheets given by integers n_a and n_b and $\hbar = n\hbar_0$. $n = n_a n_b$.

With the advent of zero energy ontology, it became clear that the notion of singular covering space of the embedding space could be only a convenient auxiliary notion. Singular means that the sheets fuse together at the boundary of multi-sheeted region. The effective covering space emerges naturally from the vacuum degeneracy of Kähler action meaning that all deformations of canonically imbedded M^4 in $M^4 \times CP_2$ have vanishing action up to fourth order in small perturbation. This is clear from the fact that the induced Kähler form is quadratic in the gradients of CP_2 coordinates and Kähler action is essentially Maxwell action for the induced Kähler form. The vacuum degeneracy implies that the correspondence between canonical momentum currents $\partial L_K/\partial(\partial_\alpha h^k)$ defining the Kähler-Dirac gamma matrices [K126] and gradients $\partial_\alpha h^k$ is not one-to-one. Same canonical momentum current corresponds to several values of gradients of embedding space coordinates. At the partonic 2-surfaces at the light-like boundaries of CD carrying the elementary particle quantum numbers this implies that the two normal derivatives of h^k are many-valued functions of canonical momentum currents in normal directions.

Multi-furcation is in question and multi-furcations are indeed generic in highly non-linear systems and Kähler action is an extreme example about non-linear system (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg` or **Fig. ??** in the appendix of this book). What multi-furcation means in quantum theory? The branches of multi-furcation are obviously analogous to single particle states. In quantum theory second quantization means that one constructs not only single particle states but also the many particle states formed from them. At space-time level single particle states would correspond to N branches b_i of multi-furcation carrying fermion number. Two-particle states would correspond to 2-fold covering consisting of 2 branches b_i and b_j of multi-furcation. $N-$particle state would correspond to N-sheeted covering with all branches present and carrying elementary particle quantum numbers. The branches coincide at the partonic 2-surface but since their normal space data are different they correspond to different tensor product factors of state space. Also now the factorization $N = n_a n_b$ occurs but now n_a and n_b would relate to branching in the direction of space-like 3-surface and light-like 3-surface rather than M^4 and CP_2 as in the original hypothesis.

In light of this the working hypothesis adopted during last years has been too limited: for some reason I ended up to propose that only N-sheeted covering corresponding to a situation in which all N branches are present is possible. Before that I quite correctly considered more general option based on intuition that one has many-particle states in the multi-sheeted space. The erratic form of the working hypothesis has not been used in applications.

Multi-furcations relate closely to the quantum criticality of Kähler action. Feigenbaum bifurcations (see `http://tinyurl.com/2swb2p`) represent a toy example of a system which via successive bifurcations approaches chaos. Now more general multi-furcations in which each branch of given multi-furcation can multi-furcate further, are possible unless on poses any additional conditions. This allows to identify additional aspect of the geometric arrow of time. Either the positive or negative energy part of the zero energy state is "prepared" meaning that single n-sub-furcations of N-furcation is selected. The most general state of this kind involves superposition of various n-sub-furcations.

6.4.3 The Relationship To The Original View About The Hierarchy Of Planck Constants

Originally the hierarchy of Planck constant was assumed to correspond to a book like structure having as pages the n-fold coverings of the embedding space for various values of n serving therefore as a page number. The pages are glued together along a 4-D "back" at which the branches of n-furcations are degenerate. This leads to a very elegant picture about how the particles belonging to the different pages of the book interact. All vertices are local and involve only particles with the same value of Planck constant: this is enough for darkness in the sense of particle physics. The interactions between particles belonging to different pages involve exchange of a particle travelling from page to another through the back of the book and thus experiencing a phase transition changing the value of Planck constant.

Is this picture consistent with the picture based on n-furcations? This seems to be the case. The conservation of energy in n-furcation in which several sheets are realized simultaneously is consistent with the conservation of classical conserved quantities only if the space-time sheet before n-furcation involves n identical copies of the original space-time sheet or if the Planck constant is $h_{eff} = nh$. This kind of degenerate many-sheetedness is encountered also in the case of branes. The first option means an n-fold covering of embedding space and h_{eff} is indeed effective Planck constant. Second option means a genuine quantization of Planck constant due to the fact the value of Kähler coupling strength $\alpha_K = g_K^2/4\pi\hbar_{eff}$ is scaled down by $1/n$ factor. The scaling of Planck constant consistent with classical field equations since they involve α_K as an overall multiplicative factor only.

6.4.4 Basic Phenomenological Rules Of Thumb In The New Framework

It is important to check whether or not the refreshed view about dark matter is consistent with existent rules of thumb.

1. The interpretation of quantized multi-furcations as WCW anyons explains also why the effective hierarchy of Planck constants defines a hierarchy of phases which are dark relative to each other. This is trivially true since the phases with different number of branches in multi-furcation correspond to disjoint regions of WCW so that the particles with different effective value of Planck constant cannot appear in the same vertex.
2. The phase transitions changing the value of Planck constant are just the multi-furcations and can be induced by changing the values of the external parameters controlling the properties of preferred extremals. Situation is very much the same as in any non-linear system.
3. In the case of massless particles the scaling of wavelength in the effective scaling of $\hbar$ can be understood if dark n-photons consist of n photons with energy E/n and wavelength $n\lambda$.
4. For massive particle it has been assumed that masses for particles and they dark counterparts are same and Compton wavelength is scaled up. In the new picture this need not be true. Rather, it would seem that wave length are same as for ordinary electron.

   On the other hand, p-adic thermodynamics predicts that massive elementary particles are massless most of the time. ZEO predicts that even virtual wormhole throats are massless. Could this mean that the picture applying on massless particle should apply to them at least at relativistic limit at which mass is negligible. This might be the case for bosons but for fermions also fermion number should be fractionalized and this is not possible in the recent picture. If one assumes that the n-electron has same mass as electron, the mass for dark single electron state would be scaled down by $1/n$. This does not look sensible unless the p-adic length defined by prime is scaled down by this fact in good approximation.

This suggests that for fermions the basic scaling rule does not hold true for Compton length $\lambda_c = \hbar/m$. Could it however hold for de-Broglie lengths $\lambda = \hbar/p$ defined in terms of 3-momentum? The basic overlap rule for the formation of macroscopic quantum states is indeed formulated for de Broglie wave length. One could argue that an $1/N$-fold reduction of density that takes place in the de-localization of the single particle states to the N branches of the cover, implies that the volume per particle increases by a factor N and single particle wave function is de-localized in a larger region of 3-space. If the particles reside at effectively one-dimensional 3-surfaces - say magnetic flux tubes - this would increase their de Broglie wave length in the direction of the flux tube and also the length of the flux tube. This seems to be enough for various applications.

One important notion in TGD inspired quantum biology is dark cyclotron state.

1. The scaling $\hbar \rightarrow k\hbar$ in the formula $E_n = (n + 1/2)\hbar eB/m$ implies that cyclotron energies are scaled up for dark cyclotron states. What this means microscopically has not been obvious but the recent picture gives a rather clearcut answer. One would have k-particle state formed from cyclotron states in N-fold branched cover of space-time surface. Each branch would carry magnetic field B and ion or electron. This would give a total cyclotron energy equal to kE_n. These cyclotron states would be excited by k-photons with total energy $E = khf$ and for large enough value of k the energies involved would be above thermal threshold. In the case of Ca^{++} one has $f = 15$ Hz in the field $B_{end} = .2$ Gauss. This means that the value of $\hbar$ is at least the ratio of thermal energy at room temperature to $E = hf$. The thermal frequency is of order 10^{12} Hz so that one would have $k \simeq 10^{11}$. The number branches would be therefore rather high.
2. It seems that this kinds of states which I have called cyclotron Bose-Einstein condensates could make sense also for fermions. The dark photons involved would be Bose-Einstein condensates of k photons and wall of them would be simultaneously absorbed. The biological meaning of this would be that a simultaneous excitation of large number of atoms or molecules can take place if they are localized at the branches of N-furcation. This would make possible coherent macroscopic changes. Note that also Cooper pairs of electrons could be $n = 2$-particle states associated with N-furcation.

There are experimental findings suggesting that photosynthesis involves de-localized excitations of electrons and it is interesting so see whether this could be understood in this framework.

1. The TGD based model relies on the assumption that cyclotron states are involved and that dark photons with the energy of visible photons but with much longer wavelength are involved. Single electron excitations (or single particle excitations of Cooper pairs) would generate negentropic entanglement (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book) automatically.
2. If cyclotron excitations are the primary ones, it would seem that they could be induced by dark n-photons exciting all n electrons simultaneously. n-photon should have energy of a visible photon. The number of cyclotron excited electrons should be rather large if the total excitation energy is to be above thermal threshold. In this case one could not speak about cyclotron excitation however. This would require that solar photons are transformed to n-photons in N-furcation in biosphere.
3. Second - more realistic looking - possibility is that the incoming photons have energy of visible photon and are therefore $n = 1$ dark photons de-localized to the branches of the N-furcation. They would induce de-localized single electron excitation in WCW rather than 3-space.

6.4.5 Charge Fractionalization And Anyons

It is easy to see how the effective value of Planck constant as an integer multiple of its standard value emerges for multi-sheeted states in second quantization. At the level of Kähler action one can assume that in the first approximation the value of Kähler action for each branch is same so that the total Kähler action is multiplied by n. This corresponds effectively to the scaling $\alpha_K \rightarrow \alpha_K/n$ induced by the scaling $\hbar_0 \rightarrow n\hbar_0$.

Also effective charge fractionalization and anyons emerge naturally in this framework.

1. In the ordinary charge fractionalization (see http://tinyurl.com/26tmhoe) the wave function decomposes into sharply localized pieces around different points of 3-space carrying fractional charges summing up to integer charge. Now the same happens at at the level of WCW ("world of classical worlds") rather than 3-space meaning that wave functions in E^3 are replaced with wave functions in the space-time of 3-surfaces (4-surfaces by holography implied by General Coordinate Invariance) replacing point-like particles. Single particle wave function in WCW is a sum of N sharply localized contributions: localization takes place around one particular branch of the multi-sheeted space time surface. Each branch carries a fractional charge q/N for teh analogs of plane waves.

   Therefore all quantum numbers are additive and fractionalization is only effective and observable in a localization of wave function to single branch occurring with probability $p = 1/N$ from which one can deduce that charge is q/N.

2. The is consistent with the proposed interpretation of dark photons/gravitons since they could carry large spin and this kind of situation could decay to bunches of ordinary photons/gravitons. It is also consistent with electromagnetic charge fractionization and fractionization of spin.

3. The original - and it seems wrong - argument suggested what might be interpreted as a genuine fractionization for orbital angular momentum and also of color quantum numbers, which are analogous to orbital angular momentum in TGD framework. The observation was that a rotation through 2π at space-time level moving the point along space-time surface leads to a new branch of multi-furcation and $N+1$: th branch corresponds to the original one. This suggests that angular momentum fractionization should take place for M^4 angle coordinate ϕ because for it 2π rotation could lead to a different sheet of the effective covering.

   The orbital angular momentum eigenstates would correspond to waves $exp(i\phi m/N)$, $m = 0, 2, ..., N-1$ and the maximum orbital angular momentum would correspond the sum $\sum_{m=0}^{N-1} m/N = (N-1)/2$. The sum of spin and orbital angular momentum be therefore fractional.

   The different prediction is due to the fact that rotations are now interpreted as flows rotating the points of 3-surface along 3-surface rather than rotations of the entire partonic surface in embedding space. In the latter interpretation the rotation by 2π does nothing for the 3-surface. Hence fractionization for the total charge of the single particle states does not take place unless one adopts the flow interpretation. This view about fractionization however leads to problems with fractionization of electromagnetic charge and spin for which there is evidence from fractional quantum Hall effect.

6.4.6 Negentropic Entanglement Between Branches Of Multi-Furcations

The application of negentropic entanglement and effective hierarchy of Planck constants to photosynthesis and metabolism (see http://tinyurl.com/yd7j9f5j) [K60] suggests that these two notions might be closely related. Negentropic entanglement is possible for rational (and even algebraic) entanglement probabilities. If one allows number theoretic variant of Shannon entropy (see http://tinyurl.com/y6v73ryc) based on the p-adic norm for the probability appearing as argument of logarithm [K72], it is quite possible to have negative entanglement entropy and the interpretation is as genuine information carried by entanglement. The superposition of state pairs $a_i \otimes b_i$ in entangled state would represent instances of a rule. In the case of Schrödinger cat the rule states that it is better to not open the bottle: understanding the rule consciously however requires that cat is somewhat dead! Entanglement provides information about the relationship between two systems. Shannon entropy represents lack of information about single particle state.

Negentropic entanglement would replace metabolic energy as the basic quantity making life possible. Metabolic energy could generate negentropic entanglement by exciting biomolecules to negentropically entangled states. ATP providing the energy for generating the metabolic entanglement could also itself carry negentropic entanglement, and transfer it to the target by the emission of large $\hbar$ photons.

How the large $\hbar$ photons could carry negentropic entanglement? There are several options to consider and at this stage it is not possible to pinpoint anyone of them as the only possible one. Several of them could also be realized.

1. In zero energy ontology large $\hbar$ photons could carry the negentropic entanglement as entanglement between positive and negative energy parts of the photon state.
2. The negentropic entanglement of large $\hbar$ photon could be also associated with its positive or energy part or both. Large $\hbar_{eff} = n\hbar$ photon with n-fold energy $E = n \times hf$ is n-sheeted structure consisting of n-photons with energy $E = hf$ de-localized in the discrete space formed by the N space-time sheets. The n single photon states can entangle and since the branches effectively form a discrete space, rational and algebraic entanglement is very natural. There are many options for how this could happen. For instance, for N-fold branching the superposition of all $N!/(N-n)!n!$ states obtained by selecting n branches are possible and the resulting state is entangled state. If this interpretation is correct, the vacuum degeneracy and multi-furcations implied by it would the quintessence of life.
3. A further very attractive possibility discovered quite recently is that large $h_{eff} = nh$ is closely related to the negentropic entanglement between the states of *two* n-furcated - that is dark - space-time sheets. In the most recent formulation negentropic entanglement corresponds to a state characterized by $n \times n$ identity matrix resulting from the measurement of density matrix. The number theoretic entanglement negentropy is positive for primes dividing p and there is unique prime for which it is maximal.

The identification of negentropic entanglement as entanglement between branches of a multi-furcation is not the only possible option.

1. One proposal is that non-localized single particle excitations of cyclotron condensate at magnetic flux tubes give rise to negentropic entanglement relevant to living matter. Dark photons could transfer the negentropic entanglement possibly assignable to electron pairs of ATP molecule.

   The negentropic entanglement associated with cyclotron condensate could be associated with the branches of the large $\hbar$ variant of the condensate. In this case single particle excitation would not be sum of single particle excitations at various positions of 3-space but at various sheet of covering representing points of WCW . If each of the n branches carries $1/n$: th part of electron one would have an anyonic state in WCW .
2. One can also make a really crazy question. Could it be that ATP and various bio-molecules form n-particle states at the n-sheet of n-furcation and that the bio-chemistry involves simultaneous reactions of large numbers of biomolecules at these sheets? If so, the chemical reactions would take place as large number of copies.

Note that in this picture the breaking of time reversal symmetry [K9] in the presence of metabolic energy feed would be accompanied by evolution involving repeated multi-furcations leading to increased complexity. TGD based view about the arrow of time implies that for a given CD this evolution has definite direction of time. At the level of ensemble it implies second law but at the level of individual system means increasing complexity.

6.4.7 Dark Variants Of Nuclear And Atomic Physics

During years I have in rather speculative spirit considered the possibility of dark variants of nuclear and atomic - and perhaps even molecular physics. Also the notion of dark cyclotron state is central in the quantum model of living matter. One such notion is the idea that dark nucleons could realize vertebrate genetic code [K120].

Before the real understanding what charge fractionization means it was possible to imagine several variants of say dark atoms depending on whether both nuclei and electrons are dark or whether only electrons are dark and genuinely fractionally charged. The recent picture however fixes these notions completely. Basic building bricks are just ordinary nuclei and atoms and they form n-particle states associated with n-branches of N-furcation with $n = 1, ..., N$. The fractionization for a single particle state de-localized completely to the discrete space of N branches as the analog of plane wave means that single branch carriers charge $1/N$.

The new element is the possibility of n-particle states populating n branches of the N-furcation: note that there is superposition over the states corresponding to different selections of these n branches. $N-k$ and k-nuclei/atoms are in sense conjugates of each other and they can fuse to form N-nuclei/N-atoms which in fermionic case are analogous to Fermi sea with all states filled.

Bio-molecules seem to obey symbolic dynamics which does not depend much on the chemical properties: this has motivated various linguistic metaphors applied in bio-chemistry to describe the interactions between DNA and related molecules. This motivated the wild speculation was that N-atoms and even N-molecules could make possible the emergence of symbolic representations with $n \leq N$ serving as a name of atom/molecule and that k- and $N-k$ atom/molecule would be analogous to opposite sexes in that there would be strong tendency for them to fuse together to form N-atom/-molecule. For instance, in bio-catalysis k- and $N-k$-atoms/molecules would be paired. The recent picture about n and $N-k$ atoms seems to be consistent with these speculations which I had already given up as too crazy. It is difficult to avoid even the speculation that bio-chemistry could replace chemical reactions with their n-multiples. Synchronized quantum jumps would allow to avoid the disastrous effects of state function reductions on quantum coherence. The second manner to say the same thing is that the effective value of Planck constant is large.

6.4.8 What About The Relationship Of Gravitational Planck Constant To Ordinary Planck Constant?

Gravitational Planck constant is given by the expression $\hbar_{gr} = GMm/v_0$, where $v_0 < 1$ has interpretation as velocity parameter in the units $c = 1$. Can one interpret also $\hbar_{gr}$ as effective value of Planck constant so that its values would correspond to multi-furcation with a gigantic number of sheets. This does not look reasonable.

Could one imagine any other interpretation for $\hbar_{gr}$? Could the two Planck constants correspond to inertial and gravitational dichotomy for four-momenta making sense also for angular momentum identified as a four-vector? Could gravitational angular momentum and the momentum associated with the flux tubes mediating gravitational interaction be quantized in units of $\hbar_{gr}$ naturally?

1. Gravitational four-momentum can be defined as a projection of the M^4-four-momentum to space-time surface. It's length can be naturally defined by the effective metric $g^{\alpha\beta}_{eff}$ defined by the anti-commutators of the modified gamma matrices. Gravitational four-momentum appears as a measurement interaction term in the Kähler-Dirac action and can be restricted to the space-like boundaries of the space-time surface at the ends of CD and to the light-like orbits of the wormhole throats and which induced 4- metric is effectively 3-dimensional.

2. At the string world sheets and partonic 2-surfaces the effective metric degenerates to 2-D one. At the ends of braid strands representing their intersection, the metric is effectively 4-D. Just for definiteness assume that the effective metric is proportional to the M^4 metric or rather - to its M^2 projection: $g^{kl}_{eff} = K^2 m^{kl}$.

   One can express the length squared for momentum at the flux tubes mediating the gravitational interaction between massive objects with masses M and m as

$$g^{\alpha\beta}_{eff} p_\alpha p_\beta = g^{\alpha\beta}_{eff} \partial_\alpha h^k \partial_\beta h^l p_k p_l \equiv g^{kl}_{eff} p_k p_l = n^2 \frac{\hbar^2}{L^2} \quad . \tag{6.4.1}$$

   Here L would correspond to the length of the flux tube mediating gravitational interaction and p_k would be the momentum flowing in that flux tube. $g^{kl}_{eff} = K^2 m^{kl}$ would give

$$p^2 = \frac{n^2\hbar^2}{K^2 L^2} \quad .$$

   $\hbar_{gr}$ could be identifed in this simplified situation as $\hbar_{gr} = \hbar/K$.

3. Nottale's proposal requires $K = GMm/v_0$ for the space-time sheets mediating gravitational interacting between massive objects with masses M and m. This gives the estimate

$$p_{gr} = \frac{GMm}{v_0}\frac{1}{L} \quad . \tag{6.4.2}$$

For $v_0 = 1$ this is of the same order of magnitude as the exchanged momentum if gravitational potential gives estimate for its magnitude. v_0 is of same order of magnitude as the rotation velocity of planet around Sun so that the reduction of v_0 to $v_0 \simeq 2^{-11}$ in the case of the 4 inner planets does not mean that the propagation velocity of gravitons is reduced.

4. Nottale's formula requires that the order of magnitude for the components of the energy momentum tensor at the ends of braid strands at partonic 2-surface should have value GMm/v_0. Einstein's equations $T = \kappa G + \Lambda g$ give a further constraint. For the vacuum solutions of Einstein's equations with a vanishing cosmological constant the value of h_{gr} approaches infinity. At the flux tubes mediating gravitational interaction one expects T to be proportional to the factor GMm simply because they mediate the gravitational interaction.

5. One can consider similar equation for gravitational angular momentum:

$$g_{eff}^{\alpha\beta} L_\alpha L_\beta = g_{eff}^{kl} L_k L_l = l(l+1)\hbar^2 \quad . \tag{6.4.3}$$

This would give under the same simplifying assumptions

$$L^2 = l(l+1)\frac{\hbar^2}{K^2} \quad . \tag{6.4.4}$$

This would justify the Bohr quantization rule for the angular momentum used in the Bohr quantization of planetary orbits.

One might counter argue that if gravitational 4- momentum square is proportional to inertial 4-momentum squared, then Equivalence Principle implies that h_{gr} can have only single value. In ZEO however all wormhole throats - also virtual - are massless and the argument fails. The varying h_{gr} can be assigned only with longitudinal and transversal momentum squared separately but not to the ratio of gravitational and inertial 4-momenta squared which both vanish.

Maybe the proposed connection might make sense in some more refined formulation. In particular the proportionality between $m_{eff}^{kl} = Km^{kl}$ could make sense as a quantum average. Also the fact, that the constant v_0 varies, could be understood from the dynamical character of m_{eff}^{kl}.

6.4.9 Hierarchy Of Planck Constants And Non-Determinism Of Kähler Action

Originally the hierarchy of Planck constant was inspired by empirical inputs from neuroscience, biology, and from Nottale's observations. That it is possible to understand the hierarchy in terms of non-determinism of Kähler action - the fundamental difference between TGD and quantum field theories and string models - is a victory for TGD approach (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg`, or **Fig. ??** in the appendix of this book).

Recall that non-determinism means that all space-time surfaces with CP_2 projection, which is Lagrangian sub-manifold (at most 2-D) of CP_2, carries a vanishing induced Kähler form and is vacuum extremal. The first guess would be that there is a finite number n of space-time sheets connecting given pair of 3-surfaces at the ends of space-time surface at the light-like boundaries of causal diamond (CD). Planck constant would be given as $h_{eff} = n \times h$ in accordance with the earlier interpretation. The degenerate extremals would have same Kähler action and conserved quantities as assumed also in the earlier approach. That the degenerate extremals co-incide at the ends of space-time surface was motivated by mathemtical aesthetics in the earlier approach but finds an interpretation in terms of non-uniqueness of the preferred extremals.

It is essential that these n degrees of freedom are regarded as genuine physical degrees of freedom, which are however discrete. Negentropic entanglement and dark matter would be associated with them naturally. The effective description would be in terms of n-fold singular covering of embedding space becoming singular at the ends of the space-time surface.

I have also assigned hierarchy of Planck constants with the quantum criticality. Quantum criticality means the existence of an entire continuous family of deformations of space-time sheet with same Kähler action and conserved quantities. The deformations would by definition vanish at the ends of space-time surface. The critical deformations would act as gauge transformations identifiable as conformal symmetries indeed expected to be presents since WCW isometries form a conformal algebra and there is also Kac-Moody type algebra present. The proposal has been that the sub-algebras of conformal algebra for which conformal weights are integer multiples of integer $n = 1, 2, ..$ defined a hierarchy of gauge algebras so that the dynamical algebra reduces to n-dimensional one.

These two identifications seem to be mutually inconsistent. The resolution of the conflict comes from the gauge invariance. For a given Kähler action and conserved quantities there would be n conformal equivalence classes of these 4-surfaces rather than n surfaces, and one would have n-fold degeneracy but for conformal equivalence classes of 4-surfaces rather than 4-surfaces. In Minkowskian regions the degenerate preferred extremals are sheets (graphs of a map from M^4 to CP_2).

6.5 Vision About Dark Matter As Phases With Non-Standard Value Of Planck Constant

6.5.1 Dark Rules

It is useful to summarize the basic phenomenological view about dark matter.

The notion of relative darkness

The essential difference between TGD and more conventional models of dark matter is that darkness is only relative concept.

1. Generalized embedding space forms a book like structure and particles at different pages of the book are dark relative to each other since they cannot appear in the same vertex identified as the partonic 2-surface along which light-like 3-surfaces representing the lines of generalized Feynman diagram meet.
2. Particles at different space-time sheets act via classical gauge field and gravitational field and can also exchange gauge bosons and gravitons (as also fermions) provided these particles can leak from page to another. This means that dark matter can be even photographed [I18]. This interpretation is crucial for the model of living matter based on the assumption that dark matter at magnetic body controls matter visible to us. Dark matter can also suffer a phase transition to visible matter by leaking between the pages of the Big Book.
3. The notion of standard value $\hbar_0$ of $\hbar$ is not a relative concept in the sense that it corresponds to rational $r = 1$. In particular, the situation in which both CD and CP_2 correspond to trivial coverings and factor spaces would naturally correspond to standard physics.

Is dark matter anyonic?

In [K85] a detailed model for the Kähler structure of the generalized embedding space is constructed. What makes this model non-trivial is the possibility that CP_2 Kähler form can have gauge parts which would be excluded in full embedding space but are allowed because of singular covering/factor-space property. The model leads to the conclusion that dark matter is anyonic if the partonic 2-surface, which can have macroscopic or even astrophysical size, encloses the tip of CD within it. Therefore the partonic 2-surface is homologically non-trivial when the tip is regarded as a puncture. Fractional charges for anyonic elementary particles imply confinement to the partonic 2-surface and the particles can escape the two surface only via reactions transforming them to ordinary particles. This would mean that the leakage between different pages of the big book is a rare phenomenon. This could partially explain why dark matter is so difficult to observe.

Field body as carrier of dark matter

The notion of "field body" implied by topological field quantization is essential. There would be em, Z^0, W, gluonic, and gravitonic field bodies, each characterized by its one prime. The motivation for considering the possibility of separate field bodies seriously is that the notion of induced gauge field means that all induced gauge fields are expressible in terms of four CP_2 coordinates so that only single component of a gauge potential allows a representation as and independent field quantity. Perhaps also separate magnetic and electric field bodies for each interaction and identifiable as flux quanta must be considered. This kind of separation requires that the fermionic content of the flux quantum (say fermion and anti-fermion at the ends of color flux tube) is such that it conforms with the quantum numbers of the corresponding boson.

What is interesting that the conceptual separation of interactions to various types would have a direct correlate at the level of space-time topology. From a different perspective inspired by the general vision that many-sheeted space-time provides symbolic representations of quantum physics, the very fact that we make this conceptual separation of fundamental interactions could reflect the topological separation at space-time level.

p-Adic mass calculations for quarks encourage to think that the p-adic length scale characterizing the mass of particle is associated with its electromagnetic body and in the case of neutrinos with its Z^0 body. Z^0 body can contribute also to the mass of charged particles but the contribution would be small. It is also possible that these field bodies are purely magnetic for color and weak interactions. Color flux tubes would have exotic fermion and anti-fermion at their ends and define colored variants of pions. This would apply not only in the case of nuclear strings but also to molecules and larger structures so that scaled variants of elementary particles and standard model would appear in all length scales as indeed implied by the fact that classical electro-weak and color fields are unavoidable in TGD framework.

One can also go further and distinguish between magnetic field body of free particle for which flux quanta start and return to the particle and "relative field" bodies associated with pairs of particles. Very complex structures emerge and should be essential for the understanding the space-time correlates of various interactions. In a well-defined sense they would define space-time correlate for the conceptual analysis of the interactions into separate parts. In order to minimize confusion it should be emphasized that the notion of field body used in this chapter relates to those space-time correlates of interactions, which are more or less *static* and related to the formation of *bound states*.

6.5.2 Phase Transitions Changing Planck Constant

The general picture is that p-adic length scale hierarchy corresponds to p-adic coupling constant evolution and hierarchy of Planck constants to the coupling constant evolution related to phase resolution. Both evolutions imply a book like structure of the generalized embedding space.

Transition to large $\hbar$ phase and failure of perturbation theory

One of the first ideas was that the transition to large $\hbar$ phase occurs when perturbation theory based on the expansion in terms of gauge coupling constant ceases to converge: Mother Nature would take care of the problems of theoretician. The transition to large $\hbar$ phase obviously reduces the value of gauge coupling strength $\alpha \propto 1/\hbar$ so that higher orders in perturbation theory are reduced whereas the lowest order "classical" predictions remain unchanged. A possible quantitative formulation of the criterion is that maximal 2-particle gauge interaction strength parameterized as $Q_1Q_2\alpha$ satisfies the condition $Q_1Q_2\alpha \simeq 1$.

A justification for this picture would be that in non-perturbative phase large quantum fluctuations are present (as functional integral formalism suggests). At space-time level this could mean that space-time sheet is near to a non-deterministic vacuum extremal -at least if homologically trivial geodesic sphere defines the number theoretic braids. At certain critical value of coupling constant strength one expects that the transition amplitude for phase transition becomes very large. The resulting phase would be of course different from the original since typically charge fractionization would occur.

One should understand why the failure of the perturbation theory (expected to occur for $\alpha Q_1Q_2 > 1$) induces the reduction of Clifford algebra, scaling down of CP_2 metric, and whether the

G-symmetry is exact or only approximate. A partial understanding already exists. The discrete G symmetry and the reduction of the dimension of Clifford algebra would have interpretation in terms of a loss of degrees of freedom as a strongly bound state is formed. The multiple covering of $M^4_\pm$ accompanying strong binding can be understood as an automatic consequence of G-invariance. A concrete realization for the binding could be charge fractionization which would not allow the particles bound on large light-like 3-surface to escape without transformation to ordinary particles.

Two examples perhaps provide more concrete view about this idea.

1. The proposed scenario can reproduce the huge value of the gravitational Planck constant. One should however develop a convincing argument why non-perturbative phase for the gravitating dark matter leads to a formation of $G_a\times$ covering of $CD\backslash M^2 \times CP_2\backslash S^2_I$ with the huge value of $\hbar_{eff} = n_a/n_b \simeq GM_1M_2/v_0$. The basic argument is that the dimensionless parameter $\alpha_{gr} = GM_1M_2/4\pi\hbar$ should be so small that perturbation theory works. This gives $\hbar_{gr} \geq GM_1M_2/4\pi$ so that order of magnitude is predicted correctly.
2. Color confinement represents the simplest example of a transition to a non-perturbative phase. In this case A_2 and $n = 3$ would be the natural option. The value of Planck constant would be 3 times higher than its value in perturbative QCD. Hadronic space-time sheets would be 3-fold coverings of $M^4_\pm$ and baryonic quarks of different color would reside on 3 separate sheets of the covering. This would resolve the color statistics paradox suggested by the fact that induced spinor fields do not possess color as spin like quantum number and by the facts that for orbifolds different quarks cannot move in independent CP_2 partial waves assignable to CP_2 cm degrees of freedom as in perturbative phase.

The mechanism of phase transition and selection rules

The mechanism of phase transition is at classical level similar to that for ordinary phase transitions. The partonic 2-surface decomposes to regions corresponding to difference values of $\hbar$ at quantum criticality in such a way that regions in which induced Kähler form is non-vanishing are contained within single page of embedding space. It might be necessary to assume that only a region corresponding to single value of $\hbar$ is possible for partonic 2-surfaces and $\delta CD\times CP_2$ so that quantum criticality would be associated with the intermediate state described by the light-like 3-surface. One could also see the phase transition as a leakage of X^2 from given page to another: this is like going through a closed door through a narrow slit between door and floor. By quantum criticality the points of number theoretic braid are already in the slit.

As in the case of ordinary phase transitions the allowed phase transitions must be consistent with the symmetries involved. This means that if the state is invariant under the maximal cyclic subgroups G_a and G_b then also the final state must satisfy this condition. This gives constraints to the orders of maximal cyclic subgroups Z_a and Z_b for initial and final state: $n(Z_{a_i})$ *resp.* $n(Z_{b_i}))$ must divide $n(Z_{a_f})$ *resp.* $n(Z_{b_f}$ or vice versa in the case that factors of Z_i do not leave invariant the states. If this is the case similar condition must hold true for appropriate subgroups. In particular, powers of prime Z_{p^n}, $n = 1, 2, ...$ define hierarchies of allowed phase transitions.

6.5.3 Coupling Constant Evolution And Hierarchy Of Planck Constants

If the overall vision is correct, quantum TGD would be characterized by two kinds of couplings constant evolutions. p-Adic coupling constant evolution would correspond to length scale resolution and the evolution with respect to Planck constant to phase resolution. Both evolution would have number theoretic interpretation.

Evolution with respect to phase resolution

The coupling constant evolution in phase resolution in p-adic degrees of freedom corresponds to emergence of algebraic extensions allowing increasing variety of phases $exp(i2\pi/n)$ expressible p-adically. This evolution can be assigned to the emergence of increasingly complex quantum phases and the increase of Planck constant.

One expects that quantum phases $q = exp(i\pi/n)$ which are expressible using only iterated square root operation are number theoretically very special since they correspond to algebraic extensions of p-adic numbers obtained by an iterated square root operation, which should emerge first. Therefore systems involving these values of q should be especially abundant in Nature. That arbitrarily high square roots are involved as becomes clear by studying the case $n = 2^k$: $cos(\pi/2^k) = \sqrt{[1 + cos(\pi/2^{k-1})]/2}$.

These polygons are obtained by ruler and compass construction and Gauss showed that these polygons, which could be called Fermat polygons, have $n_F = 2^k \prod_s F_{n_s}$ sides/vertices: all Fermat primes F_{n_s} in this expression must be different. The analog of the p-adic length scale hypothesis emerges since larger Fermat primes are near a power of 2. The known Fermat primes $F_n = 2^{2^n} + 1$ correspond to $n = 0, 1, 2, 3, 4$ with $F_0 = 3$, $F_1 = 5$, $F_2 = 17$, $F_3 = 257$, $F_4 = 65537$. It is not known whether there are higher Fermat primes. $n = 3, 5, 15$-multiples of p-adic length scales clearly distinguishable from them are also predicted and this prediction is testable in living matter. I have already earlier considered the possibility that Fermat polygons could be of special importance for cognition and for biological information processing [K80].

This condition could be interpreted as a kind of resonance condition guaranteeing that scaled up sizes for space-time sheets have sizes given by p-adic length scales. The numbers n_F could take the same role in the evolution of Planck constant assignable with the phase resolution as Mersenne primes have in the evolution assignable to the p-adic length scale resolution.

The Dynkin diagrams of exceptional Lie groups E_6 and E_8 are exceptional as subgroups of rotation group in the sense that they cannot be reduced to symmetry transformations of plane. They correspond to the symmetry group $S_4 \times Z_2$ of tetrahedron and $A_5 \times Z_2$ of dodecahedron or its dual polytope icosahedron (A_5 is 60-element subgroup of S_5 consisting of even permutations). Maximal cyclic subgroups are Z_4 and Z_5andthus their orders correspond to Fermat polygons. Interestingly, $n = 5$ corresponds to minimum value of n making possible topological quantum computation using braids and also to Golden Mean.

Is there a correlation between the values of p-adic prime and Planck constant?

The obvious question is whether there is a correlation between p-adic length scale and the value of Planck constant. One-to-one correspondence is certainly excluded but loose correlation seems to exist.

1. In [L73] the information about the number theoretic anatomy of Kähler coupling strength is combined with input from p-adic mass calculations predicting α_K to be the value of fine structure constant at the p-adic length scale associated with electron. One can also develop an explicit expression for gravitational constant assuming its renormalization group invariance on basis of dimensional considerations and this model leads to a model for the fraction of volume of the wormhole contact (piece of CP_2 type extremal) from the volume of CP_2 characterizing gauge boson and for similar volume fraction for the piece of the CP_2 type vacuum extremal associated with fermion.

2. The requirement that gravitational constant is renormalization group invariant implies that the volume fraction depends logarithmically on p-adic length scale and Planck constant (characterizing quantum scale). The requirement that this fraction in the range $(0, 1)$ poses a correlation between the rational characterizing Planck constant and p-adic length scale. In particular, for space-time sheets mediating gravitational interaction Planck constant must be larger than $\hbar_0$ above length scale which is about .1 Angstrom. Also an upper bound for $\hbar$ for given p-adic length scale results but is very large. This means that quantum gravitational effects should become important above atomic length scale [L73].

6.6 Some Applications

Below some applications of the hierarchy of Planck constants as a model of dark matter are briefly discussed. The range of applications varying from elementary particle physics to cosmology and I hope that this will convince the reader that the idea has strong physical motivations.

6.6.1 A Simple Model Of Fractional Quantum Hall Effect

The generalization of the embedding space suggests that it could possible to understand fractional quantum Hall effect [D2] at the level of basic quantum TGD. This section represents the first rough model of QHE constructed for a couple of years ago is discussed. Needless to emphasize, the model represents only the basic idea and involves ad hoc assumption about charge fractionization.

Recall that the formula for the quantized Hall conductance is given by

$$\begin{aligned} \sigma &= \nu \times \frac{e^2}{h} \ , \\ \nu &= \frac{n}{m} \ . \end{aligned} \quad (6.6.1)$$

Series of fractions in $\nu = 1/3, 2/5, 3/7, 4/9, 5/11, 6/13, 7/15...,\ 2/3, 3/5, 4/7, 5/9, 6/11, 7/13...,$ $5/3, 8/5, 11/7, 14/9...4/3, 7/5, 10/7, 13/9...,\ 1/5, 2/9, 3/13...,\ 2/7, 3/11...,\ 1/7....$ with odd denominator have been observed as are also $\nu = 1/2$ and $\nu = 5/2$ states with even denominator [D2].

The model of Laughlin [D36] cannot explain all aspects of FQHE. The best existing model proposed originally by Jain is based on composite fermions resulting as bound states of electron and even number of magnetic flux quanta [D26]. Electrons remain integer charged but due to the effective magnetic field electrons appear to have fractional charges. Composite fermion picture predicts all the observed fractions and also their relative intensities and the order in which they appear as the quality of sample improves.

The generalization of the notion of embedding space suggests the possibility to interpret these states in terms of fractionized charge, spin, and electron number. There are $2 \times 2 = 4$ combinations of covering and factors spaces of CP_2 and three of them can lead to the increase of Planck constant. Besides this one can consider two options for the formula of Planck constant so that which the very meager theoretical background one can make only guesses. In the following a model based on option II for which the number of states is conserved in the phase transition changing $\hbar$.

1. The easiest manner to understand the observed fractions is by assuming that both CD and CP_2 correspond to covering spaces so that both spin and electric charge and fermion number are fractionized. This means that e in electronic charge density is replaced with fractional charge. Quantized magnetic flux is proportional to e and the question is whether also here fractional charge appears. Assume that this does not occur.

2. With this assumption the expression for the Planck constant becomes for Option II as $r = \hbar/\hbar_0 = n_a/n_b$ and charge and spin units are equal to $1/n_b$ and $1/n_a$ respectively. This gives $\nu = nn_a/n_b$. The values $m = 2, 3, 5, 7, ..$ are observed. Planck constant can have arbitrarily large values. There are general arguments stating that also spin is fractionized in FQHE.

3. Both $\nu = 1/2$ and $\nu = 5/2$ state has been observed [D2, D16]. The fractionized charge is $e/4$ in the latter case [D16, D7]. Since $n_i > 3$ holds true if coverings and factor spaces are correlates for Jones inclusions, this requires $n_a = 4$ and $n_b = 8$ for $\nu = 1/2$ and $n_b = 4$ and $n_a = 10$ for $\nu = 5/2$. Correct fractionization of charge is predicted. For $n_b = 2$ also Z_2 would appear as the fundamental group of the covering space. Filling fraction 1/2 corresponds in the composite fermion model and also experimentally to the limit of zero magnetic field [D26]. $n_b = 2$ is inconsistent with the observed fractionization of electric charge for $\nu = 5/2$ and with the vision inspired by Jones inclusions.

4. A possible problematic aspect of the TGD based model is the experimental absence of even values of n_b except $n_b = 2$ (Laughlin's model predicts only odd values of n). A possible explanation is that by some symmetry condition possibly related to fermionic statistics (as in Laughlin model) n_a/n_b must reduce to a rational with an odd denominator for $n_b > 2$. In other words, one has $n_a \propto 2^r$, where 2^r the largest power of 2 divisor of n_b.

5. Large values of n_a emerge as B increases. This can be understood from flux quantization. One has $e \int BdS = n\hbar(M^4) = nn_a\hbar_0$. By using actual fractional charge $e_F = e/n_b$ in the flux factor would give $e_F \int BdS = n(n_a/n_b)\hbar_0 = n\hbar$. The interpretation is that each of the n_a sheets contributes one unit to the flux for e. Note that the value of magnetic field in given sheet is not affected so that the build-up of multiple covering seems to keep magnetic field strength below critical value.

6. The understanding of the thermal stability is not trivial. The original FQHE was observed in 80 mK temperature corresponding roughly to a thermal energy of $T \sim 10^{-5}$ eV. For graphene

the effect is observed at room temperature. Cyclotron energy for electron is (from $f_e = 6\times10^5$ Hz at $B = .2$ Gauss) of order thermal energy at room temperature in a magnetic field varying in the range 1-10 Tesla. This raises the question why the original FQHE requires so low temperature. The magnetic energy of a flux tube of length L is by flux quantization roughly $e^2B^2S \sim E_c(e)m_eL$ ($\hbar_0 = c = 1$) and exceeds cyclotron roughly by a factor L/L_e, L_e electron Compton length so that thermal stability of magnetic flux quanta is not the explanation. A possible explanation is that since FQHE involves several values of Planck constant, it is quantum critical phenomenon and is characterized by a critical temperature. The differences of the energies associated with the phase with ordinary Planck constant and phases with different Planck constant would characterize the transition temperature.

As already noticed, it is possible to imagine several other options and the assumption about charge fractionization -although consistent with fractionization for $\nu = 5/2$, is rather ad hoc. Therefore the model can be taken as a warm-up exercise only. In [K85], where the delicacies of Kähler structure of generalized embedding space are discussed, also a more detailed of QHE is discussed.

6.6.2 Gravitational Bohr Orbitology

The basic question concerns justification for gravitational Bohr orbitology [K100]. The basic vision is that visible matter identified as matter with $\hbar = \hbar_0$ ($n_a = n_b = 1$) concentrates around dark matter at Bohr orbits for dark matter particles. The question is what these Bohr orbits really mean. Should one in improved approximation relate Bohr orbits to 3-D wave functions for dark matter as ordinary Bohr rules would suggest or do the Bohr orbits have some deeper meaning different from that in wave mechanics. Anyonic variants of partonic 2-surfaces with astrophysical size are a natural guess for the generalization of Bohr orbits.

Dark matter as large $\hbar$ phase

D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7\pm.7$ km/s giving $v_0/c = 4.6\times10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive [K100] .

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation -or at least Bohr rules with appropriate interpretation - would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale.

Prediction for the parameter v_0

One of the key questions relate to the value of the parameter v_0. Before the introduction of the hierarchy of Planck constants I proposed that the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n-fold much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$-sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. The planetary mass ratios can be produced with an accuracy better than 10 per cent assuming ruler and compass phases.

Further predictions

The study of inclinations (tilt angles with respect to the Earth's orbital plane) leads to a concrete model for the quantum evolution of the planetary system. Only a stepwise breaking of the rotational symmetry and angular momentum Bohr rules plus Newton's equation (or geodesic equation) are needed, and gravitational Schrödinger equation holds true only inside flux quanta for the dark matter.

1. During pre-planetary period dark matter formed a quantum coherent state on the (Z^0) magnetic flux quanta (spherical cells or flux tubes). This made the flux quantum effectively a single rigid body with rotational degrees of freedom corresponding to a sphere or circle (full SO(3) or SO(2) symmetry).
2. In the case of spherical shells associated with inner planets the $SO(3) \rightarrow SO(2)$ symmetry breaking led to the generation of a flux tube with the inclination determined by m and j and a further symmetry breaking, kind of an astral traffic jam inside the flux tube, generated a planet moving inside flux tube. The semiclassical interpretation of the angular momentum algebra predicts the inclinations of the inner planets. The predicted (real) inclinations are 6 (7) resp. 2.6 (3.4) degrees for Mercury resp. Venus). The predicted (real) inclination of the Earth's spin axis is 24 (23.5) degrees.
3. The $v_0 \rightarrow v_0/5$ transition allowing to understand the radii of the outer planets in the model of Da Rocha and Nottale can be understood as resulting from the splitting of (Z^0) magnetic flux tube to five flux tubes representing Earth and outer planets except Pluto, whose orbital parameters indeed differ dramatically from those of other planets. The flux tube has a shape of a disk with a hole glued to the Earth's spherical flux shell.

   It is important to notice that effectively a multiplication $n \rightarrow 5n$ of the principal quantum number is in question. This allows to consider also alternative explanations. Perhaps external gravitational perturbations have kicked dark matter from the orbit or Earth to $n = 5k$, $k = 2, 3, ..., 7$ orbits: the fact that the tilt angles for Earth and all outer planets except Pluto are nearly the same, supports this explanation. Or perhaps there exist at least small amounts of dark matter at all orbits but visible matter is concentrated only around orbits containing some critical amount of dark matter and these orbits satisfy $n \ mod \ 5 = 0$ for some reason.
4. A remnant of the dark matter is still in a macroscopic quantum state at the flux quanta. It couples to photons as a quantum coherent state but the coupling is extremely small due to the gigantic value of $\hbar_{gr}$ scaling alpha by $\hbar/\hbar_{gr}$: hence the darkness.

   The rather amazing coincidences between basic bio-rhythms and the periods associated with the states of orbits in solar system suggest that the frequencies defined by the energy levels of the gravitational Schrödinger equation might entrain with various biological frequencies such as the cyclotron frequencies associated with the magnetic flux tubes. For instance, the period associated with $n = 1$ orbit in the case of Sun is 24 hours within experimental accuracy for v_0.

Comparison with Bohr quantization of planetary orbits

The predictions of the generalization of the p-adic length scale hypothesis are consistent with the TGD based model for the Bohr quantization of planetary orbits and some new non-trivial predictions follow.

1. The model can explain the enormous values of gravitational Planck constant $\hbar_{gr}/\hbar_0 =\simeq GMm/v_0) = n_a/n_b$. The favored values of this parameter should correspond to n_{F_a}/n_{F_b} so that the mass ratios $m_1/m_2 = n_{F_{a,1}} n_{F_{b,2}}/n_{F_{b,1}} n_{F_{a,2}}$ for planetary masses should be preferred. The general prediction $GMm/v_0 = n_a/n_b$ is of course not testable.
2. Nottale [E13] has suggested that also the harmonics and sub-harmonics of $\hbar_{gr}$ are possible and in fact required by the model for planetary Bohr orbits (in TGD framework this is not absolutely necessary [K100]). The prediction is that favored values of n should be of form $n_F = 2^k \prod F_i$ such that F_i appears at most once. In Nottale's model for planetary orbits as Bohr orbits in solar system [K100] $n = 5$ harmonics appear and are consistent with either $n_{F,a} \rightarrow F_1 n_{F_a}$ or with $n_{F,b} \rightarrow n_{F_b}/F_1$ if possible.

The prediction for the ratios of planetary masses can be tested. In the table below are the experimental mass ratios $r_{exp} = m(pl)/m(E)$, the best choice of $r_R = [n_{F,a}/n_{F,b}] * X$, X common factor for all planets, and the ratios $r_{pred}/r_{exp} = n_{F,a}(planet) n_{F,b}(Earth)/n_{F,a}(Earth) n_{F,b}(planet)$. The deviations are at most 2 per cent.

$planet$	Me	V	E	M	J
y	$\frac{2^{13}\times 5}{17}$	$2^{11}\times 17$	$2^9\times 5\times 17$	$2^8\times 17$	$\frac{2^{23}\times 5}{7}$
y/x	1.01	.98	1.00	.98	1.01
$planet$	S	U	N	P	
y	$2^{14}\times 3\times 5\times 17$	$\frac{2^{21}\times 5}{17}$	$\frac{2^{17}\times 17}{3}$	$\frac{2^4\times 17}{3}$	
y/x	1.01	.98	.99	.99	

Table 6.1: Table compares the ratios $x = m(pl)/(m(E)$ of planetary mass to the mass of Earth to prediction for these ratios in terms of integers n_F associated with Fermat polygons. y gives the best fit for the allowed factors of the known part y of the rational $n_{F,a}/n_{F,b} = yX$ characterizing planet, and the ratios y/x. Errors are at most 2 per cent.

A stronger prediction comes from the requirement that GMm/v_0 equals to $n = n_{F_a}/n_{F,b}$ $n_F = 2^k \prod_k F_{n_k}$, where $F_i = 2^{2^i} + 1$, $i = 0, 1, 2, 3, 4$ is Fibonacci prime. The fit using solar mass and Earth mass gives $n_F = 2^{254} \times 5 \times 17$ for $1/v_0 = 2044$, which within the experimental accuracy equals to the value $2^{11} = 2048$ whose powers appear as scaling factors of Planck constant in the model for living matter [K44]. For $v_0 = 4.6 \times 10^{-4}$ reported by Nottale the prediction is by a factor 16/17.01 too small (6 per cent discrepancy).

A possible solution of the discrepancy is that the empirical estimate for the factor GMm/v_0 is too large since m contains also the the visible mass not actually contributing to the gravitational force between dark matter objects whereas M is known correctly. The assumption that the dark mass is a fraction $1/(1 + \epsilon)$ of the total mass for Earth gives

$$1 + \epsilon = \frac{17}{16} \quad (6.6.2)$$

in an excellent approximation. This gives for the fraction of the visible matter the estimate $\epsilon = 1/16 \simeq 6$ per cent. The estimate for the fraction of visible matter in cosmos is about 4 per cent so that estimate is reasonable and would mean that most of planetary and solar mass would be also dark (as a matter dark energy would be in question).

That $v_0(eff) = v_0/(1 - \epsilon) \simeq 4.6 \times 10^{-4}$ equals with $v_0(eff) = 1/(2^7 \times F_2) = 4.5956 \times 10^{-4}$ within the experimental accuracy suggests a number theoretical explanation for the visible-to-dark fraction.

The original unconsciously performed identification of the gravitational and inertial Planck constants leads to some confusing conclusions but it seems that the new view about the quantization of Planck constants resolves these problems and allows to see $\hbar_{gr}$ as a special case of $\hbar_I$.

1. $\hbar_{gr}$ is proportional to the product of masses of interacting systems and not a universal constant like $\hbar$. One can however express the gravitational Bohr conditions as a quantization of circulation $\oint v \cdot dl = n(GM/v_0)\hbar_0$ so that the dependence on the planet mass disappears as required by Equivalence Principle. This would suggest that gravitational Bohr rules relate to velocity rather than inertial momentum as is indeed natural. The quantization of circulation is consistent with the basic prediction that space-time surfaces are analogous to Bohr orbits.

2. $\hbar_{gr}$ seems to characterize a relationship between planet and central mass and quite generally between two systems with the property that smaller system is topologically condensed at the space-time sheet of the larger system. Thus it would seem that $\hbar_{gr}$ is not a universal constant and cannot correspond to a special value of ordinary Planck constant. Certainly this would be the case if $\hbar_I$ is quantized as λ^k-multiplet of ordinary Planck constant with $\lambda \simeq 2^{11}$.

The recent view about the quantization of Planck constant in terms of coverings of CD seems to resolve these problems.

1. The integer quantization of Planck constants is consistent with the huge values of gravitational Planck constant within experimental resolution and the killer test for $\hbar = \hbar_{gr}$ emerges if one takes seriously the stronger prediction $\hbar_{gr} = n_{F,a}/n_{F,b}$.

2. One can also regard $\hbar_{gr}$ as ordinary Planck constant $\hbar_{eff}$ associated with the space-time sheet along which the masses interact provided each pair (M, m_i) of masses is characterized by its own sheets. These sheets could correspond to flux tube like structures carrying the gravitational flux of dark matter. If these sheets corresponds to n_{F_a}-fold covering of CD, one can understand $\hbar_{gr}$ as a particular instance of the $\hbar_{eff}$.

Quantum Hall effect and dark anyonic systems in astrophysical scales

Bohr orbitology could be understood if dark matter concentrates on 2-dimensional partonic surfaces usually assigned with elementary particles and having size of order CP_2 radius. The interpretation is in terms of wormhole throats assignable to topologically condensed CP_2 type extremals (fermions) and pairs of them assignable to wormhole contacts (gauge bosons). Wormhole throat defines the light-like 3-surface at which the signature of metric of space-time surface changes from Minkowskian to Euclidian.

Large value of Planck constant would allow partons with astrophysical size. Since anyonic systems are 2-dimensional, the natural idea is that dark matter corresponds to systems carrying large fermion number residing at partonic 2-surfaces of astrophysical size and that visible matter condenses around these. Not only black holes but also ordinary stars, planetary systems, and planets could correspond at the level of dark matter to atom like structures consisting of anyonic 2-surfaces which can have complex topology (flux tubes associated with planetary orbits connected by radial flux tubes to the central spherical anyonic surface). Charge and spin fractionization are key features of anyonic systems and Jones inclusions inspiring the generalization of embedding space indeed involve quantum groups central in the modelling of anyonic systems. Hence one has could hopes that a coherent theoretical picture could emerge along these lines.

This seems to be the case. Anyons and charge and spin fractionization are discussed in detail [K85] and leads to a precise identification of the delicacies involved with the Kähler gauge potential of CP_2 Kähler form in the sectors of the generalized embedding space corresponding to various pages of boook like structures assignable to CD and CP_2. The basic outcome is that anyons correspond geometrically to partonic 2-surfaces at the light-like boundaries of CD containing the tip of CD inside them. This is what gives rise to charge fractionization and also to confinement like effects since elementary particles in anyonic states cannot as such leak to the other pages of the generalized embedding space. G_a and G_b invariance of the states imply that fractionization occurs only at single particle level and total charge is integer valued.

This picture is much more flexible that based on G_a symmetries of CD orbifold since partonic 2-surfaces do not possess any orbifold symmetries in CD sector anymore. In this framework various astrophysical structures such as spokes and circles would be parts of anyonic 2-surfaces with complex topology representing quantum geometrically quantum coherence in the scale of say solar system. Planets would have formed by the condensation of ordinary matter in the vicinity of the anyonic matter. This would predict stars, planetary system, and even planets to have onion-like structure consisting of shells at the level of dark matter. Similar conclusion is suggested also by purely classical model for the final state of star predicting that matter is strongly concentrated at the surface of the star [K119].

Anyonic view about blackholes

A new element to the model of black hole comes from the vision that black hole horizon as a light-like 3-surface corresponds to a light-like orbit of light-like partonic 2-surface. This allows two kinds of black holes. Fermion like black hole would correspond to a deformed CP_2 type extremal which Euclidian signature of metric and topologically condensed at a space-time sheet with a Minkowskian signature. Boson like black hole would correspond to a wormhole contact connecting two space-time sheets with Minkowskian signature. Wormhole contact would be a piece deformed CP_2 type extremal possessing two light-like throats defining two black hole horizons very near to each other. It does not seem absolutely necessary to assume that the interior metric of the black-hole is realized in another space-time sheet with Minkowskian signature.

Second new element relates to the value of Planck constant. For $\hbar_{gr} = 4GM^2$ the Planck length $L_P(\hbar) = \sqrt{\hbar G}$ equals to Schwartschild radius and Planck mass equals to $M_P(\hbar) = \sqrt{\hbar/G} =$

$2M$. If the mass of the system is below the ordinary Planck mass: $M \leq m_P(\hbar_0)/2 = \sqrt{\hbar_0/4G}$, gravitational Planck constant is smaller than the ordinary Planck constant.

Black hole surface contains ultra dense matter so that perturbation theory is not expected to converge for the standard value of Planck constant but do so for gravitational Planck constant. If the phase transition increasing Planck constant is a friendly gesture of Nature making perturbation theory convergent, one expects that only the black holes for which Planck constant is such that $GM^2/4\pi\hbar < 1$ holds true are formed. Black hole entropy -being proportional to $1/\hbar$- is of order unity so that TGD black holes are not very entropic.

If the partonic 2-surface surrounds the tip of causal diamond CD, the matter at its surface is in anyonic state with fractional charges. Anyonic black hole can be seen as single gigantic elementary particle stabilized by fractional quantum numbers of the constituents preventing them from escaping from the system and transforming to ordinary visible matter. A huge number of different black holes are possible for given value of $\hbar$ since there is infinite variety of pairs (n_a, n_b) of integers giving rise to same value of $\hbar$.

One can imagine that the partonic surface is not exact sphere except for ideal black holes but contains large number of magnetic flux tubes giving rise to handles. Also a pair of spheres with different radii can be considered with surfaces of spheres connected by braided flux tubes. The braiding of these handles can represent information and one can even consider the possibility that black hole can act as a topological quantum computer. There would be no sharp difference between the dark parts of black holes and those of ordinary stars. Only the volume containing the complex flux tube structures associated with the orbits of planets and various objects around star would become very small for black hole so that the black hole might code for the topological information of the matter collapsed into it.

6.6.3 Accelerating Periods Of Cosmic Expansion As PhaseTransitions Increasing The Value Of Planck Constant

There are several pieces of evidence for accelerated expansion, which need not mean cosmological constant, although this is the interpretation adopted in [E6, E2]. Quantum cosmology predicts that astrophysical objects do not follow cosmic expansion except in jerk-wise quantum leaps increasing the value of the gravitational Planck constant. This assumption provides explanation for the apparent cosmological constant. Also planets are predicted to expand in this manner. This provides a new version of Expanding Earth theory originally postulated to explain the intriguing findings suggesting that continents have once formed a connected continent covering the entire surface of Earth but with radius which was one half of the recent one.

The four pieces of evidence for accelerated expansion

1. Supernovas of type Ia

Supernovas of type Ia define standard candles since their luminosity varies in an oscillatory manner and the period is proportional to the luminosity. The period gives luminosity and from this the distance can be deduced by using Hubble's law: $d = cz/H_0$, H_0 Hubble's constant. The observation was that the farther the supernova was the more dimmer it was as it should have been. In other words, Hubble's constant increased with distance and the cosmic expansion was accelerating rather than decelerating as predicted by the standard matter dominated and radiation dominated cosmologies.

2. Mass density is critical and 3-space is flat

It is known that the contribution of ordinary and dark matter explaining the constant velocity of distance stars rotating around galaxy is about 25 per cent from the critical density. Could it be that total mass density is critical?

From the anisotropy of cosmic microwave background one can deduce that this is the case. What criticality means geometrically is that 3-space defined as surface with constant value of cosmic time is flat. This reflects in the spectrum of microwave radiation. The spots representing small anisotropies in the microwave background temperature is 1 degree and this correspond to flat 3-space. If one had dark matter instead of dark energy the size of spot would be.5 degrees!

Thus in a cosmology based on general relativity cosmological constant remains the only viable option. The situation is different in TGD based quantum cosmology based on sub-manifold gravity and hierarchy of gravitational Planck constants.

3. The energy density of vacuum is constant in the size scale of big voids

It was observed that the density of dark energy would be constant in the scale of 10^8 light years. This length scale corresponds to the size of big voids containing galaxies at their boundaries.

4. Integrated Sachs-Wolf effect

Also so called integrated Integrated Sachs-Wolf effect supports accelerated expansion. Very slow variations of mass density are considered. These correspond to gravitational potentials. Cosmic expansion tends to flatten them but mass accretion to form structures compensates this effect so that gravitational potentials are unaffected and there is no effect of CMB. Situation changes if dark matter is replaced with dark energy the accelerated expansion flattening the gravitational potentials wins the tendency of mass accretion to make them deeper. Hence if photon passes by an over-dense region, it receives a little energy. Similarly, photon loses energy when passign by an under-dense region. This effect has been observed.

Accelerated expansion in classical TGD

The minimum TGD based explanation for accelerated expansion involves only the fact that the embeddings of critical cosmologies correspond to accelerated expansion. A more detailed model allows to understand why the critical cosmology appears during some periods.

The first observation is that critical cosmologies (flat 3-space) imbeddable to 8-D embedding space H correspond to negative pressure cosmologies and thus to accelerating expansion. The negativity of the counterpart of pressure in Einstein tensor is due to the fact that space-time sheet is forced to be a 4-D surface in 8-D embedding space. This condition is analogous to a force forcing a particle at the surface of 2-sphere and gives rise to what could be called constraint force. Gravitation in TGD is sub-manifold gravitation whereas in GRT it is manifold gravitation. This would be minimum interpretation involving no assumptions about what mechanism gives rise to the critical periods.

Accelerated expansion and hierarchy of Planck constants

One can go one step further and introduce the hierarchy of Planck constants. The basic difference between TGD and GRT based cosmologies is that TGD cosmology is quantum cosmology. Smooth cosmic expansion is replaced by an expansion occurring in discrete jerks corresponding to the increase of gravitational Planck constant. At space-time level this means the replacement of 8-D embedding space H with a book like structure containing almost-copies of H with various values of Planck constant as pages glued together along critical manifold through which space-time sheet can leak between sectors with different values of $\hbar$. This process is the geometric correlate for the phase transition changing the value of Planck constant.

During these phase transition periods critical cosmology applies and predicts automatically accelerated expansion. Neither genuine negative pressure due to "quintessence" nor cosmological constant is needed. Note that quantum criticality replaces inflationary cosmology and predicts a unique cosmology apart from single parameter. Criticality also explains the fluctuations in microwave temperature as long range fluctuations characterizing criticality.

Accelerated expansion and flatness of 3-cosmology

Observations 1) and 2) about super-novae and critical cosmology (flat 3-space) are consistent with this cosmology. In TGD dark energy must be replaced with dark matter because the mass density is critical during the phase transition. This does not lead to wrong sized spots since it is the increase of Planck constant which induces the accelerated expansion understandable also as a constraint force due to embedding to H.

The size of large voids is the characteristic scale

The TGD based model in its simplest form model assigns the critical periods of expansion to large voids of size 10^8 ly. Also larger and smaller regions can express similar periods and dark space-time sheets are expected to obey same universal "cosmology" apart from a parameter characterizing the duration of the phase transition. Observation 3) that just this length scale defines the scale below which dark energy density is constant is consistent with TGD based model.

The basic prediction is jerk-wise cosmic expansion with jerks analogous to quantum transitions between states of atom increasing the size of atom. The discovery of large voids with size of order 10^8 ly but age much longer than the age of galactic large voids conforms with this prediction. One the other hand, it is known that the size of galactic clusters has not remained constant in very long time scale so that jerk-wise expansion indeed seems to occur.

Do cosmic strings with negative gravitational mass cause the phase transition inducing accelerated expansion

Quantum classical correspondence is the basic principle of quantum TGD and suggest that the effective antigravity manifested by accelerated expansion might have some kind of concrete space-time correlate. A possible correlate is super heavy cosmic string like objects at the center of large voids which have negative gravitational mass under very general assumptions. The repulsive gravitational force created by these objects would drive galaxies to the boundaries of large voids. At some state the pressure of galaxies would become too strong and induce a quantum phase transition forcing the increase of gravitational Planck constant and expansion of the void taking place much faster than the outward drift of the galaxies. This process would repeat itself. In the average sense the cosmic expansion would not be accelerating.

6.6.4 Phase Transition Changing Planck Constant And Expanding Earth Theory

TGD predicts that cosmic expansion at the level of individual astrophysical systems does not take place continuously as in classical gravitation but through discrete quantum phase transitions increasing gravitational Planck constant and thus various quantum length and time scales. The reason would be that stationary quantum states for dark matter in astrophysical length scales cannot expand. One would have the analog of atomic physics in cosmic scales. Increases of $\hbar$ by a power of two are favored in these transitions but also other scalings are possible.

This has quite far reaching implications.

1. These periods have a highly unique description in terms of a critical cosmology for the expanding space-time sheet. The expansion is accelerating. The accelerating cosmic expansion can be assigned to this kind of phase transition in some length scale (TGD Universe is fractal). There is no need to introduce cosmological constant and dark energy would be actually dark matter.
2. The recently observed void which has same size of about 10^8 light years as large voids having galaxies near their boundaries but having an age which is much higher than that of the large voids, would represent one example of jerk-wise expansion.
3. This picture applies also to solar system and planets might be perhaps seen as having once been parts of a more or less connected system, the primordial Sun. The Bohr orbits for inner and outer planets correspond to gravitational Planck constant which is 5 times larger for outer planets. This suggests that the space-time sheet of outer planets has suffered a phase transition increasing the size scale by a factor of 5. Earth can be regarded either as n=1 orbit for Planck constant associated with outer planets or n= 5 orbit for inner planetary system. This might have something to do with the very special position of Earth in planetary system. One could even consider the possibility that both orbits are present as dark matter structures. The phase transition would also explain why n=1 and n=2 Bohr orbits are absent and one only n=3, 4, and 5 are present.
4. Also planets should have experienced this kind of phase transitions increasing the radius: the increase by a factor two would be the simplest situation.

The obvious question - that I did not ask - is whether this kind of phase transition might have occurred for Earth and led from a completely granite covered Earth - Pangeia without seas - to the recent Earth. Neither it did not occur to me to check whether there is any support for a rapid expansion of Earth during some period of its history.

Situation changed when my son visited me last Saturday and told me about a Youtube video [?] by Neal Adams, an American comic book and commercial artist who has also produced animations for geologists. We looked the amazing video a couple of times and I looked it again yesterday. The video is very impressive artwork but in the lack of references skeptic probably cannot avoid the feeling that Neal Adams might use his highly developed animation skills to cheat you. I found also a polemic article [?] of Adams but again the references were lacking. Perhaps the reason of polemic tone was that the concrete animation models make the expanding Earth hypothesis very convincing but geologists refuse to consider seriously arguments by a layman without a formal academic background.

The claims of Adams

The basic claims of Adams were following.

1. The radius of Earth has increased during last 185 million years (dinosaurs [I1] appeared for about 230 million years ago) by about factor 2. If this is assumed all continents have formed at that time a single super-continent, Pangeia, filling the entire Earth surface rather than only 1/4 of it since the total area would have grown by a factor of 4. The basic argument was that it is very difficult to imagine Earth with 1/4 of surface containing granite and 3/4 covered by basalt. If the initial situation was covering by mere granite -as would look natural- it is very difficult for a believer in thermodynamics to imagine how the granite would have gathered to a single connected continent.
2. Adams claims that Earth has grown by keeping its density constant, rather than expanded, so that the mass of Earth has grown linearly with radius. Gravitational acceleration would have thus doubled and could provide a partial explanation for the disappearance of dinosaurs: it is difficult to cope in evolving environment when you get slower all the time.
3. Most of the sea floor is very young and the areas covered by the youngest basalt are the largest ones. This Adams interprets this by saying that the expansion of Earth is accelerating. The alternative interpretation is that the flow rate of the magma slows down as it recedes from the ridge where it erupts. The upper bound of 185 million years for the age of sea floor requires that the expansion period - if it is already over - lasted about 185 million years after which the flow increasing the area of the sea floor transformed to a convective flow with subduction so that the area is not increasing anymore.
4. The fact that the continents fit together - not only at the Atlantic side - but also at the Pacific side gives strong support for the idea that the entire planet was once covered by the super-continent. After the emergence of subduction theory this evidence as been dismissed.
5. I am not sure whether Adams mentions the following objections [?]. Subduction only occurs on the other side of the subduction zone so that the other side should show evidence of being much older in the case that oceanic subduction zones are in question. This is definitely not the case. This is explained in plate tectonics as a change of the subduction direction. My explanation would be that by the symmetry of the situation both oceanic plates bend down so that this would represent new type of boundary not assumed in the tectonic plate theory.
6. As a master visualizer Adams notices that Africa and South-America do not actually fit together in absence of expansion unless one assumes that these continents have suffered a deformation. Continents are not easily deformable stuff. The assumption of expansion implies a perfect fit of *all* continents without deformation.

Knowing that the devil is in the details, I must admit that these arguments look rather convincing to me and what I learned from Wikipedia articles supports this picture.

The critic of Adams of the subduction mechanism

The prevailing tectonic plate theory [?] has been compared to the Copernican revolution in geology. The theory explains the young age of the seafloor in terms of the decomposition of the litosphere to tectonic plates and the convective flow of magma to which oceanic tectonic plates participate. The magma emerges from the crests of the mid ocean ridges representing a boundary of two plates and leads to the expansion of sea floor. The variations of the polarity of Earth's magnetic field coded in sea floor provide a strong support for the hypothesis that magma emerges from the crests.

The flow back to would take place at so called oceanic trenches [?] near continents which represent the deepest parts of ocean. This process is known as subduction. In subduction oceanic tectonic plate bends and penetrates below the continental tectonic plate, the material in the oceanic plate gets denser and sinks into the magma. In this manner the oceanic tectonic plate suffers a metamorphosis returning back to the magma: everything which comes from Earth's interior returns back. Subduction mechanism explains elegantly formation of mountains [?] (orogeny), earth quake zones, and associated zones of volcanic activity [?] .

Adams is very polemic about the notion of subduction, in particular about the assumption that it generates steady convective cycle. The basic objections of Adams against subduction are following.

1. There are not enough subduction zones to allow a steady situation. According to Adams, the situation resembles that for a flow in a tube which becomes narrower. In a steady situation the flow should accelerate as it approaches subduction zones rather than slow down. Subduction zones should be surrounded by large areas of sea floor with constant age. Just the opposite is suggested by the fact that the youngest portion of sea-floor near the ridges is largest. The presence of zones at which both ocean plates bend down could improve the situation. Also jamming of the flow could occur so that the thickness of oceanic plate increases with the distance from the eruption ridge. Jamming could increase also the density of the oceanic plate and thus the effectiveness of subduction.
2. There is no clear evidence that subduction has occurred at other planets. The usual defense is that the presence of sea is essential for the subduction mechanism.
3. One can also wonder what is the mechanism that led to the formation of single super continent Pangeia covering 1/4 of Earth's surface. How probable the gathering of all separate continents to form single cluster is? The later events would suggest that just the opposite should have occurred from the beginning.

Expanding Earth theories are not new

After I had decided to check the claims of Adams, the first thing that I learned is that Expanding Earth theory [?], whose existence Adams actually mentions, is by no means new. There are actually many of them.

The general reason why these theories were rejected by the main stream community was the absence of a convincing physical mechanism of expansion or of growth in which the density of Earth remains constant.

1. 1888 Yarkovski postulated some sort of aether absorbed by Earth and transforming to chemical elements (TGD version of aether could be dark matter). 1909 Mantovani postulated thermal expansion but no growth of the Earth's mass.
2. Paul Dirac's idea about changing Planck constant led Pascual Jordan in 1964 to a modification of general relativity predicting slow expansion of planets. The recent measurement of the gravitational constant imply that the upper bound for the relative change of gravitational constant is 10 time too small to produce large enough rate of expansion. Also many other theories have been proposed but they are in general conflict with modern physics.
3. The most modern version of Expanding Earth theory is by Australian geologist Samuel W. Carey. He calculated that in Cambrian period (about 500 million years ago) all continents were stuck together and covered the entire Earth. Deep seas began to evolve then.

Summary of TGD based theory of Expanding Earth

TGD based model differs from the tectonic plate model but allows subduction which cannot imply considerable back-flow of magma. Let us sum up the basic assumptions and implications.

1. The expansion is or was due to a quantum phase transition increasing the value of gravitational Planck constant and forced by the cosmic expansion in the average sense.
2. Tectonic plates do not participate to the expansion and therefore new plate must be formed and the flow of magma from the crests of mid ocean ridges is needed. The decomposition of a single plate covering the entire planet to plates to create the mid ocean ridges is necessary for the generation of new tectonic plate. The decomposition into tectonic plates is thus prediction rather than assumption.
3. The expansion forced the decomposition of Pangeia super-continent covering entire Earth for about 530 million years ago to split into tectonic plates which began to recede as new non-expanding tectonic plate was generated at the ridges creating expanding sea floor. The initiation of the phase transition generated formation of deep seas.
4. The eruption of plasma from the crests of ocean ridges generated oceanic tectonic plates which did not participate to the expansion by density reduction but by growing in size. This led to a reduction of density in the interior of the Earth roughly by a factor 1/8. From the upper bound for the age of the seafloor one can conclude that the period lasted for about 185 million years after which it transformed to convective flow in which the material returned back to the Earth interior. Subduction at continent-ocean floor boundaries and downwards double bending of tectonic plates at the boundaries between two ocean floors were the mechanisms. Thus tectonic plate theory would be more or less the correct description for the recent situation.
5. One can consider the possibility that the subducted tectonic plate does not transform to magma but is fused to the tectonic layer below continent so that it grows to an iceberg like structure. This need not lead to a loss of the successful predictions of plate tectonics explaining the generation of mountains, earthquake zones, zones of volcanic activity, etc...
6. From the video of Adams it becomes clear that the tectonic flow is East-West asymmetric in the sense that the western side is more irregular at large distances from the ocean ridge at the western side. If the magma rotates with slightly lower velocity than the surface of Earth (like liquid in a rotating vessel), the erupting magma would rotate slightly slower than the tectonic plate and asymmetry would be generated.
7. If the planet has not experienced a phase transition increasing the value of Planck constant, there is no need for the decomposition to tectonic plates and one can understand why there is no clear evidence for tectonic plates and subduction in other planets. The conductive flow of magma could occur below this plate and remain invisible.

The biological implications might provide a possibility to test the hypothesis.

1. Great steps of progress in biological evolution are associated with catastrophic geological events generating new evolutionary pressures forcing new solutions to cope in the new situation. Cambrian explosion indeed occurred about 530 years ago (the book "Wonderful Life" of Stephen Gould [I25] explains this revolution in detail) and led to the emergence of multicellular creatures, and generated huge number of new life forms living in seas. Later most of them suffered extinction: large number of phylae and groups emerged which are not present nowadays.

   Thus Cambrian explosion is completely exceptional as compared to all other dramatic events in the evolution in the sense that it created something totally new rather than only making more complex something which already existed. Gould also emphasizes the failure to identify any great change in the environment as a fundamental puzzle of Cambrian explosion. Cambrian explosion is also regarded in many quantum theories of consciousness (including TGD) as a revolution in the evolution of consciousness: for instance, micro-tubuli emerged at this time. The periods of expansion might be necessary for the emergence of multicellular life forms on planets and the fact that they unavoidably occur sooner or later suggests that also life develops unavoidably.
2. TGD predicts a decrease of the surface gravity by a factor 1/4 during this period. The reduction of the surface gravity would have naturally led to the emergence of dinosaurs 230 million years ago as a response coming 45 million years after the accelerated expansion ceased. Other reasons led then to the decline and eventual catastrophic disappearance of the dinosaurs.

The reduction of gravity might have had some gradually increasing effects on the shape of organisms also at microscopic level and manifest itself in the evolution of genome during expansion period.

3. A possibly testable prediction following from angular momentum conservation ($\omega R^2 = constant$) is that the duration of day has increased gradually and was four times shorter during the Cambrian era. For instance, genetically coded bio-clocks of simple organisms during the expansion period could have followed the increase of the length of day with certain lag or failed to follow it completely. The simplest known circadian clock is that of the prokaryotic cyanobacteria. Recent research has demonstrated that the circadian clock of Synechococcus elongatus can be reconstituted in vitro with just the three proteins of their central oscillator. This clock has been shown to sustain a 22 hour rhythm over several days upon the addition of ATP: the rhythm is indeed faster than the circadian rhythm. For humans the average innate circadian rhythm is however 24 hours 11 minutes and thus conforms with the fact that human genome has evolved much later than the expansion ceased.

4. Scientists have found a fossil of a sea scorpion with size of 2.5 meters [I4], which has lived for about 10 million years for 400 million years ago in Germany. The gigantic size would conform nicely with the much smaller value of surface gravity at that time. The finding would conform nicely with the much smaller value of surface gravity at that time. Also the emergence of trees could be understood in terms of a gradual growth of the maximum plant size as the surface gravity was reduced. The fact that the oldest known tree fossil is 385 million years old [I19] conforms with this picture.

Did intra-terrestrial life burst to the surface of Earth during Cambrian expansion?

The possibility of intra-terrestrial life [**?**] is one of the craziest TGD inspired ideas about the evolution of life and it is quite possible that in its strongest form the hypothesis is unrealistic. One can however try to find what one obtains from the combination of the IT hypothesis with the idea of pre-Cambrian granite Earth. Could the harsh pre-Cambrian conditions have allowed only intra-terrestrial multicellular life? Could the Cambrian explosion correspond to the moment of birth for this life in the very concrete sense that the magma flow brought it into the day-light?

1. Gould emphasizes the mysterious fact that very many life forms of Cambrian explosion looked like final products of a long evolutionary process. Could the eruption of magma from the Earth interior have induced a burst of intra-terrestrial life forms to the Earth's surface? This might make sense: the life forms living at the bottom of sea do not need direct solar light so that they could have had intra-terrestrial origin. It is quite possible that Earth's mantle contained low temperature water pockets, where the complex life forms might have evolved in an environment shielded from meteoric bombardment and UV radiation.

2. Sea water is salty. It is often claimed that the average salt concentration inside cell is that of the primordial sea: I do not know whether this claim can be really justified. If the claim is true, the cellular salt concentration should reflect the salt concentration of the water inside the pockets. The water inside water pockets could have been salty due to the diffusion of the salt from ground but need not have been same as that for the ocean water (higher than for cell interior and for obvious reasons). Indeed, the water in the underground reservoirs in arid regions such as Sahara is salty, which is the reason for why agriculture is absent in these regions. Note also that the cells of marine invertebrates are osmoconformers able to cope with the changing salinity of the environment so that the Cambrian revolutionaries could have survived the change in the salt concentration of environment.

3. What applies to Earth should apply also to other similar planets and Mars [L80] is very similar to Earth. The radius is .533 times that for Earth so that after quantum leap doubling the radius and thus Schumann frequency scale (7.8 Hz would be the lowest Schumann frequency) would be essentially same as for Earth now. Mass is.131 times that for Earth so that surface gravity would be.532 of that for Earth now and would be reduced to.131 meaning quite big dinosaurs! have learned that Mars probably contains large water reservoirs in it's interior and that there is an un-identified source of methane gas usually assigned with the presence of life. Could it be that Mother Mars is pregnant and just waiting for the great quantum leap when

it starts to expand and gives rise to a birth of multicellular life forms. Or expressing freely how Bible describes the moment of birth: in the beginning there was only darkness and water and then God saidLet the light come!

To sum up, TGD would provide only the long sought mechanism of expansion and a possible connection with the biological evolution. It would be indeed fascinating if Planck constant changing quantum phase transitions in planetary scale would have profoundly affected the biosphere.

6.6.5 Allais Effect As Evidence For Large Values Of Gravitational Planck Constant?

Allais effect [E1, E24] is a fascinating gravitational anomaly associated with solar eclipses. It was discovered originally by M. Allais, a Nobelist in the field of economy, and has been reproduced in several experiments but not as a rule. The experimental arrangement uses so called paraconical pendulum, which differs from the Foucault pendulum in that the oscillation plane of the pendulum can rotate in certain limits so that the motion occurs effectively at the surface of sphere.

Experimental findings

Consider first a brief summary of the findings of Allais and others [E24].

a) In the ideal situation (that is in the absence of any other forces than gravitation of Earth) paraconical pendulum should behave like a Foucault pendulum. The oscillation plane of the paraconical pendulum however begins to rotate.

b) Allais concludes from his experimental studies that the orbital plane approach always asymptotically to a limiting plane and the effect is only particularly spectacular during the eclipse. During solar eclipse the limiting plane contains the line connecting Earth, Moon, and Sun. Allais explains this in terms of what he calls the anisotropy of space.

c) Some experiments carried out during eclipse have reproduced the findings of Allais, some experiments not. In the experiment carried out by Jeverdan and collaborators in Romania it was found that the period of oscillation of the pendulum decreases by $\Delta f/f \simeq 5\times 10^{-4}$ [E1, E22] which happens to correspond to the constant $v_0 = 2^{-11}$ appearing in the formula of the gravitational Planck constant. It must be however emphasized that the overall magnitude of $\Delta f/f$ varies by five orders of magnitude. Even the sign of $\Delta f/f$ varies from experiment to experiment.

d) There is also quite recent finding by Popescu and Olenici, which they interpret as a quantization of the plane of oscillation of paraconical oscillator during solar eclipse [E26].

TGD based models for Allais effect

I have already earlier proposed an explanation of the effect in terms of classical Z^0 force [K15]. If the Z^0 charge to mass ratio of pendulum varies and if Earth and Moon are Z^0 conductors, the resulting model is quite flexible and one might hope it could explain the high variation of the experimental results.

The rapid variation of the effect during the eclipse is however a problem for this approach and suggests that gravitational screening or some more general interference effect might be present. Gravitational screening alone cannot however explain Allais effect.

A model based on the idea that gravitational interaction is mediated by topological light rays (MEs) and that gravitons correspond to a gigantic value of the gravitational Planck constant however explains the Allais effect as an interference effect made possible by macroscopic quantum coherence in astrophysical length scales. Equivalence Principle fixes the model to a high degree and one ends up with an explicit formula for the anomalous gravitational acceleration and the general order of magnitude and the large variation of the frequency change as being due to the variation of the distance ratio $r_{S,P}/r_{M,P}$ (S, M, and P refer to Sun, Moon, and pendulum respectively). One can say that the pendulum acts as an interferometer.

6.6.6 Applications To Elementary Particle Physics, Nuclear Physics, And Condensed Matter Physics

The hierarchy of Planck constants could have profound implications for even elementary particle physics since the strong constraints on the existence of new light particles coming from the decay widths of intermediate gauge bosons can be circumvented because direct decays to dark matter are not possible. On the other hand, if light scaled versions of elementary particles exist they must be dark since otherwise their existence would be visible in these decay widths. The constraints on the existence of dark nuclei and dark condensed matter are much milder. Cold fusion and some other anomalies of nuclear and condensed matter physics - in particular the anomalies of water- might have elegant explanation in terms of dark nuclei.

Leptohadron hypothesis

TGD suggests strongly the existence of lepto-hadron [K117]. Lepto-hadrons are bound states of color excited leptons and the anomalous production of e^+e^- pairs in heavy ion collisions finds a nice explanation as resulting from the decays of lepto-hadrons with basic condensate level $k = 127$ and having typical mass scale of one MeV. The recent indications on the existence of a new fermion with quantum numbers of muon neutrino and the anomaly observed in the decay of orto-positronium give further support for the lepto-hadron hypothesis. There is also evidence for anomalous production of low energy photons and e^+e^- pairs in hadronic collisions.

The identification of lepto-hadrons as a particular instance in the predicted hierarchy of dark matters interacting directly only via graviton exchange allows to circumvent the lethal counter arguments against the lepto-hadron hypothesis (Z^0 decay width and production of colored lepton jets in e^+e^- annihilation) even without assumption about the loss of asymptotic freedom.

PCAC hypothesis and its sigma model realization lead to a model containing only the coupling of the lepto-pion to the axial vector current as a free parameter. The prediction for e^+e^- production cross section is of correct order of magnitude only provided one assumes that lepto-pions (or electro-pions) decay to lepto-nucleon pair $e^+_{ex}e^-_{ex}$ first and that lepto-nucleons, having quantum numbers of electron and having mass only slightly larger than electron mass, decay to lepton and photon. The peculiar production characteristics are correctly predicted. There is some evidence that the resonances decay to a final state containing $n > 2$ particle and the experimental demonstration that lepto-nucleon pairs are indeed in question, would be a breakthrough for TGD.

During 18 years after the first published version of the model also evidence for colored μ has emerged [C15]. Towards the end of 2008 CDF anomaly [C7] gave a strong support for the colored excitation of τ. The lifetime of the light long lived state identified as a charged τ-pion comes out correctly and the identification of the reported 3 new particles as p-adically scaled up variants of neutral τ-pion predicts their masses correctly. The observed muon jets can be understood in terms of the special reaction kinematics for the decays of neutral τ-pion to 3 τ-pions with mass scale smaller by a factor 1/2 and therefore almost at rest. A spectrum of new particles is predicted. The discussion of CDF anomaly [K117] led to a modification and generalization of the original model for lepto-pion production and the predicted production cross section is consistent with the experimental estimate.

Cold fusion, plasma electrolysis, and burning salt water

The article of Kanarev and Mizuno [D29] reports findings supporting the occurrence of cold fusion in NaOH and KOH hydrolysis. The situation is different from standard cold fusion where heavy water D_2O is used instead of H_2O.

In nuclear string model nucleon are connected by color bonds representing the color magnetic body of nucleus and having length considerably longer than nuclear size. One can consider also dark nuclei for which the scale of nucleus is of atomic size [L1], [L1]. In this framework can understand the cold fusion reactions reported by Mizuno as nuclear reactions in which part of what I call dark proton string having negatively charged color bonds (essentially a zoomed up variant of ordinary nucleus with large Planck constant) suffers a phase transition to ordinary matter and experiences ordinary strong interactions with the nuclei at the catode. In the simplest model the final state would contain only ordinary nuclear matter. The generation of plasma in plasma electrolysis can be seen as a process analogous to the positive feedback loop in ordinary nuclear reactions.

Rather encouragingly, the model allows to understand also deuterium cold fusion and leads to a solution of several other anomalies.

1. The so called lithium problem of cosmology (the observed abundance of lithium is by a factor 2.5 lower than predicted by standard cosmology [E11]) can be resolved if lithium nuclei transform partially to dark lithium nuclei.
2. The so called $H_{1.5}O$ anomaly of water [D31, D25, D35, D18] can be understood if 1/4 of protons of water forms dark lithium nuclei or heavier dark nuclei formed as sequences of these just as ordinary nuclei are constructed as sequences of ^4He and lighter nuclei in nuclear string model. The results force to consider the possibility that nuclear isotopes unstable as ordinary matter can be stable dark matter.
3. The mysterious behavior burning salt water [D1] can be also understood in the same framework.
4. The model explains the nuclear transmutations observed in Kanarev's plasma electrolysis. This kind of transmutations have been reported also in living matter long time ago [C6, C19]. Intriguingly, several biologically important ions belong to the reaction products in the case of NaOH electrolysis. This raises the question whether cold nuclear reactions occur in living matter and are responsible for generation of biologically most important ions.

6.6.7 Applications To Biology And Neuroscience

The notion of field or magnetic body regarded as carrier of dark matter with large Planck constant and quantum controller of ordinary matter is the basic idea in the TGD inspired model of living matter.

Do molecular symmetries in living matter relate to non-standard values of Planck constant?

Water is exceptional element and the possibility that G_a as symmetry of singular factor space of CD in water and living matter is intriguing.

1. There is evidence for an icosahedral clustering in [D37] [D32]. Synaptic contacts contain clathrin molecules which are truncated icosahedrons and form lattice structures and are speculated to be involved with quantum computation like activities possibly performed by microtubules. Many viruses have the shape of icosahedron. One can ask whether these structures could be formed around templates formed by dark matter corresponding to 120-fold covering of CP_2 points by CD points and having $\hbar(CP_2) = 5\hbar_0$ perhaps corresponding color confined light dark quarks. Of course, a similar covering of CD points by CP_2 could be involved.
2. It should be noticed that single nucleotide in DNA double strands corresponds to a twist of $2\pi/10$ per single DNA triplet so that 10 DNA strands corresponding to length $L(151) = 10$ nm (cell membrane thickness) correspond to $3 \times 2\pi$ twist. This could be perhaps interpreted as evidence for group C_{10} perhaps making possible quantum computation at the level of DNA.
3. What makes realization of G_a as a symmetry of singular factor space of CD is that the biomolecules most relevant for the functioning of brain (DNA nucleotides, amino-acids acting as neurotransmitters, molecules having hallucinogenic effects) contain aromatic 5- and 6-cycles.

These observations led to an identification of the formula for Planck constant (two alternatives were allowed by the condition that Planck constant is algebraic homomorphism) which was not consistent with the model for dark gravitons. If one accepts the proposed formula of Planck constant, the dark space-time sheets with large Planck constant correspond to factor spaces of both $\hat{CD}\backslash M^2$ and of $CP_2\backslash S^2_I$. This identification is of course possible and it remains to be seen whether it leads to any problems. For gravitational space-time sheets only coverings of both CD and CP_2 make sense and the covering group G_a has very large order and does not correspond to geometric symmetries analogous to those of molecules.

High T_c super-conductivity in living matter

The model for high T_c super-conductivity realized as quantum critical phenomenon predicts the basic scales of cell membrane [K23] from energy minimization and p-adic length scale hypothesis. This leads to the vision that cell membrane and possibly also its scaled up dark fractal variants define Josephson junctions generating Josephson radiation communicating information about the nearby environment to the magnetic body.

Any model of high T_c superconductivity should explain various strange features of high T_c superconductors. One should understand the high value of T_c, the ambivalent character of high T_c super conductors suggesting both BCS type Cooper pairs and exotic Cooper pairs with non-vanishing spin, the existence of pseudogap temperature $T_{c_1} > T_c$ and scaling law for resistance for $T_c \leq T < T_{c_1}$, the role of fluctuating charged stripes which are anti-ferromagnetic defects of a Mott insulator, the existence of a critical doping, etc... [D24, D8].

There are reasons to believe that high T_c super-conductors correspond to quantum criticality in which at least two (cusp catastrophe as in van der Waals model), or possibly three or even more phases, are competing. A possible analogy is provided by the triple critical point for water vapor, liquid phase and ice coexist. Instead of long range thermal fluctuations long range quantum fluctuations manifesting themselves as fluctuating stripes are present [D24].

The TGD based model for high T_c super-conductivity [K23] relies on the notions of quantum criticality, general ideas of catastrophe theory, dynamical Planck constant, and many-sheeted space-time. The 4-dimensional spin glass character of space-time dynamics deriving from the vacuum degeneracy of the Kähler action defining the basic variational principle would realize space-time correlates for quantum fluctuations.

1. Two kinds of super-conductivities and ordinary non-super-conducting phase would be competing at quantum criticality at T_c and above it only one super-conducting phase and ordinary conducting phase located at stripes representing ferromagnetic defects making possible formation of $S = 1$ Cooper pairs.
2. The first super-conductivity would be based on exotic Cooper pairs of large $\hbar$ dark electrons with $\hbar = 2^{11}\hbar_0$ and able to have spin $S = 1$, angular momentum $L = 2$, and total angular momentum $J = 2$. Second type of super-conductivity would be based on BCS type Cooper pairs having vanishing spin and bound by phonon interaction. Also they have large $\hbar$ so that gap energy and critical temperature are scaled up in the same proportion. The exotic Cooper pairs are possible below the pseudo gap temperature $T_{c_1} > T_c$ but are unstable against decay to BCS type Cooper pairs which above T_c are unstable against a further decay to conduction electrons flowing along stripes. This would reduce the exotic super-conductivity to finite conductivity obeying the observed scaling law for resistance.
3. The mere assumption that electrons of exotic Cooper pairs feed their electric flux to larger space-time sheet via *two* elementary particle sized wormhole contacts rather than only *one* wormhole contacts implies that the throats of wormhole contacts defining analogs of Higgs field must carry quantum numbers of quark and anti-quark. This inspires the idea that cylindrical space-time sheets, the radius of which turns out to be about about 5 nm, representing zoomed up dark electrons of Cooper pair with Planck constant $\hbar = 2^{11}\hbar_0$ are colored and bound by a scaled up variant of color force to form a color confined state. Formation of Cooper pairs would have nothing to do with direct interactions between electrons. Thus high T_c super-conductivity could be seen as a first indication for the presence of scaled up variant of QCD in mesoscopic length scales.

This picture leads to a concrete model for high T_c superconductors as quantum critical superconductors [K23]. p-Adic length scale hypothesis stating that preferred p-adic primes $p \simeq 2^k$, k integer, with primes (in particular Mersenne primes) preferred, makes the model quantitative.

1. An unexpected prediction is that coherence length ξ is actually $\hbar_{eff}/\hbar_0 = 2^{11}$ times longer than the coherence length 5-10 Angstroms deduced theoretically from gap energy using conventional theory and varies in the range $1-5$ μm, the cell nucleus length scale. Hence type I super-conductor would be in question with stripes as defects of anti-ferromagnetic Mott insulator serving as duals for the magnetic defects of type I super-conductor in nearly critical magnetic field.

2. At quantitative level the model reproduces correctly the four poorly understood photon absorption lines and allows to understand the critical doping ratio from basic principles.

3. The current carrying structures have structure locally similar to that of axon including the double layered structure of cell membrane and also the size scales are predicted to be same. One of the characteristic absorption lines has energy of.05 eV which corresponds to the Josephson energy for neuronal membrane for activation potential $V = 50$ mV. Hence the idea that axons are high T_c superconductors is highly suggestive. Dark matter hierarchy coming in powers $\hbar/\hbar_0 = 2^{k11}$ suggests hierarchy of Josephson junctions needed in TGD based model of EEG [K44].

Magnetic body as a sensory perceiver and intentional agent

The hypothesis that dark magnetic body serves as an intentional agent using biological body as a motor instrument and sensory receptor is consistent with Libet's findings about strange time delays of consciousness. Magnetic body would carry cyclotron Bose-Einstein condensates of various ions. Magnetic body must be able to perform motor control and receive sensory input from biological body.

Cell membrane would be a natural sensor providing information about cell interior and exterior to the magnetic body and dark photons at appropriate frequency range would naturally communicate this information. The strange quantitative co-incidences with the physics of cell membrane and high T_c super-conductivity support the idea that Josephson radiation generated by Josephson currents of dark electrons through cell membrane is responsible for this communication [K44].

Also fractally scaled up versions of cell membrane at higher levels of dark matter hierarchy (in particular those corresponding to powers $n = 2^{k11}$) are possible and the model for EEG indeed relies on this hypothesis. The thickness for the fractal counterpart of cell membrane thickness would be 2^{44} fold and of order of depth of ionosphere! Although this looks weird it is completely consistent with the notion of magnetic body as an intentional agent.

Motor control would be most naturally performed via genome: this is achieved if flux sheets traverse through DNA strands. Flux quantization for large values of Planck constant requires rather large widths for the flux sheets. If flux sheet contains sequences of genomes like the page of book contains lines of text, a coherent gene expression becomes possible at level of organs and even populations and one can speak about super- and hyper-genomes. Introns might relate to the collective gene expression possibly realized electromagnetically rather than only chemically [K23, K24].

Dark cyclotron radiation with photon energy above thermal energy could be used for co-ordination purposes at least. The predicted hierarchy of copies of standard model physics leads to ask whether also dark copies of electro-weak gauge bosons and gluons could be important in living matter. As already mentioned, dark W bosons could make possible charge entanglement and non-local quantum bio-control by inducing voltage differences and thus ionic currents in living matter.

The identification of plasmoids as rotating magnetic flux structures carrying dark ions and electrons as primitive life forms is natural in this framework. There exists experimental support for this identification [I22] but the main objection is the high temperature involved: this objection could be circumvented if large $\hbar$ phase is involved. A model for the pre-biotic evolution relying also on this idea is discussed in [**?**].

At the level of biology there are now several concrete applications leading to a rich spectrum of predictions. Magnetic flux quanta would carry charged particles with large Planck constant.

1. The shortening of the flux tubes connecting biomolecules in a phase transition reducing Planck constant could be a basic mechanism of bio-catalysis and explain the mysterious ability of biomolecules to find each other. Similar process in time direction could explain basic aspects of symbolic memories as scaled down representations of actual events.

2. The strange behavior of cell membrane suggests that a dominating portion of important biological ions are actually dark ions at magnetic flux tubes so that ionic pumps and channels are needed only for visible ions. This leads to a model of nerve pulse explaining its unexpected thermodynamical properties with basic properties of Josephson currents making it

un-necessary to use pumps to bring ions back after the pulse. The model predicts automatically EEG as Josephson radiation and explains the synchrony of both kHz radiation and of EEG.

3. The DC currents of Becker could be accompanied by Josephson currents running along flux tubes making possible dissipation free energy transfer and quantum control over long distances and meridians of chinese medicine could correspond to these flux tubes.
4. The model of DNA as topological quantum computer assumes that nucleotides and lipids are connected by ordinary or "wormhole" magnetic flux tubes acting as strands of braid and carrying dark matter with large Planck constant. The model leads to a new vision about TGD in which the assignment of nucleotides to quarks allows to understood basic regularities of DNA not understood from biochemistry.
5. Each physical system corresponds to an onion-like hierarchy of field bodies characterized by p-adic primes and value of Planck constant. The highest value of Planck constant in this hierarchy provides kind of intelligence quotient characterizing the evolutionary level of the system since the time scale of planned action and memory correspond to the temporal distance between tips of corresponding causal diamond (CD). Also the spatial size of the system correlates with the Planck constant. This suggests that great evolutionary leaps correspond to the increase of Planck constant for the highest level of hierarchy of personal magnetic bodies. For instance, neurons would have much more evolved magnetic bodies than ordinary cells.
6. At the level of DNA this vision leads to an idea about hierarchy of genomes. Magnetic flux sheets traversing DNA strands provide a natural mechanism for magnetic body to control the behavior of biological body by controlling gene expression. The quantization of magnetic flux states that magnetic flux is proportional to $\hbar$ and thus means that the larger the value of $\hbar$ is the larger the width of the flux sheet is. For larger values of $\hbar$ single genome is not enough to satisfy this condition. This leads to the idea that the genomes of organs, organism, and even population, can organize like lines of text at the magnetic flux sheets and form in this manner a hierarchy of genomes responsible for a coherent gene expression at level of cell, organ, organism and population and perhaps even entire biosphere. This would also provide a mechanism by which collective consciousness would use its biological body - biosphere.

DNA as topological quantum computer

I ended up with the recent model of TQC in bottom-up manner and this representation is followed also in the text. The model which looks the most plausible one relies on two specific ideas.

1. Sharing of labor means conjugate DNA would do TQC and DNA would "print" the outcome of TQC in terms of mRNA yielding amino-acids in the case of exons. RNA could result also in the case of introns but not always. The experience about computers and the general vision provided by TGD suggests that introns could express the outcome of TQC also electromagnetically in terms of standardized field patterns as Gariaev's findings suggest [I15]. Also speech would be a form of gene expression. The quantum states braid (in zero energy ontology) would entangle with characteristic gene expressions. This argument turned out to be based on a slightly wrong belief about DNA: later I learned that both strand and its conjugate are transcribed but in different directions. The symmetry breaking in the case of transcription is only local which is also visible in DNA replication as symmetry breaking between leading and lagging strand. Thus the idea about *entire* leading strand devoted to printing and second strand to TQC must be weakened appropriately.
2. The manipulation of braid strands transversal to DNA must take place at 2-D surface. Here dancing metaphor for topological quantum computation [C8] generalizes. The ends of the space-like braid are like dancers whose feet are connected by thin threads to a wall so that the dancing pattern entangles the threads. Dancing pattern defines both the time-like braid, the running of classical TQC program and its representation as a dynamical pattern. The space-like braid defined by the entangled threads represents memory storage so that TQC program is automatically written to memory as the braiding of the threads during the TQC. The inner membrane of the nuclear envelope and cell membrane with entire endoplasmic

reticulum included are good candidates for dancing halls. The 2-surfaces containing the ends of the hydrophobic ends of lipids could be the parquets and lipids the dancers. This picture seems to make sense.

One ends up to the model also in top-down manner.

1. Darwinian selection for which standard theory of self-organization [B1] provides a model, should apply also to TQC programs. TQC programs should correspond to asymptotic self-organization patterns selected by dissipation in the presence of metabolic energy feed. The spatial and temporal pattern of the metabolic energy feed characterizes the TQC program - or equivalently - sub-program call.
2. Since braiding characterizes the TQC program, the self-organization pattern should correspond to a hydrodynamical flow or a pattern of magnetic field inducing the braiding. Braid strands must correspond to magnetic flux tubes of the magnetic body of DNA. If each nucleotide is transversal magnetic dipole it gives rise to transversal flux tubes, which can also connect to the genome of another cell.
3. The output of TQC sub-program is probability distribution for the outcomes of state function reduction so that the sub-program must be repeated very many times. It is represented as four-dimensional patterns for various rates (chemical rates, nerve pulse patterns, EEG power distributions, ...) having also identification as temporal densities of zero energy states in various scales. By the fractality of TGD Universe there is a hierarchy of TQC's corresponding to p-adic and dark matter hierarchies. Programs (space-time sheets defining coherence regions) call programs in shorter scale. If the self-organizing system has a periodic behavior each TQC module defines a large number of almost copies of itself asymptotically. Generalized EEG could naturally define this periodic pattern and each period of EEG would correspond to an initiation and halting of TQC. This brings in mind the periodically occurring sol-gel phase transition inside cell near the cell membrane.
4. Fluid flow must induce the braiding which requires that the ends of braid strands must be anchored to the fluid flow. Recalling that lipid mono-layers of the cell membrane are liquid crystals and lipids of interior mono-layer have hydrophilic ends pointing towards cell interior, it is easy to guess that DNA nucleotides are connected to lipids by magnetic flux tubes and hydrophilic lipid ends are stuck to the flow.
5. The topology of the braid traversing cell membrane cannot affected by the hydrodynamical flow. Hence braid strands must be split during TQC. This also induces the desired magnetic isolation from the environment. Halting of TQC reconnects them and make possible the communication of the outcome of TQC.
6. There are several problems related to the details of the realization. How nucleotides A, T, C, G are coded to strand color and what this color corresponds to? The prediction that wormhole contacts carrying quark and anti-quark at their ends appear in all length scales in TGD Universe resolves the problem. How to split the braid strands in a controlled manner? High T_c super conductivity provides a partial understanding of the situation: braid strand can be split only if the supra current flowing through it vanishes. From the proportionality of Josephson current to the quantity $sin(\int 2eVdt)$ it follows that a suitable voltage pulse V induces DC supra-current and its negative cancels it. The conformation of the lipid controls whether it it can follow the flow or not. How magnetic flux tubes can be cut without breaking the conservation of the magnetic flux? The notion of wormhole magnetic field saves the situation now: after the splitting the flux returns back along the second space-time sheet of wormhole magnetic field.

To sum up, it seems that essentially all new physics involved with TGD based view about quantum biology enter to the model in crucial manner.

Quantum model of nerve pulse and EEG

In this article a unified model of nerve pulse and EEG is discussed.

1. In TGD Universe the function of EEG and its variants is to make possible communications from the cell membrane to the magnetic body and the control of the biological body by the

magnetic body via magnetic flux sheets traversing DNA by inducing gene expression. This leads to the notions of super- and hyper-genome predicting coherent gene expression at level of organs and population.

2. The assignment the predicted ranged classical weak and color gauge fields to dark matter hierarchy was a crucial step in the evolution of the model, and led among other things to a model of high T_c superconductivity predicting the basic scales of cell, and also to a generalization of EXG to a hierarchy of ZXGs, WXGs, and GXGs corresponding to Z^0, W bosons and gluons.

3. Dark matter hierarchy and the associated hierarchy of Planck constants plays a key role in the model. For instance, in the case of EEG Planck constant must be so large that the energies of dark EEG photons are above thermal energy at physiological temperatures. The assumption that a considerable fraction of the ionic currents through the cell membrane are dark currents flowing along the magnetic flux tubes explains the strange findings about ionic currents through cell membrane. Concerning the model of nerve pulse generation, the newest input comes from the model of DNA as a topological quantum computer and experimental findings challenging Hodgkin-Huxley model as even approximate description of the situation.

4. The identification of the cell interior as gel phase containing most of water as structured water around cytoskeleton - rather than water containing bio-molecules as solutes as assumed in Hodkin-Huxley model - allows to understand many of the anomalous behaviors associated with the cell membrane and also the different densities of ions in the interior and exterior of cell at qualitative level. The proposal of Pollack that basic biological functions involve phase transitions of gel phase generalizes in TGD framework to a proposal that these phase transitions are induced by quantum phase transitions changing the value of Planck constant. In particular, gel-sol phase transition for the peripheral cytoskeleton induced by the primary wave would accompany nerve pulse propagation. This view about nerve pulse is not consistent with Hodkin-Huxley model.

The model leads to the following picture about nerve pulse and EEG.

1. The system would consist of two superconductors- microtubule space-time sheet and the space-time sheet in cell exterior- connected by Josephson junctions represented by magnetic flux tubes defining also braiding in the model of TQC. The phase difference between two super-conductors would obey Sine-Gordon equation allowing both standing and propagating solitonic solutions. A sequence of rotating gravitational penduli coupled to each other would be the mechanical analog for the system. Soliton sequences having as a mechanical analog penduli rotating with constant velocity but with a constant phase difference between them would generate moving kHz synchronous oscillation. Periodic boundary conditions at the ends of the axon rather than chemistry determine the propagation velocities of kHz waves and kHz synchrony is an automatic consequence since the times taken by the pulses to travel along the axon are multiples of same time unit. Also moving oscillations in EEG range can be considered and would require larger value of Planck constant in accordance with vision about evolution as gradual increase of Planck constant.

2. During nerve pulse one pendulum would be kicked so that it would start to oscillate instead of rotating and this oscillation pattern would move with the velocity of kHz soliton sequence. The velocity of kHz wave and nerve pulse is fixed by periodic boundary conditions at the ends of the axon implying that the time spent by the nerve pulse in traveling along axon is always a multiple of the same unit: this implies kHz synchrony. The model predicts the value of Planck constant for the magnetic flux tubes associated with Josephson junctions and the predicted force caused by the ionic Josephson currents is of correct order of magnitude for reasonable values of the densities of ions. The model predicts kHz em radiation as Josephson radiation generated by moving soliton sequences. EEG would also correspond to Josephson radiation: it could be generated either by moving or standing soliton sequences (latter are naturally assignable to neuronal cell bodies for which $\hbar$ should be correspondingly larger): synchrony is predicted also now.

6.7 Appendix

6.7.1 About Inclusions Of Hyper-Finite Factors Of Type Ii$_1$

Many names have been assigned to inclusions: Jones, Wenzl, Ocneacnu, Pimsner-Popa, Wasserman [A76]. It would seem to me that the notion Jones inclusion includes them all so that various names would correspond to different concrete realizations of the inclusions conjugate under outer automorphisms.

1. According to [A76] for inclusions with $\mathcal{M}:\mathcal{N}\leq 4$ (with $A_1^{(1)}$ excluded) there exists a countable infinity of sub-factors with are pairwise non inner conjugate but conjugate to $\mathcal{N}$.
2. Also for any finite group G and its outer action there exists uncountably many sub-factors which are pairwise non inner conjugate but conjugate to the fixed point algebra of G [A76]. For any amenable group G the inclusion is also unique apart from outer automorphism [A45].

Thus it seems that not only Jones inclusions but also more general inclusions are unique apart from outer automorphism.

Any *-endomorphism σ, which is unit preserving, faithful, and weakly continuous, defines a sub-factor of type II$_1$ factor [A76]. The construction of Jones leads to a standard inclusion sequence $\mathcal{N}\subset\mathcal{M}\subset\mathcal{M}^1\subset ....$ This sequence means addition of projectors e_i, $i<0$, having visualization as an addition of braid strand in braid picture. This hierarchy exists for all factors of type II. At the limit $\mathcal{M}^\infty=\cup_i\mathcal{M}^i$ the braid sequence extends from $-\infty$ to ∞. Inclusion hierarchy can be understood as a hierarchy of Connes tensor powers $\mathcal{M}\otimes_\mathcal{N}\mathcal{M}....\otimes_\mathcal{N}\mathcal{M}$. Also the ordinary tensor powers of hyper-finite factors of type II_1 (HFF) as well as their tensor products with finite-dimensional matrix algebras are isomorphic to the original HFF so that these objects share the magic of fractals.

Under certain assumptions the hierarchy can be continued also in opposite direction. For a finite index an infinite inclusion hierarchy of factors results with the same value of index. σ is said to be basic if it can be extended to *-endomorphisms from $\mathcal{M}^1$ to $\mathcal{M}$. This means that the hierarchy of inclusions can be continued in the opposite direction: this means elimination of strands in the braid picture. For finite factors (as opposed to hyper-finite ones) there are no basic *-endomorphisms of $\mathcal{M}$ having fixed point algebra of non-abelian G as a sub-factor [A76].

1. Jones inclusions

For hyper-finite factors of type II$_1$ Jones inclusions allow basic *-endomorphism. They exist for all values of $\mathcal{M}:\mathcal{N}=r$ with $r\in\{4cos^2(\pi/n)|n\geq 3\}\cap[4,\infty)$ [A76]. They are defined for an algebra defined by projectors e_i, $i\geq 1$. All but nearest neighbor projectors commute. $\lambda=1/r$ appears in the relations for the generators of the algebra given by $e_ie_je_i=\lambda e_i$, $|i-j|=1$. $\mathcal{N}\subset\mathcal{M}$ is identified as the double commutator of algebra generated by e_i, $i\geq 2$.

This means that principal graph and its dual are equivalent and the braid defined by projectors can be continued not only to $-\infty$ but that also the dropping of arbitrary number of strands is possible [A76]. It would seem that ADE property of the principal graph meaning single root length codes for the duality in the case of $r\leq 4$ inclusions.

Irreducibility holds true for $r<4$ in the sense that the intersection of $Q'\cap P=P'\cap P=C$. For $r\geq 4$ one has $dim(Q'\cap P)=2$. The operators commuting with Q contain besides identify operator of Q also the identify operator of P. Q would contain a single finite-dimensional matrix factor less than P in this case. Basic *-endomorphisms with $\sigma(P)=Q$ is $\sigma(e_i)=e_{i+1}$. The difference between genuine symmetries of quantum TGD and symmetries which can be mimicked by TGD could relate to the irreducibility for $r<4$ and raise these inclusions in a unique position. This difference could partially justify the hypothesis that only the groups $G_a\times G_b\subset SU(2)\times SU(2)\subset SL(2,C)\times SU(3)$ define orbifold coverings of $H_\pm=CD\times CP_2\rightarrow H_\pm/G_a\times G_b$.

2. Wasserman's inclusion

Wasserman's construction of $r=4$ factors clarifies the role of the subgroup of $G\subset SU(2)$ for these inclusions. Also now $r=4$ inclusion is characterized by a discrete subgroup $G\subset SU(2)$ and is given by $(1\otimes\mathcal{M})^G\subset(M_2(C)\times\mathcal{M})^G$. According to [A76] Jones inclusions are irreducible also for $r=4$. The definition of Wasserman inclusion for $r=4$ seems however to imply that the identity matrices of both $\mathcal{M}^G$ and $(M(2,C) \otimes \mathcal{M})^G$ commute with $\mathcal{M}^G$ so that the inclusion should be reducible for $r = 4$.

Note that G leaves both the elements of $\mathcal{N}$ and $\mathcal{M}$ invariant whereas $SU(2)$ leaves the elements of $\mathcal{N}$ invariant. $M(2,C)$ is effectively replaced with the orbifold $M(2,C)/G$, with G acting as automorphisms. The space of these orbits has complex dimension $d = 4$ for finite G.

For $r < 4$ inclusion is defined as $M^G \subset M$. The representation of G as outer automorphism must change step by step in the inclusion sequence $... \subset \mathcal{N} \subset \mathcal{M} \subset ...$ since otherwise G would act trivially as one proceeds in the inclusion sequence. This is true since each step brings in additional finite-dimensional tensor factor in which G acts as automorphisms so that although $\mathcal{M}$ can be invariant under $G_{\mathcal{M}}$ it is not invariant under $G_{\mathcal{N}}$.

These two inclusions might accompany each other in TGD based physics. One could consider $r < 4$ inclusion $\mathcal{N} = \mathcal{M}^G \subset \mathcal{M}$ with G acting non-trivially in $\mathcal{M}/\mathcal{N}$ quantum Clifford algebra. $\mathcal{N}$ would decompose by $r = 4$ inclusion to $\mathcal{N}_1 \subset \mathcal{N}$ with $SU(2)$ taking the role of G. $\mathcal{N}/\mathcal{N}_1$ quantum Clifford algebra would transform non-trivially under $SU(2)$ but would be G singlet.

In TGD framework the G-invariance for $SU(2)$ representations means a reduction of S^2 to the orbifold S^2/G. The coverings $H_\pm \to H_\pm/G_a \times G_b$ should relate to these double inclusions and $SU(2)$ inclusion could mean Kac-Moody type gauge symmetry for $\mathcal{N}$. Note that the presence of the factor containing only unit matrix should relate directly to the generator d in the generator set of affine algebra in the McKay construction. The physical interpretation of the fact that almost all ADE type extended diagrams ($D_n^{(1)}$ must have $n \geq 4$) are allowed for $r = 4$ inclusions whereas D_{2n+1} and E_6 are not allowed for $r < 4$, remains open.

6.7.2 Generalization From $Su(2)$ To Arbitrary Compact Group

The inclusions with index $\mathcal{M} : \mathcal{N} < 4$ have one-dimensional relative commutant $\mathcal{N}' \cup \mathcal{M}$. The most obvious conjecture that $\mathcal{M} : \mathcal{N} \geq 4$ corresponds to a non-trivial relative commutant is wrong. The index for Jones inclusion is identifiable as the square of quantum dimension of the fundamental representation of $SU(2)$. This identification generalizes to an arbitrary representation of arbitrary compact Lie group.

In his thesis Wenzl [A55] studied the representations of Hecke algebras $H_n(q)$ of type A_n obtained from the defining relations of symmetric group by the replacement $e_i^2 = (q-1)e_i + q$. H_n is isomorphic to complex group algebra of S_n if q is not a root of unity and for $q = 1$ the irreducible representations of $H_n(q)$ reduce trivially to Young's representations of symmetric groups. For primitive roots of unity $q = exp(i2\pi/l)$, $l = 4, 5...$, the representations of $H_n(\infty)$ give rise to inclusions for which index corresponds to a quantum dimension of any irreducible representation of $SU(k)$, $k \geq 2$. For $SU(2)$ also the value $l = 3$ is allowed for spin 1/2 representation.

The inclusions are obtained by dropping the first m generators e_k from $H_\infty(q)$ and taking double commutant of both H_∞ and the resulting algebra. The relative commutant corresponds to $H_m(q)$. By reducing by the minimal projection to relative commutant one obtains an inclusion with a trivial relative commutant. These inclusions are analogous to a discrete states superposed in continuum. Thus the results of Jones generalize from the fundamental representation of $SU(2)$ to all representations of all groups $SU(k)$, and in fact to those of general compact groups as it turns out.

The generalization of the formula for index to square of quantum dimension of an irreducible representation of $SU(k)$ reads as

$$\mathcal{M} : \mathcal{N} = \prod_{1 \leq r < s \leq k} \frac{sin^2\left((\lambda_r - \lambda_s + s - r)\pi/l\right)}{sin^2\left((s-r)n/l\right)} \quad . \tag{6.7.1}$$

Here λ_r is the number of boxes in the r^{th} row of the Yang diagram with n boxes characterizing the representations and the condition $1 \leq k \leq l-1$ holds true. Only Young diagrams satisfying the condition $l - k = \lambda_1 - \lambda_{r_{max}}$ are allowed.

The result would allow to restrict the generalization of the embedding space in such a way that only cyclic group Z_n appears in the covering of $M^4 \to M^4/G_a$ or $CP_2 \to CP_2/G_b$ factor. Be as it may, it seems that quantum representations of any compact Lie group can be realized using the generalization of the embedding space. In the case of $SU(2)$ the interpretation of higher-dimensional quantum representations in terms of Connes tensor products of 2-dimensional fundamental representations is highly suggestive.

The groups $SO(3,1) \times SU(3)$ and $SL(2,C) \times U(2)_{ew}$ have a distinguished position both in physics and quantum TGD and the vision about physics as a generalized number theory implies them. Also the general pattern for inclusions selects these groups, and one can say that the condition that all possible statistics are realized is guaranteed by the choice $M^4 \times CP_2$.

1. $n > 2$ for the quantum counterparts of the fundamental representation of $SU(2)$ means that braid statistics for Jones inclusions cannot give the usual fermionic statistics. That Fermi statistics cannot "emerge" conforms with the role of infinite-D Clifford algebra as a canonical representation of HFF of type II_1. $SO(3,1)$ as isometries of H gives Z_2 statistics via the action on spinors of M^4 and $U(2)$ holonomies for CP_2 realize Z_2 statistics in CP_2 degrees of freedom.

2. $n > 3$ for more general inclusions in turn excludes Z_3 statistics as braid statistics in the general case. $SU(3)$ as isometries induces a non-trivial Z_3 action on quark spinors but trivial action at the embedding space level so that Z_3 statistics would be in question.

Chapter 7

Mathematical Speculations about the Hierarchy of Planck Constants

7.1 Introduction

I decided to separate the purely mathematical speculations about the hierarchy of Planck constants (actually only effective hierarchy if the recent interpretation is correct) from the material describing the physical ideas, key mathematical concepts, and the basic applications. These mathematical speculations emerged during the first stormy years in the evolution of the ideas about Planck constant and must be taken with a big grain of salt. I feel myself rather conservative as compared to the fellow who produced this stuff for 7 years ago. This all is of course very relative. Many readers might experience this recent me as a reckless speculator.

The first highly speculative topic discussed in this chapter is about possible connection of the hierarchy of Planck constants with Jones inclusions.

1. The connection with Jones inclusions was originally a purely heuristic guess based on the observation that the finite groups characterizing Jones inclusion characterize also pages of the Big Book. The key observation is that Jones inclusions are characterized by a finite subgroup $G \subset SU(2)$ and that this group also characterizes the singular covering or factor spaces associated with CD or CP_2 so that the pages of generalized embedding space could indeed serve as correlates for Jones inclusions. The elements of the included algebra $\mathcal{M}$ are invariant under the action of G and $\mathcal{M}$ takes the role of complex numbers in the resulting non-commutative quantum theory.

2. The understanding of quantum TGD at parton level led to the realization that the dynamics of Kähler action realizes finite measurement resolution in terms of finite number of modes of the induced spinor field. This automatically implies cutoffs to the representations of various super-conformal algebras typical for the representations of quantum groups closely associated with Jones inclusions [K16]. The Clifford algebra spanned by the fermionic oscillator operators would provide a realization for the factor space $\mathcal{N}/\mathcal{M}$ of hyper-finite factors of type II_1 identified as the infinite-dimensional Clifford algebra $\mathcal{N}$ of the configuration space and included algebra $\mathcal{M}$ determining the finite measurement resolution. The resulting quantum Clifford algebra has anti-commutation relations dictated by the fractionization of fermion number so that its unit becomes $r = \hbar/\hbar_0$. $SU(2)$ Lie algebra transforms to its quantum variant corresponding to the quantum phase $q = exp(i2\pi/r)$.

3. Jones inclusions appear as two variants corresponding to $\mathcal{N} : \mathcal{M} < 4$ and $\mathcal{N} : \mathcal{M} = 4$. The tentative interpretation is in terms of singular G-factor spaces and G-coverings of M^4 or CP_2 in some sense. The alternative interpretation in terms of two geodesic spheres of CP_2 would mean asymmetry between M^4 and CP_2 degrees of freedom.

4. Number theoretic Universality suggests an answer why the hierarchy of Planck constants is necessary. One must be able to define the notion of angle -or at least the notion of phase and of trigonometric functions- also in p-adic context. All that one can achieve naturally is the

notion of phase defined as root of unity and introduced by allowing algebraic extension of p-adic number field by introducing the phase if needed. In the framework of TGD inspired theory of consciousness this inspires a vision about cognitive evolution as the gradual emergence of increasingly complex algebraic extensions of p-adic numbers and involving also the emergence of improved angle resolution expressible in terms of phases $exp(i2\pi/n)$ up to some maximum value of n. The coverings and factor spaces would realize these phases geometrically and quantum phases q naturally assignable to Jones inclusions would realize them algebraically. Besides p-adic coupling constant evolution based on hierarchy of p-adic length scales there would be coupling constant evolution with respect to $\hbar$ and associated with angular resolution.

There are also speculations relating to the hierarchy of Planck constants, Mc-Kay correspondence, and Jones inclusions. Even Farey sequences, Riemann hypothesis and and N-tangles are discussed. Depending on reader these speculations might be experienced as irritating or entertaining. It would be interesting to go this stuff through in the light of recent understanding of the effective hierarchy of Planck constants to see what portion of its survives.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

7.2 Jones Inclusions And Generalization Of The Embedding Space

The original motivation for the generalization of the embedding space was the idea that the pages of the Big Book would provide correlates for Jones inclusions. In the following an attempt to formulate this vision more precisely is carried out.

7.2.1 Basic Facts About Jones Inclusions

Here only basic facts about Jones inclusions are discussed. Appendix contains a more detailed discussion of inclusions of HFFs.

Jones inclusions defined by subgroups of $SL(2,C) \times SU(2)$

Jones inclusions with $\mathcal{M} : \mathcal{N} < 4$ have representation as $R_0^G \subset R^G$ with G a discrete subgroup of $SU(2)$. $SO(3)$ or $SU(2)$ can be interpreted as acting in CP_2 as rotations. On quantum spinors the action corresponds to double cover of G.

A more general choice for G would be as a discrete subgroup $G_a \times G_b \subset SL(2,C) \times SU(2) \times SU(2)$. Poincare invariance suggests that the subgroup of $SL(2,C)$ reduces either to a discrete subgroup of $SU(2)$ and in the case that the rotation are genuinely 3-dimensional (E^6, E^8), the only possible interpretation would be as isotropy group of a particle at rest. When the group acts on plane as in case of A_n and D_{2n}, it could be also assigned to a massless particle.

If the group involves boosts it contains an infinite number of elements and it is not clear whether this kind of situation is physically sensible. In this case Jones inclusion could be interpreted as an inclusion for the tensor product of G invariant algebras associated with CD and CP_2 degrees of freedom and one would have $\mathcal{M} : \mathcal{N} = \mathcal{M} : \mathcal{N}(G_a) \times \mathcal{M} : \mathcal{N}(G_b)$. Since the index increases as the order of G increases one has reasons to expect that in the case of $G_a = SL(2,C)$ $N_a = \infty$ implies larger $\mathcal{M} : \mathcal{N}(G_a) > 4$.

A possible interpretation is that the values $\mathcal{M} : \mathcal{N} \leq 4$ are analogous to bound state energies so that a discrete rotation group acting in the relative rotational degrees of freedom can act as a symmetry group whereas the values $\mathcal{M} : \mathcal{N} > 4$ are analogous to ionized states for which particles are almost freely moving with respect to each other with a constant velocity.

When one restricts the coefficients to G-invariant elements of Clifford algebra the Clifford field is G-invariant under the natural action of G. This allows two interpretations. Either the Clifford field is G invariant or that the Clifford field is defined in orbifold $CD/G_a \times CP_2/G_b$. CD/G_a is obtained by replacing hyperboloid H_a $(t^2 - x^2 - y^2 - z^2 = a^2)$ with H_a/G_a. These spaces have been considered as cosmological models having 3-space with finite volume [K101] (also a lattice like structure could be in question).

The quantum phases associated with sub-groups of $SU(2)$

It is natural to identify quantum phase as that defined by the maximal cyclic subgroup for finite subgroups of $SU(2)$ and infinite subgroups of $SL(2,C)$. Before continuing a brief summary about quantum phases associated with finite subgroups of $SU(2)$ is in order. E_6 corresponds to $N = 24$ and $n = 3$ and E_8 to icosahedron with $N = 120$, $n = 5$ and Golden mean and the minimal value of n making possible universal topological quantum computer [K5].

D_n and A_n have orders $2n$ and $n+1$ and act as symmetry groups of n-polygon and have n-element cyclic group as a maximal cyclic subgroup. For double covers the orders are twice this. Thus A_n *resp.* D_{2n} correspond to $q = exp(i\pi/n)$ *resp.* $q = exp(i\pi/2n)$. Note that the restriction $n \geq 3$ means geometrically that only non-trivial polygons are allowed.

7.2.2 Jones Inclusions And The Hierarchy Of Planck Constants

The anyonic arguments for the quantization of Planck constant suggest that one can assign separate scalings of Planck constant to CD and CP_2 degrees of freedom and that these scalings in turn reflect as scalings of $M^4\pm$ and CP_2 metrics. This is definitely not in accordance with the original TGD vision based on uniqueness of embedding space but makes sense if space-time and embedding space are emergent concepts as the hierarchy of number theoretical von Neumann algebra inclusions indeed suggests. Indeed, the scaling factors of CD and CP_2 metric remain non-fixed by the general uniqueness arguments since Cartesian product is in question.

Hierarchy of Planck constants and choice of quantization axis

Jones inclusions seem to relate in a natural manner to the selection of quantization axis.

1. In the case of CD the orbifold singularity is for all groups G_a except E_6 and E_8 the time-like plane M^2 corresponding to a radial ray through origin defining the quantization axis of angular momentum and intersecting light-cone boundary along a preferred light-like ray. For E_6 and E_8 (tetrahedral and icosahedral symmetries) the singularity consists of planes M^2 related by symmetries of G sharing time-like line M^1 and in this case there are several alternative identifications of the quantization axes as axis around which the maximal cyclic subgroup acts as rotations.
2. From this it should be obvious that Jones inclusions represented in this manner would relate very closely to the selection of quantization axes and provide a geometric representation for this selection at the level of space-time and WCW . The existence of the preferred direction of quantization at a given level of dark matter level should have observable consequences. For instance, in cosmology this could mean a breaking of perfect rotational symmetry at dark matter space-time sheets. The interpretation would be as a quantum effect in cosmological length scales. An interesting question is whether the observed asymmetry of cosmic microwave background could have interpretation as a quantum effect in cosmological length and time scales.

Do factor spaces and coverings correspond to the two kinds of Jones inclusions?

What could be the interpretation of the singular coverings and and factor spaces? If both geodesic spheres of CP_2 are allowed $\mathcal{M} : \mathcal{N} = 4$ could correspond to the allowance of cosmic strings and other analogous objects. This option is however asymmetric with respect to CD and CP_2 and the more plausible option is that the two kinds of Jones inclusions correspond to singular factor spaces and coverings.

1. Jones inclusions appear in two varieties corresponding to $\mathcal{M} : \mathcal{N} < 4$ and $\mathcal{M} : \mathcal{N} = 4$ and one can assign a hierarchy of subgroups of $SU(2)$ with both of them. In particular, their maximal Abelian subgroups Z_n label these inclusions. The interpretation of Z_n as invariance group is natural for $\mathcal{M} : \mathcal{N} < 4$ and it naturally corresponds to the coset spaces. For $\mathcal{M} : \mathcal{N} = 4$ the interpretation of Z_n has remained open. Obviously the interpretation of Z_n as the homology group defining covering would be natural.

2. For covering spaces one would however obtain the degrees of freedom associated with the discrete fiber and the degrees of freedom in question would not disappear completely and would be characterized by the discrete subgroup of $SU(2)$. For anyons the non-trivial homotopy of plane brings in non-trivial connection with a flat curvature and the non-trivial dynamics of topological QFTs. Also now one might expect similar non-trivial contribution to appear in the spinor connection of $\hat{CD}\hat{\times}G_a$ and $\hat{CP}_2\hat{\times}G_b$. In conformal field theory models non-trivial monodromy would correspond to the presence of punctures in plane. This picture is also consistent with the G singlets of the quantum states despite the fact that fermionic oscillator operators belong to non-trivial irreps of G.

Coverings and factors spaces form an algebra like structure

It is easy to see that coverings and factor spaces defining the pages of the Big Book form an algebra like structure.

1. For factor spaces the unit for quantum numbers like orbital angular momentum is multiplied by n_a *resp.* n_b and for coverings it is divided by this number. These two kind of spaces are in a well defined sense obtained by multiplying and dividing the factors of $\hat{H}$ by G_a *resp.* G_b and multiplication and division are expected to relate to Jones inclusions with $\mathcal{M}:\mathcal{N}<4$ and $\mathcal{M}:\mathcal{N}=4$, which both are labeled by a subset of discrete subgroups of $SU(2)$.

2. The discrete subgroups of $SU(2)$ with fixed quantization axis possess a well defined multiplication with product defined as the group generated by forming all possible products of group elements as elements of $SU(2)$. This product is commutative and all elements are idempotent and thus analogous to projectors. Trivial group G_1, two-element group G_2 consisting of reflection and identity, the cyclic groups Z_p, p prime, and tetrahedral, octahedral, and icosahedral groups are the generators of this algebra.

By commutativity one can regard this algebra as an 11-dimensional module having natural numbers as coefficients ("rig"). The trivial group G_1, two-element group G_2 generated by reflection, and tetrahedral, octahedral, and icosahedral groups define 5 generating elements for this algebra. The products of groups other than trivial group define 10 units for this algebra so that there are 11 units altogether. The groups Z_p generate a structure analogous to natural numbers acting as analog of coefficients of this structure. Clearly, one has effectively 11-dimensional commutative algebra in 1-1 correspondence with the 11-dimensional "half-lattice" N^{11} (N denotes natural numbers). Leaving away reflections, one obtains N^7. The projector representation suggests a connection with Jones inclusions. An interesting question concerns the possible Jones inclusions assignable to the subgroups containing infinitely manner elements. Reader has of course already asked whether dimensions 11, 7 and their difference 4 might relate somehow to the mathematical structures of M-theory with 7 compactified dimensions. One could introduce generalized configuration space spinor fields in WCW labeled by sectors of H with given quantization axes. By introducing Fourier transform in N^{11} one would formally obtain an infinite-component field in 11-D space.

Connection between Jones inclusions, hierarchy of Planck constants, and finite number of spinor modes

The original generalization of the embedding space to accommodate the hierarchy of Planck constants was based on the idea that the singular coverings and factor spaces associated with the causal diamond CD and CP_2, which appears as factors of $CD\times CP_2$ correspond somehow to Jones inclusions, and that the integers n_a and n_b characterizing the orders of maximal cyclic groups of groups G_a and G_b associated with the two Cartesian factors correspond to quantum phases $q=exp(i2\pi/n_i)$ in such a way that singular factor spaces correspond to Jones inclusions with index $\mathcal{M}:\mathcal{N}<4$ and coverings to those with index $\mathcal{M}:\mathcal{N}=4$.

Since Jones inclusions are interpreted in terms of finite measurement resolution, the mathematical realization of this heuristic picture should rely on the same concept realized also by the fact that the number of non-zero modes for induced spinor fields is finite. This allows to consider two possible interpretations.

1. The finite number of modes defines an approximation to the hyper-finite factor of type II_1 defined by WCW Clifford algebra.
2. The Clifford algebra spanned by fermionic oscillator operators is quantum Clifford algebra and corresponds to the somewhat nebulous object $\mathcal{N}/\mathcal{M}$ associated with the inclusion $\mathcal{M} \subset \mathcal{N}$ and coding the finite measurement resolution to a finite quantum dimension of the Clifford algebra. The fact that quantum dimension is smaller than the actual dimension would reflect correlations between spinor components so that they are not completely independent.

If the latter interpretation is correct then second quantized induced spinor fields should obey quantum variant of anti-commutation relations reducing to ordinary anti-commutation relations only for $n_a = n_b = 0$ (no singular coverings nor factor spaces). This would give the desired connection between inclusions and hierarchy of Planck constants. It is possible to have infinite number of quantum group like structure for $\hbar = \hbar_0$.

There are two quantum phases q and one should understand what is the phase that appears in the quantum variant of anti-commutation relations. A possible resolution of the problem relies on the observation that there are two kinds of number theoretic braids. The first kind of number theoretic braid is defined as the intersection of M_+ (or light-like curve of δM^4_+ in more general case) and of δM^4_+ projection of X^2. Second end of braid is defined as the intersection of CP_2 projection of X^2 of homologically non-trivial sphere S^2_{II} of CP_2. The intuitive expectation is that these dual descriptions apply for light-like 3-surfaces associated *resp.* co-associative regions of space-time surface and that both descriptions apply at wormhole throats. The duality of these descriptions is guaranteed also at wormhole throats if physical Planck constant is given by $\hbar = r\hbar_0$, $r = \hbar(M^4)/\hbar(CP_2)$, so that only the ratio of the two Planck constants matters in commutation relations. This would suggest that it is $q = exp(i2\pi/r)$, which appears in quantum variant of anti-commutation relations of the induced spinor fields.

The action of $G_a \times G_b$ on WCW spinors and spinor fields

The first question is what kind of measurement resolution is in question. In zero energy ontology the included states would typically correspond to insertion of zero energy states to the positive or negative part of the physical state in time scale below the time resolution defined by the time scale assignable to the smallest CD present in the zero energy state. Does the description in terms of G invariance apply in this case or does it relate only to time and length scale resolution whereas hierarchy of Planck constants would relate to angle resolution? Assume that this is the case.

The second question is how the idea about $\mathcal{M}$ as an included algebra defining finite measurement resolution and G invariance as a symmetry defining $\mathcal{M}$ as the included algebra relate to each other.

1. One cannot say that G creates states, which cannot be distinguished from each other. Rather G-invariant elements of $\mathcal{M}$ create states whose presence in the state cannot be detected.
2. For covering space option $\mathcal{M}$ represents states which are invariant under discrete subgroup of $SU(2)$ acting in the covering. States with integer spin would be below measurement resolution and only factional spins of form j/n would be observable. For factor space option $\mathcal{M}$ would represents states which are invariant under discrete subgroup of $SU(2)$ acting in H-say states with spin. States with spin which is multiple of n would be below measurement resolution. The situation would be very similar to each other. Number theoretic considerations and the fact that the number of fermionic oscillator operators is finite suggest that for coverings the condition $L_z < 1$ and for factor spaces the condition $L_z < n$ is satisfied by the generators of Clifford algebra regarded as irreducible representation of G. For factor spaces the interpretation could be in terms of finite angular resolution $\Delta\phi \leq 2\pi/n$ excluding angular momenta $L_z \geq n$. For coverings the resolution would be related to rotations (or rather, braidings) as multiples of 2π: multiples $m2\pi$ $m \geq n$ cannot be distinguished from $m \mod n$ multiples.
3. The minimal assumption is that integer orbital angular momenta are excluded for coverings and n-multiples are excluded for factors spaces. The stronger assumption would be that there is angular momentum cutoff. This point is however very delicate. The states with $j > n$ can be obtained as tensor products of representations with $j = m$. If entanglement is present one cannot anymore express the state as a product of $\mathcal{M}$ element and $\mathcal{N}$ element so that

the states $j > n$ created in this manner would not be equivalent with those with $j \ mod \ n$. The replacement of the ordinary tensor product with Connes tensor product would indeed generate automatically entangled states and one could interpret Connes tensor product as a way to create only the allowed states.

4. For quantum groups allow only finite number of representations up to some maximum spin determined by the integer n characterizing quantum phase q. This would mean angular momentum cutoff leaving only a finite number of representations of quantum group [K16]. This fits nicely with what one obtains in the case of factor spaces. For coverings the new element is that the unit of spin becomes $1/n$: otherwise the situation seems to be similar. Quantum group like structure is obtained if the fermionic oscillator operators satisfy the quantum version of anti-commutation relations. The algebra would be very similar except that the orbital angular momentum labeling oscillator operators has different unit. Oscillator operators are naturally in irreducible representations of G and only the non-trivial representations of G are allowed.
5. Besides Jones inclusions corresponding to $\mathcal{M} : \mathcal{N} < 4$ there are inclusions with $\mathcal{M} : \mathcal{N} = 4$ to which one can also assign quantum phases. It would be natural to assign covering spaces and factor spaces to these two kinds of inclusions. For the minimal option excluding only the orbital angular momentum which are integers or multiples of n the fraction of excluded states is very small for coverings so that $\mathcal{M} : \mathcal{N} = 4$ is natural for this option. $\mathcal{M} : \mathcal{N} < 4$ would in turn correspond naturally to factor spaces.
6. Since the two kinds of number theoretic braids correspond to points which belong to M^2 or S^2, one might argue that several quantum anti-commutation relations must be satisfied simultaneously. This is not the case since the eigen modes of D_{C-S} and hence also oscillator operators code information about partonic surface X^2 itself and also about $X^4(X^3_l)$ rather than being purely local objects. In the case of covering space the oscillator operators can be arranged to irreducible representations of G and in the case of factor space the oscillator operators are G-invariant.

One must distinguish between G invariance for WCW spinors and spinor fields.

1. In the case of factor spaces 3-surface are G invariant so that there is no difference between spinors and spinor fields as far as G is considered. Irreducible representations of G would correspond to the superpositions of G-transforms of oscillator operators for a fixed G-invariant X^3_l.
2. For covering space option G-invariance would mean that 3-surface is a mere G-fold copy of single 3-surface. There is no obvious reason to assume this. Hence one cannot separate spinorial degrees of freedom from WCW degrees of freedom since G affects both the spin degrees of freedom and the 3-surface. Irreducible representations of G would correspond to genuine WCW spinor fields involving a superposition of G-transforms of also X^3_l. The presence of both orbital and spin degrees of freedom could provide alternative explanation for why $\mathcal{M} : \mathcal{N} = 4$ holds true for covering space option.

If the fermionic oscillator algebra is interpreted as a representation for $\mathcal{N}/\mathcal{M}$, allowed fermionic oscillator operators belong to non-trivial irreps of G. One can however ask whether the many-fermion states created by these operators are G-invariant for some physical reason so that one would have kind of G-confinement forcing the states to be many-fermion states with standard unit of quantum numbers for coverings and integer multiples of n for factor spaces. This would conform with the ideas that anyonicity is a microscopic property not visible at the level of entire state and that many-fermion systems in the anyonic state resulting in strong coupling limit for ordinary value of $\hbar$ are in question. The processes changing the value of Planck constant would be phase transitions involving all fermions of the G-invariant state and would be slow for this reason. This would also contribute to the invisibility of dark matter.

7.2.3 Questions

What is the role of dimensions?

Could the dimensions of CD and CP_2 and the dimensions of spaces defined by the choice of the quantization axes play a fundamental role in the construction from the constraint that the fundamental group is non-trivial?

1. Suppose that the sub-manifold in question is geodesic sub-manifold containing the orbits of its points under Cartan subgroup defining quantization axes. A stronger assumption would be that the orbit of maximal compact subgroup is in question.
2. For M^{2n} Cartan group contains translations in time direction with orbit M^1 and Cartan subgroup of $SO(2n-1)$ and would be M^n so that $\hat{M^{2n}}$ would have a trivial fundamental group for $n > 2$. Same result applies in massless case for which one has $SO(1,1) \times SO(2n-2)$ acts as Cartan subgroup. The orbit under maximal compact subgroup would not be in question.
3. For CP_2 homologically non-trivial geodesic sphere CP_1 contains orbits of the Cartan subgroup. For $CP_n = SU(n+1)/SU(n) \times U(1)$ having real dimension $2n$ the sub-manifold CP_{n-1} contains orbits of the Cartan subgroup and defines a sub-manifold with codimension 2 so that the dimensional restriction does not appear.
4. For spheres $S^{n-1} = SO(n)/SO(n-1)$ the dimension is $n-1$ and orbit of $SO(n-1)$ of point left fixed by Cartan subgroup $SO(2) \times ..$ would for $n = 2$ consist of two points and S_{n-2} in more general case. Again co-dimension 2 condition would be satisfied.

What about holes of WCW ?

One can raise analogous questions at the level of WCW geometry. Vacuum extremals correspond to Lagrangian sub-manifolds $Y^2 \subset CP_2$ with vanishing induced Kähler form. They correspond to singularities of WCW ("world of classical worlds") and WCW spinor fields should vanish for the vacuum extremals. Effectively this would mean a hole in configuration space, and the question is whether this hole could also naturally lead to the introduction of covering spaces and factor spaces of the WCW s. How much information about the general structure of the theory just this kind of decomposition might allow to deduce? This kind of singularities are infinite-dimensional variants of those discussed in catastrophe theory and this suggests that their understanding might be crucial.

Are more general inclusions of HFFs possible?

The proposed scenario could be criticized because discrete subgroups of $SU(2)$ are in a preferred position. The Jones inclusions considered correspond to quantum spinor representations of various quantum groups $SU(2)_q$, $q = exp(i2\pi/n)$. This explains the result $\mathcal{M} : \mathcal{N} \leq 4$. These representations are certainly in preferred role as far as WCW spinor fields are considered but it is possible to assign a hierarchy of inclusions of HFFs labeled by quantum phase q with arbitrary representation of an arbitrary compact Lie group. These inclusions would be analogous to discrete states in the continuum $\mathcal{M} : \mathcal{N} > 4$.

Since the inclusions are characterized by single quantum phase $q = exp(i2\pi/n)$ in the case of compact Lie groups (Appendix), one can ask whether more general discrete groups than subgroups of $SU(2)$ should be allowed. The inclusions of HFFs associated with higher dimensional Lie groups have $\mathcal{M} : \mathcal{N} > 4$ and are analogous to bound states in continuum (Appendix). In the case of CP_2 this would allow to consider much more general sub-groups.

The question is therefore whether some principle selects subgroups of $SU(2)$. There are indeed good arguments supporting the hypothesis that only discrete Abelian subgroups of $SU(2)$ are possible.

1. The notion of number theoretic braid allows only the only subgroups of rotation group leaving M^2 invariant and sub-groups of $SU(3)$ leaving geodesic sphere S^2_i invariant. This would drop groups having genuinely 3-D action. In the case of $SU(3)$ discrete subgroups of $SO(3)$ or $U(2)$ remain under consideration. The geodesic sphere of type II is however analogous to North/South pole of S^2 and second phase factor associated with the coordinates (ξ^1, ξ^2) becomes redundant since $(|\xi^1|^2 + |\xi^2|^2)^{1/2}$ becomes infinite at S^2_{II} so that ξ^1/ξ^2 becomes appropriate coordinate. Hence action of $U(2)$ reduces to that of $SU(2)$ since ξ^1 and ξ^2 correspond to same value of color hyper charge associated with $U(1)$.

2. A physically attractive possibility is that $G_a \times G_b$ leaves the choice of quantization axes invariant. This condition makes sense also for coverings. This would leave only Abelian groups into consideration and drop D_{2n}, E_6, and E_8. It is quite possible that only these groups define sectors of the generalized embedding space. This means that $G_b = Z_{n_1} \times Z_{n_2} \subset U(1)_I \times U(1)_Y \subset SU(2) \times U(1)_Y$ and even more general subgroups of $SU(3)$ (if non-commutativity is allowed) are a priori possible. Again the first argument reduces the list to cyclic subgroups of $SU(2)$.
3. The products of groups Z_n are also number theoretically in a very special position since they relate naturally to the finite cyclic extensions and also to the maximal Abelian extension of rationals. With this restriction on $G_a \times G_b$ one can consider the hypothesis that elementary particles correspond are maximally quantum critical systems left invariant by all groups $G_a \times G_b$ respecting a given choice of quantization axis and implying that darkness is associated only to field bodies and Planck constant becomes characterizer of interactions rather than elementary particles themselves.

7.3 Some Mathematical Speculations

7.3.1 The Content Of Mckay Correspondence In TGD Framework

The possibility to assign Dynkin diagrams with the inclusions of II_1 algebras is highly suggestive concerning possible physical interpretations. The basic findings are following.

1. For $\beta = \mathcal{M} : \mathcal{N} < 4$ Dynkin diagrams code for the inclusions and correspond to simply laced Lie algebras. $SU(2)$, D_{2n+1}, and E_7 are excluded.
2. Extended ADE Dynkin diagrams coding for simply laced ADE Kac Moody algebras appear at $\beta = 4$. Also $SU(2)$ Kac Moody algebra appears.

Does TGD give rise to ADE hierarchy of gauge theories

The first question is whether any finite subgroup $G \subset SU(2)$ acting in CP_2 degrees of freedom could somehow give rise to multiplets of the corresponding gauge group having interactions described by a gauge theory. Orbifold picture suggests that might be the case.

1. The "sheets" for the space-time sheet forming an $N(G)$-fold cover of CD are in one-one correspondence with group G. This degeneracy gives rise to additional states and these states correspond to the group algebra having basis given by group characters $\chi(g)$. One obtains irreducible representations of G with degeneracies given by their dimensions. Altogether one obtains $N(G)$ states in this manner. In the case of $A(n)$ the number of these states is $n + 1$, the number of the states of the fundamental representation of $SU(n + 1)$. In the same manner, for D_{2n} the number of these states equals to the number of states in the fundamental representation of D_{2n}. It seems that the rule is quite general. Thus these representations would in the case of fermions give the states of the fundamental representation of the corresponding gauge group.
2. From fermion and anti-fermion states one can construct in a similar manner pairs giving $N(G)^2$ states defining in the case of $A(n)$ $n^2 - 1$-dimensional gauge boson multiplet plus singlet. Also other groups must give boson multiplet plus possible other multiplets. For instance, for $D(4)$ the number of states is 64 and boson multiplet is 8-dimensional so that many other spin 1 states result.
3. These findings give hopes that the orbifold multiplets could be modelled by a gauge theory based on corresponding gauge group. What is nice that this huge hierarchy of gauge theories is associated with dark matter so that the predictivity and falsifiability are not lost unlike in M-theory.

Does one obtain also a hierarchy of conformal theories with ADE Kac Moody symmetry?

Consider next the question Kac Moody interactions correspond to extended ADE diagrams are possible.

1. In this case the notion of orbifold seems to break down since the symmetry related points form a continuum $SU(2)$ and space-time surface would become 6-dimensional if the CD projection is 4-dimensional. If one takes space-time as something which emerges, one could take this possibility half seriously. A more natural natural possibility is that CD projection is 2-dimensional geodesic sphere in which case one would have string like objects so that conformal field theory with Kac-Moody algebra would emerge naturally.
2. The new degrees of freedom would define 2-dimensional continuum and it would not be completely surprising if conformal field theory based on ADE Kac Moody algebra could describe the situation. One possibility is that these continua for different inclusions correspond to $SU(2)$ decompose to an $N(G)$-fold covers of S^2/G orbifold so that also now groups G would be involved with the Jones inclusions, which might provide a hint about how to construct them. S^2/G would play the role of stringy world sheet for the conformal field theory in question. This effective re-arrangement of the topology S^2 might be due to the fact that conformal fields possess G symmetry which effectively groups points of S^2 to $n(G)$-multiplets. The localized representations of the Lie group corresponding to G would correspond to the multiplets obtained from the representations of group algebra of G as in previous case.
3. The formula for the scaling factor of CD metric would give infinite scaling factor if one identifies the scaling factor as maximal order of cyclic subgroup of $SU(2)$. As a matter fact there is no finite cyclic subgroup of this kind. The solution to the problem would be identification of the scaling factor as the order of the maximal cyclic subgroup of G so that the scaling factors would be same for the two situations related by McKay correspondence.

Generalization to CD degrees of freedom

One can ask whether the proposed picture generalizes formally also the case of CD.

1. In this case quantum groups would correspond to discrete subgroups $G \subset SL(2,C)$. Kac Moody group would correspond to G-Kac Moody algebra made local with respect to $SL(2,C)$ orbit in CD divided by G. These orbits are 3-dimensional hyperboloids H_a with a constant value of light cone proper time a so that the division by G gives fundamental domain H_a/G with a finite 3-volume.
2. The 4-dimensionality of space-time would require 1-dimensional CP_2 projection. Vacuum extremals of Kähler action would be in question. Robertson-Walker metric have 1-dimensional CP_2 projection and carry non-vanishing density of gravitational mass so that in this sense the theory would be non-trivial. G would label different lattice like cosmologies defined by tessellations with fundamental domain H_a/G.
3. The multiplets of G would correspond to collections of points, one from each cells of the lattice like structure. Macroscopic quantum coherence would be realized in cosmological scales. If one takes seriously the vision about the role of short distance p-adic physics as a generator of long range correlations of the real physics reflected as p-adic fractality, this idea does not look so weird anymore.

   Complexified modular group $SL(2,Z+iZ)$ and its subgroups are interesting as far as p-adicization is considered. The principal congruence subgroups $\Gamma(N)$ of $SL(2,Z+iZ)$ which are unit matrices modulo N define normal subgroups of the complex modular group and are especially interesting candidates for groups $G \subset SL(2,C)$. The group $\Gamma(N=p^k)$ labeling fundamental domains of the tessellation $H_a/\Gamma(N=p^k)$ defines a mathematically attractive candidate for a point set associated with the intersections of p-adic space-time sheets with real space-time sheets. Also analogous groups for algebraic extensions of Z are interesting.

   The simplest discrete subgroup of SL(2, C) with infinite number of elements would corresponds to powers of boost to single direction and correspond at the non-relativistic limit to multiples of basic velocity. This could also give rise to quantization of cosmic recession velocities. There is evidence for the quantization of cosmic recession velocities (for a model in which single object produces quantized redshifts see [K37]) and it is interesting to see whether they could be interpreted in terms of the lattice like periodicity in cosmological length scales implied by the effective reduction of physics to M^4_+/G_n. In [E9] the values $z = 2.63, 3.45, 4.47$ of cosmic red shift are listed. These correspond to recession velocities $v = (z^2-1)/(z^2+1)$ are (0.75, 0.85, 0.90). The corresponding hyperbolic angles are given by $\eta = acosh(1/(1-v^2))$ and the values of η are $(1.46, 1.92, 2.39)$. The differences $\eta(2) - \eta(1) = .466$ and $\eta(3) - \eta(2) = .467$ are same within experimental uncertainties. One has however $\eta(n)/(\eta(2) - \eta(1)) = (3.13, 4.13, 5.13)$ instead of $(3, 4, 5)$. A possible interpretation is in terms of the velocity of the observer with respect to the frame in which quantization of η happens.

Quantitative support for the interpretation

A more detailed analysis of the situation gives support for the proposed vision.

1. A given value of quantum group deformation parameter $q = exp(i\pi/n)$ makes sense for any Lie algebra but now a preferred Lie-algebra is assigned to a given value of quantum deformation parameter. At the limit $\beta = 4$ when quantum deformation parameter becomes trivial, the gauge symmetry is replaced by Kac Moody symmetry.
2. The prediction is that Kac-Moody central extension parameter should vanish for $\beta < 4$. There is an intriguing relationship to formula for the quantum phase q_{KM} associated with (possibly trivial) Kac-Moody central extension and the phase defined by ADE diagram

$$\begin{array}{ll} q_{KM} = exp(i\phi) \ , & \phi_1 = \frac{\pi}{k+h^v} \ , \\ q_{Jones} = exp(i\phi) \ , & \phi = \frac{\pi}{h} \end{array}$$

   In the first formula sum of Kac-Moody central extension parameter k and dual Coxeter number h^v appears whereas Coxeter number h appears in the second formula. Internal consistency requires

$$k + h^v = h \ . \tag{7.3.1}$$

   It is easy see that the dual Coxeter number h^v and Coxeter number h given by $h = (dim(g) - r)/r$, where r is the dimension of Cartan algebra of g, are identical for ADE algebras so that the Kac-Moody central extension parameter k must indeed vanish. For $SO(2n+1)$, $Sp(n)$, G_2, and F_4 the condition $h = h^v$ does not hold true but one has $h(n) = 2n = h^v + 1$ for $SO(2n+1)$, $h(n) = 2n = 2(h^v - 1)$ for $Sp(n)$, $h = 6 = h^v + 2$ for G_2, and $h = 12 = h^v + 3$ for F_4.

   What is intriguing that G_2, which seems to play a fundamental role in the dual formulation of quantum TGD based on the identification of space-times as surfaces in hyper-octonionic space M^8 [K107] is not allowed. As a matter fact, $G_2 \rightarrow SU(3)$ reduction occurs also in the dual formulation based on $G_2/SU(3)$ coset model and is required by the separate conservation of quark and lepton numbers predicted by TGD. ADE groups would be associated with the interaction between space-time sheets rather than entire dynamics and need not have anything to do with the Kac-Moody algebra associated with color and electro-weak interactions appearing in the construction of physical states [K66].
3. There seems to be a concrete connection with conformal field theories. This connection would allow to understand the emergence of quantum groups appearing naturally in these theories. Quite generally, the conformal central extension parameter for unitary Virasoro representations resulting by Sugawara construction from Kac Moody representations satisfies either of the conditions

$$\begin{aligned} c &\geq \frac{kdim(g)}{k+h^v} + 1 \ , \\ c &= \frac{kdim(g)}{k+h^v} + 1 - \frac{6}{(h-1)h} \ . \end{aligned} \tag{7.3.2}$$

   For $k = 0$, which should be interesting for $\beta < 4$, the second formula reduces to

$$c = 1 - \frac{6}{(h-1)h} \ . \tag{7.3.3}$$

The formula gives the values of c for minimal conformal field theories with finite number of conformal fields and real conformal weights. Indeed, h in this formula seems to correspond to the same h as appearing in the expression $\beta \equiv \mathcal{M} : \mathcal{N} = 4cos^2(\pi/h)$.

$\beta = 3, h = 6$ corresponds to 3-state Potts model with $c = 4/5$ which should thus have a gauge group for which Coxeter number is 6: the group should be either $SU(6)$ or $SO(8)$. Two-state Potts model, that is Ising model with $\beta = 2, h = 4$ would correspond to $c = 1/2$ and to a gauge group $SU(4)$ or $SO(4)$. For $h = 3$ ("one-state Potts model") with group $SU(3)$ one would have $c = 0$ and vanishing conformal anomaly so that conformal degrees of freedom would become pure gauge degrees of freedom.

These observations give support for the following picture.

1. Quite generally, the number of states of the generalized β-state Potts model has an interpretation as the dimension $\beta = \mathcal{M} : \mathcal{N}$ of $\mathcal{M}$ as $\mathcal{N}$-module. Besides the models with integer number of states there is an infinite number of models for which the number of states is not an integer. The conditions $c \leq 1$ guaranteeing real conformal weights and $\beta \leq 4$ correspond to each other for these models.
2. $\beta > 4$ Potts models would be formally obtained by allowing h to be imaginary in the defining formula for $\mathcal{M} : \mathcal{N}$. In this case c would be however complex so that the theory would not be unitary.
3. For minimal models with $(\beta < 4, c < 1)$ Kac-Moody central extension parameter is vanishing so that Kac Moody algebra indeed acts like gauge symmetries and gauge symmetries would be in question. $(\beta = 4, c = 1)$ would define a "four-state Potts model" with infinite-dimensional unitary group acting as a gauge group. On the other hand, the appearance of extended ADE Dynkin diagrams suggests strongly that this limit is not realized but that $\beta = \mathcal{M} : \mathcal{N} = 4$ corresponds to $k = 1$ conformal field theory allowing Kac Moody symmetries for any ADE group, which as simply-laced groups allows vertex operator construction. The appearance of $kdim(g)/(k + g)$ in the more general formula would thus code the Kac Moody group whereas for $\beta < 4$ ADE diagram codes for the preferred gauge group characterizing the minimal CFT.
4. The possibility that any ADE gauge group or Kac-Moody group can characterize the interaction between space-time sheets conforms with the idea about Universe as a Topological Quantum Computer able to simulate any conceivable quantum dynamics. Of course, one cannot exclude the possibility that only electro-weak and color symmetries are realized in this manner.

G_a as a symmetry group of magnetic body and McKay correspondence

The group $G_a \subset SU(2) \subset SL(2, C)$ means exact rotational symmetry realized in terms of CD coverings of CP_2. The 5 and 6-cycles in biochemistry (sugars, DNA, ...) are excellent candidates for these symmetries. For very large values of Planck constant, say for the values $\hbar(CD)/\hbar(CP_2) = GMm/v_0 = (n_a/n_b)\hbar_0$, $v_0 = 2^{-11}$, required by the model for planetary orbits as Bohr orbits [K100], G_a is huge and corresponds to either Z_{n_a} or in the case of even value of n_a to the group generated by Z_n and reflection acting on plane and containing $2n_a$ elements.

The notion of magnetic body seems to provide the only conceivable candidate for a geometric object possessing G_a as symmetries. In the first approximation the magnetic field associated with a dark matter system is expected to be modellable as a dipole field having rotational symmetry around the dipole axis. Topological quantization means that this field decomposes into flux tube like structures related by the rotations of Z_n or D_{2n}. Dark particles would have wave functions de-localized to this set of these flux quanta and span group algebra of G_a. Magnetic flux quanta are indeed assumed to mediate gravitational interactions in the TGD based model for the quantization of radii of planetary orbits and this explains the dependence of $\hbar_{gr}$ on the masses of planet and central object [K100].

For the model of dark matter hierarchy appearing in the model of living matter one has $n_a = 2^{11k}$, $k = 1, 2, 3, .., 7$ for cyclotron time scales below life cycle for a magnetic field $B_d = .2$ Gauss at $k = 4$ level of hierarchy (the field strength is fixed by the model for the effects of ELF em fields on vertebrate brain at harmonics of cyclotron frequencies of biologically important ions [K44]). Note that B_d scales as 2^{-11k} from the requirement that cyclotron energy is constant.

ADE correspondence between subgroups of $SU(2)$ and Lie groups in ADE hierarchy encourages to consider the possibility that TGD could mimic ADE hierarchy of gauge theories. In the case of G_a this would mean that many fermion states constructed from single fermion states, which are in one-one correspondence with the elements of G_a group algebra, would define multiplets of the gauge group corresponding to the Dynkin diagram characterizing G_a: for instance, $SU(n_a)$ in the case of Z_{n_a}. Fermion multiplet would contain n_a states and gauge boson multiplet $n_a^2 - 1$ states. This would provide enormous information processing capacity since for $n_a = 2^{11k}$ fermion multiplet would code exactly $11k$ bits of information. Magnetic body could represent binary information using the many-particle states belonging to the representations of say $SU(n_a)$ at its flux tubes.

7.3.2 Jones Inclusions, The Large N Limit Of $SU(N)$ GaugeTheories and AdS/CFT Correspondence

The framework based on Jones inclusions has an obvious resemblance with larger N limit of $SU(N)$ gauge theories and also with the celebrated AdS/CFT correspondence [B33] so that a more detailed comparison is in order.

Large N limit of gauge theories and series of Jones inclusions

The large N limit of $SU(N)$ gauge field theories has as definite resemblance with the series of Jones inclusions with the integer $n \geq 3$ characterizing the quantum phase $q = exp(i\pi/n)$ and the order of the maximal cyclic subgroup of the subgroup of $SU(2)$ defining the inclusion. Recall that all ADE groups except D_{2n+1} and E_7 are allowed (SU(2) is excluded since it would correspond to $n = 2$).

The limiting procedure keeps the value of g^2N fixed. Rather remarkably, this is equivalent with keeping αN constant but assuming $\hbar$ to scale as $n = N$. Thus the quantization of Planck constants would provide a physical laboratory for the testing of large N limit.

The observation suggesting a description of YM theories in terms of closed strings is that Feynman diagrams can be interpreted as being imbedded at closed 2-surfaces of minimal genus guaranteeing that the internal lines meet except in vertices. The contribution of genus g diagrams is proportional to N^{g-1} at the large N limit. The interpretation in terms of closed partonic 2-surfaces is highly suggestive and the N^{g-1} should come from the multiple covering property of CP_2 by N CD-points (or vice versa) with the finite subgroup of $G \subset SU(2)$ defining the Jones inclusion and acting as symmetries of the surface.

Analogy between stacks of branes and multiple coverings of CD and CP_2

An important aspect of AdS/CFT dualities is a prediction of an infinite hierarchy of gauge groups, which as such is as interesting as the claimed dualities. The prediction relies on the notion Dp-branes. Dp-branes are $p+1$-dimensional surfaces of the target space at which the ends of open strings can end. In the simplest situation one considers N parallel p-branes at the limit when the distances between branes characterized by an expectation value of Higgs fields approach zero to obtain what is called N-stack of branes. There are N^2 different strings connecting the branes and the heuristic idea is that they correspond to gauge bosons of $U(N)$ gauge theory. Note that the requirement that AdS/CFT dualities exist forces the introduction of branes and the optimistic interpretation is that a non-perturbative effect of still unknown M-theory is in question. In the limit of an ideal stack one assumes that $U(N)$ gauge theory at the brane representing the stack is obtained. The branes must also carry a p-form defining gauge potential for a closed $p+1$-form. This Ramond charge is quantized and its value equals to N.

Consider now the group $G_a \times G_b \subset SL(2,C) \times SU(2) \subset SU(3)$ defining double Jones inclusion and implying the scalings $\hbar(M^4) \rightarrow n(G_b)\hbar(M^4)$ and $\hbar(CP_2) \rightarrow n(G_a)\hbar(CP_2)$. These space-time surfaces define $n(G_a)$-fold multiple coverings of CP_2 and $n(G_b)$-fold multiple coverings of CD. In CP_2 degrees of freedom the collection of G_b-related partonic 2-surfaces (/3-surfaces/4-surfaces) is highly analogous to the stack of branes. In CD degrees of freedom the stack of copies of surface typically correspond to along a circle (A_n, D_{2n} or at vertices of tetrahedron or isosahedron.

In TGD framework the interpretation strings are not needed to define gauge fields. The group algebra of G realized as discrete plane waves at G-orbit gives rise to representations of G. The hypothesis supported by few examples is that these additional degrees of freedom allow to construct multiplets of the gauge group assignable to the ADE diagram characterizing the inclusion.

AdS/CFT duality

AdS/CFT duality is a further aspect of the brane construction. The dual description of the situation is in terms of a string theory in a background in which N-brane acts as a macroscopic object giving rise to a black-hole like object in (say) 10-dimensional target space. This background has the form $AdS_5 \times X_5$, where AdS_5 is 5-dimensional hyperboloid of M^6 and thus allows $SO(4,2)$ as isometries. X_5 is compact constant curvature space. S_5 gives rise to $N = 4$ SUSY in M^4 with M^4 interpreted as a brane. The first support for the dualities comes from the symmetries: for instance, the $N = 4$ super-symmetrized isometries of $AdS_5 \times S^5$ are same as the symmetries of 4-dimensional $N = 4$ SUSY for $p = 3$ branes. N-branes can be used as models for black holes in target space and black-hole entropy can be calculated using either target space picture or conformal field theory at brane and the results turn out be the same.

Does the TGD equivalent of this duality exists in some sense?

1. As far as partonic 2-surfaces identified as 1-branes are considered, conformal field theory description is trivially true. In TGD framework the analog of Ramond charges are the integers n_a and n_b characterizing the multipliticies of the maximal Abelian subgroups having clear topological meaning. This conforms with the observation that large N limit of the gauge field theories can be formulated in terms of closed surfaces at which the Feynman diagrams are imbedded without self crossings. It seems that the integers n_a and n_b characterizing the Jones inclusion naturally take the role of Ramond charge: this does not of course exclude the possibility they can be expressed as fluxes at space-time level as will be indeed found.
2. Conformal field theory description can be generalized in the sense that one replaces the $n(G_a) \times n(G_b)$ partonic surfaces with single one and describes the new states as primary fields arranged into representations of the ADE group in question. This would mean that the standard model gauge group extends by additional factor which is however non-trivially related to it.
3. If one can accept the idea that the conformal field theory description for partons gives rise to M^4 gauge theory as an approximate description, it is not too difficult to imagine that also ADE hierarchy of gauge theories results as a description of the exotic states. One can say that CFT in p-brane is replaced now with CFT on partonic 2-surface (1-brane) analogous to a closed string.
4. In the minimal interpretation there is no need to add strings connecting the branches of the double covering of the partonic 2-surface whose function is essentially that of making possible gauge bosons as fermion anti-fermion pairs. One could of course imagine gauge fluxes as counterparts of strings but just the fact that G-invariance dictates the configurations completely forces to question this kind of dynamics.
5. There is no reason to expect the emergence of $N = 4$ super-symmetric field theory in M^4 as in the case of super-string models. The reasons should be already obvious: super-conformal generators G anti-commute to L_0 proportional to mass squared rather than four-momentum and the spectrum extended by $G_a \times G_b$ degeneracy contains more states.

One can of course ask whether higher values of p could make sense in TGD framework.

1. It seems that the light-like orbits of the partonic 2-surfaces defining 2-branes do not bring in anything new since the generalized conformal invariance makes it possible the restriction to a 2-dimensional cross section of the light like causal determinant.
2. The idea of regarding space-time surface X^4 as a 3-brane in H in which some kind of conformal field theory is defined is in conflict with the basis ideas of TGD. The role of X^4 interior is to provide classical correlates for quantum dynamics to make possible quantum measurement theory and also introduce correlations between partonic 2-surfaces even in the case that partonic conformal dynamics reduces to a topological string theory. It is quantum classical correspondence which corresponds to this duality.

What is the counterpart of the Ramond charge in TGD?

The condition that there exist a p-form defining $p+1$-gauge field with p-charge equal to n_a or n_b is a rather stringent additional condition also in TGD framework. For $n<\infty$ this kind of charge is defined by Jones inclusion and represented topologically so that Ramond charge is not needed in $n<\infty$ case. By the earlier arguments one must however be able to assign integers n_a and n_b also to $G=SU(2)$ inclusions with Kac-Moody algebra characterized by an extended ADE diagram with the phases $q_i=exp(i\pi/n_i)$ relating to the monodromy of the theory. Since Jones inclusion does not define in this case the value of $n<\infty$ in any obvious manner, the counterpart of the Ramond charge is needed.

1. For partonic 2-surfaces ordinary gauge potential would define this form and the condition would state that magnetic flux equals to n so that the anyonic partonic two-surfaces would be homologically non-trivial in CP_2 degrees of freedom. String ends would define basic example of this situation. This would be the case also in M^4_+ degrees of freedom: the partonic 2-surface would essentially wind n_a times around the tip of δCD and the gauge field in question would be monopole magnetic field in δCD. This kind of situation need not correspond to anything cosmological since future and past light-cones appear in the basic definition of the scattering amplitudes.
2. For $p=3$ Chern-Simons action for the induced CP_2 Kähler form associated with the partonic 2-surface indeed defines this kind of charge. Ramond charge should be simply N. CP_2 type extremals or their small deformations satisfy this constraint and are indeed very natural in elementary particle physics context but too restrictive in a more general context.

Note that the light-like orbits of non-deformed CP_2 type extremals have light-like random curve as an M^4 projection and the conformal symmetries of M^4 obviously respect light-likeness property. Hence $SO(4,2)$ symmetry characterizing AdS$_5$/CFT is not excluded but would be broken by p-adic thermodynamics and by TGD based Higgs mechanism involving the identification of inertial momentum as average value of non-conserved gravitational momentum parallel to the light-like zitterbewegung orbit.

Can one speak about black hole like structures in TGD framework?

For AdS/CFT correspondence there is also a dynamical coupling to the target space metric. The coupling to H-metric is present also now since the overall scalings of the CD *resp.* CP_2 metrics by n_b *resp.* by n_a are involved. This applies to when multiple covering is used explicitly. In the description in which one replaces the multiple covering by ordinary $M^4\times CP_2$, the metric suffers a genuine change and something analogous to the black-hole type metrics encountered in AsS/CFT correspondence might be encountered.

Consider as an example an n_a-fold covering of CP_2 points by M^4 points (ADE diagram A_{n_a-1}). The n-fold covering means only $n2\pi$ rotation for the phase angle ψ of CP_2 complex coordinate leads to the original point. The replacement $\psi\rightarrow\psi/n_a$ gives rise to what would look like ordinary $M^4\times CP_2$ but with a modified CP_2 metric. The metric components containing ψ as index are scaled down by $1/n_a$ or $1/n_a^2$. Notice that Ψ effectively disappears from the dynamics at the large n_a limit.

If one uses an effective description in which covering is eliminated the metric is indeed affected at the level of embedding space black hole like structures at the level of dynamic space might make emerge also in TGD framework at large N limit since the masses of the objects in question become large and CP_2 metric is scaled by N so that CP_2 has very large size at this limit. This need not lead to any inconsistencies if these phases are interpreted as dark matter. At the elementary particle level p-adic thermodynamics predicts that p-adic entropy is proportional to thermal mass squared which implies elementary particle black-hole analogy.

Other dualities

Also quantum classical correspondence defines in a loose sense a duality justifying the basic assumptions of quantum measurement theory. The light-like orbits of 2-D partons are characterized by a generalization of ordinary 2-D conformal invariance so that CFT part of the duality would be very natural. The dynamical target space would be replaced with the space-time surface X^4 with a dynamical metric providing classical correlates for the quantum dynamics at partonic 2-surfaces. The duality in this sense cannot be however exact since classical dynamics cannot fully represent quantum dynamics.

Classical description is not expected to be unique. The basic condition on space-time surfaces assignable to a given configuration of partonic 2-surfaces associated with the surface X_V^3 defining S-matrix element are posed by quantum classical correspondence. Both hyper-quaternionic and co-hyper-quaternionic space-time surfaces are acceptable and this would define a fundamental duality.

A concrete example about this HQ-coHQ duality would be the equivalence of space-time descriptions using 4-D CP_2 type extremals and 4-D string like objects connecting them. If one restricts to CP_2 type extremals and string like objects of from $X^2 \times Y^2$, the target space reduces effectively to M^4 and the dynamical degrees of freedom correspond in both cases to transversal M^4 degrees of freedom. Note that for CP_2 type extremals the conditions stating that random light-likeness of the M^4 projection of the CP_2 type extremal are equivalent to Virasoro conditions. CP_2 type extremals could be identified as co-HQ surfaces whereas stringlike objects would correspond to HQ aspect of the duality.

HQ-coHQ provides dual classical descriptions of same phenomena. Particle massivation would be a basic example. Higgs mechanism in a gauge theory description based on CP_2 type extremals would rely on zitterbewegung implying that the average value of gravitational mass identified as inertial mass is non-vanishing and is discussed already. Higgs field would be assigned to the wormhole contacts. The dual description for the massivation would be in terms of string tension and mass squared would be proportional to the distance between G-related points of CP_2.

These observations would suggest that also a super-conformal algebra containing $SL(2,R) \times SU(2)_L \times U(1)$ or its compact version exists and corresponds to a trivial inclusion. This is indeed the case [A18]. The so called large $N = 4$ super-conformal algebra contains energy momentum current, 2+2 super generators G, $SU(2) \times SU(2) \times U(1)$ Kac-Moody algebra (both $SU(2)$ and SL(2, R) could be interpreted as acting on M^4 spin degrees of freedom, and 2 spin 1/2 fermionic currents having interpretation in terms of right handed neutrinos corresponding to two H-chiralities. Interestingly, the scalar generator is now missing.

7.3.3 Could Mckay Correspondence And Jones Inclusions Relate To Each Other?

The understanding of Langlands correspondence for general reductive Lie groups in TGD framework seems to require some physical mechanism allowing the emergence of these groups in TGD based physics. The physical idea would be that quantum dynamics of TGD is able to emulate the dynamics of any gauge theory or even stringy dynamics of conformal field theory having Kac-Moody type symmetry and that this emulation relies on quantum deformations induced by finite measurement resolution described in terms of Jones inclusions of sub-factors characterized by group G leaving elements of sub-factor invariant. Finite measurement resolution would would result simply from the fact that only quantum numbers defined by the Cartan algebra of G are measured.

There are good reasons to expect that infinite Clifford algebra has the capacity needed to realize representations of an arbitrary Lie group. It is indeed known that that any quantum group characterized by quantum parameter which is root of unity or positive real number can be assigned to Jones inclusion [A45]. For $q = 1$ this would gives ordinary Lie groups. In fact, all amenable groups define unique sub-factor and compact Lie groups are amenable ones.

It was so called McKay correspondence [A96] which originally stimulated the idea about TGD as an analog of Universal Turing machine able to mimic both ADE type gauge theories and theories with ADE type Kac-Moody symmetry algebra. This correspondence and its generalization might also provide understanding about how general reductive groups emerge. In the following I try to cheat the reader to believe that the tensor product of representations of SU(2) Lie algebras for Connes tensor powers of $\mathcal{M}$ could induce ADE type Lie algebras as quantum deformations for the direct sum of n copies of $SU(2)$ algebras This argument generalizes also to the case of other compact Lie groups.

About McKay correspondence

McKay correspondence [A96] relates discrete finite subgroups of $SU(2)$ ADE groups. A simple description of the correspondences is as follows [A96].

1. Consider the irreps of a discrete subgroup $G \subset SU(2)$ which correspond to irreps of G and can be obtained by restricting irreducible representations of $SU(2)$ to those of G. The irreducible representations of $SU(2)$ define the nodes of the graph.
2. Define the lines of graph by forming a tensor product of any of the representations appearing in the diagram with a doublet representation which is always present unless the subgroup is 2-element group. The tensor product regarded as that for $SU(2)$ representations gives representations $j-1/2$, and $j+1/2$ which one can decompose to irreducibles of G so that a branching of the graph can occur. Only branching to two branches occurs for subgroups yielding extended ADE diagrams. For the linear portions of the diagram the spins of corresponding SU(2) representations increase linearly as $.., j, j+1/2, j+1, ...$

One obtains extended Dynkin diagrams of ADE series representing also Kac-Moody algebras giving A_n, D_n, E_6, E_7, E_8. Also A_∞ and $A_{-\infty,\infty}$ are obtained in case that subgroups are infinite. The Dynkin diagrams of non-simply laced groups B_n ($SO(2n+1)$), C_n (symplectic group $Sp(2n)$ and quaternionic group $Sp(n)$), and exceptional groups G_2 and F_4 are not obtained.

ADE Dynkin diagrams labeling Lie groups instead of Kac-Moody algebras and having one node less, do not appear in this context but appear in the classification of Jones inclusions for $\mathcal{M} : \mathcal{N} < 4$. As a matter fact, ADE type Dynkin diagrams appear in very many contexts as one can learn from John Baez's This Week's Finds [A15] .

1. The classification of integral lattices in Rn having a basis of vectors whose length squared equals 2
2. The classification of simply laced semisimple Lie groups.
3. The classification of finite sub-groups of the 3-dimensional rotation group.
4. The classification of simple singularities. In TGD framework these singularities could be assigned to origin for orbifold CP_2/G, $G \subset SU(2)$.
5. The classification of tame quivers.

Principal graphs for Connes tensor powers $\mathcal{M}$

The thought provoking findings are following.

1. The so called principal graphs characterizing $\mathcal{M} : \mathcal{N} = 4$ Jones inclusions for $G = SU(2)$ are extended Dynkin diagrams characterizing ADE type affine (Kac-Moody) algebras. D_n is possible only for $n \geq 4$.
2. $\mathcal{M} : \mathcal{N} < 4$ Jones inclusions correspond to ordinary ADE type diagrams for a subset of simply laced Lie groups (all roots have same length) A_n ($SU(n)$), D_{2n} ($SO(2n)$), and E_6 and E_8. Thus D_{2n+1} ($SO(2n+2)$) and E_7 are not allowed. For instance, for $G = S_3$ the principal graph is not D_3 Dynkin diagram.

The conceptual background behind principal diagram is necessary if one wants to understand the relationship with McKay correspondence.

1. The hierarchy of higher commutations defines an invariant of Jones inclusion $\mathcal{N} \subset \mathcal{M}$. Denoting by $\mathcal{N}'$ the commutant of $\mathcal{N}$ one has sequences of horizontal inclusions defined as $C = \mathcal{N}' \cap \mathcal{N} \subset \mathcal{N}' \cap \mathcal{M} \subset \mathcal{N}' \cap \mathcal{M}^1 \subset ...$ and $C = \mathcal{M}' \cap \mathcal{M} \subset \mathcal{M}' \cap \mathcal{M}^1 \subset ....$ There is also a sequence of vertical inclusions $\mathcal{M}' \cap \mathcal{M}^k \subset \mathcal{N}' \cap \mathcal{M}^k$. This hierarchy defines a hierarchy of Temperley-Lieb algebras [A94] assignable to a finite hierarchy of braids. The commutants in the hierarchy are direct sums of finite-dimensional matrix algebras (irreducible representations) and the inclusion hierarchy can be described in terms of decomposition of irreps of k^{th} level to irreps of $(k-1)^{th}$ level irreps. These decomposition can be described in terms of Bratteli diagrams [A28].

2. The information provided by infinite Bratteli diagram can be coded by a much simpler bi-partite diagram having a preferred vertex. For instance, the number of $2k$-loops starting from it tells the dimension of k^{th} level algebra. This diagram is known as principal graph.

Principal graph emerges also as a concise description of the fusion rules for Connes tensor powers of $\mathcal{M}$.

1. It is natural to decompose the Connes tensor powers [A96] $\mathcal{M}_k = \mathcal{M}\otimes_{\mathcal{N}}...\otimes_{\mathcal{N}}\mathcal{M}$ to irreducible $\mathcal{M}-\mathcal{M}$, $\mathcal{N}-\mathcal{M}$, $\mathcal{M}-\mathcal{N}$, or $\mathcal{N}-\mathcal{N}$ bi-modules. If $\mathcal{M}:\mathcal{N}$ is finite this decomposition involves only finite number of terms. The graphical representation of these decompositions gives rise to Bratteli diagram.
2. If $\mathcal{N}$ has finite depth the information provided by Bratteli diagram can be represented in nutshell using principal graph. The edges of this bipartite graph connect $\mathcal{M}-\mathcal{N}$ vertices to vertices describing irreducible $\mathcal{N}-\mathcal{N}$ representations resulting in the decomposition of $\mathcal{M}-\mathcal{N}$ irreducibles. If this graph is finite, $\mathcal{N}$ is said to have finite depth.

A mechanism assigning to tensor powers Jones inclusions ADE type gauge groups and Kac-Moody algebras

The earliest proposals inspired by the hierarchy of Jones inclusions is that in $\mathcal{M}:\mathcal{N}<4$ case it might be possible to construct ADE representations of gauge groups or quantum groups and in $\mathcal{M}:\mathcal{N}=4$ using the additional degeneracy of states implied by the multiple-sheeted cover $H\rightarrow H/G_a\times G_b$ associated with space-time correlates of Jones inclusions. Either G_a or G_b would correspond to G. In the following this mechanism is articulated in a more refined manner by utilizing the general properties of generators of Lie-algebras understood now as a minimal set of elements of algebra from which the entire algebra can be obtained by repeated commutation operator (I have often used " Lie algebra generator" as an synonym for "Lie algebra element"). This set is finite also for Kac-Moody algebras.

1. Two observations

The explanation to be discussed relies on two observations.

1. McKay correspondence for subgroups of G ($\mathcal{M}:\mathcal{N}=4$) *resp.* its variants ($\mathcal{M}:\mathcal{N}<4$) and its counterpart for Jones inclusions means that finite-dimensional irreducible representations of allowed $G\subset SU(2)$ label both the Cartan algebra generators and the Lie (Kac-Moody) algebra generators of t_+ and t_- in the decomposition $g=h\oplus t_+\oplus t_-$, where h is the Lie algebra of maximal compact subgroup.
2. Second observation is related to the generators of Lie-algebras and their quantum counterparts (see Appendix for the explicit formulas for the generators of various algebras considered). The observation is that each Cartan algebra generator of Lie- and quantum group algebras, corresponds to a triplet of generators defining an SU(2) sub-algebra. The Cartan algebra of affine algebra contains besides Lie group Cartan algebra also a derivation d identifiable as an infinitesimal scaling operator L_0 measuring the conformal weight of the Kac-Moody generators. d is exceptional in that it does not give rise to a triplet. It corresponds to the preferred node added to the Dynkin diagram to get the extended Dynkin diagram.

2. Is ADE algebra generated as a quantum deformation of tensor powers of $SU(2)$ Lie algebras representations?

The ADE type symmetry groups could result as an effect of finite quantum resolution described by inclusions of HFFs in TGD inspired quantum measurement theory.

1. The description of finite resolution typically leads to quantization since complex rays of state space are replaced as $\mathcal{N}$ rays. Hence operators, which would commute for an ideal resolution cease to do so. Therefore the algebra $SU(2)\otimes...\otimes SU(2)$ characterized by n mutually commuting triplets, where n is the number of copies of $SU(2)$ algebra in the original situation and identifiable as quantum algebra appearing in $\mathcal{M}$ tensor powers with $\mathcal{M}$ interpreted as $\mathcal{N}$ module, could suffer quantum deformation to a simple Lie algebra with $3n$ Cartan algebra generators. Also a deformation to a quantum group could occur as a consequence.

2. This argument makes sense also for discrete groups $G \subset SU(2)$ since the representations of G realized in terms of WCW spinor s extend to the representations of SU(2) naturally.
3. Arbitrarily high tensor powers of $\mathcal{M}$ are possible and one can wonder why only finite-dimensional Lie algebra results. The fact that $\mathcal{N}$ has finite depth as a sub-factor means that the tensor products in tensor powers of $\mathcal{N}$ are representable by a finite Dynkin diagram. Finite depth could thus mean that there is a periodicity involved the kn tensor powers decomposes to representations of a Lie algebra with $3n$ Cartan algebra generators. Thus the additional requirement would be that the number of tensor powers of $\mathcal{M}$ is multiple of n.

3. Space-time correlate for the tensor powers $\mathcal{M} \otimes_{\mathcal{N}} ... \otimes_{\mathcal{N}} \mathcal{M}$

By quantum classical correspondence there should exist space-time correlate for the formation of tensor powers of $\mathcal{M}$ regarded as $\mathcal{N}$ module. A concrete space-time realization for this kind of situation in TGD would be based on n-fold cyclic covering of H implied by the $H \rightarrow H/G_a \times G_b$ bundle structure in the case of say G_b. The sheets of the cyclic covering would correspond to various factors in the n-fold tensor power of $SU(2)$ and one would obtain a Lie algebra, affine algebra or its quantum counterpart with n Cartan algebra generators in the process naturally. The number n for space-time sheets would be also a space-time correlate for the finite depth of $\mathcal{N}$ as a factor.

WCW spinors could provide fermionic representations of $G \subset SU(2)$. The Dynkin diagram characterizing tensor products of representations of $G \subset SU(2)$ with doublet representation suggests that tensor products of doublet representations associated with n sheets of the covering could realize the Dynkin diagram.

Singlet representation in the Dynkin diagram associated with irreps of G would not give rise to an SU(2) sub-algebra in ADE Lie algebra and would correspond to the scaling generator. For ordinary Dynkin diagram representing gauge group algebra scaling operator would be absent and therefore also the exceptional node. Thus the difference between ($\mathcal{M} : \mathcal{N} = 4$) and ($\mathcal{M} : \mathcal{N} < 4$) cases would be that in the Kac-Moody group would reduce to gauge group $\mathcal{M} : \mathcal{N} < 4$ because Kac-Moody central charge k and therefore also Virasoro central charge resulting in Sugawara construction would vanish.

4. Do finite subgroups of $SU(2)$ play some role also in $\mathcal{M} : \mathcal{N} = 4$ case?

One can ask wonder the possible interpretation for the appearance of extended Dynkin diagrams in ($\mathcal{M} : \mathcal{N} = 4$) case. Do finite subgroups $G \subset SU(2)$ associated with extended Dynkin diagrams appear also in this case. The formal analog for $H \rightarrow G_a \times G_b$ bundle structure would be $H \rightarrow H/G_a \times SU(2)$. This would mean that the geodesic sphere of CP_2 would define the fiber. The notion of number theoretic braid meaning a selection of a discrete subset of algebraic points of the geodesic sphere of CP_2 suggests that $SU(2)$ actually reduces to its subgroup G also in this case.

5. Why Kac-Moody central charge can be non-vanishing only for $\mathcal{M} : \mathcal{N} = 4$?

From the physical point of view the vanishing of Kac-Moody central charge for $\mathcal{M} : \mathcal{N} < 4$ is easy to understand. If parton corresponds to a homologically non-trivial geodesic sphere, space-time surface typically represents a string like object so that the generation of Kac-Moody central extension would relate directly to the homological non-triviality of partons. For instance, cosmic strings are string like objects of form $X^2 \times Y^2$, where X^2 is minimal surface of M^2 and Y^2 is a holomorphic sub-manifold of CP_2 reducing to a homologically non-trivial geodesic sphere in the simplest situation. A conjecture that deserves to be shown wrong is that central charge k is proportional/equal to the absolute value of the homology (Kähler magnetic) charge h.

6. More general situation

McKay correspondence generalizes also to the case of subgroups of higher-dimensional Lie groups [A96]. The argument above makes sense also for discrete subgroups of more general compact Lie groups H since also they define unique sub-factors. In this case, algebras having Cartan algebra with nk generators, where n is the dimension of Cartan algebra of H, would emerge in the process. Thus there are reasons to believe that TGD could emulate practically any dynamics having gauge group or Kac-Moody type symmetry. An interesting question concerns the interpretation of non-ADE type principal graphs associated with subgroups of SU(2).

7. Flavor groups of hadron physics as a support for HFF?

The deformation assigning to an n-fold tensor power of representations of Lie group G with k-dimensional Cartan algebra a representation of a Lie group with nk-dimensional Cartan algebra could be also seen as a dynamically generated symmetry. If quantum measurement is characterized by the choice of Lie group G defining measured quantum numbers and defining Jones inclusion characterizing the measurement resolution, the measurement process itself would generate these dynamical symmetries. Interestingly, the flavor symmetry groups of hadron physics cannot be justified from the structure of the standard model having only electro-weak and color group as fundamental symmetries. In TGD framework flavor group $SU(n)$ could emerge naturally as a fusion of n quark doublets to form a representation of $SU(n)$.

7.3.4 Farey Sequences, Riemann Hypothesis, Tangles, And TGD

Farey sequences allow an alternative formulation of Riemann Hypothesis and subsequent pairs in Farey sequence characterize so called rational 2-tangles. In TGD framework Farey sequences relate very closely to dark matter hierarchy, which inspires *"Platonia as the best possible world in the sense that cognitive representations are optimal"* as the basic variational principle of mathematics. This variational principle supports RH.

Possible TGD realizations of tangles, which are considerably more general objects than braids, are considered. One can assign to a given rational tangle a rational number a/b and the tangles labeled by a/b and c/d are equivalent if $ad - bc = \pm 1$ holds true. This means that the rationals in question are neighboring members of Farey sequence. Very light-hearted guesses about possible generalization of these invariants to the case of general N-tangles are made.

Farey sequences

Some basic facts about Farey sequences [A4] demonstrate that they are very interesting also from TGD point of view.

1. Farey sequence F_N is defined as the set of rationals $0 \leq q = m/n \leq 1$ satisfying the conditions $n \leq N$ ordered in an increasing sequence.
2. Two subsequent terms a/b and c/d in F_N satisfy the condition $ad - bc = 1$ and thus define and element of the modular group $SL(2, Z)$.
3. The number $|F(N)|$ of terms in Farey sequence is given by

$$|F(N)| = |F(N-1)| + \phi(N-1) \ . \tag{7.3.4}$$

   Here $\phi(n)$ is Euler's totient function giving the number of divisors of n. For primes one has $\phi(p) = 1$ so that in the transition from p to $p+1$ the length of Farey sequence increases by one unit by the addition of $q = 1/(p+1)$ to the sequence.

The members of Farey sequence F_N are in one-one correspondence with the set of quantum phases $q_n = exp(i2\pi/n)$, $0 \leq n \leq N$. This suggests a close connection with the hierarchy of Jones inclusions, quantum groups, and in TGD context with quantum measurement theory with finite measurement resolution and the hierarchy of Planck constants involving the generalization of the embedding space. Also the recent TGD inspired ideas about the hierarchy of subgroups of the rational modular group with subgroups labeled by integers N and in direct correspondence with the hierarchy of quantum critical phases [K35] would naturally relate to the Farey sequence.

Riemann Hypothesis and Farey sequences

Farey sequences are used in two equivalent formulations of the Riemann hypothesis. Suppose the terms of F_N are $a_{n,N}$, $0 < n \leq |F_N|$. Define

$$d_{n,N} = a_{n,N} - \frac{n}{|F_N|} \ .$$

In other words, $d_{n,N}$ is the difference between the n: th term of the N: th Farey sequence, and the n: th member of a set of the same number of points, distributed evenly on the unit interval. Franel and Landau proved that both of the following statements

$$\sum_{n=1,...,|F_N|} |d_{n,N}| = O(N^r) \text{ for any } r > 1/2 \ ,$$
$$\sum_{n=1,...,|F_N|} d_{n,N}^2 = O(N^r) \text{ for any } r > 1 \ . \quad (7.3.5)$$

are equivalent with Riemann hypothesis.

One could say that RH would guarantee that the numbers of Farey sequence provide the best possible approximate representation for the evenly distributed rational numbers $n/|F_N|$.

Farey sequences and TGD

Farey sequences seem to relate very closely to TGD.

1. The rationals in the Farey sequence can be mapped to the roots of unity by the map $q \rightarrow exp(i2\pi q)$. The numbers $1/|F_N|$ are in turn mapped to the numbers $exp(i2\pi/|F_N|)$, which are also roots of unity. The statement would be that the algebraic phases defined by Farey sequence give the best possible approximate representation for the phases $exp(in2\pi/|F_N|)$ with evenly distributed phase angle.
2. In TGD framework the phase factors defined by F_N corresponds to the set of quantum phases corresponding to Jones inclusions labeled by $q = exp(i2\pi/n)$, $n \leq N$, and thus to the N lowest levels of dark matter hierarchy. There are actually two hierarchies corresponding to M^4 and CP_2 degrees of freedom and the Planck constant appearing in Schrödinger equation corresponds to the ratio n_a/n_b defining quantum phases in these degrees of freedom. $Z_{n_a \times n_b}$ appears as a conformal symmetry of "dark" partonic 2-surfaces and with very general assumptions this implies that there are only in TGD Universe [K35, K33].
3. The fusion of physics associated with various number fields to single coherent whole requires algebraic universality. In particular, the roots of unity, which are complex algebraic numbers, should define approximations to continuum of phase factors (angle is not well-defined notion p-adically but trigonometric functions are if algebraic extensions involving roots of unity are allowed). The S-matrix associated with p-adic-to-padic transitions can involve only this kind of algebraic phases. One can also say that cognitive representations can involve only algebraic phases and algebraic numbers in general. For completions of algebraic extensions of rationals U-matrix, M-matrix and S-matrix would be obtained by algebraic continuation from from that in the extension of rationals. One can also say that in the intersection all parameters belong to an extension of rationals and various transition amplitudes have parameters in this intersection. The core of physics (its "genes") would be number theoretically universal [?]
4. The subgroups of the hierarchy of subgroups of the modular group with rational matrix elements are labeled by integer N and relate naturally to the hierarchy of Farey sequences. The hierarchy of quantum critical phases is labeled by integers N with quantum phase transitions occurring only between phases for which the smaller integer divides the larger one [K35].

Interpretation of RH in TGD framework

Number theoretic universality of physics suggests an interpretation for the Riemann hypothesis in TGD framework. RH would be equivalent to the statement that the Farey numbers provide best possible approximation to the set of rationals $k/|F_N|$ or to the statement that the roots of unity contained by F_N define the best possible approximation for the roots of unity defined as $exp(ik2\pi/|F_N|)$ with evenly spaced phase angles. The roots of unity allowed by the lowest N levels of the dark matter hierarchy allows the best possible approximate representation for algebraic phases represented exactly at $|F_N|$: th level of hierarchy.

A stronger statement would be that the Platonia, where RH holds true would be the best possible world in the sense that algebraic physics behind the cognitive representations would allow the best possible approximation hierarchy for the continuum physics (both for numbers in unit interval and for phases on unit circle). Platonia with RH would be cognitive paradise.

One could see this also from different view point. "Platonia as the cognitively best possible world" could be taken as the "axiom of all axioms": a kind of fundamental variational principle of mathematics. Among other things it would allow to conclude that RH is true: RH must hold true either as a theorem following from some axiomatics or as an axiom in itself.

Could rational N-tangles exist in some sense?

The article of Kauffman and Lambropoulou [A71] about rational 2-tangles having commutative sum and product allowing to map them to rationals is very interesting from TGD point of view. The illustrations of the article are beautiful and make it easy to get the gist of various ideas. The theorem of the article states that equivalent rational tangles giving trivial tangle in the product correspond to subsequent Farey numbers a/b and c/d satisfying $ad - bc = \pm 1$ so that the pair defines element of the modular group SL(2, Z).

1. Rational 2-tangles

1. The basic observation is that 2-tangles are 2-tangles in both "s- and t-channels". Product and sum can be defined for all tangles but only in the case of 2-tangles the sum, which in this case reduces to product in t-channel obtained by putting tangles in series, gives 2-tangle. The so called rational tangles are 2-tangles constructible by using addition of $\pm[1]$ on left or right of tangle and multiplication by $\pm[1]$ on top or bottom. Product and sum are commutative for rational 2-tangles but the outcome is not a rational 2-tangle in the general case. One can also assign to rational 2-tangle its negative and inverse. One can map 2-tangle to a number which is rational for rational tangles. The tangles $[0]$, $[\infty]$, $\pm[1]$, $\pm 1/[1]$, $\pm[2]$, $\pm[1/2]$ define so called elementary rational 2-tangles.
2. In the general case the sum of $M-$ and $N-$tangles is $M+N-2$-tangle and combines various $N-$tangles to a monoidal structure. Tensor product like operation giving $M+N$-tangle looks to me physically more natural than the sum.
3. The reason why general 2-tangles are non-commutative although 2-braids obviously commute is that 2-tangles can be regarded as sequences of $N-$tangles with 2-tangles appearing only as the initial and final state: N is actually even for intermediate states. Since $N > 2$-braid groups are non-commutative, non-commutativity results. It would be interesting to know whether braid group representations have been used to construct representations of $N-$tangles.

2. Does generalization to $N >> 2$ case exist?

One can wonder whether the notion of rational tangle and the basic result of the article about equivalence of tangles might somehow generalize to the $N > 2$ case.

1. Could the commutativity of tangle product allow to characterize the $N > 2$ generalizations of rational 2-tangles. The commutativity of product would be a space-time correlate for the commutativity of the S-matrices defining time like entanglement between the initial and final quantum states assignable to the N-tangle. For 2-tangles commutativity of the sum would have an analogous interpretation. Sum is not a very natural operation for N-tangles for $N > 2$. Commutativity means that the representation matrices defined as products of braid group actions associated with the various intermediate states and acting in the same representation space commute. Only in very special cases one can expect commutativity for tangles since commutativity is lost already for braids.
2. The representations of 2-tangles should involve the subgroups of N-braid groups of intermediate braids identifiable as Galois groups of N: th order polynomials in the realization as number theoretic tangles. Could non-commutative 2-tangles be characterized by algebraic numbers in the extensions to which the Galois groups are associated? Could the non-commutativity reflect directly the non-commutativity of Galois groups involved? Quite generally one can ask whether the invariants should be expressible using algebraic numbers in the extensions of rationals associated with the intermediate braids.

3. Rational 2-tangles can be characterized by a rational number obtained by a projective identification $[a, b]^T \rightarrow a/b$ from a rational 2-spinor $[a, b]^T$ to which SL(2(N-1), Z) acts. Equivalence means that the columns $[a, b]^T$ and $[c, d]^T$ combine to form element of SL(2, Z) and thus defining a modular transformation. Could more general 2-tangles have a similar representation but in terms of algebraic integers?
4. Could N-tangles be characterized by $N-1$ $2(N-1)$-component projective column-spinors $[a_i^1, a_i^2, .., a_i^{2(N-1)}]^T$, $i = 1, ...N-1$ so that only the ratios $a_i^k/a_i^{2(N-1)} \leq 1$ matter? Could equivalence for them mean that the $N-1$ spinors combine to form $N-1+N-1$ columns of $SL(2(N-1), Z)$ matrix. Could N-tangles quite generally correspond to collections of projective $N-1$ spinors having as components algebraic integers and could $ad - bc = \pm 1$ criterion generalize? Note that the modular group for surfaces of genus g is SL(2g, Z) so that $N-1$ would be analogous to g and $1 \leq N \geq 3$- braids would correspond to $g \leq 2$ Riemann surfaces.
5. Dark matter hierarchy leads naturally to a hierarchy of modular sub-groups of $SL(2, Q)$ labeled by N (the generator $\tau \rightarrow \tau + 2$ of modular group is replaced with $\tau \rightarrow \tau + 2/N$). What might be the role of these subgroups and corresponding subgroups of $SL(2(N-1), Q)$. Could they arise in "anyonization" when one considers quantum group representations of 2-tangles with twist operation represented by an N: th root of unity instead of phase U satisfying $U^2 = 1$?

How tangles could be realized in TGD Universe?

The article of Kauffman and Lambropoulou stimulated the question in what senses N-tangles could be realized in TGD Universe as fundamental structures.

1. Tangles as number theoretic braids?

The strands of number theoretical $N-$braids correspond to roots of N: th order polynomial and if one allows time evolutions of partonic 2-surface leading to the disappearance or appearance of real roots $N-$tangles become possible. This however means continuous evolution of roots so that the coefficients of polynomials defining the partonic 2-surface can be rational only in initial and final state but not in all intermediate "virtual" states.

2. Tangles as tangled partonic 2-surfaces?

Tangles could appear in TGD also in second manner.

1. Partonic 2-surfaces are sub-manifolds of a 3-D section of space-time surface. If partonic 2-surfaces have genus $g > 0$ the handles can become knotted and linked and one obtains besides ordinary knots and links more general knots and links in which circle is replaced by figure eight and its generalizations obtained by adding more circles (eyeglasses for $N-$eyed creatures).
2. Since these 2-surfaces are space-like, the resulting structures are indeed tangles rather than only braids. Tangles made of strands with fixed ends would result by allowing spherical partons elongate to long strands with fixed ends. DNA tangles would the basic example, and are discussed also in the article. DNA sequences to which I have speculatively assigned invisible (dark) braid structures might be seen in this context as space-like "written language representations" of genetic programs represented as number theoretic braids.

7.3.5 Only The Quantum Variants Of M^4 And M^8 Emerge FromLocal Hyper-Finite II_1 Factors

Super-symmetry suggests that the representations of CH Clifford algebra $\mathcal{M}$ as $\mathcal{N}$ module $\mathcal{M}/\mathcal{N}$ should have bosonic counterpart in the sense that the coordinate for M^8 representable as a particular $M^2(Q)$ element should have quantum counterpart. Same would apply to M^4 coordinate representable as $M^2(C)$ element. Quantum matrix representation of $\mathcal{M}/\mathcal{N}$ as $SL_q(2, F)$ matrix, $F = C, H$ is the natural candidate for this representation. As a matter fact, this guess is not quite correct. It is the interpretation of $M_2(C)$ as a quaternionic quantum algebra whose generalization to the octonionic quantum algebra works.

Quantum variants of M^D exist for all dimensions but only spaces M^4 and M^8 and their linear sub-spaces emerge from hyper-finite factors of type II_1. This is due to the non-associativity of the octonionic representation of the gamma matrices making it impossible to absorb the powers of the octonionic coordinate to the Clifford algebra element so that the local algebra character would disappear. Even more: quantum coordinates for these spaces are commutative operators so that their spectra define ordinary M^4 and M^8 which are thus already quantal concepts.

The commutation relations for $M_{2,q}(C)$ matrices

$$\left(\begin{array}{cc} a & b \\ c & d \end{array} \right) \ , \tag{7.3.6}$$

read as

$$\begin{array}{lll} ab = qba \ , & ac = qac \ , & bd = qdb \ , cd = qdc \ , \\ [ad, da] = (q - q^{-1})bc \ , & bc = cb \ . & \end{array} \tag{7.3.7}$$

These relations can be extended by postulating complex conjugates of these relations for complex conjugates $a^\dagger, b^\dagger, c^\dagger, d^\dagger$ plus the following non-vanishing commutators of type $[x, y^\dagger]$:

$$[a, a^\dagger] = [b, b^\dagger] = [c, c^\dagger] = [d, d^\dagger] = 1 \ . \tag{7.3.8}$$

The matrices representing M^4 point must be expressible as sums of Pauli spin matrices. This can be represented as following conditions on physical states

$$\begin{array}{rcl} O|phys\rangle & = & 0 \ , \\ O & \in & \{a - a^\dagger, d - d^\dagger, b - c^\dagger, c - b^\dagger\} \ . \end{array} \tag{7.3.9}$$

For instance, the first two conditions follow from the reality of Pauli sigma matrices $\sigma_x, \sigma_y, \sigma_z$. These conditions are compatible only if the operators O commute. This is the case and means also that the operators representing M^4 coordinates commute and it is possible to define quantum states for which M^4 coordinates have well-defined eigenvalues so that ordinary M^4 emerges purely quantally from quaternions whose real coefficients are made non-Hermitian operators to obtain operator complexificiation of quaternions. Also the quantum states in which M^4 coordinates are emerge naturally.

$M_{2,q}(C)$ matrices define the quantum analog of C^4 and one can wonder whether other linear sub-spaces can be defined consistently or whether M^4_q and thus Minkowski signature is unique. This seems to be the case. For instance, the replacement $a - \overline{a} \rightarrow a + \overline{a}$ making also time variable Euclidian is impossible since $[a + \overline{a}, d - \overline{d}] = 2(q - q^{-1})bc$ does not vanish. The observation that M^4 coordinates can be regarded as eigenvalues of commuting observables proves that quantum CD and its orbifold description are equivalent.

What about M^8: does it have analogous description? The representation of M^4 point as $M_2(C)$ matrix can be interpreted a combination of 4-D gamma matrices defining hyper-quaternionic units. Hyper-octonionic units indeed have anti-commutation relations of gamma matrices of M^8 and would give classical representation of M^8. The counterpart of $M_{2,q}(C)$ would thus be obtained by replacing the coefficients of hyper-octonionic units with operators satisfying the generalization of $M_{2,q}(C)$ commutation relations. One should identify the reality conditions and find whether they are mutually consistent.

Introduce the coefficients of E^4 gamma matrices having interpretation as quaterionic units as

$$\begin{array}{ll} a_0 = ix(a + d) \ , & a_3 = x(a - d) \ , \\ a_1 = x(b + c) \ , & a_2 = x(ib - c) \ , \\ x = \frac{1}{\sqrt{2}} \ , & \end{array}$$

and write the commutations relations for them to see how the generalization should be performed.

The selections of commutative and quaternionic sub-algebras of octonion space are fundamental for TGD and quantum octonionic algebra should reflect these selections in its structure. In the case of quaternions the selection of commutative sub-algebra implies the breaking of 4-D Lorentz symmetry. In the case of octonions the selection of quaternion sub-algebra should induce the breaking of 8-D Lorentz symmetry. Quaternionic sub-algebra obeys the commutations of $M_q(2,C)$ whereas the coefficients in in the complement commute mutually and quantum commute with the complex sub-algebra. This nails down the commutation relations completely:

$$\begin{aligned}
[a_0,a_3] &= -i(q-q^{-1})(a_1^2+a_2^2) \ , \\
[a_i,a_j] &= 0 \ , \ i,j\neq 0,3 \ , \\
a_0a_i &= qa_ia_0 \ , \ i\neq 0,3 \ , \\
a_3a_i &= qa_ia_3 \ , \ i\neq 0,3 \ .
\end{aligned} \tag{7.3.10}$$

Checking that M^8 indeed corresponds to commutative subspace defined by the eigenvalues of operators is straightforward.

The argument generalizes easily to other dimensions $D\geq 4$ but now quaternionic and octonionic units must be replaced by gamma matrices and an explicit matrix representation can be introduced. These gamma matrices can be included as a tensor factor to the infinite-dimensional Clifford algebra so that the local Clifford algebra reduces to a mere Clifford algebra. The units of quantum octonions which are just ordinary octonion units do not however allow matrix representation so that this reduction is not possible and embedding space and space-time indeed emerge genuinely. The non-associativity of octonions would determine the laws of physics in TGD Universe!

Thus the special role of classical number fields and uniqueness of space-time and embedding space dimensions becomes really manifest only when a quantal deformation of the quaternionic and octonionic matrix algebras is performed. It is possible to construct the quantal variants of the coset spaces $M^4\times E^4/G_a\times G_b$ by simply posing restrictions on the of eigen states of the commuting coordinate operators. Also the quantum variants of the space-time surface and quite generally, manifolds obtained from linear spaces by geometric constructions become possible.

Chapter 8

Philosophy of Adelic Physics

8.1 Introduction

I have developed during last 39 years a proposal for unifying fundamental interactions which I call "Topological Geometrodynamics" (TGD). During last twenty years TGD has expanded to a theory of consciousness and quantum biology and also p-adic and adelic physics have emerged as one thread in the number theoretical vision about TGD.

Since Quantum TGD and physical arguments have served as basic guidelines in the development of p-adic ideas, the best way to introduce the subject of p-adic physics, is by describing first TGD briefly.

In this article I will consider the p-adic aspects of TGD - the first thread of the number theoretic vision - as I see them at this moment.

1. I will describe p-adic mass calculations based on p-adic generalization of thermodynamics and super-conformal invariance [K66, K33] with number theoretical existence constrains leading to highly non-trivial and successful physical predictions. Here the notion of canonical identification mapping p-adic mass squared to real mass squared emerges and is expected to be key player of adelic physics and allow to map various invariants from p-adics to reals and vice versa.

2. I will propose the formulation of p-adicization of real physics and adelization meaning the fusion of real physics and various p-adic physics to single coherent whole by a generalization of number concept fusing reals and p-adics to larger structure having algebraic extension of rationals as a kind of intersection.

   The existence of p-adic variants of definite integral, Fourier analysis, Hilbert space, and Riemann geometry is far form obvious, and various constraints lead to the idea of NTU and finite measurement resolution realized in terms of number theory. Maybe the only way to overcome the problems relies on the idea that various angles and their hyperbolic analogs are replaced with their exponentials and identified as roots of unity and roots of e existing in finite-dimensional algebraic extension of p-adic numbers. Only group invariants - typically squares of distances and norms - are mapped by canonical identification from p-adic to real realm and various phases are mapped to themselves as number theoretically universal entities.

   Another challenge is the correspondence between real and p-adic physics at various levels: space-time level, embedding space level, and WCW level. Here the enormous symmetries of WCW and those of embedding space are in crucial role. Strong form of holography (SH) allows a correspondence between real and p-adic space-time surfaces induced by algebraic continuation from string world sheets and partonic 2-surface, which can be said to be common to real and p-adic space-time surfaces.

3. In the last section I will describe the role of p-adic physics in TGD inspired theory of consciousness. The key notion is Negentropic entanglement (NE) characterized in terms of number theoretic entanglement negentropy (NEN). Negentropy Maximization Principle (NMP) would force the growth of NE. The interpretation would be in terms of evolution as increase of negentropy resources - Akashic records as one might poetically say. The newest finding is

that NMP in statistical sense follows from the mere fact that the dimension of extension of rationals defining adeles increases unavoidably in statistical sense - separate NMP would not be necessary.

In the sequel I will use some shorthand notations for key principles and key notions. Quantum Field Theory (QFT); Relativity Principle (RP); Equivalence Principle (EP); General Coordinate Invariance (GCI); World of Classical Worlds (WCW); Strong Form of GCI (SGCI); Strong Form of Holography (SH); Preferred Extremal (PE); Zero Energy Ontology (ZEO); Quantum Criticality (QC); Hyper-finite Factor of Type II_1 (HFF); Number Theoretical Universality (NTU); Canonical Identification (CI); Negentropy Maximization Principle (NMP); Negentropic entanglement (NE); Number Theoretical Entanglement Negentropy (NEN); are the most often occurring acronyms.

8.2 TGD briefly

This section gives a brief summary of classical and quantum TGD, which to my opinion is necessary for understanding the number theoretic vision.

8.2.1 Space-time as 4-surface

TGD forces a new view about space-time as 4-surface of 8-D imbedding space. This view is extremely simple locally but by its many-sheetedness topologically much more complex than GRT space-time.

Energy problem of GRT as starting point

The physical motivation for TGD was what I have christened the energy problem of General Relativity [K128, L23].

1. The notion of energy is ill-defined because the basic symmetries of empty space-time are lost in the presence of gravity. The presence of matter curves empty Minkowski space M^4 so that its isometries realized as transformations leaving the distances between points and thus shapes of 4-D objects invariant are lost. Noether's theorem states that symmetries and conservation laws correspond to each other. Hence conservation laws are lost and conserved quantities are ill-defined. Usually this is not seen a practical problem since gravitation is so weak interaction.
2. The proposed way out of the problem is based on the assumption that space-times are imbeddable as 4-surfaces to some 8-dimensional space $H = M^4 \times S$ by replacing the points of 4-D empty Minkowski space with 4-D very small internal space S. The space S is unique from the requirement that the theory has the symmetries of standard model: $S = CP_2$, where CP_2 is complex projective space with 4 real dimensions [K128]. Isometries of space-time are replaced with those of imbedding space. Noether's theorem predicts the classical conserved charges for given general coordinate invariant (GCI) action principle.

Also now the curvature of space-time codes for gravitation. Equivalence Principle (EP) and General Coordinate Invariance (GCI) of GRT augmented with Relativity Principle (RT) of SRT remain the basic principles. Now however the number of solutions to field equations - preferred extremals (PEs) - is dramatically smaller than in Einstein's theory [K10, K18].

1. An unexpected bonus was geometrization classical fields of standard model for $S = CP_2$. Also the space-time counterparts for field quanta emerge naturally but this requires a profound generalization of the notion of space-time: the topological inhomogenities of space-time surface are identified as particles. This means a further huge reduction for dynamical field like variables at the level of single space-time sheet. By general coordinate invariance (GCI) only four imbedding space coordinates appear as variables analogous to classical fields: in a typical GUT their number is hundreds.

2. CP_2 also codes for the standard model quantum numbers in its geometry in the sense that electromagnetic charge and weak isospin emerge from CP_2 geometry: the corresponding symmetries are not isometries so that electroweak symmetry breaking is coded already at this level. Color quantum numbers correspond to the isometries of CP_2 defining an unbroken symmetry: this also conforms with empirical facts. The color of TGD however differs from that in standard model in several aspects and LHC has began to exhibit these differences via the unexpected behavior of what was believed to be quark gluon plasma [K74]. The conservation of baryon and lepton numbers follows as a prediction. Leptons and quarks correspond to opposite chiralities for imbedding space spinors.

3. What remains to be explained in standard model is family replication phenomenon for leptons and quarks. Both quarks and leptons appear as three families identical apart from having different masses. The conjecture was is that fermion families correspond to different topologies for 2-D surfaces characterized by genus telling the number g (genus) of handles attached to sphere to obtain the surface: sphere, torus, ... The 2-surfaces are identified as "partonic 2-surfaces" whose orbits are light-like 3-surface at which the signature of the induced metric of space-time surface transforms from Minkowskian to Euclidian. The partonic orbits replace the lines of Feynman diagrams in TGD Universe in accordance with the replacement of point-like particle with 2-surface.

   Only the three lowest genera are observed experimentally. A possible explanation is in terms of conformal symmetries: the genera $g \leq 2$ allow always Z_2 as a subgroup of conformal symmetries (hyper-ellipticity) whereas higher genera in general do not. The handles of partonic 2-surfaces could form analogs of unbound many-particle states for $g > 2$ with a continuous spectrum of mass squared but for $g = 2$ form a bound state by hyper-ellipticity [K33].

4. Later further arguments in favor of$H = M^4 \times CP_2$ have emerged. One of them relates to twistorialization and twistor lift of TGD [K114, K50, K14]. 4-D Minkowski space is unique space-time with Minkowskian signature of metric in the sense that it allows twistor structure. This is a problem in attempts to introduce twistors to General Relativity Theory (GRT) and a serious obstacle in the quantization based on twistor Grassmann approach, which has demonstrate its enormous power in the quantization of gauge theories. In TGD framework one can ask whether one could lift also the twistor structure to the level of H. M^4 has twistor structure and so does also CP_2: which is the only Euclidian 4-manifold allowing twistor space, which is also a Kähler manifold! This led to the notion of twistor lift of TGD inducing rather recent breakthrough in the understanding of TGD.

TGD can be also seen as a generalization of hadronic string model - not yet superstring model since this model became fashionable two years after the thesis about TGD [K3]. Later it has become clear that string like objects, which look like strings but are actually 3-D are basic stuff of TGD Universe and appear in all scales [K37, K10]. Also strictly 2-D string world sheets popped up in the formulation of quantum TGD (analogy with branes) [**?**]o that one can say that string model in 4-D space-time is part of TGD.

Concluding, TGD generalizes standard model symmetries and provides an incredibly simple proposal for a dynamics: only 4 classical field variables and in fermionic sector only quark and lepton like spinor fields. The basic objection against TGD looks rather obvious in the light of after-wisdom. One loses linear superposition of fields, which holds in good approximation in ordinary field theories, which are almost linear. The solution of the problem to be discussed later relies on the notion many-sheeted space-time [L23].

Many-sheeted space-time

The replacement of the abstract manifold geometry of general relativity with the geometry of 4-surfaces brings in the shape of surface as seen from the perspective of 8-D space-time as additional degrees of freedom giving excellent hopes of realizing the dream of Einstein about geometrization of fundamental interactions.

The work with the generic solutions of the field equations assignable to almost any variational principle satisfying GCI led soon to the realization that the topological structure of space-time in this framework is much more richer than in GRT.

1. Space-time decomposes into space-time sheets of finite size. This led to the identification of physical objects that we perceive around us as space-time sheets. The original identification of space-time sheet was as a surface of in H with outer boundary. For instance, the outer boundary of the table would be where that particular space-time sheet ends (what "ends" means is not however quite obvious!). We would directly see the complex topology of many-sheeted space-time! Besides sheets also string like objects and elementary particle like objects appear so that TGD can be regarded also as a generalization of string models obtained by replacing strings with 3-D surfaces.

   It turned that boundaries are probably excluded by boundary conditions. Rather, two sheets with boundaries must be glued along their boundaries together to get double covering. Sphere can be seen as simplest example of this kind of covering: northern and southern hemispheres are glued along equator together.

2. The original vision was that elementary particles are topological inhomogenities glued to these space-time sheets using topological sum contacts. This means drilling a hole to both sheets and connecting with a very short cylinder. 2-dimensional illustration should give the idea. In this conceptual framework material structures and shapes would not be due to some mysterious substance in slightly curved space-time but reduce to space-time topology just as energy- momentum currents reduce to space-time curvature in GRT.

   This view has gradually evolved to much more detailed picture. Elementary particles have wormhole contacts as basic building bricks. Wormhole contact is very small region with *Euclidian (!)* signature of the induced metric connecting two Minkowskian space-time sheets with light-like boundaries carrying spinor fields and there particle quantum numbers. Wormhole contact carries magnetic monopole flux through it and there must be second wormhole contact in order to have closed lines of magnetic flux. Particle world lines are replaced with 3-D light-like surfaces - orbits of partonic 2-surfaces - at which the signature of the induced metric changes.

   One might describe particle as a pair of magnetic monopoles with opposite charges. With some natural assumptions the explanation for the family replication phenomenon in terms of the genus g of the partonic 2-surface is not affected. Bosons emerge as fermion anti-fermion pairs with fermion and anti-fermion at the opposite throats of the wormhole contact. In principle family replication phenomenon should have bosonic analog. This picture assigns to particles strings connecting the two wormhole throats at each space-time sheet so that string model mathematics becomes part of TGD.

The notion of classical field differs in TGD framework in many respects from that in Maxwellian theory.

1. In TGD framework fields do not obey linear superposition and all classical fields are expressible in terms of four imbedding space coordinates in given region of space-time surface. Superposition for classical fields is replaced with *superposition of their effects* [K109, K128] - in full accordance with operationalism. Particle can topologically condense simultaneously to several space-time sheets by generating topological sum contacts (not stable like the wormhole contacts carrying magnetic monopole flux and defining building bricks of particles). Particle "experiences" the superposition of the effects of the classical fields at various space-time sheets rather than the superposition of the fields.

   It is also natural to expect that at macroscopic length scales the physics of classical fields (to be distinguished from that for field quanta) can be explained using only four primary field like variables. Electromagnetic gauge potential has only four components and classical electromagnetc fields give and excellent description of physics. This relates directly to electroweak symmetry breaking in color confinement which in standard model imply the effective absence of weak and color gauge fields in macroscopic scales. TGD however predicts that copies of hadronic physics and electroweak physics could exist in arbitrary long scales [K73] and there are indications that just this makes living matter so different as compared to inanimate matter.

2. The notion of induced gauge field means that one induces electroweak gauge potentials defining so called spinor connection at space-time surface (induction of bundle structure). Induction boils down locally to a projection of the imbedding space vectors representing the spinor

connection. The classical fields at the imbedding space level are non-dynamical and fixed and extremely simple: one can say that one has generalization of constant electric field and magnetic fields in CP_2. The dynamics of the 3-surface however implies that induced fields can form arbitrarily complex field patterns. This is essentially dynamics of shadows.

Induced gauge fields are not equivalent with ordinary free gauge fields. For instance, the attempt to represent constant magnetic or electric field as a space-time time surface has a limited success. Only a finite portion of space-time carrying this field allows realization as 4-surface. I call this topological field quantization. The magnetization of electric and magnetic fluxes is the outcome. Also gravitational field patterns allowing imbedding are very restricted: one implication is that topological with over-critical mass density are not globally imbeddable. This would explain why the mass density in cosmology can be at most critical. This solves one of the mysteries of GRT based cosmology [K101].

Quite generally, the field patterns are extremely restricted: not only due to imbeddability constraint but also due to the fact that by SH only very restricted set of space-time surfaces can appear solutions of field equations: I speak of preferred extremals (PEs) [K10, K18, L23]. One might speak about archetypes at the level of physics: they are in quite strict sense analogies of Bohr orbits in atomic physics: this is implies by the realization of GCI. This kind of simplicity does not conform with what we observed. The way out is many-sheeted space-time. Although fields do not superpose, particles experience the superposition of effects from the archetypal field configurations (superposition is replaced with set theoretic union).

3. The important implication is that one can assign to each material system a field identity since electromagnetic and other fields decompose to topological field quanta. Examples are magnetic and electric flux tubes and flux sheets and topological light rays representing light propagating along tube like structure without dispersion and dissipation making em ideal tool for communications [K82]. One can speak about field body or magnetic body of the system.

   Field body indeed becomes the key notion distinguishing TGD inspired model of quantum biology from competitors but having applications also in particle physics since also leptons and quarks possess field bodies. The is evidence for the Lamb shift anomaly of muonic hydrogen [C2] and the color magnetic body of u quark whose size is somewhat larger than the Bohr radius could explain the anomaly [K74]. The magnetic flux tubes of magnetic body carry monopole fluxes existing without generating currents. In cosmology the flux tubes assignable to the remnants of cosmic strings make possible long range magnetic fields in all scales impossible in standard cosmology. Also super-conductivity is proposed to rely on dark $h_{eff} = n \times h$ Cooper pairs at pairs of flux tubes carrying monopole flux.

GRT and gauge theory limit of TGD is obtained as an approximation.

1. GRT/gauge theory type description is an approximation obtained by lumping together the space-time sheets to single region of M^4, with gravitational fields and gauge potentials as sums of corresponding induced field quantities at space-time surface geometrized in terms of geometry of H. Gravitational field corresponds to the deviation of the induced metric from Minkowski metric using M^4 coordinates for space-time surface so that the description applies only in long length scale limit.

   Space-time surface has both Minkowskian and Euclidian regions. Euclidian regions are identified in terms of what I call generalized scattering/twistor diagrams. The 3-D boundaries between Euclidian and Minkowskian regions have degenerate induced 4-metric and I call them light-like orbits of partonic 2-surfaces or light-like wormhole throats analogous to blackhole horizons. The interiors of blackholes are replaced with the Euclidian regions and every physical system is characterized by this kind of region.

   Lumping of sheets together implies that global conservation laws cannot hold exactly true for the resulting GRT type space-time. Equivalence Principle (EP) as Einstein's equations stating conservation laws locally follows as a local remnant of Poincare invariance.

2. Euclidian regions are identified as slightly deformed pieces of CP_2 connecting two Minkowskian space-time regions. Partonic 2-surfaces defining their boundaries are connected to each other by magnetic flux tubes carrying monopole flux.

   Wormhole contacts connect two Minkowskian space-time sheets already at elementary particle level, and appear in pairs by the conservation of the monopole flux. Flux tube can be visualized as a highly flattened square traversing along and between the space-time sheets involved. Flux tubes are accompanied by fermionic strings carrying fermion number. Fermionic strings give rise to string world sheets carrying vanishing induced em charged weak fields (otherwise em charge would not be well-defined for spinor modes). String theory in space-time surface becomes part of TGD. Fermions at the ends of strings can get entangled and entanglement can carry information.

3. Strong form of GCI (SGCI) states that light-like orbits of partonic 2-surfaces on one hand and space-like 3-surfaces at the ends of causal diamonds on the other hand provide equivalent descriptions of physics. The outcome is that partonic 2-surfaces and string world sheets at the ends of CD can be regarded as basic dynamical objects.

   Strong form of holography (SH) states the correspondence between quantum description based on these 2-surfaces and 4-D classical space-time description, and generalizes AdS/CFT correspondence. One has huge super-symplectic symmetry algebra acting as isometries of WCW with conformal structure [K36, K95, K126], conformal algebra of light-cone boundary extending the ordinary conformal algebra, and ordinary Kac-Moody and conformal symmetries of string world sheets. This explains why 10-D space-time can be replaced with ordinary space-time and 4-D Minkowski space can be replaced with partonic 2-surfaces and string world sheets. This holography looks very much like the one we are accustomed with!

8.2.2 Zero energy ontology (ZEO)

In standard ontology of quantum physics physical states are assumed to have positive energy. In zero energy ontology (ZEO) [K76] physical states decompose to pairs of positive and negative energy states such that the net values of the conserved quantum numbers vanish. The interpretation of these states in ordinary ontology would be as transitions between initial and final states, physical events.

ZEO and positive energy ontology

ZEO is consistent with the crossing symmetry of QFTs meaning that the final states of the quantum scattering event can be described formally as negative energy states. As long as one can restrict the consideration to either positive or negative energy part of the state ZEO is consistent with positive energy ontology. This is the case when the observer characterized by a particular CD studies the physics in the time scale of much larger CD containing observer's CD as a sub-CD. When the time scale sub-CD of the studied system is much shorter that the time scale of sub-CD characterizing the observer, the interpretation of states associated with sub-CD is in terms of quantum fluctuations.

ZEO solves the problem, which emerges in any theory assuming symmetries giving rise to conservation laws. The problem is that the theory itself is not able to characterize the values of conserved quantum numbers of the initial state of say cosmology. In ZEO this problem disappears since in principle any zero energy state is obtained from any other state by a sequence of quantum jumps without breaking of conservation laws. The fact that energy is not conserved in GRT based cosmologies can be also understood since each CD is characterized by its own conserved quantities. As a matter fact, one must speak about average values of conserved quantities since one can have a quantum superposition of zero energy states with the quantum numbers of the positive energy part varying over some range.

At the level of principle the implications are quite dramatic. In quantum jump as recreation replacing the quantum Universe with a new one it is possible to create entire sub-universes from vacuum without breaking the fundamental conservation laws. From the point of view of consciousness theory the important implication is that "free will" is consistent with the laws of physics. This makes obsolete the basic arguments in favor of materialistic and deterministic world view.

Zero energy states are located inside causal diamond (CD)

By quantum classical correspondence zero energy states must have space-time and imbedding space correlates.

1. Positive and negative energy parts of zero energy state reside at future and past light-like boundaries of causal diamond (CD) identified as intersection of future and past directed light-cones and visualizable as double cone. The analog of CD in cosmology is big bang followed by big crunch. Penrose diagrams provide an excellent 2-D visualization of the notion. CDs form a fractal hierarchy containing CDs within CDs. Disjoint CDs are possible and CDs could also intersect.

   The interpretation of CD in TGD inspired theory of consciousness is as an imbedding space correlate for perceptive field of conscious entity: the contents of conscious experience is about the region defined by CD. At the level of space-time sheets the experience come from space-time sheets in the interior of CD. Whether the sheets can be assumed to continue outside CD is still unclear.

   Quantum measurement theory must be modified in ZEO since state function reduction can happen at both boundaries of CD and the reduced states at opposite boundaries are related by time reversal. One can also have quantum superposition of CDs changing between reductions at active boundary followed by localization in the moduli space of CDs with the tip of passive boundary fixed. Quantum measurement theory generalizes to a theory of consciousness with continuous entity identified as a sequence of state function reductions at active (changing) boundary of CD [K9].

2. By number theoretical universality (NTU) the temporal distances between the tips of the intersecting light-cones are assumed to come as integer multiples $T = m \times T_0$ of a fundamental time scale T_0 defined by CP_2 size R as $T_0 = R/c$. p-Adic length scale hypothesis [K78, K124] motivates the stonger hypothesis that the distances tend to come as as octaves of T_0: $T = 2^n T_0$. One prediction is that in the case of electron this time scale is .1 seconds defining the fundamental biorhythm. Also in the case u and d quarks the time scales correspond to biologically important time scales given by 10 ms for u quark and by and 2.5 ms for d quark [K12] . This means a direct coupling between microscopic and macroscopic scales.

8.2.3 Quantum physics as physics of classical spinor fields in WCW

The notions of Kähler geometry of "World of Classical Worlds" (WCW) and WCW spinor structure are inspired by the vision about the geometrization of the entire quantum theory.

Motivations for WCW

The notion of "World of Classical Worlds" (WCW) [K57, K36, K95] was forced by the failure of both path integral approach and canonical quantization in TGD framework. The idea is that the Kähler function defining WCW Kähler geometry is determined by the real part of an action S determining space-time dynamics and receiving contributions from both Minkowskian and Euclidian regions of space-time surface X^4 (note that $\sqrt{g_4}$ is proportional to imaginary unit in Minkoskian regions).

1. If S is space-time volume both canonical quantization and path integral would make sense at least formally since in principle one could solve the time derivatives of four imbedding space coordinates as functions of canonical momentum densities (general coordinate invariance allows to eliminate four coordinates). The calculation of path integral is however more or less hopeless challenge in practice.

2. A mere space-time volume as action is however not physically attractive. This was thought to leave under consideration only Kähler action S_K - Maxwell action for the induced Kähler form expressible in terms of gauge potential defined by the induced Kähler gauge potential of CP_2. This action has however a huge vacuum degeneracy. Any space-time surface with at most 2-D CP_2 projection, which is Lagrangian sub-manifold of CP_2, is vacuum extremal. Symplectic transformations acting like U(1) gauge transformations generate new vacuum extremals. They however fail to act as symmetries of non-vacuum extremals so that gauge invariance is not in question: the deviation of the induced metric from flat metric is the reason for the failure. This degeneracy is assumed to give rise to what might be called 4-D spin glass degeneracy meaning that the landscape for the maxima of Kähler function is fractal.

3. Canonical quantization fails because by the extreme non-linearity of the action principle making it is impossible to solve time derivatives explicitly in terms of canonical momentum

densities. The problem is especially acute for the canonical imbedding of empty Minkowski space to $M^4 \times CP_2$. The action is vanishing up to fourth order in imbedding space coordinates so that canonical momentum densities vanish identically and there is no hope of defining propagator in path integral approach. The mechanical analog would be criticality around which the potential reduces to $V \propto x^4$. Quantum criticality is indeed a basic aspect of TGD Universe.

The hope held for a long time was that WCW geometry allowing to get rid of path integral would solve the problems. One could however worry about vacuum degeneracy implying that WCW metric becomes extremely degenerate for vacuum extremals and also holography becomes extremely non-unique for them. Also the expected feailure of perturbative approach around M^4 is troublesome.

WCW and twistor lift of TGD

During last year this picture has indeed changed thanks to what might be called twistor lift of TGD [K114, K50, K14] inspired by twistor Grassmann approach to supersymmetric gauge theories [B18]. Remarkably, twistor lift would provide automatically the fundamental couplings of standard model and GRT and also the scale assigned to GUTs as CP_2 radius. PEs would be both extremals of Kähler action and minimal surfaces.

1. The basic observation is E^4, and its Euclidian compactification S^4 and CP_2 are completely unique in that they allow twistor space with Kähler structure [A57]. This was discovered by Hitchin at roughly the same time as I discovered TGD! This generalizes to M^4 having a generalization of ordinary Kähler structure to what I have called Hamilton-Jacobi structure by decomposition $M^4 = M^2 \times E^2$, where M^2 allows hypercomplex structure [K114, K50]. One can consider also integral distributions of tangent decompositions $M^4 = M^2(x) \times E^2(x)$, depending on position. The twistor space has a double fibration by S^2 with base spaces identifiable as M^4 and conformal compactification of M^4 for which metric is defined only up to conformal scaling. The first fibration $M^4 \times S^2$ with a well-defined metric would correspond to the classical TGD.
2. Both Newton's constant G and cosmological constant Λ emerge from twistor lift in M^4 factor. The radius of S^2 is identified in terms of Planck length $l_P = \sqrt{G}$. For CP_2 factor, the radius corresponds to the radius of CP_2 geodesic sphere. 4-D Kähler action can be lifted to 6-D Kähler action only for $M^4 \times CP_2$ so that TGD would be completely unique both mathematically and physically. The twistor space of CP_2 is flag-manifold $SU(3)/U(1) \times U(1)$ having interpretation as the space for the choices of quantization axis of color isospin and hypercharge. This choice could correspond to a selection of Eguchi-Hanson complex coordinates for CP_2 by fixing their phase angles in which isospin and hypercharge rotations induce shifts.
3. The physically motivated conjecture is that the PEs can be lifted to their 6-D twistor bundles with S^2 serving as a fiber, that one induce the twistor structure and the outcome is equal to the twistor structure of space-time surface, and that this condition is at least part of the PE property. This would correspond to the solution of massless wave equations in terms of twistors in the original twistor approach of Penrose [B41]. The analog of spontaneous compactification would lead to 4-D action equal to Kähler action plus volume term. One could of course postulate this action directly without mentioning twistors at all.

   The coefficient of the volume term would correspond to dark energy density characterized by cosmological constant Λ being extremely small in cosmological scales. It removes vacuum degeneracy although the situation remains highly non-perturbative. This can be combined with the earlier conjecture that cosmological constant Λ behaves as $\Lambda \propto 1/p$ under p-adic coupling constant evolution so that Λ would be large in primordial cosmology.
4. The generic extremals of space-time action would depend on coupling parameters, which does not fit with the number theoretic vision inspiring speculations that space-time surface can be seen as quaternionic sub-manifolds of 8-D octonionic space-time [K107], satisfying quaternion analyticity [K50], or a 4-D generalization of holomorphy. By SH the extremals are however "preferred". What could this imply?

Intriguingly, all known non-vacuum extremals and also CP_2 type vacuum extremals having null-geodesic as M^4 projection are extremals of both Kähler action and volume term separately! The dynamics for volume term and Kähler action effectively decouple and coupling constants do not appear at all in field equations. The twistor lift would only select minimal surface amongst vacuum extremals, modify the Kähler function of WCW identifiable as exponent for the real part of action, and provide a profound mathematical and physical motivation for cosmological constant Λ remaining mysterious GRT framework. One could even hope that preferred extremals are nothing but minimal surface extremals of Kähler action with the vanishing conditions for some sub-algebra of super-symplectic algebra satisfied automatically!

The analog of decoupling of Kähler action and volume term should take place also for induced spinors. This is expected if mere analyticity properties make spinor modes solutions of modified Dirac equations. This is true in 2-D case Hamilton-Jacobi structure should guarantee this in 4-D case [K126, K50].

PEs depend on coupling parameters only via boundary conditions stating the vanishing of Noether charges for a sub-algebra of super-symplectic algebra and its commutator with entire algebra. Also the conservation conditions at 3-D light-like surfaces at which the signature of metric changes imply dependence on coupling parameters. These conditions allow the transfer of classical charges between Minkowskian and Euclidian regions necessary to understand momentum exchange between particles and environment classically only if Kähler couplings strength is complex - otherwise there is no exchange of conserved quantities since their real *resp.* imaginary at the two sides [L13]. Interestingly, also in twistor Grassmann approach the massless poles in propagators are complex.

This picture conforms with the conjecture that discrete p-adic evolution of the Kähler coupling strength in subset of primes near prime powers of two corresponds to complex zeros of zeta [L13]. This conforms also with the conjectured discreteness of p-adic coupling constant evolution by phase transitions changing the values of coupling parameters. One implication is that all loop corrections in functional integral vanish.

5. In path integral approach quantum TGD would be extremely non-perturbative around extremals for which Kähler action vanishes. Same is true also in WCW approach. The cure would be provided by the hierarchy of Planck constants $h_{eff}/h = n$, which effectively scales Λ down to Λ/n. n would be the number sheets of the M^4 covering defined by the space-time surface: the action of Galois group for the number theoretic discretization of space-time surface could give rise to this covering. The finiteness of the volume term in turn forces ZEO: the volume of space-time surface is indeed finite due to the finite size of CD.

Consider now the delicacies of this picture.

1. Should assign also to M^4 the analog of symplectic structure giving an additional contribution to the induced Kähler form? The symmetry between M^4 and CP_2 suggests this, and this term could be highly relevant for the understanding of the observed CP breaking and matter antimatter asymmetry [L53]. Poincare invariance is not lost since the needed moduli space for M^4 Kähler forms would be the moduli space of CDs forced by ZEO in any case, and M^4 Kähler form would serve as the correlate for fixing rest system and spin quantization axis in quantum measurement.

2. Also induced spinor fields are present. The well-definedness of electro-magnetic charge for the spinor modes forces in the generic case the localization of the modes of induced spinor fields at string world sheets (and possibly to partonic 2-surfaces) at which the induced charged weak gauge fields and possibly also neutral Z^0 gauge field vanish. The analogy with branes and super-symmetry force to consider two options.

   Option I: The *fundamental* action principle for space-time surfaces contains besides 4-D action also 2-D action assignable to string world sheets, whose topological part (magnetic flux) gives rise to a coupling term to Kähler gauge potentials assignable to the 1-D boundaries of string world sheets containing also geodesic length part. Super-symplectic symmetry demands that modified Dirac action has 1-, 2-, and 4-D parts: spinor modes would exist at both string boundaries, string world sheets, and space-time interior. A possible interpretation for the interior modes would be as generators of space-time super-symmetries [K99].

This option is not quite in the spirit of SH and string tension appears as an additional parameter. Also the conservation of em charge forces 2-D string world sheets carrying vanishing induced W fields and this is in conflict with the existence of 4-D spinor modes unless they satisfy the same condition. This looks strange.

Option II: Stringy action and its fermionic counterpart are effective actions only and justified by SH. In this case there are no problems of interpretation. SH requires only that the induced spinor fields at string world sheets determine them in the interior much like the values of analytic function at curve determine it in an open set of complex plane. At the level of quantum theory the scattering amplitudes should be determined by the data at string world sheets. If induced W fields at string world sheets are vanishing, the mixing of different charge states in the interior of X^4 would not make itself visible at the level of scattering amplitudes! In this case 4-D spinor modes do not define space-time super-symmetries.

3. Why the string world sheets coding for effective action should carry vanishing weak gauge fields? If M^4 has the analog of Kähler structure [L53], one can speak about Lagrangian sub-manifolds in the sense that the sum of the symplectic forms of M^4 and CP_2 projected to Lagrangian sub-manifold vanishes. Could the induced spinor fields for effective action be localized to generalized Lagrangian sub-manifolds? This would allow both string world sheets and 4-D space-time surfaces but SH would select 2-D Lagrangian manifolds. At the level of effective action the theory would be incredibly simple.

   Induced spinor fields at string world sheets could obey the "dynamics of avoidance" in the sense that *both* the induced weak gauge fields W, Z^0 and induced Kähler form (to achieve this U(1) gauge potential must be sum of M^4 and CP_2 parts) would vanish for the regions carrying induced spinor fields. They would coupleonly to the *induced em field (!)* given by the R_{12} part of CP_2 spinor curvature [K19] for $D = 2, 4$. For $D = 1$ at boundaries of string world sheets the coupling to gauge potentials would be non-trivial since gauge potentials need *not* vanish there. Spinorial dynamics would be extremely simple and would conform with the vision about symmetry breaking of weak group to electromagnetic gauge group.

   The projections of canonical currents of Kähler action to string world sheets would vanish, and the projections of the 4-D modified gamma matrices would define just the induced 2-D metric. If the induced metric of space-time surface reduces to an orthogonal direct sum of string world sheet metric and metric acting in normal space, the flow defined by 4-D canonical momentum currents is parallel to string world sheet. These conditions could define the "boundary" conditions at string world sheets for SH.

This admittedly speculative picture has revolutionized the understanding of both classical and quantum TGD during last year. [K50, K14, L23]. In particular, the construction of single-sheeted PEs as minimal surfaces allows a kind of lego like engineering of more complex PEs [L19]. The minimal surface equations generalize Laplace equation of Newton's gravitational theory to non-linear massless d'Alembert equation with gravitational self-coupling. One obtains the analog of Schwartschild solution and radiative solutions describing also gravitational radiation [L23]. An open question is whether classical theory makes sense if also the analog of Kähler form in M^4 is allowed.

Identification of WCW

The notion of WCW [K57, K36, K95] was inspired by the super-space approach of Wheeler in which 3-geometries are the basic geometric entities.

1. In TGD framework 3-surfaces take this role. Einstein's program for geometrizing classical physics is generalized to a geometrization of entire quantum physics. Hermitian conjugation corresponds to complex conjugation in infinite-dimensional context so that WCW must have Kähler geometry. The geometrization of fermionic statistics/oscillator operators is in terms of gamma matrices of WCW expressible as linear combinations of oscillator operators for second quantized induced spinor field. Formally purely classical spinor modes of WCW represent many fermion states as functionals of 3-surface. One can also interpret gamma matrices as generators of super-conformal symmetries in accordance with the fact that also SUSY involves Clifford algebra.

In ZEO the entanglement coefficients between positive and negative energy parts of zero energy states determine the S-matrix so that S-matrix would be coded by the modes of WCW spinor fields. Twistor approach to TGD [K50] suggests that the S-matrix reduces completely to the symmetries defined by the multi-local (locus corresponds to partonic 2-surface) generators of the Yangian associated with the super-symplectic algebra.

2. ZEO forces to identify 3-surfaces as pairs of 3-surfaces with members at the opposite boundaries of CD. SH reduces them to a collection of partonic 2-surfaces at boundaries of CD plus number theoretic discretization in space-time interior. Basic geometric objects are pairs of initial and final states (coordinates for both in mechanical analogy) rather than initial states with initial value conditions (coordinates and velocities in mechanical analogy) and initial value problem transforms to boundary value problem. Processes rather than states become the basic elements of ontology: this has far reaching consequences in biology and neuroscience.

3. The realization of GCI requires that the definition of WCW Kähler function assigns to a "physically" 3-surface a unique 4-surface for 4-D general coordinate transformations to act: "physically" could mean "apart from transformations acting as gauge transformations" not affecting the action and conserved classical charges. The outcome is holography.

4. Strong form of holography (SH) would emerge as follows. The condition that light-like 3-surfaces defining boundaries between Euclidian and Minkowskian regions are basic geometric entities equivalent with pairs of space-like 3-surfaces at the ends of given causal diamond CD implies SH: partonic 2-surfaces and their 4-D tangent space data should code the physics. One could also speak about almost/effective 2-dimensionality. Tangent space data could in turn be coded by string world sheets. Number theoretical discretitization of space-time interior with preferred coordinates in the extension of rationals could give meaning for "almost".

5. Kähler metric is expressible both in terms of second derivatives of Kähler function K [K57] and as anticommutators of WCW gamma matrices expressible as linear combinations of fermionic oscillator operators. This suggests a close relationship between space-time dynamics and spinor dynamics.

   Super-symplectic symmetry between the action defining space-time surfaces (Kähler action plus volume term) and modified Dirac action would realize this relationship. This is achieved if the modified gamma matrices are defined by the canonical momentum currents of 2-D action associated with string world sheets. These currents are parallel to the string world sheets. This implies the analog of AdS/CFT correspondence requiring only that induced spinor modes at string world sheets determine them in space-time interior (this is like analytic continuation). The localization of spinor modes at string world sheets is *not* required as I believed first.

The geometry of loop spaces developed by Freed [A37] serves as a model in the construction of WCW Kähler geometry [K95].

1. The existence of loop space Riemann connection requires maximal isometry group identifiable as Kac-Moody group so that Killing vector fields span the entire tangent space of the loop space.

2. In TGD framework the properties of Kähler action lead to the idea that WCW is union of homogenous or even symmetric spaces of symplectic algebra acting at the boundary of $\delta CD \subset \delta CD_+ \cup \delta CD_-$, $\delta CD_\pm \subset \delta M^4_\pm \times CP_2$. ZEO requires that the conserved quantum numbers for physical states are opposite for the positive and negative energy parts of the states at the two opposite boundary parts of CD. The symmetric spaces G/H in the union are labelled by zero modes, which do not appear in the line element as differentials but only as parameters of the metric. Conserved Noether charges of isometries and symplectic invariants of examples of zero modes as also the super-symplectic Noether charges invariant under complex conjugation of WCW coordinates.

3. Homogenous spaces of the symplectic group G are obtained by dividing by a subgroup H. An especially attractive option is suggested by the fractal structure of the symplectic algebra containing an infinite hierarchy of sub-algebras G_n for which conformal weights are $n > 0$-multiples of those for G. For this option $H = G_n$ is isomorphic to G and one could have infinite hierarchies of inclusions analogous to the hierarchy of inclusions of hyperfinite factors of type II_1 (HFFs). PE property requires almost 2-dimensionality and elimination of huge

number of degrees of freedom. The natural condition is that the Noether charges of G_n vanish at the ends of CD. A stronger condition is that also the Noether charges for $[G, G_n]$ vanish. This implies effective normal algebra property and G/G_n acts effectively like group.

The inclusion of HFFs would define measurement resolution with included factor acting like gauge algebra. Measurement resolution would be naturally determined by the number theoretic discretization of the space-time surface so that physics as geometry and number theory visions would meet each other.

4. This inclusion hierarchy can be identified in terms of quantum criticality (QC). The transitions $n \to kn$ increasing the value of $n > 0$ reduce QC since pure gauge symmetries are reduced, and new physical super-symplectic degrees of freedom emerge. QC also requires that Kähler couplings strength analogous to temperature is analogous to critical temperature so that the quantum theory is uniquely defined if their is only one critical temperature. Spectrum for α_K seems more plausible and the possibility that Kähler coupling strength depends on the level of the number theoretical hierarchy defined by the allowed extensions of rationals can be considered [L13].

WCW spinor structure

The basic idea is geometrization of quantum states by identifying them as modes of WCW spinor fields [K126, K95]. This requires definition of WCW spinors and WCW spinor structure, WCW gamma matrices and Dirac operator, etc..

The starting point is the definition of WCW gamma matrices using a representation analogous to the usual vielbein representation as linear combinations of flat space gamma matrices. The conceptual leap is the observation that there is no need to assume that the counterparts of flat space gamma matrices have vectorial quantum numbers. Instead, they are identified as fermionic oscillator operators for second quantized free induced spinor fields at space-time surface.

This allows geometrization of the fermionic statistics since WVW spinors for a given 3-surface are analogous to fermionic Fock states. One can also say that spinor structure follows as a square root of metric and also that the spinor basis defines a geometric correlate of Boolean mind [K32]. The dependence of WCW spinor field on 3-surface represents the bosonic degrees of freedom not reducible to many-fermion states. For instance, most of hadron mass would be associated with these degrees of freedom.

Quantum TGD involves Dirac equations at space-time level, imbedding space level, and level of WCW. The dynamics of the induced spinor fields is related by super-symmetry to the action defining space-time surfaces as preferred extremals. [K126, K95].

1. The gamma matrices in the equation - modified gamma matrices - are determined by contractions of the canonical momentum currents of Kähler action with the imbedding space gamma matrices. The localization at string world sheets for which only induced neutral weak fields or only em field are non-vanishing is accompanied by the integrability condition that various conserved currents run along string world sheets: one can speak of sub-flow. I
2. Modified Dirac equation can be solved exactly just like in the case of string models using holomorphy and the properties of complexified modified gamma matrices. This is expected to be true also in 4-D case by Hamilton-Jacobi structure. If the dynamics of avoidance is realized the modified Dirac equation would be essentially free Dirac equation and holomorphy would allow to solve it.

At the level of WCW one obtains also the analog of massless Dirac equation as the analog of super Virasoro conditions of Super Virasoro algebra.

1. The fermionic counterparts of super-conformal gauge conditions assignable with sub-algebra G_n of supersymplectic conformal symmetry associated with the both light-cone boundary (light-like radial coordinate), with conformal symmetries of light-cone boundary, and with string world sheets.
2. The ground states of supersymplectic representations satisfy massless imbedding space Dirac equation in imbedding space so that Dirac equations in WCW, in imbedding space, and at string world sheets are involved. In twistorialization also massless M^8 Dirac equation emerges in the tangent space M^8 of imbedding space assignable to the partonic 2-surfaces

and generalizes the 4-D light-likeness with its 8-D counterpart applying to states with M^4 mass. Here octonionic representation of imbedding space gamma matrices emerges naturally and allows to speak about 8-D analogs of Pauli's sigma matrices [K114].

8.2.4 Quantum criticality, measurement resolution, and hierarchy of Planck constants

The notions of quantum criticality (QC), finite measurement resolution, and hierarchy of Planck constants proposed to give rise to dark matter as phases of ordinary matter are central for TGD [**?**, K125, K47].

These notions relate closely to the strong form of holography (SH) implied by strong form of general coordinate invariance (SGCI). In adelic physics all this would relate closely to the hierarchy of extensions of rationals serving as a correlate for number theoretical evolution.

Finite measurement resolution and fractal inclusion hierarchy of super-symplectic algebras

The fractal hierarchy of isomorphic sub-algebras of supersymplectic algebra - call it g - defines an excellent candidate for the realization of finite the measurement resolution. Similar hierarchies can be assigned also for the extended super-conformal algebra assignable with light-like boundaries of CD and with Kac-Moody and conformal algebras assignable to string world sheets.

An interesting possibility is that the the conformal weights assignable to infinitesimal scaling operator of the light-like radial coordinate of light-cone boundary correspond to zeros of Riemann zeta [K124] [L10]. A kind of dual spectrum would correspond to conformal weights that correspond to logarithms for powers of primes. One can identify the conformal weight as negative of the pole of fermionic zeta $z_F = \zeta(s)/\zeta(2s)$ natural in TGD framework. The real part of conformal weight for the generators is $h_R = -1/4$ for "non-trivial" poles and positive integer $h = n > 0$ for "trivial" poles. There is also a pole for $h = -1$. Hence one obtains tachyonic ground states, which must be assumed also in p-adic mass calculations [K66].

Also the generators of Yangian algebra [K114] integrating the algebras assignable to various partonic 2-surfaces to a multi-local algebra are labelled by a non-negative integer n analogous to conformal weight and telling the number of partonic 2-surfaces involved with the action of the generator. Also this algebra has similar fractal hierarchy of sub-algebras so that the considerations that follow might apply also to it. Now that number of partonic 2-surface would play the role of measurement resolution.

As noticed, there are also other algebras, which allow conformal hierarchy if one can restrict the conformal weights to be non-negative. The first of them generates generalized conformal transformations of light-cone boundary depending on light-like radial coordinate as parameter: also now radial conformal weights for generators can have zeros of zeta as spectrum. As a special case one obtains infinite-dimensional group of isometries of light-cone boundary. Second one corresponds to ordinary conformal and Kac-Moody symmetries for induced spinor fields acting on string world sheets. Also here similar hierarchy of sub-algebras can be considered. In the following argument one restricts to super-symplectic algebra assumed to act as isometries of WCW.

Consider now how the finite measurement resolution could be realized as an infinite hierarchy of super-symplectic gauge symmetry breakings. The physical picture relies on quantum criticality of TGD Universe. The levels of the hierarchy labelled by positive integer n and a ball at the top of ball at... serves as a convenient metaphor.

1. The sub-algebra g_n for which conformal weights of generators (whose commutators give the sub-algebra) are positive integer multiples for those of the entire algebra g defines the algebra acting as pure gauge algebra defining a sub-group of symplectic group. The action of g_n as gauge algebra would mean that it affects on degrees of freedom below the measurement resolution. One can assign to this algebra a coset space G/G_n of the entire symplectic group G and of subgroup G_n. This coset space would describe the dynamical degrees of freedom. If the subgroup were a normal subgroup, the coset space would be a group. This is not the case now since the commutator $[g, g_n]$ of the entire algebra with the sub-algebra does not belong to g_n.

However, if one poses stronger - physically very attractive - gauge conditions stating that not only g_n but also the commutator algebra $[g, g_n]$ annihilates the physical states and that corresponding classical Noether charges vanish, one obtains effectively a normal subgroup and one has good hopes that coset space acts effectively as group, which is finite-dimensional as far as conformal weights are considered.

2. $n > 0$ is essential for obtaining effective normal algebra property. Without this assumption the commutator $[g, g_n]$ would be entire g. If the spectrum of supersymplectic conformal weights is integer valued it is not obvious why one should pose the restriction $n \geq 1$.

3. In this framework pure conformal invariance could reduce to a finite-dimensional gauge symmetry. A possible interpretation would be in terms of Mc-Kay correspondence [A79] assigning to the inclusions of HFFs labelled by integer $n \geq 3$ a hierarchy of simply laced Lie-groups. Since the included algebra would naturally correspond to degrees of freedom not visible in the resolution used, the interpretation as a dynamical gauge group is suggestive. The dynamical gauge group could correspond to n-dimensional Cartan algebra acting in conformal degrees of freedom identifiable as a simply laced Lie group. This would assign a infinite hierarchy of dynamical gauge symmetries to the broken conformal gauge invariance acting as symmetries of dark matter. This still leaves infinite number of degrees of freedom assignable to the imbedding space Hamiltonians and spectrum generated by zeros of zeta but this might have interpretation in terms of gauging so that additional vanishing conditions for Noether charges are suggestive.

Dark matter as large phases with large gravitational Planck constant $h_{eff} = h_{gr}$

D. Da Rocha and Laurent Nottale [E13] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive [K100, K83].

1. The proposal is that a Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems and that only the generalizations of Bohr orbits are involved. The space-time sheets in question would carry dark matter.

2. Nottale's hypothesis would predict a gigantic value of $\hbar_{gr}$. Equivalence Principle and the independence of gravitational Compton length $\Lambda_{gr} = \hbar_{gr}/m = GM/v_0 = 2r_S/v_0$ (typically astrophysical scale) on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that $\hbar_{gr}$ would be much smaller. Large $\hbar_{gr}$ could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets, which is quantum coherent in the required time scale [K100].

   One could criticize the hypothesis since it treats the masses M and m asymmetrically: this is only apparently true [**?**].

3. It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta). The cross section of the flux tube corresponds to a sphere $S^2_i \subset CP_2$, $i = I, II$ [K14]. S^2_I is homologically non-trivial carrying Kähler magnetic monopole flux. S^2_{II} is homologically trivial carrying vanishing Kähler magnetic flux but non-vanishing electro-weak flux [K14].

   The flux tubes of type I have both Kähler magnetic energy and dark energy due to the volume action. Flux tubes of type II would have only the volume energy. Both flux tubes could be remnants of cosmic string phase of primordial cosmology. The energy of these flux quanta would be correlated for galactic dark matter and volume action and also magnetic tension would give rise to negative "pressure" forcing accelerated cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside flux tubes identifiable also as dark energy.

4. Both theoretical consistency and certain experimental findings from astrophysics [E21, E28] and biology [K31, K17] suggest the identification $h_{eff} = n \times h = h_{gr}$. The large value of h_{gr} can be seen as a manner to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description) [K95]. The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n. Macroscopic quantum coherence in astrophysical scales is implied. If also modified Dirac action is present, part of the interior degrees of freedom associated with the fermionic part of conformal algebra become physical.

   Fermionic oscillator operators could generate super-symmetries and sparticles could correspond to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to an ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) or to a bunch of n low energy gravitons.

Hierarchies of quantum criticalities, Planck constants, and dark matters

Quantum criticality is one of the corner stone assumptions of TGD. In the original approach the value of Kähler coupling strength α_K together with CP_2 radius R fixed quantum TGD and is analogous to critical temperature. Twistor lift [K14] brings in additional coupling constant Λ obeying p-adic coupling constant evolution and Planck length l_G, which like CP_2 radius would not obey coupling constant evolution (as also G). The values of these parameters should be fixed by quantum criticality. What else does quantum criticality mean is however far from obvious, and I have pondered the notion repeatedly both from the point of view of mathematical description and phenomenology [K57, K126, K95].

1. Criticality is characterized by long range correlations and sensitivity to external perturbations and living systems define an excellent example of critical systems - even in the scale of populations since without sensitivity and long range correlations cultural evolution and society would not be possible. For a physicist with the conceptual tools of existing theoretical physics the recent information society in which the actions of people at different side of globe are highly correlated, should look like a miracle.

2. The hierarchy of Planck constants with dark matter identified as phases of ordinary matter with non-standard value $h_{eff} = n \times h$ of Planck constant is one of the "almost-predictions" of TGD is definitely something essentially new physics. The phase transition transforming ordinary matter to dark matter in this sense generates long range quantal correlations and even macroscopic quantum coherence.

   Finding of a universal mechanism generating dark matter have been a key challenge during last ten years. Could quantum criticality having classical or perhaps even thermodynamical criticality as its correlate be always accompanied by the generation of dark matter? If this were the case, the recipe would be stupifyingly simple: create a critical system! Dark matter would be everywhere and we would have observed its effects for centuries! Magnetic flux tubes (possibly carrying monopole flux) define the space-time correlates for long range correlations at criticality and would carry the dark matter. They are indeed key players in TGD inspired quantum biology.

3. Change of symmetry is assigned with criticality as also conformal symmetry (in 2-D case). In TGD framework conformal symmetry is extended and infinite hierarchy of breakings of conformal symmetry so that a sub-algebras of various conformal algebras with conformal weights coming as integer multiples of integer n defining h_{eff} would occur.

4. Phase separation is what typically occurs at criticality and one should understand also this. The strengthening of this hypothesis with the assumption $h_{eff} = h_{gr}$, where $h_{gr} = GMm/v_0$ is is the gravitational Planck constant originally introduced by Nottale [K84, ?]. In the formula v_0 has dimensions of velocity, and will be proposed to be determined by a condition relating the size of the system with mass M to the radius within which the wave function of particle m with $h_{eff} = h_{gr}$ is localized in the gravitational field of M.

The condition $h_{eff} = h_{gr}$ implies that the integer n in h_{eff} is proportional to the mass of the particle. The implication is that particles with different masses reside at flux tubes with different Planck constant and separation of phases indeed occurs.

5. What is remarkable is that neither gravitational Compton length nor cyclotron energy spectrum depends on the mass of the particle. This universality could play key role in living matter. One can assign Planck constant also to other interactions such as electromagnetic interaction so that one would have $h_{em} = Z_1Z_2e^2/v_0$. The phase transition could take place when the perturbation series based on the coupling strength $\alpha = Z_1Z_2e^2/\hbar$ ceases to converge. In the new phase perturbation series would converge since the coupling strength is proportional to $1/h_{eff}$. Hence criticality and separation into phases serve as criteria as one tries to see whether the earlier proposals for the mechanisms giving rise to large h_{eff} phases make sense. One can also check whether the systems to which large h_{eff} has been assigned are indeed critical.

One example of criticality is super-fluidity. Superfluids exhibit rather mysterious looking effects such as fountain effect [D39] and what looks like quantum coherence of superfluid containers, which should be classically isolated. These findings serve as a motivation for the proposal that genuine superfluid portion of superfluid corresponds to a large h_{eff} phase near criticality at least and that also in other phase transition like phenomena a phase transition to dark phase occurs near the vicinity [**?**].

But how does quantum criticality relate to number theory and adelic physics? $h_{eff}/h = n$ has been identified as the number of sheets of space-time surface identified as a covering space of some kind. Number theoretic discretization defining the "spine" for a monadic space-time surface [L31] defines also a covering space with Galois group for an extension of rationals acting as covering group. Could n be identifiable as the order for a sub-group of Galois group? If this is the case, the proposed rule for h_{eff} changing phase transitions stating that the reduction of n occurs to its factor would translate to spontaneous symmetry breaking for Galois group and spontaneous - symmetry breakings indeed accompany phase transitions.

TGD variant of AdS/CFT duality

AdS/CFT duality [B33] has provided a powerful approach in the attempts to understand the non-perturbative aspects of super-string theories. The duality states that conformal field theory in n-dimensional Minkowski space M^n identifiable as a boundary of $n+1$-dimensional space AdS_{n+1} is dual to a string theory in $AdS_{n+1} \times S^{9-n}$.

As a mathematical discovery AdS/CFT duality is extremely interesting but it seems that it need not have much to do with physics as such. From TGD point of view the reason is obvious: the notion of conformal invariance is quite too limited. In TGD framework conformal invariance is extended to a super-symplectic symmetry in $\delta M^4_\pm \times CP_2$, whose Lie-algebra has the structure of conformal algebra. Also ordinary super-conformal symmetries associated with string world sheets are present as well as generalization of 2-D conformal symmetries to their analogs at light-cone boundary and light-like orbits of partonic 2-surfaces. In this framework AdS/CFT duality is expected to be modified.

The matrix elements $G_{K\overline{L}}$ of Kähler metric of WCW can be expressed in two manners. As contractions of the derivatives $\partial_K\partial_{\overline{L}}K$ of the Kähler function of WCW with isometry generators or as anticommutators $\{\Gamma_K, \Gamma_{\overline{L}}\}$ of WCW gamma matrices identified as supersymplectic Noether super charges assignable to fermionic strings connecting partonic 2-surfaces. Kähler function is identified as real part of the action: if coupling parameters are real it reduces to the action for the Euclidian space-time regions with 4-D CP_2 projection and otherwise contains contributions from both Minkowskian and Euclidian regions. The action defines the modified gamma matrices appearing in modified Dirac action as contractions of canonical momentum currents with imbedding space gamma matrices.

This observation suggests that there is a super-symmetry between action and modified Dirac action. The problem is that induced spinor fields naive of SH and also well-definedness of em charge demand the localization of induced spinor modes at 2-D string world sheets. This simply cannot be true. On the other hand, SH only requires that the data about induced spinor fields and space-time surface at the string world sheets is enough to construct the modes in space-time interior.

This leaves two options if one assumes that SH is exact (recall however that the number theoretic interpretation for the hierarchy of Planck constants suggests that the number-theoretic spin of monadic space-time surface represents additional discrete data needed besides that assignable to string world sheets to describe dark matter). As found in the section 8.2.3, there are two options.

Option I: The analog of brane hierarchy is realized at the level of fundamental action. There is a separate fundamental 2-D action assignable with string world sheets - area and topological magnetic flux term - as also world line action assignable to the boundaries of string world sheets. By previous argument string tension should be determined by the value of the cosmological constant Λ obeying -adic coupling constant evolution rather than by G: otherwise there is no hope about gravitationally bound states above Planck scale. String tension would appear as an additional fundamental coupling parameter (perhaps fixed by quantum criticality). This option does not quite conform with the spirit of SH.

Option II: 4-D space-time action and corresponding modified Dirac action defining fundamental actions are expressible as effective actions assignable to string world sheets and their boundaries. String world sheet effective action action could be expressible as string area for the effective metric defined by the anti-commutators of modified gamma matrices at string world sheet. If the sum of the induced Kähler forms of M^4 and CP_2 vanishes at string world sheets the effective metric would be the induced 2-D metric: this together with the observed CP breaking could provide a justification for the introduction of the analog of Kähler form in M^4. String tension would be dynamical rather than determined by l_P and depend on Λ, l_P, R and α_K. This representation of Kähler action would be one aspect of the analog of AdS/CFT duality in TGD framework.

Both options would allow to understand how strings connecting partonic 2-surfaces give rise to the formation of gravitationally bound states. Bound states of macroscopic size are possible only if one allows hierarchy of Planck constants and this is required also by the (extremely) small value of Λ (in cosmic scales).

Consider the concerete realizations for this vision.

1. SGCI requires effective 2-dimensionality. In given UV and IR resolutions partonic 2-surfaces and string world sheets are assignable to a finite hierarchy of CDs inside CDs with given CD characterized by a discrete scale coming as an integer multiple of a fundamental scale (essentially CP_2 size). Λ would closely relate to the size scale of CD. String world sheets have boundaries consisting of either light-like curves in induced metric at light-like wormhole throats and space-like curves at the ends of CD whose M^4 projections are light-like. These braids carrying fermionic quantum numbers intersect partonic 2-surfaces at discrete points.
2. This implies a rather concrete analogy with $AdS_5 \times S_5$ duality, which describes gluons as open strings. In zero energy ontology (ZEO) string world sheets are indeed a fundamental notion and the natural conjecture is that these surfaces are minimal surfaces, whose area by quantum classical correspondence depends on the quantum numbers of the external particles.

String tension of gravitational flux tubes

For Planckian cosmic strings only quantum gravitational bound states of length of order Planck length are possible. There must be a mechanism reducing the string tension. The *effective* string tension assignable to magnetic flux tubes must be inversely proportional to $1/h_{eff}^2$, $h_{eff} = n \times h = h_{gr} = 2\pi GMm/v_0$ in order to obtain gravitationally bound states in macroscopic length scales identified as structures for which partonic 2-surfaces are connected by flux tubes accompanied by fermionic strings.

The reason is that the size scale of (quantum) gravitationally bound states of masses M and m is given by grvitational Compton length $\Lambda_{gr} = GM/v_0$ [K100, K84, **?**] assignable to the gravitational flux tubes connecting the masses M and m. If the string tension is of order Λ_{gr}^2 this is achieved since the typical length of string would be Λ_{gr}. Gravitational string tension must be therefore of order $T_{gr} \sim 1/\Lambda_{gr}^2$. How could this be achieved? One can imagine several options and here only the option based on the assumptions

1. Twistor lift makes sense.
2. Fundamental action is 4-D for both space-time and fermionic degrees of freedom and 2-D string world sheet action is an effective action realizing SH. Note effective action makes also possible braid statistics, which does not make sense at fundamental level.

3. Also M^4 carries the analog of Kähler form and the sum of induced Kähler forms from M^4 and CP_2 vanishes at string world sheets and also weak gauge fields vanishes at string world sheets leaving only em field.

is considered since it avoids all the objections that I have been able to invent.

For the twistor lift of TGD [K14] predicting cosmological constant Λ depending on p-adic length scale $\Lambda \propto 1/p$ the gravitational strings would be naturally homologically trivial cosmic strings. These vacuum extremals of Kähler action transform to minimal surface extremals with string tension given by $\rho_{vac}S$, where ρ_{vac} the density of dark energy assignable to the volume term of the action and S the transverse area of the flux tube. One should have $\rho_{vac}S = 8\pi\Lambda S/G = 1/\Lambda_{gr}^2$ so that one would have

$$8\pi\Lambda S = \frac{G}{\Lambda_{gr}^2} \quad .$$

Λ for flux tubes (characterizing the size of CDs containing them) would depend on the gravitational coupling Mm.

8.2.5 Number theoretical vision

Physics as infinite-D spinor geometry of WCW and physics as generalized number theory are the two basic vision about TGD. The number theoretical vision involves three threads [K106, K107, K105].

1. The first thread [K106] involves the notion of number theoretical universality NTU: quantum TGD should make sense in both real and p-adic number fields (and their algebraic extensions induced by extensions of rationals). p-Adic number fields are needed to understand the space-time correlates of cognition and intentionality [K78, K51, K80].

   p-Adic mass calcuations lead to the notion of a p-adic length scale hierarchy quantifying the notion of the many-sheeted space-time [K78, K51]. One of the first applications was the calculation of elementary particle masses [K66]. The basic predictions are only weakly model independent since only p-adic thermodynamics for Super Virasoro algebra are involved. Not only the fundamental mass scales would reduce to number theory but also particle masses are predicted correctly under rather mild assumptions and are exponentially sensitive to the p-adic length scale predicted by p-adic length scale hypothesis. Also predictions such as the possibility of neutrinos to have several mass scales were made on the basis of number theoretical arguments and have found experimental support [K66, K33].

2. Second thread [K107] is inspired by the dimensions $D = 1, 2, 4, 8$ of the basic objects of TGD and assumes that classical number fields are in a crucial role in TGD. 8-D imbedding space would have octonionic structure and space-time surfaces would have associative (quaternionic) tangent space or normal space. String world sheets could correspond to commutative surfaces. Also the notion of M^8-H-duality is part of this thread and states that quaternionic 4-surfaces of M^8 containing preferred M^2 in its tangent space can be mapped to PEs in H by assigning to the tangent space CP_2 point parametrizing it. M^2 could be replaced by integrable distribution of $M^2(x)$. If PEs are also quaternionic one has also $H - H$ duality allowing to iterate the map so that PEs form a category. Also quaternion analyticity of PEs is a highly attractive hypothesis [K114]. For instance, it might be possible to interpret string world sheets and partonic 2-surfaces appearing in strong form of holography (SH) as co-dimension 2 surfaces analogous to poles of analytic function in complex plane. Light-like 3-surfaces might be seen as analogs of cuts. The coding of analytic function by its singularities could be seen as analog of SH.

3. The third thread [K105] corresponds to infinite primes and leads to several speculations. The construction of infinite primes is structurally analogous to a repeated second quantization of a supersymmetric arithmetic quantum field theory with free particle states characterized by primes. The many-sheeted structure of TGD space-time could reflect directly the structure of infinite prime coding it. Space-time point would become infinitely structured in various p-adic senses but not in real sense (that is cognitively) so that the vision of Leibniz about monads reflecting the external world in their structure is realized in terms of algebraic holography.

Space-time becomes algebraic hologram and realizes also Brahman=Atman idea of Eastern philosophies.

8.3 p-Adic mass calculations and p-adic thermodynamics

p-Adic mass calculations carried for the first time around 1995 were the stimulus eventually leading to the number theoretical vision as a kind dual for the geometric vision about TGD. In this secton I will roughly describe the calculations [K33, K66] and the questions and challenges raised by them.

8.3.1 p-Adic numbers

Like real numbers, p-adic numbers (`http://tinyurl.com/hmgqtoh`) can be regarded as completions of the rational numbers to a larger number field [K51]. Each prime p defines a p-adic number field allowing the counterparts of the usual arithmetic operations.

1. The basic difference between real and p-adic numbers is that p-adic topology is ultra-metric. Ultrametricity means that the distance function $d(x,y)$ (the counterpart of $|x-y|$ in the real context) satisfies the inequality

   $$d(x,z) \leq Max\{d(x,y), d(y,z)\} \ ,$$

   (Max(a, b) denotes maximum of a and b) rather than the usual triangle inequality

   $$d(x,z) \leq d(x,y) + d(y,z) \ .$$

2. The topology defined by p-adic numbers is compact-open. Hence the generalization of manifold obtained by gluing together n-balls fails because smallest open n-balls are just points and one has totally disconnected topology.
3. p-Adic numbers are not well-ordered like real numbers. Therefore one cannot assign orientation to the p-adic number line. This in turn leads to difficulties with attempts to define definite integrals and the notion of differential form although indefinite integral is well-defined. These difficulties serve as important guidelines in the attempts to understand what p-adic physics is and also how to fuse real and various p-adic physics to a larger structure.
4. p-Adic numbers allow an expansion in powers of p analogous to the decimal expansion

   $$x = \sum_{n\geq 0} x_n p^n \ ,$$

   and the number of terms in the expansion can be infinite so that p-adic number need not be finite as a real number. The norm of the p-adic number (counterpart of $|x|$ for real numbers) is defined as

   $$N_p(x) = \sum_{n\geq 0} x_n p^n = p^{-n_0} \ ,$$

   and depends only very weakly on p-adic number. The ultra-metric distance function can be defined as $d_p(x,y) = N_p(x-y)$.
5. p-Adic numbers allow a generalization of the differential calculus. The basic rules of the p-adic differential calculus are the same as those of the ordinary differential calculus. There is however one important new element: the set of the functions having vanishing p-adic derivative consists of so called pseudo constants, which are analogs of real valued piecewise constant functions. In the real case only constant functions have vanishing derivative. This implies that p-adic differential equations are non-deterministic. This non-determinism is identified as a counterpart of the non-determinism of cognition and imagination [K80].

8.3.2 Model of elementary particle

p-Adic mass calculations [K33, K66] rely heavily on a topological model for elementary particle and it is appropriate to describe it before going to the summary of calculations.

Family replication phenomenon topologically

One of the basic ideas of TGD approach to particle physics has been genus-generation correspondence: boundary components of the 3-surface should be carriers of elementary particle numbers and the observed particle families should correspond to various boundary topologies.

With the advent of zero energy ontology (ZEO) this picture has changed somewhat.

1. The wormhole throats identified as light-like 3-surfaces at with the induced metric of the space-time surface changes its signature from Minkowskian to Euclidian correspond to the light-like orbits of partonic 2-surfaces. One cannot of course exclude the possibility that also boundary components allow to satisfy boundary conditions without assuming vacuum extremal property of nearby space-time surface.

   The intersections of the wormhole throats with the light-like boundaries of causal diamonds (CDs) identified as intersections of future and past directed light cones ($CD \times CP_2$ is actually in question but I will speak about CDs) define special partonic 2-surfaces and the conformal moduli of these partonic 2-surfaces appear in the elementary particle vacuum functionals [K33] naturally. A modification of the original simple picture came from the proposed identification of physical particles as bound states of two wormhole contacts connected by tubes carrying monopole fluxes.

2. For generalized scattering diagrams stringy trouser vertices are replaced with vertices at which the ends of light-like wormhole throats meet. This vertex is the analog of 3-vertex for Feynman diagrams in particle physics lengths scales and for the biological replication (DNA and even cell) in macroscopic length scales.

   In this picture the interpretation of the analog of trouser vertex is in terms of propagation of same particle along two different paths. This interpretation is mathematically natural since vertices correspond to 2-manifolds rather than singular 2-manifolds, which are just splitting to two disjoint components. Second complication comes from the weak form of electric-magnetic duality forcing to identify physical particles as weak strings with magnetic monopoles at their ends and one should understand also the possible complications caused by this generalization.

These modifications force to consider several options concerning the identification of light fermions and bosons and one can end up with a unique identification only by making some assumptions. Masslessness of all wormhole throats - also those appearing in internal lines - and dynamical $SU(3)$ symmetry for particle generations are attractive general enough assumptions of this kind. Bosons and their possible spartners would correspond to wormhole contacts with fermion and anti-fermion at the throats of the contact. The expectation was tht free fermions and their possible spartners correspond to CP_2 type vacuum extremals with single wormhole throat. It however turned however that dynamical $SU(3)$ symmetry forces to identify massive (and possibly topologically condensed) fermions as pairs of (g, g) type wormhole contacts. The existence of higher boson families would mean breaking of quark and lepton universality and there are indications for this kind of anomaly [K73] .

The notion of elementary particle vacuum functional

Obviously one must know something about the dependence of the elementary particle state functionals on the geometric properties of the boundary component and in the sequel an attempt to construct what might be called elementary particle vacuum functionals (EPVFs), is made. The basic assumptions underlying the construction are the following ones [K33].

1. EPVFs depend on the geometric properties of the two-surface X^2 representing elementary particle.

2. EPVFs possess extended Diff invariance: all 2-surfaces on the orbit of the 2-surface X^2 correspond to the same value of the vacuum functional. This condition is satisfied if vacuum functionals have as their argument, not X^2 as such, but some 2- surface Y^2 belonging to the unique orbit of X^2 (determined by the principle selecting PE as a generalized Bohr orbit [K57, K10, K18]) and determined in general coordinate invariant manner.

3. ZEO allows to select uniquely the partonic 2-surface as the intersection of the wormhole throat at which the signature of the induced 4-metric changes with either the upper or lower

boundary of $CD\times CP_2$. This is essential since otherwise one one could not specify the vacuum functional uniquely.

4. Vacuum functionals possess conformal invariance and therefore for a given genus depend on a finite number of variables specifying the conformal equivalence class of Y^2.
5. Vacuum functionals satisfy the cluster decomposition property: when the surface Y^2 degenerates to a union of two disjoint surfaces (particle decay in string model inspired picture), vacuum functional decomposes into a product of the vacuum functionals associated with disjoint surfaces.
6. EPVFs are stable against the decay $g \rightarrow g_1 + g_2$ and one particle decay $g \rightarrow g-1$. This process corresponds to genuine particle decay only for stringy diagrams. For generalized scattering diagrams the interpretation is in terms of propagation along two different paths simultaneously.

In [K33] the construction of EPVFs is described in detail. This requires some basic concepts related to the description of the space of the conformal equivalence classes of Riemann surfaces and the concept of hyper-ellipticity. Since theta functions will play a central role in the construction of the vacuum functionals, also their basic properties are needed. Also possible explanations for the experimental absence of the higher fermion families are considered. Concerning p-adic mass calculations, the key question is how to construct p-adic variants of EPVFs.

8.3.3 p-Adic mass calculations

p-Adic thermodynamics

Consider first the basic ideas of p-adic thermodynamics.

1. p-Adic valued mass squared is identified as as thermal mass in p-adic thermodynamics. Boltzmann weights $exp(-E/T)$ do not make sense if one just replaces exponent function with the p-adic variant of its Taylor series. The reason is that $exp(x)$ has p-adic norm equal to 1 for all acceptable values of the argument x (having p-adic norm smaller than one) so that partition function does not have the usual exponential convergence property. Nothing however prevents from consider Boltzmann weights as powers p^n making sense for integer values of n. Here the p-adic norm approaches zero for $n \rightarrow +\infty$: thus the correspondences $e^{-E/T} \leftrightarrow p^{E/T_p}$.

   The values of E/T_p must be quantized to integers. This is guaranteed if E is integer valued in suitable unit of energy and $1/T_p$ has integer valued spectrum using same unit for T_p. Super-conformal invariance guarantees integer valued spectrum of E, which in the recent case corresponds to mass squared. These number theoretical conditions are very powerful and lead to the quantization of also thermal mass squared for given p-adic prime p.
2. The p-adic mass squared is mapped to real number by canonical identification $I : \sum x_n p^n \rightarrow \sum x_n p^{-n}$ or its variant for rationals. Canonical identification is continuous and maps powers of p^n to their inverses. One modification of canonical identification maps rationals m/n in their representation in which m and n have no common divisors to $I(m)/I(n)$. The predictions of calculations depend in some cases on which variant one uses but rational option looks the most reasonable choice.
3. p-Adic length scale hypothesis states that preferred p-adic primes correspond to powers of 2: $p \simeq 2^k$, but smaller than 2^k. The values of k form with $p = 2^k - 1$ is prime - Mersenne prime - are especially favored. The nearer the prime p to 2^k, the more favored p is physically. One justification for the hypothesis is that preferred primes have been selected by an evolutionary process.
4. It turns out that p-adic temperature is $T_p = 1$ for fermions. For gauge bosons $T_p \leq 1/2$ seems to be necessary assumption for gauge bosons implying that the contribution to mass squared is very small so that super-symplectic contribution assignable to the wormhole magnetic flux tube dominates for weak bosons. For canonical identification $m/n \rightarrow I(m)/I(n)$ second order contribution to fermionic mass squared is very small.
5. The large values of p-adic prime p guarantee that the p-adic thermodynamics convervges extremely rapidly. For $m/n \rightarrow I(m)/I(n)$ already the second order contribution is extremely

small since the expansion for the real mass squared is in terms of $1/p$ and for electron with $p = M_{127}$ one has $p \sim 10^{38}$. Hence the calculations are essentially exact and errors are those of the model. It is quite possible that calculations could be done exactly using exact expressions for the super-symplectic partition functions generalized to p-adic context. The success of the p-adic mass calculations is especially remarkable because p-adic length scale hypothesis $p \simeq 2^k$ predicts exponential sensitivity of the particle mass scale on k.

Symmetries

The number theoretical existence of p-adic thermodynamics requires powerful symmetries to guarantee integer valued spectrum for the thermalized contribution to the mass squared.

1. Super-conformal symmetry with integer valued conformal weights for Virasoro scaling generator L_0 is essential because it predicts in string models that mass squared is apart from ground state contribution integer valued in suitable units. In TGD framework fermionic string world sheets are characterized by super-conformal symmetry. This gives the p-adic thermodynamics assumed in the calculations. One could however assign Super Virasoro algebra also to super-symplectic algebra having its analog as sub-algebra with positive integer conformal weights. Same applies to the extended conformal algebra of light-cone boundary.

2. TGD however predicts also generalization of conformal symmetry associated with light-cone boundary involving ordinary complex conformal weights and the conformal weight associated with the light-like radial coordinate. For the latter conformal weights for the generators of supersymmetry might be given by $h = -s_n/2$. s_n zero of zeta or pole $h = -s = -1$ of zeta.

   Also super-symplectic symmetries would have similar radial spectrum of conformal weights. Conformal confinement requiring that the conformal weights of states are real implies that the spectrum of conformal weights for physical states consists of non-negative integers as for ordinary superconformal invariance.

   It is not clear whether thermalization occurs in these degrees of freedom except perhaps for trivial conformal weights. These degrees of freedom need not therefore contribute to thermal masses of leptons and quarks but would give dominating contribution to hadron masses and weak boson masses. The negative conformal weights predicted by $h = -s/2$ hypothesis predicts that ground state weight is negative for super-symplectic representations and must be compensated for massless states.

   The assumption that ground state conformal weight is negative and thus tachyonic is essential in case of p-adic mass calculations [K66], and only for massless particles (graviton, photon, gluons) it vanishes or is of order $O(1/p)$. This could be achieved if the ground state of super-symplectic representation has $h = 0$.

3. Modular invariance [K33] assignable to partonic 2-surfaces is a further assumption similar to that made made in string models. This invariance means that for a given genus the dynamical degrees of freedom of the partonic 2-surface correspond to finite-dimensional space of Teichmueller parameters. For genus $g = 0$ this space is trivial.

   Also modular invariance for string world sheets can be considered. By SH the information needed in mass calculations should be assignable to partonic 2-surfaces: the assumption is that one can assign this information to single partonic 2-surface. Stringy contribution would be seen only in scattering amplitudes.

   This might be true only effectively: the recent view about elementary particles is that they are pairs of wormhole contacts connected by flux tubes defining a closed monopole flux and wormhole throats of contact have same genus for light states. Furthermore the quantum numbers of particle are associated with single throat for fermions and with opposite throats of single contact for bosons. The second wormhole contact would carry neutralizing weak charges to realize the finite range of weak interactions as "weak confinement".

   The number of genera is infinite and one must understand why only three quark and lepton generations are observed. An attractive explanation is in terms of symmetry. For the three lowest genera the partonic 2-surfaces are always hyper-elliptic and have thus global conformal Z_2 symmetry. For higher genera this is not true always and EPVFs constructed from the assumption of modular invariance vanish for the hyper-elliptic surfaces. This suggests that the higher genera are very massive or can be interpreted as many-particle states of handles, which are not bound states but have continuous mass squared.

Contributions to mass squared

There are several contributions to the p-adic thermal mass squared come from the degrees of freedom, which are thermalized.

Super-conformal degrees of freedom associated with string world sheets are certainly thermalized. p-Adic mass calculations strongly suggest that the number of super-conformal tensor factors is $N = 5$ but also $N = 4$ and $N = 6$ can be considered marginally.

I have considered several identifications of tensor factors and not found a compelling alternative. If one assumes that super-symplectic degrees of freedom do not contribute to the thermal mass, string world sheets should explain masses of elementary fermions. Here charged lepton masses are the test bench. One other hand, if super-symplectic degrees of freedom contribute one obtains additional tensor factor assignable to $h = -s/2$, s trivial zero of zeta). Only one tensor factor emerges since Hamiltonians correspond to the products of functions of the coordinates of light-cone boundary and CP_2).

1. $SU(2)_L \times U(1)$ gives 2 tensor factors. $SU(3)$ gives 1 tensor factor. The two transversal degrees of freedom for string world sheet suggest 2 degrees of freedom corresponding to Abelian group E^2. Rotations however transforms these degrees to each other so that 1 tensor factor should emerge. This gives 4 tensor factors. Could it correspond to the degrees of freedom parallel to string at its end assignable to wormhole throat? Could normal vibrations of partonic 2-surface? This would $N = 5$ tensor factors. Another possibility is that the fifth tensor factor comes from super-symplectic Super-Virasoro algebra defined by trivial conformal weights.

2. Super-symplectic contributions need not be present for ordinary elementary fermions. For weak bosons they could give string tension assignable to the magnetic flux tube connecting the wormhole contacts. It is not clear whether this contribution is thermalized. This contribution might be present only for the phases with $h_{eff} = n \times h$. This contribution would dominate in hadron masses.

3. Color degrees of freedom contribute to the ground state mass squared since ground state corresponds to an imbedding space spinor mode massless in 8-D sense. The mass squared contribution corresponds to an eigenvalue of CP_2 spinor d'Alembertian. Its eigenvalues correspond to color multiplets and only the covariantly constant right handed neutrino is color singlet. For the other modes the color representation is non-trivial and depends on weak quantum numbers of the fermion. The construction of the massless state from a tachyonic ground state with conformal weight $h_{vac} = -3$ must involve colored super-Kac Moody generators compensating for the anomalous color charge so that one obtains color single for leptons and color triplet for quarks as massless state.

4. Modular degrees of freedom give a contribution depending on the genus g of the partonic 2-surface. This contribution is estimated by considering p-adic variants of elementary particle vacuum functionals Ω_{vac} [K66] expressible as products of theta functions with the structure of partition function. Theta functions are expressible as sums of exponent functions $exp(X)$ with X defined as a contraction of the matrix Ω_{ij} defined by Teichmueller parameters between integer valued vectors.

   In ZEO the interpretation of Ω_{vac} is as a complex square root of partition functional (quantum theory as complex square root of thermodynamics in ZEO). The integral of $|\Omega|^2$ over allowed moduli has interpretation as partition function. The exponential $exp(Re(X)) = p^{Re(X)/log(p)}$ has interpretation as an exponential of "Hamiltonian" defined by the vacuum conformal weight defined by moduli. $T = log(p)$ is identified as p-adic temperature as in ordinary p-adic thermodynamics.

   NTU requires that the integration over the moduli parameters reduces to a sum over number theoretically universal moduli parameters. The exponents $exp(X)$ must exist p-adically. PE property alone could guarantee this. The exponentials appearing in theta functions should reduce to products $p^k p^{iy} = exp(k/log(p))p^{iy}$ with k is integer and p^{iy} a root of unity. The vacuum expectation value of $Re(X)$ contributing to the mass squared is obtained from the standard formula as logarithmic temperature derivative of the "integral" $\int |\Omega_{vac}|^2$. The formula is same as for the Super-Virasoro contributions apart from the integration reducing to a sum.

The considerations of the section 8.4.2 [L10] suggest that for given p-adic prime p the exponent $k+iy$ corresponds to a linear combinations of poles of fermionic zeta $z_F(s) = \zeta(s)/\zeta(2s)$ in the class $C(p)$ with non-negative integer coefficients. This class corresponds essentially to the conformal weights of a fractal sub-algebra of super-symplectic algebra. It could give rise also to the complex values of action so that Riemann zeta would define the core of TGD.

The general dependence of the contribution of genus g to mass squared on g follows from the functional form of EPVF as a product theta functions serving as building brick partition functions apart from overall multiplicative constant and gives a nice agreement with the observed charged lepton mass ratios. The basic feature of the formula is exponential dependence on g.

5. The super-symplectic stringy contribution assignable to the magnetic flux tube dominates for weak bosons and is analogous to the stringy contribution to the hadron masses.

p-Adic mass calculations leave open several questions. What is the precise origin of preferred p-adic primes and of p-adic length scale hypothesis? How to understand the preferred number $N = 5$ of Super-Kac-Moody tensor factors? How to calculate the contribution of super-symplectic degrees of freedom - are they thermalized? Why only 3 lowest genera are light and what are the masses of the predicted bosonic higher genera implying breaking of fermion universality.

8.3.4 p-Adic length scale hypothesis

p-Adic length scale hypothesis [K1, K78] has served as a basic hypothesis of p-adic TGD for several years. This hypothesis states that the scales $L_p = \sqrt{p}l$, $l = 1.376 \cdot 10^4\sqrt{G}$ are fundamental length scale at p-adic condensate level p. The original interpretation of the hypothesis was following:

1. Above the length scale L_p p-adicity sets on and effective course grained space-time or imbedding space topology is p-adic rather than ordinary real topology. Imbedding space topology seems to be more appropriate identification.

2. The length scale L_p serves as a p-adic length scale cutoff for the quantum field theory description of particles. This means that space-time begins to look like Minkowski space so that the QFT $M^4 \to CP_2$ becomes a realistic approximation. Below this length scale string like objects and other particle like 3-surfaces are important.

3. It is un-natural to assume that just single p-adic field would be chosen from the infinite number of possibilities. Rather, there is an infinite number of cutoff length scales. To each prime p there corresponds a cutoff length scale L_p above which p-adic quantum field theory $M^4 \to CP_2$ makes sense and one has a hierarchy of p-adic QFTs. These different p-adic field theories correspond to different hierarchically levels possibly present in the topological condensate. Hierarchical ordering $< p_1 < p_2 < ...$ means that only the surface $p_1 < p_2$ can condense on the surface p_2. The condensed surface can in practice be regarded as a point like particle at level p_2 described by the p-adic conformal field theory below length scale L_{p_2}.

The recent view inspired by adelic physics is that preferred p-adic primes correspond to so called ramified primes for the algebraic extension of rationals defining the adele [K124]. Weak form of Negentropy Maximization Principle (WNMP) [K72] in turn allows to conclude that the length scales corresponding to powers of primes are preferred. Therefore p-adic length scale hypothesis generalizes. There is evidence for 3-adic time scales in biology [I20, I21] and 3-adic time scales can be also assigned with Pythagorean scale in geometric theory of harmony [K92] [L5].

8.3.5 Mersenne primes and Gaussian Mersennes are special

Mersenne primes and their complex counterparts Gaussian Mersennes pop up in p-adic mass calculations and both elementary particle physics, biology [K89], and astrophysics and cosmology [K69] provide support for them.

Mersenne primes

One can also consider the milder requirement that the exponent $\lambda = 2^{\epsilon L_0}$ represents trivial scaling represented by unit in good approximation for some p-adic topology. Not surprisingly, this is the case for $L_0 = mp^k$ since by Fermat's theorem $a^p \ mod \ p = 1$ for any integer a, in particular $a = 2$. This is also the case for $L_0 = mk$ such that $2^k \ mod \ p = 1$ for p prime. This occurs if $2^k - 1$ is Mersenne prime: in this case one has $2^{L_0} = 1$ modulo p so that the sizes of the fractal sub-algebras are exponentially larger than the sizes of $L_0 \propto p^n$ algebras. Note that all scalings a^{L_0} are near to unity for $L_0 = p^n$ whereas now only $a = 2$ gives scalings near unity for Mersenne primes. Perhaps this extended fractality provides the fundamental explanation for the special importance of Mersenne primes.

In this case integrated scalings 2^{L_0} leave the states almost invariant so that even a stronger form of the breaking of the exact conformal invariance would be in question in the super-symplectic case. The representation would be defined by the generators for which conformal weights are odd multiples of n ($M_n = 2^n - 1$) and L_{-kn}, $k > 0$ would generate zero norm states only in order $O(1/M_n)$.

Especially interesting is the hierarchy of primes defined by the so called Combinatorial Hierarchy resulting from TGD based model for abstraction process. The primes are given by $2, 3, 7 = 2^3 - 1, 127 = 2^7 - 1, 2^{127} - 1, ..$: $L_0 = n \times 127$ would correspond to M_{127}-adicity crucial for the memetic code.

Gaussian Mersennes are also special

If one allows also Gaussian primes then the notion of Mersenne prime generalizes: Gaussian Mersennes are of form $(1 \pm i)^n - 1$. In this case one could replace the scaling operations by scaling combined with a twist of $\pi/4$ around some symmetry axis: $1 + i = \sqrt{2}exp(i\pi/4)$ and generalized p-adic fractality would mean that for certain values of n the exponentiated operation consisting of n basic operations would be very near to unity.

1. The integers k associated with the lowest Gaussian Mersennes are following: $2, 3, 5, 7, 11, 19, 29, 47, 73, 79, 113$. $k = 113$ corresponds to the p-adic length scale associated with the atomic nucleus and muon. Thus all known charged leptons, rather than only e and τ, as well as nuclear physics length scale, correspond to Mersenne primes in the generalized sense.

2. The primes $k = 151, 157, 163, 167$ define perhaps the most fundamental biological length scales: $k = 151$ corresponds to the thickness of the cell membrane of about ten nanometers and $k = 167$ to cell size about 2.56 μm. This observation also suggests that cellular organisms have evolved to their present form through four basic evolutionary stages. This also encourages to think that $\sqrt{2}exp(i\pi/4)$ operation giving rise to logarithmic spirals abundant in living matter is fundamental dynamical symmetry in bio-matter.

   Logarithmic spiral provides the simplest model for biological growth as a repetition of the basic operation $\sqrt{2}exp(i\pi/4)$. The naive interpretation would be that growth processes consist of $k = 151, 157, 163, 167$ steps involving scaling by $\sqrt{2}$. This however requires the strange looking assumption that growth starts from a structure of size of order CP_2 length. Perhaps this exotic growth process is associated with pair of MEs or magnetic flux tubes of opposite time orientation and energy emergenging CP_2 sized region in a mini big bang type process and that the resulting structure serves as a template for the biological growth.

3. $k = 239, 241, 283, 353, 367, 379, 457$ associated with the next Gaussian Mersennes define astronomical length scales. $k = 239$ and $k = 241$ correspond to the p-adic time scales .55 ms and 1.1 ms: basic time scales associated with nerve pulse transmission are in question. $k = 283$ corresponds to the time scale of 38.6 min. An interesting question is whether this period could define a fundamental biological rhythm. The length scale $L(353)$ corresponds to about 2.6×10^6 light years, roughly the size scale of galaxies. The length scale $L(367) \simeq \times 3.3 \times 10^8$ light years is of same order of magnitude as the size scale of the large voids containing galaxies on their boundaries (note the analogy with cells). $T(379) \simeq 2.1 \times 10^{10}$ years corresponds to the lower bound for the order of the age of the Universe. $T(457) \sim 10^{22}$ years defines a completely superastronomical time and length scale.

8.3.6 Questions

The proposed picture leaves open several questions.

1. Could the descriptions by both real and p-adic thermodynamics be possible? Could they be equivalent (possibly in finite measurement resolution) as is suggested by NTU? The consistency of these descriptions would imply temperature quantization and p-adic length scale hypothesis not possible in purely real context.
2. What could the extension of conformal symmetry to supersymplectic symmetry mean? One possible view is that super-symplectic symmetries correspond to dark degrees of freedom and that only the super-symplectic ground states with negative conformal weights affect the p-adic thermodynamics, which applies only to fermionic degrees of freedom at string world sheets. Super-symplectic degrees of freedom would give the dominant contribution to hadron masses and could contribute also to weak gauge boson masses. $N = 5$ for the needed number of tensor factors is however a strong constraint and perhaps most naturally obtained when also the super-symplectic Virasoro associated with the trivial zeros of zeta is thermalized.
3. What happens in dark sectors. Preferred extremal property is proposed to mean that the states are annihilated by super-symplectic sub-algebra isomorphic to the original algebra and its commutator with the entire algebra. The conjecture is that this gives rise to Kac-Moody algebras as dynamical symmetries - maybe ADE type algebras, whose Dynkin diagrams characterize the inclusion of HFFs. Does this give an additional tensor factor to super-Virasoro algebra?
4. Superconformal symmetry true in the sense that Super Virasoro conditions hold true. Partition function however depends on mass squared only rather than the entire scaling generator L_0 as thought erratically in the first formulation of p-adic calculation. This does not mean breaking of conformal invariance. Super Virasoro conditions hold true although partition function is for the vibrational part of L_0 determining the mass squared spectrum.

8.4 p-Adicization and adelic physics

This section is devoted to the challenges related to p-adicization and adelization of physics in which the correspondence between real and p-adic numbers via canonical identification serves as the basic building brick. Also the problems associated with p-adic variants of integral, Fourier analysis, Hilbert space, and Riemann geometry should be solved in a way respecting fundamental symmetries and their p-adic variants must be met. The notion of number theoretical universality (NTU) plays a key role here. One should also answer to questions about the origin of preferred primes and p-adic length scale hypothesis.

8.4.1 Challenges

The basic challenges encountered are construction of the p-adic variants of real number based physics, understanding their relationship to real physics, and the fusion of various physics to single coherent whole.

The p-adicization of real physics is not just a straightforward formal generalization of scattering amplitudes of existing theories but requires a deeper understanding of the physics involved. The interpretation of p-adic physics as correlate for cognition and imagination is an important guideline and will be discussed l in more detail in separate section.

Definite integral and Fourier analysis are basic elements of standard physics and their generalization to the p-adic context defines a highly non-trivial challenge. Also the p-adic variants of Riemann geometry and Hilbert space are suggestive. There are however problems.

1. There are problems associated with p-adic definite integral. Riemann sum does not make sense since it approaches zero if the p-adic norm of discretization unit approaches zero. The problems are basically due to the absence of well-orderedness essential for the definition of definite integral and differential forms and their integrals.

   Residue integration might make sense in finite angle resolution. For algebraic extension containing $e^{i\pi/n}$ the number theoretically universal approximation $i\pi = n(e^{i\pi/n} - 1)$ could be used. In twistor approach integrations reduce to multiple residue integrations and since twistor approach generalizes in TGD framework, this approach to integration is very attractive.

Positivity is a central notion in twistor Grassmannian approach [B19]. Since canonical identification maps p-adic numbers to non-negative real numbers, there is a strong temptation to think that positivity relates to NTU [L17].

2. There are problems with Fourier analysis. The naive generalization of trigonometric functions by replacing e^{ix} with its p-adic counterpart is not physical. Same applies to e^x. Algebraic extensions are needed to get roots of unity ad e as counterparts of the phases and discretization is necessary and has interpretation in terms of finite resolution for angle/phase and its hyperbolic counterpart.

3. The notion of Hilbert space is problematic. The naive generalization of Hilbert space norm square $|x|^2 = \sum x_n \overline{x}_n$ for state $(x_1, x_2, ...)$ can vanish p-adically. Also here NTU could help. State would contain as coefficients only roots of e and unity and only the overall factor could be p-adic number. Coefficients could be restricted to the algebraic numbers generating the algebraic extension of rational numbers and would not contain powers of p or even ordinary p-adic numbers expect in the overall normalization factor.

Second challenge relates to the relationship between real and p-adic physics. Canonical identification (CI) $\sum x_n p^n \rightarrow \sum x_n p^{-n}$ or some of its variants should play an important role. CI is expected to map the invariants appearing in scattering amplitudes to their real counterparts.

1. Real and p-adic variants of space-time surfaces should exist and relate to each other somehow. Is this relationship local and involve CI at space-time level or imbedding space level? Or is it only a global and non-local assignment of preferred real extremals to their p-adic counterparts? Or is between these extreme options and involves algebraic discretization of the space-time surface weakening the strong form of SH as already proposed? How do real and p-adic imbedding spaces relate to each other and can this relationship induce local correspondence between preferred extremals (PEs) [K10, K18, K14]?

2. NTU in some sense is a highly suggestive approach to these questions and would suggest that canonical identification applies to isometry invariants whereas angles and hyperbolic angles, or rather the corresponding "phases" belonging to an extension of p-adics containing roots of e and roots of unity are mapped to themselves. Note that the roots of e define extensions of rationals, which induce finite dimensional algebraic extensions of p-adic numbers. This would make possible to define imbedding space in accordance with NTU. Also the Hilbert space could be defined by requiring that its points correspond to number theoretically universal angles expressible in terms of roots of unity.

3. What about real and p-adic variants of WCW? Are they needed at all? Or could their existence be used as a powerful constraint on real physics? The representability of WCW as a union of infinite-dimensional symmetric spaces labelled by zero modes suggests that the same description applies at the level of WCW and imbedding space.

One cannot circumvent the question about how to generalize functional integral from real WCW to p-adic WCWs. In particular, what is the p-adic variant of the action defining the dynamics of space-time surfaces. In the case of exponent of action the p-adic variant could be defined by assuming algebraic universality: again the roots of e and of unity would be in central role. Also the Kähler structure of WCW implying that Gaussian and metric determinants cancel each other in functional integral, would be absolutely crucial.

One must remember that the exponents of action for scattering amplitudes for the stationary phase extremal cancel from the path integral representation of scattering amplitudes. Also now this mechanism would allow to get rid of the poorly defined exponent for single minimum. If there is sum over scattering amplitudes assignable to different maxima, normalization sould give ratios of these exponents for different extrema/maxima and only these ratios should belong to the extension of rationals.

The zero modes of WCW metric are invariants of supersymplectic group so that canonical identification could relate their real and p-adic variants. Zero modes could break NTU and would be behind p-adic thermodynamics and dependence of mass scale on p-adic prime.

The third challenge relates to the fusion of p-adic physics and real physics to a larger structure. Here a generalization of number concept obtained by glueing reals and various p-adics together along an extension of rational numbers inducing the extensions of p-adic numbers is highly suggestive. Adeles associated with the extension of rationals are highly attractive and closely related notion. Real and various p-adic physics would be correlates for sensory and cognitive aspects of the same universal physics rather than separate physics in this framework. One important implication of this view is that real entropy and p-adic negentropies characterize the same entanglement with coefficients in an extension of rationals.

NTU for hyperbolic and ordinary phases is definetely the central idea. How the invariance of angles under conformal transformations does relate to this? Could one perhaps define a discretized version of conformal symmetry preserving the phases defined by the angles between vectors assignable with the tangent spaces of discretized geometric structures and thus respecting NTU? Of should one apply conformal symmetry at Lie algebra level only?

8.4.2 NTU and the correspondence between real and p-adic physics

p-Adic real correspondence is certainly the basic problem of p-adicization and adelization. One can make several general questions about p-adic real correspondence and canonical identification inspired by p-adic mass calculations.

How generally p-adic real correspondence does apply? Could canonical identification for group invariants combined with direct identification of ordinary and hyperbolic phases identified as roots of unity and e apply at WCW and imbedding space level having maximally symmetric geometries? Could this make sense even at space-time level as a correspondence induced from imbedding space level [L31]? Does canonical identification apply locally for the discretizations of space-time surface or only globally for the parameters characterizing PEs (string world sheets and partonic 2-surfaces by SH), which are general coordinate invariant and Poincare invariant quantities?

The following vision seems to be the most feasible one found hitherto.

1. Preservation of symmetries and continuity compete. Lorenz transformations do not commute with canonical identification. This suggests that canonical identification applies only to Lorentz invariants formed from quantum numbers. This is enough in the case of scattering amplitudes. Canonical identification applies only to isometry invariants at the level of WCW and the phases/exponents of ordinary/hyperbolic angles correspond to numbers in the algebraic extension common to extensions of rationals and various p-adics.

2. Canonical identification applies at the level of momentum space and maps p-adic Lorentz invariants of scattering amplitudes to their real counterparts. Phases of angles and their hyperbolic counterparts should correspond to parameters defining extension and should be mapped as such to their p-adic counterparts.

3. The constraints coming from GCI and symmetries do not allow local correspondence but allow to consider its discretized version at space-time leve induced by the correspondence at the level of imbedding space.

   This requires the restriction of isometries and other symmetries to algebraic subgroups defined by the extension of rationals. This would imply reduction of symmetry due to finite cognitive/measurement resolution and should be acceptable. If one wants to realize the ideas about imagination, discretization must be applied also for the space-time interior meaning partial breaking of SH and giving rise to dark matter degrees freedom in TGD sense. SH could apply in real sector for realizable imaginations only. Note that the number of algebraic points of space-time surface is expected to be relatively small.

The correspondence must be considered at the level of imbedding space, space-time, and WCW.

1. At the level of imbedding space p-adic–real correspondence is induced by points in extension of rationals and is totally discontinuous. This requires that space-time dimension is smaller than imbedding space dimension.

2. At space-time level the correspondence involves field equations derivable from a local variational principle make sense also p-adically although the action itself is ill-defined as 4-D integral. The notion of p-adic PE makes sense by strong form of holography applied to 2-surfaces in the intersection. p-Adically however only the vanishing of Noether currents for a sub-algebra of the super-symplectic algebra might make sense. This condition is stronger than the vanishing of Noether charges defined by 3-D integrals.

3. Correspondence at the level of WCW can make sense and reduces to that for string world sheets and partonic 2-surfaces by SH. Real and p-adic 4-surfaces would be obtained by algebraic continuation as PEs from 2-surfaces by assuming that the space-time surface contains subset of points of imbedding space belonging to the extension of rationals [L31]. p-Adic pseudo constants make p-adic continuation easy. Real continuation need not exist always. p-Adic WCW would be considerably larger than real WCW and make possible a predictive quantum theory of imagination and cognition.

   What I have called intersection of realities and p-adicities can be identified as the set of 2-surfaces plus algebraic discretization of space-time interior. Also the values of induced spinor fields at the points of discretization must be given. The parameters characterizing the extremals (say coefficients of polynomials) - WCW coordinates - would be in extension of rationals inducing a finite-D extension of p-adic number fields.

   The hierarchy of algebraic extensions induces an evolutionary hierarchy of adeles. The interpretation could be as a mathematical correlate for cosmic evolution realized at the level of the core of WCW defined by the intersection? 2-surfaces could be called space-time genes.

4. Also the p-adic variant Kähler action or at least the exponent of Kähler action defining vacuum functional should be obtainable by algebraic continuation. The weakest condition states that the ratios of action exponents for the maxima of Kähler function to the sum of action exponents for maxima belong to the extension. Without this condition the hopes of satisfying NTU seem rather meager.

8.4.3 NTU at space-time level

What about NTU at space-time level? NTU requires a correspondence between real and p-adic numbers and the details of this corresponds have been a long standing problem.

1. The recent view about the correspondence between real PEs to their p-adic counterparts does not demand discrete local correspondence assumed in the earlier proposal [K127]. The most abstract approach would give up the local correspondence at space-time level altogether, and restrict the preferred coordinates of WCW (having maximal group of isometries) to numbers in the extension of rationals considered. WCW would be discretized.

   Intuitively a more realistic view is a correspondence at space-time level in the sense that real and p-adic space-time sheets intersect at points belonging to the extension of rationals and defining "cognitive representations". Only some p-adic space-time surfaces would have real counterpart.

2. The strongest form of NTU would require that the allowed points of imbedding space belonging an extension of rationals are mapped as such to corresponding extensions of p-adic number fields (no canonical identification). At imbedding space level this correspondence would be extremely discontinuous. The "spines" of space-time surfaces would however contain only a subset of points of extension, and a natural resolution length scale could emerge and prevent the fluctuation. This could be also seen as a reason for why space-times surfaces must be 4-D. The fact that the curve $x^n + y^n = z^n$ has no rational points for $n > 2$, raises the hope that the resolution scale could emerge spontaneously.

3. The notion of monadic geometry discussed in detail in [L31] would realize this idea. Define first a number theoretic discretization of imbedding space in terms of points, whose coordinates in group theoretically preferred coordinate system belong to the extension of rationals considered. One can say that these algebraic points are in the intersection of reality and various p-adicities. Overlapping open sets assigned with this discretization define in the real sector a covering by open sets. In p-adic sector compact-open-topology allows to assign with each point 8^{th} Cartesian power of algebraic extension of p-adic numbers. These compact open

sets define analogs for the monads of Leibniz and p-adic variants of field equations make sense inside them.

The monadic manifold structure of H is induced to space-time surfaces containing discrete subset of points in the algebraic discretization with field equations defining a continuation to space-time surface in given number field, and unique only in finite measurement resolution. This approach would resolve the tension between continuity and symmetries in p-adic–real correspondence: isometry groups would be replaced by their sub-groups with parameters in extension of rationals considered and acting in the intersection of reality and p-adicities.

The Galois group of extension acts non-trivially on the "spines" of space-time surfaces. Hence the number theoretical symmetries act as physical symmetries and define the orbit of given space-time surface as a kind of covering space. The coverings assigned to the hierarchy of Planck constants would naturally correspond to Galois coverings and dark matter would represent number theoretical physics.

This would give rise to a kind of algebraic hierarchy of adelic 4-surfaces identifiable as evolutionary hierarchy: the higher the dimension of the extension, the higher the evolutionary level.

8.4.4 NTU and WCW

p-Adic–real correspondence at the level of WCW

It has not been obvious whether one should perform p-adicization and adelization at the level of WCW. Minimalist could argue that scattering amplitudes are all we want and that their p-adicization and adelization by algebraic continuation can be tolerated only if it can give powerful enough constraints on the amplitudes.

1. The anti-commutations for fermionic oscillator operators are number theoretically universal. Supersymmetry suggests that also WCW bosonic degrees of freedom satisfy NTU. This could mean that the coordinates of p-adic WCW consist of super-symplectic invariants mappable by canonical identification to their real counterparts plus phases and their hyperbolic counterparts expressible as genuinely algebraic numbers common to all number fields. This kind of coordinates are naturally assignable to symmetric spaces [L31].
2. Kähler structure should be mapped from p-adic to real sector and vice versa. Vacuum functional identified as exponent of action should be NTU. Algebraic continuation defined by SH involves p-adic pseudo constants. All p-adic continuations by SH should correspond to the same value of exponent of action obtained by algebraic continuation from its real value. The degeneracy associated with p-adic pseudo-constants would be analogous to gauge invariance - imagination in TGD inspired theory of consciousness.
3. Ist it possible have NTU for WCW functional integration? Or is it enough to realize NTU for scattering amplitudes only. What seems clear that functional integral must reduce to a discrete sum. Physical intuition suggests a sum over maxima of Kähler function forming a subset of PEs representing stationary points. One cannot even exclude the possibility that the set of PEs is discrete and that one can sum over all of them.

   Restriction to maximum/stationary phase approximation gives rise to sum over exponents multiplied with Gaussian determinants. The determinant of Kähler metric however cancels the Gaussian determinants, and one obtains only a sum over the exponents of action.

   The breaking of strong NTU could happen: consider only p-adic mass calculations. This breaking is however associated with the parts of quantum states assignable to the boundaries of CD, not with the vacuum functional.

NTU for functional integral

Number theoretical vision relies on NTU. In fermionic sector NTU is necessary: one cannot speak about real and p-adic fermions as separate entities and fermionic anti-commutation relations are indeed number theoretically universal.

What about NTU in case of functional integral? There are two opposite views.

1. One can define p-adic variants of field equations without difficulties if preferred extremals are minimal surface extremals of Kähler action so that coupling constants do not appear in the solutions. If the extremal property is determined solely by the analyticity properties as it is for various conjectures, it makes sense independent of number field. Therefore there would be no need to continue the functional integral to p-adic sectors. This in accordance with the philosophy that thought cannot be put in scale. This would be also the option favored by pragmatist.
2. Consciousness theorist might argue that also cognition and imagination allow quantum description. The supersymmetry NTU should apply also to functional integral over WCW (more precisely, its sector defined by CD) involved with the definition of scattering amplitudes.

1. Key observations

The general vision involves some crucial observations.

1. Only the expressions for the scatterings amplitudes should should satisfy NTU. This does not require that the functional integral satisfies NTU.
2. Since the Gaussian and metric determinants cancel in WCW Kähler metric the contributions form maxima are proportional to action exponentials $exp(S_k)$ divided by the $\sum_k exp(S_k)$. Loops vanish by quantum criticality.
3. Scattering amplitudes can be defined as sums over the contributions from the maxima, which would have also stationary phase by the double extremal property made possible by the complex value of α_K. These contributions are normalized by the vacuum amplitude.

   It is enough to require NTU for $X_i = exp(S_i)/\sum_k exp(S_k)$. This requires that $S_k - S_l$ has form $q_1 + q_2 i\pi + q_3 log(n)$. The condition brings in mind homology theory without boundary operation defined by the difference $S_k - S_l$. NTU for both S_k and $exp(S_k)$ would only values of general form $S_k = q_1 + q_2 i\pi + q_3 log(n)$ for S_k and this looks quite too strong a condition.
4. If it is possible to express the 4-D exponentials as single 2-D exponential associated with union of string world sheets, vacuum functional disappears completely from consideration! There is only a sum over discretization with the same effective action and one obtains purely combinatorial expression.

2. What does one mean with functional integral?

The definition of functional integral in WCW is one of the key technical problems of quantum TGD [K124]. NTU states that the integral should be defined simultaneously in all number fields in the intersection of real and p-adic worlds defined by string world sheets and partonic 2-surfaces with WCW coordinates in algebraic extension of rationals and allowing by strong holography continuation to 4-D space-time surface. NTU is powerful constraint and could help in this respect.

1. Path integral is not in question. Rather, the functional integral is analogous to Wiener integral and perhaps allows identification as a genuine integral in the real sector. In p-adic sectors algebraic continuation should give the integral and here number theoretical universality gives excellent hopes. The integral would have exactly the same form in real and p-adic sector and expressible solely in terms of algebraic numbers characterizing algebraic extension and finite roots of e and roots of unity $U_n = exp(i2\pi/n)$ in algebraic extension of p-adic numbers.

   Since vacuum functional $exp(S)$ is exponential of complex action S, the natural idea is that only rational powers e^q and roots of unity and phases $exp(i2\pi q)$ are involved and there is no dependence on p-adic prime p! This is true in the integer part of q is smaller than p so that one does not obtain e^{kp}, which is ordinary p-adic number and would spoil the number theoretic universality. This condition is not possible to satisfy for all values of p unless the value of Kähler function is smaller than 2. One might consider the possibility that the allow primes are above some minimum value.

   The minimal solution to NTU conditions is that the ratios of action exponentials for maxima of Kähler function to the sum of these exponentials belong to the extension of rationals considered.

2. What does one mean with functional integral? TGD is expected to be an integrable in some sense. In integrable QFTs functional integral reduces to a sum over stationary points of the action: typically only single point contributes - at least in good approximation.

   For real α_K and Λ vacuum functional decomposes to a product of exponents of real contribution from Euclidian regions ($\sqrt{g_4}$ real) and imaginary contribution Minkowskian regions ($\sqrt{g_4}$ imaginary). There would be no exchange of momentum between Minkowskian and Euclidian regions. For complex values of α_K [L13] situation changes and Kähler function as real part of action receives contributions from both Euclidian and Minkowskian regions. The imaginary part of action has interpretation as analog of Morse function and action as it appears in QFTs. Now saddle points must be considered.

   PEs satisfy extremely strong conditions [K10, K14]. All classical Noether charges for a subalgebra associated with super-symplectic algebra and isomorphic to the algebra itself vanish at both ends of CD. The conformal weights of this algebra are $n > 0$-ples of those for the entire algebra. What is fascinating that the condition that the preferred extremals are minimal surface extremals of Kähler action could solve these conditions and guarantee also NTU at the level of space-time surfaces. Supersymplectic boundary conditions at the ends of CD would however pose number theoretic conditions on the coupling parameters. In p-adic case the conditions should reduce to purely local conditions since p-adic charges are not well-defined as integrals.

3. In TGD framework one is constructing zero energy states rather calculating the matrix elements of S-matrix in terms of path integral. This gives certain liberties but a natural expectation is that functional integral as a formal tool at least is involved.

   Could one *define* the functional integral as a discrete sum of contributions of standard form for the preferred extremals, which correspond to maxima in Euclidian regions and associated stationary phase points in Minkowskian regions? Could one assume that WCW spinor field is concentrated along single maximum/stationary point.

   Quantum classical correspondence suggests that in Cartan algebra isometry charges are equal to the quantal charges for quantum states expressible in number theoretically universal manner in terms of fermionic oscillator operators or WCW gamma matrices? Even stronger condition would be that classical correlation functions are identical with quantal ones for allowed space-time surfaces in the quantum superposition. Could the reduction to a discrete sum be interpreted in terms of a finite measurement resolution?

4. In QFT Gaussian determinants produce problems because they are often poorly defined. In the recent case they could also spoil the NTU based on the exceptional properties of e. In the recent case however Gaussian determinant and metric determinant for Kähler metric cancel each other and this problem disappears. One could obtain just a sum over products of roots of e and roots of unity. Thus also Kähler structure seems to be crucial for the dream about NTU.

8.4.5 Breaking of NTU at the level of scattering amplitudes

NTU in strong sense could be broken at the level of scattering amplitudes. At space-time level the breaking does not look natural in the recent framework. Consider only p-adic mass calculations predicting that mass scale depends on p-adic prime. Also for the action strong form of NTU might fail for small p-adic primes since the value of the real part of action would be larger than than p. Should one allow this? What does one actually mean with NTU in the case of action?

Canonical identification is an important element of p-adic mass calculations and might also be needed to map p-adic variants of scattering amplitudes to their real counterparts. The breaking of NTU would take place, when the canonical real valued image of the p-adic scattering amplitude differs from the real scattering amplitude. The interpretation would be in terms of finite measurement resolution. By the finite measurement/cognitive resolution characterized by p one cannot detect the difference.

The simplest form of the canonical identification is $x = \sum_n x_n p^n \rightarrow \sum x_n p^{-n}$. Product xy and sum $x + y$ do not in general map to product and sum in canonical identification. The interpretation would be in terms of a finite measurement resolution: $(xy)_R = x_R y_R$ and $(x+y)_R = x_R + y_R$ only modulo finite measurement resolution. p-Adic scattering amplitudes are obtained by algebraic continuation from the intersection by replacing algebraic number valued parameters (such as momenta) by general p-adic numbers. The real images of these amplitudes under canonical identification are in general not identical with real scattering amplitudes the interpretation being in terms of a finite measurement resolution.

In p-adic thermodynamics NTU in the strong sense fails since thermal masses depend on p-adic mass scale. NTU can be broken by the fermionic matrix elements in the functional integral so that the real scattering amplitudes differ from the canonical images of the p-adic scattering amplitudes. For instance, the elementary particle vacuum functionals in the space of Teichmueller parameters for the partonic 2-surfaces and string world sheets should break NTU [K33].

8.4.6 NTU and the spectrum of Kähler coupling strength

During years I have made several attempts to understand coupling evolution in TGD framework. The most convincing proposal has emerged rather recently and relates the spectrum of $1/\alpha_K$ to that for the zeros of Riemann zeta [L13] and to the evolution of the electroweak U(1) couplings strength.

1. The first idea dates back to the discovery of WCW Kähler geometry defined by Kähler function defined by Kähler action (this happened around 1990) [K57]. The only free parameter of the theory is Kähler coupling strength α_K analogous to temperature parameter α_K postulated to be is analogous to critical temperature. Whether only single value or entire spectrum of of values α_K is possible, remained an open question.

   About decade ago I realized that Kähler action is *complex* receiving a real contribution from space-time regions of Euclidian signature of metric and imaginary contribution from the Minkoswkian regions. Euclidian region would give Kähler function and Minkowskian regions analog of QFT action of path integral approach defining also Morse function. Zero energy ontology (ZEO) [K128] led to the interpretation of quantum TGD as complex square root of thermodynamics so that the vacuum functional as exponent of Kähler action could be identified as a complex square root of the ordinary partition function. Kähler function would correspond to the real contribution Kähler action from Euclidian space-time regions. This led to ask whether also Kähler coupling strength might be complex: in analogy with the complexification of gauge coupling strength in theories allowing magnetic monopoles. Complex α_K could allow to explain CP breaking. I proposed that instanton term also reducing to Chern-Simons term could be behind CP breaking.

   The problem is that the dynamics in Minkowskian and Euclidian regions decouple completely and if Euclidian regions serve as space-time correlates for physical objects, there would be no exchanges of classical charges between physical objects. Should one conclude that α_K must be complex?

2. p-Adic mass calculations for 2 decades ago [K66] inspired the idea that length scale evolution is discretized so that the real version of p-adic coupling constant would have discrete set of values labelled by p-adic primes. The simple working hypothesis was that Kähler coupling strength is renormalization group (RG) invariant and only the weak and color coupling strengths depend on the p-adic length scale. The alternative ad hoc hypothesis considered was that gravitational constant is RG invariant. I made several number theoretically motivated ad hoc guesses about coupling constant evolution, in particular a guess for the formula for gravitational coupling in terms of Kähler coupling strength, action for CP_2 type vacuum extremal, p-adic length scale as dimensional quantity. Needless to say these attempts were premature and a hoc.

3. The vision about hierarchy of Planck constants $h_{eff} = n \times h$ and the connection $h_{eff} = h_{gr} = GMm/v_0$, where $v_0 < c = 1$ has dimensions of velocity [**?**] forced to consider very seriously the hypothesis that Kähler coupling strength has a spectrum of values in one-one correspondence with p-adic length scales. A separate coupling constant evolution associated with h_{eff} induced by $\alpha_K \propto 1/\hbar_{eff} \propto 1/n$ looks natural and was motivated by the idea that Nature is theoretician friendly: when the situation becomes non-perturbative, Mother Nature comes in rescue and an h_{eff} increasing phase transition makes the situation perturbative again.

   Quite recently the number theoretic interpretation of coupling constant evolution [K124] [L10] in terms of a hierarchy of algebraic extensions of rational numbers inducing those of p-adic number fields encouraged to think that $1/\alpha_K$ has spectrum labelled by primes and values of h_{eff}. Two coupling constant evolutions suggest themselves: they could be assigned to length scales and angles which are in p-adic sectors necessarily discretized and describable using only algebraic extensions involve roots of unity replacing angles with discrete phases.

4. Few years ago the relationship of TGD and GRT was finally understood [K119]. GRT space-time is obtained as an approximation as the sheets of the many-sheeted space-time of TGD are replaced with single region of space-time. The gravitational and gauge potential of sheets add together so that linear superposition corresponds to set theoretic union geometrically. This forced to consider the possibility that gauge coupling evolution takes place only at the level of the QFT approximation and α_K has only single value. This is nice but if true, one does not have much to say about the evolution of gauge coupling strengths.

5. The analogy of Riemann zeta function with the partition function of complex square root of thermodynamics suggests that the zeros of zeta have interpretation as inverses of complex temperatures $s = 1/\beta$. Also $1/\alpha_K$ is analogous to temperature. This led to a radical idea to be discussed in detail in the sequel.

   Could the spectrum of $1/\alpha_K$ reduce to that for the zeros of Riemann zeta or - more plausibly - to the spectrum of poles of fermionic zeta $\zeta_F(ks) = \zeta(ks)/\zeta(2ks)$ giving for $k = 1/2$ poles as zeros of zeta and as point $s = 2$? ζ_F is motivated by the fact that fermions are the only fundamental particles in TGD and by the fact that poles of the partition function are naturally associated with quantum criticality whereas the vanishing of ζ and varying sign allow no natural physical interpretation.

   The poles of $\zeta_F(s/2)$ define the spectrum of $1/\alpha_K$ and correspond to zeros of $\zeta(s)$ and to the pole of $\zeta(s/2)$ at $s = 2$. The trivial poles for $s = 2n$, $n = 1, 2, ..$ correspond naturally to the values of $1/\alpha_K$ for different values of $h_{eff} = n \times h$ with n even integer. Complex poles would correspond to ordinary QFT coupling constant evolution. The zeros of zeta in increasing order would correspond to p-adic primes in increasing order and UV limit to smallest value of poles at critical line. One can distinguish the pole $s = 2$ as extreme UV limit at which QFT approximation fails totally. CP_2 length scale indeed corresponds to GUT scale.

6. One can test this hypothesis. $1/\alpha_K$ corresponds to the electroweak U(1) coupling strength so that the identification $1/\alpha_K = 1/\alpha_{U(1)}$ makes sense. One also knows a lot about the evolutions of $1/\alpha_{U(1)}$ and of electromagnetic coupling strength $1/\alpha_{em} = 1/[cos^2(\theta_W)\alpha_{U(1)}$. What does this predict?

   It turns out that at p-adic length scale $k = 131$ ($p \simeq 2^k$ by p-adic length scale hypothesis, which now can be understood number theoretically [K124]) fine structure constant is predicted with .7 per cent accuracy if Weinberg angle is assumed to have its value at atomic scale! It is difficult to believe that this could be a mere accident because also the prediction evolution of $\alpha_{U(1)}$ is correct qualitatively. Note however that for $k = 127$ labelling electron one can reproduce fine structure constant with Weinberg angle deviating about 10 per cent from the measured value of Weinberg angle. Both models will be considered.

7. What about the evolution of weak, color and gravitational coupling strengths? Quantum criticality suggests that the evolution of these couplings strengths is universal and independent of the details of the dynamics. Since one must be able to compare various evolutions and combine them together, the only possibility seems to be that the spectra of gauge coupling strengths are given by the poles of $\zeta_F(w)$ but with argument $w = w(s)$ obtained by a global conformal transformation of upper half plane - that is Möbius transformation (see `http://tinyurl.com/gwjs85b`) with real coefficients (element of $GL(2, R)$) so that one as $\zeta_F((as + b)/(cs + d))$. Rather general arguments force it to be and element of $GL(2, Q)$, $GL(2, Z)$ or maybe even $SL(2, Z)$ ($ad - bc = 1$) satisfying additional constraints. Since TGD predicts several scaled variants of weak and color interactions, these copies could be perhaps parameterized by some elements of $SL(2, Z)$ and by a scaling factor K.

   Could one understand the general qualitative features of color and weak coupling contant evolutions from the properties of corresponding Möbius transformation? At the critical line there can be no poles or zeros but could asymptotic freedom be assigned with a pole of $cs + d$ and color confinement with the zero of $as + b$ at real axes? Pole makes sense only if Kähler action for the preferred extremal vanishes. Vanishing can occur and does so for massless extremals characterizing conformally invariant phase. For zero of $as + b$ vacuum function would be equal to one unless Kähler action is allowed to be infinite: does this make sense?. One can however hope that the values of parameters allow to distinguish between weak and color interactions. It is certainly possible to get an idea about the values of the parameters of the transformation and one ends up with a general model predicting the entire electroweak coupling constant evolution successfully.

To sum up, the big idea is the identification of the spectra of coupling constant strengths as poles of $\zeta_F((as + b/)(cs + d))$ identified as a complex square root of partition function with motivation coming from ZEO, quantum criticality, and super-conformal symmetry; the discretization of the RG flow made possible by the p-adic length scale hypothesis $p \simeq k^k$, k prime; and the assignment of complex zeros of ζ with p-adic primes in increasing order. These assumptions reduce the coupling constant evolution to four real rational or integer valued parameters (a, b, c, d). In the sequel this vision is discussed in more detail.

8.4.7 Other applications of NTU

NTU in the strongest form says that all numbers involved at "basic level" (whatever this means!) of adelic TGD are products of roots of unity and of power of a root of e. This is extremely powerful physics inspired conjecture with a wide range of possible mathematical applications.

1. For instance, vacuum functional defined as an exponent of action for preferred externals would be number of this kind. One could define functional integral as adelic operation in all number fields: essentially as sum of exponents of action for stationary preferred extremals since Gaussian and metric determinants potentially spoiling NTU would cancel each other leaving only the exponent.
2. The implications of NTU for the zeros of Riemann zeta [L10] will be discussed in more detail in the Appendix. Suffice it to say that the observations about Fourier transform for the distribution of loci of non-trivial zeros of zeta together with NTU leads to explicit proposal for the algebraic for of zeros of zeta. The testable proposal is that zeros decompose to disjoint classes $C(p)$ labelled by primes p and the condition that p^{iy} is root of unity in given class $C(p)$.
3. NTU generalises to all Lie groups. Exponents $exp(in_iJ_i/n)$ of lie-algebra generators define generalisations of number theoretically universal group elements and generate a discrete subgroup of compact Lie group. Also hyperbolic "phases" based on the roots $e^{m/n}$ are possible and make possible discretized NTU versions of all Lie-groups expected to play a key role in adelization of TGD.

   NTU generalises also to quaternions and octonions and allows to define them as number theoretically universal entities. Note that ordinary p-adic variants of quaternions and octonions do not give rise to a number field: inverse of quaternion can have vanishing p-adic variant of norm squared satisfying $\sum_n x_n^2 = 0$.

   NTU allows to define also the notion of Hilbert space as an adelic notion. The exponents of angles characterising unit vector of Hilbert space would correspond to roots of unity.

8.4.8 Going to the roots of p-adicity

The basic questions raised by the p-adic mass calculations concern the origin of preferred p-adic primes and of p-adic length scale hypothesis. One can also ask whether there might be a natural origin for p-adicity at the level of WCW.

Preferred primes as ramified primes for extensions of rationals?

Preferred primes as ramified primes for extensions of rationals?

The intuitive feeling is that the notion of preferred prime is something extremely deep and to me the deepest thing I know is number theory. Does one end up with preferred primes in number theory? This question brought to my mind the notion of *ramification of primes* (`http://tinyurl.com/hddljlf`) (more precisely, of prime ideals of number field in its extension), which happens only for special primes in a given extension of number field, say rationals. Ramification is completely analogous to the degeneracy of some roots of polynomial and corresponds to criticality if the polynomial corresponds to criticality (catastrophe theory of Thom is one application). Could this be the mechanism assigning preferred prime(s) to a given elementary system, such as elementary particle? I have not considered their role earlier also their hierarchy is highly relevant in the number theoretical vision about TGD.

1. Stating it very roughly (I hope that mathematicians tolerate this sloppy language of physicist): as one goes from number field K, say rationals Q, to its algebraic extension L, the original prime ideals in the so called *integral closure* (`http://tinyurl.com/js6fpvr`) over integers of K decompose to products of prime ideals of L (prime ideal is a more rigorous manner to express primeness). Note that the general ideal is analog of integer.

   Integral closure for integers of number field K is defined as the set of elements of K, which are roots of some monic polynomial with coefficients, which are integers of K having the form $x^n + a_{n-1}x^{n-1} + ... + a_0$. The integral closures of both K and L are considered. For instance, integral closure of algebraic extension of K over K is the extension itself. The integral closure of complex numbers over ordinary integers is the set of algebraic numbers.

   Prime ideals of K can be decomposed to products of prime ideals of L: $P = \prod P_i^{e_i}$, where e_i is the ramification index. If $e_i > 1$ is true for some i, *ramification* occurs. P_i:s in question are like co-inciding roots of polynomial, which for in thermodynamics and Thom's catastrophe theory corresponds to criticality. Ramification could therefore be a natural aspect of quantum criticality and ramified primes P are good candidates for preferred primes for a given extension of rationals. Note that the ramification make sense also for extensions of given extension of rationals.

2. A physical analogy for the decomposition of ideals to ideals of extension is provided by decomposition of hadrons to valence quarks. Elementary particles becomes composite of more elementary particles in the extension. The decomposition to these more elementary primes is of form $P = \prod P_i^{e(i)}$, the physical analog would be the number of elementary particles of type i in the state (`http://tinyurl.com/h9528pl`). Unramified prime P would be analogous a state with e fermions. Maximally ramified prime would be analogous to Bose-Einstein condensate of e bosons. General ramified prime would be analogous to an e-particle state containing both fermions and condensed bosons. This is of course just a formal analogy.

3. There are two further basic notions related to ramification and characterizing it. *Relative discriminant* is the ideal divided by all ramified ideals in K (integer of K having no ramified prime factors) and relative different for P is the ideal of L divided by all ramified P_i:s (product of prime factors of P in L). These ideals represent the analogs of product of preferred primes P of K and primes P_i of L dividing them. These two integers ideals would characterize the ramification.

In TGD framework the extensions of rationals (`http://tinyurl.com/h9528pl`) and p-adic number fields (`http://tinyurl.com/zq22tvb`) are unavoidable and interpreted as an evolutionary hierarchy physically and cosmological evolution would gradually proceed to more and more complex extensions. One can say that string world sheets and partonic 2-surfaces with parameters of defining functions in increasingly complex extensions of prime emerge during evolution. Therefore ramifications and the preferred primes defined by them are unavoidable. For p-adic number fields the number of extensions is much smaller for instance for $p > 2$ there are only 3 quadratic extensions.

How could ramification relate to p-adic and adelic physics and could it explain preferred primes?

1. Ramified p-adic prime $P = P_i^e$ would be replaced with its e:th root P_i in p-adicization. Same would apply to general ramified primes. Each un-ramified prime of K is replaced with $e = K : L$ primes of L and ramified primes P with $\#\{P_i\} < e$ primes of L: the increase of algebraic dimension is smaller. An interesting question relates to p-adic length scale. What happens to p-adic length scales. Is p-adic prime effectively replaced with e:th root of p-adic prime: $L_p \propto p^{1/2}L_1 \rightarrow p^{1/2e}L_1$? The only physical option is that the p-adic temperature for P would be scaled down $T_p = 1/n \rightarrow 1/ne$ for its e:th root (for fermions serving as

fundamental particles in TGD one actually has $T_p = 1$). Could the lower temperature state be more stable and select the preferred primes as maximimally ramified ones? What about general ramified primes?

2. This need not be the whole story. Some algebraic extensions would be more favored than others and p-adic view about realizable imaginations could be involved. p-Adic pseudo constants are expected to allow p-adic continuations of string world sheets and partonic 2-surfaces to 4-D preferred extremals with number theoretic discretization. For real continuations the situation is more difficult. For preferred extensions - and therefore for corresponding ramified primes - the number of real continuations - realizable imaginations - would be especially large.

   The challenge would be to understand why primes near powers of 2 and possibly also of other small primes would be favored. Why for them the number of realizable imaginations would be especially large so that they would be winners in number theoretical fight for survival?

Can one make this picture more concrete? What kind of algebraic extensions could be considered?

1. In p-adic context a proper definition of counterparts of angle variables as phases allowing definition of the analogs of trigonometric functions requires the introduction of algebraic extension giving rise to some roots of unity. Their number depends on the angular resolution. These roots allow to define the counterparts of ordinary trigonometric functions - the naive generalization based on Taylors series is not periodic - and also allows to defined the counterpart of definite integral in these degrees of freedom as discrete Fourier analysis. For the simplest algebraic extensions defined by $x^n - 1$ for which Galois group is abelian are are unramified so that something else is needed. One has decomposition $P = \prod P_i^{e(i)}$, $e(i) = 1$, analogous to n-fermion state so that simplest cyclic extension does not give rise to a ramification and there are no preferred primes.

2. What kind of polynomials could define preferred algebraic extensions of rationals? Irreducible polynomials are certainly an attractive candidate since any polynomial reduces to a product of them. One can say that they define the elementary particles of number theory. Irreducible polynomials have integer coefficients having the property that they do not decompose to products of polynomials with rational coefficients. It would be wrong to say that only these algebraic extensions can appear but there is a temptation to say that one can reduce the study of extensions to their study. One can even consider the possibility that string world sheets associated with products of irreducible polynomials are unstable against decay to those characterize irreducible polynomials.

3. What can one say about irreducible polynomials? Eisenstein criterion (`http://tinyurl.com/47kxjz` states following. If $Q(x) = \sum_{k=0,..,n} a_k x^k$ is n:th order polynomial with integer coefficients and with the property that there exists at least one prime dividing all coefficients a_i except a_n and that p^2 does not divide a_0, then Q is irreducible. Thus one can assign one or more preferred primes to the algebraic extension defined by an irreducible polynomial Q of this kind - in fact any polynomial allowing ramification. There are also other kinds of irreducible polynomials since Eisenstein's condition is only sufficient but not necessary.

   Furthermore, in the algebraic extension defined by Q, the prime ideals P having the above mentioned characteristic property decompose to an n:th power of single prime ideal P_i: $P = P_i^n$. The primes are maximally/completely ramified.

   A good illustration is provided by equations $x^2 + 1 = 0$ allowing roots $x_\pm = \pm i$ and equation $x^2 + 2px + p = 0$ allowing roots $x_\pm = -p \pm \sqrt{p}p - 1$. In the first case the ideals associated with $\pm i$ are different. In the second case these ideals are one and the same since $x_+ == -x_- + p$: hence one indeed has ramification. Note that the first example represents also an example of irreducible polynomial, which does not satisfy Eisenstein criterion. In more general case the n conditions on defined by symmetric functions of roots imply that the ideals are one and same when Eisenstein conditions are satisfied.

4. What is so nice that one could readily construct polynomials giving rise to given preferred primes. The complex roots of these polymials could correspond to the points of partonic 2-surfaces carrying fermions and defining the ends of boundaries of string world sheet. It must be however emphasized that the form of the polynomial depends on the choices of the complex

coordinate. For instance, the shift $x \rightarrow x+1$ transforms $(x^n-1)/(x-1)$ to a polynomial satisfying the Eisenstein criterion. One should be able to fix allowed coordinate changes in such a manner that the extension remains irreducible for all allowed coordinate changes.

Already the integral shift of the complex coordinate affects the situation. It would seem that only the action of the allowed coordinate changes must reduce to the action of Galois group permuting the roots of polynomials. A natural assumption is that the complex coordinate corresponds to a complex coordinate transforming linearly under subgroup of isometries of the imbedding space.

In the general situation one has $P = \prod P_i^{e(i)}$, $e(i) \geq 1$ so that aso now there are prefered primes so that the appearance of preferred primes is completely general phenomenon.

The origin of p-adic length scale hypothesis?

p-Adic length scale hypothesis emerged from p-adic length scale hypothesis. A possible generalization of this hypothesis is that p-adic primes near powers of prime are physically favored. There indeed exists evidence for the realization of 3-adic time scale hierarchies in living matter [I21] (`http://tinyurl.com/jbh9m27`) and in music both 2-adicity and 3-adicity could be present: this is discussed in TGD inspired theory of music harmony and genetic code [K92]. See also [L33, L24].

One explanation would be that for preferred primes the number of p-adic space-time sheets representable also as real space-time sheets is maximal. Imagined worlds would be maximally realizable. Preferred p-adic primes would correspond to ramified primes for extensions with the property that the number of realizable imaginations is especially large for them. Why primes satisfying p-adic length scale hypothesis or its generalization would appear as ramified primes for extensions, which are winners in number theoretical evolution?

Also the weak form of NMP (WNMP) applying also to the purely number theoretic form of NMP [K72] might come in rescue here.

1. Entanglement negentropy for a NE [K72] characterized by n-dimensional projection operator is the $log(N_p(n)$ for some p whose power divides n. The maximum negentropy is obtained if the power of p is the largest power of prime divisor of p, and this can be taken as definition of number theoretical entanglement negentropy (NEN). If the largest divisor is p^k, one has $N = k \times log(p)$. The entanglement negentropy per entangled state is $N/n = klog(p)/n$ and is maximal for $n = p^k$. Hence powers of prime are favoured, which means that p-adic length scale hierarchies with scales coming as powers of p are negentropically favored and should be generated by NMP. Note that $n = p^k$ would define a hierarchy of $h_{eff}/h = p^k$. During the first years of h_{eff} hypothesis I believe that the preferred values obey $h_{eff} = r^k$, r integer not far from $r = 2^{11}$. It seems that this belief was not totally wrong.
2. If one accepts this argument, the remaining challenge is to explain why primes near powers of two (or more generally p) are favoured. $n = 2^k$ gives large entanglement negentropy for the final state. Why primes $p = n_2 = 2^k - r$ would be favored? The reason could be following. $n = 2^k$ corresponds to $p = 2$, which corresponds to the lowest level in p-adic evolution since it is the simplest p-adic topology and farthest from the real topology and therefore gives the poorest cognitive representation of real PE as p-adic PE (Note that $p = 1$ makes formally sense but for it the topology is discrete).
3. WNMP [K72, K121] suggests a more feasible explanation. The density matrix of the state to be reduced is a direct sum over contributions proportional to projection operators. Suppose that the projection operator with largest dimension has dimension n. Strong form of NMP would say that final state is characterized by n-dimensional projection operator. WNMP allows "free will" so that all dimensions $n-k$, $k = 0, 1, ...n-1$ for final state projection operator are possible. 1-dimensional case corresponds to vanishing entanglement negentropy and ordinary state function reduction isolating the measured system from external world.
4. The negentropy of the final state per state depends on the value of k. It is maximal if $n-k$ is power of prime. For $n = 2^k = M_k + 1$, where M_k is Mersenne prime $n-1$ gives the maximum negentropy and also maximal p-adic prime available so that this reduction is favoured by NMP. Mersenne primes would be indeed special. Also the primes $n = 2^k - r$ near 2^k produce large entanglement negentropy and would be favored by NMP.

5. This argument suggests a generalization of p-adic length scale hypothesis so that $p = 2$ can be replaced by any prime.

8.5 p-Adic physics and consciousness

p-Adic physics as physics of cognition and imagination is an important thread in TGD inspired theory of consciousness. In the sequel I describe briefly the basic of TGD inspired theory of consciousness as generalization of quantum measurement theory to ZEO (ZEO), describe the definition of self, consider the question whether NMP is needed as a separate principle or whether it is implied is in statistical sense by the unavoidable statistical increase of $n = h_{eff}/h$ if identified as a factor of the dimension of Galois group extension of rationals defining the adeles, and finally summarize the vision about how p-adic physics serves as a correlate of cognition and imagination.

8.5.1 From quantum measurement theory to a theory of consciousness

The notion of self can be seen as a generalization of the poorly defined definition of the notion of observer in quantum physics. In the following I take the role of skeptic trying to be as critical as possible.

The original definition of self was as a subsystem able to remain unentangled under state function reductions associated with subsequent quantum jumps. The density matrix was assumed to define the universal observable. Note that a density matrix, which is power series of a product of matrices representing commuting observables has in the generic case eigenstates, which are simultaneous eigenstates of all observables. Second aspect of self was assumed to be the integration of subsequent quantum jumps to coherent whole giving rise to the experienced flow of time.

The precise identification of self allowing to understand both of these aspects turned out to be difficult problem. I became aware the solution of the problem in terms of ZEO (ZEO) only rather recently (2014).

1. Self corresponds to a sequence of quantum jumps integrating to single unit as in the original proposal, but these quantum jumps correspond to state function reductions to a fixed boundary of causal diamond CD leaving the corresponding parts of zero energy states invariant - "small" state function reductions. The parts of zero energy states at second boundary of CD change and even the position of the tip of the opposite boundary changes: one actually has wave function over positions of second boundary (CD sizes roughly) and this wave function changes. In positive energy ontology these repeated state function reductions would have no effect on the state (Zeno effect) but in TGD framework there occurs a change for the second boundary and gives rise to the experienced flow of time and its arrow and self: self is generalized Zeno effect.
2. The first quantum jump to the opposite boundary corresponds to the act of "free will" or birth of re-incarnated self. Hence the act of "free will" changes the arrow of psychological time at some level of hierarchy of CDs. The first reduction to the opposite boundary of CD means "death" of self and "re-incarnation" of time-reversed self at opposite boundary at which the the temporal distance between the tips of CD increases in opposite direction. The sequence of selves and time reversed selves is analogous to a cosmic expansion for CD. The repeated birth and death of mental images could correspond to this sequence at the level of sub-selves.
3. This allows to understand the relationship between subjective and geometric time and how the arrow of and flow of clock time (psychological time) emerge. The average distance between the tips of CD increases on the average as along as state function functions occur repeatedly at the fixed boundary: situation is analogous to that in diffusion. The localization of contents of conscious experience to boundary of CD gives rise to the illusion that universe is 3-dimensional. The possibility of memories made possibly by hierarchy of CDs demonstrates that this is not the case. Self is simply the sequence of state function reductions at same boundary of CD remaining fixed and the lifetime of self is the total growth of the average temporal distance between the tips of CD.

One can identify several rather abstract state function reductions selecting a sector of WCW.

1. There are quantum measurements inducing localization in the moduli space of CDs with passive boundary and states at it fixed. In particular, a localization in the moduli characterizing the Lorentz transform of the upper tip of CD would be measured. The measured moduli characterize also the analog of symplectic form in M^4 strongly suggested by twistor lift of TGD - that is the rest system (time axis) and spin quantization axes. Of course, also other kinds of reductions are possible.
2. Also a localization to an extension of rationals defining the adeles should occur. Could the value of $n = h_{eff}/h$ be observable? The value of n for given space-time surface at the active boundary of CD could be identified as the order of the smallest Galois group containing all Galois groups assignable to 3-surfaces at the boundary. The superposition of space-time surface would not be eigenstate of n at active boundary unless localization occurs. It is not obvious whether this is consistent with a fixe value of n at passive boundary.

   The measured value of n could be larger or smaller than the value of n at the passive boundary of CD but in statistical sense n would increase by the analogy with diffusion on half line defined by non-negative integers. The distance from the origin unavoidably increases in statistical sense. This would imply evolution as increase of maximal value of negentropy and generation of quantum coherence in increasingly longer scales.
3. A further abstract choice corresponds to the the replacement of the roles of active and passive boundary of CD changing the arrow of clock time and correspond to a death of self and re-incarnation as time-reversed self.

Can one assume that these measurements reduce to measurements of a density matrix of either entangled system as assumed in the earlier formulation of NMP, or should one allow both options. This question actually applies to all quantum measurements and leads to a fundamental philosophical questions unavoidable in all consciousness theories.

1. Do all measurements involve entanglement between the moduli or extensions of two CDs reduced in the measurement of the density matrix? Non-diagonal entanglement would allow final states states, which are not eigenstates of moduli or of n: this looks strange. This could also lead to an infinite regress since it seems that one must assume endless hierarchy of entangled CDs so that the reduction sequence would proceed from top to bottom. It looks natural to regard single CD as a sub-Universe.

   For instance, if a selection of quantization axis of color hypercharge and isospin (localization in the twistor space of CP_2) is involved, one would have an outcome corresponding to a quantum superposition of measurements with different color quantization axis!

   Going philosophical, one can also argue, that the measurement of density matrix is only a reaction to environment and does not allow intentional free will.
2. Can one assume that a mere localization in the moduli space or for the extension of rationals (producing an eigenstate of n) takes place for a fixed CD - a kind of self measurement possible for even unentangled system? If there is entanglement in these degrees of freedom between two systems (say CDs), it would be reduced in these self measurements but the outcome would not be an eigenstate of density matrix. An interpretation as a realization of intention would be approriate.
3. If one allows both options, the interpretation would be that state function reduction as a measurement of density matrix is only a reaction to environment and self-measurement represents a realization of intention.
4. Self measurements would occur at higher level say as a selection of quantization axis, localization in the moduli space of CD, or selection of extension of rationals. A possible general rule is that measurements at space-time level are reactions as measurements of density matrix whereas a selection of a sector of WCW would be an intentional action. This because formally the quantum states at the level of WCW are as modes of classical WCW spinor field single particle states.
5. If the selections of sectors of WCW at active boundary of CD commute with observables, whose eigenstates appear at passive boundary (briefly *passive observables*) meaning that time reversal commutes with them - they can occur repeatedly during the reduction sequence and self as a generalized Zeno effect makes sense.

If the selections of WCW sectors at active boundary do not commute with passive observables then volition as a choice of sector of WCW must change the arrow of time. Libet's findings show that conscious choice induces neural activity for a fraction of second before the conscious choice. This would imply the correspondences *"big" measurement changing the arrow of time - self-measurement at the level of WCW - intentional action* and *"small" measurement - measurement at space-time level - reaction.*

Self as a generalized Zeno effect makes sense only if there are active commuting with passive observables. If the passive observables form a maximal set, the new active observables commuting with them must emerge. The increase of the size of extension of rationals might generate them by expanding the state space so that self would survive only as long at it evolves.

Otherwise there would be only single unitary time evolution followed by a reduction to opposite boundary. This makes sense only if the sequence of "big" reductions for sub-selves can give rise to the time flow experienced by self: the birth and death of mental images would give rise to flow of time of self.

A hierarchical process starting from given CD and proceeding downwards to shorter scales and stopping when the entanglement is stable is highly suggestive and favors self measurements. What stability could mean will be discussed in the next section. CDs would be a correlate for self hierarchy. One can say also something about the anatomy and correlates of self hierarchy.

1. Self experiences its sub-selves as mental images and even we would represent mental images of some higher level collective self. Everything is conscious but consciousness can be lost or at least it is not possible to have memory about it. The flow of consciousness for a given self could be due to the quantum jump sequences performed by its sub-selves giving rise to mental images.
2. By quantum classical correspondence self has also space-time correlates. One can visualize sub-self as a space-time sheet "glued" by topological sum to the space-time sheet of self. Subsystem is not described as a tensor factor as in the standard description of subsystems. Also sub-selves of selves can entangle negentropically and this gives rise to a sharing of mental images about which stereo vision would be basic example. Quite generally, one could speak of stereo consciousness. Also the experiences of sensed presence [J10] could be understood as a sharing of mental images between brain hemispheres, which are not themselves entangled. This is possible also between different brains. In the normal situation brain hemispheres are entangled.
3. At the level of 8-dimensional imbedding space the natural correlate of self would be CD (causal diamond). At the level of space-time the correlate would be space-time sheet or light-like 3-surface. The contents of consciousness of self would be determined by the space-time sheets in the interior of CD. Without further restrictions the experience of self would be essentially four-dimensional. Memories would be like sensory experiences except that they would be about the geometric past and for some reason are not usually colored by sensory qualia. For instance .1 second time scale defining sensory chronon corresponds to the secondary p-adic time scale characterizing the size of electron's CD (Mersenne prime M_{127}), which suggests that Cooper pairs of electrons are essential for the sensory qualia.

8.5.2 NMP and self

The view about Negentropy Maximization Principle (NMP) [K72] has co-evolved with the notion of self and I have considered many variants of NMP.

1. The original formulation of NMP was in positive energy ontology and made same predictions as standard quantum measurement theory. The new element was that the density matrix of sub-system defines the fundamental observable and the system goes to its eigenstate in state function reduction. As found, the localizations at to WCW sectors define what might be called self-measurements and identifiable as active volitions rather than reactions.
2. In p-adic physics one can assign with rational and even algebraic entanglement probabilities number theoretical entanglement negentropy (NEN) satisfying the same basic axioms

as the ordinary Shannon entropy but having negative values and therefore having interpretation as information. The definition of p-adic negentropy (real valued) reads as $S_p = -\sum P_k log(|P_k|_p)$, where $|.|_p$ denotes p-adic norm. The news is that $N_p = -S_p$ can be positive and is positive for rational entanglement probabilities. Real entanglement entropy S is always non-negative.

NMP would force the generation of negentropic entanglement (NE) and stabilize it. NNE resources of the Universe - one might call them Akashic records- would steadily increase.

3. A decisive step of progress was the realization is that NTU forces all states in adelic physics to have entanglement coefficients in some extension of rationals inducing finite-D extension of p-adic numbers. The same entanglement can be characterized by real entropy S and p-adic negentropies N_p, which can be positive. One can define also total p-adic negentropy: $N = \sum_p N_p$ for all p and total negentropy $N_{tot} = N - S$.

   For rational entanglement probabilities it is easy to demonstrate that the generalization of adelic theorem holds true: $N_{tot} = N - S = 0$. NMP based on N_{tot} rather than N would not say anything about rational entanglement. For extensions of rationals it is easy to find that $N - S > 0$ is possible if entanglement probabilities are of form X_i/n with $|X_i|_p = 1$ and n integer [L27]. Should one identify the total negentropy as difference $N_{tot} = N - S$ or as $N_{tot} = N$?

   Irrespective of answer, large p-adic negentropy seems to force large real entropy: this nicely correlates with the paradoxical finding that living systems tend to be entropic although one would expect just the oppositecite [L27]: this relates in very interesting manner to the work of biologists Jeremy England [I23]. The negentropy would be cognitive negentropy and not visible for ordinary physics.

4. The latest step in the evolution of ideas NMP was the question whether NMP follows from number theory alone just as second law follows form probability theory! This irritates theoretician's ego but is victory for theory. The dimension n of extension is positive integer and cannot but grow in statistical sense in evolution! Since one expects that the maximal value of negentropy (define as $N - S$) must increase with n. Negentropy must increase in long run.

Number theoretic entanglement can be stable

Number theoretical Shannon entropy can serve as a measure for genuine information assignable to a pair of entanglement systems [K72]. Entanglement with coefficients in the extension is always negentropic if entanglement negentropy comes from p-adic sectors only. It can be negentropic if negentropy is defined as the difference of p-adic negentropy and real entropy.

The diagonalized density matrix need not belong to the algebraic extension since the probabilities defining its diagonal elements are eigenvalues of the density matrix as roots of N:th order polynomial, which in the generic case requires n-dimensional algebraic extension of rationals. One can argue that since diagonalization is not possible, also state function reduction selecting one of the eigenstates is impossible unless a phase transition increasing the dimension of algebraic extension used occurs simultaneously. This kind of NE could give rise to cognitive entanglement.

There is also a special kind of NE, which can result if one requires that density matrix serves a universal observable in state function reduction. The outcome of reduction must be an eigen space of density matrix, which is projector to this subspace acting as identity matrix inside it. This kind NE allows all unitarily related basis as eigenstate basis (unitary transformations must belong to the algebraic extension). This kind of NE could serve as a correlate for "enlightened" states of consciousness. Schrödingers cat is in this kind of state stably in superposition of dead and alive and state basis obtained by unitary rotation from this basis is equally good. One can say that there are no discriminations in this state, and this is what is claimed about "enlightened" states too.

The vision about number theoretical evolution suggests that NMP forces the generation of NE resources as NE assignable to the "passive" boundary of CD for which no changes occur during sequence of state function reductions defining self. It would define the unchanging self as negentropy resources, which could be regarded as kind of Akashic records. During the next "re-incarnation" after the first reduction to opposite boundary of CD the NE associated with the reduced state would serve as new Akashic records for the time reversed self. If NMP reduces to the statistical increase of $h_{eff}/h = n$ the consciousness information contents of the Universe increases in statistical sense. In the best possible world of SNMP it would increase steadily.

Does NMP reduce to number theory?

The heretic question that emerged quite recently is whether NMP is actually needed at all! Is NMP a separate principle or could NMP reduced to mere number theory [K72]? Consider first the possibility that NMP is not needed at all as a separate principle.

1. The value of $h_{eff}/h = n$ should increase in the evolution by the phase transitions increasing the dimension of the extension of rationals. $h_{eff}/h = n$ has been identified as the number of sheets of some kind of covering space. The Galois group of extension acts on number theoretic discretizations of the monadic surface and the orbit defines a covering space. Suppose n is the number of sheets of this covering and thus the dimension of the Galois group for the extension of rationals or factor of it.

2. It has been already noticed that the "big" state function reductions giving rise to death and reincarnation of self could correspond to a measurement of $n = h_{eff}$ implied by the measurement of the extension of the rationals defining the adeles. The statistical increase of n follows automatically and implies statistical increase of maximal entanglement negentropy. Entanglement negentropy increases in statistical sense.

   The resulting world would not be the best possible one unlike for a strong form of NMP demanding that negentropy does increaes in "big" state function reductions. n also decrease temporarily and they seem to be needed. In TGD inspired model of bio-catalysis the phase transition reducing the value of n for the magnetic flux tubes connecting reacting bio-molecules allows them to find each other in the molecular soup. This would be crucial for understanding processes like DNA replication and transcription.

3. State function reduction corresponding to the measurement of density matrix could occur to an eigenstate/eigenspace of density matrix only if the corresponding eigenvalue and eigenstate/eigenspace is expressible using numbers in the extension of rationals defining the adele considered. In the generic case these numbers belong to N-dimensional extension of the original extension. This can make the entanglement stable with respect to state the measurements of density matrix.

   A phase transition to an extension of an extension containing these coefficients would be required to make possible reduction. A step in number theoretic evolution would occur. Also an entanglement of measured state pairs with those of measuring system in containing the extension of extension would make possible the reduction. Negentropy could be reduced but higher-D extension would provide potential for more negentropic entanglement and NMP would hold true in the statistical sense.

4. If one has higher-D eigen space of density matrix, p-adic negentropy is largest for the entire subspace and the sum of real and p-adic negentropies vanishes for all of them. For negentropy identified as total p-adic negentropy SNMP would select the entire sub-space and NMP would indeed say something explicit about negentropy.

Or is NMP needed as a separate principle?

Hitherto I have postulated NMP as a separate principle [K72]. Strong form of NMP (SNMP) states that Negentropy does not decrease in "big" state function reductions corresponding to death and re-incarnations of self.

One can however argue that SNMP is not realistic. SNMP would force the Universe to be the best possible one, and this does not seem to be the case. Also ethically responsible free will would be very restricted since self would be forced always to do the best deed that is increase maximally the negentropy serving as information resources of the Universe. Giving up separate NMP altogether would allow to have also "Good" and "Evil".

This forces to consider what I christened weak form of NMP (WNMP). Instead of maximal dimension corresponding to N-dimensional projector self can choose also lower-dimensional sub-spaces and 1-D sub-space corresponds to the vanishing entanglement and negentropy assumed in standard quantum measurement theory. As a matter fact, this can also lead to larger negentropy gain since negentropy depends strongly on what is the large power of p in the dimension of the resulting eigen sub-space of density matrix. This could apply also to the purely number theoretical reduction of NMP.

WNMP suggests how to understand the notions of Good and Evil. Various choices in the state function reduction would correspond to Boolean algebra, which suggests an interpretation in terms of what might be called emotional intelligence [K121]. Also it turns out that one can understand how p-adic length scale hypothesis - actually its generalization - emerges from WNMP [K124].

1. One can start from ordinary quantum entanglement. It corresponds to a superposition of pairs of states. Second state corresponds to the internal state of the self and second state to a state of external world or biological body of self. In negentropic quantum entanglement each is replaced with a pair of sub-spaces of state spaces of self and external world. The dimension of the sub-space depends on which pair is in question. In state function reduction one of these pairs is selected and deed is done. How to make some of these deeds good and some bad? Recall that WNMP allows only the possibility to generate NNE but does not force it. WNMP would be like God allowing the possibility to do good but not forcing good deeds.

   Self can choose any sub-space of the subspace defined by $k \leq N$-dimensional projector and 1-D subspace corresponds to the standard quantum measurement. For $k = 1$ the state function reduction leads to vanishing negentropy, and separation of self and the target of the action. Negentropy does not increase in this action and self is isolated from the target: kind of price for sin.

   For the maximal dimension of this sub-space the negentropy gain is maximal. This deed would be good and by the proposed criterion NE corresponds to conscious experience with positive emotional coloring. Interestingly, there are $2^k - 1$ possible choices, which is almost the dimension of Boolean algebra consisting of k independent bits. The excluded option corresponds to 0-dimensional sub-space - empty set in set theoretic realization of Boolean algebra. This could relate directly to fermionic oscillator operators defining basis of Boolean algebra - here Fock vacuum would be the excluded state. The deed in this sense would be a choice of how loving the attention towards system of external world is.

2. A map of different choices of k-dimensional sub-spaces to k-fermion states is suggestive. The realization of logic in terms of emotions of different degrees of positivity would be mapped to many-fermion states - perhaps zero energy states with vanishing total fermion number. State function reductions to k-dimensional spaces would be mapped to k-fermion states: quantum jumps to quantum states!

   The problem brings in mind quantum classical correspondence in quantum measurement theory. The direction of the pointer of the measurement apparatus (in very metaphorical sense) corresponds to the outcome of state function reduction, which is now 1-D subspace. For ordinary measurement the pointer has k positions. Now it must have $2^k - 1$ positions. To the discrete space of k pointer positions one must assign fermionic Clifford algebra of second quantized fermionic oscillator operators. The hierarchy of Planck constants and dark matter suggests the realization. Replace the pointer with its space-time k-sheeted covering and consider zero energy energy states made of pairs of k-fermion states at the sheets of the n-sheeted covering? Dark matter would be therefore necessary for cognition. The role of fermions would be to "mark" the k space-time sheets in the covering.

The cautious conclusion is that NMP as a separate principle is not necessary and follows in statistical sense from the unavoidable increase of $n = h_{eff}/h$ identified as dimension of extension of rationals define the adeles if this extension or at least the dimension of its Galois group is observable.

8.5.3 p-Adic physics as correlate of cognition and imagination

The items in the following list give motivations for the proposal that p-adic physics could serve as a correlate for cognition and imagination.

1. By the total disconnectedness of the p-adic topology, p-adic world decomposes naturally into blobs, objects. This happens also in sensory perception. The pinary digits of p-adic number

can be assigned to a p-tree. Parisi proposed in the model of spin glass [B29] that p-adic numbers could relate to the mathematical description of cognition and also Khrennikov [J4] has developed this idea. In TGD framework that idea is taken to space-time level: p-adic space-time sheets represent thought bubbles and they correlate with the real ones since they form cognitive reprentations of the real world. SH allows a concrete realization of this.

2. p-Adic non-determinism due to p-adic pseudo constants suggests interpretation in terms of imagination. Given 2-surfaces could allow completion to p-adic preferred extremal but not to a real one so that pure "non-realizable" imagination is in question.
3. Number theoretic negentropy has interpretation as negentropy characterizing information content of entanglement. The superposition of state pairs could be interpreted as a quantum representation for a rule or abstracted association containing its instances as state pairs. Number theoretical negentropy characterizes the relationship of two systems and should not be confused with thermodynamical entropy, which characterizes the uncertainty about the state of single system.

The original vision was that p-adic non-determinism could serve as a correlate for cognition, imagination, and intention. The recent view is much more cautious. Imagination need not completely reduce to p-adic non-determinism since it has also real physics correlates - maybe as partial realizations of SH as in nerve pulse pattern, which does not propagate down to muscles.

A possible interpretation for the solutions of the p-adic field equations would be as geometric correlates of cognition, imagination, and perhaps even intentionality. Plans, intentions, expectations, dreams, and possibly also cognition as imagination in general could have p-adic cognitive space-time sheets as their geometric correlates. A deep principle seems to be involved: incompleteness is the characteristic feature of p-adic physics but the flexibility made possible by this incompleteness is absolutely essential for imagination and cognitive consciousness in general.

The most feasible view is that the intersections of p-adic and real space-time surfaces define cognitive representations of real space-time surfaces (PEs, [K18, K10, K14]). One could also say that real space-time surface represents sensory aspects of conscious experience and p-adic space-time surfaces its cognitive aspects. Both real and p-adics rather than real or p-adics.

The identification of p-adic pseudo constants as correlates of imagination at space-time level is indeed a further natural idea.

1. The construction of PEs by SH from the data at 2-surfaces is like boundary value problem with number theoretic discretization of space-time surface as additional data. PE property in real context implies strong correlations between string world sheets and partonic 2-surfaces by boundary conditions a them. One cannot choose these 2-surfaces completely independently in real context.
2. In p-adic sectors the integration constants are replaced with pseudo-constants depending on finite number of pinary digits of variables depending on coordinates normal to string world sheets and partonic 2-surfaces. The fixing of the discretization of space-time surface would allow to fix the p-adic pseudo-constants. Once the number theoretic discretization of space-time surface is fixed, the p-adic pseudo-constants can be fixed. Pseudo-constant could allow a large number of p-adic configurations involving string world sheets, partonic 2-surfaces, and number theoretic discretization but not allowed in real context.

   Could these p-adic PEs correspond to imaginations, which in general are not realizable? Could the realizable intentional actions belong to the intersection of real and p-adic WCWs? Could one identify non-realistic imaginations as the modes of WCW spinor fields for which 2-surfaces are not extendable to real space-time surfaces and are localized to 2-surfaces? Could they allow only a partial continuation to real space-time surface. Could nerve pulse pattern representing imagined motor action and not proceeding to the level of muscles correspond to a partially real PE?

   Could imagination and problem solving be search for those collections of string world sheets and partonic 2-surfaces, which allow extension to (realization as) real PEs? If so, p-adic physics would be there as an independent aspect of existence and this is just the original idea. Imagination could be realized in state function reduction, which always selects only those 2-surfaces, which allow continuation to real space-time surfaces. The distinction between only imaginable and also realizable would be the extendability by using strong form of holography.

3. An interesting question is why elementary particles are characterized by preferred p-adic primes (primes near powers of 2, in particular Mersenne primes). Could the number of realizable imaginations for these primes be especially large?

I have the feeling that this view allows respectable mathematical realization of imagination in terms of adelic quantum physics. It is remarkable that SH derivable from - you can guess, SGCI (the Big E again!), plays an absolutely central role in it.

8.6 Appendix: Super-symplectic conformal weights and zeros of Riemann zeta

Since fermions are the only fundamental particles in TGD one could argue that the conformal weight of for the generating elements of supersympelectic algebra could be negatives for the poles of fermionic zeta ζ_F. This demands $n > 0$ as does also the fractal hierarchy of supersymplectic symmetry breakings. NTU of Riemann zeta in some sense is strongly suggested if adelic physics is to make sense.

For ordinary conformal algebras there are only finite number of generating elements ($-2 \leq n \leq 2$). If the radial conformal weights for the generators of g consist of poles of ζ_F, the situation changes. ζ_F is suggested by the observation that fermions are the only fundamental particles in TGD.

1. Riemann Zeta $\zeta(s) = \prod_p(1/(1-p^{-s})$ identifiable formally as a partition function $\zeta_B(s)$ of arithmetic boson gas with bosons with energy $log(p)$ and temperature $1/s = 1/(1/2+iy)$ should be replaced with that of arithmetic fermionic gas given in the product representation by $\zeta_F(s) = \prod_p(1+p^{-s})$ so that the identity $\zeta_B(s))/\zeta_F(s) = \zeta_B(2s)$ follows. This gives

$$\frac{\zeta_B(s)}{\zeta_B(2s)} \quad .$$

   $\zeta_F(s)$ has zeros at zeros s_n of $\zeta(s)$ and at the pole $s = 1/2$ of $zeta(2s)$. $\zeta_F(s)$ has poles at zeros $s_n/2$ of $\zeta(2s)$ and at pole $s = 1$ of $\zeta(s)$.

   The spectrum of $1/T$ would be for the generators of algebra $\{(-1/2+iy)/2, n > 0, -1\}$. In p-adic thermodynamics the p-adic temperature is $1/T = 1/n$ and corresponds to "trivial" poles of ζ_F. Complex values of temperature does not make sense in ordinary thermodynamics. In ZEO quantum theory can be regarded as a square root of thermodynamics and complex temperature parameter makes sense.

2. If the spectrum of conformal weights of the generating elements of the algebra corresponds to poles serving as analogs of propagator poles, it consists of the "trivial" conformal $h = n > 0$- the standard spectrum with $h = 0$ assignable to massless particles excluded - and "non-trivial" $h = -1/4 + iy/2$. There is also a pole at $h = -1$.

   Both the non-trivial pole with real part $h_R = -1/4$ and the pole $h = -1$ correspond to tachyons. I have earlier proposed conformal confinement meaning that the total conformal weight for the state is real. If so, one obtains for a conformally confined two-particle states corresponding to conjugate non-trivial zeros in minimal situation $h_R = -1/2$ assignable to N-S representation.

   In p-adic mass calculations ground state conformal weight must be $-5/2$ [K66]. The negative fermion ground state weight could explain why the ground state conformal weight must be tachyonic $-5/2$. With the required 5 tensor factors one would indeed obtain this with minimal conformal confinement. In fact, arbitrarily large tachyonic conformal weight is possible but physical state should always have conformal weights $h > 0$.

3. $h = 0$ is not possible for generators, which reminds of Higgs mechanism for which the naïve ground states corresponds to tachyonic Higgs. $h = 0$ conformally confined massless states are necessarily composites obtained by applying the generators of Kac-Moody algebra or super-symplectic algebra to the ground state. This is the case according to p-adic mass calculations [K66], and would suggest that the negative ground state conformal weight can be associated with super-symplectic algebra and the remaining contribution comes from ordinary

super-conformal generators. Hadronic masses, whose origin is poorly understood, could come from super-symplectic degrees of freedom. There is no need for p-adic thermodynamics in super-symplectic degrees of freedom.

8.6.1 A general formula for the zeros of zeta from NTU

Dyson's comment about Fourier transform of Riemann Zeta [A62] (`http://tinyurl.com/hjbfsuv`) is interesting from the point of NTU for Riemann zeta.

1. The numerical calculation of Fourier transform for the imaginary parts iy of zeros $s = 1/2+iy$ of zeta shows that it is concentrated at discrete set of frequencies coming as $log(p^n)$, p prime. This translates to the statement that the zeros of zeta form a 1-dimensional quasicrystal, a discrete structure Fourier spectrum by definition is also discrete (this of course holds for ordinary crystals as a special case). Also the logarithms of powers of primes would form a quasicrystal, which is very interesting from the point of view of p-adic length scale hypothesis. Primes label the "energies" of elementary fermions and bosons in arithmetic number theory, whose repeated second quantization gives rise to the hierarchy of infinite primes [K105]. The energies for general states are logarithms of integers.
2. Powers p^n label the points of quasicrystal defined by points $log(p^n)$ and Riemann zeta has interpretation as partition function for boson case with this spectrum. Could p^n label also the points of the dual lattice defined by iy.
3. The existence of Fourier transform for points $log(p_i^n)$ for any vector y_a in class $C(p)$ of zeros labelled by p requires $p_i^{iy_a}$ to be a root of unity inside $C(p)$. This could define the sense in which zeros of zeta are universal. This condition also guarantees that the factor $n^{-1/2-iy}$ appearing in zeta at critical line are number theoretically universal ($p^{1/2}$ is problematic for Q_p: the problem might be solved by eliminating from p-adic analog of zeta the factor $1-p^{-s}$.
   (a) One obtains for the pair (p_i, s_a) the condition $log(p_i)y_a = q_{ia}2\pi$, where q_{ia} is a rational number. Dividing the conditions for (i, a) and (j, a) gives
   $$p_i = p_j^{q_{ia}/q_{ja}}$$
   for every zero s_a so that the ratios q_{ia}/q_{ja} do not depend on s_a. From this one easily deduce $p_i^M = p_j^N$, where M and N are integers so that one ends up with a contradiction.
   (b) Dividing the conditions for (i, a) and (i, b) one obtains
   $$\frac{y_a}{y_b} = \frac{q_{ia}}{q_{ib}}$$
   so that the ratios q_{ia}/q_{ib} do not depend on p_i. The ratios of the imaginary parts of zeta would be therefore rational number which is very strong prediction and zeros could be mapped by scaling y_a/y_1 where y_1 is the zero which smallest imaginary part to rationals.
   (c) The impossible consistency conditions for (i, a) and (j, a) can be avoided if each prime and its powers correspond to its own subset of zeros and these subsets of zeros are disjoint: one would have infinite union of sub-quasicrystals labelled by primes and each p-adic number field would correspond to its own subset of zeros: this might be seen as an abstract analog for the decomposition of rational to powers of primes. This decomposition would be natural if for ordinary complex numbers the contribution in the complement of this set to the Fourier trasform vanishes. The conditions $(i, a)$and (i, b) require now that the ratios of zeros are rationals only in the subset associated with p_i.

For the general option the Fourier transform can be delta function for $x = log(p^k)$ and the set $\{y_a(p)\}$ contains N_p zeros. The following argument inspires the conjecture that for each p there is an infinite number N_p of zeros $y_a(p)$ in class $C(p)$ satisfying

$$p^{iy_a(p)} = u(p) = e^{\frac{r(p)}{m(p)}i2\pi} \ ,$$

where $u(p)$ is a root of unity that is $y_a(p) = 2\pi(m(a) + r(p))/log(p)$ and forming a subset of a lattice with a lattice constant $y_0 = 2\pi/log(p)$, which itself need not be a zero.

In terms of stationary phase approximation the zeros $y_a(p)$ associated with p would have constant stationary phase whereas for $y_a(p_i \neq p))$ the phase $p^{iy_a(p_i)}$ would fail to be stationary. The phase e^{ixy} would be non-stationary also for $x \neq log(p^k)$ as function of y.

1. Assume that for $x = qlog(p)$, where q not a rational, the phases e^{ixy} fail to be roots of unity and are random implying the vanishing/smallness of $F(x)$.
2. Assume that for a given p all powers p^{iy} for $y \notin \{y_a(p)\}$ fail to be roots of unity and are also random so that the contribution of the set $y \notin \{y_a(p)\}$ to $F(p)$ vanishes/is small.
3. For $x = log(p^{k/m})$ the Fourier transform should vanish or be small for $m \neq 1$ (rational roots of primes) and give a non-vanishing contribution for $m = 1$. One has

$$\begin{aligned} F(x = log(p^{k/m}) &= \sum_{1\leq a\leq N(p)} e^{k\frac{M(a,p)}{mN(p)}i2\pi} u(p) \ , \\ u(p) &= e^{\frac{r(p)}{m(p)}i2\pi} \ . \end{aligned}$$

   Obviously one can always choose $N(a,p) = N(p)$.
4. For the simplest option $N(p) = 1$ one would obtain delta function distribution for $x = log(p^k)$. The sum of the phases associated with $y_a(p)$ and $-y_a(p)$ from the half axes of the critical line would give

$$F(x = log(p^n)) \propto X(p^n) \equiv 2cos(n\frac{r(p)}{m(p)}2\pi) \ .$$

   The sign of F would vary.
5. For $x = log(p^{k/m})$ the value of Fourier transform is expected to be small by interference effects if $M(a,p)$ is random integer, and negligible as compared with the value at $x = log(p^k)$. This option is highly attractive. For $N(p) > 1$ and $M(a,p)$ a random integer also $F(x = log(p^k)$ is small by interference effects. Hence it seems that this option is the most natural one.
6. The rational $r(p)/m(p)$ would characterize given prime (one can require that $r(p)$ and $m(p)$ have no common divisors). $F(x)$ is non-vanishing for all powers $x = log(p^n)$ for $m(p)$ odd. For $p = 2$, also $m(2) = 2$ allows to have $|X(2^n)| = 2$. An interesting ad hoc ansatz is $m(p) = p$ or $p^{s(p)}$. One has periodicity in n with period $m(p)$ that is logarithmic wave. This periodicity serves as a test and in principle allows to deduce the value of $r(p)/m(p)$ from the Fourier transform.

What could one conclude from the data (`http://tinyurl.com/hjbfsuv`)?

1. The first graph gives $|F(x = log(p^k)|$ and second graph displays a zoomed up part of $|F(x = log(p^k)|$ for small powers of primes in the range $[2,19]$. For the first graph the eighth peak ($p = 11$) is the largest one but in the zoomed graphs this is not the case. Hence something is wrong or the graphs correspond to different approximations suggesting that one should not take them too seriously.

   In any case, the modulus is not constant as function of p^k. For small values of p^k the envelope of the curve decreases and seems to approach constant for large values of p^k (one has $x < 15$ ($e^{15} \simeq 3.3 \times 10^6$).
2. According to the first graph $|F(x)|$ decreases for $x = klog(p) < 8$, is largest for small primes, and remains below a fixed maximum for $8 < x < 15$. According to the second graph the amplitude decreases for powers of a given prime (say $p = 2$). Clearly, the small primes and their powers have much larger $|F(x)|$ than large primes.

There are many possible reasons for this behavior. Most plausible reason is that the sums involved converge slowly and the approximation used is not good. The inclusion of only 10^4 zeros would show the positions of peaks but would not allow reliable estimate for their intensities.

1. The distribution of zeros could be such that for small primes and their powers the number of zeros is large in the set of 10^4 zeros considered. This would be the case if the distribution of zeros $y_a(p)$ is fractal and gets "thinner" with p so that the number of contributing zeros scales down with p as a power of p, say $1/p$, as suggested by the envelope in the first figure.

2. The infinite sum, which should vanish, converges only very slowly to zero. Consider the contribution $\Delta F(p^k,p_1)$ of zeros not belonging to the class $p_1 \neq p$ to $F(x = log(p^k)) = \sum_{p_i} \Delta F(p^k,p_i)$, which includes also $p_i = p$. $\Delta F(p^k,p_i)$, $p \neq p_1$ should vanish in exact calculation.

   (a) By the proposed hypothesis this contribution reads as

   $$\Delta F(p,p_1) = \sum_a cos\left[X(p^k,p_1)(M(a,p_1) + \frac{r(p_1)}{m(p_1)})2\pi)\right] \ .$$
   $$X(p^k,p_1) = \frac{log(p^k)}{log(p_1)} \ .$$

   Here a labels the zeros associated with p_1. If p^k is "approximately divisible" by p^1 in other words, $p^k \simeq np_1$, the sum over finite number of terms gives a large contribution since interference effects are small, and a large number of terms are needed to give a nearly vanishing contribution suggested by the non-stationarity of the phase. This happens in several situations.

   (b) The number $\pi(x)$ of primes smaller than x goes asymptotically like $\pi(x) \simeq x/log(x)$ and prime density approximately like $1/log(x) - 1/log(x)^2$ so that the problem is worst for the small primes. The problematic situation is encountered most often for powers p^k of small primes p near larger prime and primes p (also large) near a power of small prime (the envelope of $|F(x)|$ seems to become constant above $x \sim 10^3$).

   (c) The worst situation is encountered for $p = 2$ and $p_1 = 2^k - 1$ - a Mersenne prime and $p_1 = 2^{2^k} + 1$, $k \leq 4$ - Fermat prime. For $(p,p_1) = (2^k,M_k)$ one encounters $X(2^k,M_k) = (log(2^k)/log(2^k - 1)$ factor very near to unity for large Mersennes primes. For $(p,p_1) = (M_k,2)$ one encounters $X(M_k,2) = (log(2^k - 1)/log(2) \simeq k$. Examples of Mersennes and Fermats are $(3,2),(5,2),(7,2),(17,2),(31,2),(127,2),(257,2),...$ Powers 2^k, $k = 2,3,4,5,7,8,..$ are also problematic.

   (d) Also twin primes are problematic since in this case one has factor $X(p = p_1 + 2,p_1) = \frac{log(p_1+2)}{log(p_1)}$. The region of small primes contains many twin prime pairs: (3,5), (5,7), (11,13), (17,19), (29,31),....

   These observations suggest that the problems might be understood as resulting from including too small number of zeros.

3. The predicted periodicity of the distribution with respect to the exponent k of p^k is not consistent with the graph for small values of prime unless the periodic $m(p)$ for small primes is large enough. The above mentioned effects can quite well mask the periodicity. If the first graph is taken at face value for small primes, $r(p)/m(p)$ is near zero, and $m(p)$ is so large that the periodicity does not become manifest for small primes. For $p = 2$ this would require $m(2) > 21$ since the largest power $2^n \simeq e^{15}$ corresponds to $n \sim 21$.

To summarize, the prediction is that for zeros of zeta should divide into disjoint classes $\{y_a(p)\}$ labelled by primes such that within the class labelled by p one has $p^{iy_a(p)} = e^{(r(p)/m(p))i2\pi}$ so that has $y_a(p) = [M(a,p) + r(p)/m(p))]2\pi/log(p)$.

8.6.2 More precise view about zeros of Zeta

There is a very interesting blog post by Mumford (`http://tinyurl.com/zemw27o`), which leads to much more precise formulation of the idea and improved view about the Fourier transform hypothesis: the Fourier transform or its generalization must be defined for all zeros, not only the non-trivial ones and trivial zeros give a background term allowing to understand better the properties of the Fourier transform.

Mumford essentially begins from Riemann's "explicit formula" in von Mangoldt's form.

$$\sum_p \sum_{n\geq 1} log(p)\delta_{p^n}(x) = 1 - \sum_k x^{s_k - 1} - \frac{1}{x(x^2-1)} \ ,$$

where p denotes prime and s_k a non-trivial zero of zeta. The left hand side represents the distribution associated with powers of primes. The right hand side contains sum over cosines

$$\sum_k x^{s_k-1} = 2\frac{\sum_k cos(log(x)y_k)}{x^{1/2}} ,$$

where y_k ithe imaginary part of non-trivial zero. Apart from the factor $x^{-1/2}$ this is just the Fourier transform over the distribution of zeros.

There is also a slowly varying term $1-\frac{1}{x(x^2-1)}$, which has interpretation as the analog of the Fourier transform term but sum over trivial zeros of zeta at $s=-2n$, $n>0$. The entire expression is analogous to a "Fourier transform" over the distribution of all zeros. Quasicrystal is replaced with union on 1-D quasicrystals.

Therefore the distribution for powers of primes is expressible as "Fourier transform" over the distribution of both trivial and non-trivial zeros rather than only non-trivial zeros as suggested by numerical data to which Dyson [A62] referred to (`http://tinyurl.com/hjbfsuv`). Trivial zeros give a slowly varying background term large for small values of argument x (poles at $x=0$ and $x=1$ - note that also $p=0$ and $p=1$ appear effectively as primes) so that the peaks of the distribution are higher for small primes.

The question was how can one obtain this kind of delta function distribution concentrated on powers of primes from a sum over terms $cos(log(x)y_k)$ appearing in the Fourier transform of the distribution of zeros.

Consider $x=p^n$. One must get a constructive interference. Stationary phase approximation is in terms of which physicist thinks. The argument was that a destructive interference occurs for given $x=p^n$ for those zeros for which the cosine does not correspond to a real part of root of unity as one sums over such y_k: random phase approximation gives more or less zero. To get something nontrivial y_k must be proportional to $2\pi \times n(y_k)/log(p)$ in class $C(p)$ to which y_k belongs. If the number of these y_k:s in $C(p)$ is infinite, one obtains delta function in good approximation by destructive interference for other values of argument x.

The guess that the number of zeros in $C(p)$ is infinite is encouraged by the behaviors of the densities of primes one hand and zeros of zeta on the other hand. The number of primes smaller than real number x goes like

$$\pi(x) = N(primes < x) \sim \frac{x}{log(x)}$$

in the sense of distribution. The number of zeros along critical line goes like

$$N(zeros < t) = (t/2\pi) \times log(\frac{t}{2\pi})$$

in the same sense. If the real axis and critical line have same metric measure then one can say that the number of zeros in interval T per number of primes in interval T behaves roughly like

$$\frac{N(zeros < T)}{N(primes < T)} = log(\frac{T}{2\pi}) \times \frac{log(T)}{2\pi}$$

so that at the limit of $T \rightarrow \infty$ the number of zeros associated with given prime is infinite. This asumption of course makes the argument a poor man's argument only.

8.6.3 Possible relevance for TGD

What this speculative picture from the point of view of TGD?

1. A possible formulation for NTU for the poles of fermionic Riemann zeta $\zeta_F = \zeta(s)/\zeta(2s)$ could be as a condition that is that the exponents $p^{ks_a(p)/2} = p^{k/4}p^{iky_a(p)/2}$ exist in a number theoretically universal manner for the zeros $s_a(p)$ for given p-adic prime p and for some subset of integers k. If the proposed conditions hold true, exponent reduces $p^{k/4}e^{k(r(p/m(p)i2\pi}$ requiring that k is a multiple of 4. The number of the non-trivial generating elements of super-symplectic algebra in the monomial creating physical state would be a multiple of 4. These monomials would have real part of conformal weight -1. Conformal confinement suggests that these monomials are products of pairs of generators for which imaginary parts cancel.

2. Quasi-crystal property might have an application to TGD. The functions of light-like radial coordinate appearing in the generators of supersymplectic algebra could be of form r^s, s zero of zeta or rather, its imaginary part. The eigenstate property with respect to the radial scaling rd/dr is natural by radial conformal invariance.

   The idea that arithmetic QFT assignable to infinite primes is behind the scenes in turn suggests light-like momenta assignable to the radial coordinate have energies with the dual spectrum $log(p^n)$. This is also suggested by the interpretation of ζ as square root of thermodynamical partition function for boson gas with momentum $log(p)$ and analogous interpretation of ζ_F.

   The two spectra would be associated with radial scalings and with light-like translations of light-cone boundary respecting the direction and light-likeness of the light-like radial vector. $log(p^n)$ spectrum would be associated with light-like momenta whereas p-adic mass scales would characterize states with thermal mass. Note that generalization of p-adic length scale hypothesis raises the scales defined by p^n to a special physical position: this might relate to ideal structure of adeles.

3. Finite measurement resolution suggests that the approximations of Fourier transforms over the distribution of zeros taking into account only a finite number of zeros might have a physical meaning. This might provide additional understand about the origins of generalized p-adic length scale hypothesis stating that primes $p \simeq p_1^k$, p_1 small prime - say Mersenne primes - have a special physical role.

Chapter 9

The Recent View about SUSY in TGD Universe

What SUSY is in TGD framework is a longstanding question, which found a rather convincing answer rather recently. In twistor Grassmannian approach to $\mathcal{N} = 4$ SYM [B21, B15, B16, B18, B45, B22, B5] twistors are replaced with supertwistors and the extreme elegance of the description of various helicity states using twistor space wave functions suggests that super-twistors are realized both at the level of M^8 geometry and momentum space.

In TGD framework $M^8 - H$ duality allows to geometrize the notion of super-twistor in the sense that at the level of M^8 different components of super-field correspond to components of super-octonion each of which corresponds to a space-time surfaces satisfying minimal surface equations with string world sheets as singularities - this is geometric counterpart for masslessness.

9.0.1 New view about SUSY

The progress in understanding of $M^8 - H$ duality [L85] throws also light to the problem whether SUSY is realized in TGD [L92] and what SUSY breaking cold mean. It is now rather clear that sparticles are predicted and SUSY remains exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them. Super-octonion components of polynomials have different orders so that also the extension of rational assignable to them is different and therefore also the ramified primes so that p-adic prime as one them can be different for the members of SUSY multiplet and mass splitting is obtained.

The question how to realize super-field formalism at the level of $H = M^4 \times CP_2$ led to a dramatic progress in the identification of elementary particles and SUSY dynamics. The most surprising outcome was the possibility to interpret leptons and corresponding neutrinos as local 3-quark composites with quantum numbers of anti-proton and anti-neutron. Leptons belong to the same super-multiplet as quarks and are antiparticles of neutron and proton as far quantum numbers are considered. One implication is the understanding of matter-antimatter asymmetry. Also bosons can be interpreted as local composites of quark and anti-quark.

Hadrons and perhaps also hadronic gluons would still correspond to the analog of monopole phase in QFTs. Homology charge could appear as a space-time correlate for color at space-time level and explain color confinement. Also color octet variants of weak bosons, Higgs, and Higgs like particle and the predicted new pseudo-scalar are predicted. They could explain the successes of conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC).

One ends up with an improved understanding of quantum criticality and the relation between its descriptions at M^8 level and H-level. Polynomials describing a hierarchy of dark matters describe also a hierarchy of criticalities and one can identify inclusion hierarchies as sub-hierarchies formed by functional composition of polynomials: the criticality is criticality for the polynomials interpreted as p-adic polynomials in $O(p) = 0$ approximation meaning the presence of multiple roots in this approximation.

9.0.2 Connection of SUSY and second quantization

The linear combinations monomials of theta parameters appearing in super-fields are replaced in case of hermitian H super coordinates consisting of combinations of monomials with vanishing quark number. For super-spinors of H the monomials carry odd quark number with quark number 1. Monomials of theta parameters are replaced by local monomials of quark oscillator operators labelled besides spin and weak isospin also by points of cognitive representation with embedding space coordinates in an extension of rationals defining the adele. Discretization allows anti-commutators which are Kronecker deltas rather than delta functions. If continuum limit makes sense, normal ordering must be assumed to avoid delta functions at zero coming from the contractions. The monomials (not only the coefficients appearing in them) are solved from generalized classical field equations and are linearly related to the monomials at boundary of CD playing the role of quantum fields and classical field equations determine the analogs of propagators.

The Wick contractions of quark-antiquark monomials appearing in the expansion of super-coordinate of H could define the analog of radiative corrections in discrete approach. $M^8 - H$ duality and number theoretic vision require that the number of non-vanishing Wick contractions is finite. The number of contractions is bounded by the finite number of points in cognitive representation and increases with the degree of the octonionic polynomial and gives rise to a discrete coupling constant evolution parameterized by the extensions of rationals. The polynomial composition hierarchies correspond to inclusion hierarchies for isomorphic sub-algebras of super-symplectic algebra having interpretation in terms of inclusions of hyper-finite factors of type II_1.

Quark oscillator operators in cognitive representation correspond to quark field q. Only terms with quark number 1 appear in q and leptons emerge in Kähler action as local 3-quark composites. Internal consistency requires that q must be the super-spinor field satisfying super Dirac equation. This leads to a self-referential condition $q_s = q$ identifying q and its super-counterpart q_s. Also super-coordinate h_s must satisfy analogous condition $(h_s)_s = h_s$, where $h_s \rightarrow (h_s)_s$ means replacement of h in the argument of h_s with h_s.

The conditions have an interpretation in terms of a fixed point of iteration and expression of quantum criticality. The coefficients of various terms in q_s and h_s are analogous to coupling constants can be fixed from this condition so that one obtains discrete number theoretical coupling constant evolution. The basic equations are quantum criticality condition $h_s = (h_s)_s$, $q = q_s$, $D_{\alpha,s}\Gamma_s^\alpha = 0$ coming from Kähler action, and the super-Dirac equation $D_s q = 0$.

9.0.3 Proposal for S-matrix

One also ends up to the first completely concrete proposal for how to construct S-matrix directly from the solutions of super-Dirac equations and super-field equations for space-time super-surfaces.

1. The idea inspired by WKB approximation is that the exponent of the super variant of Kähler function including also super-variant of Dirac action defines S-matrix elements as its matrix elements between the positive and negative energy parts of the zero energy states formed from the corresponding vacua at the two boundaries of CD annihilated by annihilation operators and *resp.* creation operators. The states would be created by the monomials appearing in the super-coordinates and super-spinor.

2. Super-Dirac equation implies that super-Dirac action vanishes on-mass-shell. The proposed construction however allows to get also scattering amplitudes between all possible states using the exponential of super-Kähler action. Super-Dirac equation however makes possible to express derivatives of the quark oscillator operators (values of quark field at points of cognitive representation) so that one can use only the points of cognitive representation without introducing lattice discretization. Discrete coupling constant evolution follows from the fact that the contractions of oscillator operators occur at the boundary of CD and their number is limited by the finite number of points of cognitive representation.

3. S-matrix is trivial unless CD contains the images of 6-D analogs of branes as universal special solutions of the algebraic equations determining space-time surfaces at the level of M^8. 4-D

space-time surfaces representing particle orbits meet at the partonic 2-surfaces associated with the 3-D surfaces at $t = r_n$ hyper-surfaces of M^4. The values of $t = r_n$ correspond to the roots of the real polynomial with rational coefficients determining the space-time surface. These transitions are analogs of weak measurements, and in TGD theory of consciousness they give rise to the experience flow of time and can be said to represent "very special moments" in the life of self [L83].

4. The creation and annihilation operators at vertices associated with the monomials would be connected to the points assignable to cognitive representations at opposite boundaries of CD and also to partonic 2-surfaces in the interior of CD possibly accompanied by sub-CDs. This would give analogs of twistor Grassmannian diagrams containing finite number of partonic 2-surfaces as topological vertices containing in turn finite number ordinary vertices defined by the monomials. The diagrams would be completely classical objects in accordance with the fact that quantum TGD is completely classical theory apart from state function reduction.

5. This view allows also a formulation of continuum theory since the monomials appearing in the action density in the interior of CD are linear superposition of the monomials at the points of boundary of CD involving 3-D integral so that contractions of oscillator operators only reduce one integration without introducing divergence. One can also normal order the monomials at boundary of CD serving as initial values. If preferred extremals are analogs of Bohr orbits, one can express extremals using either boundary as the seat of initial data.

9.1 How to formulate SUSY at the level of $H = M^4 \times CP_2$?

In the following I will represent the recent trial for constructing SUSY at the level of $H = M^4 \times CP_2$. The first trial replaced theta parameters of SUSY with quark oscillator operators labelled by spin and isospin and had rather obvious shortcomings: in particular, one did not obtain many-quark states with large quark numbers. The second trial allows quark oscillator operators to have as labels also the points of space-time surface in cognitive representation and thus having coordinates of H belonging to an extension of rationals defining the adele [?]

9.1.1 First trial

If SUSY is realized at the level of M^8, it should have a formulation also at the level of H. The basic elements of the first trial form part of also second trial. The basic modification made in the second trial is that finite number of theta parameters replaced with the fermionic oscillator operators labelled by the points of cognitive representations so that they are analogous to fermion fields in lattice, and only local composites of the oscillator operators appear in the super coordinates and super-spinors. This means that SUSY is essential element of the second quantization of fermions in TGD.

1. $M^8 - H$ duality is non-local and means that the dynamics at the level of H is not strictly local but dictated by partial differential equations for super-fields having interpretation as describing purely local many-fermion states made of fundamental fermions with quantum numbers of leptons and quarks (quarks do not possess color as spin like quantum number) ad their antiparticles.

2. Classical field equations and modified Dirac equation must result from this picture. Induction procedure for the spinors of H must generalize so that spinors are replaced by super-spinors Ψ_s having multi-spinors as components multiplying monomials of theta parameters θ. The determinant of metric and modified gamma matrices depend on embedding space coordinates h replaced with super coordinates h_s so that monomials of θ appear in two different ways. Hermiticity requires that sums of monomial and its hermitian conjugate appear in h_s. Monomials must also have vanishing fermion numbers. Otherwise one can obtain fermionic states propagating like bosons. For Dirac action one must assume that Ψ_s involves only odd monomials of θ with quark number 1 involving monomials appearing in h_s to get only states with quark number 1 and correct kind of propagators.

3. One Taylor expands both bosonic action density (6-D Kähler action dimensionally reducing to 4-D Kähler action plus volume term) and Super-Dirac action with respect to the super-coordinates h_s. In Super-Dirac action one has also the expansion of super-spinor in odd monomials with total quark number 1. The coefficients of the monomials of θ:s are obtained are partial derivatives of the action. Since the number of θ parameters is finite and corresponds to the number of spin-weak-isopin states of quarks and leptons, the number of terms is finite if the θ parameters anti-commute to zero. If not, one can get an infinite number of terms from the Taylor series for the action to the coefficient given monomial. Number theoretical considerations do not favor this and there should exist a cancellation mechanism for the radiative corrections coming from fermionic Wick contractions if thetas correspond to fermionic oscillator operators as it seems to be.

4. One can interpret the superspace as the exterior algebra of the spinors of H. This reminds of the result that the sections of the exterior algebra of Riemann manifold codes for the Riemann geometry (see `http://tinyurl.com/yxrcr8xv`). This generalizes the observation that one can hear the shape of a drum since the sound spectrum is determined by its frequency spectrum defined by Laplacian.

   Super-fields define a Clifford algebra generated by θ parameters as a kind of square root of exterior algebra which corresponds to the Clifford algebra of gamma matrices. Maybe this algebra could code also for the spinor structure of embedding space or even that of space-time surface so that the super-fields could be seen as carriers of geometric information about space-time surface as a preferred extremal. In 8-D case there is also $SO(1,8)$ triality suggesting that corresponding three Clifford algebras correspond to exterior algebra fermionic and anti-fermionic algebras.

What about the situation at the level of M^8?

1. At M^8 level the components of super-octonion correspond to various derivatives of the basic polynomial $P(t)$ so that space-time geometry correlates with the quantum numbers assignable to super-octonion components - this is in accordance with QCC (quantum-classical correspondence). This is highly desirable at the level of H too.

   Could the space-time surface in M^8 be same for super-field components with degree $d < d_{max}$ in some special cases? The polynomial associated with super octonion components are determined by the derivatives of the basic polynomial $P(t)$ with order determined by the degree of the super-monomial. If they have decomposition $P(t) = P_1^k(t)$, the monomials with degree $d < k$ the roots corresponding to the roots $P_1(t)$ co-incide. Besides this there are additional roots of $d^r P_1/dt^r$ for super-octonion component with r θ parameters.

   A possible interpretation could be as quantum criticality in which there is no SUSY breaking for components having $d < k$ (masses in p-adic thermodynamics could be the same since the extension defined by P_1 and corresponding ramified primes would be same). This would conform with the general vision about quantum criticality.

2. Usual super-field formalism involves Grassmann integration over θ parameters to give the action. M^8 formalism does not involve the θ integral at all. Should this be the case also at the level of H? This would guarantee that different components of H- coordinates as super-field would give rise to different space-time surface and QCC would be realized. θ integration produces SUSY invariants naturally involved with the definition of vertices involving components of super-fields. Also vertices involving fermionic and bosonic states emerge since bosonic super-field components appear in super-coordinates in super-Dirac action.

This approach does not say anything about second quantization. There is a strong temptation to replace the theta parameters with fermionic oscillator operators. One cannot however obtain second quantization of fermions in this manner since the maximal quark number (and lepton number if leptons are present as fundamental fermions) of the states is 4. To achieve second quantization, one must replace the theta parameters with fermionic oscillator operators labelled besides spin and weak isospin by the coordinates of points of 3-surface, most naturally the points belonging to a cognitive representation characterizing space-time surface for given extension of rationals.

9.1.2 Second trial

I have already earlier considered a proposal for how SUSY could be realized in TGD framework. As it often happens, the original proposal was not quite correct. The following discussion gives a formulation solving the problems of the first proposal and suggests a concrete formulas for the scattering amplitudes in ZEO based on super-counterparts of preferred extremals. In the sequel I will talk about super Kähler function as functional of 3-surfaces and - super Kähler function action. By holography allowing to identify 3-surfaces with corresponding space-time surfaces as analogs of Bohr orbits, these notions have the same meaning.

Could the exponent of super-Kähler function as vacuum functional define S-matrix as its matrix elements

Consider first the key ideas - some of them new - formulated as questions.

1. Could one see SUSY in TGD sense as a counterpart for the quantization in the sense of QFT so that oscillator operators replace theta parameters and would become fermionic oscillator operators labelled by spin and electroweak spin - as proposed originally - and by selected points of 3-surface of light-cone boundary with embedding space coordinates in extension of rationals? One would have analog of fermion field in lattice identified as a number theoretic cognitive representation for given extension of rationals. The new thing would be allowance of local composites of oscillator operators having interpretation in terms of analogs for the components of super-field.

   SUSY in TGD sense would be realized by allowing local composites of oscillator operators containing 4+4 quark oscillator operators at most. At continuum limit normal ordering would produce delta functions at origin unless one assumes normal ordering from beginning. For cognitive representations one would have only Kronecker deltas and one can consider the possibility that normal ordering is not present. The vanishing of normal ordering terms above some number of them suggested to be the dimension for the extension of rationals would give rise to a discrete coupling constant evolution due to the contractions of fermionic oscillator operators.

2. What is dynamical in the superpositions of oscillator operator monomials? Are the coefficients dynamical? Or are the oscillator operators themselves dynamical - this would mean a QFT type reduction to single particle level? The latter option seems to be correct. Oscillator operators are labelled by points of cognitive representation and in continuum case define an analog of quantum spinor field, call it q. This suggests that this field satisfies the super counter part of modified Dirac equation and must involve also super part formed from the monomials of q and $\overline{q}$. This however requires the replacement of q with q_s in super-Dirac operator and super-coordinates h_s and one ends up with an iteration $q \to q_s \to ...$

   The only solution to the paradoxical situation is that one has self-referential equation $q = q_s$ having interpretation in terms of quantum criticality fixing the coefficients of terms in $q = q_s$. Analogous condition $h_s = (h_s)_s$ must be satisfied by h_s under substitution $h_s \to (h_s)_s$. These conditions fix coefficients of terms in H super-coordinate h_s and q_s interpreted as coupling constants so that quantum criticality implying a discrete coupling constant evolution as function of extension of rationals follows. Also super-Dirac equation $D_s q_s = 0$ and field equations $D_{s,\alpha}\Gamma^{\alpha,s} = 0$ for Kähler action guaranteeing hermiticity are satisfied.

3. Could one interpret the time reversal operation taking creation- and annihilation operators to each other as time reflection permuting the points at the opposite boundaries of CD? The positive *resp.* negative energy parts of zero energy states would be created by creation *resp.* annihilation operators from respective vacuums assigned to the opposite boundaries of CD.

4. Could one regard preferred extremal regarded as 4-surface in super embedding space parameterized by the hermitian embedding coordinates plus the coefficients of the monomials of quarks and antiquarks with vanishing quark number, whose time evolution follows from dimensionally reduced 6-D super-Kähler action? Could one assume similar interpretation for super spinors consisting of monomials with total quark number equal to 1 and appearing in super-Dirac action?

5. In WKB approximation the exponent of action defines wave function. In QFTs path integral is defined by an exponent of action and scattering operator can be formally defined as action exponential. Could the matrix elements for the exponent of the super counterpart of Kähler function plus super Dirac action between states at opposite boundaries of CD between positive and negative energy parts of zero energy states define S-matrix? Could the positive and negative energy parts of zero energy states be identified as many particles states formed from the monomials associated with embedding space super-coordinates and super-spinors?

6. Could the construction of S-matrix elements as matrix elements of super-action exponential reduce to classical theory? Super-field monomials in the interior of CD would be linear superpositions of super-field monomials at boundary of CD. Note that oscillator operator monomials rather than their coefficients would be the basic entities and the dynamics would reduce to that for oscillator operators as in QFTs. The analogs of propagators would relate the monomials to those at boundary ly to the monomials at the boundary of CD, and would be determined by classical field equations so that in this sense everything would be classical. Note however that the fixed point condition $q = q_s$ and super counterpart of modified Dirac equation are non-linear.

   Vertices would be defined by monomials appearing in super-coordinate and super-spinor field appearing in terms of those at boundary of CD. If two vertices at interior points x and y of CD are connected there is line leading from x to a point z at boundary of CD and back to y and one would have sum over points z in cognitive representation. This applies also to self energy corrections with $x = y$. At the possibly existing continuum limit integral would smoothen the delta function singularities and in presence of normal ordering at continuum would eliminate them.

   In the expressions for the elements of S-matrix annihilation operators appearing in the monomials would be connected to the passive boundary P of CD and creation operators to the active boundary. If no partonic 2-surfaces appear as topological vertices in the interior of CD, this would give trivial S-matrix!

   $M^8 - H$ duality however predicts the existence of brane like entities as universal 6-D surfaces as solutions of equations determining space-time surfaces. Their M^4 projection is $t = r_n$ hyperplane, where r_n corresponds to a root of a real polynomial with algebraic coefficients giving rise to octonion polynomial, and is mapped to similar surface in H. 4-D space-time surfaces representing incoming and outgoing lines would meet along their ends at these partonic 2-surfaces.

   Partonic 2-surfaces at these hyper-surfaces would contain ordinary vertices as points in cognitive representation. Given vertex would have at most 4+4 incoming and outgoing lines assignable to the monomial defining the vertex. This picture resembles strongly the picture suggested by twistor Grassmannian approach. In particular the number of vertices is finite and their seems to be no superposition over different diagrams. In this proposal, the lines connecting vertices would correspond to 1-D singularities of the space-time surfaces as minimal surfaces in H. Also stringy singularities can be considered but also these should be discretized.

   By fixing the set of monomials possibly defining orthonormal state basis at both boundaries one would obtain given S-matrix element. S-matrix elements would be matrix elements of the super-action exponential between states formed by monomials of quark oscillator operators. Also entanglement between the monomials defining initial and finals states can be allowed. Note that this in principle allows also initial and final states not expressible using monomials but that monomials are natural building bricks as analogs of field operators in QFTs.

7. The monomials associated with embedding space coordinates are embedding space vectors constructible from Dirac currents (left- or right-handed) with oscillator operators replacing the induced spinor field and its conjugate. The proposed rules for constructing S-matrix would give also scattering amplitudes with odd quark number at boundaries of CD. Could the super counterpart of the bosonic action (super Kähler function) be all that is needed to construct the S-matrix?

   In fact, classically Dirac action vanishes on mass shell: if this is true also for super-Dirac action then the addition of Dirac action would not be needed. The super-Taylor expansion of super- Kähler action gives rise to the analogs of perturbation theoretic interaction terms so that one has perturbation theory without perturbation theory as Wheeler might state it. The detailed study of the structure of the monomials appearing in the super-Kähler action shows that they have interpretation as currents assignable to gauge bosons and scalar and pseudo-scalar Higgs.

Super Dirac action is however needed. Super-Dirac equation for q and $D_{\alpha,s}\Gamma^{\alpha}_s = 0$ allow to reduce ordinary divergences $\partial_\alpha j^\alpha$ of fermionic currents appearing in super-Kähler action to commutators $[A_{\alpha,s} j^\alpha]$. Therefore no information about q at nearby points is needed and one avoids lattice discretization: cognitive representation is enough.

8. Topological vertices represent discontinuities of the space-time surface bringing strongly in mind the non-determinism of quantum measurement, and one can ask whether the 3-branes and associated partonic 2-surfaces. Could the state function reductions analogous to weak measurements correspond to these discontinuities? Ordinary state function reductions would change the arrow of time and the roles of active and passive boundaries of CD [L77]. In TGD inspired theory of consciousness these time values would correspond to "very special moments" in the life of self [L83].

9. The unitarity of S-matrix can be understood from the structure of the exponent of Kähler action. The exponent decomposes to a sum of real and purely imaginary parts. The exponent of the hermitian imaginary part is a unitary operator for a given space-time surface. Real exponent containing also radiative corrections from the normal ordering gives exponent of Kähler function as vacuum functional in WCW (sum in the case of cognitive representations) and by choosing the normalization factor of the state appropriately one obtains unitary S-matrix.

9.1.3 More explicit picture

The following sketch tries to make the picture of the second trial more explicit.

1. The construction of S-matrix should reduce to super-geometry coded by super Kähler function determined by the 6-D Kähler action for twistor lift by dimensional reduction. This might be possible since zero energy states have vanishing total conserved charges and exponent of super-Kähler function has matrix elements only between states at opposite boundaries of CD having same total charges.

2. Construction should reduce to preferred extremals and their super-deformations determined by variational principle with boundary conditions. The boundary values of super-deformations at either boundary could be also interpreted as initial values for preferred extremals analogous to Bohr orbits. The expectations for the super action with fixed initial values between positive and negative energy parts would give the scattering amplitudes assignable to a given space-time surface. There would be functional integral over space-time surfaces using exponent of Kähler function as weight. In number theoretic vision this would reduce to sum over preferred extremals labelled by cognitive representations serving as WCW coordinates.

3. Number theoretic vision suggests a discretization in terms of cognitive representation consisting of points with coordinates in extension of rationals defining the adele. This representation could be associated with the boundaries of CD and possibly with M^4 time=constant hyper-planes assignable with the universal special solutions in M^8. At the partonic 2-surfaces associated with these hyper-planes 4-D extremals would meet along their ends: topological particle vertices would be in question. Is string world sheets and partonic 2-surfaces correspond to singularities, the boundaries of strings world sheets as intersections of the string world sheets and orbits of partonic 2-surfaces should represent fermion lines.

4. Creation operators would be assigned with the passive boundary of CD - call it P - and annihilation operators as their conjugates would act as creation operators at the opposite boundary, active boundary - call it A. Time reversal symmetry of CD suggests that annihilation operator as conjugate of creation operator labelled by the a point of boundary of CD corresponds to the same point in common coordinates for light-cone boundary. This would conform also with the basic character of the half-algebras associated with super-symplectic symmetries.

The original proposal was that oscillator operators have only spin and electroweak spin as indices but the standard view about spin and statistics requires that also the points of the 3-surface must label them. Also the fact that the total quark number can be larger than 4 of course requires this too. Algebraically the only difference with respect to this proposal is that one allows also the points of 3-surface at the boundary of CD as labels.

5. Number theoretical vision requires that only points of 3-surface having embedding space coordinates in the extension of rationals defining the adelic physics are allowed. In the generic case the number of points in the cognitive representation would be finite and would increase with the dimension of extension so that at the limit of algebraic numbers they form a dense set of 3-surface.

   Since action has infinite expansion in powers of super coordinates the contractions of oscillator operators would give rise to a renormalization of the coefficients of the monomials and of classical action. For cognitive representations one would avoid normal ordering problems sine the number of contractions is limited by the number of points in cognitive representation. This would give rise to discrete coupling constant evolution as function of the extension of rationals.

6. In continuum theory all points of 3-D boundary would label quark oscillator operators and one must normal order the oscillator operators in given local monomial. Also now the idea about connecting creation and annihilation operators to opposite boundaries of CD would allow to get rid of infinities due to contractions.

The action exponential would lead to a rather concrete proposal for the coefficients of the monomials appearing in super-fields.

1. The deformations of embedding space coordinates would be expressible as WCW-local superpositions of isometry generators or as WCW-global superpositions of Hamiltonian currents contracted with the coordinate deformations. The latter would conform with super-symplectic symmetries of WCW. CP_2 Hamiltonian currents would give color quantum numbers. S^2 Hamiltonian currents would be also present. One could see space-time local Kac-Moody symmetries assignable to light-like partonic orbits and string world sheets as a dual representations at space-time level of symplectic symmetries at embedding space level.

2. Spinor modes would be expressible as superpositions of embedding space spinor modes having expansion as super-Taylor series at the boundaries of CD. This would give spin and electroweak quantum numbers.

Does one really obtain description of gauge bosons and gravitons by using the exponent?

1. Could the coefficients of super-monomials at boundary of CD allow interpretation in terms of gauge bosons? These entities could have well-defined quantum numbers so that this might be possible. Quark spin and isospin would represent additional spin degrees of freedom. The Hamiltonians of H of CP_2 expressible for given 3-surface as local superpositions of SU(3) Killing vector fields would represent color degrees of freedom.

   For string world sheets one would naturally have transversal M^4 super-coordinates and CP_2 super-coordinates as analogs of fields. Could this allow to get gauge bosons as excitations of strings as in string theories.

2. Gauge bosons could be also bi-local composites of fermion and anti-fermion at opposite boundaries of wormhole contact or at opposite wormhole contacts of wormhole flux tube. Gravitons could be 4-local composites. Baryons and mesons could be this kind of non-local composites. One can consider also the analog of monopole phase of QFTs in which particles would be multilocal composites.

3. The bosonic action is for induced metric and induced Kähler form. QFT wisdom would suggest that their super-analogs could correspond to external particles. One could indeed take the induced gauge potentials or -fields at boundary and form their contractions with Killing vectors of isometries to obtain general coordinate invariant quantities and form their super-analogs as normal ordered local composites. One can consider the same idea for induced gravitational field or its deviation from Minkowski metric.

Formally this would correspond to an addition to the action exponential of perturbative terms of type jA appearing in QFTs representing coupling to external currents and take the limit $j \rightarrow 0$. In QFT picture this works since various gauge fields are functionally independent but in TGD framework this is not the case. Second problem is to to construct a complete orthonormalized set of states in this manner. Therefore it seems this description can make sense only at QFT limit of TGD.

Dimensionally reduced 6-D Kähler action as an analog of SYM action

The 6-D dimensionally reduced Kähler action reduces to a sum of 4-D Kähler action and volume term and will be simply referred to as Kähler action. The super variant of this action is obtained by replacing embedding space coordinates with their super counterparts. Super-Kähler action is analogous to pure SYM action.

1. Space-time would be super-surface in super counterpart of $H = M^4 \times CP_2$ with coordinates h^k having super components proportional to multi-spinors multiplying the monomials of oscillator operators. The ocillator operator monomials rather than only the multi-spinor coefficients of the oscillator monomials transforming like vectors of H are regarded as analogs of quantum fields expressible by classical field equations as linear superpositions of their values at the boundary of CD for preferred extremals. The dynamics of monomials would reduce to that for oscillator operators labelled by points of cognitive representation and having interpretation as restriction of quantized quark field satisfying super-Dirac equation and the quantum criticality condition $q = q_s$.
2. Fermionic creation operators and annihilation operators labelled not only by spin and weak isospin as in the original proposal but also by the finite number of points of the cognitive representation. Therefore oscillator operators are analogous to the values of fermion field in discretization obeying super variant of modified Dirac equation. Both leptonic and quark like oscillator operators corresponding to two different H-chiralities and having different couplings to Kähler gauge potential could be present but octonionic triality allows only quarks. The vacuum expectation value of the action action exponentials contains only monomials with vanishing B (and L if leptons are present as fundamental fields). The matrix elements between positive and negative energy parts of zero energy states gives S-matrix.

   Real super-coordinates can be assumed to be hermitian and thus contain only sums of monomials and their conjugates having vanishing fermion numbers. This guarantees supersymmetrization respecting bosonic statistics at the level of propagators since all kinetic terms involve two covariant derivatives - one can indeed transform ordinary derivatives of monomials coming from the Taylor expansion to covariant derivatives involving also the coupling to Kähler form since the total Kähler charge of terms vanishes.

The lack of anti-commutativity of fermionic oscillator operators implies the presence of terms resulting in contractions.

1. The super-Taylors series would involve a finite number of partial derivatives of action. Wick contractions of oscillator operators would give rise to an infinite number of terms in continuum case. The appearance of infinite Taylor series defining the coefficients of super-polynomial is however troublesome from the point of view of number theoretic vision since there is no guarantee that the coefficients are rational functions. The finite number of points in the cognitive representation implying finite number of oscillator operators however allows only finite number of terms in the super-Taylor expansion.

   The monomials appearing in action in the interior of CD can be expressed as linear superpositions of those at boundary also in continuum case. Therefore each monomial is 3-D integral over the monomials at the boundary of CD. As a consequence, the contractions giving delta functions only eliminate one integration but do not give rise to infinities. A general solution to the divergence problems emerges.

   This is actually nothing new: one of the key ideas behind the notion of WCW is that path integral over space-time surfaces is replaced by a functional integral over 3-surfaces in WCW holographically equivalent with preferred extremals as analogs of Bohr orbits. The nonlocality of the theory due to the replacement of point-like particles with 3-surfaces would solve the divergence problems.

An interesting possibility in line with the speculations of Nima-Arkani Hamed and others is that the action defining space-time as a 4-surface of embedding space could emerge from the anti-commutators of the oscillator operator monomials as radiative corrections so that the bosonic action would vanish when the super-part of h_s vanishes.

Super-Dirac action

Before doing anything one can recall what happens in the case of modified Dirac action.

1. One has separate modified Dirac actions $\overline{\Psi}D\Psi$, $D = \Gamma^\alpha D_\alpha$ for quarks and leptons (later it will be found that modified Dirac action for quarks might be enough) and the covariant derivatives differ since there is a coupling to n-ple of included Kähler potential. For leptons one has $n = -3$ and for quarks $n = 1$. This guarantees that em charges come out correctly. This coupling appears in the covariant derivative D_α of fermionic super field.

2. One obtains modified Dirac equations for quarks and leptons by variation with respect to spinors. The variation with respect to the embedding space coordinates gives quantized versions of classical conservation laws with respect to isometries. One also obtains and infinite number of super-currents as contractions of modes of the modified Dirac operator with Ψ.

3. Classical field equations for the space-time surface emerge as a consistency condition guaranteeing that the modified Dirac operator is hermitian: canonical momentum currents of classical action must be conserved and define conserved quantum when contracted with Killing vectors of isometries. Quantum-classical correspondence (QQC) requires than for Cartan algebra of symmetry algebra the classical Noether charges are same as the fermionic Noether charges.

   It turns out that the super-symmetrization of modified Dirac equation gives only fermions and they fermionic superpartners in this manner if one requires that propagators are consistent with statistics.

Consider first the situation without the quantum criticality condition $q = q_s = \Psi_s$. H coordinates are super-symmetrized and induced spinor field becomes a super-spinor $\Psi_s = \Psi^N O_N(q, \overline{q})$ with Ψ_N depending on h_s (summation over N is understood).

1. As in the case of bosonic action the vacuum expectation value gives modified Dirac action conserving fermion numbers but one could assume that the monomials in the leptonic (quark) modified Dirac action have either non-vanishing L (B) and vanishing B (L). It seems that the lepton (baryon -) number of monomials can vary from 1 to maximum value. A more restrictive condition would be that the value is 1 for all terms.

2. Super-Dirac spinor is expanded in monomials $O_N(q, \overline{q})$ of q and its conjugate $\overline{q}$, whose anti-commutator is non-trivial. One can equally well talk about quark like oscillator operators. The sum $\Psi = \Psi^N O_N$ defining super-spinor field. The multi-spinors Ψ_N are functions of space-time coordinates, which are ordinary numbers. Quark oscillator operators are same as appearing in the embedding space super-coordinates. Only monomials O_N having total quark number equal to 1 are allowed. Super-spinor field however contains terms involving quark pairs giving rise to spartners of multiquark states with fixed quark number. The conjugate of super-spinor is defined in an obvious manner.

3. The metric determinant and modified gamma matrices appearing in the Dirac action are expanded as Taylor series in hermitian super-coordinate $h_s + \overline{h}_s$ with $h = h^N O_N$. This as as in the case of bosonic action.

   There are also couplings to gauge potentials defined by the spinor connection of CP_2 and the expansion of them with respect to the embedding space coordinates gives at the first step rise covariant derivatives of gauge potentials giving spinor curvature. At next steps one obtains covariant derivatives of spinor curvature, which however vanish so that the number of terms coming from the dependence of spinor connection on CP_2 coordinates is expected to be finite. Constant curvature property of CP_2 is therefore be essential (not that also M^4 would have covariantly constant spinor curvature in twistor lift and give rise to CP breaking).

The super-coordinate expansion of the metric determinant $\sqrt{g}$ and modified gamma matrices Γ^α and covariant derivatives D_α involving dependence on H coordinates give additional monomials of q parameters appear as hermitian monomials. Classical field equations correspond to $D_\alpha \Gamma^\alpha = 0$ guaranteeing the hermiticity of $D = \Gamma^\alpha D_\alpha$.

4. When super-coordinates of H are replaced with ordinary embedding space coordinates the only Wick contractions are between O^N and $\overline{O}^N$ in the vacuum expectation of Dirac action, and the action reduces to super-Dirac action with components satisfying modified Dirac equation. Propagator is Dirac propagator for all terms and the presence of only odd components in Ψ with quark number 1 and even components in h^s guarantees that Fermi statistics is not violated at the level of propagators. The dependence on h_s induces coupling between different components of the super-spinor. The components of super-spinor are interpreted as second quantized objects.

5. The terms in the action would typically involve n-tuples of partial derivatives $L_{k_1\alpha_1 \ldots k_n 1\alpha_n}$ defined earlier for $L = \sqrt{g}$ coming from super-Taylor expansions. Similar derivatives come from the modified gamma matrices Γ^α.

   Also now one obtains loops from the self contractions in the terms coming from the expression of action and gamma matrices. These terms should vanish and as already found this would requires vanishing of currents perhaps identifiable as Noether currents of symmetries. This guarantees that the Taylor expansion contains only finite number of terms as required by number theoretic vision.

The multi-fermion vertices defined by the action would be non-trivial but involve always contraction of all fermion indices between monomials formed from oscillator operators in Ψ and their conjugates in $\overline{\Psi}$ if the loop contractions sum up to zero. One could interpret these super-symmetric vertices as a redistribution of fermions of a local many-fermion state between external local many-fermion states particles represented by the monomials appearing in the vertices. The fermions making the initial state would be same as in final state and all distributions of fermion number between sfermion lines would be allowed. The action obtained by contraction would has SUSY as symmetry but the propagation of different sfermions is fermionic and does not look like that for ordinary spartners.

The quantum criticality condition $q = q_s$ makes the situation non-linear and should fix the coefficients of various terms in super-Taylor expansions as fixed point values of coupling constants.

Could super-Kähler action alone give fermionic scattering amplitudes?

The concrete study of the super-counterpart of Kähler action led to a realization of an astonishing possibility: super-Kähler action alone could give also fermionic scattering amplitudes.

1. In principle this is possible if in S-marix one has contractions of quark creation operator and annihilation operator appearing in quark-antiquark bilinear with different partonic 2-surfaces. This would give fermionic line connecting the points of the cognitive representation at the boundary of CD with points at partonic 2-surfaces in $t = r_n$ hyper-planes in the interior of CD or at the opposite boundary of CD.

   As a matter of fact, this must be the case if the exponent for the sum of super-Kähler and super-Dirac action gives the scattering amplitudes as its matrix elements! The reason is that super-Dirac action vanishes or its solutions.

   The super-Dirac equation must be however present and corresponding variational principle must be satisfied. The hermiticity of the modified Dirac operator requires the vanishing of the covariant derivatives of the modified gamma matrices meaning that bosonic field equations are satisfied. This must be true also for the super variants of the modified gamma matrices.

   If super-Dirac equation is satisfied, the action of modified Dirac operator without connection (ordinary rather than covariant derivative) terms on the discretized quark fields can be expressed in terms of spinor connection as $\Gamma^\alpha - s\partial_\alpha \Psi = \Gamma^\alpha_s A_{\alpha,s} \Psi$ and there is no need for explicit information about the behavior of quark field in the nearby points so that cognitive representation is enough. Otherwise one must have the usual lattice type discretization.

2. The super expansion of super-Kähler action contains only ordinary derivatives of 4-currents defined by quark bi-linears. If the quark field operators with continuous arguments are behind

those with discretized arguments and satisfy modified Dirac equation, one can transform the action on quark and antiquark fields to a multiplication with induced gauge potential. This gives nothing but the coupling terms to the gauge potentials in the standard perturbation theory, where one assumes free solutions of Dirac action as approximate solutions. One therefore obtains on mass shell variant of the perturbation theory! Perturbation theory without perturbation theory, might Wheeler say. Or more concretely: the fact that one can treat super-coordinates only perturbatively.

3. The natural guess is that all terms in the expansion of super-Kähler can be transformed to interaction terms and super-Kähler action gives the analog of perturbation theory as a discretized version. The leptonic terms associated with $(3,3)$ term in super-Kähler action should transfrom to the analog of interaction terms for leptonic Dirac action. Whether Kähler gauge potential and spinor connection are developed in super-Taylor series in ordinary manner or remains an open questions.

9.1.4 What super-Dirac equation could mean and does one need super-Dirac action at all?

What does super-Dirac equation actually mean? Super Dirac action vanishes on mass shell and super-Kähler action would give all scattering amplitudes. Are super-Dirac action and super-spinor field needed at all? Should one interpret the oscillator operators defining analog of quark field q as the super-Dirac field Ψ_s as conceptual economy suggests. But doesn't this imply $q = q_s$?

One can consider 3 options as an attempt to answer these questions. Options I and II are not promising. Option III leads to very nice concrete realization of quantum criticality.

Option I: No super-Dirac action and constant oscillator operators

1. If oscillator operators can be regarded as constant, the super Taylor expansion for super Kähler action would give ordinary divergences of the fermionic currents and the action of derivative would be on modified gamma matrices and charge matrix A commutator of $[A_\alpha, \Gamma^\alpha Q]$ and the outcome would be non-vanishing so that one would obtain the coupling terms also now. Could the commutator $[A_\alpha, \Gamma^\alpha]$ be interpreted in terms of gravitational interaction and the commutator $[A_\alpha, Q]$ as electro-weak interaction? In any case, there would be no need for super-Dirac action!

2. There is however an objection. Quark oscillator operators are labelled by the points of cognitive representation and in continuum case they are analogous to the values of quantized spinor field. Should one identify this spinor field with super-spinor field and solve it using a generalization of modified Dirac equation to super-Dirac equation? Can one argue that oscillator operators labelled by points represent superpositions of constant oscillator operators involving integration over 3-D surface at light-cone boundary and are indeed constant?

This option does not look promising.

Option II: q satisfies ordinary Dirac equation

1. Could one assume that the solution q_0 of ordinary Dirac equation defines the solution to be used as q in the super-Kähler action. The coupling terms of super-Kähler action obtained using $D_0 q_0 = 0$ would be proportional to the classical spinor connection. Classical Kähler action does not involve gauge potentials so that internal consistency would not be lost at this level. The super-variant of Kähler action however involves derivatives of the analogs of fermion currents and there transformation to purely local objects requires the introduction of electroweak gauge potentials so that the symmetry between super-Kähler and super-Dirac would be lost.

2. This would save from developing gauge potentials A_k to super Taylor series - as found this would give only 2 terms by the covariant constancy of spinor curvature. The divergence would reduce to a term involving only a commutator $[A_{alpha}, Q]$, where A_α is purely classical. If Q is Kähler charge, this commutator would vanish, which looks strange since electroweak hyper-charge is proportional to Q_K. This could be seen as a failure. If Kähler gauge potential

is replaced with its super-variant $A_\alpha + J_{\alpha l}\delta h^l_s$ the commutator is non-vanishing as it should be.

3. Leptons would not appear in $q = q_0$ but since the exponent of super-Kähler action would define the scattering amplitudes by the vanishing of (super-)Dirac action, one could say that leptons emerge as 3-quark composites. SUSY would be dynamical after all!

   Mathematically this option looks awkward and must be dropped from consideration.

Option III: q is a solution of super-Dirac equation

It is best to start from an objection.

1. Assume that q is given Super-Dirac equation

$$D_s(q)q = 0 \ .$$

   This non-linear equation involves powers of q and its conjugate. The problem is that super-Dirac equation is non-linear in q and there are actually 7 separate equations for the part of q with quark number one. 7 equations is too much. The only manner to solve the problem is to replace q with q_s to get $D_s q_s = 0$. But this would require replacing q with q_s in $D_s(q)$ and it would seem that one has an infinite recursion.

2. Could q be self-referential in the sense that one has

$$q_s = q \ . \tag{9.1.1}$$

   q would be invariant under iteration $q \to q_s$. This would give excellent hopes of fixing q uniquely. This allows also physical interpretation. The fixed points of iteration give typically fractals and quantum criticality means indeed fractality. This condition could therefore realize quantum criticality, and would give hopes about unique solution for $q = q_s$ for given extension of rationals.

   Also h_s should satisfy similar self-referentiality condition expressing quantum criticality:

$$h_s = (h_s)_s \ . \tag{9.1.2}$$

   The general ansatz for h_s involves analogs of electroweak vector currents formed from quark field and lepton field as its local composites. q_s has analogous structure. The currents contracted with the Hamiltonian vector fields of symplectic transformations of light-cone boundary appear in the Minkowski salars and have some coefficients having an interpretation as coupling constants. $q = q_s$ condition defining quantum criticality would fix the values of these coupling parameters for given extension of rationals and would realize discrete coupling constant evolution.

   The general ansatz for h^k_s involves analogs of electroweak vector currents formed from quark field and lepton field as its local composites. q_s has analogous structure. The currents contracted with the Hamiltonian vector fields of symplectic transformations of light-cone boundary appear in the Minkowski salars and have some coefficients having an interpretation as coupling constants. $q = q_s$ condition defining quantum criticality would fix the values of these coupling parameters for given extension of rationals and would realize discrete coupling constant evolution.

3. Many consciousness theorists love the idea of self-referentiality described by Douglas Hofstadter in fascinating manner in his book "Gödel, Escher, Bach". They might get enthusiastic about the naïve identification of q_s and h_s with field of consciousness. In TGD inspired theory of consciousness the self-referentiality of consciousness is understood in different manner but $q = q_s$ and $h_s = (h_s)_s$ as quantum correlated for the self-referentiality is certainly a fascinating possibility.

   Consider now a more detailed picture.

1. What does one really mean with q_s? q_s could contain parts with quark number 1 and 3 but a very natural requirement is that it has well-defined fermion number and thus has only a part with quark number 1. The part with quark number 3 is not needed since super-Kähler action would contain it: leptons would emerge as local 3-quark composites from super-Kähler action.

2. Super-Dirac equation would be given by

$$\begin{aligned} D_s(q)q &= 0 \ , \\ D_s(q) &= \Gamma^{\alpha,s}(q)D_{\alpha,s}(q) \ . \end{aligned} \tag{9.1.3}$$

   $D_s(q)$ is super-Dirac operator and

$$\Gamma^{\alpha}_s = T^{\alpha k}_s \gamma_k \tag{9.1.4}$$

   are super counterparts of the modified gamma matrices $\Gamma^{\alpha} = T^{\alpha k}\gamma_k$ defined by the contractions of canonical momentum currents of Kähler action with the gamma matrices γ_k of H:

$$T^{\alpha}_k = \frac{\partial L_K}{\partial(\partial_{\alpha} h^k)} \ . \tag{9.1.5}$$

   One would have $\gamma_{k,s} = \gamma_k$ by covariant constancy. L_K denotes Kähler action density, which is sum of 4-D Kähler action and volume term. The field equations of super Kähler action give

$$D_{\alpha,s}\Gamma^{\alpha}_s = 0 \tag{9.1.6}$$

   guaranteeing the hermiticity of the super Dirac operator.

3. The basic equations would thus reduce to

$$\begin{aligned} q = q_s \ , \\ D_{\alpha,s}\Gamma^{\alpha}_s = 0 \ , \\ D_s(q)q = 0 \ . \end{aligned} \tag{9.1.7}$$

In the continuum case one could think of solving the field equations iteratively.

1. One would first by solve $q = q_0$ for classical modified Dirac operator $D(h_0)$ defined by the ordinary coordinates h_0 of H. Next one would solve $q_1 = q_0 + \Delta q_1$ for the super version $D_1 = D(q_0)$. This would allow to solve next iterate $h_1 = h_0 + \Delta h_1$ using $D(q_1)$. One could continue this process in the hope that the iteration converges. At each step one have group of equations $D_n q_n = 0$ for q_n and for h_{n+1}.

2. An objection is that the iteration could lead outside the extension of rationals if it involves infinite number of iterates. This could occur for space-time surface itself if the normal ordering terms affect the classical action and force to modify the preferred extremal and also cognitive representation at each step. Remaining inside the extension of rationals could also mean that the coefficients of the monomials at points of cognitive representation belong to the extension.

   It is not of course completely clear whether these equations make sense in the interior of CD or can be solved unlike the lowest equation. It however seems that for each independent monomial m_n the equation would be of form $D_0 m_n = ...$ so that other terms would define kind of sources term and the equation super-Dirac equation could be written as non-linear equation $D_0 q = -\Delta D(q)q$.

3. Each order of bosonic monomials would give its own group of equations making sense also for the cognitive representations and the same would be true for quark monomials and monomials of different orders would be coupled but different quark numbers in q (quarks and leptons) would decouple. These equations are analogous to those appearing in QFT in a gauge theory involving gauge fields and fermion fields.

For cognitive representations the situation is much simpler.

1. All that is needed is the transformation of the ordinary divergences of fermionic currents to a form in which derivative ∂_α is replaced with the linear action of super-gauge potential $A_{\alpha,s}$. Therefore there is no need to solve the non-linear modified Dirac equation in this case and it would become necessary only at the continuum limit. The full solution of non-linear super-Dirac equation would be necessary only in the continuum theory.
2. Could one think that q has vanishing derivatives at the points of cognitive representation: $\partial_\alpha q = 0$ implying $\Gamma^\alpha A_\alpha q = 0$ If the condition holds true then q would be effectively constant for cognitive representations and the situation would effectively reduce to that for option I. This condition is is diffeo-invariant but not gauge invariant. If the points of cognitive representation correspond to singularities of the space-time surface at which several roots of the octonionic polynomial co-incide, the tangent space at the level of M^8 parameterized by a point of CP_2 is not unique and the singular point is mapped to several points in H, and the conditions $\partial_\alpha q = 0$ would make sense at the level of M^8 at least.
3. If one assumes that the quarks correspond to singular points defined by intersections of roots also in the continuum case, one obtains discretization also in this case irrespective of whether one assumes $\partial_\alpha q = 0$ at singularities. Allowing analytic functions with rational Taylor coefficients one obtains also now roots which can be however transcendental and one can identify intersections of roots in the similar manner.

To sum up, there are many uncertainties involved but to my opinion the most satisfactory option is Option III. If one assumes that condition at continuum case, one would obtain also now the discretization.

What information is needed to solve the scattering amplitudes?

One can look the situation also from a more practical point of view. Are there any hopes of actually calculating something? Is it possible to have the information needed?

1. The condition that super-Dirac equation is satisfied would remove the need to have a lattice and cognitive representation would be enough. If the condition $\partial_\alpha q = 0$ holds true, the situation simplifies even more but this condition is not essential. The condition that the points of the cognitive representation assignable to quark oscillator operators correspond to singularities of space-time surface at which several space-time sheets intersect, would make the identification of these points of cognitive representation easier. Note that the notion of singular point makes sense also at the continuum limit giving cognitive representation even in this case in terms of possibly transcendental roots of octonion analytic functions.

   If the singular points correspond to solution to 4 polynomial conditions on octonionic polynomials besides the 4 conditions giving rise to the space-time surfaces. The intersections for two branches representing two roots of polynomial equation for space-time surface indeed involve 4 additional polynomial conditions so that the points would have coordinates in an extension of rationals, which is however larger than for the roots $t = r_n$. One could of course consider an additional condition requiring that the points belong to the extension defined by r_n but this seems un-necessary.

   The octonionic coordinates used at M^8-side are unique apart from a translation of real coordinate and value of the radial light-like coordinate $t = r_n$ corresponds to a root of the polynomial defining the octonionic polynomial as its algebraic continuation. At this plane the space-time surfaces corresponding to polynomials defining external particles as space-time surfaces would intersect at partonic 2-surfaces containing the shared singular points defined as intersections.

2. The identification of cognitive representations goes beyond the recent knowhow in algebraic geometry. I have considered this problem in [L88] in light of some recent number theoretic ideas. If the preferred extremals are images of octonionic polynomial surfaces and M^8-H duality the situation improves, and one might hope of having explicit representation of the images surfaces in H-side as minimal surfaces defined by polynomials.

9.1.5 About super-Taylor expansion of super-Kähler and super-Dirac actions

The study of the details of of the general vision reveals several new rather elegant features and clarifies the connections with QFT picture.

About the structure of bosonic and fermionic monomials

The super part of the embedding space coordinates is H-vector and this allows to pose strong conditions on the form of the monomials.

1. One can construct the simplest monomials as bilinears of quarks and anti-quarks. Since oscillator operators are analogs of quark fields, one can construct analogs of left- and right-handed electroweak currents $\overline{q}(1\pm\gamma_5)\gamma^kQq$ involving charge matrix Q naturally assignable to electroweak interactions. The charge matrices Q should reflect the structure of CP_2 spinor connection so that analogs of electroweak currents would be in question. One can multiply the objects Hamiltonians HA_A of the isometries and even symplectic transformations at the boundary of CD.

2. One can obtain higher monomials of q and $\overline{q}$ by multiplying these vectorial currents by bilinears, which are scalars and pseudo-scalars obtained by contracting some symmetry related vector field j^k_A of H with gamma matrices of H to give $\overline{q}(1\pm\gamma_5)j^k_AQ\gamma_kq$ giving rise to analogs of scalar and pseudoscalar Higgs. The Killing vector fields of isometries of H and symplectic vector fields assignable to the Hamiltonians of $\delta CD\times CP_2$ are a natural choice for j^k_A.

   One can construct also scalar currents for which gamma matrices contract with gradient of Hamiltonian to give $\overline{q}(1\pm\gamma_5)\gamma^k\partial_kH_AQ\gamma_kq$ as kind of duals of symplectic currents. These do not define symplectic transformations.

   These vector fields make sense at the boundaries of CD and this is enough (they could make sense also at shifted boundaries) since the field equations would allow to express monomials as linear superpositions of the monomials at boundary of CD. Oscillator would always be assigned with the boundaries of CD.

3. If the spin of graviton is assigned with spinor indices, the vector nature of the monomials excludes the analog of graviton. One can however consider also the possibility that the second spin index of graviton like state corresponds to the Hamilton of a symplectic isometry of S^2: for small enough size scales of CD this angular momentum would look like spin. In CP_2 degrees this would give rise to an analog of gluon. Also gluon with spin zero would be obtained.

   An alternative option is to assume that graviton corresponds to a non-local state with vectorial excitations at opposite throats of wormhole contact or at different wormhole contacts of closed flux tube. All these states are in principle possible and the question is which of them correspond to ordinary gravitons.

The super counterpart of Dirac spinor consists of odd monomials of quark spinor. Well-defined fermion number allows only monomials with quark number 1 and with definite H-chirality. Quark spinors allow leptons like stats as local 3-quark composites appearing in the super-Kähler action determining the scattering amplitudes since super-Dirac action vanishes at mass shell.

1. In the bosonic case one has vectorial entities and now it is natural to require that one has an object transforming like spinor of H. This poses strong conditions on the monomials since one should have spin 1/2-isospin 1/2 representation.

2. The lowest monomial correponds to quark-antiquark current. What about leptonic analog. The number of oscillator operators at given point is 4+4=8. Leptonic part of super-Kähler

action must have 3+3 indices. Therefore also leptonic bilinear seems to be possible and pairs of quarks and lepton like states are possible.

Intuitively it is clear that leptonic term exists and corresponds to an entity completely anti-symmetric in spin-isospin index pairs (s_3, i_3) of quark spinors. The construction of baryons without color symmetry indeed gives proton and neutron. In order to obtain Δ resonance from u and d quarks, one must have color degrees of freedom and perform anti-symmetrization in these.

The general condition is that the tensor product of 3 8-D spin representation of $SO(1,7)$ contains 8-D representation in its decomposition. The existence of lepton representation is clear from the fact that the completely antisymmetric representation formed from 4 quarks is $SO(1,7)$ singlet and is product of lepton representation with 3 fold tensor product which must therefore contain spin-isospin 4-plet . The coupling to Kähler gauge potential would correspond to leptonic coupling, which is 3 times the quark coupling.

3. Quarks and lepton monomials have also satellites obtained by adding scalars and pseudo-scalars constructible as quark-anti-quark bi-linears in the manner already discussed. The interpretation as analogs of Higgs fields might make sense.

Normal ordering terms from contractions of oscillator operators

Normal ordering terms from contractions of oscillator operators is a potential problem. In the discretization based on cognitive representations this problem disappears.

1. Contraction terms could induce discrete coupling constant evolution by renormalizing the local monomials. Infinite number these terms would spoil number theoretical vision since a sum over infinite number of terms in general leads outside the extension of rationals involved. If the number of contractions is finite, there are no problems. This is the case in the number theoretical vision since contraction involves always a pair of points. If the rule for construction of S-matrix holds true these points are at opposite boundaries of CD. In the general case they can be at the same boundary. The number of contracted points cannot be larger than the number of points in cognitive representation, which is finite in the generic situation.

   This would give discrete coupling constant evolution as function of extension of rationals since the contractions renormalize the coefficients of the 4+4 terms in the local composites of oscillator operators. The original proposal that additional symmetries are needed to obtain discrete coupling constant evolution is not needed.

2. One could argue that algebraic numbers as a limit for extension is enough to get the continuum limit since the points of cognitive representation would be dense subset of 3-surface. For continuum theory 3-D delta functions would replace Kronecker deltas in anti-commutators implying in ordinary QFT divergences coming as powers of 3-D delta function at zero.

   In the proposed vision one can allow contractions even in the continuum case. The monomials in the interior are linear multilocal composites of those at either boundary of CD involving 3-D integration over boundary points. Contractions associated with two monomials in the interior means an appearance of delta function cancelling the second integration so that there is no divergence.

About the super-Taylor expansions of spinor connection and -curvature

There are also questions related to the details of the expansion of of spinor connection and -curvature in powers of monomials of quark oscillator operators.

1. The rule is that one develops Kähler function as Taylor series with argument shifted by super-part of the super-coordinate. This involves expansion in powers of coordinate gradients and also the expansion of Kähler gauge potential. In the case of modified Dirac action one must expand also the spinor connection of CP_2.

   A potential problem is that the Taylor expansions of Kähler gauge potential and spinor connection have infinite number of terms. Since the monomials in the interior can be expressed linearly in terms of those at boundary of CD by classical field equations, number theoretic discretization based on cognitive representation implies that only a finite number of terms are obtained by using normal ordering and the fact that the number of oscillator operators at same point is 4+4=8. Normal ordering terms would represent radiative corrections giving rise to renormalization depending on the extension of rationals.

2. Is this enough or should one modify the Taylor expansion of Kähler gauge potential A? The idea that $A_k dh^k$ is the basic entity suggests that one must form super Taylor series for both A_k and dh^k. This would give $A_k dh^k \rightarrow A_k \partial_k \delta h^k + A_l \partial_(\delta h^l))dh^k$. By performing an infinitesimal super gauge transformation $A_l \rightarrow A_l + \partial_l(A_l \delta h^k)$ one obtains $A_k \rightarrow A_k + J_{kl}\Delta h_s^k$, where Δh_s^k denotes super part of super-coordinate. The next term would vanish by covariant constancy of J_{kl}.

   The same trick could be applied to spinor connection and since also spinor curvature is covariantly constant, one would obtain only 2 terms in the expansion also in the continuum case. This provides an additional reason for why S $(= CP_2)$ must be constant curvature space.

   This applies also to M^4: in fact, twistor approach strongly suggests that also M^4 has the analog of covariantly constant Kähler form. This conforms with the breakdown of Poincare symmetry at M^8 level forced by the selection of the octonion structure. Poincare invariance is gained by integrating over the moduli space of octonion structures in the construction of scattering amplitudes. What is remarkable that one could use the irreps of Lorentz group at boundaries of CD, which for obvious reasons are much more natural than than those of Poincare group.

3. In the case of embedding metric the same trick would give only the c-number term and only the gradients of embedding space coordinates would contribute to the super counterpart of the induced metric. In this case general gauge super-coordinate transformation would allow to treat the components of metric as constants.

What is the role of super-symplectic algebra?

This picture is not the whole story yet. Super-symplectic approach predicts that the super-symplectic algebra (SSA) generated essentially by the Hamiltonians of $S^2 \times CP_2$ assignable to the representations of $SO(3) \times SU(3)$ localized with the respect to the light-like radial coordinate of light-cone boundary characterize the states besides electro-weak quantum numbers. Color quantum numbers would correspond to Hamiltonians in octet representation. This would predict huge number of additional states.

There are however gauge conditions stating that sub-algebra of SSA having radial conformal weights coming as n-ples of SSA and isomorphic to SSA and its commutator with SSA annihilate physical states. This reduces the degrees of freedom considerably but the number of symplectic Hamiltonians is still infinite: measurement resolution very probably makes this number to finite.

9.2 Other aspects of SUSY according to TGD

In this section other aspects of SUSY according to the present proposal are discussed.

9.2.1 $M^8 - H$ duality and SUSY

$M^8 - H$ duality and $h_{eff}/h_0 = n$ hypothesis pose strong constraints on SUSY in TGD sense.

1. $h_{eff}/h_0 = n$ interpreted as dimension of extension of rationals gives constraints. Galois extensions are defined by irreducible monic polynomials $P(t)$ extended to octonionic polynomials, whose roots correspond to 4-D space-surfaces and in special case 6-spheres at 7-D light-cones of M^8 taking the role of branes.

   The condition that the roots of extension defined by Q are preserved for larger extension $P \circ Q$ is satisfied if P has zero as root:

$$P(0) = 0 \ .$$

   This simple observation is of crucial importance, and suggests an evolutionary hierarchy $P \circ Q$ with simplest possible polynomials Q at the bottom of the hierarchy are very naturally assignable to elementary particles. These polynomials have degree two and are of form $Q = x^2 \pm n$. Discriminant equals to $D = 2n$ and has the prime factors of n as divisors defining ramified primes identified as p-adic primes assignable to particles.

Remark: Also polynomials $P(t) = t - c$ are in principle possible. The corresponding space-time surfaces at the level of H would be M^4 and CP_2 and they are extremals of Kähler action but do not have particle interpretation.

It turns out the normal ordering of oscillator operators renormalizes the coefficients of P. In particular P can be shifted by a constant term and this deforms the roots of the real polynomial. Also the action principle to be discussed allows $RE(P) = c$ and $IM(P) = c$ surfaces as solutions.

2. The key idea is that the powers o^n of octonion are associative. If the coefficients of $P(o)$ are real or possibly even complex rationals $m + in$ commuting with octonions, associativity is not lost. Octonion o would be replaced by super-octions o_s with (possibly complex-) rational coefficients. o_s is octonion shifted by oscillator operator polynomial analogous to a real number. The conjugate octonion $\overline{o}$ would be treated analogously. Associativity would be preserved.

3. One could assign oscillator operators to both leptons and quarks but the option identifying leptons as local 3-quark local composites and in this sense spartners of quarks allows only baryon number zero composites of quarks and anti-quarks to appear in the octonionic polynomial, which is also hermitian. This would conform with $SO(1,7)$ triality.

   Remark: Anti-leptons are spartners of quarks in the sense of being their local composites but not in the sense that they would appear as local composites in q_s. Leptonic currents can appear in super-Kähler action so that anti-leptons are spartners of quarks in this sense.

   Oscillator operators would transform like components of 8-D spinor *resp.* its conjugate and have interpretation as quark *resp.* anti-quark like spinors. $SO(1,7)$ triality allows only leptonic or quark-like spinors and quark-like spinors are the only physical choice. Also the super-quark q_s which must satisfy self-referential condition $q_s = q$ must have components behaving like $8 - D$ spinors with quark number 1. o_s should satisfy analogous condition $o_s = (o_s)_s$.

4. Super-polynomial $P_s(o)$ would be defined by super-analytic continuation as $P(o_s)$ by Taylor expanding it with respect to the super-part of o_s. The outcome is super-polynomial with coefficients of oscillator operator monomials containing k quark-antiquark pairs given by ordinary octonionic polynomials $P_{n-k}(o)$. Each $P_{n-k}(o)$ obtained by algebraically continuing the k:th derivative of the real polynomial $P(t)$ would define 4-surface by requiring that the imaginary or real part of $P_{n-k}(o)$ (in quaternionic sense) vanishes or is constant. Normal ordering of oscillator operators renormalizes the coefficients of P_{n-k}. The interpretation would be as radiative corrections.

   Octonionic super-polynomials obtained from octonionic polynomials of degree n as super-Taylor series decompose to a sum of products of octonionic polynomials $P_k(o)$ with degree $k = n-d$ with oscillator operator monomials consisting of d quark-antiquark pairs. If the degree n of the octonionic polynomial is smaller than the maximal number $N = 4$ of oscillator operator pairs in super-polynomial, only a fraction of spartners are possible. SUSY is realized only partially and one can say that part of spartners are absent at the lowest levels of evolutionary hierarchy. At the lowest level of hierarchy corresponding to $n = 2$ only fermions (quarks) would be present as local states and would form non-local states such as baryons and mesons. Gauge bosons and Higgs like state would be bi-local states and graviton 4-local state.

   Remark: Gauge bosons and Higgs like states as local fermion-anti-fermion composites at level $n = 2\times 2$. For the option involving only quarks (color is not spin like quantum number). Note that the value of $n_0 = 3 \times 2 = 6$ in $h = n_0 \times h_0$ suggested by the findings of Randel Mills [L26, L60] would allow the known elementary particles.

5. The geometric description of SUSY would be in terms of super-octonions and polynomials and the components of SUSY multiplet would correspond to components of a real polynomial continued to that of super-octonion and would in general give rise to minimal space-time surfaces as their roots: one space-time sheet for each component of the super-polynomial.

The components would have different degrees so that the minimal extensions defined by the roots would be different. Therefore also the p-adic primes characterizing corresponding particles could be different as ramified primes of extension and in p-adic mass calculations this would mean different p-adic mass scales and breaking of SUSY although the mass formulas would be same for the members of SUSY multiplet. The remaining question is how the ramified prime defining the p-adic prime is selected. The components of super-polynomial would have different degrees so that the extensions defined by the roots would be different. Therefore also the p-adic primes characterizing corresponding particles would be different as ramified primes of extension and in p-adic mass calculations this would mean different p-adic mass scales and breaking of SUSY although the mass formulas would be same for the members of SUSY multiplet. The remaining question is how the ramified prime defining the p-adic prime is selected.

9.2.2 Can one construct S-matrix at the level of M^8 using exponent of super-action?

The construction of S-matrix in H picture in terms of exponential of action defining Kähler function of WCW forces to ask whether M^8 really is an alternative picture as the term "duality" would suggest or is it only part of a description necessitating both M^8 and H. If the duality holds true in strict sense the proposed construction of S-matrix at the level of H should make sense also at the level of M^8. Is this possible at all or could it be that S-matrix emerges the level of H and that M^8 level provides only a tool to describe preferred extremals in H by using what I have called M^8 duality? In the sequel I will look what one obtains if the duality holds true in strict sense.

1. The original idea was to identify space-time-surfaces in M^8 as roots of polynomial equations generalizing ordinary polynomial conditions. Could this makes sense also when octonions are replaced by super-octonions and what super-octonions and quark oscillator operators could mean?

2. The oscillator operators are interpreted as a discretized version of second quantized quark field q allowing local composites of q defining analogs of SUSY multiplets. One can indeed define second quantization for cognitive representations also now. Quark oscillator operators would be analogs of complex coefficients commuting with octonionic units ($i = \sqrt{-1}$ commute with them). The gamma matrices appearing in the quark-antiquark bi-linears would be ordinary gamma matrices of M^8.

   Remark: I have also considered the possibility that M^8 spinors correspond to octonionic spinors with octonionic units defining sigma matrices.

3. One could define simplest contribution the octonionic super-coordinate o_s as sum of M^8 octonion and super-part defined as contraction of 8-component quark current $\overline{q}\gamma^k q$ with contracted with octonionic units e_k to give $\Delta o_s = \overline{q}\gamma^k Q q e_k$. Charge matrices Q are linear combinations of sigma matrices of M^8 in the currents. Gamma matrices should be ordinary gamma matrices and q would transform like ordinary M^8 spinor. The entity $o_s = o + \Delta o_s$ would replace octonionic coordinate o in polynomial equations expressing the vanishing of the real or imaginary part (in quaternionic sense) for $P(0_s)$.

   The contractions of Killing vector fields of translations with gamma matrices would give scalars $j^k\gamma_k$ giving in turn scalars $S = \overline{q}j^k\gamma_k Q q$ and these could be used to build higher monomials. Octonion analyticity in the proposed sense does not allow to use Killing vector fields of rotations and symplectic currents. On the other hand, for cognitive representations these vector fields are restricted to single point of cognitive representation: could this mean that one can allow also the more general scalars.

   Leptons should emerge from o_s. This is the case if one allows also higher monomials in o_s. Also leptonic tri-linears and their conjugate could be built and these would give leptonic bi-linears $\overline{L}\gamma^k Q L$. Therefore all (covariantly) constant contributions to super-octonion are possible. The coefficients of various monomials in o_s would be derivatives of polynomial P since they are obtained as super-Taylor series and the coefficients of these polynomials would have interpretation as coupling constants.

4. At the level of H one can construct much larger number of monomials of quark oscillator operators transforming like vector in H. The scalars and pseudo-scalars constructed from the Killing vector fields and symplectic currents can be used to build higher monomials. At the level of H the super-symplectic Hamiltonian currents except those associated with isometries could however annihilate physical states.

   The quark currents defined by symplectic isometries are however not constant so that there seems to be a slight inconsistency. Could one assume that also color isometries at the level of H annihilate states quite generally as also S^2 isometries associated with the "heavenly" sphere S^2 in the decomposition $\delta M^4_+ = S^2 \times R_+$? Or can one argue that the restriction to translations is enough because one considers only points of cognitive representation?

5. What about quantum super-spinors q_s (analog of quantized quark field). q would be ordinary rather than octonionic spinor. q_s would be constructed using q and the scalars already discussed. These monomials would carry information about couplings constants. If they are identifiable as the spinors appearing in o_s, one must have $q = q_s$ realizing quantum criticality in quark sector. This would pose strong conditions on the coefficients of the monomials appearing in q interpreted as coupling constants. The conditions would depend on the extension of rationals defined by the polynomia $P(o)$.

   The discretization by cognitive representations at the level of H is made possible by super-Dirac equation. At M^8 level there is no need to get rid of partial derivatives acting on currents and super-Dirac equation is not needed.

6. The polynomial equations are purely local algebraic equations and the notions of propagation and boundary value problem do not make sense at the level of M^8. $M^8 - H$ correspondence should lead to the emergence of these notions by mapping surfaces to minimal surfaces natural by quantum criticality. Octonion analyticity and associativity of tangent or normal space inducing dynamics should induce M^8 analog of propagation.

Could one imagine a counterpart for the action exponential and a construction of S-matrix similar to that in the case of H?

1. The action principle should be purely local involving no derivatives of the super-octonionic polynomial $P(o_s)$. It should produce $RE(P) = 0$ and $IM(P) = 0$ as solutions. One might allow also solution $RE(P) = c$, where c is rational number. This would shift of the real polynomial continued algebraically to octonionic polynomial modifying the roots. One should obtain also 6-spheres as universal solutions and identifiable as subsets of 7-D light cones. Now one would have $IM(P) = 0$, $RE(P) = c$ modifying the roots $t = r_n$ defining hyper-surfaces in M^4.

2. Action should be sum over contributions over the points of cognitive representation, perhaps identifiable as the set of singular points at which two roots co-incide.

   (a) Could one minimize the action with respect to the components of $RE(P)$ or $IM(P)$? If this were the case one obtains one would have either $RE(P) = 0$ or $IM(P) = 0$. Surfaces with associative tangent and normal space should have different action and this does not look nice.

   (b) Could one require stationarity of the action with respect to the small deformations of the points of cognitive representation so that they would represent local extrema of action density? These points indeed change, when the polynomial is modified. Since only the deformations of these points are the visible trace of variation for cognitive representations, one could require that the value of action is stationary against these variations rather than variations of the values of $RE(P)$ and or $IM(P)$. This would give rise a condition involving derivatives of $RE(P)$ and $IM(P)$ at singular points with respect to space-time components of octonion. This option will be considered in the sequel.

3. The action density should be finite, and allow both solution types. One can imagine two options.

   Option I: If one requires that the action density is dimensionless, the simplest guess for the "action density" L is

$$L = \frac{(RE, IM)}{[(RE, RE) + (IM, IM)]} \quad ,$$

where one has $RE \equiv RE(P(o))$ and $IM \equiv IM(P(o))$ and the inner product is quaternionic inner product. The problem is that denominator gives infinite series giving rise to infinite number of normal ordering terms which may lead out of extension. For exceptional solutions $RE = 0, IM = 0$ the denominator also diverges.

Option II: The alternative avoiding these problems is analogous to the action density of completely local free field theory given by

$$L = K(RE, IM) \ . \tag{9.2.1}$$

K is constant with dimensions of inverse length squared and should relate to the CP_2 length squared. This is not dimensionless but can remain bounded if the quantity (RE, IM) remains bounded for large values of $(RE, RE) + (IM, IM)$.

4. For **Option I** L is a generalization of conformally invariant action from 2-D complex case, in which L reduces to $L = w_1w_2/(w_1^2 + w_2^2) = sin(\phi)cos(\phi)$, $w_1 = Re(w(z))$,$w_2 = Im(w(z))$. (ϕ) is the conformally invariant direction angle associated with w.

   The variation of 2-D action with respect to position of the point of cognitive representation gives

   $$\frac{[(\partial_u w_1 w_2 + w_1 \partial_u w_2)(w_1^2 + w_2^2) + w_1w_2(w_1\partial_u w_1 + w_2\partial_u w_2)]}{(w_1^2 + w_2^2)^2} \ , \quad u \in \{x, y\} \ .$$

   The general solutions are $w_i = c_i \neq 0$, where c_i are constant rational numbers.

   The criticality of the action density (maybe it could be seen as a manifestation of quantum criticality) is essential and means that the graph of L as function of w_1 and w_2 is analogous to saddle $w_1w_2/((w_1^2 + w_2^2)$. The condition that L is well-defined requires $c_1 \neq 0$. c_1 could in principle depend on point of cognitive representation. **Option II** gives the same equations in complex case.

5. For **Option II** one obtains 8 equations in the octonionic case and the outcome is that the derivatives of RE or IM or both with respect to components of o vanish. One can have $RE(P(o)) = c_1 \neq 0$ or $IM(P(o)) = c_2 \neq 0$, where c_i is rational. Both conditions are true for the special 6-D solution at 7-D light-cone boundary. Also now both options give the same equations.

What about the super variant of the variational principle?

1. Super-Taylor expansion must be carried out and normal ordering reduces the action to 5 independent terms according to the number $k \in \{0, ..., 4\}$ of quark pairs involved. It seems that only **Option II** is free of number theoretical problems due to normal ordering. Also in this case one has renormalization corrections to various terms in RE and IM. Inner product does not however give rise to additional terms. The degree of the polynomial $P_{n-k}(o_s)$ is equal to $n - k$ and decreases as the degree h of the monomial increases and normal ordering terms are present.

2. One can decompose action action density as $L = \sum L_k$ corresponding to different numbers k of quark pairs. The stationarity conditions hold true for the polynomial coefficient $P_{n-k}(o)$ of each oscillator operator monomial appearing in RE and IM. One has both $RE(P_{n-k}) = c_k \neq$ and $IM(P_{n-k}) = c_k \neq 0$ options. Both conditions are true for the special solutions. Without further conditions the option can depend on k and on the point of cognitive representation. $c_k \neq 0$ for some values of k guarantees that L to be non-vanishing so that the exponential of S can define a non-trivial S-matrix.

   Since an approximation of continuous case should be in question, the options should be same all points of the cognitive representation. In the lowest order approximation one obtains $k = 0$ solution obtained without super-symmetry. Normal ordering terms however modify the coefficients of $P(o)$ so that this solution is not exact.

3. Each monomial $P_{n-k}(o)$ defines its own space-time surface and conditions should hold true independently for each super-component L_k. Second option would be to consider vacuum expectation value of the action in which case one would have only single surface.

4. One would have purely local free field theory and the construction of S-matrix would be extremely simple. One could introduce CDs and the identification of hermitian conjugates of fermionic oscillator operators labelled by points at given boundary of CD as creation operators at time reflected points at opposite boundary. If one can talk about sub-CDs assignable to partonic 2-surfaces in M^8 picture one obtains similar identification for them. Also leptons would emerge from S-matrix.

To sum up, the second trial has a generalization although octonionic picture allows only the Killing vectors of translations of E^8 in the construction of o_s and q_s. The action principle replaces the earlier ansatz with solution in which one has roots of polynomials of $RE(P)$ and $IM(P)$ shifted by rational number. Also a renormalization of P takes place.

9.2.3 How the earlier vision about coupling constant evolution would be modified?

In [L81, L73] I have considered a vision about coupling constant evolution assuming twistor space $T(M^4) = M^4 \times S^2$. In this model the interference of the Kähler form made possible by the same signature of $S^2(M^4)$ and $S^2(CP_2)$ gives rise to a length scale dependent cosmological constant appearing defining the running mass squared scale of coupling constant evolution.

For $T(M^4)$ identified as $CP_3(3,h)$ the signatures of twistor spheres are opposite and Kähler forms differ by factor i (imaginary unit commuting with octonion units) so that the induced Kähler forms do not interfere anymore. The evolution of cosmological constant must come from the evolution of the ratio of the radii of twistor spaces (twistor spheres). This forces to modify the earlier picture.

1. $M^8 - H$ duality has two alternative forms with $H = CP_{2,h} \times CP_2$ or $H = M^4 \times CP_2$ depending on whether one projects the twistor spheres of $CP_{3,h}$ to $CP_{2,h}$ or M^4. Let us denote the twistor space $SU(3)/U(1) \times U(1)$ of CP_2 by F.
2. The key idea is that the p-adic length scale hierarchy for the size of 8-D CDs and their 4-D counterparts is mapped to a corresponding hierarchy for the sizes of twistor spaces $CP_{3,h}$ assignable to M^4 by $M^8 - H$-duality. By scaling invariance broken only by discrete size scales of CDs one can take the size scale of CP_2 as a unit so that $r = R^2(S^2(CP_{3,h})/R(S^2(F))$ becomes an evolution parameter.

   Coupling constant evolution must correspond to a variation for the ratio of $r = R^2(S^2(CP_{3,h})/R(S^2(F))$ and a reduction to p-adic length scale evolution is expected. A simple argument shows that Λ is inversely proportional to constant magnetic energy assignable to $S^2(X^4)$ divided by $1/\sqrt{g_2(S^2)}$ in dimensional reduction needed to induce twistor structure. Thus one has $\Lambda \propto 1/r^2 \propto 1/L_p^2$. Preferred p-adic primes would be identified as ramified primes of extension of rationals defining the adele so that coupling constant evolution would reduce to number theory.
3. The induced metric would vanish for $R(S^2(CP_{3,h}) = R(S^2(F))$. Λ would be infinite at this limit so that one must have $R(S^2(CP_{3,h}) \neq R(S^2(F))$. The most natural assumption is that one $R(S^2(CP_{3,h}) > R(S^2(F))$ but one cannot exclude the alternative option. Λ behaves like $1/L_p^2$. Inversions of CDs with respect to the values of the cosmological time parameter $a = L_p$ would produce hierarchies of length scales, in particular p-adic length scales coming as powers of $\sqrt{p}$. CP_2 scale and the scale assignable to cosmological constant could be seen as inversions of each other with respect to a scale which is of order 10^{-4} meters defined by the density of dark energy in the recent Universe and thus biological length scale.
4. The original model for the length scale evolution of coupling parameters [L81] would reduce to that along paths at $S^2(CP_2)$ and would depend on the ends points of the path only. This picture survives as such. Also in the modified picture the zeros of Riemann zeta could naturally correspond to the quantum critical points as fixed points of evolution defining the coupling constants for a given extension of rationals.

   Space-time surfaces the level of M^8 would be determined by octonionic polynomials determined by real polynomials with rational coefficients. The non-critical values of couplings might correspond to the values of the couplings for space-time surfaces associated with octonion analytic functions determined by real analytic functions with rational Taylor coefficients.

9.2.4 How is the p-adic mass scale determined?

p-Adic prime identified as a ramified prime of extension of rationals is assumed to determine the p-adic mass scale. There are however several ramified primes and somehow the quantum numbers of particle should dictate with ramified prime is chosen. There are two options to consider depending on whether both the extension and ramified prime are same for all spartners Option 1) or whether spartners can have different ramified primes (Option 2)). There also options depending on whether both leptons and quarks appear in their own super-Dirac actions (Option a) or whether only quarks appear in super-Dirac action (Option b implied by quark number conservation) . Call the 4 composite options Option 1a), 2a), 1b), 2b) respectively.

1. Consider first Options 1a) and 1b). The ramified prime is same for all states corresponding to the same degree of θ monomial and thus same value of $F+\overline{F}$. At the lowest $k=2$ level containing only fermions as local states the p-adic thermal masses of quarks and leptons are same for Option 1a) at least for single generation and for all generations if Q_2 does not depend on the genus g of the partonic 2-surface. For Option 1b) the masses would *not* be same for leptons and quarks since they would correspond to different degrees of super-octonionic polymials. For both options would have $n=n(g)$.

2. For Option 2 ramified prime depends on the state of the SUSY multiplet. This would require that for fermions with $k=2$ the integer n in $Q_2(x)=x^2\pm n$ has the p-adic primes assignable to leptons and quarks as factors.

   There are 6 different quarks and 6 different leptons with different p-adic mass scales. For Option 2a) n should have 12 prime factors which are near to power of 2. For leptons the factors correspond to Mersenne primes M_k, $k\in\{107,127\}$ and Gaussian Mersenne $k=113$. Gaussian Mersenne is complex integer. TGD requires complexification of octonions with imaginary unit i commuting with octonionic units so that also Gaussian primes are possible. This would resolve the question whether $P(t)$ can have complex coefficients $m+in$.

   For option 2b) quarks and leptons as local proton and neutron would have different extensions since the polynomials would be different. The p-adic primes for 6 quark states quarks would depend on genus. The value of n need not depend on genus g since the ramified primes p depends on g: $p=p(g)$.

   Since the polynomials describing higher levels of the dark hierarchy would be composites $P\circ Q_2$ with $P(0)=0$, Q_2 would be a really fundamental polynomial in TGD Universe. For Option 2b) it would be associated with quarks and would code for the elementary particles physics. The higher levels such as leptons would represent dark matter levels.

3. The crucial test is whether the mass scales of gauge bosons can be understood. If one assumes additivity of p-adic mass squares so that the masses for 2-local bosons would be p-adically sums of mass squared at the "ends" of the flux tube. If the discriminant $D=2n$ of Q_2 contains high enough number of factors this is possible. The value of the factor p for photon would be rather larger from the limits on photon mass. For graviton the value p would be even larger.

To sum up, the vision about dark phases suggests that the monopole phase is possible already for the minimal value $n=2$ involving only fundamental quarks for Option 2b), which is the simplest one and could solve the probelm of matter antimatter asymmetry. Bosons and leptons as purely local composites of quarks are possible for $n=6$. Rather remarkably, also empirical constraints [L26, L60] led to the conclusion $h=6h_0$. The condition is actually weaker: $h/h_0 \ mod \ 6=0$.

9.2.5 Super counterpart for the twistor lift of TGD

Twistor lift of TGD is now relatively well understood. I have made somewhat adhoc attempts to construct TGD analog of the Grassmannian approach so super-twistors. The proposed formalism for constructing scattering amplitudes seems to generalize as such to the twistor lift of TGD.

Could twistor Grassmannian approach make sense in TGD?

By M^8-H duality [L40] there are two levels involved: M^8 and H. These levels are encountered both at the space-time level and momentum space level. Do super-octonions and super-twistors make sense at M^8 level?

1. At the level of M^8 the high uniqueness and linearity of octonion coordinates makes the notion of super-octonion natural. By $SO(8)$ triality octonionic coordinates (bosonic octet 8_0), octonionic spinors (fermionic octet 8_1), and their conjugates (anti-fermionic octet 8_{-1}) would for triplet related by triality. A possible problem is caused by the presence of separately conserved B and L. Together with fermion number conservation this would require $\mathcal{N}=4$ or even $\mathcal{N}=4$ SUSY, which is indeed the simplest and most beautiful SUSY.
2. At the level of the 8-D momentum space octonionic twistors would be pairs of two quaternionic spinors as a generalization of ordinary twistors. Super octo-twistors would be obtained as generalization of these.

Also Grassmannian is replaced with super-Grassmannian and super-coordinates as matrix elements of super matrices are introduced.

1. The integrand of the Grassmannian integral defining the amplitude can be expanded in Taylor series with respect to θ parameters associated with the super coordinates C as rows of super $G(k,n)$ matrix.
2. The delta function $\delta(C,Z)$ factorizing into a product of delta functions is also expanded in Taylor series to get derivatives of delta function in which only coordinates appear. By partial integration the derivatives acting on delta function are transformed to derivatives acting on integrand already expanded in Taylor series in θ parameters. The integration over the θ parameters using the standard rules gives the amplitudes associated with different powers of θ parameters associated with Z and from this expression one can pick up the scattering amplitudes for various helicities of external particles.

The super-Grassmannian formalism is extremely beautiful but one must remember that one is dealing with quantum field theory. It is not at all clear whether this kind of formalism generalizes to TGD framework, where particle are 3-surfaces [L40]. The notion of cognitive representation effectively reducing 3-surfaces to a set of point-like particles strongly suggests that the generalization exists.

The progress in understanding of M^8-H duality throws also light to the problem whether SUSY is realized in TGD and what SUSY breaking does mean. It seems now clear that sparticles are predicted and SUSY remains in the simplest scenario exact but that p-adic thermodynamics causes thermal massivation: unlike Higgs mechanism, this massivation mechanism is universal and has nothing to do with dynamics. This is due to the fact that zero energy states are superpositions of states with different masses. The selection of p-adic prime characterizing the sparticle causes the mass splitting between members of super-multiplets although the mass formula is same for all of them.

The increased undestanding of what twistorialization leads to an improved understanding of what twistor space in TGD could be. It turns out that the hyperbolic variant $CP_{3,h}$ of the standard twistor space CP_3 is a more natural identification than the earlier $M^4\times S^2$ also in TGD framework but with a scale corresponding to the scale of CD at the level of M^8 so that one obtains a scale hierarchy of twistor spaces [L97]. Twistor space has besides the projection to M^4 also a bundle projection to the hyperbolic variant $CP_{2,h}$ of CP_2 so that a remarkable analogy between M^4 and CP_2 emerges. One can formulate super-twistor approach to TGD using the same formalism as will be discussed in this article for the formulation at the level of H. This requires introducing besides 6-D Kähler action and its super-variant also spinors and their super-variants in super-twistor space. The two formulations are equivalent apart from the hierarchy of scales for the twistor space. Also M^8 allows analog of twistor space as quaternionic Grassmannian HP_3 with signature (6,6). What about super- variant of twistor lift of TGD? consider first the situation before the twistorialization.

1. The parallel progress in the understanding SUSY in TGD framework [L92] leads to the identification of the super-counterparts of M^8, H and of twistor spaces modifying dramatically the physical interpretation of SUSY. Super-spinors in twistor space would provide the description of quantum states. Super-Grassmannians would be involved with the construction

of scattering amplitudes. Quaternionic super Grassmannians would be involved with M^8 description.

2. In fermionic sector only quarks are allowed by $SO(1,7)$ triality and that anti-leptons are local 3-quark composites of quarks. Gauge bosons, Higgs and graviton would be also spartners and assignable to super-coordinates of embedding space expressible as super-polynomials of quark oscillator operators. Super-symmetrization means also quantization of fermions allowing local many-quark states.

3. SUSY breaking would be caused by the same universal mechanism as ordinary massivation of massless states. The mass formulas would be supersymmetric but the choice of p-adic prime identifiable as ramified prime of extension of rationals would depend on the state of super-multiplet. ZEO would make possible symmetry breaking without symmetry breaking as Wheeler might put it.

Super-counterpart of twistor lift using the proposed formalism

The construction of super-coordinates and super-spinors suggests a straightforward twistorialization. One would only replace the super-embedding space and super-spinors with super-twistor space and corresponding super-spinors. Dimensional reduction should give essentially the 4-D theory apart from the variation of the radius of the twistor space predicting variation of cosmological constant. The size scale of CD would correspond to the size scale of the twistor space for M^4 and for CP_2 the size scale would serve as unit and would not vary.

1. Replace the coordinates of twistor space with superspinors expressed in terms of quark and anti-quark spinors lifted to the corresponding spinors of twistor space. Express 6-D Kähler action in terms of super-coordinates.

2. Replace H-spinors with the spinors of 12-D twistor space and assume only quark chirality. By the bundle property of the twistor space one can express the spinors as tensor products of spinors of the twistor spaces $T(M^4)$ and $T(CP_2)$. One can express the spinors of $T(M^4)$ tensor products of spinors of M^4 - and S^2 spinors locally and spinors of $T(CP_2)$ as tensor products of CP_2 - and S^2 spinors locally. Chirality conditions should reduce the number of 2 spin components for both $T(M^4)$ and $T(CP_2)$ to one so that there are no additional spin degrees of freedom.

   The dimensional reduction can be generalized by identifying the two S^2 fibers for the preferred extremals so that one obtains induced twistor structure. In spinorial sector the dimensional reduction must identify spinorial degrees of freedom of the two S^2s by the proposed chirality conditions also make them non-dynamical. The S^2 spinors covariantly constant in S^2 degrees of freedom.

   Define the twistor counterpart of the analog of modified Dirac action using same general formulas as in case of H.

3. Identify super spinors as sum of odd monomials of theta parameters with quark number 1 identified as oscillator operators. Identify super-Dirac action for twistor space by replacing $T(H)$ coordinates with their super variants and Dirac spinors with their super variants.

9.3 Are quarks enough to explain elementary particle spectrum?

TGD based SUSY involves super-spinors and super-coordinates. Suppose that one has a cognitive representation defined by the points of space-time surface with coordinates in an extension of rationals defining adele and belonging to the partonic 2-surfaces defined by the intersections of 6-D roots of octonionic polynomials with 4-D roots. This representation has H counterpart.

Cognitive representation gives rise to a tensor product of these algebras and the oscillator operators define a discretized version of fermionic oscillator operator algebra of quantum field theories. One would have interpretation as many-fermion states but the local many-fermion states would have particle interpretation. This would replace fermions of the earlier identification of elementary particles with SUSY multiplets in the proposed sense. This brings in large number of new particles. One can however ask whether the return to the original picture in which single partonic 2-surface corresponds to elementary particle could be possible. Certainly it would simplify the picture dramatically.

Could this picture explain elementary particle spectrum and how it would modify the recent picture?: these are the questions.

9.3.1 Attempt to gain bird's eye of view

Rather general arguments suggest that SYM action plus Super-Dirac action could explain elementary particle spectrum. Some general observations help to get a bird's eye of view about the situation.

1. The antisymmetric tensor products for fermions and anti-fermions produce states with same spectrum of electro-weak quantum numbers irrespectively of whether the fermion and anti-fermion are at same point or at different points. Which option is correct or are these options correspond analogous to two different phases of lattice gauge theory in which nodes *resp.* links determine the states? Only multi-local states containing fermions with identical spin and weak isospin at different points are not possible as local states.

   There is no point in denying the existence of either kind of states. What suggests itself is the generalization of electric-magnetic duality relating perturbative Coulomb phase in which ordinary particles dominate and the non-perturbative phase in which magnetic monopoles dominate. I have considered what I have called weak form of electic-magnetic duality already earlier [K126] but as a kind of self-duality stating that for homologically charged partonic 2-surfaces electric and magnetic fluxes are identical. The new picture would conform with the view of ordinary QFT about this duality.

2. The basic distinction between TGD and standard model is that color is not spin-like quantum number but represented as color partial waves basically reducing to the spinor harmonics plus super-symplectic generators carrying color quantum numbers. Spinor harmonics as such have non-physical correlation between color and electro-weak quantum numbers [K66] although quarks and leptons correspond to triality $t = 1$ and triality $t = 0$ states.

3. It turns out that one could understand quarks, leptons, and electro-weak gauge bosons and their spartners as states involving only single partonic 2-surface [K33]: this would give essentially the original topological model for family replication in which partonic 2-surfaces were identified as boundary components of 3-surface. In principle one can allow also quarks and gluons with unit charge matrix with color partial waves defining Lie-algebra generator as bosonic states. Could these states correspond to free partons for which perturbative QCD applies at high energies?

   Also color octet partial waves of electro-weak bosons and Higgs and the predicted additional pseudo-scalar - something totally new - are possible as both local and bi-local states. There would be no mixing of $U(1)_Y$ state and neutral $SU(2)_w$ states for color octet gluon. In this sense electro-weak symmetry breaking would be absent.

4. Electro-weak group as holonomy group of CP_2 can be mapped to the Cartan group of color group, and electro-weak and color quantum numbers would relate like spin and angular momentum to each other. This encourages to think that there are deep connections between electro-weak physics and color physics, which have remained hidden in standard model.

   The conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC) of hadron physics suggests a strong connection between color physics and electro-weak physics. There is also evidence for so called X bosons with mass 16.7 MeV [C12] [L34] suggesting in TGD framework that weak physics could have fractally scaled down copy in hadronic and even nuclear scales.

   Could ordinary gluons be responsible for CVC whereas colored variants of weak bosons and Higgs/pseudo-scalar Higgs would be responsible for PCAC? Usually strong force in hadronic sense is assigned with pion exchange. This approach does not work perturbatively. Could one assign strong force with the exchange of pseudo-scalar, and colored variants of gluons, pseudo-scalar, and Higgs?

5. Hitherto it has been assumed that homology charges (Kähler magnetic charges) characterize flux tubes connecting the two wormhole throats associated with the monopole flux of elementary particle. Could one understand the bi-local or multi-local objects of this kind as exotic phase analogous to magnetic monopole dominated phase of gauge theories as dual of Coulomb phase?

   Hadrons would certainly be excellent candidates for monopole dominated phase. Gluons would be pairs of quarks associated with homologically charged partonic 2-surfaces with opposite homology charges. Gluons would literally serve as "glue" in the spirit of lattice QCD. Gluons and hadrons would be multi-local states made from quarks and gluons as homologically trivial configurations with vanishing total homology charge.

6. Is there a correlation between color hyper-charge and homology charge forcing quarks and gluons to be always in this phase and forcing leptons to be homologically neutral? This could provide topological realization of color confinement. The simplest option is that valence quarks have homology charges $2, -1, -1$ summing up to zero. This was one of the first ideas in TGD about 38 years ago.

   One can also imagine that the homological quark charges $(3, -2, -1)$ summing up to zero define a classical correlate for the color triplet of quarks, a realization of Fermi statistics, and allow to understand color confinement topologically. The color partial waves in H would emerge at the embedding space level and characterize the ground states of super-symplectic representations. Color triplets of quarks and antiquarks could thus correspond to homology charges $(3, -2, -1)$ and $(-3, 2, 1)$ and neutral gluons could be superpositions of pairs of form $(q, -q)$, $q = 3, -1, -1$. Charged gluons as flux tubes would not be possible in the confined phase.

7. Is monopole phase possible also for leptons as general QFT wisdom suggests? For instance, could Cooper pairs could be flux tubes having members of Cooper pair - say electrons - at its ends and photons in this phase be superposition of fermion and anti-fermion at the ends of the flux tube and monopole confinement would make the length of flux tube short and photon massive in superconducting phase.

9.3.2 Comparing the new and older picture about elementary particles

The speculative view held hitherto about elementary particles in TGD Universe correspond to the TGD analog of the magnetic monopole dominated phase of QFTs. This view is considerably more complicated than the new view and involves unproven assumptions.

1. Identification of elementary particles

   Old picture: Ordinary bosons (and also fermions) are identified as multilocal many-fermion states. The fermions and anti-fermions would reside at different throats of the 2 wormhole contacts associated with a closed monopole flux tube associated with the elementary particle and going through wormhole contact to second space-time sheet. All elementary particles are analogous to hadron-like entities involving closed monopole flux tubes.

   One can raise objections against this idea. Leptons are known to be very point-like. One must also assume that the topologies of monopole throats are same for given genus in order that p-adic mass calculations make sense. The assumption that quarks correspond to monopole pairs makes things unnecessarily complex: it would would be enough to assume that they correspond to partonic 2-surfaces with monopole charge at the "ends" of flux tubes at given space-time sheet.

   One must assume that the genus of the 4 throats is same for known elementary particles: this assumption looks rather natural but can be criticized. The correlations forced by preferred extremal property should of course force the genera of wormhole throats to be identical.

   New picture: Elementary fermions would be partonic 2-surfaces. Leptons would have vanishing homology charge. Elementary bosons could be simply pairs of fermion anti-fermion located at the opposite ends of flux tubes. This would dramatically simplify the topological description of particle reactions. In the case of quarks however the homological space-time correlate of color confinement is attractive and would force monopole flux tubes. It turns out that this picture corresponds to the simplest level in the $h_{eff} = nh_0$ hierarchy. One could also see leptons and quarks as analogs of perturbative and non-perturbative monopole dominated phases of gauge theories.

Flux tubes could allow to understand phases like super-conductivity involving massivation of photons (Meissner effect). For instance, Cooper pairs could correspond closed flux tubes involving charged fermions at their "ends". In high Tc super-conductivity Cooper pairs in this sense would be formed at higher critical temperature and at lower critical temperature they would form quantum coherent phase [K90, K91]. Flux tube picture could also allow to understand strongly interacting phases of electrons.

2. Electroweak massivation

   Old picture: Electro-weak massivation has been assumed to involve screening of electroweak isospin by a neutrino pair at the second wormhole contact. The screening is not actually necessary in p-adic thermodynamics in its recent form since the thermal massivation is due to the mixing of different mass eigenstates.

   New picture: There is no need to add pairs of right- and left-handed neutrino to screen the weak charges in the scale of flux tube.

3. Identification of vertices

   Old picture: In old picture one could do almost without vertices: in the simplest proposal particle reactions would correspond to re-arrangements of fermions and antifermions so that fermion and antifermion number would be conserved separately. Therefore one needs an analog of vertex in which partonic 2-surface turns back in time in order to describe creation of particle pairs and emission of bosons identified as fermion-antifermion pairs.

   New picture: In vertices fermions and antifermions assignable to super spinor component would be redistributed between different orbits of partonic 2-surfaces meeting along their ends at the 6-D braney object in M^8 picture or turn backwards in time - the interpretation for this might be in terms of interaction with classical induce gauge field. What is new are the new vertices corresponding to the monomials of oscillator operators in the super-spinor. The original identification of particles (given up later) as single partonic 2-surface predicts genus-generation correspondence without additional assumptions. Both old and new picture predict also higher gauge boson genera for which some evidence exists: TGD predictions for the masses are correct [K73].

9.3.3 Are quarks enough as fundamental fermions?

For the first option - call it Option a) - quarks and leptons would define their own super-spinors. Whether only quark or lepton-like spinors are enough remains still an open question.

1. I have also considered the possibility that quarks are actually anti-leptons carrying homology charge and have anomalous em charge equal to $-1/3$ units. One might perhaps say that quarks are kind of anyonic states [K85]. It is however difficult to understand how the coupling to Kähler form could be dynamical and have values $n = -3$ and $n = 1$ for homologically neutral and charged states respectively. This would mean that only lepton like θ parameters appear in super-coordinates and only leptonic Dirac action is needed.
2. For this option proton would be bound state of homologically charged leptons. This in principle allows decays of type $p \rightarrow e^+ ...$ and $p \rightarrow e^+ + e^+ + \overline{\nu}$ requiring that the 3 partonic 2-surfaces fused with non-trivial homology charges fuse to single homologically trivial 2-surface. This form of proton instability would be different from that of GUTs. The topology changing process is expected to be slow. Is the introduction of two super-octonionic θ parameters natural assignable to B and L or is single parameter enough?
3. The coupling to Kähler form is not explicitly visible on the bosonic action but is visible in modified Dirac action. Could leptonic modified Dirac action transform to quark type modified Dirac action? This does not seem plausible.

The super-Dirac action for quarks however suggests another option, call it Option b). Leptons could be local 3-quark states.

1. Could one identify leptons as local 3 quark composites - essentially anti-baryons as far as quantum numbers are considered - but with different p-adic scale and emerging from the

super-Dirac action for quarks as purely local states with super-degree $d = 3$? Could one imagine totally new approach to the matter antimatter asymmetry?

Leptons would be purely local 3-quark composites and baryons non-local 3-quark composites so that charge neutrality alone would would guarantee matter-antimatter symmetry at fundamental level. Anti-quark matter would slightly prefer to be purely local and quark matter 3-local. The small CP violation due to the M^4 part of Kähler action forced by twistor lift should explain this asymmetry.

Leptons and anti-leptons would drop from thermal equilibrium with quarks at some stage in very early cosmology. The reason would be the slowness of the reactions producing local 3-quark composites from quarks. This slowness is required also by the stability of proton. Opposite matter anti-matter asymmetries at the level of both leptons and quarks would have been generated at this stage by CP violation and would have become visible after annihilation.

2. The local baryons would have much simpler spectrum and would correspond for given genus g (lepton generation) to the baryons formed from u and d quarks having however no color. There would be no counterparts for higher quarks. This would suggests that (L, ν_L) could be local analog of (p, n).

   For ordinary baryons statistics is a problem and this led to the introduction of quark color absent for local states. The isospin structure of the local analogs of p and n is not a problem. In uud (udd) type states allowed by statistics the spins of the u (d) quarks must have opposite spin. The analogs of Δ resonances are not possible so that one would obtain only the analogs of p and n!

3. The widely different mass scales for leptons and quarks would be due to locality making possible different ramified primes for the extension of rationals. The widely differing p-adic length scales of leptons and neutrinos could be undersood if the ramified prime for given extension can be different for the particles super-multiplets with same degree of octonionic polynomial. This could be caused by electroweak symmetry breaking. The vanishing electroweak quantum numbers of right-handed neutrino implies a dynamics in sharp contrast with that of neutron, whose dynamics would be dictated by non-locality.

   Also local pions are possible. The lepto-pions of lepto-hadron hypothesis [K117] could correspond to either local pions or to pion-like bound states of lepton and anti-leptons. There is evidence also for the muon- and tau-pions.

4. This idea might provide a mathematically extremely attractive solution to the matter antimatter asymmetry: matter and antimatter would be staring us directly into eyes. The alternative TGD inspired solution would be that small CP breaking would induce opposite matter-antimatter asymmetries inside long cosmic strings and in their exteriors so that annihilation period would lead to the observed asymmetry.

The decay $p \rightarrow e^+ + X$ could in principle take place and also the reverse decay $e^+ \rightarrow p + X$ can be considered in higher energy collisions of electron. The life-time for the decay modes predicted by GUTs is extremely long - longer than 1.67×10^{34} years (see `http://tinyurl.com/nqco2j7`). This fact provides a killer test for the proposal.

One should estimate the life-time of proton in number theoretic approach. The corresponding SUSY vertex corresponds to a Wick contraction involving 4 terms in super-Dirac action: the trilinear term for quarks and 3 linear terms.

1. The vertex would associated with a partonic 2-surface at which 3 incoming quark space-time sheets and outgoing electron space-time sheet meet. At quark level the vertex means an emanation of 3 quark lines from single 3-quark line at a point of partonic 2-surface in the intersection of the ends of 4 space-time surfaces with 6-sphere $t = r_n$ defining a universal root of octononic polynomial $P(o)$. t is M^4 time coordinate [L85]. The vertex itself does not seem to be small.

2. A fusion of 3 homologically non-trivial partonic 2-surfaces to single partonic 2-surface with trivial homology charge cannot occur since partonic 2-surfaces with different homology charge cannot co-incide.

   The reaction $p \rightarrow e^+ + ..$ can occur only if the quark-like partonic 2-surface fuse first to single homologically trivial partonic 2-surface: this would correspond to de-confinement phase transition for quarks. After that the 3 quark lines would fuse to single e^+ line.

(a) To gain some intuition consider two oppositely oriented circles around a puncture of a plane with opposite homology charges. The circles can reconnect to homologically trivial circle. Instead of circles one would now have 3 homologically trivial quark-like 2-surfaces at three light-like boundaries between Minkowskian and Euclidian regions of the space-time surface representing proton. First 2 quark-like 2-surfaces would touch and develop a wormhole contact connecting them. After that the resulting di-quark 2-surface and third quark 2-surface would fuse. The 3 quarks would be now analogous to de-confined quarks.

(b) At the next step the 3 separate quark lines would fuse to single one. This process must occur in single step since di-quark cannot correspond to single point because the Dirac super-polynomial is odd in oscillator operators and has quark number 1. The fusion point would correspond to 3 degenerate roots of the octonionic polynomial associated with the partonic 2-surface. This partonic 2-surface would be associated with $t = r_n$ hyperplane of M^4 and it would become leptonic 3-surface.

(c) 3 4-D sheets defined by the roots of the octonionic polynomial should meet at the vertex assignable to $t = r_n$ hyper-plane. This gives 2 additional conditions besides the conditions defining space-time sheets. This for both the protonic and positronic space-time sheets. One would have double quantum criticality. The tip of a cusp catastrophe serves as an analog. Since the coefficients of the octonionic polynomial are rational numbers, it might be possible to estimate the probability for this to occur: the probability could be proportional to the ratio N_2/N_0 of the number N_2 of doubly critical points to the number N_0 of all points with coordinates in the extension. This could make the process very rare.

It must be however emphasized that also the option in which also leptons are fundamental fermions cannot be excluded.

9.3.4 What bosons the super counterpart of bosonic action predicts?

It has been already noticed that the spectra of fermion-antifermion states are identical for local and bi-local states if one assumes that the wave function in the relative coordinate of fermion and anti-fermion is symmetric. This does not yet imply that the particle spectrum is realistic in the case of the bosonic action.

The situation is simplified considerably by the facts that color is not spin-like quantum number but analogous to momentum and can therefore be forgotten, family replication can be explained topologically, and depending B and L are separately conserved for Option a) but for Option b) L reduces to B since leptons would be local 3-quark composites. Let us restrict first the considered to Option b).

1. What kind of spectrum would be predicted? Consider first quark Clifford algebra formed by the oscillator operators defining the spartners of quark without any conditions on total quark number of the monomial Forgetting color, one has 8 states coming from left and right handed weak doublet and their anti-doublets. The numbers of elements $N(k)$ in Clifford algebra with given quark number $B = k = N(q) - N(\overline{q})$ is given by $N(k) = \sum 0 \leq q \leq 4 - kB(4, q + k) \times B(4, q)$ in terms of binomial coefficients.

   For $B = 0$ one obtains $N(0) = \sum 0 \leq q \leq 4B(4, q)^2 = 70$ states. The states corresponding to the same degree of oscillator operator polynomial and therefore having fixed $q + \overline{q} = B + \overline{B}$ have same masses. For $q - \overline{q} = 0$ bosonic state having $q = \overline{q} = 0$ with fixed k one has $q + \overline{q} = 4 + k$ so that one has $N(k) = B(4, k)^2$ ($N(k)$ states with same mass even after p-adic massivation). The numbers $N(k)$ are $(1, 4^2 = 16, 6^2 = 36, 4^2 = 16, 1)$.

2. The number of $q\overline{q}$ type states in super-Kähler action is 16. If one considers super-symmetrization of the bosonic action, these states would correspond to bosons. Could these states allow an interpretation in terms of the known gauge bosons and Higgs? Weak bosons correspond to 4 helicity doublets giving 8 states. Higgs doublet corresponds to doublet and its conjugate. There is also a pseudo-scalar doublet and its conjugate.

   Gluon cannot belong to this set of states, which actually conforms with the fact that gluon corresponds to CP_2 isometries rather than holonomies and gluon corresponds to CP_2 partial wave since color is not spin-like quantum number. Known particle would give 8+2+2=12 states and pseudo-scalar doublets the remaining 4. This kind of pseudo-scalar states are predicted both as local and the bi-local states. As already explained, one can however also understand gluons in this picture as octet color partial waves. Also color octet variants of $SU(2)_w$ weak bosons are predicted.

3. There are actually some indications for a Higgs like state with mass 96 GeV (see http://tinyurl.com/yxnmy8c7) . Could this be the pseudo-scalar state. Higgs mass 125 GeV is very nearly the minimal mass for $k = 89$. The minimal mass for $k = 90$ would be 88 GeV so that the interpretation as pseudo-scalar with $k = 90$ might make sense. The proposal that gluons could have also weak counterparts suggests that also the pseudo-scalar could have this kind of counterpart. The scaling of the mass of the Higgs like state with $k = 90$ to $k = 112$ ($k = 113$ corresponds to nuclear p-adic scale) would give mass $m(107) = 37.5$ MeV. Kh.U. Abraamyan *et al* have found evidence for pion like boson with mass 38 MeV [C4, C5, C9] (see http://tinyurl.com/y7zer8dw).

4. For Option b) only monomials with $N(q) - N(\overline{q}) = k = 1$ are allowed in q_s and leptons would be local 3-quark states and currents formed from them would appear in super-Kähler action. One would obtain $N(k = 1) = \sum 0 \leq q \leq 3B(4, q + 1) \times B(4, q) = 56$ statesi quark multiplet. There would be no doubling gauge bosons since only one H-chirality would be present. The observed bosons would be basically superpositions of quark-anti-quark pairs - either local or non-local.

Option b) involving only quarks as fundamental fermions does not predict unobserved gauge bosons whereas Option a) involving both leptons and quarks as fundamental fermions does so.

1. For Option a) taking into account quarks and restricting to electro-weak bosonic states to those with ($B = L = 0$) leads to a doubling of bosonic states at $k = 2$ level. The couplings of gauge bosons require that the states are superpositions of quark and lepton pairs with coefficients proportional to the coupling parameters. There are two orthogonal superpositions of quark and lepton pairs having orthogonal charge matrices with inner product defined by trace for the product. Ordinary gauge bosons correspond to the first combination.

   The orthogonality of charge matrices gives a condition on them. The charged matrices having vanishing trace can be chosen that they have opposite signs for opposite H-chiralities. For charge matrices involving unit matrix one must have charge matrices proportional to (-3,1) for (L,q) one must have (1,3) for second state. For gluons there is no condition if one treats color octet as Lie algebra generator with vanishing trace. The problem is that there is no experimental evidence for these bosons.

2. For Option b) leptons would be local 3-quark states and spartners of quarks. There would be no doubling gauge bosons since only one H-chirality would be present. The observed bosons would be basically superpositions of quark-anti-quark pairs - either local or non-local.

3. Option b) predicts that given quark with given isospin and M^4 helicity L or R), say u_L, has 5 spartners with same quantum numbers given by $u_L u_R \overline{u}_L$, $u_L d_R \overline{d}_L$, $u_L d_L \overline{d}_R$; $u_R d_L \overline{d}_L$; and $d_L d_R \overline{u}_L$. These 6 states cannot correspond to quark families and SUSY breaking due to the possibility of having different p-adic scale (ramified prime) making the mass scale of the spartners large is suggestive.

There would be two phases of matter corresponding to local and bi-local states (baryons would be 3-local states).

1. For both phases electro-weak bosons and also gluons with electro-weak charge matrix 1 to bosonic super action as states involving only single partonic 2-surface. As already mentioned, also color counterparts of $SU(2)_w$ bosons are possible. Also graviton could correspond to spartner for bosonic super-action. This would give essentially the original model for family replication. 2-surfaces would be homologically trivial in this phase analogous to Coulomb phase.

2. In the dual phase the bi-local states would correspond to non-vanishing homology charges for quarks at least. In this phase one should assign also to leptons 2 wormhole contacts. In super-conducting phase it could the second electron of Cooper pair. Massive photons in this phase would consist of homologically charged fermion pairs. Lepton could also involve screening lepton-neutrino pair at second wormhole contact.

The universality of gauge boson couplings provides a test for the model.

1. In bi-local model gauge bosons would correspond to representations of a dynamical symmetry group $SU(3)_g$ associated with the 3 genera [K33]. Bosons would correspond to octet and singlet representations and one expects that the 3 color neutral states are light. This would give 3 gauge boson generations. Only the couplings of the singlet representation of $SU(3)_g$ would be universal and higher generations would break universality both for both gluons and electro-weak bosons. There is evidence the breaking of universality as also for second and third generation of some weak bosons and the mass scales assigned with Mersenne primes above M^{89} are correct [K73].

2. If also fermions correspond to closed flux tubes with 2 wormhole contacts, the fermion boson couplings would correspond to the gluing of two closed flux tube strings along their both "ends" defined by wormhole contacts. A pair of 3-vertices for Feynman diagrams would be in question. If fermions are associated with single wormhole contact, its is not so easy to imagine how the closed bosonic flux tube could transform to single wormhole contact in the process. The wormhole contacts that meet and have opposite fermion numbers should disappear. This is allowed in the scenario involving 6-branes if the magnetic flux is trivial as it must be. For quarks and gluons the homology charges must be opposite if wormhole contact is to disappear.

3. If gauge bosons correspond to local fermion pairs, the most natural boson states have fixed value of g apart from topological mixing giving rise to CKM mixing just like fermions and universality is not natural. One can of course assume topological mixing guaranteeing it. Ordinary gauge bosons should be totally de-localized in the space of 3 lowest genera [K33] (analogous to constant plane waves) in order to have universality. The vertices could be understood as a fusion of partonic 2-surfaces. One should however understand why the mixing is so different for fermions and bosons. SUSY would suggest identical mixings.

The simplest model corresponds to quarks as fundamental fermions. Leptons and various bosons would be local composites in perturbative phase. In monopole dominate phase hadronic quarks would have homology charges and gluons would be pairs of quark and anti-quark at opposite throats of closed monopole flux tube. Basically particle reaction vertices would correspond to gluing of 3-surfaces along partonic 2-surfaces at 3-spheres defining $t = r_n$ hyperplanes of M^4.

9.4 Is it possible to have leptons as (effectively) local 3-quark composites?

The idea about leptons as composites of 3 quarks is strongly suggested by the mathematical structure of TGD. In [L92] a proposal that leptons are local composites of quarks. In [L112, L100, L101] a more general idea that leptons look like local composites of quarks in scale longer than CP_2 scale defining the scale of partonic 2-surface assignable to the particle.

A strong mathematical motivation for the proposal is that quark oscillator operators are enough to construct the gamma matrices of the "world of classical worlds" (WCW) and leptonic oscillator operators corresponding to opposite chirality for $H = M^4 \times CP_2$ spinors are somehow superfluous.

The proposal has profound consequences. One might say that SUSY in the TGD sense has been below our nose for more than a century. The proposal could also solve matter-antimatter asymmetry since the twistor-lift of TGD predicts the analog of Kähler structure for Minkowski space and a small CP breaking, which could make possible a cosmological evolution in which quarks prefer to form baryons and antiquarks to form leptons.

The objection against the proposal is that the leptonic analog of Δ might emerge. One must explain why this state is at least experimentally absent. In [L92] I did not develop a detailed argument for the intuition that one indeed avoids the leptonic analog of Δ. In this article the construction of leptons as effectively local 3 quark states allowing effective description in terms of the modes of leptonic spinor field in $H = M^4 \times CP_2$ having H-chirality opposite to quark spinors is discussed in detail.

9.4.1 Some background

Some background is necessary.

1. In TGD color is not spin-like quantum number but corresponds to color partial waves in CP_2 for H-spinors describing fundamental fermions distinguished from fermions as elementary particles.

   Different chiralities of H-spinors were identified in the original model as leptons and quarks. If quarks couple to $n = 1$ Kähler gauge potential of CP_2 and leptons to its $n = 3$ multiple, ew quantum numbers of quarks and leptons come out correctly and lepton and quark numbers are separately conserved.

2. Few years ago emerged the idea that fundamental leptons to be distinguished from physical leptons are bound states of 3-quarks. They could be either local composites or look like local composites in scales larger than CP_2 size scale assignable to partonic 2-surface associated with the lepton.

3. The spin, ew quantum numbers associated with $SU(2)_L \times U(1)_R$ are additive and these quantum numbers should come out correctly for states with leptonic spin and ew numbers.

   Fundamental leptons/quarks are not color singlets/triplets although have vanishing triality. The color quantum numbers also correlate with ew quantum numbers and M^4 helicity/handedness. Only the right-handed neutrino ν_R is a color singlet. The mass squared values of the resulting states deducible from the massless Dirac equation in H are non-vanishing since CP_2 partial waves carry mass of order CP_2 mass.

   The application of color octet generators of super-symplectic algebra (SSA) of super-Kac-Moody algebra (SKMA) with non-vanishing conformal weight contributing to mass squared can guarantee that color quantum numbers are those of physical leptons and quarks. In p-adic mass calculations one must assume negative half-integer valued ground state conformal weight $h_{vac} < 0$.

There are two challenges.

1. One must construct leptons as local of the effectively local 3-quark composites. The challenge is to prove that the resulting states with spin and ew quantum numbers possess the color quantum numbers of fundamental leptons.

2. A priori one cannot exclude leptonic analog of Δ resonance obtained in the quark model of baryons as states for which the wave functions in spin and ew spin degrees of freedom are completely symmetric. The color wave function would be indeed completely antisymmetric also for the leptonic Δ. The challenge is to explain why they do not exist or are not observed.

9.4.2 Color representations and masses for quarks and leptons as modes of $M^4 \times CP_2$ spinor field

It would be also highly desirable to obtain for the masses of 3-quark states the same expressions as embedding space Dirac operator predicts for leptonic masses. The masses depend on ew spin but are same for right and left-handed modes except in the case of right-handed neutrino. This could fixes the value of h_{vac} for leptons if it is assumed to be representable as 3-quark state. Empirical data are consistent with its absence from the spectrum.

The color representations associated with quark and lepton modes of $M^4 \times CP_2$ spinor fields were originally discussed by Hawking and Pope [A56] and are considered from TGD point of view in [K66].

Consider first quarks. For U_R the representations $(p+1,p)$ with triality 1 are obtained and $p = 0$ corresponds to color triplet 3. For D_R the representations $(p,p+2)$ are obtained and color triplet is missing from the spectrum ($p = 0$corresponds to $\overline{6}$). The representations and masses are the same for the left handed representations in both cases since the left handed modes are obtained by applying CP_2 Dirac operator to the right-handed modes.

The CP_2 contributions to the quark masses are given by the formula

$$
\begin{aligned}
m^2(U,p) &= \frac{m_1^2}{3}\left[p^2+3p+2\right] \quad , \quad p \geq 0 \ , \\
m^2(D,p) &= \frac{m_1^2}{3}\left[p^2+4p+4\right] = \frac{m_1^2}{3}(p+2)^2 \quad , \quad p \geq 0 \ , \\
m_1^2 &\equiv 2\Lambda \ .
\end{aligned} \tag{9.4.1}
$$

Here Λ is cosmoloigal constant characterizing the CP_2 metric. The mass squared splitting between U and D type states is given by

$$
\Delta m^2(D,U) = m^2(D,p) - m^2(U,p) = \frac{m_1^2}{3}(p+2) \ . \tag{9.4.2}
$$

Consider next leptons. Right handed neutrino ν_R corresponds to (p,p) states with $p \geq 0$ with mass spectrum

$$
m^2(\nu) = \frac{m_1^2}{3}\left[p^2+2p\right] \quad , \quad p \geq 0 \ . \tag{9.4.3}
$$

Charged handed charged leptons L correspond to $(p,p+3)$ states with mass spectrum

$$
m^2(L) = \frac{m_1^2}{3}\left[p^2+5p+6\right] \quad , \quad p \geq 0 \ . \tag{9.4.4}
$$

$(p,p{+}3)$ instead of (p,p) reflects the fact that leptons couple to 3-multiple of Kähler gauge potential. Right-handed neutrino has however vanishing total coupling.

Left handed solutions are obtained by operating with CP_2 Dirac operator on right handed solutions with one exception: the action of the Dirac operator on the covariantly constant right handed neutrino ($(p,p)=(0,0)$ state) annihilates it.

The mass splitting between charged leptons and neutrinos is given by

$$
\Delta m^2(L,\nu) = m^2(L,p) - m^2(\nu,p) = m_1^2(p+2) = 3\Delta m^2(D,U) \ , \tag{9.4.5}
$$

and is 3 times larger than the corresponding mass splitting. The mass splitting for leptons as states of type UUD and UDD is however different. If mass squared is additive as assumed in p-adic mass calculations one has $\Delta m^2(UDD,UUD) = \Delta m^2(D,U)$. The condition that the mass splitting for lepton states is the same as predicted by the identification as 3-quark states requires that the scale factor m_1^2 for 3 quarks states is 3 times larger than for quarks:

$$
m_1^2(L) = 3m_1^2(q) \ . \tag{9.4.6}
$$

9.4.3 Additivity of mass squared for quarks does not give masses of lepton modes

It would be natural that the same values for the leptons as 3-quark composites are same as for leptons as fundamental fermions. It is interesting to see whether the additivity of the mass squared values conforms with this hypothesis.

The sums of mass squared values for UUD (charged lepton) and UDD (neutrino) type states are given by

$$
\begin{aligned}
m^2(UUD) &= 2m(U)^2 + m(D)^2 = 3p^2+10p+8 \ , \\
m^2(UDD) &= 2m(D)^2 + m(U)^2 = 3p^2+11p+10 \ .
\end{aligned} \tag{9.4.7}
$$

These mass squared values are not consistent with the values proportional to the mass squared values proportional to p^2+5p+6 for L and to $p(p+2)$ for neutrinos. Covariantly constant right handed neutrino is not possible as a 3-quark state and this conforms with empirical facts.

The working hypothesis that mass squared is additive can be of course given up and a more general condition could be formulated in terms of four-momenta:

$$\begin{aligned} & p_1(U)+p_2(U)+p(D))^2 \\ & = 2m(U)^2+m(D)^2+2\sum[p_1(U)\cdot p_2(U)+(p_1(U)+p_2(U))\cdot p(D)] = km(L)^2 \ , \\ & (p(U)+p_1(D)+p_2(D))^2 \\ & = m(U)^2+2m(D)^2+2\sum[p_1(D)\cdot p_2(D)+(p_1(D)+p_2(D))\cdot p(U)] = km(\nu)^2 \ . \end{aligned} \tag{9.4.8}$$

k is proportionality constant. These condition give single constraint in the 9-dimensional 3-fold Cartesian power of 3-D mass shells. The constraint is rather mild.

9.4.4 Can one obtain observed leptons and avoid leptonic Δ?

The antisymmetry of the wave function under exchange of quark states gives a strong constraint and fixes the allowed states. Does one obtain states with the quantum numbers of observed leptons as color singlets, and can one avoid the leptonic analogue of Δ?

1. For ordinary leptons complete color antisymmetry would require a complete symmetry under permutations of spin-ew quantum numbers: there are four states altogether. Antisymmetrization would be completely analogous to that occurring for baryons as 3-quark states and would require that fundamental leptons are antisymmetric color singlets.

2. The standard quark model picture natural for strong isospin does not conform with spin-ew symmetries and the resulting states need not allow an interpretation as effective modes of fundamental leptonic spinors. For $SU(2)_L\times U(1)_R$ the situation changes since right-handed helicities are $SU(2)_L$ singlets. The states of form $U_LD_RU_R$ (L_R) and $D_LD_RU_R$ (ν_R) could correspond to right-handed leptons and states of form $U_LD_RU_R$ (L_L) and $D_LD_RU_R$ (ν_L) to left-handed leptons.

3. The manipulation of Yang Tableaux (`https://cutt.ly/Ik9SGuU`) allow to see when a color singlet is contained in all 3-fold tensor products - that is $3\otimes 3\times 3$, $3\times 3\times \overline{6}$, $3\times\overline{6}\times\overline{6}$, and $\overline{6}\times\overline{6}\times\overline{6}$ - formed from the representations 3 and $\overline{6}$.

   One has $3\otimes 3=\overline{3}+6$ and $\overline{6}\otimes\overline{6}=6+15_1+15_2$. Both $\overline{3}\otimes 3=1\oplus 2\times 8\oplus 10$ and $\overline{6}\otimes 6=1\oplus 8\oplus 27$ contain singlet and octet.

   Therefore both $3\otimes 3\times 3$ (UUU) and $\overline{6}\times\overline{6}\times\overline{6}$ (DDD) contain 1 and 8. $3\otimes 3\otimes\overline{6}$ (UUD corresponding to charged lepton) contains $6\otimes\overline{6}$ and therefore both 1 and 8. However, $3\otimes\overline{6}\otimes\overline{6}$ (neutrino as UDD) contains neither singlet nor octet.

4. The singlet contained in $\overline{6}\otimes 6$ should be also antisymmetric under the permutations of the color partial waves of quarks in 6. The singlet state has representation of the form $B_{KLM}A^KA^LA^M$, where $A^K=A^K_{rs}q^rq^s$ is the representation of $\overline{6}$ in terms of color triplet q^i. The tensor G_{KLM} should be antisymmetric. Since the singlet comes from Yang diagram as a vertical column, which corresponds to an anti-symmetric representation of S_3t, it seems that it is indeed antisymmetric.

   If this is the case, UUU and DDD singlets are indeed antisymmetric with respect to the exchange of quarks, and the state in spin-ew degrees of freedom can be totally symmetric.

5. As found, $\overline{6}\otimes 3\times 3$) (charged lepton as UUD) contains both 1 and 8 and 1 is antisymmetric as a full vertical column in the Yang diagram. If charged lepton corresponds to 1 it is analogous to proton in these degrees of freedom.

   $\overline{6}\otimes\overline{6}\times 3$) (neutrino as DDU) contains neither 1 nor 8. In both cases an entanglement between color and spin-ew degrees of freedom is implied.

Remark: Baryonic quarks reside at distinct partonic 2-surfaces and allow separate color neutralization by SSA or SKMA generators and are color triplets so that the standard picture about color confinement prevails in the baryonic sector.

6. If the 3-quark state is not a color octet, the operators needed to cancel the negative conformal weight must consist of at least two SSA or SKMA operators, which are color octets. UUD contains 8 and 1 but UDD does not. For neutrinos which cannot be color octets or singlets, at least 2 color octet generators are required to neutralize the color. For color singlet charged lepton this is not needed since p-adic thermodynamics allows a massless ground state. The difference charged leptons and neutrinos might relate to the fact that the long p-adic length scales for neutrinos are so long as compared to those for charged leptons.

   As has become clear, the neutral Δ type state UDD is not possible since color singlet and octet are not allowed and the neutralization of the negative conformal weight using at least two color generators as in the case of neutrino. Also for other components of Δ color singlet-ness requires at least two generators whereas octet requires only one generator. For color octets a complete symmetry in spin-ew degrees of freedom is not possible.

The conclusion is that charged lepton and charged components of Δ allow for color singlet completely symmetric wave function in spin-ew degrees of freedom unentangled from color. Neutrino and neutral Δ require entanglement between color and spin-ew degrees of freedom.

9.4.5 Are both quarks and leptons or only quarks fundamental fermions?

One of the longstanding open problems of TGD has been which of the following options is the correct one.

1. Quarks and leptons are fundamental fermions having opposite H-chiralities. This predicts separate conservation of baryon and lepton numbers in accordance with observations.
2. Leptons correspond to bound states of 3 quarks in CP_2 scale. This option is simple but an obvious objection is that they are expected to have mass of order CP_2 mass. Baryons could decay to 3 leptons. Also GUTs have this problem. This scenario also allows the existence of exotic leptons as analogs of Delta resonances for baryons.

I haven't been able to answer this question yet and several arguments supporting the quarks + leptons option have emerged.

Consider first what is known.

1. Color is real and baryons are color singlets like leptons.
2. In QCD, it is assumed that quarks are color triplets and that color does not correlate with electroweak quantum numbers, but this is only an assumption of QCD. Because of quark confinement, we cannot be sure of this.

The TGD picture has two deviations from the QCD picture, which could also cause problems.

1. The fundamental difference is that color and electroweak quantum numbers are correlated for the spinor harmonics of H in both the leptonic and quark sector. In QCD, they are not assumed to be correlated. Both u and d quarks are assumed to be color triplets in QCD, and charged lepton L and ν_L are color singlets.
   (a) Could the QCD picture be wrong? If so, the quark confinement model should be generalized. Color confinement would still apply, but now the color singlet baryons would not be made up of color triplet quark states, but would be more general irreducible representations of the color group. This is possible in principle, but I haven't checked the details.
   (b) Or can one assume, as I have indeed done, that the accompanying color-Kac Moody algebra allows the construction of "observed" quarks as color triplet states. In the case of leptons, one would get color singlets. I have regarded this as obvious. One should carefully check out which option works or whether both might work.

2. The second problem concerns the identification of leptons. Are they fundamental fermions with opposite H-chirality as compared to quarks or are they composites of three antiquarks in the CP_2 scale (wormhole contact). In this case, the proton would not be completely stable since it could decay into three antileptons.

   (a) If leptons are fundamental, color singlet states must be obtained using color-Kac-Moody. It must be admitted that I am not absolutely sure that this is the case.

   (b) If leptons are states of three antiquarks, then first of all, other electroweak multiplets than spin and isospin doublets are predicted. There are 2 spin-isospin doublets (spin and isospin 1/2) and 1 spin-isospin quartets (spin and isospin 3/2). This is a potential problem. Only one duplicate has been detected.

   (c) Limitations are brought by the antisymmetrization due to Fermi statistics, which drops a large number of states from consideration. In addition, masses are very sensitive to quantum numbers, so it will probably happen that the mass scale is the CP_2 mass scale for the majority of states, perhaps precisely for the unwanted states.

It is good to start by taking a closer look at the tensor product of the irreducible representations (irreps) of the color group [K66].

1. The irreps are labeled by two integers (n_1, n_2) by the maximal values of color isospin and hypercharge. The integer pairs (n_1, n_2) are not additive in the tensor product, which splits into a direct sum of irreducible representations. There is however a representation for which the weights are obtained as the sum of the integer pairs (n_1, n_2) for the representations appearing in the tensor product.

   Rotation group presentations simplified example. We get the impulse moment $j_1 + j_2, ...|j_1 - j_2|$. Further, three quarks make a singlet.

2. On basis of the triality symmetry, one expects that, by adding Kac-Moody octet gluons, the states corresponding to (p,p+3)-type and (p,p)-type representations can be converted to each other and even the conversion to color singlet (0,0) is possible. This is the previous assumption that I took for granted and there is no need to give it up.

Let's look at quarks and baryons first.

1. U type spinor harmonics correspond to (p+1,p) type color multiplets, while D type spinor harmonics correspond to (p,p+2) type representations. From these, quark triplets can be obtained by adding Kac-Moody gluons and the QCD picture would emerge. But is this necessary? Could one think of using only quark spinor harmonics?

2. The three-quark state UUD corresponds to irreducible representations in the decomposed tensor product. The maximum weight pair is (3p+2,3p+2) if p is the same for all quarks, while UDD with this assumption corresponds to the maximum weights (3p+1,3p+1+3). The value of p may depend on the quark, but even then we get (P,P) and (P,P+3) as maximal weight pairs. UUU and DDD states can also be viewed.

   Besides these, there are other pairs with the same triallity and an interesting question is whether color singlets can be obtained without adding gluons. This would change the QCD picture because the fundamental quarks would no longer be color triplets and the color would depend on the weak isospin.

3. The tensor product of a (p,p+3)-type representation and (possibly more) gluon octets yields also (p,p)-type representations. In particular, it should be possible to get (0,0) type representation.

Consider next the identification of leptons.

1. For leptons, neutrino nu$_L correspondstoa(p,p){-}typerepresentationandchargedlepton$L$toa(p+3,p) - typerepresentation.$

2. Could the charged antilepton correspond to a representation of the type UDD and antineutrino to a representation of the type UUD?

Here comes the cold shower! This assumption is inconsistent with charge additivity! UDD is neutral and corresponds to (p,p+3) rather than (p,p). You would expect the charge to be 1 if the correspondence for color and electroweak quantum numbers is the same as for the lepton + quark option!

UUD corresponds to (p,p) rather than (p,p+3) and the charge is 1. You would expect it to be zero. Lepton charges cannot be obtained correctly by adding charge +1 or -1 to the system.

In other words, the 3-quark state does not behave for its quantum numbers like a lepton, i.e. an opposite spinor with H-chirality as a spinor harmonic.

Therefore bound states of quarks cannot be approximated in terms of spinor modes of H for purely group-theoretic reasons. The reason might be that leptonic and quark spinors correspond to opposite H-chiralities. Of course, it could be argued that since the physical leptons are color singlets, this kind of option could be imagined. Aesthetically it is an unsatisfactory option.

To sum up, the answers to the questions posed above would therefore be the following:

1. Quark spinor harmonics can be converted into color triplets by adding gluons to the state (Kac-Moody). Even if this is not done, states built from three non-singlet quarks can be converted into singlets by adding gluons.
2. The states of the fundamental leptons can be converted into color singlets by adding Kac-Moody gluons. Therefore the original scenario, where the baryon and lepton numbers are preserved separately, is group-theoretically consistent.
3. Building of analogs of leptonic spinor harmonics from antiquarks is not possible since the correlation between color and electroweak quantum numbers is not correct. I should have noticed this a long time ago, but I didn't. In any case, there are also other arguments that support the lepton + quark option. For example, symplectic *resp.* conformal symmetry representations could involve only quarks *resp.* leptons.

9.5 Appendix: Still about the topology of elementary particles and hadrons

In its recent form TGD allows several options for the model of elementary particles [L92]. I wrote this piece of text because I got worried about details of the definition of wormhole contact appearing as basic building brick of elementary particle.

1. Wormhole contacts in 4-D sense (having Euclidian signature of induced metric) modellable as deformed pieces of CP_2 type extremals connecting Minkowskian space-time sheets (representable as graphs of a map $M^4 \to CP_2$) are identified basic building bricks of elementary particles. 3-D light-like orbits of 2-D wormhole throats- partonic 2-surfaces - at which the signature of induced metric changes from Euclidian to Minkowskian - partonic orbits - are assumed to be carriers of elementary particle quantum numbers localized at points representing intersections of fermionics string world sheets with the partonic 2-surfaces.
2. One can identify simplest wormhole contact as topological sum: two surfaces touch each other. Remove 3-D regions from both space-time sheets and connecting the topologically identical boundaries with a cylinder $X^2 \times D^1$, where X^2 has the topology of the boundary characterized by genus. The assumption that X^2 is boundary requires that its projection to CP_2 is homologically trivial.

   This is not consistent with the assumption that the flux tube carries monopole flux. These wormhole contacts are unstable and must be distinguished from wormhole contacts mediating monopole flux. I have not however defined the notion precisely enough.
3. One can consider two situations in which homologically non-trivial wormhole contact appears.

   Option I: Assume that the 3-D time=constant sections of two Minkowskian space-time sheets are glued together along their boundaries to form a closed 2-sheeted surface and the throats of wormhole contact - partonic 2-surfaces - serve as magnetic charges creating opposite fluxes. One can say that the two throats have opposite homology charges and therefore form a homologically trivial 2-surface to which one can glue the wormhole contact along its boundaries. The flux at sheet B could be seen as return flux from sheet A and the throat could be seen as very short monopole flux tube.

Option II: Assume no gluing along boundaries for the 3-D time=constant sections of two Minkowskian space-time sheets. In this case one must assume at least two wormhole contacts to get vanishing homology charges at both sheets. At both space-time sheets the throats of the contacts with opposite homology charges would be connected by monopole fluxes flowing through the wormhole contacts identifiable as a very short monopole flux tube. This makes sese also for the Option I and might be required since is not clear whether space-time having boundaries carrying monopole flux can be glued together.

Remark: One can also consider the light-like orbit of partonic 2-surface connecting its ends (the minimal distance between partonic 2-surfaces vanishes). The homology charges of ends are opposite in ZEO.

The proper identification of the model of elementary particles remains still open [L92] [K73]. What relevance do these two options this picture have to the model of elementary particles?

1. For Option I leptons and gauge bosons could be identified as single wormhole contact carrying non-trivial homology flux. The size scale of the closed space-time sheet would correspond to the Compton wavelength of the particle. This model is the simplest one at the level of scattering diagrams and was re-considered in [L92].

   Even Euclidian regions of single space-time sheet with vanishing homology charge can be considered as a model for leptons and gauge bosons. In this case it is however not clear how to understand how the size scale of the particle as Compton length could be understood at space-time level. This model was one of the first models. I have also considered the identification of the particle as boundary component of Minkowskian space-time surface.

2. Option II was assumed in the model following the original model for leptons and gauge bosons. It was also proposed that electroweak confinement as dual description of massivation takes place in the sense that the weak charges associated with the two wormhole contacts cancel each other. The size scale of flux tube at given sheet would correspond to the Compton length assignable to the particle. In this case scattering amplitudes are more complex topologically.

What about baryons?

1. The simplest model assumes that quarks do not differ from leptons and gauge bosons in any manner. The contribution of the quarks to masses of hadrons is very small fraction of total mass, which suggests that color flux tubes carrying also homology charge are present and give the dominating contribution.

   One can also consider a structure formed by color magnetic monopole flux tubes carrying most of the hadron mass with Minkowskian signature carrying flux of 2 units branching to two flux tubes carrying 1 unit each. The flux tubes would have length given by hadronic p-adic length scale. The ends of flux tubes would be wormhole throats connected by wormhole contacts to the mirror image of this structure. One can say that homology charges 2,-1,-1 assignable to the throats of single space-time sheet sum up to zero. This brings in mind color hypercharge. Could color confinement have vanishing of homology charge as classical space-time correlate?

2. In this article I have considered two alternative identification of leptons. Leptons and quarks could correspond to the different chiralities of $M^4 \times CP_2$ spinors and lepton and baryon numbers would be separately conserved. For second option leptons would b local 3-quark composites and therefore analogous to spartners of quarks: this option is possible only in TGD framework and the reason is that color is not spin-like quantum number in TGD framework. Baryon and lepton numbers would not be separately conserved.

   One can ask what could be the simplest mechanism inducing the decay of baryon as 3-quark composite involving only 3 wormhole contacts and giving lepton as a local 3-quark composite plus something. Wormhole throats of 3 quarks carrying the quark quantum numbers should fuse together to form a leptonic wormhole throat, and the 3 quark lines representing boundaries of string world sheets should fuse to single line. If the sum of quark homology charges is vanishing, lepton must have a vanishing homology charge unless the reaction involves also a step taking care of the conservation of homology charge as a decay of the resulting wormhole contact with vanishing monopole flux to two wormhole contacts with opposite monopole fluxes. Already the first step of the decay process is quite complex, and one can hope that the rate for the reaction is slow enough.

Part II

SOME APPLICATIONS

Chapter 10

Quantum Criticality and Dark Matter: part I

10.1 Introduction

Quantum criticality is one of the corner stone assumptions of TGD. The value of Kähler coupling strength fixes quantum TGD and is analogous to critical temperature. TGD Universe would be quantum critical. What does this mean is however far from obvious and I have pondered the notion repeatedly both from the point of view of mathematical description and phenomenology [K57, K126].

1. Criticality is characterized by long range correlations and sensitivity to external perturbations and living systems define an excellent example of critical systems - even in the scale of populations since without sensitivity and long range correlations cultural evolution and society would not be possible. For a physicist with the conceptual tools of existing theoretical physics the recent information society in which the actions of people at different side of globe are highly correlated, should look like a miracle.
2. The hierarchy of Planck constants with dark matter identified as phases of ordinary matter with non-standard value $h_{eff} = n \times h$ of Planck constant is one of the "almost-predictions" of TGD is definitely something essentially new physics. The phase transition transforming ordinary matter to dark matter in this sense generates long range quantal correlations and even macroscopic quantum coherence.

   Finding of a universal mechanism generating dark matter have been a key challenge during last ten years. Could it be that criticality is always accompanied by the generation of dark matter? If this is the case, the recipe would be stupifuingly simple: create a critical system! Dark matter would be everywhere and we would have observed its effects for centuries! Magnetic flux tubes (possibly carrying monopole flux) define the space-time correlates for long range correlations at criticality and would carry the dark matter. They are indeed key players in TGD inspired quantum biology.
3. Change of symmetry is assigned with criticality as also conformal symmetry (in 2-D case). In TGD framework conformal symmetry is extended and infinite hierarchy of breakings of conformal symmetry so that a sub-algebras of various conformal algebras with conformal weights coming as integer multiples of integer n defining h_{eff} would occur.
4. Phase separation is what typically occurs at criticality and one should understand also this. The strengthening of this hypothesis with the assumption $h_{eff} = h_{gr}$, where $h_{gr} = GMm/v_0$ is is the gravitational Planck constant originally introduced by Nottale. In the formula v_0 has dimensions of velocity, and will be proposed to be determined by a condition relating the size of the system with mass M to the radius within which the wave function of particle m with $h_{eff} = h_{gr}$ is localized in the gravitational field of M.
5. The condition $h_{eff} = h_{gr}$ implies that the integer n in h_{eff} is proportional to the mass of particle. The implication is that particles with different masses reside at flux tubes with different Planck constant and separation of phases indeed occurs.

6. What is remarkable is that neither gravitational Compton length nor cyclotron energy spectrum depends on the mass of the particle. This universality could play key role in living matter. One can assign Planck constant also to other interactions such as electromagnetic interaction so that one would have $h_{em} = Z_1Z_2e^2/v_0$. The phase transition could take place when the perturbation series based on the coupling strength $\alpha = Z_1Z_2e^2/\hbar$ ceases to converge. In the new phase perturbation series would converge since the coupling strength is proportional to $1/h_{eff}$. Hence criticality and separation into phases serve as criteria as one tries to see whether the earlier proposals for the mechanisms giving rise to large h_{eff} phases make sense. One can also check whether the systems to which large h_{eff} has been assigned are indeed critical.

The motivation for this work came from super-fluidity. Superfluids exhibit rather mysterious looking effects such as fountain effect and what looks like quantum coherence of superfluid containers, which should be classically isolated. These findings serve as a motivation for the proposal that genuine superfluid portion of superfluid corresponds to a large h_{eff} phase near criticality at least and that also in other phase transition like phenomena a phase transition to dark phase occurs near the vicinity.

10.1.1 Summary about applications of hierarchy of Planck constants to quantum criticality

During years I have proposed several examples about systems to which I have assigned non-standard value of Planck constant $h_{eff} = n \times h$. If the hypothesis about the connection with criticality is correct they should exhibit criticality and if $h_{eff} = h_{gr}$ hypothesis is true, also phase separation. Also the proposed mechanisms to generate dark matter should involve generation of criticality.

Particle Physics

In particle physics there are some possible applications for the new view about dark matter.

1. The perturbative expansion of scattering amplitudes in terms of gauge coupling strength or gravitational coupling strength ceases to converge at some critical value of the coupling parameter. This can be regarded as a critical phenomenon since a transition to strongly coupled phase with different properties takes place. For instance, in gauge theories according to the electric-magnetic duality the magnetic monopoles replaces charged particles as natural basic entities. The original proposal indeed was that the transition to large h_{eff} phase takes place when the perturbation theory in terms of say electromagnetic coupling strength $Z_1Z_2e^2/\hbar c$ ceases to converge. By replacing h with $h_{em} = Z_1Z_2/e^2h_{eff}$ the convergence is achieved and v_0/c replaces gauge coupling strength as coupling constant. A stronger hypothesis is that $h_{eff} = h \times h = h_{em}$ would connect this hypothesis with generalized conformal invariance and its breaking.
2. One of the earliest applications of TGD notion of color (associated not only with quarks and gluons but also leptons through color partial waves) was to explain anomalous production of electron-positron pairs in heavy ion collisions just above the Coulomb wall [C13, C10, C11, C14]. The TGD inspired hypothesis [K117] was that the electron positron pairs result from the decays of leptopions, which are pion-like color singlet bound states of color octet excitations of electron and positron but one could consider also other options. The identification as positronium is excluded since in this case direct decays would not be kinematically possible. The objection against postulating new elementary light particles is that they should make themselves visible in the decay widths of weak bosons.

   One manner to escape the problem is that spartners are heavy so that the decays of weak bosons to spartner pairs are not possible. Another explanation could be that the exotic particles involved correspond to non-standard value of Planck constant. As a matter fact, these particles could be very massive but due to the large value of h_{eff} would appear as effectively massless particles below the scaled-up Compton length.

   One can consider also other identifications for the new particles possibly involved. TGD predicts that right handed covariantly constant neutrino generates $\mathcal{N} = 2$ supersymmetry.

An elegant universal explanation for the absence of spartners would be that they are heavy but can make themselves visible as dark variants in scales below scaled up Compton length. Maybe the lepto-electrons are selectrons possibly moving in color octet partial wave!

This explanation would apply to all elementary elementary particles and predict that these particles can be produced only in critical systems. This would solve the puzzle created by the non-observation of standard $\mathcal{N} = 1$ SUSY and at LHC. Leptopion production indeed takes place at criticality: just above the Coulomb wall, when the incoming nucleus becomes able to collide directly with the target. It should be noticed that there is experimental evidence also for the leptopions associated with muon and tau [K117].

3. RHIC and later LHC found that the de-confinement phase transition (criticality is obviously involved!) supposed to lead to QCD plasma produced something different. The phase in question has long range correlations and exhibits the presence of string like structures decaying to ordinary hadrons. There is also evidence for strong parity breaking in the system and it is involved with the magnetic fields present [C1]. TGD interpretation could be in terms of a criticality in which long range correlations are generated as dark matter is created. Since strong parity breaking is involved, it seems that the dark particles must be associated with the weak length scale characterized by Mersenne prime M_{89}, which characterizes also the "almost-predicted"! scaled up copy of ordinary hadron physics characterize by Mersenne prime M_{107}. The mass scale is 512 times higher than for ordinary hadrons. Due to darkness the Compton scales of M_{89} hadrons and also weak bosons would be scaled up to about M_{107} p-adic scale if $h_{eff}/h = 2^9$ holds true.

Condensed Matter Physics

By its nature condensed matter physics provides rich repertoire of critical phenomena.

1. Different phases of same substance, say water, can be in phase equilibrium at criticality and dark matter. There are critical regions of parameter space -critical lines and critical points, in which the transitions between different phases are possible. Long range thermodynamical correlations are associated with these systems and the association with dark matter would suggest that dark matter could appear in these critical systems.
2. Different substances can form mixtures (`http://tinyurl.com/286nqx`). For instance, oil can mix to water in some parameter regions. This kind of systems are good candidates for critical systems. There is actually rich spectrum of mixtures. Solutions (`http://tinyurl.com/yz3hvq`), colloids (`http://tinyurl.com/yabljt8l`), dispersions (`http://tinyurl.com/bq3vm2m`) and the substances can be also in different phases (gas, liquid, solid) so that very rich spectrum of possibilities emerges. Is the generation of dark matter involved only with the phase transitions between different types of mixed phases or between mixed and non-mixed phase? Are some phases like gel inherently critical?
3. One example about criticality is phase transition to super-fluidity or super-conductivity. In the transition from super-conducitivity the value of specific hearts diverges having the shape of greek letter λ: hence the name lambda point. This suggests that in transition point the specific heat behaves like N^2 due to the quantum coherence instead of proportionality to N as usually. The strange properties of super-fluid, in particular fountain effect, could be understood in terms of $h_{eff} = h_{gr}$ hypothesis as will be discussed.

Living Matter

Biology is full of critical systems and criticality makes living matter highly sensitive to the external perturbations, gives maximal richness of structure, and makes them quantum coherent in macroscopic scales. Therefore it is not difficult to invent examples. The basic problem is whether the criticality is associated only with the transitions between different systems or with the systems themselves.

1. Sols and gels are very important in biology. Sol is definition a micture solid grains and liquid (say blood of cell liquid). Gel involves fixed solid structure and liquid. Sol-gel phase transition of the cell fluid takes place when nerve pulse travels along axon leading to the expansion of

the cell. Is the dark phase generated with the sol-gel transition or does it characterized sol. Perhaps the most logical interpretation is that it is involved with the phase transition.

2. Pollack's fourth phase of water resembles gel [L7]. Charge separation implying that the exclusion zones are negatively charged takes place. Charging takes place because part of protons goes to outside of EZ. TGD proposal is that protons go to magnetic flux tubes outside the region or to flux tubes which are considerably larger than EZ that most of their wave functions is located outside the EZ. Is fourth phase is permanently quantum critical? Or is the quantum criticality associated only with the transition so that magnetic flux tubes would carry protons but they would not be dark after the phase transition. EZs have a strange property that impurities flow out of them. Could the presence of dark flux tubes and $h_{eff} = h_{gr}$ forces the separation of particles with different masses?

3. The chirality selection of bio-molecules is a mystery from the point of view of standard physics. Large h_{eff} phase with so large value of Planck constant that the Compton length of weak bosons defines nanoscale, could explain this: weak bosons would be effectively massless and mediate long range interactions below the scaled up Compton scale. This phase transition could also force phases separation if $h_{gr} = h_{eff}$ holds true. If the masses of biomolecules with different handedness are slightly different also the values of h_{gr} would differ and the molecules would go to flux tubes with different value of h_{eff} - at least in the phase transition. The value of $\hbar_{gr} = GMm/v_0$ is in the range $10^{10} - 10^{11}$ for biomolecules so that the $\Delta n/n \simeq \Delta m/m \simeq 10^{-10} - 10^{-11}$ would be needed: this would correspond to an energy of eV which corresponds to the energy scale of bio-photons and visible light.

4. Neuronal membrane could be permanently a critical system since the membrane potential is slightly above the threshold for nerve pulse generation. Criticality might give rise to the dark magnetic flux tubes connecting lipids to the DNA nucleotides or codons assumed in the model of DNA as topological quantum computer. The braiding of the flux tubes would represent the effect of the nerve pulse patterns and would be generated by the 2-D flow of the lipids of the membrane forming a liquid crystal.

Fringe Physics

If one wants the label of crackpot it is enough to study critical phenomena. Those who try to replicate (or usually, to non-replicate) the claimed findings fail (or rather manage) easily since criticality implies careful tuning of the external parameters to demonstrate the phenomenon. Therefore the tragedy of fringe physicist is to become a victim of the phenomenon that he is studying.

1. Cold fusion involves bombarding of target consisting of Palladium target doped with deuterium using hydrogen atoms as projectiles. Cold fusion is reported to occur in a critical range of doping fraction. This suggests quantum criticality and large h_{eff} phase. One of the TGD based models generalizes the model of Widom and Larsen [C3]. The model assumes that weak interactions involving emission of W boson neutralizing the incoming proton makes possible to overcome the Coulomb wall. What would make the system critical? Does criticality make Palladium a good catalyst? Could the Palladium and with a large surface area define nano-scale variant of partonic 2-surface and large area which quite generally would make it effective as catalyst? Certainly this could hold true for bio-catalysts. Could Pd target be permanently in critical state? Effectiveness of catalyst might mean quantum coherence making chemical reaction rates proportional to N^2 instead of N, which could be the number of reactants of particular kind.

2. Di-electric breakdown in given medium occurs when the electric field strength is just above the critical value. A lot of strange claims have been assigned to these systems by non-professionals: in academic environment these phenomena are kind of taboo. Tesla studied them and was convinced that these phenomena involve new physics [K8]. The basic finding was that charges appeared everywhere: this certainly conforms with long range fluctuations and emergence of flux tubes carrying charged particles as dark matter to the environment. Unfortunately, recent day physicist regards Tesla's demonstrations as a mere entertainment and does not bother to ponder whether Maxwell's theory really explains what happens. It is tragic that the greatest intellectual achievements stop thinking for centuries. $h_{gr} = h_{eff}$

hypothesis allows even to estimate the length scales range in which these phenomena should appear.

Ball lightning (http://tinyurl.com/5jxd7k) is also a good candidate for an analogous phenomenon and has been admitted to be a real phenomenon after sixties even by skeptics.

C. Seward has discovered that di-electric breakdowns generate rather stable torus-like magnetic flux tubes around the breakdown current [H1] (http://tinyurl.com/ybdrpqju), which he calls ESTSs (Electron Spiral Toroid Spheromak) and proposed that ball lightnings might correspond to rotating ESTSs.

In TGD framework the stability might be understood if the toroid corresponds to a magnetic flux tube carrying monopole flux. This would allow to understand stability of the configuration and of ball lightning. Monopole flux tubes could also provide a solution to the plasma confinement problem plaguing hot fusion. Also ordinary lightnings involve poorly understood aspect such as gamma and X-ray bursts and high energy electrons. The common mystery is how the dissipation in atmosphere could allow this phenomena. A possible explanation would be in terms of dark flux tubes generated near criticality to the generation of lightning.

3. So called free energy systems [H6] (for TGD inspired view see the book [K103] include many phenomena claimed to involve a liberation of surplus energy. To my opinion, it is quite possible that over-unity energy production is a transient phenomenon and the dreams about final solution of energy problems will not be fulfilled. What makes these phenomena so interesting to me is that they might involve new physics predicted or at least allowed by TGD.

   The splitting of water represents besides magnetic motors (to be discussed below) a key example of free energy phenomena. In the splitting of water to oxygen and hydrogen the formation of Brown's gas [H6] (Wikipedia article about Brown's gas http://tinyurl.com/5tyl92 provides an amusing example full of "fringe science"s about how skeptic writes about something inducing cognitive dissonance in skeptic's mind) with strange properties was reported long time ago. For instance, Brown gas is reported to melt metals whose melting temperature is thousands of degrees although the Brown's gas itself has temperature of order 100 degrees Celsius.

   I have proposed an interpretation as large h_{eff} phase containing dark proton sequences at magnetic flux tubes and responsible for the liberation of energy as this phase transforms to ordinary one. Brown's gas could be essentially the fourth phase of water containing exclusion zones (EZs) discovered by Pollack [L7]. The TGD inspired model for them [L7] involves magnetic flux tubes at which part of protons in EZ is transferred and forms dark proton sequences-essentially dark protons. There a many ways to generate Brown's gas: for instance, cavitation due to the mechanical agitation and application of electric fields could do it. The expanding and compressing bubble created by acoustic wave in sono-luminescence and reported to have a very high temperature and maybe even allowing nuclear fusion, could be also EZ.

4. Water memory [I8, I9, I2] is one of the curse words of skeptic and related to scientific attempts to understand the claimed effects of homeopathy, which defines even stronger curse word in the vocabulary of skeptic - of equal strength as "remote mental interaction". The simple idea that the mere presence of original molecules could be replaced by electromagnetic representation of relevant properties of the molecule is utterly impossible for a skeptic to grasp - despite that also skeptic lives in information society. I have developed a model for water memory explaining also claimed homeopathic effects [K55] and this process has been extremely useful for the development of the model of living matter. Same mechanisms that apply to the model of living matter based on the notion of magnetic body, apply also to water memory and remote mental interactions.

   The key idea is that low energy frequency spectrum provides a representation for the bio-active molecules. The spectrum could be identified as cyclotron frequency spectrum associated with the magnetic bodies of EZs and allow them to mimic the bio-active molecule as far as the effects on living matter are considered. The mechanical agitation of the homeopathic remedy could generate EZs just as it generates cavitation. The model for dark proton sequences yields counterparts of DNA, RNA, amino-acids and even tRNA and genetic code based primitive life would be realized at fundamental particle level with biological realization serving as a higher level representation.

The above sections only list examples about systems where dark matter in TGD sense could appear. A lot of details remain to be understood. The basic question whether some of these systems are permanently near critical state or only in phase transitions between different phases.

Proposed Mechanisms For Generating Large h_{eff} Phase

I have proposed several mechanisms, which might generate large h_{eff} phase, and an interesting question is whether these mechanisms generate criticality.

1. Generation of strong electric fields near criticality for the di-electric breakdown is consistent with criticality and living matter would provide a key example in this respect. Teslas's strange findings support the view about presence of dark matter phases.
2. The findings of Cyril Smith [I5] suggesting a pairing between low and high em frequencies such that low frequency irradiation of bio-matter creates regions to which one can assign high frequency and corresponding wavelength as a size scale. TGD explanation would be that the ratio f_h/f_l of high and low frequencies equals to the $h_{eff}/h = n$, and there is a criticality in the sense that for integer values of this frequency ratio a phase transition transforming dark low energy photons to high frequency of same energy or vice versa can take place. The reverse transition might be interpreted as an analog of Bose-Einstein condensation for low frequency photons (recall the n-fold covering property). The criticality would thus be associated with the formation of the analog of Bose-Einstein condensate.
3. I have proposed that rotating systems could in certain circumstances make a transition to a critical state in which large h_{eff} phase is generated.
   (a) First motivation comes from a model for the findings reported by Russian experimentalists Roschin and Godin [H8] who studied a rotating magnetic system probably inspired by the work of british inventor Searl. The experimenters claim several unexpected effects near criticality for mechanical breakdown of the system. For instance, cylindrical magnetic walls of thickness of few centimeters with distance of order.5 meters are formed. The system starts to accelerate spontaneously. Cooling of the nearby environment is reported. Also visible light probably due to di-electric breakdown - another critical phenomenon - are reported.
   One of the proposed TGD inspired explanations [K11] suggests that there is energy and angular momentum transfer from the magnetic walls which could contain dark matter. Dark photons at cyclotron frequencies but possessing energies of visible photons could make the energy transfer very effective. One possibility is the change of direction for spontaneous dark magnetization emitting large amount of energy. Also collective cyclotron transitions reducing the angular momentum of Bose-Einstein condensate like state can be considered.
   (b) Second motivation comes from the magnetic motor of Turkish inventor Yildiz [H5, H9], which run for hours in a public demonstration. I have developed a model of magnetic motor, which might contain the essential elements of the motor of Yildiz.
   The key idea is that radial permanent magnets generate magnetic monopole flux tubes emanating radially through the stator and rotor returning back along z-axis. Monopole character implies that no current to preserve the magnetic field. This I think is essential. If the rotor consist of magnets tangential to a circle, a constant torque is generated. Angular momentum and energy conservation of course requires a feed of energy and angular momentum. If dark matter phase is generated, it could come from some magnetic body containing charged particles with spontaneous magnetization and carrying both spin and energy. Also angular momentum of cyclotron Bose-Einstein condensate can be considered. One possibility is that the dark matter associated with Earth estimated later to be a fraction of about $.2\times 10 \times -4$ of Earth's mass is the provider of angular momentum and energy. The system is certainly critical in the sense that it is near the mechanical breakdown and in some demonstrations the breakdown has also occurred. This of course raises the possibility that the energy feed comes from mechanical tensions.
   (c) Third motivation comes from a model of a rotating system to which constant torque is applied. This situation can be described in terms of potential function $V = \tau\phi$ and

modelled using Schrödinger equation [K61]. Since V is not periodic function of ϕ, the solution cannot be periodic if τ lasts forever. It is however possible to have a situation in which the duration T of τ is finite. In this case one can consider the possibility that the phase space which is in the simplest situation circle is replaced with its n-fold covering and solutions are periodic with period $n \times 2\pi$ during the period T and before it energy eigenstates for a free system. The average energy for the final state would be differ from that for the initial state and the difference would be the energy fed to the system equal to $\Delta E = \tau \Delta\phi$ classically. During energy feed the systems wave functions have $1/n$-fractional angular momenta unless one assumes $h_{eff} = n \times h$ phase.
What is intriguing that also stationary solutions are obtained: the equation reduces to that for Airy functions in this case. These solutions do not however satisfy periodicity condition for any finite n. Solutions located in a finite covering of circle cannot be energy eigenstates. Could the constancy of energy mean that no dissipation takes place and no energy is feed to the system.
This description brings in mind the general view about large h_{eff} phases as being associated with the breaking of conformal invariance. n could characterize the number of sheets of the covering of S^2. What does criticality correspond to now? Why should angular momentum and energy feed require or imply criticality? There is also a criticality associated with the change of n as the minimum number of periods that τ lasts. If this is the correct identification, the value of n would increase after every turn in positive energy ontology. In ZEO it would be pre-determined and determined by the duration of τ.
The motivation for the model comes from the ATPase molecule (http://tinyurl.com/y9jxsvr5), which is a basic tool in energy metabolism. ATPase can be regarded as a molecular motor taking its energy from the change of the energy of protons as they flow through the cell membrane. Three ADPs are transformed to ATP during single turn by giving them phosphate molecule. What could make the system critical? The system in question is not neuronal membrane but there is tendency to consider the possibility that also the mitochondrial membrane potential is near to breakdown value and the flow of protons through it is the counterpart for nerve pulse.

4. TGD inspired model [L6] for the recent findings about microtubules by the group of the group of Bandyonopadhyay. [J1, J8] is based on the assumption that the oscillatory em perturbation of the system induces generation of A type microtubules not present in Nature by a phase transition from B type microtubules. This phenomenon would take for a critical frequency and $f_h/f_l = n$ condition is suggestive. The proposal is that large h_{eff} phase is generated and gives rise to long range correlations at the level of microtubule so that 13-tubulin units combine to form longer units and the broken helical symmetry becomes un-broken symmetry. Quite recently also an observation of short lasting (nanoseconds) super-conductivity at room temperature (http://tinyurl.com/prvjp6y) induced by irradiation of high temperature super conductor with infrared light. The mechanism could be similar and involve $f_h/f_l = n$ condition.

In the first part of the chapter the general ideas about quantum criticality and phase transitions inspired by the TGD view about dark matter are discussed with some general applications. A good example is ferromagnetism and the phase transition to ferromagnetic state. In the remaining 3 parts more specific applications, mostly to various anomalies, are discussed.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at http://tgdtheory.fi/tgdglossary.pdf [L4].

10.2 Criticality In TGD Framework

In the following the proposal that quantum criticality or even criticality (with thermodynamical criticality included) could in TGD framework correspond to phase transition generating dark matter identified as phases of the ordinary matter with non-standard value $h_{eff} = n \times h$ of Planck constant and residing at dark magnetic flux tubes is discussed.

The precise meaning of quantum criticality has remained frustratingly fuzzy since the long range fluctuations and possible quanta associated with them do not correspond to any of the co-existing phases naturally but rather to transitions between them. Here Zero Energy Ontology (ZEO) in which basic objects are time evolutions suggests an elegant description: the ends of space-time surfaces at opposite boundaries of CD correspond to different values of h_{eff}. This would also give a connection with inclusions of hyper-finite factors: the integer m characterizing the inclusion equals to the radion $m = h_{eff}(f)/h_{eff}(i)$ of Planck constants for final and initial phases.

10.2.1 Mathematical Approach To Criticality

Concerning the understanding of criticality one can proceed purely mathematically. Consider first 2-dimensional systems and 4-D conformal invariance of Yang-Mills theories.

1. In 2-dimensional case the behavior of the system at criticality is universal and the dependence of various parameters on temperature and possible other critical parameters can be expressed in terms of critical exponents predicted in the case of effectively 2-dimensional systems by conformal field theory discovered by Russian theoreticians Zamolodchikov, Polyakov and Belavin [B8]. To my opinion, besides twistor approach this is one of the few really significant steps in theoretical physics during last forty years.
2. Twistors discovered by Penrose relate closely to 4-D conformal invariance generalized to Yangian symmetry [A17] [B21, B15, B16] in the approach developed by Nima Arkani-Hamed and collaborators recently. 2-dimensional conformal field theories are relatively well-understood and classified. String models apply the notions and formalism of conformal field theories.
3. The notion of conformal symmetry breaking emerges from basic mathematics and is much deeper than its variant based on Higgs mechanism able to only reproduce the mass spectrum but not to predict it: in p-adic thermodynamics based on super-conformal invariance prediction becomes possible [K77].

Basic Building Bricks Of TGD Vision

The big vision is that 2-D conformal invariance generalizes to 4-D context [K36, K57] and the conjecture is that it can be extended to Yangian symmetry assignable - not to finite-D conformal algebra of Minkowski space - but to the infinite-D generalization of 2-D conformal algebra to 4-D context. The details of this generalization are not understood but the building bricks have been identified.

1. One building brick is the infinite-D group of symplectic symmetries of $\delta M^4 - + \times CP_2$ having the structure of conformal algebra but the radial light-like coordinate r_M of δM^4_+ replacing complex coordinate z: r_M presumably allows a continuation to a hyper-complex analog of complex coordinate. One can say that finite-D Lie algebra defining Kac-Moody algebrais replaced with an infinite-D symplectic algebra of $S^2 \times CP_2$ and made local with respect to r_M
2. Second building brick is defined by the conformal symmetries of S^2 depending parametrically on r_M and are due to metric 2-dimensionality of δM^4_+. These symmetries are possible only in 4-D Minkowski space. The isometry algebra of δM^4_+ is isomorphic with that of ordinary conformal transformations (local radial scaling compensates the local conformal scaling).
3. Light-like orbits of the partonic 2-surfaces have also the analog of the extended conformal transformations as conformal symmetries and respect light-likeness.
4. At least in space-time regions with Minkowskian signature of the induced metric spinor modes are localized to string 2-D world sheets from the condition that electric charge is well-defined for the modes. This guarantees that weak gauge potentials are pure gauge at string world sheets and eliminates coupling of fermions to classical weak fields which would be a strong arguments against the notion of induced gauge field. Whether string world sheets and partonic 2-surfaces are actually dual as far as quantum TGD is considered, is still an open question.

The great challenge is to combine these building bricks to single coherent mathematical whole. Yangian algebra, which is multi-local with locus generalized from a point to partonic 2-surface would be the outcome. Twistors would be part of this vision: M^4 and CP_2 are indeed the unique 4-D manifolds allowing twistor space with Kähler structure [A57]. Number theoretic vision involving classical number fields would be part of this vision. 4-dimensionality of space-time surfaces would follow from associativity condition stating that space-time surfaces have associative tangent - or normal space as surfaces in 8-D embedding space endowed with octonionic tangent space structure. 2-dimensionality of the basic dynamical objects would follow from the condition that fundamental objects have commutative tangent - or normal space. String world sheets/partonic 2-surfaces would be commutative/co-commutative or vice versa.

Hierarchy Of Criticalities And Hierarchy Breakings Of Conformal Invariance

The TGD picture about quantum criticality connects it to the failure of classical non-determinism for Kähler action defining the space-time dynamics. A connection with the hierarchy of Planck constants [K47] and therefore dark matter in TGD sense emerges: the number n of conformal equivalence classes for space-time surfaces with fixed ends at the boundaries of causal diamond corresponds to the integer n appearing in the definition of Planck constant $h_{eff} = n \times h$.

A more detailed description for the breaking of conformal invariance is as follows. The statement that sub-algebra V_n of full conformal algebra annihilates physical states means that the generators L_{kn}, $k > 0$, $n > 0$ fixed, annihilate physical states. The generators L_{-kn}, $k > 0$, create zero norm states. Virasoro generators can be of course replaced with generators of Kac-Moody algebra and even those of the symplectic algebra defined above.

Since the action of generators L_m on the algebra spanned by generators $Ln + m$, $m > 0$, does not lead out from this algebra (ideal is in question), one can pose a stronger condition that all generators with conformal weight $k \geq n$ annihilate the physical states and the space of physical states would be generated by generators L_k, $0 < k < n$. Similar picture would hold for also for Kac-Moody algebras and symplectic algebra of $\delta M^4_+ \times CP_2$ with light-like radial coordinate of δM^4_+ taking the role of z. Since conformal charge comes as n-multiples of $\hbar$, one could say that one has $h_{eff} = n \times h$.

The breaking of conformal invariance would transform finite number of gauge degrees to discrete physical degrees of freedom at criticality. The long range fluctuations associated with criticality are potentially present as gauge degrees of freedom, and at criticality the breaking of conformal invariance takes place and these gauge degrees of freedom are transformed to genuine degrees of freedom inducing the long range correlations at criticality.

Changes of symmetry are assigned with criticality since Landau. Could one say that the conformal subalgebra defining the genuine conformal symmetries changes at criticality and this makes the gauge degrees of freedom visible at criticality?

Emergence Of The Covering Spaces Associated With The Hierarchy Of Planck Constants

The original vision was that the hierarchy of Planck constants corresponds to a hierarchy of n-fold singular coverings of the embedding space - or more precisely given causal diamond (CD) forming a book-like structure with pages labelled by the effective value of Planck constant $h_{eff}/h = n$. This view allowed to understand the basic aspects of the hierarchy: in particular, the relative darkness of phases associated with different values of n. The generalization of embedding space is however un-necessary. The non-determinism of Kähler action allows to replace singular coverings of embedding space with the identification of space-time surfaces with their singular coverings. Space-like 3-surfaces at the opposite boundaries of CD are connected by a multi-sheeted covering with sheets co-inciding at the ends.

How does this picture relate to the breaking of conformal symmetry? The idea is simple. One goes to n-fold covering space by replacing z coordinate by $w = z^{1/n}$. With respect to the new variable w one has just the ordinary conformal algebra with integer conformal weights but in n-fold singular covering of complex plane or sphere. Singularity of the generators explains why $L_k(w)$, $k < n$, do not annihilate physical states anymore. Sub-algebra would consist of non-singular generators and would act as symmetries and also the stronger condition that Lk, $k \geq n$, annihilates the physical states could be satisfied. Classically this would mean that the corresponding classical Noether charges for Kähler action are non-vanishing.

Another manner to look the same situation is to use z coordinate. Now conformal weight is fractionized as integer multiples of $1/n$ and since the generators with fractional conformal weight are singular at origin, one cannot assume that they annihilate the physical states: fractional conformal invariance is broken. Quantally the above conditions on physical states would be satisfied. Sphere - perhaps the sphere assigned with the light-cone boundary or geodesic sphere of CP_2 - would be effectively replaced with its n-fold covering space, and due to conformal invariance one would have n additional discrete degrees of freedom.

These discrete degrees of freedom would define n-dimensional Hilbert space space by the n fractional conformal generators. One can also second quantize by assigning oscillator operators to these discrete degrees of freedom. In this picture the effective quantization of Planck constant would result from the condition that conformal weights for the physical states are integers.

Other Connections

The values of effective Planck constants seems to have profound connections to several key ideas of TGD.

1. As already found, the connection with the hierarchy of broken conformal symmetries is highly suggestive. The integer $h_{eff}/h = n$ would characterize the sub-algebra of gauge conformal symmetries.
2. There seems to be a connection with negentropic entanglement [K72] associated with the density matrix of the state resulting in state function reduction, which is proportional to unit matrix - projector to an eigen space of density matrix. Negentropic entanglement would occur in the new discrete degrees of freedom most naturally. In the special 2-particle case negentropic entanglement corresponds to unitary entanglement encountered in quantum computation: large h_{eff} makes possible long-lived entanglement and its negentropic character implies that Negentropy Maximization Principle [K72] favors its generation. An interesting hypothesis to be killed is that the p-adic prime characterizing the space-time sheet string world sheet or partonic 2-surface divides n.
3. The realization of number theoretic univarsality in terms of strong form holography assumes that string world sheets and partonic 2-surfaces serve as "space-time genes" allowing continuation to preferred extremals. These 2-surfaces are characterized by parameters, which belong to an extension of rationals inducing extensions of p-adic number fields. One has a hierarchy of extensions of increasing complexity. Given extension is characterized by preferred primes known as ramified primes with the property that their decomposition to a products of of primes of extension contains higher powers than one. The product n of ramified rational primes characterizes the extension and is an integer.

   p-Adic continuations identifiable as imaginations would be due to the existence of p-adic pseudo-constants. The continuation could fail for most configurations of partonic 2-surfaces and string world sheets in the real sector: the interpretation would be that some space-time surfaces can be imagined but not realized [K80]. For certain extensions the number of realizable imaginations could be exceptionally large. These extensions would be winners in the number theoretic fight for survivalandcorresponding ramified primes would be preferred p-adic primes. NMP implies a generalization of p-adic length scale hypothesis stating that primes near but below powers of prime are physically favored and thus selected in number theoretic evolution.

   $h_{eff}/h = n$ gives the number of sheets of covering and a more plausible identification is as the dimension of covering assignable to number theoretic discretization of space-time surface [L31]. This dimension is the dimension of Galois group for the extension of rationals or its factor is highly suggestive and would lead to a direct connection with the number theoretic view about evolution.

Hierarchy of Planck constants, space-time surfaces as covering spaces, and adelic physics

From the beginning it was clear that $h_{eff}/h = n$ corresponds to the number of sheets for a covering space of some kind. First the covering was assigned with the causal diamonds. Later I assigned it with space-time surfaces but the details of the covering remained unclear. The final identification emerged only in the beginning of 2017.

Number theoretical universality (NTU) leads to the notion of adelic space-time surface (monadic manifold) involving a discretization in an extension of rationals defining particular level in the hierarchy of adeles defining evolutionary hierarchy. The first formulation was proposed in [K127] and more elegant formulation in [L31].

The key constraint is NTU for adelic space-time containing sheets in the real sector and various padic sectors, which are extensions of p-adic number fields induced by an extension of rationals which can contain also powers of a root of e inducing finite-D extension of p-adic numbers (e^p is ordinary p-adic number in Q_p).

One identifies the numbers in the extension of rationals as common for all number fields and demands that embedding space has a discretization in an extension of rationals in the sense that the preferred coordinates of embedding space implied by isometries belong to extension of rationals for the points of number theoretic discretization. This implies that the versions of isometries with group parameters in the extension of rationals act as discrete versions of symmetries. The correspondence between real and p-adic variants of the embedding space is extremely discontinuous for given adelic embedding space (there is hierarchy of them with levels characterized by extensions of rationals). Space-time surfaces typically contain rather small set of points in the extension ($x^n + yn^2 = z^n$ contains no rationals for $n > 2!$). Hence one expects a discretization with a finite cutoff length at space-time level for sufficiently low space-time dimension $D = 4$ could be enough.

After that one assigns in the real sector an open set to each point of discretization and these open sets define a manifold covering. In p-adic sector one can assign 8:th Cartesian power of ordinary p-adic numbers to each point of number theoretic discretization. This gives both discretization and smooth local manifold structure. What is important is that Galois group of the extension acts on these discretizations and one obtains from a given discretization a covering space with the number of sheets equal to a factor of the order of Galois group.

$h_{eff}/h = n$ was identified from the beginning as the dimension of poly-sheeted covering assignable to space-time surface. The number n of sheets would naturally a factor of the order of Galois group implying $h_{eff}/h = n$ bound to increase during number theoretic evolution so that the algebraic complexity increases. Note that WCW decomposes into sectors corresponding to the extensions of rationals and the dimension of the extension is bound to increase in the long run by localizations to various sectors in self measurements [K72]. Dark matter hierarchy represents number theoretical/adelic physics and therefore has now rather rigorous mathematical justification. It is however good to recall that $h_{eff}/h = n$ hypothesis emerged from an experimental anomaly: radiation at ELF frequencies had quantal effects of vertebrate brain impossible in standard quantum theory since the energies $E = hf$ of photons are ridiculously small as compared to thermal energy.

Indeed, since n is positive integer evolution is analogous to a diffusion in half-line and n unavoidably increases in the long run just as the particle diffuses farther away from origin (by looking what gradually happens near paper basket one understands what this means). The increase of n implies the increase of maximal negentropy and thus of negentropy. Negentropy Maximization Principle (NMP) follows from adelic physics alone and there is no need to postulate it separately. Things get better in the long run although we do not live in the best possible world as Leibniz who first proposed the notion of monad proposed!

10.2.2 Phenomenological Approach To Criticality

These statements do not have any obvious content for an experimentalist. One should have also a more concrete view about criticality. Theoretician would call this phenomenology.

1. Phase transitions and criticality are essential piece of being alive. Criticality means high sensitivity to signals and makes sensory perception possible. Criticality implies also long range correlations making us coherent units. The long range correlations between people who have never seen each other, like most of us, make possibly society, and demonstrate that the criticality appears also at collective levels of life and consciousness: usually biologists dismiss this. For physicist - at least me - the correlation between behaviors of him and his cat looks like a miracle!

2. Self-organization takes place by phase transitions and criticality with long range correlations.

In zero energy ontology (ZEO) self-organisation is however self-organisation for entire temporal patterns of space-time dynamics characterised by the 3-surfaces at the ends of causal diamond so that behaviours rather than states emerge. Also the synergy is made possible by criticality.

3. Criticality appears only in a very narrow range of control parameters and is therefore difficult to produce critical systems tend to fall off from criticality: good example is our society which is all the time at the verge of some kind of catastrophe.

One can build refined and highly predictive conformal field theory models but they do not tell what are the microscopic mechanisms behind criticality.

1. What are the space-time correlates for criticality and long range correlations? Something must quite concretely connect the sub-systems, bind them to single coherent unit at criticality. Magnetic flux tubes is of course the TGD based answer! But this is not enough. The long range correlations must be quantal and this requires that Planck constant is large: $h_{eff} = n/times; h$! Dark matter! The emergence of dark matter phase makes system critical! TGD Universe is critical at fundamental level and this implies that this dark matter is present at all length scales.

2. Long range interactions certainly define a basic characteristic of criticality. How do they emerge? Does some universal mechanism exist? $h_{eff} = n \times h$ hypothesis and p-adic length cale hypothesis allow to understand this. Weak bosons are effectively massless below weak boson Compton length - about 10^{-17} meters. When h_{eff} is scaled up by n, this Compton length is scaled up by n too. Weak interactions would become long ranged below much longer length scale, say even cellular scale and among other things explain chiral selection of biomolecules. Similar argument can be carried out for gluons and dark/p-adically scaled down) quarks and gluons would also appear in living matter.

3. Phase separation is key feature of criticality. How does this separation take place? Is there a universal mechanism as suggested by the fact that at criticality everything is universal. The answer relies on the notion of many-sheeted space-time, $h_{eff} = n \times h$ hierarchy, and the notion of gravitational Planck constant $\hbar_{gr} = GMm/v_0$ introduced originally by Nottale [E13]. The additional hypothesis [K100]

$$h_{eff} = h_{gr}$$

brings in gravitational interaction: the gravitational Planck constant is assigned with gravitation mediated by magnetic flux tubes connecting the two dark systems. The hypothesis predicts that h_{eff} is proportional to particle mass. This means each particle type is at its own dark flux tube/quantum nicely separated from each other. This would explain the phase separation at criticality even if the phase transformed after criticality to ordinary $h_{eff} = h$ phase. Pollack's exclusion zones (EZs) [L7] show the effect too: charge separation occurs and impurities in EZ get put of it. $h_{eff} = h_{gr}$ hypothesis implies that the scaled up Compton length becomes $\lambda_{gr} = GM/v_0$ and does not depend on particle mass at all: and ideal outcome concerning collective quantum coherence. In living matter with dynamics characterized by phase transitions this phase separation of different biologically important molecules would be in crucial role. The cell would not be anymore a random soup of huge number of different biomolecules but nicely arranged archive.

Critical reader - and even me after 9 ears of work! - can of course ask what the mass M appearing in the formula for h_{gr} really is. The logical answer is that it is the portion of matter that is dark: to this dark particles couple. In the Nottale's original model M and in TGD generalization of this model M corresponds to the entire mass of say Sun. This makes sense only if the approximate Bohr orbits in solar system reflect the situation when most of the matter in solar system was dark. Nowadays this is not the case anymore. For Earth the portion of dark matter in TGD sense should be something like 4×10^{-4} as becomes clear by just looking the values of the energies associated with dark cyclotron photons and requiring that they are in the range of bio-photon energies (dark photons would transforming to ordinary photons produce bio-photons). Without this assumption the range of bio-photon energies would be above 40 keV.

Besides dark matter also p-adically scaled up variants of weak interaction physics are possible: now weak bosons would be light but not massless above the Compton length which would be scaled up. In the TGD based model of living matter both dark matter and p-adically scaled up variants of particles appear and both are crucial for understanding metabolism. Both kind of phases could appear universally in critical systems. Dark matter would be a critical phenomenon and appear also in thermodynamical phase transitions, not only in quantum phase transitions.

Also so called free energy phenomena, cold fusion, remote mental interactions, etc are critical phenomena and therefore very difficult to replicate unless one knows this so that it is very easy to label researchers of these phenomena crackpots. The researchers in these fields could be seen as victims of the phenomenon they are studying! Life of course is also a critical phenomenon but even the vulgar skeptics are living and conscious beings and usually do not try to deny this!

10.2.3 Do The Magnetic Flux Quanta Associated With Criticality Carry Monopole Flux?

TGD allows the possibility that the magnetic flux quanta associated with criticality carry monopole flux. In Maxwellian electrodynamics this is not possible. These flux tubes are associated with elementary particles: in this case they have open string like portions at parallel space-time sheets connected at their ends by wormhole contacts to form a closed two-sheeted loop. Since the magnetic monopole flux is conserved along the flux tube, one has full reason to wonder whether these closed magnetic flux tubes can be created from vacuum.

One can imagine two ways to create flux loops: in a continuous energy conserving manner classically or by quantum jump in which quantum sub-Universe associated with given causal diamond (CD) is re-created (recall that causal diamonds define the observable Universes and they have finite size as intersections of future and past directed light-ones)

Consider for simplicity flux tubes which are circular. How the flux tubes can be generated?

1. One possibility is that an existing circular flux tube splits into two. This would take place by self-reconnection: circular flux tubes evolves first a figure eight shape, and after that self-reconnects and splits to two circular flux tubes. Figure eight shape is necessary because the direction of the conserved magnetic flux defines orientation and flux tube portions with opposite orientations cannot join. This mechanism allows replication of flux tubes and could be behind the $1 \rightarrow 2$ decays of elementary particles and the reverse reactions. It could be also behind biological replication at both DNA and cell level, and even higher levels. The reconnection of U-shaped flux tubes for two systems so that they become connected by a pair of flux tubes is the reverse of this process and is proposed to define fundamental mechanism of directed attention.

2. Can one imagine a purely classical mechanism in which flux tubes would be generated from nothing? An idealization as a closed string allows to imagine a closed string which begins from point and expands: in string models this kind of closed strings indeed pop up from vacuum. Energy conservation however forbids the classical occurrence of this process. Therefore this process is possible only in path integral formalism which allows processes, which are classically impossible.

   In TGD framework space-time surfaces appearing in the functional integral are extremals of Kähler action and conserve energy so that this kind of process is impossible. It is difficult to say what happens when the string is replaced with a flux tube having a a finite thickness: could this make it possible an energy conserving process in which initial state would not contain flux tubes but final would contain flux tubes? At elementary particle level this would mean generation of a particle or a pair from vacuum but this does not take place. Note that the development of Higgs expectation can be interpreted as generation of new vacuum state which contains Higgs bosons: TGD counterpart of the ground state would be a superposition of states containing various numbers of flux loops.

3. One can however consider a *quantum jump* generating flux tube from nothing. The sequence of quantum jumps consist of sub-sequences consisting of state function reductions to a fixed boundary of CD ("upper" or "lower"). A sub-sequence defining self corresponds to a sequence of repeated quantum measurements having no effect on the state in ordinary quantum measurement theory. In TGD state function reduction has effect on the second boundary. Or to

be precise, on the wave function in the moduli space associated with the second boundary with moduli characterising among other things the temporal distance from the fixed boundary. This effect gives rise to the experienced flow of time as increase of the average temporal distance between the tips of CD and also to its arrow.

These state function sequences do not last for ever (self has finite lifetime!): Negentropy Maximization Principle (NMP) eventually forces state function reduction at the opposite boundary of CD. The new state can contain flux loops which did not exist in the initial state. These flux loops could exists also outside the CD but this is not relevant for the physics experienced by the conscious observer associated with given CD.

The generation of this kind of monopole flux loops from nothing could be seen as a direct proof for macroscopic quantum jumps re-creating the Universe. Penrose proposed something similar in Shadows of Mind: quasicrystals are non-periodic lattices which look like lattices but - unlike ordinary crystals - cannot be generated by gradual lattice growth but must pop up in quantal manner to existence.

10.3 What's New In TGD Inspired View About Phase Transitions?

The comment of Ulla mentioned Kosterlitz-Thouless phases transition and its infinite order. I am not a condensed matter physicist so that my knowledge and understanding are rather rudimentary and I had to go to Wikipedia (see `http://tinyurl.com/ybevezgf`). I realized that I have not payed attention to the classification of types of phase transitions, while speaking of quantum criticality [?]. Also the relationship of ZEO inspired description of phase transitions to that of standard positive energy ontology has remained poorly understood. In the following I try to represent various TGD inspired visions about phase transitions and criticality in organized manner and relate them to the standard description.

10.3.1 About Thermal And Quantum Phase Transitions

It is good to beging with something concrete. Wikipedia article lists examples about different types of phase transitions. These phase transitions are thermodynamical.

1. In first order phase thermodynamical phase transitions heat is absorbed and phases appear as mixed. Melting of ice and boiling of water represent the basic examples. Breaking of continuous translation symmetry occurs in crystallization and symmetry is smaller at low temperature. One speaks of spontaneous symmetry breaking: thermodynamical fluctuations are not able to destroy the configuration breaking the symmetry.

2. Second order phase transitions are also called continuous and they also break continuous symmetries. Susceptility diverges, correlation range is infinite, and power-law behaviour applies to correlations. Ferromagnetic, super-conducting, and superfluid transitions are examples. Conformal field theory predics power-law behavior and infinite correlation length. Infinite susceptility means that system is very sensitive to external perturbations. First order phase transition becomes second order transition at critical point. Here the reduction by strong form of holography might make sense for high Tc superconductors at least (they are effectively 2-D).

3. Infinite order phase transitions are also possible. Kosterlitz-Thouless phase transition occurring in 2-D systems allowing conformal symmetries represents this kind of transition. These phase transitions are continuous but do not break continuous symmetries as usually.

4. There are also liquid-glass phase transitions. Their existence is hypothetical. The final state depends on the history of transition. Glass state itself is more like an on going phase transition rather than phase.

These phase transitions are thermal and driven by thermal fluctuations. Also quantum phase transitions (see `http://tinyurl.com/yblptwr6`) are possible.

1. According to the standard definition they are possible only at zero temperature and driven by quantum fluctuations. For instance, gauge coupling strength would be analogous to quantum temperature. This is a natural definition in standard ontology, in which thermodynamics and quantum theory are descriptions at different levels.

   Quantum TGD can be seen as a square root of thermodynamics in a well-defined sense and it makes possible to speak about quantum phase transitions also at finite temperature if one can identify the temperature like parameter characterizing single particle states as a kind of holographic representations of the ordinary temperature.

2. The traces of quantum phase transitions are argued to be visible also at finite temperatures if the energy gap is larger than the thermal energy: $\hbar\omega >\geq T$. In TGD framework Planck constant has a spectrum $h_{eff}/h = n$ and allows very large values. This allows quantum phase transitions even at room temperature and TGD inspired quantum biology relies crucially on this. What is of special interest that also ordinary thermal phase transitions might be accompanied by quantum phase transitions occurring at the level of magnetic body and perhaps even inducing the ordinary thermal phase transition.

3. Quantum critical phase transitions occur at critical point and are second order phase transitions so that susceptibility diverges and system is highly sensitive to perturbations and so in wide range around critical temperature (zero in standard theory). Long range fluctuations are generated and this conforms with the TGD vision about the role of large h_{eff} phases and generalized conformal symmetry: which also implies that the region around criticality is wide (exponentially decaying correlations replaced with power law correlations).

10.3.2 Some Examples Of Quantum Phase Transitions In TGD Framework

TGD suggests some examples of quantum phase transition like phenomena.

1. Bose-Einstein (BE) condensate consisting of bosons in same state would represent a typical quantum phase. I have been talking a lot about cyclotron BE condensates at dark magnetic flux tubes [K63, K90, K91]. The bosonic particles would be in the same cyclotron state. One can consider also the analogs of Cooper pairs with members at flux tubes of a pair of parallel flux tubes with magnetic fields in same or opposite direction. One member at each tube having spin 1 or zero. This would give rise to high T_c superconductivity.

2. One natural mechanism of quantum phase transition would be condensation to a new single particle state. The rate for an additions of new particle to condensate is proportional to $N+1$ and disappearance of particle from it to N, where N is the number of particles in condensate. The net rate for BE condensation is difference of these and non-vanishing.

   Quantum fluctuations induce phase transition between states of this condensate at criticality. For instance, cyclotron condensate could make a spontaneous phase transition to a lower energy state by a change of cyclotron energy state and energy would be emitted as a dark cyclotron radiation. This kind of dark photon radiation could in turn induce cyclotron transition to a higher cyclotron state at some other flux tube. If NMP holds true it could pose restrictions for the occurrence of transitions since one expects that negentropy is reduced. The transitions should involve negentropy transfer from the system.

   The irradiation of cyclotron BE condensate with some cyclotron frequency could explain cyclotron phase transition increasing the energy of the cyclotron state. This kind of transition could explain the effecs of ELF em fields on vertebrate brain [J6] in terms of cyclotron phase transition and perhaps serving as a universal communication and control mechanism in the communications of the magnetic body with biological body and other magnetic bodies [K44]. The perturbation of microtubules by an oscillating voltage [J1] (see `http://tinyurl.com/ze366ny`) has been reported by the group of Bandyonophyay [J8] to induce what I have interpreted as quantum phase transition [L18] (see `http://tinyurl.com/yatfreqe`).

   External energy feed is essential and dark cyclotron radiation or generalized Josephson radiation from cell membrane acting as generalized Josephson junction and propagating along flux tubes could provide it. Cyclotron energy is scaled up by h_{eff}/h and would be of the order of biophoton energy in TGD inspired model of living matter and considerably above thermal energy at physiological temperature.

3. Also quantum phase transitions affecting the value of h_{eff} are possible [K89] When h_{eff} is reduced and frequency is not changed, energy is liberated and the transition proceeds without external energy feed (NMP might pose restrictions). Another option is increase of h_{eff} and reduce the frequency in such a way that single particle energies are not changed. One can imagine many other possibilities since also p-adic length scale leading to a change of mass scale could change. A possible biological application is to the problem of understanding how biomolecules find each other in the molecular soup inside cell so that catalytic reactions can proceed. Magnetic flux tubes pairs connecting the biomolecules would be generated in the reconnection of U-shaped tentacle like flux tubes associated with the reactants, and the reduction of h_{eff} for the flux tube pair would contract it and force the biomolecules near each other.

4. The model for cold fusion in TGD Universe relies on a process, which is analogous to quantum phase transition [L9] [K30]. Protons from the exclusion zones (EZs) of Pollack [L7] [L7] are transferred to dark protons at magnetic flux tubes outside EZ and part of dark protons sequences transform by dark weak decays and dark weak boson exchanges to neutrons so that beta stable dark nuclei are obtained with binding energy much smaller than nuclear binding energy. This could be seen as dark nuclear fusion and quantum analog of the ordinary thermal nuclear fusion. The transformation of dark nuclei to ordinary nuclei by h_{eff} reducing phase transition would liberate huge energy if allowed by NMP [K72] and explain the reported biofusion.

5. Energetics is clearly an important factor (in ordinary phase transitions for open system thermal energy feed is present). The above considerations assume that ordinary positive energy ontology effectively applies. ZEO [K72] allows to consider a more science fictive possibility. In ZEO energy is conserved when one considers single zero energy state as a time evolution of positive energy state. If single particle realizes square root of thermodynamics, one has superposition of zero energy states for which single particle states appear as pairs of positive and negative energy states with various energies: each state in superposition respects energy conservation. In this kind of situation one can consider the possibility that temperature increases and average single particle energy increases. In positive energy ontology this is impossible without energy feed but in ZEO it is not excluded. I do not understand the situation well enough to decide whether some condition could prevent this. Note however that in TGD inspired cosmology energy conservation holds only in given scale (given CD) and apparent energy non-conservation would result by this kind of mechanism.

10.3.3 ZEO Inspired View About Phase Transitions

This section begins with questions related to TGD based description of phase transitions, discusses the TGD view about the role of symmetries in phase transitions, and asks what new ZEO can give to the description of phase transitions.

Question related to TGD inspired description of phase transitions

The natural questions are for instance following ones.

1. The general classification of thermodynamical phase transitions is in terms of order: the order of the lowest discontinuous derivative of the free energy with respect to some of its arguments. In catastrophe theoretic description one has a hierarchy of criticalities of free energy as function of control variables (also other behavior variables than free energy are possible) and phase transitions with phase transitions corresponding to catastrophe containing catastrophe.... such that the order increases. For intance, for cusp catatrophe one has lambda-shaped critical line and critical point at its tip. Thom's catastrophe theory description is mathematically very attractive but I think that it has problems at experimental side. It indeed applies to flow dynamics defined by a gradient of potential and thermodynamics is something different.

In TGD framework the sum of Kähler function defined by real Kähler action in Euclidian space-time regions and imaginy Kähler action from Minkowskian space-time regions defining a complex quanty replaced free energy. This is in accordance with the vision that quantum TGD can be seen as a complex square root of thermodynamics. Situation is now infinite-dimensional and catastrophe set would be also infinite-D. The hierarchy of isomorphic super-conformal algebras defines an infinite hierarchy of criticalities with levels labelled by Planck constants and catastrophe theoretic description seems to generalize.

Does this general description of phase transitions at the level of dark magnetic body (field body is more general notion but I will talk about magnetic body (MB) in the sequel) allow to understand also thermodynamical phase transitions as being induced from those for dark matter at MB?

2. Quantum TGD can be formally regarded as a square root of thermodynamics. Does this imply "thermal holography" meaning that single particle states can represent ensemble state as square root of the thermal state of ensemble. Could one unify the notions of thermal and quantum phase transition and include also the phase transitions changing h_{eff}? Could MB make this possible?

3. How does the TGD description relate to the standard description? TGD predicts that conformal gauge symmetries correspond to a fractal hierarchy of isomorphic conformal sub-algebras. Only the lowest level with maximal conformal symmetry matters in standard theory. Are the higher "dark" levels something totally new or do they appear in the description of also ordinary phase transitions? What is the precise role of symmetries and symmetry changes in TGD description and is this consistent with standard description. Here the notion of field body is highly suggestive: the dynamics of field body could induce the dynamics of ordinary matter also in phase transitions.

There is a long list of questions related to various aspects of TGD based description of phase transitions.

1. In TGD framework NMP applying to single system replaces second law applying to ensemble as fundamental description. Second law follows from the randomness of the state function reduction for ordinary matter and in long length and time scales from the ultimate occurrence of state function reductions to opposite boundary of CD in ensemble. How does this affect the description of phase transitions? NMP has non-trivial implications only for dark matter at MB since it NMP does favor preservation and even generation of negentropic entanlement (NE). Does NMP imply that MB plays a key role in all phase transitions?

2. Does strong form of holography of TGD reduce all transitions in some sense to this kind of 2-D quantum critical phase transitions at fundamental level? Note that partonic 2-surfaces can be seen as carriers of effective magnetic charges and string world sheets carrying spinor modes accompany magnetic flux tubes. Could underlying conformal gauge symmetry and its change have practical implications for the description of all phase transitions, even 3-D and thermodynamical phase transitions?

3. Could many-sheetedness of space-time - in particular the associated p-adic length scale hierarchy - be important and could one identify the space-time sheets whose dynamics controls the transition? Could the fundamental description in terms of quantum phase transitions relying on strong form of holography apply to all phase transitions? Could dark phases at MB be the key to the description of also ordinary thermodynamical phase transitions? Could one see dark MB as master and ordinary matter as slave and redue the description of all phase transitions to dark matter level.

Could the change of h_{eff} for dark matter at field body accompany any phase transition - even thermodynamical - or only quantum critical phase transition at some level in the hierarchy of space-time sheets? Or are also phase transitions involving no change of h_{eff} possible? Do ordinary phase transitions correspond to these. What is the role of h_{eff} changing "transitons" and their dynamical symmetries?

4. The huge vacuum degeneracy of Kähler action implies that any space-time surface with CP_2 projection that is Lagrangian manifold and has therefore dimension not larger than two, is vacuum extremal. The small deformations of these vacuum extremals define preferred

extremals. One expects that this vacuum degeneracy implies infinite number of ground states as in the case of spin glass (magnetized system consisting of regions with different direction of magnetization). One can speak of 4-D spin glass. It would seem that the hierarchy of Planck constants labelling different quantum phases and the phase transitions between these phases can be interpreted in terms of 4-D spin glass property? Besides phases one would have also phase transitions having "transitons" as building bricks.

It seems that one cannot assign 4-D spin glass dynamics to MB. If magnetic flux tubes are carriers of monopole flux, they cannot be small local deformations of vacuum extremals for which Kähler form vanishes. Hence 4-D spin glass property can be assigned to flux tubes carrying vanishing magnetic flux. Early cosmology suggests that cosmic strings as infinitely flux tubes having 2-DCP_2 projection and carrying monopole flux are deformed to magnetic flux tubes and suffer topological condensation around vacuum extremals and deform them during the TGD counterpart of inflationary period.

Remark: Glass state looks like a transition rather than state and ZEO and 4-D spin glass description would seem to fit naturally to his situation: glass would be a 4-D variant of spin glass. The time scale of transition is long and one might think that h_{eff} at the space-time sheet "controlling" transition is rather large and also the change of h_{eff} is large.

Symmetries and phase transitions

The notion of symmetry is considerably more complex in TGD framework than in standard picture based on positive energy ontology. There are dynamical symmetries of dark matter states located at the boundaries of CD. For space-time sheets describing phase transitions there are also dynamical symmetries but they are different. In standard physics one has just states and their symmetries. Conformal gauge symmetries forming a hierarchy: conformal field theories this symmetry is maximal and the hierarchy is absent.

1. There is importance and very delicate difference between thermal and thermodynamical symmetries. Thermal symmetries are due to thermal equilibrium implying symmetries in *statistical sense*. Quantal symmetries correspond to representations of symmetry group and are possible if thermal fluctuations do not transform the states of the representations the states of other representation.

   Dark dynamical symmetries are quantum symmetries. The breaking of thermal translational symmetry of liquid leads to discrete translational symmetry of crystal having interpretation as quantum symmetry. The generation of continuous thermal translational symmetry from discrete quantum symmetry means loss of quantum symmetry. To my opinion, standard thinking is sloppy here.

2. For thermodynamical phase transitions temperature reduction induces spontaneous breaking of symmetry: consider only liquid-to-crystal transition. Analogously, in gauge theories the reduction gauge coupling strength leads to spontaneous symmetry breaking: quantum fluctuations combine representation of sub-group to a representation of larger group. It would seem that spontaneous symmetry breaking actually brings in a symmetry and the ubroken symmetry is "thermal" or pure gauge symmetry. QCD serves as an example: as strong coupling strength (analogous to temperature) becomes large confinement occurs and color symmetry becomes pure gauge symmetry.

3. In TGD the new feature is that there are two kinds of symmetries for dark conformal hierarchies. Symmetries are either pure gauge symmetries or genuine dynamical symmetries affecting the dark state at field body physically. As h_{eff} increases, the conformal pure gauge symmetry is reduced (the conformal gauge algebra annilating the states becomes smaller) but dynamical symmetry associated with the degrees of freedom above measurement resolution increases. In ordinary conformal theories pure gauge conformal symmetry is always maximal so that this phenomenon does not occur.

   The intuitive picture is that the increase of dynamical symmetry induced by the reduction of pure gauge conformal symmetry occurs as temperature is lowered and quantum coherence in longer scales becomes possible. This conforms with the thermodynamical and gauge theory views if pure gauge symmetry is identified as counterpart of symmetry as it is understood in thermodynamics and gauge theories.

The dynamical symmetry of dark matter however increases. This symmetry is something new and would be genuine quantum symmetry in the sense that quantum fluctuations respect the representations of this group. The increase of h_{eff} indeed implies reduction of Kähler coupling strength analogous to reduction of temperature so that these quantum symmetries can emerge.

4. There is also a dynamical symmetry associated with phase transitions $h_{eff}(f) = m \times h_{eff}(i)$ such that m would define the rank of ADE Lie group G classifying states of "transitons". Lie groups with ranks $n_{eff}(i)$ and $n_{eff}(f)$ would be ranks for the Lie group G in the initial and final states. G would correspond to either gauge (not pure gauge) or Kac-Moody symmetry as also for corresponding dynamical symmetry groups associated with phases.

5. An interesting question relates to Kosterlitz-Thouless Thouless phase transition (see `http://tinyurl.com/yce24jr9`), which is 2-D and for which symmetry is not changed. Could one interpret it as a phase transition changing h_{eff} for MB: symmetry group as abstract group would not change although the scale in which acts would change: this is like taking zoom. The dynamical symmetry group assignable to dark matter at flux tubes would however change but remain hidden.

To sum up, the notion of magnetic (field) body might apply even to the ordinary phase transitions. Dark symmetries - also discrete translational and rotational symmetries - would be assigned with dark MB possibly present also in ordinary phases. The dynamical symmetries of MB would bring a new element to the description. Ordinary phase transitions would be induced by those of MB. This would generalize the vision that MB controls biological body central for TGD view about living matter. In the spirit of slaving hierarchy and TGD inspired vision about quantum biology, ordinary matter would be slave and MB the master and the description of the phase transitions in terms of dynamics of master could be much more simpler than the standard description. This would be a little bit like understanding technical instrument from the knowledge of its function and from control level rather than from the mere physical structure.

Quantum phase transitions and 4-D spin glass energy landscape

TGD has led to two descriptions for quantum criticality. The first one relies on the notion of 4-D spin glass degeneracy and emerged already around 1990 when I discovered the unique properties of Kähler action. Second description relies on quantum phases and quantum phase transitions and I have tried to explain my understanding about it above. The attempt to understand how these two approaches relate to each other might provide additional insights.

1. Vacuum degeneracy of Kähler action is certainly a key feature of TGD and distinguishes it from all classical field theories. Small deformations of the vacua probably induced by gluing of magnetic flux tubes (primordially cosmic strings) to these vacuum space-time sheets deforms them slightly and would give rise to TGD Universe analogous to 4-D spin glass. The challenge is to relate this description to the vision provided by quantum phases and quantum phase transitions.

2. In condensed matter physics one speaks of fractal spin glass energy landscape with free energy minima inside free energy minima. This landscape obeys ultrametric topology: p-adic topologies are ultra metric and this was one of the original motivations for the idea that p-adic physics might be relevant for TGD. Free energy is replaced with the sum of Kähler function - Kähler action of Euclidian space-time regions and imaginary Kähler action from Minkowskian space-time regions.

3. In the fractal spin glass energy landscape there is an infinite number of minima of free energy. The presence of several degenerate minima leads to what is known as frustration. In TGD framework all the vacuum extremals have the same vanishing action so that there is infinite degeneracy and infinite frustration (also created by the attempt to understand what this might imply physically!). The diffeomorphisms of M^4 and symplectic transformations of CP_2 map vacuum extremals to each other and acts therefore as gauge symmetries. Symplectic transformations indeed act as U(1) gauge transformations. Besides this each Lagrangian sub-manifold of CP_2 defines its own space of vacuum-extremals as orbit of this symplectic group.

As one deforms vacuum extremals slightly to non-vacuum extremals, classical gravitational energy becomes non-vanishing and Kähler action does not vanish anymore and the above gauge symmetries become dynamical symmetries. This picture serves as a useful guideline in the attempts to physically interpret. In TGD inspired quantum biology gravitation plays indeed fundamental role (gravitational Planck constant h_{gr}).

4. Can one identify a quantum counterpart of the degeneracy of extremals? The notion of negentropic entanglement (NE) is cornerstone of TGD. In particular, for maximal negentropic entanglement density matrix is proportional to unit matrix so that states are degenerate in the same sense as the states with same energy in thermodynamics. Energy has Kähler function as analogy now: hence the degeneracy of density matrix could correspond to that for Kähler function. More general NE corresponds to algebraic entanglement probabilities and allows to identify unique basis of eigenstates of density matrix. NE is favored by NMP and serves key element of TGD inspired theory of consciousness.

   In standard physics degeneracy of density matrix is extremely rare phenomenon as is also entanglement with algebraic entanglement probabilities. These properties are also extremely unstable. TGD must be somehow special. The vacuum degeneracy of Kähler action indeed distinguishes TGD from quantum field theories, and an attractive idea is that the degeneracy associated with NE relates to that for extremals of Kähler action. This is not enough however: NMP is needed to stabilize NE and this occurs only for dark matter ($h_{eff}/h > 1$ equals to the dimension of density matrix defining NE).

   The strong form of holography takes this idea further: 2-D string world sheets and partonic 2-surfaces are labelled by parameters, which belong to algebraic extension of rationals. This replaces effectively infinite-D WCW with discrete spaces characterized by these extensions and allows to unify real and p-adic physics to adelic physics. This hierarchy of algebraic extensions would be behind various hierarchies of quantum TGD, also the hierarchy of deformations of vacuum extremals.

5. In 3-D spin glass different phases assignable to the bottoms of potential wells in the fractal spin energy landscape compete. In 4-D spin glass energy of TGD also time evolutions compete, and degeneracy and frustration chacterize also time evolutions. In biology the notions of function and behavior corresponds to temporal patterns: functions and behaviors are fighting for survival rather than only organisms.

   At quantum level the temporal patterns would correspond to phase transitions perhaps induced by quantum phase transitions for dark matter at the level of magnetic bodies. Phase transitions changing the value of h_{eff} would define correlates for "behaviors" and the above proposed description could apply to them.

6. Conformal symmetries (the shorthand "conformal" is understood in very general sense here) allow to understand not only quantum phases but also quantum phase transitions at fundamental level and "transitons" transforming according to representations of Kac-Moody group or gauge group assignable to the inclusion of hyperfinite factors characterized by the integer m in $h_{eff}(f) = m \times h_{eff}(i)$ could allow precise quantitative description. Fractal symmetry breaking leads to conformal sub-algebra isomorphic with the original one

   What could this symmetry breaking correspond in spin energy landscape? The phase transition increasing the dynamical symmetry leads to a bottom of a smaller well in spin energy landscape. The conformal gauge symmetry is reduced and dynamical symmetry increased, and the system becomes more critical. Indeed, the smaller the potential well, the more prone the system is for being kicked outside the well by quantum fluctuations. The smaller the well, the large the value of h_{eff}. At space-time level this corresponds to a longer scale. At the level of WCW (4-D spin energy landscape) this corresponds to a shorter scale.

What ZEO can give to the description of criticality?

One should clarify what quantum criticality exactly means in TGD framework. In positive energy ontology the notion of state becomes fuzzy at criticality. It is difficult to assign long range fluctuations and associated quanta with any of the phases co-existent at criticality since they are most naturally associated with the phase change. Hence Zero Energy Ontology (ZEO) might show its power in the description of (quantum) critical phase transitions.

1. Quantum criticality could correspond to zero energy states for which the value of h_{eff} differs at the opposite boundaries of causal diamond (CD). The space-time surface between boundaries of CD would describe the transition classically. If so, then quanta for long range fluctuations would be genuinely 4-D objects - "transitons" - allowing proper description only in ZEO. This could apply quite generally to the excitations associated with quantum criticality. Living matter is key example of quantum criticality and here "transitons" could be seen as building bricks of behavioral patterns. Maybe it makes sense to speak even about Bose-Einstein condensates of "transitons".

2. Quantum criticality would be associated with the transition increasing $n_{eff} = h_{eff}/h$ by integer factor m or its reversal. Large h_{eff} phases as such would not be quantum critical as I have sloppily stated in several contexts earlier. $n_{eff}(f) = m \times n_{eff}(i)$ would correspond to a phase having longer long range correlations as the initial phase. Maybe one could say that at the side of criticality (say the "lower" end of CD) the $n_{eff}(f) = m \times n_{eff}(i)$ excitations are pure gauge excitations and thus "below measurement resolution" but become real at the other side of criticality (the "upper" end of CD)? The integer m would have clear geometric interpretation: each sheet of n_i-fold coverings defining space-time surface with sheets co-inciding at the other end of CD would be replaced with its m-fold covering. Several replications of this kind or their reversals would be possible.

3. The formation of m-fold covering could be also interpreted in terms of an inclusion of hyper-finite factors labelled by integer m. This suggests a deep connection with symmetries of dark matter. Generalizing the McKay correspondence between finite subgroups of SU(2) characterizing the inclusions and ADE type Lie groups, the Lie group G characterizing the dynamical gauge group or Kac-Moody group for the inclusion of HFFs characterized by m would have rank given by m (the dimension of Cartan algebra of G).

   These groups are expected to be closely related to the inclusions for the fractal hierarchy of isomorphic sub-algebras of super-symplectic subalgebra. $h_{eff}/h = n$ could label the sub-algebras: the conformal weights of sub-algebra are n-multiples of those of the entire algebra. If the sub-algebra with larger value of n_{eff} annihilates the states, it effectively acts as normal subgroup and one can say that the coset space of the two super-conformal groups acts either as gauge group or (perhaps more naturally) Kac-Moody group. The inclusion hierarchy would allow to realize all ADE groups as dynamical gauge groups or more plausibly, as Kac-Moody type symmetry groups associated with dark matter and characterizing the degrees of freedom allowed by finite measurement resolution.

4. If would be natural to assign "transitions" with light-like 3-surfaces representing parton orbits between boundaries of CD. I have indeed proposed that Kac-Moody algebras are associated with parton orbits where super-symplectic algebra and conformal algebra of light-one boundary is associated with the space-like 3-surfaces at the boundaries of CD. This picture would provide a rather detailed view about symmetries of quantum TGD.

The number-theoretic structure of h_{eff} reducing transitions is of special interest.

1. A phase characterized by $h_{eff}/h = n_{eff}(i)$ can make a phase transition only to a phase for which $n_{eff}(f)$ divides $n_{eff}(i)$. This in principle allows purely physics based method of finding the divisors of very large integers (gravitational Planck constant $\hbar_{gr} = GMm/v_0 = \hbar_{eff} = n \times \hbar$ defines huge integer).

2. In TGD inspired theory of consciousness a possible application is to a model for how people known as idiot savants unable to understand what the notion of prime means are able to decompose large integers to prime factors [K97]. I have proposed that the division to prime factors is a spontaneous process analogous to the splitting of a periodic wave characterized by wave length $\lambda/\lambda_0 = n_i$ to a wave with wavelength $\lambda/\lambda_0 = n_{eff}(f)$ with $n_{eff}(f)$ a divisor of $n_{eff}(i)$. This process might be completely spontaneous sequence of phase transitions reducing the value of n_{eff} realized geometrically as the number of sheets of the singular covering defining the space-time sheet and somehow giving rise to a direct sensory percept.

10.3.4 Maxwell's lever rule and expansion of water in freezing: two poorly understood phenomena

The view about condensed matter as a network with nodes identifiable as molecules and bonds as flux tubes is one of the basic predictions of TGD and obviously means a radical modification of the existing picture. In the sequel two old anomalies of standard physics are explained in this conceptual framework.

Maxwell's lever rule as an indication for the presence of magnetic flux tubes

Van der Waals equation of state (`http://tinyurl.com/yayjgehm`) is a simple model for two phase system used for mostly pedagogical purposes. The model is not realistic. In particular, it has difficulties in the critical region, where two phases are present. The latter difficulties are actually encountered also in the partition function approach of statistical mechanics.

1. Van der Waals equation of state

Consider first the van der Waals equation of state.

1. Van der Waals equation of state has variables (n, T) so that the natural thermodynamical function is free energy F. The equation is of form

$$P = (\frac{\partial F}{\partial V})_T = \frac{n}{1-nb_1}T - a_1 n^2 \quad . \tag{10.3.1}$$

   Here one has $n = N/V$, where N is particle number and constant parameter. $(b_1 = 0, a_1 = 0)$ gives the equation of state for ideal gas. The interpretation of the parameters is discussed in `http://tinyurl.com/yayjgehm`.

2. To deduce free energy F and internal energy E one would need also the partial derivative of free energy

$$S = (\frac{\partial F}{\partial T})_V \quad , \tag{10.3.2}$$

   so that $dF = SdT - pdV$ could be integrated. The information about entropy is not included in van der Waals.

3. The expressions of both E and F can be fixed by assuming that E is a homogenous funktion of (S, T, P, V):

$$E = TS - PV \quad . \tag{10.3.3}$$

   This is additional assumption, which of course need not be true.

   (a) The assumption would give for the free energy per particle the expression

$$f = \frac{F}{N} = \frac{E - TS}{N} = \frac{PV}{N} = \frac{P}{n} \tag{10.3.4}$$

   In the case of van der Waals one obtains by using the expression for the pressure already given

$$f = \frac{P}{n} = \frac{T}{1-nb_1} - a_1 n \quad . \tag{10.3.5}$$

   (b) The entropy per particle is given by

$$s = \frac{S}{N} = \frac{(\frac{\partial p}{\partial T})_V}{n} = \frac{1}{1-nb_1} \quad . \tag{10.3.6}$$

   $s = S/N$ does not depend on temperature at all.

   (c) For single particle energy $e = E/N$ one has

$$e = \frac{TS-PV}{N} = a_1 n \quad . \tag{10.3.7}$$

   Also e depends on n only.

4. Van der Waals indeed allows 2 phases and they appear simultaneously in the critical region. The equation of state can be written as a condition for the vanishing of 3rd degree polynomial $P_3(n,T)$ as a function of n

$$\begin{array}{lll} P_3(n,t) = \sum_{k=0}^{3} p_k^{n^k} = 0 & p_3 = 1 \ , & p_2 = -\frac{1}{b_1} \ , \\ p_1 = \frac{P}{a_1 b_1} + \frac{T}{a_1 b_1} \ , & p_0 = -\frac{P}{a_1 b_1} \ . & \end{array} \tag{10.3.8}$$

The number of the real roots for n is odd: either 3 or 1. In the critical region, which corresponds to a cusp catastrophe (see `http://tinyurl.com/ngfa9t3`) having n as behaviour variable, the number of real roots is 3, call them $n_{max} \geq n_0 \geq n_{min}$. The largest root n_{max} and smallest root n_{min} correspond to liquid and gas phases. The middle root n_0 is unstable if the polynomial equation is interpreted as a vanishing of the derivative of a fourth-order polynomial of n having p and T as control parameters. It has no physical identification.

The projection of the cusp (see `http://tinyurl.com/ngfa9t3`) to (p,T) has shape of V with curved edges. The tip of V corresponds to critical point and at the edges of V a phase transition takes place between vapour phase and critical phase or liquid phase and critical phase. Above the tip one cannot say whether the phase is gas or liquid and the continuous transformation of gas to liquid can be also regarded as poorly understood.

At the right (left) hand side of V there is single real root n_G (n_L). $n_G < n_L$ allows the interpretation in terms of gas and liquid phases.

2. The problems of van der Waals

Consider now the problems of van der Waals in the critical region.

1. Van der Waals allows besides gas phase also liquid phase but the model does now work well in liquid phase. In the critical region where both gas and liquid phases are possible, the model works badly. Equation of state forces the system to a 2-dimensional surface in (n,p,T) space ($n = N/V$, also V can be used as variable since N is constant parameter).

   The standard interpretation is that both phases are present as pure phases and only their fractions vary. The intermediate phase allowed by van der Walls is not present. The empirical finding that the pressure for given temperature does not depend on V. $p(V,T) = p(T)$ condition states that the pressures of the two phases are same: this can be interpreted as equilibrium condition. It follows from van der Waals naturally for different roots n for the equation of state.

2. Already Maxwell proposed a modification of van der Waals. Area rule (for a visualization see `http://tinyurl.com/ycabjdhh`) tells how the oscillatory behaviour of $p(T,V)$ as function of V as one moves in transversal direction (in which p varies) to V along cusp from lowest sheet of cusp (n_-) to the highest sheet (n_+) by increasing V is replaced with constant behavior. In other words, the curve along cusp connecting constant T curves connecting the points at upper and lower sheet of cusp with the same value of p is replaced with a straight vertical line. The condition is that the area below the line is same as the area below the oscillatory curve of constant p.

   Lever rule (`http://tinyurl.com/ybuq7aye`) is needed to understand the proportions of the two phases. Usually the rule is applied to metal alloys. Consider two pure phases α ja β and their mixture γ. Let the fractions of phases α and β be X_α and X_β. Assume that the phases contain two "elements" A and B. Let the proportions of B be a in α, b in β and c in γ. The lever rule

$$X_\alpha = \frac{c-b}{a-b} \tag{10.3.9}$$

   follows trivially from the fact that in mixed phase γ one has $c = X_\alpha a + (1 - X_\alpha b = X_\alpha(a-b) + b$. In critical regions a and b should vary. To me this picture however represents a problem. What the two "elements" are in the case of say water? If water molecule corresponds to A, what does B correspond to? A different state of water molecule? Or does the system contain also some other "element" than water molecule?

As a consequence of this problem the working models are numerical since analytical models cannot explain the lever rule. This problem is not only the problem of van der Waals but quite general problem of statistical models relying on partition function giving free energy F.

3. TGD based explanation of the lever rule

The TGD interpretation for the situation could be following.

1. In the liquid phase molecules can be connected by flux tubes. They are also possible in gas phase but their number is smaller. In particular, in vapour phase intermediate between liquid and gas also gas molecules can be connected by flux tubes to form connected networks.Only single connected network could be present in liquid phase.

   The number of flux tubes per particle can depend on the thermodynamical parameters (V, T) and is expected to be considerably smaller in gas phase in regions where one can distinguish liquid phase from vapor phase (not below the tip of V).

   In liquid state the flux tubes could be shorter than in gas phase. In liquid phase there are large connected structures - maybe only single one - whereas in gas phase these structures are smaller. At criticality they might correspond to vapour droplets. Gas phase would be different from gas phase far from criticality.

2. In critical region there are regions, which form connected networks differing with respect to the number of bonds per particle characterizing the networking. The volume of the mixed phase depends on the relative volumes of these two phases since they have widely differing densities. Large number of networked molecules gives a smaller volume. The pressures in these two kinds of regions are same in mechanical equilibrium.

3. What could be the counterparts for the two "elements" A and B? Could A correspond water molecule and B to flux tube? The portion of flux tubes would distinguish between the two phases at criticality. They are present also in gas phase unless one has $b = 0$ identically. In this case a must however vary inside critical region. For $b = 0$, perhaps realized far from the left edge of V, gas phase would have no flux tubes. In liquid phase to the right from V but not below the tip of V a would be large. At tip and below it one would have $a = b$ along some line and one can say that gas phase transforms to liquid phase. As one goes around the tip the fraction a in liquid phase becomes b for gas phase.

4. What distinguishes liquid and gas phases? What suggests itself is that when the number N_b of flux tube bonds per molecule is above critical value $N_{b,crit}$, a transition to liquid phase takes place and the density is reduced to that of liquid. Below the tip of V and left to V this phase transition does not take place. To the right of the edge of we it would take place. Inside V there are both kinds of regions. What this means that the parameters a and b are new parameters characterizing the state of liquid and gas phases. This could allow better understanding of vapour phase.

5. The appearance of flux tubes could be understood in two ways. .
   (a) New flux tube pairs could emerge by reconnection of flux tube loops associated with molecules.
   (b) Already existing long flux tubes (or flux tube pairs) between molecules shorten in a phase transition reducing $h_{eff} = n \times h$ to its standard value and forces the molecules connected by them to become close together. Since the phases with non-standard value of Planck constant quite generally have higher energy (for instance, bond energies are higher and atomic binding energies lower) this implies that energy is liberated in this connection process.

It seems that flux tube picture could explain the lever rule, which works but cannot be understood in thermodynamics and statistical physics. This would be seen as a direct indication for the reality of flux tubes.

Strangeness in the freezing of water

Water has hundreds of anomalies as one learns from the excellent web pages of Martin Chaplin (see `http://www1.lsbu.ac.uk/water/`). I have discussed these anomalies in [K45]. One of them relates to the freezing of water. Usually the volume per particle is reduced in freezing but now it increases. Second biologically enormously important anomaly is the decrease of the molecular volume instead of increase as the temperature grows form 0 °C to 4 °C.

In TGD framework the anomalies of water can be seen as a support for the existence of two phases in water: dark phase identified as a phase with non-standard value $h_{eff}/h = n$ of Planck constant [?] and ordinary phase. On basis of the model explaining Maxwell's rule at criticality, one can ask whether the dark and ordinary phases correspond to those for the flux tubes rather than molecules. In the case of water the flux tubes could be assigned to hydrogen bonds, which could have quite long lengths for large values of $h_{eff}/h = n$. They would be present also for other liquids. Maybe the flux tubes carrying $h_{eff}/n = n$ dark protons associated with hydrogen bonds distinguish water from other liquids.

Dark states have higher energy than ordinary ones so that the formation of dark phase requires energy. The natural assumption is that the dark phase transforms to ordinary one in the freezing of water. Long dark flux tubes would get shorter. Alternatively, dark flux loops reconnect and form short flux tube pairs between molecules assignable to hydrogen bonds. Why this should lead to an expansion of the molecular volume?

To answer this question it is useful consider first the second anomaly. Why the volume increases as one reduces temperature from 4 °C to 0 °C? As $h_{eff}/h = n$ for flux tube or reconnecting flux tubes decreases, the length of flux tube as quantal length shortens and the result could be a rather rigid short stick. There exists a proposal that these rigid flux tubes reduce the motility of water molecules belonging to the water clusters, which correspond to connected flux tube networks. Since molecules cannot move freely anymore, empty volume is generated. The outcome is an increase of the average molecular volume.

What about freezing?

1. Above boiling point water has 3.4 hydrogen bonds to its neighbors, which is nearly the maximal number 4 realized for ice (see `http://tinyurl.com/ydcedet4`). Either all existing long flux tubes would have shortened or all loops would have shortened and reconnected to flux tube pairs.

2. In freezing the dark energy is liberated so that the latent heat should be higher when a phase with a non-standard value of $h_{eff}/h = n$ is present in the liquid phase. This could explain the especially high latent heat 334 kJ/kg for water.

3. Only ammonia NH^3 (see `http://tinyurl.com/yc6zcl6o`) has comparable latent heat 332.17 kJ/kg (see `http://tinyurl.com/h3lvm43`) . Interestingly, also NH_3 molecules form hydrogen bonds and for this reason ammonium is easily miscible to water. This property might relate also to the biological importance of NH_3 and nitrogen and hydrogen containing molecules.

4. Also O and F form hydrogen bonds. More generally, any atom containing lone electron pairs, that is pairs of valence electrons, which do not belong to valence bonds, can form hydrogen bonds. A possible explanation is that the lone pair goes to a flux tube pair associated with the hydrogen bond and gives rise to a Cooper pair making possible high T_c superconductivity by the mechanism discussed in [K90, K91]. Flux tube pair would contain also the dark proton delocalized to both flux tubes.

10.3.5 TGD based view about ferromagnetism

I received a link to a highly interesting popular article (`https://tinyurl.com/y8b86df3`) about ferromagnetism. According to the article, Yi Li, a physicist working at John Hopkins University and his two graduate students, Eric Bobrow and Keaton Stubis, seem to have made a considerable progress in understanding how the system of electron spins in lattice ends up to a ferromagnetic sate [D13] (`https://tinyurl.com/y9ycj3nt`). This ferromagnetism is known as itinerant ferromagnetism and involves vacancies, sites without electron, which can be moved freely without affecting the energy of the state. This article inspired train of thought allowing to develop TGD view about ferromagnetism.

The ideas related to the work of Li *et al*

The problem considered by Li *et al* is how the ferromagnetic state could emerge from an arbitrary state with some numbers of spin up and spin down states at lattice sites connected by edges.

1. Permutation of electrons with same spin leave the ferromagnetic state invariant and does not cost energy while permutations in arbitrary configuration can do so.
2. Li *et al* [D13] considered a simple 4×4 lattice with single vacancy and noticed a connection with so called 15-puzzle involving 15 tiles and single vacancy with neighboring tiles of vacancy able to move to its position. The observation is following. If one has spin lattice containing single vacancy, one can number the sites by a number running from 1 to N (now 15) in arbitrary manner. If so called connectedness condition holds true one can realize any permutation of these numbers. This means that 15-puzzle has always a solution. In particular, one can arrange the situation that the numbers form an ordered sequence from 1 to N so that numbers n and $n+1$ are nearest neighbors.

   The result found by Li *et al* first for 2-D 4×4 lattice with single vacancy generalizes to lattices, which are non-separable in the sense the removal of a lattice site does not separate any pair of spins - they are still connected by an edge-loop.
3. The curve solving the 15-puzzle goes through all points of the 4×4 lattice and is generally known as Hamiltonian curve. It becomes Hamiltonian cycle if the numbers 1 and N are nearest neighbors.
4. The basic problem of this approach is that the theorem is true only for single vacancy and does not allow generalization to a larger number of vacancies. It is however known that ferromagnetism is possible up to fraction 1/3 for vacancies. The challenge is to generalize the result of Li *et al.*

Some reasons to get interested

In TGD framework there are good reasons of getting interested on these results.

1. The result of Li *et al* states that ferromagnetic phase transition might be understood in terms of shifting of lattice vacancy if the lattice with single defect allows deformation of any configuration of spin labelled by numbers N running from 1 to N to a closed curve connecting nearest neighbors along which N increases. Could there be a connection with Hamiltonian curves making sense for lattice like structures (actually all graphs)? Could Hamiltonian curve have some deeper physical meaning or is it only an auxialiary notion useful for representing the possibility to realize all permutations for the points of a lattice with vacancy by shifting it suitably?

   Hamiltonian curve connects neighboring points of a lattice and goes through all points without self-intersections. Icosahedral geometry appears in biology and one can ask whether this kind of cycles could be actually realized physically - say as flux tubes at icosahedron and tetrahedron, which play key role in TGD inspired biology. Flux tube are actually fundamental objects in TGD Universe in all scales. For instance, final states of stars could correspond to flux tube spaghettis consisting of single volume filling flux tube [L78] (`https://tinyurl.com/tkkyyd2`).
2. If the Hamiltonian cycle is something physical it could correspond to flux tube. The notion of magnetic flux tube central in TGD might allow application to ferromagnetism. TGD predicts two kinds of flux tubes: Maxwellian ones and monopole flux tubes with magnetic fields requiring no currents to generate them: they are not not allowed by Maxwell's theory.

   The preservation of the Earth's magnetic field predicted to decay rather rapidly as currents generating it dissipate supports the view that it contains monopole flux part which from biological input would correspond to endogenous magnetic field B_{end}, which is a fraction 2/5 about the nominal value of $B_E = .5$ Gauss. The presence of magnetic fields in cosmological scales is also a mystery finding a solution in terms of monopole flux tubes.
3. Monopole flux tubes must be closed. Closed non-intersecting flux tubes connecting nearest neighbors in lattice would correspond to Hamiltonian cycles. In TGD inspired biology Hamiltonian cycles associated with icosahedron and tetrahedron provide a realization of the

vertebrate genetic code [L5] (https://tinyurl.com/yad4tqwl) but it is still somewhat of mystery why the points of icosahedron and tetrahedron, which are lattices (tessellations) at sphere, would be connected by a curve. Quantum classical correspondence suggests that magnetization corresponds to flux tubes connecting magnetic dipoles as formal analogs of monopole-antimonopole pairs. Could magnetic flux tubes provide a concrete realizaon for these Hamiltonan cycles?

4. Closed monopole flux tubes seem to be unrealistic for the description of ferromagnetism, which suggests the presence of N parallel flux tubes carrying magnetization M and defining a braid connecting opposite ends of ferromagnet. The monopole fluxes could arrive as single flux along parallel space-time sheet carrying field H defined by single thick flux tube. Test particle would experience $B = M + H$.

The following considerations are not much more than first impressions and probably require updating.

TGD based view

Flux tubes are the new element of condensed matter physics predicted by TGD. Could they provide insights into ferromagnetism?

1. Starting from text book picture about ferromagnetism

To develop TG view about ferromagnetism it is best to start from the text book picture.

1. In the standard model of ferromagnetism one assumes the presence of field B identified as sum $B = M + H$ of magnetization and field H equal to B outside the magnet. M is due to magnetic dipoles besides magnetic field B and the interaction of spins with H is important. B is usually regarded as the fundamental field M and and H appear as auxiliary notions and their relation to B requires a model for the system: typically H, M, and B are assumed to be linearly related.
2. The field M could be naturally assigned with a flux tube connecting the spins - perhaps at nearest neighbor lattice points. What about H? In standard model H and B are parallel for the ferromagnetic configuration. If B is assigned with the flux tube connecting the magnetic moments and B is parallel to H, this would suggest a flux tube consisting of long straight portions parallel to each other.

   In the many-sheeted space-time of TGD M and H can reside at different space-time sheets, which are parallel so that they are on top of each other in $H = M^4 \times CP_2$. H is at large space-time sheet including also the environment. The decomposition to sum would have representation as a set theoretic union.

   The test particle would experience the sum of the magnetic fields associated with the two sheets. Could M and H as the return flux associated with M and superposing with the external contribution to H correspond to these two space-time sheets so that particle would experience their sum as $B = M+H$? If so, ferromagnetism could be seen as a direct signature of many-sheeted space-time.

2. Could also monopole flux tubes be important?

There is still one important aspect related to the TGD view about magnetic field which might play important role. TGD predicts two kind of flux tubes. The first kind of flux tubes could be called Maxwellian and the corresponding magnetic fields require current to generate them. There are also flux tubes having closed cross section and carrying monopole fluxes. No currents are required to generate corresponding magnetic fields. Could also these flux tubes having no current as sources be present? This would mean new physics.

1. The first thing to notice is that the interpretation of magnetization M is as a magnetic field generated by magnetic moments. The usual interpretation is that spins are analogous to magnetic moments created by currents consisting of rotating charge. For spin there is no such rotating charge. Second interpretation is as magnetic moments identifiable as infinitesimal monopole pairs.

2. Could one think that the flux tubes containing sequence magnetic moments correspond to monopole flux and that closing this loop could give rise to monopole magnetic field? Ordinary Maxwellian part could be also present and have current as source. How M and H would relate to these. Could M correspond to the monopole part and H the Maxwellian part?

   Are spins necessary for the existence of a monopole flux tube? Could quantum classical correspondence require this? Could dark charged matter assigned with the monopole flux tubes correspond to the magnetic moments of say dark valence electrons with non-standard value of $h_{eff} = nh_0$ so that M would be represented by monopole flux tubes classically? If the return flux represented by H is absent, flux tube must give rise to a Hamiltonian cycle. If H is present, it would be enough to have flux tubes representing N braid strands fusing to single monopole flux carrying the return flux.

   Formation of a flux loop defining Hamiltonian cycle would be a new kind of phenomenon analogous to spontaneous magnetization requiring no external field H. Spontaneous magnetization would be however something different. A trivial braid consisting of N parallel strands representing M and parallel to it locally with return flux arriving along single large flux tube carrying H would be formed in ferromagnetic transition and also in spontaneous magnetization.

3. Bringing in thermodynamics

One can try to make this more concrete by bringing in thermodynamics.

1. Assume that there exists single flux tube - connecting all the lattice points (magnetic moments) or possibly N flux tubes parallel to local magnetization M and giving rise to a braid like structure representing the topology of flux lines of M connecting opposite boundaries of magnet.

2. In the general case, the points of the lattice could be connected by a flux tubes connecting points, which need not be nearest neighbors. The first guess is that the magnetic interaction energy of spins at the ends of the flux tube portion connecting them decreases with the distance between spins. There should be also magnetic energy associated with the field H at the space-time sheet carrying the return flux. Thermodynamics would bring in entropy and free energy $F = E - TS$ would be mimimized. Entropy maximization would favor long random flux tubes and energy minimization short flux tubes.

   One expects that flux tube has free energy F increasing with flux tube length. If one does not allow self-intersections - as suggested by repulsive Coulomb interaction and Fermi statistics - the flux tube could be either Hamiltonian cycle or consist of analogs of braid strands: in the case of ferromagnetism the strands would be parallel to each other. The interaction energy would be same for all Hamiltonian cycles if determined by nearest neighbour interactions.

3. In the general case with lattice replaced by graph one expects that a large number of Hamiltonian cycles not related by rotation to each other exists so that one would have large number of states with same minimum energy. Could this somehow correspond to spin glass state allowing large number of degenerate states? The flux tube need not be closed. In ferromagnetic configuration this would be the case.

4. How would the assignment of spin direction to the lattice points affect the situation? Could the numbers N_+ and N_- of spin up and spin down electrons determine the flux tube configuration (say braid) by (Gibbs) energy minimization?

4. Could 2-braid describe the transition to ferromagnetism?

In the work of Li *et al* discussed in the article, the permutations of lattice points are induced by moving the vacancy around. This picture inspired the considerations above but is too limited. In fact the work of Li *et al* only directed attention to Hamiltonian cycles and braids formed by the non-closed analogs.

1. TGD picture brings in mind braid-knot connection. One can replace the braid associated with M with a knot by connecting the magnetic moments at the opposite ends of the braid by strands of a trivial braid at parallel space-time sheet. This trivial braid would carry the return flux having interpretation in terms of field H.

The flux tubes of trivial braid could also fuse to single thicker flux tube or flow to a larger space-time sheet carrying the total return flux associated with M. This would conform with the idea that H provides a description of the system in longer length scale being analogous to a smoothed out total magnetic field acting as self-consistent background.

Remark: Could one assume that only H assignable to big flux tube has constant direction and magnitude and that M is represented as flux tubes connecting dipoles can in principle correspond to any permutation of atoms? For this option the spontaneous magnetization would correspond to a superposition of different configurations with same weights and would be invariant under permutations as in the argument of Li *et al* involving no flux tubes. This option does not look attractive.

2. What braid picture allows to say about the transition to ferromagnetism? Could the transition be realized by deforming the flux tubes associated with M and forming a non-trivial braid be induced by permutation of the lattice points taking the non-trivial braid to trivial one? This would be like opening the braid. The lattice points in the initial and final state would correspond to the ends of a dynamical evolution. The permutation would be realized as a time-like braiding with braid strands in time direction.

   Mathematically braid group corresponds to the covering group of permutation group and quantum group representations correspond to the representations of braid groups. The description of the transition could provide a new application of quantum groups.

The description as time-like braiding is not however complete since there is an additional structure involved: the flux tubes connecting the magnetic dipoles in lattice and defining a braid or even more complex configuration having flux tube connections between non-neighboring magnetic moments.

1. If there is no return flux assignable to H, M corresponds to a closed flux tube carrying monopole flux the dynamical time-like dynamical braiding would lead to a Hamiltonian cycle in this case and the number of final state configurations would be finite, there is degeneracy. Could spin glass phase correspond to this situation?

2. In ferromagnetism final state would contain N parallel strands carrying the monopole flux assignable to M and the return flux H would arrive along parallel thick flux tube. In general configuration these strands can be braided. The transition to ferromagnetism would represent time-like braiding of an ordinary 3-D braiding of flux tube strands connecting the opposite boundaries of ferromagnetic. In the initial state braid would be non-trivial and the flux tubes of braid would not have minimal length and minimum energy. In the final ferromagnetic state braid would be trivial with parallel flux tubes.

   Mathematically this process would correspond to what is called 2-braiding: I have proposed that 2-braidings are important in TGD inspired biology as a topological description of dynamical processes. An interesting interpretation is as a topological analog for problem solving. I have also proposed that in bio-systems topological quantum computation programs are represented as this kind 2-braidings for flux tubes [K4, K120] (`https://tinyurl.com/ycvgjccq` and `https://tinyurl.com/ydylud6c`.)

   Ferromagnetism would correspond to an opening of a non-trivial braid. If the return flux arrives along flux tubes this is possible smoothly only if the knot defined in this manner is trivial. To achieve opening, the 2-braiding must involve reconnections, which correspond to cutting the knot strand and reconnecting the pieces in new manner: this is how Alexander opened his knot. Fermi statistics and repulsive Coulomb interaction do not favour this mechanism. If the return flux arrives along single flux tube, the opening could correspond to a smooth deformation without reconnections transferring the braiding to the parallel space-time sheet, where it is "neutralized" by fusing the flux tubes to single flux tube.

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1. The result of Li *et al* states that ferromagnetic phase transition might be understood in terms of shifting of lattice vacancy if the lattice with single defect allows deformation of any configuration of spin labelled by numbers N running from 1 to N to a closed curve connecting nearest neighbors along which N increases. Could there be a connection with Hamiltonian curves making sense for lattice like structures (actually all graphs)? Could Hamiltonian curve have some deeper physical meaning or is it only an auxiliary notion useful for representing the possibility to realize all permutations for the points of a lattice with vacancy by shifting it suitably?

   Hamiltonian curve connects neighboring points of a lattice and goes through all points without self-intersections. Icosahedral geometry appears in biology and one can ask whether this kind of cycles could be actually realized physically - say as flux tubes at icosahedron and tetrahedron, which play key role in TGD inspired biology. Flux tube are actually fundamental objects in TGD Universe in all scales. For instance, final states of stars could correspond to flux tube spaghettis consisting of single volume filling flux tube [L78] (`https://tinyurl.com/tkkyyd2`).
2. If the Hamiltonian cycle is something physical it could correspond to flux tube. The notion of magnetic flux tube central in TGD might allow application to ferromagnetism. TGD predicts two kinds of flux tubes: Maxwellian ones and monopole flux tubes with magnetic fields requiring no currents to generate them: they are not not allowed by Maxwell's theory.

   The preservation of the Earth's magnetic field predicted to decay rather rapidly as currents generating it dissipate supports the view that it contains monopole flux part which from biological input would correspond to endogenous magnetic field B$_{end}, whichisafraction2/5aboutthenominalvalueofB_E =$
   $.5Gauss.Thepresenceofmagneticfieldsincosmologicalscalesisalsoamysteryfindingasolutionintermsofmonopolef$

3. Monopole flux tubes must be closed. Closed non-intersecting flux tubes connecting nearest neighbors in lattice would correspond to Hamiltonian cycles. In TGD inspired biology Hamiltonian cycles associated with icosahedron and tetrahedron provide a realization of the vertebrate genetic code [L5] (`https://tinyurl.com/yad4tqwl`) but it is still somewhat of mystery why the points of icosahedron and tetrahedron, which are lattices (tessellations) at sphere, would be connected by a curve. Quantum classical correspondence suggests that magnetization corresponds to flux tubes connecting magnetic dipoles as formal analogs of monopole-antimonopole pairs. Could magnetic flux tubes provide a concrete realizaon for these Hamiltonan cycles?
4. Closed monopole flux tubes seem to be unrealistic for the description of ferromagnetism, which suggests the presence of N parallel flux tubes carrying magnetization M and defining a braid connecting opposite ends of ferromagnet. The monopole fluxes could arrive as single flux along parallel space-time sheet carrying field H defined by single thick flux tube. Test particle would experience $B = M + H$.

The following considerations are not much more than first impressions and probably require updating.

TGD based view

Flux tubes are the new element of condensed matter physics predicted by TGD. Could they provide insights into ferromagnetism?

1. Starting from text book picture about ferromagnetism

To develop TG view about ferromagnetism it is best to start from the text book picture.

1. In the standard model of ferromagnetism one assumes the presence of field B identified as sum $B = M + H$ of magnetization and field H equal to B outside the magnet. M is due to magnetic dipoles besides magnetic field B and the interaction of spins with H is important. B is usually regarded as the fundamental field M and and H appear as auxiliary notions and their relation to B requires a model for the system: typically H, M, and B are assumed to be linearly related.
2. The field M could be naturally assigned with a flux tube connecting the spins - perhaps at nearest neighbor lattice points. What about H? In standard model H and B are parallel for the ferromagnetic configuration. If B is assigned with the flux tube connecting the magnetic moments and B is parallel to H, this would suggest a flux tube consisting of long straight portions parallel to each other.

   In the many-sheeted space-time of TGD M and H can reside at different space-time sheets, which are parallel so that they are on top of each other in $H = M^4 \times CP_2$. H is at large space-time sheet including also the environment. The decomposition to sum would have representation as a set theoretic union.

   The test particle would experience the sum of the magnetic fields associated with the two sheets. Could M and H as the return flux associated with M and superposing with the external contribution to H correspond to these two space-time sheets so that particle would experience their sum as $B = M + H$? If so, ferromagnetism could be seen as a direct signature of many-sheeted space-time.

2. Could also monopole flux tubes be important?

There is still one important aspect related to the TGD view about magnetic field which might play important role. TGD predicts two kind of flux tubes. The first kind of flux tubes could be called Maxwellian and the corresponding magnetic fields require current to generate them. There are also flux tubes having closed cross section and carrying monopole fluxes. No currents are required to generate corresponding magnetic fields. Could also these flux tubes having no current as sources be present? This would mean new physics.

1. The first thing to notice is that the interpretation of magnetization M is as a magnetic field generated by magnetic moments. The usual interpretation is that spins are analogous to magnetic moments created by currents consisting of rotating charge. For spin there is no such

rotating charge. Second interpretation is as magnetic moments identifiable as infinitesimal monopole pairs.

2. Could one think that the flux tubes containing sequence magnetic moments correspond to monopole flux and that closing this loop could give rise to monopole magnetic field? Ordinary Maxwellian part could be also present and have current as source. How M and H would relate to these. Could M correspond to the monopole part and H the Maxwellian part?

   Are spins necessary for the existence of a monopole flux tube? Could quantum classical correspondence require this? Could dark charged matter assigned with the monopole flux tubes correspond to the magnetic moments of say dark valence electrons with non-standard value of $h_{eff} = nh_0$ so that M would be represented by monopole flux tubes classically? If the return flux represented by H is absent, flux tube must give rise to a Hamiltonian cycle. If H is present, it would be enough to have flux tubes representing N braid strands fusing to single monopole flux carrying the return flux.

   Formation of a flux loop defining Hamiltonian cycle would be a new kind of phenomenon analogous to spontaneous magnetization requiring no external field H. Spontaneous magnetization would be however something different. A trivial braid consisting of N parallel strands representing M and parallel to it locally with return flux arriving along single large flux tube carrying H would be formed in ferromagnetic transition and also in spontaneous magnetization.

3. Bringing in thermodynamics

One can try to make this more concrete by bringing in thermodynamics.

1. Assume that there exists single flux tube - connecting all the lattice points (magnetic moments) or possibly N flux tubes parallel to local magnetization M and giving rise to a braid like structure representing the topology of flux lines of M connecting opposite boundaries of magnet.

2. In the general case, the points of the lattice could be connected by a flux tubes connecting points, which need not be nearest neighbors. The first guess is that the magnetic interaction energy of spins at the ends of the flux tube portion connecting them decreases with the distance between spins. There should be also magnetic energy associated with the field H at the space-time sheet carrying the return flux. Thermodynamics would bring in entropy and free energy $F = E - TS$ would be mimimized. Entropy maximization would favor long random flux tubes and energy minimization short flux tubes.

   One expects that flux tube has free energy F increasing with flux tube length. If one does not allow self-intersections - as suggested by repulsive Coulomb interaction and Fermi statistics - the flux tube could be either Hamiltonian cycle or consist of analogs of braid strands: in the case of ferromagnetism the strands would be parallel to each other. The interaction energy would be same for all Hamiltonian cycles if determined by nearest neighbour interactions.

3. In the general case with lattice replaced by graph one expects that a large number of Hamiltonian cycles not related by rotation to each other exists so that one would have large number of states with same minimum energy. Could this somehow correspond to spin glass state allowing large number of degenerate states? The flux tube need not be closed. In ferromagnetic configuration this would be the case.

4. How would the assignment of spin direction to the lattice points affect the situation? Could the numbers N_+ and N_- of spin up and spin down electrons determine the flux tube configuration (say braid) by (Gibbs) energy minimization?

4. Could 2-braid describe the transition to ferromagnetism?

In the work of Li *et al* discussed in the article, the permutations of lattice points are induced by moving the vacancy around. This picture inspired the considerations above but is too limited. In fact the work of Li *et al* only directed attention to Hamiltonian cycles and braids formed by the non-closed analogs.

1. TGD picture brings in mind braid-knot connection. One can replace the braid associated with M with a knot by connecting the magnetic moments at the opposite ends of the braid

by strands of a trivial braid at parallel space-time sheet. This trivial braid would carry the return flux having interpretation in terms of field H.

The flux tubes of trivial braid could also fuse to single thicker flux tube or flow to a larger space-time sheet carrying the total return flux associated with M. This would conform with the idea that H provides a description of the system in longer length scale being analogous to a smoothed out total magnetic field acting as self-consistent background.

Remark: Could one assume that only H assignable to big flux tube has constant direction and magnitude and that M is represented as flux tubes connecting dipoles can in principle correspond to any permutation of atoms? For this option the spontaneous magnetization would correspond to a superposition of different configurations with same weights and would be invariant under permutations as in the argument of Li *et al* involving no flux tubes. This option does not look attractive.

2. What braid picture allows to say about the transition to ferromagnetism? Could the transition be realized by deforming the flux tubes associated with M and forming a non-trivial braid be induced by permutation of the lattice points taking the non-trivial braid to trivial one? This would be like opening the braid. The lattice points in the initial and final state would correspond to the ends of a dynamical evolution. The permutation would be realized as a time-like braiding with braid strands in time direction.

   Mathematically braid group corresponds to the covering group of permutation group and quantum group representations correspond to the representations of braid groups. The description of the transition could provide a new application of quantum groups.

The description as time-like braiding is not however complete since there is an additional structure involved: the flux tubes connecting the magnetic dipoles in lattice and defining a braid or even more complex configuration having flux tube connections between non-neighboring magnetic moments.

1. If there is no return flux assignable to H, M corresponds to a closed flux tube carrying monopole flux the dynamical time-like dynamical braiding would lead to a Hamiltonian cycle in this case and the number of final state configurations would be finite, there is degeneracy. Could spin glass phase correspond to this situation?

2. In ferromagnetism final state would contain N parallel strands carrying the monopole flux assignable to M and the return flux H would arrive along parallel thick flux tube. In general configuration these strands can be braided. The transition to ferromagnetism would represent time-like braiding of an ordinary 3-D braiding of flux tube strands connecting the opposite boundaries of ferromagnetic. In the initial state braid would be non-trivial and the flux tubes of braid would not have minimal length and minimum energy. In the final ferromagnetic state braid would be trivial with parallel flux tubes.

   Mathematically this process would correspond to what is called 2-braiding: I have proposed that 2-braidings are important in TGD inspired biology as a topological description of dynamical processes. An interesting interpretation is as a topological analog for problem solving. I have also proposed that in bio-systems topological quantum computation programs are represented as this kind 2-braidings for flux tubes [K4, K120] (`https://tinyurl.com/ycvgjccq` and `https://tinyurl.com/ydylud6c`.)

   Ferromagnetism would correspond to an opening of a non-trivial braid. If the return flux arrives along flux tubes this is possible smoothly only if the knot defined in this manner is trivial. To achieve opening, the 2-braiding must involve reconnections, which correspond to cutting the knot strand and reconnecting the pieces in new manner: this is how Alexander opened his knot. Fermi statistics and repulsive Coulomb interaction do not favour this mechanism. If the return flux arrives along single flux tube, the opening could correspond to a smooth deformation without reconnections transferring the braiding to the parallel space-time sheet, where it is "neutralized" by fusing the flux tubes to single flux tube.

Chapter 11

Quantum Criticality and Dark Matter: part II

11.1 Introduction

Quantum criticality is one of the corner stone assumptions of TGD. The value of Kähler coupling strength fixes quantum TGD and is analogous to critical temperature. TGD Universe would be quantum critical. What does this mean is however far from obvious and I have pondered the notion repeatedly both from the point of view of mathematical description and phenomenology [K57, K126].

1. Criticality is characterized by long range correlations and sensitivity to external perturbations and living systems define an excellent example of critical systems - even in the scale of populations since without sensitivity and long range correlations cultural evolution and society would not be possible. For a physicist with the conceptual tools of existing theoretical physics the recent information society in which the actions of people at different side of globe are highly correlated, should look like a miracle.

2. The hierarchy of Planck constants with dark matter identified as phases of ordinary matter with non-standard value $h_{eff} = n \times h$ of Planck constant is one of the "almost-predictions" of TGD is definitely something essentially new physics. The phase transition transforming ordinary matter to dark matter in this sense generates long range quantal correlations and even macroscopic quantum coherence.

   Finding of a universal mechanism generating dark matter have been a key challenge during last ten years. Could it be that criticality is always accompanied by the generation of dark matter? If this is the case, the recipe would be stupifuingly simple: create a critical system! Dark matter would be everywhere and we would have observed its effects for centuries! Magnetic flux tubes (possibly carrying monopole flux) define the space-time correlates for long range correlations at criticality and would carry the dark matter. They are indeed key players in TGD inspired quantum biology.

3. Change of symmetry is assigned with criticality as also conformal symmetry (in 2-D case). In TGD framework conformal symmetry is extended and infinite hierarchy of breakings of conformal symmetry so that a sub-algebras of various conformal algebras with conformal weights coming as integer multiples of integer n defining h_{eff} would occur.

4. Phase separation is what typically occurs at criticality and one should understand also this. The strengthening of this hypothesis with the assumption $h_{eff} = h_{gr}$, where $h_{gr} = GMm/v_0$ is is the gravitational Planck constant originally introduced by Nottale. In the formula v_0 has dimensions of velocity, and will be proposed to be determined by a condition relating the size of the system with mass M to the radius within which the wave function of particle m with $h_{eff} = h_{gr}$ is localized in the gravitational field of M.

5. The condition $h_{eff} = h_{gr}$ implies that the integer n in h_{eff} is proportional to the mass of particle. The implication is that particles with different masses reside at flux tubes with different Planck constant and separation of phases indeed occurs.

6. What is remarkable is that neither gravitational Compton length nor cyclotron energy spectrum depends on the mass of the particle. This universality could play key role in living matter. One can assign Planck constant also to other interactions such as electromagnetic interaction so that one would have $h_{em} = Z_1Z_2e^2/v_0$. The phase transition could take place when the perturbation series based on the coupling strength $\alpha = Z_1Z_2e^2/\hbar$ ceases to converge. In the new phase perturbation series would converge since the coupling strength is proportional to $1/h_{eff}$. Hence criticality and separation into phases serve as criteria as one tries to see whether the earlier proposals for the mechanisms giving rise to large h_{eff} phases make sense. One can also check whether the systems to which large h_{eff} has been assigned are indeed critical.

The motivation for this work came from super-fluidity. Superfluids exhibit rather mysterious looking effects such as fountain effect and what looks like quantum coherence of superfluid containers, which should be classically isolated. These findings serve as a motivation for the proposal that genuine superfluid portion of superfluid corresponds to a large h_{eff} phase near criticality at least and that also in other phase transition like phenomena a phase transition to dark phase occurs near the vicinity.

11.1.1 Some applications to condensed matter physics

In the second part of the chapter the applications to condensed matter physics are discussed. By its nature condensed matter physics provides rich repertoire of critical phenomena.

1. Different phases of same substance, say water, can be in phase equilibrium at criticality and dark matter. There are critical regions of parameter space -critical lines and critical points, in which the transitions between different phases are possible. Long range thermodynamical correlations are associated with these systems and the association with dark matter would suggest that dark matter could appear in these critical systems.
2. Different substances can form mixtures (`http://tinyurl.com/286nqx`). For instance, oil can mix to water in some parameter regions. This kind of systems are good candidates for critical systems. There is actually rich spectrum of mixtures. Solutions (`http://tinyurl.com/yz3hvq`), colloids (`http://tinyurl.com/yabljt8l`), dispersions (`http://tinyurl.com/bq3vm2m`) and the substances can be also in different phases (gas, liquid, solid) so that very rich spectrum of possibilities emerges. Is the generation of dark matter involved only with the phase transitions between different types of mixed phases or between mixed and non-mixed phase? Are some phases like gel inherently critical?
3. One example about criticality is phase transition to super-fluidity or super-conductivity. In the transition from super-conducitivity the value of specific hearts diverges having the shape of greek letter λ: hence the name lambda point. This suggests that in transition point the specific heat behaves like N^2 due to the quantum coherence instead of proportionality to N as usually. The strange properties of super-fluid, in particular fountain effect, could be understood in terms of $h_{eff} = h_{gr}$ hypothesis as will be discussed.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

11.2 Liquids, superconductivity and superfluidity

11.2.1 Mysterious Action At Distance Between Liquid Containers

This section as also the consideration of the idea that criticality could involve a phase transition transforming ordinary matter to dark matter was inspired by a link sent by Ulla. The link was to a popular article (`http://tinyurl.com/yaaqnebp`) telling about mysterious looking action at a distance between liquid containers.

For several years it has been that superfluid helium in reservoirs next to each other with distance of few micrometers acts collectively, even when the channels connecting them are so thin and long that substantial flow of matter between them is not possible. The article mentions a theoretical model [B4] developed by a team of scientists include those from the Instute of Physical Chemistry of the Polish Academy of Sciences in Warzaw (IPCPACW). According to the article the model reveals that the phenomenon is much more general than previously thought and could take place also systems which are usually regarded as classical (what this actually means in quantum world is not quite clear!). The reading of the abstract of the article (http://tinyurl.com/y7pmbw2k) shows that only Monte Carlo studies are done so that "predicts" is more appropriate than "reveals".

According to the article, the first report about "action at a distance" was between superfluid reservoirs was published in 2010 in Nature Physics [D34] (http://tinyurl.com/y7pfc9a9). The team from the University of Buffalo and the State University of New York created an array of tens of millions of cubical reservoirs containing liquid healiium on a silicon plate. The centres of reservoirs had distance of 6 μ and the reservoirs had edge length of 2 μm so that the width of the horizontal gap between reservours was 4 μm. The reservoirs were covered with another silicon plate with a very thin gap above the reservoirs allowing to fill them with liquid helium. The thickness of this vertical gap was $d = 32$ nm - in TGD language this is $d = 3.2L(151)$, where the p-adic length scale $L(151) = 10$ nm defines the thickness of cell membrane The gap was so thin that it did not allow a significant flow of liquid helium between the different reservoirs.

Remark: To be precise, $L(151)$ should be called the Compton length of electron if it would correspond to Gaussian Mersenne $MG, k = 151 = (1+i)^k - 1$ and is $L_e(151) = \sqrt{5} \times L(151)$, where $L(151)$ would be the genuine p-adic length scale. For brevity I often call L_e just p-adic length scale and drop the subscript "e".

The expectation was that different reservoirs would behave like independent systems without interactions. In particular, the specific heat of the whole system would be sum over the specific heats of individual systems, which were identical. This was not the case. An excess of specific heat was observed in the system. The super-fluid helium was acting as a physical whole.

The natural explanation would be in terms of the superfluid character of the systems. Still the absence of the direct contact - say thin "threads" connecting the reservoirs - makes one to wonder whether the situation can be understood in the framework of conventional quantum physics.

In co-operation with Prof. Douglas Abraham from Oxford University, Dr. Maciolek from (IPCPACW) has developed a theory to exlain the observations [B4] (http://tinyurl.com/y7pmbw2k). The new theory predicts that the effect of "action at a distance" does not require quantum physics and can also occur in classical one-component fluids, as well as its mixtures. The article says that this theory is confirmed by computer simulations carried out by Oleg Vasilyev from the Max-Planck Insitute für Intelligente Systeme. I would be here a little bit skeptical: experiments conform, computer simulations only allow to calculate!

The theory makes certain predictions.

1. Super-fluidity is not a necessary condition. The phenomenon can occur if the system is near criticality and thus involves at least two different phases of matter. Therefore low temperatures a not necessary. For instance, water and lutidine - a model mixture of water and oil - mix only in certain temperature range and "action at a distance" appears only in this range. On basis of the popular article it remains clear whether this is a prediction or an experimental fact.
2. The dimensions of the reservoins and the connecting channels are also important. The phenomenon ceas the distances are significantly larger than the size of human cells.

Some comments from TGD based view about criticality already summarize are in order.

1. The notion of "classical" can be misleading. One can model physical phenomena classically - thermodynamical phase transitions are basic example of this but the microscopic - and also non-microscopic physics of long range correlations - can be actually quantal. Basically all physics is quantal and during last years people have begun to learn that even macroscopic physical can behave non-classically. In TGD framework however quantum physics as classical space-time correlates and this brings a new element.
2. The key question is what makes the superfluids closed in the reservoirs to behave like single quantum coherent system in the first experiment. TGD based view about space-time correlates of criticality and long range correlations associated with suggests that magnetic flux tubes or sheets connecting the superfluid reservoirs are essential. Even more, these flux quanta - possible carrying monople fluxes - would be universal space-time correlates of any critical

phenomenon. In separate section I will discuss a model for the fountain effect exhibited by ^4He based on the notion of flux quantum carrying the genuine super fluid (normal and superfluid component are involved) having non-standard value of Planck constant, which is rather large so that the gravitational Compton length is macroscopic length and the effects of gravitation the wave function are very small and the super-fluid apparently defies gravitational force.

3. Second question is why cell length scale of few microns would serve as a prerequisite for the phenomenon. The length scale range 10 nm-2.5 μm involves as many as four p-adic length scales labelled by Gaussian Mersennes ($k = 151, 157, 163, 167$ and corresponds to length scale range between thickness of cell membrane and cell nucleus size. TGD suggests strongly dark variants of weak with $h_{eff} = n \times h$ and also strong physics with corresponding gauge bosons being effective as massless particles below these length scales. The exchange of these massless bosons would generate long range correlations at criticality. Also p-adic variants of these physics with mass scales of weak bosons reduced to a range varying in 1-100 eV range would be involved if TGD vision is correct. Hence criticality would involve quantum physics and even dark matter!

4. Phase separation - be it separation of particles in mixture or phases of say water - is very relevant of criticality. How this happens. The TGD answer already considered relies on the notion of hierarchy of Planck constants $h_{eff} = n \times h$ and universality of cyclotron frequencies associated with magnetic flux tubes and due to the identification $\hbar_{eff} = \hbar_{gr} = GMm/v_0$ already discussed. The large mass M is the mass of the dark fraction of the Earth's mass. This implies that Planck constant characterizes particle and also that the gravitational Compton length is same for all particles and the energy spectrum of cyclotron radiation is universal and in the range of visible and UV energies associated with bio-photons.

   All these predictions conform nicely with the universality of criticality. The prediction is that bio-photons would accompany any Earthly critical system. What of course raises the eyebrows of skeptics is the proposed dependence of critical phenomena on the dark gravitational mass of the planet or system which the system is part of.

11.2.2 The Behavior Of Superfluids In Gravitational Field

Superfluids apparently defy gravitational force as so called fountain effect (`http://tinyurl.com/kx3t52r`) demonstrates. In the following TGD inspire model based on the hypothesis that the genuine superfluid part of superfluid at least near criticality corresponds to large h_{eff} phase is considered.

Fountain effect

In an arrangement involving a vessel of superfluid inside another one such that the levels of superfluids are different in the two vessels, the superfluid flows spontaneously along the walls of the vessels as a superfluid film. The flow is from the vessel in which the level of superfluid is higher until the heights are equal or *all* fluid has left the other container. For illustrations see the pictures of the article (see `http://tinyurl.com/h66hydb`) [D39] "Why does superfluid helium leak out of an open container?".

What is strange that all the fluid flows from the vessel to another one it the height of vessel is high enough. According to the prevailing wisdom super-fluid actually consists of ordinary fluid and genuine superfluid. The fluid flows from the vessel as a genuine superfluid so that the process must involve a phase transition transforming the ordinary fluid component present in the fluid to superfluid keeping superfluid fraction constant. A further strange feature is that the superfluid flows as a film covering the inner (and also outer) surface of entire container so that return flow is not possible. This suggests interpretation as a macroscopic quantum phenomenon

According to the article of Golovko the existing wisdom about flow is that it corresponds to wetting. This would however predict that the phenomenon takes place also above the critical point (λ point) for the ordinary fluid. This is not the case. Secondly, the force responsible for the sucking the superfluid from the container would act only at the boundary of the film. As the film covers both the interior and exterior walls of the container the boundary vanishes, and therefore also the force so that the flow of the superfluid to another container should stop. The amount of the superfluid leaving the container should be small and equal to the amount of super-fluid in the film: this is not the case. Hence the conventional explanation does not seem to work.

TGD inspired model for the fountain effect

What could be the TGD explanation for fountain effect?

1. Macroscopic quantum coherence in the scale of the film is suggestive and hierarchy of Planck constants $h_{eff} = n \times h$ and magnetic flux quanta suggest themselves. Whether this notion is relevant also for the description of the super-fluid itself is not of course obvious and one might argue that standard description is enough. Just for fun, we can however for a moment assume that the super-fluid fraction could correspond to a dark phase of 4^He located at flux quanta. The natural candidates for the flux quanta is a flux sheet connecting the vessel to the external world or smaller vessel and larger vessel to each other. The flux sheet would accompany the film covering the inside and outside walls.

2. The fact that the flow seems to defy gravitational force suggests that macroscopic quantum coherence is involved in these degrees of freedom and that one should describe the situation in terms of wave function for super-fluid particles in the gravitational potential of Earth. For ordinary value of Planck constant one cannot of course expect macroscopic quantum coherence since coherence length is not expected to be much larger than Compton length.

3. TGD predicts that h_{eff} characterizes Kähler magnetic flux tubes at which the dark Bose-Einstein condensate resides. There can consider several identifications of h_{eff} corresponding to gravitational interaction, electromagnetism, or long range Z^0 interaction possible in TGD and originally associated with super-fluidity interpreted as Z^0 superconductivity.

   (a) For gravitational interaction, the identification is $h_{eff} = h_{gr}$, where h_{gr} is what I call gravitational Planck constant of Nottale [E13]

   $$\hbar_{gr} = \frac{GMm}{v_0} = \frac{r_S m}{2\beta_0} \quad , \quad \beta_0 = \frac{v_0}{c} \tag{11.2.1}$$

   In the expression of h_{gr} M is the "large" mass - naturally Earth's mass M_E. m would be the mass of ^4He atom. $r_S = 2GM/c$ denotes Schwartschild radius of Earth, which from $M_E = 3 \times 10^{-6} M_{Sun}$ and from $r_S(Sun) = 3$ km is 4.5 mm. v_0 could be some characteristic velocity for Earth-super fluid system and the rotation velocity $v_0 = 465.1$ m/s of Earth is the first guess. However, the model for cyclotron frequencies of dark ions favor rather large value of v_0, say $v_0/c = 1/2$.
   The gravitational Compton length $\lambda_{gr} = h_{gr}/m$ does not depend on the mass of the particle and equals to $\lambda_{gr} = GM/v_0 \simeq 645$ meters for $v_0 = 465.1$ m/s. The scale of the superfluid system would be much smaller than the minimum coherence length, which does not look plausible.

   (b) The generalization of Nottale's formula to the electromagnetic case would be $\hbar_{em} = Q_{em}e^2/v_0 > \hbar$. Q_{em} should refer to the em charge of super-fluid but this is essentially zero so that this option is not feasible.

   (c) According to the standard model, the nuclear Z^0 charges are indeed non-vanishing in the standard model, where Z^0 quanta are massive. In TGD framework, one can however argue that they are actually screened above weak boson Compton length. For $\hbar_{eff} = \hbar_Z$ the intermediate boson Compton length could be scaled up by the factor $\hbar_Z/\hbar$ so that Z^0 quanta would be effectively massless below this screening scale.
   The generalization of the Nottale's formula would read as $\hbar_Z = Q_Z g_Z^2/v_0$, where ZQ_Z refers to the total Z^0 charge of the super-fluid phase. Cell scale would require $h_Z/h \sim 10^{11}$. Screening scale is atomic for $h_Z/h \sim 10^7$: this value turns out to be too large. A value of order 2^8 is required. If the screening of nuclear charges in the scale of ^4He nuclei, one would have $h_Z/h \sim 100$. This value of h_Z/h is however not consistent with the Nottale's hypothesis and with the idea about scaled up weak boson Compton length.

A possible way to solve this problem is to assume that weak bosons can also appear at electroweak p-adic scales which differ from intermediate boson length scale [K63] and have scaled down masses. If this scale corresponds to the atomic p-adic length scale $L(k = 137)$ as the size scale of Cooper pairs suggests, then $h_Z \simeq 2^8$ would correspond to the p-adic length scale $L(153)$ ($L(151) \simeq 10$ nm corresponds to cell membrane thickness).

A simple model for the situation discussed would rely on Schrödinger equation at the flux quantum which is locally a thin hollow cylinder turning around at the top of the wall of the container.

1. One obtains 1-dimensional Schrödinger equation

$$(-\frac{\hbar_{eff}^2\partial_z^2}{2m} + mg_{eff}z)\Psi = E\Psi \ , \quad h_{eff} = nh_0 = \frac{nh}{6} \ . \tag{11.2.2}$$

It is easy to see that the energy spectrum is invariant under the scaling $h \rightarrow h_{eff} = xh$ and $z \rightarrow z/x$. One has $\Psi_{xh,g_{eff}=g/x}(z) = \Psi_{h,g}(z/x)$ so that simple scaling of the argument z in question. The energy of the solution is same. If the ordinary solution has size scale L, the scaled up solution has size scale xL.

The height for a trajectory in gravitational field of Earth is scaled up for a given initial vertical velocity v_i is scaled as $h \rightarrow xh$ so quantum behavior corresponds to the classical behavior and de-localization scale is scaled up. Could this happen at various layers of magnetic body for dark particles so that they would be naturally at much higher heights. Cell scale would be scaled to Earth size scale of even larger sizes for the values of $\hbar_{eff}/h = n$ involved.

For classical solution with initial initial vertical velocity $v_i = 1$ m/s the height of the upwards trajectory is $h = v_i^2/2g$ 5 cm. Quantum classical correspondence would be given by $E = mv_i^2/2$ and this allows to look the delocalization scale of a solution.

2. One can introduce the dimensionless variable u (note that one has $g_{eff}/g = 1/x$, $x = h/h_{eff}$) as

$$u = \frac{z - \frac{E}{mg_{eff}}}{z_0} \ , \qquad z_0 = \left[\frac{2m^2 g_{eff}}{\hbar_{eff}^2}\right]^{-1/3} = \frac{h_{eff}}{h}(\frac{m}{m_p})^{2/3} \times (\frac{g}{L_p^2})^{-1/3} \simeq \frac{h_{eff}}{h} \times (\frac{m}{m_p})^{2/3} \times 2.4\ mm$$

$$L_p = \frac{\hbar c}{m_p} = \simeq 2.1 \times 10^{-16} \text{ m} \ ,$$

(11.2.3)

Here m_p denotes proton mass and L_p proton Compton length. z_0 scales as $\hbar_{eff}$ as one might expect. z_0 characterizes roughly the scale of the s lution. From the scale of the fountain effect about 1 meter, one can conclude that one should have $h_{eff}/h \sim 2^8$.

This allows to cast the equation to the standard form of the equation for Airy functions encountered in WKB approximation

$$-\frac{d^2\Psi}{du^2} + u\Psi = 0 \ . \tag{11.2.4}$$

Remark: Note that the classical solution depends on m. In central force problem with $1/r$ and $h_{eff} = GMm/v_0$ the binding energy spectrum $E = E_0/n^2$ has scale $E_0 = v_0^2 m$ and is universal.

3. The interesting solutions correspond to Airy functions $Ai(u)$ which approach rapidly zero for the values of $u > 1$ and oscillate for negative values of u. These functions $Ai(u + u_1)$ are orthogonal for different values of u_1. The values of u_1 correspond to different initial kinetic energies for the motion in vertical direction. In the recent situation these energies correspond to the initial vertical velocities of the super-fluid in the film. $u = u_0 = 1$ defines a convenient estimate for the value of z coordinate above which wave function approaches rapidly to zero.

For classical solution with initial initial vertical velocity $v_i = 1$ m/s the height of the upwards trajectory is $h = v_i^2/2g$ 5 cm. Quantum classical correspondence would be given by $E = mv_i^2/2 = E$ and this allows to look the delocalization scale of a solution.

The Airy function $Ai(u)$ approaches rapidly to zero (see the graph of https://en.wikipedia.org/wiki/Airy_function) and one can say that above $u_0 = 3$ the function vanishes. Already at $u_0 = 1$ wave function is rather small as compared with its value at $u = 0$. This condition translates to a condition for z as

$$z_0 = z_{cl} + u_0 z_0 \ , \quad z_{cl} = \frac{E}{m g_{eff}} \ , \quad z_0 = \frac{h_{eff}}{h}\left[\frac{\hbar^2}{2m^2 g}\right]^{1/3} \ . \qquad (11.2.5)$$

The condition is consistent with the classical picture and the classical height z_{cl} scales like h_{eff}/h. The parameter $u_0 z_0$ defines the de-localization scale consistent with the expectations. Below z_{cl} the wave function oscillates which intuitively corresponds to the sum of waves in upwards and downwards directions.

What can one conclude about the value of $x = h_{eff}/h_0$ in the case of super-fluidity?

1. Using the previous formula, the condition that z_0 is of order 1 meter fixes its value to $h_{eff}/h_0 \sim 2^8$. Could super-fluidity correspond to the value of $h_{eff} = h_{em} > h$ assignable to electromagnetic flux tubes? The generalization $h_{em} = Ze^2/v_0$ of the Nottale's formula would require that the super fluid phase has a large total em charge Z. The Cooper pairs are however neutral. This leaves under consideration only the old idea that super-fluidity corresponds to Z^0 super-conductivity inspired by the idea that TGD predicts long range Z^0 fields and by the fact that nuclei carry indeed carry non-vanishing Z^0 charge mostly due to neurons.

2. Both $\hbar_{eff}(gr)/\hbar \simeq 2^{24}$ and $\hbar_{gr} = GMm/v_0$ given by Nottale's hypothesis give quite too large value of z_0.

   The gravitational Compton length λ_{gr} is given by $\lambda_{gr} = GM_e/v_0 = r_S/2v_0$ and - in accordance with the Equivalence Principle - does not depend on m. The Schwartschild radius of Earth is $r_S = .9$ cm. One could argue that λ_{gr} is a reasonable lower bound for z_0 if h_{gr} appears in the gravitational Schrödinger equation. For $v_0/c = 2^{-11}$ required by the Bohr orbit model for the 4 inner planets, this would give $\lambda_{gr} = 9$ m. The energy scale of dark cyclotron states comes out correctly if one has $v_0/c = 1/2$ giving the lower bound $z_0 \geq r_S = .9$ cm.

   However, the proportionality of z_0 to h_{eff}/h implies that the z_0 is scaled by a factor of order $2GM_E m_p/v_0 \sim 10^{14}$ from its value $z_0 = .2$ mm and would be gigantic. It seems that this option indeed fails.

3. Could the fountain effect be due to the reduction of g in principle possible if G is prediction and CP_2 length replaces Planck length as fundamental scale? If one assumes $h_{eff} = h$ and scaled down value of g corresponding to $G_{eff} = R^2/\hbar_{gr}$ such that $\hbar_{gr}$ is scaled from its normal value: $\hbar_{gr} \rightarrow y\hbar_{gr}$, $G_{eff} \rightarrow G_{eff}/y$. This would give the scaling of $z_0 \propto g^{-1/3}$ as $z_0 \rightarrow y^{1/3} z_0$ giving $z_0 \simeq .2$ mm should be scaled up to about 1 mm which would give $y \sim 10^9$. This would mean a huge breaking of Equivalence Principle.

Superfluids dissipate!

People in Aalto University - located in Finland by the way - are doing excellent work: there is full reason to be proud! I learned from the most recent experimental discovery by people working in Aalto University from Karl Stonjek. The title of the popular article (see http://tinyurl.com/yagsttw7) is "*Friction found where there should be none—in superfluids near absolute zero*".

In rotating superfluid one has vortices and they should not dissipate. The researchers of Aalto University however observed dissipation: the finding by J Mäkinen *et al* is published in in Phys Rev B [D30] (see http://tinyurl.com/y7dtsdys). Dissipation means that they lose energy to environment. How could one explain this?

What comes in mind for an inhabitant of TGD Universe, is the hierarchy of Planck constants $h_{eff} = n \times h$ labelling a hierarchy of dark matters as phases of ordinary matter. The reduction of

Planck constant h_{eff} liberates energy in a phase transition like manner giving rise to dissipation. This kind of burst like liberation of energy is mentioned in the popular article "glitches" in neutron stars). I have already earlier proposed an explanation of fountain effect of superfluidity in which superfluid flow seems to defy gravity. The explanation is in terms of large value of h_{eff} implying delocalization of superfluid particles in long length scale [?] (see `http://tinyurl.com/y8xhvwt2`).

Remark: Quite generally, binding energies are reduced as function of $h_{eff}/h = n$. One has $1/n^2$ proportionality for atomic binding energies so that atomic energies defined as rest energy minus binding energy indeed increase with n. Interestingly, dimension 3 of space is unique in this respect. Harmonic oscillator energy and cyclotron energies are in turn proportional to n. The value of n for molecular valence bonds depends on n and the binding energies of valence bonds decrease as the valence of the atom with larger valence increases. One can say that the valence bonds involving atom at the right end of the row of the periodic table carry metabolic energy. This is indeed the case as one finds by looking the chemistry of nutrient molecules.

The burst of energy would correspond to a reduction of n at the flux tubes associated with the superfluid. Could the vortices decompose to smaller vortices with a smaller radius, maybe proportional to n? I have proposed similar mechanism of dissipation in ordinary fluids for more than two decades ago. Could also ordinary fluids involve hierarchy of Planck constants and could they dissipate in the same manner?

In biology liberation of metabolic energy - say in motor action - would take place in this kind of "glitches". It would reduce h_{eff} resources and thus the ability to generate negentropy: this leads to smaller negentropy resources and one gets tired and thinking becomes fuzzy.

An improvement of the flux tube picture

The above arguments related to fountain effect cry for a more precise formulation of the rather loose ideas about how gravitational interaction is mediated by flux tubes.

It has been assumed that $\hbar_{gr} = GMm/v_0$ characterizes the flux tubes connecting mass M - say solar mass - to a smaller mass m. This assumption cannot be correct as such.

1. The assumption treats the two masses asymmetrically.

2. A huge number of flux tubes is needed since every particle pair $M - m$ would involve a flux tube. It would be also difficult to understand the fact that one can think the total gravitational interaction in Newtonian framework as sum over interactions with the composite particles of M. In principle M can be decomposed into parts in many ways - elementary particles and their composites and larger structures formed from them: there must be some subtle difference between these different compositions - all need not be possible - not seen in Newtonian and GRT space-time but maybe having representation in many-sheeted space-time and involving h_{gr}.

3. Flux tube picture in the original form seems to lead to problems with the basic properties of the gravitational interaction: namely superposition of gravitational fields and absence or at least smallness of screening by masses between M and m. One should assume that the ends of the flux tubes associated with the pair pair $M - m$ move as m moves with respect to M. This looks too complex.

   Linear superposition and absence of screening can be understood in the picture in which particles form topological sum contacts with the flux tubes mediating gravitational interaction. This picture is used to deduce QFT-GRT limit of TGD. Note that also other space-time sheets can mediate the interaction and pairs of MEs and flux tubes emanating from M but not ending to m are one possible option. In the following I however talk about flux tubes.

These problems find a solution if h_{gr} characterizes the *magnetic body* (MB) of a particle with mass m topologically condensed to a flux tube carrying total flux M. m can also correspond to a mass larger than elementary particle mass. This makes the situation completely symmetric with respect to M and m. The essential point is that the interaction takes place via touching of MB of m with flux tubes from M.

1. In accordance with the fractality of the many-sheeted space-time, the elementary particle fluxes from a larger mass M can combine to a sum of fluxes corresponding to masses $M_i < M$ with $\sum M_i = M$ at larger flux tubes with $\hbar_{gr} = GMM_i/v_{0,i} \geq \hbar$. This can take place in many ways, and in many-sheeted space-time gives rise to different physical situations.

   Due to the large value of h_{gr} it is possible to have macroscopic quantum phases at these sheets with a universal gravitational Compton length $L_{gr} = GM_i m/v_0$. Here m can be also a mass larger than elementary particle mass. In fact, the convergence of perturbation theory indeed makes the macroscopic quantum phases possible. This picture holds true also for the other interactions. Clearly, many-sheeted space-time brings in something new, and there are excellent reasons to believe that this new relates to the emergence of complexity - say via many-sheeted tensor networks [L25].

2. This picture implies that fountain effect is a kind of antigravity effect for dark matter - maybe even for non-microscopic masses m - since the larger size of MB implies larger average distance from the source of the gravitational flux. This might have technological applications some day.

This picture is a considerable improvement but there are still problems to ponder. In particular, one should understand why the integer $n = h_{eff}/h = h_{gr}/h$ interpreted as a number of sheets of the singular covering space of MB of m emerges topologically. The large value of h_{gr} implies a huge number of sheets.

Could the flux sheet covering associated with M_i code the value of M_i using as unit Planck mass as the number of sheets of this covering? One would have $N = M/M_{Pl}$ sheeted structure with each sheet carrying Planckian flux. The fluxes experienced by the MB of m in turn would consist of sheets carrying fusion $n_m = M_{Pl}v_0/m$ Planckian fluxes so that the total number of sheets would be reduced to $n = N/m = GMm/v_0$ sheets.

Why this kind of fusion of Planck fluxes to larger fluxes should happen? Could quantum information theory provide clues here? And why v_0 is involved?

What about Sun?

Just for interest one can also look what one obtains in the case of Sun: this also leads to a guess for a general formula for the parameter v_0.

1. The replacement of Earth-particle system with particle-Sun system requires scaling r_S by a factor $10^6/3$, the scaling of R_E by factor about 110, and scaling of v_0/c by factor 4.3 if v_0 is identified as solar rotation velocity. The resulting value of z_0 is 1.7×10^{10} m whereas the distance of Earth from Sun is $R = 1.5 \times 10^{11}$ m, roughly 10 times larger than z_0.

2. On the other hand, of one uses the value $v_0/c \simeq 2^{-11}$ needed in the model of inner planetary orbits as Bohr orbits, one obtains $z_0 = 7.3 \times 10^8$ m to be compared with the value of solar radius $R_S = 6.96 \times 10^8$ meters. For this value of v_0 the gravitational Compton length is $\lambda_{gr} = 6 \times 10^6$ meters, which happens to be rather near to the Earth's radius.

3. The challenge is to predict the value of the parameter v_0. The above observation suggests that one could pose the consistency consistency condition $R = z_0$ to fix the value of v_0. This would give the formula

$$\beta_0 = (\frac{r_s}{4\pi R})^{1/2} \quad .$$

   This scales up β_0 from 1.6×10^{-6} to 2.3×10^{-6} by a factor $1.41 \simeq \sqrt{2}$. For Sun one obtains $\beta_0 = 5.85 \times 10^{-4}$ consistent with the value required by Bohr quantization.

Evidence for macroscopic quantum coherence of fluid flow at criticality

Evidence for the hierarchy of Planck constants implying macroscopic quantum coherence in quantum critical systems is rapidly accumulating. Also people having the courage to refer to TGD in their articles are gradually emerging. A series of fluid dynamics experiments (`http://tinyurl.`

com/jpf5f5j) providing this kind of evidence is performed by Yves Couder and Emmanuel Fort (see for instance [D5]). Mathematician John W. M. Bush has commented [D27] (http://tinyurl.com/jskythl) these findings in the Proceedings of National Academy of Sciences and the article provides references to a series of papers by Couder and collaborators.

The system studied consist of a tray containing water at a surface, which is oscillating. The intensity of vibration is just below the critical value inducing so called Faraday waves (http://tinyurl.com/hwuloet) at the surface of water. Although the water surface is calm, water droplet begins to bounce and generates waves propagating along the water surface - "walkers". Walkers behave like classical particles at Bohr orbits. As they pass through a pair of slits they behave they choose random slit but several experiments produce interference pattern. Walkers exhibit an effect analogous to quantum tunneling and even the analogs of quantum mechanical bound states of walkers realized as circular orbits emerge as the water tray rotates!

The proposed interpretation of the findings is in terms of Bohm theory (http://tinyurl.com/homasgz). Personally I find it very difficult to believe in this since Bohm's theory has profound mathematical difficulties. Bohm's theory was inspired by Einstein's belief on classical determinism and the idea that quantum non-determinism is not actual but reduces to the presence of hidden variables. Unfortunately, this idea led to no progress.

TGD is analogous to Bohm's theory in that classical theory is exact but quantum theory is now only an exact classical correlate: there is no attempt to eliminate quantum non-determinism. Quantum jumps are between superpositions of entire classical time evolutions rather than their time=constant snapshots: this solves the basic paradox of Copenhagen interpretation. A more refined formulation is in terms of zero energy ontology, which in turn forces to generalize quantum measurement theory to a theory of consciousness.

Macroscopic quantum coherence associated with the behavior of droplets bouncing on the surface of water is suggested by the experiments. For instance, quantum measurement theory seems to apply to the behavior of single droplet as it passes through slit. In TGD the prerequisite for macroscopic quantum coherence would be quantum criticality at which large $h_{eff} = n \times h$ is possible. There indeed is an external oscillation of the tray containing water with an amplitude just below the criticality for the generation of Faraday waves at the surface of water. Quantum classical correspondence states that the quantum behavior should have a classical correlate. The basic structure of classical TGD is that of hydrodynamics in the sense that dynamics reduces to conservation laws plus conditions expressing the vanishing of an infinite number of so called super-symplectic charges - the conditions guarantee strong form of holography and express quantum criticality. The generic solution of classical field equations could reduce to Frobenius integrability conditions guaranteeing that the conserved isometry currents are integrable and thus define global coordinates varying along the flow lines [K95]. The correlate for quantum criticality would be classical criticality and criticalities for various hydrodynamical stabilities would serve as excellent candidates for the situation in which large h_{eff} should become manifest.

One should of course be very cautious. For ordinary Schrödinger equation the system is closed. Now the system is open. This is not a problem if the only function of external vibration is to induce quantum criticality. The experiment brings in mind the old vision of Frölich about external vibrations as induced of what looks like quantum coherence. In TGD framework this coherence would be forced coherence at the level of visible matter but the oscillation itself would correspond to genuine macroscopic quantum coherence and large value of h_{eff} [L16]. A standard example is provided by penduli, which gradually start to oscillate in unisono in presence of weak synchronizing signal. In brain neurons would start to oscillator synchronously by the presence of dark photons with large h_{eff}.

11.2.3 New findings related to high Tc super-conductivity

I learned simultaneously about two findings related to high Tc super-conductivity. The first finding [D28] provides further evidence for high Tc superconductivity at room temperature and pressure. Skinner has made a strange observation about magnetic susceptibility as a function of temperature for two values of external magnetic field [D3] (see http://tinyurl.com/yaxtjjp5). What looks like noise is essentially same for the curves at the level of detail. Unless only pseudonoise is in question, the finding forces to ask whether the data are manipulated. TGD inspired explanation involving so called de Haas-van Alphen effect allows to understand how pseudo noise for certain pairs of value of external magnetic field could have same shape.

Second finding provides evidence for positive feedback in the transition to high Tc superconductivity. This inspires a proposal of a general TGD based mechanism of bio-control in which small signal can serve as a control knob inducing phase transition producing macroscopically quantum coherent large h_{eff} phases in living matter.

I have added to the text the discovery of BCS type super-conductivity in lantanium hydroxide at temperature of 250 K towards the end of 2018 together with TGD based explanation in terms of $h_{eff} = n \times h_0$ hypothesis.

High Tc superconductivity at room temperature and pressure

Indian physicists Kumar Thapa and Anshu Pandey have found evidence for superconductivity at ambient (room) temperature and pressure in nanostructures [D28] (see `http://tinyurl.com/ybqybvap`). There are also earlier claims about room temperature superconductivity that I have discussed in my writings [K23, K90, K91].

1. *The effect and its TGD explanation*

Here is part of the abstract of the article of Kumar Thapa and Anshu Pandey.

We report the observation of superconductivity at ambient temperature and pressure conditions in films and pellets of a nanostructured material that is composed of silver particles embedded into a gold matrix. Specifically, we observe that upon cooling below 236 K at ambient pressures, the resistance of sample films drops below 10^{-4} *Ohm, being limited by instrument sensitivity. Further, below the transition temperature, samples BCSome strongly diamagnetic, with volume susceptibilities as low as -0.056. We further describe methods to tune the transition to temperatures higher than room temperature.*

During years I have developed a TGD based model of high Tc superconductivity and of bio-superconductivity [K23, K90, K91] (see `http://tinyurl.com/yazy5kwt` and `http://tinyurl.com/y7dd4f9m`).

Dark matter is identified as phases of ordinary matter with non-standard value $h_{eff}/h = n$ of Planck constant [?, K84] ($h = 6h_0$ is the most plausible option [L26, L60]). Charge carriers are $h_{eff}/h_0 = n$ dark macroscopically quantum coherent phases of ordinary charge carriers at magnetic flux tubes along which the supra current can flow. The only source of dissipation relates to the transfer of ordinary particles to flux tubes involving also phase transition changing the value of h_{eff}.

This superconductivity is essential also for microtubules exhibit signatures for the generation of this kind of phase at critical frequencies of AC voltages serving as a metabolic energy feed providing for charged particles the needed energy that they have in $h_{eff}/h_0 = n$ phase [L6].

Large h_{eff} phases with same parameters than ordinary phase have typically energies large than ordinary phase. For instance. Atomic binding energies scale like $1/h_{eff}^2$ and cyclotron energies and harmonic oscillator energies quite generally like h_{eff}. Free particle in box is however quantum critical in the sense that the energy scale $E = \hbar_{eff}^2/2mL^2$ does not depend on the h_{eff} if one has $L \propto h_{eff}$. At space-time level this is true quite generally for external (free) particles identified as minimal 4-surfaces. Quantum criticality means independence on various coupling parameters.

What is interesting is that Ag and Au have single valence electron. The obvious guess would be that valence electrons BCSome dark and form Cooper pairs in the transition to superconductivity. What is interesting that the basic claim of a layman researcher David Hudson is that ORMEs or mono-atomic elements as he calls them include also Gold [H2]. These claims are not of course taken seriously by academic researchers. In the language of quantum physics the claim is that ORMEs behave like macroscopic quantum systems. I decided to play with the thought that the claims are correct and this hypothesis served later one of the motivations for the hypothesis about dark matter as large h_{eff} phases [K23, K45]: this hypothesis follows from adelic physics [L50, L49] (see `http://tinyurl.com/ycbhse5c`, which is a number theoretical generalization of ordinary real number based physics.

TGD explanation of high Tc superconductivity and its biological applications strongly suggest that a feed of "metabolic" energy is a prerequisite of high Tc superconductivity quite generally. The natural question is whether experimenters might have found something suggesting that the external energy feed - usually seen as a prerequisite for self-organization - is involved with high T_c

superconductivity. During same day I got FB link to another interesting finding related to high Tc superconductivity in cuprates and suggesting positive answer to this question!

2. The strange observation of Brian Skinner about the effect

After writing the above comments I learned from a popular article (see `http://tinyurl.com/ybm8perx`) about and objection (see `http://tinyurl.com/yaxtjjp5`) by Brian Skinner [D3] challenging the claimed discovery [D28] (see `http://tinyurl.com/ybqybvap`). The claimed finding received a lot of attention and physicist Brian Skinner in MIT decided to test the claims. At first the findings look quite convincing to him. He however decided to look for the noise in the measured value of volume susceptibility χ_V. χ_V relates the magnetic field B in superconductor to the external magnetic field B_{ext} via the formulate $B=(1+\chi_V)B_{ext}$ (in units with $\mu_0=1$ one has $B_{ext}=H$, where H is used usually).

For diamagnetic materials χ_V is negative since they tend to repel external magnetic fields. For superconductors one has $\chi_V=-1$ in the ideal situation. The situation is not however ideal and stepwise change of χ_V from $\chi_V=0$ to χ_V to some negative value but satisfying $|\mu_V|<1$ serves as a signature of high Tc superconductivity. Both superconducting and ordinary phase would be present in the sample.

Figure 3a of the article of authors gives χ_V as function of temperature for some values of B_{ext} with the color of the curve indicating the value of B_{ext}. Note that μ_V depends on B_{ext}, whereas in strictly linear situation it would not do so. There is indeed transition at critical temperature $T_c=225$ K reducing $\chi_V=0$ to negative value in the range $\chi_V\in[-0.05,-.06]$ having no visible temperature dependence but decreasing somewhat with B_{ext}.

The problem is that the fluctuations of χ_V for green curve ($B_{ext}=1$ Tesla) and blue curve ($B_{ext}=0.1$ Tesla) have the same shape. With blue curve only only shifted downward relative to the green one (shifting corresponds to somewhat larger dia-magnetism for lower value of B_{ext}). If I have understood correctly, the finding applies only to these two curves and for one sample corresponding to $Tc=256$ K. The article reports superconductivity with Tc varying in the range [145,400] K.

The pessimistic interpretation is that this part of data is fabricated. Second possibility is that human error is involved. The third interpretation would be that the random looking variation with temperature is not a fluctuation but represents genuine temperature dependence: this possibility looks infeasible but can be tested by repeating the measurements or simply looking whether it is present for the other measurements.

3. TGD explanation of the effect found by Skinner

One should understand why the effect occurs only for certain pairs of magnetic fields strengths B_{ext} and why the shape of pseudo fluctuations is the same in these situations.

Suppose that B_{ext} is realized as flux tubes of fixed radius. The magnetization is due to the penetration of magnetic field to the ordinary fraction of the sample as flux tubes. Suppose that the superconducting flux tubes assignable 2-D surfaces as in high Tc superconductivity. Could the fraction of super-conducting flux tubes with non-standard value of h_{eff} - depends on magnetic field and temperature in predictable manner?

The pseudo fluctuation should have same shape as a function temperature for the two values of magnetic fields involved but not for other pairs of magnetic field strengths.

1. Concerning the selection of only preferred pairs of magnetic fields de Haas-van Alphen effect gives a clue. As the intensity of magnetic field is varied, one observes so called de de Haas-van Alphen effect (`http://tinyurl.com/hoywcnq`) used to deduce the shape of the Fermi sphere: magnetization and some other observables vary periodically as function of $1/B$ (for a model for the quantum critical variant of the effect see [D23]). In particular, this is true for χ_V.

   The value of P is

$$P_{H-A}\equiv\frac{1}{B_{H-A}}=\frac{2\pi e}{\hbar S_e}\ , \tag{11.2.6}$$

where S_e is the extremum Fermi surface cross-sectional area in the plane perpendicular to the magnetic field and can be interpreted as area of electron orbit in momentum space (for illustration see http://tinyurl.com/y9zxhu9o).

De Haas-van Alphen effect can be understood in the following manner. As B increases, cyclotron orbits contract. For certain increments of $1/B$ $n+1$:th orbit is contracted to n:th orbit so that the sets of the orbits are identical for the values of $1/B$, which appear periodically. This causes the periodic oscillation of say magnetization. From this one learns that the electrons rotating at magnetic flux tubes of B_{ext} are responsible for magnetization.

2. One can get a more detailed theoretical view about de Haas-van Alphen effect from the article of Lifschitz and Mosevich (see http://tinyurl.com/yay3pg9b). In a reasonable approximation one can write

$$\begin{array}{ll} P=\frac{e\hbar}{m_eE_F}=\frac{4\alpha}{3^{2/3}\pi^{1/3}}\times\frac{1}{B_e} \ , & B_e\equiv\frac{e}{a_e}^2=\frac{1}{x^2}\times 16 \text{ Tesla} \ , \\ a_e=(\frac{V}{N})^{1/3}=xa \ , & a=10^{-10} \text{ m} \ . \end{array} \quad (11.2.7)$$

Here N/V corresponds to valence electron density assumed to form free Fermi gas with Fermi energy $E_F=\hbar^2(3pi^2N/V)^{2/3}/2m_e$. $a=10^{-10}$ m corresponds to atomic length scale. $\alpha\simeq 1/137$ is fine structure constant. For P one obtains the approximate expression

$$P\simeq .15x^2 \text{ Tesla}^{-1} \ .$$

If the difference of $\Delta(1/B_{ext})$ for $B_{ext}=1$ Tesla and $B_{ext}=.1$ Tesla correspond to a k-multiple of P, one obtains the condition

$$kx^2\simeq 60 \ .$$

3. Suppose that $B_{ext,1}=1$ Tesla and $B_{ext,1}=.1$ Tesla differ by a period P of de Haas-van Alphen effect. This would predict same value of χ_V for the two field strengths, which is not true. The formula used for χ_V however holds true only inside given flux tube: call this value $\chi_{V,H-A}$.

The fraction f of flux tubes penetrating into the superconductor can depend on the value of B_{ext} and this could explain the deviation. f can depend also on temperature. The simplest guess is that two effects separate:

$$\chi_V=\chi_{V,H-A}(\frac{B_{H-A}}{B_{ext}})\times f(B_{ext},T) \ . \quad (11.2.8)$$

Here $\chi_{V,H-A}$ has period P_{H-A} as function of $1/B_{ext}$ and f characterizes the fraction of penetrated flux tubes.

4. What could one say about the function $f(B_{ext},T)$? $B_{H-A}=1/P_{H-A}$ has dimensions of magnetic field and depends on $1/B_{ext}$ periodically. The dimensionless ratio $E_{c,H-A}/T$ of cyclotron energy $E_{c,H-A}=\hbar eB_{H-A}/m_e$ and thermal energy T and B_{ext} could serve as arguments of $f(B_{ext},T)$ so that one would have

$$f(B_{ext},T)=f_1(B_{ext})f_2(x) \quad , \quad x=\frac{T}{E_{H-A}(B_{ext}))} \ . \quad (11.2.9)$$

One can consider also the possibility that $E_{c,H-A}$ is cyclotron energy with $\hbar_{eff}=nh_0$ and larger than otherwise. For $h_{eff}=h$ and $B_{ext}=1$ Tesla one would have $E_c=.8$ K, which is same order of magnitude as variation length for the pseudo fluctuation. For instance, periodicity as a function of x might be considered.

If $B_{ext,1} = 1$ Tesla and $B_{ext,1} = .1$ Tesla differ by a period P one would have

$$\frac{\chi_V(B_{ext,1},T)}{\chi_V(B_{ext,2},T)} = \frac{f_1(B_{ext,1})}{f_1(B_{ext,2})} \tag{11.2.10}$$

independently of T. For arbitrary pairs of magnetic fields this does not hold true. This property and also the predicted periodicity are testable.

Transition to high Tc superconductivity involves positive feedback

The discovery of positive feedback in the transition to hight Tc superconductivity is described in the popular article "*Physicists find clues to the origins of high-temperature superconductivity*" (see `http://tinyurl.com/ybo89asd`). Haoxian Li *et al* at the University of Colorado at Boulder and the Ecole Polytechnique Federale de Lausanne have published a paper [D12] on their experimental results obtained by using ARPES (Angle Resolved Photoemission Spectroscopy) in Nature Communications (see `http://tinyurl.com/y7z2lbh7`).

The article reports the discovery of a positive feedback loop that greatly enhances the superconductivity of cupra superconductors. The abstract of the article is here.

> *Strong diffusive or incoherent electronic correlations are the signature of the strange-metal normal state of the cuprate superconductors, with these correlations considered to be undressed or removed in the superconducting state. A critical question is if these correlations are responsible for the high-temperature superconductivity. Here, utilizing a development in the analysis of angle-resolved photoemission data, we show that the strange-metal correlations don't simply disappear in the superconducting state, but are instead converted into a strongly renormalized coherent state, with stronger normal state correlations leading to stronger superconducting state renormalization. This conversion begins well above Tc at the onset of superconducting fluctuations and it greatly increases the number of states that can pair. Therefore, there is positive feedback––the superconductive pairing creates the conversion that in turn strengthens the pairing. Although such positive feedback should enhance a conventional pairing mechanism, it could potentially also sustain an electronic pairing mechanism.*

The explanation of the positive feedback in TGD TGD framework could be following. The formation of dark electrons requires "metabolic" energy. The combination of dark electrons to Cooper pairs however liberates energy. If the liberated energy is larger than the energy needed to transform electron to its dark variant it can transform more electrons to dark state so that one obtains a spontaneous transition to high Tc superconductivity. The condition for positive feedback could serve as a criterion in the search for materials allowing high Tc superconductivity.

The mechanism could be fundamental in TGD inspired quantum biology. The spontaneous occurrence of the transition would make possible to induce large scale phase transitions by using a very small signal acting therefore as a kind of control knob. For instance, it could apply to bio-superconductivity in TGD sense, and also in the transition of protons to dark proton sequences giving rise to dark analogs of nuclei with a scaled down nuclear binding energy at magnetic flux tubes explaining Pollack effect [L7] [L7]. This transition could be also essential in TGD based model of "cold fusion" [L37] based also on the analog of Pollack effect. It could be also involved with the TGD based model for the finding of macroscopic quantum phase of microtubules induced by AC voltage at critical frequencies [L6] (see `http://tinyurl.com/y6vxplt3`).

BCS super conductivity at almost room temperature

Towards the end of year 2018 I learned about the discovery of BCS type (ordinary) superconductivity at temperature warmer than that at North Pole (see `http://tinyurl.com/ybgphjmd`). The compound in question was Lantanium hydride LaH_{10}. Mihail Eremets and his colleagues found that it BCSame superconducting at temperature -23 C and high pressure 170 GPa about 1.6 million times the atmospheric pressure [D11].

The popular article proposed an intuitive explanation of BCS superconductivity, which was new to me and deserves to be summarized here. Cooper pairs would surf on sound waves. The position would correspond to a constant phase for the wave and the velocity of motion would be the phase velocity of the sound wave. The intensity of sound wave would be either maximum or minimum corresponding to a vanishing force on Cooper pair. One would have equilibrium position changing adiabatically, which would conform with the absence of dissipation.

This picture would conform with the general TGD based vision inspired by Sheldrakes's findings and claims related to morphic resonance [L22] , and by the conjectured general properties of preferred extremals of the variational principle implied by twistor lift of TGD [L75]. The experimental discovery is of course in flagrant conflict with the predictions of the BCS theory. As the popular article tells, before the work of Eremets *et al* the maximum critical temperature was thought to be something like 40 K corresponding to -233 °C.

The TGD based view is that Cooper pairs have members (electrons) at parallel flux tubes with opposite directions of magnetic flux and spin and have non-standard value of Planck constant $h_{eff} = n \times h_0 = n \times h/6$ [L26, L60], which is higher than the ordinary value, so that Cooper pairs can be stable at higher temperatures. The flux tubes would have contacts with the atoms of the lattice so that they would experience the lattice oscillations and electrons could surf at the flux tubes.

The mechanism binding electrons to a Cooper pair should be a variant of that in BCS model. The exchange of phonons generates an attractive interaction between electrons leading to the formation of the Cooper pair. The intuitive picture is that the electrons of the Cooper pair can be thought of lying on a mattress and creating a dip towards which the other electron tends to move. The interaction of the flux tubes with the lattice oscillations inducing magnetic oscillations should generate this kind of interaction between electrons at flux tubes and induce a formation of a Cooper pair.

Isotope effect is the crucial test: the gap energy and therefore critical temperature are proportional the oscillation frequency ω_D of the lattice (Debye frequency) proportional to $1/\sqrt{M}$ of the mass M of the molecule in question and decreases with the mass of the molecule. One has lantanium-hydroxide, and can use an isotope of hydrogen to reduce the Debye frequency. The gap energy was found to change in the expected manner.

Can TGD inspired model explain the isotope effect and the anomalously high value of the gap energy? The naïve order of magnitude estimate for the gap energy is of form $E_{gap} = x\hbar_{eff}\omega_D$, x a numerical factor. The larger the value of $h_{eff} = n \times h_0 = n \times h/6$, the larger the gap energy. Unless the high pressure increases ω_D dramatically, the critical temperature 253 K would require $n/6 \sim T_{cr}/T_{max}(BCS) \sim 250/40 \sim 6$. Note that for this value the cyclotron energy $E_c = h_{eff}f_c$ is much below thermal energy for magnetic fields even in Tesla range so that the binding energy must be due to the interaction with phonons.

The high pressure is needed to keep lattice rigid enough at high temperatures so that indeed oscillates rather than "flowing". I do not see how this could prevent flux tube mechanism from working. Neither do I know, whether high pressure could somehow increase the value of Debye frequency to get the large value of critical temperature. Unfortunately, the high pressure (170 GPa) makes this kind of high Tc superconductors unpractical.

11.2.4 The mysterious dichloromethane droplet, which refuses to sink in water and begins to spin

I received from Resonance Foundation a link to an interesting article "*Chemists baffled by droplet spiraling to its doom*" (see `http://tinyurl.com/y4f46jh4`) telling about the strange behavior of droplets of dichloromethane (DCM) at the surface of water. DCM is heavier than water and one would expect it to sink down but it doesn't: it floats and starts to spin emitting smaller droplets from its boundary so that it eventually decays compeltely. This is like evaporation process said to resemble the behavior of spiral galaxy.

I could understand why the droplet floats - by creating a film acting as a boat - and Marangoni effect causing the droplet to decay by emitting smaller droplets (being due to the reduction of string tension at droplet water interface causing radial outwards directed tangential force).

But I could not understand how droplet could start to rotate? Where is the opposite angular momentum. Does water below the droplet rotate in opposite direction?

One strategy in TGD framework is to proceed in general manner.

1. Self-organization process is in question and the rotation could have interpretation as generation long length scale motion requiring long range correlations. Also the build-up of a "boat" as liquid layer coming from droplet allowing the rest of droplet to float instead of sinking to the water would be part of this process.

2. Energy feed is always involved with all self-organization processes. In TGD Universe one can ask whether self-organizing systems are analogous to living systems (see `http://tinyurl.com/y3xbkokb`). For living systems in TGD Universe the metabolic energy feed is needed to keep the distribution of subsystems with Planck constant $h_{eff} = nh_0$ larger than its standard value and responsible for long range quantum correlations. This because the larger the value of h_{eff} characterizing the phase of ordinary matter is, the larger the energy of the system is, and because h_{eff} tends to decrease spontaneouly. Phase of ordinary mattter with nonstandard value of h_{eff} has interpretation as dark matter.

   Quantum scales are typically proportional to h_{eff} and large value means long scale of quantum coherence at the level of magnetic body which serves as a "boss" of ordinary matter in master slave hierarchy. If so, dark matter in TGD Universe would be visible through self-organization processes: quantum coherence in long lengths cales would force self-organization and generation of long range correlations.

3. This picture would suggestthat also now magnetic body carrying dark matter phases is present. Could the angular momentum opposite to that of the rotating droplet be assigned with the rotating dark matter at magnetic body? I have proposed analogous explanation for the spontaneous accelerration rotation in rottatin magnetic systems reported by Godin and Roschin: (see `http://tinyurl.com/yyzld5ml`) . It should be easy to kill this hypothesis: the alternative standard physics option is that it is in the liquid below the droplet so that it would rotate in opposite direction.

4. Where does the needed "metabolic" energy feed come from. The decay of the droplet means that its surface tension is reduced. Surface tension measures the surface energy density so that surface energy must be lost. Could part of this energy go to the self-organization as "metabolic" energy for magnetic body and for rotational. Part of energy would go to the radial motion of smaller droplets emanating from the droplet.

5. Surface tension is thought to be due to cohesive forces. In case of water and some other liquids hydrogen bonds would be responsible for them. London forces would be attractive interactions between dipoles of polarizable molecules and would contribute for polar molecules. Surface tension characterizes surface energy density and thus energy could come from this as the droplet decays to smaller ones. But does this make sense?

   One can argue that this decay to smaller droplets increases surface area so that the process would not liberate energy but require it. The droplet is believedto float by reducing its density (Arhimedes law). This must increases its volume. Could the emitted droplets have the original original density so that their volume would be smaller and surface area too. Could this reduce the total surface energy in the process? The rest would go to rotation?

In FB Wes Johnson suggested that water droplet could act as a propeller. The droplet indeed looks like a propeller. Probably you mean that propeller property causes lift so that buoynancy would not be needed? This looks like a good idea. There would be two models. Propellor model and buoynancy model.

1. For the buoynancy model the expansion of the droplet could provide the needed buoyancy: the density of the droplet should become smaller than that of water. Droplet would generate a membrane serving as a boat. The droplet would expand and this expansion would store energy, which would be liberated. Where do the energy and angular momentum come from?

2. Both options lead to the same question. Were do energy angular momentum come? Could the energy and angular momentum come from water or from the flux tubes/sheets of the magnetic body of water as h_{eff} decreases and liberates energy? Flux tubes in water accompany hydrogen bonds and even long flux tubes could correspond to hydrogen bonds. Magnetic body of water would save the droplet from drowning!

3. This argument involves only energy. Entropy is also involved. I just wrote a little article about application of minimization of Gibbs energy G to water in TGD sense. One has $\Delta G = \Delta H - T\Delta S \leq 0$, H is enthalpy, the heat used or liberated. It is safest to have $\Delta S > 0$. If flux tubes in water shorten, long range order reduced to that in shorter length scale so that entropy is generated. This liberates also energy if long dark flux tubes behave like hydrogen bonds.

11.3 Deviations from Maxwell's electrodynamics

11.3.1 Does The Physics Of SmB_6 Make The Fundamental Dynamics Of TGD Directly Visible?

The group of Suchitra Sebastian has discovered very unconventional condensed matter system, which seems to be simultaneously both insulator and conductor of electricity but only in presence of magnetic field. Science article is entitled "Unconventional Fermi surface in an insulating state" [L14] (see `http://tinyurl.com/y79qo7lp`). There is also a popular article with title "Paradoxical Crystal Baffles Physicists" in Quanta Magazine summarizing the findings (see `http://tinyurl.com/qhwdmxj`). I learned about the finding first from the blog posting of Lubos Motl (see `http://tinyurl.com/yacm6bj7`).

Observations

The crystal studied at superlow temperatures was Samarium hexaboride - briefly SmB_6. The high resistance implies that electron cannot move more that one atom's width in any direction. Sebastian et al however observed electrons traversing over a distance of millions of atoms- a distance of orde 10^{-4} m, the size of a large neuron. So high mobility is expected only in conductors. SmB_6 is neither metal or insulator or is both of them! The finding is described by Sebastian as a "big schock" and as a "magnificent paradox" by condensed matter theorists Jan Zaanen. Theoreticians have started to make guesses about what might be involved but accorrding to Zaanen there is no even remotely credible hypothesis has appeared yet.

On basis of its electronic structure SmB_6 should be a conductor of electricity and it indeed is at room temperature: the average number conduction electrons per SmB_6 is one half. At low temperatures situation however changes. At low temperatures electrons behave collectivily. In superconductors resistance drops to zero as a consequence. In SmB_6 just the opposite happens. Each Sm nucleus has the average 5.5 electrons bound to it at tight orbits. Below 223 degrees of Celsius the conduction electrons of SmB_6 are thought to "hybridize" around samarium nuclei so that the system becomes an insulator. Various signatures demonstrate that SmB_6 indeed behaves like an insulator.

During last five years it has been learned that SmB_6 is not only an insulator but also so called topological insulator. The interior of SmB_6 is insulator but the surface acts as a conductor. In their experiments Sebastian *et al* hoped to find additional evidence for the topological insulator property and attempted to measure quantum oscillations in the electrical resistance of their crystal sample. The variation of quantum oscillations as sample is rotated can be used to map out the Fermi surface of the crystal. No quantum oscillations were seen. The next step was to add magnetic field and just see whether something interesting happens and could save the project. Suddenly the expected signal was there! It was possible to detect quantum oscillations deep in the interior of the sample and map the Fermi surface! The electrons in the interior travelled 1 million times faster than the electrical resistance would suggest. Fermi surface was like that in copper, silver or gold. A further surprise was that the growth of the amplitude of quantum oscillations as temperature was decreased, was very different from the predictions of the universal Lifshitz-Kosevich formula for the conventional metals.

Could TGD help to understand the strange behavior of SmB_6?

There are several indications that the paradoxical effect might reveal the underlying dynamics of quantum TGD. The mechanism of conduction must represent new physics and magnetic field must play a key role by making conductivity possible by somehow providing the "current wires". How? The TGD based answer is completely obvious: magnetic flux tubes - one of the basic distinctions between electrodynamics of Maxwell and its TGD variant! Also the failure of Lischitz-Kosevich formulas should be understood.

1. Single sheet of many-sheeted space-time resembles topological insulator

One should also understand topological insulator property at deeper level, that is the conduction along the boundaries of topological insulator. One should understand why the current runs along 2-D surfaces. In fact, many exotic condensed matter systems are 2-dimensional in good approximation. In the models of integer and fractional quantum Hall effect electrons form a 2-D system with braid statistics possible only in 2-D system. High temperature super-conductivity is also an effectively 2-D phenomenon. By strong form of holography these aspects are also key aspects of quantum TGD at the fundamental level of single space-time sheet when the approximation replacing many-sheeted space-time with that of GRT and standard model does not mask the simplicity of the fundamental dynamics.

1. Many-sheeted space-time is second fundamental prediction TGD. The dynamics of single sheet of many-sheeted space-time should be very simple by the strong form of holography implyhing effective 2-dimensionality. The standard model description of this dynamics masks this simplicity since the sheets of many-sheeted space-time are replaced with single region of slightly curved Minkowski space with gauge potentials sums of induced gauge potentials for sheets and deviation of metric from Minkowski metric by the sum of corresponding deviations for space-time sheets. Could the dynamics of exotic condensed matter systems give a glimpse about the dynamics of single sheet? Could topological insulator and anyonic systems [K85] provide examples of this kind of systems?

2. Second basic prediction of TGD is strong form of holography: string world sheets and partonic 2-surfaces serve as kind of "space-time genes" and the dynamics of fermions is 2-D at fundamental level. It must be however made clear that at QFT limit the spinor fields of embedding space replace these fundamental spinor fields localized at 2-surface. One might argue that the fundamental spinor fields do not make them directly visible in condensed matter physics. Nothing however prevents from asking whether in some circumstances the fundamental level could make itself visible.

   In particular, for large h_{eff} dark matter systems (, whose existence can be deduced from the quantum criticality of quantum TGD) the partonic 2-surfaces with CP_2 size could be scaled up to nano-scopic and even longer size scales. I have proposed this kind of surfaces as carriers of electrons with non-standard value of h_{eff} in QHE and FQHE [K85].

   The long range quantum fluctuations associated with large, $h_{eff} = n \times h$ phase would be quantum fluctuations rather than thermal ones. In the case of ordinary conductivity thermal energy makes it possible for electrons to jump between atoms and conductivity becomes very small at low temperatures. In the case of large scale quantum coherence just the opposite happens as observed. One therefore expects that Lifshitz-Kosevich formula for the temperature dependence of the amplitude does not hold true.

   The generalization of Lifschitz-Kosevich formula to quantum critical case deduced from quantum holographic correspondence by Hartnoll and Hofman [D23] (`http://tinyurl.com/ybednd85` is expected to hold true qualitatively also for quantum criticality in TGD sense. The first guess is that by underlying super-conformal invariance scaling laws typical for critical systems hold true. In the proposed formula the dependence on temperature is via a power of dimensionless parameter $x = T/mu;$, where μ is chemical potential for electron system. As a matter fact, exponent of power of x appears and reduces to first for Lifshitz-Konsevich formula. Since magnetic field is important, one also expects that the ratio of cyclotron energy scale $E_c \propto \hbar_{eff} eB/m_e$ to Fermi energy appears in the formula. One can even make an order of magnitude guess for the value of $h_{eff}/h \sim 10^6$ from the facts that the scale of conduction and conduction velocity were millions times higher than expected.

Strings are 1-D systems and strong form of holography implies that fermionic strings connecting partonic 2-surfaces and accompanied by magnetic flux tubes are fundamental. At light-like 3-surfaces fermion lines can give rise to braids. In TGD framework AdS/CFT correspondence generalizes since the conformal symmetries are extended. This is possible only in 4-D space-time and for the embedding space $H = M^4 \times CP_2$ making possible to generalize twistor approach [K114].

3. Topological insulator property means from the perspective of modelling that the action reduces to a non-abelian Chern-Simons term. The quantum dynamics of TGD at space-time level is dictated by Kähler action. Space-time surfaces are preferred extremals of Kähler action and for them Kähler action reduces to Chern-Simons terms associated with the ends of space-time surface opposite boundaries of causal diamond and possibly to the 3-D light-like orbits of partonic 2-surfaces. Now the Chern-Simons term is Abelian but the induced gauge fields are non-Abelian. One might say that single sheeted physics resembles that of topological insulator.

4. The effect appears only in magnetic field. I have been talking a lot about magnetic flux tubes carrying dark matter identified as large h_{eff} phases: topological quantization distinguishes TGD from Maxwell's theory: any system can be said to possess "magnetic body", whose flux tubes can serve as current wires. I have predicted the possibility of high temperature super-conductivity based on pairs of parallel magnetic flux tubes with the members of Cooper pairs at the neighboring flux tubes forming spin singlet or triplet depending on whether the fluxes are have same or opposite direction.

   Also spin and electric currents assignable to the analogs of spontaneously magnetized states at single flux tube are possible. The obvious guess is that the conductivity in question is along the flux tubes of the external magnetic field. Could this kind of conductivity explains the strange behavior of SmB_6. The critical temperature would be that in which the parallel flux tubes are stable. The interaction energy of spin with the magnetic field serves as a possible criterion for the stability if the presence of dark electrons stabilizes the flux tubes.

2. Magnetic flux tubes as dark current carriers in quantum criticality

The following represents an extremely childish attempt of a non-specialist to understand how the conductivity might be understood. The current carrying electrons at flux tubes near the top of Fermi surface are current carriers. $h_{eff} = n \times h$ and magnetic flux tubes as current wires bring in the new elements. Also in the standard situation one considers cylinder symmetric solutions of Schrödinger equation in external magnetic field and introduces maximal radius for the orbits so that formally the two situations seem to be rather near to each other. Physically the large h_{eff} and associated many-sheeted covering of space-time surface providing the current wire makes the situation different since the collisions of electrons could be absent in good approximation so that the velocity of charge carriers could be much higher than expected as experiments indeed demonstrate.

Quantum criticality is the crucial aspect and corresponds to the situation in which the magnetic field attains a value for which a new orbit emerges/disappears at the surface of the flux tube: in this situation dark electron phase with non-standard value of h_{eff} can be generated. This mechanism is expected to apply also in bio-superconductivity and to provide a general control tool for magnetic body.

1. Let us assume that flux tubes cover the whole transversal area of the crystal and there is no overlap. Assume also that the total number of conduction electrons is fixed, and depending on the value of h_{eff} is shared differently between transversal and longitudinal degrees of freedom. Large value of h_{eff} squeezes the electrons from transversal to longitudinal flux tube degrees of freedom and gives rise to conductivity.

2. Consider first Schrödinger equation. In radial direction one has harmonic oscillator and the orbits are Landau orbits. The cross sectional area behaves like $\pi R^2 = n_T h_{eff}/2m\omega_c$ giving

$n_T \propto 1/h_{eff}$. Increase of the Planck constant scales up the radii of the orbits so that the number of states in cylinder of given radius is reduced. Angular momentum degeneracy implies that the number of transversal states is $N_T = n_T^2 \propto 1/h_{eff}^2$. In longitudinal direction one has free motion in a box of length L with states labelled by integer n_L. The number of states is given by the maximum value N_L of n_L.

3. If the total number of states is fixed to $N = N_L N_T$ is fixed and thus does not depend on h_{eff}, one has $N_L \propto h_{eff}^2$. Quanta from transversal degrees of freedom are squeezed to longitudinal degrees of freedom, which makes possible conductivity.

4. The conducting electrons are at the surface of the 1-D "Fermi-sphere", and the number of conduction electrons is $N_{cond} \simeq dN/d\epsilon \times \delta\epsilon \simeq dN/d\epsilon T = NT/2\epsilon_F \propto 1/h_{eff}^4$. The dependence on h_{eff} does not favor too large values of h_{eff}. On the other hand, if scattering of electrons at flux tubes could be absent. The assumption $L \propto h_{eff}$ increases the range over which current can flow.

5. To get a non-vanishing net current one must assume that only the electrons at the second end of the 1-D Fermi sphere are current carriers. The situation would resemble that in semiconductor. The direction of electric field would induce symmetry breaking at the level of quantum states. The situation would be like that for a mass in Earth's gravitational field treated quantally and electrons would accelerate freely. Schrödinger equation would give rise to Airy functions as its solution.

3. Quantum critical quantum oscillations

What about quantum oscillations in TGD framework?

1. Quantum oscillation refers to de Haas-van Alphen effect (`http://tinyurl.com/yaaljv9j`) - an oscillation of the induced magnetic moment as a function of $1/B$ with period $\tau = 2\pi e/\hbar A$, where A is the area of the extremal orbit of the Fermi surface, in the direction of the applied field. The effect is explained to be due to the Landau quantization of the electron energy. I failed to really understand the explanation of this source and in my humble opinion the following arguments provide a clearer view about what happens.

2. If external magnetic field corresponds to flux tubes, Fermi surface decomposes into cylinders parallel to the magnetic field since the motion in transversal degrees of freedom is along circles. In the above thought experiment also a quantization in the longitudinal direction occurs if the flux tube has finite length so that Fermi surface in longitudinal direction has finite length. One expects on basis of Uncertainty Principle that the area S of the cross section of Fermi cylinder in momentum space is given by $S \propto h_{eff}^2/\pi R^2$. This follows also from the equation of motion of electron in magnetic field. As the external magnetic field B is increased, the radii of the orbits decrease inside the flux tube, and in momentum space the radii increase.

3. Why does the induced magnetic moment (magnetization) and other observables oscillate?
   (a) The simplest manner to understand this is to look at the situation at space-time level. Classical orbits are harmonic oscillator orbits in radial degree of freedom. Suppose that the area of flux tube is fixed and B is increased. The orbits have radius $r_n^2 = (n+1/2) \times \hbar/eB$ and shrink. For certain field values the flux $eBA = n\hbar$ corresponds to an integer multiple of the elementary flux quantum . of a new orbit at the boundary of the flux tube emerges if the new orbit is near the boundary of Fermi sphere providing the electrons. This is clearly a critical situation.
   (b) In de Haas- van Alphen effect the orbit $n+1$ for B has same radius as the orbit n for $1/B + \Delta(1/B)$: $r_{n+1}(1/B) = r_n(1/B + \Delta(1/B))$. This gives approximate differential equation with respect to n and one obtains $(1/B)(n) = (n+1/2) \times \Delta(1/B)$. $\Delta(1/B)$ is fixed from the condition the flux quantization. When largest orbit is at the surface of the flux, tube the orbits are same for $B(n)$ and $B(n+1)$, and this gives rise to the de Haas - van Alphen effect.

(c) It is not necessary to assume finite radius for the flux tube, and the exact value of the radius of the flux tube does not play and important role. The value of flux tube radius can be estimated from the ratio of the Fermi energy of electron to the cyclotron energy. Fermi energy about .1 eV depending only on the density of electrons in the lowest approximation and only very weakly on temperature. For a magnetic field of 1 Tesla cyclotron energy is .1 meV. The number of cylinders defined by orbits is about $n = 10^4$.

4. What happens in TGD Universe in which the areas of flux tubes identifiable as space-time quanta are finite? Could quantum criticality of the transition in which a new orbit emerges at the boundary of flux tube lead to a large h_{eff} dark electron phase at flux tubes giving rise to conduction?

(a) The above argument makes sense also in TGD Universe for the ordinary value of Planck constant. What about non-standard values of Planck constant? For $h_{eff}/h = n$ the value of flux quantum is n-fold so that the period of the oscillation in de Haas - van Alphen effect becomes n times shorter. The values of the magnetic field for which the orbit is at the surface of the flux tube are however critical since new orbit emerges assuming that the cyclotron energy corresponds is near Fermi energy. This quantum criticality could give rise to a phase transition generating non-standard value of Planck constant.

What about the period $\Delta(1/B)$ For $h_{eff}/h = n$? Modified flux quantization for extremal orbits implies that the area of flux quantum is scaled up by n. The flux changes by n units for the same increment of $\Delta(1/B)$ as for ordinary Planck constant so that de Haas -van Alphen effect does not detect the phase transition.

(b) If the size scale of the orbits is scaled up by $\sqrt{n}$ as the semiclassical formula suggests the number of classical orbits is reduced by a factor $1/n$ if the radius of the flux tube is not changed in the transition $h \rightarrow h_{eff}$ to dark phase. n-sheetedness of the covering however compensates this reduction.

(c) What about possible values of h_{eff}/h? The total value of flux seems to give the upper bound of $h_{eff}/h = n_{max}$, where n_{max} is the value of magnetic flux for ordinary value of Planck constant. For electron and magnetic field for $B = 10$ Tesla and has $n \leq 10^5$. This value is of the same order as the rough estimate from the length scale for which anomalous conduction occurs.

Clearly, the mechanism leading to anomalously high conductivity might be the transformation of the flux tubes to dark ones so that they carry dark electrons currents. The observed effect would be dark, quantum critical variant of de Haas-van Alphen effect!

Also bio-superconductivity is quantum critical phenomenon and this observation would suggests sharpening of the existing TGD based model of bio-super-conductivity. Super-conductivity would occur for critical magnetic fields for which largest cyclotron orbit is at the surface of the flux tube so that the system is quantum critical. Quantization of magnetic fluxes would quantify the quantum criticality. The variation of magnetic field strength would serve as control tool generating or eliminating supra currents. This conforms with the general vision about the role of dark magnetic fields in living matter.

To sum up, a breakthrough of TGD is continuing. I have written about thirty articles during this year - more than one article per week. There is huge garden there and trees contain fruits hanging low! It is very easy to pick them: just shatter and let them drop to the basket! New experimental anomalies having a nice explanation using TGD based concepts appear on weekly basis and the mathematical and physical understanding of TGD is taking place with great leaps. It is a pity that I must do all alone. I would like to share. I can only hope that colleagues could take the difficult step: admit what has happened and make a fresh start.

11.3.2 Are monopoles found?

LNC scientist report of having discovered magnetic monopoles (see http://tinyurl.com/ppquxyy and http://tinyurl.com/y95zbuew). The claim that free monopoles are discvoered are discovered is to my opinion too strong.

TGD allows monopole fluxes but no free monopoles. Wormhole throats however behave effectively like monopoles when looked at either space-time sheet, A or B. The first TGD explanation that comes in mind is in terms of 2-sheeted structures with wormhole contacts at the ends and monopole flux tubes connecting the wormhole throats at A and B so that closed monopole flux is the outcome. All elementary particles are predicted to be this kind of structures in the scale of Compton length. First wormhole carries throat carries the elementary particle quantum numbers and second throat neutrino pair neutralizing the weak isospin so that weak interaction is finite ranged. Compton length scales like h_{eff} and can be nano-scopic or even large for large values of h_{eff}. Also for abnormally large p-adic length scale implying different mass scale for the particle, the size scale increases.

How to explain the observations? Throats with opposite apparent quantized magnetic charges at given space-time sheet should move effectively like independent particles (although connected by flux tube) in opposite directions to give rise to an effective monopole current accompanied by an opposite current at the other space-time sheet. This is like having balls at the ends of very soft strings at the two sheets. One must assume that only the current only at single sheet is detected. It is mentioned that ohmic component corresponds to effectively free monopoles (already having long flux tubes connecting throats with small magnetic string tension). In strong magnetic fields shorter pairs of monopoles are reported to become "ionized" and give rise to a current increasing exponentially as function of square root of external magnetic field strength. This could correspond to a phase transition increasing h_{eff} with no change in particle mass. This would increase the length of monopole flux tube and the throats would be effectively free magnetic charges in much longer Compton scale.

The analog of color de-confinement comes in mind and one cannot exclude color force since non-vanishing Kähler field is necessarily accompanied by non-vanishing classical color gauge fields. Effectively free motion below the length scale of wormhole contact would correspond to asymtotic freedom. Amusingly, one would have zoomed up representation of dynamics of colored objects! One can also consider interpretation in terms of Kähler monopoles: induced Kähler form corresponds to classical electroweak U(1) field coupling to weak hypercharge but asymptotic freedom need not fit with this interpretation. Induced gauge fields are however strongly constrained: the components of color gauge fields are proportional to Hamiltonians of color rotation and induced Kähler form. Hence it is difficult to draw any conclusions.

11.3.3 Badly behaving photons and space-time as 4-surface

There was an interesting popular article with title *Light Behaving Badly: Strange Beams Reveal Hitch in Quantum Mechanics* (see http://tinyurl.com/hefhdad). The article told about a discovery made by a group of physicists at Trinity College Dublin in Ireland in the study of helical light-beams with conical geometry. These light beams are hollow and have the axis of helix as a symmetry axis. The surprising finding was that according to various experimental criteria one can say that photons have spin $S = \pm/1/2$ with respect to the rotations around the axis of the helix [L21] (see http://tinyurl.com/zoro4gz).

The first guess would be that this is due to the fact that rotational symmetry for the spiral conical beam is broken to axial rotational symmetry around the beam axis. This makes the situation 2-dimensional. In $D = 2$ one can have braid statistics allowing fractional angular momentum for the rotations around a hole - now the hollow interior of the beam. One can however counter argue that photons with half odd integer braid spin should obey Fermi statistics. This would mean that only one photon with fixed spin is possible in the beam. Something seems to go wrong with the naïve argument. It would seem that the exchange of photons does not seem to correspond to 2π rotation as a homotopy would be the topological manner to state the problem.

The authors of the article suggest that besides the ordinary conserved angular momentum one can identify also second conserved angular momentum like operator.

1. The conserved angular momentum is obtained as the replacement

$$J = L + S \rightarrow J_\gamma = L + \gamma S \ . \tag{11.3.1}$$

2. The eigenvalue equation for j_γ for a superposition of right and left polarizations with $S = \pm 1$

$$a_1 \times e_R exp(il_1\theta) + a_2 \times e_L exp(il_2\theta) \ , \tag{11.3.2}$$

where l_i and also $s_z = \pm 1$ are integers, makes sense for

$$\gamma = \frac{(l_1 - l_2)}{2} \ , \tag{11.3.3}$$

and gives eigenvalue

$$j_\gamma = \frac{l_1 + l_2}{2} \ . \tag{11.3.4}$$

Since l_1 and l_2 are integers by the continuity of the wave function at 2π (even this can be questioned in hollow conical geometry) $(l_1 + l_2)/2$ and $(l_1 - l_2)/2$ are either integers or half integers. For $l_1 - l_2 = 1$ the one has $J_\gamma = J_{1/2} = L + S/2$, which is half odd integer. The stronger statement would be that 2-D $S_\gamma = S/2$ is half-odd integer.

There is an objection against this interpretation. The dependence of the angular momentum operator on the state of photon implied by $\gamma = (l_1 - l_2)/2$ is a highly questionable feature. Operators should not depend on states but define them as their eigenstates. Could one understand the experimental findings in some different manner? Could the additional angular momentum operator allow some natural interpretation? If it really generates rotations, where does it act?

In TGD framework this question relates interestingly to the assumption that space-time is 4-surface in $M^4 \times CP_2$. Could X^4 and M^4 correspond to the two loci for the action of rotations? One can indeed have two kinds of photons. Photons can correspond to space-time sheets in $M^4 \times CP_2$ or they can correspond to space-time sheets topologically condensed to space-time surface $X^4 \subset M^4 \times CP_2$. For the first option one would have ordinary quantization of angular momentum in M^4. For the second option quantization in X^4 angular momentum, which using the units of M^4 angular momentum could correspond to half-integer or even more general quantization.

1. For the first (photons in M^4) option angular momentum $J(M^4) = L(M^4) + S(M^4)$ acts at point-like limit on a wave function of photon in M^4. $J(M^4)$ acts as a generator of rotations in M^4 should have the standard properties: in particular photon spin is $S = +/-1$.

2. For topologically condensed photons at helix the angular momentum operator $J(X^4) = L(X^4)+S(X^4)$ generates at point-like limit rotations in X^4. If M^4 coordinates - in particular angle coordinate ϕ around helical axis - are used for X^4, the identifications

$$J(X^4) = kJ(M^4) \ , \quad L(X^4) = kL(M^4) \ , \quad S(X^4) = kS(M^4) \ . \tag{11.3.5}$$

are possible.

3. In the recent case X^4 corresponds to effectively a helical conical path of photon beam, which is effectively 2-D system with axial $SO(2)$ symmetry. The space-time surface associated with the helical beam is analogous to a covering space of plane defined by Riemann surface for $z^{1/n}$ with origin excluded (hollowness of the spiral beam is essential since at z-axis various angles ϕ correspond to the same point and one would obtain discontinuity). It takes n full turns before one gets to the original point. This implies that $L(X^4) = kL(M^4)$ can be fractional with unit $\hbar/n$ meaning $k = 1/n$ when the angle coordinate of M^4 serves as angle coordinate of X^4.

4. For $n = 2$ one has $k = 1/2$ and 4π rotations in Minkowski space interpreted as shadows of rotations at X^4 must give a phase equal to unity. This would allow half integer quantization for $J(X^4), L(X^4)$ and $S(X^4)$ of photon in M^4 units. $S(X^4)$ corresponds to a local rotation in tangent space of X^4. The braid rotation defined by a path around the helical axis corresponds to a spin rotation and by $k = 1/2$ to $S(X^4) = S(M^4)/2 = 1/2$. Hence one has effectively $S(M^4) = \pm 1/2$ for the two circular polarizations and thus $\gamma = \pm 1/2$ independently of l_i: in the above model $\gamma = (l_1 - l_2)/2$ can have also other values. Now also other values of n besides $n = 2$ are predicted.

   l_i can be both integer and half odd integer valued. One can reproduce the experimental findings for integer valued l_1 and l_2. One has $j = l_1 + 1/2 = l_2 - 1/2$ from condition that superpositions of both right and left-handed spiral photons are possible. If j is half-odd integer, $l_1 + l_2 = 2j$ is odd integer. For instance, $S(X^4) = 1/2$ gives $l_1 - l_2 = -1$ consistent with integer/half-odd integer property of both l_1 and l_2. For $j = 1/2$ one has $l_1 + l_2 = 1$ and $l_1 - l_2 = -1$ giving $(l_1, l_2) == (0, 1)$.

5. Is there something special in $n = 2$. In TGD elementary particles have wormhole contacts connecting two space-time sheets as building bricks. If the sheets form a covering of M^4 singular along plane M^2 one has $n = 2$ naturally.

One can worry about many-sheeted statistics. The intuitive view is that one just adds bosons/fermions at different sheets and each sheet corresponds to a discrete degree of freedom.

1. Statistics is not changed to Fermi statistics if the exchange interpreted at X^4 corresponds to $n \times 2\pi$ rotation. For $n = 2$ a possible modification of the anti-commutation relations would be doubling of oscillator operators assigning $a_k(i)$, $i = 1, 2$ to the 2 sheets and formulating braid anti-commutativity as

$$\begin{array}{lll} \{a_k(1), a_l(2)\} = 0 \ , & \{a_k^\dagger(1), a_l^\dagger(2)\} = 0 \ , & \{a_k^\dagger(1), a_l(2)\} = 0 \ . \\ [a_k(i), a_l(i)] = 0 & \left[a_k^\dagger(i), a_l^\dagger(i)\right] = 0 & \left[a_k^\dagger(i), a_l(i)\right] = \delta_{k,l} \ . \end{array} \tag{11.3.6}$$

   This would be consistent with Bose-Einstein statistics. For n-sheeted case the formula replacing pair $(1, 2)$ with any pair $(i, j \neq i)$ applies. One would have two sets of mutually commuting (creation) operators and these sets would anti-commute and Bose-Einstein condensates seem to be possible.

2. One can worry about the connection with the hierarchy of Planck constants $h_{eff} = n \times h$, which is assigned with singular n-sheeted covering space. The 3-D surfaces defining ends of the covering at the boundaries of causal diamond (CD) would in this case co-incide. This might be the case now since the photon beam is assumed to be conical helix. Space-time surface would be analogous to n 3-D paths, which co-incide at their ends at past and future boundaries of CD.

   Does the scaling of Planck constant by n compensate for the fractionization so that the only effect would be doubled Bose-Einstein condensate. It woud seem that these condensates need not have same numbers of photons. The scaling of cyclotron energies by n is central in the application of $h_{eff} = nh$ idea. It could be interpreted by saying that single boson state is replaced with n-boson state with the same cyclotron frequency but n-fold energy.

3. In the fermionic case on obtain n additional degrees of freedom and ordinary single fermion state would be replaced with a set of states containing up to n fermions. This would lead to a kind of breakdown of fermion statistics possibly having interpretation in terms of braid statistics. And old question is whether one could understand quark color as $h_{eff}/h = n = 3$ braid statistics for leptons. At the level of CP_2 spinors em charge corresponds to sum of vectorial isospin and of anomalous color hypercharge which is for leptons $n = 3$ multiple of that for quarks. This could be perhaps interpreted in terms of scaling in hypercharge degree of freedom due to 3-sheeted covering. This picture does not seem however to work.

To sum up, also M^4 angular momentum and spin make sense and are integer valued but for the system identifiable as topological condensed photon plus helix rather than topological condensed photon at helix. Many-sheeted space-time can in principle rise to several angular momenta of this kind. Symmetry breaking go $SO(2)$ subgroup is however involved. The general prediction is $1/n$ fractionization.

Remark: I encountered a popular article (see `http://tinyurl.com/ybovyxd3`)about strange halving of photon angular momentum unit two years after writing the above comments. The immediate reaction was that the finding could be seen as a direct proof for $h_{eff} = nh_0$ hierarchy, where h_0 is the minimal value of Planck constants, which need not be ordinary Planck constant h as I have often assumed in previous writings.

Various arguments indeed support for $h = 6h_0$. This hypothesis would explain the strange findings about hydrogen atom having what Mills calls hydrino states having larger binding energy than normal hydrogen atom [L26] (see `http://tinyurl.com/goruuzm`): the increase of the binding energy would follow from the proportionality of the binding energy to $1/h_{eff}^2$. For $n_0 = 6 \rightarrow n < 6$ the binding energy is scale up as $(n/6)^2$. The values of $n = 1, 2, 3$ dividing n are preferred. Second argument supporting $h = 6h_0$ comes from the model for the color vision [L60] (see `http://tinyurl.com/y9jxyjns`).

What is the interpretation of the ordinary photon angular momentum for $n = n_0 = 6$? Quantization for angular momentum as multiples of $\hbar_0$ reads as $l = l_0\hbar_0 = (l_0/6)\hbar$, $l_0 = 1, 2...$ so that fractional angular momenta are possible. $l_0 = 6$ gives the ordinary quantization for which the wave function has same value for all 6 sheets of the covering. $l_0 = 3$ gives the claimed half-quantization.

11.3.4 Non-local production of photon pairs as support for $h_{eff}/h = n$ hypothesis

Again a new anomaly! Photon pairs have been created by a new mechanism. Photons emerge at different points (see `http://tinyurl.com/lseqyrq`).

Could this give support for the TGD based general model for elementary particle as a string like object (flux tube) with first end (wormhole contact) carrying the quantum numbers - in the case of gauge boson fermion and antifermion at opposite throats of the contact. Second end would carry neutrino-right-handed neutrino pair neutralizing the possible weak isospin. This would give only local decays. Also emissions of photons from charged particle would be local.

Could the bosonic particle be a mixture of two states. For the first state flux tube would have fermion and antifermion at the same end of the fluxtube: only local decays. For the second state fermion and antifermion would reside at the ends of the flux tubes residing at throats associated with different wormhole contacts. This state in state would give rise to non-local two-photon emissions. Mesons of hadron physics would correspond to this kind of states and in old-fashioned hadron physics one speaks about photon-vector meson mixing in the description of the photon-hadron interactions. If the Planck constant $h_{eff}/h = n$ of the emitting particle is large, the distance between photon emissions would be long. The non-local days could make the visible both exotic decay and allow to deduce the value of n! This would how require the transformation of emitted dark photon to ordinary (same would happen when dark photons transform to biophotons)

Can one say anything about the length of fux tube? Magnetic flux tube contains fermionic string. The length of this string is of order Compton length and of the order of p-adic length scale.

What about photon itself - could it have non-local fermion-antifermion decays based on the same mechanism? What the length of photonic string is is not clear. Photon is massless, no scales! One identification of length would be as wavelength defining also the p-adic length scale.

To sum up: the nonlocal decays and emissions could lend strong support for both flux tube identification of particles and for hierarchy of Planck constants. It might be possible to even measure the value of n associated with quantum critical state by detecting decays of this kind.

11.4 Thermodynamical surprises

11.4.1 Quantization of thermal conductance and quantum thermodynamics

The finnish research group led by Mikko Möttönen working at Aalto University has made several highly interesting contributions to condensed matter physics during last years (see `http://tinyurl.com/yartleg2` about condensed matter magnetic monopoles and `http://tinyurl.com/jd26rhy` about tying quantum knots: both contributions are interesting also from TGD point of view). This morning I read about a new contribution published in Nature [D19] (see `http://tinyurl.com/y7bfzsnh`). One can find also a popular article telling about the finding (see `http://tinyurl.com/yba239d7`).

What has been shown in the recent work is that quantal thermal conductivity is possible for wires of 1 meter when the heat is transferred by photons. This length is by a factor 10^4 longer than in the earlier experiments. The improvement is amazing and the popular article tells that it could mean a revolution in quantum computations since heat spoling the quantum coherence can be carried out very effectively and in controlled manner from the computer (see `http://tinyurl.com/yba239d7`). Quantal thermal conductivity means that the transfer of energy along wire takes place without dissipation.

To understand what is involved consider first some basic definitions. Thermal conductivity k is defined by the formula $j = k\nabla T$, where j is the energy current per unit area and T the temperature. In practice it is convenient to use thermal power obtained by integrating the heat current over the transversal area of the wire to get the heat current dQ/dt as analog of electric current I. The thermal conductance g for a wire allowing approximation as 1-D structure is given by conductivity divided by the length of the wire: the power transmitted is $P = g\Delta T$, $g = k/L$.

One can deduce a formula for the conductance at the limit when the wire is ballistic meaning that no dissipation occurs. For instance, superconducting wire is a good candidate for this kind of channel and is used in the measurement. The conductance at the limit of quantum limited heat conduction (see `http://tinyurl.com/y7dtfrvt`) is an integer multiple of conductance quantum $g_0 = k_B^2\pi^2T/3h$: $g = ng_0$. Here the sum is over parallel channels. What is remarkable is quantization and independence on the length of the wire. Once the heat carriers are in wire, the heat is transferred since dissipation is not present.

A completely analogous formula holds true for electric conductance along ballistic wire (see `http://tinyurl.com/y8gheqw6`): now g would be integer multiple of $g_0 = \sigma/L = 2e^2/h$. Note that in 2-D system quantum Hall conductance (not conductivity) is integer (more generally some rational) multiple of $\sigma_0 = e^2/h$. The formula in the case of conductance can be "derived" heuristically from Uncertainty Principle $\Delta E\Delta t = h$ plus putting $\Delta E = e\Delta V$ as difference of Coulomb energy and $\Delta t = q/I = qL/\Delta V = e/g_0$.

The essential prerequisite for quantal conduction is that the length of the wire is much shorter than the wavelength assignable to the carrier of heat or of thermal energy: $\lambda \gg L$. It is interesting to find how well this condition is satisfied in the recent case.

The wavelength of the photons involved with the transfer should be much longer than 1 meter. An order of magnitude for the energy of photons involve and thus for the frequency and wavelength can be deduced from the thermal energy of photons in the system. The electron temperatures considered are in the range of 10-100 mK roughly. Kelvin corresponds to 10^{-4} eV (this is more or less all that I learned in thermodynamics course in student days) and eV corresponds to 1.24 microns. This temperature range roughly corresponds to energy range of $10^{-6} - 10^{-5}$ eV. The wave wavelength corresponding to maximal intensity of blackbody radiation is in the range of 2.3-23 centimeters. One can of course ask whether the condition $\lambda \gg L = 1$ m is consistent with this. A specialist would be needed to answer this question. Note that the gap energy .45 meV of superconductor defines energy scale for Josephson radiation generated by super-conductor: this energy would correspond to about 2 mm wavelength much below one 1 meter. This energy does not correspond to the energy scale of thermal photons.

I am of course unable to say anything interesting about the experiment itself but cannot avoid mentioning the hierarchy of Planck constants. If one has $E = h_{eff}f$, $h_{eff} = n \times h$ instead of $E = hf$, the condition $\lambda \gg L$ can be easily satisfied. For superconducting wire this would be true for superconducting magnetic flux tubes in TGD Universe and maybe it could be true also for photons, if they are dark and travel along them. One can even consider the possibility that quantal heat conductivity is possible over much longer wire lengths than 1 m. Showing this to be the case, would provide strong support for the hierarchy of Planck constants.

There are several interesting questions to be pondered in TGD framework. Could one identify classical space-time correlates for the quantization of conductance? Could one understand how classical thermodynamics differs from quantum thermodynamics? What quantum thermodynamics could actually mean? There are several rather obvious ideas.

1. Space-time surfaces are preferred extremals of Kähler action satisfying extremely powerful conditions boiling down to strong form of holography stating that string world sheets and partonic 2-surfaces basically dictate the classical space-time dynamics [K10, K126, K95]. Fermions are localized to string world sheets from the condition that electromagnetic charge is well-defined for spinor modes (classical W fields must vanish at the support of spinor modes).

   This picture is blurred as one goes to GRT-standard model limit of TGD and space-time sheets are lumped together to form a region of Minkowski space with metric which deviates from Minkowski metric by the sum of the deviations of the induced metrics from Minkowski metric. Also gauge potentials are defined as sums of induced gauge potentials. Classical thermodynamics would naturally correspond to this limit. Obviously the extreme simplicity of single sheeted dynamics is lost.

2. Magnetic flux tubes to which one can assign space-like fermionic strings connecting partonic 2-surfaces are excellent candidates for the space-time correlates of wires and at the fundamental level the 1-dimensionality of wires is exact notion at the level of fermions. The quantization of conductance would be universal phenomenon blurred by the GRT-QFT approximation.

   The conductance for single magnetic flux tube would be the conductance quantum determined by preferred extremal property, by the boundary conditions coded by the electric voltage for electric conduction and by the temperatures for heat conduction. The quantization of conductances could be understood in terms of preferred extremal property. m-multiple of conductance would correspond to m flux tubes defining parallel wires. One should check whether also fractional conductances coming as rational m/n are possible as in the case of fractional quantum Hall effect and assignable to the hierarchy of Planck constants $h_{eff} = n \times h$ as the proportionality of quantum of conductance to $1/h$ suggests.

3. One can go even further and ask whether the notion of temperature could make sense at quantum level. Quantum TGD can be regarded formally as a "complex square root" of thermodynamics. Single particle wave functions in Zero Energy Ontology (ZEO) can be regarded formally as "complex square roots" of thermodynamical partition functions and the analog of thermodynamical ensemble is realized by modulus squared of single particle wave function.

   In particular, p-adic thermodynamics used for mass calculations can be replaced with its "complex square root" and the p-adic temperature associated with mass squared (rather than energy) is quantized and has spectrum $T_p = log(p)/n$ using suitable unit for mass squared [K66].

   Whether also ordinary thermodynamical ensembles have square roots at single particle level (this would mean thermodynamical holography with members of ensemble representing ensemble!) is not clear. I have considered the possibility that cell membrane as generalized Josephson junction is describable using square root of thermodynamics [L7]. In ZEO this would allow to describe as zero energy states transitions in which initial and final states of event corresponding to zero energy state have different temperatures.

Square root of thermodynamics might also allow to make sense about the idea of entropic gravity, which as such is in conflict with experimental facts [K119] .

11.4.2 Deviation from the prediction of standard quantum theory for radiative energy transfer in faraway region

I encountered in FB a highly interesting finding discussed in two popular articles (see http://tinyurl.com/yc64fmoo and http://tinyurl.com/yc9fwvnh). The original article (see http://tinyurl.com/y9kafhme) is behind paywall but one can find the crucial figure 5 online (see http://tinyurl.com/ybr87h7u) . It seems that experimental physics is in the middle of a revolution of century and theoretical physicists straying in superstring landscape do not have a slightest idea about what is happening.

The size scale of objects studied - membranes in temperature of order room temperature 300 Kfor instance - is about 1/2 micrometers: cell length scale range is in question. They produce radiation and other similar object is heated if there is temperature difference between the objects. The heat flow is proportional to the temperature difference and radiative conductance G_{rad} characterizes the situation. Planck's black body radiation law, which initiated the development of quantum theory for more than century ago, predicts G_{rad} at large enough distances.

1. The radiative transfer is larger than predicted by Planck's radiation law at *small distances* (nearby region) of order average wavelength of thermal radiation deducible from its temperature. This is not a news.

2. The surprise was that radiative conductance is 100 times larger than expected from Planck's law at *large distances* (faraway region) for small objects with size of order .5 micron. This is a really big news.

The obvious explanation in TGD framework is provided by the hierarchy of Planck constants. Part of radiation has Planck constant $h_{eff} = n \times h_0$, which is larger than the standard value of $h = 6h_0$ (a good guess for atoms [L26, L60, L61]). This scales up the wavelengths and the size of nearby region scales like n. Faraway region can become effectively nearby region and conductance increases.

My guess is that this unavoidably means beginning of the second quantum revolution brought by the hierarchy of Planck constants. These experimental findings cannot be put under the rug anymore.

11.4.3 Time crystals, macroscopic quantum coherence, and adelic physics

Time crystals were (see http://tinyurl.com/jbj5j68 were proposed by Frank Wilzek in 2012. The idea is that there is a periodic collective motion so that one can see the system as analog of 3-D crystal with time appearing as fourth lattice dimension. One can learn more about real life time crystals at http://tinyurl.com/zy73t6r.

The first crystal was created by Moore *et al* (see http://tinyurl.com/js2h6b4) and involved magnetization. By adding a periodic driving force it was possible to generate spin flips inducing collective spin flip as a kind of domino effect. The surprise was that the period was twice the original period and small changes of the driving frequency did not affect the period. One had something more than forced oscillation - a genuine time crystal. The period of the driving force - Floquet period- was 74-75 μs and the system is measured for N=100 Floquet periods or about 7.4-7.5 milliseconds (1 ms happens to be of same order of magnitude as the duration of nerve pulse). I failed to find a comment about the size of the system. With quantum biological intuition I would guess something like the size of large neuron: about 100 micrometers.

Second law does not favor time crystals. The time in which single particle motions are thermalized is expected to be rather short. In the case of condensed matter systems the time scale would not be much larger than that for a typical rate of typical atomic transition. The rate for 2P → 1S transition of hydrogen atom estimated at http://tinyurl.com/jtze3kg gives a general idea. The decay rate is proportional to $\omega^3 d^2$, where $\omega = \Delta E/\hbar$ is the frequency difference corresponding to the energy difference between the states, d is dipole moment proportional to αa_0,

a_0 Bohr radius and $\alpha \sim 1/137$ fine structure constant. Average lifetime as inverse of the decay rate would be 1.6 ns and is expected to give a general order of magnitude estimate.

The proposal is that the systems in question emerge in non-equilibrium thermodynamics, which indeed predicts a master-slave hierarchy of time and length scales with masters providing the slowly changing background in which slaves are forced to move. I am not a specialist enough to express any strong opinions about thermodynamical explanation.

First TGD based impressions

What does TGD say about the situation?

1. So called Anderson localization (see `http://tinyurl.com/z9ems4o`) is believed to accompany time crystal. In TGD framework this translates to the fusion of 3-surfaces corresponding to particles to single large 3-surface consisting of particle 3-surfaces glued together by magnetic flux tubes. On can say that a relative localization of particles occurs and they more or less lose the relative translation degrees of freedom. This effect occurs always when bound states are formed and would happen already for hydrogen atom.

   TGD vision would actually solve a fundamental problem of QED caused by the assumption that proton and electron behave as independent point like particles: QED predicts a lot of non-existing quantum states since Bethe-Salpeter equation assumes degrees of freedom, which do not actually exist. Single particle descriptions (Schrödinger equation and Dirac equation) treating proton and electron effectively as single particle geometrically (rather than independent particles) having reduced mass gives excellent description whereas QED, which was thought to be something more precise, fails. Quite generally, bound states are not properly understood in QFTs. Color confinement problem is second example about this: usually it is believed that the failure is solely due to the fact that color interaction is strong but the real reason might be much deeper.

2. In TGD Universe time crystals would be many-particle systems having collection of 3-surfaces connected by magnetic flux tubes (tensor network in terms of condensed matter complexity theory). Magnetic flux tubes would carry dark matter in TGD sense having $h_{eff}/h = n$ increasing the quantal scales - both spatial and temporal - so that one could have time crystals in long scales.

   Biology could provide basic examples. For instance, EEG resonance frequency could be associated with time crystals assignable to the magnetic body of brain carrying dark matter with large $h_{eff}/h = n$ - so large that dark photon energy $E = h_{eff}f$ would correspond to an energy above thermal energy. If bio-photons result from phase transitions $h_{eff}/h = n \rightarrow 1$, the energy would be in visible-UV energy range. These frequencies would in turn drive the visible matter in brain and force it to oscillate coherently.

3. The time crystals claimed by Monroe and Lurkin to be created in laboratory demand a feed of energy (see `http://tinyurl.com/zm4m5v9`) unlike the time crystals proposed by Wilzek. The finding is consistent with the TGD based model. In TGD the generation of large h_{eff} phase demands energy. The reason is that the energies of states increase with h_{eff}. For instance, atomic binding energies decrease as $1/h_{eff}^2$. In quantum biology this requires feeding of metabolic energy. Also now interpretation would be analogous to this.

4. Standard physics view would rely in non-equilibrium thermodynamics whereas TGD view about time crystals would rely on dark matter and hierarchy of Planck constants in turn implied by adelic physics suggested to provide a coherent description fusing real physics as physics of matter and various p-adic physics as physics of cognition.

   Number theoretical universality (NTU) leads to the notion of adelic space-time surface (monadic manifold) involving a discretization in an extension of rationals defining particular level in the hierarchy of adeles defining evolutionary hierarchy. $h_{eff}/h = n$ has been identified from the beginning as the dimension of poly-sheeted covering assignable to space-time surface. The action of the Galois group of extensions indeed gives rise to covering space. The number n of sheets would be the dimension of the extension implying $h_{eff}/h = n$, which is bound to increase during evolution so that the complexity increases.

Indeed, since n is positive integer evolution is analogous to a diffusion in half-line and n unavoidably increases in the long run just as the particle diffuses farther away from origin (by looking what gradually happens near paper basket one understands what this means). The increase of n implies the increase of maximal negentropy and thus of negentropy. Negentropy Maximization Principle (NMP) follows from adelic physics alone and there is no need to postulate it separately. Things get better in the long run although we do not live in the best possible world as Leibniz who first proposed the notion of monad proposed!

Second look a couple of years later

Further comments on time crystals

Google has reported about a realization of a time crystal as a spin system. A rather hypish layman article (`https://cutt.ly/5QWZLWk`) creates the impression that perpetuum mobile has been discovered. Also the Quanta Magazine article (`https://cutt.ly/6QWZXfT`) creates this impression. The original research article [D15] (`https://cutt.ly/RQWZCSR`) provides a realistic description.

It is interesting to look at the situation in the TGD framework. From the abstract of the article also from the Wikipedia article about time crystals one learns that the system has periodic energy feed and is therefore not closed so that the finding is not in conflict with the second law and perpetuum mobile is not in question.

What is time crystal? The notion of time crystal (`https://cutt.ly/2n65xOk`) is a temporal analog of ordinary crystals in the sense that there is temporal periodicity, was proposed by Frank Wilczeck in 2012. Experimental realization was demonstrated in 2016-2017 but not in the way theorized by Wilczek. Soon also a no-go theorem against the original form of the time crystal emerged and motivated generalizations of Wilzeck's proposal.

The findings reported by Google are however extremely interesting. Very concisely, researchers study a spin system, which has two directions of magnetization and the external laser beam induces the system to oscillate between the two magnetization directions with a period, which is a multiple of the period of the laser beam. It is interesting to consider the system in TGD framework and I have actually discussed time crystals briefly in a recent article [L125].

1. Space-time surfaces as periodic minimal surfaces as counterparts of time crystals In TGD, classical physics is an exact part of quantum theory and quantum classical correspondence holds true. Hence it is interesting to consider first the situation at the classical space-time level. In TGD time crystals have as classical correlates space-time surfaces which are periodic minimal surfaces. It is possible to have analogs of time-crystals and also more general structures built as piles of lego like basic pieces in time direction bringing in mind sentences of language and DNA, which is quasi-periodic structure and more general than crystal.

2. What about thermodynamics of time crystals? Could the time crystal be possible also in thermodynamic sense and even for thermodynamically closed systems? In the TGD frameswoork, Negentropy Maximization Principle (NMP) [L118] and zero energy ontology (ZEO) [L91, L119] forces to generalize thermodynamics to allow both time arrows. ZEO is forced by TGD inspired theory of consciousness and solves the basic paradox of quantum measurement theory. The arrow of time would change in ordinary ("big") state function reduction (BSFR) and would remain unaffected in "small" SFR (SSFR). Second law holds true at the level of real physics but in the cognitive sector information increases and NMP holds true.

Also new quantum theory might be needed to explain why the period is multiple of the driving period. The first possibly needed new element is hierarchy of effective Planck constants $h_{eff} = n \times h_0$ having number theoretical interpretation. h_{eff} measures the scale of quantum coherence and has also interpretation as the order of Galois group for a polynomial defining the space-time surface in M^8 mapped to $M^4 \times CP_2$ by $M^8 - H$ duality [L100, L101, L117].

The replacement of $h_{eff} \to kh_{eff}$ scales the periods by k and keeps energies unchanged. In TGD inspired biology h_{eff} hierarchy is in a crucial role and its levels behave relative to each other like dark matter.

In the recent case, the magnetic body (MB)of the spin system controlling its behavior would have $h_{eff} = nh_0$. Each period would be initiated by BSFR at the level of MB and change the arrow of time and induce effective change of it also at the level of the ordinary matter.

In ZEO, time crystal-like entities, which live in cycle by extracting back part of the energy that they have dissipated in a time reversed mode, are in principle possible. System "breathes". Various bio-rhythms could correspond to time crystals. The biological analogy is obvious and we know that life requires a metabolic energy feed: in TGD Universe it prevents the decrease of h_{eff} [L140].

For a thermodynamically open system, part of the dissipated energy leaks into the external world during each half cycle. Same happens in the time reversed mode and would mean that the system apparently receives positive energy also from the external world. Could this energy feed compensate for the energy loss to the external world by dissipation so that no external energy feed would be needed? Perhaps this might be the case in the ideal situation. One would have almost a perpetuum mobile! Periodic driving feeding energy to the system would be needed to take care that h_{eff} is not reduced.

11.5 Some condensed matter anomalies

11.5.1 Exciton-polariton Bose-Einstein condensate at room temperature and h_{eff} hierarchy

Ulla gave in my blog (see `https://goo.gl/Yo3zQG`) a link to a very interesting work about Bose-Einstein condensation of quasi-particles known as exciton-polaritons. The `goo.gl/eKg13S`popular article tells about a research article [D20] (see `https://goo.gl/bZ6LFs`) published in Nature by IBM scientists.

Bose-Einstein condensation happens for exciton-polaritons at room temperature, this temperature is four orders of magnitude higher than the corresponding temperature for crystals. This puts bells ringing. Could $h_{eff}/h = n$ be involved?

One learns from Wikipedia (see `https://goo.gl/jLU7QG`) that exciton-polaritons are electron hole pairs- photons kick electron to higher energy state and exciton is created.These quasiparticles would form a Bose-Einstein condensate with large number of particles in ground state. The critical temperature corresponds to the divergence of Boltzmann factor given by Bose-Einstein statistics.

1. The energy of excitons must be of order thermal energy at room temperature: IR photons are in question. Membrane potential happens to corresponds to this energy. That the material is organic, might be of relevance. Living matter involves various Bose-Einstein condensate and one can consider also excitons.

   As noticed the critical temperature is surprisingly high. For crystal BECs it is of order .01 K. Now by a factor 30,000 times higher!

2. Does the large value of $h_{eff} = n \times h$ visible make the critical temperature so high?

   Here I must look at same Wikipedia article for BEC of quasiparticles. Unfortunately the formula for the density of quasiparticle density dn/dV at criticality is copied from source and contains several errors. Dimensions are completely wrong. The formulas should read

   $$(\frac{dn}{dV})^{1/3} = \frac{(m_{eff}T_{cr})^{1/2}}{\hbar} \quad .$$

   (One can put Boltzmann constant $k_B = 1$ by using for temperature same units as for energy).

3. The correct formula for the critical temperature T_{cr} reads as

   $$T_{cr} = \frac{\hbar^2(\frac{dn}{dV})^{2/3}}{m_{eff}} \quad .$$

4. In TGD one can generalize by replacing $\hbar$ with $\hbar_{eff} = n \times \hbar$ so that one has

$$T_{cr} \to n^2 T_{cr} .$$

Critical temperature would be proportional to n^2 and the high critical temperature (room temperature) could be understood. In crystals the critical temperature is very low but in organic matter a large value of $n \sim 100$ could change the situation. $n \sim 100$ would scale up the atomic scale of 1 Angstrom as a coherence length of valence electron orbitals to cell membrane thickness about 10 nm. There would be one dark electron-hole pair per volume taken by dark valence electron: this would look reasonable.

One must consider also the conservative option $n = 1$. T_{cr} is also proportional to $(dn/dV)^2$, where dn/dV is the density of excitons and to the inverse of the effective mass m_{eff}. m_{eff} must be of order electron mass so that the density dn/dV or n is the critical parameter. In standard physics so high a critical temperature would require either large density dn/dV about factor 10^6 higher than in crystals.

Is this possible?

1. Fermi energy E_F is given by almost identical formula but with factor 1/2 appearing on the right hand side. Using the density dn_e/dV for electrons instead of dn/dV gives an upper bound for $T_{cr} \leq 2E_F$. E_F varies in the range 2-10 eV. The actual values of T_{cr} in crystals is of order 10^{-6} eV so that the density of quasi particles must be very small for crystals: $dn_{cryst}/dV \simeq 10^{-9} dn_e/dV$.

2. For crystal the size scale L_{cryst} of the volume taken by quasiparticle would be 10^{-3} times larger than that taken by electron, which varies in the range $10^{1/3} - 10^{2/3}$ Angstroms giving the range $(220 - 460)$ nm for L_{cryst}.

3. On the other hand, the thickness of the plastic layer is $L_{layer} = 35$ nm, roughly 10 times smaller than L_{cryst}. One can argue that $L_{plast} \simeq L_{layer}$ is a natural order of magnitude for L_{cryst} for quasiparticle in plastic layer. If so, the density of quasiparticles is roughly 10^3 times higher than for crystals. The $(dn/dV)^2$-proportionality of T_{cr} would give the factor $T_{cr,plast} \simeq 10^6 T_{cr,cryst}$ so that there would be no need for non-standard value of h_{eff}!

   But is the assumption $L_{plast} \simeq L_{layer}$ really justified in standard physics framework? Why this would be the case? What would make the dirty plastic different from super pure crystal?

The question which option is correct remains open: conservative would of course argue that the now-new-physics option is correct and might be right.

11.5.2 Quantum scarring from TGD point of view

I learned about very interesting phenomenon serving as a challenge for TGD. In quantum scarring the system does not thermalize as one might expect as the popular article "*Quantum scarring appears to defy universe's push for disorder*" describes (see `http://tinyurl.com/y2bo8r8y`). The experimental article by Bernien *et al* with title *Probing many-body dynamics on a 51-atom quantum simulator* [D9] (see `http://tinyurl.com/yykagmeu`) has the following abstract.

> *Controllable, coherent many-body systems can provide insights into the fundamental properties of quantum matter, enable the realization of new quantum phases and could ultimately lead to computational systems that outperform existing computers based on classical approaches. Here we demonstrate a method for creating controlled many-body quantum matter that combines deterministically prepared, reconfigurable arrays of individually trapped cold atoms with strong, coherent interactions enabled by excitation to Rydberg states. We realize a programmable Ising-type quantum spin model with tunable interactions and system sizes of up to 51 qubits. Within this model, we observe phase transitions into spatially ordered states that break various discrete symmetries, verify the high-fidelity preparation of these states and investigate the dynamics across the phase*

transition in large arrays of atoms. In particular, we observe robust many-body dynamics corresponding to persistent oscillations of the order after a rapid quantum quench that results from a sudden transition across the phase boundary. Our method provides a way of exploring many-body phenomena on a programmable quantum simulator and could enable realizations of new quantum algorithms.

There are many theoretical articles about MBQS. As an example I include the abstract of the article "*Quantum scarred eigenstates in a Rydberg atom chain: entanglement, breakdown of thermalization, and stability to perturbations*" by Turner *et al* [D22] (see `http://tinyurl.com/y54unclz`) serving as basis of TGD inspired considerations.

Recent realization of a kinetically constrained chain of Rydberg atoms by Bernien et al., [Nature (London) 551, 579 (2017)] resulted in the observation of unusual revivals in the many-body quantum dynamics. In our previous work [C. J. Turner et al., Nat. Phys. 14, 745 (2018)], such dynamics was attributed to the existence of "quantum scarred" eigenstates in the many-body spectrum of the experimentally realized model. Here, we present a detailed study of the eigenstate properties of the same model.
We find that the majority of the eigenstates exhibit anomalous thermalization: the observable expectation values converge to their Gibbs ensemble values, but parametrically slower compared to the predictions of the eigenstate thermalization hypothesis (ETH). Amidst the thermalizing spectrum, we identify non-ergodic eigenstates that strongly violate the ETH, whose number grows polynomially with system size. Previously, the same eigenstates were identified via large overlaps with certain product states, and were used to explain the revivals observed in experiment.
Here, we find that these eigenstates, in addition to highly atypical expectation values of local observables, also exhibit sub-thermal entanglement entropy that scales logarithmically with the system size. Moreover, we identify an additional class of quantum scarred eigenstates, and discuss their manifestations in the dynamics starting from initial product states.
We use forward scattering approximation to describe the structure and physical properties of quantum scarred eigenstates. Finally, we discuss the stability of quantum scars to various perturbations. We observe that quantum scars remain robust when the introduced perturbation is compatible with the forward scattering approximation. In contrast, the perturbations which most efficiently destroy quantum scars also lead to the restoration of "canonical" thermalization.

The systems exhibiting quantum scarring (QS) thermalize very slowly or do not thermalize at all. Instead, the system returns to its original state periodically. This behavior does not conform with ergodicity stating that the system goes through all possible state during time evolution.

There are a lot of systems, which fail to be ergodic.

1. In integrable systems - for which TGD is an excellent candidate - all states starting from energy eigenstate have this recurrence property as isolated systems if the energies are commeasurate (rational multiples of same unit of energy). In the recent case only preferred states have this recurrence property.

   In the experimental situation one considers a quenched system: the initial state can be modelled as energy eigenstate of some Hamiltonian H_0 which is replaced with $H = H_0 + H_1$ so that the state is not energy eigenstate anymore. Periodic behavior requires that the state is superposition of finite number of state with commensurate energies in the resolution considered. In the ideal situation the eigenstates of H are integer spaced so that they have the period of the ground state as common periodicity. Period increases if there are states with energies close to each other since states E and $E + \Delta E$ must satisfy $ET = n \times 2\pi$ and $\Delta E \times T = m \times 2\pi$ giving $T = m2\pi/\Delta E = (m/n) \times (E/\Delta E$.

2. For spin glass [B39] the energy landscape is a fractal with valleys inside valleys, and the system ends down to some valley as it dissipates. The mountains of the energy landscape force the localization and the thermalization is prevented.

Some kind of dynamical localization is expected place in the situations in which only preferred states give rise to a quantum scar. Dynamical localization could due to genuinely quantal state repulsive exchange forces depending on the relative direction of spins of valence electrons of Rydberg atoms.

One can distinguish between quantum scarring (QS) and quantum many-body scarring (MBQS).

1. In QS the wave function of the particle concentrates along unstable periodic classical orbit. The less unstable the orbit is, the stronger the scarring is. The classical orbit makes itself visible as a quantum scar.

2. In MBQS scarring is a generalization of quantum scarring and the state of many-particle system returns to the original one. In principle one can describe many-particle system wave-mechanically as single particle state in a higher-D configuration space so that in principle this does not bring anything new. MBQS has been observed in a 1-D lattice formed by Rydberg atoms and ordinary atoms so that configuration space is effectively discrete. Some atoms of this system at very low temperature are excited to what are believed to be Rydberg atoms with large value of principal quantum number n_P and therefore large radius. This requires energy because bound states energies are proportional to $1/n_P^2\hbar^2$.

   Remark: "Believed to be" sounds strange but in TGD framework atoms for which valence electrons have nonstandard value of Planck constant $h_{eff} = nh_0$ can look like Rydberg atoms. For $h = 6h_0$ suggested by experiments of Randell Mills [L26] one would have $h_{eff} = nh/6$ so that one could have one can have fractional principal quantum number $n_{P,eff} = (nn_P)/6$: this provides a test for h_{eff} hypothesis using irradiation with corresponding frequencies. For $n = 6n_1$ one might fail to distinguish these states from Rydberg states since the radii of the states scale like $n_{P,eff}^2$. Large value of h_{eff} would make possible quantum coherence in long length scales and this could be highly relevant for integrability.

Eigenstate thermalization (EST) is an important notion. Eigenstate thermalization takes place by unitary time evolution, which usually generates a superposition of large number of states with same total quantum numbers, in particular energy. Single particle states have however varying energies and in the superposition single particle states get entangled. For sub-systems the density matrix is assumed to develop to a thermal density matrix. In particular, entanglement entropy is identified as thermal entropy. For QS and MBQS EST would occur very slowly or not at all.

In TGD framework one can consider two approaches to MBQS and QS. The general approach starting from the key ideas of TGD and the approach starting directly from the special properties of Rydberg atoms and their possible analogs with non-standard value of $h_{eff} > h$. The key question is whether MBQS is analogous to the periodicity in integrable systems with commeasurate energies.

General TGD based considerations

In the sequel I will briefly discuss some aspects of the basic principles of TGD with some associations to MBQS. Reader can however skip directly to the concrete proposal if this looks easier.

1. TGD as generalization of Wheeler's superspace approach and as geometrization of quantum physics

One could see TGD as a generalization of Wheeler's superspace approach and generalization of Einstein's geometrization program for physics. Integrability, quantum criticality, quantum classical correspondence, zero energy ontology, and hierarchy of Planck constants are the aspects of TGD, which seem to be relevant for MBQS.

1. There are excellent reasons to believe that TGD Universe is integrable and quantum critical system [K95, **?**] in very general sense. Also MBQS are conjectured to possess these properties. Quantum criticality would be responsible for the ground state degeneracy characterizing the model Hamiltonian of Turner *et al* [D22].

2. TGD generalizes Einstein's vision about the geometrization of physics to the level of quantum physics. The basic geometric object is the "world of classical worlds" (WCW) consisting of pairs of 3-surfaces with members at opposite boundaries of a causal diamond (CD) and connected by preferred extremal of the action which for the twistor lift of TGD decomposes to a sum of so called Kähler action analogous to Maxwell action and a volume term, whose coefficient corresponds to cosmological constant.

   General Coordinate invariance implies holography in the sense that the these pairs of 3-surfaces as analogs of Bohr orbits are equivalent with the 4-D preferred extremals connecting them. Classical theory is an exact part of quantum TGD. Preferred extremals are minimal surfaces which fail to be such only at 2-D singular surfaces having identification as string world sheets and representing orbits of folds of a 3-surface [L75, L81].

   In zero energy ontology (ZEO) quantum state - called zero energy state - is a superposition of deterministic preferred extremals. Simplest zero energy states are superpositions with same eigenvalues of observables and total quantum numbers are conserved.

   Remark: Wave functions concentrated along periodic unstable classical orbits is central to QS. Superposition should be along unstable classical orbit. One could imagine that the state is superposition of 3-surfaces along classical orbit defined as slices obtained by intersecting with translate of either boundary of CD.

3. Zero modes are a key element of TGD and correspond to the degrees of freedom, which do not contribute to WCW metric, which is thus degenerate. There would be states with the same total quantum numbers but different values of zero modes so that ground state degeneracy of the model of Turner *et al* [D22] could correspond to wave function in zero modes.

Also fermionic degrees of freedom are geometrized.

1. Fermions are geometrized in terms of WCW spinor structure [K126] with WCW gamma matrices expressible as linear combinations of fermionic oscillator operators for second quantized induced spinor fields. Many-fermion states correspond to the modes of WCW spinor field. This implies what I call super-symplectic symmetry as an extension of the symplectic symmetry acting as isometries of WCW necessary for the existence of Riemann connection in infinite-D context [K36, **?**] (for loop spaces this was shown by Freed [A37]). Formally many-fermion states are just modes of classical spinor field in WCW.

2. Quantum-classical correspondence (QCC) implies that classical conserved Cartan charges and total fermionic charges are identical. Each particle in many-particle state corresponds near the boundaries of CD to "free particle" having single particle preferred extremal as correlate. One would have superposition of the collections of preferred extremals in the initial state. Superposition in entangled many-fermion state would correspond to a superposition of unions of corresponding 3-surfaces differing by translation and by properties correlating with other single particle quantum numbers.

   Quantum state with given total quantum numbers such as energy as for (ETH) is superposition of several many-particle states in general since total quantum numbers are sums of those with varying single particle quantum numbers. At fundamental level this would hold true in fermionic degrees of freedom (bosons are composites of fermions and antifermions in TGD Universe). For MBQS there would be only 2 different orbits corresponding to ground state of atom and Rydberg atom: the electronic Bohr orbits as pieces of space-time surface would be different for these. Therefore the situation would be rather simple classically.

3. The space-time surface - as opposed to 3-surfaces at the ends of CD - associated with many-particle system would be connected as analog of connected Feynman diagram and correspond to a formation of magnetic flux tubes between atoms as correlates of entanglement. The periodicity of the entanglement would correspond to periodic generation and disappearance of entanglement and flux tube - kind of breathing consisting of phase transitions between gas phase and liquid phase. Somewhat similar situation is encountered in simple systems consisting of plastic balls exhibiting basic aspects of life [L46].

2. Number theoretical vision

Number theoretical vision is second thread of TGD besides the vision about geometrization of physics.

1. p-Adic physics and their fusion to form a hierarchy of adelic physics characterized by a hierarchy of extensions of rational numbers inducing in turn extensions of various p-adic number fields [L49, L50]. Classical number fields represent second key aspect of number theoretical vision [L40].

   Adelic physics predicts a hierarchy $h_{eff} = nh_0$ ($h = 6h_0$ is a good guess [L26]) of effective values of Planck constant assumed to label a hierarchy of phases behaving like dark matter and having an interpretation as a dimension for extension of rationals.

2. One can ask whether non-standard value of h_{eff} guaranteeing quantum coherence in scales longer than expected is involved with MBQS. One can ask whether Rydberg atoms be actually atoms with valence electrons, which are dark for some value of h_{eff} and have scaled orbits with scaling factor $(h_{eff}/h)^2 = (n/6)^2$. If n is not a multiple of 6, one can speak of fractional principal quantum number $n_P = n/6$ and this might allow to test the hypothesis. For $h_{eff} > h$ pseudo Rydberg electrons could form a nanoscopic quantum system.

   MBQS is observed in very low temperatures and one can argue that the ordinary value of Planck constant is enough. One can however wonder whether MBQS is possible at higher temperatures for non-standard value of h_{eff} just like high Tc superconductivity if it is due to large h_{eff}.

3. If the presence of flux tube connections is necessary for large scale quantum coherence in the scale of the entire system needed and serves also as a correlate for entanglement, one can argue $h_{eff} > h$ is needed. Otherwise one expects thermalization to occur since the system decomposes to smaller quantum-coherent systems.

3. ZEO and generalization of quantum measurement theory

ZEO forces to generalize quantum measurement theory. One could also say that the need to solve the basic paradox of quantum measurement theory forces ZEO.

1. In ZEO state function reduction is replaced with the counterpart of ordinary state function reduction- "big state function reduction" (BSR) and the counterpart of weak measurement - "small state function reduction" (SSR) . The unitary evolution of state corresponds in TGD sequence of unitary evolutions followed by SSR affecting only the states at the active boundary of CD and also de-localizing the active boundary whereas passive boundary and members of state pairs at it would remain unaffected.

   SSR would localize the active boundary so that one has only single CD in superposition and mean also time measurement with time defined as the distance between the tips of CD. BSRs would change the roles of passive and active boundaries of CD and change the arrow of time assignable to the state by passive-active characterization.

2. Are SSRs or BSRs associated with the reduction of entanglement and return to the initial state in MBQS? SSR looks a more plausible interpretation. BSR would reduce the entanglement at the active boundary making it passive and change the arrow of time and next BSR would bring back the original arrow of time and CD boundary would be slightly shifted towards future. It is not clear whether the entanglement is small in the beginning of sequence of SSRs.

A concrete TGD inspired model for MBQS

The fact that MBQS occurs only for special initial states forces to ask whether it reflects the special properties of the system considered or some general properties such as integrability for a system with commeasurate energies. Or is MBQS something between these two cases: could the property of having energy spectrum with energies coming as rational multiples of a fundamental be dynamically generated (localization)?

1. System could be an integrable system for which the evolution is periodic if energies are commensurate. The spectrum should not differ too much from harmonic oscillator spectrum since small energy differences tend to spoil the periodicity. There are excellent reasons to expect that TGD is integrable theory but the behavior resembling harmonic oscillator is not obvious.

   The system is unstable and should be therefore critical and possess zero modes generating long range quantum fluctuations for which large h_{eff} phases can serve as correlates. This is achieved if ground state has a large degeneracy with respect to energy. Small perturbations can be always described in terms of harmonic oscillators. The frequencies of harmonic oscillators should be expressible as multiples of fundamentals whose ratios are rational numbers.

2. In TGD framewor the large value of h_{eff} makes possible quantum coherence in longer length scales and commeasurate integrability in such a way that eigen-energies resemble harmonic oscillator spectrum coming as integer multiples of rather few rationally related fundamentals.

3. Space-time sheet is a natural candidate for a quantum coherent structure and if the space-time sheet decomposes into smaller disjoint sheets also coherence would be lost. Magnetic flux tubes connecting smaller space-time sheets to larger units would be natural correlates of quantum coherence and carry large h_{eff} phases. One could perhaps speak of dynamically generated quantum coherence and integrability with small number of fundamental energies.

4. Dynamical localization should occur and could be due to interatomic forces. Exchange forces due to the Fermi statistics generate spin-dependent interactions, which are short ranged and repulsive for parallel spins. The exchange forces are excellent candidates for inducing the localization.

   Dark valence electrons with large h_{eff} would have stronger exchange forces. This would promote the localization since one could not have effective Rydberg atoms (ERAs) with too small distance between them. If one has a system consisting of ordinary atoms (OAs) plus ERAs, the dark valence electrons could form a macroscopic quantum having MBQS states for this reason.

   The physical picture is that states in which ERAs have too small mutual distance are not possible. This gives a constraint to the dynamics. Typically the "spin flip" giving rise to an ERA can occur only for atoms with sufficiently large distances to the nearest ERAs. This constraint dynamics forces localization inducing periodicity.

1. About intermolecular -, van der Waals -, and exchange forces

Intermolecular forces (see `http://tinyurl.com/mmxnctm`) include exchange forces due to Pauli exclusion principle, electrostatic interactions between permanent electric and magnetic multipoles, which can be both attractive and repulsive, and attractive interactions between permanent and induced multipoles - induction -, and between induced multipoles - so called dispersion forces.

In standard QFT van der Waals force-London dispersion force comes from interaction with zero point energy and analogous to Casimir force. London dispersion force is proportional to the product of ionization energies of atoms divided by their sum and product of polarizabilities and therefore proportional to $1/h_{eff}^2$ and would weaken for large h_{eff}.

Lennard-Jones potential (see `http://tinyurl.com/y9bjcxn5`) provides the simplest parameterization of these forces. There is attractive $1/r^6$ term representing dispersion forces and repulsive $1/r^{12}$ term interpreted in terms of exchange forces repulsive/attractive for parallel/opposite spins of electrons. This follows from antisymmetry of the wave function. The dispersion force if proportional to the energy scale of atom and therefore to $1/h_{eff}^2$ so that its scale decreases for large h_{eff}.

The strength of the exchange force is proportional to the inner product of spins and therefore proportional to h_{eff}^2 and increases with h_{eff}^2. This make increase the range of this force and together with the weakening of the dispersion force would make the radius at which the van der Waals force becomes repulsive larger. This would promote dynamical localization.

2. Consistency with the model of MBQS of Turner et al

In the model of MBQS discussed by by Turner *et al* [D22] (see http://tinyurl.com/y54unclz) the situation is indeed very much like proposed above. One considers a model Hamiltonian H having decomposition $H = H_0 + H_1$. Ground state and Rydberg state are formally described as two possible states of spin.

The first part in the Hamilton is sum $H_0 = k\sum X_i$ over single particle terms X_i analogous to paramagnetic spin flip term in the interaction of spins with an external magnetic field. It acts on single particle transforming ordinary atom in ground state to Rydberg atom or vice versa.

Second part $H_1 = \sum i \neq jV_{ij}Q^iQ^j$ of the Hamiltonian describes repulsive interatomic forces and is associated with pairs of particles at different sites. Individual terms are proportional to the projectors Q_i and Q_j to Rydberg states at neighboring sites i and j and the parameter V_{ij} describing interaction strength assumed to behave like $1/|i-j|^n$, $n = 6$, at the limit $i \rightarrow j$. Lennard-Jones potential would suggest $n = 12$ but this is not essential for the model since one considers an approximation in which only nearest neighbour interactions are considered. This part of the Hamiltonian is the large part non-perturbative and spin-flip term is treated as a small perturbation, which suggests that harmonic oscillator type approximation is good.

In nearest neighbour approximation the large part H_1 is proportional to a sum over terms $V_{i,i+1}Q_iQ_{i+1}$ over nearest neighbour pairs. In the states with minimum energy the positive interaction term (somewhat ironically) vanishes: this is guaranteed if all Rydberg sites have ground states as neighbours. One can introduce to the Hamilton this constraint explicitly, and by a scaling ends up to a Hamiltonian which is just the small paramagnetic spin flip term X_i multiplied from left *resp.* right side by projector P_{i-1} *resp.* P_{i+1} to the subspace satisfying the constraint.

The effect of this Hamiltonian is to induce "spin flips" such that the constraint is respected. The outcome is entangled state and the localization caused by the constraint induces the periodic dynamics and failure of ETH for preferred states.

The entanglement between dark and ordinary states makes sense: $h_{eff} = nh_0$ corresponds at space-time level n-sheeted covering of space-time. One must however assume that the entanglement coefficients are in the extension of rationals associated with the smaller value of n (n_1) belonging to that assignable to the larger value of n (n_2): therefore n_1 divides n_2.

If the effective spin-spin interaction is a sensible model for the situation, the value of h_{eff} affects only the parameters determining the spin-spin interaction. The excitation of ERAs requires energy but so does also the excitation of ordinary Rydberg atoms so that this cannot be used as an objection against the model.

11.5.3 Three surprising condensed matter findings

I learned about 3 surprising findings related to condensed matter physics and defying standard quantum theory and having a natural explanation in TGD framework.

The strange behavior of light

Light does not behave quite in the manner expected (see http://tinyurl.com/sjc9rpu). What was studied was splitting of photons to entangled pairs of photons in the crystal beam entering a crystal. Quantum field theory based on the idea of completely point-like particle predicts that photon pairs should be created at single point. What was observed that members of entangled photon pairs can be also created at separate points. The distances of these points can be about 1/100 microns- which happens to the size scale of cell membrane and fundamental scale in living matter. This length scale is about 100 times the atomic length scale.

Researchers argue that this findings supports new kind of Uncertainty Principle. I do not feel quite easy with this proposal unless it is taken to mean that particle has geometric size to be distinguished from Compton length.

1. In TGD Universe geometric size would be due to the fact that particles are not point-like but correspond to 3-D surfaces whose "orbits" define basic building bricks of space-time as 4-D surface in 8-D space-time $H = M^4 \times CP_2$. Particles can exiss superpositions of their variants with different size scales.

2. p-Adic physics for various primes p fusing together with real number based physics to what I call adelic physics would provide physical correlates of cognition and sensory experience.

The number theoretic vision assigns to each particle extension of rationals characterized by so called ramified primes, which are excellent candidates for defining preferred p-adic length scales. The dimension n of extension defining a measure for algebraic complexity and serving as a kind of universal IQ has interpretation as effective Planck constant $h_{eff}/h_0 = n$ so that a connection with quantum physics - or rather its TGD based generalization - emerges.

3. p-adic mass calculations rely on p-adic length scale hypothesis stating that primes near powers of 2 are especially interesting physically and massive elementary particles and also hadrons correspond to this kind of primes. p-Adic mass scale would be proportional to $p^{1/2}$.

A lot of new physics is predicted.

1. TGD predicts scaled variants of strong and weak interaction physics corresponding to different values of p and LHC provides handful of bumps having identification as scaled variants of ordinary hadrons and having mass which is 512 higher [K73].

2. For given particle several mass scales are in principle allowed. Quite generally, particle can correspond to several p-adic primes and therefore can exist in states with different masses differing by power of $2^{1/2}$. The existence of this kind of states in the case of neutrinos would solve some problems related to neutrinos and their masses.

3. In the case of massless particles different p-adic mass scales do not mean that masses are different (or more precisely, the masses depend on p but are extremely small and below measurement resolution so that mass differences cannot be detected). The p-adic length scale defines the geometric size of the particle as 3-surface to be distinguished from quantum size defined by Compton length. Quantum classical correspondence (QCC) strongly suggests that these two scales are same or at least closely correlated.

The hierarchy of Planck constants $h_{eff} = n \times h_0$ having an interpretation in terms of dark variants of ordinary particles predicts second kind of scale hierarchy.

1. The mass of the dark variant of elementary particle would not differ from the mass of ordinary particle but Compton size for a dark particle is proportional to n - a good guess is that $n = 6$ would correspond to ordinary particle and ordinary value h of h_{eff}.

2. The scales defined by dark matter hierarchy could relate to p-adic length scales. There could be kind of resonance coupling for massless particles: dark massless particle labelled by n and particle labelled by p-adic prime p could transform to each other with high rate if the p-adic and dark length scales are nearly the same. This could be very relevant for biology.

The experimental findings could be understood if photons can correspond to several p-adic length scales. The length scale 10 nm defining the upper bound for distance between members of entangled photon pair in experiments would correspond to p-adic length scale $L(151)$, which corresponds to Gaussian Mersenne prime $p = (1+i)^{151} - 1$. A simple model for photon could be as a closed flux tube like structure of this length. Also $k = 157, 163$, and 167 define Gaussian Mersenne primes, which is a number theoretical miracle. What is fascinating that these scales are fundamental biological length scales assignable to the basic structures of DNA.

New surprises related to super-conductors

So called Anderson's theorem applying to the conventional super-conductors (BCS) states that the addition of non-magnetic impurities does not destroy super-conductivity. It has been however found (see `http://tinyurl.com/vq2do7f`) that this is not the case for iron based high Tc super-conductors. This gives a valuable hints in still-continuing to attempts to understand high Tc super-conductivity.

I have been preaching for fifteen years new kind of super-conductivity explaining high Tc superconductivity making living systems high Tc superconductors [K90, K91].

1. The TGD view about magnetic fields differs from Maxwellian view. The counterparts of Maxwellian magnetic fields are flux quanta, flux tubes or sheets realized as space-time surfaces (or regions of them). Besides counterparts of ordinary magnetic fields there are also monopole flux tubes and they appear in all scales and form the basis of entire TGD view of Universe. They carry dark matter as $h_{eff} = n \times h_0$ phases and for large value of $h_{eff} > h$ there is quantum coherence in long scales making possible super-conductivity along dark magnetic flux tubes. This could explain also high Tc superconductivity in iron based super-conductors.

2. What was found that the addition of Cobalt atoms destroys the super-conductivity by inducing quantum phase transition. Anderson's theorem for ordinary super-conductivity however states that non-magnetic perturbations do not affect superconductivity. In TGD framework the natural interpretation would be that the quantum phase transition reduces the value of $h_{eff}/h_0 = n$ and thus also the quantum coherence length meaning that flux tube length is reduces and super-conductivity is possible only in short scales. Note that dark matter is identified as phases with non-standard value of h_{eff} different from h.

3. Also the nature of so called energy gap assignable to super-conductors was modified as Cobalt atoms were gradually added to destroy super-conductivity. This is not surprising if the value of h_{eff} was reduced. The reduction of h_{eff} in general decreases energies for other parameters kept constant and now it would mean reduction of energy gap and loss of superconductivity.

Conductors of electricity, which are poor conductors of heat

The so called Wiedemann-Franz Law states that good conductors of electricity are also good conductors of heat. The two conductivities are proportional to each other. The metal found 2017 however violates this law (see `http://tinyurl.com/w4t9vdx`) Vanadium dioxide VO_2 transforms from insulator to a conductive metal at 67 degrees Celsius. The experimenters argue that this property could make possible new technologies. For instance, conversion of wasted heat from engines could be transformed to electricity.

Electrons are found to move in coordinated, synchronous manner and this would explain the reduction of heat conductivity to 1/10 of the expected value. There is no super-conductivity however. TGD explanation would be in terms of coherence and synchrony induced from the quantum coherence of dark phases of matter having $h_{eff}/h_0 = n$ residing at the magnetic body of the system controlling it.

This forced coherence would be also crucial in living matter: ordinary living matter would not be quantum coherent but the magnetic body carrying dark matter would force the coherence. In fact, all self-organization processes could involve magnetic body and dark matter.

11.5.4 Fractons and TGD

In Quanta Magazine there was a highly interesting article about entities known as fractons (`https://cutt.ly/kQPph8n`).

There seems to be two different views about fractons as one learns by going to Wikipedia. Fracton can be regarded as a as self-similar particle-like entity (`https://cutt.ly/KQPadQL` or as "sub-dimensional" particle unable to move in isolation (`https://cutt.ly/yQPayJt`). I do not understand the motivation for "sub-dimensional". It is also unclear whether the two notions are related. The popular article assigns to the fractons both the fractal character and the inability to move in isolation.

The basic idea shared by both definitions is however that discrete translational symmetry is replaced with a discrete scaling invariance. The analog of lattice which is invariant under discrete translations is fractal invariant under discrete scalings.

One can also consider the possibility that the time evolution operator acts as a scaling rather than translation. At classical level this would produce scaled versions of the system in discrete steps. This is something totally new from quantum field theory (QFT) point of view and it is not clear whether QFT can provide a description of fractons. In QFTs energy corresponds to time translational symmetry and Hamiltonian generates infinitesimal translations. In string models the analog of stringy Hamiltonian is the infinitesimal scaling operator, Virasoro generator L_0. Energy eigenstates would be replaced by scaling eigenstates with energy replaced with conformal weight.

In TGD the extension of physics to adelic physics provides number theoretic and geometric descriptions as dual descriptions of physics [L49, L100, L101, L117]. This approach also provides insights about what fractons as scale invariant (or covariant) entities might be.

1. The extension of conformal invariance to its 4-D analog is key element of TGD and leads to the notion of super-symplectic invariance and to an extension of conformal and Kac-Moody symmetries with two coordinates analogous to the complex coordinate z for ordinary conformal symmetry. Second coordinate is light-like and the fact that light-like 3-surfaces are effectively 2-dimensional is absolutely essential for this approach. The existence of extended conformal symmetries makes the space-time dimension $D = 4$ unique whereas the twistor lift of TGD fixes H to be $H = M^4 \times CP_2$.

2. The predicted cosmological expansion is not smooth but occurs by discrete scalings as rapid jerks in which the size scale of 3-space as 3-surface increases. Actually they would correspond to discrete quantum jumps but in zero energy ontology (ZEO) in which quantum state are superpositions of space-time surfaces, their classical correlates are smooth time evolutions.

   Scalings by power of 2 are p-adically preferred [L64, L124]. $M^8 - H$ duality allows us to imagine what this means at M^8-level [L125]. This proposal conforms with the puzzling observation that also astrophysical objects participate in cosmological expansion by comoving with it, they do not expand themselves.

3. The analog of a unitary time evolution between "small" state function reductions (SSFRs) as the TGD counterparts of weak measurements, is generated by the exponential of the infinitesimal scaling operator, Virasoro generator L_0. One could imagine fractals as states invariant under discrete scalings defined by the exponential of L_0. They could be counterparts of lattices but realized at the level of space-time surfaces having quite concrete fractal structure.

4. In p-adic mass calculations the p-adic analog of thermodynamics for infinitesimal scaling generator L_0 proportional to mass squared operator M^2 replaces energy. This approach is the counterpart of the Higgs mechanism which allows only to reproduce masses but does not predict them. I carried out the calculations already around 1995 and the predictions were amazingly successful and eventually led to adelic physics fusing real and various p-adic physics [K77].

5. Long range coherence and absence of thermal equilibrium are also mentioned as properties of fractons (at least those of the first kind). Long range coherence could be due to the predicted hierarchy of Planck constants $h_{eff} = n \times h_0$ assigned with dark matter and predicting quantum coherence in arbitrarily long scales and associated with what I called magnetic bodies.

   If translations are replaced by discrete scalings, the analogs of thermodynamic equilibria would be possible for L_0 rather than energy. Fractals would be the analogs of thermodynamic equilibria. In p-adic thermodynamics, elementary particles are thermodynamic equilibria for L_0 but it is not clear whether the fractal analogy with a plane wave in lattice makes sense.

   An attractive identification of the fractal counterpart of an energy eigenstate created in the unitary evolution preceding SSFR is as a scaling eigenstate defined as a superposition of scaled variants of space-time surface obtained by discrete scalings. Energy eigenvalue would be replaced with conformal weight. In zero energy ontology (ZEO), the counterpart of a fractal quantum state could be a superposition over zero energy states located inside the scaled variants of a causal diamond (CD).

   The ZEO based proposal is that each unitary evolution preceding SSFR creates a superposition of scaled variants of CD and that the SSFR induces a localization to single CD [L91, L3]. The interpretation would be as a time measurement determined by the scale of the CD.

Second definition assumes that fractons are able to move only in combinations. This need not relate to the scaling invariance. Color confinement comes to mind as an analogy. Quarks are unable to exist as isolated entities, not only to move as in isolated entities.

In the TGD framework, the number theoretical vision leads to the notion of Galois confinement analogous to color confinement [L114]. The Galois group of a given extension of rationals indeed acts as a symmetry at the space-time level. In the TGD inspired biology Galois groups would play a fundamental role [L115]. For instance, dark analogs of genetic codons, codon pairs, and genes would be singlets (invariant) under an appropriate Galois group and therefore behave as a single quantum coherent dynamical and informational unit [L140, L116].

Suppose that one has a system - say a fractal analog of a lattice consisting of Galois singlets. Could fracton be identified as a state which is analogous to quark or gluon and therefore not invariant under the Galois group. The physical states could be formed from these as Galois singlets and are like hadrons.

Chapter 12

Quantum Criticality and Dark Matter: part III

12.1 Introduction

Quantum criticality is one of the corner stone assumptions of TGD. The value of Kähler coupling strength fixes quantum TGD and is analogous to critical temperature. TGD Universe would be quantum critical. What does this mean is however far from obvious and I have pondered the notion repeatedly both from the point of view of mathematical description and phenomenology [K57, K126].

1. Criticality is characterized by long range correlations and sensitivity to external perturbations and living systems define an excellent example of critical systems - even in the scale of populations since without sensitivity and long range correlations cultural evolution and society would not be possible. For a physicist with the conceptual tools of existing theoretical physics the recent information society in which the actions of people at different side of globe are highly correlated, should look like a miracle.

2. The hierarchy of Planck constants with dark matter identified as phases of ordinary matter with non-standard value $h_{eff} = n \times h$ of Planck constant is one of the "almost-predictions" of TGD is definitely something essentially new physics. The phase transition transforming ordinary matter to dark matter in this sense generates long range quantal correlations and even macroscopic quantum coherence.

   Finding of a universal mechanism generating dark matter have been a key challenge during last ten years. Could it be that criticality is always accompanied by the generation of dark matter? If this is the case, the recipe would be stupifuingly simple: create a critical system! Dark matter would be everywhere and we would have observed its effects for centuries! Magnetic flux tubes (possibly carrying monopole flux) define the space-time correlates for long range correlations at criticality and would carry the dark matter. They are indeed key players in TGD inspired quantum biology.

3. Change of symmetry is assigned with criticality as also conformal symmetry (in 2-D case). In TGD framework conformal symmetry is extended and infinite hierarchy of breakings of conformal symmetry so that a sub-algebras of various conformal algebras with conformal weights coming as integer multiples of integer n defining h_{eff} would occur.

4. Phase separation is what typically occurs at criticality and one should understand also this. The strengthening of this hypothesis with the assumption $h_{eff} = h_{gr}$, where $h_{gr} = GMm/v_0$ is is the gravitational Planck constant originally introduced by Nottale. In the formula v_0 has dimensions of velocity, and will be proposed to be determined by a condition relating the size of the system with mass M to the radius within which the wave function of particle m with $h_{eff} = h_{gr}$ is localized in the gravitational field of M.

5. The condition $h_{eff} = h_{gr}$ implies that the integer n in h_{eff} is proportional to the mass of particle. The implication is that particles with different masses reside at flux tubes with different Planck constant and separation of phases indeed occurs.

6. What is remarkable is that neither gravitational Compton length nor cyclotron energy spectrum depends on the mass of the particle. This universality could play key role in living matter. One can assign Planck constant also to other interactions such as electromagnetic interaction so that one would have $h_{em} = Z_1 Z_2 e^2/v_0$. The phase transition could take place when the perturbation series based on the coupling strength $\alpha = Z_1 Z_2 e^2/\hbar$ ceases to converge. In the new phase perturbation series would converge since the coupling strength is proportional to $1/h_{eff}$. Hence criticality and separation into phases serve as criteria as one tries to see whether the earlier proposals for the mechanisms giving rise to large h_{eff} phases make sense. One can also check whether the systems to which large h_{eff} has been assigned are indeed critical.

The motivation for this work came from super-fluidity. Superfluids exhibit rather mysterious looking effects such as fountain effect and what looks like quantum coherence of superfluid containers, which should be classically isolated. These findings serve as a motivation for the proposal that genuine superfluid portion of superfluid corresponds to a large h_{eff} phase near criticality at least and that also in other phase transition like phenomena a phase transition to dark phase occurs near the vicinity.

12.1.1 Some applications to living Matter

Biology is full of critical systems and criticality makes living matter highly sensitive to the external perturbations, gives maximal richness of structure, and makes them quantum coherent in macroscopic scales. Therefore it is not difficult to invent examples. The basic problem is whether the criticality is associated only with the transitions between different systems or with the systems themselves.

1. Sols and gels are very important in biology. Sol is definition a micture solid grains and liquid (say blood of cell liquid). Gel involves fixed solid structure and liquid. Sol-gel phase transition of the cell fluid takes place when nerve pulse travels along axon leading to the expansion of the cell. Is the dark phase generated with the sol-gel transition or does it characterized sol. Perhaps the most logical interpretation is that it is involved with the phase transition.

2. Pollack's fourth phase of water resembles gel [L7]. Charge separation implying that the exclusion zones are negatively charged takes place. Charging takes place because part of protons goes to outside of EZ. TGD proposal is that protons go to magnetic flux tubes outside the region or to flux tubes which are considerably larger than EZ that most of their wave functions is located outside the EZ. Is fourth phase is permanently quantum critical? Or is the quantum criticality associated only with the transition so that magnetic flux tubes would carry protons but they would not be dark after the phase transition. EZs have a strange property that impurities flow out of them. Could the presence of dark flux tubes and $h_{eff} = h_{gr}$ forces the separation of particles with different masses?

3. The chirality selection of bio-molecules is a mystery from the point of view of standard physics. Large h_{eff} phase with so large value of Planck constant that the Compton length of weak bosons defines nanoscale, could explain this: weak bosons would be effectively massless and mediate long range interactions below the scaled up Compton scale. This phase transition could also force phases separation if $h_{gr} = h_{eff}$ holds true. If the masses of biomolecules with different handedness are slightly different also the values of h_{gr} would differ and the molecules would go to flux tubes with different value of h_{eff} - at least in the phase transition. The value of $\hbar_{gr} = GMm/v_0$ is in the range $10^{10} - 10^{11}$ for biomolecules so that the $\Delta n/n \simeq \Delta m/m \simeq 10^{-10} - 10^{-11}$ would be needed: this would correspond to an energy of eV which corresponds to the energy scale of bio-photons and visible light.

4. Neuronal membrane could be permanently a critical system since the membrane potential is slightly above the threshold for nerve pulse generation. Criticality might give rise to the

dark magnetic flux tubes connecting lipids to the DNA nucleotides or codons assumed in the model of DNA as topological quantum computer. The braiding of the flux tubes would represent the effect of the nerve pulse patterns and would be generated by the 2-D flow of the lipids of the membrane forming a liquid crystal.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

12.2 Basic notions and ideas

12.2.1 Worrying About The Consistency With The TGD Inspired Quantum Biology

The life of theoretician trying to be worth of his salt is full of worrying: it is always necessary to make internal consistency checks. One of the worries is whether the hypothesis $\hbar_{eff} = n \times \hbar = \hbar_{gr} = GMm/v_0$ is really consistent with TGD inspired quantum biology or has wishful thinking made its way to the arguments? More precisely, does the nominal value $B_{end} = .2 \times 10^{-4}$ Tesla of "endogenous" magnetic field suggested by the effects of ELF em fields on brain give electron cyclotron energy $E = h_{eff} eB_{end}/2\pi m$ in few eV range for the value of n in question?

At the end of worry-filled section I have included two pieces of reckless speculation as the final relief.

Some background

First some background.

1. The identification $h_{eff} = h_{gr}$, where h_{gr} is what I call gravitational Planck constant

$$h_{gr} = \frac{GMm}{2\pi v_0} = \frac{r_S m}{4\pi\beta_0} \ , \ \beta_0 = \frac{v_0}{c} \tag{12.2.1}$$

   makes the model quantitive. In the expression of h_{gr} M is the "large" mass - naturally Earth's mass M_E. m would be the mass of ^4He atom. $r_S = 2GM/c$ denotes Schwartschild radius of Earth, which from $M_E = 3 \times 10^{-6} M_{Sun}$ and from $r_S(Sun) = 3$ km is 4.5 mm. v_0 would be some characteristic velocity for Earth-superfluid system and the rotation velocity $v_0 = 465.1$ m/s of Earth is a good candidate in this respect. Also the radius of Earth $R_E = 6.38 \times 10^6$ meters will be needed.

2. One could fix the value of v_0 in the following manner. Consider the Schrödinger equation for particle in gravitational field of a massive object at vertical flux tubes carrying the gravitational interaction. The solutions are Airy functions which decay very fast above some critical distance z_0. Require that z_0 is apart from a numerical factor equal to Earth radius. This condition predicts the value of v_0 which is consistent in the case of Earth and Sun with earlier hypothesis about their values. For Sun v_0 would be 5.65×10^{-4}c and for Earth orbital rotation velocity β_0 scaled up from 1.6×10^{-6} to 2.3×10^{-6} by a factor $1.41 \simeq \sqrt{2}$.

3. In TGD inspired biology the hypothesis $h_{gr} = h_{eff} = n \times h$ plays a key role. One of the basic implications is that the energies of cyclotron photons associated with magnetic flux tubes have universal energy spectrum since the dependence on the mass of the charged particle disappears. Also the gravitational Compton length. The gravitational Compton length $\lambda_{gr} = h_{gr}/m$ does not depend on the mass of the particle and equals to $\lambda_{gr} = GM/v_0 \simeq 645$ meters in the recent case. The scale of the superfluid system is thus much smaller than the coherence length.

4. Note that the nominal value of B_{end} is definitely not the only value in the spectrum of B_{end}. Already the model of hearing forces to allowing spectrum of about 10 octaves (3 orders of magnitude) corresponding the spectrum of audible frequencies. Also the geometric model of harmony correlating music and genetic code requires this.

Does $h_{gr} = h_{eff}$ hypothesis predict that the energy range of dark photons is that of biophotons?

Consider now the question whether the predicted value of n is consistent with the assumption that dark cyclotron photons have energies in visibleandUV range.

1. The value of integer n in $h_{eff} = n \times n$ equals to the ratio of gravitational and ordinary Compton lengths

$$n = \frac{h_{eff}}{h} = \frac{\lambda_{gr}}{\lambda_c} \quad .$$

   For electron one obtains $n = .6 \times 10^{15}$. In the case of proton the frequency the ratio would be by a factor about 2×10^3 higher.

   The value of n is much higher than the lower bound $10^9/6$ given as the ratio of visible photon frequency about 10^{14} Hz and cyclotron frequency $f = 6 \times 10^5$ Hz of electron in the magnetic field having the nominal value $B_{end} = .2$ Gauss of endogenous magnetic field. The discrepancy is six orders of magnitude. Desired value would be correspond to magnetic field strengths of order B_{end} in $B_{gal} = 1$ nT range which corresponds to the order of magnitude for galactic magnetic fields.

   The value of n would give for B_{end} and an ion with 10 Hz cyclotron frequency (say Fe^{++} ion) energy of visible photon. The condition $\frac{h_{eff}}{h}$ predicts a value which is at least by a factor $m_p/m_e \simeq 2^{11}$ higher and one must also now assume galactic magnetic field strength to obtain a sensible result.

2. The naïve expectation was that $B_{end} = .2 \times 10^{-4}$ Tesla should give energy in few eV range. Something goes definitely wrong since the magnetic fields in this value range should be in key role. Either the hypothesis $h_{eff} = h_{gr}$ is wrong or the model is somehow wrong.

3. It is of course very naïve to assume that only single value of magnetic field is important. In fact, precognitve events are found to occur most frequently almost in the middle of sideral day which could be explained as being due to the involvement of galactic magnetic field.

Should one modify the $h_{gr} = h_{eff}$ hypothesis?

If one wants bio-photon spectrum to be in visible-UV range assuming that bio-photons correspond to cyclotron photons, one must reduce the value of $r = h_{gr}B_{end}/mv_0$ for Earth particle system by a factor of order $k = 2 \times 10^{-4}$. r does not depend on the mass of the charged particle. One can replace B_{end} with some other magnetic field having value which is considerably smaller. One can also increase the value of v_0.

1. For h_{gr} determined by Earth's mass and $v_0 = v_{rot}$, where $v_{rot} \simeq 1.55 \times 10^{-6}c$ is the rotation velocity of Earth around its axis and for $B_{end} \to B_{gal} = 1$ nT, where B_{gal} is typical strength of galactic magnetic field, the energy of dark cyclotron energy is 45 eV (UV extends to 124 eV). This is roughly by a factor 50 higher than the lower bound for the range of bio-photon energies. One possibility is that B_{gal} defines the upper limit of the dark photon energies and has variation range of at least 7 octaves with lower limit roughly 1/50 nT.

   One can also consider the possibility B_{gal} defines lower bound for the magnetic field strengths involved and one has $v_0 > v_{rot}$. For sun the rotation velocity at Equator is $v_{rot} = 2 \times 10^{-5}$ m/s and v_0 is $v_0 \simeq 5.8 \times 10^{-4}c$. One has $v_0/v_{rot} \simeq 29.0$. If same is true in case of Earth, the value of the energy comes down from 25 eV to 1.6 eV which corresponds to visible wave length.

The assignment of B_{gal} to gravitational flux tubes is very natural. Now however the frequencies of dark variants of bio-photons would not be in EEG range: 10 Hz frequency would correspond to 5×10^{-4} Hz with period of 42 min. The time scale of 42 min is however very natural concerning consciousness and could be involved with longer bio-rhythms. Scaled EEG spectrum with alpha band around 46 min naturally assignable to diurnal sub-rhythms could be a testable prediction. Natural time would be sidereal (galactic) time with slightly different length of day and this allows a clear test. Recall the mysterious looking finding of Spottiswoode that precognition seems to be enhanced at certain time of sidereal day [J9]. Cyclotron frequency 1 Hz would correspond to 7 hours. One can ask whether 12 hours (25) is the natural counterpart for the cyclotron frequency 1 Hz assignable to DNA. This would correspond to lower bound $B_{gal} \rightarrow 7B_{gal}/12 \simeq .58$ nT or to $v_0 \rightarrow 1.7v_0$.

2. The idea has been that it is dark EEG photons, which correspond to bio-photons. Could one assign bio-photons also to dark EEG so that magnetic fields of Earth and galaxy would correspond to two different control levels? If $B_{end} = .2$ Gauss is assumed to determine the scale of the magnetic field associated with the flux tubes carrying gravitational flux tubes, one must reduce h_{gr}. The reduction could be due to $M \rightarrow M_D = kM$ and due to the change of v_0. k could characterize the dark matter portion of Earth but this assumption is not necessary.

   This would require $k = M_{dark,E}/M_E \simeq 5 \times 10^{-5}$ if one does not change the value of v_0. This value of k equals to the ratio of B_{gal}/B_{end} and would be 1/4:th of $k = 2 \times 10^{-4}$. One might argue that it is indeed dark matter to which the gravitational flux tubes with large value of Planck constant connect biomatter.

3. Suppose that one does not give up the idea that also Earth mass gives rise to h_{gr} and scaled analog of EEG. Then M_D must correspond to some mass distinguishable from and thus outside Earth. The simplest hypothesis is that a spherical layer around Earth is in question. TGD based model for spherical objects indeed predict layered structures [K119]. There are two separate anomalies in the solar system supporting the existence of a spherical layer consisting of dark mass and with radius equal to the distance of Moon from Earth equal to 60.3 Earth radii [K100]. The first anomaly is so called Flyby anomaly and second one involves a periodic variation of both the value of the measured Newton's constant at the surface of Earth and of the length of the day. The period is about 6 years and TGD predicts it correctly.

   One can imagine that dark particles reside at the flux tubes connecting diametrically opposite points of the spherical layer. Particles would experience the sum of gravitational forces summing up to zero in the center of Earth. Although the layer would be almost invisible (or completely invisible by argument utilizing the analogy with conducting shell) gravitationally in its interior, $h_{gr} = M_D m/v_0$ would make itself visible in the dynamics of dark particles! This layer could represent magnetic Mother Gaia and EEG would take care of communications to this layer.

   The rotation velocity $v_{rot,M} \simeq 2.1 \times v_{rot,E}$ of Moon around its axis is the first guess for the parameter v_0 identifiable perhaps as rotation velocity of the spherical layer. A better guess is that the ratio $r = v_0/v_{rot,M}$ is the same as for Sun and as assumed above for Earth. This would give for the ratio of cyclotron frequency scales $r = (B_{end}/B_{gal}) \times 2.1$. 66.7 min, which corresponds to $B_{gal} = .63$ nT, would correspond to .1 s. For this choice 1 Hz DNA cyclotron frequency would correspond 11.7 h rather near to 12 h. This encourages the hypothesis that 72 min is the counterpart of .1 s cyclotron time. The cyclotron time of DNA (very weakly dependent on the length of DNA double strand) in B_{gal} (or its minimum value) would be 12 h.

Did animal mitochondrial evolution have a long period of stagnation?

I encountered an interesting popular article (see `http://tinyurl.com/y9akruh`) telling about findings challenging Darwin's evolutionary theory. A technical representation can be found from the original article of Stoeckle and Thaler (see `http://tinyurl.com/ycsdc279`).

The conclusion of the article is that almost all animals, 9 out of 10 animal species on Earth today, including humans, would have emerged about 100,000 200,000 years ago. According to Wikipedia all animals are assumed to have emerged about 650 million years ago from a common ancestor. Cambrian explosion began around 542 million years ago and meant a sudden emergence of complex life forms. What happened looks like a mystery. TGD based explanation involving TGD based new physics is discussed [L64]. According to Wikipedia Homo Sapiens would have emerged 300,000-800,000 years ago.

On basis of Darwin's theory based on survival of the fittest and adaptation to a new environment, one would expect that the species such as ants and humans with large populations distributed around the globe become genetically more diverse over time than the species living in the same environment. The study of so called neutral mutations not relevant for survival and assumed to occur with some constant rate however finds that this is not the case. The study of so called mitochondrial DNA barcodes across 100,000 species showed that the variation of neutral mutations became very small about 100,000-200,000 years ago. One could say that the evolution differentiating between them began (or effectively began) after this time. As if mitochondrial clocks for these species would have been reset to zero at that time as the article states it. This is taken as a support for the conclusion that all animals emerged about the same time as humans.

The proposal of (at least) the writer of popular article is that the life was almost wiped out by a great catastrophe and extraterrestrials could have helped to start the new beginning. This brings in mind Noah's Ark scenario. But can one argue that humans and the other animals emerged at that time: were they only survivors from a catastrophe. One can also argue that the rate of mitochondrial mutations increased dramatically for some reason at that time.

Could one think that great evolutionary leap initiating the differentiation of mitochondrial genomes at that time and that before it the differentiation was very slow for some reason? Why this change would have occurred simultaneously in almost all animals? Something should have happened to the mitochondria and what kind of external evolutionary pressure could have caused it?

1. To me the idea about ETs performing large scale genetic engineering does not sound very convincing. That only a small fraction of animals survived the catastrophe sounds more plausible idea. Was it great flood? One can argue that animals living in water would have survived in this case. Could some cosmic event such as nearby supernova have produced radiation killing most animals? But is mass extinction really necessary? Could some evolutionary pressure without extinction caused the apparent resetting of mitochondrial clock?

2. In TGD based quantum biology the great leaps could be caused by quantum criticality perhaps induced by some evolutionary pressure due to some kind of catastrophe. The value of $h_{eff} = nh_0$ (h_0 is the minimal value of Planck constant) - kind of IQ in very general sense - in some part of mitochondria could have increased and also its value would have fluctuated. Did a new longer length scale relevant to the functioning of mitochondrias emerge? Did the mitochondrial size increase? Here I meet the boundaries of my knowledge about evolutionary biology!

3. Forget for a moment the possibility of mass extinction. Could the rate of mutations, in particular the rate of neutral mutations, have increased as a response to evolutionary pressure? Just the increased ability to change helps to survive. This rate would become high at quantum criticality due to the presence of large quantum fluctuations (variations of h_{eff}). If the mitochondria were far from quantum quantum criticality before the catastrophe, the rate of mutations would have been very slow. Animal kingdom would have lived a period of stagnation. The emerging quantum criticality - forced by a catastrophe but not involving an extinction - could have increased the rate dramatically. This picture would provide formulation for the notion of punctuated equilibria in terms of quantum criticality.

An island at which body size shrinks

I encountered in Facebook an article claiming that the bodies of animals shrink at the island of Flores belonging to Indonesia (see `http://tinyurl.com/ycluutnql`). This news is not Dog's days news (Dog's days news is a direct translation from the finnish synonym for fake news).

Both animals and humans really are claimed to have shrinked in size. The bodies of both hominins (precedessors of humans, humans, ane even elephants) have shrinked at Flores.

1. In 2003, researchers discovered in a mountain cave in the island of Flores fossils of tiny, humanlike individual. It had chimp sized brain and was 90 cm tall. Several villages at the area are inhabited by people with average body height about 1.45 meters.

2. Could the small size of the recent humans at Flores be due to interbreeding between modern humans with Homo Florensiensis (HF) occurred long time ago? The hypothesis could be tested by studying the DNA of HF. Since the estimate age of fossils of HF was 10,000 years, researchers hoped that they could find some DNA to HF. DNA was not found but researchers realized that if HF as interbreeded with humans, this DNA could show itself in DNA of modern humans at Flores. It was found that this DNA can be identified but differs insignificantly from that of modern humans. It was also found that the age of the fossils was about 60,000 years.

3. Therefore it seems that the interbreeding did not cause the reduction in size. The study also showed that at least twice in the ancient history of humans and their relatives arrived as Flores and then grew shorter [I17] (see `http://tinyurl.com/y9th5zne`). This happened also for elephants that arrived to Flores at twice.

This looks really weird! Weirdness in this proportion allows some totally irresponsible speculation.

1. The hierarchy of Planck constants $h_{eff} = nh_0$ ($h = 6h_0$ is a good guess [L26, L60, L21]) assigned with dark matter as phases of ordinary matter and responsible for macroscopic quantum coherence is central in TGD inspired biology. Quantum scales are proportional to or its power (h_{eff}^2 for atoms, h_{eff} for Compton length, and $h_{eff}^{1/2}$ for cyclotron states).

2. The value of gravitational Planck constant h_{gr} ($= h_{eff}$) at the flux tubes mediating gravitational interaction could determine the size scale of the animals. Could one consider a local anomaly in which the value of h_{gr} is reduced and leads to a shrinkage of also body size?

3. h_{gr} is of form $\hbar_{gr} = GM_D m/v_0$, where v_0 a velocity parameter [?, K84] [L57] (see `http://tinyurl.com/y8xhvwt2`, `http://tinyurl.com/yaattlzm`, and `http://tinyurl.com/y8vnyppq`). M_D is a large dark mass of order 10^{-4} times the mass of Earth. Gravitational Compton length $\Lambda_{gr} = h_{gr}/m = GM_D/v_0$ for a particle with mass m. $\Lambda_{gr} = h_{gr}/m$ does not depend on the mass of the particle - this conforms with Equivalence Principle. The estimate of [L57] gives $\Lambda_{gr} = 2\pi GM_D/v_0 = 2.9 \times r_S(E)$, where the Schwartshild radius of Earth is $r_S(E) = 2GM_E = .9$ mm. This gives $\Lambda_{gr} = 2.6$ mm, which corresponds to p-adic length scale $L(k = 187)$. Brain contains neuron blobs with this size scale. The size scale of organism is expected to be some not too large multiple of this scale.

   Could one think that v_0 at Flores is larger than normally and reduces the value of Λ_{gr} so that the size for the gravitational part of the magnetic body of any organism shrinks, and that this gradually leads to a reduction of the size of the biological body. Second possibility is that the value of dark mass M_D is at Flores smaller than elsewhere: one would have a dark analogy of ordinary local gravitational anomaly. The reduction of h_{gr} should be rather large so that the first option looks more plausible.

12.2.2 Why metabolism and what happens in bio-catalysis?

TGD view about dark matter gives also a strong grasp to metabolism and bio-catalysis - the key elements of biology.

Why metabolic energy is needed?

The simplest and at the same time most difficult question that innocent student can make about biology class is simple: "Why we must eat?". Or using more physics oriented language: "Why we must get metabolic energy?". The answer of the teacher might be that we do not eat to get energy but to get order. The stuff that we eat contains ordered energy: we eat order. But order in standard physics is lack of entropy, lack of disorder. Student could get nosy and argue that excretion produces the same outcome as eating but is not enough to survive.

We could go to a deeper level and ask why metabolic energy is needed in biochemistry. Suppose we do this in TGD Universe with dark matter identified as phases characterized by $h_{eff}/h = n$.

1. Why metabolic energy would be needed? Intuitive answer is that evolution requires it and that evolution corresponds to the increase of $n = h_{eff}/h$. To see the answer to the question, notice that the energy scale for the bound states of an atom is proportional to $1/h^2$ and for dark atom to $1/h_{eff}^2 \propto n^2$ (do not confuse this n with the integer n labelling the states of hydrogen atom!).

2. Dark atoms have smaller binding energies and their creation by a phase transition increasing the value of n demands a feed of energy - metabolic energy! If the metabolic energy feed stops, n is gradually reduced. System gets tired, loses consciousness, and eventually dies.

   What is remarkable that the scale of atomic binding energies decreases with n only in dimension $D = 3$. In other dimensions it increases and in $D = 4$ one cannot even speak of bound states! This can be easily found by a study of Schrödinger equation for the analog of hydrogen atom in various dimensions. Life based on metabolism seems to make sense only in spatial dimension $D = 3$. Note however that there are also other quantum states than atomic states with different dependence of energy on h_{eff}.

Conditions on bio-catalysis

Bio-catalysis is key mechanism of biology and its extreme efficacy remains to be understood. Enzymes are proteins and ribozymes RNA sequences acting as biocatalysts.

What catalysis demands?

1. Catalyst and reactants must find each other. How this could happen is very difficult to understand in standard biochemistry in which living matter is seen as soup of biomolecules. I have already already considered the mechanisms making it possible for the reactants to find each other. For instance, in the translation of mRNA to protein tRNA molecules must find their way to mRNA at ribosome. The proposal is that reconnection allowing U-shaped magnetic flux tubes to reconnect to a pair of flux tube connecting mRNA and tRNA molecule and reduction of the value of $h_{eff} = n \times h$ inducing reduction of the length of magnetic flux tube takes care of this step. This applies also to DNA transcription and DNA replication and bio-chemical reactions in general.

2. Catalyst must provide energy for the reactants (their number is typically two) to overcome the potential wall making the reaction rate very slow for energies around thermal energy. The TGD based model for the hydrino atom having larger binding energy than hydrogen atom claimed by Randell Mills [D17] suggests a solution [L26]. Some hydrogen atom in catalyst goes from (dark) hydrogen atom state to hydrino state (state with smaller h_{eff}/h and liberates the excess binding energy kicking the either reactant over the potential wall so that reaction can process. After the reaction the catalyst returns to the normal state and absorbs the binding energy.

3. In the reaction volume catalyst and reactants must be guided to correct places. The simplest model of catalysis relies on lock-and-key mechanism. The generalized Chladni mechanism forcing the reactants to a two-dimensional closed nodal surface is a natural candidate to consider. There are also additional conditions. For instance, the reactants must have correct orientation. For instance, the reactants must have correct orientation and this could be forced by the interaction with the em field of ME involved with Chladni mechanism.

4. One must have also a coherence of chemical reactions meaning that the reaction can occur in a large volume - say in different cell interiors - simultaneously. Here MB would induce the coherence by using MEs. Chladni mechanism might explain this if there is there is interference of forces caused by periodic standing waves themselves represented as pairs of MEs.

Phase transition reducing the value of $h_{eff}/h = n$ as a basic step in bio-catalysis

Hydrogen atom allows also large $h_{eff}/h = n$ variants with $n > 6$ with the scale of energy spectrum behaving as $(6/n)^2$ if the $n = 4$ holds true for visible matter. The reduction of n as the flux tube contracts would reduce n and liberate binding energy, which could be used to promote the catalysis.

The notion of high energy phosphate bond is somewhat mysterious concept and manifests as the ability provide energy in ATP to ADP transition. There are claims that there is no such bond. I have spent considerable amount of time to ponder this problem. Could phosphate contain (dark) hydrogen atom able to go to the a state with a smaller value of h_{eff}/h¡ and liberate the excess binding energy? Could the phosphorylation of acceptor molecule transfer this dark atom associated with the phosphate of ATP to the acceptor molecule? Could the mysterious high energy phosphate bond correspond to the dark atom state. Metabolic energy would be needed to transform ADP to ATP and would generate dark atom.

Could solar light kick atoms into dark states and in this manner store metabolic energy? Could nutrients carry these dark atoms? Could this energy be liberated as the dark atoms return to ordinary states and be used to drive protons against potential gradient through ATP synthase analogous to a turbine of a power plant transforming ADP to ATP and reproducing the dark atom and thus the "high energy phosphate bond" in ATP? Can one see metabolism as transfer of dark atoms? Could possible negentropic entanglement disappear and emerge again after ADP→ ATP.

Here it is essential that the energies of the hydrogen atom depend on $\hbar_{eff} = n \times h$ in as $\hbar_{eff}^m$, $m = -2 < 0$. Hydrogen atoms in dimension D have Coulomb potential behaving as $1/r^{D-2}$ from Gauss law and the Schrödinger equation predicts for $D \neq 4$ that the energies satisfy $E_n \propto (h_{eff}/h)^m$, $m = 2+4/(D-4)$. For $D = 4$ the formula breaks since in this case the dependence on $\hbar$ is not given by power law. m is negative only for $D = 3$ and one has $m = -2$. There $D = 3$ would be unique dimension in allowing the hydrino-like states making possible bio-catalysis and life in the proposed scenario.

It is also essential that the flux tubes are radial flux tubes in the Coulomb field of charged particle. This makes sense in many-sheeted space-time: electrons would be associated with a pair formed by flux tube and 3-D atom so that only part of electric flux would interact with the electron touching both space-time sheets. This would give the analog of Schrödinger equation in Coulomb potential restricted to the interior of the flux tube. The dimensional analysis for the 1-D Schrödinger equation with Coulomb potential would give also in this case $1/n^2$ dependence. Same applies to states localized to 2-D sheets with charged ion in the center. This kind of states bring in mind Rydberg states of ordinary atom with large value of n.

The condition that the dark binding energy is above the thermal energy gives a condition on the value of $h_{eff}/h = n$ as $n \leq 32$. The size scale of the dark largest allowed dark atom would be about 100 nm, 10 times the thickness of the cell membrane.

How molecules in cells "find" one another and organize into structures?

The title of the popular article "How molecules in cells 'find' one another and organize into structures?" (see `http://tinyurl.com/ydbznknn`) expresses an old problem of biology. Now the group led by Amy S. Gladfelter has made experimental progress in this problem. The work has been published in Science [L67] (see `http://tinyurl.com/ybwyugho`).

It is reported that RNA molecules recognize each other to condense into the same droplet due to the specific 3D shapes that the molecules assume. Molecules with complementary base pairing can find each other and only similar RNAs condense on same droplet. This brings in mind DNA replication, transcription and translation. Furthermore, the same proteins that form liquid droplets in healthy cells, solidify in diseases like neurodegenerative disorders.

Some kind of phase transition is involved with the process but what brings the molecules together remains still a mystery. The TGD based solution of this mystery is one of the first applications of the notion of many-sheeted space-time in biology, and relies on the notion of magnetic flux tubes connecting molecules to form networks.

Consider first the TGD based model about condensed and living matter. As a matter fact, the core of this model applies in all scales. What is new is there are not only particles but also bonds connecting them. In TGD they are flux tubes which can carry dark particles with nonstandard value $h_{eff}/h = n$ of Planck constant. In ER-EPR approach in fashion they would be wormholes connecting distance space-time regions. In this case the problem is instability: wormholes pinch and split. In TGD monopole magnetic flux takes care of the stability topologically.

The flux tube networks occur in all scales but especially important are biological length scales.

1. In chemistry the flux tubes are associated with valence bonds and hydrogen bonds [L44] (see `http://tinyurl.com/ycg94xpl`). In biology genetic code would be realized as dark nuclei formed by sequences of dark protons at magnetic flux tubes. Also RNA, amino-acids, and even tRNA could have dark counterparts of this kind [L20] (see `http://tinyurl.com/jgfjlbe`). Dark variants of biomolecules would serve as templates for their ordinary variants also at the level of dynamics. Biochemistry would be shadow dynamics dictated to high degree by the dark matter at flux tubes.

2. Dark valence bonds can have quite long length and the outcome is entangled tensor net [L39](see `http://tinyurl.com/y9kwnqfa`). These neuronal nets serve as correlates for cognitive mental images in brain (see `http://tinyurl.com/yczv2o5b`) emotional mental images in body [L62] (see `http://tinyurl.com/ydhxen4g`). Dark photons propagating along flux tubes (more precisely topological light rays parallel to them) would be the fundamental communication mechanism [K17] (see `http://tinyurl.com/ydx9dq6x`). Transmitters and nerve pulses would only change the connectedness properties of these nets.

The topological dynamics of flux tubes has two basic mechanisms (I have discussed this dynamics from the point of view of AI [L36] (see `http://tinyurl.com/y75246rk`).

1. Reconnection of flux tubes serves is the first basic mechanism in the dynamics of flux tube networks and would give among other things rise to neural nets. The connection between neurons would correspond basically to flux tube pair which can split by reconnection. Also two flux tube pairs can reconnect forming Y shaped structures. Flux tube pairs could be quite generally associated with long dark hydrogen bonds scaled up by $h_{eff}/h = n$ from their ordinary lengths. Flux tube pairs would carry besides dark protons also supra phases formed by the lone electron pairs associated quite generally with hydrogen bonding atoms. Also dark ions could appear at flux tubes.

   Biomolecules would have flux loops continually scanning the environment and reconnecting if they meet another flux loop. This however requires that magnetic field strengths are same at the two loops so that a resonance is achieved at level of dark photon communications. This makes possible recognition by cyclotron frequency spectrum serving as signature of the magnetic body of the molecule.

   Water memory [K21] (see `http://tinyurl.com/ycqy837a`) would rely on this recognition mechanism based on cyclotron frequencies and also immune system would use it at basic level (here one cannot avoid saying something about homeopathy although I know that this spoils the day of the skeptic: the same mechanism would be involved also with it). For instance, dark DNA strand accompanying ordinary DNA and dark RNA molecules find each other by this mechanism (see `http://tinyurl.com/yalny39x`). Same applies to other reactions such as replication and translation .

2. Shortening of the flux tubes h_{eff}/h reducing phase transition is second basic mechanism explaining how biomolecules can find each other in dense molecular soup. It is essential that the magnetic fields at flux tubes are nearly the same for the reconnection to form. A more refined model for the shortening involves two steps: reconnection of flux tubes leading to a formation of flux tube pair between molecules and shortening by h_{eff}/h reducing phase transition.

Also ordinary condensed matter phase transitions involve change of the topology of flux tube networks and the model for it allows to put the findings described in the article in TGD perspective.

1. Quite recently I wrote an article [L69] (see `http://tinyurl.com/ydhknc2c`) about a solution of two old problems of hydrothermodynamics: the behavior of liquid-gas system in the critical region not consistent with the predictions of statistical mechanics (known already at times of Maxwell!) and the behavior of water above freezing point and in freezing. Dark flux tubes carrying dark protons and possibly electronic Cooper pairs made from so called lone electron pairs characterizing atoms forming hydrogen bonds.

2. The phase transition from gas to liquid occurs when the number of flux tubes per molecule is high enough. At criticality both phases are in mechanical equilibrium - same pressure. Most interestingly, in solidification the large h_{eff} flux tubes transform to ordinary ones and liberate energy: this explains anomalously high latent heats of water and ammonia. The loss of large h_{eff} flux tubes however reduces "IQ" of the system.

The phase transitions changing the connectedness of the flux tube networks are fundamental in TGD inspired quantum biology.

1. Sol-gel transition would correspond to this kind of biological phase transitions. Protein folding [K7] (see `http://tinyurl.com/y9lqmtea`) - kind of freezing of protein making it biologically inactive - and unfolding would be second basic example of this transition. The freezing would involve formation of flux tube bonds between points of linear protein and assignable to hydrogen bonds. External perturbations induce melting of the proteins and they become biologically active as the value of $h_{eff}/h = n$ characterizing their maximal possible entanglement negentropy content (molecular IQ) increases. External perturbation feeds in energy acting as metabolic energy. I have called this period molecular summer.

2. Solidification of proteins reducing is reported to be associated with diseases such neurodegenerative disorders. In TGD picture this would reduce the molecular IQ since the ability of system to generate negentropy would be reduced when $h_{eff}$for the flux tubes decreases to its ordinary value. What brings molecules together is not understood and TGD provides the explanation as h_{eff} reducing phase transition for flux tube pairs.

The evidence for the hierarchy of Planck constants $h_{eff}/h = n$ labelling dark matter as phases with non-standard value of Planck constant [?] is accumulating. The latest piece of evidence for the hierarchy of Planck constants comes from the well-known mystery (not to me until now!) related to rare Earth metals. Some valence electrons of these atoms mystically "disappear" when the atom is heated. This transition is known as Lifshitz transition. The popular article "*Where did those electrons go? Decades-old mystery solved*" (see`http://tinyurl.com/ychzjg8d`) claims that the mystery of disappearing valence electrons is finally resolved. The popular article is inspired by the article "*Lifshitz transition from valence fluctuations in YbAl3*" by Chatterjee *et al* published in Nature Communications [L47] (see `http://tinyurl.com/ybejzq87`).

The finding [D14] (see `http://tinyurl.com/ybntawq4`) about misbehaving Ruthenium atoms supports the view that covalent bonds involve dark valence electrons. Pairs of Ru atoms were expected to transform to Ru dimers in thermodynamical equilibrium but this did not happen. This suggests that valence electrons associated with the valence bond of Ru dimers are dark in TGD sense and the valence bonded Ru dimer has a higher energy than a pair of free Ru atoms. The alternative option is that darkness makes the decay of Ru pairs to Ru dimers with smaller energy very slow.

Are mysteriously disappearing electrons dark in TGD sense?

The mysterious disappearance of valence electrons brings in mind dark atoms with Planck constant $h_{eff} = n \times h$. Dark matter corresponds in TGD Universe to a hierarchy with levels labelled by the value of h_{eff}. One prediction is that the binding energy of dark atom is proportional to $1/h_{eff}^2$ and thus behaves like $1/n^2$ and decreases with n.

$n = 1$ is the first guess for ordinary atoms but just a guess. The claim of Randell Mills is that hydrogen has exotic ground states with larger binding energy. A closer examination suggests $n = n_0 = 6$ for ordinary states of atoms. The exotic states would have $n < 6$ and therefore higher binding energy scale [L26, L45] (see `http://tinyurl.com/goruuzm` and `http://tinyurl.com/y7sc98lz`).

This leads to a model of biocatalysis in which reacting molecules contain dark hydrogen atoms with non-standard value of n larger than usual so that their binding energy is lower. When dark atom or electron becomes ordinary binding energy is liberated and can kick molecules over the potential wall otherwise preventing the reaction to occur. After that the energy is returned and the atom becomes dark again. Dark atoms would be catalytic switches. Metabolic energy feed would take care of creating the dark states. In fact, $h_{eff}/h = n$ serves as a kind of intelligence quotient for a system in TGD inspired theory of consciousness.

Could the heating of the rare earth atoms transform some valence electrons to dark electrons with $h_{eff}/h = n$ larger than for ordinary atom? The natural guess is that thermal energy kicks the valence electron to a dark orbital with a smaller binding energy? The prediction is that there should be critical temperatures behaving like $T_{cr} = T_0(1-n_0^2/n^2)$. Also transitions between different dark states are possible. These transitions might be also induced by irradiating the atom with photons with the transition energy between different dark states having same quantum numbers.

This picture leads to a new formulation of valence bond theory. The lengths of molecular bonds vary in rather narrow range whereas Schrödinger equation suggests that the bond lengths r should scale as $r \propto m^2/Z^2$ for $n = 1$ (m labels the rows of the periodic table). Closed shell electrons screen Z to $Z_{eff} = n_V$, n_V the number of valence electrons so that the formula $e = n^2m^2/Z_{eff}^2$ is a more natural starting point, and conforms with the basic idea about periodic system. This leads to a model allowing to estimate the value of n for given bond allowing also qualitative picture about electro-negativities of valence bonds. Also a comparison with bio-chemistry becomes possible. Hydrogen bond can be understood in terms of de-localization of proton.

About possible implications

The proposed explanation of the disappearing valence electrons allows to sharpen the hypothesis for dark ions. Actually dark atoms with some dark valence electrons would be in question.

1. ORMEs as one manner to end up with $h_{eff}/h = n$ hypothesis

I ended up to the discovery of dark matter hierarchy and eventually to adelic physics [L50], where $h_{eff}/h = n$ has number theoretic interpretation along several roads starting from anomalous findings. One of these roads began from the claim about the existence of strange form of matter by David Hudson. Hudson associated with these strange materials several names: White Gold, monoatomic elements, and ORMEs (orbitally re-arranged metallic elements). Any colleague without suicidical tendencies would of course refuse to touch anything like White Gold even with a 10 meter long pole but I had nothing to lose anymore.

My question was how to explain these elements if they are actually real [K23, K45]. If all valence electrons of this kind of element are dark these element have effectively full electron shells as far as ordinary electrons are considered and behave like noble gases with charge in short scales and do not form molecules. Therefore "monoatomic element" is justified. Of course, only the electrons in the outermost shell could be dark and in this case the element would behave chemically and also look like an atom with smaller atomic number Z. So called Rydberg atoms for which valence electrons are believed to reside at very large orbitals could be actually dark atoms in the proposed sense.

Obviously also ORME is an appropriate term since some valence electrons have re-arranged orbitally. White Gold would be Gold but with dark valence electron. The electron configuration of Gold is $[Xe]4f^{14}5d^{10}6s^1$. There is single unpaired electron with principal quantum number $m = 6$ and this would be dark for White Gold and chemically like Platinum (Pt), which indeed has white color.

2. Biologically important ions as analogs of ORMEs?

In TGD inspired biology the biologically important atoms H^+, Li^+, Na^+, K^+, Ca^{++}, Mg^{++} are assumed to be dark in the proposed sense. But I have not specified darkness in precise sense. Could these ions have dark valence electrons with scaled up Compton length and forming macroscopic quantum phases. For instance, Cooper pairs could become possible and make possible high Tc superconductivity with members of Cooper pair at parallel flux tubes. The earlier proposal that dark hydrogen atoms make possible biocatalysis becomes more detailed: at higher evolutionary levels also the heavier dark atoms behaving like noble gases would become important in bio-catalysis. Interestingly, Rydberg atoms have been proposed to be important for biology and they could be actually dark atoms [L16].

To sum up, if TGD view is correct, an entire spectroscopy of dark atoms and partially dark molecules is waiting to be discovered and irradiation by light with energies corresponding to excitation energies of dark states could be the manner to generate dark atomic matter. Huge progress in quantum biology could also take place. But are colleagues mature enough to check whether the TGD view is correct?

Misbehaving Ruthenium atoms

In Facebook I received a link to a highly interesting article (see `http://tinyurl.com/ybntawq4`) with title "*Breakthrough could launch organic electronics beyond cell phone screens*" tailored to catch the attention of techno-oriented leader. My attention was however caught for different reasons. The proposed technology would rely on the observation that Ruthenium atoms do not behave as they are expected to behave.

Ru atoms appear as dimers of two Ru atoms in the system considered. Free Ru atoms with one valence electron are however needed: they would become ions by giving up their valence electrons, and these electrons would serve as current carriers making the organic material in question semi-conductor. Irradiation by UV light was found to split Ruthenium dimers to single Ru atoms. If the total energy of Ru dimer is smaller than that for two Ru atoms, thermodynamics predicts that the Ru atoms recombine to dimers after the irradiation ceases. The did not however happen!

Can one understand the mystery in TGD framework?

1. Ru atoms have one outer s-electron at 5:th shell. One would expect that Ru dimer has valence bond with shared 5s electrons. I recently learned about mysteriously disappearing valence electrons of rare Earth metals caused by heating [L47]) (see `http://tinyurl.com/y7cxs8uz`). This gives strong support for the idea that valence electrons of free atoms can become dark in TGD sense: that is their Planck constant increases and the orbitals become large. The analogy with Rydberg atoms is obvious and it could be that Rydberg atoms in some case have dark valence electrons. Since electron's binding energy scale scales like $1/h_{eff}^2$ $h_{eff}/h = n \times h$, the creation of these states requires energy and therefore heating is required. Also irradiation by photons with energy equal to energy difference between ordinary and dark states should give rise to the same phenomenon. This would provide a way to create dark electrons and a new technology.

2. This also inspired the proposal that valence bond (thought to be understood in chemistry with inspiration coming from the reductionistic dogma) involves flux tube pair and $h_{eff}/h = n$ which is larger than for ordinary quantum theory. This provides new very concrete support for the view that thetransitions from atomic physics to chemistry and from chemistry to organic chemistry could involve new physics provided by TGD [L51] (see `http://tinyurl.com/yaq3459e`).

   The step from atomic physics to chemistry with valence bond would involve new physics: the delocalization of valence electrons to flux tubes due to the increase of h_{eff}! Valence electrons would be dark matter in TGD sense! The step from chemistry to organic chemistry would involve delocalization of proton as dark proton by similar mechanism and give rise to hydrogen bond and also many other new phenomena.

3. The increase of h_{eff} would reduce the binding energy from the expected. This would be the case for so called (and somewhat mysterious) high energy phosphate bond. This picture conforms with the fact that biological energy storage indeed relies on valence bonds.

   If this vision is correct, the breaking of valence bond would split the flux tube pair between two Ru atoms by reconnection to flux loops associated with Ru atom. The resulting pair of free Ru atoms would have lower energy than Ru dimer and would be favored by thermodynamics. The paradox would disappear.

A couple of critical questions are in order.

1. Why irradiation would be needed at all? Irradiation would kick the dimer system over a potential wall separating it from a state two free Ru atoms. Also the magnetic energy of the flux tube would contribute to the energy of dimer and make it higher than that of free state.

2. Why Ru dimers would not decay spontanously to pairs of free Ru atoms? This is the case if the energy needed to overcome the potential wall is higher than thermal energy at temperatures considered. One could also argue that electronic states with different values of $h_{eff}/h = n$ are not in thermal equilibrium: one has far-from-equilibrium thermodynamical state. These electrons would represent dark matter in TGD sense and interact rather weakly with ordinary matter so that it would take time for thermal equilibrium to establish itself.

   TGD indeed leads to the proposal that the formation of states regarded as far-from-thermal equilibrium states in standard physics approach means formation of flux tubes networks with $h_{eff}/h = n$ larger than for the original state [L46] (see `http://tinyurl.com/yamdwop9` and `http://tinyurl.com/y8f95b5z`). If this interpretation is correct, then one can also consider the possibility that the energy of the free state is higher than that of the dimer as assumed by the experimenters.

12.2.3 Does valence bond theory relate to the hierarchy of Planck constants?

The idea that valence bonds, or at least some of them, correspond to non-standard value of $h_{eff}/h = n$ [L46] is very attractive. It could allow to understand what chemical bonds really are and allow a detailed view about how reductionism fails in the sequence of transitions from atomic physics to molecular physics to chemistry to biochemistry.

1. The standard value of n, call it n_{min} need not correspond to $n_{min} = 1$ and the findings of Randell Mills [D17] [L26] suggesting that hydrogen atom and possibly also other atoms can have binding energies coming as k^2 multiples of ordinary ones with $k = 2, 3, 6$, suggests that $n_{min} = 6$ could correspond to the standard value of h_{eff} for atoms. $n > n_{min}$ would mean reduced binding energy and this would mean the possibility of high energy valence bonds.

2. The binding energy of atom would scale as $1/n^2$ so that for non-standard values of $n > n_{min}$ would correspond to smaller binding energy scale. The finding that heating of rare-earth atoms leads to a disappearance of some valence electrons [L47] suggests that the value of n for some valence electrons increases from n_{min} in these situation. The same effect might be achieved by irradiation at suitable photon energies corresponding to energy difference between ordinary state and dark state of electrons. An entire spectroscopy of atoms with dark valence electrons would be waiting to be discovered.

3. $n > n_{min}$ would explain why valence bonds are carriers of metabolic energy liberated in catabolic part of metabolism. The temporary reduction of of n would induce a temporary localication by shortening of flux tubes and in turn make possible bio-catalysis by kicking the reactants over the potential wall making the reaction slow. The shortening of long flux tube bonds between reacts as the value of n is reduced could explain why bio-molecules are able to find each other in the molecular crowd.

4. The Bohr radii of valence electrons of atoms scale as $a_B \propto m^2/Z_{eff}^2$, where m (usually denoted by n) is the principal quantum number determining the value of energy in the model based on Schrödinger equation. Z_{eff} is in good approximation equal to the unscreened nuclear charge $Z_{eff} = n_V$ equal to the number of valence electrons. If the superposition of atomic orbitals restricted to valence bonds is the essence in the formation of molecules, one an argue that the lengths of bonds and radii of molecules should decrease rapidly with Z_{eff}. However, the empirical fact is that the bond lengths vary in a rather narrow range, roughly by factor 2!

   The solution of the problem looks rather unique.

1. The value of n assignable to the valence bond is scaled so that nm/Z_{eff} is near to unity so that the Bohr radius is near to that for hydrogen atom. Z_{eff} is naturally the charge

unscreened by the closed electron shells and equal to the number $Z_{eff} = n_V$ of valence electrons. This conforms with the periodicity of the periodic table. Since the value of n is same for both bonded atoms, the value of Bohr radii differ which implies that electronic charge is shifted towards the atom with larger n_V and electro-negativities of atoms parameterizing this behavior are different for the atoms of the bond. This conforms qualitatively with the valence bond theory.

For $n > n_{min}$ one would have $a_B \propto (n^2/n_{min})^2 m^2/Z_{eff}^2$, and if $nm/(n_{min}Z_{eff})$ is constant in reasonable approximation, the estimate for bond length does not depend much on Z. Could the weak variation of bond lengths be a direct indication that the reduction of molecular physics to atomic physics fails? Also the size of atoms in lattice about $2a_B(H)$ (one Angström) depends only weakly on Z_{eff}: could the constancy of $nm/(n_{min}Z_{eff})$ be true in reasonable approximation also for lattice bonds?

2. The predicted lengths of valence bonds should be realistic: this forces $n > n_H$ and $n \propto Z_{eff}$ is a rough guess. One should also understand the values of electro-negativities $\chi(X)$ allowing quantitative understanding about the distribution of charge along the bond. The bond lengths assignable to the bonded atoms are in general different and the one one with shorter bond length for electrons is expected to be more electronegative since the electrons for it are less de-localized.

Transition from atomic physics to molecular physics and chemistry

The transitions from atomic physics to chemistry and from chemistry to organic and bio-chemistries are poorly understood and the reductionistic dogma remain a mere belief. Could the valence bonds associated with magnetic flux tubes in TGD Universe and correspond to a non-standard value of n scaling up the value of Bohr radius by n^2? Could valence electron pairs form analogs of Cooper pairs with the length of bond defining the size scale of the Cooper pair. This could happen in aromatic cycles playing crucial role in molecular biology. Could various high energy valence bonds making possible the storage of metabolic energy correspond to valence bonds with $n > n_{min}$ possessing therefore smaller binding energy.

One has several options.

1. U-shaped flux tube are along single space-time sheet. U-shape would minimize magnetic energy.

2. One could have closed flux tube going along first space-time sheet A, going to second sheet B through extremely short wormhole contact of size of order CP_2 radius, and returning back along B and back to A through wormhole contact. One would have a pair of flux tubes with opposite values of magnetic fields on top of each other in CP_2 direction. The net magnetic field experienced by a charged particle at QFT limit would vanish: I have called this structure wormhole magnetic field. For wormhole magnetic field the average magnetic field determining the magnetic field at QFT-GRT limit of TGD would vanish in good approximation.

3. One could have single flux tube at sheet A going to B through wormhole contact and returning back along different route along B and returning back through wormhole contact. For a network of flux tubes one could have closed magnetic paths. In this case, charged particles would experience the magnetic field of only single flux tube. This option looks very attractive and one could realize Cooper pairs having members at different space-time sheets. The flux could be also monopole flux possible in TGD Universe thanks to the homology of CP_2.

First and third option look natural in the chemistry of valence bond. The prediction would be that valence electrons are de-localized along these bonds. If the wave function behaves like hydrogen atom wave function it decays exponentially with distance from each atom and a superposition of orbitals would be in question. The Bohr radius would be proportional to n^2 implying longer de-localization scale.

For hydrogen bonds proton would be de-localized as dark proton. This could represent transition from inorganic chemistry to organic chemistry. In TGD inspired quantum biology also other ions can be de-localized at magnetic flux tubes and these de-localizations represent a further steps away from atomic physics.

In biology n would serve as a kind of IQ for a system: understanding why this should be the case requires adelic physics serving as fusion of ordinary physics and physics of cognition represented by p-adic physics [L49] [L50]. The larger the value of n, the larger the maximal value of p-adic counterpart of entanglement negentropy, which is an analog of Shannon entropy but with algebraic number valued probability P appearing in $log(P)$ replaced by its p-adic norm $|P|_p$ for a suitable algebraic extension of rationals. This entropy can be negative and has in this case interpretation as information. The sum of real and p-adic entropies tends to be negative and has interpretation as a measure for conscious information.

Valence bond theory very briefly

How to test this hypothesis about valence bonds? Electronegativity and oxidation/reduction serve as the basic notions in valence bond theory (see `http://tinyurl.com/y8wyd9zm`). Valence rule tells which bonds are favored. Bond lengths and electro-negativities are basic parameters characterizing bonds. Can one interpret these notions in terms of $n = h_{eff}/h$ hierarchy of dark matters?

1. For atom, call it A, bonded to atoms B, C,.. the sum of valences of B, C,.. is the negative of the valence of A. For H-Cl and Na-Cl the valences are +1 and -1. C as valence 4 (or equivalently -4) and CH_4 represents example of this compensation. For O_2= O=O one as double valence bond.

2. Bond length is the first key parameter allowing to get idea about valence bond. The table of Wikipedia article about the notion of bond length gives the bond lengths of C with other elements (see `http://tinyurl.com/ya4md73c`). Interestingly, C-H, C-C, C-O, C-N, C-S, C-Se bond lengths vary, which might have interpretation in terms of varying value of n: all bonds are important in biology. An alternative explanation for the variation would be that there are also other atoms involved.

   The range of variation is [106, 112] pm for C-H; [120, 154] pm for C-C (the upper limit is achieved for diamond but even longer bond lengths are known), [147, 270] pm for C-N , [143, 215] pm for C-O (note that bond length for C-O-H is thus longer than for C-H), and [181, 255] pm for C-S.

   Average bond lengths tend to decrease along the row of the periodic table and increase along column. The C-X bonds in hydrocarbons (alkenes, alkynes) are shorter than in organic polymers in general, which supports the view that they have as organic but non-living materia lower value of n than organic compounds in living matter. The bond lengths for C-metal bonds are rather long, for instance for C-Mg bond length is 207 pm, roughly twice C-H bond length.

3. Electronegativity χ is second key parameter and allows a quantitative description of valence bonds. The rule is that the electrons of an atom with smaller electronegativity χ , call it A, tend to be nearer to those of the atom B with higher value of χ: one says that B oxidizes A and B is reduced. Both oxidation and reduction occur always and one talks about redox reactions, which are fundamental in biology. The term oxidation follows from the fact that oxygen O_2 is the best known oxidant.

   The values of electronegativity for various elements are listed in Wikipedia article (see `http://tinyurl.com/pbh6r6c`) and give a rough idea about what happens for the valence electrons in various bonds. The reduction to two-atom level is only an approximation since the presence of other atoms modifies χ. For instance, the electro-negativities of C for C=O and C-(O-H) are different.

   For instance, one has $\chi(X) \in \{2.20, .98, .93, 1.00\}$ for $X \in \{H, Li, Na, Ca\}$ with $(m, Z) \in \{(1, 1), (2, 7), (3, 11), (4, 20)\}$. Clearly, one has $\chi(H) \sim 2\chi(X)$.

   A naïve expectation is that the atom with the smaller value of n/Z is more electronegative (note that valence rule must be satisfied). Indeed, electronegativity increases along the row of the periodic table. Electronegativity decreases slowly along the column of periodic table except for the metals in the columns containing Cr, Mn, Fe, Co, Ni, Cu, Zn at top row. Understanding the explicit dependence between $\chi(X)$ and $a_B(x)$ and other parameters involved would require a more detailed model.

Deducing an estimate for the value of $n = h_{eff}/h$ from bond lengths

Valence bond lengths provide information allowing to estimate the value of $n = h_{eff}/h$.

1. The expectation is that the bond length for bond A-B scales as the minimum of Bohr radius for the two atoms that is minimum value of $a_B \propto n^2m^2/Z^2$ for atoms A and B. Here one has $n = h_{eff}/h$, m (usually n) denotes the principal quantum number of valence electron, and Z the charge of the atomic nucleus. The atom with smaller value of m/Z should dictate the bond length.

2. If bond length assumed to be of order Bohr radius as function of (Z_{eff}, m), its reduction as function of m/Z_{eff} is quite too slow to be consistent with m^2/Z_{eff}^2 behavior expected for ordinary Planck constant (see the table of `http://tinyurl.com/pbh6r6c`). The formula $a_B \propto n^2m^2/Z^2_{eff^2}$ and the increase of n as function of Z_{eff} compensating the reduction of a_B due to the increase of Z_{eff} for valence bonds is suggestive.

   The first guess is that the formula $a_B(nZ_{eff}/m) = a_H$ holds true apart from factor of order 2. This would explain why valence bond lengths vary in so narrow length scale range. This fact could be even seen as argument against the reduction of chemistry to atomic physics.

The model is based on the following arguments.

1. The value of n is same for both atoms at the ends of the bond. Since the Bohr radius of atom with smaller value of nm/Z_{eff} gives rise to a smaller de-localization length of orbitals, the value of n for heavier atom, call it X, determines the length of flux tube which should be of order $2 \times a(X)$. Since the Bohr radius of the atoms with larger value of nm/Z_{eff} is longer, the electrons of this atom are more de-localized and tend to be nearer to atom with the smaller value of nm/Z_{eff}. The higher the value of Z with same value of m for both atoms the higher the electronegativity. This conforms with empirical facts.

2. The electronegativity of H is roughly twice the electronegativity of the alkali-atoms in the above example. The naïve application of the above argument this would suggest that nm/Z_{eff} for alkali atoms must be larger than n so that de-localization of electron of alkali atom would make hydrogen atom more electronegative. This of course cannot be the case. The solution of the problem is that one cannot apply the rule without taking into account valence rule. For C, N, O, F and S, Cl the electro-negativities are higher than for H. Note that one has $\chi(P) = 2.19 \sim \chi(H) = 2.20$. Interestingly, P occurs with valence 5 in phosphate.

3. $n(H) = 6$ suggested by the findings of Mills [D17] [L26] and will be assumed.

With these assumptions, one can consider two options fixing the value of $n(X)$ using as a guideline empirical data about bond lengths telling that they vary in rather narrow rang $[2, 6]a_H$.

1. For Option I one would have $a(X) = a_B(H)$ implying that all Bohr radii and bond lengths are same and equal to those for hydrogen. Bond length would be in good approximation twice the hydrogen atom Bohr radius: $r = 2a_H$. This condition is satisfied approximately for quite a number of bond lengths. The radii however vary roughly in the range $[1, 3] \times 2a_H$.

   Option I would give

$$n^2(X) = (\frac{Z_{eff}}{m})^2 n^2(H) \ .$$

   For given row characterized by the value of m one would have

$$n(Z, m) = \frac{Z_{eff}n_H}{m} = \frac{6Z_{eff}}{m} \ .$$

2. $n_H = 6$ proportionality for n allows besides $n = Z_{eff}n_H/m$ also more general option: call it Option II. One can have

$$n(X, \frac{k}{l}) = \frac{l}{k} \times n(Z_{eff}, m) = \frac{l}{k} \times \frac{6Z_{eff}}{m} \ ,$$

where $k \in \{2, 3, 6\}$ is non-trivial divisor of $n_H = 6$ besides. This scales the Bohr radius $a(mn/Z_{eff}) = a_H$ to

$$a(mn/Z_{eff}) = (l/k)^2 a_H \ .$$

For instance, $l/k = 3/2$ would give Bohr radius $a(X) = 9a_H/4$ somewhat above $2a_H$. $l/k = 4/3$ would give Bohr radius $16a_H/9$ and $l/k = 5/3$ would give Bohr radius $a(X) = 25a_H/9$ slightly below $2a_H$. The largest bond lengths are about $6a_H$. These two mechanisms could explain the variation of the bond length. This option would explain the bond lengths which $1 - 3$ times the minimal bond length $r = 2a_H$.

3. The value of the Bohr radius is not affected much if $n(Z_{eff}, m)$ is replaced with the nearest integer. This because for large enough n one the relative change $\Delta r/r = \Delta a_B/a_B$ satisfies $\Delta r/r \simeq 2\Delta n/n = (2m/Z_{eff}n_H)\Delta n = (m/3Z_{eff})\Delta n$. This allows fine tuning of the bond length for both options.

Consider now different rows of the periodic table for Option I. The lengths for Option II can be deduced from this option by scaling by $(k/l)^2$, $l = 2, 3, 6$.

1. $m = 2$: For $X \in \{Li, Be, B, C, N, O, F\}$ with $Z_{eff} \in \{1, ..., 7\}$ and $m = 2$ one $n(Z_{eff}, m) = 3Z_{eff} \in \{3, 6, ..., 18, 21, \}$. The highest values of n are in this row and this might be of biological significance. Indeed, large n means large metabolic energy and C,N, and O are fundamental in metabolism.

2. $m = 3$: For $X \in \{Na, Mg, Al, Si, P, S, Cl\}$ one has $m = 3$, $Z_{eff} \in \{1, .., 7\}$. One $n(Z_{eff}, m) = 2Z_{eff}\ n(X) \in \{2, 4, ..., 14\}$. The common values of n are in $n = 6$ corresponding to Be and Al and to $n = 12$ corresponding to C and S: note that also S corresponds to large metabolic energy. Note that P and S with $n = 12, 14$ are also important in metabolism. Whereas the lighter atoms serve control purposes.

3. $m = 4$: $X \in \{K, Ca, Sc\}$ have $Z_{eff} \in \{1, 2, 3\}$, metals $\{Ti, V, Cr, Mn, Fe, Co, Ni, Cu, Zn\}$ have $Z_{eff} \in \{4, ..., 12\}$ and $\{Ga, Ge, As, Se, Br\}$ have $Z_{eff} \in \{13, ..., 17\}$ have $m = 4$. One has $n(Z_{eff}, m) = 3Z_{eff}/2$ having also half odd-integer values. This gives $\{1, 3/2, 2\}$ for $X \in \{K, Ca, Sc\}$ and $\{5/2, ..., 17/2\}$ for $\{Ti, V, Cr, Mn, Fe, Co, Ni, Cu, Zn\}$ and $\{9, ..., 11\}$ for $\{Ga, Ge, As, Se, Br\}$. The total variation range for n is $[1/2, 11]$.

   Alkali atoms K, Ca and metals Mn, Fe, Co, Ni, Cu, Zn are biologically important. The corresponding metabolic energies are however not so large as for lower rows and this ions indeed seem to serve for control purposes. (Li,Be,B) and (V,Cu,Ga) have same values of n as also (Na,Mg,Al,Si,P) and (Sc,Mn,Co,Cu,As). Interestingly As is reported to play the role of P in some exotic metabolism.

   One could understand the deviations of bond length from the ideal value by allowing small variations of $n(Z_{eff}, m)$. In particular, the replacement of half-odd integer with integer would not considerably affect the bond length.

4. $m = 5$: For $m = 5$ the values of $n(Z_{eff}, m)$ are not integers for the proposed model unless Z_{eff} is divisible by 5. One has $Z_{eff} \in \{1, ..18\}$. The maximum integer value of n is 3. This allows only Nb, Pd,Sb. Could this relate to the fact that heavier atoms are not so important biologically? The atoms near these 3 atoms

5. $m = 6$: For $n_H = 6$ one has $n(Z_{eff}, m) = Z$ for $m = 6$ atoms. Could integer valuedness mean that these atoms might be somehow special?

6. The scale of variation of n degreases with m and this suggests smaller scale of variation for both valence bond length and electron-negativity. The range for the variation of latter indeed decreases along the columns of the periodic table. Also the values of n decrease along the colum of the periodic table: also this conforms with the empirical facts.

To sum up, it seems that one can understand bond lengths quite satisfactory and deduce from the values of n if the proposed model is accepted. The most important outcome would be explanation for the fact that bond lengths do not scale like $(m/Z_{eff})^2$ as standard quantum theory would suggest.

About biological interpretation

$h_{eff}/n = n$ for valence bond serves as a kind of IQ and also for the metabolic energy carried by molecule in valence bonds. This suggests that biologically important molecules should have larger value n. One can test this hypothesis.

1. Difference in χ means that valence electrons are shifted toward the more electro-negative atom of the valence bond. As found, the larger value of $a_B(nm/Z_{eff}))$ for the atom with smaller value of nm/Z_{eff} allows to understand why it is more electronegative.

2. By the above proposal large value of mn/Z_{eff} corresponds to long valence bond and therefore de-localization of valence electron to long scales. The length of valence bond presumably depends also on other parameters. In any case, bond length could be taken as a rough indication about the value of n associated with the bond and the above estimate for n can serve as a starting point. The variation of bond length might allow interpretation in terms of variation of n.

3. The shortest bond lengths would correspond to smallest value of n possibly assignable to what is identified as waste in metabolic reactions. Small value of n also means large binding energy so that the waste molecules in cellular respiration should have short valence bonds. High energy phosphate bond (...P-O-P...) between two phosphates - say in ADP and ATP - would correspond to a large value of n and the bond should be long. Note that P behaves as valence 5 element in phosphate.

Consider the possible implications in more detail.

1. $h_{eff}/n = n$ for valence bond serves as a kind of IQ and also for the metabolic energy carried by molecule in valence bonds. This suggests that biologically important molecules should have larger value n. One can test this hypothesis.

   (a) Difference in χ means that valence electrons are shifted toward the more electro-negative atom of the valence bond. As found, the larger value of $a_B(nm/Z_{eff}))$ for the atom with smaller value of nm/Z_{eff} allows to understand why it is more electronegative.

   (b) By the above proposal large value of mn/Z_{eff} corresponds to long valence bond and therefore de-localization of valence electron to long scales. The length of valence bond presumably depends also on other parameters. In any case, bond length could be taken as a rough indication about the value of n associated with the bond and the above estimate for n can serve as a starting point. The variation of bond length might allow interpretation in terms of variation of n.

   (c) The shortest bond lengths would correspond to smallest value of n possibly assignable to what is identified as waste in metabolic reactions. Small value of n also means large binding energy so that the waste molecules in cellular respiration should have short valence bonds. High energy phosphate bond (...P-O-P...) between two phosphates - say in ADP and ATP - would correspond to a large value of n and the bond should be long. Note that P behaves as valence 5 element in phosphate.

2. C and Si atoms are expected to form linear polymer-like structures with long valence bonds and large and varying value of n proportional to $Z_{eff} \simeq n_V$, n_V the number of valence

electrons and guaranteeing that the bond length has correct value varying in rather narrow range. Of course, bonding of atoms with additional atoms as occurs in multi-phosphates could allow also linear structures. This could partially explain why life is Carbon based.

3. The proposed valence theory allows a view about the role of C. The length of C-H bond is determined by $n(C)$ with larger $n_V(=4)$ so that the valence electron tends to be nearer to C: $\chi(C) = 2.55 \geq \chi(H) = 2.2$ conforms with this. $n(C-H) = n(C-C)$ predicted. The high negentropy of C-H bond could explain why also C-H bonds are so typical in biology. Note that petroleum consists of carbohydrates and liberates energy, supporting the view that non-standard value of n is associated with the valence bonds also in this case.

   Graphite is obtained by putting 2-D graphene layers on each other. In graphite C has valence bonds to 4 C:s and in graphene to 3 C:s. An interesting question is whether this might relate to some very special properties of graphene and whether it might correspond to larger than usual value of n. Not that these structures, in particular graphite, are much less dynamical than polymers.

4. Si is second candidate for the basic atom of life. The values of n for C-H bonds and C-C bonds are in good approximation proportional to n_V/m and in ratio $n(m=2, Z_{eff})/n(m=3, Z_{eff}) \simeq 3/2$. $n(C) > n(Si)$ implies that C-H and C-C bonds are more negentropic and energetic favoring Carbon based life. Also the C-N and C-O bonds are more negentropic and energetic than Si-P and Si-S bonds.

   Note also that $\chi(Si) = 1.9$ is smaller $\chi(H) = 2.20$ so that the electron is nearer to H making it effectively negatively charged. Silanes (hydrosilicons) are very reactive. Also this could relate to the fact that Si based life is not realized.

1. Redox reaction and energy metabolism

Oxidation means the transfer of electrons towards more electronegative atom - not necessary oxygen - and means shorter de-localization scale for electrons and shorter bond unless the value of n increases.

1. Oxidation happens when nutrients are catabolized so that they give the metabolic energy stored into the valence bonds, which could be rather stable due to the non-standard value of n. This rule would hold true quite generally. Bio-catalysis would temporarily reduce the value of n for various bonds and liberate energy allowing to kick the reacting molecules over the potential wall preventing the reaction.

2. Aerobic cell respiration relies on oxygen. At the bottom of the catabolic cascade is glucose C_6H_{12} O_6 decomposed into CO_2 =O=C=O and water. The liberated energy is used to transform ADP to ATP. C=O group acts as the functional group because C is effectively slightly positively charged and attracts negative ions and negative parts of molecules so that it is highly reactive. C=O bond length is 116 pm and considerably shorter than C-O bond length (in say C-O-H) (about 143 pm in paraffin, see `http://tinyurl.com/y95bkooa`). This conforms with the assumption that n is smallest in CO_2 so that in this sense it would be waste.

   Note however that the value of n is higher than $n = n_H = 6$ (taking seriously the findings of Mills [D17] [L26] also for O in CO_2. This should relate to the special role of CO_2 and H_2(O) concerning life. In fact, in both cases the value of $n(O)$ for bonds involved is almost maximal possible in the entire periodic table. Only F has larger n but is too reactive. Hence O is optimal choice both negentropically and energetically. S is second candidate for the role of O but $n(S)$ is by factor 2/3 smaller. Hence $m = 2$ row of the periodic table is optimal for life.

   Remark: A rough estimate of the proposed valence bond theory for the ratio of the values of n for C-C and C-O bonds assuming that all bonds have the same length would be $n_V(O)/n_V(C) = 3/2$.

3. The waste products of metabolism should consist of compounds, which do not have C-C bonds, in particular molecules having only single C atom. The flux tubes associated with the

valence bonds should have low value of n and correspond to low molecular IQ. Bond lengths and de-localization lengths should be short. The molecules should involve typically single carbon atom or no carbon atoms.

Carbon di-oxide CO_2 with valence structure O=C=O represents basic example about outcome of oxidation. CO_2 is the basic organic waste product of metabolism and indeed has especially short bond length. C=O group is functional since oxidation makes C slightly positively charged so that it attracts negative ions and negatively charged parts of molecules. Also ammonium NH_3 is waste product and now electrons are shifted towards N as one finds by comparing the electronegative of H and N appearing in the table of Wikipedia article (see `http://tinyurl.com/pbh6r6c`). As already noticed, the notion of "waste" is only relative notion.

Urea (H_2-N)-C(=O)-(N-H_2) is second waste product. As a matter of fact, liver forms urea and water by combing CO_2 with two NH_3 (ammonium) molecules. Liver puts two waste molecules to single packet.

4. Also H_2O has high negentropy and energy contents although it appears as "waste" in cell respiration. The presence of hydrogen bonds between water molecules gives rise to dark protons, which also affects the situation. Photosynthesis indeed has water and CO_2 input elements so that it makes sense for them to have high negentropy and energy content.

   Remark: Solar light would generate negatively charged exclusion zones of Pollack [L7] [L7] crucial for life.

One can look the situation also from the view point of storage of metabolic energy and negentropy.

1. Hydroxy group O-H (see `http://tinyurl.com/y7uv924k`) attached to C appears in sugars with chemical formula $C_nH_{2n}O_n$. For the simplest hydrocarbons one would have formula C_nH_{2n} apart from boundary corrections at the ends of the polymer. Sugars store more metabolic energy than hydrocarbons and the valence theory should allow to understand this. The rough estimate of the proposed valence bond theory for the ratio of the values of n for C-O and C-H assuming that all bonds have the same length would be $n_V(O)/n_V(C) = 3/2$. This predicts that sugars are more negentropic and energetic than hydrocarbons.

2. C-O bond length is in the range [143,215] pm and C-H bond in the range [106,112] pm. The value of n for C-O-H bond must be higher than predicted by the assumption that the bonds have equal lengths. The replacement $n(O) \rightarrow 3/2n(O)$ allowed by $n_H = 6$ would predict that the ratio of C-H and C-O bond lengths is 9/4. Smaller variations of $n(O)$ are also possible. This would increase further the negentropy and energy contents of O and conform with Negentropy Maximization Principle (NMP), whose statistical form is a prediction of adelic TGD [L50] [L49].

3. The Wikipedia article about hydroxy group tells that compounds containing hydroxyl groups (O-H) tend to form hydrogen bonds forcing them to stick together. This would mean formation of dark protons and suggests formation of flux tube networks, which could be also behind the formation of water molecule clusters and be fundamental aspect in the formation of systems with life-like properties [L46] (see `http://tinyurl.com/yassnhzb`). O-H is thus favored over H also for this reason.

4. One can understand also the somewhat mysterious high energy phosphate bond. Phosphate has chemical formula (P=O)O_3^-. In phosphate the contents of metabolic energy and negentropy are maximized for the proposed model for valence bonds since only F has higher value of n than O in the periodic table assuming that bond lengths are identical. The actual bond lengths require that the value of $n(O)$ is even higher than this.

2. DNA, RNA, and amino-acids

What about other biomolecules such as DNA and amino-acids?

1. DNA (see `http://tinyurl.com/cpndtse`) involves the backbone consisting of a sequence of phosphates $(P{=}O)O_3^-$ and ribose molecules. The 6-cycles of ribose molecules contain 5 carbon atoms and one oxygen atom. As already noticed, phosphate has very high energy and negentropy contents. P has very nearly the same electronegativity (2.19) as H (2.20) and O has electronegativity 3.44 so that the P-O bonds resemble H-O bonds as far electronegativity is considered.

   The aromatic 5- and 6-cycles of DNA involving de-localized electrons contain two N atoms besides C atoms. The electro-negativities are $\xi(C) = 2.55$ and and $\xi(N) = 3.04$ so that electrons should be nearer to N. The length of C-N bond is longer than C-C bond so that the values of n could be the same and n for C-N bond could be even higher than for C-C bond so that it would be more negentropic. This could explain why nitrogens are present in DNA rings rather than only carbon atoms. Note that DNA strands are connected by N$\cdots$N and N$\cdots$O hydrogen bonds possibly involving dark protons.

   In RNA (see `http://tinyurl.com/cmvyw2r`) one C-H in the ribose is replaced with C-O-H in pentose ring. A, T, C, G are replaced with A, U, C, G (T is methylated form of U obtained by replacing -H with $-CH_3$). Only short strands of RNA appear and RNA does not have double stranded form but has single stranded form forming double helix. An interesting question is why the replacement of C-H with C-O-H in the pentose inducing change in electronic charge distribution affects so dramatically the properties of DNA. O-H group is functional and involved with the formation of hydrogen bonds. Maybe quantum criticality of ribose has something to do with the widely different properties of DNA and RNA.

2. Amino-acids (see `http://tinyurl.com/jsphvgt`) have structural formula H_2N- ((C-H)-R)-((C=O)H)-OH, where R is the residue responsible for the functional properties of the amino-acid. Amino-acid polymers have backbone involving N-C bonds formed between amino-group $N\text{-}H_2$ and carboxyl group (C(=O)H)-OH by hydrolysis giving rise to peptide bond ...(C(=O)H)-NH... plus H_2O. Therefore the backbone consists of ...-(C(=O)H))-(NH)-...-(C(=O)H))-(NH)... sequence containing ((C-H)-R) between molecules of the backbone.

   Assuming same N-C and C-C bond lengths the proposed valence band theory predicts $n(N) = (n_V(N)/n_V(C)) = 5n(C)/4$ implying that higher content of metabolic energy and negentropy favors C-N bonds instead of C-C bonds. That the catabolism of peptides to sugars liberates metabolic energy conforms with this.

About the biological role of low valence ions

A comment about the role of biologically important ions is in order. As a rule they tend to have low valence, especially those whose cyclotron frequencies for $B_{end} = .2$ Gauss seem to be important biologically. The possibly existing valence bonds between atoms towards the left end of the rows of the periodic table (Li,Na,K,Ca, Mg,..) - if they even exist at all - have low valence and low value n satisfying $n \geq 6$ (note that the valence of the bond is the valence of the atom with higher valence).

1. The potential negentropy content of low valence bonds is low and also metabolic energy content defined as difference of energy from the situation in which one has $n = 6$ derived from the experiments of Randell Mills [L26]. Thus low valence bonds are not important for metabolism.

2. Low valence ions have however different role: they appear as biologically important positive ions important for the communications to and control by MB. For instance, dark photons with cyclotron frequencies in magnetic fields of flux tubes would be involved with control by dark photons. These dark photons could also transfer energy to MB. The values of n for dark photons can be as high as $n \sim 10^{12}$ or even higher from the condition that the energies of dark photons with frequencies in EEG range are above thermal energy or even in visible and UV range for bio-photons.

   Values of n for dark ions could be thus much higher than for electrons at valence bonds if their cyclotron energies correspond to dark photon energies. Dark photons and dark cyclotron condensates would represent a higher level of evolutionary hierarchy and control and coordination in quite long length scales responsible for the quantum coherence of living matter.

Remark: Recall that the assumption $\hbar_{eff} = \hbar_{gr} = GMm/v_0$, where $v_0 > c$ has dimensions of velocity, m mass of the charged particle, and M some large mass, guarantees universality of cyclotron energy spectrum (spectrum of bio-photons in visible and UV range). This gives $n \sim 10^{13}$ for 10 Hz cyclotron frequency photon energy about 1 eV. Fe^{++} has $f_c = 10$ Hz in $B_{end} = .2$ Gauss.

1. Some examples about biologically important ions

One can consider some examples about biologically important ions.

1. In TGD protons H^+ appear as dark protons. The small value of the atomic binding energy would explain why hydrogen appears as ion: dark atoms with this value of n would have extremely large size. Dark protons need not of course have the value of n characterizing dark EEG photons. Rather, entire hierarchy of frequency scales is expected ranging down to the energies of IR photons still above the thermal energy.

2. Hydrogen bond carrying de-localized proton would serve as the simplest example and be associated with magnetic flux tube. Hydrogen bonded water molecule clusters are crucial for life. Hydrogen bonds are also formed between OH groups of say water and some other high valence atoms.

   Dark DNA/RNA/amino-acid/tRNA realized as dark proton sequences at magnetic flux tubes and realizing vertebrate genetic code (prediction) would be second realization giving rise to dark nuclei [L20]. Cell membrane as generalized Josephson junction would involve electronic Cooper pairs and dark protons or even their Cooper pairs [K93]. At microscopic level membrane proteins defining various ion channels and pumps would act as generalized Josephson junctions.

3. What about heavier ions? Their dark variants appear at MB and play a key role in TGD based model for quantum biology and neuroscience. They appear at flux tubes assignable to generalized Josephson junctions and at layers of MB in much longer scales (note that hydrogen bond is analogous to Josephson junction). Dark ions carrying much more information in BE condensates than dark valence electrons would serve control purposes whereas dark electrons at valence bonds would carry metabolic energy.

4. What about noble gases? Can one say that they have maximal valence or do they have vanishing valence and therefore $n = 6$ as the findings of Mills suggest? If they had maximal valence they should be biologically important but they are not: thus $n = 6$ identification is feasible. Ions at the right end of the rows of periodic table, say Cl^- ion, are like noble gas atoms as far as valence is considered. The electronic negentropy of H-Cl understood as H^+ bonded with Cl^- (ionic bond rather than valence bond) and metabolic energy content would be minimal. Cl^- could however form cyclotron condensates with a large value of n, which would explain its biological importance.

   Amusingly, plastic balls in plasma of Ar^+ ions appear in the experiment demonstrating life like properties of this system ("breathing") [L46]. Ar^+ would have maximal possible valence and thus maximal value of n and would appear at flux tubes.

2. How the ionization is possible in living matter?

The appearance of ions in living matter looks mysterious. Same is true concerning ions in electrolytes. It is easy to talk about cold plasma but much more difficult to answer the question how this cold plasma can be created. Usually the formation of plasma involves ionization, which requires high temperature of order of the atomic binding energy for the valence electrons of the atom. For hydrogen atom the binding energy is around 13 eV, which corresponds to a temperature of roughly 130,000 Kelvin! This is three orders of magnitude higher than room temperature! In electrolyte the presence of rather weak electric fields cannot explain why the ionization takes place.

For some reason chemists and biologists do not spend much time in pondering fundamentals and theoreticians enjoying monthly salary have a highly irreverent attitude to these disciplines as an intellectual entertainment of lower life-forms. Therefore also this question has been guided under the rug and stayed there.

TGD based explanation for the paradox is simple. If the value of $h_{eff}/h = n$ for valence electrons is high enough, the binding energy, which is proportional to $1/n^2$, becomes so high that a photon with rather low energy, say infrared (IR) photon, can be ionize the dark atom. One can say that the atoms in this state are quantum critical, a small perturbation can ionize them.

1. In the TGD based model of cold fusion as dark nucleosynthesis the atoms would have $n = 2^{12} = 2056$ and the ionization would create dark nuclei as sequences of dark protons at magnetic flux tubes [L37].

2. In the TGD based model for the analogs of DNA/RNA/amino-acid sequences/tRNA as dark proton sequences the value of n would be of the order of 10^6 higher so that the distance between dark protons, would be a same as between DNA letters, about 3 Angstroms. For these values of n dark atoms are unstable at room temperatures.

3. In Pollack effect [L7] the irradiation by IR light or visible light or by pumping energy to the system by some other means produces negatively charged exclusion zones (EZs) in which water molecules form hexagonal layers and obey the effective stoichiometry $H_{3/2}$ O. Part of protons (every fourth) goes to magnetic flux tubes as dark protons. How it is possible to create this state by IR radiation?

   (a) The original assumption was that in second OH bond of water molecule is excited to a high energy state near to ionization energy so that IR photon can split the bond. The question is: why and how? Do the UV photons from solar radiation cause the excitation?

   (b) A more elegant option is that the value of n for the second O-H bond is so large that the bond binding energy is so small that IR photon can split the bond. Solar UV photons could induce the dark excitation. Taking 5 eV as rough estimate for the bond binding energy in the normal state water, this requires the reduction of energy by a factor of order 10^3 to give IR energy .05 eV (energy scale assignable to the membrane potential eV). n would increase by factor of order $2^5 = 32$ from its value for O-H bond according to standard chemistry. A small push by absorption of IR photon can split the O-H bond and create dark proton at flux tube. Any perturbation feeding to the system this energy can induce the kicking of dark proton to flux tube. The generalization of this mechanism to various atoms could be one of the basic mechanisms of quantum criticality in living matter.

12.3 Revolution in chemistry

This section was inspired by a link to a very interesting article with title "Rules of attraction: Strange chemical bonds that defy the textbooks" (https://cutt.ly/Jner49B) telling of new chemistry. Unfortunately, a subscription to New Scientist was required. It was however easy to find in the web several popular articles telling about the changing views concerning chemical bonds.

The article "This weird chemical bond acts like a mash-up of hydrogen and covalent bonds" (https://cutt.ly/Snetr52) tells about hybrids of hydrogen and and covalent bonds. For short bond lengths these bonds become strong valence bonds and for long bond lengths weak hydrogen bonds which can even have length of 3 Angstrom.

The article "Strange bonds entirely new to chemists predicted in ammonia hydrides" (https://cutt.ly/1netzra) tells that ammonium NH_3 can form in the presence of hydrogen in very high pressure an exotic compound NH_7, which can decay to NH_4^+ + H_2+ H^-. NH_4^+ is also exotic.

The article "Sticking together: Another look at chemical bonds and bonding" https://cutt.ly/cnetPvG discusses the theory of chemical bonds proposed by Prof. David Brown, which has turned out to be very successful.

12.3.1 Two theoretical views about chemical bonds

In the following the bond theory of David Brown and the TGD view about chemical bonds are discussed and compared.

The bond theory of David Brown

The bond theory of David Brown is published as an article with title "Another look at bonds and bonding" in Structural Chemistry 31(1), 2019 [D6] (`https://cutt.ly/eneociZ`).

1. The theory involves the notion of electric flux as a purely classical element. The delocalization of valence electrons is of course a non-classical element and one can argue that this aspect is not well-understood in standard chemistry. In the TGD framework, the counterpart of electric flux is a flux tube carrying magnetic flux, which can be monopole flux. The tube can also carry an electric flux and a simple modification of purely magnetic flux tubes gives tubes carrying also an electric flux.

2. The key concept besides the notions of valence defined as the number N_v of valence electrons belonging to bonds, and the number of valence bonds N_b, is valence strength defined as $S_v \equiv N_v/N_b$. The total electric flux is the sum of fluxes assignable to the bonds and equals to the total electric charge -N_ve of valence electrons.

   By flux conservation, the electric fluxes at the ends of a given bond are opposite and this gives a strong constraint on the model. This condition is new from the point of standard bond theory and is purely classical.

3. The configurations with minimum energy are expected to be symmetric. In this case, the electric fluxes for the bonds are expected to be identical and proportional to the common bond strength.

   (a) An important implication of flux conservation in the symmetric case is that the valence strengths must be the same for bonded atoms. This condition excludes a large number of candidates.

   (b) If N_b is larger than N_v, the flux is fractional. This would represent an exotic situation. An interesting question is whether the flux could correspond to a quark pair or two quark pairs possible in the TGD framework in long scales. The idea that atoms could involve quarks looks of course rather outlandish from the standard model perspective. In this case the flux would be 1/3:rd or 2/3:rd of the flux associated with a single valence electron.

4. The model works for many kinds of bonds, and is claimed to work even for hydrogen bonds, and can be used to predict possible bonding structures. What is remarkable, that the notion of conserved electric flux assignable to chemical bonds resonates with the TGD view that non-trivial space-time topology behind the notion of flux tube is directly visible at the level of chemistry.

TGD view about chemical bonds

I remember the time when I realized that TGD suggests a description of the chemical bond in terms of the space-time topology. Could chemistry books be wrong, was the question, which I barely dared to articulate.

Gradually I learned that chemistry books do not really allow any deeper understanding of chemical bonds. One just says that they follow from Schodinger equation but computational complexity prevents proving this.

1The TGD based view about valence bonds

TGD indeed implies a revolution in chemistry. Chemical bonds are accompanied by flux tubes possibly carrying dark particles with effective Planck constant $h_{eff} = h_0 > h = 6h_0$. Valence electrons of the less electronegative atom would get to the flux tube and become dark. This leads to a model of valence bonds [L44] predicting that the value of $h_{eff}/h_0 = n$ increases as one moves to the right along the row of the periodic table. This implies a delocalization of the valence electrons to longer scale scaling like h_{eff}^2 for the Bohr model and this is essential for the delocalization. This delocalization would be essential for chemistry of valence bonds and for biochemistry in particular. In metals delocalization would occur in the scale of lattice.

Also $h_{eff} < h$ bonds are in principle possible. Randell Mills has found evidence for a variant of hydrogen for which energies are scaled by a factor 1/4: this would mean $h_{eff} = h/2$ [D17] [L26].

The strange disappearance of the valence electrons of some transition metals in heating has been also known for decades [D14, L47] [L47]: heating would provide the energy needed to increase h_{eff} for valence electrons so that they become dark relative to us? Note that in TGD based biology metabolic energy would be used to increase h_{eff}, which serves as a kind of universal IQ as a measure of algebraic complexity.

An interesting possibility is that in the past scaled down atoms with $h_{eff} = h/2$ have existed [L124]. Could they correspond to most of the dark matter, the primordial dark matter? In the same article, it is also proposed that the Cambrian explosion involving a dramatic leap in biological evolution involved a phase transition in the Earth's scale doubling the radius of the "atomic" flux tubes with thickness of order atomic scale or order 1 Angstrom.

The presence of the "atomic" flux tubes makes it possible to understand the scale of atoms which depends only weakly on atoms although the Bohr radii of valence electrons are typically considerably smaller than atomic scale. The doubling of the flux tube radius by a factor two would have induced the doubling of the atomic size scale. This would have induced the doubling of h_{eff} for valence electrons. This phase transition would have led to the emergence of biochemistry.

2. Hydrogen bonds in the TGD framework

The popular article (`https://cutt.ly/Jner49B`) also mentions bonds without electrons. These bonds would make possible entanglement between atoms.

Hydrogen bond is an example of non-valence bond. Hydrogen bonds are special in that they can be as long as 3 Angstroms. The theory proposed by Brown would describe hydrogen bonds in terms of electron's delocalization.

In TGD framework it would be a proton that becomes dark delocalized to a flux tube accompanying the hydrogen bond, and has therefore $h_{eff} > h$ [L140]. In water one could have a spectrum of h_{eff} values with various bond lengths and this would give water its very special properties [L69]. Even flux tubes without any particles but serving as topological space-time correlates and even prerequisistes of entanglement between atoms are possible.

The hydrogen atom can form a bond between two atoms. Usually, the hydrogen forms a hydrogen bond with the first atom and valence bond with the second atom. This applies for large distances for which hydrogen bond is weak. At short distances hydrogen bond becomes stronger and it has been found that hybrids of hydrogen- and valence bonds between between two fluorine atoms - hydrogen-mediated chemical bonds - have been observed (`https://cutt.ly/Snetr52`).

In the TGD framework, the hybrids could be understood as a delocalization of both electron and proton to the two bonds involved. For short bond lengths, the proton would not be delocalized and for long bond lengths the electron would not be delocalized.

12.3.2 Water oxidation and photosynthesis in TGD framework

These comments were inspired by an interesting article "Isolating an elusive missing link" (`https://cutt.ly/dng7My6`) about water oxidation. It came as a surprise to me that water oxidation is still a poorly understood piece of biochemistry. Bio-chemists believe that they understand various aspects of the reaction reasonably well with one exception, which is the formation of the O_2 molecule in water oxidation and the article tells about progress in this respect.

The water oxidation reaction (WOR) is one of the most important reactions on the planet since it is a key step in photosynthesis and is also the source of nearly all the atmosphere's oxygen. What is so beautiful is that both photosynthesis as a chemical storage of the solar energy and water oxidation producing oxygen essential for aerobic respiration to utilize the stored energy, are parts of the same process.

Understanding the intricacies of WOR can hold the key to improve the efficiency of the reaction which could be utilized to produce hydrogen. As the article tells, the reaction's chemical mechanisms are complex and the intermediates highly unstable. This makes their isolation and characterisation extremely challenging. To overcome this, scientists are using molecular catalysts as models to understand the fundamental aspects of water oxidation — particularly the oxygen-oxygen bond-forming reaction.

WOR (see `https://cutt.ly/Jng72R8` and `https://cutt.ly/Ang781b`) forms an essential part of photosynthesis (`https://cutt.ly/Tng77R4`). What happens in WOR is that two water molecules split $4H^+$,$4e^-$, and O_2. The reaction mechanism is not completely understood. Somehow the solar radiation induces the process in which two H_2O molecules split to $4H^+$, $4e^-$, and O_2. The 4 electrons are utilized in photosynthesis in the KOK cycle.

In the sequel a general mechanism of catalysis inspired by zero energy ontology (ZEO) [L91] is discussed. In this approach biocatalysis involves two "big" state function reductions (BSFRs) changing the arrow of time. The first BSFR induces time reversed time evolution leading from the final of a sub-reaction kicking the reactants over the potential wall preventing the reaction from occurring. After the second BSFR the time evolution continues from the initial state of time reversed time evolution in standard time direction. The catalytic process takes place spontaneously and the role of the catalyst is to make the spontaneous occurrence in the reversed time direction possible and probable enough. Quantum coherence in long scales is necessary and the identification of dark matter as $h_{eff} = nh_0$ phases implied by adelic physics predicts it [L86].

12.3.3 Basic facts about photosynthesis and water splitting

Photosynthesis involves two parts. The first part does not involve photons and leads to the splitting of water producing from two water molecules 4 protons and 4 electrons plus O_2 molecule. The second part involving photons stores their energy chemically. The first part occurs in oxygen evolbing complex (EOC), known also as water splitting complex, and acting as a cofactor of photosystem II enzyme in which the photosynthesis proper takes place.

Oxygen evolving (water splitting) complex (OEC)

OEC is the cofactor of photosystem II enzyme. OEC has an inorganic core obeying the empirical chemical formula $Mn_4Ca_{1O}O_xCl_{1-2}(HCO_3)_y$ core. Core is surrounded by 4 protein subunits of photosystem II at membrane-lumen interface.

OEC functions as follows.

1. The extraction of 4 electrons and and hydrogen ions from 2 water molecules produces O_2 molecule as a kind of waste.

2. OCE transfers 4 electrons, one at a time, to photosystem II via a tyrosine residue in the reaction center. Photosystem II must store the energy of 3 photons before the fourth one provides sufficient energy for water oxidation. Kok theory states that OEC can exist in 5 states $S0, ..., S4$. $S4$ since OEC has lost 4 electrons. $S0$ is the most reduced.

   $S4$ is unstable to reset to ground state $S0$ and reacts with water producing free oxygen. OEC receives the 4 electrons and returns to state S_0.

3. After that photons from photosystem II drive the system from $S0$ to $S4$. The electrons from OEC are transferrred to photosystem II one-by-one. Photons from photosystem II energize electrons which are driven through the a variety of coenzymes and cofactors to reduce plastoquinone to plastoquinol.

4. The 4 hydrogen ions are used to create a proton gradient. This means that they are driven against the membrane potential gradient and gain potential energy liberated later as the protons return back and provide electrostatic energy used to by ATP synthase to transfrom ADP to ATP.

The energetics of photosynthesis

Consider first the energetics of photosynthesis (`https://cutt.ly/Tng77R4`).

1. As far the energetics is considered, the process of photosynthesis is equivalent to CO_2+ $2H_2O$ +$\gamma \rightarrow$ CH_2O + O_2 + H_2O. What happens to the 4 electrons and protons produced in the splitting of water?

2. CO_2 loses one O and CO combines with two protons and electrons to form CH_2O. This requires a catalyst to temporarily kick out O from CO_2. This energy is returned to the catalyst when two electrons and protons combine with O to form H_2O. The binding energy of C=O bond and H=O bond are indeed nearly the same.

3. Water splitting requires energy $E = 4E_B(O-H) + 4E_B(e-p)$= 4 ×(5.15 + 13.6) = eV. The formation of $O = O$ provides energy $E_B(O = O)$ = 5.13 eV. In the formation of H_2O 2 × (5.15 + 13.6) eV is returned to the catalyst but this energy has been already taken into account and compensates for the energy needed to split CO_2. Therefore the energy $E_1 = 4(E_B(O-H) + E_B(e-p)) - E_B(O = O)$= [4 × (5.15 + 13.6) - 5.13] eV needed from catalyst must correspond the energy liberated in the formation of CH_2O. This is true assuming that the binding energy of e and p associated in the O-H valence bond is the sum of atomic binding energy and O-H bond energy.

   The formation of CH_2O involves combination of 2 protons and electrons to form two hydrogens, the atomic binding energy $2E_B$(e-p)= 2 × 13.6 eV is liberated in the formation of CH_2O and compensates the same energy appearing in E_1. Hence the atomic binding energies can be forgotten in the energy budget and it is enough to compare only the molecular binding energy E_2 in E_1 with E_B. The molecular contribution to E_1 is $E_2 = 4\times 5.15 - 5.13$ eV = 15.48 eV.

4. The formation of CH_2O means a generation of molecular binding energy which is approximately the sum of O=CH_2 binding energy and C-H binding energies. Besides this atomic binding energy of two hydrogen atoms is liberated. The liberated molecular binding energy is $E_B(CH_2O) = E_B(O = CH_2) + 2 \times E_B(C-H)$= (7.75+2× 4.28) eV= 16.31 eV. The amount of the liberated molecular binding energy is $E_B(CH_2O)$ = 16.31 eV and is by .83 eV larger than E_2 = 15.48 eV. One must drive 4 protons against a potential gradient and this requires energy $4 \times .07eV$ = .28 eV which is smaller than this energy. 4 photons are used and if their energies are about 2 eV they provide 8 eV.

5. This estimate does not take thermodynamics and second law into account. Since pressure and temperature can be assumed to stay constant in the process, the thermodynamical approach using Gibbs free energy $G = E + pV - TS$ as thermodynamical function is natural. $dG = VdP - SdT + \sum_i \mu_i dN_i$ reduces to $dG = \sum_i \mu_i dN_i$ if pressure and temperature are constant.

6. The overall process can be written as $6CO_2$ + $6H_2O$ + light $\rightarrow$ $C_6H_{12}O_6$ + 6O2. This form corresponds to a polymerization of 6 CH_2O molecules to form sugar $C_6H_{12}O_6$.

7. Several separate steps are involved with photosynthesis besides the splitting of water; there are 4 steps corresponding to the transfer of electron decomposing to substeps related to the photosynthesis proper. Each of this steps involves catalysis.

That two water molecules are involved in the basic process, could also be essential. The hydrogen bond between the water molecules or its dark variant could play some role.

12.3.4 TGD view about water photosynthesis and water oxidation

TGD provides a new view about bio-catalysis in which the magnetic body (MB) acts as a controller. One might hope that at this level the description of the biocatalysis is much simpler than at the level of biochemistry. Therefore one can ask whether a simple overall view based on the energetics and the notion of MB carrying dark matter at its flux tubes could help. Since 5 different kinds of valence bonds must be temporarily split in the overall reaction, a catalyst providing the needed energy to temporarily break the valence bonds is needed.

There are many other steps involving catalysis and the actual situation taking into account reactions involving photons is extremely complex. From the foregoing it is clear that the splitting of water requires 75 eV energy. Most of this energy is related to the ionization of hydrogen atoms. Note however that valence electrons in the valence bonds are approximated as ordinary atomic electrons. Unless quantum tunnelling is involved, this energy must be provided by some source.

Does time reversal provide a general mechanism of bio-catalysis?

In the TGD framework one can consider 3 general mechanisms of biocatalysis and both mechanisms could be involved.

The basic problem in the understanding of catalysis is to identify the mechanism kicking the reactants over the potential wall making the reaction extremely slow. How to get over this potential wall is even rid of it?

Zero energy ontology (ZEO) [L91, L86, L118] suggests a completely general mechanism of biocatalysis.

1. ZEO is behind the TGD inspired quantum measurement theory and makes it possible to resolvin the basic paradox of standard quantum measurement theory and gives rise to a quantum theory of consciousness. The new element is that the arrow of time changes in ordinary ("big") state function reduction (BSFR), whereas it remains unchanged in "weak" measurements, "small" state function reductions (SSFR).

   In ZEO, BSFR creates a superposition of time reversed deterministic classical time evolutions analogous to Bohr orbits and leading from the final state to the geometric past. The findings of Minev *et al* [L77] support the notion of BSFR [L77].

2. BSFR has far reaching implications. Since water splitting has turned out to be very difficult to understand, a natural question is whether BSFR could explain it. If water splitting occurs as BSFR, it would correspond to a spontaneously occurring process 4e+4p+ $O_2 \rightarrow H_2O$ in reversed time direction requiring no catalysis. Second BSFR would mean a return to a moment of geometric time where one as 4e+4p+O_2.

3. BSFRs could be involved also with the other steps involving the splitting of chemical bonds and make it possible to kick the reactants over the potential wall. In a reversed time direction this process would take place spontaneously and lead from free atoms to their bound states by generation of atomic and molecular bonds.

4. BSFR is especially natural in the situations in which the reverse process occurs spontaneously meaning that only the process but not its reversal involves potential wall. This seems to be the case in bio-catalysis. In nuclear reactions the situation is different since both the process and its reversal involve overcoming of a Coulomb wall.

The ZEO based vision provides only a general idea about what is involved with a given catalytic step of a given step of say photosynthesis but tells nothing of its chemical details, in particular of the role of MB and dark particles at it. The chemistry of water oxidation catalyst - oxygen evolving complex - is extremely complex but a general principle might considerably facilitate its understanding.

How to model the reverse time evolution behind catalysis as ordinary time evolution?

The most plausible option is that water oxidation and also other steps involving liberation of atoms from bound states take place via BSFR. The ZEO based view about biocatalysis does not however exclude the modelling of the time reversed process as occurring in the standard direction of time and being based on some catalytic mechanism. The presence of the catalyst would make the BSFR possible. This kind of modelling would help to characterize the catalyst and the superposition of time reversed time evolutions leading to the final state as time evolutions with standard time direction.

1. The first mechanism involves a reduction of h_{eff} for a magnetic flux tube liberating energy temporarily kicking the system over the potential wall. That energy is liberated follows from the fact that energies in general increase with h_{eff}. The reduction of the cyclotron energy

proportional to h_{eff} of the charged particles at the flux tube would liberate the needed energy.

This mechanism might make sense in the case of molecular binding energies of order few eV which is the energy scale for the cyclotron energies in the magnetic field $B_{end} = .25$ eV for $h_{eff} = h_{gr} = GMm/v_0$.

This mechanism could apply to the catalytic steps of photosynthesis separately. If it applies to water oxidation, the energy 75 eV provided by flux tubes would be returned gradually to the catalyst (shortened magnetic flux tube) during photosynthesis proper as electrons and protons bind to the hydrogen atoms of COH_2 and H_2O during the process.

2. Second, much more speculative (even in TGD framework), mechanism would be a phase transition increasing the thickness of a magnetic flux tube liberating magnetic and volume energy [L124] suggested to be behind the proposed rather rapid expansion of the Earth increasing its radius by factor 2 and leading to Cambrian explosion. This transition would scale up the value of h_{eff} for valence electrons by factor of 2 and reduce the bond energies by factor 1/4 and also the energy of electron of hydrogen atom.

   This mechanism could be behind the proposed model of quantum tunneling innuclear reactions [L89]. The quantum tunnelling would involve BSFR as a phase transition increasing the value of h_{eff} and size scale of the colliding nuclear strings. These dark nuclei would have dramatically reduced binding energy and increased spatial scale making it possible to overcome the Coulomb barrier. The reactions would proceed fast in the dark phase and the second BSFR would transform dark nuclei to ordinary nuclei.

   "Cold fusion" would rely on this mechanism except that now the protons would be transformed to dark protons by the analog of Pollack effect, and there is no need to break nuclear bonds as in ordinary nuclear reactions [L37, L105]. Also in the case of nuclear quantum tunnelling the phase transition would be scaling of flux tubes.

   The basic objection is that the phase transition would reduce the energy scale of valence electrons of all atoms involved. This does not look realistic.

1. Shortening of flux tubes by a reduction of h_{eff}

How could the bio-catalysis assisted by MB proceed? MB should provide the energy needed to split the bonds of various molecules. Could the reduction of h_{eff} for cyclotron states at dark magnetic flux tubes liberate cyclotron energy of charged particles at them proportional to h_{eff}?

1. The magnetic cyclotron energy $E_c = \hbar_{eff} ZeB/m$ is large for large $\hbar_{eff}$. Magnetic energies for $h_{eff} = h$ are rather small. Also for relatively small values of $h_{eff} > h$ assignable to valence bonds, the cyclotron energies for proton and electron are much smaller than atomic binding energies for reasonable values of the magnetic field.

2. The basic proposal which led to the h_{eff} vision is that in TGD framework large values of h_{eff} allowing cyclotron energy in an endogenous magnetic field $B_{end} \simeq .2$ Gauss to be in visible and UV range are possible. The value of h_{eff}/h of order 10^{14} is needed. Nottale hypothesis

$$\hbar_{eff} = \hbar_{gr} = \frac{GMm}{v_0} ,$$

   One has $v_0 = 1/2$ in the simplest model - implying that gravitational Compton length equals to the Schwartschild radius $r_S \simeq 1$ cm of the Earth - implies that cyclotron energies do not depend on the mass of small particle, in particular they are same for all charged particle with given charge if the value of B is same.

3. Dark protons at magnetic flux tube are a basic building brick of TGD inspired quantum biology. Could the liberation of cyclotron energy for dark protons (or for dark ions) allow to break O-H bonds and ionize hydrogens? Could some dark protons (or ions) transform temporarily to ordinary protons and liberate their cyclotron energy propto h_{gr} so that MB could act as a bio-catalyst.

4. The general view of bio-catalysis suggests that the eventual formation of the final state molecules liberates energy, which increases the value of h_{eff} of dark flux tubes to its original value. Note that the energy of the final state is by .83 eV smaller than that of the initial state so that dissipative losses are possible even if the entire energy of the photon is stored as metabolic energy.

2. Could the thickening of flux tubes liberate energy and reduce atomic binding energy scale?

Although this option does not look realistic, it deserves a more detailed treatment.

1. The phase transition, which increases the radius of magnetic flux tubes by a power of 2 (by p-adic length scale hypothesis), liberates energy and could induce an increase of h_{eff} at the level of valence electrons and reduce the binding energy scale of the valence electrons proportional to $1/h_{eff}^2$ by a factor $1/2^{2k}$. This mechanism was discussed in [L124].

2. The energy of 75 eV would be reduced to about $75/2^{2k}$ eV. $k = 1$ would give 18.8 eV and $k = 2$ would give 4.7 eV. The reduction of the binding energy would be due to the energy liberated in the thickening of the magnetic flux tubes. Water would be special in the sense that these phase transitions could occur for the magnetic flux tubes assignable to water and explain its multiphase character responsible for its numerous thermodynamic anomalies.

3. What is the scale of the flux tubes involved? Suppose that a flux tube portion of radius L and roughly the same length is involved. If the energy E of is of order $E \sim \hbar_{eff}/L$, the length scale would be about $L = (h/h_{eff} \times 1.6$ nm. For $h_{eff} = h$, the length scale corresponds to cell membrane thickness. The outcome of this process would be a reduction of the scale of valence electron binding energies and of bond energies. In this phase the required reactions could proceed easily.

12.4 Biocontrol and supraphases

12.4.1 A New Control Mechanism Of TGD Inspired Quantum Biology

The idea that TGD Universe is quantum critical, is the corner stone of quantum TGD and fixes the theory more or less uniquely since the only coupling constant parameter of the theory - Kähler coupling strength - is analogous to critical temperature. Also more than one basic parameters are in principle possible - maximal quantum criticality fixes the values of all of them - but it seems that only Kähler coupling strength is needed. TGD Universe is a quantum critical fractal: like a ball at the top of hill at the top of hill at.... Quantum criticality allows to avoid the fine tuning problems plaguing as a rule various unified theories.

Quantum criticality

The meaning of quantum criticality at the level of dynamics has become only gradually clearer. The development of several apparently independent ideas generated for about decade ago have led to the realization that quantum criticality is behind all of them. Behind quantum criticality are in turn number theoretic vision and strong forms of general coordinate invariance and holography.

1. The hierarchy of Planck constants defining hierarchy of dark phases of ordinary matter corresponds to a hierarchy of quantum criticalities assignable to a fractal hierarchy of sub-algebras of super-symplectic algebra for which conformal weights are n-ples of those for the entire algebra, n corresponds to the value of effective Planck constant $h_{eff}/h = n$. These algebras are isomorphic to the full algebra and act as gauge conformal algebras so that a broken super-conformal invariance is in question.

2. Quantum criticality in turn reduces to the number theoretic vision about strong form of holography. String world sheets carrying fermions and partonic 2-surfaces are the basic objects as far as pure quantum description is considered. Also space-time picture is needed

in order to test the theory since quantum measurements always involve also the classical physics, which in TGD is an exact part of quantum theory.

Space-time surfaces are continuations of collections of string world sheets and partonic 2-surfaces to preferred extremals of Kähler action for which Noether charges in the sub-algebra of super-symplectic algebra vanish. This condition is the counterpart for the reduction of the 2-D criticality to conformal invariance. This eliminates huge number of degrees of freedom and makes the strong form of holography possible.

3. The hierarchy of algebraic extensions of rationals defines the values of the parameters characterizing the 2-surfaces, and one obtains a number theoretical realization of an evolutionary hierarchy. One can also algebraically continue the space-time surfaces to various number fields - reals and the algebraic extensions of p-adic number fields. Physics becomes adelic. p-Adic sectors serve as correlates for cognition and imagination. One can indeed have string world sheets and partonic 2-surfaces, which can be algebraically continued to preferred extremals in p-adic sectors by utilizing p-adic pseudo constants giving huge flexibility. If this is not possible in the real sector, figment of imagination is in question! It can also happen that only part of real space-time surface can be generated: this might relate to the fact that imaginations can be seen as partially realized motor actions and sensory perceptions.

Quantum criticality and TGD inspired quantum biology

In TGD inspired quantum biology quantum criticality is in crucial role. First some background.

1. Quantum measurement theory as a theory of consciousness is formulated in zero energy ontology (ZEO) and defines an important aspect of quantum criticality. Strong form of NMP states that the negentropy gain in the state function reduction at either boundary of causal diamond (CD) is maximal. Weak form of NMP allows also quantum jumps for which negentropic entanglement is not generated: this makes possible ethics (good and evil) and morally responsible free will: good means basically increase of negentropy resources.

2. Self corresponds to a sequence state function reductions to the same boundary of CD and h_{eff} does not change during that period. The increase of h_{eff} (and thus evolution!) tends to occur spontaneously, and can be assigned to the state function reduction to the opposite boundary of CD in zero energy ontology (ZEO). The reduction to the opposite boundary means death of self and living matter is fighting in order to avoid this even. To me the only manner to make sense about basic myth of Christianity is that death of self generates negentropy.

3. Metabolism provides negentropy resources for self and hopefully prevents NMP to force the fatal reduction to the opposite boundary of CD. Also homeostasis does the same. In this process self makes possible evolution of sub-selves (mental images dying and re-incarnating) state function by state function reduction so that the negentropic resources of the Universe increase.

A new mechanism of quantum criticality

Consider now the mechanisms of quantum criticality. The TGD based model [L14] [?] (`http://tinyurl.com/y8oblpl9`) for the recent paradoxical looking finding [L14] (`http://tinyurl.com/y79qo7lp`) that topological insulators can behave like conductors in external magnetic field led to a discovery of a highly interesting mechanism of criticality, which could play a key role in living matter.

1. The key observation is that magnetic field is present. In TGD framework the obvious guess is that its flux tubes carry dark electrons giving rise to anomalous currents running in about million times longer time scales and with velocity, which is about million times higher than expected. Also supra-currents can be considered.

   The currents can be formed of the cyclotron energies of electrons are such that they correspond to energies near the surface of the Fermi sphere: recall that Fermi energy for electrons is determined by the density of conduction electrons and is about 1 eV. Interestingly, this energy is at the lower end of bio-photon energy spectrum. In the field of 10 Tesla the cyclotron energy of electron is .1 mV so that the integer characterizing cyclotron orbit must be $n \simeq 10^5$ if conduction electron is to be transferred to the cyclotron orbit.

2. The assumption is that external magnetic field is realized as flux tubes of fixed radius, which correspond to space-time quanta in TGD framework. As the intensity of magnetic field is varied, one observes so called de Haas-van Alphen effect (`http://tinyurl.com/ych7b9n8`) used to deduce the shape of the Fermi sphere: magnetization and some other observables vary periodically as function of $1/B$ (for a model for the quantum critical variant of the effect see [D23]).

   This can be understood in the following manner. As B increases, cyclotron orbits contract. For certain increments of $1/B$ $n+1$:th orbit is contracted to n:th orbit so that the sets of the orbits are identical for the values of $1/B$, which appear periodically. This causes the periodic oscillation of say magnetization.

3. For some critical values of the magnetic field strength a new orbit emerges at the boundary of the flux tube. If the energy of this orbit is in the vicinity of Fermi surface, an electron can be transferred to the new orbit. This situation is clearly quantum critical.

   If the quantum criticality hypothesis holds true, $h_{eff}/h = n$ dark electron phase can be generated for the critical values of magnetic fields. This would give rise to the anomalous conductivity perhaps involving spin current due to the spontaneous magnetization of the dark electrons at the flux tube. Even super-conductivity based on the formation of parallel flux tube pairs with either opposite or parallel directions of the magnetic flux such that the members of the pair are at parallel flux tubes, can be considered and I have proposed this a mechanism of bio-superconductivity and also high Tc super-conductivity.

A new mechanism of quantum bio-control

The quantum criticality of the process in which new electron orbit emerges near Fermi surface suggests a new mechanism of quantum bio-control by generation of super currents or its reversal.

1. In TGD inspired quantum biology magnetic body uses biological body as motor instrument and sensory receptor and EEG and its fractal variants with dark photons with frequencies in EEG range but energy $E = h_{eff}f$ in the range of bio-photon energies make the necessary signalling possible.

2. Flux tubes can become braided and this makes possible quantum computation like processes [K4]. Also so called 2-braids - defined by knotted 2-surfaces imbedded in 4-D space-time surface - are possible for the string world sheets defined by flux tubes identified to be infinitely thin, are possible. As a matter fact, also genuine string world sheets accompany the flux tubes. 2-braids and knots are purely TGD based phenomenon and not possible in superstring theory or M-theory.

3. It is natural to speak about motor actions of the magnetic body. It is assumed that the flux tubes of the magnetic body connect biomolecules to form a kind of Indra's web explaining the gel like character of living matter. h_{eff} reducing phase transitions contract flux tubes connecting biomolecules so that they can find each other by this process and bio-catalysis becomes possible. This explains the mysterious looking ability of bio-molecules to find each other in the dense molecular soup. In fact the dark matter part is far from being soup! The hierarchy of Planck constants and $h_{eff} = h_{gr}$ hypothesis imply that dark variants of various particles with magnetic moment are neatly at their own flux tubes like books in shelf.

   Reconnection of the U-shaped flux tubes emanating from two subsystems generates a flux tube pair between them and gives rise to supracurrents flowing between them. Also cyclotron radiation propagating along flux tubes and inducing resonant transitions is present. This would be the fundamental mechanism of attention.

4. I have proposed that the variation of the thickness of the flux tubes could serve as a control mechanism since it induces a variation of cyclotron frequencies allowing to get in resonance or out of it. For instance, two molecules could get in flux tube contact when the cyclotron frequencies are identical and this can be achieved if they are able to vary their flux tube thickness. The molecules of immune system are masters in identifying alien molecules and the underlying mechanism could be based on cyclotron frequency spectrum and molecular attention. This would be also the mechanism behind water memory and homeopathy (http://tinyurl.com/yda3d6se [K55] which still is regarded as a taboo by mainstreamers.

5. Finally comes the promised new mechanism of bio-control. The variation of the magnetic field induced by that of flux tube thickness allows also to control whether there is quantum criticality for the generation of dark electron supra currents of electrons. The Fermi energy of the conduction electrons at the top of Fermi sphere is the key quantity and dictated by the density of these electrons. This allows to estimate the order of magnitude of the integers N characterizing cyclotron energy for ordinary Planck constant and the maximal value of $h_{eff}/h = n$ cannot be larger than N.

12.4.2 Are Bacteria Able To Induce Super-Fluidity?

Nature News (see http://tinyurl.com/nhcmkle) tells t hat a team led by Auradou *et al* reports in the article "Turning Bacteria Suspensions into Superfluids" [I7] published in Phys Rev Letters (see http://tinyurl.com/ycfm8gnn) that bacterium swimming in fluid do not only reduce its viscosity associated with shear stress (viscous force parallel to the surface) but makes it to behave in super-fluid like manner above a critical concentration of bacteria.

As the number of bacteria (E. coli) was increased, the viscosity associated with shear stress (the viscous force parallel to the surface) dropped: this in accordance with theoretical expectations. Adding about 6 billion cells (the fluid volume is not mentioned but it seems that the effect occurs above critical density of bacteria), the apparent viscosity dropped to zero - or more precisely, below the experimental resolution. The super-fluid like behavior was preserved above the critical concentration. What is important that this did not happen for dead bacteria: bacteria play an active role in the reduction of viscosity.

Researchers are not able to identify the mechanism leading to the superfluid-like behavior but some kind of collective effect is believed to be in question. The findings suggest that the flagellae - kind of spinning hairs used by the bacteria to propel themselves - should play an essential part in the phenomenon. As bacteria swim, they fight against current, decreasing the local forces between molecules that determine the fluid's viscosity. Above critical density the local effects would somehow become global.

Cates *et al* have proposed this kind of phenomenon: see the article "Shearing Active Gels Close to the Isotropic-Nematic Transition" (see http://tinyurl.com/y9e3xl9v) [I11]. The authors speak in the abstract about zero apparent viscosity.

1. The title of the article of Cates *et al* tells that the phenomenon occurs near isotropic-nematic transition. Nematic is defined as a liquid crystal for which the molecules are thread-like and parallel. I dare guess that in the recent case the approximately parallel flagellae would be modelled as liquid crystal like 2-D phase at the surface of bacterium. In the isotropic phase the orientations of the flagellae would be uncorrelated and long range orientational correlations would emerge in the phase transition to nematic phase.

2. Also the notions of contractile and extensile gels are introduced. Contraction and extension of gels are though to occur through molecular motors. The transformation of the fluid to apparent superfluid would require energy to run the molecular motors using metabolic energy and ordinary superfluidity would not be in question.

3. The model predicts divergence of viscosity for contractile gels. For extensile gels a zero of apparent viscosity is predicted. There is a hydrodynamical argument for how this would occur but I did not understand it. The active behavior of the bacteria would means that the gel like surface phase (nematic liquid crystal) formed by the flagellae extends to reduce viscosity. If I have understood correctly, this applies only to the behavior of single bacterium and is about the reduction of viscosity in the immediate vicinity of cell.

My deep ignorance about rheology allows me freedom to speculate freely about the situation in TGD framework.

1. In TGD inspired biology gel phase corresponds to a phase, which involves flux tube connections between basic units. Flux tubes contain dark matter with non-standard value $h_{eff} = n \times h$. The h_{eff} changing phase transitions scaling the lengths of flux tubes proportional to h_{eff} are responsible for the contractions and extensions of gel.

   The extension of the gel should lead to a reduction of viscosity since one expects that dissipative effects are reduced as h_{eff} increases and quantum coherence is established in longer scales. Large h_{eff} phases are associated with criticality. Now the criticality would be associated with isotropic-nematic phase transition. The parallelization of flagellae would be due to the quantum coherence assignable with the flagellae.

   Note that the mechanism used by bacteria to control the liquid flow would be different since now molecular motors are replaced by h_{eff} changing phase transitions playing key role in TGD inspired view about biochemistry. For instance, reacting biomolecules find each other by h_{eff} reducing phase transition contracting the flux tubes connecting them.

2. This model does not yet explain the reduction of apparent viscosity to zero in the entire fluid occurring above a critical density of bacteria. What could happen could be analogous to the emergence of high T_c superconductivity according to TGD [K91] (`http://tinyurl.com/ycm6ao76`). Below pseudo gap temperature the emergence of magnetic flux tube pairs makes possible super-conductivity in short scales. At critical temperature a phase transition in which flux tubes reconnect to form larger thermodynamically stable networks occurs. One can speak about quantum percolation.

   The reduction of viscosity for a single bacterium could be based on the phase transition of liquid molecules to dark molecules flowing along the resulting flux tubes with very small friction (large $heff$) but only below certain scale smaller than the typical distance between bacteria. This would be the analog for what happens below pseudo gap. Above critical density he magnetic flux tubes associated with bacteria would reconnect and forming a net of connected flux tube paths at scale longer than inter-bacterial distances. This would be the counterpart for the emergence of superconductivity by percolation in long scales.

12.4.3 Bacteria behave like spin system: Why?

In Physorg there was an interesting article titled "Bacteria streaming through a lattice behave like electrons in a magnetic material" (see `http://tinyurl.com/hysxsl6`). The popular article tells about article with title Ferromagnetic and antiferromagnetic order in bacterial vortex lattices by Dunkel *et al* [I13] (see `http://tinyurl.com/ydbzvmcc`) . The following summarizes what has been studied and observed.

1. The researchers have studied a square lattice of about 100 wells with well radius below 50 microns and well depth about 18 microns. The wells are connected by thin channels. Also triangular lattice has been studied.

2. Below a critical radius about 35 microns an ordered flow is generated. The flow involves interior flow and edge flow in opposite direction consisting of single bacterium layer. One can understand this from angular momentum conservation. The coherence of this flow is however surprising. If one believes that each bacterium in principle chooses its swimming direction, one must understand what forces bacteria to select the same swimming direction.

3. Below a critical radius of channel about d=4 microns the flow directions in the neighboring wells are opposite for the square lattice. One has superposition of lattice and its dual with opposite flow directions. In the case of triangular lattice analogous cituation is encountered. In this situation there is no flow between the wells but there is an interaction. The minimization of dissipative losses requires minimization of velocity gradients inside channels. made possible by same local flow direction for the edge currents of neighboring wells.

4. Above the critical radius the flow changes its character. The flows synchronize and the interior flows rotate in the same direction as do also edge flows which occur also between the neighboring channels and give rise to closed flows around the boundaries of square like regions behind wells having larger scale. This flow pattern is consistent with angular momentum conservation: the angular momenta of lattice and its dual cancel each other.

5. The phase transition is analogous to that from antiferromagnetism to ferromagnetism. The total angular momenta of bacteria, their colonies, are analogous to spins. The situation can be modelled as 2-D Ising model consisting of lattice of spins with nearest neighbor interactions. Usually the spins are assigned with electrons but now they are assigned with bacteria.

This raises interesting questions. Bacteria swim by using flagellae. They can decide the swimming direction and control it by controlling the flagellae. Bacteria are living organisms and have a free will. Why would bacterium colory behave like quantal many-spin system. What happens when the swimming direction becomes same for the bacteria inside single well: does the colony become an entity with collective consciousness and do bacteria obey "social pressure". Does this happen also for the colony formed by these colonies in transition to ferromagnetism like state?

If one takes TGD inspired quantum biology as starting point, one can represent more concrete questions and possible answers to them.

1. Magnetic body (MB) controls the biological body (BB) be it organism or part of it [**?**]. MB contains dark matter as cyclotron Bose-Einstein condensates of bosonic ions. Pairs of parallel flux tubes could also contain members of Cooper pairs whose spin depends on whether the magnetic fields at flux tubes are parallel or antiparallel [K90, K91].

2. What could be the mechanism of control? MB is assumed to send dark photon signals from MB to biological body to control it and an attractive idea is that control is by angular momentum conservation. Since the angular momentum transfer involve is due to a phase transition analogous to the change of the direction of magnetization or generation of magnetization the angular momentum transfer is large irrespective of the value of unit of angular momentum for dark photon (see discussion below). This large angular momentum could be transformed to angular momentum of ordinary matter and in recent case be responsible for generating the rotational motion of bacterium or larger unit.

   The transfer of dark photons induced by a phase transition changing the direction of dark magnetization might thus induce a large transfer of angular momentum to BB and generate macroscopic rotation. If this were the case the rotational state of dark MB of bacterium would serve as a template for bacterium.

   The bacterium colony associated with the well below critical size would correspond to super-organism having MB whose rotational state could serve as template for the bacterial MBs in turn serving as a similar template for the bacteria.

3. If the net angular momenta of MB and corresponding BB (bacterium, well colony, colony of these) vanish separately, the model is consistent with the model of the article in which local considerations determine the rotational directions. In this case the MBs of well colonies would behave like spins with nearest neighbor interactions.

   One can also consider the possibility that at quantum criticality long range quantum fluctuations occur and the local equilibrium conditions do not hold true. Even more, the net angular momenta of MB and BB would cancel each other but would not necessarily separately. This would imply apparent non-conservation of angular momentum at the level of bacterium colony at criticality and might allow to find experimental support for the notion of magnetic body. The proof of MB carrying dark matter as a concept would be very much like that of neutrino the existence of which was deduced from apparenent energy non-conservation in beta decays.

The model has a problem to worry about. I still am not quite sure whether $h_{eff}/h = n$ means that the unit of spin is scaled up by n or that a fractionization of angular momentum by $1/n$ for single sheet of associated n-fold covering of space-time surface takes place. The control mechanism based on angular momentum conservation could however be at work in both cases. The option assuming fractionization seems to be the realistic one and only this will be considered in the following. Reader can ponder the option assuming scaled up unit of angular momentum (the scaling up of angular momentum of dark photon is not in coherence with the assumption that dark photon has same four-momentum as ordinary photon to which it should be able to transform).

1. Consider first the simplest variant for the effective fractionization of quantum numbers. If one has n-fold covering singular at the boundaries of CD then spin fractionization can be considered such that one has effectively n spin $1/n$ photons - one per sheet - and the net spin is just the standard spin. This picture fits with the vision that the n-fold covering means that one must make n full 2π turns before turning to the original point at space-time sheet: this allows at space-time surface wave functions with fractional spin which would be many-valued in Minkowski space. Similar fractionization would occur to other quantum numbers such as four-momentum so that net four-momentum would not change. The wavelength of these building bricks of dark photon analogous to Bose-Einstein condensate have frequencies scaled down by factor $1/n$.

   In this case the direct decay to single ordinary photon interpreted as bio-photon is allowed by conservation laws. Of course, also decays to several ordinary photons are possible. The decay to a bunch of n ordinary photons with total momenta $1/n$ times that of dark photon is possible if the spins of ordinary photons sum up to the spin of dark photon.

   The total angular momentum liberated from the cyclotron Bose-Einstein condensate spin could be transferred to spin of ordinary particles, say proton or ion for which the natural scale of orbital angular momentum is much larger (proportional to the rest energy). Simple order of magnitude estimate for orbital angular momentum with respect to the symmetry axis of possibly helical magnetic flux tube shows that in this case the spin could be transformed to angular momentum in the scale of organism and to the motion of organism itself.

   Note that dark photon could also decay to a bunch of ordinary photons with momentum scaled down by $1/n$ since the spins of the photons can sum up to spin 1.

2. A many-sheeted analog of second quantization generalizes the above picture. The n space-time sheets can be labelled by an integer $m = 1, ..., n$ defining an analog of discrete position variable. One can second quantize the fundamental fermions in this discrete space so that one has not only the ordinary many fermion states with $N = 0/1$ fermions in given mode but also states with fractionization of fermion number and other quantum numbers by $q = m/n < 1$ in a given mode. This would induce fractionization of bosons identified as fractional many-fermion states.

   Particle with fractional spin cannot decay directly to ordinary particle unless one has m=n: this correspond to the first option. Fractional particles characterized by q and 1-q can however fuse to ordinary particle. An attractive additional hypothesis is that the net quantum numbers are integer multiples of the basic unit.

   I have discussed the possibility of molecular sex: the opposite molecular sexes would have fractional charges summing up to ordinary charges. If magnetic bodies with opposite molecular sexes are paired they have ordinary total quantum numbers and can control ordinary matter by the proposed mechanism based on conservation of angular momentum (or some other charges). Dark matter would serve as template for ordinary matter and dark phase transitions would induce those of visible matter. The proposal that DNA, RNA, tRNA, and amino-acids are accompanied by dark proton sequences (or more general dark nuclei) could realize this picture. DNA double strand could be seen as an outcome of a molecular marriage in this framework! At higher level brain hemispheres might be a seen as a dark matter marriage. This picture can be also seen as emergence of symbols and dynamics based on symbol sequences at the molecular level with molecular marriage making possible very precise selection rules.

12.5 Dark variants of basic information molecules

Two highly interesting findings providing insights about the origins of life have emerged and it is interesting to see how they fit to the TGD inspired vision.

The group led by Thomas Carell has made an important step in the understanding the origins of life. They have identified a mechanism leading to the generation of purines A and G which besides pyrimidines A,T (U) are the basic building bricks of DNA and RNA. The crucial step is to make the solution involved slightly acidic by adding protons. For year later I learned that a variant of Urey-Miller experiment with simulation of shock waves perhaps generated by extraterrestrial impacts using laser pulses generates formamide and this in turn leads to the generation of all 4 RNA bases.

These findings represent a fascinating challenge for TGD inspired quantum biology. The proposal is that formamide is the unique amide, which can form stable bound states with dark protons and crucial for the development of life as dark matter-visible matter symbiosis. Pollack effect would generate electron rich exclusions zones and dark protons at magnetic flux tubes. Dark protons would bind stably with unique amine leaving its chemical properties intact. This would lead to the generation of purines and the 4 RNA bases. This would be starting point of life as symbiosis of ordinary matter and dark matter as large $h_{eff}/h = n$ phases of ordinary matter generated at quantum criticality induced by say extraterrestrial impacts. The TGD based model for cold fusion and the recent results about superdense phase of hydrogen identifiable in TGD framework as dark proton sequences giving rise to dark nuclear strings provides support for this picture.

There is however a problem: a reductive environment (with ability to donate electrons) is needed in these experiments: it seems that early atmosphere was not reductive. In TGD framework one can imagine two - not mutually exclusive - solutions of the problem. Either life evolved in underground oceans, where oxygen concentration was small or Pollack effect gave rise to negatively charged and thus reductive exclusion zones (EZs) as protons were transferred to dark protons at magnetic flux tubes. The function of UV radiation, catalytic action, and of shock waves would be generation of quantum criticality inducing the creation of EZs making possible dark $h_{eff}/h = n$ phases.

The first step: binding of dark protons to formamido-pyrimidine

I learned about very interesting discovery related to the problem of understanding how the basic building bricks of life might have emerged. RNA (DNA) has nucleotides A,G,C,U (T) as basic building bricks.

The first deep question is how the nucleotides A,G,C,U, and T emerged.

1. There are two types of nucleotides. Pyrimidines C and T/U (see `http://tinyurl.com/k3vxl9b`) have single carbon 6-cycle. Purines A and G (see `http://tinyurl.com/odvqw2p`) in turn have single 6-single and 5-cycle fused attached together along one side. Purines are clearly more complex than pyrimidines.

2. U.K. chemist John Sutherland demonstrated a plausible sequence of steps leading to the emergence of pyrimidines. Purines turned out to be more problematic. Leslie Orgel and colleagues suggested a possible pathway but it produces purines in too tiny amounts.

Now a group led by Thomas Carell in Ludwig Maximilian University have found a more plausible mechanism [I10] (see `http://tinyurl.com/z65kpyo`).

1. Carell and colleagues studied the interaction of biomolecule formamido-pyrimidine (FaPy) with DNA and found that it also reacts to produce purines. Could FaPys have served as precedessors of purines? (For formamide see `http://preview.tinyurl.com/lwqyqnu` and for the class of chemical compounds known as amines see `http://tinyurl.com/mad6c2u`).

2. The first step would have been a copious production of amino-pyrimidines containing several chemical groups known as amines. The problem is that the are so many amines and they normally react indiscriminantly to produce many different compounds. One wants mostly purines so that only one critical amine is wanted.

3. When Carell and his team added some acid to the solution to decrease its pH, a miracle happened. The extra protons from acid attached to the amines of the amino-pyrimidine and made them non-reactive. There was however one exception: just the amine giving rise to

purine in its reactions! The reactive amine also readily bonded with formic acid acid (see http://tinyurl.com/lmstt7n) or formamide. Hence it seems that one big problem has been solved.

The second challenge is to understand how the building bricks of RNA and DNA combined to form longer polymers and began to replicate.

1. One prevailing vision is that so called RNA world preceded the recent biology dominated by DNA. The goal has been to achieve generation of RNA sequence in laboratory. Unlike DNA RNA sequences are not stable and long sequences are difficult to generate. DNA in turn replicates only inside cell and the presence of what is known as ordered water seems to be essential for this.

2. This step might involve new physics and chemistry and I have considered the possibility that the new physics involves magnetic bodies and dark proton sequences as a representation of the genetic code at the level of dark nuclear physics. There is no need to add that the fact that dark proton states provide representations for RNA, DNA, tRNA, and amino-acids [K55, L1] looks like a miracle and I find still difficult to believe that it is true and for genetic code. Also the representation of vertebrate code emerges in terms of correspondences of dark proton states.

   This suggests that the replication of DNA and takes place at the level of dark proton sequencies - dark nuclear strings - serving as a dynamical template for the biological replication. Also transcription and translation would be induced by dark process. Actually all biochemical processes could have as template the dynamics of molecular magnetic bodies and biochemistry would be kind of shadow of deeper dynamics.

3. There is actually support for dark proton sequences. Quite recently I learned about the article of Leif Holmlid and Bernhard Kotzias [L32] (see http://tinyurl.com/hxbvfc7) about the superdense phase of hydrogen. In TGD superdense phase has interpretation as dark proton sequences at magnetic flux tubes with the Compton length of dark proton coded by $h_{eff}/h \simeq 2^{11}$ to electron's Compton length [L9]. Remarkably, it is reported that the superdense hydrogen is super-conductor and super-fluid at room temperatures and even above: this is just what TGD predicts.

   The dark protons in TGD inspired quantum biology [L16] should have much longer Compton length of order of the distance between nucleotides in DNA sequences in order to serve as templates for chemical DNA. This gives a dark Compton length of order $\simeq 3.3$ Angstroms from the fact that there are 10 codons per 10 nm. This gives $h_{eff}/h \simeq 2^{18}$.

One can return back to the first step in the genesis of DNA and RNA. The addition of protons to the solution used to model prebiotic environment to make it slightly acidic was the key step. Why?

1. Here cold fusion might help. Cold fusion is claimed to take place in electrolysis involving ionization and charge separation. The electric fields used in electrolysis induce ionization and thus charge separation. For me it has however remained a mystery how electric fields, which are extremely tiny using the typical strength of molecular electric field as standard are able to induce a charge separation. Of course, every chemist worth of his salt regards this as totally trivial problem. I am however foolish enough to consider the possibility that some new physics might be involved.

2. The mechanism causing charge separation could be analogous to or that discovered by Pollack as he irradiated water bounded by a gel phase [L7] [L7]: in the recent case the electric field would take the role of irradiation as a feeder of energy. Negatively charged exclusion zones (EZs) were formed and 1/4 of protons went somewhere.

   The TGD proposal is that part of protons went to magnetic flux tubes and formed dark proton sequences identifiable as dark nuclear strings. The scaled down nuclear binding energy favours the formation of dark nuclear strings perhaps proceeding as analog of nuclear chain reaction. This picture allows to ask whether dark proton sequences giving rise to a fundamental representation of the genetic code could have been present already in water [L16]!

3. How DNA/RNA could have then formed? Could the protons making the solution acidic be dark so that the proton attaching to the amine would be dark? Could it be that for all amines except the right one the proton transforms to ordinary proton and destroys the chemical reactivity. Could the attached dark proton remain dark just for the correct amine so that the amine would remain reactive and give rise to purine in further reactions? Could A,G,C,T and U be those purines and pyrimidines - or even more general biomolecules - for which the attachment to dark proton does not transform it to ordinary proton and in this manner affect dramatically the chemical properties of the molecule? What is the condition for the preservation of the darkness of the proton?

Second step: Could shock waves due to extraterrestrial impacts have produced RNA bases?

About year later I learned about a further interesting finding related to the prebiotic evolution (see the popular article at `http://tinyurl.com/m8npeor`). The conclusion of the research article (see [I14]) is that that the extraterrestrial impacts on Earth's early atmosphere might have generated all 4 RNA bases (see `http://tinyurl.com/kxxc7db`). Also now the formamide is involved and my layman guess is that the motivation for this comes from the experiment of Carell *et al* [I10] (see `http://tinyurl.com/z65kpyo`) discussed above. If formamide is generated then it becomes possible to generate formamido-pyridine and from this the RNA bases can be generated.

The experiment was a modern version of Urey-Miller experiment originally intended to simulate the situation at the surface of the early atmosphere modelled as a mixture a water H_2O, carbon-monoxide CO, and ammonium NH_3. The shock waves generated by the impacts were modelled in the experiment using terawatt laser pulses.

In the original Urey-Miller experiment amino-acids were generated. In the modern version of the experiment it was found that also formamide $CONH_3$ is formed, whose presence under suitable circumstances can lead to the generation of all 4 RNA bases. The presence of UV radiation, shock waves caused by extraterrestrial collisions, or of catalyst is the necessary condition.

In TGD Universe the additional condition could guarantee quantum criticality accompanied by dark $h_{eff}/h = n$ phases leading to the generation of dark protons and their stable binding with formamido-pyrimidine. The stable binding would not be possible for other amido-pyrimidines since dark protons would transform to ordinar protons for them. All 4 RNA bases would emerge from formamido-pyrimidine. All basic molecules of life could be produced in the reductive atmosphere.

The atmosphere was assumed to be reductive and this is a problem: the best that one can hope is that the early atmosphere was weakly reductive. Chemical compound is reductive (see `http://tinyurl.com/m9cqnob`) if it tends to donate electron. Reduction means receiving electron - and in chemistry hydrogen atom. To obtain a reducing atmosphere (see `http://tinyurl.com/lx4tat2`) one should remove oxygen from it. It however seems that the early atmosphere has contained oxygen and was oxidative rather than reductive. How could one overcome the problem?

1. In the experiment of Carell *et al* protons were added to reduce the pH of water. The basic experimental rule is that this makes the environment more reductive. The TGD proposal is that it led to a formation of dark proton-amine pair for the amine leading to the formation of purine. Charge separation by Pollack effect [L7] [L16] leading to the generation of dark proton sequences (dark nuclei) at magnetic flux tubes could have been due to the IR radiation, and maybe also by UV radiation, catalytic action, or by shock waves. The presence of electrons in the exclusion zones (EZs) could have made them electron donors and therefore reductive.

   The addition of protons in the experiment of Carell reducing the pH of water could have induced a transformation of dark protons at magnetic flux tube to ordinary protons. Dark protons bound to the amines would have transformed to ordinary protons and inducing their chemical inactivity. Only for the amine formamide serving as a precursor of purine the dark proton-amine bound state was stable and remained chemically reactive since dark proton did not affect the properties of visible matter part of the compound. Symbiosis between dark and ordinary matter began. This view conforms also with the vision about the pairing of DNA/RNA and dark DNA/RNA formed by sequences of proton triplets representing DNA/RNA codons [L20]. DNA is indeed negatively charged and dark proton could neutralize it but allow it to remain chemically active.

2. Second possibility is suggested by the conjecture that prebiotic life evolved in the crust of Earth, perhaps in the underground oceans or regions related to volcanoes [L64, L16]. The content of oxygen of this environment could have been much lower than at the surface making it reductive: it would not be possible to even talk about atmosphere. But where did the metabolic energy come from? Could volcanic energy emitted as dark long wave photons with energies in the range of bio-photon energies help here? There are indeed a theories assuming that first life forms emerged from volcanoes. These problems are discussed in [L64, L16] from TGD viewpoint. Note that these two explanations do not exclude each other.

12.5.1 Could the replication of mirror DNA teach something about chiral selection?

I received a link to a very interesting popular article (see `http://tinyurl.com/zqgutdv`) from which I learned that short strands of mirror DNA and mirror RNA - known as aptamers - have been be produced commercially for decades - a total surprise to me. Aptamers bind to targets like proteins and block their activity and this ability can be utilized for medical purposes.

Now researchers at Tsinghua University of Beijing have been able to create a mirror variant of an enzyme - DNA polymeraze - catalyzing the transcription of mirror DNA to mirror RNA also replication of mirror DNA [I26]. What is needed are the DNA strand to be replicated or transcribed, the mirror DNA nucleotides, and short primer strand (see `http://tinyurl.com/j3o8cyx)`) since the DNA polymeraze starts to work only if the primer is present. This is like recalling a poem only after hearing the first few words.

The commonly used DNA polymerase containing about 600 amino-adics is too long to be built up as a right-handed version and researchers used a much shorter version: African swine fever virus having only 174 amino-acids. The replication turned out to be very slow. A primer of 12 nucleotides was extended to a strand of 18 nucleotides in about 4 hours: 3/2 nucleotides per hour. The extension to a strand of 56 nucleotides took 36 hours making 44/36= 11/9 nucleotides per hour. DNA and its mirror image co-existed peacefully in a solution. One explanation for the absence of mirror life is that the replication and transcription of mirror form was so slow that it lost the fight for survival. Second explanation is that the emergence of mirror forms of DNA polymerase and other enzymes was less probable.

Can one learn anything about this?

1. Chiral selection is one of the deep mysteries of biology. Amino-acids are left-handed and DNA and RNA double strands form a right-handed screw. One can assign handedness with individual DNA nucleotides and with DNA double strand but web sources speak only about the chirality of double strand. If the chirality of the DNA nucleotides were not fixed, it would have been very probably discovered long time ago as an additional bit doubling the number of DNA letters.

2. What could be the origin of the chirality selection? Second helicity could have been loser in the fight for survival and the above finding supports this: fast ones eat the slow ones like in market economy. There must be however a breaking of mirror symmetry. Weak interactions break of mirror symmetry but the breaking is extremely small because the weak bosons mediating weak interaction are so massive that the length scale in which the breaking of mirror symmetry matters is of order 1/100 times proton size. This breaking is quite too small to explain chiral selection occurring in nano-scales: there is discrepancy of 8 orders of magnitude. The proposal has been that the breaking of mirror symmetry has been spontaneous and induced by a very small seed. As far as I know, no convincing candidate for the seed has been identified.

According to TGD inspired model chiral selection would be induced from that in dark matter sector identified in terms of phases of ordinary matter with non-standard value of Planck constant $h_{eff}/h = n$ [**?**, K84]. In living matter dark matter would reside at magnetic flux tubes and control ordinary matter. TGD predicts standard model couplings, in particular weak parity breaking. For $h_{eff}/h = n$ the scale below which weak bosons behave as massless particles implying large parity breaking is scaled up by n. Large parity breaking for dark matter becomes possible in even biological length scales for large enough h_{eff}.

The crucial finding is that the states of dark proton regarded as part of dark nuclear string can be mapped naturally to DNA, RNA, tRNA, and amino-acid molecules and that vertebrate genetic code can be reproduced naturally [K55]. This suggests that genetic code is realized at the level of dark nuclear physics and induces its chemical variant. More generally, biochemistry would be kind of shadow of dark matter physics. A model for dark proton sequences and their helical pairing is proposed and estimates for the parity conserving and breaking parts of Z^0 interaction potential are deduced.

Dark matter and chirality selection

In TGD framework the hierarchy of Planck constants suggests an explanation for the chirality selection.

1. In TGD Universe the new physics of quantum biology involves magnetic bodies and dark proton sequences as a representation of the genetic code at the level of dark nuclear physics. The crucial observation is that dark proton states provide representations for RNA, DNA, tRNA, and amino-acids [K55, L1] and there is also natural map between DNA and amino-acid type states giving rise to vertebrate genetic code. This looks like a miracle and I find still difficult to believe that it is true. A The extreme slowness of the wrong-handed DNA replication as compared to the ordinary replication means large breaking of parity symmetry. This is possible to understand in terms of weak interactions only if they are dark in DNA length scales so that weak bosons are effectively massless and weak interactions are as strong as electromagnetic interactions.

   This suggests that the replication of DNA and takes place at the level of dark proton sequencies - dark nuclear strings - serving as a dynamical template for the biological replication. Also transcription and translation would be induced by dark processes. Actually all biochemical processes could have as template the dynamics of molecular magnetic bodies and biochemistry would be kind of shadow of dark matter physics.

   If this is the case, then chiral selection would take place the selection at the level of dark nuclear strings and induce that the level of biochemistry. If dark and ordinary chiralities fit together like hand and glove. Dark matter at magnetic bodies could control the behavior of ordinary matter. By parity breaking the dark weak binding energy between members of proton pairs in the dark DNA strand consisting of a pair of helical dark proton strings is higher for the second helical chirality and would favour this chirality. A very naïve thermodynamical estimate is that the ratio of the densities of two chiralities is proportional to the Boltzmann exponent $exp(-\Delta E_B/T)$. The transition to thermodynamical equilibrium can be however very slow so that thermodynamical argument need not make sense.

2. There is experimental support for dark proton sequences. Leif Holmlid and Berhard Kotzias [L32] (see `http://tinyurl.com/hxbvfc7`) have published an article about the superdense phase of hydrogen proposed to make possible to overcome the Coulomb wall making cold fusion impossible in the textbook Universe. In TGD superdense phase has interpretation as dark proton sequences at magnetic flux tubes with the Compton length of dark proton coded by $h_{eff}/h = n_{eff} \simeq 2^{11}$ to electron's Compton length [L9]. Remarkably, it is reported that the superdense hydrogen is super-conductor and super-fluid at room temperatures and even above: this is just what TGD predicts.

   The dark protons in TGD inspired quantum biology (see `http://tinyurl.com/lwxdl7y`) should have much longer Compton length of the order of the distance between nucleotides in DNA sequences in order to serve as templates for chemical DNA. This gives a dark Compton length of order $\simeq 3.3$ Angstroms from the fact that there are 10 codons per 10 nm. This would give $n_{eff,p} \simeq 2^{18}$. The safest manner to estimate the dark binding energy is by scaling the binding energy about $E_B \simeq 7$ MeV per nucleon by $1/n_{eff,p}$ to give $E_{B,d} = E_B/n_{eff,p} = 28$ eV.

3. Further evidence for the importance of dark protons in biology comes from the recent finding of the group led by Thomas Carell related to the understanding the origins of life [I10] (see `http://tinyurl.com/z65kpyo`). For TGD inspired model see [L30], [?]. Carell *et al*

have identified a mechanism leading to the generation of purines A and G, which besides pyrimidines A,T (U) are the basic building bricks of DNA and RNA. The crucial step is to make the solution involved slightly acidic by adding protons.

In TGD inspired quantum biology this suggest that the protons in the acidic water are dark and that the attachment of the dark protons to the amines of the amino-pyrimidine transforms them to ordinary protons and makes the amino-pyrimidine non-reactive. There would be however one exception: the amine which reacts further to give purines as a reaction product. In this case the proton would remain dark and the chemical properties of the amine would remain intact. This suggests that DNA nucleotides and DNA strands can attach to dark protons or are accompanied by them.

Model for the replication of DNA

One can consider a detailed model for the replication as induced by the addition of dark protons to dark proton sequence representing dark DNA strand. The added dark protons would be accompanied or attached with the DNA nucleotides as suggested by the work of Carell *et al.*

1. In the replication and transcription of DNA the basic step would be the addition of dark proton to an increasing dark proton sequence. The need for primer means that there must already exist a dark proton sequence. In the presence of prime the attractive dark nuclear binding energy of the added dark proton with the prime would make the dark fusion rate higher. The addition of dark protons could proceed like a dark nuclear chain reaction. It would be made possible by the dark nuclear binding energy per proton scaling like $1/h_{eff,p}$.

   For the ordinary nuclei the binding energy per nucleon would be of the order of 7 MeV (note that charge independence of strong interactions holds in good approximation). The scaling down by $h_{eff}/h = 2^{18}$ would give $E_B \simeq 4$ eV, which corresponds to UV photon energy. Note that bio-photons assumed to correspond dark photons with same energy have energies in visible and UV range.

2. Dark nuclear energy cannot explain parity breaking. The axial part of dark weak energy between dark protons belonging to dark strand and its conjugate and having nuclei acids and its conjugate as a chemical "shadow" must be also involved. Two values of h_{eff} are involved: $h_{eff,p}$ assignable to the flux tubes containing dark protons parallel to DNA strands and $h_{eff,W}$ assignable to the transversal flux tube connecting dark protons associated with different dark strands.

   One of the assumptions of the TGD inspired model of cold fusion [L9, L32] is that the weak scale is scaled up from weak boson Compton length to about atomic length scale. This would require $h_{eff,W}/h = n_{eff,W}$ for weak bosons to be roughly

   $$n_{eff,W} \simeq \frac{m_Z}{m_p} \times n_{eff,p} \simeq 91 \times n_{eff,p}$$

   so that one would have $n_{eff,W} \simeq 2^{25}$. If this is the case weak interactions are of essentially same strength as em interaction below the scaled up Compton scale of order 3 Angstroms. This makes it possible to talk about classical Z^0 Coulomb potential and about spin dependent parity breaking Z^0 force. These two interaction energies sum up and this reduces the binding energy per proton in double strand for the other chirality.

3. The parity conserving Z^0 Coulomb interaction energy between two protons at different strands connected by a flux tube is given by the expression

   $$\begin{aligned} &V_{PC}(r_{12}) = -kV(r_{12}) \ , \quad V(r_{12}) = \frac{\hbar}{r_{12}} \ , \\ &k = \alpha_Z Q_Z^2(p) \ , \quad \alpha_Z = \frac{\alpha}{sin^2(\theta_W)cos^2(\theta_W)} \ , \quad Q_Z(p) = 1/4 - sin^2(\theta_W) \ . \end{aligned} \tag{12.5.1}$$

Here units $\hbar = 1$, $c = 1$ are used. r_{12} refers to the distance between dark protons at magnetic flux tubes assignable to DNA strands. Base pair thickness is about .34 nm and thickness of DNA double strand is about 2 nm. r_{12} could be between these two limits.

4. The spin dependent and parity non-conserving Z^0 interaction potential for Dirac spinors proportional to the gradient of the Z^0 Coulomb potential can be written as

$$V_{PNC} = \alpha_Z Q_Z^A(p) Q_Z^V(p)\gamma_5 V(r_{12}) \ . \qquad (12.5.2)$$

Here $Q_Z^A = I_{3,A}/2 = 1/4$ is the axial weak charge of proton. The vectorial charge of proton is $Q_Z(p) = 1/4 - sin^2(\theta_W) \simeq 0.02$ so that it is much smaller than $Q_Z^A(p)$. Hence the axial force dominates by a factor $10^2/8 \sim 12.5$ for a given relative position. Usually the axial part becomes very small by symmetries as one estimates quantum averages but in the recent situation one cannot expect this since the positions of dark protons are in the first approximation fixed.

5. Using non-relativistic correspondence following from $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$ and $(\gamma_5)^2 = -1$: this equation holds true also for $(\gamma^0\gamma^k p_k(m)$, and one has

$$\gamma_5 \rightarrow \frac{\overline{\sigma}\cdot p}{m_p} \ .$$

Here $\overline{\sigma}$ denotes Pauli sigma matrices expressible as $\gamma^0\gamma^i$). Using the replacement $p \leftrightarrow i\hbar_{eff,W}\nabla$ one can write V_{PNC} as the sum of the axial energies of the two protons

$$\begin{aligned} V_{s_1,s_2} &= V_{s_1} + V_{s_2} \ , \\ V_{s_i} &= \frac{\hbar_{eff,W}}{m_p}\overline{\sigma}_i\cdot\nabla_i V_{PC}(r_{12}) = (-1)^i\frac{kn_{eff,W}\hbar}{m_p}\hbar\frac{\overline{\sigma}_i\cdot\overline{r}_{12}}{r_{12}^2} \ . \ i = 1,2 \ . \end{aligned} \qquad (12.5.3)$$

The parity breaking part of Z^0 force is proportional to $n_{eff,W}$ from the expression of momentum operator in terms of gradient operator so that dark matter physics makes itself visible and increases further the magnitude of parity breaking. The potential energy changes sign in reflection $\overline{r}_{12} \rightarrow -\overline{r}_{12}$. This gives

$$\begin{aligned} V_{s_1,s_2} &= -\frac{\alpha_Z}{4}(\frac{1}{4} - sin^2(\theta_W))\frac{n_{eff,W}\hbar}{m_p r_{12}}\frac{(\overline{\sigma}_1 - \overline{\sigma}_2)\cdot\overline{r}_{12}}{r_{12}}\frac{\hbar}{r_{12}} \\ &= \frac{1}{4}\frac{1}{(\frac{1}{4} - sin^2(\theta_W)}\frac{n_{eff,W}\hbar}{m_p r_{12}}\frac{(\overline{\sigma}_1 - \overline{\sigma}_2)\cdot\overline{r}_{12}}{r_{12}}V_{PC}(r_{12}) \ . \end{aligned} \qquad (12.5.4)$$

6. For the vectorial part one has

$$V_{PC} = -\alpha_Z(\frac{1}{4} - sin^2(\theta_W))^2 V(r_{12}) \ . \qquad (12.5.5)$$

The order of magnitude is about $V_Z = .16/x$ eV.

7. The condition that r_{12} corresponds to dark Compton length of proton implies in the first approximation $\frac{n_{eff,p}}{m_p r_{12}} = 1$ so that $n_{eff,W}$ proportionality gives factor $m_Z/m_p \simeq 91$. The order of magnitude parity breaking potential is the value potential at distance in the range $r_{12} \in [3.4, 2]$ nm. Let us expres the horizontal distance between the paired dark protons as $r_{12} = x$ Angstroms. This gives for the axial part

$$\begin{aligned} V_{s_1,s_2} &= \frac{1}{4}\frac{1}{(\frac{1}{4} - sin^2(\theta_W))}\frac{m_Z}{m_p}(\overline{\sigma}_1 - \overline{\sigma}_2) \cdot \frac{\overline{r}_{12}}{r_{12}} V_{PC}(r_{12}) \\ &\simeq .5 \times 10^2 \times 91 \times \frac{V_{PC}(r_{12})}{x} \times (\overline{\sigma}_1 - \overline{\sigma}_2) \cdot \frac{\overline{r}_{12}}{r_{12}} \quad . \end{aligned} \tag{12.5.6}$$

The order or magnitude for the axial part is roughly $4550/x$ times larger than for the vectorial part. V_{PNC} is proportional to $1/x^2$ and V_{PC} to $1/x$. The condition that the states are spin eigenstates requires that spin quantization axes must be chosen along the flux tube connecting the dark protons. This is rather natural choice.

This would give for the axial part order of magnitude $V_{PNC} \sim 728/x^2$. For 2 nm distance one would obtain $V_{PNC} \sim 1.82$ eV. For 1 nm distance one would have $x = 10$ and this would give $V_{PNC} \simeq 7.28$ eV. For this value $V_{PC} \simeq 16$ meV, which is of same order of magnitude as thermal energy $kT/2$ at room temperature.

8. The process of adding dark protons to the increasing DNA sequence must be possible irrespectively of the direction of spin. The spin eigenvalue in the direction of the horizontal axis connecting the members of dark proton pair is assumed to be opposite for the members of the dark proton pairs of dark double strand. This assumption comes from the model of the dark genetic code. This demands that V_{NPC} is considerably smaller than strong binding energy E_B. For 1 nm distance one has $V_{PNC} \simeq 7.28$ eV considerably smaller than $E_B \simeq 28$ eV.

9. What is the relation of the fermionic chirality to the geometric chirality? The reflection for dark protons induces the reflection of the entire helix turning also its direction. The reflection permutes the dark protons of each pair since their positions are related by reflection in the plane orthogonal to z-axis $(x_2, y_2) = (-x_1, -y_1)$. One has $(x_1, y_1, z) \leftrightarrow (x_2, y_2, -z)$. A further rotation of π in say (x, z)-plane around say y-axis is symmetry and gives $(x_2, y_2, -z) \rightarrow (-x_2, y_2, z) = (x_1, -y_1, z)$. Hence the net effect is $(x_1, y_1, z) \rightarrow (x_1, -y_1, z)$ and DNA strand with an opposite screw direction is generated.

   The model of dark genetic code motivates the assumption that the dark protons of the pair are spin eigenstates for the spin projection along the axis connecting the members of the pair. The direction of the spin quantization axis changes in reflection from that given by (x_1, y_1) to that given by $(x_1, -y_1)$ so that the states are not anymore eigenstates of the spin projection along this axis. Thus the fermionic chirality indeed correlates with the chirality of double strand and the two chiralities are in physically different position.

   What happens at the level of classical fields? Kähler magnetic field transforms like angular momentum in reflections and rotations as is easy to see from its expression in terms of vector potential. Hence it does not change its direction in reflection but changes its direction in the rotation. Hence the magnetic flux along flux tube changes to opposite in the reflection. This also affects the physics and induces effects at the level of dark strong interactions. The magnetic energy is of form $s \cdot B$ and vanishes classically. Quantum mechanically it does not vanish since s is operator and one can wonder what this implies physically.

Differences between standard model and TGD based description

The above estimate relies on standard model, which is quantum field theory in Minkowski space, and one can wonder what new elements TGD brings in. I do not try to estimate the effects in TGD framework but just list the differences.

1. In TGD framework space-time is 4-surface in $M^4 \times CP_2$ and this description must be replaced with a description using 8-D embedding spinors. At space-time level massive M^4 Dirac equation $p_k\gamma^k\Psi = m\Psi$ is replaced by 8-D chiral symmetry implying separate conservation of quark and lepton numbers with the analog of massless Dirac equation for the Kähler-Dirac gamma matrices, which are superpositions of M^4 and CP_2 gamma matrices. K-D gamma matrices are contractions of canonical momentum current densities of Kähler action with the embedding space gamma matrices. If the action is volume term, one obtains induced gamma matrices. The twistorialization of TGD by replacing the embedding space with the product of twistor spaces of M^4 and CP_2 and lifting space-time surfaces to their twistor spaces with induced twistor structure leads to the addition of volume term to Kähler action [K50]. This term corresponds to cosmological constant and is extremely small in the recent cosmology.

2. One can decompose K-D gamma matrices to their M^4 and CP_2 parts: $\Gamma^\alpha = \Gamma^\alpha_{M^4} + \Gamma^\alpha_{CP_2}$ and write the K-D equation as $\Gamma^\alpha_{M^4} D_\alpha\Psi = -\Gamma^\alpha_{CP_2}\Psi$. The presence of $\Gamma^\alpha_{CP_2}$ parts breaks conservation of M^4 chirality and serves as a signal for massivation. This operator is kind of mass operator acting non-trivial in electroweak spin degrees of freedom assignable to CP_2 and the action of its square is analogous to the action of mass squared operator.

   The understanding of particle massivation at this level does not seem however possible and the proper approach relies of p-adic thermodynamics for super-Virasoro representations for which ground states are characterized by the modes of embedding space spinors which are massless in 8-D sense and are eigenstates of M^4 mass squared operator with eigenvalues determined by CP_2 spinor Laplacian [K66]. Its action on M^4 chirality is same as action of mass in massive Dirac equation in M^4.

3. In the case of M^4 Dirac equation the multiplication of massive Dirac equation with γ_5 using anti-commutativity of γ_5 and γ_k gives $\gamma^k p_k\gamma_5\Psi = -m\gamma_5\Psi$ instead of$p_k\gamma^k\Psi = m\Psi$. TGD framework γ_5 anti-commutes with $\Gamma^\alpha_{M^4}$ but commutes with $\Gamma^\alpha_{CP_2}$ so that also now one has similar equation $\Gamma^\alpha_{M^4} D_\alpha\Psi = +\Gamma^\alpha_{CP_2}\Psi$.

12.5.2 Is dark DNA dark also in TGD sense?

I encountered a highly interesting article about "dark DNA" hitherto found in the genome of gerbils and birds, for instance in the genome of of the sand rat living in deserts (see `http://tinyurl.com/y8zdgnej`). The gene called Pdxl related to the production of insulin seems to be missing as also 87 other genes surrounding it! What makes this so strange that the animal cannot survive without these genes! Products that the instructions from the missing genes would create are however detected!

According to the ordinary genetic, these genes cannot be missing but should be hidden, hence the attribute "dark" in analogy with dark matter. The dark genes contain A lot of G and C molecules and this kind of genes are not easy to detect: this might explain why the genes remain undetected.

A further interesting observation is that one part of the sand rat genome has many more mutations than found in other rodent genomes and is also GC rich. Could the mutated genes do the job of the original genes? Missing DNA are found in birds too. For instance, the gene for leptin - a hormone regulating energy balance - seems to be missing.

The finding is extremely interesting from TGD view point, where dark DNA has very concrete meaning. Dark matter at magnetic flux tubes is what makes matter living in TGD Universe. Dark variants of particles have non-standard value $h_{eff} = n \times h$ of Planck constant making possible macroscopic quantum coherence among other things. Dark matter would serve as template for ordinary matter in living systems and biochemistry could be kind of shadow of the dynamics of dark matter. What I call dark DNA would correspond to dark analogs of atomic nuclei realized as dark proton sequences with entangled proton triplet representing DNA codon. The model predicts correctly the numbers of DNA codons coding for given amino-acid in the case of vertebrate genetic code and therefore I am forced to take it very seriously [L20, L12] (see `http://tinyurl.com/jgfjlbe` and `http://tinyurl.com/ydb2tfy8`).

The chemical DNA strands would be attached to parallel dark DNA strands and the chemical representation would not be always perfect: this could explain variations of DNA. This picture inspires also the proposal that evolution is not a passive process occurring via random mutations with survivors selected by the evolutionary pressures. Rather, living system would have R&D lab as one particular department. Various variants of DNA would be tested by transcribing dark DNA to ordinary mRNA in turn translated to amino-acids to see whether the outcome survives. This experimentaiton might be possible in much shorted time scale than that based on random mutations. Also immune system, which is rapidly changing, could involve this kind of R&D lab.

Also dark mRNA and amino-acids could be present but dark DNA is the fundamental information carrying unit and it would be natural to transcribe it to ordinary mRNA. Of course, also dark mRNA could be produced and translated to amino-acids and even dark amino-acids could be transformed to ordinary ones. This would however require additional machinery.

What is remarkable is that the missing DNA is indeed associated with DNA sequences with exceptionally high mutation rate. Maybe R&D lab is there! If so, the dark DNA would be dark also in TGD sense! Why GC richness should relate to this, is an interesting question.

12.5.3 Clustering of RNA polymerase molecules and Comorosan effect

Once again I had good luck: I received a link (see `http://tinyurl.com/y7bego83`) to a highly interesting popular article telling about the work by Ibrahim Cisse at MIT and his colleagues [I12] (see `http://tinyurl.com/y9wzt5yl`) about the clustering of RNA polymerase proteins in the transcription of RNA. Similar clustering has been observed already earlier and interpreted as a phase separation giving rise to protein droplets [L67]. Now this interpretation is not proposed by experiments but they say that it is quite possible but they cannot prove it.

I have already earlier discussed the coalescence of proteins into droplets as this kind of process in TGD framework [?] [L67]. The basic TGD based ideas is that proteins - and biomolecules in general - are connected by flux tubes characterized by the value of Planck constant $h_{eff} = n \times h_0$ for the dark particles at the flux tube. The higher the value of n is the larger the energy of given state. For instance, the binding energies of atoms decrease like $1/n^2$. Therefore the formation of the molecular cluster liberates energy usable as metabolic energy.

Remark: h_0 is the minimal value of h_{eff}. The best guess is that ordinary Planck constant equals to $h = 6h_0$ [L26, L60] (see `http://tinyurl.com/goruuzm` and `http://tinyurl.com/y9jxyjns`.

TGD view about the findings

Gene control switches - such as RNA II polymerases in DNA transcription to RNA - are found to form clusters called super-enhancers. Also so called Mediator proteins form clusters. In both cases the number of members is in the range 200-400. The clusters are stable but individual molecules spend very brief time in them. Clusters have average lifetime of $5.1 \pm .4$ seconds.

Why the clustering should take place? Why large number of these proteins are present although single one would be enough in the standard picture. In TGD framework one can imagine several explanations. One can imagine at least following reasons.

1. If the initiation of transcription is quantum process involving state function reduction, clustering could allow to make this process process deterministic at the level of single gene in spite of the non-determinism of state function reduction. Suppose that the initiation of transcription is one particular outcome of state function reduction. If there is only single RNA II polymerase able to make only single trial, the changes to initiate the transcription are low. This could be the case if the protein provides metabolic energy to initiate the process and becomes too “tired” to try again immediately. In nerve pulse transmission there is analogous situation: after the passing of the nerve pulse generation the neuron has dead time period. As a matter of fact, it turns out that the analogy could be much deeper.

   How to achieve the initiation with certainty in this kind of situation? Suppose that the other outcomes do not affect the situation appreciably. If one particular RNA polymerase fails to initiate it, the others can try. If the number of RNA transcriptase molecule is large enough, the transcription is bound to begin eventually! This is much like in fairy tales about princess and suitors trying to kill the dragon to get the hand of princess. Eventually comes the penniless swineherd.

2. If the initiation of transcription requires large amount of metabolic energy then only some minimal number of N of RNA II polymerase molecules might be able to provide it collectively. The collective formed by N molecules could correspond to a formation of magnetic body (MB) with a large value of $h_{eff} = n \times h_0$ and controlling the molecules and inducing its coherent behavior. The molecules would be connected by magnetic flux tubes.

3. If the rate for occurrence is determined by an amplitude which is superposition of amplitudes assignable to individual proteins the rate is proportional to N^2, N the number of RNA II polymerase molecules. The process for the cluster is reported to be surprisingly fast as compared to the expectations - something like 20 seconds. The earlier studies have suggests that single RNA polymerase stays at the DNA for minutes to hours.

   Clustering could allow to speed up bio-catalysis besides the mechanism allowing to find molecules to find by a reduction of $h_{eff}/h = n$ for the bonds connecting the reactants and the associated liberation of metabolic energy allowing to kick the reactants over the potential wall hindering the reaction.

Concerning the process of clustering there are two alternative options both relying on the model of liquid phase explaining Maxwell's rule assuming the presence of flux tube bonds in liquid and of water explaining its numerous anomalies in terms of flux tubes which can be also dark (see `http://tinyurl.com/ydhknc2c`).

1. **Option I**: Molecules could form in the initial situation a phase analogous to vapour phase and there would be very few flux tube bonds between them. The phase transition would create liquid phase as flux tube loops assignable to molecules would reconnect form flux tube pairs connecting the molecules to a tensor network giving rise to quantum liquid phase. The larger then value of n, the longer the bonds between molecules would be. This kind of model [L46] (see `http://tinyurl.com/yassnhzb`) is used to explain the strange findings that a system consisting of plastic balls seems to show primitive features of life such as metabolism.

2. **Option II**: The molecules are in the initial state connected by flux tubes and form a kind of liquid phase and the clustering reduces the value of $h_{eff}/h = n$ and therefore the lengths of flux tubes. This would liberate dark energy as metabolic energy going to the initiation of the transcription. One could indeed argue that connectedness in the initial state with large enough value of n is necessary since the protein cluster must have high enough "IQ" to perform intelligent intentional actions.

Protein blobs are said to be drawn together by the "floppy" bits (pieces) of intrinsically disordered proteins. What could this mean in the proposed picture? Disorder would mean absence of correlations between building bricks of floppy parts of the proteins in translational degrees of freedom.

1. Could floppiness correspond to low string tension assignable to long flux loops with large n assignable to the building bricks of "floppy" pieces of protein? Could reconnection for these loops give rise to pairs of flux tubes connecting the proteins in the transition to liquid phase (Option I)? Floppiness would also make possible to scan the environment by flux loops to get in touch with the flux loops of other molecules and in the case of hit (cyclotron resonance) induce reconnection.

2. In spite of floppiness in this sense, one could have quantum correlations between the internal quantum numbers of the building bricks of the floppy pieces. This would also increase the value of n serving as molecular IQ and provide molecule with higher metabolic energy liberated in the catalysis.

About Comorosan effect and clustering of RNA II polymerase proteins

What about the interpretation of the time scales τ equal 5, 10, and 20 seconds appearing in the clustering of RNA II polymerase proteins and Mediator proteins? What is intriguing that so called Comorosan effect [I24, I6] involves time scale of 5 seconds and its multiples claimed by Comorosan long time ago to be universal time scales in biology. The origin of these time scales has remained more or less a mystery although I have considered several TGD inspired explanations for this time scale is based on the notion of gravitational Planck constant [K129] (see http://tinyurl.com/yb8fw3kq).

One can consider several starting point ideas, which need not be mutually exclusive.

1. The time scales τ associated with RNA II polymerase and perhaps more general bio-catalytic systems as Comorosan's claims suggest could correspond to the durations of processes ending with "big" state function reduction. In zero energy ontology (ZEO) there are two kinds of state function reductions [L52]. "Small" state function reductions - analogs of weak measurements - leave the passive boundary of causal diamond (CD) unaffected and thus give rise to self as generalized Zeno effect. The states at the active boundary change by a sequence of unitary time evolutions followed by measurements inducing also time localization of the active boundary of CD but not affecting passive boundary. The size of CD increases and gives rise to flow of time defined as the temporal distance between the tips of CD. Large reductions change the roles of the passive and active boundaries and mean death of self. The process with duration of τ could correspond to a life-time of self assignable to CD.

   Remark: It is not quite clear whether CD can disappear and generated from vacuum. In principle this is possible and the generation of mental images as sub-selves and sub-CDs could correspond to this kind of process.

2. In [K129] I proposed that Josephson junctions are formed between reacting molecules in bio-catalysis. These could correspond to the shortened flux tubes. The difference $E_J = ZeV$ of Coulomb energy of Cooper pair over flux tube defining Josephson junction between molecules would correspond to Josephson frequency $f_J = 2eV/h_{eff}$. If this frequency corresponds to $\tau_J = 5$ seconds, h_{eff} should be rather large since E_J is expected to be above thermal energy at physiological temperature.

   Could Josephson radiation serve as a kind of of synchronizing clock for the state function reductions so that its role would be analogous to that of EEG in case of brain? A more plausible option is that Josephson radiation is a reaction to the presence of cyclotron radiation generated at MB and performing control actions at the biological body (BB) defined in very general sense. In the case of brain dark cyclotron radiation would generate EEG rhythms responsible for control via genome and dark generalized Josephson radiation modulated by nerve pulse patterns would mediate sensory input to the MB at EEG frequencies.

   A good guess motivated by the proposed universality of the Comorosan periods is that the energy in question does not depend on the catalytic system and corresponds to Josephson energy for protein through cell membrane acting as Josephson junction and giving to ionic channel or pump. The flux tubes themselves have universal properties.

3. The hypothesis $\hbar_{eff} = \hbar_{gr} = GMm/\beta_0 c$ of Nottale [E13] for the value of gravitational Planck constant [K100, K83, K84, **?**] gives large $\hbar$. Here $v_0 = \beta_0 c$ has dimensions of velocity. For dark cyclotron photons this gives large energy $E_c \propto \hbar_{gr}$ and for dark Josephson photons small frequency $f_J \propto 1/h_{gr}$. Josephson time scale τ_f would be proportional to the mass m of the charged particle and therefore to mass number A of ion involved: $f_J \propto A$ possibly explaining the appearance of multiples of 5 second time scale. Cyclotron time scale does not depend on the mass of the charged particle at all and now sub-harmonics of τ_c are natural.

The time scales assignable to CD or the lifetime-time of self in question could correspond to either cyclotron of Josephson time scale τ.

1. If one requires that the multiplies of the time scale 5 seconds are possible, Josephson radiation is favoured since the Josephson time scale proportional to $h_{gr} \propto m \propto A$, A mass number of ion.

   The problem is that the values $A = 2, 3, 4, 5$ are not plausible for ordinary nuclei in living matter. Dark nuclei at magnetic flux tubes consisting of dark proton sequences could however have arbitrary number of dark protons and if dark nuclei appear at flux tubes defining Josephson junctions, one would have the desired hierarchy.

2. Although cyclotron frequencies do not have sub-harmonics naturally, MB could adapt to the situation by changing the thickness of its flux tubes and by flux conservation the magnetic field strength to which f_c is proportional to. This would allow MB to produce cyclotron radiation with the same frequency as Josephson radiation and MB and BB would be in resonant coupling.

Consider now the model quantitatively.

1. For $\hbar_{eff} = \hbar_{gr}$ one has

$$r = \frac{\hbar_{gr}}{\hbar} = \frac{GM_D m}{c\beta_0} = 4.5 \times 10^{14} \times \frac{m}{m_p}\frac{y}{\beta_0} \quad .$$

Here $y = M_D/M_E$ gives the ratio of dark mass M_D to the Earth mass M_E. One can consider 2 favoured values for m corresponding to proton mass m_p and electron mass m_e.

2. $E = h_{eff}f$ gives the concrete relationship $f = (E/eV) \times 2.4 \times 10^{14} \times (h/h_{eff})$ Hz between frequencies and energies. This gives

$$x = \frac{E}{eV} = 0.4 \times r \times \frac{f}{10^{14}Hz} \quad .$$

3. If the cyclotron frequency $f_c = 300$ Hz of proton for $B_{end} = .2$ Gauss corresponds to biophoton energy of x eV, one obtains the condition

$$r = \frac{GM_D m_p}{\hbar\beta_0} \simeq .83 \times 10^{12}x \quad .$$

Note that the cyclotron energy does not depend on the mass of the charged particle. One obtains for the relation between Josephson energy and Josephson frequency the condition

$$x = \frac{E_J}{eV} = 0.4 \times .83 \times 10^{-2} \times \frac{m}{m_p} \times x\frac{f_J}{Hz} \quad , \quad E_J = ZeV \quad .$$

One should not confuse eV in ZeV with unit of energy. Note also that the value of Josephson energy does not depend on h_{eff} so that there is no actual mass dependence involved.

For proton one would give a hierarchy of time scales as A-multiples of $\tau(p)$ and is therefore more natural so that it is natural to consider this case first.

1. For $f_J = .2$ Hz corresponding to the Comorosan time scale of $\tau = 5$ seconds this would give $ZeV = .66x$ meV. This is above thermal energy $E_{th} = T = 27.5$ meV at $T = 25$ Celsius for $x > 42$. For *ordinary* photon ($h_{eff} = h$) proton cyclotron frequency $f_c(p)$ would correspond for $x > 42$ to EUV energy $E > 42$ eV and to wavelength of $\lambda < 31$ nm.

   The energy scale of Josephson junctions formed by proteins through cell membrane of thickness $L(151) = 10$ nm is slightly above thermal energy, which suggests $x \simeq 120$ allowing to identify $L(151) = 10$ nm as the length scale of the flux tube portion connecting the reactants. This would give $E \simeq 120$ eV - the upper bound of EUV range. For $x = 120$ one would have $GM_E m_p y/v_0 \simeq 10^{14}$ requiring $\beta_0/y \simeq 2.2$. The earlier estimates [?] for the mass M_D give $y \sim 2 \times 10^{-4}$ giving $\beta_0 \sim 4.4 \times 10^{-4}$. This is rather near to $\beta_0 = 2^{-11} \sim m_e/m_p$ obtained also in the model for the orbits of the 4 inner planets as Bohr orbits.

   For ion with mass number A this would predict $\tau_A = A \times \tau_p = A \times 5$ seconds so that also multiples of the 5 second time scale would appear. These multiples were indeed found by Comoran and appear also in the case of RNA II polymerase.

2. For proton one would thus have 2 biological extremes - EUV energy scale associated with cyclotron radiation and thermal energy scale assignable to Josephson radiation. Both would be assignable to dark photons with $h_{eff} = h_{gr}$ with very long wavelength. Dark and ordinary photons of both kind would be able to transform to each other meaning a coupling between very long lengths scales assignable to MB and short wavelengths/time scales assignable to BB.

   The energy scale of dark Josephson photons would be that assignable with Josephson junctions of length 10 nm with long wavelengths and energies slightly above E_{th} at physiological temperature. The EUV energy scale would be 120 eV for dark cyclotron photons of highest energy would be fixed by flux tube length of 10 nm.

   For lower cyclotron energies forced by the presence of bio-photons in the range containing visible [K17, K31] and UV and obtained for B_{end} below .2 Gauss, the Josephson photons would have energies below E_{th}. That the possible values of B_{end} are below the nominal value $B_{end} = .2$ Gauss deduced from the experiments of Blackman [J6] does not conform with the earlier ad hoc assumption that B_{end} represents lower bound. This does not change the earlier conclusions.

   Could the 120 eV energy scale have some physical meaning in TGD framework? The corresponding wavelength for ordinary photons corresponds to the scale $L(151) = 10$ nm which correspond to the thickness of DNA double strand. Dark DNA having dark proton triplets as codons could correspond to either $k = 149$ or $k = 151$. The energetics of Pollack effect suggests that $k = 149$ is realized in water even during prebiotic period [L54] (see `http://tinyurl.com/yalny39x`). In the effect discovered by Blackman the ELF photons would transform dark cyclotron photons having $h_{eff} = h_{gr}$ and energy about .12 keV. They would induce cyclotron transitions at flux tubes of B_{end} with thickness of order cell size scale. These states would decay back to previous states and the dark photons transformed to ordinary photons absorbed by ordinary DNA with coil structure with thickness of 10 nm. Kind of standing waves would be formed. These waves could transform to acoustic waves and induce the observed effects. Quite generally, dark cyclotron photons would control the dynamics of ordinary DNA by this mechanism.

   It is natural to assume that $B_{end} = .2$ Gauss corresponds to the upper bound for B_{end} since magnetic fields are expected to weaken farther from the Earth's surface: weakening could correspond to thickening of flux tubes reducing the field intensity by flux conservation. The model for hearing [K92] requires cyclotron frequencies considerably above proton's cyclotron frequency in $B_{end} = .2$ Gauss. This requires that audible frequencies are mapped to electron's cyclotron frequency having upper bound $f_c(e) = (m_p/m_e)f_c(p) \simeq 6 \times 10^5$ Hz. This frequency is indeed above the range of audible frequencies even for bats.

For electron one has $h_{gr}(e) = (m_e/mp) \times h_{gr}(p) \simeq 5.3 \times 10^{-4} h_{gr}(p)$, $\hbar_{gr}(p)/\hbar = 4.5 \times 10^{14}/\beta_0$. Since Josephson energy remains invariant, the Josephson time scales up from $\tau(p) = 5$ seconds to $\tau(e) = (m_e/m_p)\tau(p) \simeq 2.5$ milliseconds, which is the time scale assignable to nerve pulses [K93, K44].

To sum up, the model suggests that the idealization of flux tubes as kind of universal Josephson junctions. The model is consistent with bio-photon hypothesis. The constraints on $h_{gr} = GM_Dm/v_0$ are consistent with the earlier views and allows to assign Comorosan time scale 5 seconds to proton and nerve pulse time scale to electron as Josephson time scales. This inspires the question whether the dynamics of bio-catalysis and nerve pulse generation be seen as scaled variants of each other at quantum level? This would not be surprising if MB controls the dynamics. The earlier assumption that $B_{end} = 0.2$ Gauss is minimal value for B_{end} must be replaced with the assumption that it is maximal value of B_{end}.

Chapter 13

Quantum Criticality and Dark Matter: part IV

13.1 Introduction

Quantum criticality is one of the corner stone assumptions of TGD. The value of Kähler coupling strength fixes quantum TGD and is analogous to critical temperature. TGD Universe would be quantum critical. What does this mean is however far from obvious and I have pondered the notion repeatedly both from the point of view of mathematical description and phenomenology [K57, K126].

1. Criticality is characterized by long range correlations and sensitivity to external perturbations and living systems define an excellent example of critical systems - even in the scale of populations since without sensitivity and long range correlations cultural evolution and society would not be possible. For a physicist with the conceptual tools of existing theoretical physics the recent information society in which the actions of people at different side of globe are highly correlated, should look like a miracle.

2. The hierarchy of Planck constants with dark matter identified as phases of ordinary matter with non-standard value $h_{eff} = n \times h$ of Planck constant is one of the "almost-predictions" of TGD is definitely something essentially new physics. The phase transition transforming ordinary matter to dark matter in this sense generates long range quantal correlations and even macroscopic quantum coherence.

   Finding of a universal mechanism generating dark matter have been a key challenge during last ten years. Could it be that criticality is always accompanied by the generation of dark matter? If this is the case, the recipe would be stupifuingly simple: create a critical system! Dark matter would be everywhere and we would have observed its effects for centuries! Magnetic flux tubes (possibly carrying monopole flux) define the space-time correlates for long range correlations at criticality and would carry the dark matter. They are indeed key players in TGD inspired quantum biology.

3. Change of symmetry is assigned with criticality as also conformal symmetry (in 2-D case). In TGD framework conformal symmetry is extended and infinite hierarchy of breakings of conformal symmetry so that a sub-algebras of various conformal algebras with conformal weights coming as integer multiples of integer n defining h_{eff} would occur.

4. Phase separation is what typically occurs at criticality and one should understand also this. The strengthening of this hypothesis with the assumption $h_{eff} = h_{gr}$, where $h_{gr} = GMm/v_0$ is is the gravitational Planck constant originally introduced by Nottale. In the formula v_0 has dimensions of velocity, and will be proposed to be determined by a condition relating the size of the system with mass M to the radius within which the wave function of particle m with $h_{eff} = h_{gr}$ is localized in the gravitational field of M.

5. The condition $h_{eff} = h_{gr}$ implies that the integer n in h_{eff} is proportional to the mass of particle. The implication is that particles with different masses reside at flux tubes with different Planck constant and separation of phases indeed occurs.

6. What is remarkable is that neither gravitational Compton length nor cyclotron energy spectrum depends on the mass of the particle. This universality could play key role in living matter. One can assign Planck constant also to other interactions such as electromagnetic interaction so that one would have $h_{em} = Z_1 Z_2 e^2/v_0$. The phase transition could take place when the perturbation series based on the coupling strength $\alpha = Z_1 Z_2 e^2/\hbar$ ceases to converge. In the new phase perturbation series would converge since the coupling strength is proportional to $1/h_{eff}$. Hence criticality and separation into phases serve as criteria as one tries to see whether the earlier proposals for the mechanisms giving rise to large h_{eff} phases make sense. One can also check whether the systems to which large h_{eff} has been assigned are indeed critical.

The motivation for this work came from super-fluidity. Superfluids exhibit rather mysterious looking effects such as fountain effect and what looks like quantum coherence of superfluid containers, which should be classically isolated. These findings serve as a motivation for the proposal that genuine superfluid portion of superfluid corresponds to a large h_{eff} phase near criticality at least and that also in other phase transition like phenomena a phase transition to dark phase occurs near the vicinity.

13.1.1 Miscellanous applications including fringe physics

If one wants the label of crackpot it is enough to study critical phenomena. Those who try to replicate (or usually, to non-replicate) the claimed findings fail (or rather manage) easily since criticality implies careful tuning of the external parameters to demonstrate the phenomenon. Therefore the tragedy of fringe physicist is to become a victim of the phenomenon that he is studying.

1. Cold fusion involves bombarding of target consisting of Palladium target doped with deuterium using hydrogen atoms as projectiles. Cold fusion is reported to occur in a critical range of doping fraction. This suggests quantum criticality and large h_{eff} phase. One of the TGD based models generalizes the model of Widom and Larsen [C3]. The model assumes that weak interactions involving emission of W boson neutralizing the incoming proton makes possible to overcome the Coulomb wall. What would make the system critical? Does criticality make Palladium a good catalyst? Could the Palladium and with a large surface area define nano-scale variant of partonic 2-surface and large area which quite generally would make it effective as catalyst? Certainly this could hold true for bio-catalysts. Could Pd target be permanently in critical state? Effectiveness of catalyst might mean quantum coherence making chemical reaction rates proportional to N^2 instead of N, which could be the number of reactants of particular kind.

2. Di-electric breakdown in given medium occurs when the electric field strength is just above the critical value. A lot of strange claims have been assigned to these systems by non-professionals: in academic environment these phenomena are kind of taboo. Tesla studied them and was convinced that these phenomena involve new physics [K8]. The basic finding was that that charges appeared everywhere: this certainly conforms with long range fluctuations and emergence of flux tubes carrying charged particles as dark matter to the environment. Unfortunately, recent day physicist regards Tesla's demonstrations as a mere entertainment and does not bother to ponder whether Maxwell's theory really explains what happens. It is tragic that the greatest intellectual achievements stop thinking for centuries. $h_{gr} = h_{eff}$ hypothesis allows even to estimate the length scales range in which these phenomena should appear.

   Ball lightning (`http://tinyurl.com/5jxd7k`) is also a good candidate for an analogous phenomenon and has been admitted to be a real phenomenon after sixties even by skeptics.

   C. Seward has discovered that di-electric breakdowns generate rather stable torus-like magnetic flux tubes around the breakdown current [H1] (`http://tinyurl.com/ybdrpqju`), which he calls ESTSs (Electron Spiral Toroid Spheromak) and proposed that ball lightnings might correspond to rotating ESTSs.

In TGD framework the stability might be understood if the toroid corresponds to a magnetic flux tube carrying monopole flux. This would allow to understand stability of the configuration and of ball lightning. Monopole flux tubes could also provide a solution to the plasma confinement problem plaguing hot fusion. Also ordinary lightnings involve poorly understood aspect such as gamma and X-ray bursts and high energy electrons. The common mystery is how the dissipation in atmosphere could allow this phenomena. A possible explanation would be in terms of dark flux tubes generated near criticality to the generation of lightning.

3. So called free energy systems [H6] (for TGD inspired view see the book [K103] include many phenomena claimed to involve a liberation of surplus energy. To my opinion, it is quite possible that over-unity energy production is a transient phenomenon and the dreams about final solution of energy problems will not be fulfilled. What makes these phenomena so interesting to me is that they might involve new physics predicted or at least allowed by TGD.

   The splitting of water represents besides magnetic motors (to be discussed below) a key example of free energy phenomena. In the splitting of water to oxygen and hydrogen the formation of Brown's gas [H6] (Wikipedia article about Brown's gas `http://tinyurl.com/5tyl92` provides an amusing example full of "fringe science"s about how skeptic writes about something inducing cognitive dissonance in skeptic's mind) with strange properties was reported long time ago. For instance, Brown gas is reported to melt metals whose melting temperature is thousands of degrees although the Brown's gas itself has temperature of order 100 degrees Celsius.

   I have proposed an interpretation as large h_{eff} phase containing dark proton sequences at magnetic flux tubes and responsible for the liberation of energy as this phase transforms to ordinary one. Brown's gas could be essentially the fourth phase of water containing exclusion zones (EZs) discovered by Pollack [L7]. The TGD inspired model for them [L7] involves magnetic flux tubes at which part of protons in EZ is transferred and forms dark proton sequences- essentially dark protons. There a many way to generate Brown's gas: for instance, cavitation due to the mechanical agitation and application of electric fields could do it. The expanding and compressing bubble created by acoustic wave in sono-luminescence and reported to have a very high temperature and maybe even allowing nuclear fusion, could be also EZ.

4. Water memory [I8, I9, I2] is one of the curse words of skeptic and related to scientific attempts to understand the claimed effects of homeopathy, which defines even stronger curse word in the vocabulary of skeptic - of equal strength as "remote mental interaction". The simple idea that the mere presence of original molecules could be replaced by electromagnetic representation of relevant properties of the molecule is utterly impossible for a skeptic to grasp - despite that also skeptic lives in information society. I have developed a model for water memory explaining also claimed homeopathic effects [K55] and this process has been extremely useful for the development of the model of living matter. Same mechanisms that apply to the model of living matter based on the notion of magnetic body, apply also to water memory and remote mental interactions.

   The key idea is that low energy frequency spectrum provides a representation for the bio-active molecules. The spectrum could be identified as cyclotron frequency spectrum associated with the magnetic bodies of EZs and allow them to mimic the bio-active molecule as far as the effects on living matter are considered. The mechanical agitation of the homeopathic remedy could generate EZs just as it generates cavitation. The model for dark proton sequences yields counterparts of DNA, RNA, amino-acids and even tRNA and genetic code based primitive life would be realized at fundamental particle level with biological realization serving as a higher level representation.

The above sections only list examples about systems where dark matter in TGD sense could appear. A lot of details remain to be understood. The basic question whether some of these systems are permanently near critical state or only in phase transitions between different phases.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at http://tgdtheory.fi/tgdglossary.pdf [L4].

13.2 The analogs of CKM mixing and neutrino oscillations for particle and its dark variants

In TGD Universe dark matter in TGD sense corresponds to $h_{eff}/h_0 = n$, $h = 6h_0$ is a good guess [L26, L60, L44] phases of ordinary matter associated with magnetic flux tubes. These flux tubes would be n-sheeted covering spaces, and n would correspond to the dimension of the extension of rationals in which Galois group acts. The evidence for this interpretation of dark matter is accumulating. I have already earlier discussed [L65] one of the latest anomalies - so called 21-cm anomaly. This finding motivates a more detailed model for the interaction between different levels of dark matter hierarchy and in the sequel I will propose this kind of model.

13.2.1 21-cm anomaly as a motivation for the model of the interaction between different levels of h_{eff} hierarchy

Sabine Hossenfelder (see http://tinyurl.com/y7h5ys2r) told about the article [E15] discussing the possible interpretation (see http://tinyurl.com/yasgfgq8) of so called 21-cm anomaly associated with the hyperfine transition of hydrogen atom and observed by EDGES collaboration [E16].

The EDGES Collaboration has recently reported the detection of a stronger-than-expected absorption feature in the global 21-cm spectrum, centered at a frequency corresponding to a redshift of $z \sim 17$. This observation has been interpreted as evidence that the gas was cooled during this era as a result of scattering with dark matter. In this study, we explore this possibility, applying constraints from the cosmic microwave background, light element abundances, Supernova 1987A, and a variety of laboratory experiments. After taking these constraints into account, we find that the vast majority of the parameter space capable of generating the observed 21-cm signal is ruled out. The only range of models that remains viable is that in which a small fraction, $\sim 0.3-2$ per cent, of the dark matter consists of particles with a mass of $\sim$ 10-80 MeV and which couple to the photon through a small electric charge, $\epsilon \sim 10^{-6}-10^{-4}$. Furthermore, in order to avoid being overproduced in the early universe, such models must be supplemented with an additional depletion mechanism, such as annihilations through a $L_\mu - L_\tau$ gauge boson or annihilations to a pair of rapidly decaying hidden sector scalars.

What has been found is an unexpectedly strong absorption feature in 21-cm spectrum: the redshift is about $z = \Delta f/f \simeq v/c \simeq 17$, which from Hubble law $v = HD$ corresponds to a distance $D \sim 2.3\times 10^{11}$ ly. Dark matter interpretation would be in terms of scattering of the baryons of gas from dark matter at lower temperature. The anomalous absorption of 21 cm line could be explained with the cooling of gas caused by the flow of energy to a colder medium consisting of dark matter. If I understood correctly, this would generate a temperature difference between background radiation and gas and consequent energy flow to gas inducing the anomaly.

The article excludes large amount of parameter space able to generate the observed signal. The idea is that the interaction of baryons of the gas with dark matter. The interaction would be mediated by photons. The small em charge of the new particle is needed to make it "dark enough". My conviction is that tinkering with the quantization of electromagnetic charge is only a symptom about how desperate the situation is concerning interpretation of dark matter in terms of some exotic particles is. Something genuinely new physics is involved and the old recipes of particle physicists do not work.

In TGD framework the dark matter at lower temperature would be $h_{eff}/h = n$ phases of ordinary matter residing at magnetic flux tubes. This picture follows from what I call adelic physics [L50, L49]. This kind of energy transfer between ordinary and dark matter is a general signature of dark matter in TGD sense, and there are indications from some experiments relating to primordial life forms for this kind of energy flow in lab scale [L46] (see http://tinyurl.com/yassnhzb) .

The ordinary photon line appearing in the Feynman diagram describing the exchange of photon would be replaced with a photon line containing a vertex in which the photon transforms to dark photon. The coupling in the vertex - call it m^2 - would have dimensions of mass squared.

This would transform the coupling e^2 associated with the photon exchange effectively to e^2m^2/p^2, where p^2 is photon's virtual mass squared. The slow rate for the transformation of ordinary photon to dark photon could be see as an effective reduction of electromagnetic charge for dark matter particle from its quantized value.

Remark: In biological systems dark cyclotron photons would transform to ordinary photons and would be interpreted as bio-photons with energies in visible and UV.

The importance of this finding is that it supports the view about dark matter as ordinary particles in a new phase. There are electromagnetic interactions but the transformation of ordinary photons to dark photons slows down the process and makes these exotic phases effectively dark.

The above picture motivates the attempt to construct a model for the mixing of not only ordinary photons but any particle with its dark variants with various values of $h_{eff}/h_0 = n$ by generalizing the formalism developed for the mixing of neutrinos and their oscillations. Also now oscillations are predicted and they could serve as a test for TGD based model of dark matter. Also the description at the level of Feynman diagrams is briefly summarize. This picture in principle allows the modelling of the energy transfer between ordinary and dark sectors.

13.2.2 Mixing and oscillations of dark photons

In TGD framework dark matter corresponds to phases of ordinary matter with non-standard value of Planck constant $h_{eff}/h_0 = n$ [?]. Here $h = 6h_0$ is a good guess [L26, L60]. It has been assumed that only the reaction vertices would be between particles with same value of $h_{eff}/h = n$, whereas the transformation changing the value of n during propagation is assumed to be possible. For instance, biophotons would be ordinary photons emerging when dark photons transform to ordinary photons. Therefore the mixing of ordinary particles with their dark variants can be considered.

This allows to deduce the general form of propagator which is simple for the mixed mass squared eigenstates in terms of mass squared matrix. There is however a problem associated with photons. They must have extremely small mass although p-adic mass calculations suggests that photon has very small p-adic thermal mass squared [K66]. Are they exactly massless and what conditions masslessness poses on mixing? It turns out that the eigenstates of n most naturally have same mass and the mixing makes other state massless so that ordinary photon would not have minimal value of n - presumably $n = 6$ - during propagation but in absorption the state would be projected to $n = 6$.

Mixing and oscillations of ordinary and dark particles

Could the analog of CKM mixing take place for ordinary and dark photons? Is the analog of neutrino oscillations possible for photon and dark photon? Could these oscillations occur also for neutrinos besides ordinary neutrino oscillations? The model for the analog of ordinary-dark oscillations could be essentially the same as that for neutrino oscillations (see `http://tinyurl.com/oov344k`) and consist of the following pieces.

In the case of neutrino mixing involving 3 neutrinos the calculation gives the result given in Wikipedia article (see `http://tinyurl.com/oov344k`). Since the formula does not depend on the number of flavors, it easily generalize to the case that one has arbitrary number N of values of $h_{eff}/h_0 = n$, which mix. The analog of CKM matrix describing the mixing of neutrinos, the mass squared differences, and the distance L between source and receiver determines the oscillation dynamics and generalizes as such to the description of mixing and oscillation of particles with different values of h_{eff}. For N values of n including $n = n_0 = 6$ assigned with ordinary matter, the analog of CKM matrix is $N \times N$ unitary matrix.

This matrix, call it C, is completely determined by the mass squared matrix with non-diagonal components. Mass squared eigenstates are superpositions of states with well-defined value of n_{eff} having the rows of this matrix as coefficients. Therefore the non-diagonal component of mass squared matrix, to called K^2, describing the mixing of different values of n determines both mixing and oscillations.

A non-trivial modification of the formula for the neutrino oscillations comes from the fact that plane wave factor s $exp([iE_i - p)L/\hbar_{eff}(\alpha)]$ depend on the value of $\hbar_{eff}(\alpha) = n^\alpha \hbar_0$.

The following model applies to any particle species.

1. The mixing of ordinary and dark particles would be an analog of CKM mixing for quarks and leptons. Now ordinary particleand its dark variants would mix with each other. Note that given value of n can correspond to several extensions of rationals. In principle also this degeneracy must be also be taken into account.

2. The analog of neutrino oscillations would mean that ordinary particles disappear from beam by transforming to dark particles and can be regenerated. The formalism for neutrino oscillations seems to generalize almost as such to ordinary-dark particle oscillations. Oscillations could be used as test for TGD view about dark matter.

3. In the initial and final state the particle would be either ordinary or dark with some value of n being analogous to a flavor eigenstate for neutrino. These states are not eigenstates of mass and energy and it convenient to express them as mass squared eigenstates related by CKM matrix to eigenstates of n. During propagation states can be regarded as superpositions of eigenstates of mass squared operator M^2. This hermitian operator is sum of ordinary mass squared operators for the sectors labelled by n but there are non-diagonal term is causing the mixing.

4. One has on mass shall condition in momentum space which can be written as

$$(p^2 - M_{op}^2)\Psi = 0 \ . \tag{13.2.1}$$

   p^2 represents four momentum square in various sectors labelled by n^α and can be regarded as direct sum $p^2 = \oplus p^2(n^\alpha)$.

   For given value of 3-momentum the situation is identical for a system consisting of N coupled harmonic oscillators and the situation is mathematically equivalent to the diagonalization of the system by finding the eigenmodes and eigenfrequencies.

5. Mass squared operator is direct sum

$$M_{op}^2 = \oplus_{n^\alpha} m^2(n^\alpha) + K^2 \ . \tag{13.2.2}$$

   $K^2_{\alpha\beta}$ is non-diagonal coupling different sectors n^α and thus mixing of partial waves with different values of n. The assumption has been that $m^2(n^\alpha)$ does not depend on n^α. The presence of the non-diagonal mixing term $K^2_{\alpha\beta}$ causes mass squared eigenstates to have different masses.

   M_{op}^2 would have for $N = 2$ (ordinary particle and its dark variant with single value of n) the form

$$M_{op}^2 = \left[\begin{array}{cc} m^2 & K^2 \\ \hline K^2 & m^2 \end{array} \right] \ . \tag{13.2.3}$$

   Note that one K^2 can be also complex.

6. In this form the value of $\hbar_{eff}$ is not visible at all in p^2. At the space-time level $p^2 = E^2 - p_3^2$ must be however expressed as d'Alembert operator via the usual rules $E \rightarrow i\hbar_{eff}\partial_t$ and $p \rightarrow i\hbar_{eff}\partial_z$ so that one has

$$\begin{array}{ll} (-\Box - M_{op}^2)\Psi = 0 \ , & \Box = \oplus_\alpha \Box_{n^\alpha} \ , \\ \Box_n = n^2\hbar_0^2\Box \ , & \Box = \partial_t^2 - \partial_x^2 - \partial_y^2 - \partial_z^2 \ . \end{array} \tag{13.2.4}$$

Plane wave solutions are of form $exp(i(E-p)z/n\hbar_0)$ and differ by a scaling of the argument. This applies also to general solutions. One has fractally scaled variants of the solution and K^2 matrix defines coupling between them.

7. This formulation generalizes trivially to general 4-D case solutions and to general solutions of d'Alembert type field equations. In QFT language one has an analog of N-component scalar field for which mass squared matrix M_{op}^2 containing quadratic couplings between field components. The generalization seems obvious also for more general fields such as spinor fields and gauge fields. For instance, for gauge fields one would have N copies of gauge fields with non-diagonal couplings. The invariants $F_{n^\alpha}^{\mu\nu}F_{n^\beta,\mu\nu}$ are suggestive for gauge invariant couplings.

   The new element is that these N fields have different value of h_{eff} and the solutions are fractally scaled variants of each other.

8. The eigenstates $|i\rangle$ of the d'Alembert type operator are eigenstates of M_{op}^2 and eigenvalues are mass squared eigenvalues m_i^2. $|i\rangle$ are superpositions states with fixed value of n with coefficients, which are the components of the analog C of CKM matrix:

$$|i,x\rangle = C_{i\alpha}e^{i\frac{p_4\cdot x}{n^\alpha\hbar_0}}|n^\alpha\rangle \quad . \tag{13.2.5}$$

   Here one has summation of the repeated index α appearing as both upper and lower index. This holds quite generally for Fourier basis. Therefore the non-diagonal part of mass squared operator determines the C as a prediction.

The S-matrix for the effectively 2-D system considered is needed to deduce oscillation probabilities. One has a beam of particles with momentum p independent of value of n travelling distance L along line $z=t$. The mass parameter $m^2(n)$ is independent of n.

1. To deduce S-matrix start from the expression of the identity operator Id as

$$Id = |i,t=0\rangle\langle i,t=0|$$

   acting at the end $z=0$. The states $|i,t=0\rangle$ correspond to the starting point $z=0$ of propagation. The notation $|n^\alpha,t=0\rangle = |n^\alpha\rangle$ will be used. Time evolution shifts the states $|i,t=0\rangle = C_{i\alpha}|n^\alpha\rangle$ to $t=z=L$) by the above time evolution.

2. S-matrix is obtained by translating the states $|i,t=0\rangle$ appearing in the identity operator to $(t=L, z=L)$.

$$S = \sum_i |i,t=L\rangle\langle i,t=0| \quad . \tag{13.2.6}$$

3. One can find the expression of S in the basis $|i,t=L\rangle$ by writing $|i,t=L\rangle$ as a superposition of states $|n^\alpha\rangle$:

$$\begin{aligned} &|i,t=0\rangle = C_{i\alpha}|n^\alpha\rangle \quad \rightarrow \quad |i,t=L\rangle = C_{i\alpha}U_\alpha^i|n^\alpha\rangle \quad , \\ &U_\alpha^i = e^{i\frac{(E_i-p)L}{n^\alpha\hbar_0}} \quad , \qquad E_i = \sqrt{p^2+m_i^2} \quad . \end{aligned} \tag{13.2.7}$$

   Using this formula one can express S using basis $|n^\alpha|$.

$$S = S_{\alpha\beta}|n^\alpha\rangle\langle n^\beta| \ ,$$
$$S_{\alpha\beta} = \overline{C}^i_\alpha C_{i\beta} U^i_\alpha \ . \tag{13.2.8}$$

Here the summation convention for the repeated index i applies.

What are needed are the oscillation probabilities $P_{\alpha\beta}$.

1. The probabilities that an eigenstate $|n^\alpha\rangle$ transforms to eigenstate $|n^\beta\rangle$ during the travel are given by

$$P_{\alpha\beta} = |S_{\alpha\beta}|^2 = Y_{\alpha\beta ij}U^{ij}_{\alpha\beta} \ , \qquad Y_{\alpha\beta ij} = \overline{C}_{i\alpha}C_{i\beta}C_{j\alpha}\overline{C}_{j\beta}$$
$$U^{ij}_{\alpha\beta} = U^i_\alpha\overline{U^j_\beta} = cos(X^{ij}_{\alpha\beta}) + isin(X^{ij}_{\alpha\beta}) \ , \quad X^{ij}_{\alpha\beta} = \frac{(E_i-p)L}{n^\alpha\hbar_0} - \frac{(E_j-p)L}{n^\beta\hbar_0} \ . \tag{13.2.9}$$

2. One can decompose $P_{\alpha\beta}$ as

$$P_{\alpha\beta} = Re[Y_{\alpha\beta ij}]cos(X^{ij}_{\alpha\beta}) - Im[Y_{\alpha\beta ij}]sin(X^{ij}_{\alpha\beta}) \ , \tag{13.2.10}$$

and apply trigonometric formula $cos(2x) = 1 - 2sin^2(x)$, and decompose the summation to indices to 3 groups with $i < j$, $j < i$ and $i = j$ to get

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{i<j} Re[Y_{\alpha\beta ij}]sin^2(\frac{X^{ij}_{\alpha\beta}}{2}) - 2\sum_{i<j} Im[Y_{\alpha\beta ij}]sin(X^{ij}_{\alpha\beta}) \ . \tag{13.2.11}$$

Note that $\sum_\beta P_{\alpha\beta} = 1$ holds true since in the summation second term vanishes due to unitary condition $U^\dagger U = 1$ and $i > j$ condition in the formula.

3. In the completely relativistic situation $p >> m_i$ one can make the analog of non-relativistic approximation as $E_i = p + m_i^2/2p$. In this case one has

$$X^{ij}_{\alpha\beta} = \frac{(E_i-p)L}{n^\alpha\hbar_0} - \frac{(E_j-p)L}{n^\beta\hbar_0} \simeq \frac{m_i^2 L}{pn^\alpha\hbar_0} - \frac{m_j^2 L}{pn^\beta\hbar_0} \ . \tag{13.2.12}$$

4. For given 3-momentum p $P_{\alpha\beta}$ is a sum over $N \times (N-1)$ periodic functions of L with periods

$$\lambda^{ij}_{\alpha\beta} = \frac{2\pi}{X^{ij}_{\alpha\beta}} \ . \tag{13.2.13}$$

5. At the limit of large L the trigonometric factors oscillate rapidly and in the averaging over sources region. The term proportional to $sin(x)$ gives zero whereas $sin^2(x)$ gives average 1/2. The probabilities for various transitions induced by the oscillations depend on the analog of CKM matrix only. If the distance L is very large and the dependence on the mass squared differences and distance disappears in the averaging over the source region and one obtains

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 2Re[Y_{\alpha\beta ij}] \ . \tag{13.2.14}$$

Some general comments are in order.

1. The oscillation is detectable if the size of the non-diagonal part K^2 of the mass matrix is large enough as compared to the diagonal part. It is not clear whether this condition holds true for say fermions. The absence of tachyons requires that the value of m^2 (no dependence on n) is positive. m^2 could be interpreted as thermal mass squared in terms of p-adic mass calculations [K77, K66]. In the case of massless particles the mixing during propagation can however make the mass arbitrarily small as will be found.

2. What can be measured is the diagonal probability P_{11}, where $\alpha = 1$ corresponds to $h_{eff} = h$. The formula reduces to that for neutrino oscillations or its generalization to N flavors since $h_{eff} = h$ holds true now:

$$X_{11}^{ij} = \frac{(E_i - p)L}{\hbar} - \frac{(E_j - p)L}{\hbar} \simeq \frac{(m_i^2 - m_j^2)L}{p\hbar} \quad . \tag{13.2.15}$$

   Remark: The part of P_{11} proportional to sine function has sine opposite to that in the formula of Wikipedia article (see `http://tinyurl.com/oov344k`): the reason is that the definition of $Y_{\alpha\beta ij}$ used here is complex conjugate of that used in Wikipedia formula.

3. Mass squared matrix and mixing matrix are not uniquely determined by the mass squared eigenvalues. Any unitary transform $M_D^2 \rightarrow UM_D^2U^\dagger$ of the mass matrix M_D^2 has the same eigenvalues. If the states with well-defined h_{eff} have the same mass in absence of mixing, $UM_D^2U^\dagger$ must have diagonal part equal to m^2Id.

   This gives N conditions on U in both real and complex case. The conditions are however not dependent since the trace of M_D^2 equal to Nm^2 is preserved in the transformation so that there are only $N-1$ conditions in both real and complex case.

   Since the number of the independent elements of a unitary matrix with unit determinant is $N^2 - 1$, this leaves in complex case $(N-1)^2$ parameter set of mass matrices with the same eigenvalues. Orthogonal matrix has $(N-1)N/2$ independent elements so that one has $(N-2)(N-1)/2$ parameters in the real case. For $N = 2$ complex case one has 1-parameter set of solutions corresponding to the phase of K^2, in the real $N = 2$ case one has two solutions corresponding to two signs for K^2. For $N = 3$ one has 4 parameters in complex case and 1 parameter in real case.

Mass squared matrix for photons

What can one say about mass squared matrix for photons? Consider a situation in which only two photons are mixed.

1. The most general form of mass matrix is in the case of single value of n given by $M_{op}^2 = [m^2, K^2; \overline{K^2}, m^2]$. Note that the diagonal element is assumed to be nonvanishing: this allows to avoid tachyonic mass squared eigenstate. The eigen valuels of M_{op}^2 are given by

$$M_\pm^2 = m^2 \pm |K|^2 \quad . \tag{13.2.16}$$

2. The condition $M_-^2 \geq 0$ gives $m^2 \geq |K|^2$. For the general mass squared matrix $M_{op}^2 = [m_1^2, K^2; \overline{K^2}, m_2^2]$ the condition reads $m_1m_2 \geq |K|^2$. If m_1 is very small, m_2 must be large in the scale defined by $|K|$.

   One can argue that this form of mass squared matrix is the only reasonable option. If $n = 6$ photon is massless one obtains photons with masses $m^2 = \pm K^2$ and tachyonic photon is physically very problematic. It must be remembered that fort wistor lift of TGD all particles are massless in 8-D sense and can be massive in 4-D sense. Thereforethe assumption that "free" photon is massive need not lead to problems.

3. The mass of what we identify as ordinary photon and identified now as a mixed photon with lowest mass is extremely small: the recent upper bound is 7×10^{17} eV, which corresponds to Compton length of 10^{11} meters, which is of the order one astronomical unit AU: this probably relates to the measurement method. Photons thus behave like massless particles in the scale of Sun-Earth system. Therefore the approximation would $m^2 = |K|^2$ is excellent. The masses would be $M_-^2 = 0$ and $M_+^2 = 2m^2$.

Dark photons in TGD sense play a key role in TGD inspired model of living matter. Bio-photons would result in the transformation of dark photons to ordinary photons. Mass squared eigenstates of photons have mass spectrum and a natural question is whether dark photon mass relevant to biology corresponds to a Compton length scale relevant to biology. In p-adic physics Compton lengths correspond to p-adic length scales which by p-adic length scale hypothesis correspond to primes $p \simeq 2^k$ near power of 2 (slightly below it).

Mersenne primes and their Gaussian analogs are especially interesting physically and in the length scale range 10 nm (neural membrane thickness) and 2.5 μ (size scale of nucleus) there are as many as 4 Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ corresponding to $k \in \{151, 157, 163, 167\}$. Could the p-adic mass scales $m/m_e = 2^{(k-127)/2}$ associated with these length scales be especially important in biology. More generally all p-adic mass scales assignable to these two kinds of Mersenne primes could be important as mass scales of mixed photons.

Could the mixing with dark photons provide and additional contribution to particle masses?

p-Adic thermodynamics [K66] provides an excellent description of particle massivation in the fermionic sector. It assumes only p-adic thermodynamics and superconformal invariance with partition functions determined by it, p-adic length scale hypothesis, and canonical identification $x = \sum x_n p^n \rightarrow \sum x_n p^{-n}$ mapping p-adic thermodynamical mass squared expectations to their real counterparts.

This need not however be the entire story. It is not clear whether one can really understand most of the hadron mass in this manner and whether gauge boson masses involving in the usual approach Higgs mechanism can be completely understood in this manner. Therefore one can ask whether the mixing of particles with their dark variants could contribute to the particle masses. In case of gauge bosons this contribution could be significant.

Description of ordinary-dark scattering diagrams

One would like also to develop a model for the scattering of ordinary and dark particles via exchange of ordinary photons transforming to dark photons or vice versa. Here one must be satisfied to phenomenological description although it is clear that there are non-trivial issues related to the gauge invariance in presence of massivation. The general TGD picture strongly suggests that these problems can be solved. In twistor lift of TGD particles become massless in 8-D sense and can be massive in 4-D sense.

The simplest assumption is that the massless photon propagator $D = P/p^2 - i\epsilon$, where P is a projector to the space of physical polarizations, is replaced with matrix propagator

$$D = [\frac{P}{p^2 Id - M^2(op)}]_{ij} = \frac{P}{p^2}\sum_{n\geq 0}[\frac{M^2(op)}{p^2}]_{ij}^n \quad . \tag{13.2.17}$$

For the mass squared eigenstates this gives diagonal matrix with poles corresponding to mass squared eigenvalues. What looks problematic is that the projector P for massive states projects to a 3-D space of polarization and for massless states to 2-D space of polarization. If also ordinary photon has very small mass as p-adic mass calculations strongly suggest, also it has longitudinal polarization and all projectors are 3-D.

The reaction vertices are possible only between particles with same value of n so that the propagator must be replaced in this basis by $C^\dagger DC$, where C is the analog of CKM mixing matrix mediating transition to mass eigenstates.

13.3 TGD Inspired View About Blackholes And Hawking Radiation

The most recent revelation of Hawking was in Hawking radiation conference held in KTH Royal Institute of Technology in Stockholm. The title of the posting of Bee (see http://tinyurl.com/yakcmrza) telling about what might have been revealed is "Hawking proposes new idea for how information might escape from black holes". Also Lubos (see http://tinyurl.com/ydg78w92) has - a rather aggressive - blog post about the talk. A collaboration of Hawking, Andrew Strominger and Malcom Perry is behind the claim and the work should be published within few months.

This inspired a fresh discussion of the notions of blackhole and Hawking radiation in TGD framework. The intention is to demonstrate that a pseudo problem following from the failure of General Relativity below black hole horizon is in question. There are several new elements involved but concerning black holes the most relevant new element is the assignment of Euclidian space-time regions as lines of generalized Feynman diagrams implying that also blackhole interiors correspond to this kind of regions. Negentropy Maximization Principle is also an important element and predicts that number theoretically defined black hole negentropy can only increase. The real surprise was that the temperature of the variant of Hawking radiation at the flux tubes of proton Sun system is room temperature! Could TGD variant of Hawking radiation be a key player in quantum biology?

13.3.1 Is Information Lost Or Not In Blackhole Collapse?

The basic problem is that classically the collapse to blackhole seems to destroy all information about the matter collapsing to the blackhole. The outcome is just infinitely dense mass point. There is also a theorem of classical GRT stating that blackhole has no hair: blackhole is characterized only by few conserved charges.

Hawking has predicted that blackhole loses its mass by generating radiation, which looks like thermal. As blackhole radiates its mass away, all information about the material which entered to the blackhole seems to be lost. If one believes in standard quantum theory and unitary evolution preserving the information, and also forgets the standard quantum theory's prediction that state function reductions destroy information, one has a problem. Does the information really disappear? Or is the GRT description incapable to cope with the situation? Could information find a new representation?

Superstring models and AdS/CFT correspondence have inspired the proposal that a hologram results at the horizon and this hologram somehow catches the information by defining the hair of the blackhole. Since the radius of horizon is proportional to the mass of blackhole, one can however wonder what happens to this information as the radius shrinks to zero when all mass is Hawking radiated out.

What Hawking suggests is that a new kind of symmetry known as super-translations - a notion originally introduced by Bondi and Metzner - could somehow save the situation. Andrew Strominger has recently discussed the notion [B27] (http://tinyurl.com/ycdv9e7y). The information would be "stored to super-translations". Unfortunately this statement says nothing to me nor did not say to Bee and New Scientist reporter. The idea however seems to be that the information carried by Hawking radiation emanating from the blackhole interior would be caught by the hologram defined by the blackhole horizon.

Super-translation symmetry acts at the surface of a sphere with infinite radius in asymptotically flat space-times looking like empty Minkowski space in very distant regions. The action would be translations along sphere plus Poincare transformations.

What comes in mind in TGD framework is conformal transformations of the boundary of 4-D lightcone, which act as scalings of the radius of sphere and conformal transformations of the sphere. Translations however translate the tip of the light-cone and Lorentz transformations transform the sphere to an ellipsoid so that one should restrict to rotation subgroup of Lorentz group. Besides this TGD allows huge group of symplectic transformations of $\delta CD \times CP_2$ acting as isometries of WCW and having structure of conformal algebra with generators labelled by conformal weights.

13.3.2 What Are The Problems?

My fate is to be an aggressive dissident listened by no-one, and I find it natural to continue in the role of angry old man. Be cautious, I am arrogant, I can bite, and my bite is poisonous!

1. With all due respect to Big Guys, to me the problem looks like a pseudo problem caused basically by the breakdown of classical GRT. Irrespective of whether Hawking radiation is generated, the information about matter (apart from mass, and some charges) is lost if the matter indeed collapses to single infinitely dense point. This is of course very unrealistic and the question should be: how should we proceed from GRT.

   Blackhole is simply too strong an idealization and it is no wonder that Hawking's calculation using blackhole metric as a background gives rise to blackbody radiation. One might hope that Hawking radiation is genuine physical phenomenon, and might somehow carry the information by being not genuinely thermal radiation. Here a theory of quantum gravitation might help. But we do not have it!

2. What do we know about blackholes? We know that there are objects, which can be well described by the exterior Schwartschild metric. Galactic centers are regarded as candidates for giant blackholes. Binary systems for which another member is invisible are candidates for stellar blackholes. One can however ask wether these candidates actually consist of dark matter rather than being blackholes. Unfortunately, we do not understand what dark matter is!

3. Hawking radiation is extremely weak and there is no experimental evidence pro or con. Its existence assumes the existence of blackhole, which presumably represents the failure of classical GRT. Therefore we might be seeing a lot of trouble and inspired heated debates about something, which does not exist at all! This includes both blackholes, Hawking radiation and various problems such as firewall paradox.

There are also profound theoretical problems.

1. Contrary to the intensive media hype during last three decades, we still do not have a generally accepted theory of quantum gravity. Super string models and M-theory failed to predict anything at fundamental level, and just postulate effective quantum field theory limit, which assumes the analog of GRT at the level of 10-D or 11-D target space to define the spontaneous compactification as a solution of this GRT type theory. Not much is gained.

   AdS/CFT correspondence is an attempt to do something in absence of this kind of theory but involves 10- or 11- D blackholes and does not help much. Reality looks much simpler to an innocent non-academic outsider like me. Effective field theorizing allows intellectual laziness and many problems of recent day physics will be probably seen in future as being caused by this lazy approach avoiding attempts to build explicit bridges between physics at different scales. Something very similar has occurred in hadron physics and nuclear physics and one has kind of stable of Aigeias to clean up before one can proceed.

2. A mathematically well-defined notion of information is lacking. We can talk about thermodynamical entropy - single particle observable - and also about entanglement entropy - basically a 2-particle observable. We do not have genuine notion of information and second law predicts that the best that one can achieve is no information at all!

   Could it be that our view about information as single particle characteristic is wrong? Could information be associated with entanglement and be 2-particle characteristic? Could information reside in the relationship of object with the external world, in the communication line? Not inside blackhole, not at horizon but in the entanglement of blackhole with the external world?

3. We do not have a theory of quantum measurement. The deterministic unitary time evolution of Schrödinger equation and non-deterministic state function reduction are in blatant conflict. Copenhagen interpretation escapes the problem by saying that no objective reality/realities exist. Easy trick once again! A closely related Pandora's box is that experienced time and geometric time are very different but we pretend that this is not the case.

The only way out is to bring observer part of quantum physics: this requires nothing less than quantum theory of consciousness. But the gurus of theoretical physics have shown no interest to consciousness. It is much easier and much more impressive to apply mechanical algorithms to produce complex formulas. If one takes consciousness seriously, one ends up with the question about the variational principle of consciousness. Yes, your guess was correct! Negentropy Maximization Principle! Conscious experience tends to maximize conscious information gain. But how information is represented?

13.3.3 TGD View About Black Holes And Hawking Radiation

My own basic strategy is to not assume anything not necessitated by experiment or not implied by general theoretical assumptions - these of course represent the subjective element.

The basic ideas of TGD relevant for blackhole concept

The basic assumptions/predictions of TGD relevant for the recent discussion are following.

1. Space-times are 4-surfaces in $H = M^4 \times CP_2$ and ordinary space-time is replaced with many-sheeted space-time. This solves what I call energy problem of GRT by lifting gravitationally broken Poincare invariance to an exact symmetry at the level of embedding space H.

   GRT type description is an approximation obtained by lumping together the space-time sheets to single region of M^4, with various fields as sums of induced fields at space-time surface geometrized in terms of geometry of H.

   Space-time surface has both Minkowskian and Euclidian regions. Euclidian regions are identified in terms of what I call generalized Feynman/twistor diagrams. The 3-D boundaries between Euclidian and Minkowskina regions have degenerate induced 4-metric and I call them light-like orbits of partonic 2-surfaces or light-like wormhole throats analogous to blackhole horizons and actually replacing them. The interiors of blackholes are replaced with the Euclidian regions and every physical system is characterized by this kind of region.

   Euclidian regions are identified as slightly deformed pieces of CP_2 connecting two Minkowskian space-time regions. Partonic 2-surfaces defining their boundaries are connected to each other by magnetic flux tubes carrying monopole flux.

   Wormhole contacts connect two Minkowskian space-time sheets already at elementary particle level, and appear in pairs by the conservation of the monopole flux. Flux tube can be visualized as a highly flattened square traversing along and between the space-time sheets involved. Flux tubes are accompanied by fermionic strings carrying fermion number. Fermionic strings give rise to string world sheets carrying vanishing induced em charged weak fields (otherwise em charge would not be well-defined for spinor modes). String theory in space-time surface becomes part of TGD. Fermions at the ends of strings can get entangled and entanglement can carry information.

2. Strong form of General Coordinate Invariance (GCI) states that light-like orbits of partonic 2-surfaces on one hand and space-like 3-surfaces at the ends of causal diamonds on the other hand provide equivalent descriptions of physics. The outcome is that partonic 2-surfaces and string world sheets at the ends of CD can be regarded as basic dynamical objects.

   Strong form of holography states the correspondence between quantum description based on these 2-surfaces and 4-D classical space-time description, and generalizes AdS/CFT correspondence. Conformal invariance is extended to the huge super-symplectic symmetry algebra acting as isometries of WCW and having conformal structure. This explains why 10-D space-time can be replaced with ordinary space-time and 4-D Minkowski space can be replaced with partonic 2-surfaces and string world sheets. This holography looks very much like the one we are accustomed with!

3. Quantum criticality of TGD Universe fixing the value(s) of the only coupling strength of TGD (Kähler coupling strength) as analog of critical temperature. Quantum criticality is realized in terms of infinite hierarchy of sub-algebras of super-symplectic algebraas isometries

of WCW, the "world of classical worlds" consisting of 3-surfaces or by holography preferred extremals associated with them.

Given sub-algebra is isomorphic to the entire algebra and its conformal weights are $n \geq 1$-multiples of those for the entire algebra. This algebra acts as conformal gauge transformations whereas the generators with conformal weights $m < n$ act as dynamical symmetries defining an infinite hierarchy of simply laced Lie groups with rank $n-1$ acting as dynamical symmetry groups defined by Mac-Kay correspondence so that the number of degrees of freedom becomes finite. This relates very closely to the inclusions of hyper-finite factors - WCW spinors provide a canonical representation for them.

This hierarchy corresponds to a hierarchy of effective Planck constants $h_{eff} = n \times h$ defining an infinite number of phases identified as dark matter. For these phases Compton length and time are scale up by n so that they give rise to macroscopic quantum phases. Superconductivity is one example of this kind of phase - charge carriers could be dark variants of ordinary electrons. Dark matter appears at quantum criticality and this serves as an experimental manner to produce dark matter. In living matter dark matter identified in this manner would play a central role. Magnetic bodies carrying dark matter at their flux tubes would control ordinary matter and carry information.

4. I started the work with the hierarchy of Planck constants from the proposal of Nottale stating that it makes sense to talk about gravitational Planck constant $\hbar_{gr} = GMm/v_0$, $v_0/c \leq 1$ (the interpretation of symbols should be obvious). Nottale found that the orbits of inner and outer planets could be modelled reasonably well by applying Bohr quantization to planetary orbits with tge value of velocity parameter differing by a factor 1/5. In TGD framework h_{gr} would be associated with magnetic flux tubes mediating gravitational interaction between Sun with mass M and planet or any object, say elementary particle, with mass m. The matter at the flux tubes would be dark as also gravitons involved. The Compton length of particle would be given by GM/v_0 and would not depend on the mass of particle at all.

   The identification $h_{gr} = h_{eff}$ is an additional hypothesis motivated by quantum biology, in particular the identification of biophotons as decay products of dark photons satisfying this condition. As a matter fact, one can talk also about h_{em} assignable to electromagnetic interactions: its values are much lower. The hypothesis is that when the perturbative expansion for two particle system does not converge anymore, a phase transition increasing the value of the Planck constant occurs and guarantees that coupling strength proportional to $1/h_{eff}$ decreases. This is one possible interpretation for quantum criticality. TGD provides a detailed geometric interpretation for the space-time correlates of quantum criticality.

   Macroscopic gravitational bound states not possible in TGD without the assumption that effective string tension associated with fermionic strings and dictated by strong form of holography is proportional to $1/h_{eff}^2$. The bound states would have size scale of order Planck length since for longer systems string energy would be huge. $h_{eff} = h_{gr}$ makes astroscopic quantum coherence unavoidable. Ordinary matter is condensed around dark matter. The counterparts of black holes would be systems consisting of only dark matter.

5. Zero energy ontology (ZEO) is central element of TGD. There are many motivations for it. For instance, Poincare invariance in standard sense cannot make sense since in standard cosmology energy is not conserved. The interpretation is that various conserved quantum numbers are length scale dependent notions.

   Physical states are zero energy states with positive and negative energy parts assigned to ends of space-time surfaces at the light-like boundaries of causal diamonds (CDs). CD is defined as Cartesian products of CP_2 with the intersection of future and past directed lightcones of M^4. CDs form a fractal length scale hierarchy. CD defines the region about which single conscious entity can have conscious information, kind of 4-D perceptive field. There is a hierarchy of WCWs associated with CDs. Consciously experienced physics is always in the scale of given CD.

   Zero energy states identified as formally purely classical WCW spinor fields replace positive energy states and are analogous to pairs of initial and final, states and the crossing symmetry of quantum field theories gives the mathematical motivation for their introduction.

6. Quantum measurement theory can be seen as a theory of consciousness in ZEO. Conscious observer or self as a conscious entity becomes part of physics. ZEO gives up the assumption about unique universe of classical physics and restricts it to the perceptive field defined by CD.

   In each quantum jump a re-creation of Universe occurs. Subjective experience time corresponds to state function reductions at fixed, passive bounary of CD leaving it invariant as well as state at it. The state at the opposite, active boundary changes and also its position changes so that CD increases state function by state function reduction doing nothing to the passive boundary. This gives rise to the experienced flow of geometric time since the distance between the tips of CD increases and the size of space-time surfaces in the quantum superposition increases. This sequence of state function reductions is counterpart for the unitary time evolution in ordinary quantum theory.

   Self "dies" as the first state function reduction to the opposite boundary of CD meaning re-incarnation of self at it and a reversal of the arrow of geometric time occurs: CD size increases now in opposite time direction as the opposite boundary of CD recedes to the geometric past reduction by reduction.

   Negentropy Maximization Principle (NMP) defines the variational principle of state function reduction. Density matrix of the subsystem is the universal observable and the state function reduction leads to its eigenspaces. Eigenspaces, not only eigenstates as usually.

   Number theoretic entropy makes sense for the algebraic extensions of rationals and can be negative unlike ordinary entanglement entropy. NMP can therefore lead to a generation of NE if the entanglement correspond to a matrix proportional to a unitary matrix so that the density matrix of the final state is higher-D unit matrix. Another possibility is that entanglement matrix is algebraic but that its diagonalization in the algebraic extension of rationals used is not possible. This is expected to reduce the rate for the reduction since a phase transition increasing the size of extension is needed.

   The weak form of NMP does not demand that the negentropy gain is maximum: this allow the conscious entity responsible for reduction to decide whether to increase maximally NE resources of the Universe or not. It can also allow larger NE increase than otherwise. This freedom brings the quantum correlates of ethics, moral, and good and evil. p-Adic length scale hypothesis and the existence of preferred p-adic primes follow from weak form of NMP and one ends up naturally to adelic physics.

Could electric-magnetic duality allow to understand $1/h_{eff}^2$ dependence of the effective string tension?

Electric-magnetic duality (possibly the TGD counterpart of AdS/CFT duality) might allow to understand the proportionality of effective string tension to $1/h_{eff}^2$.

1. The *effective* string tension assignable to fermionic strings accompanying magnetic flux tubes and allowing to express Minkowskian Kähler as stringy action must be inversely proportional to $1/h_{eff}^2$ in order to obtain gravitationally bound states in macroscopic length scales identified as structure for which partonic 2-surfaces are connected by strings accompanying flux tubes. This requirement is not easy to prove since $1/\alpha_K$ is proportional to h_{eff}. Could electric-magnetic duality imply this formula with the interpretation that the effective string tension corresponds to Kähler action for string like object?

2. The Dirac condition would give

   $$\frac{g_m g_K}{2\pi} = z \in Z$$

   giving

   $$\frac{1}{\alpha_m} = \frac{4\alpha_K}{z^2} = \frac{\pi}{2qz^2} \quad .$$

if one accepts the argument of [K124] requiring that Kähler action for CP_2 type vacuum extremal is rational number $q = m/n$ guaranteeing that the exponent of Kähler action for Euclidian space-time regions of preferred extremals belongs to an a finite-dimensional extension of p-adic numbers generated by a root of e (note that e is adelically completely unique). This argument implies $\alpha_K = \pi/8q$ (note that this result is in conflict with earlier ideas about the algebraic structure of α_K [L73] based on much more ad hoc argumens).

This would give

$$\frac{1}{2g_m^2} = \frac{1}{16qz^2} \quad .$$

The contribution of the action from Minkowskian regions would be proportional to π and in case of string like objects string area A should be a rational number. The value of string tension would be reduced by a factor

$$\frac{g_K^2}{g_m^2} = \frac{4\alpha_K^2}{z^2} \quad .$$

This is inconsistent with the model of cosmic strings [K37] predicting much larger tension (consistency would require $g_m = g_K$), and would lead to problems in the model of galactic dark matter assuming that galactic strings are like pearls in necklace around single cosmic string. The duality can thus hold only for M^4 type regions of space-time surface.

3. The formula $g_K^2/g_m^2 = 4\alpha_K^2/z^2$ implies $1/h_{eff}^2$ proportionality for the effective string tension if the formula $h_{eff} = z \times h$ makes sense. z would correspond to the number of sheets for the magnetic flux tubes defining covering of M^4.

The analogs of blackholes in TGD

Could blackholes have any analog in TGD? What about Hawking radiation? The following speculations are inspired by the above general vision.

1. Ordinary blackhole solutions are not appropriate in TGD. Interior space-time sheet of *any* physical object is replaced with an Euclidian space-time region. Also that of blackhole by perturbation argument based on the observation that if one requires that the radial component of blackhole metric is finite, the horizon becomes light-like 3-surface analogous to the light-like orbit of partonic 2-surface and the metric in the interior becomes Euclidian.

2. The analog of blackhole can be seen as a limiting case for ordinary astrophysical object, which already has blackhole like properties due to the presence of $h_{eff} = n \times h$ dark matter particles, which cannot appear in the same vertices with visible manner. Ideal analog of blackhole consist of dark matter only, and is assumed to satisfy the $h_{gr} = h_{eff}$ already discussed. It corresponds to region with a radius equal to Compton length for arbitrary particle $R = GM/v_0 = r_S/2v_0$, where r_S is Schwartschild radius. Macroscopic quantum phase is in question since the Compton radius of particle does not depend on its mass. Blackhole limit would correspond to $v_0/c \rightarrow 1$ and dark matter dominance. This would give $R = r_S/2$. naïve expectation would be $R = r_S$ (maybe factor of two is missing somewhere: blame me!).

3. NMP implies that information cannot be lost in the formation of blackhole like state but tends to increase. Matter becomes totally dark and the NE with the partonic surfaces of external world is preserved or increases. The ingoing matter does not fall to a mass point but resides at the partonic 2-surface which can have arbitrarily large surface. It can have also wormholes connecting different regions of a spherical surface and in this manner increase its genus. NMP, negentropy , negentropic entanglement between $h_{eff} = n \times h$ dark matter systems would become the basic notions instead of second law and entropy.

4. There is now a popular article (`http://tinyurl.com/o6n3k4y` explaining the intuitive picture behind Hawking's proposal. The blackhole horizon would involve tangential flow of light and particles of the infalling matter would induce supertranslations on the pattern of this light thus coding information about their properties to this light. After that this light would be radiated away as analog of Hawking radiation and carry out this information.

   The objection would be that in GRT horizon is no way special - it is just a coordinate singularity. Curvature tensor does not diverge either and Einstein tensor and Ricci scalar vanish. This argument has been used in the firewall debates to claim that nothing special should occur as horizon is traversed. So: why light would rotate around it? No reason for this!

   The answer in TGD would be obvious: horizon is replaced for TGD analog of blackhole with a light-like 3-surface at which the induced metric becomes Euclidian. Horizon becomes analogous to light front carrying not only photons but all kinds of elementary particles. Particles do not fall inside this surface but remain at it!

   The objection now is that photons of light front should propagate in direction normal to it, not parallel. The point is however that this light-like 3-surface is the surface at which induced 4-metric becomes degenerate: hence massless particles live on it.

5. The replacement of second law with NMP leads to ask whether a generalization of blackhole thermodynamics (`http://tinyurl.com/y7pvj23x`) does make sense. Since blackhole thermodynamics characterizes Hawking radiation, the generalization could make sense at least if there exist analog for the Hawking radiation (`http://tinyurl.com/md6mmvg`). Note that also geometric variant of second law makes sense.

   Could the analog of Hawking radiation be generated in the first state function reduction to the opposite boundary, and be perhaps be assigned with the sudden increase of radius of the partonic 2-surface defining the horizon? Could this burst of energy release the energy compensating the generation of gravitational binding energy? This burst would however have totally different interpretation: even gamma ray bursts from quasars could be considered as candidates for it and temperature would be totally different from the extremely low general relativistic Hawking temperature of order

$$T_{GR} = \frac{\hbar}{8\pi GM} ,$$

   which corresponds to an energy assignable to wavelength equal to 4π times Schwartschild radius. For Sun with Schwartschild radius $r_S = 2GM = 3$ km one has $T_{GR} = 3.2 \times 10^{-11}$ eV.

One can of course have fun with formulas to see whether the generalizaton assuming the replacement $h \rightarrow h_{gr}$ could make sense physically. Also the replacement $r_S \rightarrow R$, where R is the real radius of the star will be made.

1. Blackhole temperature can be formally identified as surface gravity

$$T = \frac{\hbar_{gr}}{\hbar}\frac{\hbar GM}{2\pi R^2} = \frac{m}{8\pi v_0}\frac{r_S^2}{R^2} .$$

   For Sun with radius $R = 6.96 \times 10^5$ km one has $T/m = 3.2 \times 10^{-11}$ giving about 3×10^{-2} eV for proton. This is by 9 orders higher than ordinary Hawking temperature. Amazingly, this temperature equals to room temperature! Is this a mere accident? If one takes seriously TGD inspired quantum biology in which quantum gravity plays a key role [K84], this does not seem to be the case. Note that for electron the temperature would correspond to energy $3/2 \times 10^{-5}$ eV which corresponds to 4.5 GHz frequency for ordinary Planck constant.

   It must be however made clear that the value of v_0 for dark matter could differ from that deduced assuming that entire gravitational mass is dark. For $M \rightarrow M_D = kM$ and $v_0 \rightarrow \sqrt{k}v_0$ the orbital radii remain unchanged but the velocity of dark matter object at the orbit scales to $\sqrt{k}v_0$. This kind of scaling is suggested by the fact that the value of h_{gr} seems to be too large as compared to that deduced by the identification of biophotons as decay results of dark photons with $h_{eff} = h_{gr}$ (some arguments suggest the value $k \simeq 2 \times 10^{-4}$) [?].

Note that for the radius $R = r_S/2\sqrt{v_0\pi}$ the thermal energy exceeds the rest mass of the particle. For neutron stars this limit might be achieved.

2. Blackhole entropy

$$S_{GR} = \frac{A}{4\hbar G} = 4\pi \frac{GM^2}{\hbar} = 4\pi \frac{M^2}{M_{Pl}^2}$$

would be replaced with the negentropy for dark matter making sense also for systems containing both dark and ordinary matter. The negentropy $N(m)$ associated with a flux tube of given type would be a fraction h/h_{gr} from the total area of the horizon using Planck area as a unit:

$$N(m) = \frac{h}{h_{gr}} \times \frac{A}{4\hbar G} = \frac{h}{h_{gr}} \times \frac{R^2}{r_S^2} S_{GR} = v_0 \frac{M}{m} \frac{R^2}{r_S^2} \ .$$

The dependence on m makes sense since a given flux tube type characterized by mass m determining the corresponding value of h_{gr} has its own negentropy and the total negentropy is the sum over the particle species. The negentropy of Sun is numerically much smaller that corresponding blackhole entropy.

3. Horizon area is proportional to $(GM/v_0)^2 \propto h_{eff}^2$ and should increase in discrete jumps by scalings of integer and be proportional to n^2.

How does the analog of blackhole evolve in time? The evolution consists of sequences of repeated state function reductions at the passive boundary of CD followed by the first reduction to the opposite boundary of CD followed by a similar sequence. These sequences are analogs of unitary time evolutions. This defines the analog of blackhole state as a repeatedly re-incarnating conscious entity and having CD, whose size increases gradually. During given sequence of state function reductions the passive boundary has constant size. About active boundary one cannot say this since it corresponds to a superposition of quantum states.

The reduction sequences consist of life cycles at fixed boundary and the size of blackhole like state as of any state is expected to increase in discrete steps if it participates to cosmic expansion in average sense. This requires that the mass of blackhole like object gradually increases. The interpretation is that ordinary matter gradually transforms to dark matter and increases dark mass $M = R/G$.

Cosmic expansion is not observed for the sizes of individual astrophysical objects, which only co-move. The solution of the paradox is that they suddenly increase their size in state function reductions. This hypothesis allows to realize Expanding Earth hypothesis in TGD framework [L64]. Number theoretically preferred scalings of blackhole radius come as powers of 2 and this would be the scaling associated with Expanding Earth hypothesis.

13.3.4 More About BMS Supertranslations

Bee (see `http://tinyurl.com/z4p9h7l`) had a blog posting about the new proposal of Hawking, Perry and Strominger (HPS, see `http://tinyurl.com/z6tpzar`) [B27] to solve the blackhole information loss problem. In the article Maxwellian electrodynamics is taken as a simpler toy example.

1. One can assign to gauge transformations conserved charges. Gauge invariance tells that these charges vanish for all gauge transformations, which approach trivial transformation at infinity. Now however it is assumed that this need not happen. The assumption that action is invariant under these gauge transformations requires that the radial derivative of the function Φ defining gauge transformation approaches zero at infinity but gauge transformation can be non-trivial in the angle coordinates of sphere S^2 at infinity. The allowance of these gauge

transformations implies infinite number of conserved charges and QED is modified. The conserved gauge charges are generalizations of ordinary electric charged defined as electric fluxes (defining zero energy photons too) and reduce to electric gauge fluxes with electric field multiplied by Φ.

2. For Maxwell's theory the ordinary electric charged defined as gauge flux must vanish. The coupling to say spinor fields changes the situation and due to the coupling the charge as flux is expressible in terms of fermionic oscillator operators and those of U(1) gauge field . For non-constant gauge transformations the charges are at least formally non-trivial even in absence of the coupling to fermions and linear in quantized U(1) gauge field.

3. Since these charges are constants of motion and linear in bosonic oscillator operators, they create or annihilate gauge bosons states with vanishing energy: hence the term soft hair. Holographists would certainly be happy since the charges could be interpreted as representing pure information. If one considers only the part of charge involving annhilation operators one can consider the possibility that in quantum theory physical states are eigenstates of these "half charges" and thus coherent states which are the quantum analogs of classical states. Infinite vacuum degeneracy would be obtained since one would have infinite number of coherent states labelled by the values of the annihilation operator parts of the charges. A situation analogous to conformal invariance in string models is obtained if all these operators either annihilate the vacuum state or create zero energy state.

4. If these U(1) gauge charges create new ground states they could carry information about matter falling into blackhole. Particle physicist might protest this assumption but one cannot exclude it. It would mean generalization of gauge invariance to allow gauge symmetries of the proposed kind. What distinguishes U(1) gauge symmetry from non-Abelian one is that fluxes are well-defined in this case.

5. In the gravitational case the conformal transformations of the sphere at infinity replace U(1) gauge transformations. Usually conformal invariance would requite that almost all conformal charges vanish but now one would not assume this. Now physical states would be eigentates of annihilation operator parts of Virasoro generators L_n and analogous to coherent states and code for information about the ground state. In 4-D context interpretation as strong form of holography would make sense.The critical question is why should one give up conformal invariance as gauge symmetry in the case of blackholes.

It is is interesting to look TGD analogy for BMS supertranslation symmetries. Not for solving problems related to blackholes - TGD is not plagued by these problems - but because the analogs of these symmetries are very important in TGD framework.

1. In TGD framework conformal transformations of boundary of causal diamond (CD) correspond to the analogs of BMS transformations. Actually conformal transformations of not only sphere (with constant value of radial coordinate labeling points of light rays emerging from the tip of the light-cone boundary) but also in radial degrees of freedom so that conformal symmetries generalize. This happens only in case of 4-D Minkowski space and also for the light-like 3-surfaces defining the orbits of partonic 2-surfaces. One actually obtains a huge generalization of conformal symmetries. As a matter of fact, Bee wondered whether the information related to radial degrees of freedom is lost: one might argue that holography eliminates them.

2. Amusingly, one obtains also the analogs of U(1) gauge transformations in TGD! In TGD framework symplectic transformations of light-cone boundary times CP_2 act like U(1) gauge transformations but are not gauge symmetries for Kähler action except for vacuum extremals! This is assumed in the argument of the article to give blackhole its soft hair but without any reasonable justification. One can assign with these symmetries infinite number of non-trivial conserved charges: super-symplectic algebra plays a fundamental role in the construction of the geometry of "World of Classical Worlds" (WCW).

   At embedding space level the counterpart for the sphere at infinity in TGD with the sphere at which the lightcone-boundaries defining the boundary of causal diamond (CD) intersect.

At the level of space-time surfaces the light-like orbits of partonic 2-surfaces at which the signature of the induced metric changes are the natural counterparts of the 3-surface at infinity.

In TGD framework Noether charges vanish for some subalgebra of the entire algebra isomorphic to it and one obtains a hierarchy of quantum states (infinite number of hierarchies actually) labelled by an integer identifiable in terms of Planck constant $h_{eff}/h = n$. If colleagues managed to realize that BMS has a huge generalization in the situation when space-times are surface in $H = M^4 \times CP_2$, floodgates would be open.

One obtains a hierarchy of breakings of superconformal invariance, which for some reason has remained un-discovered by string theorists. The natural next discovery would be that one indeed obtains this kind of hierarchy by demanding that conformal gauge charges still vanish for a sub-algebra isomorphic with the original one. Interesting to see who will make the discovery. String theorists have failed to realize also the completely unique aspects of generalized conformal invariance at 3-D light-cone boundary raising dimension $D = 4$ to a completely unique role. To say nothing about the fact that M^4 and CP_2 are twistorially completely unique. I would continue the list but it seems that the emergence super string elite has made independent thinking impossible, or at least the communications of the outcomes of independent thinking.

Does one obtain the analogs of generalized gauge fluxes for Kähler action in TGD framework?

1. The first thing to notice is that Kähler gauge potentials are not the primary dynamical variables. This role is taken by the embedding space coordinates. The symplectic transformations of CP_2 act like gauge transformations mathematically but affect the induced metric so that Kähler action does not remain invariant. The breaking is small due to the weakness of the classical gravitation. Indeed, if symplectic transformations are to define isometries of WCW, they cannot leave Kähler action invariant since the Kähler metric would be trivial! One can deduce symplectic charges as Noether charges and they might serve as analogs fo the somewhat questionable generalized gauge charges in HPS proposal.

2. If the counterparts of the gauge fluxes make sense they must be associated with partonic 2-surfaces serving as basic building bricks of elementary particles. Field equations do not follow from independent variations of Kähler gauge potential but from that of embedding space coordinates. Hence identically conserved Kähler current does not vanish for all extremals. Indeed, so called massless extremals (MEs) [K18] can carry a non-vanishing light-like Kähler current, whose direction in the general case varies. MEs are analogous to laser beams and if the current is Kähler charged it means that one has massless charged particle.

3. Since Kähler action is invariant also under ordinary gauge transformations one can formally derive the analog of conserved gauge charge for non-constant gauge transformation Φ. The question is whether this current has any physical meaning.

   One obtains current as contraction of Kähler form and gradient of Φ:

$$j^{\alpha}_{\Phi} = J^{\alpha\beta}\partial_{\beta}\Phi \ , \tag{13.3.1}$$

   which is conserved only if Kähler current vanishes so that Maxwell's equations are true or if the contraction of Kähler current with gradient of Φ vanishes:

$$j^{\alpha}_{\Phi}\partial_{\alpha}\Phi = 0 \ . \tag{13.3.2}$$

   The construction of preferred extremals leads to the proposal that the flow lines of Kähler current are integrable in the sense that one can assign a global coordinate Ψ with them. This means that Kähler current is proportional to gradient of scalar function Ψ:

$$j^{\alpha}_{\Phi} = g^{\alpha\beta}\partial_{\beta}\Psi \quad . \tag{13.3.3}$$

This implies that the gradients of Φ and Ψ are orthogonal. If Kähler current is light-like as it is for the known extremals, Φ is superposition of light-like gradient of Ψ and of two gradients in a sub-space of tangent space analogous to space of two physical polarizations. Essentially the local variant of the polarization-wave vector geometry of the modes of radiative solutions of Maxwell's equations is obtained. What is however important that superposition is possible only for modes with the same local direction of wave vector ($\nabla\Psi$) and local polarization.

Kähler current would be scalar function k times gradient of Ψ :

$$j^{\alpha}_{\Phi} = kg^{\alpha\beta}\partial_{\beta}\Psi \quad . \tag{13.3.4}$$

The proposal for preferred extremals generalizing at least MEs leads to the proposal that the extremals define two light-like coordinates and two transversal coordinates.

4. The conserved current decomposes to a sum of interior and boundary terms. Consider first the boundary term. The boundary contributions to the generalized gauge charge is given by the generalized fluxes

$$Q_{\delta,\Phi} = \oint J^{tn}\Phi g^{1/2} \tag{13.3.5}$$

over partonic 2-surfaces at which the signature of the induced metric changes from Euclidian to Minkowskian. These contributions come from both sides of partonic 2-surface corresponding to Euclidian and Minkowskian metric and they differ by a imaginary unit coming from $g^{1/2}$ at the Minkoskian side. $Q_{\delta,\Phi}$ could vanish since $g^{1/2}$ approaches zero because the signature of the induced metric changes at the orbit of the partonic 2-surfaces. What happens depends on how singular the electric component of gauge potential is allow to be. Weak form of electric magnetic duality proposed as boundary condition implies that the electric flux reduces to magnetic flux in which case the result would be magnetic flux weighted by Φ.

5. Besides this there is interior contribution, which is Kähler current multiplied by -Φ:

$$Q_{int,\Phi} = \int j^t\Phi g^{1/2} \quad . \tag{13.3.6}$$

This contribution is present for MEs.

6. Could one interpret these charges as genuine Noether charges? Maybe! The charges seem to have physical meaning and they depend on extremals. The functions Φ could even have some natural physical interpretation. The modes of the induced spinor fields are localized at string world sheets by strong form of holography and by the condition that electric charge is well defined notion for them. The modes correspond to complex scalar functions analogous to powers z^n associated with the modes of conformal fields. Maybe the scalar functions could be assigned to the second quantized fermions. Note that one cannot interpret these contributions in terms of oscillator operators since the second quantization of the induced gauge fields does not make sense. This would conform with strong form of holography which in TGD framework sense that the descriptions in terms of fundamental fermions and in terms of classical dynamics of Kähler action are dual. This duality suggest that the quantal variants of generalized Kähler charges are expressible in terms of fermionic oscillator operators generating also bosonic states as analogs of bound states. The generalized charge eigenstates might be also seen as analogs of coherent states.

13.4 How to demonstrate quantum superposition of classical gravitational fields?

There was rather interesting article in Nature [B12] (see http://tinyurl.com/ybylck8m)) by Marletto and Vedral about the possibility of demonstrating the quantum nature of gravitational fields by using weak measurement of classical gravitational field affecting it only very weakly. There is also an article in arXiv by the same authors [B11] (see http://tinyurl.com/ybylck8m). The approach relies on quantum information theory.

The gravitational field would serve as a measurement interaction and the weak measurements would be applied to gravitational witness serving as probe - the technical term is ancilla. Authors claim that weak measurements giving rise to analog of Zeno effect could be used to test whether the quantum superposition of classical gravitational fields (QSGR) does take place. One can however argue that the extreme weakness of gravitation implies that other interactions and thermal perturbations mask it completely in standard physics framework. Also the decoherence of gravitational quantum states could be argued to make the test impossible.

One must however take these objections with a big grain of salt. After all, we do not have a theory of quantum gravity and all assumptions made about quantum gravity might not be correct. For instance, the vision about reduction to Planck length scale might be wrong. There is also the mystery of dark matter, which might force considerable motivation of the views about dark matter. Furthermore, General Relativity itself has conceptual problems: in particular, the classical conservation laws playing crucial role in quantum field theories are lost. Superstrings were a promising candidate for a quantum theory of gravitation but failed as a physical theory.

In TGD, which was born as an attempt to solve the energy problem of TGD and soon extended to a theory unifying gravitation and standard model interactions and also generalizing string models, the situation might however change. In zero energy ontology (ZEO) the sequence of weak measurements is more or less equivalent to the existence of self identified as generalized Zeno effect! The value of $h_{eff}/h = n$ characterizes the flux tubes mediating various interactions and can be very large for gravitational flux tubes (proportional to GMm/v_0, where $v_0 < c$ has dimensions of velocity, and M and m are masses at the ends of the flux tube) with $Mm > v_0 m_{Pl}^2$ (m_{Pl} denotes Planck mass) at their ends. This means long coherence time characterized in terms of the scale of causal diamond (CD). The lifetime T of self is proportional to h_{eff} so that for gravitational self T is very long as compared to that for electromagnetic self. Selves could correspond sub-selves of self identifiable as sensory mental images so that sensory perception would correspond to weak measurements and for gravitation the times would be long: we indeed feel the gravitational force all the time. Consciousness and life would provide a basic proof for the QSGR (note that large neutron has mass of order Planck mass!).

13.4.1 Is gravitation classical or quantal?

The conflict between general relativity (GRT) in which gravitation has classical description in terms of geometry and quantum theory was noticed very early, certainly already by Einstein, which explains his refusal to accept quantum theory. Feynman crystallized the problem [B12] and was led to suggest that gravitation must be described quantally. The following arguments suggests that classical gravitational fields reducing to space-time geometries in GRT are necessary to describe gravitationally bound states.

1. The electron in atom is de-localized so that one must have also quantum superposition superposition of classical gravitational fields associated with it. This requires allowance of a space of classical gravitational fields, where one has Schrödinger amplitudes. In GRT framework this means allowance of the space of space-time geometries or at least the space of 3-geometries and Wheeler indeed proposed this notion (super-space).

   The same is true in electrodynamics, where pure QED gives wrong predictions for hydrogen atom but the simple model based on classical em fields gives excellent predictions. This can be understood in TGD in terms of the notion of bound states involving fusion of 3-surfaces to single 3-surface connected by magnetic flux tubes serving also as correlates for quantum entanglement.

From TGD point of view the path integral quantization of quantum field theories was a mistake and prevented the discovery geometrization of field concept in terms of sub-manifold geometry and the notion of WCW generalizing the geometrization of physics program of Einstein to the entire quantum theory.

2. The quantization of gravitation as quantum field theory (QFT) in flat Minkowski space background is not enough. One must replace world with WCW as done in TGD, where worlds correspond to space-time surfaces in $M^4 \times CP_2$. The induction of process for metric and spinor connection geometrizes various fields and the classical worlds are space-time surfaces.

This leads to a completely new vision about gravitation and other interactions consistent with the standard model leading to notions like hierarchy of Planck constants allowing quantum coherence in even astrophysical length and time scales, p-adic physics, and eventually adelic physics as physics of sensory experience and cognition. What is remarkable that the TGD counterpart of ER-EPR correspondence discovered much before ER-EPR states that magnetic flux tubes serve as correlates for negentropic entanglement and are accompanied by fermionic strings.

1. Each interaction is characterized by its own magnetic flux tubes and by the value of Planck constant $h_{eff} = n \times h$ labelling phases of ordinary matter identified as dark matter. h_{eff} actually has number theoretic interpretation in adelic physics [L48, L49].

2. The Planck constant associated with the magnetic flux tube is proportional to the product of corresponding charges at its ends [K47, ?, K100, K83]. For gravitational interaction one has $\hbar_{eff} = \hbar_{gr} = GMm/v_0$, where M and m are masses at the ends of the flux tube and $v_0 < c$ is parameter with dimensions of velocity. For electromagnetic interaction one has $\hbar_{em} = Ze^2Q_1Q_2/v_0$. The value of h_{gr} is much larger than h and h_{em} if one has $Mm/v_0 > m_{Pl}^2$ (m_{Pl} denotes Planck mass). For $Mm < v_0 m_{Pl}^2$ one has $h_{eff} = h_{gr} = h$.

   The large values of h_{gr} suggests that gravitational quantum coherence is possible even in astrophysical scales: Nottale [E13] indeed proposed that one can regard planetary orbits as Bohr orbits. The fountain effect of superfluidity could be one example of this [?]. h_{gr} would be also in key role in living matter.

This argument relies on mere logic and to my opinion makes the notion of WCW (or some analog of it) compelling if one accepts geometrization of gravitation.

13.4.2 Zeno effect and weak measurements

The proposal of Marletto and Vedral [B11, B11] is inspired by quantum information theory. Some of the basic notions involved ancilla or probe, gravitational witness, and weak measurement giving rise to an analog of Zero effect.

Can one test quantum character of gravitation experimentally?

One can also approach the situation purely experimentally by trying to find effects demonstrating the quantum character of gravitation. The basic problem is the extreme weakness of gravitation. It seems that quantum gravitational effects are masked by other interactions and thermodynamical effects.

1. The simplest question is whether particles in gravitational field of say Earth behave quantally in analogy with the behavior in electromagnetic fields. This is found to be case by studying neutrons in the Earth's gravitational field. This finding by the way killed the idea about entropic gravity identifying gravity as thermodynamical effect [K119]. This experiment does not however say anything about whether classical gravitational fields form quantum superpositions.

2. The emission rate of gravitons by elementary particles are extremely low. Hence on cannot test the theory at elementary particle level by measuring graviton emission and absorption or by studying the gravitational counterparts of bound states such as atoms - this if one assumes standard value of Planck constant only. Also the graviton interference effects and

effects like Bose-Einstein condensation seem to be impossible to test. In the early Universe strong gravitational fields exist and inflationary period could show quantum gravitational effects. This kind of tests are however indirect. In TGD framework the cosmic string based model for galactic dark can be seen as support for quantum gravitation.

3. The situation changes if one allows the hierarchy of Planck constants. In this case one can have the analogs of atoms as planetary systems. One can argue that quantum character of gravitational bound states solves the analog of infrared catastrophe of hydrogen atom, which led to the birth of atomic physics. The formation of blackhole would correspond to infrared catastrophe. Dark gravitons with large h_{gr} have large energies $E = h_{gr}f$ and one can even speculate with the possibility of direct observation of low energy dark gravitons as they transform to bunches of $h_{gr}/h = n$ ordinary gravitons [K83].

The experiment testing QSGR should generate a de-localized particle - say superposition of two sharply localized states. This kind of de-localized states appear in atomic and molecular physics and superconductors and Bose-Einstein condensates provide macroscopic variants of these states. One should test whether these states involve QSGR. Standard physics says that for larger objects this kind of de-localized states are not possible.

If one allows hierarchy of Planck constants, in particular $h_{gr} = h_{eff}$ hypothesis, the situation changes. The fountain effect of super-fluidity could be a representative example [?]. The de-localization of particles at magnetic flux tubes in the phase transition generating dark matter would directly affect the gravitational field created by system and it might be possible detect this change. Quite generally, quantum critical systems would be excellent candidates for demonstrating quantum superposition of classical gravitational fields.

The challenge is how to demonstrate the existence of QSGR. Gravitational interaction is too weak but could hierarchy of Planck constants change the situation somehow?

The notion of weak measurement

Contrary to my prejudice, the notion of weak measurement (see `http://tinyurl.com/zt36hpb`) makes sense mathematically and is different from the notion of weak values (see `http://tinyurl.com/yc63pygw`), which to my opinion are mathematical nonsense. The idea of weak measurement is to entangle the weakly measured system with probe and measure the state of the probe rather than system repeatedly. If the initial state of probe is strongly localized to some value of the observable measured, the sequence of measurements does not affect much the weakly measured system and one can monitor it.

1. One has tensor product $A \otimes B$ of two systems and weak interaction entangling them and realized by interaction Hamiltonian H, whose exponential gives rise to time evolution. Time evolution consists of periods Δt_n ending by a measurement of some observable x for B giving eigenvalue q. Each period induces a unitary evolution of A by a Hamiltonian, which does not commute with x. The weakness of the interaction and strong localization of the initial state of B imply that A is only weakly perturbed and ancilla B follows its state in good accuracy.

2. In the simplest situation both system A and B are characterized by single commuting observable x analogous to position operator and its conjugate p. A is the system to be monitored and B is the ancilla. The initial state $|\Psi(0)$ of A is arbitrary and the initial state $\Phi(0)$ of the probe B can be assumed to be Gaussian (for instance): harmonic oscillator could be in question.

3. The canonical choice for the interaction Hamiltonian would be $H = kx \otimes p$. Quantum measurement of x for the ancilla (probe) B after time Δt implies a localization to the state $|q\rangle$. After than unitary evolution induces again de-localization and until new position measurement occurs. One can solve the Schrödinger equation and express the outcome of the measurement of the position x for B as

$$\begin{aligned}\Psi(\Delta t)\Phi(\Delta t) &= M_q\Psi(0)\otimes|q\rangle \ , \\ M_q &= \frac{1}{N}\times exp(-ik\Delta tx\otimes p) \ , \\ N &= \sqrt{\langle\Psi(0)|M_q^\dagger M_q|\Psi(0)\rangle} \ . \end{aligned} \tag{13.4.1}$$

The unitary operator M_q - Kraus operator - depends on position operator x. It has the eigenvalue q as a parameter.

For Gaussian initial state $\Psi(0)$ one has

$$M_q = \frac{1}{(2\pi\sigma^2)^{1/4}} exp((-q-x)^2/4\sigma^2) \ . \tag{13.4.2}$$

If the measured state is localized around the eigenvalue of x_0 of x, this distribution is peaked around x_0 and also the eigenvalue of the ancilla position q remains near it. One might say that ancilla follows the state of the weakly measurement system. Note that H is only interaction Hamiltonian and contains also part associated with the weakly measured system.

It is important that the unitary evolution induced by $H = kx\otimes p$ does not leave the eigenstate of q invariant but induces shift by x. Therefore the repeated measurements of q imply a stepwise motion in q-space inducing a similar motion for Ψ in A.

Weak measurement brings strongly in mind Zeno effect in which repeated measurement leave the state unaffected. In the recent case this is not the case since H does not commute with position operator of ancilla. Remarkably, the weak measurement is highly analogous to the generalized Zeno effect in zero energy ontology (ZEO) defining self as sequence of “small state function reductions” at the active boundary of causal diamond (CD) and giving rise to the experience about flow of time.

1. Weak measurement could serve as a model for sensory perception following monitoring target. Self indeed consists of sequences of unitary time evolutions in which system entangles with external world although the its state about the passive boundary of CD representing the unchanging part of self is unaffected. Magnetic flux tubes serve as correlates for both entanglement and attention.

2. The members of state pairs at passive boundary of CD remain unaffected. The sequence of small state function reductions ends up with the death of self as ”big” state function reduction at opposite boundary of CD takes place. For sub-selves defining mental images this would mean that attention ceases. Self can be said to performing weak measurements as long it lives! As self dies a time reversed self assignable to the opposite boundary of CD is created.

Remark: There is also so called interaction free measurement (see `http://tinyurl.com/y7zq97q2`), which I considered for some years ago as counterpart of self. This hypothesis turned out to be un-necessary. Interaction free measurement does not seem to be quite same as weak measurement.

Weak measurement induced by measurement of classical gravitational fields

How could one apply weak measurement to monitor gravitational fields and their quantum superpositions?

1. One could consider replacing x and p by some components of classical gravitational field and their canonical conjugates at some point of space. If one could arrange the measurement interaction to be of the form described above, one could follow the state of classical gravitational field and also the quantum superposition for the values of classical gravitational field - say in given position. The expectation of the operator M_q for the state would reveal the distribution in coordinate x characterizing the value of gravitational field. This would however require successful quantization of gravitational fields. Second problem relates to the measurement interaction: how could one arrange it to be of the desired form.

2. Could the measurement interaction be taken to be the gravitational interaction between A and B? Now the positions for two masses m_A and m_B would become observables and measurement interaction would induce motion in A and the distribution of position for the mass m_A would be visible in the unitary operator M_q acting on state Ψ of the target.

   The extreme weakness of gravitational interaction indeed makes it an obvious candidate for witness interaction. Most importantly, the classical gravitational field created by the target at the position of the ancilla appears in the measurement interaction. The weakness however suggests that gravitation as a measurement interaction is masked by other interactions and by thermal noise. The analog of Zeno period is expected to be very short in standard quantum theory.

If I understood correctly, the authors suggests that the occurrence of the analog of Zeno period is used as a way to demonstrate the superposition of classical gravitational fields. I could not quite follow the argument. Zeno period should be present also when there is no de-localization of masses m_A and m_B. Information about M_q is needed in order to deduce whether de-localization and superposition of classical gravitational fields is present. If gravitational field is purely classical, one cannot even talk about weak measurement.

What about the situation in TGD?

In TGD situation changes. ZEO and TGD inspired theory of consciousness enter into play. It would be enough to prove experimentally that the notion of self, which is analog of weak measurement period and an outcome of TGD based view about quantum gravitation relying on the notion of WCW and ZEO, makes sense.

1. Not surprisingly, the hierarchy of Planck constants would play a key role. The lifetime T of self is proportional to $h_{eff}/h = n$, and for gravitational flux tubes one has $\hbar_{eff} = \hbar_{gr} = GMm/v_0$. h_{gr} is much much larger than $h_{eff} = h_{em}$ for the flux tubes mediating electromagnetic interactions (note that flux tubes gives rise to the analog of ER-EPR correspondence which I proposed much before ER-EPR).

   For $Mm > v_0 m_{Pl}^2$ dark matter with $h_{gr} > h$ is possible. Interestingly, Planck mass corresponds in living matter to a water blob with size of large neuron. For $v_0 < c$ neurons could define a pair of systems allowing to test the superposition of classical gravitational fields. One could consider de-localization of neurons or systems associated with them. The de-localization of dark particles at magnetic flux tubes might help here since it would redistribute part of the matter affecting the gravitational field created by it. The detection of gravitational field of neuron might allow to detect this phase transition: neuron would apparently lose part of its weight.

2. The phase transition generating dark matter in say neuronal system might allow detection via the emergence of generalized Zeno effect. Generalized Zeno effect - identifiable as lifetime of self - would serve as a signature of gravitational entanglement. Gravitational Zeno effect - maybe identifiable in terms of sensory perception of gravitational field of target - lasts much longer than its electromagnetic counterpart and its existence would demonstrate that QSGR is real! This would also demonstrate that TGD inspired theories of consciousness and quantum biology, where h_{gr} plays a key role, might have something to do with reality!

The problem of the proposal is that we are not yet able to detect and manipulate dark matter in laboratory for the simple reason that we do not understand it (maybe we do it routinely at the level of biology!).

1. The TGD based conjecture [?] is that dark matter as $h_{eff}/h = n$ phases of ordinary matter emerges at quantum criticality. Large h_{eff} would make possible long range quantum fluctuations and correlates by scaling up various quantum lengths typically by h_{eff}/h. Therefore the ability to create and control quantum critical systems would be the prerequisite for the proposed test.

2. Various macroscopic quantum systems are excellent candidates for quantum criticality. Superfluids exhibiting fountain effects apparently defying gravitation could be such systems too [?]. Note that gravitational Compton length $\hbar_{gr}/m = GM/v_0$ does not depend on the m at all (this is implied by Equivalence Principle) so that particles with different masses could form gravitationally quantum coherent state.

3. In biology this kind of systems could be created for some critical values of parameters: living system would be almost by definition quantum critical and metabolic energy feed would be necessary to induce quantum criticality since in general the energies of various quantum states are larger for $h_{eff} > h$, in particular atomic binding energies behave like $1/h_{eff}^2$. DNA, proteins, cell membrane, axonal membranes, and microtubules would represent examples of critical systems. Nervous system would be such system in longer length scale.

There was a very interesting link to an article telling about the category theoretical description of topological order [B44] (see `http://tinyurl.com/y9c29yly`). The description of non-Abelian Quantum Hall in terms of patterns of zeros of multi-electron wave function and using so called Z_n current algebra states is considered in [B24].

Topological order means emergence of discrete degrees of freedom implying ground state degeneracy and long range correlations, even long range entanglement. Topological order appears in 2+1-D systems. Braiding and braid statistics characterized by R-matrix are central elements. There is also a connection with integrable 2-D quantum field theories. The generalization of R-matrix defines 2-particle S-matrix defining the building brick of N-particle S-matrix in 2-D integrable quantum field theories: the basic interaction is passing-by inducing a phase lag. For braids the exchange is a continuous homotopy and braiding dynamics could make possible topological quantum computation [K4].

One cannot avoid the feeling that topological order is exactly the mathematical tool needed in quantum TGD. On basis of what I have learned recently [L14, L8] (see `http://tinyurl.com/yafezdrm`) and `http://tinyurl.com/ydc33gcs`), condensed matter physicists might be discovering many-sheeted space-time and exotic effects predicted by quantum TGD without realizing what they are doing! I have believed hitherto that this would be something for elementary particle physicists but they are sunken into the multiverse muds of M-theory landscape.

There are several reasons to believe that the notion of topological order in TGD could be very useful in more concrete formulation of quantum TGD.

1. TGD can be seen as almost topological QFT. 3-D surfaces are by holography equivalent with 4-D space-time surfaces and by strong form of holography equivalent with string world sheets and partonic 2-surfaces. What make this duality possible is super-symplectic symmetry [K36, K35] realizing strong form of holography and quantum criticality realized in terms of hierarchy of Planck constants characterizing hierarchy of phases of ordinary matter identified as dark matter. This hierarchy is accompanied by a fractal hierarchy of sub-algebras of supersymplectic algebra isomorphic to the entire algebra [K124]: Wheeler would talk about symmetry breaking without symmetry breaking.

2. $h_{eff} = n \times h$ hierarchy corresponds to n-fold singular covering of space-time surface for which the sheets of the covering co-incide at the boundaries of the causal diamond (CD), and the n sheets together with superconformal invariance give rise n additional discrete topological degreees of freedom - one has particles in space with n points. Kähler action for preferred extremals reduces to Abelian Chern-Simons terms characterizing topological QFT. Furthermore, the simplest example of topological order - point like particles, which can be connected by links - translates immediately to the collections of partonic 2-surfaces and strings connecting them.

3. There is also braiding of fermion lines/magnetic flux tubes and Yangian product and co-product defining fundamental vertices, quantum groups associated with finite measurement resolution and described in terms of inclusions of hyper-finite factors [K125] .

Number theoretic vision [K124] - in particular adelic physics - is an additional building brick in TGD. It would be nice to see what comes out from the combination of topological order with the hierarchy of algebraic extensions of rationals and associated extensions of p-adic number fields by extending the physics to adelic physics. The existence of this extension must pose powerful constraints on physics.

In this article topological order and its category theoretical description are considered from TGD point of view - category theoretical notions are indeed very natural in TGD framework. The basic finding is that the concepts developed in condensed matter physics (topological order, rough description of states as tangles (graphs imbedded in 3-D space), ground state degeneracy, surface states protected by symmetry or topology) fit very nicely to TGD framework and has interpretation in terms of the new space-time concept. This promises applications also in the conventional areas of condensed matter physics such as more precise description of solid, liquid, and gas phases.

The following considerations can be blamed to be "just philosophy" since I am not a condensed matter physics and do not try to pretend being computational virtuoso. What I dare argue that TGD allows much more wider perpective than is possible inside the boundaries posed by specialization. My hope is that the reader would realize that TGD provides fascinating challenges and inspiration for theoretical physicist - even those working in condensed matter physics.

13.4.3 What Does Topological Order Mean?

Topological order is something not describable by local order parameters allowing to characterize different phases by their different symmetries using Landau theory. Fractional Quantum Hall effective is simplest example of this: all phases have the same symmetries. One signature is the existence of several degenerate ground states.

As already noticed, in the fractal Universe of TGD one has a hierarchy of quantum criticalities with levels labelled by $h_{eff} = n \times h$ giving rise to "symmetry breaking without symmetry breaking" in terms of an inclusion hierarchy of isomorphic mutually isomorphic subalgebras of super-symplectic algebra. Could this hierarchy lurk behind the existence of phases with identical symmetries? This hierarchy makes sense also for the ordinary conformal invariance, which is much smaller symmetry than super-symplectic one and replaces AdS/CFT duality with more physical looking duality defined by strong form of holography.

For some reason colleagues have not noticed the possibility of this kind of conformal symmetry breaking. This is not the only rather trivial fact that has escaped the attention of hasty colleagues during last decades. The completely unique role of 4-D space-time, the twistorial uniqueness of $M^4 \times CP_2$ [K114], and the fact that CP_2 codes for standard model symmetries, have also remained un-noticed.

The article *Detecting topological order in a ground state wave function* (see `http://tinyurl.com/y78j4f3v`) by Levin and Wen [B37] gives an idea about what topological order is. The simplest situation in which topological order is encountered, is when one has a set of objects such that each pair can be connected by link. The pair can be characterized by "spin" telling whether its members are connected or not. In condensed matter physics one could have lattice like structure with link between given neighboring points or not. This is very special situation. In principle all possible configurations involving links between objects are possible. One could of course pose additional conditions such as as embedding of the vertices as lattice, restriction of the links to nearest neighbour links, allowance of only single link between members of pair, and some maximum number of links emanating from given object.

What does topological order mean in quantum theory?

1. In topological quantum computation each braid topology defines unitary S-matrix and one has only single braid topology. Topology is still classical and fixed although the dynamics in this fixed topology is quantal.

2. There is however no deep reason to assume localization into a single topology. This mixing could occur already in particle physics. The TGD based explanation of family replication

phenomenon [K33] assumes that quantum superpositions of the topologies of partonic 2-surfaces characterized by genus and that CKM matrix reflects different topological mixings for U and D type quarks [K79]. Ground state wave function would be quantum superposition of graph topologies. Even more: for given graph one would have also a superposition of different embeddings to 3-space as tangles characterized by knotting and linking.

One can formally describe the topology in terms of "topological spins".

1. For a quantum graph each topological configuration of the system is quantum superposition of graphs with some pairs of vertices connected by link or not. What is fixed are the vertices. One can assign to each pair "spin" -1/1 telling whether the connecting link is present or not. One could assume that each vertex is connected to at least one vertex to exclude lonely vertices. This gives a large number of graphs and ground state is quantum superposition of these graphs. This brings in the long range quantum entanglement between pairs. Some kind of reference configuration could be a graph in which all objects are connected to every other object once.

2. The embedding of graph to 3-D space gives tangle. Tangle consists of several groups of vertices from which connecting links emerge. By fractality one can also tangles within tangles. Tangle can be characterized by its projection to a suitably chosen plane. In the projection two tangle strands cross and there are two different crossings depending which strand is above which. This defines second spin like variable characterizing tangles.

3. In TGD space-time also 2-braiding is possible. 2-braid can be thought of as an evolution of ordinary knot giving rise to 2-D surface in 4-D space-time. One an have un-knotting or its reversal of knots by a violent manner: the braid strands go simply through each other. Knot invariants are actually constructed by performing this violent un-knotting step by step. A spin like variable telling whether this occurs for a pair of braid strands appearing in 2-knot is needed.

The article (see `http://tinyurl.com/y78j4f3v`) considers a lattice in which links are possible between neighboring lattices points. The ground state is a superposition over all link paths as a state with long range entanglement: the product of spins equals to 1 for all closed loops crossing a given curve since the loops intersect the curve always even number of times (this is where topology shows itself!) Could this kind quantum superposition be the first principle approach when one wants to describe many particle system? Liquid, gas, and solid phases would be of course hugely simplified descriptions in this picture. The basic unpleasant question is obvious: can long links be really thermally stable in standard physics?

13.4.4 Topological Order And Category Theory

The article (see `http://tinyurl.com/y7qjl4bv`) summarizes the proposal to describe topological order in terms of category theory. In reductionistic approach one decomposes the object to smaller and smaller pieces. In particle physics the actions of symmetries on object characterize the object in terms of quantum numbers. In category theoretical approach one describes the system in terms of its relations with other systems. Relations corresponds to morphisms mathematically and are deduced by studying the interactions with other systems. How particle interacts with the other particles defines what particle is.

At the level of topology the braiding of object with other objects provides this kind of basic morphism. Fusion or stacking with other objects defines second morphism. The integer valued coefficients of fusion telling which quantum objects appear in the stacking of the object with another object provide information about objects via its relations. Fusion has splitting as its reversal. Algebraically product and co-product correspond to these operations and I have proposed that zero energy states as transition amplitudes represents sequences algebraic operations - product and co-product identified essentially as 3-particle vertices - in Yangian algebra closely related to category theoretical approach [K114]. Particle vertices would represent additional morphisms besides braiding.

Category theoretical approach can be made quantitative in terms of integers N_k^{ij} telling the multiplicity for representation k in the fusion of representations i and j and fractionals spins s_i

characterizing the braid statistics. The category in question must involve also the counterpart of tensor product since in physics one must engineer more complex systems from simpler ones. One speaks of tensor category.

One can define stacking of topological orders serving as the counterpart for tensor product and making topological orders a monoid. Stacking is not ordinary tensor product since there is some inherent entanglement always present. I dare to guess that a special case of Connes tensor product is in question [K125]. This inherent entanglement eliminates a lot of states from the ordinary tensor product. Stacking is interpreted in condensed matter context as formation of multilayers.

If stacking by a given topological order leaves other topological orders as such, the topological order is trivial. A non-trivial topological order can have an inverse: this is equivalent with having no topological excitations. The inverse of the topological order is obtained by time reversal operation acting as symmetry. Non-invertible topological orders correspond to non-Abelian braid statistics.

The basic result of article does not say at the first glance too much to a non-specialist. *Up to an invertible topological order 2+1-D fermionic/bosonic topological orders with/without symmetry are classified by modular braided fusion categories (BFC) over symmetric BFC, where symmetric BFC describes product state with/without symmetry.*

I understand that symmetric BFC corresponds to invertible topological orders acting via the stacking and not affecting the topological order: this is like multiplying vector with scalar in projective space.

13.4.5 Category Theoretical Description Of Topological Order In TGD

Much of the philosophy and mathematical building bricks of this vision are shared by quantum TGD. The notions of topological order, stacking, and gapless states represent however something new and are highly interesting concerning the more detailed formulation of quantum TGD. This kind of approach is not all that is needed in TGD but could give the tools needed to build the roughest topological characterization of spinor fields in the "world of classical worlds" (WCW) at many-particle level.

Topological order in TGD

In quantum TGD combinatorial description in terms of graphs would give the roughest topological description of the ground state in terms of partonic 2-surfaces (vertices) and fermionic strings or magnetic flux tubes (links) connecting them. It must be made clear, that topological order in TGD sense means radical deviation from the standard model thinking in which space-time is fixed background. This goes also beyond the descriptive powers of the long length scale limit of string models assuming that space-time serves as arena of dynamics.

There are two basic topological elements besides many-sheetedness: the graph structure characterized by telling which partonic 2-surfaces are connected by strings/flux tubes and the tangle structure present because there exists infinite number of topologically non-equivalent embeddings of the graph to 3-D space. 4-D space-time thus allows richest possible topological order besides gigantic super-symplectic symmetries.

1. The strings/flux tubes could connect different partonic surfaces and also return back to the same partonic 2-surface but at different point carrying fermion number. Strings and flux tubes get knotted and linked in 2+1 dimensional situation. The outcome is tangle. If there are only two partonic 2-surface no self-entangling one has braid.

2. For partonic 2-surfaces carrying several fermions also self-tangles are possible and one can have quantum superposition of different self-tangles. Flux tubes of dipole magnetic field serve as an illustration.

3. Also the many-sheeted character of space-time gives additional topological degree of freedom in TGD framework. In TGD Universe even elementary particles are structures with at least two space-time sheets since they consist of a pair of wormhole contacts connecting two space-time sheets and wormhole throats at both sheets are connected by flux tubes carrying monopole flux and fermionic strings. For large values of h_{eff} the size of these structure

is scaled up so that one could electrons with size scale of cell! As discussed below, many-sheetedness could correspond to what is called stacking of topological orders.

Topological order defined by links is robust and not affected by thermal fluctuations unless the links are thermally unstable. Thermal stability at high temperatures can be argued to be an ad hoc assumption in standard physics. In TGD framework the thermal stability of long links would be due to the hierarchy of Planck constants $h_{eff} = n \times h$. This could make possible long range quantum entanglement between distant topological spins possible in high temperatures.

What about applications? Can one apply the notion of topological order only to low exotic condensed matter systems at low temperature? TGD suggests that applications are possible even at room temperatures.

1. The distinction between liquids and gases is not really well-understood in text book statistical physics missing strings as fundamental objects so that one has only the point particles - partonic 2-surfaces in TGD - and potential function modelling the interactions between them. Topological order replacing potential function with strings/flux tubes should allow an improved understanding the distinction between fluids and gases.

2. The clusters of water molecules are problematic in the standard model description of water, and are crucial in the physics of living matter (consider only the fourth phase of water discovered by Pollack). The existence of strings connecting partonic 2-surfaces would make the clusters of liquid molecules in TGD framework. There is also a connection with $h_{eff} = n \times h$ hypothesis made rigorous by the hierarchy of quantum criticalities explaining dark matter. The longer the flux tubes defining the link needed for clustering are, the larger the value of h_{eff} must be, and the value of h_{eff} characterizes the length scale in which quantum coherence is present.

3. Reductionist finds it convenient to assume that nuclear physics is totally isolated from the condensed matter physics. There are anomalies challenging this hypothesis. For instance, X rays from Sun with energies in the energy scale of transition energies of heavier ions are found to affect the nuclear decay rates so that they vary periodically with period of year [L1]. Could condensed matter transitions do the same trick?

   The claims about cold fusion represents second example [L1] Most main streamers refuse to even consider cold fusion as a possibly real phenomenon. The flux tubes carrying dark quarks with large h_{eff} would bind nucleons to form nuclei and they could be so long as to make possible interactions with condensed matter. They could explain several other anomalies such as the anomalous value of proton radius.

Stacking, time reversal, and gapless states in TGD framework?

Stacking can be seen as a constrained tensor product. It could have several interpretations in TGD framework.

1. Stacking might correspond to a formation of quantum states assignable to many-sheeted structures formed from single sheeted structures? Stacking would occur already as one forms elementary particles as double-sheeted structures. Could it be involved with the formation of n-sheeted coverings associated with $h_{eff}/h = n$ and quantum criticality?

2. Topological condensation of a smaller space-time sheet to a larger space-time sheet might have interpretation in terms of stacking? Topologically condensed space-time sheet cannot be represented as a tensor factor in TGD framework. Can the situation be described as a pair of included and including factors with included factor defining measurement resolution for the including factor? Connes tensor product is indeed associated with the inclusion?

3. Many-sheeted space-time suggests the rather exotic looking possibility that two disjoint space-time sheets can have topologically condensed smaller space-time sheets (like liquid drops of the wall) connected to each other by thin flux tubes not visible in the scale of bigger space-time sheets - entanglement would be a resolution dependent notion. In the scale of the bigger space-time sheet one would have ordinary tensor product without entanglement.

In the scale of smaller space-time sheets one would have entanglement: subsystems of un-entangled systems would entangle. This has a direct application in TGD inspired theory of consciousness: sub-selves (mental images) of self can fuse to stereo mental image shared by the selves although selves do not entangle and remained separate conscious entities [K97].

Could this be described in the formalism based on categories? Is the notion of resolution inherent to this description? The inclusions of hyper-finite factors can be interpreted in terms of finite measurement resolution, and the description of inclusions indeed involves quantum groups as also topological order. The larger space-time sheet seen in the resolution defined by topological condensed space-time sheets would be characterized by quantum space with fractional quantum dimension resulting by modding out the degrees of freedom of topologically condensed space-time sheets.

4. One can imagine a further interpretation for stacking. Negentropic entanglement between states associated with separated space-time sheets could also give rise to a restricted tensor product [K72]. Negentropic entanglement (NE) can be algebraic such that the coefficients belong to the algebraic extension of rationals characterizing the adele but entanglement probabilities are outside this extension, which encourages the hypothesis that diagonalization is not possible and this kind of NE is stable. NE can also correspond to a projector in which case state function reduction need not lead to an eigen ray since the whole sub-space is eigenspace of density matrix.

Time reversal defines inverse topological order provided one can regard it as a symmetry. For instance, time reversal symmetry protects topological insulator. More generally, one can have symmetry protected topological order SPT (see `http://tinyurl.com/ycjasy6b`), which is actually trivial topological order but without long range entanglement. Symmetry protected states do not lead to emergent fractional charge, fractional statistic, nor emergent gauge gauge theory unlike topological order. In TGD framework the emergent gauge symmetry could be identified as a symmetry associated with the action of included hyperfinite factor, which indeed causes no measurable effects in the resolution used.

Here an interesting delicacy appears. Is its particle physicist's time reversal, which is slightly broken symmetry? Or is it time reversal in the sense of TGD inspired theory of quantum measurement and consciousness bringing in the arrow of time (or thermodynamics)? Time reversal in the latter sense cannot be interpreted as a symmetry. For instance, time reversal in the latter sense involves state function reduction at opposite boundary of CD, which is dynamical and non-deterministic process leading to death of self and its re-incarnation as time reversed self. Note that time reversal is not allowed for non-Abelian braid statistics and although Kähler action is abelian the vierbein group of CP_2 is non-Abelian and can give rise to non-Abelian braiding by electroweak gauge group.

Gapless boundary excitations implying ground state degeneracy are also an important part of picture.

1. In the case of topological order they are robust against all local perturbations and protected by topology. Systems described by topological QFTs provide a basic example about non-trivial topological order. In the case of SPTs one has only robustness against local perturbations that do not break symmetries.

2. Super-symplectic algebra provides a concretization of the situation in TGD context. The sub-algebra of supersymplectic algebra with conformal weights, which are $h_{eff}/h = n$-ples of those for entire algebra act as gauge transformations and are thus perturbations, which do not change the state: one could say that there is symmetry protection. This differs from topological protection since not all deformations of 3-surfaces at the ends of space-time at boundaries of CD act like gauge symmetries. Indeed, the remaining generators of super-symplectic algebra act as genuine dynamical symmetries and if the generators with conformal weights $0 \leq k \leq n-1$ create physical states one indeed has finite degeneracy of states (this if the conformal weights of the super-symplectic algebra are integers). This gives just the n-fold degeneracy corresponding to singular n-sheeted covering property of space-time surface. Of course, the is a huge difference: usually one deals with finite-D or even discrete groups whereas super-symplectic group is really huge.

To test TGD one must be able to see the physics of single space-time sheet. The difficulty is that usually this physics is masked experimentally: usually we see only the superposition of effects from several sheets. It is also masked theoretically in the approximation based on the space-time of General Relativity and standard model since it is obtained by replacing many-sheeted space-time by a slightly curved region of Minkowski space involving replacement of induced gauge potentials resp. gravitational fields of space-time sheets with their sum defining the gauge potentials of standard model resp. gravitational field of GRT, replacing partonic 2-surfaces by point like particles, and describing fermionic strings in terms of interaction potentials. Condensed matter physicists might be already occasionally seeing the physics of single space-time sheet.

Category theory and TGD

Category theoretical thinking is part of TGD [K27].

1. In reductionistic approach particles are fundamental building bricks. The idea about an isolated particle must be given up in TGD. The strings connecting partonic 2-surfaces are present from beginning rather than only the partonic 2-surfaces, which are the counterparts of particles in the reductionistic approach. Note that in string models one has strings but no partonic two-surfaces so that one still remains in the framework of reductionism!

   This has highly non-trivial implications for the understanding of the formation of gravitational bound states and from TGD point of view the failure of superstring models in long length scales is trivial to understand: superstring description of gravitational interactions makes sense only in Planck length scale: the rest is - not history but - wishful thinking eventually leading to landscape and multiverse [K95](`http://tinyurl.com/y95qojt7`).

2. Zero Energy Ontology (ZEO) [K76, K72] is very category theoretical approach. One gives up the notion of positive energy state in ZEO. Positive energy states are replaced with zero energy states, which are pairs of positive and negative energy states at opposite boundaries of causal diamond (CD) and have opposite quantum numbers. Zero energy state is analogous to event in standard ontology consisting of initial and final state. Object is replaced with a relation between objects, one might say.

   Zero energy states are described by M-matrices (M-matrix is expressible as products of square root of density matrix and unitary S-matrix). Dynamics is coded by unitary U-matrix expressible in terms of M-matrices so that states code the dynamics in their representation. ZEO shows its power in TGD inspired theory of consciousness and allows to replace observer as an outsider of the physical world with the notion of self, a conscious entity describable in terms of quantum physics.

13.5 Deconstruction And Reconstruction In Quantum Physics And Conscious Experience

Deconstruction means roughly putting something into pieces. One could also speak about deconstruction followed by a reconstruction since deconstruction creates the impressions that something is just destroyed. Often deconstruction is thought to involve the reconstruction. This process is applied in deconstructivistic architecture (`http://tinyurl.com/y9quf3x4`) as one can learn by going to Wikipedia and also cubism brings in mind this kind of approach. In this process one organizes typical features of given style in new - one might even say “crazy” manner. There can be even a kind of social interaction between buildings: as if they were communicating by exchanging features.

Postmodernism is a closely related movement and claims that truths are socially constructed: great narratives are dead. Nothing could irritate more physicist who has learned how much mistakes and hard work are needed to distill the truth! Everything does not simply go! On the other hand, one can argue the recent sad state of super string theories and frontier theoretical physics in general suggests that postmodernists are right. Superstrings and multiverse are definitely purely social constructs: they were the only games in the town but now American Mathematical Society warns that super string theoreticians are spoiling the public image of science. Multiverse lived only few years. Certainly one great narrative - the story or reductionism and materialism thought to find its final culmination as M-theory - is dead. It is however nonsense to claim that all great narratives are dead. That telling the alternative great narratives in respected journals is impossible does not mean that they are dead!

But is not wise throw the big ideas of deconstruction and reconstruction away. Rather, one can ask whether they could be made part of a new great narrative about physical world and consciousness.

13.5.1 Deconstruction And Reconstruction In Perception, Condensed Matter Physics And In TGD Inspired Theory Of Consciousness

Deconstruction and reconstruction appear in the construction of percepts, in condensed matter physics, and are also part of TGD inspired theory of consciousness.

Perception

The very idea of deconstruction in architectural sense is highly interesting from the perspective of both quantum physics and consciousness.

The buildup of our perception involves very concretely deconstruction process. First the sensory input is decomposed into features. Edges, corners, positions, motions analyzed to direction and velocity, colors,... Objects are replaced with collections of attributes: position, motion, shape, surface texture, color,.... Deconstruction occurs at lower cortical layers. After this reconstruction takes place: various kinds of features are combined together through a mysterious looking process of binding - and the outcome is a percept.

Reconstruction can occur also in "wrong" manner. This occurs in hallucinations, delusions, and dreams. Humour is based on association of "wrong" things from different categories together. Synesthesia involves association between different sensory modalities: note with a given pitch have characteristic color or numbers correspond to colors or shapes. I remember an article telling about how subject persons in hypnosis can experience what circle with four corners looks like. Some attribute can be lacking from the reconstruction: person can perceive the car as object but not its motion. Car is there now. Moment later it is here. Nothing between.

Also non-standard reconstructions are possible. Could these non-standard reconstructions define a key aspect of creativity. Could reconstruction represent in some lucky situations new idea rather than hallucination or delusion?

For few years ago I listened a radio document about a professional, who builds soundscapes to movies and learned that the construction of soundscape is deconstruction followed by reconstruction. One starts from natural sounds but as such they are not very impressive: driving by car over some-one does not create any dramatic effect- just "splat" - nothing else. Thisit is so non-dramatic that it can be used to create comic. In order to cure the situation the real sounds are analyzed to features and then reconstructed by amplifying some features and by throwing away the unessential ones. The output sounds much more real than the real input. Of course, actors are masters of this technique and this is why videos about ordinary people doing something funny is like looking autistic ghosts. And if you look at the collection of modules of video game you see modules with name "Aargh", "Auch", "Bangggg", etc..

Association is the neuroscientist's key notion and allows to get an idea about what happens in reconstruction. Reconstruction involves association of various features to form the final percepts. First this process occurs for various sensory modalities. Sensory percepts from various sensory modalities are then combined to full percepts in association regions.

But what associations are at deeper level. What features are? Heretic could ask whether they could correspond to conscious experiences not conscious to us but conscious at lower level. Reader probably noticed that reconstruction-deconstruction took place here: the student is not supposed to ask this question since the theories of consciousness for some funny reason - maybe a pure accident - almost as a rule make the assumption that consciousness has no structure- no selves with subselves with sub-selves with... How these features bind to our conscious percepts? Neuroscience alone cannot tell much about this since it is based on physicalism: "hard problem" serves the articulation of this problem.

The following considerations represent deconstructions and reconstructions, and I will not explicitly mention when this happens. I just warn the reader. Do not stop reading however!

Condensed matter physics

One must bring in some basic notions of quantum theory if one wants to reduce deconstruction and reconstruction to quantum physics. The key mathematical fact is that in quantum theory each particle in many-particle state corresponds to a tensor factor. This notion is very difficult to eplain without actually having a lecture series about quantum tneory but I can try.

1. The basic idea is that one can build Hilbert spaces by forming their tensor products of them. If you have Hilbert spaces of dimensions n_1 and n_2, the tensor product has dimension $n_1 \times n_2$. Hilbert spaces represent physical systems: say electron and proton. To describe word consisting of proton and electron you from the tensor product of these Hilbert spaces. This is like playing with legos.

   Now I must be honest, I was cheating a little bit. Life is not quite so simple. One can also form bound states of two systems - say hydrogen atom from proton and electron, and the bound states of hydrogen atom represent only a sub-space of the tensor product. Connes tensor product is more exotic example: it represents only a sub-space of the entire tensor product: only certain kind of entangled states for which the composites are strongly correlated are allowed. As a matter fact, gluing the legos together creates strong correlations between them so that it serves as a good analogy for Connes tensor product and tensor product assignable to bound states.

2. Even elementary particles have several degrees of freedom -say spin and charge - to which one can assign Hilbert spaces decomposing formally into tensor product of Hilbert spaces associated with these degrees of freedom. Sub-space of the full tensor product is allowed, and one can purely formally say that elementary particle is a bound state of even more elementary particles. Somewhat like written word having meaning to us consists of letters, which as such represent nothing to us (but could represent something to lower level conscious entities). Could it be possible to apply deconstruction to elementary particles?

Now comes the surprise: condensed matter physicists have discovered deconstruction long time ago!

1. Electron in the valence band of conductor has three kinds of degrees of freeedom labelled by spin, charge and orbital state- state of electron in atom - characterizing the valence band. One can velocity to both spin, charge and orbital state. The state of electron decomposes in purely formal sense to a bound state of spinon, chargon, and holon. The question is whether one could have a situation deconstructing this bound state to its composites moving with different velocities. One would have effectively three particles and quantally three waves moving with same velocity. For free electrons obeying Dirac equation this is not possible. But this could be (and is!) possible in condensed matter. This deconstruction is mathematically like ionizing an atom: ion and electron are the outcome.

2. Instead of single wave motion there can be three free wave motions occurring with different velocities (wave vectors) corresponding to spinon, chargon and holon. In popular articles this process is called "splitting" of electron. This term is optimal choice if the purpose is to create profound mis-understandings in layman reader associating naturally splitting with a geometric process of putting tiny ball into pieces. As already explained, it is Hilbert space which is decomposed into tensor factors, not a tiny ball. The correlations between factors forced by bound state property are broken in this divorce beteen degrees of freedom.

3. What condensed matter theorist propose is roughly following. The consideration is restricted to effectively one-dimensional systems, call them wires. Atoms along line and electrons at atoms, which can be in conduction bands and give rise to a current. Electron has spin, charge, and orbital degrees of freedom if in conduction band and delocalized and thus shared by the atoms. The spin direction of the electron can vary along wire, and electron can excited to a higher orbital in atom and this exciattion can also vary along wire. These degrees of

freedom define tensor factors. Usually these degrees of freedom are bound to single entity free electrons and interacting electrons usually move as a single entity with charge, spin, and orbital excitation.

The holy trinity of charge, spin, and orbital degrees of freedom can be however split under some circumstances prevailing in condensed matter. The phase of the spinor representing electron can vary along wire and defines wave motion with some velocity/wave vector assignable with the ordinary electric current. The spin of electron can rotate at each point. Also the phase of this rotation can vary along wire so that a wave moving along wire with velocity different from that for charge: this is spin wave having as classical analog the rotation of bicycle pedals. If electron moves in a linear lattice of atoms, the orbital excitation can also vary along the wire and a third wave moving with its own velocity is possible. One has three free particle like entities moving with different velocities! This kind of waves are certainly not possible for the solutions of Dirac equation representing freely moving fermions and particle physicists do not encounter them.

4. These wave motions are different from the wave motions associated with phonons and magnons. For sound it is periodic oscillation for the position of atom, which propagates in sound wave. For magnon it is change of spin direction which propagates and defines a spin 1 collective excitation. Spinon as a quasiparticle has spin 1/2 so that spinon and magnon are different things. Spinon is formal constituent of electron made visible by the condensed matter environment. Magnon is collective excitation of condensed matter system.

   Spin currents provide an example of a situation in which spin and charge currents can flow at different speeds and are becoming important in a new technology known as spintronics. Spin currents have very low resistance and the speculation is that they might relate to high T_c super conductivity.

From the articles that I have seen one might conclude that deconstruction is in practice possible only for effectively 1-dimensional systems. I do not see any obvious mathematical reason why the deconstruction could not occur also in higher-dimensional systems.

It is however true that 1-dimensional systems are very special physically and mathematically ans super string theorists know. Braid statistics replaces ordinary statistics at for them and this brings in a lot of new effects. Furthermore, 2-D integrable gauge theories allow interactions as permutations of quantum numbers and lead to elegant models describing deconstructed degrees of fields as quantum fields in 2-D Minkowski space with interactions reducing to 2-particle interactions describable in terms of R-matrix satisfying the Yang-Baxter equations. It is difficult to say how much the association of deconstruction to 1-D systems is due the fact that they are mathematically easier to handle than higher-D ones.

The rise and fall of superstring models certainly was due to this technical easiness. I learned that the easiest manner to kill the idea that fundamental objects are 3-D was to say that superconformal invariance of super-string models is lost and the theory is not calculable. It took ineeed long time to realize that super-conformal has huge generalization when space-time is 4-D and embedding space has Minkowski space as its factor. Twistorial considerations fixed the whole scheme uniquely. Theoretician should be patient.

TGD inspired theory of consciousness

The believer in quantum consciousness of course wonders what could be the quantum counterparts of deconstruction and reconstruction. It would seem that analysis and synthesis of the sensory input deconstructs the mental image associated with it to features - simpler fundamental mental images- and reconstruct from these the percept as mental image. What does this correspond at the level of physics?

Before one can really answer one must understand what the quantum physical correlates of mental image are. How mental images die and are born? What features are as mental images? What their binding to sensory percepts does mean physically?

Here I can answer only on my own behalf and to do it I must introduce the basic notions and ideas of TGD inspired theory of consciousness. I will not go to details here because I have done this so many times and just suggest that the reading of some basic stuff about TGD inspired theory of consciousness. Suffice it to list just the basic ideas and notions.

1. Zero energy ontology and causal diamonds and hierarchy of Planck constants assignable to quantum criticality are basic notions. Number theoretic vision is also central. In particular, adelic physics fusing real physics and various p-adic physics as correlates for cognition is also basic building brick.

2. Consciousness theory is generalization of quantum measurement theory constructed to solve the basic problems of ordinary quantum measurement theory: observer becomes self described by physics rather than being outsider of the physical world. Negentropy Maximization Principle (NMP) defines the basic variational principle and state that the negentropy gain in state function reduction is maximal.

   Self hierarchy is the basic notion of TGD inspired theory of consciousness. Self experiences subselves as mental images. Self corresponds to a state function reduction sequence to the same boundary of causal diamond (CD). In standard quantum measurement theory this sequence does not change the state but in TGD framework the state at the opposite boundary of CD and even opposite boundary changes. This gives rise to the experience flow of time having the increases of the temporal distance between the tips of CD as a geometric correlate. Self dies as the first reduction to the opposite boundary takes place and re-incarnates at the opposite boundary as its time reversal. Negentropy Maximization Principle forces it to occur sooner or later. The continual birth and death of mental images supports this view if one accepts the idea about hierarchy. One can also consider identification for what the change of the arrow of time means for mental image.

3. Magnetic bodies carrying dark matter identified as $h_{eff} = n \times h$ phases of ordinary matter define quantum correlates for selves. Magnetic body has hierarchical onion-like structure and it communicates with biological body using dark photons propagating along magnetic flux tubes. EEG and its fractal generalization make both communication from/control of biological body to/by magnetic body. Dark matter hierarchy can be reduced to quantum criticality and this in turn has deep roots in the adelic physics.

What reconstruction could mean in TGD inspired theory of consciousness?

1. The restriction of deconstruction to the degrees of freedom of elementary particle is unnecessary restrictive. One can consider also larger units such a as molecules, cells, etc.. and their representations using tensor products.

2. Besides bound state formation also negentropic entanglement (NE) allows reconstruction of states which are almost stable with respect to NMP. There are two kinds of NE. which can be metastable with respect to NMP. In the first case density matrix is a projector with n identical eigenvalues. This state can result in a state function reduction since it is an eigenstate of the fundamental observable defined by density matrix. It can also happen that the eigenvalues of density matrix having matrix elements in algebraic extension algebraic extension of rationals characterizing the system in the evolutionary hierarchy do not belong to the extension. One can argue that since diagonalization is not possible in the extension, also state function reduction is impossible without a phase transition extending the extension and identifiable as a kind of evolutionary step.

   Both kinds of NEs might be involved. The first option would correspond to a kind of enlightened consciousness since any orthonormal state basis would define eigenstate basis of density matrix. Schrödinger cat would be half alive and half dead or half of X and half of Y, where X and Y are any orthonormal superpositions of alive and dead. For the second option there would be a unique state basis. For instance, cat could be $1/\sqrt{2}$ alive and $1 - 1/\sqrt{2}$ dead. This could correspond to a state of rational mind discriminating between things. If a phase transition bringing in $\sqrt{2}$ takes place, state function reduction makes cat fully alive or dead.

3. In condensed matter example the velocity of quantal wave motion serves as a criterion allowing to tell whether the degrees of freedom bind or not. Electron velocity is obviously too

limited as a signature for binding or its absence. In neuroscience the coherence of EEG is seen as a signature of binding and this suggests that oscillation with same EEG frequency is the signature of binding of mental images to a larger one. In TGD inspired theory of consciousness EEG frequencies correspond to differences of generalized Josephson frequencies that is sums of Josephson frequency for the resting potential and of the difference of cyclotron frequencies for ions at different sides of cell membrane [K44, K90, K91].

4. At the level of magnetic flux tubes binding would correspond to a reconnection of magnetic flux tubes of synchronously firing region to form a larger structure for which the magnetic field strength is same for the composites and therefore also cyclotron frequencies are identical. Reconstruction would have a concrete geometric correlate at the level of magnetic flux tubes as reconnection. Different parts of brain containing quantum states serving as features of mental image would connected by flux tubes of the magnetic body and binding of mental images would take place.

5. In TGD inspired quantum biology dark matter identified as large $h_{eff} = n \times h$ phases give rise to a deconstruction if one accepts the hypothesis $\hbar_{eff} = \hbar_{gr} = GMm/v_0$, where M represents mass of dark matter and m particle mass. Here h_{gr} is assigned with a flux tube connecting masses M and m and v_0 is a velocity parameter characterizing the system. This hypothesis implies that dark cyclotron energy is proportional to $h_{gr} f_c$, where f_c is cyclotron frequency is independent of particle mass: universal cyclotron energy spectrum is the outcome. The dark cyclotron photons can transform to ordinary photons identified as bio-photons.

   What makes this so remarkable is that particles with magnetic dipole moment possessing different masses correspond to different values of h_{eff} and reside at different magnetic flux tubes. This is mass spectroscopy - or deconstruction of charged particles matter by taking the particles with different masses to their own dark worlds! Dark living matter would not be a random soup of particles: each charged particle (also neutral particles with magnetic dipole moment) sits neatly at its own shelf labelled by h_{gr}! In TGD inspired theory of consciousness magnetic flux tubes can be associated with magnetic bodies serving as correlates of selves so that deconstruction for mental images would reduce to this process with each charged particle representing one particular combination and perhaps also a quale [K52].

What about re-construction in this framework?

1. In reconstruction flux tube connections between two subsystems representing sub-selves (experienced by self as mental images) would be formed so that they would fuse to single system characterized by the same cyclotron frequency. Flux tube connection would be formed by the reconnection of U-shaped flux tubes to form single pair of connecting flux tubes connecting the systems. Resonant exchange of dark cyclotron photons and also dark super-conductivity would accompany this process. This process would represent a correlate for directed attention and would take place already at bio-molecular level. For instance, I have proposed that biomolecules with aromatic rings in which circulating electron pair currents generate magnetic bodies are especially important and in some sense fundamental level of the self hierarchy at molecular level. In brain different brain regions could connect to single coherently firing region in this manner.

2. The magnetic bodies associated with brain regions representing features could be connected in this manner to larger sub-selves. Negentropic quantum entanglement - a purely TGD based notion - could define a further correlate for the binding. This entanglement could take place in discrete degrees of freedom related to the hierarchy $h_{eff} = n \times h$ of Planck constants having no correlate in standard physics. The discrete degree of freedom would correspond to n sheets of singular coverings representing space-time surfaces. The sheets would co-incide at the ends of causal diamonds (CDs): on possible interpretation (holography allows many of them) could be that entire closed 3-surfaces formed by space-like 3-surfaces and light-like 3-surface connecting them can be seen as basic objects.

3. Reconstruction by negentropic quantum entanglement and flux tube connections inducing resonance could also lead to non-standard composites. Synesthesia could be understood

in this manner and even the sensory experience about circle with four corners could be understood. The binding of left and right brain visual experiences to single one could take place through negentropic entanglement and effectively generate the third dimension. The dimensions would not however simply add: 3-D experience instead of 4-D. Could sensory perception of higher than 3-D objects be possible by a reconstruction fusing several visual percepts - maybe even from different brains - together? Could higher levels of self hierarchy carry out this kind of reconstruction? Could Mother Gaia fuse our experiences to single experience about what it is to be a human kind, species, or bio-sphere?

13.5.2 Could Condensed Matter Physics And Consciousness Theory Have Something To Share?

Magnetic bodies are present in all scales and one can to ask whether consciousness theory condensed matter physics might have something in common. Could the proposed description apply even at the level of condensed matter? Could construction and reconstruction of mental images identifiable as sub-selves take place already at this level and have interpretation in terms of primitive information processing building standardized primitive mental images?

Deconstruction need not be restricted to electron and velocity could be replaced by oscillation frequency for various fields: at quantum level there is not actually real distinction since in quantum theory velocity defines wave vector. Also more complex objects, atoms, molecules, etc. could be deconstructed and the process could occur at the level of magnetic bodies and involve in essential manner reconnection and other "motor actions" of flux tubes. The notions of quasi-particle and collective excitation would generalized dramatically and the general vision about basic mechanism might help to understand this zoo of exotics.

Future condensed matter theorists might also consider the possibility of reconstruction in new manner giving rise to the analogs of synesthesia. Could features from different objects be recombined to form exotic quasi-objects having parts all around. Could dark matter in TGD sense be involved in essential manner: could cyclotron resonance or its absence serve as a correlate for the binding. The disjoint regions of space would be in well-defined sense near to each other in the reconstructed state. Topology would be different: p-adic topology could provide a natural description for a situation: in p-adic topology systems at infinite distance in real sense can be infinitesimally close to each other p-adically.

One can build many-particle states free many-particle states using tensor products of these primitive tensor factors. Bound states are clearly new kinds of particle like entities. Under additional constraints one obtains bound states. Could deconstruction in physical sense mean the decomposition of this kind of bound states to effectively free many-particle states? Can one see reconstruction the reversal of these process? And is it possible that tensor factors are combined in a totally new manner somewhat like basic geometric features in deconstructivistic architecture?

Chapter 14

About the Nottale's formula for h_{gr} and the relation between Planck length and CP_2 length

14.1 Introduction

This chapter is about two topics: about the identification of the parameter v_0 with dimensions of velocity appearing in the Nottale's formula for gravitational Planck constant [L56], and about possible TGD explanation for the observed variation of gravitational constant assuming that Planck length l_P is actually CP_2 radius R as the condition that TGD as a TOE has only one fundamental length requires, and that the formula $G = R^2/\hbar_{eff}$ holds true meaning that Newton's constant is different for various levels in dark matter hierarchy [L68].

14.1.1 About the physical interpretation of the velocity parameter in the formula for the gravitational Planck constant

Nottale's formula [E13] for the gravitational Planck constant $\hbar_{gr} = GMm/v_0$ involves parameter v_0 with dimensions of velocity. I have worked with the quantum interpretation of the formula [K100, K83, K84, ?] but the physical origin of v_0 - or equivalently the dimensionless parameter $\beta_0 = v_0/c$ (to be used in the sequel) appearing in the formula has remained open hitherto. In the following a possible interpretation based on many-sheeted space-time concept, many-sheeted cosmology, and zero energy ontology (ZEO) is discussed.

A generalization of the Hubble formula $\beta = L/L_H$ for the cosmic recession velocity, where $L_H = c/H$ is Hubble length and L is radial distance to the object, is suggestive. This interpretation would suggest that some kind of expansion is present. The fact however is that stars, planetary systems, and planets do *not* seem to participate cosmic expansion. In TGD framework this is interpreted in terms of quantal jerk-wise expansion taking place as relative rapid expansions analogous to atomic transitions or quantum phase transitions. The TGD based variant of Expanding Earth model assumes that during Cambrian explosion the radius of Earth expanded by factor 2 [L64] [L64, L63, L72].

There are two measures for the size of the system. The M^4 size L_{M^4} is identifiable as the maximum of the radial M^4 distance from the tip of CD associated with the center of mass of the system along the light-like geodesic at the boundary of CD. System has also size L_{ind} defined defined in terms of the induced metric of the space-time surface, which is space-like at the boundary of CD. One has $L_{ind} < L_{M^4}$. The identification $\beta_0 = L_{M^4}/L_H < 1$ does not allow the identification $L_H = L_{M^4}$. L_H would however naturally corresponds to the size of the magnetic body of the system in turn identifiable as the size of CD.

One can deduce an estimate for β_0 by approximating the space-time surface near the light-cone boundary as Robertson-Walker cosmology, and expressing the mass density ρ defined as $\rho = M/V_{M^4}$, where $V_{M^4} = (4\pi/3)L_{M^4}^3$ is the M^4 volume of the system. ρ can be expressed as a fraction

ϵ^2 of the critical mass density $\rho_{cr} = 3H^2/8\pi G$. This leads to the formula $\beta_0 = \sqrt{r_S/L_{M^4}} \times (1/\epsilon)$, where r_S is Schwartschild radius.

This formula is tested for planetary system and Earth. The dark matter assignable to Earth can be identified as the innermost part of inner core with volume, which is .01 per cent of the volume of Earth. Also the consistency of the Bohr quantization for dark and ordinary matter is discussed and leads to a number theoretical condition on the ratio of the ordinary and dark masses.

$\beta_0/4\pi$ is analogous to gravitational fine structure constant for $h_{eff} = h_{gr}$. Could one see it as fundamental coupling parameter appearing also in other interactions at quantum criticality in which ordinary perturbation series diverges? Remarkably, the value of G does not appear at all in the perturbative expansion in quantum critical phase! Could G can have several values?

There is also a problem: the twistorialization of TGD [K98] leads to the conclusion that the radius of twistor sphere for M^4 is given by Planck length l_P so that - contrary to the view held for decades - one would have two fundamental lengths - l_P and CP_2 radius R and there is no idea about how they are related. Quantum criticality cannot relate them since they are not coupling parameters.

The formula for $G = l_P^2/\hbar$ however suggests a generalization $G = R^2/h_{eff}$ with h_{eff}/h_0 having value in the range $10^7 - 10^8$: one would have $l_P = R$! Also classical gravitation could tolerate the spectrum of G since Newton's equations in gravitational field is invariant under scaling $h_{eff} \to xh_{eff}$ inducing $G \to G/x$ and $t \to t/x, r \to r/x$ with scales up the size scale of space-time sheets as the proportionality of Compton length to h_{eff} requires.

14.1.2 Is the hierarchy of Planck constants behind the reported variation of Newton's constant?

Nowadays it is fantastic to be a theoretical physicists with a predictive theory. Every week I get from FB links to fascinating experimental findings crying for explanation (I am grateful for people providing these links). The last link of this kind was to a popular article (see `http://tinyurl.com/ya2wekch`) telling about the article [E29] (see `http://tinyurl.com/yanvzxj6`) reporting measurements of Newton's constant G carried out by Chinese physicists Shan-Qing Yang, Cheng-Gang Shao, Jun Luo and colleagues at the Huazhong University of Science and Technology and other institutes in China and Russia. The outcomes of two experiments using different methods differ more than the uncertainties in the experiments, which forces to consider the possibility that G can vary.

The experiments

The experiments use torsion pendulum: this method was introduced by Henry Cavendish in 1978.

Remark: A remark about terminology is in order. Torque $\tau = F \times r$ on particle has dimensions Nm. Torsion (see `http://tinyurl.com/q8esymu`) in solid is essentially the density of torque per volume and has dimensions N/m^2. Twist angle is induced by torsion in equilibrium. The situation is governed by the theory of elasticity.

Basically one has torsion balance in which the gravitational torque produced by two source masses on masses associated with a torsion pendulum - dumbbell shaped system having identical masses at the ends of a bar and hanging from a thread at the middle point of the bar. As the source masses are rotated a twist of the thread emerges and twist angle corresponds to an equilibrium in which the torsion of the thread compensates the torque produced by gravitational interaction with source masses. Cavendish achieved 1 per cent accuracy in his measurements.

Refined variations of these measurements have been developed during years and the current precision is 47 parts per million (ppm). In some individual experiments the precision is 13.7 ppm. Disagreements larger than 500 ppm are reported, which suggests that new physics might be involved.

The latest experiments were made by the above mentioned research group. Two methods are used. TOS (Time Of Swing) and AAF (Angular Acceleration Feedback). AAF results deviates from the accepted value whereas TOS agrees. The accuracies were 11.64 ppm and 11.61 ppm in TOS and AAF respectively. AAF however gave by 45 ppm larger value of G.

In TOS technique the pendulum oscillates. The frequency of oscillation is determined by the positions of the external masses and G can be deduced by comparing frequencies for two different mass configurations. There are two equilibrium positions. The pendulum is either parallel to the line connecting masses relatively near to each other ("near" position). The pendulum orthogonal to the line connecting masses in "far" position. By measuring the different oscillation frequencies one can deduce the value of G.

Angular-acceleration feedback (AAF) method involves rotating the external masses and the pendulum on two separate turn tables. Twist angle is kept zero by changing the angular velocity of the other turn table: thus feedback is involved. If I have understood correctly, the torsion induced by gravitational torque compensates the torsion created by twisting of the thread around its axis in opposite direction and from the value of torsion for zero twist angle one deduces G. One could perhaps say that in AAF torsion is applied actively whereas in TOS it appears as reaction.

Why the measured value obtained for G would be larger for AAF? Could the active torsion inducing compensating twisting of the torsion pendulum actually increase G?

A possible TGD explanation for the variation of G

In TGD framework the hierarchy of Planck constants $h_{eff} = nh_0$, $h = 6h_0$ together with the condition that theory contains CP_2 size scale R as only fundamental length scale, suggest the possibility that Newtons constant is given by $G = R^2/\hbar_{eff}$, where R replaces Planck length ($l_P = \sqrt{\hbar G} \rightarrow l_P = R$) and $\hbar_{eff}/h$ is in the range $10^6 - 10^7$. The spectrum of Newton' constant is consistent with Newton's equations if the scaling of $\hbar_{eff}$ inducing scaling G is accompanied by opposite scaling of M^4 coordinates in $M^4 \times CP_2$: dark matter hierarchy would correspond to discrete hierarchy of scales given by breaking of a continuous scale invariance to a discrete one.

In the special case $h_{eff} = h_{gr} = GMm/v0$ - gravitational Planck constant originally introduced by Nottale [E13]- assignable to quantum critical dynamics gravitational fine structure constant $\alpha_{gr} = GMm/(4\pi\hbar_{gr}) = (v_0/c)/4\pi$ serves as coupling constant and has no dependence of the value of G or masses M and m in accordance with the universality of quantum critical dynamics.

In this chapter I consider a possible interpretation for the finding of a Chinese research group measuring two different values of G differing by 47 ppm in terms of varying h_{eff}. Also a model for fountain effect of superfluidity as de-localization of wave function and increase of the maximal height of vertical orbit due to the change of the gravitational acceleration g at surface of Earth induced by a change of h_{eff} due to super-fluidity is discussed. Also Podkletnov effect is considered. TGD inspired theory of consciousness allows to speculate about levitation experiences possibly induced by the modification of G_{eff} at the flux tubes for some part of the magnetic body accompanying biological body in TGD based quantum biology.

14.2 About the physical interpretation of the velocity parameter in the formula for the gravitational Planck constant

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There is also a problem: the twistorialization of TGD [K98] leads to the conclusion that the radius of twistor sphere for M^4 is given by Planck length l_P so that - contrary to the view held for decades - one would have two fundamental lengths - l_P and CP_2 radius R and there is no idea about how they are related. Quantum criticality cannot relate them since they are not coupling parameters.

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14.2.1 Formula for the gravitational Planck constant and some background

The formula

$$\hbar_{gr} = \frac{GMm}{v_0} \tag{14.2.1}$$

for the gravitational Planck constant was originally introduced by Nottale [E13]. Here v_0 is a parameter with dimensions of velocity.

The formula is expected to hold true at the magnetic flux tubes mediating gravitational interaction and obeying also the general formula

$$h_{gr} = h_{eff} \ , \quad h_{eff} = nh_0 \ , \quad h = 6h_0 \ . \tag{14.2.2}$$

The support for the formula $h = 6h_0$ is discussed in [L26, L60]. The value of h_{gr} can be very large unlike the value of h_{eff} associated with say valence bonds.

There are two kinds of flux tubes - homologically non-trivial and trivial ones corresponding to two kinds of geodesic spheres of CP_2, and they seem to correspond to small and large values of h_{eff}.

1. Since the Kähler magnetic energy of homologically non-trivial flux tubes carrying monopole magnetic flux is large, the natural expectation is that gravitation and presumably also other long range interactions mediated by massless particles - with color interactions perhaps forming an exception - correspond to homologically trivial flux tubes for which only the volume

energy due to cosmological constant is non-vanishing. Massive particles would correspond to flux tubes carrying monopole magnetic flux associated with homologically non-trivial flux tubes. Homology could therefore define a key difference between massive and massless bosons at space-time level.

2. One can argue the flux tubes accompanying flux tubes with non-trivial homological charge are relatively short: since the length of the flux tube is expected to be proportional to h_{eff} or its positive power, this would suggest small values of h_{eff} for them. For instance, valence bonds for which non-standard value of h_{eff} is suggestive could correspond to relatively flux tubes carrying monopole flux [L44].

3. Suppose that the value of exponent of Kähler function for the "world of classical worlds" (WCW) is exponent of Kähler function expressible as the 6-D variant of Kähler action for the twistor lift of 4-D Kähler action reducing to the sum of 4-D Kähler action and volume term in the dimensional reduction of the 6-surface to S^2 bundle over space-time surface required by the induction of twistor structure [K114, K98, K14]. If so, the shortness of homologically non-trivial flux tubes could be forced by the large values of Kähler magnetic action and energy making the exponent small.

14.2.2 A formula for β_0 from ZEO

I have made some attempts relate the value of $\beta_0 = v_0/c$ appearing in the formula for h_{gr} to some typical rotation velocity in the system [K100, K83] but although orders of magnitude are reasonable, these attempts have not led to a prediction of v_0. It might be that the explanation is hidden at deeper level and involves many-sheeted space-time and the view about quantum theory based on zero energy ontology (ZEO) in an essential manner.

A generalization of the Hubble formula $\beta = L/L_H$ for the cosmic recession velocity, where $L_H = c/H$ is Hubble length and L is radial distance to the object, is suggestive. Some kind of expansion suggests itself. The fact is however that stars, planetary systems, and planets do *not* seem to participate cosmic expansion. In TGD framework this is interpreted in terms of quantal jerk-wise expansion taking place as relative rapid expansions analogous to atomic transitions or quantum phase transitions. The TGD based variant of Expanding Earth model assumes that during Cambrian explosion the radius of Earth expanded by factor 2 [L64] [L64, L63, L72].

The interpretation of the velocity parameter β_0 to be discussed involves in an essential manner ZEO based quantum measurement theory giving rise to a quantum theory of consciousness [L52]. The causal diamond CD assignable to given conscious entity expands state function reduction by state function and this expansion is very much analogous to cosmic expansion.

In TGD inspired theory of consciousness, which is essentially quantum measurement theory in ZEO [L52], self as a conscious entity corresponds to a sequence of analogs of weak measurements changing the members of state pairs at active boundary of CD and increasing the size of CD by shifting the active boundary farther away from the passive boundary. Passive boundary and the members of state pairs at it remain invariant. This produces a generalized Zeno effect leaving both passive boundary and states at it invariant. This gives the unchanging contribution to the consciousness that one might call "soul". Experienced time corresponds to the increasing distance between the tips of CD and experienced time to the sequence of weak measurements. Active boundary gives rise to changing part in the contents of consciousness. Self dies and reincarnates in opposite time direction when the big state function reduction changing the roles of the boundaries of CD occurs and CD begins to increase in opposite time direction.

To make progress one must consider more precisely what space-time as 4-surface property means in ZEO. The unchanging part of the consciousness corresponds to the passive light-like boundary of CD and various constant parameters should be assigned with the quantum state at it.

There are two measures for the size of the system at the passive boundary and also a measure for the size of its magnetic body mediating gravitational interactions.

1. One can identify M^4 size L_{M^4} as the maximum of the radial M^4 distance from the tip of CD associated with center of mass of the system to the boundary of the system along the light-like geodesic at the passive boundary of CD.

2. System has also size L_{ind} defined as the maximum distance in the induced metric of the space-time surface, which is space-like at the boundary of CD. L_{ind} cannot correspond to Hubble length L_H since this would give $\beta > 0$.

3. A reasonable option is that L_H corresponds to the size scale of the part of the magnetic body of the system responsible for mediation of gravitational interactions. L_H would thus correspond to effective range of gravitational interactions. The simplest guess is that L_H corresponds the maximal radial size of CD given as $L_H = T/2$, where T is the temporal distance between the tips of the CD.

One can deduce an estimate for β_0 by approximating the space-time surface near the passive boundary of CD as Robertson-Walker cosmology. This approximation is indeed natural since space-time surface is small deformation of future/past light-cone near the boundary. The assumption about RW cosmology is *not* needed elsewhere inside CD. This conforms with the holography.

This estimate is only an approximation involving the ratio $\epsilon^2 = \rho/\rho_{cr} < 1$ of the average mass density ρ to the critical mass density

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

besides H. One can consider at least two options.

1. Option I: ρ corresponds to the average density $\rho = M/V_{M^4}$ within M^4 volume $V_{M^4} = (4\pi/3)L^3_{M^4}$ at the passive boundary. The condition $\rho = \epsilon^2\rho_{cr}$ allows to solve $\beta = L/L_H$ as

$$\beta_0 = \frac{L_{M^4}}{L_H} = \frac{1}{\epsilon}\sqrt{\frac{r_S}{L_{M^4}}} \quad , \quad r_S = 2GM \quad . \tag{14.2.3}$$

   Here r_S is Schwartschild radius. As noticed, a reasonable identification for L_H would be as the size scale of the gravitational magnetic body given by the size $L_H = T/2$. It turns that this formula is rather reasonable and consistent with earlier results in the case of planetary system and Earth.

2. Option II gives up completely the attempt to interpret the situation in terms of Hubble constant and identifies $\beta_0 = L_{ind}/L_{M^4} < 1$. In this case the expression in terms of mass density in terms of critical mass density does not help to obtain a more detailed formula. If one requires consistency with the previous formula, one obtains L_{ind} as pr $L_{ind} = \sqrt{r_S L_{M^4}}/\epsilon$. For $\epsilon = 1$ one has geometric mean.

14.2.3 Testing the model in the case of Sun and Earth

One can test these equations for Sun and Earth to see whether they could make sense. The restriction to the option I with volume V identified as the volume in the induced metric at the passsive boundary of CD. Option II is obtained at the limit $\epsilon_! = 1$.

Consider first Sun.

1. In the case of Sun the model for the Bohr quantization of planetary orbits was originally proposed by Nottale [E13] and was developed further in TGD framework in [K100, K83] assuming that genuine quantum coherence in astrophysical scales possible for dark matter is in question. The value of β_0 is in a reasonable approximation $\beta_0(inner) = 2^{-11}$ for the 4 inner planets and $\beta_0(out) = \beta_0(inner)/5$ for the outer planets.

2. For the 4 inner planets, the distance of Earth given by astronomical unit $AU = .149 \times 10^9$ km is the natural estimate for L_H so that one has $L_H = AU$. For outer planets the natural choice is of the order of the orbit of the outer planet with largest orbital radius, which is Neptune with distance of 30 AU for Neptune. The prediction of the model for the orbital radius of Neptune is 25 AU so that the estimate looks reasonable. Note that the radii in Bohr model are proportional to $h_{gr}^2 n^2$, n the principal quantum number, so that the scaling $v_0 \rightarrow v_0/5$ scales the radius by factor 5^2. This also means that scaling $n \rightarrow kn$ and scaling $v_0 \rightarrow v_0/k$ produces the same scaled orbital radius.

3. For the 4 inner planets one obtains

$$\beta_0 = \frac{r_S}{L_H} \times \frac{1}{\epsilon} = 1.1 \times 10^{-4} \times \frac{1}{\epsilon} \ .$$

The value co-incides with $\beta_0 = 2^{-11}$ providing a reasonable approximation in Nottale model for $r = 4.55$. This leaves open the fraction $\epsilon^2 = \rho/\rho_{crit}$. One would have $\epsilon^2 = .048$. The size scale of CD would be about $1/\beta = 2^{11}$ using AU as a unit.

Consider next Earth. One can consider two choices for L.

1. Case I: Earth radius $R_E = 6.371 \times 10^3$ km is the first candidate: this choice might be relevant for the applications at Earth's surface such as fountain effect in super-fluidity.

2. Case II: The distance $d_M = 60.3 R_E$ of Moon, is second choice for the scale L. The Schwartschild radius of Earth is $r_S = 9$ mm.

The value of β_0 in these two cases is given by.

$$\beta_0(I) = \sqrt{\frac{r_S}{R_E}}\frac{1}{\epsilon} = .38 \times 10^{-4}\frac{1}{\epsilon} \ ,$$
$$\beta_0(II) = \sqrt{\frac{r_S}{d_M}}\frac{1}{\epsilon} = .04 \times 10^{-4}\frac{1}{\epsilon} \ .$$

The condition $\beta_0(I) = 2^{-11}$ is marginally consistent with the biology related considerations of [L57] and requires $r = 13.16$. The size of the CD would be about $2^{11} R_E$ for option I.

For the same value of r for both I and II one has $\beta(I) = 7.76\beta(II) \simeq 8\beta(II)$ so that option II could be obtained from option I by the scaling $\beta(I) \rightarrow \beta/8$ inducing the scaling $R_E \rightarrow 64 R_E > 60.3 R_E$. By the proportionality of Bohr orbit radius to $1/\beta^2$, the ratio $r(II)/r(I) = \sqrt{64/60.33} = 1.030$ would compensate this error. The mass mass of the moon is $M_M = .012 M_E$ so that the replacement of M_E with the $M_E + M_M$ would produce correction factor 1.012 which is by 2 per cent smaller than the required correction factor.

14.2.4 Under what conditions the models for dark and ordinary Bohr orbits are consistent with each other?

Under what conditions the Bohr orbitologies for dark and ordinary matter are consistent with each other?

1. The condition $v^2 = GM/r$ determines the relationship between velocity and radius in Newtonian theory. The values of v and r cannot therefore change for ordinary matter, which must coupled to all matter - both ordinary and dark matter of the central system.

2. A natural assumption is that dark matter couples only to the dark matter within the volume closed by its orbit. If dark object corresponds to an object modellable as point-like object (the alternative option is that dark matter is along a closed flux tubes along Bohr orbit) then the above condition reads $v_D^2 = GM_D/r$ so that one has

$$\frac{v_D}{v} = \sqrt{\frac{M_D}{M}} \ . \tag{14.2.4}$$

There seems to be no reason why the velocities of dark matter and ordinary matter could not be different. In the case of dark matter there is also Bohr orbit condition giving for gravitational Bohr radius as a generalization of $a_0 = \hbar/\alpha m_e \rightarrow a_{gr} = \hbar_{gr}/\alpha_{gr} m$ with $\alpha = e^2/4\pi\hbar \rightarrow \alpha_{gr} = GMm/4\pi\hbar_{gr} = v_0/4\pi$. This gives

$$a = a_{gr,D} n_D^2 \ , \quad a_{gr} = \frac{4\pi G M_D}{v_0^2} \ . \tag{14.2.5}$$

This formula should be consistent with the formula originally derived for matter and motivated by the idea that ordinary matter forms bound states with dark matter. I have considered also the option that dark matter is delocalized along the flux tube associated with the orbit of the planet.

3. The two formulas make sense simultaneously only if one can interpret the Bohr orbit for M_D as Bohr orbit for M having same radius. This condition gives $M_D n_D^2 = Mn^2$ giving

$$n_D^2 = \frac{M}{M_D} n^2 \ . \tag{14.2.6}$$

Therefore M/M_D should be square of integer, which is rather strong constraint.

One can test this formula in the case of planetary system and for Earth.

1. The first guess is that the inner core of Sun with radius in the range $.2R_S$ and $.25R_S$ corresponds mostly to dark matter. Solar core contains about 34 cent of solar mass (see http://tinyurl.com/nrcojr2). This gives in excellent approximation $M/M_D = 3$, which is however not square. $M/M_D = 4$ would satisfy the condition and would have $n_D = 2n$.

   Since dark matter corresponds to extensions of rationals, one can ask whether one could allow for dark matter algebraic integers as values of n_D so that $n_D = \sqrt{3}n$ would be allowed for an extension containing $\sqrt{3}$. This would be a number theoretic generalization of quantization in terms of in terms of integers somewhat analogous to that associated with quantum groups.

2. For Earth the estimate [L57] gives $M/M_D \simeq .5 \times 10^4$ giving $\beta_0 = 4.4 \times 10^{-4}$ rather near to $\beta_0 = 2^{-11} \simeq 5 \times 10^{-4}$. It is enough to find integer sufficiently near to 5000 having the property that it is square. One has $70^2 = 4900$ and $71^2 = 5041$.

   One would have $n_D \simeq 5000 \times n$ and consistency with the formula. Earth has outer core occupying 15 cent of its volume, inner core occupying 1 cent of the volume and innermost inner core with radius 300 km occupying fraction 10^{-4} of the volume (see http://tinyurl.com/y8vf7vc3) suggests that the innermost inner core consists of dark mass with density twice the average density.

   Remark: I have considered for M_D a probably too science fictive identification in terms of possibly existing gravitational analog of Dirac monopole. The gravitational flux would emanate radially from the center of the Earth along flux tubes carrying magnetic monopole flux and turn back at certain distance and return back along second space-time sheet and back to the original space-time sheet at wormhole like structure. This field would not be visible at large enough distances.

   If one has $M_D = 2 \times 10^{-4} M_E$, the density of the innermost inner core would be 2ρ, where ρ is the average density of Earth. From Wikipedia (see http://tinyurl.com/ma6xqnh) one learns that the average density ρ_E of Earth is $5.52 \times \rho_W$, ρ_W= kg/dm^3 and the density in the inner core varies in the range $\rho/\rho_w \in [12.6 - 13.0]$. The lower limit is approximately $2 \times \rho$. This suggests that the density of the innermost inner core is somewhat larger than 2ρ.

14.2.5 How could Planck length be actually equal to much larger CP_2 radius?!

The following argument stating that Planck length l_P equals to CP_2 radius R: $l_P = R$ and Newton's constant can be identified $G = R^2/\hbar_{eff}$. This idea looking non-sensical at first glance was inspired by an FB discussion with Stephen Paul King.

First some background.

1. I believed for long time that Planck length l_P would be CP$_2$ length scale R squared multiplied by a numerical constant of order $10^{-3.5}$. Quantum criticality would have fixed the value of l_P and therefore $G = l_P^2/\hbar$.

2. Twistor lift of TGD [K114, K14, K98, L75] led to the conclusion that that Planck length l_P is essentially the radius of twistor sphere of M^4 so that in TGD the situation seemed to be settled since l_P would be purely geometric parameter rather than genuine coupling constant. But it is not! One should be able to understand why the ratio l_P/R but here quantum criticality, which should determine only the values of genuine coupling parameters, does not seem to help.

   Remark: M^4 has twistor space as the usual conformal sense with metric determined only apart from a conformal factor and in geometric sense as $M^4 \times S^2$: these two twistor spaces are part of double fibering.

Could CP_2 radius R be the radius of M^4 twistor sphere, and could one say that Planck length l_P is actually equal to R: $l_P = R$? One might get $G = l_P^2/\hbar$ from $G = R^2/\hbar_{eff}$!

1. It is indeed important to notice that one has $G = l_P^2/\hbar$. $\hbar$ is in TGD replaced with a spectrum of $\hbar_{eff} = n\hbar_0$, where $\hbar = 6\hbar_0$ is a good guess [L26, L60]. At flux tubes mediating gravitational interactions one has

$$\hbar_{eff} = \hbar_{gr} = \frac{GMm}{v_0} \ ,$$

   where v_0 is a parameter with dimensions of velocity. I recently proposed a concrete physical interpretation for v_0 [L56] (see `http://tinyurl.com/yclefxb2`). The value $v_0 = 2^{-12}$ is suggestive on basis of the proposed applications but the parameter can in principle depend on the system considered.

2. Could one consider the possibility that twistor sphere radius for M^4 has CP_2 radius R: $l_P = R$ after all? This would allow to circumvent introduction of Planck length as new fundamental length and would mean a partial return to the original picture. One would $l_P = R$ and $G = R^2/\hbar_{eff}$. $\hbar_{eff}/\hbar$ would be of $10^7 - 10^8$!

The problem is that $\hbar_{eff}$ varies in large limits so that also G would vary. This does not seem to make sense at all. Or does it?!

To get some perspective, consider first the phase transition replacing $\hbar$ and more generally $\hbar_{eff,i}$ with $\hbar_{eff,f} = h_{gr}$.

1. Fine structure constant is what matters in electrodynamics. For a pair of interacting systems with charges Z_1 and Z_2 one has coupling strength $Z_1Z_2e^2/4\pi\hbar = Z_1Z_2\alpha$, $\alpha \simeq 1/137$.

2. As shown in [K100, K83, K84, **?**] one can also define gravitational fine structure constant α_{gr}. Only α_{gr} should matter in quantum gravitational scattering amplitudes. α_{gr} wold be given by

$$\alpha_{gr} = \frac{GMm}{4\pi\hbar_{gr}} = \frac{v_0}{4\pi} \ . \tag{14.2.7}$$

   $v_0/4\pi$ would appear as a small expansion parameter in the scattering amplitudes. This in fact suggests that v_0 is analogous to α and a universal coupling constant which could however be subject to discrete number theoretic coupling constant evolution.

3. The proposed physical interpretation is that a phase transition $\hbar_{eff,i} \rightarrow \hbar_{eff,f} = h_{gr}$ at the flux tubes mediating gravitational interaction between M and m occurs if the perturbation series in $\alpha_{gr} = GMm/4\pi/\hbar$ fails to converge ($Mm \sim m_{Pl}^2$ is the naïve first guess for this value). Nature would be theoretician friendly and increase h_{eff} and reducing α_{gr} so that perturbation series converges again.

   Number theoretically this means the increase of algebraic complexity as the dimension $n = h_{eff}/h_0$ of the extension of rationals involved increases fron n_i to n_f [L40] and the number

n sheets in the covering defined by space-time surfaces increases correspondingly. Also the scale of the sheets would increase by the ratio n_f/n_i.

This phase transition can also occur for gauge interactions. For electromagnetism the criterion is that $Z_1Z_2\alpha$ is so large that perturbation theory fails. The replacement $\hbar \rightarrow Z_1Z_2e^2/v_0$ makes $v_0/4\pi$ the coupling constant strength. The phase transition could occur for atoms having $Z \geq 137$, which are indeed problematic for Dirac equation. For color interactions the criterion would mean that $v_0/4\pi$ becomes coupling strength of color interactions when α_s is above some critical value. Hadronization would naturally correspond to the emergence of this phase.

One can raise interesting questions. Is v_0 (presumably depending on the extension of rationals) a completely universal coupling strength characterizing any quantum critical system independent of the interaction making it critical? Can for instance gravitation and electromagnetism are mediated by the same flux tubes? I have assumed that this is not the case. It it could be the case, one could have for $GMm < m_{Pl}^2$ a situation in which effective coupling strength is of form $(GmMm/Z_1Z_2e^2)(v_0/4\pi)$.

The possibility of the proposed phase transition has rather dramatic implications for both quantum and classical gravitation.

1. Consider first quantum gravitation. v_0 does not depend on the value of G at all! The dependence of G on $\hbar_{eff}$ could be therefore allowed and one could have $l_P = R$. At quantum level scattering amplitudes would not depend on G but on v_0. I was of course very happy after having found the small expansion parameter v_0 but did not realize the enormous importance of the independence on G! Quantum gravitation would be like any gauge interaction with dimensionless coupling, which is even small! This might relate closely to the speculated TGD counterpart of AdS/CFT duality between gauge theories and gravitational theories.

2. What about classical gravitation? Here G should appear. What could the proportionality of classical gravitational force on $1/\hbar_{eff}$ mean? The invariance of Newton's equation

$$\frac{d\overline{v}}{dt} = -\frac{GM\overline{r}}{r^3} \tag{14.2.8}$$

under $h_{eff} \rightarrow xh_{eff}$ would be achieved by scaling $\overline{r} \rightarrow \overline{r}/x$ and $t \rightarrow t/x$. Note that these transformations have general coordinate invariant meaning as scalings of Minkowski coordinates of M^4 in $M^4 \times CP_2$. This scaling means the zooming up of size of space-time sheet by x, which indeed is expected to happen in $h_{eff} \rightarrow xh_{eff}$!

What is so intriguing that this connects to an old problem that I pondered a lot during the period 1980-1990 as I attempted to construct to the field equations for Kähler action approximate spherically symmetric stationary solutions [K119]. The naïve arguments based on the asymptotic behavior of the solution ansatz suggested that the one should have $G = R^2/\hbar$. For a long time indeed assumed $R = l_P$ but p-adic mass calculations [K66] and work with cosmic strings [K37] forced to conclude that this cannot be the case. The mystery was how $G = R^2/\hbar$ could be normalized to $G = l_P^2/\hbar$: the solution of the mystery is $\hbar \rightarrow \hbar_{eff}$ as I have now - decades later - realized!

14.3 Is the hierarchy of Planck constants behind the reported variation of Newton's constant?

Nowadays it is fantastic to be a theoretical physicists with a predictive theory. Every week I get from FB links to fascinating experimental findings crying for explanation (I am grateful for people providing these links). The last link of this kind was to a popular article (see `http://tinyurl.com/ya2wekch`) telling about the article [E29] (see `http://tinyurl.com/yanvzxj6`) reporting measurements of Newton's constant G carried out by Chinese physicists Shan-Qing Yang, Cheng-Gang Shao, Jun Luo and colleagues at the Huazhong University of Science and Technology and other institutes in China and Russia. The outcomes of two experiments using different methods differ more than the uncertainties in the experiments, which forces to consider the possibility that G can vary.

In the sequel I consider a possible interpretation for the finding of a Chinese research group measuring two different values of G differing by 47 ppm in terms of varying h_{eff}. Also a model for fountain effect of superfluidity as de-localization of wave function and increase of the maximal height of vertical orbit due to the change of the gravitational acceleration g at surface of Earth induced by a change of h_{eff} due to super-fluidity is discussed. Also Podkletnov effect is considered. TGD inspired theory of consciousness allows to speculate about levitation experiences possibly induced by the modification of G_{eff} at the flux tubes for some part of the magnetic body accompanying biological body in TGD based quantum biology.

14.3.1 The experiments

The experiments use torsion pendulum: this method was introduced by Henry Cavendish in 1978.

Remark: A remark about terminology is in order. Torque $\tau = F \times r$ on particle has dimensions Nm. Torsion (see `http://tinyurl.com/q8esymu`) in solid is essentially the density of torque per volume and has dimensions N/m^2. Twist angle is induced by torsion in equilibrium. The situation is governed by the theory of elasticity.

Basically one has torsion balance in which the gravitational torque produced by two source masses on masses associated with a torsion pendulum - dumbbell shaped system having identical masses at the ends of a bar and hanging from a thread at the middle point of the bar. As the source masses are rotated a twist of the thread emerges and twist angle corresponds to an equilibrium in which the torsion of the thread compensates the torque produced by gravitational interaction with source masses. Cavendish achieved 1 per cent accuracy in his measurements.

Refined variations of these measurements have been developed during years and the current precision is 47 parts per million (ppm). In some individual experiments the precision is 13.7 ppm. Disagreements larger than 500 ppm are reported, which suggests that new physics might be involved.

The latest experiments were made by the above mentioned research group. Two methods are used. TOS (Time Of Swing) and AAF (Angular Acceleration Feedback). AAF results deviates from the accepted value whereas TOS agrees. The accuracies were 11.64 ppm and 11.61 ppm in TOS and AAF respectively. AAF however gave by 45 ppm larger value of G.

In TOS technique the pendulum oscillates. The frequency of oscillation is determined by the positions of the external masses and G can be deduced by comparing frequencies for two different mass configurations. There are two equilibrium positions. The pendulum is either parallel to the line connecting masses relatively near to each other ("near" position). The pendulum orthogonal to the line connecting masses in "far" position. By measuring the different oscillation frequencies one can deduce the value of G.

Angular-acceleration feedback (AAF) method involves rotating the external masses and the pendulum on two separate turn tables. Twist angle is kept zero by changing the angular velocity of the other turn table: thus feedback is involved. If I have understood correctly, the torsion induced by gravitational torque compensates the torsion created by twisting of the thread around its axis in opposite direction and from the value of torsion for zero twist angle one deduces G. One could perhaps say that in AAF torsion is applied actively whereas in TOS it appears as reaction.

Why the measured value obtained for G would be larger for AAF? Could the active torsion inducing compensating twisting of the torsion pendulum actually increase G?

14.3.2 TGD based explanation in terms of hierarchy of Newton's constants

Some time ago I added a piece to an article telling about change in my view about Planck length [L56] (see `http://tinyurl.com/yclefxb2`). In TGD hierarchy of Planck constants is predicted: $\hbar_{eff} = nh_0$ is integer multiple of $h_0 = h/6$. During writing this, it became clear that h_0 need not be minimal value h_{min} of h_{eff} as I have assumed for some time (the first guess was that h is the minimal value).

This suggests also a hierarchy of Newton's constants $G_{eff} = l_P^2/\hbar_{eff}$ as subharmonics of l_P^2, where Planck length l_P is now re-identified as $l_P = R$, where R is CP_2 "radius" which for $G_{eff} = G$ is about $10^{3.5}$ larger than ordinary Planck length $l_P = \sqrt{\hbar G}$. The corresponding value of $\hbar_{eff}$, call it $\hbar_{eff}(gr)$, would be $\hbar_{eff}(gr)/h_{min} \simeq 2^{24}$. $\hbar_{eff}(gr)$ should not be confused with $\hbar_{gr} = GMm/v_0$ proposed by Nottale [E13] which for $M = M_E$ and $m = 2m_p$ is much larger.

Remark: This raises a problem to be discussed in the application to fountain effect. $h_{eff}(gr)$ is by factor of order 2^{24} larger than h, which looks strange since it involves delocalization of wave function to 2^{24} larger scale.

Could the variation of G - or better to call it G_{eff} - correspond to a variation of $h_{eff}/h = n$ in G_{eff}? Newton's constant for dark matter would be different from that for ordinary matter and vary in huge limits.

1. This looks non-sensical at first but would guarantee that one can scale up the solutions to Newton's equations by $h_{eff}/\hbar$ by scaling lengths by $n/n_0 = n/6$ [L26, L60, L61]: one would have thus scaling symmetry scaling also G_{eff} as is natural since it is dimensional parameter. Dark matter would be in rather precise sense zoomed up variants of ordinary matter and n would label the possible zoom ups.

2. h_{eff} has spectrum and as a special case one has $\hbar_{eff} = h_{gr} = GMm/v_0$. Is this case case the gravitational coupling become $G_{eff}Mm = v_0$ and does not depend on masses or G at all. In quantum scattering amplitudes a dimensionless parameter $(1/4\pi)v_0/c$ would appear in the role of gravitational fine structure constant and would be obtained from $\hbar_{eff} = h_{gr} = GMm/v_0$ consistent with Equivalence Principle. The miracle would be that G_{eff} would disappear totally from the perturbative expansion in terms of GMm as one finds by looking what $\alpha_{gr} = GMm/\hbar_{gr}$ is! This picture would work when GMm is larger than perturbative expansion fails to converge. For Mm above Planck mass squared this is expected to be the case. What happens below this limit is yet unclear (n is integer).

   Could v_0 be fundamental coupling constant running only mildly? This does not seem to be the case: Nottale's original work proposing $\hbar_{gr}$ proposes that v_0 for outer planets is by factor 1/5 smaller than for the 4 inner planets [K100, K84].

3. This picture works also for other interactions [**?**] Quite generally, nature would be theoretician friendly and induce a phase transition increasing $\hbar$ when the coupling strength exceeds the value below which perturbation series converges so that perturbation series converges. In adelic physics this would mean increase of the algebraic complexity since $h_{eff}/h = n$ is the dimension of extension of rationals inducing the extensions of various p-adic number fields and defining the particular level in the adelic hierarchy [L49, L50]. The parameters characterizing space-time surfaces as preferred extremals of the action principle would be numbers in this extension of rationals so that the phase transition would have a well-defined mathematical meaning. In TGD the extensions of rationals would label different quantum critical phases in which coupling constants would not run so that coupling constant evolution would be discrete as function of the extension.

4. This vision allows also to understand discrete coupling constant evolution replacing continuous coupling constant evolution of quantum field theories as being forced by the convergence of perturbation expansion and induced by the evolution defined by the hierarchy of extensions of rationals. When convergence is lost, a phase transition increasing algebraic complexity takes place and increases n. Extensions of rationals have also other characteristics than the dimension n.

   For instance, each extension is characterized by ramified primes and the proposal is that favoured p-adic primes assignable to cognition and also to elementary particles and physics in general correspond to so called ramified primes analogous to multiple zeros of polynomials. Therefore number theoretic evolution would also give rise to p-adic evolution as analog of ordinary coupling constant evolution with length scale.

   At quantum criticality coupling constant evolution is trivial and in QFT context this would mean that loops vanish separately or at least they sum up to zero for the critical values of coupling constants. This argument however seems to make the whole argument about convergence of coupling constant expansion obsolete unless one allows only the quantum critical values of coupling constants guaranteeing that quantum TGD is quantum critical. There are strong reasons to believe that the TGD analog of twistor diagrammatics involves only tree diagrams and there are strong number theoretic argument for this: infinite sum of diagrams does not in general give a number in given extension of rationals. Quantum criticality would be forced by number theory.

5. This would solve a really big conceptual problem, which I did not realize as I discovered the twistor lift of TGD making the choice $M^4\times CP_2$ unique [K114, K98] [L56]. The usual Planck length $l_P=\sqrt{\hbar G}$ as the radius of the M^4 twistor sphere would separate length scale from CP_2 scale R it is not a coupling constant like parameter and quantum criticality does not allow even in principle its understanding. The presence of two separate fundamental length scales in a theory intended to be unification does simply not make sense.

The variability of G with $\hbar_{eff}$ could explain the variation of G in various experiments since for gravitational flux tubes $\hbar_{eff}/\hbar\sim 10^7$ would be true. The smallest variation would be of order 10^{-7} as n varies by one unit. This is a testable prediction (see `http://tinyurl.com/yclefxb2`).

As already explained, the maximum for the variation of G is 500 ppm $=5\times 10^{-4}$. This would correspond to $\Delta n\sim 5\times 10^3$. The difference between TOS and AAF is 47 ppm and would correspond to $\Delta n\sim 470$. The variation could be also due to a small variation, say $k\rightarrow k+1$, for a prime factor k of n. 47 ppm would give $k\simeq 2,128$. For $k=2^{11}\rightarrow k-1$ in TOS to AAF and favored by number theoretic considerations would give $\Delta k/k=49$ ppm.

Why small variations for the factors of n would be favored? If one assumes that number theoretical evolution corresponds to the increasing order of the Galois group such that the new Galois group contains the earlier Galois group as a subgroup (this would serve as an analogy for conserved genes in biological evolution). Larger Galois groups would naturally contain the "standard" Galois group associated with N as a sub-group. From number theoretic point of view the proposal $\hbar_{eff}/\hbar=N=2^{24}$ is perhaps the simplest one since all Galois groups appearing as its sub-groups would have order with is 6×2^k for $h=6h_0$. Larger values of $h_{eff}/\hbar$ should have N as a factor.

Why the presence of of the feedback torque on the torsion pendulum would reduce the value of $\hbar_{eff}/h=n$ by about 5×10^3 units in AAF for the gravitational flux tubes connecting the source masses to the masses of torsion pendulum from that in TOS? Somehow the value of h_{eff} should be reduced.

14.3.3 A little digression: Galois groups and genes

As found, the question about possible variations of G_{eff}, leads to the idea that subgroups of Galois group could be analogous to conserved genes in that they could be conserved in number theoretic evolution. In small variations such as above variation Galois subgroups as genes would change only a little bit. For instance, the dimension of Galois subgroup would change.

The analogy between subgoups of Galois groups and genes goes also in other direction. I have proposed long time ago that genes (or maybe even DNA codons) could be labelled by $h_{eff}/h=n$. This would mean that genes (or even codons) are labelled by a Galois group of Galois extension (see `http://tinyurl.com/zu5ey96`) of rationals with dimension n defining the number of sheets of space-time surface as covering space. This could give a concrete dynamical and geometric meaning for the notin of gene and it might be possible some day to understand why given gene correlates with particular function. This is of course one of the big problems of biology.

One should have some kind of procedure giving rise to hierarchies of Galois groups assignable to genes. One would also like to assign to letter, codon and gene and extension of rationals and its Galois group. The natural starting point would be a sequence of so called intermediate Galois extensions E^H leading from rationals or some extension K of rationals to the final extension E. Galois extension has the property that if a polynomial with coefficients in K has single root in E, also other roots are in E meaning that the polynomial with coefficients K factorizes into a product of linear polynomials. For Galois extensions the defining polynomials are irreducible so that they do not reduce to a product of polynomials.

Any sub-group $H\subset Gal(E/K))$ leaves the intermediate extension E^H invariant in element-wise manner as a sub-field of E (see `http://tinyurl.com/y958drcy`). Any subgroup $H\subset$

$Gal(E/K)$) defines an intermediate extension E^H and subgroup $H_1 \subset H_2 \subset ...$ define a hierarchy of extensions $E^{H_1} > E^{H_2} > E^{H_3}...$ with decreasing dimension. The subgroups H are normal - in other words $Gal(E)$ leaves them invariant and $Gal(E)/H$ is group. The order $|H|$ is the dimension of E as an extension of E^H. This is a highly non-trivial piece of information. The dimension of E factorizes to a product $\prod_i |H_i|$ of dimensions for a sequence of groups H_i.

Could a sequence of DNA letters/codons somehow define a sequence of extensions? Could one assign to a given letter/codon a definite group H_i so that a sequence of letters/codons would correspond a product of some kind for these groups or should one be satisfied only with the assignment of a standard kind of extension to a letter/codon?

Irreducible polynomials define Galois extensions and one should understand what happens to an irreducible polynomial of an extension E^H in a further extension to E. The degree of E^H increases by a factor, which is dimension of E/E^H and also the dimension of H. Is there a standard manner to construct irreducible extensions of this kind?

1. What comes into mathematically uneducated mind of physicist is the functional decomposition $P^{m+n}(x) = P^m(P^n(x))$ of polynomials assignable to sub-units (letters/codons/genes) with coefficients in K for a algebraic counterpart for the product of sub-units. $P^m(P^n(x))$ would be a polynomial of degree $n+m$ in K and polynomial of degree m in E^H and one could assign to a given gene a fixed polynomial obtained as an iterated function composition.

   Intuitively it seems clear that in the generic case $P^m(P^n(x))$ does not decompose to a product of lower order polynomials. One must be however cautious here. It can be shown (see `https://arxiv.org/pdf/1511.06446.pdf`) that the probability that a random polynomial with rational coefficients is irreducible behaves as $O(logN/N)$, where N is upper bound for the magnitude of coefficients. On the other hand, the probability that a random monic polynomial (integer coefficients and unit constant coefficient) is not irreducible (factorizes) goes as $O(1/N)$. It is also shown that by their special properties permutation groups S_n are strongly favoured as Galois groups.

   One could use also polynomials assignable to codons or letters as basic units. Also polynomials of genes could be fused in the same manner.

   The choice of polynomials P^n is rather free since for given order of Galois group there are only finite number of finite groups and the number of polynomials is infinite. The first cautious guess is that the Galois group depends rather weakly on the rational coefficients regarded as real numbers.

2. If the iteration of polynomial maps indeed gives a Galois extensions, the dimension m of the intermediate extension should be same as the order of its Galois group. Composition would be non-commutative but associative as the physical picture demands. The longer the gene, the higher the algebraic complexity would be. Could functional decomposition define the rule for who extensions and Galois groups correspond to genes? Very naïvely, functional decomposition in mathematical sense would correspond to composition of functions in biological sense.

3. This picture would conform with $M^8 - M^4 \times CP_2$ correspondence [L40] in which the construction of space-time surface at level of M^8 reduces to the construction of zero loci of polynomials of octonions, with rational coefficients. DNA letters, codons, and genes would correspond to polynomials of this kind.

Could one say anything about the Galois groups of DNA letters?

1. Since $n = h_{eff}/h$ serves as a kind of quantum IQ, and since molecular structures consisting of large number of particles are very complex, one could argue that n for DNA or its dark variant realized as dark proton sequences can be rather large and depend on the evolutionary level of organism and even the type of cell (neuron viz. soma cell). On the other, hand one could argue that in some sense DNA, which is often thought as information processor, could be analogous to an integrable quantum field theory and be solvable in some sense. Notice also that one can start from a background defined by given extension K of rationals and consider polynomials with coefficients in K. Under some conditions situation could be like that for rationals.

2. The simplest guess would be that the 4 DNA letters correspond to 4 non-trivial finite groups with smaller possible orders: the cyclic groups Z_2, Z_3 with orders 2 and 3 plus 2 finite groups of order 4 (see the table of finite groups in `http://tinyurl.com/j8d5uyh`). The groups of order 4 are cyclic group $Z_4 = Z_2 \times Z_2$ and Klein group $Z_2 \oplus Z_2$ acting as a symmetry group of rectangle that is not square - its elements have square equal to unit element. All these 4 groups are Abelian. Polynomial equations of degree not larger than 4 can be solved exactly in the sense that one can write their roots in terms of radicals.

3. Could there exist some kind of connection between the number 4 of DNA letters and 4 polynomials of degree less than 5 for whose roots one an write closed expressions in terms of radicals as Galois found? Could it be that the polynomials obtained by a a repeated functional composition of the polynomials of DNA letters have also this solvability property?

   This could be the case! Galois theory states that the roots of polynomial are solvable by radicals if and only if the Galois group is solvable meaning that it can be constructed from abelian groups using Abelian extensions (see `http://tinyurl.com/ybcua92y`).

   Solvability translates to a statement that the group allows so called sub-normal series $1 < G_0 < G_1 ... < G_k$ such that G_{j-1} is normal subgroup of G_j and G_j/G_{j-1} is an abelian group. An equivalent condition is that the derived series $G \triangleright G^{(1)} \triangleright G^{(2)} \triangleright ...$ in which $j+1$:th group is commutator group of G_j ends to trivial group. If one constructs the iterated polynomials by using only the 4 polynomials with Abelian Galois groups, the intuition of physicist suggests that the solvability condition is guaranteed!

4. Wikipedia article also informs that for finite groups solvable group is a group whose composition series has only factors which are cyclic groups of prime order. Abelian groups are trivially solvable, nilpotent groups are solvable, p-groups (having order, which is power prime) are solvable and all finite p-groups are nilpotent.

   Every group with order less than 60 elements is solvable. Fourth order polynomials can have at most S_4 with 24 elements as Galois groups and are thus solvable. Fifth order polynomials can have the smallest non-solvable group, which is alternating group A_5 with 60 elements as Galois group and in this case are not solvable. S_n is not solvable for $n > 4$ and by the finding that S_n as Galois group is favored by its special properties (see `http://tinyurl.com/y6wyq9v2`).

   A_5 acts as the group icosahedral orientation preserving isometries (rotations). Icosahedron and tetrahedron glued to it along one triangular face play a key role in TGD inspired model of bio-harmony and of genetic code [L5, L71]. The gluing of tetrahedron increases the number of codons from 60 to 64. The gluing of tetrahedron to icosahedron also reduces the order of isometry group to the rotations leaving the common face fixed and makes it solvable: could this explain why the ugly looking gluing of tetrahedron to icosahedron is needed? Could the smallest solvable groups and smallest non-solvable group be crucial for understanding the number theory of the genetic code.

An interesting question inspired by $M^8 - H$-duality [L40] is whether the solvability could be posed on octonionic polynomials as a condition guaranteeing that TGD is integrable theory in number theoretical sense or perhaps following from the conditions posed on the octonionic polynomials. Space-time surfaces in M^8 would correspond to zero loci of real/imaginary parts (in quaternionic sense) for octonionic polynomials obtained from rational polynomials by analytic continuation. Could solvability relate to the condition guaranteeing M^8 duality boiling down to the condition that the tangent spaces of space-time surface are labelled by points of CP_2. This requires that tangent or normal space is associative (quaternionic) and that it contains fixed complex subspace of octonions or perhaps more generally, there exists an integrable distribution of complex subspaces of octonions defining an analog of string world sheet.

14.3.4 Does fountain effect involve non-standard value of G or delocalization due to a large value of h_{eff}?

Deviations in the value of G are not new, and I have written about several gravitational anomalies. This could mean also anti-gravity effects in a well-defined sense which is however not the same as often thought (negative gravitational masses or repulsive gravitational force).

In particular, in the well-known fountain effect (`http://tinyurl.com/kx3t52r`) of superfluidity, superfluid seems to defy gravitation. I have asked whether $h_{eff}/h_0 = n$ increases at superfluid flux tubes to h_{gr} and this gives to the effect as a de-localization in much longer scale [?]. The delocalization could be also due to the reductionof h_{em} or possibly h_Z assignable to long range classical Z^0 force predicted by TGD.

If G is reduced - this means violation of Equivalence Principle in its standard form - the effect would be possible also classically. Since in superfluidity one has h_{eff} larger than usually, this might happen if gravitons travel also along flux tubes at which super fluid flows.

A simple model for the situation discussed in [?] would rely on Schrödinger equation at the flux quantum which is locally a thin hollow cylinder turning around at the top of the wall of the container.

1. One obtains 1-dimensional Schrödinger equation

$$(-\frac{\hbar_{eff}^2\partial_z^2}{2m} + mg_{eff}z)\Psi = E\Psi \ , \quad h_{eff} = nh_0 = \frac{nh}{6} \ . \tag{14.3.1}$$

   It is easy to see that the energy spectrum is invariant under the scaling $h \rightarrow h_{eff} = xh$ and $z \rightarrow z/x$. One has $\Psi_{xh,g_{eff}=g/x}(z) = \Psi_{h,g}(z/x)$ so that simple scaling of the argument z in question. The energy of the solution is same. If the ordinary solution has size scale L, the scaled up solution has size scale xL.

   The height for a trajectory in gravitational field of Earth is scaled up for a given initial vertical velocity v_i is scaled as $h \rightarrow xh$ so quantum behavior corresponds to the classical behavior and de-localization scale is scaled up. Could this happen at various layers of magnetic body for dark particles so that they would be naturally at much higher heights. Cell scale would be scaled to Earth size scale of even larger sizes for the values of $\hbar_{eff}/h = n$ involved.

   For classical solution with initial initial vertical velocity $v_i = 1$ m/s the height of the upwards trajectory is $h = v_i^2/2g$ 5 cm. Quantum classical correspondence would be given by $E = mv_i^2/2$ and this allows to look the delocalization scale of a solution.

2. One can introduce the dimensionless variable u (note that one has $g_{eff}/g = 1/x$, $x = h/h_{eff}$) as

$$u = \frac{z - \frac{E}{mg_{eff}}}{z_0} \ , \qquad z_0 = \left[\frac{2m^2 g_{eff}}{\hbar_{eff}^2}\right]^{-1/3} = \frac{h_{eff}}{h}(\frac{m}{m_p})^{2/3} \times (\frac{g}{L_p^2})^{-1/3} \simeq \frac{h_{eff}}{h} \times (\frac{m}{m_p})^{2/3} \times 2.4\ mm$$

$$L_p = \frac{\hbar c}{m_p} = \simeq 2.1 \times 10^{-16} \text{ m} \ , \tag{14.3.2}$$

Here m_p denotes proton mass and L_p proton Compton length. z_0 scales as $\hbar_{eff}$ as one might expect. z_0 characterizes roughly the scale of the s lution. From the scale of the fountain effect about 1 meter, one can conclude that one should have $h_{eff}/h \sim 2^8$.

This allows to cast the equation to the standard form of the equation for Airy functions encountered in WKB approximation

$$-\frac{d^2\Psi}{du^2} + u\Psi = 0 \ . \tag{14.3.3}$$

Remark: Note that the classical solution depends on m. In central force problem with $1/r$ and $h_{eff} = GMm/v_0$ the binding energy spectrum $E = E_0/n^2$ has scale $E_0 = v_0^2 m$ and is universal.

3. The interesting solutions correspond to Airy functions $Ai(u)$ which approach rapidly zero for the values of $u > 1$ and oscillate for negative values of u. These functions $Ai(u + u_1)$ are orthogonal for different values of u_1. The values of u_1 correspond to different initial kinetic energies for the motion in vertical direction. In the recent situation these energies correspond to the initial vertical velocities of the super-fluid in the film. $u = u_0 = 1$ defines a convenient estimate for the value of z coordinate above which wave function approaches rapidly to zero.

   For classical solution with initial initial vertical velocity $v_i = 1$ m/s the height of the upwards trajectory is $h = v_i^2/2g$ 5 cm. Quantum classical correspondence would be given by $E = mv_i^2/2 = E$ and this allows to look the delocalization scale of a solution.

   The Airy function $Ai(u)$ approaches rapidly to zero (see the graph of `https://en.wikipedia.org/wiki/Airy_function`) and one can say that above $u_0 = 3$ the function vanishes. Already at $u_0 = 1$ wave function is rather small as compared with its value at $u = 0$. This condition translates to a condition for z as

$$z_0 = z_{cl} + u_0 z_0 \quad , \quad z_{cl} = \frac{E}{mg_{eff}} \quad , \quad z_0 = \frac{h_{eff}}{h}\left[\frac{\hbar^2}{2m^2g}\right]^{1/3} \quad . \tag{14.3.4}$$

   The condition is consistent with the classical picture and the classical height z_{cl} scales like h_{eff}/h. The parameter $u_0 z_0$ defines the de-localization scale consistent with the expectations. Below z_{cl} the wave function oscillates which intuitively corresponds to the sum of waves in upwards and downwards directions.

What can one conclude about the value of $x = h_{eff}/h_0$ in the case of super-fluidity?

(a) Using the previous formula, the condition that z_0 is of order 1 meter fixes its value to $h_{eff}/h_0 \sim 2^8$. Could super-fluidity correspond to the value of $h_{eff} = h_{em} > h$ assignable to electromagnetic flux tubes? The generalization $h_{em} = Ze^2/v_0$ of the Nottale's formula would require that the super fluid phase has a large total em charge Z. The Cooper pairs are however neutral. This leaves under consideration only the old idea that super-fluidity corresponds to Z^0 super-conductivity inspired by the idea that TGD predicts long range Z^0 fields and by the fact that nuclei carry indeed carry non-vanishing Z^0 charge mostly due to neurons.

(b) Both $\hbar_{eff}(gr)/\hbar \simeq 2^{24}$ and $\hbar_{gr} = GMm/v_0$ given by Nottale's hypothesis give quite too large value of z_0.

    The gravitational Compton length λ_{gr} is given by $\lambda_{gr} = GM_e/v_0 = r_S/2v_0$ and - in accordance with the Equivalence Principle - does not depend on m. The Schwartschild radius of Earth is $r_S = .9$ cm. One could argue that λ_{gr} is a reasonable lower bound for z_0 if h_{gr} appears in the gravitational Schrödinger equation. For $v_0/c = 2^{-11}$ required by the Bohr orbit model for the inner planets, this would give $\lambda_{gr} = 9$ m. The energy scale of dark cyclotron states comes out correctly if one has $v_0/c = 1/2$ giving the lower bound $z_0 \geq r_S = .9$ cm.

    However, the proportionality of z_0 to h_{eff}/h implies that the z_0 is scaled by a factor of order $2GM_E m_p/v_0 \sim 10^{14}$ from its value $z_0 = .2$ mm and would be gigantic. It seems that this option indeed fails.

(c) Could the fountain effect be due to the reduction of g in principle possible if G is prediction and CP_2 length replaces Planck length as fundamental scale? If one assumes $h_{eff} = h$ and scaled down value of g corresponding to $G_{eff} = R^2/\hbar_{gr}$ such that $\hbar_{gr}$ is scaled from its normal value: $\hbar_{gr} \rightarrow y\hbar_{gr}$, $G_{eff} \rightarrow G_{eff}/y$. This would give the scaling of $z_0 \propto g^{-1/3}$ as $z_0 \rightarrow y^{1/3} z_0$ giving $z_0 \simeq .2$ mm should be scaled up to about 1 mm which would give $y \sim 10^9$. This would mean a huge breaking of Equivalence Principle.

14.3.5 Does Podkletnov effect involve non-standard value of G?

Podkletnov observed [H7] at eighties a few percent reduction of gravity: he immediately lost his job in Tampere University in Finland. It was regarded as a scandalous event. Something new might have been discovered in finnish laboratory!

I have considered a possible mechanism explaining the finding of Podkletnov [L11]. One could however ask whether the presence of a superconductor involving also the presence of phase with non-standard value of Planck constant could also affect the value of h_{eff} assignable to the flux tubes of the Kähler magnetic field? The mechanism could be the same as in the fountain effect. The non-standard value of h_{em} could induce delocalization and reduction of g. Now also a small change g from its normal value can be considered and would have been few per cent in this case. This would mean a small breaking of

14.3.6 Did LIGO observe non-standard value of G and are galactic blackholes really supermassive?

Also smaller values of G than the G_N are possible and in fact, in condensed matter scales it is quite possible that $n = R^2/G$ is rather small. Gravitation would be stronger but very difficult to detect in these scales. Neutron in the gravitational field of Earth might provide a possible test. The general rule would be that the smaller the scale of dark matter dynamics, the larger the value of G and maximum value would be $G_{max} = R^2/h_0$, $h = 6h_0$.

Are the blackholes detected by LIGO really so massive?

LIGO (see `http://tinyurl.com/bszfs29`) has hitherto observed 3 fusions of black holes giving rise to gravitational waves. For TGD view about the findings of LIGO see [L29, L23] (see `http://tinyurl.com/y79yqw6q` and `http://tinyurl.com/ya8ctxgc`). The colliding blackholes were deduced to have unexpectedly larger large masses: something like 10-40 solar masses, which is regarded as something rather strange.

Could it be that the masses were actually of the order of solar mass and G was actually larger by this factor and h_{eff} smaller by this factor? The mass of the colliding blackholes could be of order solar mass and G would larger than its normal value - say by a factor in the range (10,50). If so, LIGO observations would represent the first evidence for TGD view about quantum gravitation, which is very different from superstring based view. The fourth fusion was for neutron stars rather than black holes and stars had mass of order solar mass.

This idea works if the physics of gravitating system depends only on $G(M + m)$. That classical dynamics depends on $G(M + m)$ only, follows from Equivalence Principle. But is this true also for gravitational radiation? If the power of gravitational radiation distinguishes between different values of M when GM is kept constant, the idea is dead.

(a) If the power of gravitational radiation distinguishes between different values of M+m, when $G(M + m)$ is kept constant, the idea is dead. This seems to be the case. The dependence on $G(M+m)$ only leads to contradiction at the limit when M+m approaches zero and $G(M + m)$ is fixed. The reason is that the energy emitted per single period of rotation would be larger than M+m. The natural expectation is that the radiated power per cycle and per mass M+m depends on $G(M + m)$ only as a dimensionless quantity.

(b) From arXiv one can find an article (see `http://tinyurl.com/y99j3fpr`) in which the energy per unit solid angled and frequency radiated in collision of blackholes is estimated. The outcome is proportional to $E^2G(M + m)^2$, where E is the energy of the colliding blackhole.

    The result is proportional mass squared measured in units of Planck mass squared as one might indeed naïvely expect since $G(M+m)^2$ is analogous to the total gravitational charge squared measured using Planck mass.

The proportionality to E^2 comes from the condition that dimensions come out correctly. Therefore the scaling of G upwards would reduce mass and the power of gravitational radiation would be reduced down like $M+m$. The power per unit mass depends on $G(M+m)$ only. Gravitational radiation allows to distinguish between two systems with the same Schwartschild radius, although the classical dynamics does not allow this.

(c) One can express the classical gravitational energy E as gravitational potential energy proportional to GM/R This gives only dependence on GM as also Equivalence Principle for classical dynamics requires and for the collisions of blackholes R is measured by using $G(M+m)$ as a natural unit.

Remark: The calculation uses the notion of energym which in general relativity is precisely defined only for stationary solutions. Radiation spoils the stationarity. The calculations of the radiation power in GRT is to some degree artwork feeding in the classical conservation laws in post-Newtonian approximation lost in GRT. In TGD framework the conservation laws are not lost and hold true at the level of $M^4 \times CP_2$.

What about supermassive galactic blacholes?

What about supermassive galactic black holes in the centers of galaxies: are they really super-massive or is G super-large! The mass of Milky Way super-massive blackhole is in the range 10^5-10^9 solar masses. Geometric mean is $n=10^7$ solar masses and of the order of the standard value of $R^2/G_N = n \sim 10^7$. Could one think that this blackhole has actually mass in the range 1-100 solar masses and assignable to an intersection of galactic cosmic string with itself! How galactic blackholes are formed is not well understood. Now this problem would disappear. Galactic blackholes would be there from the beginning!

The general conclusion is that only gravitational radiation allows to distinguish between different masses $M+m$ for given $G(M+m)$ in a system consisting of two masses so that classically scaling the opposite scalings of G and $M+m$ is a symmetry.

14.3.7 Is it possible to determine experimentally whether gravitation is quantal interaction?

Marletto and Vedral have proposed an interesting method for measuring whether gravitation is quantal interaction (see `https://arxiv.org/pdf/1707.06036.pdf`).

I tried to understand what the proposal suggests and how it translates to TGD language.

(a) If gravitational field is quantum it makes possible entanglement between two states. This is the intuitive idea but what it means in TGD picture? Feynman interpreted this as entanglement of gravitational field of an objects with the state of object. If object is in a state, which is superposition of states localized at two different points x_i, the classical gravitational fields ϕ_{gr} are different and one has a superposition of states with different locations

$$|I\rangle = \sum_{i=1,2} |m_i \ at \ x_i > |\phi_{gr,x_i}\rangle \equiv |L\rangle + |R\rangle \ .$$

(b) Put two such de-localized states with masses m_i at some distance d to get state $|1\rangle|2\rangle$, $|i\rangle = |L\rangle_i + |R\rangle >_i$. The 4 components pairs of the states interact gravitationally and since there are different gravitational fields between different states the states get different phases, one can obtain entangled state.

Gravitational field would entangle the masses. If one integrates over the degrees of freedom associated with gravitational field one obtains density matrix and the density matrix is not pure if gravitational field is quantum in the sense that it entangles with the particle position.

That gravitation is able to entangle the masses would be a proof for the quantum nature of gravitational field. It is not however easy to detect this. If gravitation only serves as a parameter in the interaction Hamiltonian of the two masses, entanglement can be generated but does not prove that gravitational interaction is quantal. It is required that the only interaction between the systems is gravitational so that other interactions do not generate entanglement. Certainly, one should use masses having no em charges.

(c) In TGD framework the view of Feynman is natural. One has superposition of space-time surfaces representing this situation. Gravitational field of particle is associated with the magnetic body of particle represented as 4-surface and superposition corresponds to a de-localized quantum state in the "world of classical worlds" with x_i representing particular WCW coordinates.

I am not specialist in quantum information theory nor as quantum gravity experimentalist, and hereafter I must proceed keeping fingers crossed and I can only hope that I have understood correctly. To my best understanding, the general idea of the experiment would be to use interferometer to detect phase differences generated by gravitational interaction and inducing the entanglement. Not for photons but for gravitationally interacting masses m_1 and m_2 assumed to be in quantum coherent state and be describable by wave function analogous to em field. It is assumed that gravitational interact can be describe classically and this is also the case in TGD by quantum-classical correspondence.

(a) Authors think quantum information theoretically and reduce everything to qubits. The de-localization of masses to a superposition of two positions correspond to a qubit analogous to spin or a polarization of photon.

(b) One must use and analog of interferometer to measure the phase difference between different values of this "polarization".

In the normal interferometer is a flattened square like arrangement. Photons in superpositions of different spin states enter a beam splitter at the left-lower corner of interferometer dividing the beam to two beams with different polarizations: horizontal (H) and vertical (V). Vertical (horizontal) beam enters to a mirror which reflects it to horizontal (vertical beam). One obtains paths V-H and H-V meeting at a transparent mirror located at the upper right corner of interferometer and interfere.

There is detector D_0 resp. D_1 detecting component of light gone through in vertical resp. horizontal direction of the fourth mirror. Firing of D_1 would select the H-V and the firing of D_0 the V-H path. This thus would tells what path (V-H or H-V) the photon arrived. The interference and thus also the detection probabilities depend on the phases of beams generated during the travel: this is important.

(c) If I have understood correctly, this picture about interferometer must be generalized. Photon is replaced by mass m in quantum state which is superposition of two states with polarizations corresponding to the two different positions. Beam splitting would mean that the components of state of mass m localized at positions x_1 and x_2 travel along different routes. The wave functions must be reflected in the first mirrors at both path and transmitted through the mirror at the upper right corner. The detectors D_i measure which path the mass state arrived and localize the mass state at either position. The probabilities for the positions depend on the phase difference generated during the path. I can only hope that I have understood correctly: in any case the notion of mirror and transparent mirror in principle make sense also for solutions of Schrödinger eequation.

(d) One must however have two interferometers. One for each mass. Masses m_1 and m_2 interact quantum gravitationally and the phases generated for different polarization states differ. The phase is generated by the gravitational interaction. Authors estimate that phases generate along the paths are of form

$$\Phi_i = \frac{m_1 m_2 G}{\hbar d_i}\Delta t \quad .$$

$\Delta t = L/v$ is the time taken to pass through the path of length L with velocity v. d_1 is the smaller distance between upper path for lower $mass m_2$ and lower path for upper mass m_1. d_2 is the distance between upper path for upper mass m_1 and lower m_2. See Figure 1 of the article (see `https://arxiv.org/pdf/1707.06036.pdf`).

What one needs for the experiment?

(a) One should have de-localization of massive objects. In atomic scales this is possible. If one has $h_{eff}/h_0 > h$ one could also have zoomed up scale of de-localization and this might be very relevant. Fountain effect of superfluidity pops up in mind.

(b) The gravitational fields created by atomic objects are extremely weak and this is an obvious problem. Gm_1m_2 for atomic mass scales is extremely small: since Planck mass m_P is something like 10^{19} proton masses and atomic masses are of order 10-100 atomic masses.

    One should have objects with masses not far from Planck mass to make Gm_1m_2 large enough. Authors suggest using condensed matter objects having masses of order $m \sim 10^{-12}$ kg, which is about 10^{15} proton masses 10^{-4} Planck masses. Authors claim that recent technology allows de-localization of masses of this scale at two points. The distance d between the objects would be of order micron.

(c) For masses larger than Planck mass one could have difficulties since quantum gravitational perturbation series need not converge for $Gm_1m_2 > 1$ (say). For proposed mass scales this would not be a problem.

What can one say about the situation in TGD framework?

(a) In TGD framework the gravitational Planck $h_{gr} = Gm_1m_2/v_0$ assignable to the flux tubes mediating interaction between m_1 and m_2 as macroscopic quantum systems could enter into the game and could reduce in extreme case the value of gravitational fine structure constant from $Gm_1m_2/4\pi\hbar$ to $Gm_1m_2/4\pi\hbar_{eff} = \beta_0/4\pi$, $\beta_0 = v_0/c < 1$. This would make perturbation series convergent even for macroscopic masses behaving like quantal objects. The physically motivated proposal is $\beta_0 \sim 2^{-11}$. This would zoom up the quantum coherence length scales by $h_{gr}/\hbar$.

(b) What can one say in TGD framework about the values of phases Φ?

    i. For $\hbar \to \hbar_{eff}$ one would have

    $$\Phi_i = \frac{Gm_1m_2}{\hbar_{eff} d_i}\Delta t \quad .$$

    For $\hbar \to \hbar_{eff}$ the phase differences would be reduced for given Δt. On the other hand, quantum gravitational coherence time is expected to increase like h_{eff} so that the values of phase differences would not change if Δt is increased correspondingly. The time of 10^{-6} seconds could be scaled up but this would require the increase of the total length L of interferometer arms and/or slowing down of the velocity v.

    ii. For $\hbar_{eff} = \hbar_{gr}$ this would give a universal prediction having no dependence on G or masses m_i

    $$\Phi_i = \frac{v_0\Delta t}{d_i} = \frac{v_0}{v}\frac{L}{d_i} \quad .$$

If Planck length is actually equal to CP_2 length $R \sim 10^{3.5}\sqrt{G_N\hbar}$, one would has $G_N = R^2/\hbar_{eff}$, $\hbar_{eff} \sim 10^7$. One can consider both smaller and larger values of G and for larger values the phase difference would be larger. For this option one would obtain $1/\hbar_{eff}^2$ scaling for Φ. Also for this option the prediction for the phase difference is universal for $h_{eff} = h_{gr}$.

iii. What is important is that the universality could be tested by varying the masses m_i. This would however require that m_i behave as coherent quantum systems gravitationally. It is however possible that the largest quantum systems behaving quantum coherently correspond to much smaller masses.

14.3.8 Fluctuations of Newton's constant in sub-millimeter scales

Sabine Hossenfelder had a post with link to an article "*Hints of Modified Gravity in Cosmos and in the Lab?*" [E23] (see `http://tinyurl.com/y6j8sntw`). Here is the part of abstract that I find the most interesting.

On sub-millimeter scales we show an analysis of the data of the Washington experiment (Kapner et al. (2007) searching for modifications of Newton's Law on sub-millimeter scales and demonstrate that a spatially oscillating signal is hidden in this dataset. We show that even though this signal cannot be explained in the context of standard modified theories (viable scalar tensor and f(R) theories), it is a rather generic prediction of nonlocal gravity theories.

What is interesting from TGD point of view that the effect - if it is indeed real - appears in scale of .085 mm about 10^{-4} μm, which is the scale defined by the density of dark energy in recent universe and thus by cosmological constant. This is also size scale of large neuron.

Findings

Washington group studied gravitational torque on torque pendulum for sub-millimeter distances of masses involved [E17] (see `http://tinyurl.com/y2un6686`). Figure 19 of [E23] (see `http://tinyurl.com/y6j8sntw`) illustrates data points representing the deviation of the gravitational torque from the Newtonian prediction as a function of distance in the range .05-10 mm.

The deviation can parameterized in terms of effective scaling $G \to kG$ of Newton's constant, which is assumed to be predictable rather than due to fluctuations and depend on the distance only

$$k = 1 + xcos(\frac{2\pi r}{\lambda} + \frac{3\pi}{4}) \ .$$

x is a numerical parameter. The highly non-trivial assumption is that Newton's potential is modified by an oscillating term, which must go to zero at large distances: its amplitude could approach to zero like $1/r$. The model predicts an anomalous gravitational torque $\Delta\tau$ proportional to $k-1$ and having the form

$$\Delta\tau = acos(\frac{2\pi r}{\lambda} + \frac{3\pi}{4}) \ ,$$

where r is the distance between the masses. The parameter $\lambda = \hbar/m$ is formally analogous to Compton length for imaginary mass m.

The finding is that the statistical significance for the best fit to the data is $(a,\lambda) = (0.004 \ \ fNm, 65 \ \ mm^{-1})$ is more than 3σ, where a is the amplitude of the deviation. The highly non-trivial problem is however that one obtains also other minima of χ^2 measuring the goodness of the fit with different values of the parameter λ.

I am not specialist but while looking at the data, I cannot avoid the feeling that the fit does not make much sense and reflects theoretical prejudices (belief in modified gravity of some kind) rather than reality. My first impression that fluctuations in the value of Newton's constant

G are in question. The value of G is indeed known to vary from experiment to experiment and the variation is too large to be explained in terms of measurement inaccuracies [?]see `http://tinyurl.com/yanvzxj6`).

Could it be that the value of G fluctuates, and for some reason in the length scale range around .1 mm the fluctuations are especially large meaning different values of G are large? Could some kind of criticality enhanced rather dramatically below .1 mm be involved?

Could fluctuations in the value of G explain the findings?

Twistor lift of TGD [K114, K98, K14, L75] predicts that cosmological constant is length scale dependent and that Newton's constant G has a spectrum reflecting the spectrum of effective Planck constant $h_{eff} = nh_0$ ($h = 6h_0$ is a good guess [L26]): dark matter would correspond to $h_{eff} = nh_0$ phases of ordinary matter.

p-Adic length scale hypothesis allows to assign to cosmological constant Λ two length scales: the cosmological p-adic scale defined by Λ itself and the short p-adic length scale determined by the density of dark energy so that physics is cosmological scales and physics in microscopic scales reflect each other.

This encourages the idea that one might understand the experimental findings in terms of fluctuations of G induced by quantum fluctuations of h_{eff} at quantum criticality.

(a) TGD suggests a spectrum for the values of G. The starting points is the expression for the effective Planck constant ass $h_{eff} = n \times h_0$. In adelic physics the value of n is identified as the number of sheets for the space-time surface as covering space and would correspond to the order of Galois group of extension of rationals inducing the extensions of p-adic number fields appearing in the adele [L49, L50].

(b) An additional hypothesis is that space-time surface can be regarded as covering of both M^4 and CP_2 with numbers of sheets equal to n_1 and n_2: $n = n_1n_2$. The number of sheets over M^4 would be n_1 so that CP_2 coordinates would be n_1-valued functions of M^4 coordinates. The number of sheets over CP_2 would be n_2 and one would have effective n_2 copies of n_1 valued regions in M^4.

The gravitational Planck constant $\hbar_{gr} = GMm/v_0$ originally introduced by Nottale [E13] is proposed to correspond to $\hbar_{eff} = \hbar_{gr} = n_1n_2\hbar_0$. The real Planck length $l_P(real)$ would correspond to $l_P(real) = R$, the CP_2 size scale identified as geodesic length, and Newton's constant would correspond to

$$G = \frac{R^2}{\hbar_1} = \frac{R^2}{n_1\hbar_0} \quad .$$

One would have $n_1 \sim 6 \times 10^7$ from $l_P^2/R^2 \sim 10^7$.

(c) The value of n_1 can fluctuate and induce fluctuations of G. The fluctuations could be even large. One can even ask whether the fountain effect of superfluidity involves a large value of n_1 responsible for macroscopic quantum coherence and due to the increase of the value of $\hbar eff$ caused the increase of n_1 in turn reducing the value of G [?].

Could the fluctuations of n_1 explain the findings about the value of G deduced from Washington experiment? The appearance of several values for parameter λ might signal about fluctuations of G rather than modification of the radial dependence of gravitational potential.

Why the fluctuations in the value of G would be so large in sub-millimeter length scales?

(a) Cosmological constant $\Lambda \simeq 1.1 \times 10^{-52}$ m^{-2} has dimension of $1/L^2$, L length scale. The density of dark energy $\rho_{vac} = \Lambda/8\pi G$ has dimensions of $\hbar/L^4$. One can assign to Λ very long p-adic length scale $L(k_1) = 2^{k_1/2}R$ ($p_1 \simeq 2^{k_1}$), and to $\rho_{vac}/\hbar$ rather short p-adic length scale $L(k_2) = 2^{k_2/2}R$. One has

$$\frac{\rho_{vac}}{\hbar} = \frac{x_2}{L(k_2)^4} = \frac{x_1}{8\pi} \frac{1}{l_P^2 L(k_1)^2} \quad ,$$

where x_1 and x_2 are numerical constants not far from unity. This would give

$$L(k_2) = (8\pi \frac{x_2}{x_1})^{1/4} (L(k_1) l_P)^{1/2} \quad .$$

$L(k_2)$ would be proportional the geometric mean of $L(k_1)$ and l_P. This implies

$$2^{2k_2} = \frac{x_2}{x_1} \times 8\pi \times (\frac{l_P}{R})^2 2^{k_1} \quad .$$

Very roughly, $k_1 \sim 2k_2 - 26$ would hold true for $x_2/x_1 \sim 1$. It turns out that k_2 corresponds to a p-adic length scale about 10^{-4} meters, which happens to be the size of large neuron suggesting that quantum gravitation is indeed highly relevant to biology but in manner different from that speculated by Penrose.

(b) p-Adic fractality suggests that cosmological constant is not actually constant or even time varying but depends on p-adic length scales so that the values are indeed extremely large as one approaches CP_2 scale and get very small as one approaches cosmological scales. This would solve the cosmological constant problem. The dependence would be $\Lambda(k) \propto 1/L(k)^2$, where $L(k)$ is the p-adic length scale characterizing the size of the space-time sheet. There would be a sequence of phase transition reducing Λ and these phase transition would involve quantum criticality and long length scale fluctuations possibly assignable to those of h_{eff} and thus of n_2 and G.

If one assumes that k_2 corresponds to preferred p-adic lengths scales assignable to elementary particles, nuclei, atomic physics and biology, one obtains a prediction that the corresponding p-adic length scales correspond to cosmologically important length scales via $k_1 \sim 2k_2$. One could study cosmology by studying gravitation in laboratory scales!

In these scales quantum phase transitions changing cosmological constant could make themselves visible via microscopic physics. Phase transitions involve long length scale fluctuations characteristic for criticality. In TGD these quantum fluctuations correspond to fluctuations of h_{eff} since Compton lengths scale like h_{eff}. The fluctuations of n_1 in $n = \hbar_{eff}/\hbar = n_1 n_2$ would induce fluctuations of G.

(c) Especially interesting are the p-adic length scales which are biologically important. The number theoretical miracle is that there are as many as 4 very closely located Gaussian Mersenne primes $M_{G,n} = (1+i)^m - 1$ in the range of cell membrane thickness and size of cell nucleus corresponding to $k = 151, 157, 163, 167$. The corresponding p-adic length scales $L(k) = 2^{(k-151)/2} L(151)$, $L(151) \simeq 10$ nm could be also gravitationally especially interesting. The hierarchical coiling of DNA might relate to the hierarchy of Gaussian Mersennes and phase transitions changing cosmological constant and the density of magnetic and volume energies assignable to the magnetic flux tubes playing key role in TGD inspired biology. These phase transitions would scale the thickness of the flux tubes determined by p-adic lengths scale.

It should be relatively easy to check whether the p-adic length scale hierarchy up to biological length scales has scaled variant in astrophysical and cosmological scales.

14.3.9 Conscious experiences about antigravity

Conscious experiences about anti-gravitational effects have been also reported and since I have nothing to lose as a happy pensioner and consciousness theorist [L52] I can take the liberty to talk also about these effects, even at personal level.

(a) There are stories about flying yoga masters. I am skeptic but I know from my own experience that out-of-body and levitation experiences - I mean indeed *experiences* - feel very real. I have proposed a model explaining them based on the notion of magnetic body as intentional agent carrying dark matter and using biological body as sensory receptor and motor instrument.

(b) I have indeed spent at younger age many moments in a kind of between away-and-sleep state in the roof of bedroom trying to prove myself that I really am there and then suddenly returned back to normal in wake-up state. Even the matresse behaved how it is expected to behave as some-one falls on it. Maybe part of my magnetic body was out-of-biological body after having experienced $h_{eff}/h = n$ increasing phase transition! Sometimes I have experienced wakeup quite concretely as a kind of contraction in which I have returned to my body: reduction of $h_{eff}/h = n$ for some part of magnetic body would explain this.

(c) I have had also altered states of consciousness between wake-up and sleep in which I felt my body like oscillating and being attracted by refrigerator, whose sound had started to amplify. I experienced the refrigerator as a living being and I was afraid that it intended engulf my consciousness! I had to decide whether I let it go but did not have courage to do it and I returned to the normal state.

(d) In dreams I have been also routinely flying and with somewhat childish narcissism pretended to the other people in dream that this is perfectly normal for me, it just occurred me that it would be fun to fly but honestly: I did not realize that it might make you scared! What was remarkable that I never got above about 10 meters: could this correspond to jumping in air in a reduced gravitational field? As a matter of fact, in dream I was typically going down in stairs and then decided to fly. I often landed at the end of stairs. This would fit with reduced gravity implying weaker downwards gravitational acceleration.

14.4 Three alternative generalizations of Nottale's hypothesis in TGD framework

Gravitational Planck constant $h_{gr} = GMm/v_0$ was originally introduced by [E13] and its form realizes Equivalence Principle (EP) in its Newtonian form (gravitational acceleration does not depend on mass m). The generalization of the idea was formulated in the TGD framework in [K100, K83]. $h_{eff} = nh_0 = h_{gr}$ would characterize the U-shaped flux tube tentacles emanating from M and mediating gravitational interaction.

One implication is that the parameter $v_0/c = \beta_0 < 1$ appears as a natural expansion parameter of the gravitational scattering amplitudes in the perturbative expansion replacing GMm. There is no dependence of GMm. Note that $\hbar_{gr} \geq h$ requires $GMm \geq v_0$.

$v_0 \simeq 2^{-11}$ suggested by the Nottale's Bohr orbit model for the 4 inner planets and is consistent with the model for the fountain effect of superfluidity [K39]. Indeed, the gravitational Compton length of the superfluid particle is $GM/v_0 \simeq 10$ m, which makes sense.

However, the model has a problem. For $M = M_E$, the cyclotron energies $\hbar_{gr}eB_{end}/m$ of dark ions in the endogenous magnetic field $B_{end} = 2/5B_E = .2$ Gauss explaining the findings of Blackman [J6] in terms of the $h_{eff} = nh_0 = h_{gr}$ hypothesis would be given by $E_c = GM_E/v_0 \times ZeB_{end}$ and would not depend on the mass m of the charged particle. For $\beta_0 \simeq 2^{-11}$ E_c would be in keV range and 3-4 orders of magnitude above visible range. Biophoton energies are however in visible and UV range.

14.4.1 Three ways to solve the problem of the too large cyclotron energy scale

One can imagine three ways to solve the too large cyclotron energy scale.

(a) The dark mass M_D is by 3-4 orders of magnitude smaller than the mass M_E of Earth. Here one should be able to understand why dark particles couple only to a part of M_E.

(b) Gravitational constant G_D for dark mass is by 3-4 orders of magnitude smaller than G. This would mean a violation of Equivalence Principle (EP). In the TGD framework, G indeed follows as a prediction and might vary [K13]. This could also provide an alternative explanation for the fountain effect.

(c) The velocity parameter $\beta_0 = v_0/c \leq 1$ has the value $\beta_0 = 1$ or is near to but below this value. For instance $\beta_0 = 1/2$ is enough. This option is favored by the Nottale's Bohr orbit model of the planetary system. The outer planets β_0 indeed varies: one has $\beta_0(outer) = \beta_0(inner)/5$. One has also $M = M_D$ and $G = G_D$.

Can dark mass M_D be smaller than the total mass M?

The model [K89] for the effects of ELF radiation on vertebrate brain [J6] led to a generalization of Nottale's hypothesis by replacing the total mass M in the case of Earth by $M_D \simeq 10^{-4} M_E$ suggesting that in this case the dark particles involved couple only to a part of mass identifiable as dark mass M_D.

A possible interpretation is that at long distance from mass M the flux tubes fused to larger flux tubes and the gravitational mass M_D interacting with the test particle increases to M at large distances. This might be in conflict with known facts.

The dark mass M_D appearing in the gravitational Planck constant $h_{eff} = h_{gr} = GM_D m/v_0$ must at short distances depend approximately linearly on the distance between the masses M_D and m . In the average sense. M_D would depend linearly on distance r. This is required by the condition that the Bohr radii correspond to the classical radii in the average sense. The actual dependence of M_D on r is expected to be a staircase like function.

At the quantum level, this effectively eliminates the average gravitational force in scales below the critical radius r_{cr} above which $M_D = M$ is true. Indeed, due to the average $M_D \propto r$ dependence, gravitational potential would be constant on the average. Could one regard this effective elimination of the gravitational force as a kind of Quantum EP or as an analog of asymptotic freedom?

Or could the value of G be reduced to $G_D < G$?

The reduction of $\hbar_{gr}$ could be also due to the reduction G to G_D. This is because only the parameter GM appears in the basic formulas.

(a) In the TGD framework Planck length as a fundamental length is replaced by CP_2 length R and Planck length or rather, Newton's constant G follows as a prediction. One can write

$$G = \frac{\hbar_{gr}\beta_0}{Mm} \quad .$$

(b) In the number theoretic vision about h_{eff} one can identify $\hbar_{gr}$ as the dimension of the Galois group of an extension of rationals [L102]. Since one has in the general case extension of extension of ...of rationals, one has a factorization $\hbar_{gr}/h_0 = \prod_i n_i$ where n_i are dimensions of extensions of extensions in the sequence.

This suggests that Mm corresponds to an integer in suitable units, say CP_2 length $R \simeq 10^{3.5} m_{Pl}$ and β_0 could also correspond to inverse of integer. $\hbar_{gr}$ would correspond to an integer and the reduction of G to G_D would correspond to a dropping of integer factor from this integer.

(c) $\hbar_{gr}$ would factorize to integers assignable to M and m and the integer assignable to G would be reduced in $G \rightarrow G_D$. If this integer factorized into a product assignable to M and m characterizing their gravitational couplings, one could understand why the reduction of G occurs only for superfluidity and dark phases in living matter. No additional assumptions about flux tube distribution would be needed.

Does variable β option make sense?

The third option would assume $\beta_0 = 1$ or near to but of order $\beta_0 = 2^{-11}$ for the dark ions in living matter. This conforms with the idea that dark dark matter interacts with all matter and satisfies EP.

The value of β_0 could be seen as the property of the dark matter particle and depend on the particle or on the distance from the central object as in the case of the solar system.

Gravitational Compton length l_{gr} Bohr orbit radius a_{gr} are given by $l_{gr} = GM/v_0 = r_S/2v_0$ and $a_{gr} = 2\pi r_S/v_0^2$. The reduction of β_0 scales up the quantum scale considered. Could this give some idea about how the value of β_0 relates to the size scale of the system considered? For the dark ions at magnetic flux tubes the l_{gr} would be about $r_S/2 \simeq .45$ cm, which is a biological scale. Could it correspond to the size scale of some structure of the vertebrate brain, say pineal gland with radius .37 cm?

In the sequel these options will be considered. I try to not take any of these options as a favorite but I must admit that the last option looks the most plausible one - at least now.

14.4.2 Could $M_D < M$ make sense?

For the generalization of the Nottale hypothesis discussed in the introduction, the gravitational Planck constant $\hbar_{gr} = GM_Dm/v_0$ introduced by Nottale [E13] is proportional to dark mass M_D which is in general would be smaller than the entire mass M.

Remark: As noticed in the introduction, it is GM that appears in h_{gr}, so that an alternative option is that G is reduced G_D. It would naturally characterize mass m rather than flux tube. Violation of Equivalence Principle would be in question.

Dark cyclotron energies $E_c = \hbar_{gr}eB_{end}/m = GM_DeB/v_0$ do not depend on the mass of the particle. The condition that the cyclotron frequencies in EEG range correspond to biophoton energy scale in visible and UV range for $B_{end} = .2$ Gauss, gives the estimate $M_D \simeq 2 \times 10^{-4} M_E < M_E$. One proposal is that M_D corresponds to the mass of the inner-inner core of Earth: see the appendix of [L123].

This raises the question about how the gravitational flux tubes emanating from mass M and connecting it to small masses m - say elementary particles, atoms of ions - are distributed. At short distances, the entire mass would not be connected to a given mass m by this kind of flux tubes. Does the amount M_D of the mass connected to mass m depend on the distance between m and M? How the allowed values of m are distributed and do they depend on distance? For instance, the condition $GM_Dm/v_0 > \hbar$ must be satisfied.

Remark: One can argue that radial magnetic flux tubes are not realistic. One can also consider the possibility that U-shaped flux tubes acting as kind of tentacles in TGD inspired quantum biology, are in question so that magnetic flux would return back. The fusion of flux tubes to larger flux tubes at longer distances makes sense also now.

Some guide lines

There are several hints, which suggest answers to some of these questions.

(a) In the TGD variant [K100] of the Bohr model for the planetary orbits [E13] around Sun, the dark mass M_D for Sun equals to solar mass: $M_D = M_{Sun}$. This suggests that at large enough distances M_D approaches the total mass M of the object. One can imagine that the flux tubes from M fuse to larger flux tubes so that m experiences $h_{gr} \propto M_{Sun}$ at large distances.

(b) In the Bohr orbit model of the planetary system in the gravitational potential of mass M_D, the gravitational binding energy of mass m at the lowest Bohr orbit with $n = 1$ is proportional to $\alpha_{gr}^2 m/2 = mv_0^2/8\pi^2$ ($\alpha_{gr} = v_0/4\pi$) and does not depend on M_D at all. This is true also for higher orbits with $n > 1$.

The consistency with the classical formula for the potential energy $V_{gr}(r) = GM_Dm/r$ suggests that M_D is in average sense proportional to the distance between M and m at small distances.

The radius r_B of the gravitational Bohr orbit is $r_B = \hbar_{gr}/\alpha_{gr}m = 4\pi GM_D/v_0^2$ and does not depend on m at all (note that $2GM_D$ is the Schwartchild radius associated with M_D). The larger the value of M_D, the larger the distance of m to M. This supports $M_D \propto r$ proportionality at small distances in average sense. There is some distance at which the value of M_D reaches M and does not grow anymore.

These arguments suggest that $M_D \propto r$ holds true in a reasonable approximation and that the gravitational flux tubes from smaller parts of M fuse to form larger flux tubes corresponding to the sum of the masses. A particle at a small distance would experience only part of the gravitational force created by M.

M_D/r would be constant on the average sense below the critical radius R_{cr} at which M_D becomes M and the values of M_D would form a linear staircase. At a given step of the staircase, the value of M_D would be constant and M_D/r would decrease. The radial gravitational force averaged over the staircase would vanish. In the average sense, one would have a free particle in a box.

Taking seriously the identification of M_D at the surface of Earth as the mass of the inner-inner core of the Earth, leads to ask whether the gravitational staircase could correlate with the layered structure of the Earth's interior.

Gravitational force is effectively eliminated below R_{cr}. Could this be interpreted in terms of Quantum Equivalence Principle? Asymptotic freedom is another analogy that pops in mind.

Magnetospheric sensory representations as a test of the proposal

This proposal can be tested in the TGD based model for sensory representations realized at magnetosphere [K65, K63].

(a) The proposal is that the magnetosphere of Earth defines sensory representations for the life forms at the surface of Earth. The communication and control would rely on dark photons with energies $E = h_{gr}f$ above thermal energy at physiological temperatures. For energies in visible and UV range dark photons can induce molecular transitions crucial for biochemistry by transforming to ordinary photons identifiable as biophotons [K17, K31].

(b) The energetic condition should be true near the surface of Earth, inside the rotating inner magnetosphere, and also in the outer magnetosphere extending to the distance of order $200R_E$. In plasma sheet, the order of magnitude for B is $B \sim 10-20$ nTesla. One has $B/B_{end} \sim 5 \times 10^{-4}$ for $B = 10$ nTesla.

(c) The cyclotron energies are given by $E_c = GM_DeB/v_0$ and do not depend on m. At the surface of Earth one has $M_D \simeq 2 \times 10^{-4} M_E$. At large enough distances one has $M_D = M_E$. In the outer magnetosphere this is expected to be true.

This would give $E_c(outer) = (M/M_D) \times (B/B_{end})E_c(Earth) \simeq 2.5E_c(Earth)$. The cyclotron energies would be of the same order of magnitude as required.

(d) Note that the values of v_0 are assumed to the same in inner and outer magnetosphere. In the Nottale's Bohr orbit model for the planetary orbits, outer planets and the 4 inner planets have different value of v_0: $v_0(outer) = v_0(inner)/5$. This would scale down the gravitational binding energy for outer planets by factor 1/25, which is reasonable. Scaling of v_0 in the case of Earth would increase cyclotron energy scale.

Critical summary

It must be admitted that I have not been able to develop the generalization of Nottale's hypothesis in a completely satisfactory form and it is best still to summarize the essentials. There is an excruciating uncertainty about the details related to the hypothesis.

(a) The hypothesis involves two parameters: $M_D \leq M$ and $\beta_0 = v_0/c$. The integer n labelling the Bohr orbit is an additional parameter. The critical question is whether M_D can really differ from M.

(b) Bohr orbit conditions expressing Newton's equation for circular orbit and angular momentum quantization in units of $\hbar_{gr}$ gives for the orbital radius T and velocity v the expressions in terms of the basic parameters.

$$\begin{aligned} R(n) &= n^2\frac{GM_D}{\beta_0^2} = \frac{GM_D}{v^2} \quad , \\ v &= \frac{\beta_0}{n} \quad , \\ E &= \frac{mv_0^2}{8\pi^2n^2} \quad . \end{aligned} \tag{14.4.1}$$

What is remarkable and perhaps strange looking is that velocity and binding energy are independent of the value of M_D. If one knows the orbital parameters, such as radius and period T one can

One can use various inputs in an attempt to fix the parameters of the model.

(a) In the case of the Sun, the radii and the velocities of the orbits of planets provide the information which allows to determine these parameters. $\beta_0(outer) = \beta_0(inner)/5$ relates the inner and outer planets. The value of n and $\beta_0(inner) \simeq 2^{-11}$ are determined by the planetary velocities. $M_D = M$ is implied by the known orbital radii.

(b) In the case of Earth there is no analog of planetary data available. The situation should look classical so that the values of n involved are large unlike in the case of Sun.

If the orbit of a stationary satellite is regarded as a Bohr orbit, one can get an estimate for n. In this case $v = v_0/n$ can be deduced from the period T and radius $R(n)$ of the orbit. For the stationary orbit, one has $R/R_E \simeq 6.62$. Newton's equation gives $GM_D/R = \beta^2$ so that $M_D = M$ must be true. If M_D depends on distance, $M_D \simeq M_E$ must hold true at distance about $6R_E$.

For $M_D = M$ and $\beta_0 = 2^{-11}$, $\beta = \beta_0/n$ gives $n \simeq 50$. Bohr orbit with Earth radius would have $n \simeq 19$. The reduction of M_D to $2\times10^{-4}M_E$ while keeping the radius of the Bohr orbit same, would require $n = 19 \rightarrow 1343$.

The above considerations are consistent with $M_D = M$. The hypothesis $M_D \simeq 2\times10^{-4}M_D$ deserves a critical discussion.

(a) The condition that the cyclotron frequencies in the endogenous magnetic field $B_{end} =$ .2 Gauss postulated to explain the findings of Blackman and others correspond to $h_{eff} = h_{gr}$ for which the frequencies at EEG frequency range correspond to the energies in the energy range of biophotons. This gives $M_D \sim 2\times10^{-4}M_E$ and the proposed identification is as the mass of the inner-inner core of Earth. Its radius is roughly 5 per cent of the radius of Earth. The model for the fountain effect of super-fluidity is consistent with this estimate of M_D.

(b) If M_D really varies, the small masses m cannot couple to the entire mass of (say) Earth: this could be perhaps understood in the flux tube picture in the proposed way.

14.4.3 What about the reduction of G to G_D?

As noticed in the introduction, it is actually the parameter GM_D that appears in Bohr conditions. Could it be G is replaced with G_D and one has $M = M_D$? In TGD the value of G indeed comes out as a prediction. CP_2 length R defines the counterpart of Planck length l_P and Newton's constant G is predicted to be $G\hbar = R^2/n_1$, where $n_1 =\sim 10^7$.

One can also write $G = \frac{\hbar_{gr}\beta_0}{Mm}$. Could the value of n_1 increase so that the value of G is reduced to G_D?

(a) The condition is that also the new value divides $\hbar_{gr}$ or more precisely, the integer assignable to G in the decomposition of $\hbar_{gr}$ to a product of integers.

(b) n_1 has a number theoretic interpretation [K13] as a factor of the order of the Galois group assignable to $\hbar_{gr} = n_{gr}h_0$. The variation of n_1 is in principle possible and there is evidence for small variations of G perhaps assignable to that of n_1.

(c) The increase of n_1 by a factor about $10^4/2$ is in principle possible: one would have $G_D = 2 \times 10^{-4}G$. The new value of n_1 should also divide n_{gr}. This kind of reduction of G for the superfluid phase could also explain the fountain effect as a dramatic weakening of the Earth's gravitation at the gravitational flux tubes connecting Earth to superfluid.

(d) Why would not G be reduced for the ordinary matter? It seems that the superfluid-/dark particle property must change the coupling to gravity? The factorization of $\hbar_{gr} = G_D Mm/v_0$ would naturally correspond to the factorization of n_{gr} to a product of factors characterizing masses M, m and the flux tube?

If $G\hbar$ - when expressed using CP_2 length as unit - factorizes to product of integers assignable to M and m, then the integer associated with m would be reduced so that the reduction of G would characterize the dark particle with mass m.

Note that also Podkletnov effect [H7, H3] discussed from the TGD point of view in [L11] suggests a few per cent reduction of G.

(e) A geometric interpretation suggests itself [L102] . The basic factorization would correspond to a decomposition to $n_{gr} = n_1 n_2$. n_1 would correspond to the number of sheets of space-time surface as a covering of M^4 and n_2 as covering of CP_2: the interpretation as a quantum coherent flux tube bundle of n_2 tubes is suggestive. The values of n_2 would be larg and correspond to the factor Mm or Mm/v_0. n_1 would be relatively small and could correspond to G or its factorization to a produce of integers assignable to M and m. This makes sense since the coupling of m to gravitational flux tubes is assumed to be by touching.

To sum up, it seems that one should improve the physical understanding of the Galois group of extension, which in general is extension of extension of ... so that its dimension n is the product of dimensions of extensions involved. Do these dimensions correspond to effective Planck constants assignable to various interactions as suggested in [K13]?

14.4.4 The option based on variable value of β_0

The motivations for the model with a variable value of $\beta_0 = v_0/c$ have been already explained. In the sequel I will develop a model for the communications between dark matter phases with $h_{eff} = nh_0$ satisfying $h_{eff} = h_{gr}$. One can consider two options for the communications depending on whether the value of h_{eff} changes as (for instance) in the communications between dark and ordinary matter or whether it is preserved.

(a) If the value of h_{eff} can change, energy conservation for $E = h_{eff}f$ allows energy resonance whereas the frequency changes. The simplest option is that the dark photon transforms to say ordinary photon with the same amplitude

(b) If the value h_{eff} is preserved, one has both energy and frequency resonance. In the case of cyclotron radiation, the simultaneous occurrence of energy and frequency resonances poses strong conditions on the values of the magnetic fields, the values of charged particle masses, and the parameter β_0 at the ends of the communication line.

Conditions for frequency - and energy resonance

The condition that the frequency is the same at both ends implies for cyclotron frequencies $f_c = ZeB/2\pi m$ the condition

$$\frac{Z_1 B_1}{m_1} = \frac{Z_2 B_2}{m_2} \quad . \tag{14.4.2}$$

For $h_{eff} = h_{gr}$ the condition that the cyclotron energy $E_c = GMZeB/v_0$ at both ends is same implies

$$\frac{Z_1 B_1}{v_{0,1}} = \frac{Z_2 B_2}{v_{0,2}} \quad . \tag{14.4.3}$$

Together these conditions give

$$\frac{m_1}{m_2} = \frac{Z_1 B_1}{Z_2 B_2} = \frac{\beta_{0,1}}{\beta_{0,2}} \quad . \tag{14.4.4}$$

4. For instance, if the two particles are proton and electron, one obtains

$$\frac{\beta_{0,1}}{\beta_{0,2}} \simeq \frac{m_e}{m_p} \quad .$$

This ratio is is consistent with the values $\beta_{0,2} = 1$ and $\beta_{0,1} = 2^{-11}$ in the accuracy considered. Is this a mere accident?

Resonance conditions for communications from the Earth's surface to the magnetosphere?

The simplest option is that the interacting particles have the same values of mass and β_0 and magnetic fields are identical. This is achieved if the flux tubes have constant thickness. Whether this is the case is not clear.

However, the idea that the flux tube picture about magnetic fields is locally consistent with the Maxwellian view inspires the question whether also the magnetic field strength at the flux tubes of B_{end} behaves like $B_{end} \propto 1/r^3$ as B_E in dipole approximation behaves.

B_{end} is by flux conservation proportional to $1/S$, where S is the area of the flux tube. sOne would have $S \propto r^3$. The constancy of B_{end}/m would suggest $m \propto 1/r^3$. If the charged particles are ions characterized by the A/Z ratio.

This would suggest that the regions of tubes/sheets in frequency resonance are at distances

$$\frac{r}{r_0} = (\frac{Z}{Z_0})^{-1/3}(\frac{A_0}{A})^{-1/3}$$

for ions Z_0, A_0 at the surface of the Earth. The heaviest ions would be nearest to the surface of Earth. Energy resonance condition

$$\frac{B_{end}(r)}{\beta_{0,2}} = \frac{B_{end}(R_E)}{v_{0,1}}$$

would give the additional condition

$$\frac{\beta_{0,2}}{\beta_{0,1}} = (\frac{R_E}{r})^3 = \frac{Z}{Z_0} \times \frac{A_0}{A} \quad .$$

β_0 would be quantized and would decrease with the distance.

Magnetosphere as sensory canvas

TGD leads to a model of the "personal" magnetic body (MB) as being associated with the Earth's MS. Different regions of the body and brain would be mapped to regions of the MS, which would give rise to sensory representations at the personal MB [K65, K63]. Personal MB, which would have size scale of at least of the Earth's MS, would also control biological body.

1. An interesting finding relates to the values of the magnetic field $B_{end} \simeq 2B_E/5$ (perhaps identifiable as the monopole flux part of B_E) and the value of $B \sim 10$ nT in the magnetotail at the night-side of the Earth.

   One has $B/B_{end} \sim 2^{-11}$ so that for dark proton-dark electron communications between the Earth's surface and this region of outer MS the resonance conditions would be satisfied for $\beta_0 = x$ and $\beta_0 = 2^{-11}x$, where $x < 1$ not far from unity.

2. Could the parameter β_0 characterize particles and act as a tunable control parameter allowing to achieve energy resonance? Also the values of B are tunable by changing the thickness of the flux tubes as a kind of motor action of MB.

This idea can be applied to the h_{eff} preserving communications between biological body and the MS of the Earth.

1. The quantum coherence condition suggests that the communications are optimal when the wavelength of dark photon is larger than the distance considered: $\lambda > r$ or equivalently the frequency satisfies $f \leq c/r$ (one has $c = 1$ in the units used). If the structure of the MS has distances from the Earth's surface below r_{max} then the frequencies $f \leq 1/r_{max}$ are optimal.

2. Given the distance r_{max} and assuming $B = B_{end}$ at the surface of Earth, one obtains for the cyclotron frequencies the condition

   $$f_c = \frac{ZeB_{end}}{2\pi m} \leq \frac{1}{r_{max}} \quad .$$

   For instance, EEG frequency 10 Hz corresponds to 3×10^7 m. The cyclotron frequency of DNA sequence does not depend on its length and composition since DNA has constant charge per unit length. One has $f_c \simeq 1$ Hz so that the corresponding distance is $r = 3 \times 10^8$ m, that is $r = 46.9R_E$.

   Remark: B_{end} probably has a spectrum. Music experiences relies on frequency scale and if the audible frequencies correspond to cyclotron frequencies then eB_{end}/m is variable. This suggests that the spectrum of B_{end} covers at least the range of the audible frequencies spanning roughly 10 octaves [K92].

14.5 Can TGD predict the value of Newton's constant?: the view two years later

Newton's constant G cannot be a fundamental constant in TGD framework . G has dimensions of length squared divided by Planck constant and CP_2 length R is the only fundamental length in TGD Universe. The analog of Newton's constant $G = R^2/\hbar$ is too larger by factor of order $10^7 - 10^8$: the previous estimate gives for this factor the value $2^{24} = 16,777,216 = 1.6777216 \times 10^7$.

The first guess was that one must modify the formula by replacing $\hbar$ with h_{eff} with $h_{eff} = nh_0$, $h = 6h_0$: $G = R^2/\hbar_{eff}$ (see [L26, L60, L108]).

n has however has arbitrarily values and the proposal cannot be correct as such if one accepts the notion of gravitational Planck constant $\hbar_{gr} = GMm/v_0 = h_{eff} = nh_0$: here M and m are masses of systems having gravitational interaction and $v_0 < c$ is velocity parameter having value $v_0/c \simeq 2^{-11}$ for inner planets [E13] [K100, K83, K84]. h_{gr} is assigned with the flux tubes mediating gravitational interaction.

One can assign also to other interactions corresponding effective Planck constants - for instance, $h_{em} = Z_1 Z_2 e^2/\beta_0$, $\beta = v_0/c < 1$ to electromagnetic interactions. The general idea is that when the value of coupling strength $Q_1 Q_2 g^2/\hbar$ for a two particle system becomes so large that perturbation theory fails, Planck constant is replaced with h_{eff} and perturbation theory works again. Topologically this means a phase transition replacing space-time sheets with their n-fold coverings.

14.5.1 Development of ideas

More general formula for G

The more general proposal is that h_{eff} in the formula for G must be replaced with $h_{gr,1} = n_{gr}h_0$, where n_{gr} is closely related to the $n = h_{eff}/h_0$ but not equal to it. The estimate $G\hbar/R^2 \simeq 1.6 \times 107$ and $\hbar = 6\hbar_0$ gives the estimate $n_{gr} = 6 \times 2^{24} \simeq 1.00663296 \times 10^8$.

To make the continuation easier, it is good to express the idea in more detail.

1. CP_2 "radius" R identified in terms of geodesic length $l = 2\pi R$ is the fundamental geometrically realized unit of length measurement, and takes the role of Planck length $l_P^2 = G\hbar$ having only dimensional analytic justification. G is now the prediction and the first guess is $G = R^2/\hbar_{gr,1}$, where $\hbar_{gr,1} = n_{gr} \times \hbar_0$ is effective Planck constant with n_{gr} identified as dimension of "gravitational" extension of rationals.

   $n = h_eff/h_0$ is the number of sheets of covering of space-time surface transformed to each other by Galois group. Since l_P/R is in the range $10^{-7} - 10^{-8}$, one must have $\hbar_{gr,1}/\hbar$ in the range $10^7 - 10^8$.

2. In principle $n_{gr}/6$ could have values $10^{-7} - 10^{-8}$ times smaller than the value associated with G. If so, G could be up to factor $10^7 - 10^8$ times larger than the standard value $G = G_N$. The downwards fluctuations of $\hbar_{eff}$ strengthen the gravitational attraction. One cannot exclude even large fluctuations of G.

The first attempt to identify n_{gr} fails

The motivation for this article came from an attempt to understand the value of gravitational constant G as a prediction of TGD - I have already earlier developed a model in which gravitational constant is predicted in terms of CP_2 radius R and a number related to effective Planck constant $h_{eff} = nh_0$.

1. The first proposal was that one can write $h_{eff}/h_0 = n$ as $n = n_1 \times n_2$, where n_1 is the number of sheets of space-time surface as a covering M^4 (space-time points with same CP_2 coordinates) and n_2 is the number of sheets as covering CP_2 (space-time points with same M^4 coordinates). There would be n_1 different space-time sheets for given M^4 projection - this corresponds to the idea about many-sheeted space-time. There would be n_2 different regions of space-time for given region of CP_2 projection. One can imagine n_2 parallel flux tubes in M^4 forming a coherent structure. This intuitive picture could but need not survive in more precise formulation.

2. The improved formula would be $G = R^2/n_{gr}\hbar_0$, where one as either *a)*: $n_{gr} = n_1$ or *b)*: $n_{gr} = n_2$. Which option - if either of them - is the correct one? Note that the option $n_{gr} = n$ is not possible since b can have huge values and G would approach to zero for dark matter in long length scales: with the recent understanding of physics this does not look plausible.

The limit $n_{gr} < n_{nmax} \sim 10^8$ means a bound on the number of space-time sheets over M^4 or CP_2.

1. For option *a)* with $n_{gr} = n_1 < n_{max} \sim 10^8$ one can imagine that the Galois group corresponds to a discrete finite sub-group of $SU(3)$, analogous to the isometry groups of Platonic solids. In the case of $SO(3)$ the order of this group is bounded to the order 60 of isosahedral group unless the group is Abelian. The largest discrete sub-group of $SU(3)$ analogous to icosahedral group has order 1080 and is too small by several orders of magnitude.

   Remark: The number parallel flux tubes could be arbitrarily large for tis option - a possible interpretation would be that gravitational quantum coherence is true in very long length scales.

2. For option *b)* with $n_{gr} = n_2 < n_{max}$ would state that the number of parallel flux tubes forming a coherent structure is bounded. The number of space-time sheets over M^4 could be arbitrarily large. The only natural symmetry group for M^4 is discrete sub-group of $SO(3)$. For the icosahedral group the order is 60 and quite too small.

Both options fail.

A modified formula for G

The failure forces to consider a more general formula for G. The outcome is the following argument.

1. TGD predicts a hierarchy of effective Planck constants $h_{eff}/h_0 = n$, where n is the order of Galois group of Galois extension defining extension of rationals [L49, L50] [?, K84]. Dimension n of extension factorizes to a product $n = n_1 n_2 ...$ for extension E_1 of extension E_2 of .... rationals. $M^8 - H$ correspondence allows to associate the Galois group with an irreducible polynomial characterizing space-time surface as an algebraic surface in M^8. The gradual increase of extension by forming a functional composite of a new polynomial with the already existing one ($P \to P_{new} \circ P$) would be analogous to the evolution of genome: earlier extensions would be analogous to conserved genes.

2. The proposal modifying the earlier proposal is $G = R^2/n_{gr}\hbar_0$, where n_{gr} is the order of Galois group G_{gr} "at the bottom" of the hierarchy of extensions, and one has $\hbar = 6h_0$. One would have $n = n_1 n_2 ... n_{gr}$. G_{gr} "at the bottom" is proposed to represent number theoretically geometric information about the embedding space by providing a discretization for the product of maximal finite discrete sub-group of isometries and tangent space rotations of embedding space.

3. By $M^8 - H$ duality these sub-groups should be identical for H and M^8. The prediction is that maximal G_{gr} is product of icosahedral group I with 3 copies of coverings $\overline{I}$. Rather remarkably, the prediction for G is correct if one assumes that the value of R is what p-adic mass calculation for electron mass gives.

Since the hierarchy of Planck constants relates to number theoretical physics proposed to describe the correlates of cognition, the connection with cognition strongly suggests itself. Icosahedral and tetrahedral geometries occur also in the TGD based model of genetic code in terms of bio-harmony [L5], which suggests that genetic code represents geometric information about embedding space symmetries. These connections are discussed in detail.

14.5.2 A formula for G in terms of order of gravitational Galois group and implications

In the sequel the formula $G = R^2/n_{gr}\hbar_0$ will be deduced from number theoretical vision based on adelic physics [L49, L50] and $M^8 - H$ duality [L41, L42, L43, L85]. The prediction allows variation of G - G is indeed known to vary more than expected. These "small" variations and also possible large variations are discussed. The successful prediction forces to consider seriously the connections between quantum gravitation, cognition, and quantum biology, in particular genetic code.

An improved attempt to identify n_{gr}

The original proposal for a formula of $G = R^2/n_{gr}\hbar_0$ failed and one must try something more general.

1. The Galois group of Galois extension has a decomposition in terms of a hierarchy of normal sub-groups. G can be represented as product of maximal normal sub-group H and group G/H. H in turn has similar decomposition and the process can be continued to get a hierarchical decomposition. This leads to a concrete model for "small" state function reduction (SSFR) as a cascade of cognitive measurements. Some special normal sub-group in the hierarchy relevant for gravitation is a good candidate for "gravitational" Galois group G_{gr}, whose order is n_{gr}.

   An attractive assumption is that the Galois group assignable to gravitational interactions is fundamental in the sense that it corresponds to the lowest step of the Galois ladder. The vision about evolution inspired by M^8-H duality is as an increasing hierarchy of polynomials P with rational coefficients defining space-time surfaces as algebraic surfaces in complexified M^8: their real projections would define 4-D space-time surfaces mapped to $H = M^4 \times CP_2$ by $M^8 - H$ duality [L41, L42, L43].

   Polynomials P would be functional composites as a generalization of abstraction process as statements about statements and evolution would proceed as sequence of abstraction steps $P \to P_{new} \circ P$. This step preserve the roots of P if the polynomials involved vanish at origin: $P(0) = 0$. Besides Galois groups the roots associated with earlier steps would be evolutionary invariants analogous to conserved genes. If one has decomposition $P = P_1 \circ P_2 ... \circ P_{gr}$, one could understand why why n_{gr} is almost universal constant.

2. Gravitation relates to space-time geometry and a good guess is that G_{gr} provides a representation for a discrete finite sub-group of the isometry group of embedding space and perhaps also for the sub-group $SO(3) \subset SO(3,1)$ acting in $E^3 \subset M^4$ or its lift to $SU(2)$. Octonionic structure in M^8 indeed selects unique rest system and even the spatial origin of the linear coordinate system is fixed. This would reduce the attempt to identify n_{gr} to a study of finite discrete sub-groups of embedding space isometries and spin covering of its tribein rotation group.

   It must be made clear that G_{gr} would be associated with the space-time sheets mediating gravitational interactions: this would include gravitational flux tubes with $\hbar_{gr} = GMm/v_0$. For flux tubes mediating - say - electromagnetic interaction the counterpart of G_{gr} could be much smaller, it would however include the group $Z_2 \times Z_3$, which is center for $SU(2) \times SU(3)$ predicting $h = 6h_0$ suggested by some empirical findings [L26, L60, L108].

3. By $M^8 - H$ duality one must consider the isometry groups and 3-D tangent space-groups of both M^8 and H to see whether n_{gr} could find a natural identification. $M^8 - H$ duality requires that the gravitational sub-group is same for M^8 and H options.

   The group $SO(3) \times U(2)$ is shared by $SO(3) \times SU(3)$ for H and $SO(3) \times SO(4)$. Tangent space group of $E^3 \subset M^4$ is $SU(2)$ and tangent space group of CP_2 is $U(2)$ in the two cases and if only maximally non-Abelian groups are accepted $U(2)$ effectively reduces to $SU(2)$, which can however correspond to a non-trivial sub-group of $U(2)$. This would mean that the maximal finite discrete sub-group of isometries and vielbein groups is direct product of 4 groups, which are icosahedral groups I or their coverings $\overline{I}$.

   The orders of icosahedral group I without reflection *resp.* its covering $\overline{I}$ is 60 *resp.* 120. $SU(2)$ for the tangent space groups is natural hypothesis since one has also fermions. For $I \times \overline{I}^3$ one would have $n_{gr} = 8 \times 6^4 \times 10^4 \simeq 32^{11} \times 15^4 = 1.0368 \times 10^8$. This is to be compared with the rough estimate $n_{gr} = 6 \times 2^{24} \simeq 1.00663296 \times 10^8$. The proposal works amazingly well!

4. Also other Platonic groups assignable to Platonic solids (tetrahedron, cube and octahedron, icosahedron and dodecahedron) are in principle possible: actually all discrete and finite sub-groups of $SU(2)$ can be considered. The non-Platonic groups however act on plane polygons, and might be more naturally assignable other than gravitational interactions. They are also

associated with Mac-Kay correspondence [L95] assigning to these finite groups ADE Lie groups/Kac-Moody algebras. This hierarchy is also associated with inclusions of hyperfinite factors of type II_1 (HFFs) proposed in TGD framework to provide a representation for finite measurement resolution [K125, K48].

Could Newton's constant vary and what about formulas for other coupling strengths?

The proposed formula for G forces to consider the possibility of large variations of G due to the variation of n_{gr} as the order of gravitational Galois group G_{gr}. This group is fixed by $M^8 - H$ duality to be a product of finite discrete sub-groups of $SO(3)$ with 3 discrete sub-groups of $SU(2)$. By loosening the conditions one can however think also the possibility of other choices

1. The allowance of only Platonic solids would make possible to understand possible large increases of G but not its reduction. What is interesting is that the increase of G implies increase of gravitational Compton length $\Lambda_{gr} = G(M+m)/v_0$ unless G/v_0 is constant.

2. If one accepts also the non-Platonian finite sub-groups of $SU(2)$ with representations are realized as 2-D polygons, the range of values of G is much larger and both large and small variations of G from the preferred value become possible as variations of n_{gr}.

3. If one wants to explain the reported small but theoretically too large variations of G allowing only Platonic solids, one must allow superpositions of space-time surfaces with different values of n_{gr}. In general $\langle n_{gr} \rangle$ would be smaller than the maximal value and $\langle G \rangle$ would increase. Large variations decreasing G cannot be explained in terms of Platonic solids or their superpositions.

4. If one gives up $M^8 - H$ duality larger variations of G downwards become possible. For instance, $\overline{I}$ in the case of $SU(3)$ isometries could be replaced with $\Sigma(1080)$ with 1080 elements (`http://tinyurl.com/uq3nxko`). This would reduce G by factor $1080/120 = 9$. More generally, in $SU(3)$ there are following analogs of Platonic groups labelled as $\Sigma(n)$, $n \in \{60, 168, 36 \times 3, 72 \times 3, 21 \times 3, 72 \times 3 = 216, 216 \times 3 = 648, 360 \times 3 = 1080\}$. Also the semi-direct products $\Sigma(60) \times Z_3$ and $\Sigma(168) \times Z_3$ belong to the list.

The counterpart of ADE hierarchy for $SU(3)$ is obvious interest from the point of view of color interactions if one allows the breaking of $M8 - H$ duality. There is an article by Ludl in arXiv [B40] (`http://tinyurl.com/uq3nxko`) about the finite discrete sub-groups of $SU(3)$. Table 1 of the article provides a summary of the discrete sub-groups.

1. There are 3 series parameterized by several integers with no general formula for the order. The are however infinite series of groups which belong to these series and have unbounded order. These groups are semi-direct products, which makes their representability as Galois groups of Galois extensions possible.

   Could these groups be associated with the flux tubes mediating color interactions? Could colour coupling strength be expressible as $\alpha_s = g_s^2/4\pi\hbar_s$, where $\hbar_s = n_s\hbar_0$? Could the value of g_s^2 be equal to the square g_K^2 of Kähler coupling defining fundamental constant. Could similar expression hold true also for electroweak coupling strengths. Could the breaking of gauge and gravitational symmetries be coded by different values of n_s,$n_{SU(2)_{ew}}$, $n_{U(1)}$, and n_{gr}.

2. There are also following exceptional groups analogous to Platonic groups for $SU(3)$ and labelled as $\Sigma(n)$, $n \in \{60, 168, 36\times3, 72\times3, 21\times3, 72\times3 = 216, 216\times3 = 648, 360\times3 = 1080\}$. Also the semi-direct products $\Sigma(60) \times Z_3$ and $\Sigma(168) \times Z_3$ belong to the list. The largest order for this series is 1080. The smallest order is 60 and corresponds to icosahedral group.

The discrete sub-groups of $SO(4)$ are interesting in M^8 picture and could contain also semi-direct products as sub-groups for products of sub-groups of $SO(3)$ and $SU(2)$. These sub-groups are listed in the appendix of the article by de Medeiros and Figueroa-O'Farrill (`http://tinyurl.com/tyagn3c`).

What do experiments say?

What do experiments say? Various experiments have been already discussed.

1. Several experiments suggests small variations of G, which are however too large theoretically. There are experiments in millimeter scales and also Podkletnov's experiment [H7, H4] [L11].

2. Could the fountain effect of super-fluidity be understood as a large reduction the value of G. It seems that a more elegant explanation is in terms of macroscopic quantum coherence due to the large value of $h_{gr} = GMm/v_0$ for space-time sheets mediating gravitation in the case of super-fluid [**?**].

3. The findings reported by Martin Grusenick [K104] - *if true* - would suggests a huge increase of G by a factor of order 10^5 if the increase of spatial lengths in the direction of the Earth's magnetic field causes the effect. The variation is too large to have an explanation allowing only Platonic solids alone. The effect could be due to the contraction of the measurement apparatus under its own weight.

   Perhaps a more elegant explanation for Grusenick's claim would be in terms of warping of space-time surface possible even in absence of gravitational field predicted by TGD. Warping means that the space-time surface has metric isometric with Minkowski metric but when the M^4 coordinates of $M^4 \subset M^4 \times CP_2$ are used, there is a scaling of the metric in various directions since CP_2 projection of the embedding is not a point but geodesic circle. This would modify the propagation velocity in radial direction.

4. One can also ask whether the unexpected mass for the blackhole candidates observed by LIGO could be due to anomalously large value of G. In TGD framework the view about blackhole like entities is much more detailed than in GRT and one could understand them also without variation of G.

Since consciousness, cognition, and gravitation are closely related in TGD Universe, one cannot avoid association with the claims made by meditators about levitation. Could the experience about levitation mean a genuine levitation of dark matter at the level of magnetic body (MB), which corresponds to a higher level cognitive consciousness and naturally gravitational consciousness by huge values of $\hbar_{gr}$.

1. Could G be reduced producing anti-gravitational effect at MB? If one allows only $M^8 - H$ duality and Platonic solids G is smallest possible and cannot be reduced. Allowing also polygons would allow arbitrary small values of G. This option does not look however plausible since one can argue that the experience would reduce from 3-D for Platonic solids to 2-D for regular polygons.

2. Perhaps a more elegant explanation is that levitation experiences and out-of-body experiences [K108] (OBEs, which I have had also myself), are due to the delocalization of particles of "personal" MB due to the large value of h_{gr}. One could perhaps say that the active flux tubes of MB correspond to those mediating gravitational interaction and having $h_{gr} = GMm/v_0$. Ironically, gravitational consciousness would be experience of no having no weight.

A connection gravitation and genetic code?

A deep connection between gravitation and genetic code suggests itself.

1. TGD suggests at least two fundamental representations of genetic code besides the usual chemical representation. The first representation is terms of dark nuclei consisting of sequences dark proton triplets representing codons [L20, L54]. Both DNA, RNA, tRNA and amino-acids have analogs as dark proton sequences. Second representation is in terms of dark photon triplets defining what I call bio-harmony [L5]. Basic objection against emission of 3-dark photons simultaneously is that the process is extremely probable. If one has however Galois confinement in the sense that only Galois singlets appear as asymptotic states, the assumption that dark photons are Z_3 triplets allows only the emission of triplets [L108].

2. What is fascinating that both icosahedral and tetrahedral groups appear in the model for the genetic code in terms of bio-harmony [L5, L71, L76]. Could genes and associated molecules DNA, RNA, tRNA, and amino-acids code for information about the geometry of embedding space in some sense? DNA codons correspond to 20 triangular faces of icosahedron (3 Hamiltonian cycles are used obtain 202+20+20=60 codons) and 4 triangular faces of tetrahedron to get the remaining 4 codons. By icosahedral-dodecahedral duality gene as a sequence of these faces defines a path at dodecahedron - two subsequent codon of gene would not however map to nearest points at dodecahedron. What could this mean if anything?

3. Genes code for information and therefore could relate to cognition, and the proposed representations of genetic code would mean that genes emerge already at the fundamental level: chemical representation would be only mimicry of the dark nuclear code at higher, chemical level. The hierarchy of Planck constants relates also directly to information and h_{eff} can be seen as a kind of "IQ".

   The dependence of G on n_{gr} suggest that also gravitation relates to cognition. This would not be surprising since the long-ranged non-screened character of gravitation could make possible quantum coherence in astrophysical scales: the value $h_{eff}/h_0 = n = h_{gr}/h_0$ is indeed a direct measure of the evolutionary level.

   The connection with cognition could also explain why ancient mathematicians managed to discover the mathematical structures encountered two millenia later in theories trying to unify fundamental interactions.

Could Newton's constant relate to cognition?

After having discovered the above argument fixing n_{gr} from M^-H duality, I could have written conclusions of the paper. The emphasis however shifted to TGD based view about evolution and cognition and its connection with gravitation. h_{eff} indeed closely relates to an evolutionary hierarchy of cognition via the idea that gravitational/geometric part of Galois group is fundamental and "at the bottom" of the hierarchy of Galois extensions of rationals. Extensions of rationals would define cognitive representations representing discretizations of spaces of various dimensions as subsets of reals or complex numbers, and also allow to represent discrete sub-groups approximating continuous groups as Galois groups.

The most fundamental physics related groups to be approximated as Galois groups would relate to the isometries and vielbein rotations of embedding space. The maximally compact subgroup would be in question both cases. The important point would be that these groups would act on extension of rationals providing cognitive representation as subset of reals/complex numbers rather than in embedding space. This kind of representation would be analogous to a linguistic, linear representation of geometric object as opposed to concrete geometric representation in embedding space.

14.5.3 Could gravitation and geometric cognition relate?

It has been already demonstrated how one can predict the value G correctly as in TGD framework. The emphasis of this section is on geometric cognition and the possibility that the value of G directly reflects this connection.

Hierarchy of effective Planck constants and Galois extensions of rationals

In adelic vision [L49, L50, L41, L42, L42] about TGD $n = h_{eff}/h_0$ corresponds to the dimension of extension of rationals characterizing space-time surface. n is also the order of Galois group of extension for Galois extensions. Recall that Galois extension has the nice property that the order of Galois group equals to the dimension of the extension. Galois extension can be regarded as extension of extension of....rationals an there is a hierarchy of Galois group such that the included sub-groups are normal sub-groups. One can express n as a product $n = n_1 n_2..$ of the dimensions of these extensions.

This leads to the vision about the reduction of evolution to a hierarchy of Galois extensions such that evolution means increase of the extension and therefore number theoretical complexity and of h_{eff} meaning increase of quantum coherence scale.

If the extensions tend to emerge as further extensions preserving the earlier extensions - as is natural to think -, the extension "at the bottom" of the hierarchy of extensions is rather stable. Since the geometric cognitive consciousness can be argued to be fundamental, the dimension of Galois group corresponds to n_{gr} in $n = n_1 n_2 .... n_{gr} = m \times n_{gr}$. n_{gr} would be rather stable factor of n.

n_{gr} would be analogous to the conserved genes of primary life form from which evolution started. The change of genome at this level would induce dramatic changes making the survival of the new life form implausible. This alone would not predict unique value for n_{gr} but only that its value is dynamically rather stable. One must of course understand why this particular value of n_{gr} would be selected. What distinguishes this extension from a general extension? The groups in question allow infinite number of finite discrete sub-groups but $M^8 - H$ duality would select highly unique sub-groups as common to both. Only groups, which are products of 4 isometry groups of Platonic solids or there double coverings and maximal order for the group minimizing G would leave only the icosahedral group I and its coverings into consideration.

$M^8 - H$ duality and representation of space-time surfaces in M^8 as algebraic surfaces assignable to polynomials with rational coefficients

$M^8 - H$ duality [L85] provides a concrete realization of the number theoretic vision in terms of space-time surfaces, and also allows to realize the view about number theoretical evolution in terms of a hierarchy of polynomials obtained by functional composition of polynomials.

The articles [L41, L42, L43] contain a detailed description of $M^8 - H$ duality. The article [L104] described a possible connection with chaos theory and Mandelbrot/Julia fractals based on the possibility that time evolution by "small" state function reductions (SSFRs) correspond in good approximation iteration of polynomial. The article [L111] describes a model of SSFR as a cognitive measurement identified as a reduction cascade in the group algebra of Galois group having a decomposition in terms of normal sub-groups.

Basic vision

Consider first what TGD space-time is.

1. In TGD framework space-times can be regarded 4-surfaces in $H = M^4 \times CP_2$ or in complexification of octonionic M^8. Linear Minkowski coordinates or Robertson-Walker coordinates for light-cone (used in TGD based cosmology) provide highly unique coordinate choice and this problem disappears.

2. The solutions of field equations are preferred extremals satisfying extremely powerful additional conditions giving rise to a huge generalization of the ordinary 2-D conformal symmetry to 4-D context. In fact, twistor twist of TGD predicts that one has minimal surfaces, which are also extremals of 4-D Kähler action apart from 2-D singularities identifiable as string world sheets and partonic 2-surfaces having a number theoretical interpretation. The huge symmetries act as maximal isometry group of "world of classical worlds" (WCW) consisting of preferred extremals connecting pair of 3-surfaces, whose members are located at boundaries of causal diamond (CD). These symmetries strongly suggest that TGD represents completely integrable system and thus non-chaotic and diametrical opposite of a chaotic system. Therefore the chaos - if present - would be something different.

$M^8 - H$ duality suggests an analogous picture at the level of M^8. $M^8 - H$ duality in itse most restrictive form states that space-time surfaces are characterized by "roots" of rational polynomials extended to complexified octonionic ones by replacing the real coordinate by octonionic coordinate o [L41, L42, L43].

1. One can define the imaginary and real parts $IM(P)$ and $RE(P)$ of $P(o)$ in octonionic sense by using the decomposition of octonions $o = q_1 + I_4 q_2$ to two quaternions so that $IM(P)$ and $RE(P)$ are quaternion valued. For 4-D space-time surfaces one has either $IM(P) = 0$ or $RE(P) = 0$ in the generic case. The curve defined by the vanishing of imaginary or real part of complex function serves as the analog.

2. If the condition $P(0) = 0$ is satisfied, the boundary of δM^8_+ of M^8 light-cone is special. By the light-likeness of δM^8_+ points the polynomial $P(o)$ at δM^8_+ reduces to ordinary real polynomial $P(r)$ of the radial M^4 coordinate r identifiable as linear M^4 time coordinate t: $r = t$.

   Octonionic roots $P(o) = 0$ at M^8 light-cone reduce to roots $t = r_n$ of the real polynomial $P(r)$ and give rise to 6-D exceptional solutions with $IM(P) = RE(P) = 0$ vanish. The solutions are located to δM^8_+ and have topology of 6-sphere S^6 having 3-balls B^3 with $t = r_n$ as of M^4_+ projections. The "fiber" at point of B^3 with radial M^4 coordinate $r_M \leq r_n$ is 3-sphere $S^3 \subset E^4 \subset M^8 = M^4 \times E^4$ contracting to point at the δM^4_+.

   These 6-D objects are analogous to 5-branes in string theory and define "special moments in the life of self". At these surfaces the 4-D "roots" for $IM(P)$ or $RE(P)$ intersect and intersection is 2-D partonic surface having interpretation as a generalization of vertex for particles generalized to 3-D surfaces (instead of strings). In string theory string world sheets have boundaries at branes. Strings are replaced with space-time surfaces and branes with "special moments in the life of self".

   Quite generally, one can consider gluing 4-D "roots" for different polynomials P_1 and P_2 at surface $t = r_n$ when r_n is common root. For instance, P and its iterates $P^{\circ N}$ having r_n and the lower inverse iterates as common roots can be glued in this manner.

3. It is possible complexify M^8 and thus also r. Complexification is natural since the roots of P are in general complex. Also 4- space-time surface is complexified to 8-D surface and real space-time surface can be identified as its real projection.

To sum up, space-time surfaces would be coded a polynomial with rational or at most algebraic coefficients. Essentially the discrete data provided by the roots r_n of P would dictate the space-time surface so that one would have extremely powerful form of holography.

Should one allow also transcendental extensions?

One can consider generalizations of the simplest picture.

1. One can also consider a generalization of polynomials to general analytic functions F of octonions obtained as octonionic continuation of a real function with rational Taylor coefficients: the identification of space-time surfaces as "roots" of $IM(F)$ or $RE(F)$ makes sense.

2. What is intriguing that for space-time surfaces for which $IM(F_1) = 0$ and $IM(F_2) = 0$, one has $IM(F_1F_2) = RE(F_1)IM(F_2) + IM(F_1)RE(F_2) = 0$. One can multiply space-time surfaces by multiplying the polynomials. Multiplication is possible also when one has $RE(F_1) = 0$ and $IM(F_2) = 0$ or $RE(F_2) = 0$ or $IM(F_1) = 0$ since one has $RE(F_1F_2) = RE(F_1)RE(F_2) - IM(F_1)IM(F_2) = 0$.

   For $IM(F) = 0$ type space-time surfaces one can even define polynomials analytic functions of the space-time surface with rational Taylor coefficients. One could speak of functions having space-time surface as argument, space-time surface itself would behave like number.

3. One can also form functional composites $P \circ Q$ (also for analytic functions with complex coefficients). Since $P \circ Q$ at $IM(Q) = 0$ surface is quaternionic, its image by P is quaternionic and satisfies $IM(P \circ Q) = 0$ so that one obtains a new solution. One can iterate space-time surfaces defined by $Im(P) = 0$ condition by iterating these polynomials to give $P, P^{circ2}, ..., P^{\circ N}$... From $IM(P) = 0$ solutions one obtains a solutions with $RE(Q) = 0$ by multiplying the M^8 coordinates with I_4 appearing in $o = q_1 + I_4 q_2$.

   The $Im(P) = 0$ solutions can be iterated to give $P \rightarrow P \circ P \rightarrow ..$, which suggests that the sequence of SSFRs could at least approximately correspond to the dynamics of iterations and generalizations of Mandelbrot and Julia sets and other complex fractals and also their space-time counterparts. Chaos (or rather, complexity theory) including also these fractals could be naturally part of TGD!

Evolution of cognition

Polynomials in M^8 obtained as continuation of real polynomials with rational (or perhaps even algebraic) coefficients and vanishing at origin define a concrete representation for the extensions of rationals. There is infinite number of polynomials realizing the same extension. The interpretation is as an evolutionary hierarchy.

Since the number of extensions larger than given extension is larger than those smaller than it, the sequence of BSFRs changing the extension leads unavoidably to evolution as a statistical increase of the dimension of extension. The functional composition of polynomials which vanish at origin gives rise to evolutionary hierarchies for which the number theoretical complexity increases as one climbs up in the hierarchy. Extensions in these hierarchies are analogous to conserved genes if the replacement of extension F in BSFR can only extend F to larger extension E. This might be true in statistical sense.

Extensions could increase statistically also in SSFRs. In [L104] I considered the possibility that the sequence of SSFRs could correspond in reasonable approximation to an iteration of polynomial P. This would give direct connection with the Mandelbrot and Julia fractals.

The basic question is whether the number theoretical vision based on M^8 and adelic physics could be seen as exact dual of the geometric vision based on $H = M^4 \times CP_2$ and the notion of "WCW" (WCW) or does number theoretical view describe cognitive representations as approximate mimicry of actual physics so that the duality would be many-to-1.

The latter option seems to more plausible. Evolution leads to an improved representations but 1-1 correspondence is not reached even at the level of algebraic numbers allowing cognitive representations dense at space-time surface, but might be reached by accepting transcendental extensions replacing polynomials with analytic functions with rational (or even algebraic) coefficients to guarantee the continuation to p-adic number fields. One argument in favor of transcendentals is that exponential functions and trigonometric functions should be possible. Exponential functions would force e which however defines finite-D extension of p-adic numbers. The roots of trigonometric functions would bring in π and is powers.

General ideas about cognition and cognitive representations

Consider first cognitive representations at space-time level.

1. Cognitive representations at the level space-time surfaces would be provided by the points of space-time surface with embedding space coordinates in extension of rationals considered. One the coordinates of embedding space are fixed, these discretization are unique. The selection of coordinates is in the octonionic case highly unique. Only time translation in the rest system defined by the linear octonion coordinates is allowed. Also in case H the coordinates are unique apart from color rotations. Also vielbein/spin rotation group of 3-surface could have representation as a Galois group.

2. Galois group would act on the cognitive representation at space-time level and in general would not leave it invariant so that one would obtain new space-time surface. The wave functions in the space of space-time surfaces would correspond to wave functions in the space of cognitive representations which would correspond to elements of Galois group or factor space if sub-group of Galois group leaves the representation invariant. Wave functions would be elements of the group algebra of Galois group with possible conditions corresponding of invariance with respect to sub-group restricting the function to coset space effectively. This picture leads to a vision about "small" state function reductions (SSFRs) as cascades of measurements leading to a tensor product of states in the hierarchy of normal sub-groups of Galois group [L111]. The interpretation would be as cognitive measurements.

3. What about fermions? Fermionic Fock states have in TGD framework interpretation in terms of quantum variant Boolean algebra realized in terms of multi-qubits. One can say that the spinor structure of space is kind of square root of metric and describes correlates of logic [L109]. This would apply even at the level of WCW.

   What could finite measurement and cognitive resolution for fermions mean? The natural hypothesis is that the group algebras of Galois groups generated by wave functions in Galois group and having dimension n equal that for extension of rationals describe bosonic degrees of freedom and that fermionic state correspond to the spinors in this algebra- possible restrictions come from chirality restrictions. The dimension of the spinor space would be at most 2^n.

Cognitive representations at space-time level would be rather concrete. But is it possible to realize mathematical imagination, is it possible to imagine higher-D spaces?

1. Cognitive representations would indeed occur already at the level of number system. The extension of rationals can be regarded as n-D space over rationals instead of reals and would be mapped to a dense subset of real variant of n-D space. One can say that subset of real (or complex) numbers represents cognitively the higher-D space. The Galois group would represent discretization for the symmetries of these n-D space and from this one can say something about the possible isometry group of the corresponding real or complex space.

   This ability to imagine real and complex spaces of arbitrary dimension and might be fundamental aspect of mathematical consciousness.

2. If one takes seriously the idea about the connection with Newton's constant G, one can ask whether the evolution of the mathematical cognition proceeded via the gradual increase of the order of G_{gr} and meant gradual reduction of G in rather dramatic steps if only Platonic groups are allowed.

   Remark: Nottale's proposal for h_{gr} implies that gravitational Compton length for two particle system is $G(M+m)/v_0$ and increase with G since h_{gr} increases. If the velocity parameter v_0 and G do not correlate, larger value of G and therefore smaller value of n_{gr} and lower level of space-time consciousness would mean longer gravitational Compton length as a measure for quantum coherence and higher level of consciousness. This looks somewhat strange. Should one conclude that v_0 and G correlate: for instance, could G/v_0 be independent of G_{gr}?

How could mathematical physics as correlation between cognitive/imagined and sensory worlds have emerged?

1. Somehow the idea that we live in Euclidian 3-space emerged and later emerged special relativity, general relativity and its followers. It seems essential that the cognitive representations at the level of number field found counterparts at the level of sensory world represented as 3-space and eventually space-time and embedding space.

   Quaternions and octonions are naturally assignable to M^8, M^4 and H. Quaternions have $SO(3)$ as the analog of Galois group with concrete geometric interpretation. The discovery would be that this group acts on the object of sensory world. Could it be that these two equivalent choices of embedding space are the only ones for which this consciousness about this sensory-cognitive correspondence can evolve? The essential point would be that the symmetry groups of physics would be sub-groups of automorphism groups for octonions and quaternions.

   Remark: The extension allowing discrete sub-group of $SO(3)$ as Galois group must be distinguished from much smaller extension needed to represent this sub-group as 3×3 orthogonal matrices.

2. Could the emergence of the idea of Platonic solids - say in mathematics of ancient Greece - correspond to a step in evolution in which this sensory-cognitive correspondence emerged. Cognitive and sensory started to resonate, as one might say.

Could Galois groups provide a representation for the discrete sub-groups of isometries and tangent space rotations of embedding space?

I have already earlier considered the possibility that Galois groups could provide representations for the finite sub-groups of isometry groups of $H = M^4 \times CP_2$ and $M^8 = M^4 \times E^4 = M^2 \times E^2 \times E^4$, see for instance [L111].

1. A natural looking assumption is that only finite discrete sub-groups having a hierarchical decomposition in terms of normal sub-groups characterizing Galois extensions and having thus order equal to dimension of extension would be allowed.

   In case of sub-groups of the rotation group, one can of course consider also sub-group generated as products of discrete sub-groups but they have infinite number of elements, which does not conform with the idea about finiteness of cognition. For instance, one can take Platonic groups and groups C_n and D_{2n} such that there rotation axis does not go through a point of Platonic solid and generate the product group. This group would have the product of Galois groups as Galois group. One could think that also these are allowed if one has finite measurement resolution and cognitive resolution. This brings in the notion of approximation, which might have emerged in cognitive evolution too.

2. In terms of polynomials defining the space-time surface in M^8 as algebraic surface, one would have $P = P_1 \circ ....P_N \circ P_{gr}$. The Galois group associated with gravitational polynomial P_{gr} of degree n_{gr} would be normal sub-group of the entire Galois group and the Galois group of $P_1 \circ ....P_N$ would be factor group. This polynomial would correspond to higher evolutionary level and perhaps consciousness not directly related to embedding space geometry.

   G_{gr} would be sub-group of embedding space isometries and vielbein rotations and therefore have the characteristic decomposition to a direct product. Direct product decomposition could be replaced with sub-direct product decomposition for sub-groups of direct product. Product- or semi-direct product decomposition would correspond to that assumed for the original proposal and interpreted in terms of many-sheetedness over M^4 *resp.* CP_2 (flux tube bundles in M^4.

3. $M^8 - H$ duality forces the identification of the direct product as four-fold product of discrete sub-groups of $SU(2)$ appearing in McKay correspondence and to the special role of icosahedral group and its covering. As found in the introduction, the condition that the Gal_{gr} is discrete finite sub-group of product of M^8 and H isometries leads to a unique identification for this group as $I \times \overline{I} \times \overline{I} \times \overline{I}$, where I is icosahedral group and $\overline{I}$ its covering, and predicts correctly the value of G.

The assumption that the product of discrete isometry groups of the factors of embedding space is representable as Galois group of Galois extension representable in terms of a polynomial can be criticized. Can the Galois group for Galois extension of rationals defined by irreducible polynomial be a direct product of Galois groups for extensions?

1. The answer to the question can be found from web (`http://tinyurl.com/sj26xrc`): it is found that this is possible for Galois extensions if the product of extensions is the extension and the intersection of extensions consists of rationals. This question is physically highly relevant since Z_6 should have representation as Galois group having interpretation as direct product of centers of $SU(2)$ and $SU(3)$.

2. If this were not the case, one would be in trouble since this would exclude representations of the products $G_1 \times G_2$ of discrete sub-groups associated with isometries H and M^8 as Galois groups. One can of course think of having discrete sub-groups of $G_1 \times G_2$ having a lower order with direct products of sub-groups of G_i excluded. These are possible. $Z_2 \times Z_2$ allows the sub-groups $\{(0,0),(1,0)\}$, $\{(0,0),(0,1)\}$, and$\{(0,0),(1,1)\}$ and these are not products.

3. More generally, one could have a semi-direct product of normal sub-groups of $H_1 \subset G_1$ and $H_2 \subset G_2$ (`http://tinyurl.com/zhx5xpz`). This implies a correlation between the discrete isometries of the factors of embedding space, which would have physical interpretation. Semi-direct product allows surjective projections to $p_i : G_i \rightarrow H_i$ with normal sub-groups N_i as kernels. The product group $G_1/N_1 \times G_2/N_2$ is the graph of isomorphism $G_1/N_1 \equiv G_2/N_2$. This obviously poses strong conditions on the groups. For $G_1 = G_2$ one can would have $N_1 = N_2$. Since Z_2 is always normal sub-group, one would obtain an acceptable group in this manner if both factors have even order, and the order would be reduced by factor 1/4. The orders of the acceptable sub-groups are factors of $ord(G_1) \times ord(G_2)$.

Remark: One should be of course be very cautious in considering the isometry groups. For instance, could the discrete sub-groups automorphism group G_2 of octonions be relevant in M^8 picture? One can also ask whether the finite discrete sub-groups of $SO(7)$ as maximal compact subgroup of $SO(1,7)$ might be relevant.

Genetic code and geometric consciousness

TGD predict at least two representations of genetic code. The first representation is in terms of dark photon triplets and second representation in terms of dark proton triplets.

TGD based model for genetic code based on bio-harmony realizes genetic code as a code for communications by dark photons. Triplet of dark photons having interpretation as 3-chord of bio-harmony is the basic idea. Icosahedral and tetrahedral geometries connect bio-harmony with geometry [L5, L76].

1. 12-note scale is represented as Hamiltonian cycle at icosahedron having 12 vertices. By assigning to edge of the Hamiltonian cycle quint (scaling of frequency by factor 3/2), the Hamiltonian cycle defines a harmony with 20 3-chords assignable to the triangular faces of the icosahedron. Hamiltonian cycles are characterized by their symmetry group S, which is Z_6, Z_4 and Z_2 (here one has two variants one depending on whether Z_2 represents reflection or rotation by π) or Z_1 (no symmetry, disharmony). By combining 3 Hamiltonian cycles with symmetries Z_6, Z_4, and Z_2 one obtains 60 3-chords.

2. One can assign to given 3-chord DNA codon and the analog amino-acid as the orbit of this chord under the symmetry group of the cycle. One almost obtains vertebrate genetic code with correct number of DNA codons associated with given amino-acid as number of faces at the orbit associated with it. Only 4 amino-acids and 4 DNA codons are missing. Tetrahedral harmony defined by unique Hamilton cycle gives the remaining 4 chords assignable to the triangular faces of tetrahedron. The outcome is vertebrate genetic code.

3. Icosahedron is in a unique position. Icosahedron has 17 Hamiltonian cycles whereas tetrahedron cube and dodecahedron have only 1 and octahedron 2. In case of dodecahedron the Hamiltonian cycle divides the dodecahedron to two identical parts with 6 pentagons suggesting that the the symmetry group is Z_6 and the number of amino-acids is 2.

4. There is large number of bioharmonies obtained by combining unique Z_6 harmony with pairs of Z_4 and Z_2 harmonies. Since music expresses and induces emotions, the identification would be as correlates for fundamental emotion/moods appearing already at molecular level, and perhaps even at deeper levels [L62]. The interpretation of codon as 6-bit would correspond to the standard reductionistic view about information represented as bit sequences. Harmony would code for the holistic aspects of information. These two views would correspond to intelligence in the usual sense and emotional intelligence.

Second representation of genetic code is in terms of dark nuclei consisting of sequences of dark protons triplets [L20, L108]. Codon corresponds to an entangled state of 3 dark protons forming a linear or circular structure with ordering of protons. The dark protons sequences associated with flux tubes parallel to ordinary DNA double strands would provide pairing of dark and ordinary DNA. Also RNA, tRNA, and amino-acids would be represented as dark proton triplets and DNA-amino-acid correspondence has a natural description.

One can raise questions about the interpretation of these two representations of the genetic code (and also about chemical representation).

1. Could genetic code be represented in terms of bio-harmony provide a quantum representation for two Platonic solids: icosahedron and tetrahedron, perhaps their product in $M^4 \times CP_2$. This would answer the question why both icosahedron and tetrahedron. An alternative interpretation is that one has product of isometries and tangent space rotations for M^4 (or CP_2).

   Could genes somehow represent concretely information about embedding space geometry and its symmetries - could one even imagine that genes are kind of statements? Could also dark proton representation have interpretation as a concrete representation in sensory realm.

2. One can raise questions about the bio-harmony. Why just 3 Hamiltonian cycles at icosahedron plus tetrahedral cycle? Could these 4 factors correspond to the 2+2 factors due to the $M^4 \times CP_2$ isometries and tangent space rotations. One would have representation for all these factors. But why one of them would be tetrahedron rather than icosahedron in which case one would have 80 codons? Why the symmetry groups S of Hamiltonian cycles would be Z_6, Z_4 and Z_2?

   Remark: Tetrahedral symmetries and orientation preserving octahedral symmetries are sub-groups of icosahedral symmetries (`http://tinyurl.com/vav2n2r`).

3. What about representation of color symmetries of CP_2 Platonic solid in terms of dark codons? Could one assign to dark codon formed by protons a representation in $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$ to get colored variants of genetic code. Genes would have vanishing total color. Can one consider representation of color as a subgroup of Galois group. Also more general Galois groups can be considered and genes as units would be defined as Galois singlets [L108].

Could the notion of genetic code generalize to the level of more general Galois groups.

1. Could one consider a generalization of the genetic code to cognitive representations based on Galois group and its coset groups. Restrict first the consideration to any finite discrete subgroup of isometries of H or M^8. Represent it physically in M^4 or CP_2 as a discrete structure analogous to Platonic solid. Form all Hamiltonian paths in the discretization and identify the n-D basic cells of this n-D structure as basic entities - analogs of DNA codons/chords. Identify the orbits of these entities under symmetry group of the cycle as analogs of amino-acids. Define the analog of genetic code as in the case of ordinary genetic code.

2. Could one imagine cognitive representation of arbitrary Galois group in terms of wave functions in group or its coset space. Could one consider generalization of bio-harmony in terms of Hamiltonian cycles in this coset space. Could one assign analogs of DNA codons to the faces of the polyhedron and could amino-acids correspond to the orbits of the faces under symmetries of the Hamiltonian cycle? Amino-acid wave functions would be constant at the orbits of the symmetry group of the cycle.

3. The relation to the model of "small" state function reductions (SSFRs) [L111] is interesting. SSFRs would have an interpretation as cognitive measurements in Galois group of extension. Let E be the extension of rationals and F the largest sub-field of E: let the corresponding Galois groups be G and H. The reduction would be a cascade starting with a reduction of the wave function in Galois group of E/F to a product of wave functions in G/H and H. At the next step same would take place for H and after finite number of steps one would have full reduction [L111].

   These reduction cascades provide a model for cognitive processing as cognitive quantum measurements. This process brings in mind the translation of DNA to amino-acids. Could map to amino-acid involving transition from I to sub-group I/S, S the symmetry group of bio-harmony, be analogous to a state function reduction.

14.6 TGD inspired solution to three cosmological and astrophysical anomalies

I learned within a period of week about two cosmological anomalies new to me. The first anomaly is 160 minute oscillations discovered by Kotov and associated with a wide range of astrophysical systems. Second anomaly is the ionization of the interstellar gas. There might be a connection between these anomalies.

14.6.1 Could 160 minute oscillation affecting Galaxies and the Solar System correspond to cosmic "alpha rhythm"?

Kotov has discovered that many celestial objects involve 160 minute oscillation, whose origin is not identified. There is an overwhelming evidence that a non-local phenomenon is in question.

TGD suggests an explanation as a kind of cosmic alpha rhythm. Fe^{2+} has 10 Hz alpha frequency, which is fundamental biorhythm as cyclotron frequency in .2 Gauss magnetic field assigned as endogenous magnetic field to living matter in TGD based quantum model of living matter. In .2 nT magnetic field which is consistent with empirically estimated values of interstellar magnetic field the cyclotron period is 160 minutes.

This co-incidence suggests that dark cyclotron photons with large value of Planck constant $h_{eff} = nh_0$ assigned with the phases of ordinary matter identifiable as dark matter and residing at magnetic flux tubes - in particular those carrying dark gravitons - induces the oscillations. The quantum coherence of dark matter would induce the coherence of oscillations in astrophysical length scales. The quantum effects on visible matter could be non-trivial since the energy $E = h_{eff}f$ of dark photons can be above thermal threshold. The same mechanism is central in TGD based quantum model for the control of visible bio-matter by dark matter.

Observations

The blog posting in Tallbloke's talkshop titled "*Evidence for a 160 minute oscillation affecting Galaxies and the Solar System*" [L79] (see `http://tinyurl.com/y5en9cxz`) tells about the finding by Valery Kotov that many celestial objects have parameters, which correspond to a fundamental frequency of 160.0101 minutes. There is an overwhelming evidence that a non-local phenomenon is in question. For instance, Earth day is 9 times 160 minutes.

The blog articles [L79, E7] give a long list of links to the works demonstrating the presence of this period: see for instance [E4, E5].

160 minute period occurs in many contexts.

1. Infrasonic oscillations, measured by Doppler effect, on the surface of Sun corresponds to a period of 160,01 minutes. These oscillations were discovered by Severny, Kotov, and Tsapp [E4, E5] and independently by Brookes *et al.* They were later conformed by two other teams - for references see the article "*Solar Activity, Wave of Kotov and Strange Coincidences*" [E7] (see `http://tinyurl.com/y6bfzy4q`). The following properties of Kotov waves are listed.

    (a) These waves are perfectly periodic and regular: no break of phase was observed over more than thirty years of observations

    (b) There are periods when the oscillation becomes blurred for the benefit of it's lobe in 159.956 minutes (modulation in 400 days).

    (c) The mode of vibration is badly identified.

    (d) The mechanism is not understood. V. Kotov proposes the influence of gravitational waves to explain the phenomenon but this explanation seems unrealistic.

2. The 160.0102 ± 0.0002 minutes appears also in solar eruptions.

3. There is a variation of the luminosity of Sun with period about 160/and or 80 minutes of Sun

4. The period of variations of luminosity of Delta Scuti stars has been found to be $162\pm$ 4 min and RR Lyrae stars 161.4 ± 1.6 minutes.

5. Kotov waves have been reported to occur even in quasars such as NGC 4151 and 3C 273 (see `http://tinyurl.com/yxcwh4rl`).

A possible TGD based explanation of Kotov waves

This finding relates in an interesting manner to the TGD based model of living systems in which cyclotron frequencies in endogenous magnetic field of $B_{end} = .2$ Gauss $= .2 \times 10^{-4}$ Tesla play a key role. The nominal value for the strength of the magnetic field of Earth varies since the value of B_E depends on position on surface of Earth. I have taken it as $B_E = .5$ Gauss but also $B_E = .3$ Gauss is mentioned. Whether $B_{end} = B_E$ can be assumed, is not clear.

1. For iron the cyclotron frequency of Fe^{2+} ion playing crucial role in oxygen based life is around 10 Hz, which serves as a fundamental biorhythm - alpha rhythm.

2. 160 min cyclotron frequency for Fe would correspond to magnetic field of .2 nT.

3. Interstellar or galactic magnetic field strengths are not far from this strength.

   - 1 nT for galactic magnetic field is claimed (see `http://tinyurl.com/yzesn4k`). This would give 32 min period.

   - For interstellar magnetic field the value 0.1 nTesla for interstellar magnetic field is claimed (see `http://tinyurl.com/y45hq72k`). Also the value .3 nT is claimed (see `http://tinyurl.com/glj8gvu`).

The proposed value .2 nT is half-way between these two values. Maybe there is fundamental biorhythm in cosmic scales! This is more or less predicted by TGD based vision about quantum coherence in all length scales made possible by the hierarchy $h_{eff} = n \times h_0$ of Planck constants predicted to define phases of ordinary matter identifiable as dark matter.

1. For large values of h_{eff} predicted by TGD the energies of the dark cyclotron photons can be above thermal threshold in living matter. This implies that the dark cyclotron radiation can have non-trivial effects on living manner: this kind of effects actually led to the idea about hierarchy of Planck constants. Now it can be deduced from what I call adelic physics [L49] (see `http://tinyurl.com/ycbhse5c`). The proposal is that bio-photons covering at least visible and UV range - the range of molecular transition energies - result as dark photons with say EEG frequencies transform to ordinary photons [K17].

2. In TGD inspired biology the cyclotron frequencies define coordinating rhythms [K89, K87] and the recent proposal [L96] (see `http://tinyurl.com/y4vtcv8u`) is that both sensory perception and motor actions and long term memory rely on a universal mechanism based on formation of holograms and their reading using dark cyclotron photon beam as reference beam. Could this mean that this mechanism is used even in galactic and cosmic scales so that life would be everywhere as TGD based theory of consciousness predicts?

3. If quantum coherence in astrophysical scales is involved, the values of h_{eff} would be very large and given by the Nottale formula $h_{eff} = h_{gr}0GMm/v_0$, where $v_0 < c$ is velocity parameter and M and m are the masses connected by the magnetic flux tubes carrying gravitons [L55]. The dark photons involved would have large energies $E = h_{eff}f$ and could therefore energies in the range of molecular transition energies and have effects on the dynamics of astrophysical system just as they would have on the physiology of brain behavior [J6].

4. Note that the magnetic flux tubes as parts of topologically highly non-trivial space-time surface would have sphere rather than disk as cross section. Although the value of Kähler magnetic and ordinary magnetic fields are non-vanishing at it, the total flux vanishes so that there is no observable magnetic field in the scale of cross section. No current is needed to generate the magnetic field in question. This kind of flux tubes are not possible in Maxwell's theory.

In this framework Kotov waves could be seen as a direct support for the magnetic flux tubes along which gravitons propagate. The control action forcing the synchronous oscillations would be by dark matter at gravitational flux tubes and the large value of h_{gr} would make possible coherent oscillations with 160 minute period to have effect on ordinary matter.

14.6.2 26 second pulsation of Earth: an analog of EEG alpha rhythm?

There is an interesting article in Discover Magazine with title "*The Earth Is Pulsating Every 26 Seconds, and Seismologists Don't Agree Why*" (`https://cutt.ly/ogI6soU`). That mini earthquakes would appear with a period of 26 seconds is a rather fascinating possibility and one can ask what the TGD based explanation for the poorly understood origin of the rhythm might be.

What has been observed?

The pulsations are Rayleigh waves in which the motion of the mass is vertical. The source of these pulsations can be located near the coast of the Gulf of Guinea. The amplitude of pulsations is largest during storms and during summer time, which suggests that ocean waves feed energy to some kind of waves. The first proposal is that deep ocean waves striking at the shore are the source of the pulsations. The problem is that the periods of these waves vary up to 20 s and shorter than the period 26 s of the pulsations.

Second hypothesis suggests that these microseisms are a form of harmonic tremor associated with the magmatic activity beneath the South Atlantic Ocean. The source is located suspiciously near a large volcano on the island of Sao Tome in the Bight of Bonny proposed to be the source. Also some other volcanoes are accompanied by microseism but the problem is why not all volcanoes would serve as sources.

The popular article talks about periodic pulsations and calls them mini earthquakes. What does this imply if one assumes that the author of the article is using the words in precise sense?

1. Stresses in the Earth's crust are involved with seismic waves. There are three basic kinds of stresses. The stress can due to the compression or stretching: in this case one speaks of tension. This could cause an oscillation. Oscillating string is a very simple example. Pulsations would be oscillations in the vertical direction. This phenomenon could be purely classical and involve no quantum jumps.

2. Ordinary earthquakes are however generated by shear stress: in an earthquake two parallel layers of rock touch each other in a fault. Faults need not be non-horizontal. When a large enough external force parallel to the fault acts on the second layer, the friction fails to keep the pieces together, and the layers start to slip. This event would be naturally quantum jump by its discontinuity. A phenomenological description is in terms of catastrophe theory but there is no proper classical description for what really happens when slippage starts.

3. Periodic mini earthquakes result if these slippages are induced by a periodic force acting on the other piece of rock in the direction of the fault. The analog of local pulsation would require a nearly vertical fault. The challenge would be to explain this periodic force. Standard physics might satisfactorily explain the periodic force and provide an estimate for the period but the description of the discontinuous transition might require TGD based quantum theory.

For the purpose of building a simple mental model, consider a 2-D lattice like structure consisting of cylindrical tectonic plates touching each other. At the border of the abyss at which the water depth suddenly increases deep ocean waves would act as an oscillating pressure to a cylinder and force it to oscillate.

If pulsations are indeed in question, the resulting horizontal motion of cylinders should be transformed to vertical motion. How this could be achieved? The pressure of ocean waves causes a compression in the horizontal direction. Since the material in question is incompressible and therefore preserves its volume, the cylinder must stretch in the vertical direction. The non-linearity of the coupling making possible period doubling could be due to the fact that the vertical stretching is a secondary effect. In the situation considered the coupling could be especially strong and make possible period doubling. The nearness of the volcano could increase the strength of coupling.

Could period doubling be involved?

Pulsations represent a special case of microseismic waves.

The microseism spectrum involves two parts: first part the period extends to 15 s as for deep ocean waves and for the second the frequencies are above 30 s and extend to 300s. However, 30 s is rather near to 26 seconds. If there is a coupling of deep ocean waves arriving at shore with microseism waves, one must explain how the almost period doubling results. In general linear coupling between oscillations preserves frequency so that non-linearity suggests itself. What comes in mind is that the system exhibits for frequency around $T = 13$ s a period doubling occurring universally in non-linear systems near chaos. Originally closed orbits in the configuration space of the system with period T are transformed in bifurcation to orbits with period $2T$. Why should $T = 13$ s be so special? In the TGD Universe, magnetic body carrying dark matter as $h_{eff} = nh_0$ phases acts as master controlling ordinary matter. The basic rule is that $h_{eff} \rightarrow nh_{eff}$ scales the energies $E = h_{eff}f$ of say phonons by n. The frequencies for the transitions preserving energy are scaled by $1/n$. Could the period doubling correspond to a transition $h_{eff} \rightarrow 2h_{eff}$ at MB and occur for $T = 13$ s, which could correspond to a cyclotron frequency of 1/13 Hz for MB. Quite generally, the cyclotron frequencies of MB of Earth would couple resonantly to various frequencies appearing in the dynamics of ordinary matter with $h_{eff} = h = 6h_0$. This would make the control possible. For $B = 2^{-7}B_{end}$ with $B_{end}02/B_E/5$, $B_E = .5$ Gauss, the cyclotron period of iron ion would be near 13 s. 25.6 Hz is rather near to 26 Hz and corresponds to 2^8:th sub-harmonic of the alpha rhythm 10 Hz, which suggests period doubling appearing in the approach to chaos as an explanation: 8^{th} period doubling of EEG alpha frequency could be in question!

Trying to understand the pulsation frequency

Could one understand the origin of the frequency 26 s in TGD framework as reflecting the presence of magnetic body (MB)? First some background about TGD.

1. TGD based quantum theory relies on zero energy ontology [L91] (https://cutt.ly/jgI6du1) and predicts quantum coherence in all scales being assignable to the magnetic bodies of systems consisting of ordinary matter. MBs would carry dark matter as $h_{eff} = n \times h_0$ macroscopically quantum coherent phases.

2. Ordinary ("big") state function reductions (BSFRs) would change the arrow of time and this implies that they look like deterministic smooth time evolutions leading to the final state of BSFR. The world would be quantum coherent but look classical in all scales! The change of the arrow of time leads to a radically new view about self-organization and about biology and also self-organized quantum criticality emerges naturally and leads to the emergence of "breathing systems" so that the applications to living systems are natural. In fact, evidence for very simple "breathing" systems is emerging [L86] (https://cutt.ly/QgI6fuE).

   Earthquakes have some strange features and this led to the proposal that earth quarks could involve BSFR in macroscopic scales at the level of MB of Earth [L82] (https://cutt.ly/ogI6gc3). Could also these mini earthquakes involve BSFRs? Could they be interpreted as a sequence of life cycles for a conscious entity with a life time of about 26 seconds assignable to Earth?

3. It is known that electromagnetic activity accompanies Earth quarks and this activity is such that the interpretation in terms of time reversal suggests itself. Could 26 seconds define a period for an analog of alpha rhythm in EEG? There is also another strange rhythm with a period of 160 minutes assignable to astrophysical systems and I have proposed an interpretation as a "cosmic" alpha rhythm [L79] (https://cutt.ly/SgI6h92).

This picture leads to ask whether the p-adic length scale hierarchy predicted by TGD could provide some understanding concerning the period of $T = 26$ seconds associated with the pulsations.

1. TGD predicts a hierarchy of p-adic length scales $L_p \propto p^{1/2}$, $p \simeq 2^k$, $k > 0$ preferred integer, coming as half octaves. TGD does not deny the possibility of scaled variants of various particles. For instance, electron could correspond to several integers k with masses proportional to $2^{k/2}$).

2. Secondary p-adic length scales correspond to scales $p^{1/2}L_p \propto p$. There also tertiary etc. time scales forming a fractal hierarchy coming in powers of$p^{1/2}$ and by p-adic length scales as preferred half octaves.

3. For instance, electron corresponds to p-adic prime $p = 2^{127} - 1$ (the largest Mersenne prime, which does not yet correspond to super-astrophysical length scale). Secondary p-adic length scale corresponds to a period $T_e \simeq .1$ seconds. This is a fundamental biorhythm appearing in alpha band of EEG. Also quarks correspond to secondary p-adic length scales which correspond to human time scales.

   $T = 26$ seconds is rather precisely equal to $2^8 \times T_e$, $T_e = .1$ seconds: the relative error is 1/64 or about 2 per cent. A scaled version of electron with mass $m = m_e/2^4 \simeq 32$ keV would correspond to 25.6 seconds. The p-adic prime $p \simeq 2^k$, $k = 127 + 8 = 135$ defining p-adic scale about .4 Angstrom. This is not far from Bohr radius $a_B = .53$ Angstrom for hydrogen atom.

Of course, the new dark particle need not be electron. One can consider more detailed attempts to understand the situation.

Option I:

The first attempt involves the notion of electropion or more generally, leptopion, see [K117] (`http://tgdtheory.fi/pdfpool/leptc.pdf`) for which there is empirical support and empirical evicence that ordinary pion allows p-adically scaled up variants.

1. The scenaro would be based on axion-like states proposed also as candidates for dark matter predicted by TGD. They would be indeed dark also in TGD but in TGD sense being particles having $h_{eff} = n \times h_0 > h$. This would explain why they are not seen in decay widths in particle accelerators (and excluding them).

2. There is evidence for electropion with mass $2 \times m_e$ (already from 1970's) decaying to an electron-positron pair but forgotten since it does not conform with the standard model (it would increase decay widths of weak bosons). TGD provides a model for this state and predicts similar states for muon and tau and evidence also for these states have been found but also forgotten.

TGD also suggest fractally scaled variants of pion states with different p-adic length scales $p \propto 2^k$ and there is empirical evidence for these states with masses both larger and smaller than pion mass.

1. One can also imagine scaled variants of electropion with different p-adic lengths scales. The primary p-adic time scale assignable to electropion scales corresponds to $k \leq 127$. How to estimate k?

   If the mass squared (conformal weight is additive in p-adic mass calculations then mass squared of electropion is $m^2 = 2m_e^2$ giving $m = 2^{1/2} \times m_e$ for $k = 127$. Correct mass requires $k_e = 127 \to 126$. Compton time of electropion would be $T(electropion, 126) = T_c(126, e)/2$, where $T_c(126, e)$ is the Compton time of electron with $k = 126$.

   The secondary p-adic time Compton time associated with the scaled variant of $k = 126$ electropion corresponds to $T(electropion, 126 + \Delta k) = 2^{\Delta k}T_e/2$. One must have $\Delta k = 8 + 2 = 10$ and $k = 137$. Amusingly, $k = 137$ corresponds to atomic length scale and to fine structure constant. This co-incidence could be regarded as a cosmic joke.

Why this atomic length scale, or rather the corresponding secondary p-adic length scale of scaled electropion, would be associated with the Earth's pulsations? Electropions should be dark and perhaps form a coherent state as in the model for the production of anomalous electron-positron pairs based on electropion involving in an essential manner non-orthogonal electric and magnetic fields of colliding nuclei?

Option II: The second proposal is based on TGD inspired quantum biology involving Bose-Einstein condensates of Cooper pairs of electrons, protons, and fermionic ions and also of bosonic ions at magnetic flux tubes and characterized by effective Planck constant $h_{eff} = nh_0$, $h = 6h_0$, making possible quantum coherence in length scales longer than Compton length.

1. Consider the Bose-Einstein condensate of electron Cooper pairs. Electron Cooper pairs has Compton length equal to $L_{2e} = L_e/2$, L_e the electronic Compton length. Secondary

Compton time equals to $T_{2e}^{2)} = 2^{127/2}T_e/2 = .05$ s. Superconductivity in longer length scales than Compton length requires $h_{eff} > h$. The scaled up Compton scale $L_{n,2e} = nL^{2e}$ gives the coherence length of a superconductor and the secondary Compton time scales to $nT_{2e}^{2)} = .05n$ s. This time equals to $T = 25.6$ s for $n = 2^9$. The interpretation in terms of period doubling can be considered.

2. The general hypothesis [K63] is that there is resonance between dark and p-adic length scales so that this dark scale would correspond to identical p-adic length scale which would correspond to $L(k = 127 + 18 = 145) \sim 1.25$ nm equal to the transversal length scale for DNA.

3. TGD predicts that ordinary dark DNA in aqueous environment is accompanied by dark DNA realized as flux tubes carrying dark proton triplets realizing genetic code. Also amino-acids would be accompanied by these dark proton triplets and electrons would neutralize proteins charge which would be 3 proton charges per amino-acid. This would suggest that this scale relates to dark DNA, RNA, and proteins, which would involve space-time sheets which are electronic super conductors, and that the 26 second rhythm reflects the presence of water.

Option III: This alternative is nearest the idea about 260 Hz rhythm as analog of alpha rhythm. Iron ion has cyclotron frequency 10 Hz in B_{end}. Period doublings could correspond to the scalings of B_{end} by powers 2^{-n} of two scaling the cyclotron frequency by factor 2^{-2n}. The area of the flux tube would be scaled up by 2^n. If h_{eff} is scaled by 2^n, the energies are unaffected. For $n = 8$ the cyclotron frequency of iron ion would be near to 25.6 s. Could also the powers $2^{-n} \times 10$ Hz appear in the microseismic spectrum as period doubled alpha rhythm in the approach to chaos?

Could 26 second rhythm be kind of a bio-rhythm for Earth analogous to heart-beat or breathing? These two rhythms are highly varying and assignable to self-organization. EEG alpha rhythm is however universal. Could the Earthly bio-rhythm be analogous to the alpha band in the analog of EEG of Earth with frequencies scaled down by factor 1/256?

Each period would correspond to a mini earth quake. Also the ordinary EEG would involve similar BSFRs as an analog of sleep-awake rhythms and all bio-rhythms could be this kind of sleep-awake rhythms. One could of course check whether the 26 second rhythm has an electromagnetic analog?

There exists also another analogous rhythm, the 160 minute rhythm assignable to many astrophysical objects. I have proposed an interpretation as a kind of cosmic alpha rhythm.

1. 160 minute period is obtained from 26 second rhythm by scaling by a factor about $369 \simeq 2^{8.5}$ with error of 2 per cent - half octave again.

2. For the electro-pion option, one can think that one scales electropion with $k = 127$ having mass $2^{1/2} \times m_e$ to $k = 127 \rightarrow 127 + 17 = 144$ to get secondary Compton time scale $2^{16+1/2)}T_e = 154.5$ minutes not too far from 160 seconds. The interpretation as 17^{th} period doubling for $k = 127$ electro-pion with $T_c = \sqrt{2}T_e$ could make sense. There is indeed evidence for the period doubling of pion-like state. $f_c = f_e/\sqrt{2} \simeq 7.1$ Hz is lower than the nominal value $f_S = 7.8$ Hz of the lowest Schumann frequency. The cyclotron frequency of K^+ in B_{end} is 7.7 Hz and rather near to f_S.

3. For the Cooper pair option one could argue that since h_{eff} is integer valued, one can allow a value of n near to $2^{17.5} \simeq 185364$: this would give p-adic length scale $L(162)$, $L(163)$, which corresponds to one of the miracle length scales $k \in \{151, 157, 163, 167\}$ defining scales assignable to DNA coiling, would have been a more desired outcome.

14.6.3 Why is intergalactic gas ionized?

I became aware about new-to-me cosmological anomaly (see `http://tinyurl.com/y6ps6tb8`). FB really tests by tolerance threshold but it is also extremely useful. The news is that the sparsely distributed hot gas in the space between galaxies is ionized. This is difficult to understand: as universe cooled below the temperature at which hydrogen atoms became stable, it should neutralized in standard cosmology.

In bio-systems there is similar problem. Why biologically important ions are indeed ions at physiological temperatures? Even the understanding of electrolytes is plagued by a similar problem. It sounds like sacrilege to even mention to a fashionable deeply-reductionistic popular physicist talking fluently about Planck scale physics, multiverses, and landscape about the scandalous possibility that electrolytes might involve new physics! The so called cold fusion is however now more or less an empirical fact [L37] (see `http://tinyurl.com/y7u5v7j4`) and takes place in electrolytes - also living matter is an electrolyte.

TGD explanation is based on the hierarchy of Planck constants $h_{eff} = n \times h_0$ predicted by adelic physics as kind of IQ of the system.

1. The energy of radiation with very low frequencies - such as EEG frequencies - can be in the range of ionisation energies of atoms by $E = h_{eff} \times f$ - typically in UV range. Hence interaction between long and short length scales characterized by different values of h_{eff} becomes possible and in TGD magnetic body (MB) in long scales would indeed control biomatter at short scales in this manner. Cyclotron radiation from magnetic flux tubes of MB carrying dark ions would be used as control tool and Josephson radiation from cell membrane would be utilized to transfer sensory input to MB.

2. TGD variant of Nottale's hypothesis predicts really large values of h_{eff}. One would have $h_{eff} = h_{gr} = GMm/v_0$ at the magnetic flux tubes connecting masses M and m and carrying gravitons ($v_0 < c$ is a parameter with dimensions of velocity) [L55] (see `http://tinyurl.com/y63l7624`). What is important that at gravitational flux tubes cyclotron frequencies would not depend on m being thus universal. For instance, bio-photons with energies in UV and visible range would result from dark photons with large $h_{eff} = h_{gr}$ for frequencies even in EEG range and below.

The ordinary photons resulting from dark photons would ionize biologically important atoms and molecules. In the interstellar space the situation would be the same: dark photons transforming to ordinary higher energy photons would ionize the interstellar gas.

This relates closely to another cosmological mystery.

1. Standard model based cosmology cannot explain the origin of magnetic fields appearing in all scales. Magnetic fields require in Maxwell's theory current and in cosmology thermal equilibrium does not allow any currents in long length scales. In TGD however magnetic flux tubes carrying monopole fluxes are possible by the topology of CP_2. They would have closed 2-surface as cross section rather than disk. They are stable and do not require current to generate the magnetic field. These flux tubes would be carriers of dark matter generating the dark cyclotron radiation ionizing interstellar gas in the scale of wavelength, which would be astrophysical.

2. There are also another kind of magnetic flux tubes for which cross section is sphere but the flux vanishes since the sphere is contractible. hese flux tubes are not stable against splitting. There would be no magnetic field in the scale of flux tube. Magnetic field is however non-vanishing and ions in it generate dark cyclotron radiation. These flux tubes would naturally carry gravitons and photons. These flux tubes could could mediate gravitational and electromagnetic interactions: gravitons and photons (also dark) would propagate along them.

3. This picture leads to a model for the formation of galaxies as tangles of long monopole flux carrying cosmic strings looking like dipole field in the region of galaxy (for TGD based model of quasars [L74] see `http://tinyurl.com/y2jbru4k`): the energy of these tangle would transform to ordinary matter as the cosmic strings would gradually thicken - this corresponds to cosmic expansion. The process would be the analog of inflation in TGD. Also stars and even planets could be formed in this manner, and thickened cosmic strings would be carriers of dark matter in TGD sense. The model explains the flat galactic rotation curves trivially.

4. Dark ions responsible for the intergalactic ionization could reside at these monopole flux tubes or at the flux tubes which vanishing magnetic flux carrying mediating gravitational interactions. Which option is correct? Or can one consider both options?

   There might be a connection with the $T = 160$ minute period appears in astrophysics in many scales from stars to quasars. The observation is hat dark cyclotron photons created by Fe^{2+} ions in interstellar magnetic field about .2 nT have period of 160 minutes.

   (a) In TGD inspired biology the endogenous magnetic field is about .2 Gauss and now the time scale is $t = .1$ seconds which corresponds to alpha rhythm, the fundamental bio-rhythm. 160 minutes would correspond to cosmic alpha rhythm! Also cyclotron photons with this frequency could induce ionization of interstellar scales. This would require h_{gr} which is by a factor $T/t = 10^5$ higher. For ordinary alpha frequency M is naturally proportional to the mass of Earth: $M = k_E M_E$. Solar mass is 3.33×10^5 times higher than the solar mass M_S, which suggests that the flux tubes of system with mass of Sun are involved. Could the dark matter in question be associated with the flux tubes connecting Sun to smaller masses m mediating gravitational interaction? The ratio of Planck constants would be

   $$\frac{h_{gr,S}}{h_{gr,E}} = \frac{k_S}{k_E} \times \frac{v_{0,E}}{v_{0,S}} \times \frac{M_S}{M_E} \quad .$$

   This would demand

   $$\frac{k_S}{k_E} \times \frac{v_{0,E}}{v_{0,S}} = \frac{1}{3.33} \simeq 3 \quad .$$

   (b) Note that the 160 minute period was discovered in the dynamics of Sun: no mechanism is not know for an oscillation coherent in so long length scale. Could this mean that the MB of Sun controls dynamics of Sun just as the MB of Earth controls the dynamics of biosphere? Is Sun a conscious, intelligent, entity?

14.7 Fast radio wave bursts: is life a cosmic fractal?

I encountered a highly interesting popular article with title "*Mysterious 'fast radio burst' detected closer to Earth than ever before*" (`https://cutt.ly/QdNX5Xc`)

Fast radio wave bursts (FRBs) arrive from a distance of hundreds of millions of light years - the scale of a large void. If the energy of FRBs is radiated isotropically in all directions - an assumption to be challenged below - the total energy is of the same order of magnitude that the energy of the Sun produced during a century. There are FRBs repeating with a period of 16 days located to a distance of 500 million light years from Earth.

The latest bursts arrive from a distance of only about 30 thousand light years from our own galaxy Milky Way described in the popular article can be assigned with magnetar (see `https://cutt.ly/udNMKRF`), which is a remnant of neutron start and has extremely strong magnetic field of about 10^{11} Tesla.

14.7.1 Basic findings

Below is the abstract of the article [E19] (`https://cutt.ly/sdNX69z`) reporting the discovery.

> *We report on International Gamma-Ray Astrophysics Laboratory (INTEGRAL) observations of the soft γ ray repeater SGR 1935+2154 performed between 2020 April 28 and May 3. Several short bursts with fluence of $\sim 10^{-7}$–10^{-6} erg cm^{-2} were detected by the Imager on-board INTEGRAL (IBIS) instrument in the 20–200 keV range. The burst with the hardest spectrum, discovered and localized in real time by the INTEGRAL Burst Alert System, was spatially and temporally coincident with a short and*

very bright radio burst detected by the Canadian Hydrogen Intensity Mapping Experiment (CHIME) and Survey for Transient Astronomical Radio Emission 2 (STARE2) radio telescopes at 400–800 MHz and 1.4 GHz, respectively.

Its lightcurve shows three narrow peaks separated by $\sim$ 29 *ms time intervals, superimposed on a broad pulse lasting* $\sim$ *0.6 s. The brightest peak had a delay of 6.5* $\pm$ *1.0 ms with respect to the 1.4 GHz radio pulse (that coincides with the second and brightest component seen at lower frequencies). The burst spectrum, an exponentially cutoff power law with photon index* $\Gamma = 0.7^{+0.4}_{-0.2}$ *and peak energy* $E_p = 65 \pm 5$ *keV, is harder than those of the bursts usually observed from this and other magnetars.*

By the analysis of an expanding dust-scattering ring seen in X-rays with the Neil Gehrels Swift Observatory X-ray Telescope (XRT) instrument, we derived a distance of $4.4^{+2.8}_{-1.3}$ *kpc for SGR 1935+2154, independent of its possible association with the supernova remnant G57.2+0.8. At this distance, the burst 20–200 keV fluence of* $(6.1 \pm 0.3) \times 10^{-7}$ *erg cm*$^{-2}$ *corresponds to an isotropic emitted energy of* $\sim 1.4 \times 10^{39}$ *erg. This is the first burst with a radio counterpart observed from a soft* γ *ray repeater and it strongly supports models based on magnetars that have been proposed for extragalactic fast radio bursts.*

What could be the interpretation of the finding in the TGD framework? The weirdest feature of the FRB is its gigantic total energy assuming that the radiation is isotropic during the burst. This assumption can be challenged in the TGD framework, where the stellar systems are connected to a monopole flux tube network and radiation flows along flux tubes, which can also branch. This brings strongly in mind the analog of a nervous system in cosmic scales and this analogy is used in what follows.

14.7.2 TGD based model for the FRBs

TGD based model is motivated by the fractality of the TGD Universe and zero energy ontology (ZEO) based view about quantum measurement theory predicting that self-organization correspond in all scales corresponds to a formation systems living in at least primitive sense.

An essential element is the hierarchy of effective Planck constants $h_{eff} = nh_0$ implied by adelic physics formulating the number theoretic vision about TGD. h_{eff} labels phases of ordinary particles behaving like dark matter and n corresponds to the dimension of extension of rationals. The first generalization of Nottale's hypothesis $\hbar_{gr} = GMm/v_0$ to be discussed below in more detail was to $h_{eff} = \hbar_{gr}$. The recent form of the hypothesis is that $\hbar_{gr}$ corresponds to a large integer factor of $h_{eff}/h_0 = n$.

The differences between TGD based view about classical fields lead to the notion of magnetic body consisting of flux quanta. Entire Universe would be a fractal network of nodes (say stars, planets, etc... identifiable as flux tube tangles identifiable as spaghetti like structures) connected by flux tubes, which can come in two varieties depending on whether the magnetic flux associated with them vanishes or is monopole flux.

14.7.3 Heuristic picture

With this background in mind one can start the heuristic model building.

1. The duration of pulses is few milliseconds: the duration of nerve pulses is the same. Is this a wink-wink to the Poirots of astrophysics?

2. Bursts can arrive regularly - for instance with a period of $T = 16.35$ days [E12] (`https://cutt.ly/xdNMjQK`). This brings in the mind of astro-Poirot biorhythm, in particular EEG rhythms. This would not be the only such rhythms: also the period of $T_{alpha} = 160$ minutes, for which have proposed an interpretation as a cosmic analog of alpha rhythm is known [L79]. The ratio $T/T_\alpha = 147.15$ would give for the analogous brain rhythm the value of 14.7 seconds.

3. Let us assume that stellar systems indeed form an analog of neural network connected by flux and assume that the topology of this network is analogous to that defined by axons. In TGD framework neural communications between neurons occur actually by using dark

photons with effective Planck constant $h_{eff} = nh_0$ along the flux tubes with the velocity of light so that feedback from brain and even from the magnetic body of brain back to sensory organs as virtual sensory input becomes possible. The function of nerve pulses is to connect the outgoing branch of the flux tube associated with the axon and those associated with dendrites of the post-synaptic neuron to a longer flux tubes by using neurotransmitters as relays.

4. The stellar object as an analog of a neuron would send its dark photon signals along the flux tube assignable to a single axon. Axon would later branch to dendrites arriving to other stellar systems and eventually perhaps to planets as analogs of synaptic contacts. An interesting question is whether also the analogs of nerve pulses and neurotransmitters acting as relays in the synaptic contacts defined by planets could make sense. What could nerve pulses propagating along the flux tube correspond to?

   Remark: In the TGD based model of brain there would be also flux tube network analogous to the meridian system of Eastern medicine and responsible for the holistic and spatial aspects of consciousness since more than one flux tube can emanate from a given node making possibly non-linear networks [L39]. Nervous system with tree- like structure would be responsible for the linear and temporal aspects of conscious experience. Meridian system would be a precedessor of the neural system.

5. The distances of FRBs are of the order of large voids having galaxies at their boundaries and forming lattice-like networks possibly assignable to the tessellations of 3-D hyperbolic space defining cosmic time= constant surfaces. This kind of tessellations could accompany also brain [L103]. In the fractal Universe of TGD one can wonder whether these voids are analogs of cells or even neurons and form cosmic biological organisms with flux tubes forming a network allowing communications.

14.7.4 The total emitted energy if it is analogous to nerve pulse pattern along flux tube directed to solar system

The basic implication is that the energy of the emitted radiation could be dramatically smaller than that predicted by am isotropic radiation burst. It is interesting to look whether the proposed picture survives quantitative modelling.

1. The reduction factor r for the total emitted energy would be essentially $r = S/A$, where S is the area of the "axonal" flux tube and $A = 4\pi R^2$ is the surface area of the magnetar. One must estimate the value of r.

2. Flux quantization for a single sheet of the many-sheeted magnetic flux tube involved would give $eBS = \hbar_0$ $h = 6h_0$ [L26, L60]. The general order of magnitude estimate is $eB \sim \hbar_0/S$. If each sheet carries out the same energy, the number of sheets is $n = h_{eff}/h_0$ and the effective area of a flux tube is $S = \hbar_0/eB$. Does the magnetic field assigned with magnetar correspond to a single sheet or to all sheets? If the field is measured from cyclotron energies assuming $h_{eff} = h$ it would correspond to all sheets and the measured magnetic field would be the effective magnetic field $B_{eff} = nB/6$ for $h = 6h_0$.

3. The branching of the flux tube could correspond to the splitting of the many-sheeted flux tube to tubes with smaller number of sheets and involve reduction of h_{eff}. This would give the estimate $r = \hbar_0/eBA$. Magnetic field of 1 Tesla corresponds to a unit flux quantum with radius - magnetic length - about 2.6×10^{-8} meters. Assuming the estimate $R = 20$ km for the magnetar radius, one has $r \sim 10^{-25}/6$.

4. The estimate for the total emitted energy assuming isotropic radiation is the energy radiated by the Sun during a century. Sun transforms roughly $E_{100} = 1.3\times10^{19}$ kg of mass to radiation during a century. This gives for the energy emitted in FRB the estimate $E = rE_{100} \sim 10^{-6}/6$ kg which is roughly 7.5 Planck masses $m_{Pl} \simeq 2.2 \times 10^{-8}$ kg $= 1.2 \times 10^{19}$ GeV. The order of magnitude is Planck mass. The estimate is of course extremely rough.

   In any case, the idea that pulses could have mass of order few Planck masses is attractive. Note that a large neuron with radius about 10^{-4} meters has a mass of order Planck mass [L99].

5. From the total detected energy $dE/dS = 6.1 \times 10^{-7}$ erg m^{-2} =$3.8\times 10^9 eVm^{-2}$ and total radiated energy $E = 7.5 m_{Planck}$ one can estimate the total area S covered by the branched energy flux if it covers the entire area with a shape of disk of radius R. This gives some idea about how wide the branching is. The total energy is $E = (dE/dS) \times \pi R^2$ giving $R = \sqrt{E/\pi(dE/dS)} \simeq .9 \times 10^9$ m. The equitoral radius of the Sun is $R_{Sun} = .7 \times 10^9$ m. $R_{Sun} \sim .78R$ This conforms with the idea that the radiation arrives along the axon-like flux tube connecting Sun and the magnetar branching so that it covers entire Sun.

14.7.5 Is the ratio $\hbar_{gr}/\hbar$ equal to the ratio of the total emitted energy to the total energy received by Sun?

The ratio h_{eff}/h should be of the same order of magnitude as the ratio $X = E/E_{rad}$, where E_{rad} is the energy of the radio wave photon with frequency 1.4 GHz for $h_{eff} = h$: $X \sim h_{eff}/h$. The ratio $Y = X/(h_{eff}/h)$ should satisfy $Y \sim 1$.

1. To proceed further, one can use the TGD variant of Nottale's hypothesis. The hypothesis states that one can assign to gravitational flux tubes gravitational Planck constant $\hbar_{gr}$. The original hypothesis was $\hbar_{eff} = \hbar_{gr}$ and the more recent form inspired by the adelic vision states that h_{gr} corresponds to a large integer factor of h_{eff}. One has $\hbar_{gr} = GMm/v_0 = r_S m/2v_0$. Here M is the mass of the large object - now that of magnetar. m is the mass of the smaller quantum coherent object in contact with the gravitational flux tube mediating gravitational interaction as dark graviton exchanges.

   v_0 is a velocity parameter, which for Sun would be $\beta_{0,S} = v_0/c \simeq 2^{-11}$ from the model for the inner planets as Bohr orbits [E13] [K100, K83, K84, **?**].

2. The Planckian educated guess is $m \sim m_{Pl}$ so that one would have $\hbar_{gr}/\hbar = r_S(M)/(2L_{Pl}\beta_0)$, where L_{Pl} is Planck length and $r_S(M)$ is the Schwartshild radius of the magnetar. This would give $Y = X/(h_{gr}/\hbar) = .4$ if one has $r_S = 3$ km as for the Sun. r_S is probably large but smaller than magnetar radius about 20 km. The masses of the magnetars are in the range 1-2 solar masses. For $M = 2M_S$ one obtains $Y = .8$

   The rough estimate is not far from $Y = 1$ and suggests that the interacting quantum units at the receiving end have mass of order Planck mass. Interestingly, the mass of a large neuron with radius 10^{-4} m is about Planck mass [L99], which supports the view that quantum gravitation in the TGD sense is fundamental for life - even in the cosmic scales.

14.7.6 The parameter v_0 as analog of nerve pulse conduction velocity?

The physical interpretation of the velocity parameter v_0 is one of the key challenges of TGD.

1. The order of magnitude of v_0 is the same as for the rotational velocities in the solar system. I have considered a geometry based interpretation in [L56, L55] [K13].

2. The analogy with the neural system encourages the question whether v_0 could have a concrete interpretation as the analog of the nerve pulse conduction velocity assignable to the dark magnetic flux tubes connecting distant systems.

   In TGD framework nerve pulses [K93] are proposed to be induced by Sine-Gordon solitons for the generalized Josephson junctions assignable to the cell membrane and identifiable as transversal flux tubes assignable to various membrane proteins such as ion channels and pumps. The dark variants of the biologically important ions would give rise to the supra currents.

   Could the gravitational flux tubes analogous to axons have this kind of structure and give rise to generalized Josephson junctions with ions serving also in this case as current carriers?

To sum up, the proposed interpretation as cosmic neural networks conforms with the basic assumptions of TGD. Most importantly, quantitative predictions are correct. The picture is of course not deduce from axioms: this is pattern recognition with basic principles predicting a lot of new physics.

14.8 Appendix: About the dependence of scattering amplitudes on $\hbar_{eff}$

In TGD $\hbar$ is replaced with $\hbar_{eff} = nh_0 = nh/6$ [L26, L60, L61], and it is important to know the general dependence of scattering amplitudes on $\hbar_{eff}$. In QFT formalism the standard choice of units is $\hbar = 1, c = 1$ so that it requires some work to deduce the general dependence of the scattering amplitudes and rate on $\hbar_{eff}$. One must also check whether this dependence is consistent TGD with view about coupling constant evolution as a discrete sequence of phase transitions between quantum critical states.

14.8.1 General observations about the dependence of n-particle scattering amplitudes on $\hbar$

The "*Quantum Field Theory*" by Itzykson and Zuber [B35] provides the information about the general dependence of scattering amplitudes on $\hbar$ albeit in implicit form since units $\hbar = 1, c = 1$ are used.

1. Since putting $\hbar = 1$ is not possible in TGD framework, one must carefully check how the scattering amplitudes and rates depend on $\hbar$. In this respect tree scattering amplitudes in Abelian gauge theory like QED are characterized by the number of vertices. Each vertex involves g. Besides this there are delta functions expressing on mass shell conditions and momentum conservation.

   The amplitude involving n gauge boson-fermion vertices is proportional to g^n and scattering rate is proportional to g^{2n}. g^2 has dimension of $\hbar$ so that the condition that the coupling parameters give dimensionless factor requires additional power of $\hbar$ giving rise to α^{2n} factor, where $\alpha = g^2/4\pi$ is the analog of fine structure constant.

2. The general rule must be that gFF vertex involves factor $g/\sqrt{4\pi\hbar}$. The origin of $1/\sqrt{4\pi\hbar}$ factor can be traced out to the dimensions $[\sqrt{\hbar}/L]$ of scalar and vector boson fields, and the dimension $[\sqrt{\hbar}/L^{3/2}]$ spinor fields following from the condition that Hamiltonian for free fields has dimension $[\hbar/L]$ of energy. This implies that in gauge boson-fermion vertex one has $g/\sqrt{\hbar}$ and in a gauge theory having no dimensional couplings $g/\sqrt{\hbar}$ appears as coupling constant quite generally. In non-abelian gauge theory 3-boson vertices involving g and 4-boson vertices involving g^2 are also present and this rule gives power α^n, $n = n_3 + 2n_4$, where n_3 is the number of 3-vertices (BBB and BBF) and n_4 is the number of bosonic 4-vertices.

   This is however gauge theory limit at which particles become points-like and the flux tubes giving rise to a tensor network are neglected. In this framework one could interpret $g^2/4\pi\hbar$ as coupling parameter assignable to the flux tube connecting particles and this is indeed more natural number theoretically since $\hbar_{eff}/h_0$ is integer. In case of gravitation this seems to be the only possibility.

3. The density of states factor appearing in the rate does not depend on $\hbar$. In particle-in-the box quantization momenta are given by $p = n\hbar/L$ and density of states is $d^3n = Vd^3p/\hbar^3$. When one scales up $\hbar$ also V is scaled so that d^3n remains invariant.

One can now look the scattering amplitudes and rates in more detail. The "*Quantum Field Theory*" by Itzykson and Zuber [B35] provides examples of practical calculations and allows to deduce simple rules for $\hbar$ dependence of scattering amplitudes and rates.

1. For fermion-fermion scattering in Abelian gauge theories in the lowest order $2 \to 2$ scattering $\hbar$ disappears from the scattering cross section, and one obtains just the classical result. For instance, electrodynamics lowest order scattering cross sections - say for Compton scattering or electro-electron scattering - are proportional to α^2/m^2 in units $\hbar = 1$, $c = 1$. Putting in $\hbar$ one obtains $\alpha^2\hbar^2/m^2$. $\alpha = e^2/4\pi\hbar$ implies that $\hbar$ disappears so that its value does not matter. Therefore there is strong dependence on $\hbar_{eff}$ for fermion-fermion in gauge theory in tree approximation. For the radiative corrections to 2-2 scattering coming in powers of α

the value of $\hbar$ matters and the larger its value the smaller the corrections are and this gives hopes about the convergence of the perturbation theory. The theoretician friendly Nature would induce a phase transition increasing h_{eff} to guarantee the convergence of perturbation series.

2. For a gauge theory scattering of type $2 \rightarrow n > 2$ via tree diagrams there are n vertices and the total scattering cross section is proportional to α^n/m^2 and thus depends on $\hbar$ for $n > 2$. The rate for production of states with higher particle number decrease with $\hbar_{eff}$. Hence $\hbar$ is measurable also in this manner.

3. For particle decays the rate is proportional to $1/\hbar_{eff}$: $\alpha^2 m$ is the basic dependence from dimensional analysis. Increase of $\hbar_{eff}$ scales up life-time as one might expect. For the decay of positronium non-perturbative effects due to bound state nature bring in additional power of α and the life time scales like a higher power of $\hbar_{eff}$.

4. It is often sloppily argued that classical limit corresponds to the limit $\hbar = 0$. This limit however completely fails as an approximation in situations in which $\hbar \rightarrow 0$ limit does not make sense. For instance, for atoms bound state energies are proportional to $1/\hbar^2$ and approach to infinite value as $\hbar$ goes to zero.

Clearly, $2 \rightarrow 2$ scattering for massive particles is very special in that for tree diagrams in QED and gauge theories the outcome does not depend on $\hbar_{eff}$ at all. It is intriguing that $2 \rightarrow 2$ scattering is main provider of information. This leaves room for the possibility of $\hbar_{eff}$ hierarchy.

14.8.2 Photon-photon scattering as objection against TGD view about discrete coupling constant evolution

Twistor approach suggests in TGD framework that perturbative corrections for a given extension of rationals vanish altogether [K50, K114, K98].

1. The weak form of the proposal is that this occurs only for critical values of coupling constants so that the sum over loop diagrams would vanish in these cases. Coupling constants would depend on extension of rationals and coupling constant evolution would be induced by the hierarchy of these extensions and coupling constant evolution would be discrete. This picture follows if space-time surfaces correspond to zero loci for real or imaginary parts of octonionic polynomials at M^8 side of $M^8 - H$ duality [L40].

   One could argue that the hierarchy of extensions of rationals defines a hierarchy of cognitive resolutions obtained by approximation analytic functions of octonions at M^8 side of $M^8 - H$ duality with polynomials. For space-time surfaces represented as zero loci of real or imaginary part of an *arbitrary* analytic function, the radiative corrections would not vanish.

2. Strong form of the proposal would mean that individual loop corrections vanish identically.

An objection against vanishing of loops is photon-photon scattering, which occurs via box diagram at QFT limit of TGD. This gives for sigma the behavior α^4/E^2 by dimensional argument. The rate is proportional to $1/\hbar_{eff}^2$. Photon-photon scattering is observed and QED predictions are correct.

What the vanishing of loops - in particular box diagrams - at QFT limit TGD could mean for photon-photon scattering? Does this kill the idea about the reduction of scattering amplitudes to tree level?

1. TGD description is based on many-sheeted space-time and the fundamental scattering events in twistor diagrams are for fermions. It is this level at which one would have only the analogs of tree diagrams. QFT limit is only an effective description, and the action is expected to be standard model action in a good approximation. If so, the problem disappears.

2. How photon-photon scattering could emerge at the fundamental level? TGD picture relies on twistor diagrams rather than Feynman diagrams. The proposal is that at fundamental

level twistor diagrams at $M^4 \times CP_2$ side of $M^8 - H$ duality involve only fermions and their bound states.

At M^8 side of $M^8 - H$ duality the geometric variant of approach would be realized. Components of super field would correspond to components of super-octonion and polynomial of super-octonion would be analogs of super-field. The vanishing of the real or imaginary part (in quaternionic sense) for the component polynomials would assign to each component of this super-polynomial a space-time surface in M^8.

For twistor diagrams the analogs of virtual particles are possible but they would have on-mass-shell complex momenta. Photon-photon scattering could occur as on-mass-shell process in this sense and involve the decay of photon to fermion antifermion pair with complex momenta. Second incoming photon would absorb the antifermion with complex momentum. The reaction would proceed in the similar manner in the remaining two vertices.

14.8.3 What about quantum gravitation for dark matter with large enough h_{eff}?

It is interesting to look what h_{gr} hypothesis implies for quantum gravitation for dark matter. Does the QFT type description for quantum gravitation of dark matter make sense in TGD framework?

1. One can consider two identifications for the fundamental parameter as either G or l_P^2. These identifications lead to same predictions as far the dependence of scattering amplitudes on $\hbar_{eff}$ is considered.

   (a) G is the fundamental parameter GMm has same dimension $[hbar]$ as $Z_1Z_2e^2$ and thus one can define the analog of gravitational fine structure constant as GM_P^2. The 2-2 scattering cross section is completely analogous to that for Coulomb scattering and does not depend on $\hbar_{eff}$ at all. This result is rather satisfactory.

   (b) Second option is that Planck length l_P defines fundamental length and G is identified as $G = l_p^2/\hbar_{eff}$. This gives $GMm = l_P^2 Mm/\hbar_{eff}$ with Plack length identified as CP_2 radius R: $l_P = R$ [L68]. The independence of the cross section or $2 \rightarrow 2$ scattering on $\hbar_{eff}$ in lowest order holds true also now. $\sqrt{\hbar_{eff}}M/M_P = \sqrt{G}M = Ml_P/\sqrt{\hbar}$ would serve as analog of e now.

2. In the lowest order the scattering amplitude for $2 \rightarrow 2$ scattering by graviton exchange should be essentially Fourier transform of Newton's gravitational potential at the static limit. The independence of $2 \rightarrow 2$ scattering cross section on h_{eff} looks a natural condition since in the lowest order the scattering would not depend at all on the value of $\hbar_{eff}$. Coupling strength GMm is analogous to $Z_1Z_2e^2$ and both have dimension $[\hbar]$. Therefore the cross section for $2 \rightarrow 2$ scattering does not depend on $\hbar$ if one expresses $G = l_P^2/\hbar_{eff}$, $l_P = R$. This implies that QFT type description with point-like particles can serve as an approximate description of gravitational interaction.

   This and Nottale's proposal [E13] would require that $GMm/\hbar_{eff}$ serves as dimensionless coupling parameter. Coupling strength α_{gr} would characterize pair of interacting particles rather than particle and would be naturally associated with flux tube mediating the interaction as graviton exchange and has an interpretation as generalization of string model picture. This picture makes sense also for gauge bosons.

3. Does the description of two-particle system with masses M and m make sense using Schrödinger equation? De-localization might cause problems and TGD proposal is that only the de-localization of dark matter occurs and also this takes place only on flux tubes along the orbits of planets [K100, K83, K84].

   The first observation is that the parameter $GMm/\hbar$ is for planetary systems so huge so that perturbation series fails. $Mm = m_P^2 = \hbar/l_P^2$ serves as an estimate for the upper bound of Mm. For $\hbar_{gr}$ situation changes and one can write the gravitational analog of Schrödinger equation as

$$(-\frac{\nabla_u^2}{2} + \frac{\beta_0^2}{u})\Psi = e\Psi \ , \quad e = \frac{E\beta_0^2}{m} \ , \quad u = GM = \frac{r_S}{2} \ . \qquad (14.8.1)$$

$\beta_0 = v_0/c = v_0$ for $c = 1$ clearly occurs in the role of e and the scaling $E = me/\beta_0^2$.

4. If gravitational Shrödinger equation makes sense, the gravitational analogs of atomic transitions should also make sense. For h_{gr} huge pulses of gravitational radiation would accompany the transitions of the gravitational analog of hydrogen atom since binding energies are proportional to mv_0^2/n^2, m the mass of the planet. What would happen would be emission of dark graviton with energy equal to say energy difference of initial and final states (planetary Bohr orbits), which would then decay to a bunch of ordinary gravitons [K83].

   One could estimate the rate of transitions using the existing results from atomic physics. One can also try to estimate the transition rate from a generalization of Uncertainty Principle (UP): $\Delta T = \hbar_{gr}/\Delta E$. Order of magnitude is about GMn^2/v_0^3 ($c = 1$). This gives $10^5 n^2$ seconds for $v_0/c = 2^{-11}$. This time is of order 30 hours! The transition would be associated with dark matter. This looks totally unrealistic. This estimate makes sense only if there is de-localization of dark matter to analogs of hydrogen orbitals.

   A better estimate should include the interaction with dark graviton field rather than mere UP. Here one can use Fermi's Golden Rule (see `http://tinyurl.com/yblec2on`). The change of energy would be huge and therefore also graviton's energy and momentum. Wave vector however matters and would be give by $k = p/\hbar_{gr}$ and de Broglie wavelength would be of order of planetary orbit so that the analog of dipole approximation $exp(ik.x) = 1 + ik \cdot x$ would make sense. The time for transition would be about $\Delta T = \hbar_{gr}\Delta E/E^2$ and of the same order of magnitude as previous estimate. This does not make sense. De-localization of dark parts of planets in the scale of solar system would lead to surreal effects.

5. In TGD picture the dark matter is assumed to be de-localized only at the flux tubes associated with planetary orbits. TGD approach relies on zero energy ontology (ZEO) in which quantum states correspond to quantum superpositions of preferred extremals of action (sum of Kähler action and volume term proportional to cosmological constant). The transition would involve classical orbits transforming to each other by dark graviton emission. The transition would occur as a replacement of flux tube trajectory with given energy with a trajectory having lower energy. If one assumes Bohr quantization for the trajectories, the energy liberated as dark graviton in the transition is huge using normal standards for quantum transitions.

   The basic condition is that the trajectories intersect. For instance, if the original trajectory is circle, the final trajectory could be ellipsoidal trajectory with a lower energy and located inside the circular trajectory and touching it at diametrically opposite points. A natural expectation is that the transition rate is proportional to $P = (V_{12}/\sqrt{V_1V_2})^2$, where V_{12} is the volume shared by the two flux tubes V_i are flux tube volumes. The square roots $\sqrt{V_i}$ of the flux tube volumes would correspond to normalization factors for dark matter wave functions at flux tubes. The square of this factor would give a very small coefficient and make the transition very slow despite the factor that the dimensionless coupling analogous to α would be $\beta_0/4\pi$.

   One would have $V_{12} \sim d^3$, where d is flux tube thickness. Flux tube volume would be $2\pi^2 Rd^2$ so that one would have orer of magnitude estimate $P \sim (1/4\pi^4)(d/R)^2$ determined by the ratio of the thickness of the flux tube to the area of the orbit determined by it. If the thickness of the flux tube is of the order of planet radius, P for Earth has order of magnitude 10^{-11}. By multiplying the estimate about 30 hours given by Uncertainty Principle would would obtain a rough estimate 10^9 years for the lifetime of the flux tube orbit of Earth.

   This kind of transitions should correspond to "big" state function reductions analogous to ordinary quantum measurements rather than "small" state function reductions having so called weak measurements (see `http://tinyurl.com/zt36hpb`) as analogs. In "big" state function reductions the arrow of geometric time changes in the sense that the roles of passive and active boundary of causal diamond (CD) change and the sequence of weak measurements occurs at opposite boundary of CD shifting farther away from the passive boundary, which was active boundary before the "big" state function reduction. Note that the temporal distance between the tips of CD increases and gives rise to clock time as a counterpart of experienced time defined by the sequence of "small" state function reductions)

6. For QFT description of quantum gravitation $\sqrt{\hbar}E/M_P = El_P/\sqrt{\hbar} = E\sqrt{G}$ would serve the role of the coupling parameter analogous to e. To get some idea what happens one can look graviton-graviton scattering amplitude for 4 gravitons having all 2 positive 2 negative helicities and known as M^{--++}. Lowest order calculations without loops at Minkowski limit (tree diagrams, see `http://tinyurl.com/y82rsw9y`) give an expression as a sum of terms proportional to x^2, where the dimensionless variable x is $x = El_P/\sqrt{\hbar_{eff}}$: E is energy scale. Amplitude is proportional $1/\hbar_{eff}$ and the scattering amplitude approaches zero for large values of h_{eff}.

14.8.4 A little sidetrack: How a finite number of terms in perturbation expansion can give a good approximation although perturbation series fails to converge?

The perturbative expansion of electrodynamics does not converge. This looks paradoxical since the predictions of QED are extremely accurate. This statement is of course somewhat sloppy since there are many notions of convergence. For instance, converge could occur in some kinematical regions and fail to do so in some other regions.

If convergence does not occur in kinematically important regions, how can then apply the perturbative expansion at all? Part of the explanation is certainly that in $2 \rightarrow 2$ scattering the lowest order does not depend on $\hbar$ at all so that it could be calculated by using so large a value of $\hbar$ that convergence occurs. Could one take the convergent result cut to a finite number of powers of α in convergence region and continue it by replacing α with its actual value to region where the convergence fails? Finite cutoffs would not deviate much from the correct result but the remainder would be infinite.

Chapter 15

TGD View about Quasars

The work of Rudolph Schild and his colleagues Darryl Letier and Stanley Robertson (among others) suggests that quasars are not supermassive blackholes but something else - MECOs, magnetic eternally collapsing objects having no horizon and possessing magnetic moment. Schild et al argue that the same applies to galactic blackhole candidates and active galactic nuclei, perhaps even to ordinary blackholes as Abhas Mitra, the developer of the notion of MECO proposes.

In the sequel TGD inspired view about quasars relying on the general model for how galaxies are generated as the energy of thickened cosmic strings decays to ordinary matter is proposed. Quasars would not be be blackhole like objects but would serve as an analog of the decay of inflaton field producing the galactic matter. The energy of the string like object would replace galactic dark matter and automatically predict a flat velocity spectrum.

TGD is assumed to have standard model and GRT as QFT limit in long length scales. Could MECOs provide this limit? It seems that the answer is negative: MECOs represent still collapsing objects. The energy of inflaton field is replaced with the sum of the magnetic energy of cosmic string and positive volume energy, which both decrease as the thickness of flux tube increases. The liberated energy transforms to ordinary particles and their dark variants in TGD sense. Time reversal of blackhole would be more appropriate interpretation. One can of course ask, whether the blackhole candidates in galactic nuclei are time reversals of quasars in TGD sense.

The writing of the article led also to a considerable understanding of two key aspects of TGD. The understanding of twistor lift and p-adic evolution of cosmological constant improved considerably. Also the understanding of gravitational Planck constant and the notion of space-time as a covering space became much more detailed in turn allowing much more refined view about the anatomy of magnetic body.

15.1 Introduction

The work of Rudolph Schild and his colleagues Darryl Letier and Stanley Robertson (among others) suggests that quasars are not supermassive blackholes but something else [E27] (see `http://tinyurl.com/y9uyzjlp`)- MECOs, magnetic eternally collapsing objects. There is a popular article about the claim (see `http://tinyurl.com/ydcurslo`). Schild *et al* argue that the same applies to galactic blackhole candidates and active galactic nuclei, perhaps even to ordinary blackholes as Abhas Mitra, the developer of the notion of MECO proposes.

15.1.1 Could quasars be MECOs rather than supermassive blackholes?

The basic claim of Schild *et al* is that quasars are not blackholes but eternally collapsing magnetic objects. This claim is based on long lasting study of quasar Q0957+561.

Methods

Before the publication of their article [E27] authors studied single quasar - Q0957+561 at distance of about billion light years for more than two decades. They also speak of Q0957+561 A,B referring to the two images of this quasar produced by gravitational lensing made possible by the fact that there happens to be a galaxy between us and Q0957+561. This lucky co-incidence has made possible to deduce detailed information about the structure and dynamics of the quasar. Besides galactic lense effect there is micro-lensing caused by the start of galaxy moving between the quasar and galaxy and leading to a variation of the measured luminosity - flickering.

The information about the quasar's structure and dynamics is deduced from the time dependence of the spectrum of the galaxy at various frequencies. Autocorrelation functions provide information about the dynamics of quasar and turn to have a period of about 10 days independent of frequencies. This period must be related to the dynamics and geometry of the quasar and the distance travelled by light in this time must define a basic scale of the quasar.

The repetitions of almost similar temporal patters - features - suggest an interpretation in terms of signal generated in quasar and then reflected as it encounters second part of quasar. Also fluorescence would generate secondary radiation. The time lapse gives direct information about the size and the shape of the structure. Combined with theoretical considerations this this gives a rather detailed view about the geometry and dynamics of the quasar. The fluctuations of the luminosity provide also information.

Findings and interpretation as MECOs

The quasars would indeed differ from blackholes. Quasars would have magnetic moment unlike ordinary blackholes but lack event horizon. Quasars would have relatively complex geometric structure and dynamics. Authors describe their findings in terms of Schild-Vakulik structure (see `http://tinyurl.com/y92m2tah`) with the following anatomy.

1. A central object analogous to blackhole in that the radius is essentially Schwartschild radius r_S (or gravitational radius R_g as authors prefer to call it). The mass of this object is estimated to be $M \sim 3.6 \times 10^9 M_{sun}$. The corresponding Schwartschild radius r_S is by scaling from that of Sun equal to $r_{S,Sun} \simeq 3$ km equal to $r_S = 1.1 \times 10^{10}$ km. Note that the mass of the proposed supermassive black hole in the core of Milky Way is about 4.1 million solar masses and 3 orders of magnitude smaller. Could this mean that that quasar center loses its mass in the process and generates in this way the galaxy so that a kind of time reversal of blackhole would be in question? Note that the mass of the visible part of Milky Way itself is of order 10^{12} solar masses.

2. An empty disk around the central object would be caused by magnetic propellor effect: the radial Lorentz force overcoming gravitational attraction would sweep charged particles from the disk. This effect is possibly inside magnetosphere, where magnetic pressure dominates over the ordinary pressure. Lorentz force would dominate over the gravitational force. An objection against this proposal (see `http://tinyurl.com/ycwd2nho`) is that the gas in this region could be filled with very hot, tenuous gas, which would not radiate much.

3. An inner luminous ring at the inner boundary of the accretion disk having radius $R \sim 74R_g$ would be the luminous object producing the radiation. Instead of r_S authors talk about gravitational radius R_g of the central object, which would be slightly larger than Schwartschild radius. The inner radius would be about $(3.9 \pm .16) \times 10^{11}$ km. The diameter d characterizing the thickness of the inner ring is estimated to be about $d = 5.4 \times 10^9$ km. Note that d is roughly one half of r_S.

   The radius of the disk defines the size of the magnetosphere of the object. Few per cent fluctuation in the luminosity with variance increasing linearly with time has been observed - the radiation from accretion disk would increase like t^2 or t^3 depending on whether it is optically thick or thin. This observation has motivated the assignment of the luminosity to the ring.

   The fluctuations must be generated by some events. The proposed interpretation is that the flow of the matter to the central object causes these events. Second possibility is that the fluctuations are associated with outwards mass flow from the central object colliding with the accretion disk.

4. In the accretion disk gravitation and pressure dominate over magnetic forces and there is a competition between pressure and gravitation. This structure is also associated also with ordinary blackholes. The mass flow could be outwards in the disk.

5. The outer ring as boundary of the accretion disk is called Elvis structure: the name derives from Martin Elvis, who has also studied the structure of quasars [E25] (see `http://tinyurl.com/yd5j9uno`). In the abstract of the article it is stated that a funnel shaped thin shell creates various structures in the inner regions of quasar. The identification of this structure would be in terms of the base of the funnel from which the matter flows out. Funnel has opening angle about 60 degrees. The outflow leads to ask whether the net flow of matter matter from the quasar is outwards rather than inwards. There are also illustrations of the 3-D structure of quasars (see `http://tinyurl.com/y755gc4a`).

   The size R_e of and the vertical location H_e of the Elvis structure above disk are estimated to be $R_e = 2 \times 10^{12}$ km and $H_e = 5 \times 10^{11}$ km. The radial width of UV-luminous Elvis structure would be $\Delta R_e = 4 \times 10^{11}$ km .

   There is also a structure emitting radio waves. Its size R_r and vertical location H_r are estimated to be $R_r = 2 \times 10^{11}$ km and $H_r = 9 \times 10^{11}$ km .

6. The strength of the magnetic field B at the gravitational radius $R_g \simeq r_S$ of the central object is estimated on basis if MECO to be $2.5\times 10^9 \sqrt{7M_{Sun}/M} \simeq 4.4\times 10^4$ Tesla. The dependence of the magnetic field on distance far from the dipole core is $(R_g/R)^3$. The estimate for the observed magnetic field strength extrapolated to $R = R_g$ is given in Table 2 and equals to .77 Tesla being much smaller than 4.4×10^4 Tesla. The latter field correspond to a magnetic field obtained from MECO solution for stellar object by scaling.

The authors propose that a solution of field equations of general relativity found by Abhas Mitra, called (M)ECO ((magnetic) eternally collapsing object) [E10] could provide a model for the empirical findings about the structure and dynamics of the quasars. The original proposal of Mitra is that (M)ECOs could replace blackholes.

Mitra's general argument against blackholes is that the formation of ordinary blackholes is not possible since the collapsing matter should move with superluminal velocity. There are however objections against this argument (see `http://tinyurl.com/ycwd2nho`). (M)ECOs would be free of horizons and represent eternal collapse: at Eddington limit the radiation pressure inside the object would halt the collapse. (M)ECOs can have hair, in particular magnetic moment.

15.1.2 TGD view

In the sequel TGD inspired view about quasars relying on the general model for how galaxies are generated as the energy of thickened cosmic strings decays to ordinary matter is proposed. Quasars would not be be blackhole like objects but would serve as an analog of the decay of inflaton field producing the galactic matter. The energy of the string like object would replace galactic dark matter and automatically predict a flat velocity spectrum.

TGD is assumed to have standard model and GRT as QFT limit in long length scales. Could MECOs provide this limit? It seems that the answer is negative: MECOs represent still collapsing objects. The energy of inflaton field is replaced with the sum of the magnetic energy of cosmic string and positive volume energy (essentially magnetic energy from 6-D perspective), which both decrease as the thickness of flux tube increases. The liberated energy transforms to ordinary particles and their dark variants in TGD sense. Time reversal of blackhole would be more appropriate interpretation. One can of course ask, whether the blackhole candidates in galactic nuclei are time reversals of quasars in TGD sense.

I am not specialist so that I must concentrate on just what I see the most essential aspects and considerations rely crucially on the general TGD inspired vision about formation of galaxies. Furthermore, quasar dynamics is not a mere straightforward application of TGD but has proceeded through highs and lows - almost moments of total despair! The understanding of the twistor lift of TGD, of cosmological constant, of hierarchy of Planck constants and the notion of gravitational Planck constant are far from complete, and the information coming from the quasar dynamics has provided a valuable input allowing to solve some key puzzles involved.

15.2 Background about TGD

To develop TGD view about quasars, one must first summarize general vision about the formation of galaxies in TGD Universe. The starting point is the twistor lift of TGD and cosmic strings and their deformations as basic dynamical objects. A further key notion is the hierarchy of Planck constant predicted by adelic physics [L49, L50]. The notion of gravitational Planck constant is still only partially understood and this work forced to develop a more precise view allowing to overcome various objections.

All applications make me aware of some poorly understood aspects of TGD and quasar model was not an exception. It forced to clarify some details related to twistor lift and answer what covering space property and the notion gravitational Planck constant do really mean in TGD. Also the details related to the understanding of cosmological constant emerging from twistor lift of TGD naturally have been clarified considerably.

15.2.1 General vision

Consider first the general vision about galaxy formation in TGD Universe.

1. In TGD Universe quasars would represent the analog of the decay of inflaton field to matter [L58]. Galaxies associated with long cosmic string would be like pearls in necklace [L65]. The long string like object - magnetic flux tube - would have what I have called knots or tangles along it. The gravitational force created by the long string would automatically explain the flat velocity spectrum of distant stars and galactic dark matter would correspond to the energy assignable to this long string like object: there would be no halo.

   That galaxies are assignable to long linear structures have been known for decades [E30] but for some reason this message has not been taken by the theoreticians believing in dark matter halo. The number of conflicts of the halo model with empirical facts has increased steadily and it now seems that dark matter halo is empirically excluded.

   The galactic tangle would contain stars and even planets as sub-tangles. The topology of the flux tube structure would be analogous to the field line topology of magnetic field field, in reasonable approximation a dipole field in the case of quasar. Knotting and linking would be possible.

2. The dynamics of the flux tubes structures relies on the twistor lift of TGD [K114, K98, K14] predicting that the dimensional reduction of 6-D Kähler action defining twistor structure at space-time surface as twistor structure induced from that of $H = M^4 \times CP_2$ and having the crucial Kähler structure only for this choice of H. Space-time surfaces correspond to the base-spaces of their 6-D twistor spaces as induced twistor structures with S^2 fiber. 8-D twistor structure solves one of the basic problems of ordinary twistor approach due to the condition that particles must be massless. Now particles must be massless in 8-D sense and can therefore be massive in 4-D sense.

   The dimensionally reduced action contains besides 4-D Kähler action also a volume term analogous to cosmological constant term. The interpretation of field equations is as a 4-D generalization of equations of motion for point-like particle with Kähler charge natural since particles are indeed replaced with 3-surfaces in TGD.

   Cosmic strings identifiable as 4-surfaces having string world sheets as M^4 projection and complex 2-surface Y^2 as CP_2 projection belong to the basic extremals [K10, K18]. These surfaces are unstable against thickening of 2-D M^4 projection to 4-dimensional one and one can speak of flux tubes.

   There are two kinds of flux tubes: those for with Y^2 carries homological charge having interpretation as magnetic charge so that these flux tubes carry monopole flux and those for which Y^2 has vanishing homological charge. The flux tubes of first kind are of special interest as far as formation of galaxies is considered. Whatever happens to this flux tubes, the quantized magnetic flux - homology charge - is conserved.

3. The flux tubes of the tangle like structures along the long cosmic string would increase in thickness so that by flux conservation they would liberate magnetic energy as ordinary

particles and their dark variants since magnetic energy density per length behaves like $1/S$, S cross-sectional area. On the other hand, the volume energy proportional to S increases and there is some flux tube radius at which the energy is minimum and expansion cannot continue anymore. This process would eventually give rise to the formation of the galaxy.

If cosmological constant depends on p-adic length scale like $1/L^2(k)$, one has hierarchy of limiting radii for flux tubes. Interestingly, for the cosmological constant in cosmological scales the flux tube radius deduced from the density of volume energy is about 1 mm, a biological scale, which means connection between cosmology and biology.

Remark: The volume energy is indeed positive since it is magnetic energy associated with twistor sphere S^2 for dimensionally reduced 6-D Kähler action.

15.2.2 Twistor lift of TGD

Twistor lift of TGD led to a dramatic progress in the understanding of TGD but also created problems with previous interpretation. The new element was that Kähler action as analog of Maxwell action was replaced with dimensionally reduced 6-D Kähler action decomposing to 4-D Kähler action and volume term having interpretation in terms of cosmological constant.

One can of course ask whether the resulting induced twistor structure is acceptable. Certainly it is not equivalent with the standard twistor structure. In particular, the condition $J^2 = -g$ is lost. In the case of induced Kähler form at X^4 this condition is also lost. For spinor structure the induction guarantees the existence and uniqueness of the spinor structure, and the same applies also to the induced twistor structure being together with the unique properties of twistor spaces of M^4 and CP_2 the key motivation for the notion.

There are some potential problems related to the definition of Kähler function. The most natural identification is as 6-D dimensionally reduced Kähler action.

1. WCW metric must be Euclidian - that positive definite. Since it is defined in terms of second partial derivatives of the Kähler function with respect to complex WCW coordinates and their conjugates, the preferred extremals must be completely stable to guarantee that this quadratic form is positive definite. This condition excludes extremals for which this is not the case. There are also other identifications for the preferred extremal property and stability condition would is a obvious additional condition. Note that at quantum criticality the quadratic form would have some vanishing eigenvalues representing zero modes of the WCW metric.

2. Vacuum functional of WCW is exponent of Kähler function identified as negative of Kähler action for a preferred extremal. The potential problem is that Kähler action contains both electric and magnetic parts and electric part can be negative. For the negative sign of Kähler action the action must remain bounded, otherwise vacuum functional would have arbitrarily large values. This favours the presence of magnetic fields for the preferred extremals and magnetic flux tubes are indeed the basic entities of TGD based physics.

3. One can ask whether the sign of Kähler action for preferred extremals is same as the overall sign of the diagonalized Kähler metric: this would exclude extremals dominated by Kähler electric part of action or at least force the electric part be so small that WCW metric has the same overall signature everywhere.

If one accepts the proposal that the preferred extremals are minimal surfaces (the known extremals are), extremal property is satisfied for both 4-D Kähler action and volume term separately except at finite set of singular points at which there is transfer of conserved charges between the two degrees of freedom. In this principle this would allow the identification of Kähler function as either 4-D Kähler function or 4-D volume term (actually magnetic S^2 part of 6-D Kähler action). This option looks however rather ad hoc.

15.2.3 Is the cosmological constant really understood?

The interpretation of the coefficient of the volume term as cosmological constant has been a long-standing interpretational issue and caused many moments of despair during years. The intuitive picture has been that cosmological constant obeys p-adic length scale scale evolution meaning that Λ would behave like $1/L_p^2 = 1/p \simeq 1/2^k$ [K14].

This would solve the problems due to the huge value of Λ predicted in GRT approach: the smoothed out behavior of Λ would be $\Lambda \propto 1/a^2$, a light-cone proper time defining cosmic time, and the recent value of Λ - or rather, its value in length scale corresponding to the size scale of the observed Universe - would be extremely small. In the very early Universe - in very short length scales - Λ would be large.

A simple solution of the problem would be the p-adic length scale evolution of Λ as $\Lambda \propto 1/p$, $p \simeq 2^k$. The flux tubes would thicken until the string tension as energy density would reach stable minimum. After this a phase transition reducing the cosmological constant would allow further thickening of the flux tubes. Cosmological expansion would take place as this kind of phase transitions (for a mundane application of this picture see [L64]).

This would solve the basic problem of cosmology, which is understanding why cosmological constant manages to be so small at early times. Time evolution would be replaced with length scale evolution and cosmological constant would be indeed huge in very short scales but its recent value would be extremely small.

I have however not really understood how this evolution could be realized! Twistor lift seems to allow only a very slow (logarithmic) p-adic length scale evolution of Λ [L73]. Is there any cure to this problem?

1. The magnetic energy decreases with the area S of flux tube as $1/S \propto 1/p \simeq 1/2^k$, where $\sqrt{p}$ defines the transversal length scale of the flux tube. Volume energy (magnetic energy associated with the twistor sphere) is positive and increases like S. The sum of these has minimum for certain radius of flux tube determined by the value of Λ. Flux tubes with quantized flux would have thickness determined by the length scale defined by the density of dark energy: $L \sim \rho_{vac}^{-1/4}$, $\rho_{dark} = \Lambda/8\pi G$. $\rho_{vac} \sim 10^{-47}$ GeV^4 (see http://tinyurl.com/k4bwlzu) would give $L \sim 1$ mm, which would could be interpreted as a biological length scale (maybe even neuronal length scale).

2. But can Λ be very small? In the simplest picture based on dimensionally reduced 6-D Kähler action this term is not small in comparison with the Kähler action! If the twistor spheres of M^4 and CP_2 give the same contribution to the induced Kähler form at twistor sphere of X^4, this term has maximal possible value!

   The original discussions in [K114, K14] treated the volume term and Kähler term in the dimensionally reduced action as independent terms and Λ was chosen freely. This is however not the case since the coefficients of both terms are proportional to $(1/\alpha_K^2)S(S^2)$, where $S(S^2)$ is the area of the twistor sphere of 6-D induced twistor bundle having space-time surface as base space. This are is same for the twistor spaces of M^4 and CP_2 if CP_2 size defines the only fundamental length scale. I did not even recognize this mistake.

The proposed fast p-adic length scale evolution of the cosmological constant would have extremely beautiful consequences. Could the original intuitive picture be wrong, or could the desired p-adic length scale evolution for Λ be possible after all? Could non-trivial dynamics for dimensional reduction somehow give it? To see what can happen one must look in more detail the induction of twistor structure.

1. The induction of the twistor structure by dimensional reduction involves the identification of the twistor spheres S^2 of the geometric twistor spaces $T(M^4) = M^4 \times S^2(M^4)$ and of T_{CP_2} having $S^2(CP_2)$ as fiber space. What this means that one can take the coordinates of say $S^2(M^4)$ as coordinates and embedding map maps $S^2(M^4)$ to $S^2(CP_2)$. The twistor spheres $S^2(M^4)$ and $S^2(CP_2)$ have in the minimal scenario same radius $R(CP_2)$ (radius of the geodesic sphere of CP_2. The identification map is unique apart from $SO(3)$ rotation R of either twistor sphere possibly combined with reflection P. Could one consider the possibility that R is not trivial and that the induced Kähler forms could almost cancel each other?

2. The induced Kähler form is sum of the Kähler forms induced from $S^2(M^4)$ and $S^2(CP_2)$ and since Kähler forms are same apart from a rotation in the common S^2 coordinates, one has $J_{ind} = J + RP(J)$, where R denotes a rotation and P denotes reflection. Without reflection

one cannot get arbitrary small induced Kähler form as sum of the two contributions. For mere reflection one has $J_{ind} = 0$.

Remark: It seems that one can do with reflection if the Kähler forms of the twistor spheres are of opposite sign in standard spherical coordinates. This would mean that they have have opposite orientation.

One can choose the rotation to act on (y, z)-plane as $(y, z) \rightarrow (cy + sz, -sz + cy)$, where s and c denote the cosines of the rotation angle. A small value of cosmological constant is obtained for small value of s. Reflection P can be chosen to correspond to $z \rightarrow -z$. Using coordinates $(u = cos(\Theta), \Phi)$ for $S^2(M^4)$ and (v, Ψ) for $S^2(CP_2)$ and by writing the reflection followed by rotation explicitly in coordinates (x, y, z) one finds $v = -cu - s\sqrt{1-u^2}sin(\Phi)$, $\Psi = arctan[(su/\sqrt{1-u^2}cos(\Phi) + ctan(\Phi)]$. In the lowest order in s one has $v = -u - s\sqrt{1-u^2}sin(\Phi)$, $\Psi = \Phi + scos(\Phi)(u/\sqrt{1-u^2})$.

3. Kähler form J^{ind} is sum of unrotated part $J(M^4) = du \wedge d\Phi$ and $J(CP_2) = dv \wedge d\Psi$. $J(CP_2)$ equals to the determinant $\partial(v, \Psi)/\partial(u, \Phi)$. A suitable spectrum for s could reproduce the proposal $\Lambda \propto 2^{-k}$ for Λ. The S^2 part of 6-D Kähler action equals to $(J^{ind}_{\theta\phi})^2/\sqrt{g_2}$ and in the lowest order proportional to s^2. For small values of s the integral of Kähler action for S^2 over S^2 is proportional to s^2.

   One can write the S^2 part of the dimensionally reduced action as $S(S^2) = s^2F^2(s)$. Very near to the poles the integrand has $1/[sin(\Theta) + O(s)]$ singularity and this gives rise to a logarithmic dependence of F on s and one can write: $F = F(s, log(s))$. In the lowest order one has $s \simeq 2^{-k/2}$, and in improved approximation one obtains a recursion formula $s_n(S^2, k) = 2^{-k/2}/F(s_{n-1}, \log(s_{n-1})$ giving renormalization group evolution with k replaced by anomalous dimension $k_{n,a} = k + 2log[F(s_{n-1}, log(s_{n-1})]$ differing logarithmically from k.

4. The sum $J^{ind} = J + RP(J)$ defining the induced Kähler form in $S^2(X^4)$ is covariantly constant since both terms are covariantly constant by the rotational covariance of J.

5. The embeddings of $S^2(X^4)$ as twistor sphere of space-time surface to both spheres are holomorphic since rotations are represented as holomorphic transformations. Also reflection as $z \rightarrow 1/z$ is holomorphic. This in turn implies that the second fundamental form in complex coordinates is a tensor having only components of type $(1, 1)$ and $(-1, -1)$ whereas metric and energy momentum tensor have only components of type $(1, -1)$ and $(-1, 1)$. Therefore all contractions appearing in field equations vanish identically and $S^2(X^4)$ is minimal surface and Kähler current in $S^2(X^4)$ vanishes since it involves components of the trace of second fundamental form. Field equations are indeed satisfied.

6. The solution of field equations becomes a family of space-time surfaces parameterized by the values of the cosmological constant Λ as function of S^2 coordinates satisfying $\Lambda/8\pi G = \rho_{vac} = J \wedge (*J)(S^2)$. In long length scales the variation range of Λ would become arbitrary small.

7. If the minimal surface equations solve separately field equations for the volume term and Kähler action everywhere apart from a discrete set of singular points, the cosmological constant affects the space-time dynamics only at these points. The physical interpretation of these points is as seats of fundamental fermions at partonic 2-surface at the ends of light-like 3-surfaces defining their orbits (induced metric changes signature at these 3-surfaces). Fermion orbits would be boundaries of fermionic string world sheets.

   One would have family of solutions of field equations but particular value of Λ would make itself visible only at the level of elementary fermions by affecting the values of coupling constants. p-Adic coupling constant evolution would be induced by the p-adic coupling constant evolution for the relative rotations R combined with reflection for the two twistor spheres. Therefore twistor lift would not be mere manner to reproduce cosmological term but determine the dynamics at the level of coupling constant evolution.

8. What is nice that also $\Lambda = 0$ option is possible. This would correspond to the variant of TGD involving only Kähler action regarded as TGD before the emergence of twistor lift. Therefore the nice results about cosmology [K101] obtained at this limit would not be lost.

15.2.4 Does p-adic coupling constant evolution reduce to that for cosmological constant?

One of the chronic problems if TGD has been the understanding of what coupling constant evolution could be defined in TGD.

Basic notions and ideas

Consider first the basic notions and ideas.

1. The notion of quantum criticality is certainly central. The continuous coupling constant evolution having no counterpart in the p-adic sectors of adele would contain as a sub-evolution discrete p-adic coupling constant evolution such that the discrete values of coupling constants allowing interpretation also in p-adic number fields are fixed points of coupling constant evolution.

   Quantum criticality is realized also in terms of zero modes, which by definition do not contribute to WCW metric. Zero modes are like control parameters of a potential function in catastrophe theory. Potential function is extremum with respect to behavior variables replaced now by WCW degrees of freedom. The graph for preferred extremals as surface in the space of zero modes is like the surface describing the catastrophe. For given zero modes there are several preferred extremals and the catastrophe corresponds to the regions of zero mode space, where some branches of co-incide. The degeneration of roots of polynomials is a concrete realization for this.

   Quantum criticality would also mean that coupling parameters effectively disappear from field equations. For minimal surfaces (generalization of massless field equation allowing conformal invariance characterizing criticality) this happens since they are separately extremals of Kähler action and of volume term.

   Quantum criticality is accompanied by conformal invariance in the case of 2-D systems and in TGD this symmetry extends to its 4-D analogas isometries of WCW.

2. In the case of 4-D Kähler action the natural hypothesis was that coupling constant evolution should reduce to that of Kähler coupling strength $1/\alpha_K$ inducing the evolution of other coupling parameters. Also in the case of the twistor lift $1/\alpha_K$ could have similar role. One can however ask whether the value of the 6-D Kähler action for the twistor sphere $S^2(X^4)$ defining cosmological constant could define additional parameter replacing cutoff length scale as the evolution parameter of renormalization group.

3. The hierarchy of adeles should define a hierarchy of values of coupling strengths so that the discrete coupling constant evolution could reduce to the hierarchy of extensions of rationals and be expressible in terms of parameters characterizing them.

4. I have also considered number theoretical existence conditions as a possible manner to fix the values of coupling parameters. The condition that the exponent of Kähler function should exist also for the p-adic sectors of the adele is what comes in mind as a constraint but it seems that this condition is quite too strong.

   If the functional integral is given by perturbations around single maximum of Kähler function, the exponent vanishes from the expression for the scattering amplitudes due to the presence of normalization factor. There indeed should exist only single maximum by the Euclidian signature of the WCW Kähler metric for given values of zero modes (several extrema would mean extrema with non-trivial signature) and the parameters fixing the topology of 3-surfaces at the ends of preferred extremal inside CD. This formulation as counterpart also in terms of the analog of micro-canonical ensemble (allowing only states with the same energy) allowing only discrete sum over extremals with the same Kähler action [L70].

5. I have also considered more or less ad hoc guesses for the evolution of Kähler coupling strength such as reduction of the discrete values of $1/\alpha_K$ to the spectrum of zeros of Riemann zeta or actually of its fermionic counterpart [L13]. These proposals are however highly ad hoc.

Could the area of twistor sphere replace cutoff length?

As I started once again to consider coupling constant evolution I realized that the basic problem has been the lack of explicit formula defining what coupling constant evolution really is.

1. In quantum field theories (QFTs) the presence of infinities forces the introduction of momentum cutoff. The hypothesis that scattering amplitudes do not depend on momentum cutoff forces the evolution of coupling constants. TGD is not plagued by the divergence problems of QFTs. This is fine but implies that there has been no obvious manner to define what coupling constant evolution as a continuous process making sense in the real sector of adelic physics could mean!

2. Cosmological constant is usually experienced as a terrible head ache but it could provide the helping hand now. Could the cutoff length scale be replaced with the value of the length scale defined by the cosmological constant defined by the S^2 part of 6-D Kähler action? This parameter would depend on the details of the induced twistor structure. It was shown above that if the moduli space for induced twistor structures corresponds to rotations of S^2 possibly combined with the reflection, the parameter for coupling constant restricted to that to $SO(2)$ subgroup of $SO(3)$ could be taken to be taken $s = sin(\epsilon)$.

3. RG invariance would state that the 6-D Kähler action is stationary with respect to variations with respect to s. The variation with respect to s would involve several contributions. Besides the variation of $1/\alpha_K(s)$ and the variation of the $S(^2)$ part of 6-D Kähler action defining the cosmological constant, there would be variation coming from the variations of 4-D Kähler action plus 4-D volume term . This variation vanishes by field equations. As matter of fact, the variations of 4-D Kähler action and volume term vanish separately except at discrete set of singular points at which there is energy transfer between these terms. This condition is one manner to state quantum criticality stating that field equations involved no coupling parameters.

   One obtains explicit RG equation for α_K and Λ having the standard form involving logarithmic derivatives. The form of the equation would be

$$\frac{dlog(\alpha_K)}{ds} = -\frac{S(S^2)}{S_K(X^4) + S(S^2)} \frac{dlog(S(S^2))}{ds} \quad . \tag{15.2.1}$$

   It should be noticed that the choices of the parameter s in the evolution equation is arbitrary so that the identification $s = sin(\epsilon)$ is not necessary.

   The equation contains the ratio $S(S^2)/(S_K(X^4) + S(S^2))$ of actions as a parameter. This does not conform with idea of micro-locality. One can however argue that this conforms with the generalization of point like particle to 3-D surface. For preferred extremal the action is indeed determined by the 3 surfaces at its ends at the boundaries of CD. This implies that the construction of quantum theory requires the solution of classical theory.

   In particular, the 4-D classical theory is necessary for the construction of scattering amplitudes, and one cannot reduce TGD to string theory although strong form of holography states that the data about quantum states can be assigned with 2-D surfaces. Even more: $M^8 - H$ correspondence implies that the data determining quantum states can be assigned with discrete set of points defining cognitive representations for given adel This set of points depends on the preferred extremal!

4. How to identify quantum critical values of α_K? At these points one should have $dlog(\alpha_K)/ds = 0$. This implies $dlog(S(S^2)/ds = 0$, which in turn implies $dlog(\alpha_K)/ds = 0$ unless one has $S_K(X^4) + S(S^2) = 0$. This condition would make exponent of 6-D Kähler action trivial and the continuation to the p-adic sectors of adele would be trivial. I have considered also this possibility [L73].

   The critical values of coupling constant evolution would correspond to the critical values of S and therefore of cosmological constant. The basic nuisance of theoretical physics would determine the coupling constant evolution completely! Critical values are in principle possible. Both the numerator $J^2_{u\Phi}$ and the numerator $1/\sqrt{det(g)}$ increase with ϵ. If the rate for the variation of these quantities with s vary it is possible to have a situation in which the one has

$$\frac{dlog(J^2_{u\Phi})}{ds} = -\frac{dlog(\sqrt{det(g)})}{ds} \quad . \tag{15.2.2}$$

5. One can make highly non-trivial conclusions about the evolution at general level. For the extremals with vanishing action and for which α_K is critical (vanishing derivate), also the second derivative of $d^2S(S^2)/ds^2 =$ holds true at the critical point. The QFT analogs of these points are points at which beta function develops higher order zero. The tip of cusp catastrophe is second analogy.

   The points at which that the action has minimum are also interesting. For magnetic flux tubes for which one has $S_K(X^4) \propto 1/S$ and $S_{vol} \propto S$ in good approximation, one has $S_K(X^4) = S_{vol}$ at minimum (say for the flux tubes with radius about 1 mm for the cosmological constant in cosmological scales). One can write

$$\frac{dlog(\alpha_K)}{ds} = -\frac{1}{2}\frac{dlog(S(S^2))}{ds} \quad , \tag{15.2.3}$$

   and solve the equation explicitly:

$$\frac{\alpha_{K,0}}{\alpha_K} = \frac{S(S^2)}{S(S^2)_0})^x \quad , \quad x = 1/2 \quad . \tag{15.2.4}$$

   A more general situation would correspond to a model with $x \neq 1/2$: the deviation from $x = 1/2$ could be interpreted as anomalous dimension. This allows to deduce numerically a formula for the value spectrum of $\alpha_{K,0}/\alpha_K$ apart from the initial values.

6. One should demonstrate that the critical values of s are such that the continuation to p-adic sectors of the adele makes sense. For preferred extremals cosmological constant appears as a parameter in field equations but does not affect the field equations expect at the singular points. Singular points play the same role as the poles of analytic function or point charges in electrodynamics inducing long range correlations. Therefore the extremals depend on parameter s and the dependence should be such that the continuation to the p-adic sectors is possible.

   A naïve guess is that the values of s are rational numbers. Above the proposal $s = 2^{-k/2}$ motivated by p-adic length scale hypothesis was considered but also $s = p^{-k/2}$ can be considered. These guesses might be however wrong, the most important point is that there is that one can indeed calculate $\alpha_K(s)$ and identify its critical values.

7. What about scattering amplitudes and evolution of various coupling parameters? If the exponent of action disappears from scattering amplitudes, the continuation of scattering amplitudes is simple. This seems to be the only reasonable option. In the adelic approach [L49] amplitudes are determined by data at a discrete set of points of space-time surface (defining what I call cognitive representation) for which the points have M^8 coordinates belong to the extension of rationals defining the adele.

   Each point of $S^2(X^4)$ corresponds to a slightly different X^4 so that the singular points depend on the parameter s, which induces dependence of scattering amplitudes on s. Since coupling constants are identified in terms of scattering amplitudes, this induces coupling constant evolution having discrete coupling constant evolution as sub-evolution.

Could the critical values of α_K correspond to the zeros of Riemann Zeta?

Number theoretical intuitions strongly suggests that the critical values of $1/\alpha_K$ could somehow correspond to zeros of Riemann Zeta. Riemann zeta is indeed known to be involved with critical systems.

The naïvest ad hoc hypothesis is that the values of $1/\alpha_K$ are actually proportional to the non-trivial zeros $s = 1/2 + iy$ of zeta [L13]. A hypothesis more in line with QFT thinking is that they correspond to the imaginary parts of the roots of zeta. In TGD framework however complex values of α_K are possible and highly suggestive. In any case, one can test the hypothesis that the values of $1/\alpha_K$ are proportional to the zeros of ζ at critical line. Problems indeed emerge.

1. The complexity of the zeros and the non-constancy of their phase implies that the RG equation can hold only for the imaginary part of $s = 1/2 + it$ and therefore only for the imaginary part of the action. This suggests that $1/\alpha_K$ is proportional to y. If $1/\alpha_K$ is complex, RG equation implies that its phase RG invariant since the real and imaginary parts would obey the same RG equation.

2. The second - and much deeper - problem is that one has no reason for why $dlog(\alpha_K)/ds$ should vanish at zeros: one should have $dy/ds = 0$ at zeros but since one can choose instead of parameter s any coordinate as evolution parameter, one can choose $s = y$ so that one has $dy/ds = 1$ and criticality condition cannot hold true. Hence it seems that this proposal is unrealistic although it worked qualitatively at numerical level.

It seems that it is better to proceed in a playful spirit by asking whether one could realize quantum criticality in terms of of the property of being zero of zeta.

1. The very fact that zero of zeta is in question should somehow guarantee quantum criticality. Zeros of ζ define the critical points of the complex analytic function defined by the integral

$$X(s_0, s) = \int_{C_{s_0\rightarrow s}} \zeta(s)ds \ \ , \tag{15.2.5}$$

where $C_{s_0\rightarrow s}$ is any curve connecting zeros of ζ, a is complex valued constant. Here s does not refer to $s = sin(\epsilon)$ introduced above but to complex coordinate s of Riemann sphere.

By analyticity the integral does not depend on the curve C connecting the initial and final points and the derivative $dS_c/ds = \zeta(s)$ vanishes at the endpoints if they correspond to zeros of ζ so that would have criticality. The value of the integral for a closed contour containing the pole $s = 1$ of ζ is non-vanishing so that the integral has two values depending on which side of the pole C goes.

2. The first guess is that one can define S_c as complex analytic function $F(X)$ having interpretation as analytic continuation of the S^2 part of action identified as $Re(S_c)$:

$$\begin{aligned} S_c(S^2) &= F(X(s, s_0)) \ \ , \qquad X(s, s_0) = \textstyle\int_{C_{s_0\rightarrow s}} \zeta(s)ds \ \ , \\ S(S^2) &= Re(S_c) = Re(F(X)) \ \ , \\ \zeta(s) &= 0 \ \ , \qquad Re(s_0) = 1/2 \ \ . \end{aligned} \tag{15.2.6}$$

$S_c(S^2) = F(X)$ would be a complexified version of the Kähler action for S^2. s_0 must be at critical line but it is not quite clear whether one should require $\zeta(s_0) = 0$.

The real valued function $S(S^2)$ would be thus extended to an analytic function $S_c = F(X)$ such that the $S(S^2) = Re(S_c)$ would depend only on the end points of the integration path $C_{s_0\rightarrow s}$. This is geometrically natural. Different integration paths at Riemann sphere would correspond to paths in the moduli space $SO(3)$, whose action defines paths in S^2 and are indeed allowed as most general deformations. Therefore the twistor sphere could be identified Riemann sphere at which Riemann zeta is defined. The critical line and real axis would correspond to particular one parameter sub-groups of $SO(3)$ or to more general one parameter subgroups.

One would have

$$\frac{\alpha_{K,0}}{\alpha_K} = (\frac{S_c}{S_0})^{1/2} \quad . \tag{15.2.7}$$

The imaginary part of $1/\alpha_K$ (and in some sense also of the action $S_c(S^2)$) would determined by analyticity somewhat like the real parts of the scattering amplitudes are determined by the discontinuities of their imaginary parts.

3. What constraints can one pose on F? F must be such that the value range for $F(X)$ is in the value range of $S(S^2)$. The lower limit for $S(S^2)$ is $S(S^2) = 0$ corresponding to $J_{u\Phi} \to 0$.

   The upper limit corresponds to the maximum of $S(S^2)$. If the one Kähler forms of M^4 and S^2 have same sign, the maximum is $2 \times A$, where $A = 4\pi$ is the area of unit sphere. This is however not the physical case.

   If the Kähler forms of M^4 and S^2 have opposite signs or if one has RP option, the maximum, call it S_{max}, is smaller. Symmetry considerations strongly suggest that the upper limit corresponds to a rotation of 2π in say (y, z) plane ($s = sin(\epsilon) = 1$ using the previous notation).

   For $s \to s_0$ the value of S_c approaches zero: this limit must correspond to $S(S^2) = 0$ and $J_{u\Phi} \to 0$. For $Im(s) \to \pm\infty$ along the critical line, the behavior of $Re(\zeta)$ (see http://tinyurl.com/y7b88gvg) strongly suggests that $|X| \to \infty$. This requires that F is an analytic function, which approaches to a finite value at the limit $|X| \to \infty$. Perhaps the simplest elementary function satisfying the saturation constraints is

$$F(X) = S_{max} tanh(-iX) \quad . \tag{15.2.8}$$

   One has $tanh(x + iy) \to \pm 1$ for $y \to \pm\infty$ implying $F(X) \to \pm S_{max}$ at these limits. More explicitly , one has $tanh(-i/2-y) = [-1+exp(-4y)-2exp(-2y)(cos(1)-1)]/[1+exp(-4y)-2exp(-2y)(cos(1)-1)]$. Since one has $tanh(-i/2+0) = 1-1/cos(1) < 0$ and $tanh(-i/2+\infty) = 1$, one must have some finite value $y = y_0 > 0$ for which one has

$$tanh(-\frac{i}{2} + y_0) = 0 \quad . \tag{15.2.9}$$

   The smallest possible lower bound s_0 for the integral defining X would naturally to $s_0 = 1/2 - iy_0$ and would be below the real axis.

4. The interpretation of $S(S^2)$ as a positive definite action requires that the sign of $S(S^2) = Re(F)$ for a given choice of $s_0 = 1/2 + iy_0$ and for a propertly sign of $y - y_0$ at critical line should remain positive. One should show that the sign of $S = a \int Re(\zeta)(s = 1/2 + it)dt$ is same for all zeros of ζ. The graph representing the real and imaginary parts of Riemann zeta along critical line $s = 1/2 + it$ (see http://tinyurl.com/y7b88gvg) shows that both the real and imaginary part oscillate and increase in amplitude. For the first zeros real part stays in good approximation positive but the amplitude for the negative part increase be gradually. This suggests that S identified as integral of real part oscillates but preserves its sign and gradually increases as required.

   A priori there is no reason to exclude the trivial zeros of ζ at $s = -2n$, $n = 1, 2, ...$

1. The natural guess is that the function $F(X)$ is same as for the critical line. The integral defining X would be along real axis and therefore real as also $1/\alpha_K$ provided the sign of S_c is positive: for negative sign for S_c not allowed by the geometric interpretation the square root would give imaginary unit. The graph of the Riemann Zeta at real axis (real) is given in MathWorld Wolfram (see `http://tinyurl.com/55qjmj`).

2. The functional equation

$$\zeta(1-s) = \zeta(s)\frac{\Gamma(s/2)}{\Gamma((1-s)/2)} \quad (15.2.10)$$

allows to deduce information about the behavior of ζ at negative real axis. $\Gamma((1-s)/2)$ is negative along negative real axis (for $Re(s) \leq 1$ actually) and poles at $n+1/2$. Its negative maxima approach to zero for large negative values of $Re(s)$ (see `http://tinyurl.com/clxv4pz`) whereas $\zeta(s)$ approaches value one for large positive values of s (see `http://tinyurl.com/y7b88gvg`). A cautious guess is that the sign of $\zeta(s)$ for $s \leq 1$ remains negative. If the integral defining the area is defined as integral contour directed from $s<0$ to a point s_0 near origin, S_c has positive sign and has a geometric interpretation.

3. The formula for $1/\alpha_K$ would read as $\alpha_{K,0}/\alpha_K(s=-2n) = (S_c/S_0)^{1/2}$ so that α_K would remain real. This integration path could be interpreted as a rotation around z-axis leaving invariant the Kähler form J of $S^2(X^4)$ and therefore also $S = Re(S_c)$. $Im(S_c) = 0$ indeed holds true. For the non-trivial zeros this is not the case and $S = Re(S_c)$ is not invariant.

4. One can wonder whether one could distinguish between Minkowskian and Euclidian and regions in the sense that in Minkowskian regions $1/\alpha_K$ correspond to the non-trivial zeros and in Euclidian regions to trivial zeros along negative real axis. The interpretation as different kind of phases might be appropriate.

What is nice that the hypothesis about equivalence of the geometry based and number theoretic approaches can be killed by just calculating the integral S as function of parameter s. The identification of the parameter s appearing in the RG equations is no unique. The identification of the Riemann sphere and twistor sphere could even allow identify the parameter t as imaginary coordinate in complex coordinates in $SO(3)$ rotations around z-axis act as phase multiplication and in which metric has the standard form.

Some guesses to be shown to be wrong

The following argument suggests a connection between p-adic length scale hypothesis and evolution of cosmological constant but must be taken as an ad hoc guess: the above formula is enough to predict the evolution.

1. p-Adicization is possible only under very special conditions [L49], and suggests that anomalous dimension involving logarithms should vanish for $s = 2^{-k/2}$ corresponding to preferred p-adic length scales associated with $p \simeq 2^k$. Quantum criticality in turn requires that discrete p-adic coupling constant evolution allows the values of coupling parameters, which are fixed points of RG group so that radiative corrections should vanish for them. Also anomalous dimensions Δk should vanish.

2. Could one have $\Delta k_{n,a} = 0$ for $s = 2^{-k/2}$, perhaps for even values $k = 2k_1$? If so, the ratio c/s would satisfy $c/s = 2^{k_1} - 1$ at these points and Mersenne primes as values of c/s would be obtained as a special case. Could the preferred p-adic primes correspond to a prime near to but not larger than $c/s = 2^{k_1} - 1$ as p-adic length scale hypothesis states? This suggest that we are on correct track but the hypothesis could be too strong.

3. The condition $\Delta d = 0$ should correspond to the vanishing of dS/ds. Geometrically this would mean that $S(s)$ curve is above (below) $S(s) = xs^2$ and touches it at points $s = x2^{-k}$, which would be minima (maxima). Intermediate extrema above or below $S = xs^2$ would be maxima (minima).

15.2.5 What does one really mean with gravitational Planck constant?

There are important questions related to the QFT-GRT limit of TGD.

What does one mean with space-time as covering space?

The central idea is that space-time corresponds to n-fold covering for $h_{eff} = n \times h_0$. It is not however quite clear what this statement does mean.

1. How the many-sheeted space-time corresponds to the space-time of QFT and GRT? QFT-GRT limit of TGD is defined by identifying the gauge potentials as sums of induced gauge potentials over the space-time sheets. Magnetic field is sum over its values for different space-time sheets. For single sheet the field would be extremely small in the present case as will be found.

2. A central notion associated with the hierarchy of effective Planck constants $h_{eff}/h_0 = n$ giving as a special case $\hbar_{gr} = GMm/v_0$ assigned to the flux tubes mediating gravitational interactions. The most general view is that the space-time itself can be regarded as n-sheeted covering space. A more restricted view is that space-time surface can be regarded as n-sheeted covering of M^4. But why not n-sheeted covering of CP_2? And why not having $n = n_1 \times n_2$ such that one has n_1-sheeted covering of CP_2 and n_2-sheeted covering of M^4 as I indeed proposed for more than decade ago [K85] but gave up this notion later and consider only coverings of M^4? There is indeed nothing preventing the more general coverings.

3. $n = n_1 \times n_2$ covering can be illustrated for an electric engineer by considering a coil in very thin 3 dimensional slab having thickness L. The small vertical direction would serve and as analog of CP_2. The remaining 2 large dimensions would serve as analog for M^4. One could try to construct a coil with n loops in the vertical direction direction but for very large n one would encounter problems since loops would overlap because the thickness of the wire would be larger than available room L/n. There would be some maximum value of n, call it n_{max}.

   One could overcome this limit by using the decomposition $n = n_1 \times n_2$ existing if n is prime. In this case one could decompose the coil into n_1 parallel coils in plane having $n_2 \geq n_{max}$ loops in the vertical direction. This provided n_2 is small enough to avoid problems due to finite thickness of the coil. For n prime this does not work but one can of also select n_2 to be maximal and allow the last coil to have less than n_2 loops.

   An interesting possibility is that preferred extremal property implies the decomposition $n_{gr} = n_1 \times n_2$ with nearly maximal value of n_2, which can vary in some limits. Of course, one of the n_2-coverings of M^4 could be in-complete in the case that n_{gr} is prime or not divisible by nearly maximal value of n_2. We do not live in ideal Universe, and one can even imagine that the copies of M^4 covering are not exact copies but that n_2 can vary.

4. In the case of $M^4 \times CP_2$ space-time sheet would replace single loop of the coil, and the procedure would be very similar. A highly interesting question is whether preferred extremal property favours the option in which one has as analog of n_1 coils n_1 full copies of n_2-fold coverings of M^4 at different positions in M^4 and thus defining an n_1 covering of CP_2 in M^4 direction. These positions of copies need not be close to each other but one could still have quantum coherence and this would be essential in TGD inspired quantum biology [L59].

   Number theoretic vision [L49, L50] suggests that the sheets could be related by discrete isometries of CP_2 possibly representing the action of Galois group of the extension of rationals defining the adele and since the group is finite sub-group of CP_2, the number of sheets would be finite.

   The finite sub-groups of $SU(3)$ are analogous to the finite sub-groups of $SU(2)$ and if they action is genuinely 3-D they correspond to the symmetries of Platonic solids (tetrahedron,cube,octahedron, icosahedron, dodecahedron). Otherwise one obtains symmetries of polygons and the order of group can be arbitrary large. Similar phenomenon is expected now. In fact the values of n_2 could be quantized in terms of dimensions of discrete coset spaces associated with discrete sub-groups of $SU(3)$. This would give rise to a large variation of n_2

and could perhaps explain the large variation of G identified as $G = R^2(CP_2)/n_2$ suggested by the fountain effect of superfluidity [L68].

5. There are indeed two kinds of values of n: the small values $n = h_{em}/h_0 = n_{em}$ assigned with flux tubes mediating em interaction and appearing already in condensed matter physics [L44, L60, L26] and large values $n = h_{gr}/h_0 = n_{gr}$ associated with gravitational flux tubes. The small values of n would be naturally associated with coverings of CP_2. The large values $n_{gr} = n_1 \times n_2$ would correspond n_1-fold coverings of CP_2 consisting of complete n_2-fold coverings of M^4. Note that in this picture one can formally define constants $\hbar(M^4) = n_1\hbar_0$ and $\hbar(CP_2) = n_2\hbar_0$ as proposed in [K85] for more than decade ago.

Planck length as CP_2 radius and identification of gravitational constant G

There is also a puzzle related to the identification of gravitational Planck constant. In TGD framework the only theoretically reasonable identification of Planck length is as CP_2 length $R(CP_2)$, which is roughly $10^{3.5}$ times longer than Planck length [L68]. Otherwise one must introduce the usual Planck length as separate fundamental length. The proposal was that gravitational constant would be defined as $G = R^2(CP_2)/\hbar_{gr}$, $\hbar_{gr} \simeq 10^7\hbar$. The G indeed varies in un-expectedly wide limits and the fountain effect of superfluidity suggests that the variation can be surprisingly large.

There are however problems.

1. Arbitrary small values of $G = R^2(CP_2)/\hbar_{gr}$ are possible for the values of $\hbar_{gr}$ appearing in the applications: the values of order $n_{gr} \sim 10^{13}$ are encountered in the biological applications. The value range of G is however experimentally rather limited. Something clearly goes wrong with the proposed formula.

2. Schwartschild radius $r_S = 2GM = 2R^2(CP_2)M/\hbar_{gr}$ would decrease with $\hbar_{gr}$. One would expect just the opposite since fundamental quantal length scales should scale like $\hbar_{gr}$.

3. What about Nottale formula [E13] $\hbar_{gr} = GMm/v_0$? Should one require self-consistency and substitute $G = R^2(CP_2)/\hbar_{gr}$ to it to obtain $\hbar_{gr} = \sqrt{R^2(CP_2)Mm/v_0}$. This formula leads to physically un-acceptable predictions, and I have used in all applications $G = G_N$ corresponding to $n_{gr} \sim 10^7$ as the ratio of squares of CP_2 length and ordinary Planck length.

Could one interpret the almost constancy of G by assuming that it corresponds to $\hbar(CP_2) = n_2\hbar_0$, $n_2 \simeq 10^7$ and nearly maximal except possibly in some special situations? For $n_{gr} = n_1 \times n_2$ the covering corresponding to $\hbar_{gr}$ would be n_1-fold covering of CP_2 formed from n_1 n_2-fold coverings of M^4. For l $n_{gr} = n_1 \times n_2$ the covering would decompose to n_1 disjoint M^4 coverings and this would also guarantee that the definition of r_S remains the standard one since only the number of M^4 coverings increases.

If n_2 corresponds to the order of finite subgroup G of $SU(3)$ or number of elements in a coset space G/H of G (itself sub-group for normal sub-group H), one would have very limited number of values of n_2, and it might be possible to understand the fountain effect of superfluidity [L68] from the symmetries of CP_2, which would take a role similar to the symmetries associated with Platonic solids. In fact, the smaller value of G in fountain effect would suggest that n_2 in this case is larger than for G_N so that n_2 for G_N would not be maximal.

15.3 TGD view about quasars

TGD based model for quasar does not identify it as a blackhole like entity digesting matter around it but identified it as source of matter and energy resulting in the decay of the magnetic field of the flux time representing thickened cosmic string liberating also gravitational energy since the volume energy is indeed negative for a positive sign of volume action.

15.3.1 Overall view about the model

Consider now the basic picture about quasars and galaxies provided by TGD.

1. The authors still believe in restricted blackhole paradigm and assume that this structure "digests" matter from surroundings. The unit would have mass $10^{-3}M_{Sun}$ or $10^{-5}M_{Sun}$ - depending on estimate. Here TGD based view differs: the quasar need not digest matter around it but to feed it to the surroundings!

   The cleaning of charged matter from the inner disk would achieved if the total current vanishes so that the rotation velocities are opposite for charges with opposite sign and the directions of the Lorentz force are same, outwards or inwards both. The horizonal ring like structure would be a closed magnetic flux tube along which charged particles would rotate in the field created by the flux tubes of dipole field.

   The matter would flow into the central object if the Lorentz force is opposite: this is the case if the rotation velocities are opposite. Time reversal of this object analogous to blackhole would be in question and quasar could perhaps be seen as time reversed blackhole like entity analogous to time reversal of MECO. Note that in TGD time reversal symmetry T (and CP) are slightly broken. TGD predicts time reversals of the conscious entities assignable to cosmologies (and sub-cosmologies in Russian doll cosmology of TGD) and for them things would happen in opposite time direction fin the standard time frame [L58]: this cosmology is in some aspects analogous to the cosmology proposed by Penrose.

2. TGD based model leads to the proposal that the cylindrical magnetic dipole in the central region could (but certainly need not) be many-sheeted structure - n_1-sheeted covering of CP_2 consisting of disjoint flux tubes and n_2-sheeted covering of M^4 Minkowski space with $n_2 \simeq 10^7$ assigned with the Newtonian value G_N of G identified as $G_N = R^2(CP_2)/n_2\hbar$. This entity would be completely analogous to what I call magnetic body distinguishing between Maxwell's theory TGD (in many-sheeted space-time any system has field identity - field body - in the sense that its fields are associated with different space-time sheets than those of other systems).

   Magnetic body would serve as intentional agent in living systems and would be characterized by a large value of gravitational Planck constant (the notion is originally due to Nottale [E13]) $\hbar_{gr} = GMm(CP_2)/v_0 = (n_{gr}/6) \times \hbar$, $\hbar = 6 \times \hbar_0$. n_{gr} characterizes in adelic TGD [L50, L48] the algebraic complexity as dimension of extension of rationals.

   In the case of quasar M would be the mass of the central blackhole like object - about 3.6×10^9 solar masses as also the candidate for the galactic blackhole in Milky Way. $m(CP_2)$ is CP_2 mass about $10^{-3.5}$ Planck masses and would take the role of Planck mass. G would be identified as $G = R^2(CP_2)/\hbar_2$ rather than $R^2(CP_2)/\hbar_{gr}$ as in [L68], where it was assumed that $n_1 = 1$ so that one indeed had $n_{gr} = n_2$. n_{gr} would have a spectrum realized as a discrete scaling invariance in M^4 such that scaling acts also in G. It remains to be shown that the modification of the formula $G = R^2(CP_2)/\hbar_{gr}$ to $G = R^2(CP_2)/\hbar_2$ preserves the argued scaling invariance in M^4. The interpretation $v_0 < c$ is discussed in [L56].

   It was already discussed how one can understand the approximate constancy of G in this framework. In the simplest situation the coverings involved is $n_{gr} = n_1 \times n_2$ covering such that there is n_1-fold covering of CP_2 correspond to disjoint flux tubes in M^4 and n_2-fold M^4 covering associated with each flux tube. $n_2 \simeq 10^7$ would predict that gravitational constant $G = R^2/\hbar$ is near to its Newtonian value G_N.

3. The algebraic complexity of the galactic magnetic body identified as the cylindrical dipole part of the dipole field represented as flux tubes would be huge. The return flux outside the dipole would consist of simpler structures having smaller number of sheets and fusing to the dipole structure at the galactic nucleus. The flux tubes could wander to rather large distances and the stars would correspond to looped sub-tangles with flux tube structure mimicking the topology of field lines of dipole field.

   n_{gr} plays the role of IQ in TGD based model of living matter as governed by magnetic body. This forces to consider the possibility that quasars and galaxies are living organisms - much above us in hierarchy - having stars, planets,.... , us,... as sub-systems, sub-selves representing their mental images. One can say that the galactic dipole would represent the brain of galaxy. It is needless to say that this would completely revolutionize our world view.

We would not be desperate cosmic loners anymore but children of the Universe living and conscious in all scales.

Is there any empirical support for this speculative picture? There is evidence that galactic day as opposed to solar is period for precognitive events studied by people taking seriously "paranormal" phenomena which they prefer to call remote mental interactions [K21]. The reason would be that galactic magnetic field and therefore galactic magnetic body with strength of order nanoTesla is involved.

15.3.2 Estimate for the strength of the poloidal component B_θ of the magnetic field just below r_S

The estimate for the strength of the poloidal component of the magnetic field deduced from MECO solution just below $R_g \simeq r_S$ is $2.5 \times 10^9 \sqrt{7M_{Sun}/M} \simeq 4.4 \times 10^4$ Tesla. Could one say something about this field in TGD framework.

1. Flux quantization in the dipole core where the return flux of looping long cosmic string enters repeatedly to the dipole region and has the same direction would be integer multiple of unit flux assignable in the simplest case also to the long cosmic string: also this flux is quantized as integer multiples of a basic flux and predicts that the velocity is quantized as $\sqrt{n}$ if the contribution of volume term to the string tension is negligible. This is indeed expected due the smallness of Λ. Note that the long cosmic string makes the looping and after than continues.

   As already described one would have naturally $n_2 \sim 10^7$-fold covering of M^4 for the Newtonian value of G. Therefore one would have n_{gr}/n_2 disjoint flux tubes forming quantum coherent unit, the magnetic body of the quasar.

2. The Nottale proposal for the gravitational Plack constant $\hbar_{gr} = \hbar_{eff} = n_{gr}\hbar_0$, $\hbar = 6h_0$ suggest that the dipole has unit flux but with unit $\hbar_{gr} = GMm(CP_2)/v_0$, $m_{CP_2} = \hbar/R(CP_2)$. In the most original form of the hypothesis the second mass m was any mass but one can argue that since h_{gr} cannot be smaller than h, one must assume that m must have $m(CP_2)$ as lower bound. This leads also to other problems if m is too small. The interpretation in terms of quantum coherent structures with mass coming as multiple of $m(CP_2)$ is discussed in [L59].

3. This allows to estimate the value of the magnetic field from $eBS = n_{gr}\hbar_0 = n_{gr}\hbar/6$. Substituting the estimate $R_{CP_2} = 10^{3.5} l_P \simeq 5.1 \times 10^{-32}$ m, $r_S = 10^{10}$ km. Assume first that one has $n_2 = 1$ - no covering over M^4 so that one has disjoint flux tubes. A monopole flux through a closed surface is in question and there is no boundary and the area should be replaced with area of a topological sphere, which is the CP_2 geodesic sphere deformed in M^4 direction and having area $S = 4\pi R^2$ rather than $S = \pi R^2$ for a disk like cross section of flux tube. Spherical deformation is of course idealized assumption and the area could larger if the sphere is not spherical.

$$eB = \frac{GMm(CP_2)\hbar}{8v_0\pi r_S^2} = \frac{1}{8\pi v_0}\frac{\hbar}{X} \ ,$$
$$X = R(CP_2) r_S \simeq 10^{-18}\ m^2 \ . \qquad (15.3.1)$$

This gives for the magnetic length $l_B = \sqrt{\hbar/eB}$ and magnetic field B the expressions

$$l_B = \sqrt{\frac{\hbar}{eB}} = \sqrt{8\pi v_0 \times X} \simeq \sqrt{\pi v_0}\ nm \ ,$$
$$\frac{B}{Tesla} = (\frac{l_B}{26\ nm})^{-2} = \frac{26^2}{8\pi\beta_0} \ . \qquad (15.3.2)$$

The smallest value is obtained at the limit $\beta = v_0/c = 1$ and equals $B_{min} = 26.9$ Tesla.

4. From the conclusions section of the article one learns that the the poloidal component of the magnetic field just below radius $R_g \simeq r_S$ is estimated to be about $B \sim \sqrt{7M_{Sun}/M}10^{13}$ Gauss giving $B \sim 4.4\times 10^4$ Tesla. This gives $v_0 \simeq 6\times 10^{-4}$. This is quite near to the estimate $v_0 = 2^{-11} \simeq 4.9 \times 10^{-4}$ obtained from the Bohr orbit model for the inner planet orbits in solar system [K100, ?].

   This estimate was for $n_2 = 1$ but this is very special situation. For $n_2 = 10^7$ the value of the magnetic field for single sheet one would have $B \rightarrow B/n_2 \simeq 44$ Gauss. This option is the realistic one in the proposed framework. For $n_2 = n_{gr}$ corresponding to single flux tube with this number of sheets over M^4 $B \rightarrow B/n_{gr}$ which is extremely weak field.

   The effective value $B_{eff} \sim 4.4 \times 10^4$ of the magnetic field would correspond to the sum of n_{gr} copies of this field over all sheets of all tubes. If one has quantum coherence, this field value appears in the formula for cyclotron energies and this formula is crucial in biological applications allowing to have cyclotron energies in visible and UV range for dark photons with cyclotron frequency in EEG range.

15.3.3 Intelligent blackholes?

I received from Nikolina Benedikovic an interesting link to Leonard Susskinds's interview (see `http://tinyurl.com/yco7pd55` and for arousing my curiosity. In the link one learns that Leonard Susskind has admitted that superstrings do not provide a theory of everything. This is actually not a mind blowing surprise since very few can claim that the news about the death of superstring theory would be premature. Congratulations in any case to Susskind: for a celebrated super string guru it requires courage to change one's mind publicly. I will not discuss in the following the tragic fate of superstrings. Life must continue despite the death of superstring theory and there are much more interesting ideas to consider.

Susskind is promoting an idea about growing blackholes increasing their volume as they swallow matter around them (see `http://tinyurl.com/ybw78hpn`). The idea is that the volume of the blackhole measures the complexity of the blackhole and from this its not long way to the idea that information - may be conscious information (I must admit that I cannot imagine any other kind of information) - is in question.

Some quantum information theorists find this idea attractive. Quantum information theoretic ideas find a natural place also in TGD. Magnetic flux tubes would naturally serving as space-time correlates for entanglement (the p-adic variants of entanglement entropy can be negative and would serve as measures of conscious information) and this leads to the idea about tensor networks formed by the flux tubes [L25] (see `http://tinyurl.com/y9lmfrbz`). So called strong form of holography states that 2-D objects - string world sheets and partonic 2-surfaces as sub-manifolds of space-time surfaces carry the information about space-time surface and quantum states. $M^8 - M^4 \times CP_2$ correspondence [L40] would realize quantum information theoretic ideas at even deeper level and would mean that discrete finite set of data would code for the given space-time surface as preferred extremal.

In TGD Universe long cosmic strings thickened to flux tubes would be key players in the formation of galaxies and would contain galaxies as tangles along them. These tangles would contain sub-tangles having interpretation as stars and even planets could be such tangles.

In the proposed model quasars need not be blackholes in GRT sense but have structure including magnetic moment (blackhole has no hair), an empty disk around it created by the magnetic propeller effect caused by radial Lorentz force, a luminous ring and accretion disk, and so called Elvis structure involving outwards flow of matter. One could call them quasi- blackholes - I will later explain why.

1. Matter would not fall in blackhole but magnetic and volume energy in the interior would transform to ordinary matter and mean thickening of the flux tubes forming a configuration analogous to flow lines of dipole magnetic fields by looping. Think of formation of dipole field by going around flux line replaced by flux tube, returning and continuing along another flux line/tube.

2. The dipole part of the structure would be cylindrical volume in which flux tubes would form structure consisting analogous to a coil in which one makes $n_2 \simeq 10^7$ $(G_N = R^2/n_2h_0)$

windings in CP_2 direction and continues in different position in M^4 and repeats the same. This is like having a collection of coils in M^4 but each in CP_2 direction. This collection of coils would fill the dipole cylinder having the case of quasar studied a radius smaller than the Schwartshild radius $r_S \simeq 5 \times 10^9$ km but with the same order of magnitude. The wire from given coil would continue as a field line of the magnetic dipole field and return back at opposite end of dipole cylinder and return along it to opposite pole. The total number of loops in the collection of n_1 dipole coils with n_2 windings in CP_2 direction is $n_1 \times n_2$.

3. What is unexpected that although the volume contribution to action assignable to cosmological constant is positive as it must be, the energy is negative (I have checked this many times but cannot find mistake)! Could the expansion of flux tubes liberating ordinary and dark matter particles (in TGD sense) as analog of the decay of inflaton field continue without limit? At certain flux tube radius the total energy becomes zero - this corresponds roughly to a biological length scale about 1 mm for the value of cosmological constant in the length scale of the observed universe. Could the string tension become negative so that ordinary matter could be created without limit? It is quite possible that preferred extremal property prevents negative values of string tension but I have not found a good argument for this.

   Remark: Note that the twistor lift of TGD allows to consider entire hierarchy of cosmological constants behaving like $1/L(k)^2$, where $L(k)$ is p-adic length scale corresponding to $p \simeq 2^k$.

4. Cosmological expansion would naturally relate to the thickening of the flux tubes, and one can also consider the possibility that the long cosmic string gets more and more looped (dipole field gets more and more loops) so that the quasi-blackhole would increase in size by swallowing more and more of long cosmic string spaghetti to the dipole region and transforming it to the loops of dipole magnetic field.

5. The quasar (and also galactic blackhole candidates and active galactic nuclei) would be extremely intelligent fellows with number theoretical intelligence quotient (number of sheets of the space-time surfaces as covering) about

$$\frac{h_{eff}}{h} = \frac{n}{6} = \frac{n_1 \times n_2}{6} \geq \frac{GMm(CP_2)}{v_0} \times \hbar = \frac{r_S}{R(CP_2)} \times \frac{1}{2\beta_0} \ ,$$

   where one has $\beta_0 = v_0/c$, where v_0 is roughly of the order $10^{-3}c$ is a parameter with dimensions of velocity, r_S is Schwartschild radius of quasi-blackhole of order 5×10^9 km, and $R(CP_2)$ is CP_2 radius of order 10^{-32} meters. If this blackhole like structure is indeed cosmic string eater, its complexity and conscious intelligence increases and it would represent the brains of the galaxy as a living organism. This picture clearly resembles the vision of Susskind about blackholes.

6. This cosmic spaghetti eater has also a time reversed version for which the magnetic propellor effect is in opposite spatial direction: mass consisting of ordinary particles flows to the interior. Could this object be the TGD counterpart of blackhole? Or could one see both these objects as e blackholes dual to each other (maybe as analogs of white holes and blackholes)? The quasar like blackhole would eat cosmic string and its time reversal would swallow from its environment the particle like matter that its time reversed precedessor generated. Could one speak of breathing? Inwards breath and outwards breath would be time reversals of each other. This brings in mind the TGD inspired living cosmology based on zero energy ontology (ZEO) [L58] as analog of Penrose's cyclic cosmology, which dies and re-incarnates with opposite arrow of time again and again.

A natural question is whether also the ordinary blackholes are quasi-blackholes of either kind. In the fractal Universe of TGD this would look extremely natural.

1. How to understand the fusion of blackholes (or neutron stars, I will however talk only about blackholes in the sequel) to bigger blackhole observed by LIGO if quasi-blackholes are in question? Suppose that the blackholes indeed represent dipole light tangles in cosmic string. If they are associated with the same cosmic string, they collisions would be much more probable than one might expect. One can imagine two extreme cases for the motion of the blackholes. There are two options.

(a) Tangles plus matter move along string like along highway. The collision would be essentially head on collision.

(b) Tangles plus matter around them move like almost free particles and string follows: this would however still help the blackholes to find each either. The observed collisions can be modelled as a formation of gravitational bound state in which the blackholes rotate around each other first.

The latter option seems to be more natural.

2. Do the observed black-hole like entities correspond to quasar like objects or their time reversals (more like ordinary blackholes). The unexpectedly large masses would suggests that they have not yet lost their mass by thickening as stars usually so that they are analogs of quasars. These objects would be cosmic string eaters and this would also favour the collisions of blackhole like entities associated with the same cosmic string.

3. This picture would provide a possible explanation for the evidence for gravitational echoes and evidence for magnetic fields in the case of blackholes formed in the fusion of blackholes in LIGO [L29] (see `http://tinyurl.com/y79yqw6q`). The echoes would result from the repeated reflection of the radiation from the inner blackhole like region and from the ring bounding the accretion disk.

   Note that I have earlier proposed a model of ordinary blackholes in which there would be Schwartschild radius but at some radius below it the space-time surface would become Euclidian. In the recent case the Euclidian regions would be however associated only with wormhole contacts with Euclidian signature of metric bounded by light-like orb its of partonic 2-surfaces and might have sizes of order Compton length scaled up by the value of h_{eff}/h for dark variants of particle and therefore rather small as compared to blackhole radius.

4. The latest news tells that 28 August 2019 LIGO observed two gravitational waves with a time lapse of 21 minutes in the same direction (see `http://tinyurl.com/yxpblf4p`). The events are christened as S190828j and S190828l. This suggests that the signals coule originate from same event. Gravitational lense effect could be one explanation.

   TGD suggests an alternative explanation based on the notion of gravitational flux tubes. Magnetic flux tubes, in particular gravitational flux ones, form loops. The later signal could have spent 21 minutes by rotating around this kind of loop. This rotation can occur several times but the intensity of signal is expected to diminish exponentially if only a constant fraction remains in loop at each turn.

   This sticking of radiation inside magnetic loops predicting echo like phenomenon is a general prediction of TGD and I have considered the possible occurrence of this phenomenon for cosmic gamma rays arriving in solar solar system in a model for solar cycle [L90] (see `http://tinyurl.com/y2nltfpz`.

   This kind of repetition of the signal has been observed already earlier for gravitational waves and has been dubbed "blackhole echoes" (see `http://tinyurl.com/yahxk2ca`this) but in a time scale of .1 seconds (fundamental bio-rhythm by the way). I have considered possible TGD based explanations of blackhole echoes in [L29] (see `http://tinyurl.com/y79yqw6q`) and [K83] (see `http://tinyurl.com/yy5f6wll`).

   The two time scales differ by four orders of magnitude but one cannot exclude same explanation. With light velocity Earth sized loop would correspond to a time lapse of about .1 seconds. Light travels in 21 minutes over a distance of 378 million kilometers to be compared with astronomical unit AU = 150 million kilometers defining the distance of Earth from Sun. Therefore loops in the scale of Earth's orbit around Sun could be involved and perhaps associated with the magnetic body of the collapsed system. .1 seconds defining the time scale for the blackhole echoes in turn corresponds to a circumference of order Earth circumference.

15.4 Appendix: Explicit formulas for the evolution of cosmological constant

What is needed is induced Kähler form $J(S^2(X^4)) \equiv J$ at the twistor sphere $S^2(X^4) \equiv S^2$ associated with space-time surface. $J(S^2(X^4))$ is sum of Kähler forms induced from the twistor spheres $S^2(M^4)$ and $S^2(CP_2)$.

$$J(S^2(X^4) \equiv J = P[J(S^2(M^4)) + J(S^2(CP_2))] \ , \qquad (15.4.1)$$

where P is projection taking tensor quantity T_{kl} in $S^2(M^4) \times S^2(CP_2)$ to its projection in $S^2(X^4)$. Using coordinates y^k for $S^2(M^4)$ or $S^(CP_2)$ and x^μ forS^2, P is defined as

$$P : T_{kl} \rightarrow T_{\mu\nu} = T_{kl}\frac{\partial y^k}{\partial x^\mu}\frac{\partial y^l}{\partial x^\nu} \ . \qquad (15.4.2)$$

For the induced metric $g(S^2(X^4)) \equiv g$ one has completely analogous formula

$$g = P[g(J(S^2(M^4)) + g(S^2(CP_2))] \ . \qquad (15.4.3)$$

The expression for the coefficient K of the volume part of the dimensionally reduced 6-D Kähler action density is proportional to

$$L(S^2) = J^{\mu\nu}J_{\mu\nu}\sqrt{det(g)} \ . \qquad (15.4.4)$$

(Note that $J_{\mu\nu}$ refers to S^2 part 6-D Kähler action). This quantity reduces to

$$L(S^2) = (\epsilon^{\mu\nu}J_{\mu\nu})^2\frac{1}{\sqrt{det(g)}} \ . \qquad (15.4.5)$$

where $\epsilon^{\mu\nu}$ is antisymmetric tensor density with numerical values +,-1. The volume part of the action is obtained as an integral of K over S^2:

$$S(S^2) = \int_{S^2} L(S^2) = \int_{-1}^{1} du \int_0^{2\pi} d\Phi \frac{J^2_{u\Phi}}{\sqrt{det(g)}} \ . \qquad (15.4.6)$$

$(u, \Phi) \equiv (cos(\Theta, \Phi)$ are standard spherical coordinates of S^2) varying in the ranges $[-1, 1]$ and $[0, 2\pi]$.

This the quantity that one must estimate.

15.4.1 General form for the embedding of twistor sphere

The embedding of $S^2(X^4) \equiv S^2$ to $S^2(M^4) \times S^2(CP_2)$ must be known. Dimensional reduction requires that the embeddings to $S^2(M^4)$ and $S^2(CP_2)$ are isometries. They can differ by a rotation possibly accompanied by reflection

One has

$$(u(S^2(M^4)), \Phi(S^2(M^4)) = (u(S^2(X^4), \Phi(S^2(X^4)) \equiv (u, \Phi) \ ,$$

$$\left[u(S^2(CP_2)), \Phi(S^2(CP_2))\right] \equiv (v, \Psi) = RP(u, \Phi)$$

where RP denotes reflection P following by rotation R acting linearly on linear coordinates (x,y,z) of unit sphere S^2). Note that one uses same coordinates for $S^2(M^4)$ and $S^2(X^4)$. From this action one can calculate the action on coordinates u and Φ by using the definite of spherical coordinates.

The Kähler forms of $S^2(M^4)$ resp. $S^2(CP_2)$ in the coordinates $(u = cos(\Theta), \Phi)$ resp.(v, Ψ) are given by $J_{u\Phi} = \epsilon = \pm 1$ resp. $J_{v\Psi} = \epsilon = \pm 1$. The signs for $S^2(M^4)$ and $S^2(CP_2)$ are same or opposite. In order to obtain small cosmological constant one must assume either

1. $\epsilon = -1$ in which case the reflection P is absent from the above formula (RP $\rightarrow$ R).
2. $\epsilon = 1$ in which case P is present. P can be represented as reflection $(x, y, z) \rightarrow (x, y, -z)$ or equivalently $(u, \Phi) \rightarrow (-u, \Phi)$.

Rotation R can represented as a rotation in (y,z)-plane by angle ϕ which must be small to get small value of cosmological constant. When the rotation R is trivial, the sum of induced Kähler forms vanishes and cosmological constant is vanishing.

15.4.2 Induced Kähler form

One must calculate the component $J_{u\Phi}(S^2(X^4)) \equiv J_{u\Phi}$ of the induced Kähler form and the metric determinant $det(g))$ using the induction formula expressing them as sums of projections of M^4 and CP_2 contributions and the expressions of the components of $S^2(CP_2)$ contributions in the coordinates for $S^2(M^4)$. This amounts to the calculation of partial derivatives of the transformation R (or RP) relating the coordinates (u, Φ) of $S^2(M^4)$ and to the coordinates (v, Ψ) of $S^2(CP_2)$.

In coordinates (u, Φ) one has $J_{u\Phi}(M^4) = \pm 1$ and similar expression holds for $J(v\Psi)S^2(CP_2)$. One has

$$J_{u\Phi} = 1 + \frac{\partial(v, \Psi)}{\partial(u, \Phi)} \quad . \tag{15.4.7}$$

where right-hand side contains the Jacobian determinant defined by the partial derivatives given by

$$\frac{\partial(v,\Psi)}{\partial(u,\Phi)} = \frac{\partial v}{\partial u}\frac{\partial \Psi}{\partial \Phi} - \frac{\partial v}{\partial \Phi}\frac{\partial \Psi}{\partial u} \quad . \tag{15.4.8}$$

15.4.3 Induced metric

The components of the induced metric can be deduced from the line element

$$ds^2(S^2(X^4) \equiv ds^2 = P[ds^2(S^2(M^4)) + ds^2(S^2(CP_2))] \quad .$$

where P denotes projection. One has

$$P(ds^2(S^2(M^4))) = ds^2(S^2(M^4)) = \frac{du^2}{1-u^2} + (1-u^2)d\Phi^2 \quad .$$

and

$$P[ds^2(S^2(CP_2))] = P[\frac{(dv)^2}{1-v^2} + (1-v^2)d\Psi^2] \quad ,$$

One can express the differentials $(dv, d\Psi)$ in terms of $(du, d\Phi)$ once the relative rotation is known and one obtains

$$P[ds^2(S^2(CP_2))] = \frac{1}{1-v^2}[\frac{\partial v}{\partial u}du + \frac{\partial v}{\partial \Phi}d\Phi]^2 + (1-v^2)[\frac{\partial \Psi}{\partial u}du + \frac{\partial \Psi}{\partial \Phi}d\Phi]^2 \quad .$$

This gives

$$\begin{aligned} &P[ds^2(S^2(CP_2))] \\ &= [(\tfrac{\partial v}{\partial u})^2\tfrac{1}{1-v^2} + (1-v^2)(\tfrac{\partial \Psi}{\partial u})^2]du^2 \\ &+[(\tfrac{\partial v}{\partial \Phi})^2\tfrac{1}{1-v^2} + (\tfrac{\partial \Psi}{\partial \Phi})^2 1-v^2]d\Phi^2 \\ &+2[\tfrac{\partial v}{\partial u}\tfrac{\partial v}{\partial \Phi}\tfrac{1}{(1-v^2)} + \tfrac{\partial \Psi}{\partial u}\tfrac{\partial \Psi}{\partial \Phi}(1-v^2)]dud\Phi \quad . \end{aligned}$$

From these formulas one can pick up the components of the induced metric $g(S^2(X^4)) \equiv g$ as

$$g_{uu} = \frac{1}{1-u^2} + (\frac{\partial v}{\partial u})^2\frac{1}{1-v^2} + (1-v^2)(\frac{\partial \Psi}{\partial u})^2] \ ,$$

$$g_{\Phi\Phi} = 1-u^2 + (\frac{\partial v}{\partial \Phi})^2\frac{1}{1-v^2} + (\frac{\partial \Psi}{\partial \Phi})^2(1-v^2) \quad (15.4.9)$$

$$g_{u\Phi} = g_{\Phi u} = \frac{\partial v}{\partial u}\frac{\partial v}{\partial \Phi}\frac{1}{(1-v^2)} + \frac{\partial \Psi}{\partial u}\frac{\partial \Psi}{\partial \Phi}(1-v^2) \ .$$

The metric determinant $det(g)$ appearing in the integral defining cosmological constant is given by

$$det(g) = g_{uu}g_{\Phi\Phi} - g_{u\Phi}^2 \ . \quad (15.4.10)$$

15.4.4 Coordinates (v,Ψ) in terms of (u,Φ)

To obtain the expression determining the value of cosmological constant one must calculate explicit formulas for (v,Ψ) as functions of (u,Φ) and for partial derivations of (v,Ψ) with respect to (u,Φ).

Let us restrict the consideration to the RP option.

1. P corresponds to $z \rightarrow -z$ and to

$$u \rightarrow -u \ . \quad (15.4.11)$$

2. The rotation R $(x,y,z) \rightarrow (x',y',z')$ corresponds to

$$x' = x, \quad y' = sz + cy = su + c\sqrt{1-u^2}sin(\Phi) \ , \quad z' = v = cu - s\sqrt{1-u^2}sin(\Phi) \quad (15.4.12)$$

Here one has $(s,c) \equiv (sin(\epsilon), cos(\epsilon)$, where ϵ is rotation angle, which is extremely small for the value of cosmological constant in cosmological scales.

From these formulas one can pick v and $\Psi = arctan(y'/x)$ as

$$v = cu - s\sqrt{1-u^2}sin(\Phi) \quad \Psi = arctan[\frac{su}{\sqrt{1-u^2}}cos(\Phi) + tan(\Phi)] \ . \quad (15.4.13)$$

3. RP corresponds to

$$v = -cu - s\sqrt{1-u^2}sin(\Phi) \quad \Psi = arctan[-\frac{su}{\sqrt{1-u^2}}cos(\Phi) + tan(\Phi)] \ . \quad (15.4.14)$$

15.4.5 Various partial derivatives

Various partial derivates are given by

$$\frac{\partial v}{\partial u} = -1 + s\frac{u}{\sqrt{1-u^2}}sin(\Phi) \ ,$$

$$\frac{\partial v}{\partial \Phi} = -s\frac{u}{\sqrt{1-u^2}}cos(\Phi) \ ,$$

$$\frac{\partial \Psi}{\partial \Phi} = (-s\frac{u}{\sqrt{1-u^2}}sin(\Phi) + c)\frac{1}{X} \ , \quad (15.4.15)$$

$$\frac{\partial \Psi}{\partial u} = \frac{scos(\Phi)(1+u-u^2)}{(1-u^2)^{3/2}}\frac{1}{X} \ ,$$

$$X = cos^2(\Phi) + [-s\frac{u}{\sqrt{1-u^2}} + csin(\Phi)]^2 \ .$$

Using these expressions one can calculate the Kähler and metric and the expression for the integral giving average value of cosmological constant. Note that the field equations contain S^2 coordinates as external parameters so that each point of S^2 corresponds to a slightly different space-time surface.

15.4.6 Calculation of the evolution of cosmological constant

One must calculate numerically the dependence of the action integral S over S^2 as function of the parameter $s = sin(\epsilon))$. One should also find the extrema of S as function of s.

Especially interesting values are very small values of s since for the cosmological constant becomes small. For small values of s the integrand (see Eq. 15.4.6) becomes very large near poles having the behaviour $1/\sqrt{g} = 1/(sin(\Theta) + O(s))$ coming from $\sqrt{g}$ approaching that for the standard metric of S^2. The integrand remains finite for $s \neq 0$ but this behavior spoils the analytic dependence of integral on s so that one cannot do perturbation theory around $s = 0$. The expected outcome is a logarithmic dependence on s.

In the numerical calculation one must decompose the integral over S^2 to three parts.

1. There are parts coming from the small disks D^2 surrounding the poles: these give identical contributions by symmetry. One must have criterion for the radius of the disk and the natural assumption is that the disk radius is of order s.

2. Besides this one has a contribution from S^2 with disks removed and this is the regular part to which standard numerical procedures apply.

One must be careful with the expressions involving trigonometric functions which give rise to infinite if one applies the formulas in straightforward manner. These infinities are not real and cancel, when one casts the formulas in appropriate form inside the disks.

1. The limit $u \to \pm 1$ at poles involves this kind of dangerous quantities. The expression for the determinant appearing in $J_{u\Phi}$ remains however finite and $J^2_{u\phi}$ vanishes like s^2 at this limit. Also the metric determinant $1/\sqrt{g}$ remains finite expect at $s = 0$.

2. Also the expression for the quantity X in $\Psi = arctan(X)$ contains a term proportional to $1/cos(\Phi)$ approaching infinity for $\Phi \to \pi/2, 3\pi/2$. The value of $\Psi = arc(tan(X)$ remains however finite and equal to $\pm\Phi$ at this limit depending on on the sign of us.

Concerning practical calculation, the relevant formulas are given in Eqs. 15.4.5, 15.4.6, 15.4.7, 15.4.8, 15.4.9, 15.4.10, and 15.4.15.

The calculation would allow to test/kill the key conjectures already discussed.

1. There indeed exist extrema satisfying $dS(S^2)/ds = 0$.

2. These extrema are in one-one correspondence with the zeros of zeta.

There are also much more specific conjctures to be killed.

1. These extrema correspond to $s = sin(\epsilon) = 2{-}k$ or more generally $s = p^{-k}$. This conjecture is inspired by p-adic length scale hypothesis but since the choice of evolution parameter is to high extent free, the conjecture is perhaps too spesific.

2. For certain integer values of integer k the integral $S(S^2)$ of Eq. 15.4.6 is of form $S(S^2) = xs^2$ for $s = 2^{-k}$, where x is a universal numerical constant.

   This would realize the idea that p-adic length scales realized as scales associated with cosmological constant correspond to fixed points of renormalization group evolution implying that radiative corrections otherwise present cancel. In particular, the deviation from $s = 2^{-d/2}$ would mean anomalous dimension replacing $s = 2^{-d/2}$ with $s^{-(d+\Delta d)/2}$ for $d = k$ the anomalies dimension Δd would vanish.

   The condition $\Delta d = 0$ should be equivalent with the vanishing of the dS/ds. Geometrically this means that $S(s)$ curve is above (below) $S(s) = xs^2$ and touches it at points $s = x2^{-k}$, which would be minima (maxima). Intermediate extrema above or below $S = xs^2$ would be maxima (minima).

Chapter 16

Solar Surprise

16.1 Introduction

Sabine Hossenfelder gave a link to a popular article (see http://tinyurl.com/y6mpuggu) telling about rather shocking new findings about Sun. There are 5 times more gamma rays than expected and the spectrum has a deep and narrow dip in 30-50 GeV range. Spectrum continues to much higher energies than expected, at least up to 100 GeV. One proposal is that there could be dark matter in the interior of Sun yielding the gamma rays but is unclear how they could get to the surface without experiencing the same fate as the ordinary gammas from nuclear reactions.

There is also a correlation with sunspot cycle (see http://tinyurl.com/aqw2hmz). Basic data and observations related to correlations with the solar cycle are described in the article [E20] (see http://tinyurl.com/yxajyzp8 and [E18] (see http://tinyurl.com/y2qlaaa2).

1. Power law spectrum is harder than for cosmic rays: spectral indices are $n = -2.2$ and $n = -2.7$ respectively (one has power law behavior E^n for the flux). The spectral intensity at 100 GeV is very nearly the maximum flux predicted by the model assuming that reflection of cosmic gamma rays explains the gammas.

2. The spectrum has two components: poloidal component farther from equator and equatorial component largest during sunspot minimum. The equatorial contribution is maximal at solar minimum. The spectral index of the equatorial contribution is harder and higher energies are present. The energy range is maximal during spot minima. Gamma flux is reduced during sun spot maxima.

How the observed gamma rays could be produced in TGD Universe?

1. Gamma rays cannot cannot be produced by nuclear reactions as ordinary gammas since nuclear energy scale is much below the scale of gamma rays extending to 100 GeV at least. Even the hadronic energy scale is too low. The gamma rays could be cosmic rays having already high energies: the spectral indices are however different. This leaves acceleration of charged particles producing gamma rays as the most plausible mechanism irrespective of whether the charged particles come from solar core or are cosmic rays.

2. Dark magnetic flux tubes are basic notion of TGD and could serve as the channels along which charged particles could propagate to the surface without losing their energies in collisions. An interesting hypothesis considered already earlier is that solar magnetic field are what I call wormhole magnetic fields [K129] consisting of closed monopole flux tubes with flux and return flux at different space-time sheets connected by tiny wormhole contacts. This would predict that the flow is not evenly distributed but reflects the structure of the flux tube distribution. If the flux tubes have same M^4 projection they cause no effects on test particle and behave like dark energy creating only long range gravitational fields.

   Charged particles could accelerate in the electric field of flux tube as they travel along flux tubes and generate gamma rays by some mechanism. The energy would be the increment of Coulomb energy if dissipation is neglected. A simple modification of flux tube type extremals allows the presence of helical magnetic and electric fields along flux tube orthogonal to each other. I have proposed the same mechanism to explain the gamma rays and high energy electrons at MeV energies associated with lightnings [K24]: in standard physics framework dissipative losses do not allow them.

3. What could be the production mechanism of gamma rays? If flux tubes have sharp kinks, charged particles should experience large deceleration in the kinks and could emit high energy gamma ray in the process. The highly relativistic charge particle itself could leak out (one cannot exclude nuclei from solar core). Large deflection angles however requires transfer of momentum also to flux tube degrees of freedom.

4. What could be the origin of the tip around 30-50 GeV? If the acceleration takes place in the electric fields assignable to the closed flux tubes assignable to solar dipolar magnetic field, the charged particle could travel several times around the loop giving rise to several energy bands explaining the gap and suggesting several of them. The flux loop would act as a particle accelerator.

5. The charged particles could be provided by the solar core or they could be cosmic rays. The order of magnitude for gamma ray intensity is 5 times larger than in cosmic ray model, which encourages the identification as cosmic rays (see `http://tinyurl.com/psdp99h`). The origin of cosmic rays is however also a mystery and neutron stars, supernovae, active galactic nuclei, quasars, and gamma-ray bursts have been proposed as sources of cosmic rays.

   A possible mechanism producing cosmic rays could be pair-annihilation of pairs of M_{89} pions with mass about 70 GeV [K73] to gamma ray pairs or charged particles with energies 35 MeV. Could the dip observed in the energy range around 30-50 GeV somehow relate to the charged decay products of M_{89} pions accelerating in the electric fields of flux tubes? Could the dip be gap without the decays of M_{89} pions?

   In TGD the model for the formation of galaxies, quasars, and active galactic nuclei, and even stars, and planets relies on the formation of looped tangles along long thickening cosmic strings with topology resembling that of dipole magnetic field. Galactic matter would be produced by the decay of the flux tube energy to particles as analog of the decay of inflaton field. This could generate both charged particles and gamma radiation in the solar core and in neutron stars. The acceleration could be much more effective due to the strong magnetic and electric fields involved. Also charged particles can leak out from the flux tubes and cosmic rays could be produced by this mechanism. Cosmic rays could move along the highways defined by the long magnetic flux tubes connecting galaxies.

The understanding of the correlations with the solar cycle requires a model for the polarization flip. One can consider several options but the model based on reconnection splitting dipole loops from the flux tube tangle representing the analog dipole field is the simplest one. The simplest variant of the model requires zero energy ontology (ZEO) and quantum coherence at dark flux tubes in solar length scales and that long galactic string defines wormhole magnetic field with two sheets (type I and II) connected by wormhole contacts separated from each other in the sense that M^4 projections are disjoint.

1. Let us denote the numbers of dipole loops of type $i = I, II$ by n_i. Assume that in the initial situation one has $(n_I = n_{max}, n_{II} = 0)$. B as maximum value B_{max}. The arrows of time at the two sheets are assumed to be opposite during cycles.

2. The transition leading $B = B_{max}$ to $B = 0$ would be "big" state function reduction (BSR) changing the arrow of time at sheets of both type I and II. BSR would generate maximum number of new dipole flux loops of type II: $n_{II} \to n_{max}$ so that one has $n_I = n_{II} = n_{max}$ and $B = 0$.

3. After that dipole loops of type I begin to split away by reconnections in "small" state function reductions (SSRs) so that n_I decreases. They split further in pieces and leak out from Sun whereas n_{II} remains unchanged since it corresponds to the passive boundary of CD - this is essential. Net B increases until one has $B = -B_{max}$.

4. Next occurs BSR generating maximum number of new flux loop portions of type I leading $n_I = n_{II} = n_{max}$ and $B = 0$ and same is repeated except that now n_{II} decreases.

5. One can understand the sunspot cycle in terms of split dipole loops leaving the Sun: their intersection with the solar surface would define sunspot pair and the distance of members of the pair would decrease to zero during the cycle.

The model leads to rather dramatic predictions.

1. Various magnetic structures are predicted to appear in pairs with members related by an approximate Z_2 symmetry. For the magnetic field of the Sun this symmetry would be naturally inversion symmetry with respect to the dipole core. Also reflection symmetry can correspond to Z_2. This symmetry should be universal and the predictions are in sharp contrast with the locality principle of classical physics. One could even understand the mysterious "Axis of Evil" associated as anomaly of CMB and apparently giving special role for solar system (see `http://tinyurl.com/yb6nabw4`).

2. Also unexpected connections with TGD inspired views about biology and consciousness emerge. Magnetic body (MB) is the intentional agent in living system Z_2 realized as inversion could related the parts of MB in the interior and exterior of Earth's dipole core: could the idea about intra-terrestrial life introduced originally half-jokingly [**?**]ake sense - at the level of MBs at least? ZEO based theory of consciousness predicts that conscious entities can have both arrows of time and death means reincarnation with opposite arrow of time. But where do these ghostly selves with opposite arrow of time reside? Could Z_2 - possibly realized as inversion - relate these selves to each other.

16.2 TGD based model for the solar magnetic field, solar cycle, and gamma ray emission

An attempt to understand the situation in TGD framework. One can of course consider several alternative models but the constraints from solar spot cycle and observation exclude most proposals. The following proposal seems to survive the most obvious killer arguments.

16.2.1 How the magnetic fields of galaxies and stars are generated?

To get a general enough perspective about the generation of time dependent B, one must consider the general model for how the magnetic fields of galaxies, stars, and planets are generated.

1. The magnetic fields of galaxies, stars, and planets would have formed as tangles along cosmic strings thickened to magnetic flux tubes carrying monopole flux. . Tangles would be formed by the flux tubes forming knotty structures with flux tubes defining analog for subset of flux lines of dipole field. The flux tubes can organize in several ways.

   Cosmic strings would be wormhole magnetic fields carrying opposite monopole fluxes at space-time sheets connected by wormhole contacts (in principle it is possible to consider also single-sheeted monopole fluxes). I will talk about sheets of type I and II. If the flux tubes are on top of each other in the sense that M^4 projections are identical, the magnetic field experienced by test particle touching both flux tubes would vanish. The fact that the energy of the flux tubes gives rise to gravitational field can be used to argue that one can talk about dark energy in this case. The flux tubes can be connected by extremely short wormhole contacts at places, where they are on top of each other. If the Euclidian wormhole contacts can have tube-like M^4 projection, they would be also flux tube like.

2. It is not clear whether the flux tubes of both type I and II are inside the volume bounded by Earth's B or whether second type of flux tubes are outside Earth. This gives rise to several options for how B can be realized as flux tube field and how the time dependence of B is obtained.

3. One can imagine two options, which apply to both types of fluxes separately. For the most general option (Option I) the incoming flux tube can divide to smaller flux tubes going both to the interior and exterior of the dipole core. The extreme options (Option II and II) are that it flows entirely to the dipole core or divides to flux tubes travelling outside the dipole core (this situation is analogous to hydrodynamical flow past obstacle). It will be found that option II is most attractive one.

4. Incoming flux long tube at given sheet forms a tangle. Consider first the tangle formed by the incoming long flux tube of given type at fixed space-time sheet, for definiteness restriction the consideration to flux of type I..

   (a) For Option I the neighbouring flux portions of the flux tube portions inside and outside dipole core can have random orientations: this would be like random spin system without any magnetization. The average observed field would be random. For Options II and III this kind of situation is not possible.

   (b) The flux tube in the tangle can also arrange like spins in spontaneous magnetization so that neighboring portions of the flux tube are parallel both inside the core and outside it. The flux and return flux would be at different sides of the dipole core. This could give rise to an analog of say dipole field. For instance, dipole core could correspond to a spherical volume bounded by the Earth's surface. The extreme situation would correspond to Option II or III.

5. For Option I the polarity of observed B could be due to a process analogous to spontaneous magnetization, whose degree can vary. The degree of magnetization would be determined by the ratio of the incoming fluxes going to the interior and exterior of the dipole core. The total flux Φ flowing inside dipole core is $\Phi = (p_1 - p_2)\Phi_{in}$, where p_i are the fractions of incoming fluxes going inside the dipole core and outside it. If the ratio equals to unity the net B vanishes in long enough scales. For Options II and II one cannot have time varying B unless the number n_i, $i \in \{I, II\}$ of dipole loops can vary.

   Polarization reversal could be a dynamical process. For the analog of hydrodynamical flow the portions of the flow going through the dipole core and its exterior could change, and the fraction of these portions is the parameter determining the strength B. Oscillating B would mean oscillation of this fraction. Also the numbers n_i change and induce change of B.

6. If the flux tubes of both types are in the volume carrying B, more possibilities arise for Option I since the flux tube portions of type I and II can have magnetizations of varying degree and these can be parallel or opposite inside (outside) dipole core.

7. For Options II and III the magnetization direction cannot vary unless n_i can change and the total average magnetic field would vanish for $n_I = n_{II}$. n_i can however change if dipole loops split away by reconnection. It turns out that option II is the most promising one.

16.2.2 A model of solar magnetic field in terms of monopole flux tubes

The model relies on the notion wormhole magnetic field with flux tubes carrying electric fields, the notion of reconnection, and the theory of quantum measurement based on zero energy ontology (ZEO) [K76] and extending to a theory of consciousness [L52].

Also hydrodynamic analogy, the analogies with ferromagnetic hysteresis cycle, spontaneous magnetization, and de-magnetization, the analogy with the Meissner effect explaining solar spots as magnetic flux branching from the dipole axis of solar magnetic field, and Lenz principle (induction law) stating that magnetic field generates ohmic current in turn generating magnetic field opposing the change of the magnetic field , are used as guidelines.

1. One can argue that the magnetic fields in question correspond to flux tube portions carrying monopole flux. The empirical support for the hypothesis comes from the fact that monopole fluxes need no currents to generate them. Cosmology is indeed full of long range magnetic fields whose presence is mystery in Maxwellian electrodynamics.

2. Interaction of two kinds of magnetic fields would be involved. The first magnetic field identified as solar magnetic field, call it B, is assumed to have flux tubes wormhole magnetic field carrying monopole fluxes. No current is needed to create the magnetic flux: something impossible for ordinary Maxwellian fields. Note also that the cross section of flux tube is closed 2-D surface. One could call B topological magnetic field. Mathematically B could be seen as an analog of the external magnetic field H generating as a response total magnetic field as a sum of H and magnetization M.

   Second magnetic field, call it B_1 would be Maxwellian and generated by Faraday induction. By Lenz principle it opposes the change of the magnetic flux associated with B and has roughly the same direction. B_1 would correspond to M. In the proposed framework the induced currents j would generate B_1 and it would be regarded as secondary rather than primary field.

   Remark: The flux tubes of B_1 would be obtained from closed string like objects with CP_2 projection which geodesic sphere S^2 by replacing S^2 with disk D^2, by deforming to get flux tube, and gluing it to a large background space-time sheet along D^2. The current creating B_1 would be associated with the boundary of D^2.

One cannot of course exclude the Maxwellian option for B.

1. The portion of flux tubes of B identifiable as analog of the dipole core of Maxwellian dipole field would consist of particles with magnetic moment whereas for monopole flux no magnetic moment is needed. Magnetic moment could be due to spin or orbitals motion.

   Remark: One could wonder whether quantum-classical correspondence (QCC) requires that the monopole flux has as quantum counterpart magnetization representable in terms of fermions.

2. The contribution of the spin to magnetic field is rather small so that the idea about spontaneous magnetization at flux tubes defining dipole does not look promising. Note however that the large value of h_{eff} together with proportionality of $\mu \propto \hbar_{eff}/m$ could change the situation. Macroscopic quantum coherence making possible quantum states with macroscopic radius for the orbits could be considered and would conform with the idea that the flow of currents generates B. B could be of course generated also classically.

16.2.3 Are wormhole magnetic fields really needed?

The additional assumption is that wormhole magnetic fields involving two space-time sheets connected by wormhole contacts appear in the volume containing B. More generally, fundamental magnetic fields would be wormhole magnetic fields. This additional hypothesis is necessary in the recent model of elementary particles and p-adic fractality suggests that the property holds true also astrophysical scales.

1. In elementary particle scales monopole flux tubes associated with wormhole magnetic fields must be closed and involve return flux along second space-time sheet. If the two space-time sheets have same M^4 projection, the test particle touches both sheets and experiences essentially no gauge fields. At QFT limit one would have no fields. Therefore the M^4 projections of the flux tubes at the two sheets must be disjoint in order that one has normal magnetic field in operational sense.

   The energies of both flux tubes however sum up and the wormhole flux tube pair has long range gravitational interactions. The attractive interpretation is that if the volumes in which the sheets have same M^4 projection, the energy of flux tube pair corresponds to dark energy. The portions giving rise to tangles in which the flux sheets have separate projections give rise to ordinary matter. This would give rise to galaxies, stars, and planets and even smaller objects in various scales. Flux tubes would thicken and their energy would decay to ordinary and dark matter.

2. Wormhole magnetic fields could define pairs of systems. The understanding of the geometric correlates for the hierarchy of Planck constants have already led to the realization that many-sheeted space-time means that one space-time surface can be regarded as n_1-fold covering

of CP_2 and n_2-fold covering of M^4 such that one has $h_{eff}/h_0 = n = n_1 n_2$ holds true. For n_1-fold covering of CP_2 the sheets can be disjoint regions of M^4. Although the regions are disjoint, they are physically closely correlated. This is classical correlate for macroscopic quantum coherence coded also by the large value of n.

For $n_1 = 2$ one obtains the simplest pairs. Also even values of $n_1 = 2m_1$ are of course and would describe a pair of structures with m_1 components. The components would be most naturally flux tubes fusing to larger flux tube fractally.

3. This view becomes understandable if one takes CP_2 coordinates or $M^2 \times CP_2$ coordinates as a coordinate system so that the roles of space-time and fields are changed or partially changed. At the level of wormhole contacts the change of the roles of M^4 and CP_2 is necessary. For string like objects $M^2 \times S^2$ replaces M^4. This corresponds to that part of TGD, which does not allow description in terms of GRT.

Playing with the ideas generates questions and new ideas, not always realistic. At this time the question is following.

1. Could the Euclidian region associated with wormhole contact and connecting wormhole throats at the two sheets connect two disjoint, even distant regions of M^4? If so, the wormhole contact would be analogous to Einstein-Rosen bridge except that it has Euclidian signature of the induced metric.

   Could one identify the wormhole contact as a space-time correlate for entanglement or pre-requisite for it? There would be no signal involved since in Euclidian space-time regions one cannot talk about propagation. Euclidian flux tubes are in central role in p-adic mass calculations [K66] but they are extremely short.

   I have assumed that time-like flux tubes can serve as correlates of entanglement. Could one can think that Minkowskian flux tubes would allow classical signalling and Euclidian flux tubes would serve as classical correlates for entanglement. Could both aspects be involved with quantum communications?

   Remark: One can obtain Euclidian space-time region from piece of M^4 by performing a large enough deformation in CP_2 directions and also this could give rise to Euclidian induced metric. One can also have cosmic string with piece of M^2 as string world sheet and deformed such that one has flat E^2. The deformation of this string world sheet would represent Euclidian flux tube.

2. Here one must be however extremely cautious. Hitherto I have regarded shortness of flux tubes as obvious, and might have been right. One cannot however exclude the possibility that also Euclidian wormhole contacts are involved but they do not seem to be necessary: one could have wormhole magnetic fields with wormhole contacts only in the regions where M^4 projections overlap. All depends on the properties of preferred extremals.

16.2.4 How to understand the solar cycle?

Sunspot cycle (see http://tinyurl.com/y2qlaaa2) has period of 22 years and consists of two 11 year half-periods during which opposite polarity of B. The understanding of the mechanism causing the flip of the polarity looks the most difficult part of the problem - at least from TGD point of view. Each half cycle starts from a situation in which the dipole part of B vanishes and sunspots appear at opposite sides of equator at symmetrically related positions at mid-latitudes (about 30 degrees from equator).

Sunspots (see http://tinyurl.com/y2qlaaa2) carry intense magnetic fields (fields strength is about 2 Tesla in the vicinity of Sunspot according to Wikipedia) and they have lower temperature than surroundings due to the magnetic pressure. During the half-cycle Sunspots drift towards equator and maintain their polarity. The diagrammatic description of the time evolution at the solar surfaces is known as butterfly diagram. The natural interpretation is that the sunspots at opposite sides are connected by flux loops.

During the cycle the dipole field with opposite polarity as compared to previous cycle is generated and towards the end of the cycle there is a period in which no sun-spots are observed:

they would be near equator if present. The spots could be present but the density of elementary flux tubes could be too low to give rise to average field strength enough to cause an observable reduction of temperature.

Polarity reversal of B

What could be behind polarity reversal. First some guiding ideas.

1. An analogy with ferromagnetic hysteresis circuit suggests itself. B generates B_1 having opposite direction. When the value of B_1 is critical it induces a phase transition in which the direction of Kähler flux is changed at flux tubes. Second half of the 22 year sunspot cycle would start. The ohmic current j generated by B would change and this would induce the magnetic turbulence accompanying solar spots.

   This analogy is not quite complete since the generation of B with opposite sign occurs slowly whereas the vanishing of magnetic field isa fast process. De-magnetizing phase transitions seems therefore a natural analog for the dis-appearance of B.

2. What the analog of spin flip means is highly non-trivial question when the size of the analog of spinning particle is of the size scale of Sun. Quantal and topological effect in solar scales could be in question and involve both TGD view about space-time and fields as well as hierarchy of Planck constants as description of dark matter. The model to be described in the sequel applies universally in TGD Universe and leads to quite dramatic and testable implications.

Consider next general TGD inspired ideas relating to the change of the polarity of B in TGD framework. A general model based on the formation of flux tube tangle as a representation of the say dipole field looks like a safe starting point and provides also a general model for the change of the polarity. An essential element is the distribution of incoming flux of long cosmic string like object to fluxes going through the interior and exterior of the dipole core and return back through exterior and interior. The fractions going through interior and exterior determine the strength of observed B. Whether both kinds of flux tubes are present or not, depends on model.

The first model, call it Model I, is classical. Now one could do using only single flux tube type, say type I, which however must divide to flux tubes travelling both inside and outside the dipole core.

1. The decay of B would correspond to option I involving the change of fractions p_1 and $p_2 = 1-p_2$ of the flux tube portions going through the dipole core reducing the parameter p_1-p_2 to zero. The permutations of flux tube portions inside and outside core must lead to $p_1-p_2 = 0$ and one expects that this process continues and changes the sign of $p_1 - p_2$ and therefore induce polarization reversal. The duration of the process taking $p_1 - p_2$ to zero is rather short as compared to the duration of the half-cycle. The duration of the sunspot minimum is about 10 per cent of that for the entire half cycle. In the hydrodynamical analogy the process would be redistribution of the incoming flow and could be modelled phenomenologically as a change of flow resistances associated with the two channels involved.

2. This model does not involve reconnection process and does not provide any obvious explanation for the appearance of sunspots nor for the reconnection process associated with the reversal of the polarization of B. Therefore Model I is not promising.

Second model, call it Model II, is quantum mechanical and involves ZEO in an essential manner and one could assume that incoming flux tube enters to the dipole core entirely (option II).

1. Dipole winding number n_i characterizes the situation for a given type of flux tube. The larger the value of n_i, the larger the dipole strength. n_i could change by reconnection process in which entire dipole loop reconnects and snips away. This followed by further splitting to flux loops would correspond to the emission of magnetic loops from the Sun.

   The opposite process would correspond to a fusion of flux loop with a long flux loop but looks thermodynamically implausible. Also a fusion of a short flux loop with long flux loop and the growth of the reconnected part to large dipole loop looks implausible.

2. Could ZEO based quantum TGD allowing temporary time reversals come in rescue? At dark space-time sheets one can indeed imagine the possibility of time reversals. Ordinary matter would be controlled by dark matter with larger value of $h_{eff}/h_0 = n$ serving as an IQ in TGD inspired theory of consciousness, and would be forced to follow the leader in conflict with its thermodynamical instincts. Could the process involve "big" state function reduction (BSR) and could the dominance of flux tubes of type I and II correspond to different arrows of time at the level of dark flyx tubes? Reconnections for flux loops of say type II would occur in time direction opposite to the standard direction of time but second law would hold true in generalized sense.

3. The simplest option is that all incoming flux enters to the interior of the dipole core ($p_{2,I} = 0$ identically) or to its exterior ($p_{1,I} = 0$) identically. The first looks more plausible. The integers n_i, $i = \{I, II\}$ characterize the numbers of dipole flux loops carrying magnetic fields with opposite polarizations. Dipole strength is proportional to $n_I - n_{II}$. The arrows of time at the two sheets are assumed to be opposite for flux tube of type I and II.

4. Consider now a model for the first half-cycle.

   (a) Assume for definiteness that in the initial situation one has ($n_I = n_{max}, n_{II} = 0$). B as maximum value B_{max}.

   (b) The transition leading $B = B_{max}$ to $B = 0$ would be "big" state function reduction (BSR) changing the arrow of time at sheets of both type I and II. BSR would generate maximum number of new dipole flux loops of type II: $n_{II} \rightarrow n_{max}$ so that one has $n_I = n_{II} = n_{max}$ and $B = 0$.

   This transition is clearly a quantum analog of spontaneous magnetization in sector II. Could one say that a spontaneous magnetization already present in sector I induces opposite spontaneous magnetization in sector II?

   Quantum classical correspondence (QCC) inspires the question about there is in the fermionic sector genuine spontaneous magnetization involving fermion spins. Could a formation cyclotron condensate of spin zero Cooper pairs with members at flux tubes of type I and II and having opposite spins accompany this process?

   (c) After that dipole loops of type I begin to split away by reconnections in "small" state function reductions (SSRs) so that n_I decreases. They split further in pieces and leak out from Sun. Net B increases until one has $B = -B_{max}$. This process is analogous to gradual decay of magnetization.

   (d) What looks strange that n_{II} would remains unchanged during this process. In ZEO this makes sense: it would corresponds to the passive boundary of causal diamond (CD). One would have two CDs having common portion of boundary, call it δCD. Since the arrows of time are opposite, $\delta CD \subset \delta CD_{II}$ would be passive and experience generalized Zeno effect whereas $\delta CD \subset \delta CD_I$ for CD_I would be active experiencing gradual decay of magnetization in the sequence of "small" state function reductions (SSRs).

   (e) Topologically one can understand the sunspot cycle in terms of split dipole loops leaving the Sun: their intersection with the solar surface would define sunspot pair and the distance of members of the pair would decrease to zero during the cycle.

5. The model for the second half-cycle is identical. First occurs BSR generating maximum number of new flux loop portions of type I leading $n_I = n_{II} = n_{max}$ and $B = 0$ and same is repeated except that now n_{II} decreases.

The classically highly counter-intuitive aspect of this picture is that dipole loops would appear in BSR as quantum leap in astrophysical scales. There would be no continuous time evolution generating additional dipole loops. Their dis-appearance by reconnections would correspond to classical time evolution. If one performs time reversal for thermodynamic intuition, there is nothing mystical involved.

Model II looks to me more promising -if not even the only possibility - although conservative colleague can criticize it for the speculative new physics features: these features are however basic elements of new physics predicted by TGD.

Sunpots as intersections of split dipole flux loops with the Earth's surface?

How could sunspots be understood in the picture suggested by Model II?

1. BSR would induce the cancellation of B. Sunspots should emerge after the cancellation and serve as a signature of BSR inducing change of the arrow of time at flux tube space-time sheets. The usual statement is that the density of the elementary flux tubes composing the the split flux loop is high enough the average magnetic pressure lowers the temperature so much that the solar spot becomes visible.

   Could the local reduction of temperature inside sunspots, something not expected in the naïve thermodynamical thinking be forced by the change of the arrow of time at dark flux tubes? One would have leveling of temperature differences but in opposite time direction induced by dark flux tubes having arrow of time opposite to the standard one: by dark flux tubes of type I during first half-cycle and flux tubes of type II during second half-cycle.

2. The appearance of sunspots would relate naturally to the reconnection process leading to the disappearance of the dipole loops Do the snipped flux loops, which can split further to pieces eventually leaving Sun, intersect its surface at the sunspots so that the formation of sunspot and its disappearance would correspond to a splitting of closed dipole loop by reconnection and further splitting to smaller loops.

   The motion of sunspots towards equator would correspond to the outwards motion of the split flux dipole loop and solar spots would represent its intersection with solar surface. This also explains why the number of sunpots is gradually reduced during the half-cycle.

3. The fact that sunspots emerge first at latitudes $\pm\pi/6$ means that the split dipole flux loop intersects Earth's surface at positions with distance $h = R_E/2$ from equator. Since the distance is reduced after that, the outward motion of the loop requires that dipole core has height smaller than R_E.

Also in the case of Earth's magnetic field an analogous quantum picture might apply [L15] and solar spots might have "Earth spots" as magnetic anomalies. What is fascinating that the reversals of the Earth's magnetic field would be quantum processes in the scale of entire Earth and the magnetic field would go to zero instantaneously. What this means for living systems is an interesting question to ponder.

Does the polarity inversion involve spatial inversion?

Assume that the flux tubes correspond to monopole flux tube, which defines two-sheeted wormhole magnetic field. There is a strong temptation to assume that the members of the pairs defined by portions of flux tubes of given type (I or II) in the interior and exterior of dipole core are related by an approximate symmetry. If so, one would have doubles or mirror pairs of systems. What kind of symmetry polarity inversion for the solar B could correspond?

1. Assume that the two flux tube sheets of wormhole magnetic field have M^4 projections with empty intersection. Polarization reversal could permute the positions M^4 projections of the two sheets of flux tubes turning the direction of the magnetic flux. If the space-time surface representable as a map from CP_2 to M^4, the flip could be understood as a reflection in CP_2 degrees of freedom permuting the M^4 images and represented also as a reflection or inversion in M^4. In adelic physics [L50, L49] Z_2 has interpretation as subgroup of Galois group.

2. Could the solar magnetic field be doublet structure mapped to itself under Z_2? The identification of the pair as being formed by symmetry related parts of the flux dipole tubes in the interior of the dipole core and outside it is what comes naturally in mind. The symmetry could be realized as inversion with respect to the surface of dipole core mapping inside and

outside to each other. Inversions are indeed symmetries of Maxwell's theory, gauge theories, and of twistor Grassmannian approach. For the magnetic field of an infinitely long cylinder carrying rotating current at its surface the symmetry is exact. Also for $n_1 = 2m_1$ m_1 could correspond to a subgroup of CP_2. One would have double of bundles formed from m_1 flux tubes: dipole flux tuve consisting of m_1 elementary flux tubes.

3. The symmetry involved need not always be inversion. It could be also spatial reflection. The possibility of higher values of $n = n_1 n_2$, $n_1 = 2m_1$ suggests the possibility of long range correlations between m_1 pairs in astrophysical scales manifesting themselves quite concretely.

4. The representability of the group permuting flux tubes as finite discrete subgroups of $SO(3)$ acting as symmetries of Platonic solids would be very natural, and one can ask whether the appearance of Platonic solids in biology reflects this. This might allow to get some idea about why icosahedral model of harmony in terms of Hamiltonian cycles leading to the notion of bio-harmony predicts correctly genetic code [L5].

16.2.5 Trying to understand solar gamma ray spectrum in TGD Universe

One can try to understand the observations about gamma rays [E20, E18] (see `http://tinyurl.com/yxajyzp8` and `http://tinyurl.com/y2qlaaa2`) in the proposed picture. Some kind of acceleration mechanism suggests itself strongly.

1. An electric field associated with flux tubes with helical magnetic field is the simplest option. TGD allows simple deformations of flux tube like solutions [K62] in which Kähler magnetic and electric fields are orthogonal and helical and one can hope that they define preferred extremals.

   What about the electric force experienced by a test particle when the flux tubes of type I and II having same M^4 projection? The identification these objects in terms of dark energy would suggests that also the net electric force cancels and this kind of flux tube pair serves as a kind of superconducting wire.

2. If the flux tubes and gamma rays are dark with large $h_{eff}/h_0 = n = n_1 n_2$, they can propagate without interactions with ordinary matter. The dissipation would be solely due to curvature, in particular the kinks of the flux tube but would not be present at rectilinear portions of the flux tube. Therefore the amount of dissipation would be small.

   Forgetting the losses caused by the curvature of the flux tube, there would be maximum energy $E = ZeV$, V the voltage along flux tube section to which the particles such as protons can be accelerated, and this would define cutoff energy for the emitted gamma rays. I have proposed that this kind of model explains also the gamma rays associated with lightnings [K24].

3. The dip in the spectrum suggests at least two energy scales for accelerated particles emitting gammas as brehmstrahlung and defining the endpoint of the brehmstrahlung spectrum. The explanation that comes in mind is that particles can go through several cycles of acceleration along closed dipole flux tubes and emit gamma rays at kinks. This would give rise to energy bands labelled by the number of acceleration cycle. The possibility of saturation looks plausible. One would have particle accelerator analogous to storage ring. What would be new as compared to LHC would be quantum coherence in the scale of accelerator. For the values of h_{eff} involved the dark particles would have Compton lengths of the order of the size of Sun.

4. How could the charged particle and gamma rays emerge from the flux tubes? One can start from everyday experience. Car can fall off the road in sharp curve. Now the sharp curve would correspond to a kink in flux tube. By momentum conservation there should be a large exchange of momentum with the flux tube to keep the charged particle at the flux tube and this is improbable for sharp kinks. Since the charged particles are relativistic and gamma rays must be directed to the observer, the change of momentum direction must be large. In any case, this requires a large exchange of momentum with the collective flux tubes degrees

of freedom. It is quite possible that several gamma rays are emitted at the kink. The charged particle can also leak out.

A proper description of the situation might be in terms of dark cyclotron states. If the TGD view about dark matter as $h_{eff}/h_0 = n = n_1 n_2$ phases is true on can treat the bundle of flux tubes as single quantum coherent entity. In particular, the solar spots could be identified as this kind of quantum coherent flux tube bundles and n_2 could correspond to the number of elementary flux tubes.

5. The sharp kinks appear at two places. Near the North pole where dipole field lines/flux tubes make a sharp kink. Due to differential rotation the flux tubes associated with the dipole contribution follow the rotation of equator and develop tentacles. The shape of strongly flattened square implies instability against splitting of the tentacles and decay to flux loops by reconnection. This part of the magnetic field decays and leads to magnetic turbulence. Also in the standard picture differential rotation is expected to induce reconnections of field lines. The kinks at the ends would induce emission of gammas and leakage of charged particles. Even single gamma ray could be enough.

   Gamma radiation indeed has two components. Polar component is roughly constant and the equatorial component having sharp maximum during sunspot minimum.

   Spectral index is different for the energy distributions for cosmic rays and gamma rays from Sun: solar distributions are harder. Also the equatorial distribution is harder than polar distribution. One expects that the distribution depends on the energy of the gamma ray and on the sharpness of the kink. In the case of polar distribution two gammas is minimum whereas for equatorial distribution single ray can be enough. This softens the polar distribution as compared to equatorial one. Since several loops are possible even the cosmic ray distribution for charged particles can harden.

Where could the charged particles originate?

1. The basic observation is that flux of gammas is 5 times higher than predicted by the model identifying them as cosmic rays reflected in solar magnetic field fails. Roughly the same order of magnitude suggests that cosmic gamma rays could be the origin. Spectral distribution does not support this idea.

2. Charged particles could come from the solar core or along the long thickened cosmic string continuing as flux tubes of the magnetic field. Cosmic string would not accelerate the charged particles but only feed in the particles beams as kind of supra currents. Also cosmic rays could enter the flux tubes as assumed in the original model: in fact, cosmic rays would naturally arrive along the long flux tubes connecting Sun to sources of cosmic rays.

   This could explain why the upper bound for gamma ray energies for cosmic rays equals to the maximal detected energy (100 GeV). Instead of being reflected cosmic rays could rotate possibly several times around dipole flux tube and leak out in the kink. The emission of gamma rays at kinks reduces the energy gain for simple loop and for higher number of loops the reduction is larger. Saturation is quite possible.

3. The origin of galactic rays is still a mystery (see `http://tinyurl.com/psdp99h`). One proposal is that they originate from neutron stars. The proposed acceleration mechanism could be at work in the case of neutron stars so that neutron star could indeed provide the charged particles. As discussed there are also other options.

16.2.6 Empirical support for the confinement of radiation to monopole flux tubes

In the following some pieces of support for the confinement of radiation to closed monopole flux tubes are discussed. I learned about a very interesting result related to early cosmology and challenging the standard cosmology. The result is described in popular article "*Early opaque universe linked to galaxy scarcity*" (see `http://tinyurl.com/y74xe4jr`). The original article "*Evidence for*

Large-scale Fluctuations in the Metagalactic Ionizing Background Near Redshift Six" of Becker *et al* [E14] is published in Astrophysical Journal (see `http://tinyurl.com/y7ho454e`).

The abstract of the article is following.

> *The observed scatter in intergalactic Lyα opacity at $z \leq 6$ requires large-scale fluctuations in the neutral fraction of the intergalactic medium (IGM) after the expected end of reionization. Post-reionization models that explain this scatter invoke fluctuations in either the ionizing ultraviolet background (UVB) or IGM temperature. These models make very different predictions, however, for the relationship between Lyα opacity and local density. Here, we test these models using Lyα-emitting galaxies (LAEs) to trace the density field surrounding the longest and most opaque known Lyα trough at $z < 6$. Using deep Subaru Hyper Suprime-Cam narrowband imaging, we find a highly significant deficit of $z \simeq 5.7$ LAEs within 20 $\mathrm{h}^{-1}\,\mathrm{Mpc}$ of the trough. The results are consistent with a model in which the scatter in Lyα opacity near $z \sim 6$ is driven by large-scale UVB fluctuations, and disfavor a scenario in which the scatter is primarily driven by variations in IGM temperature. UVB fluctuations at this epoch present a boundary condition for reionization models, and may help shed light on the nature of the ionizing sources.*

The basic conclusion is that the opaque regions of the early Universe about 12.5 billion years ago (redshift $z \sim 6$) correspond to small number of galaxies. This is in contrast to standard model expectations. Opacity is due to the absorption of radiation by atoms and the UV radiation generated by galaxies, which ionizes atoms and makes Universe non-transparent. In standard cosmology the radiation would arrive from rather large region. The formation of galaxies is estimated to have begun .5 Gy years after Big Bang but there is evidence for galaxies already for .2 Gy after Big Bang (see `http://tinyurl.com/y9c75t2b`). Since the region studied corresponds to a temporal distance about 12.5 Gly and the age of the Universe is around 13.7 Gy, UV radiation from a region of size about 1 Gly should have reached the intergalactic regions and have caused the ionization.

Second conclusion is that there are large fluctuations in the opacity. What is suggested is that either the intensity of the UV radiation or that the density of intergalactic gas fluctuates. The fluctuations in the intensity of UV radiation could be understood if the radiation from the galaxies propagates only to finite distance in early times. Why this should be the case is difficult to understand in standard cosmology.

Could TGD provide the explanation.

1. In TGD framework galaxies would have born as cosmic strings thickened to flux tubes. This causes reduction of the string tension as energy per unit length. The liberated dark energy and matter transformed to ordinary matter and radiation. Space-time emerges as thickened magnetic flux tubes. Galaxies would correspond to knots of cosmic strings and stars to their sub-knots.

2. If the UV light emerging from the galaxies did not get far away from galaxies, the ionization of the intergalactic gas did not occur and these regions became opaque if distance to nearest galaxies was below critical value.

3. Why the UV radiation at that time would have been unable to leave some region surrounding galaxies? The notion of many-sheeted space-time suggests a solution. Simplest space-time sheets are 2-sheeted structure if one does not allow space-time to have boundaries. The members of the pair with boundary are glued to together along their common boundary. The radiation would have left this surface only partially. Partial reflection should occur as the radiation along first member of pair is reflected as a reflected signal propagating along second member. This model could explain the large fluctuations in the opacity as fluctuations in the density of galaxies.

4. A more concrete confinement mechanism would be based on the propagation of light from galaxy along magnetic monopole flux loops. If the loop is closed, it can confine the radiation. This confinement could occur also at the level of stars. The model for the solar cycle and observed anomalously high emission of gamma rays from Sun in 1-100 GeV range involves

confinement of charged particles to dipole loops represented as space-time surfaces. The confinement is possible also for gamma rays.

5. Cosmic expansion occurring in TGD framework in jerk-wise manner as rapid phase transitions would have expanded the galactic space-time sheets and in the recent Universe this confinement of UV radiation would not occur and intergalactic space would be homogenously ionized and transparent.

The echo phenomenon could be completely general characteristic of the many-sheeted space-time.

1. The popular article "*Evidence in several Gamma Ray Bursts of events where time appears to repeat backwards*" (see `http://tinyurl.com/y89j6u2y`) tells about the article "*Smoke and Mirrors: Signal-to-Noise and Time-Reversed Structures in Gamma-Ray Burst Pulse Light Curve*" of Hakkila *et al* (see `https://arxiv.org/pdf/1804.10130.pdf`). The study of gamma ray bursts (GRBs) occurring in the very early Universe with distance of few billion light years (smaller than for opacity measurements by an order of magnitude) has shown that the GRB pulses have complex structures suggesting that the radiation is reflected partially back at some distance and then back in core region. The duration of these pulses varies from 1 ms to 200 s.

   Could also this phenomenon be caused by the finite size of the space-time sheets assignable to the object creating GRBs? Perhaps the simplest explanation would be in terms of confinement of gamma rays inside monopole flux loops asociated with the source of the radiation such as quasar of blackhole. This predict periodic re-appearence of pulses.

2. There is also evidence for blackhole echoes, which could represent example of a similar phenomenon. Sabine Hossenfelder (see `http://tinyurl.com/ybd9gswm`) tells about the new evidence for blackhole echoes in the fusion of blackholes for GW170817 event observed by LIGO reported by Niayesh Afshordi, Professor of astrophysics at Perimeter Institute in the article "*Echoes from the Abyss: A highly spinning black hole remnant for the binary neutron star merger GW170817*" (see `https://arxiv.org/abs/1803.10454`). The earlier 2.5 sigma evidence has grown into 4.2 sigma evidence. 5 sigma is regarded as a criterion for discovery.

   TGD based comments can be found in [L23] (see `http://tinyurl.com/y9suamjl`). The confinement of gravitational radiation inside monopole flux loops asociated with blackhole like object would explain the findings. This however forces to replace the standard view about blackholes having no hair with TGD based view [L74] allowing magnetic fields represented in terms of monopole flux tubes.

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   The confinement of gravitational radiation inside monopole flux loops asociated with blackhole like object would explain the findings. This however forces to replace the standard view about blackholes having no hair with TGD based view [L74] allowing magnetic fields represented in terms of monopole flux tubes.

Fermi bubbles (see `http://tinyurl.com/yaj3l2rp`) are observed above and below Milky Way at X-ray and gamma ray energies and have radii about 11.5 thousand light years ($\sim 10^{20}$ meters). They might be due the leakage of dark photons from the dark flux tubes of the magnetic field assignable to what is identified as galactic blackhole with mass about $4.5 \times 10^6 M_{Sun}$ and Schwartschild radius about 1.35×10^{10} m. Dark photons would transform to ordinary photons [K100].

Could one understand the radius of Fermi bubbles in TGD framework?

1. According to the proposal of [L68] Planck constant decomposes as $h_{eff}/h_0 = n_1 n_2$, where n_1 and n_2 are the number of sheets as coverings of M^4 and CP_2. $n_2 \simeq 10^7$ is needed to produce Newton's constant G from CP_2 length R taking in TGD the role of Planck length: one would have as $G = R^2/\hbar_2 = G/n_2\hbar$. The variation of n_2 would explain the variation of G.

2. Gravitational Compton length for a particle of mass m $\Lambda_{gr} = GM/v_0$ does not depend on m (Equivalence Principle). $v_0 \simeq 2^{-11}$ is a reasonable guess from solar system [K100] and would give $\Lambda_{gr} \simeq 1.35 \times 10^{13}$m. The scale defined as $L_{gr} = n_2\Lambda_{gr}$ equals to $L_{gr} \sim 10^{20}$ m, which is the radius of Fermi bubbles, and might have interpretation as the size of magnetic body (MB) of the blackhole like entity [L74] associated with galaxy.

3. What does comparison with Sun give? For Sun with Schwartschild radius 3 km the same formula would give $L_{gr} = 3 \times 10^{10}$ m $=$ 2AU, the diameter of the Earth's orbit. For Mars the distance is 1.5 AU and it has very weak magnetic field now. Could this be regarded as a reasonable identification for the size of the solar MB or of its important layer. Note also that one has $\Lambda_{gr} \simeq 6 \times 10^6$ m to be compared with the radius of Earth $R_E \simeq 6.37 \times 10^6$ m.

16.2.7 Surprises in the physics at the boundary of the heliosphere

I learned from interesting results about cosmic rays and behavior of magnetic field at the boundary of heliosphere (see the article "*Voyager Mission Reveals Unexpected Pressure at The Edge of The Solar System*" (see http://tinyurl.com/y474zww4). The article "*Pressure Runs High at Edge of Solar System*" (see http://tinyurl.com/y5t258c8) gives a more precise description of the findings.

There were two spacecrafts. Voyager2 was inside heliopause ad Voyager1 slightly outside it. They experienced different kind of reduction in cosmic ray flux. I picked up the following piece of text explaining the basic findings.

> *The scientists noted that the change in galactic cosmic rays wasn't exactly identical at both spacecraft. At Voyager 2 inside the heliosheath, the number of cosmic rays decreased in all directions around the spacecraft. But at Voyager 1, outside the solar system, only the galactic cosmic rays that were traveling perpendicular to the magnetic field in the region decreased. This asymmetry suggests that something happens as the wave transmits across the solar system's boundary.*

Consider first TGD based view about magnetosphere of Sun.

1. TGD allows two kinds of magnetic fields: those for which flux tubes carry monopole flux and those for which they do not. Monopole flux tubes are impossible in Maxwellian world and solve several problems related to magnetic fields such as the existence of magnetic fields in cosmic scales, and the maintenance problem of the Earth's magnetic field [L15].

   One of the latest applications is to the undertanding of the weird properties of the magnetic field of Mars identified in the model as consisting of monopole flux tubes [L80] and thus visible only through northern and southern lights involving reconnections of the monopole flux tubes. Also Mercury has unexpectely strong magnetic field and it could correspond to monopole flux tube tangle associated with flux tubes from Sun.

   The latest application is to a model of earthquakes and volcanic eruptions [L82] known to be induced by cosmic rays but quite too deep for them to penerate to the depths required. There is strong correlation with solar minima and it has turned out that the solar minimum corresponds to maximum of magnetic field. There is also a causal anomaly: electromagnetic fluctuations in upper atmosphere precede rather than follow these event. The new view about magnetic fields and zero energy ontology predicting that arrow of time changes in "big" (ordinary) state function reductions explains these anomalies. Causal anomalies involving change of also thermodynamical arrow of time are a generic signature of macroscopic state function reductions in TGD Universe.

2. Also a new view about cosmic rays emerges. Cosmic rays would travel along flux tubes of a gigantic fractal flux tube network defining analog of nervous system for the Universe [L90]. This picture leads to a rather detailed model for the formation of galaxies, stars and even planets as tangles along the flux tubes of this network having same topological structure as dipole magnetic field but with flux tubes carrying monopole flux [L74].

3. In TGD framework heliosphere corresponds to magnetically to U-shaped tentacles from Sun - flux tubes emanating from Sun radially and returning back to Sun and carrying solar wind and also cosmic rays. They look locally like parallel flux tubes carrying opposite magnetic fluxes. Flux tubes would extend to the heliopause and turn back and emit by reconnection narrow rectangle shaped closed flux tubes.By fractality these tentacles appear in all scales and are in crucial role in understanding of bio-catalysis and basic biochemical reactions like DNA replication, transcription of DNA to RNA, and translation of RNA to polypeptides.

4. Cosmic rays can travel as dark particles along them in TGD sense meaning that they would have effective Planck constant $h_{eff} = n \times h_0$, where h_0 is minimal value of h_{eff}. The flux tubes from Sun would thus bring dark particles along flux tubes. Suppose that the flux of cosmic rays arrive along these flux tubes, perhaps as dark particles.

Next one must translate various words to physical concepts in TGD framework.

1. Heliosheath (Voyager 2) is expected to be a turbulent boundary region. Magnetic turbulence means that the directions of U-shaped flux tubes coming from Sun are random. This is magnetic counterpart of a boiling liquid.

   Closed U-shaped flux tubes from Sun reach the heliopause before reconnection meaning emission of closed flux tubes looking like narrow rectangles travelling in radial direction: the direction of the flux is assumed to be along the radial flux tube and two directions are possible.

2. The region outside heliopause (Voyager 1) contains two kinds of monopole flux tubes, which need no current for their existence. Those of galactic magnetic field locally parallel to heliopause like in liquid flow around obstacle plus the closed flux tubes as outcomes of reconnection. They are assumed to be narrow rectangle-like objects in radial direction coming from the heliopause. There are also flux quanta of ordinary magnetic field generated by currents.

3. The wave called global merged interaction region (GMIR) caused by the activity of Sun means reconnections for the U-shaped flux tubes from the Sun at solar surface generating ordinary magnetic fields giving rise to sunspots. This reduces the number of U-shaped flux tubes and therefore also solar wind and the amount of cosmic rays arriving along them. Thus the reduction of solar wind and of cosmic rays both inside and outside heliosphere.

4. If the local directions of solar flux U-shaped tubes inside heliosheath are random by turbulence the reduction of flux takes place in all directions. It the long sides of closed flux tube rectangles are radial (orthogonal to the dominating galactic magnetic field), the reduction of flux takes place only in directions orthogonal to the galactic magnetic field. This was observed.

5. The high pressure could be due to the presence of closed flux tubes formed in reconnection and would represent the contribution of solar wind.

16.3 About general implications of the pairing hypothesis

If wormhole magnetic fields appear in all scales, flux tube pairs and more general $n_1 = 2m_1$ multiplets of flux tubes decomposing to m_2 pairs should be universal aspect of the dynamics of TGD Universe. In the following the implications are considered only briefly. The basic consequence is of course that Universe becomes in all scales a quantum coherent object and the locality hypothesis of classical physics would be simply wrong.

16.3.1 Elementary particle physics

Wormhole magnetic fields appear already in elementary particle physics. Elementary particles correspond to at least 2-sheeted flux tube structures with wormhole throats containing the boundaries of string world sheets carrying fundamental fermions. I have already earlier considered the possibility that the M^4 projections of the sheets are disjoint.

Remark: In the general case one would have $n_1 = 2m_1$. Color symmetry for quarks could have as a remnant $m_1 = 3m_3$. For leptons m_1 would not be divisible by 3. Since n_1 corresponds to discrete subgroup for $SU(3)$, m_1 could correlate with the triality of $SU(3)$ partial wave defining the color quantum numbers of the particle.

16.3.2 Astrophysics and cosmology

The predictions in astrophysics and cosmology are in strong conflict with the locality principle of classical physics.

1. The model for magnetic spin flips in solar cycle leads to the conclusion that solar magnetic field could have doublet structure with parts related by inversion with respect to solar surface. Could the entire MB of Sun have copy somewhere. In principle this is an experimental question. The copy would be connected to Sun by wormhole magnetic flux tubes and this suggests long range correlations.

   Stars indeed very often appear as binaries (see `http://tinyurl.com/oooagma`). Could these pairs be related by approximate CP_2 symmetry inducing reflection of inversion in M^4? Could the planets of mirror paired stars be related by Z_2? Could there be correlations between the rotation planes for instance.

2. What about Earth could be invariant under inversion so that the radius of Earth could define the radius remaining invariant under inversion. This could make Earth so special as far as life is considered.

   Could Earth have a double in longer length scale? The least science fictive candidate would be another planet.

   Mars (see `http://tinyurl.com/mttm7h8`) has radius $.53R_E$, which is the radius that Earth would have had before the Cambrian Explosion according to TGD inspired variant of Expanding Earth model [L64]. Mass is 11 per cent of the Earth's mass. There are indications for life in Mars. Venus (see `http://tinyurl.com/72rz2g2`) has characteristics surprisingly near to those of Earth except that rotation is in opposite direction than for Earth: the rotation period is -243.025 days. The distances from Sun for (Venus,Earth,Mars) triplet are $(.72, 1.00, 1.52)$ AU. Could Venus and Mars form a mirror pair with respect to inversion at radius R_E.

   Recently Nasa found an exoplanet christened as Gliese 581d (see `http://tinyurl.com/yxdmpnbj` and `http://tinyurl.com/y2bwco6q`) located in constellation Lyra at distance of only 20.4 light years. The planet is almost exact copy of Earth as far the prerequisites of life are considered. Semimajor axis of the orbit is .22 from that of Earth. Mass is about 6.98 times higher than Earth mass, the radius is $2.20R_E$. The Sun of the planet could be mirror image of Earth: if this is the case, the should be correlations such as common rotation planes.

3. I have considered [L15] also a model for the changes of the orientation of Earth's magnetic field involving the interaction of monopole flux tubes and ordinary magnetic field via magnetic torques, and the solar model probably generalizes almost as such. Now however the orientation of the magnetic field can vary. This could relate to the fact that the axis of rotation differs from the magnetic axis. Again inversion as an approximate symmetry is suggestive.

4. The most intriguing finding about CMB spectrum is anomaly known as "Axis of Evil" (see `http://tinyurl.com/yb6nabw4`). The anomaly appears to give for the plane of planetary system of Sun and the location of Sun a greater significance that one might expect by change.

This violates the Copernican Principle. The effect resembles selection of spin quantization axis in quantum measurement of spin performed by the measurer. A possible explanation at the level of space-time is that by $h_{eff}/h_0 = n$ hierarchy disjoint space-time sheets even in cosmic length scales are related by discrete CP_2 symmetries implying correlations.

16.3.3 Biology

The binary structures populating biology might correspond to pairs of monopole flux tubes. The original motivation for the proposal that they are important comes from p-adic length scale hypothesis: primes $p \simeq 2^{k+2}$ and $p \simeq 2^k$, where k and $k+2$ are twin primes, could define structures with size scale $L(k+2)$ decomposing to a pair of structures with size scale $L(k)$ [K25]. The structures of twin pair would form quantum entangled structures.

1. DNA and RNA double strands are basic examples of these structures. Even single DNA and RNA molecules form mirror pairs with their conjugates and could be connected by long wormhole contacts. This would make them quantum coherent structures making possible the mysterious ability of bio-molecules to find each other in the molecular crowd. Bio-systems would be extremely organized structure rather than a soup of randomly moving molecules. Could this kind of symmetries characterize all molecules that are paired or form higher structures with $n_1 = 2m_1$?

2. Cell membranes are formed by pair of lipid layers and also these could be twin pair. Epithelial sheets consist of two cell layers. At the level of body and brain there is also a pairing of subs-structures in left and right brain. Pineal gland is a connected structure could itself be a pair. Also brain hemispheres form a pair. Even married (or even non-married!) couple could form this kind of pair and what looks like a random personal relationship could be something much deeper.

3. All multi-molecular structures in living matter at least could correspond to groups of n_1 disjoint space-time sheets, perhaps magnetic flux tubes. The value of n_1 would serve as a measure for the scale of coherence and complexity.

4. Inversion corresponds to the inversion of the polarity of the Earth's magnetic field but might happen also at the cell level. In biology involution turning cell inside-out occurs during the gastrulation phase (see `http://tinyurl.com/y4pvpxyr`) of the embryonic development and leads to a development of 2 (ectoderm,endoterm) or 3 cell layers (ectoderm, mesoderm, endeterm) giving later rise to different types of tissues. This process looks rather mysterious - at least to me. Could involution be induced by the inversion of the magnetic body of the developing embryo?

5. MB controls (also our) biological body (BB) and uses scaled variants of EEG consisting of dark photons for this purpose [?] It is natural to assume that our MB corresponds to the part of MB above the Earth's surface or dipole core. If Z_2 acts as inversion with respect to the surface of the dipole core then also the part of MB below the surface of the dipole core should correspond to an intentional agent.

   Could these MBs be associated intra-terrestrials ITs or could they control same BBs as our usual MBs? Here one must consider the precise definition of inversion: is it with respect to the surface of Earth or of the dipole core of the Earth's B? Taking inversion in the first sense of the definition very literally, one could argue that plants having also roots are inversion invariant with respect to the Earth surface but animals are strictly speaking not inversion invariant in either sense. Could we have separate personal mirror MBs and also BBs: analogs of Dr. Jekyll and Mr. Hyde? In fact, I have have-jokingly considered a model for crop circles, and this led to a crazy idea about IT life [?] Could this idea be not so crazy as it looks first? Accepting dark matter as $h_{eff}/h_0 = n$ phases, the high temperature in Earth interior ceases to be an objection.

6. $n_1 = 2m_1$ implies also that conscious entity can have n_1 disjoint pieces. They could be MBs controlling the same BB (multiple personality disorder) or maybe even separate BBs. Could these possibly distinct BBs locate at different sides of globe or even cosmos? What comes

in mind Kieslowski's trilogy "Three colors". When the connection between hemispheres is destroyed, brain hemispheres controlling different body halves would live effectively separate lives, and could even fight for the control of BB. This gives some ideas as one tries to image what it is to have several BBs. It is interesting that in dreams we often have different identities than in wake-up state.

16.3.4 Consciousness

The existence of twin pairs might have profound implications for consciousness [L52, L66].

1. I proposed for about 2 decades ago what I called magnetospheric consciousness [K65, K63, K41, K42]. The MB of not only Earth but also our MB would have parts assignable to the interior and exterior of the Earth. Even the structures of brain should have a scaled up MB image at both levels. The approximate inversion symmetry brings in exciting additional aspects. Maybe this division could provide the physical correlates for the Heaven-Hell dualism of religions and "as above-so below" dualism of perennial world views and mysticism.

2. Interior-exterior divisions are central for consciousness and the hierarchy of conscious entities in correspondence with the hierarchy of space-time sheets inspires the question whether also our biological bodies and environment could be related by an approximate symmetry at the level of MB at least so that one could speak of MBs assignable to the interior and exterior of BB. The sensory representations would reflect this approximate symmetry. Subsystem able to remain entangled at the passive boundary of CD defines the permanent part of self. But also its complement remains unentangled and should define permanent part of self: does this mean that the world outside me is a conscious entity?

3. One of the most dramatic predictions of TGD inspired theory of consciousness based on zero energy ontology (ZEO) is re-incarnation of self in death as a time-reversed self. There is indirect support for this: for instance, mental images identified as sub-selves die and re-incarnate and the period during which they are absent would correspond to the life with opposite arrow of time.

   Where could these ghostly time-reversed re-incarnations live? Or putting it more formally: what regions of space-time surface do these entities control and receive sensory input from? Could inversion with respect to Earth's surface relate the space-time regions associated with self and its time reversal. If personal MB is part of MB above the Earth's surface, its inversion would be the part of MB below it. When we die we get buried. Could this ritual reflect the sub-conscious idea that our life continues as IT lifeform?

Chapter 17

Holography and Quantum Error Correcting Codes: TGD View

17.1 Introduction

Strong form of holography is one of the basic tenets of TGD, and I have been working with topological quantum computation in TGD framework with the braiding of magnetic flux tubes defining the space-time correlates for topological quantum computer programs [K4]. Flux tubes are accompanied by fermionic strings, which can become braided too and would actually represent the braiding at fundamental level. Also time like braiding of fermionic lines at light-like 3-surfaces and the braiding of light-like 3-surfaces themselvs is involved and one can talk about space-like and time-like braidings. These two are not independent being related by dance metaphor (think dancers at the parquette connected by threads to a wall generating both time like and space-like braidings). I have proposed that DNA and the lipids at cell membrane are connected by braided flux tubes such that the flow of lipids in lipid layer forming liquid crystal would induce braiding storing neural events to memory realized as braiding.

I have a rather limited understanding about error correcting codes. Therefore I was happy to learn that there is a conference in Stanford in which leading gurus of quantum gravity and quantum information sciences are talking about these topics. The first lecture that I listened was about a possible connection between holography and quantum error correcting codes. The lecturer was Preskill and the title of the talk was "Holographic quantum error-correcting codes: Toy models for the bulk/boundary correspondence" (see `https://www.youtube.com/watch?v=SW2rlQVfnK0` and `http://tinyurl.com/z8fsfh8`). A detailed representation can be found in the article of Preskill *et al* [B26] (see `http://arxiv.org/pdf/1503.06237.pdf`).

The idea is that time= constant section of AdS, which is hyperbolic space allowing tessellations, can define tensor networks. So called perfect tensors are building bricks of the tensor networks providing representation for holography. There are three observations that put bells ringing and actually motivated this article.

1. Perfect tensors define entanglement which TGD framework corresponds negentropic entanglement playing key role in TGD inspired theory of consciousness and of living matter.

2. In TGD framework the hyperbolic tesselations are realized at hyperbolic spaces $H_3(a)$ defining light-cone proper time hyperboloids of M^4 light-cone.

3. TGD replaces AdS/CFT correspondence with strong form of holography.

17.1.1 Could one replace AdS/CFT correspondence with TGD version of holography?

One can criticize AdS/CFT based holography because it has Minkowski space only as a rather non-unique conformal boundary resulting from conformal compactification. Situation gets worse as one starts to modify AdS by populating it with blackholes. And even this is not enough: one can imagine anything inside blackhole interiors: wormholes connecting them to other blackholes, anything. Entire mythology of mystic creatures filling the white (or actually black) areas of the map. Post-modernistic sloppiness is the problem of recent day theoretical physics - everything goes - and this leads to inflationary story telling. Minimalism would be badly needed.

AdS/CFT is very probably mathematically correct. The question is whether the underlying conformal symmetry - certainly already huge - is large enough and whether its proper extension could allow to get rid of admittedly artificial features of AdS/CFT.

In TGD framework conformal symmetries are generalized thanks due to the metric 2-dimensionality of light-cone boundary and of light-like 3-surfaces in general. The resulting generalization of Kac-Moody group as super-symplectic group replaces finite-dimensional Lie group with infinite-dimensional group of symplectic transformations and leads to what I call strong form of holography in which AdS is replaced with 4-D space-time surface and Minkowski space with 2-D partonic 2-surfaces and their light-like orbits defining the boundary between Euclidian and Minkowskian space-time regions: this is very much like ordinary holography. Also embedding space $M^4 \times CP_2$ fixed uniquely by twistorial considerations plays an important role in the holography.

AdS/CFT realization of holography is therefore not absolutely essential. Even better, its generalization to TGD involves no fictitious boundaries and is free of problems posed by closed time-like geodesics.

17.1.2 Perfect tensors and tensor networks realized in terms of magnetic body carrying negentropically entangled dark matter

Preskill *et al* suggest a *representation* of holography in terms of tensor networks associated with the tesselations of hyperbolic space and utilizing perfect tensors defining what I call negentropic entanglement. Also Minkowski space light-cone has hyperbolic space as proper time=constant section (light-cone proper time constant section in TGD) so that the model for the tensor network realization of holography cannot be distinguished from TGD variant, which does not need AdS at all.

The interpretational problem is that one obtains also states in which interior local states are non-trivial and are mapped by holography to boundary states are: holography in the standard sense should exclude these states. In TGD this problem disappears since the macroscopic surface is replaced with what I call wormhole throat (something different as GRT wormhole throat for which magnetic flux tube is TGD counterpart) can be also microscopic.

17.1.3 Physics of living matter as physics condensed dark matter at magnetic bodies?

A very attractive idea is that in living matter magnetic flux tube networks defining quantum computational networks provide realization of tensor networks realizing also holographic error correction mechanism: negentropic entanglement - perfect tensors - would be the key element! As I have proposed, these flux tube networks would define kind of central nervous system make it possible for living matter to experience consciously its biological body using magnetic body.

These networks would also give rise to the counterpart of condensed matter physics of dark matter at the level of magnetic body: the replacement of lattices based on subgroups of translation group with infinite number of tesselations means that this analog of condensed matter physics describes quantum complexity.

I am just a novice in the field of quantum error correction (and probably remain such) but from experience I know that the best way to learn something new is to tell the story with your own words. Of course, I am not at all sure whether this story helps anyone to grasp the new ideas. In any case, if one have a new vision about physical world, the situation becomes considerably easier since creative elements enter to this story re-telling. How these new ideas could be realized in the theory world? This question I try to answer in the following.

The goal in the sequel is therefore an attempt to formulate the connection between quantum holography and error correcting codes in TGD framework bringing in new features relating to the new views about space-time, quantum theory, and living matter and consciousness in relation to quantum physics.

17.2 Holography

In the following I summarize my understanding about holography.

17.2.1 Holographies

Holography has become a key notion in attempts to understand gauge theories and gravity. One variant of holography suggests that blackhole horizons containg the information about quantum state assignable to blackhole. The naïve picture is that area unit defined by Planck length corresponds to single bit.

There is also second form of holography. AdS/CFT correspondence states that conformal field theory at the boundary of $AdS_n \times S^{10-n}$ is dual to a string theory or gravitational theory in 10-dimensional space. AdS_n has Minkowski space as $n-1$ dimensional boundary and for conformal field theory in 4-D Minkowski space one would have AdS_5. AdS_n is Minkowski space with 2 time like dimensions realized as a surface $t_1^2+t_2^2-x_1^2-x_2^2-x_3^2-x_4^2 = R^2$, where R is radius of curvature.

I cannot avoid some nitpicking.

1. AdS is somewhat problematic physically since it has closed time-like geodesics as is rather clear from the defining condition (see `http://tinyurl.com/h7xltde`) and also from the $(1,1,-1,-1,-1)$ signature of the metric. The second problem with AdS is that it is rather formal construct.

2. One speaks also about AdS boundary. AdS does not however have an actual boundary. A more precise term is conformal boundary resulting in conformal compactification transforming the metric to Minkowski metric apart from conformal factor. This transformation is fixed apart from a conformal transformation of AdS. The existence of the transformation follows from the conformal flatness of AdS which for dimensions not smaller than four can be formulated in terms of the vanishing of Weyl tensor (see `http://tinyurl.com/y7fsnzk8`). Every manifold with constant sectional curvature is conformally flat. Note that CP_2 is not conformally flat.

   Conformal boundary corresponds to vanishing value of z in the representation of $ds^2 = (ds^2(M^{n-1}) + dz^2)/z^2$ with $z > 0$ giving the half-space and $z = 0$ the $n-1$-D Minkowski space. The metric as making explicit the conformal flatness of AdS, where $\Omega^2 = 1/z^2$ is the conformal scaling factor which becomes infinite in subspace, which is Minkowski space. A second kind of singular behavior would be vanishing of Ω^2 $(z = \infty)$ and also this subspace could be identified as conformal boundary.

   Conformal compactification is a map of original space with the property that it performs conformal scaling for the metric tensor. Conformal compactification requires finding of a coordinate transformation taking the metric to the desired form. Note that in the case of Minkowski space with spherical coordinates with metric $dt^2 - dr^2 - r^2d\Omega^2$ the conformal compactification is induced by the compactifying conformal mapping of $(t,r) \rightarrow (T,R)$ satisfying therefore $g_{TT} = -g_{RR} = \Omega^2$ and $r^2 = \Omega^2R^2$. The map can be explicitly constructed and maps (t,r) plane to a triangle. Ω^2 vanishes or diverges at the conformal boundary. This can be done also for AdS so that the resulting conformal boundary is Minkowski space and has Minkowski metric with vanishing scale factor.

   To my humble opinion, procedures of this kind should be avoided in fundamental theory. On the other hand, the twistor space of Minkowski space used in twistorialization is also conformal compactification. In this case one however has what is known as double fibration [B43] (see `http://tinyurl.com/yb4bt74l`) meaning that one has fibration from $M^4 \times S^2$ - the trivial S^2 bundle defining the geometric twistor space to the twistors space identified as complex projective space defining conformal compactification of M^4. Double fibration is essential in the twistorialization of TGD [K50].

3. Even worse - to my opinion - is that people are not happy with AdS as such but add to AdS all kinds of stuff such as blackholes making varying assumptions about their interiors. This leads easily to an inflation of sloppily defined notions. Just this happened in super string theory and eventually led to the multiverse mania. Eventually the proposal emerged that

the requirement that physical theories should be able to predict something should be given up since the exceptional "beauty" of some of these baroque constructions would be enough to justify them as the only possible theory of everything.

In TGD framework AdS/CFT correspondence is replaced with the strong form of holography.

1. Ordinary holography would state that preferred 3-surfaces - either space-time 3-surfaces at the boundaries of CD or light-like 3-surfaces connecting them - carry same quantum information as space-time surface. Strong form of holography (SH) follows from the conditions that these two identifications of 3-surfaces are equivalent and says that 2-D partonic 2-surfaces and string world sheets carry the quantal information. SHS is very similar to ordinary holography and to e blackhole holography.

   There is no need for conformal compactification and conformal boundary since boundary is something very real being identified as the light-like orbit of partonic 2-surfaces defining the boundary between Euclidian and Minkowskian space-time regions. 4-D space-time surface replaces the 10-D bulk of AdS/CFT correspondence and partonic 2-surfaces and their light-like orbits replace the space-time at boundary of AdS, where one has conformal quantum field theory (CFT).

   In analogy with conformal boundary, 4-D space-time metric becomes singular (determinant vanishes) at the light-like orbits of wormhole throats meaning that the dimension of tangent space degenerates to 4 to 3. These objects are not fictive but completely physical and can be seen as analogs of blackhole horizons making sense as analogs of boundaries for objects of any size. The new element is that the interior has Euclidian signature of induced metric and is interpreted as geometric counterpart for the line of generalized Feynman diagram or twistor diagram representing the orbit of particle [K50].

2. Classical holography means that space-time surfaces can be constructed as continuations of string world sheets and partonic 2-surfaces (and possibly also their light-like orbits giving rise to discrete number of non-gauge degrees of freedom) serving as space-time genes. Space-time surfaces are thus preferred extremals of the basic action principle. The preferred extremals would be thus 2-dimensional in information theoretic sense.

   This boils down to the condition that a sub-algebra super-symplectic algebra isomorphic to the algebra itself annihilates the physical states and that the classical Noether charges for the sub-algebra vanish. This huge number of conditions makes space-time surfaces as preferred extremals analogous to Bohr orbits. This picture leads also to the hierarchy of Planck constants $h_{eff} = n \times h$ assignable to the fractal hierarchy of isomorphic subalgebras of the super-symplectic and other conformal algebars. This implies a generalization of quantum theory crucial for biology and quantum computation since large value of h_{eff} means macroscopic quantum coherence.

3. Conformal invariance is extended dramatically and the Lie group defining Kac-Moody group becomes the symplectic group of $\delta M^4_\pm \times CP_2$, $\delta M^4_\pm$ denotes light-cone boundary. The light-like radial coordinate of light-cone boundary takes the role of additional complex coordinate z. Note that the conformal invariance is really huge at the fundamental level. This is what allows to replaced the 10-dimensional space of AdS/CFT with 4-D space-time surface and make the holography free of non-physical auxiliary constructs.

   There is also second difference: embedding space $H = M^4 \times CP_2$ - unique by twistorial considerations - enters into the game too and is expected to take some roles of AdS at the level of embedding space. The hyperbolic character of time=constant sections together with the findings of Preskill *et al* [B26] suggests how this might happen. This will be the main theme of this article.

4. In AdS-CFT correspondence radial direction from boundary is interpreted as renormalization scale. I have considered analogous interpretation for the direction normal to the partonic 2-surfaces in TGD: the idea was that any light-like 3-surface in the slicing by light-like 3-surfaces "parallel" to the orbit of partonic 3-surface is physically equivalent representation for holography. An alternative and perhaps more plausible interpretation is that RG scale

corresponds to the proper-time for light-cone boundary labelling the slices $H_3(a)$. It could also naturally correspond to the size scale of CD, perhaps the value of a for which the size of the tessellation contained by CD is largest.

Quantum criticality is basic aspects of quantum TGD and predicts that various coupling strengths have a spectrum of critical values labelled by the p-adic primes $p \simeq 2^k$, k prime: an attractive conjecture is that their values relate in simple manner to the zeros of Riemann zeta [L13]. These length scales in turn would relate to the size scales of CDs. Coupling constant evolution would become discrete. Perhaps one could interpret the preferred role of the boundaries between Minkowskian and Euclidian regions as counterpart for fixed point property of critical values of coupling constants under RG evolution.

17.2.2 Blackholes and wormholes

The many-sheeted space-time distinguishes TGD from GRT. GRT is obtained from TGD as an approximation in which the sheets are replaced by single region of Minkowski space with deformed metric obtained by adding the deviations of the metrics of the sheets with their sum representing the deviation of the metric from Minkowski metric. Components of spinor connection are added in the same manner to obtain electroweak gauge fields of the standard model. Classical color gauge potentials are identified as projections of color Killing vector fields and siimlar description applies to them. This picture follows by considering test particle as a small surface having topological contacts with various sheets of the many-sheeted space-time: the classical forces experienced by it are sums over contributions from various sheets.

Blackholes and wormholes are basic notion of GRT and they have also TGD counterparts.

1. Blackholes play akey role in the attempts to relate quantum gravity, holography, and quantum information theory in GRT framework. I am happy to see that theoreticians like Susskind (see `http://tinyurl.com/y927h695`) have the courage to imagine freely. From my own experience there is of course a risk that this leads to endless loose speculations getting us nowhere. To me blackhole interior represents the failure of GRT and the new theory must replace it with something less singular.

   In TGD framework blackholes are not fundamental and the Euclidian space-time regions with 4-D CP_2 projection representing orbits of wormhole contacts between to space-time sheets replace them. Wormhole contacts become basic building bricks of elementary particles and simplest elementary particles consist of a pair of wormhole contacts with magnetic monopole flux flowing from throat to the other one, through contact to second space-time sheet and back.

   These Euclidian regions define the analogs of lines of Feynman diagrams, and one can speak of generalized Feynman (or twistor-) diagrams represented at the level of space-time geometry and topology. There is no need to perform Wick rotation to get well-defined path integral: Euclidian regions provide to the vacuum functional automatically the real exponent guaranteing the mathematical existence of the functional integral.

2. Wormhole is second key notion of GRT. ER-EPR correspondence is a proposal that wormhole throats - kind of flux tubes between distant regions of space-time - serve as correlates of entanglement. An even stronger conjecture is that space-time somehow emerges from entanglement - I cannot get enthusiastic about this idea. Wormhole throats in GRT sense are not however stable but suffer a pinch splitting them.

   I have made an analogous proposal much earlier in TGD framework. Magnetic flux tubes would take the role or wormhole throats. Flux tubes would carry Kähler magnetic flux monopole Kähler magnetic flux making them stable.

   Remark: To avoid confusion let us make clear that wormhole throat in TGD framework means partonic 2-surface, which can be regarded either as the end of magnetic flux tube replacing wormhole throat understood in GRT sense.

3. Partonic 2-surface is the key notion in TGD, and it turns out that this could define the microscopic representation of 2-surface. The macroscopic continuous 2-surface would be

replaced with a union of partonic 2-surfaces and the generalization of Ruy-Taganaki formula [B42] for entanglement entropy would be formulated in terms of partonic 2-surfaces.

17.2.3 Hyperbolic tessellations are possible for both AdS and Minkowski space

What makes AdS_n so interesting from the point of view of holography and error correcting codes is that the hyperbolic tessellations for time= constant surfaces defining hyperbolic spaces can be used to define quantum holographic codes and states.

1. AdS has a slicing by hyperbolic spaces $t_2^2 - x_1^2 - .. = R^2 - t_1^2$ with slices labelled by $t_1^2 \leq R^2$. Hyperbolic spaces have infinite number of tessellations known also as tilings. The tile is defined as a double coset space $\Gamma \backslash SO(1,n)/SO(n)$, where Γ is an infinite discrete subgroup of $SO(1,n)$ with discontinuous action. Γ is analogous to a discrete subgroup of translations in Euclidian space.

   One can imagine of connecting each face of a given tile to the faces of neighboring tiles and interpret the connection as inputs and outputs and assign to each face an isometry at the level of state space. The tessellation would define a tensor network mapping the inputs assignable to the interior tiles to the boundary states of the network.

2. Remarkably, also Minkowski space appearing as factor of H allows slicing by 3-D hyperbolic spaces $H_3(a)$ defined as hyperboloids $t^2 - r^2 = a^2$ (mass shell in particle physics). This slicing is central in TGD based cosmology with a defining the size scale of the universe. I have already earlier proposed that these hyperbolic tessellations must have fundamental role in TGD.

   For instance, they could naturally define a discretization for $a = constant$ hyperboloids assignable to either tip of the causal diamond CD defined as intersection of future and past directed light-cones. The discretization would be naturally associated with the second boundary of CD and there are indications that astrophysical objects could be at cells of this kind of tessellation analogous to condensed matter lattices. Recession velocity equivalent to equivalently cosmic redshift and by Hubble's law equivalent to distance would be quantized. Evidence for this quantization exists [K69].

3. What is even more significant, a very large fraction of 3-manifolds are known to be hyperbolic manifolds in the sense that they allow hyperbolic metric [K69] [A6]. Thus one can say that they can be mapped to pieces of hyperbolic tessellation. The induce metric of the 3-surface is of course not hyperbolic in general. However, if 3-surface representable as graph from hyperbolic space $H_3(a)$ to CP_2 one can assign a hyperbolic metric to it by considering the metric for the projection to $H_3(a)$ as metric of the 3-surface.

   This seems to work for all 3-surfaces representable as graphs $H_3 \rightarrow CP_2$ but there could be problems due to the presence of boundary. If one allows the boundary of 3-surface to contain broken tiles, there seems to be a very large number of tessellations. The tessellation would be unique only if boundary tiles are required to be unbroken: this condition also implies that the tessellation need not exist.

   Accepting broken boundary tiles, one could say that the hyperbolic tessellation is induced to space-time surface and that space-time surface representable as graph $M^4 \rightarrow CP_2$ allows a slicing by hyperbolic tessellations. One can of course ask, whether the preferred extremal property allows only unbroken tiles. In this case the boundary of tessellation could correspond to light-like 3-surface at which the signature of the induced metric changes to Euclidian. This would fit nicely with the notion of SH.

   The 3-volume of hyperbolic manifold defined by the hyperbolic metric is a topological invariant. The Minkowskian 3-volume would be indeed same for all allowed deformations with fixed $H_3(a)$ projection. Hence all deformations preserving the graph property really correspond to the same hyperbolic 3-manifold whose topology would reduce to its boundary topology. Note that preferred extremal property implies effective 2-dimensionality and this might restrict dramatically the number of allowed 3-surfaces. For unbroken tiles the boundary of the 3-surface contributes to the topology. The absence of genuine boundaries apart from the causal boundaries implied by the change of the metric signature is strongly suggested by boundary conditions, and could be achieved by gluing of piece of tessellation and its deformed copy along boundaries and also this brings in non-trivial topology.

4. CP_2 as a compact coset space allows also tessellations by its discrete subgroups with finite number of tiles. These are analogous to Platonic solids. These tessellations might be interesting for 3-surfaces not allowing a representation as a graph of a map from $H_3(a)$ to CP_2. The Euclidian regions obtained as deformations of CP_2 type vacuum extremals are good candidates in this respect. Deformations of string like objects $M^2 \times S^2$, S^2 a geodesic sphere in CP_2 would allow naturally tessellations defined by the tessellations of H_1 and S^2. These objects would be rather simple from the point of view of complexity theory.

These observations suggest that the hyperbolic tessellations realizing the holography can be actualized in quantum TGD and that they could even define quantum holographic codes.

1. The lines connecting faces of the tiles serve as correlates for entanglement. If magnetic flux tubes mediating monopole flux take the role of lines, the only possibility is that tiles and magnetic flux tubes connecting the centers of tiles form a network. The braiding of the flux tubes is also possible and would make possible topological quantum computation.

2. Tiles could contain quantum states and dark matter realized as $h_{eff} = n \times h$ phases for fundamental quarks and leptons serving as building bricks of elementary particles defined a good candidate for these phases. What simplifies the situation is that there are good reasons for the localization of the modes of induced spinor fields at string world sheets and string world sheets can be associated with flux tubes.

3. tessellation should be seen as an idealized model only. Only the topology of the graph defined by the tessellation and perfect tensor character of entanglement seem to be important. Magnetic body could be thus dynamical. Flux tubes could get braided, they could reconnect, they could experience phase transitions changing the value of $h_{eff} = n \times h$ inducing reduction or increase of length also 2-knotting and braiding can be imagined in 4-D space-time and could bring in totally new kind of topological dynamics. These phase transitions are crucial in TGD inspired quantum biology.

   Flux tube networks could suffer phase transitions in which the character of the tile as hyperbolic manifold changes just like the condensed matter lattice can change its character and in living matter this kind of phase transitions might be important. This suggests that quantum complexity theory could be seen as counterpart of condensed matter physics at the level of magnetic body.

4. An interesting question is what happens to the tessellations as one approaches boundary of CD, which is part of light-cone boundary. The study of the metric demonstrates that the tessellation degenerates to the tip of CD at this limit and all information is lost. This suggests that the representative capacity of tessellation as a function of light-cone proper time a is measured by the 3-volume of the tessellation in hyperbolic metric. The condition that this volume is restricted inside CD restricts the range of radial coordinate r in $r_M = ar$ and the volume has maximum for some value $0 < a < T$, where T is the distance between the tips of CD. Light-boundary is metrically sphere: could Platonic solids define tessellations at this limit? Could one see this limit as the limit at which ordinary condensed matter physics emerges as the quantum information associated with hyperbolic tessellations disappears.

There are indications that in living matter and brain this kind of networks are realized and I have proposed that magnetic flux tubes define this kind of quantum computational networks in living matter. One could even say that space-time becomes sensorily conscious about itself by building this kind of networks analogous to coordinate grids.

These observations suggest that quantum holography and error correction codes might be realized at the level of magnetic bodies dynamically in TGD framework. They would not be essential for defining quantum TGD itself but living and intelligent systems could develop the tessellations to develop bodily sensory consciousness.

17.3 Entanglement and Physics of Quantum Complexity

Both Susskind (see `http://tinyurl.com/y927h695`) and Preskill [B26] emphasize entanglement as the key notion distinguishing between quantum and classical. Entanglement brings in complexity as an exponential decrease of the dimension of the state space and thus the number of states due to the possibility of entanglement. For instance, for classical 3-D spins the configurations space of N spins has dimension $3 \times N$. For quantal spins the complex dimension of state space is 2^N and exponentially larger. It is not perhaps exaggeration to say that the science of complex systems is science of entanglement. Quantum information science has started to seriously study various aspects of entanglement and build simple models.

The notion of entanglement is rather abstract. One can even entangle mutually non-interacting CFTs. This brings in mind Connes tensor product and corresponding entanglement relating to the inclusions of hyper-finite factors [K125]. This entanglement is not forced by interactions but by internal consistency and one might regard it as kinematical. Negentropic entanglement (NE) central for TGD and number theoretic vision is also this kind of kinematic entanglement having very little to do with ordinary dynamics. Surprisingly this is nothing but the entanglement associated with perfect tensors used in the construction of Preskill. Note however that NMP favors its generation.

17.3.1 Some general results

The understanding of the relationship between holography, geometry, and entanglement has evolved dramatically during the last decade and it was nice to become aware of the work done and find that it complements nicely my own work work done in the framework provided by TGD theory of consciousness as a generalization of quantum measurement theory to ZEO.

1. Ryu-Takayanagi formula [B42] for the entanglement entropy as area of minimal surface is key formula and generalizes the area-entropy relationship for blackholes having also interpretation as entanglement entropy. Another key idea is ER-EPR correspondence stating that entanglement as wormholes connecting the entangled systems as a correlate: magnetic flux tubes as correlate for entanglement is an old idea of TGD.

   I have also considered the possibility that the braiding of magnetic flux tubes could serve as correlate for entanglement. If flux tubes appear as pairs with opposite directions of magnetic flux then entanglement might have braiding as correlate. Braiding could be also - and probably is - something independent.

2. Tensor networks constructed using perfect tensors as a representation for strongly interacting systems is a key idea in the article of Preskill *et al* [B26]. The king idea is the notion of perfect tensor with $2n$ components defining entangled system which in TGD framework would be negentropically entangled in the special sense that all n-dimensional subsystems would be maximally entangled with their complement. These tensors can be also used to define quantum error correcting codes and hyperbolic tessellations allow to construct this kind of codes.

Multiscale Entanglement Renormalization Ansatz (MERA) [B30] (`http://tinyurl.com/y9avp92y`) allows to study numerically hierarchical systems with long range entanglement described by local scale-invariant Hamiltonians. MERA networks allow to estimate the ground state of the system.

1. Hierarchical arrays of tensors represent entanglement in different length scales. MERA can be see as a more general isometry from bulk to boundary than the tensor networks proposed by Preskill *et al.* MERA involves also unitary transformations induced de-entanglement in vertical direction. The isometries $V \to V \otimes V$ (see Fig. 1 of the article) can be made unitary by extension $|0\rangle \otimes V \to V \otimes V$. The added tensor factor $|0\rangle$ is analogous to ancilla state in quantum computation. The inverse of this map can be regarded as roughening operation and eventually all entanglement is eliminated as one goes upwards in MERA network.

2. AdS/CFT- MERA connection is proposed first by Swingle [B7] and tensor networks as its realization is mentioned also in the [B26]. AdS/CFT-MERA connection is discussed by Bao *et al* in [B23] (`http://tinyurl.com/y9a9lr29`). AdS/MERA correspondence is suggested to work only in scales longer than AdS scale, and argued to fail to be complete even above AdS scale. The argument starts from the observation that correspondence requires the assignment of length scales to the vertical and horizontal bonds of the MERA network (dimension is 2) and somekind of lattice in AdS is proposed. Authors are not yet aware about the possibility that hyperbolic tessellations might provide an elegant geometrization of MERA network utilizing the discrete symmetries of hyperbolic space and provide the needed scales: scaling invariance allows continuum of scale choices and AdS radius does not seem to matter.

   An open question (to me at least) is whether the unitary transformations are necessary for MERA as claimed by Bao *et al*, and whether the unitary steps are realizable in terms of tessellations: Preskill *et al* do not seem to have them in their construction.

Does MERA have a natural TGD counterpart?

1. The tessellations of H_3 induce tessellations of 3-surfaces in TGD framework and tessellation allows geometrization of the tensor networks as mere graphs.

2. tessellations are scaling invariant but one can ask whether there are natural length scales. Minkowski space does not have any intrinsic scale but the geometric twistor space $M^4 \times S^2$ of M^4 provides such a scale as S^2 radius given by Planck length if the view about twistorialization of TGD is correct [K50]. The physical picture suggests that already CP_2 scale (roughly 10^4 times Planck length) defines a lower bound for scales in which the analog of AdS/MERA correspondence can make sense.

3. p-Adic length scale hierarchy and hierarchy of Planck constants with scaled up versions of p-adic length scales are quantitative formulations for the hierarchies at space-time level and give rise to many-sheeted space-time. These hierarchies are accompanied by several other fractal hierarchies: for instance, in TGD inspired theory of consciousness conscious entities form a fractal hierarchy. A natural guess would be that p-adic length scale and their dark scaled up variants define a hierarchy of length scales below which holography cannot be realized exactly.

From TGD point of view hyperbolic tessellations and MERA have clearly been missing pieces of a puzzle.

17.3.2 Entanglement in TGD Universe

There are several notions related to entanglement and inspired by the construction of TGD inspired theory of consciousness.

1. Zero Energy Ontology (ZEO) is the basic new element forcing to reconstruct quantum measurement theory. This leads to a theory of consciousness and allows to identify what life and death of conscious entity be from the point of view of quantum physics [K9].

   For dark matter the outcome of state function reduction would not be random below the duration of the sequence of state function reductions to the same boundary of causal diamond (CD) defining the lifetime of conscious entity: life could be seen as generalized Zeno effect. The first reduction to the opposite boundary of CD forced by NMP to eventually occur would mean death of the conscious entity and subsequent re-incarnation at the opposite boundary of CD. This conclusion is not just an idea which happened to pass by: reaching it took almost a quarter of century!

2. The goal of complexity science is to understand and control complex quantum systems and the basic challenge is to overcome quantum decoherence. Here TGD view about dark matter suggests a solution. TGD predicts a hierarchy of quantum phases of ordinary matter labelled by the value of Planck constant $h_{eff} = n \times h$ for which quantum scales are scaled up so that macroscopic quantum coherence becomes possible [K47, ?, K84]. The time scale for decoherence is expected also to scale up.

This hierarchy defines a hierarchy of conformal symmetry breakings possible also for ordinary conformal symmetry but for some reason remained un-recognized. This makes possible fractal symmetry breaking: the symmetry broken sub-algebra is isomorphic to the original one: symmetry breaking without symmetry breaking!

Second mystery of theoretical physics of last decades is the failure to realize that conformal invariance has a generalization to 4-D context due to the fact that light-cone boundary in 4-D Minkowski space is metrically 2-D. This symmetry is crucial for the construction of TGD as Kähler geometry and spinor structure of "World of Classical Worlds" (WCW). Unfortunately, my attempts to communicate this simple fact have been fruitless. Maybe the reasons are basically sociological: "people from Harward" simply refuse to take seriously what lower level organisms try to explain to them.

3. A further new element is Negentropy Maximization Principle (NMP) [K72], which serves as the basic variational principle of TGD inspired quantum measurement theory and also that of TGD inspired theory of consciousness. NMP states roughly that the negentropy gain in state function reduction is maximal. Mathematically NMP is very much like second law but applies to number-theoretic entanglement negentropy characterizing information content of the entire entangled system rather than ensemble entropy characterizing either member of of entangled pair so that no conflict with second law need to be implied. It indeed seems, that second law becomes scale dependent notion holding true only above the life-time of conscious entities involved.

4. NE is a further new element is brought by number theoretical vision. NE is possible if entanglement probabilities belong to an algebraic extension of rationals to which the parameters characterizing string world sheets and partonic 2-surfaces are assumed to belong.

   Remark: The restriction of parameters to be in algebraic extension means realization of finite measurement resolution at the level of WCW allowing to get rid of the problems related to breaking of basic symmetries in the discretization realized at space-time level.

   NE is defined as entangelement negentropy is defined as p-adic variant of entanglement entropy for a prime for which the entanglement entropy is minimal. This makes sense if the probabilities belong to algebraic extension.

   The p-adic entanglement entropy defined by the same formula as Shannon entropy but replacing ordinary logarithm of probability with the logarithm of the p-adic norm of probability can be negative so that it is better to talk about negentropy. The entanglement negentropy has interpretation as a measure of coscious information about the state of entangled system rather as entropy characterizing the lack of information about the state of either sub-system defining thermodynamical entropy.

   An important special case corresponds to NE for which density matrix is proportional to unit matrix: in this case the entanglement negentropy is maximal. The interpretation is that entanglement negentropy accompanies positively colored conscious experiences like experience of understanding and love. In the case of unit matrix it would correspond to meditative experience in which all distinctions are reported to disappear: this could relate to the fact that in this case any state basis is eigenstate of the density matrix.

5. Which states are characterized by NE and thus by algebraic entanglement probabilities? Could all states associated with the values $h_{eff}/h > 1$ be such states? This would make sense if the entanglement is in the discrete degrees of freedom implied by the n-dimensional covering of space-time surface.

6. There is a nice connection to quantum error correcting codes. Preskill *et al* [B26] introduce the notion of perfect tensor as a building brick of the tensor networks which they propose to give rise to a representation of holography using hyperbolic tessellations. Tensors define entangled states and perfect tensors have $2n$ k-valued indices such that any decomposition of the system to $2n$ states possesses maximal entanglement. This system has also the property that there is isometric embedding of any $n-k$ states to the state space spanned by $n+k$ states in the complement. This kind of system is ideal for error correcting quantum code

and would represent the stable subspace of states for which error correction is possible. The definition of the negentropically entangled system stable under is essentially the same!

7. NMP favors generation of NE. NE can be transferred between systems and can thus reduce for a given subsystem (just like thermodynamical entropy), it tends to increase for the entire Universe. NE is assigned with dark matter with non-standard value of Planck constant. This forces to reconsider the possibility that second law, which could be seen as a consequence of the randomness of the outcome of state function reduction, holds only for the visible matter with standard value of Planck constant and characterized by ordinary entanglement. A more precise formulation is that second law holds true in time scales longer than the time scale of the living systems involved: the death of conscious entity means ordinary state function at the opposite boundary of CD and therefore generates ensemble entropy. For NE the outcome is not random during the reduction sequence defining self as Zeno effect.

If this vision is on the correct track, the science of complexity would be also physics of dark matter, living matter, consciousness, and cognition.

17.4 Quantum Error Correcting Codes, Holography, and Tensor Networks

Quantum computer must be isolated from the environment that is keep the state of the computing system pure. In the world described by standard quantum theory alone it is however extremely difficult to prevent the generation of entanglement with environment in turn implying that the state obtained by tracing over the environmental degrees of freedom is non-pure: entanglement entropy is generated and decoherence occurs.

To avoid this error correction mechanisms have been developed.

1. The basic idea is that one has system, environment, and auxiliary system called ancilla. In the initial state the systems are unentangled but perturbations generate entanglement between system and environment (why not between ancilla and environment?). One can however perform a unitary transformation for the whole system transferring the entanglement from system-environment to environment-ancilla pair by performing a unitary transformation and by replacing ancilla essentially with a new one. Ancilla would function as trash bin.

2. The mirror image of this idea is actually familiar from TGD inspired theory of consciousness. NMP predicts that negentropy tends to increase in quantum jumps. The negentropy of individual systems can be however reduced. In particular, NE from the system can be stolen: this would define the quantum correlate for stealing as crime! Religious myths catch this idea. Eve could not avoid the temptation to eat the fruit from the tree of Good and Bad Knowledge! The error correcting code would perform just the opposite by cleaning away the parasitic entanglement.

3. In TGD based biological quantum computation this mechanism could be used to transfer conscious information from quantum computing system to other system. Cleaning would become picking of fruits! I have proposed that the fundamental function of metabolism is basically transfer of NE from nutrients to organism [K84].

In he ordinary computation error correcting codes replace bits with bit sequences for which additional bits serve as check bits allowing to detect whether one or more bits have changed. If the number of bits is below some number defining the code, the errors can be corrected. For an unpractical person like me, it was quite a surprise to learn that without this kind of error correction mechanisms IT technology would not be possible. Errors are not rare events but do occur all the time.

In quantum situation bits are replaced with qubits and error correcting code is defined by sub-space of the full code space. Quantum error correcting code maps logical qubits by Hilbert space isometry to an entangled state formed by a larger number of physical qubits. The image under the isometry defines code subpace.

A kind of quantum hologram in which the information is distributed between larger number of qubits is created. If the number of erratic qubits is below some threshold characterizing the code, the errors can be corrected by applying a unitary transformation determined by the erased bits. This is highly analogous to the properties of the ordinary hologram. These codes allow also also to protect some maximal number of logical bits and also to detect whether quantal eves-dropping has taken place.

This suggests a beautiful correspondence between the classical and quantal holographies in TGD framework.

1. Classical holography would be realized in terms of string world sheets and partonic 2-surfaces serving as holograms coding space-time surfaces as preferred extremals whereas quantum logical hologram would be realized in terms of fermions whose many-particle states at string world sheets and partonic 2-surfaces define quantum Boolean logic.

   WCW gamma matrices are constructed as combinations of the fermionic oscillator operators so that the classical and quantum holographies would correspond to WCW orbital degrees of freedom and spin degrees of freedom. Note also that the modes of WCW spinor fields are formally classical. In fact super-conformal symmetries relate the orbital and spin degrees of freedom of WCW so that also the two holographies might be closely related at deeper level.

2. This observation raises a question related to the interpretation and fundamental formulation of TGD [K126, K95]. The recent formulation assumes that Kähler-Dirac action is defined at entire space-time but that the solutions are localized to string world sheets: this guarantees the well-definedness of em charge if induced W gauge fields vanish at string world sheets. There are several other excellent reasons for assuming the localization.

   Could it be that also the induced spinor fields in the complement of partonic 2-surfaces and string world sheets are present just like interior degrees of freedom for space-time surfaces? Should one one follow supersymmetric instincts and holograpically continue also the spinor modes at string world sheets to the space-time interior just as one continues the string world sheets themselves? The holographic localization of the modes at string world sheets making possible conformal invariance at them would mean that all information about many-fermion states are carried by string world sheets.

17.4.1 Tensor networks

The concept of tensor network relies on the notion of isometry between Hilbert spaces, to the notion of perfect tensor defining a basic building brick for the networks defining isometry of logical qubits in the interior of 3-surface with the entangled states formed by physical qubits at the boundary. Hyperbolic tessellation is a further key concept. Two key results are that the isometry defining quantum holographic mapping of interior quantum states to boundary states can be realized in terms of tensor network assignable to the tessellation and entanglement entropy can be expressed as area for the surface separating subset of the boundary from its complement.

Hyperbolicity means negative curvature and for hyperbolic spaces means the existence of infinite number of tessellations with infinite number of cells allowing to build tensor networks. In 2-D case this is basically due to the negative sign of the angle deficit for a geodesic polygon: this favors regular tiles with large number of faces.

Group-theoretically this means that the discrete group defining the tessellation is infinite and there is infinite number of them. For constant curvature spaces with positive curvature this group is finite as also tessellation itself. Also the number of tessellations is finite - at least in the case of sphere (the 5 Platonic solids). By the way, Platonic solids might be interesting at the light-like boundary of CD resulting as a limit of hyperbolic space since it reduces to sphere metrically.

17.4.2 Isometries and perfect tensors

Unitary transformations of Hilbert space satisfying $UU^\dagger = Id$ are isometries preserving the inner product. The isometries from n-dimensional Hilbert space H_n to $n+k$-dimensional Hilbert space

H_{n+k} are possible and define embeddings.The inverse transformation is defined only in the subspace defined by the n-dimensional image and has k-dimensional kernel. If there are unitary transformations U and U' of the two Hilbert spaces satisfying $TU = U'T$ then $T^\dagger T \propto Id_n$ is satisfied.

For an isometry $T : H_1 \otimes H_2 \to H$ one can construct isometry $\tilde{T} : H_1 \to H \otimes H_2$ such that $\tilde{T}^\dagger \tilde{T} = dim(H_2)Id$ by simply moving a_2 from $|a_2a_1\rangle$ to the right-hand side in the formula

$$T : |a_2a_1\rangle \to \sum_b |b\rangle T_{ba_2a_1} \quad ,$$

and by tracing to get

$$\tilde{T} : |a_1\rangle \to \sum_{ba_2} |ba_2\rangle T_{ba_2a_1} \quad .$$

Tensor $T_{a_1....a_{2n}}$ defines entangled states by a contraction with quantum states in labelled by a_i defining the set of $2n$ indices with v values.

Perfect tensors have the defining property that for any decomposition of the index set to two parts A and its complement A^c, $|A| \leq |A^c|$ the map defined by the tensor from A to A^c is isometry. Furthermore, perfect tensor maps $m \leq n$ spins isometrically to $2n - m$ spins. This obviously defines code as mapping of $m \leq n$- spin states to $2n - m$-spin states. In particular, single spin is map to $2n-1$ spins isometrically. For any (n, n) decomposition perfect tensor defines absolutely maximally entangled states.

In the case considered spin is coded by a linear map to entangled $2n - 1$-spins state of physical spins and there is 1 protected logical spin. There is stability against the erasure of $n - 1$ physical spins. A party holding n spins has complete information about logical spin because the erasure of $n - 1$ spins is correctable. Party holding $n - 1$ spins has no information about the full state. One could say that local physics in interior is coded to non-local physics at boundary, which is just what happens also in the ordinary hologram.

This picture suggest a deep connection between quantum information theory, quantum measurement theory according to ZEO, and consciousness: NE is optimal from the point of view of error correction and therefore it is natural to assume NMP and adelic physics bringing in various p-adic physics as correlates for cognition.

17.4.3 Hyperbolic tessellations and holographic quantum states and codes

The key results of the work of Preskill *et al* [B26] is what they call toy model for the realization of holography at the level of quantum states using tensor networks associated with tessellations of 2-D hyperbolic space forming time= constant section of AdS_3. The results are believed to hold true also in higher dimensions.

There is infinite number of tessellations labelled by the discrete infinite groups of Lorentz group so that the number of tensor networks is infinite reflecting faithfully the fact that entanglement can be equated with quantum complexity. The tensor network mapping the localized interior states isometrically to entangled boundary states is highly non-unique. For instance, one can choose the part of boundary involved in many ways and there is infinite number of tessellations. This suggests also a physical definition of complexity. Complexity should be defined in terms of the minimal tensor network allowing to realize this isometry.

Preskill *et al* define holographic quantum states and quantum codes. For holographic quantum states each tile of the tessellation carries quantum numbers meaning that not all of the spins are contracted. One could interpret these states as an explicit realization of the isometric mapping of localized interior states to boundary states with their image defining the stable code space. These states should not be however possible if one interprets holography in strict sense. This forces to consider whether the notion of holography used is general enough - this will be considered later from TGD point of view. For quantum codes all interior spins are contracted and the state is pure boundary state. Two examples about tessellations involving hexagons and pentagons are discussed explicitly.

17.4.4 Entanglement structure of holographic states

In the case considered 3-surface is replaced with 2-surface and in this case Ryu-Takayanagi formula [B42] represents the entanglement entropy between sets A and A^c at the boundary of 2-surface as a length of a minimal geodesic of hyperbolic plane connecting the boundary points of A. In 3-D case one would have minimal 2-surface.

The unit of "area" for d-dimensional boundary used is $4G_{d+2}$, where G_D is Newtons's constant of D-dimensional gravitational theory. The formula follows by assuming Einstein's equations at low energies. The argument of Ryu and Takayanagi involves AdS/CFT duality and introduction of AdS blackhole. One has string theory description of gravitation in AdS, one takes long length scale limit of this theory by making AdS dynamical and then introduces blackholes: reader can decide whether to take these steps leading to the desired outcome seriously. The source of my own skepticism is that the long length scale limit of string theory at long length scales involves too much hand waving and has led to the landscape problem in super string models.

Remark: In TGD framework strong form of holography implies that for $d = 2$ AdS_4 is replaced by 4-D space-time surface. One avoids the S^6 factor of 10-D fictive embedding space altogether. 8-D embedding space is present but is completely non-dynamical. Possible blackhole like entities (the light-like orbits of partonic 2-surfaces) are associated in TGD based holography with space-time surface. Strings in AdS correspond to string worlds sheets in space-time. Clearly there are strong similarities but also crucial differences.

Preskill *et al* [B26] represent an argument for how Ryu-Takayanagi formula could be understood in terms of the isometry relating local states in interior and non-local states at boundary. The minimal geodesic defines what is called causal wedge. The local operators within the causal wedge can be mapped isometrically to those at the boundary. Or stated differently: logical qubits in interior can be isometrically mapped to physical many-qubit states at the boundary.

As far as I understand, only the graph property of the tessellation and perfect tensor serving as building brick of the isometry are essential so that deformations of the tessellations are possible. In the argument the minimal curve reduces to the number of tiles along the boundary of the causal wedge and the formula expresses only upper bound for the entanglement entropy. The formula is purely combinatorial and based on the number of spin states at each node and on the number of nodes along the minimal geodesic.

Also an algorithm for finding the mininal geodesic is considered. So called greedy geodesic constructible numerically by starting from a given perfect tensor at boundary and pushing it to interior by isometries defined by perfect tensor as far as this is possible is discussed. For tessellations the greedy geodesics associated with A and A^c co-incide and define the minimal geodesic.

The notion of entanglement wedge is also considered. Within it the local operators in interior are mappable by isometry to the boundary operators. The authors consider multipartite entanglement and entanglement distillation but I am not specialist enough to attempt to relate this to TGD. Quantum error correction in holographic codes is also discussed in more detail. Some comments about information theoretic interpretation of blackholes are also represented and the problematic interpretation of holographic states is considered. Creation of blackhole would be analogous to an elimination of tiles in the interior and generating in this manner free induces and disjoint component to boundary.

17.5 TGD View about the Holographic States and Codes

17.6 TGD View about the Holographic States and Codes

The considerations of the article generalize to TGD rather easily. In TGD one has 3-D hyperbolic space H^3 as a sub-manifold of future light-cone (actually part of H^3 as sub-manifold of CD) so that possible problems due to the existence of closed time-like geodesics are avoided as also the objections due to the artificial nature of the conformal boundary. I have already described the geometric ideas.

17.6.1 Realization of the holographic states in terms of flux tube networks

For TGD point of view the most interesting question is whether also holographic states de
ning the code explicitly are possible as genuine physical states: it would be disappointing if the beautiful mathematics of hyperbolic 3-manifolds would not be realized physically. One could indeed consider of putting many-fermion states to the nodes of the tessellation in which the faces of tiles are connected by flux tubes and this would give kind of physical realization of holography.

Dark matter at magnetic flux tubes would represent higher level able to concretely represent physics at the lower level by realizing strong form of holography. Dark matter would have kind of sensory representations about the matter at lower levels of dark matter hierarchy. Biological systems would be this kind of systems since they build representations about environment - send themselves - some of them even about laws of physics (!)-, and there is evidence that in living matter kind of coordinate grids are realized [K88]: the proposed TGD interpretation is in terms of flux tube networks defining tensor networks mapping boundary states to interior states.

In TGD framework magnetic flux tubes carrying monopole Kahler magnetic flux are correlates for entanglement and could also define the edges of the tensor networks based on H^3 tessellations. Partonic 2-surfaces are however not homologically trivial and therefore not boundaries of anything: this is what makes them behave as Kähler monopoles and stabilizes both wormhole contacts and magnetic flux tubes. As a consequence, they appear as throats of wormhole contacts behaving like opposite sign magnetic charges. Even this is not enough: by the fact that magnetic flux lines must be closed, one must have pairs of wormhole contacts as a minimal structure and elementary particles are indeed modelled in this manner.

17.6.2 Generalization of the area formula for entanglement entropy

How the area law for entanglement entropy could generalize?

1. The replacement of entropy with its number theoretical variant, which becomes negative for NE, looks very natural in living matter. The growth of the network during time evolution discussed by Susskind in this lecture (see `http://tinyurl.com/y8zhhca5`) would not correspond to approach to chaos of thermal equilibrium but generation of NE and evolution at dark matter level! Second law would be manifested only at the level of visible matter since entanglement is always entropic for it.

   The phase transitions increasing Planck constant would generate NE and would scale up quantum length by $h_{eff}/h = n$ so that the networks would increase.

   Remark: It has become clear that instead of h one must have minimal value h_0 of h_{eff} which could be smaller than h [L26, L60].

   Susskind assigns the growth of the network to blackhole interiors containing kind of invisible growing part of the network. In TGD Universe the emergence of wormholes connected by flux tubes would correspond to the growth of the tensor network. The hyperbolic tessellations of H^3 defining hyperbolic structure at 3-surfaces representable as graphs $H^3 \rightarrow CP_2$ have scaling invariance as a symmetry.

   The growth of the tensor network would occur in quantum critical phase transitions increasing the value of Planck constant. It is known that there is no cosmic expansion in local scales: this seems to be conflict with expansion in cosmological scales. The explanation could be that the expansion in given scale occurs in jerkwise manner - perhaps via phase transitiond increasing h_{eff} permanently or possiblty temporarily followed by a phase transition increasing p-adic length scale and reducing h_{eff} back to the standard value so that the scale is not changed. I have actually proposed a model for Expanding Earth (motivation comes from the observations that continents fit nicely together if the radius of Earth is by a factor 1/2 smaller than it is now) in terms of a phase transition increasing Earth radius by a factor of two. Sudden increase of the information content of flux tube network would be in question - kind of eureka experience of Mother Gaia [K5]! The model explains also the mysterious emergence of highly evolved multicellular lifeforms in Cambrian explosion as life forms evolved in the underground oceans and burst to the surface of Earth in the phase transition creating also the oceans.

Before Cambrian Explosion Earth would have been like Mars, which by the way has radius equal to 1/2 of the Earth's radius.

2. Tensor network discretizes the continuum view: in particular, the continuous minimal surface is replaced with a set consisting of tiles of the tessellation. Could this description emerge from TGD as a genuine microscopic description in which macroscopic area consists of sum over microscopic areas? Wormhole throats from which flux tubes defining the links of the tensor network emerge could indeed specify the boundary of 3-D surface at microscopic level. Consider a subset A consisting of wormholes throats and the complement A^c of this set.

3. Ryu-Takayanagi formula involves a 2-surface, whose boundary is same as the boundary between A and A^c. What is essential that this surface is separating. The entanglement entropy for $A - A^c$ pair is given by the area of the separating surface and the discretized version of this can be understood quite concretely by studying the tensor network. One should identify a separating 2-surface for this set and its complement in TGD framework.

   To separate the throat from environment one must cut both the throat and flux tubes emerging from it. One could do this for all flux tubes in the set leading to complement of the set - or to environment. The entanglement entropy (or maybe negentropy for dark particles) would be proportional to the sum of the cross sectional areas divided by some unit for area. This could be the area the area of CP_2 geodesic sphere. One can argue that the cutting of monopole flux tube is not possible physically since it would create two opposite monopole charges. One can certainly imagine open strings like objects with CP_2 projection, which is homologically trivial but preferred extremal property and boundary conditions probably do not allow this. A more plausible realization would be as a pair of parallel flux tubes with opposite directions of magnetic fluxes with are identical. Reconnection for this pair would cut the flux tube pair. This kind of U-shaped flux tube pairs are central in TGD model for living matter. They act as kind of tentacles sniffing the environment, and when two flux tubes pairs of this kind meet, they can reconnect if the fluxes are identical and magnetic field strengths are suciently near to each other - the fluxes are quantized by the monopole character of flux.

This microscopic picture seems to be considerably more flexible than the picture based on the consideration of the 3-surfaces with macroscopic boundary. For instance, the entanglement negentropy of blackhole horizon (or actually the surface defining the causal horizon of any astrophysical object as surface at which the signature of the induced metric changes) with environment could be expressed as sum of cross-sectional areas of flux tubes connecting the horizon to the environment. These flux tubes would mediate gravitational interaction. It would be essential that the networks emerge at the level of magnetic body and dark matter, not the ordinary matter.

What is somewhat troubling in the construction of Preskill *et al* involving interior spins is that these should not be present in macroscopic holography. Authors suggest the interpretation of the holes of the tensor network as blackholes giving rise to horizon as new part of boundary. In TGD framework spins as physical states would always have as building bricks wormhole throats so that one would have counterparts of boundary states also now so that this problem disappears.

TGD interpretation generalizes the interpretation of Preskill *et al* by allowing single wormhole throat as a minimal blackhole like entity. In fact, the counterparts of blackholes would in TGD framework correspond to macroscopic wormhole throats. Also anyonic systems in condensed matter physics could correspond to this kind of systems elementary particles would be glued to this large boundary surface by 3-D topological condensation somewhat like various objects like plants to the Earth's surface [K85].

17.6.3 Summary

AdS/CFT correspondence is not essential for the realization of tensor networks. TGD based holography works equally well and the hyperbolic space H^3 emerges naturally in this framework. TGD approach allows also to get rid of various problematic aspects of AdS/CFT correspondence (artificial character of AdS, time-like closed geodesics, AdS boundary is not real) and a microscopic generalization of the Ruy-Taganaki formula is possible. Only the combinatorial structure of the tessellation and the notion of perfect tensor matter from information theoretic point of view (Ryu-Takayanagi formula however requires Einstein's equations) and this allows considerable flexibility in the realization of tensor works. There are many ways to induce the tessellation from M^4 since boundary tiles need not be complete. Holography might allow much more general representations than ideal tessellations since all that matters is the topological structure of the graph involved and that the tensors used as building bricks are perfect. The maps expressing holography can be also composed from a large variety of different perfect tensors.

The tensor networks might be realized in TGD inspired quantum biology and rely on NE assignable to discrete dark matter degrees of freedom. I have already earlier considered the hypothesis that the coordinate grid like structure formed from flux tubes could define kind of template for the self-organization of biosystem in 4-D sense implied by ZEO. Quantum complexity could force a generalization of condensed matter physics to that associated with tessellations of H^3, the number of which is infinite!

There are many interesting questions not discussed. Quantum TGD can be regarded as "complex square root" of thermodynamics. What can one say about the first law of this quantum thermodynamics? What about the analog of the first law for the area law?

17.7 Tensor Networks and S-matrices

The concrete construction of scattering amplitudes has been the toughest challenge of TGD and the slow progress has occurred by identification of general principles with many side tracks. One of the key problems has been unitarity. The intuitive expectation is that unitarity should reduce to a local notion somewhat like classical field equations reduce the time evolution to a local variational principle. The presence of propagators have been however the obstacle for locally realized unitarity in which each vertex would correspond to unitary map in some sense.

TGD suggests two approaches to the construction of S-matrix.

1. The first approach is generalization of twistor program [K114]. What is new is that one does not sum over diagrams but there is a large number of equivalent diagrams giving the same outcome. The complexity of the scattering amplitude is characterized by the minimal diagram. Diagrams correspond to space-time surfaces so that several space-time surfaces give rise to the same scattering amplitude. This would correspond to the fact that the dynamics breaks classical determinism. Also quantum criticality is expected to be accompanied by quantum critical fluctuations breaking classical determinism. The strong form of holography would not be unique: there would be several space-time surfaces assignable as preferred extremals to given string world sheets and partonic 2-surfaces defining "space-time genes".

2. Second approach relies on the number theoretic vision and interprets scattering amplitudes as representations for computations with each 3-vertex identifiable as a basic algebraic operation [K114]. There is an infinite number of equivalent computations connecting the set of initial algebraic objects to the set of final algebraic objects. There is a huge symmetry involved: one can eliminate all loops moving the end of line so that it transforms to a vacuum tadpole and can be snipped away. A braided tree diagram is left with braiding meaning that the fermion lines inside the line defined by light-like orbit are braided. This kind of braiding can occur also for space-like fermion lines inside magnetic flux tubes and defining correlate for entanglement. Braiding is the TGD counterpart for the problematic non-planarity in twistor approach.

Third approach involving local unitary as an additional key element is suggested by tensor networks relying on the notion of perfect entanglement discussed by Preskill *et al* [B26].

1. Tensor networks provide an elegant representation of holography mapping interior states isometrically (in Hilbert space sense) to boundary states or vice versa for selected subsets of states defining the code subspace for holographic quantum error correcting code. Again the tensor net is highly non-unique but there is some minimal tensor net characterizing the complexity of the entangled boundary state.

2. Tensor networks have two key properties, which might be abstracted and applied to the construction of S-matrix in zero energy ontology (ZEO): perfect tensors define isometry for any subspace defined by the index subset of perfect tensor to its complement and the non-unique graph representing the network. As far as the construction of Hilbert space isometry between local interior states and highly non-local entangled boundary states is considered, these properties are enough.

One cannot avoid the question whether these three constructions could be different aspects of one and same construction and that tensor net construction with perfect tensors representing vertices could provide and additional strong constraint to the long sought for explicit recipe for the construction of scattering amplitudes.

17.7.1 Objections

It is certainly clear from the beginning that the possibly existing description of S-matrix in terms of tensor networks cannot correspond to the perturbative QFT description in terms of Feynman diagrams.

1. Tensor network description relates interior and boundary degrees in holography by a isometry. Now however unitary matrix has quite different role. It could correspond to U-matrix relating zero energy states to each other or to the S-matrix relating to each other the states at boundary of CD and at the shifted boundary obtained by scaling. These scalings shifting the second boundary of CD and increasing the distance between the tips of CD define the analog of unitary time evolution in ZEO. The U-matrix for transitions associated with the state function reductions at fixed boundary of CD effectively reduces to S-matrix since the other boundary of CD is not affected.

   The only manner one could see this as holography type description would be in terms of ZEO in which zero energy states are at boundaries of CD and U-matrix is a representation for them in terms of holography involving the interior states representing scattering diagram in generalized sense.

2. The appearance of small gauge coupling constant tells that the entanglement between "states" in state spaces whose coordinates formally correspond to quantum fields is weak and just opposite to that defined by a perfect tensor. Quite generally, coupling constant might be the fatal aspect of the vertices preventing the formulation in terms of perfect entanglement.

   One should understand how coupling constant emerges from this kind of description - or disappears from standard QFT description. One can think of including the coupling constant to the definition of gauge potentails: in TGD framework this is indeed true for induced gauge fields. There is no sensical manner to bring in the classical coupling constants in the classical framework and the inverse of Kähler coupling strength appears only as multiplier of the Kähler action analogous to critical temperature.

   More concretely, there are WCW spin degrees of freedom (fermionic degrees of freedom) and WCW orbital degrees of freedom involving functional integral over WCW. Fermionic contribution would not involve coupling constants whereas the functional integral over WCW involving exponential of vacuum functional could give rise to the coupling constants assignable to the vertices in the minimal tree diagram.

3. The decomposition $S = 1 + iT$ of unitary S-matrix giving unitarity as the condition $-i(T - T^\dagger) + T^\dagger T = 0$ reflects the perturbative thinking. If one has only isometry instead of unitary transformation, this decomposition becomes problematic since T and $T^\dagger$ whose some appears in the formula act in different spaces. One should have the generalization of Id as a "trivial" isometry. Alternatively, one should be able to extend the state space H_{in} by adding a tensor factor mapped trivially in isometry.

4. There are 3- and 4-vertices rather than only -say, 3-vertices as in tensor networks. For non-Abelian Chern-Simons term for simple Lie group one would have besides kinetic term only 3-vertex $Tr(A \wedge A \wedge A)$ defining the analog of perfect tensor entanglement when interpreted as

co-product involving 3-D permutation symbol and structure constants of Lie algebra. Note also that for twistor Grassmannian approach the fundamental vertices are 3-vertices. It must be however emphasized that QFT description emerges from TGD only at the limit when one identifies gauge potentials as sums of induced gauge potentials assignable to the space-time sheets, which are replaced with single piece of Minkowski space.

5. Tensor network description does not contain propagators since the contractions are between perfect tensors. It is to make sense propagators must be eliminated. The twistorial factorization of massless fermion propagator suggest that this might be possible by absorbing the twistors to the vertices.

These reasons make it clear that the proposed idea is just a speculative question. Perhaps the best strategy is to look this crazy idea from different view points: the overly optimistic view developing big picture and the approach trying to debunk the idea.

17.7.2 The overly optimistic vision

With these prerequisites on one can follow the optimistic strategy and ask how tensor networks could the allow to generalize the notion of unitary S-matrix in TGD framework.

1. Tensor networks suggests the replacement of unitary correspondence with the more general notion of Hilbert space isometry. This generalization is very natural in TGD since one must allow phase transitions increasing the state space and it is quite possible that S-matrix represents only isometry: this would mean that $S^\dagger S = Id_{in}$ holds true but $SS^\dagger = Id_{out}$ does not even make sense. This conforms with the idea that state function reduction sequences at fixed boundary of causal diamonds defining conscious entities give rise evolution implying that the size of the state space increases gradually as the system becomes more complex. Note that this gives rise to irreversibility understandandable in terms of NMP [K72]. It might be even impossible to formally restore unitary by introducing formal additional tensor factor to the space of incoming states if the isometric map of the incoming state space to outgoing state space is inclusion of hyperfinite factors.

2. If the huge generalization of the duality of old fashioned string models makes sense, the minimal diagram representing scattering is expected to be a tree diagram with braiding and should allow a representation as a tensor network. The generalization of the tensor network concept to include braiding is trivial in principle: assign to the legs connecting the nodes defined by perfect tensors unitary matrices representing the braiding - here topological QFT allows realization of the unitary matrix. Besides fermionic degrees of freedom having interpretation as spin degrees of freedom at the level of "World of Classical Worlds" (WCW) there are also WCW orbital degrees of freedom. These two degrees of freedom factorize in the generalized unitarity conditions and the description seems much simpler in WCW orbital degrees of freedom than in WCW spin degrees of freedom.

3. Concerning the concrete construction there are two levels involved, which are analogous to descriptions in terms of boundary and interior degrees of freedom in holography. The level of fundamental fermions assignable to string world sheets and their boundaries and the level of physical particles with particles assigned to sets of partonic 2-surface connected by magnetic flux tubes and associated fermionic strings. One could also see the ends of causal diamonds as analogous to boundary degrees of freedom and the space-time surface as interior degrees of freedom.

The description at the level of fundamental fermions corresponds to conformal field theory at string world sheets.

1. The construction of the analogs of boundary states reduces to the construction of N-point functions for fundamental fermions assignable to the boundaries of string world sheets. These boundaries reside at 3-surfaces at the space-like space-time ends at CDs and at light-like 3-surfaces at which the signature of the induced space-time metric changes.

2. In accordance with holography, the fermionic N-point functions with points at partonic 2-surfaces at the ends of CD are those assignable to a conformal field theory associated with the union of string world sheets involved. The perfect tensor is assignable to the fundamental 4-fermion scattering which defines the microscopy for the geometric 3-particle vertices having twistorial interpretation and also interpretation as algebraic operation.

   What is important is that fundamental fermion modes at string world sheets are labelled by conformal weights and standard model quantum numbers. No four-momenta nor color quantum numbers are involved at this level. Instead of propagator one has just unitary matrix describing the braiding.

3. Note that four-momenta emerging in somewhat mysterious manner to stringy scattering amplitudes and mean the possibility to interpret the amplitudes at the particle level.

Twistorial and number theoretic constructions should correspond to particle level construction and also now tensor network description might work.

1. The 3-surfaces are labelled by four-momenta besides other standard model quantum numbers but the possibility of reducing diagram to that involving only 3-vertices means that momentum degrees of freedom effectively disappear. In ordinary twistor approach this would mean allowance of only forward scattering unless one allows massless but complex virtual momenta in twistor diagrams. Also vertices with larger number of legs are possible by organizing large blocks of vertices to single effective vertex and would allow descriptions analogous to effective QFTs.

2. It is highly non-trivial that the crucial factorization to perfect tensors at 3-vertices with unitary braiding matrices associated with legs connecting them occurs also now. It allows to split the inverses of fermion propagators into sum of products of two parts and absorb the halves to the perfect tensors at the ends of the line. The reason is that the inverse of massless fermion propagator (also when masslessness is understood in 8-D sense allowing M^4 mass to be non-vanishing) to be express as bilinear of the bi-spinors defining the twistor representing the four-momentum. It seems that this is absolutely crucial property and fails for massive (in 8-D sense) fermions.

17.7.3 Twistorial and number theoretic visions

Both twistorial and number theoretical ideas have given a strong boost to the development of ideas.

1. With experience coming from twistor Grassmannian approach, twistor approach is conjectured to allow an extension of super-symplectic and other superconformal symmetry algebras to Yangian algebras by adding a hierarchy of multilocal generators [K114]. The twistorial diagrams for $\mathcal{N} = 4$ SUSY can be reduced to a finite number and there is large number of equivalent diagrams. One expects that this is true also in TGD framework.

   Twistorial approach is extremely general and quite too demanding to my technical skills but its is a useful guideline. An important outcome of twistor approach is that the intermediate states are massless on-mass-shell states but with complex momenta. Does this generalize and could each vertex define unitary scattering event with complex four-momenta in possibly complexified Minkowski space? Or could even real momenta be possible for massive particles, which would be massless in 8-D sense thanks to the existence of octonionic tangent space structure of 8-D embedding space? And what is the role of the unique twistorial properties of M^4 and CP_2?

2. Number theoretical vision suggests that the scattering amplitudes correspond to sequences of algebraic operations taking inputs and producing outputs, which in turn serve as inputs for a neighboring node [K114]. The vertices form a diagram defining a network like structure defining kind of distributed computations leading from given inputs to given outputs. A computation leading from given inputs to given outputs is suggestive. There exists an infinite number of this kind of computations and there must be the minimal one which defines the complexity of the scattering. The maximally simplifying guess is that this diagram would

correspond to a braided tree diagram. At space-time level these diagrams would correspond to different space-time surfaces defining same physics: this is because of holography meaning that only the ends of space-time surfaces at boundaries of CD matter.

This vision generalizes of the old-fashioned stringy duality. It states that all diagrams can be reduced to minimal diagrams. This is achieved by by moving the ends of internal lines so that loops becomes vacuum tadpoles and can be snipped off. Tree diagrams must be however allowed to braid and outside the vertices the diagrams look like braids. Braids for which threads can split and glue together is the proper description for what the diagrams could be. Braiding would provide the counterpart for the non-planar twistor diagrams.

The fermion lines inside the light-like 3-surfaces can get braided. Smaller partonic 2-surfaces can topologically condense at given bigger partonic 2-surface (electronic parton surface can topologically condense to nano-scopic parton surface) and the orbits of the condensed partonic 2-surfaces at the light-like orbit of the parton surface can get braided. This gives rise to a hierarchy of braids with braids.

17.7.4 Generalization of the notion of unitarity

The understanding of unitarity has been the most difficult issue in my attempts to understand S-matrix in TGD framework. When something turns out to be very difficult to understand, it might make sense to ask whether the definition of this something involves un-necessary assumptions. Could unitarity be this kind of notion?

The notion of tensor network suggests that unitarity can generalized and that this generalization allows the realization of unitarity in extremely simple manner using perfect tensors as building bricks of diagrams.

1. Both twistorial and number theoretical approaches define M-matrix and associated S-matrix as a map between the state spaces H_{in} and H_{out} assignable to the opposite boundaries of CD - say positive and negative energy parts of zero energy state. In QFT one has $H_{in} = H_{out}$ and the map would be Hilbert space unitary transformation satisfying $SS^\dagger = S^\dagger S = Id$.

2. The basic structure of TGD (NMP favoring generation of negentropic entanglement, the hierarchy of Planck constants, length scale hierarchies, and hierarchy of space-time sheets) suggests that the time evolution leads to an increasingly complex systems with higher-dimensional Hilbert space so that $H_{in} = H_{out}$ need not hold true but is replaced with $H_{in} \subset H_{out}$. This view is very natural since one must allow quantum phase transitions increasing the value of h_{eff} and the value of p-adic prime defining p-adic length scale.

   S-matrix would thus define isometric map $H_{in} \subset H_{out}$. Isometry property requires $U^\dagger U = Id_{in}$. If the inclusion of H_{in} to H_{out} is a genuine subspace of H_{out}, the condition $UU^\dagger = Id_{out}$ does not make sense anymore. This means breaking of reversibility and is indeed implied by the quantum measurement theory based on ZEO.

3. It would be at least formally possible to fuse all state spaces to single very large state space by replacing isometry $H_{in} \subset H_{out}$ with unitary map $H_{out} \to H_{out}$ by adding a tensor factor in which the map acts as identity transformation. This is not practical since huge amounts of redundant information would be introduced. Also the information about hierarchical structure essential for the idea of evolution would be lost. This hierarchical of inclusions should also be crucial for understanding the construction of S-matrix or rather, the hierarchy of S-matrices of isometric inclusions including as a special case unitary S-matrices.

4. There is also a further intricacy, which might prevent the formal unitarization by the addition of an inert tensor factor. I have talked a lot about HFFs referring to hyper-finite factors of type II_1 (possibly also of type III_1) and their inclusions [K125]. The reason is that WCW spinors form a canonical representation for these von Neumann algebras.

   Could the isometries replacing unitary S-matrix correspond to inclusions of HFFs? In the recent interpretation the included factor (now H_{in}) corresponds to the degrees of freedom below measurement resolution. Certainly this does not make sense now. The interpretation in terms of finite measurement resolution need not however be the only possible interpretation and the interpretation in terms of measurement resolution might of course be wrong. Therefore one can ask whether the relation between H_{in} and H_{out} could be more complex than just $H_{out} = H_{in} \otimes H_1$ so that formal unitarization would fail.

17.7.5 Scattering diagrams as tensor networks constructed from perfect tensors

Preskill's tensor network construction [B26] realizes isometric maps as representations of holography and as models for quantum error correcting codes. These tensor networks have remarkable similarities with twistorial and number theoretical visions, which suggests that it could be used to construct scattering amplitudes. A further idea inspired by holography is that the description of scattering amplitudes in terms of fundamental fermions and physical particles are dual to each other.

1. In the construction of quantum error codes tensor network defines an isometric embedding of local states in the interior to strongly entangled non-local states at boundary. Their vertices correspond to tensors, which in the proposal of Preskill *et al* [B26] are perfect tensors such that one can take any m legs of the vertex and the tensor defines isometry from the state space of m legs to that of $n-m$ legs. When the number of indices is $2n$, the entanglement defined by perfect tensor between any n-dimensional subspace and its complement is maximal

   TGD framework maximal entanglement corresponds to negentropic entanglement with density matrix proportional to identity matrix. What is important that the isometry is constructed by composing local isometries associated with a network. Given isometry can be constructed in very many ways but there is some minimal realization.

2. The tensor networks considered in [B26] are very special since they are determined by tessellations of hyperbolic space H_2. This kind of tessellations of H_3 could be crucial for understanding the analog of condensed matter physics for dark matter and could appear in biology [L25]. What is crucial is that only the graph property and perfect tensor property matter as far as isometricity is considered so that it is possible to construct very general isometries by using tensor networks.

17.7.6 Eigenstates of Yangian co-algebra generators as a way to generate maximal entanglement?

Negentropically entangled objects are key entities in TGD inspired theory of consciousness and also of tensor networks, and the challenge is to understand how these could be constructed and what their properties could be. These states are diametrically opposite to unentangled eigenstates of single particle operators, usually elements of Cartan algebra of symmetry group. The entangled states should result as eigenstates of poly-local operators. Yangian algebras involve a hierarchy of poly-local operators, and twistorial considerations inspire the conjecture that Yangian counterparts of super-symplectic and other algebras made poly-local with respect to partonic 2-surfaces or end-points of boundaries of string world sheet at them are symmetries of quantum TGD [K50]. Could Yangians allow to understand maximal entanglement in terms of symmetries?

1. In this respect the construction of maximally entangled states using bi-local operator $Q^z = J_x \otimes J_y - J_x \otimes J_y$ is highly interesting since entangled states would result by state function. Single particle operator like J_z would generate un-entangled states. The states obtained as eigenstates of this operator have permutation symmetries. The operator can be expressed as $Q^z = f^z_{ij} J^i \otimes J^j$, where f^A_{BC} are structure constants of SU(2) and could be interpreted as co-product associated with the Lie algebra generator J^z. Thus it would seem that unentangled states correspond to eigenstates of J^z and the maximally entangled state to eigenstates of co-generator Q^z. Kind of duality would be in question.

2. Could one generalize this construction to n-fold tensor products? What about other representations of SU(2)? Could one generalize from SU(2) to arbitrary Lie algebra by replacing Cartan generators with suitably defined co-generators and spin 1/2 representation with

fundamental representation? The optimistic guess would be that the resulting states are maximally entangled and excellent candidates for states for which negentropic entanglement is maximized by NMP [K72].

3. Co-product is needed and there exists a rich spectrum of algebras with co-product (quantum groups, bialgebras, Hopf algebras, Yangian algebras). In particular, Yangians of Lie algebras are generated by ordinary Lie algebra generators and their co-generators subject to constraints. The outcome is an infinite-dimensional algebra analogous to one half of Kac-Moody algebra with the analog of conformal weight N counting the number of tensor factors. Witten gives a nice concrete explanation of Yangian [B15] for which co-generators of T^A are given as $Q^A = \sum_{i<j} f^A_{BC} T^B_i \otimes T^C_j$, where the summation is over discrete ordered points, which could now label partonic 2-surfaces or points of them or points of string like object (see http://tinyurl.com/y727n8ua). For a practically totally incomprehensible description of Yangian one can look at the Wikipedia article (see http://tinyurl.com/y7heufjh).

4. This would suggest that the eigenstates of Cartan algebra co-generators of Yangian could define an eigen basis of Yangian algebra dual to the basis defined by the totally unentangled eigenstates of generators and that the quantum measurement of poly-local observables defined by co-generators creates entangled and perhaps even maximally entangled states. A duality between totally unentangled and completely entangled situations is suggestive and analogous to that encountered in twistor Grassmann approach where conformal symmetry and its dual are involved. A beautiful connection between generalization of Lie algebras, quantum measurement theory and quantum information theory would emerge.

17.7.7 Two different tensor network descriptions

The obvious question is whether also unitary S-matrix of TGD could be constructed using tensor network built from perfect tensors. In ZEO the role of boundary would be taken by the ends of the space-time at upper and lower light-like boundaries of CD carrying the particles characterized by standard model quantum numbers. Strong form of holography would suggest that partonic surfaces and strings at the ends of CD provide information for the description of zero energy states and therefore of scattering amplitudes. The role of interior would be taken by the space-time surface - in particular the light-like orbits of partonic surfaces carrying the fermion lines identified as boundaries of string world sheets. Conformal field theory description would apply to fermions residing at string world sheets with boundaries at light-like orbits of partonic 2-surfaces.

In QFT Feynman diagrammatics one obtains a sum over diagrams with arbitrary numbers of loops. In both twistorial and number theoretic approach however only a finite number of diagrams with possibly complex on mass shell massless momenta are needed. If the vertices are however such that particles remain on-mass-shell but are allowed to have complex four-momenta then the integration over internal momenta (loops) is not present and tensor network description could make sense. This encourages the conjecture that tensor networks could be used to construct the scattering amplitudes in TGD framework.

What could perfect tensor property mean for the vertices identified as nodes of a tensor network? There are two levels to be considered: the geometric level identifying particles as 3-surfaces with net quantum numbers and the fermion level identifying particles as fundamental fermions at the boundaries of string world sheets.

1. At the geometric level vertices corresponds to light-like orbits of partonic 2-surfaces meeting at common end which is partonic 2-surface. This is 3-D generalization of Feynman diagram as a geometric entity. At the level of fermion lines associated with the light-like 3-surfaces one the basic interaction corresponds to the scattering of 2-fermions leading to re-sharing of fermion lines between outgoing light-like 3-surfaces, which include also representations for virtual particles. One has 4-fermion vertex but not in the sense that it appears in the interaction of weak interactions at low energies.

   Geometrically the basic vertex could be 3-vertex: $n > 3$-vertices are unstable against deformation to lower vertices. For 3-vertex perfect tensor property means that the tensor defining the vertex maps any 1-particle subspaces to 2-particle subspace isometrically. The geometric vertices define a network consisting of 3-D “lines” and 2-D vertices but one cannot tell what is within the 3-D lines and what happens in the 2-D nodes. The lines would consist of braided fundamental fermion lines and in nodes the basic process would be 2+2 scattering for fermions. In the case of 3-vertex momentum conservation would effectively eliminate the four-momentum and the state spaces associated with vertex would be effectively discrete. This is p-adically of utmost importance.

2. At the level of fundamental fermion lines in the interior of particle lines one would have 4-vertices and if a perfect tensor describes it, it gives rise to a unitary map of any 2-fermion subspace to its complement plus isometric maps of 1-fermion subspaces to 3-fermion subspaces. In this case momenta cannot act as labels of fermion lines for rather obvious reasons: the solution of the problem is that conformal weights label fundamental fermion lines

   The conservation of discrete quark and lepton numbers allows only vertices of type $qL \rightarrow qL$ and its variants obtained by crossing. In this case the isometries might allow realization. The isometries must be defined to take into account quark and lepton number conservation by crossing replacing fermion with antifermion. By allowing the states of Hilbert space in node to be both quarks and leptons, difficulties can be avoided.

Tensor network description in terms of fundamental fermions and CFT

Consider first fundamental fermions. What are the labels characterizing the states of fundamental fermions fermions propagating along the lines? There are two options: the labels are either conformal weights or four-momenta.

1. Since fermions corresponds to strings defining the boundaries of string world sheets and since strong form of holography implies effective 2-dimensionality also in fermion sector, the natural guess is that the conformal weights plus some discrete quantum numbers - standard model quantum numbers at least - are in question. The situation would be well-defined also p-adically for this option. In this case one can hope that conformal field theory at partonic 2-surface could define the fermionic 4-vertex more or less completely. There would be no need to assign propagators between different four-fermion vertices. The scattering diagram would define a composite formed from light-like 3-surfaces and one would have single isometry build from 4-fermion perfect tensors. There would be no integrations over internal momenta.

2. Second option is that fundamental fermions are labelled by four-momenta. The outgoing four-momenta in 4-vertices would not be completely fixed by the values of the incoming momenta and this extends the state space. Concerning p-adicization this integral is not desirable and this forces to consider seriously discrete labelling. The unitarity condition for 2+2 scattering would involve integral over 2-sphere. Four-fermion scattering must be unitary process in QFT so that this condition might be possible to satisfy. The problem would be how to fix this fundamental scattering matrix uniquely. This option does not look attractive number theoretically.

The most plausible option is that holography means that conformal field theory describes the scattering of fundamental fermions and QFT type description analogous to twistorial approach describes the scattering of physical fermions. If only 3-vertices are allowed, and if masslessness corresponds to masslessness in 8-D sense, one obtains non-trivial scattering vertices (for ordinary twistor approach all massless momenta would be collinear if real).

Tensor network description for physical particles

Could the twistorial description expected to correspond to the description in terms of particles allow tensor network description?

1. Certainly one must assign four-momenta to incoming *physical* particles - also fermions - but they correspond to pairs of wormhole contacts rather than fundamental fermions at the boundaries of string world sheets. It would be natural to assign four-momenta also to the virtual *physical* fermions appearing in the diagram and the geometric view about scattering would allow only 3-vertices so that momentum conservation would eliminate momentum degrees of freedom effectively. This would be a p-adically good news.

2. At the level of fundamental fermions entanglement is described as a tensor contraction of the CFT vertices. This locality is natural since the vertices are at null distance from each other. At QFT limit the entanglement between the ends of the line is characterized the propagator.

   One must get rid of propagators in order to have tensor network description. The inclusion of propagators to the fundamental tensor diagrams would break the symmetry between the legs of vertex since the propagator cannot be included to its both ends. Situation changes if one can represent the propagator as a bilinear of something more primitive and include the halves to the opposite ends of the line. Twistor representation of four-momentum indeed defines this kind of representation as a bilinear $p^{a\tilde{b}} = \lambda\tilde{\mu}^{\tilde{b}}$ of twistors λ and $\tilde{\mu}$. There is problem due to the diverging $1/p^2$ factor but residue integral eliminates this factor and one can write directly the fermionic propagator factors as $p^{a\tilde{b}}$.

3. In QFT description the perturbative expansion is in powers of coupling constant. If the reduction to braided tree diagrams analogous to twistor diagrams occurs, power g^{N-2} of coupling constant is expected to factorize as a multiplier of a tree diagram with N external legs. One should understand this aspect in the tensor net-work picture.

   For $\mathcal{N} = 4$ SUSY there is coupling constant renormalization. Similar prediction is expected from TGD. Coupling constant evolution is expected to be discrete and induced by the discrete evolution of Kähler coupling strength defined by the spectrum of its critical values. The conjecture is that critical values are naturally labelled by p-adic primes $p \simeq 2^k$, k prime, labelling p-adic length scales. Therefore one might hope that problems could be avoided.

These observations encourage the expectation that twistorial approach involving only 3-vertices allows to realize tensor network idea also at the level of physical particles. It might be essential that twistors can be generalized to 8-D twistors. Octonionic representation of gamma matrices might make this possible. Also the fact twistorial uniqueness of M^4 and CP_2 might be crucial.

Gauge theory follows as QFT limit of TGD so that one cannot in principle require that gauge theory vertices satisfy the isometricity conditions. Nothing however prevents from checking whether gauge theory limit might inherit this property.

1. For instance, could 3-vertices of Yang-Mills theory define isometric embedding of 1-particle states to 2 particle states? For a given gauge boson there should exist always a pair of gauge bosons, which can fuse to it. Consider a basis for Lie-algebra generators of the gauge group. If the generator T is such that there exists no pair $[A, B]$ with the property $[A, B] = T$, Jacobi identity implies that T must commute with all generators and one has direct sum of Lie algebras generated by T and remaining generators.

2. In the case of weak algebra $SU(2) \times U(1)$ the weak mixing of Y and I_3 might allow the isometric embeddings of type $1 \rightarrow 2$. Does this mean that Weinberg angle must be non-vanishing in order to have consistent theory? A realistic manner to get rid of the problem is to allow at QFT limit the lines to be also fermions so that also U(1) gauge boson can be constructed as fermion pair.

How the two tensor network descriptions would be related?

There are two descriptions for the zero energy states providing representation of scattering amplitudes: the CFT description in terms of fundamental fermions at the boundaries of string world sheets, and the description in terms of physical particles to which one can assign light-like 3-surfaces as virtual lines and total quantum numbers.

1. CFT description in terms of fundamental fermions in some aspects very simple because of its 2-dimensionality and conformal invariance. The description is in terms of physical particles having light-like 3-surfaces carrying some total quantum numbers as correlates and is simpler in different sense. These descriptions should be related by an Hilbert space isometry.

2. The perfect tensor property for 4-fermion vertices makes fundamental fermion states analogous to physical states realizing logical qubits as highly entangled structures. Geometric

description in terms of 3-surfaces is in turn analogous to the description in terms of logical qubits.

3. Holography-like correspondence between these descriptions of zero energy states (scattering diagrams) should exist. Physical particles should correspond to the level, at which resolution is smaller and which should be isometrically mapped to the strongly entangled level defined by fundamental fermions and analogous to boundary degrees of freedom (fundamental fermions *are* at the boundaries of string world sheets!).

   The map relating the two descriptions seems to exist. One can assign four-momenta to the legs of conformal four-point function as parameters so that one obtains a mapping from the states labelled by conformal weights to the states labelled by four-momenta! The appearance of 4-momenta from conformal theory is somewhat mysterious looking phenomenon but this duality makes it rather natural.

17.7.8 Taking into account braiding and WCW degrees of freedom

One must also take intro account braiding and orbital degrees of freedom of WCW. The generalization of tensor network to braided tensor network is trivial. Thanks to the properties of tensor network orbital and spinor degrees of freedom factorize so that also the treatment of WCW degrees of freedom seems to be possible.

What about braiding?

The scattering diagrams would be tree diagrams with braiding of fermionic lines along light-like 3-surfaces - dance of fundamental quarks and leptons at parquette defined by the partonic 2-surface one might say. Also space-like braiding at magnetic flux tubes at the ends of CD is possible and its time evolution between the ends of space-time surfaces defines 2-braiding which is generalization of the ordinary braiding but will not be discussed here. This gives rise to a hierarchy of braidings. One can talk about flux tubes within flux tubes and about light-like 3-surface within light-like 3-surfaces. The smaller light-like 3-surface would be glued by a wormhole contact to the larger one and contact could have Euclidian signature of induced metric.

How can one treat the braiding in the tensor network picture? The answer is simple. Braiding corresponds to an element of braid group and one can represent it by a unitary matrix as one does in topological QFT as one constructs knot invariants. In particular, the trace of this unitary matrix defines a knot invariant. The generalization of the tensor network is simple. One attaches to the links connecting two nodes unitary transformation defining a representation of the braid involved. Local variant of unitarity would mean isometricity at nodes and unitarity at links.

What about WCW degrees of freedom?

The above considerations are about fermions that its WCW spinor degrees of freedom and the space-time surface itself has been regarded as a fixed background. How can one take into account WCW degrees of freedom?

The scattering amplitude involves a functional integral over the 3-surfaces at the ends of CD. The functional integration over WCW degrees of freedom gives an expression depending on Kähler coupling strength α_K and determines the dependence on various gauge coupling strengths expressible in terms of α_K. This makes it possible to have the tensor network description in fermionic degrees of freedom without losing completely the dependence of the scattering amplitudes on gauge couplings. By strong form of holography the functional integral should reduce to that over partonic 2-surfaces and strings connecting them. Number theoretic discretization with a cutoff determined by measurement resolution forces the parameters characterizing the 2-surfaces to belong to an algebraic extension of rationals and is expected to reduce functional integral to a sum over discretized WCW so that it makes sense also in p-adic sectors [K95, K124].

A brief summary of quantum measurement theory in ZEO is necessary. The repeated state function reduction shifts active boundary A of CD and affects the states at it. The passive boundary of CD- call it P - and the states at it - remain unaffected. The repeated state function reductions leaving P unaffected and giving usually rise to Zeno effect, correspond now to the TGD counterpart of unitary time evolution by shifts between subsequent state function reductions. Call A and its shifted version A_{in} and A_{out} and the corresponding state spaces H_{in} and H_{out}. The unitary (or more generally isometric) S matrix represents this shift. This is the TGD counterpart of a unitary evolution of QFTs. S forms a building brick of a more general unitary matrix U acting in the space of zero energy states but U is not considered now.

Consider now the isometricity conditions.

1. Unitarity conditions generalized to isometricity conditions apply to S. Isometricity conditions $S^\dagger S = Id_{in}$ can be applied at A_{in}. The states appearing in the isometry conditions as initial and final states correspond to A_{in} and A_{out}. There is a trace over WCW spin indices (labels for many-fermion states) of H_{out} in the conditions $S^\dagger S = Id_{in}$. Isometricity conditions involve also an integral over WCW orbital degrees of freedom at both ends: these degrees of freedom are strongly correlated and for a strict classical determinism the correlation between the ends is complete. If the tensor network idea works, the summation over spinor degrees of freedom at A_{out} gives just a unit matrix in the spinor indices at A_{in} and leaves only the WCW orbital degrees of freedom in consideration. This factorization of spinor and orbital WCW degrees of freedom simplifies the situation dramatically.

2. One can express isometricity conditions for modes with $\Psi_{in,M}$ and $\Psi_{out,N}$ at A_{in} and A_{out}: this requires functional integration over 3-surfaces WCW at A_{in} and A_{out}. The conditions are formulated in terms of the labels - call them M_{in}, N_{in} - of WCW spinor modes at A_{in} including standard model quantum numbers and labels characterizing the states of supersymplectic and super-conformal representations. The trace is over the corresponding indices R_{out} at A_{out}. The WCW functional integrals in the generalized unitarity conditions are therefore over A_{in} and A_{out} and should give Kronecker delta $\sum_{R_{out}} S^\dagger_{M_{in}R_{out}} S_{R_{out}N_{in}} = \delta_{M_{in},N_{in}}$.

3. The simplest view would be that Kähler action with boundary conditions implies completely deterministic dynamics. The conditions expressing strong form of holography state that sub-algebras of super-symplectic algebra and related conformal algebras isomorphic to the entire algebra give rise to vanishing Noether charges. Suppose that these conditions posed at the ends of CD are so strong that they fix the time evolution of the space-time surface as preferred extremal completely when posed at either boundary. In this case the isometricity conditions would be so strong that the double functional integration appearing in the matrix product reduces to that at A_{in} and the isometricity conditions would state just the orthonormality of the basis of WCW spinor modes at A_{in}.

4. Quantum criticality and in particular, the hierarchy of Planck constants providing a geometric description for non-deterministic long range fluctuations, does not support this view. Also the fact that string world sheets connect the boundaries of CD suggests that determinism must be broken. The inner product defining the completeness of the WCW state basis in orbital degrees of freedom can be however generalized to a bi-local inner product involving functional integration over 3-surfaces at both A_{in} and A_{out}. There is however a very strong correlation so that integration volume at A_{out} is expected to be small. This also suggests that one can have only isometricity conditions.

17.7.9 How do the gauge couplings appear in the vertices?

Reader is probably still confused and wondering how the gauge couplings appear in the vertices from the functional integral over WCW degrees of freedom. In twistorial approach, the vanishing of loops in $\mathcal{N} = 4$ SYM theory gives just g^N, N the number of 3-vertices. Each vertex should give gauge coupling. Or equivalently, each propagator line connecting vertices should give α_K. The functional integral should give this factor for each propagator line. Generalization of conformal invariance is expected to give this picture.

To proceed some basic facts about N-point functions of CFTs are needed.

1. In conformal field theory the functional form of two-point function is completely fixed by conformal symmetry:

$$
\begin{aligned}
G^{(2)}(z_i,\overline{z}_i) &= \frac{C_{12}}{z_{12}^{2h}\overline{z}_{12}^{2\overline{h}}} \ , \\
z_{ij} &= z_i - z_j \ , \ \overline{z}_{ij} = \overline{z}_i - \overline{z}_j \ , \\
h_1 = h_2 = h &= h_a + ih_b \ , \ \overline{h} = \overline{h}_a + i\overline{h}_b \ .
\end{aligned}
\tag{17.7.1}
$$

$h_1 = h_2 \equiv h$ and its conjugate $\overline{h}$ are conformal weights of conformal field and its conjugate. Note that the conformal weights of conformal fields Φ_1 and Φ_2 must be same. In TGD context C_{12} is expected to be proportional to α_K and this would give to each vertex g_K when couplings are absorbed into vertices.

2. The 3-point function for 3 conformal fields Φ_i, $i = 1,2,3$ is dictated by conformal symmetries apart from constant C_{123}:

$$
G^{(3)}(z_i,\overline{z}_i) = C_{123} \times \frac{1}{z_{12}^{h_1+h_2-h_3} z_{23}^{h_2+h_3-h_1} z_{31}^{h_3+h_1-h_2}} \times \frac{1}{\overline{z}_{12}^{\overline{h}_1+\overline{h}_2-\overline{h}_3} \overline{z}_{23}^{\overline{h}_2+\overline{h}_3-\overline{h}_1} \overline{z}_{31}^{\overline{h}_3+\overline{h}_1-\overline{h}_2}} \ .
\tag{17.7.2}
$$

Here C_{123} should bef fixed by super-symplectic and related symmetries and determined the numerical coefficients various couplings when expressed in terms of g_K.

3. 4-point functions have analogous form

$$
\begin{aligned}
G^{(4)}(z_i,\overline{z}_i) &= f_{1234}(x,\overline{x}) \prod_{i<j} z_{ij}^{-(h_i+h_j)+h/3} \prod_{i<j} \overline{z}_{ij}^{-(\overline{h}_i+\overline{h}_j)+\overline{h}/3} \ , \\
h &= \sum_i h_i \ ,
\end{aligned}
\tag{17.7.3}
$$

but are proportional to an arbitrary function f_{1234} of conformal invariant $x = z_{12}z_{34}/z_{13}z_{24}$ and its conjugate.

If only 3-vertices appear/are needed for physical particles - as both twistorial and number theoretic approaches strongly suggest - the conformal propagators and vertices are fixed apart from constants C_{ijk}, which in turn should be fixed by the huge generalization of conformal symmetries. α_K emerges in the expected manner.

This picture seems to follow from first principles.

1. One can fix the partonic 2-surfaces at the boundaries of CD but there is a functional integral over partonic 2-surfaces defining the vertices: their deformations induce deformations of the legs. One can expand the exponent of Kähler action and in the lowest order the perturbation term is trilinear and non-local in the perturbations. This gives rise to 3-point function of CFT nonlocal in z_i. The functional integral over perturbations gives the propagators in legs proportional to α_K in terms of two point function of CFT. Note that the external propagator legs can be eliminated in S-matrix.

2. The cancellation of higher order perturbative corrections in WCW functional integral is required by the quantum criticality and means trivial coupling constant evolution for α_K and other coupling constants. Coupling constant evolution is discretized with values of α_K analogous to critical temperatures and should correspond to p-adic coupling constant evolution [L13].

3. This picture leaves a lot of details open. An integration over the values of z_i is needed and means a kind of Fourier analysis leading from complex domain. The analog of Fourier analysis would be for deformations of partonic 2-surface labelled by some natural labels. Conformal weights could be natural labels of this kind.

It is easy to get confused since there are several diagrammatics involved: the topological diagrammatics of 3-surfacse assignable to the physical particles with partonic 2-surfaces as vertices, the diagrammatics associated with the perturbative functional integral for the Kähler action, and the fermionic diagrammatics suggested to reduce to tensor network. The conjectures are as follows.

1. The "primary" vertices $G^{(n)}$, $n > 3$ assignable to single partonic 2-surface and coming from a functional integral for Kähler action vanishes. This corresponds to quantum criticality and trivial RG evolution.

2. $G^{(n)}$, $n > 3$ in the sense of topological diagrammatics without loops and involving n partonic 2-surfaces do not vanish. One can construct the analog of $G^{(4)}$ from two $G^{(3)}$:s at different partonic 2-surfaces and propagator defined by 2-point function connecting them as string diagram.

   Also topological variant of $G^{(4)}$ assignable to single partonic 2-surface can be constructed by allowing the 3-D propagator "line" to return back to the partonic 2-surface. This would correspond to an analog of loop. Similar construction applies to "primary" $G^{(n)}$,$n > 4$. In number theoretic vision these loops are eliminated as redundant representations so that one has only braided tree diagrams. Also twistor Grassmann approach supports this view.

To sum up, the tensor network description would apply to fermionic degrees of freedom. In bosonic degrees of freedom functional integral would give CFT picture with 3-vertex as the only "primary" vertex and from this twistorial and number theoretic visions follow via the super-symplectic symmetries of the vertex coefficients C_{ijk} extended to Yangian symmetries.

Chapter i

Appendix

A-1 Introduction

The great dream of a physicist believing in reductionism and TGD would be a formalism generalizing Feynman diagrams allowing any graduate student to compute the predictions of the theory. TGD has forced myself to give up naive reductionism but I believe that TGD allows generalization of Feynman diagram in such a way that one gets rid of the infinities plaguing practically all existing theories. The purpose of this chapter is to develop general vision about how this might be achieved. The vision is based on generalization of mathematical structures discovered in the construction of topological quantum field theories (TQFT), conformal field theories (CQFT). In particular, the notions of Hopf algebras and quantum groups, and categories are central. The following gives a very concise summary of the basic ideas.

In introduced the original version of this chapter in extended form long time ago having also remarkably long title. I ended up with diagrams which had category theoretical meaning and had huge symmetries generalizing the duality symmetry of the hadronic string model and allowing to reduce their number dramatically leaving only tree diagrams still having symmetries. These diagrams were however not Feynman diagrams and this made me scared. After the emergence of twistor Grassmann approach and twistor diagrams the situation changed completely and it might be that I was on the correct track after all. This chapter is what was left from my adventure.

1. Feynman diagrams as generalized braid diagrams

The first key idea is that generalized Feynman diagrams with diagrams analogous to knot and link diagrams in the sense that diagrams involving loops are equivalent with tree diagrams. This would be a generalization of duality symmetry of string models.

TGD itself provides general arguments supporting same idea. The identification of preferred extremal of Kähler action (perhaps absolute minimum in Eucdlian space-time regions) as a four-dimensional Feynman diagram characterizing particle reaction means that there is only single Feynman diagram instead of functional integral over 4-surfaces: this diagram is expected to be minimal one. S-matrix element as a representation of a path defining continuation of configuration space spinor field between different sectors of it corresponding different 3-topologies leads also to the conclusion that all continuations and corresponding Feynman diagrams are equivalent. Universe as a compute metaphor idea allowing quite concrete realization by generalization of what is meant by space-time point leads to the view that generalized Feynman diagrams characterize equivalent computations.

2. Coupling constant evolution from infinite number of critical values of Kähler coupling strength

The basic objection that this vision does not allow to understand coupling constant evolution involving loops in an essential way can be circumvented.

Quantum criticality requires that Kähler coupling constant α_K is analogous to critical temperature (so that the loops for configuration space integration vanish). The hypothesis motivated by the enormous vacuum degeneracy of Kähler action is that gauge couplings have an infinite number of possible values labelled by p-adic length scales and probably also by the fractal dimensions of effective tensor factors defined hierarchy of II_1 factors (so called Beraha numbers).

The dependence on p-adic length scale L_p corresponds to the usual renormalization group evolution whereas the latter dependence would correspond to angular resolution and finite-dimensional extensions of p-adic number fields R_p. Finite resolution and renormalization group evolution are forced by the algebraic continuation of rational number based physics to real and p-adic number fields since p-adic and real notions of distance between rational points differ dramatically.

TGD suggests discrete p-adic coupling constant evolution in which coupling constants are renormalization group invariants for the evolution associated with given p-adic prime p. This would mean vanishing of loops obtained also in $\mathcal{N} = 4$ SUSY allowing twistorialization. Gauge couplings could depend on prime p characterizing the p-adic length scale. The p-adic prime and therefore also the length scale and coupling constants characterizing the dynamics for given CD would vary wildly as function of integer characterizing CD size scale. This could mean that the CDs whose size scales are related by multiplication of small integer are close to each other. They would be near to each other in logarithmic sense and logarithms indeed appear in running coupling constants. This "prediction" is of course subject to criticism.

The discrete p-adic coupling constant evolution should relate to the continuous RG evolution of QFTs. This requires understanding of how this space-time corresponds to the many-sheeted space-time of TGD. GRT space-time as effective space-time obtained by replacing the many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of space-time sheets from Minkowski metric. Also gauge potentials of standard model would correspond classically to superpositions of induced gauge potentials over space-time sheets. Gravitational constant, cosmological constant, and various gauge couplings emerge as predictions. Ordinary continuous coupling constant evolution would follow only at GRT-QFT limit.

3. Equivalence of loop diagrams with tree diagrams from the axioms of generalized ribbon category

A further was that Hopf algebra related structures and appropriately generalized ribbon categories could provide a concrete realization of this picture. Generalized Feynman diagrams which are identified as braid diagrams with strands running in both directions of time and containing besides braid operations also boxes representing algebra morphisms with more than one incoming and outgoing strands describing particle reactions (3-particle vertex should be enough). In particular, fusion of 2-particles and decay of particle to two would correspond to generalizations of algebra product μ and co-product Δ to morphisms of the category defined by the super-symplectic algebras associated with 3-surfaces with various topologies and conformal structures. The basic axioms for this structure generalizing Hopf algebra axioms state that diagrams with self energy loops, vertex corrections, and box diagrams are equivalent with tree diagrams.

To sum up, I could not develop this project further - mainly by the lack of needed mathematical knowhow, and this chapter is more like an Appendix. It might however be that twistorial approach could allow to concretize the idea about equivalence of loop diagrams with tree diagrams for generalization of Feynman diagrams. What is certain, that the notion of coupling constant evolution aat the level of many-sheeted space-time, should be very simple in TGD- maybe trivial for a given algebraic extension of given p-adic number field. Only at the QFT limit the lumping of sheets to single one is expected to induce complexities.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L4].

A-2 Hopf Algebras And Ribbon Categories As Basic Structures

In this section the basic notions related to Hopf algebras and categories are discussed from TGD point of view. Examples are left to appendix. The new element is the graphical representation of the axioms leading to the idea about the equivalent of loop diagrams and tree diagrams based on general algebraic axioms.

A-2.1 Hopf Algebras And Ribbon Categories Very Briefly

An algebraic formulation generalizing braided Hopf algebras and related structures to what might be called quantum category would involve the replacement of the co-product of Hopf algebras with morphism of quantum category having as its objects the Clifford algebras associated with WCW spinor structure for various 3-topologies. The corresponding Fock spaces would would define algebra modules and the objects of the category would consists of pairs of algebras and corresponding modules. The underlying primary structure would be second quantized free induced spinor fields associated with 3-surfaces with various 3-topologies and generalized conformal structures.

1. Bi-algebras

Bi-algebras have two algebraic operations. Besides ordinary multiplication $\mu : H \otimes H \rightarrow H$ there is also co-multiplication $\Delta : H \rightarrow H \otimes H$. Algebra satisfies the associativity axiom (Ass): $a(bc) = (ab)c$, or more formally, $\mu(id \otimes \mu) = \mu(\mu \otimes id)$, and the unit axiom (Un) stating that there is morphism $\eta : k \rightarrow A$ mapping the unit of A to the unit of field k. Commutativity axiom (Co) $ab = ba$ translates to $\mu \otimes \tau \equiv \mu^{op} = \mu$, where τ permutes factors in tensor product $A \otimes A$.

Δ satisfies mirror images of these axioms. Co-associativity axiom (Coass) reads as $(\Delta \otimes id)\Delta = (id \otimes \Delta)\Delta$, co-unit axiom (Coun) states existence of morphism $\epsilon : k \rightarrow C$ mapping the unit of A to that of k, and co-commutativity (Coco) reads as $\tau \circ \Delta \equiv \Delta^{op} = \Delta$. For a bi-algebra H also additional axioms are satisfied: in particular, Δ (μ) acts as algebra (bi-algebra) morphism. When represented graphically, this constraint states that a box diagram is equivalent to a tree diagram as will be found and served as the stimulus for the idea that loop diagrams might be equivalent with tree diagrams.

Left and right algebra modules and algebra representations are defined in an obvious manner and satisfy associativity and unit axioms. A left co-module corresponds a pair (V, Δ_V) where the co-action $\Delta_N : V \rightarrow A \otimes V$ satisfies co-associativity and co-unit axioms. Right co-module is defined in an analogous manner.

Particle fusion $A \otimes B \rightarrow C$ corresponds to μ: $A \otimes B \rightarrow C = AB$. Co-multiplication Δ corresponds time reversal $C \rightarrow A \otimes B$ of this process, which is kind of a time-reversal for multiplication. The generalization would mean that μ and Δ become morphisms $\mu : B \otimes C \rightarrow A$ and $\Delta : A \rightarrow B \otimes C$, where A, B, C are objects of the quantum category. They could be either representations of same algebra or even different algebras.

2. Drinfeld's quantum double

Drinfeld's quantum double [A29] is a braided Hopf algebra obtained by combining Hopf algebra $(H, \mu, \Delta, \eta, \epsilon, S, R)$ and its dual $H^\star$ to a larger Hopf algebra known as quasi-triangular Hopf algebra satisfying $\Delta = R\Delta^{op}R^{-1}$, where $\Delta^{op}(a)$ is obtained by permuting the two tensor factors. Duality means existence of a scalar product and the two algebras correspond to Hermitian conjugates of each other.

In TGD framework the physical states associated with these algebras have opposite energies since in TGD framework antimatter (or matter depending on the phase of matter) corresponds to negative energy states. The states of the Universe would correspond to states with vanishing conserved quantum numbers, and in concordance with crossing symmetry, particle reactions could be interpreted as transitions generating zero energy states from vacuum.

The notion of duality [A29] is needed to define an inner product and S-matrix. Essentially Dirac's bra-ket formalism is in question. The so called evaluation map $ev : V \otimes V^\star \rightarrow k$ defined as $ev(v^i \otimes v_j) = \langle v^i, v_j \rangle = \delta_{ij}$ defines an inner product in any Hopf algebra module. The inverse of this map is the linear map $k \rightarrow V$ defined by $\delta_v(1) = v_i \otimes v^i$. For a tensor category with unit I, field k is replaced with unit I, and left duality these maps are replaced with maps $b_V : I \rightarrow V \otimes V^\star$ and $d_V = V \otimes V^\star \rightarrow I$. Right duality is defined in an analogous manner. The map d_V assigns to a given zero energy state S-matrix element. Algebra morphism property $b_V(ab) = b_V(a)b_V(b)$ would mean that the outcome is essentially the counterpart of free field theory Feynman diagram. This diagram is convoluted with the S-matrix element coded to the entanglement coefficients between positive and negative energy particles of zero energy state.

3. Ribbon algebras and ribbon categories

The so called ribbon algebra [A29] is obtained by replacing one-dimensional strands with ribbons and adding to the algebra the so called twist operation θ acting as a morphism in algebra and in any algebra module. Twist allows to introduce the notion of trace, in particular quantum trace.

The thickening of one-dimensional strands to 2-dimensional ribbons is especially natural in TGD framework, and corresponds to a replacement of points of time=constant section of 4-surface with one-dimensional curves along which the S-matrix defined by R-matrix is constant. Ribbon category is defined in an obvious manner. There is also a more general definition of ribbon category with objects identified as representations of a given algebra and allowing morphisms with arbitrary number of incoming and outgoing strands having interpretation as many-particle vertices in TGD framework. The notion of quantum category defined as a generalization of a ribbon category involving the generalization of algebra product and co-product as morphisms between different objects of the category and allowing objects to correspond different algebras might catch the essentials of the physics of TGD Universe.

A-2.2 Algebras, Co-Algebras, Bi-Algebras,And Related Structures

It is useful to formulate the notions of algebra, co-algebra, bi-algebra, and Hopf algebra in order to understand how they might help in attempt to formulate more precisely the view about what generalized Feynman diagrams could mean. Since I am a novice in the field of quantum groups, the definitions to be represented are more or less as such from the book "Quantum Groups" of Christian Kassel [A29] with some material (such as the construction of Drinfeld double) taken from [A51]. What is new is a graphical representation of algebra axioms and the proposal that algebra and co-algebra operations have interpretation in terms of generalized Feynman diagrams.

In the following considerations the notation id_k for the isomorphism $k \rightarrow k \otimes k$ defined by $x \rightarrow x \otimes x$ and its inverse will be used.

Algebras

Algebra can be defined as a triple (A, μ, η), where A is a vector space over field k and $\mu : A \otimes A \rightarrow A$ and $\eta : k \rightarrow A$ are linear maps satisfying the following axioms (Ass) and (Un).

(Ass): The square

$$\begin{array}{ccc} A\otimes A\otimes A & \stackrel{\mu\otimes id}{\longrightarrow} & A\otimes A \\ \downarrow id\otimes\mu & & \downarrow\mu \\ A\otimes A & \stackrel{\mu}{\longrightarrow} & A \end{array} \qquad \text{(A-2.1)}$$

commutes.

(Un): The diagram

$$\begin{array}{ccccc} k\otimes A & \stackrel{\eta\otimes id}{\longrightarrow} & A\otimes A & \stackrel{id\otimes\eta}{\longleftarrow} & A\otimes k \\ & \searrow\cong & \downarrow\mu & \cong\swarrow & \\ & & A & & \end{array} \qquad \text{(A-2.2)}$$

commutes. Note that η imbeds field k to A.

(Comm) If algebra is commutative, the triangle

$$\begin{array}{ccc} A\otimes A & \stackrel{\tau_{A,A}}{\longrightarrow} & A\otimes A \\ & \mu\searrow \quad \swarrow\mu & \\ & A & \end{array} \qquad \text{(A-2.3)}$$

commutes. Here $\tau_{A,A}$ is the flip switching the factors: $\tau_{A,A}(a \otimes a') = a' \otimes a$.

A morphism of algebras $f : (A, \mu, \eta) \to (A', \mu', \eta')$ is a linear map $A \to A'$ such that

$$\mu' \circ (f \otimes f) = f \circ \mu, \quad \text{and} \quad f \circ \eta = \eta' \ .$$

A graphical representation of the algebra axioms is obtained by assigning to the field k a dashed line to be referred as a vacuum line in the sequel and to A a full line, to η a vertex $\times$ at which k-line changes to A-line. The product μ can be represented as 3-particle vertex in which algebra lines fuse together. The three axioms (Ass), (Un) and (Comm) can are expressed graphically in figure **??**.

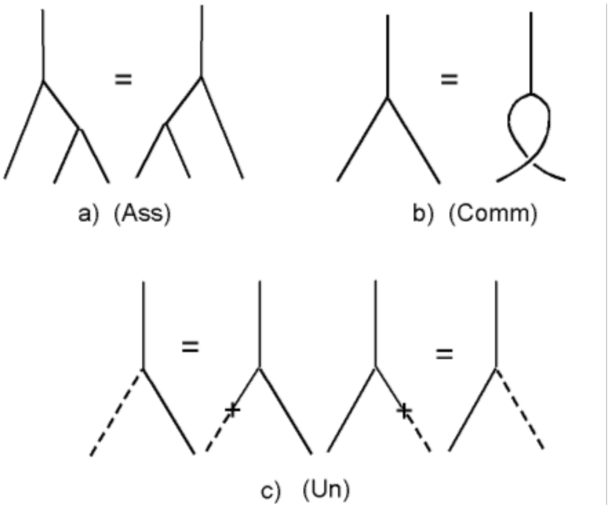

Figure 1: Graphical representation for the axioms of algebra. a) $a(bc) = (ab)c$, b) $ab = ba$, c) $ka = \mu(\eta(k), a)$ and $ak = \mu(a, \eta(k))$.

Note that associativity axiom implies that two tree diagrams not equivalent as Feynman diagrams are equivalent in the algebraic sense.

Co-algebras

The definition of co-algebra is obtained by systematically reversing the directions of arrows in the previous diagrams.

A co-algebra is a triple(C, Δ, ϵ), where C is a vector space over field k and $\Delta : C \to C \otimes C$ and $\epsilon : C \to k$ are linear maps satisfying the following axioms (Coass) and (Coun).

(Coass): The square

$$\begin{array}{ccc} C & \stackrel{\Delta}{\longrightarrow} & C \otimes C \\ \downarrow\Delta & & \downarrow id \otimes \Delta \\ C \otimes C & \stackrel{\Delta \otimes id}{\longrightarrow} & C \otimes C \otimes C \end{array} \tag{A-2.4}$$

commutes.

(Coun): The diagram

$$\begin{array}{ccccc} k \otimes C & \underset{\epsilon \otimes id}{\longleftarrow} & C \otimes C & \stackrel{id \otimes \epsilon}{\longrightarrow} & C \otimes k \\ & \nwarrow \cong & \Delta\uparrow & \cong \nearrow & \\ & & C & & \end{array} \tag{A-2.5}$$

commutes. The map Δ is called co-product or co-multiplication whereas ϵ is called the counit. The commutative diagram state that the co-product is co-associative and that co-unit commutes with co-product.

(Cocomm) If co-algebra is commutative, the triangle

$$\begin{array}{ccc} & C & \\ {\scriptstyle\Delta}\swarrow & & \searrow{\scriptstyle\Delta} \\ C\otimes C & \xrightarrow{\tau_{C,C}} & C\otimes C \end{array} \qquad \text{(A-2.6)}$$

commutes. Here $\tau_{C,C}$ is the flip switching the factors: $\tau_{C,C}(c \otimes c') = c' \otimes c$.

A morphism of co-algebras $f : (C, \Delta, \epsilon) \rightarrow (C', \Delta', \epsilon')$ is a linear map $C \rightarrow C'$ such that

$$(f \otimes f) \circ \Delta = \Delta' \circ f \quad , \quad \text{and} \quad \epsilon = \epsilon' \circ f \quad .$$

It is straightforward to define notions like co-ideal and co-factor algebra by starting from the notions of ideal and factor algebra. A very useful notation is Sweedler's sigma notation for $\Delta(x)$, $x \in C$ as element of $C \otimes C$:

$$\Delta(x) = \sum_i x'_i \otimes x''_i \equiv \sum_{\{x\}} x' \otimes x'' \quad .$$

Also co-algebra axioms allow graphical representation. One assigns to ϵ a vertex $\times$ at which C-line changes to k-line: the interpretation is as an absorption of a particle by vacuum. The co-product Δ can be represented as 3-particle vertex in which C-line decays to two C-lines. The graphical representation of the three axioms (Coass), (Coun), and (Cocomm) is related to the representation of algebra axioms by "time reversal", that is turning the diagrams for the algebra axioms upside down (see figure **??**).

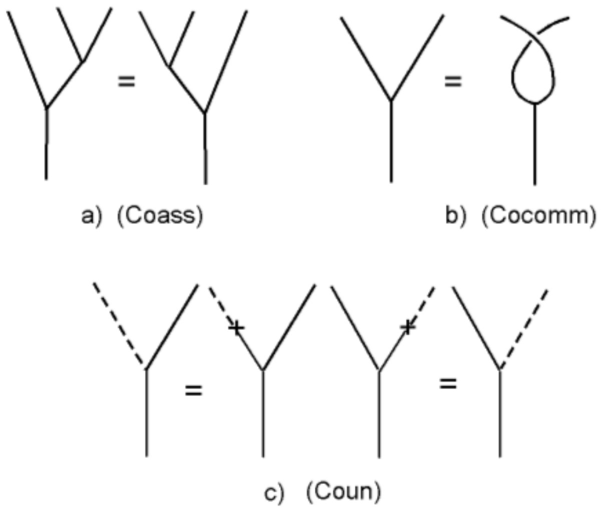

Figure 2: Graphical representation for the axioms of co-algebra is obtained by turning the representation for algebra axioms upside down. a) $(id \otimes \Delta)\Delta = (\Delta \otimes id)\Delta$, b) $\Delta = \Delta^{op}$, c) $(\epsilon \otimes id) \circ \Delta = (id \otimes \epsilon) \circ \Delta = id$.

Bi-algebras

Consider next a vector space H equipped simultaneously with an algebra structure (H, μ, η) and a co-algebra structure (H, Δ, ϵ). There are some compatibility conditions between these two structures. $H \otimes H$ can be given the induced structures of a tensor product of algebras and of co-algebras.

The following two statements are equivalent.

1. The maps μ and η are morphisms of co-algebras. For μ this means that the diagrams

$$\begin{array}{ccc} H\otimes H & \xrightarrow{\mu} & H \\ \Big\downarrow{\scriptstyle (id\otimes\tau\otimes id)\otimes(\Delta\otimes\Delta)} & & \Big\downarrow{\scriptstyle\Delta} \\ (H\otimes H)\otimes(H\otimes H) & \xrightarrow{\mu\otimes\mu} & H\otimes H \end{array} \tag{A-2.7}$$

and

$$\begin{array}{ccc} H\otimes H & \xrightarrow{\epsilon\otimes\epsilon} & k\otimes k \\ \Big\downarrow{\scriptstyle\mu} & & \Big\downarrow{\scriptstyle id} \\ H & \xrightarrow{\epsilon} & k \end{array} \tag{A-2.8}$$

commute. For η this means that the diagrams

$$\begin{array}{ccc} k & \xrightarrow{\eta} & H \\ \Big\downarrow{\scriptstyle id} & & \Big\downarrow{\scriptstyle\Delta} \\ k\otimes k & \xrightarrow{\eta\otimes\eta} & H\otimes H \end{array} \qquad \begin{array}{ccc} k & \xrightarrow{\eta} & H \\ & {\scriptstyle id}\searrow \quad \swarrow{\scriptstyle\epsilon} & \\ & k & \end{array} \tag{A-2.9}$$

commute.

2. The maps Δ and ϵ are morphisms of algebras.
For Δ this means that diagrams

$$\begin{array}{ccc} H\otimes H & \xrightarrow{\Delta\otimes\Delta} & (H\otimes H)\otimes(H\otimes H) \\ \Big\downarrow{\scriptstyle\mu} & & \Big\downarrow{\scriptstyle(\mu\otimes\mu)(id\otimes\tau\otimes id)} \\ H & \xrightarrow{\Delta} & H\otimes H \end{array} \tag{A-2.10}$$

and

$$\begin{array}{ccc} k & \xrightarrow{\eta} & H \\ \Big\downarrow{\scriptstyle id} & & \Big\downarrow{\scriptstyle\Delta} \\ k\otimes k & \xrightarrow{\eta\otimes\eta} & H\otimes H \end{array} \tag{A-2.11}$$

commute.

For ϵ this means that the diagrams

$$\begin{array}{ccc} H\otimes H & \xrightarrow{\epsilon\otimes\epsilon} & k\otimes k \\ \downarrow{\scriptstyle \mu} & & \downarrow{\scriptstyle id} \\ H & \xrightarrow{\epsilon} & k \end{array} \qquad \begin{array}{ccc} k & \xrightarrow{\eta} & H \\ & {\scriptstyle id}\searrow \quad \swarrow{\scriptstyle \epsilon} & \\ & k & \end{array} \tag{A-2.12}$$

commute. The proof of the theorem involves the comparison of the commutative diagrams expressing both statements to see that they are equivalent.

The theorem inspires the following definition.

Definition: A bi-algebra is a quintuple $(H, \mu, \eta, \Delta, \epsilon)$, where (H, μ, η) is an algebra and (H, Δ, ϵ) is co-algebra satisfying the mutually equivalent conditions of the previous theorem. A morphisms of bi-algebras is a morphism for the underlying algebra and bi-algebra structures.

An element $x \in H$ is known as primitive if one has $\Delta(x) = 1 \otimes x + x \otimes 1$ and have $\epsilon(x) = 0$. The subspace of primitive elements is closed with respect to the commutator $[x, y] = xy - yx$. Note that for primitive elements $\mu \circ \Delta = 2id_H$ holds true so that $\mu/2$ acts as the left inverse of Δ.

Given a vector space V, there exists a unique bi-algebra structure on the tensor algebra $T(V)$ such that $\Delta(v) = 1 \otimes v + v \otimes 1$ and $\epsilon(v) = 0$ for any element v of V. By the symmetry of Δ this bi-algebra structure is co-commutative and corresponds to the "classical limit". Also the Grassmann algebra associated with V allows bi-algebra structure defined in the same manner.

Figure **??** provides a representation for the axioms of bi-algebra stating that Δ and ϵ act as algebra morphisms of algebra and or equivalent that μ and η act as co-algebra morphisms. The axiom stating that Δ (μ) is algebra (co-algebra) morphism implies that scattering diagrams differing by a box loop are equivalent. The statement that μ is co-algebra morphism reads $(id \otimes \mu \otimes id)(\Delta \otimes \Delta) = \Delta \circ \mu$ whereas the mirror statement $\Delta(ab) = \Delta(a)\Delta(b)$ for Δ reads as $\Delta \circ \mu = \mu(\Delta \otimes \Delta)$ and gives rise to the same graph.

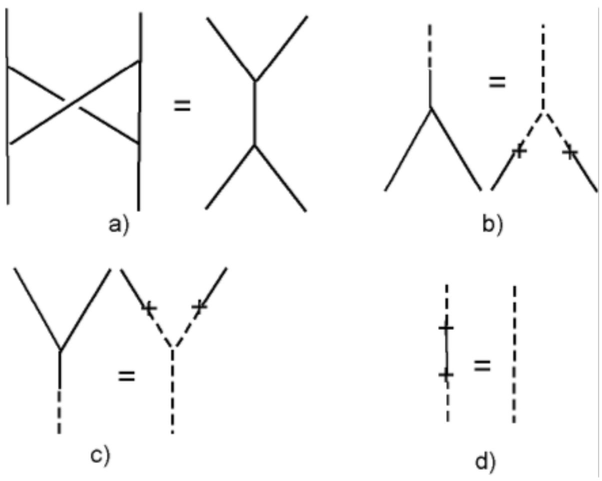

Figure 3: Graphical representation for the conditions guaranteeing that μ and η (Δ and ϵ) act as homomorphisms of co-algebra (algebra). a)$(id \otimes \mu \otimes id)(\Delta \otimes \Delta) = \Delta \circ \mu$, b) $\epsilon \circ \mu = id \circ (\epsilon \otimes \epsilon)$, c) $\Delta \circ \eta = \mu \otimes \circ id_k$, d) $\epsilon \circ \eta = id_k$.

Hopf algebras

Given an algebra (A, μ, η) and co-algebra (C, Δ, ϵ), one can define a bilinear map, the convolution on the vector space $Hom(C, A)$ of linear maps from C to A. By definition, if f and g are such linear maps, then the convolution $f \star g$ is the composition of the maps

$$C \underset{\Delta}{\rightarrow} C\otimes C \underset{f\otimes g}{\rightarrow} A\otimes A \underset{\mu}{\rightarrow} A \tag{A-2.13}$$

Using Sweedler's sigma notion one has

$$f\star g(x) = \sum_{\{x\}} f(x')g(x'') \ . \tag{A-2.14}$$

It can be shown that the triple $(Hom(C,A),\star,\Delta,\eta\circ\epsilon)$ is an algebra and that the map $\Lambda_{C,A}$: $A\otimes C^\star \rightarrow Hom(C,A)$ defined as

$$\Lambda_{C,A}(a\otimes\gamma)(c) = \gamma(c)a$$

is a morphism of algebras, where $C^\star$ is the dual of the finite-dimensional co-algebra C.

For $A = C$ the result gives a mathematical justification for the crossing symmetry inspired re-interpretation of the unitary S-matrix interpreted usually as an element of $Hom(A,A)$ as a state generated by element of $A\otimes A^\star$ from the vacuum $|vac\rangle = |vac_A\rangle\otimes|vac_{A^\star}\rangle$. This corresponds to the interpretation of the reaction $a_i|vac_A\rangle \rightarrow a_f|vac_A\rangle$ as a transition creating state $a_i\otimes a_f^\star|vac\rangle$ with vanishing conserved quantum numbers from vacuum.

With these prerequisites one can introduce the notion of Hopf algebra. Let $(H,\mu,\eta,\Delta,\epsilon)$ be a bi-algebra. An endomorphism S of H is called an antipode for the bi-algebra H if

$$S\star id_H = id_H\star S = \eta\circ\epsilon \ .$$

A Hopf algebra is a bi-algebra with an antipode. A morphism of a Hopf algebra is a morphism between the underlying bi-algebras commuting with the antipodes.

The graphical representation of the antipode axiom is given in the figure below.

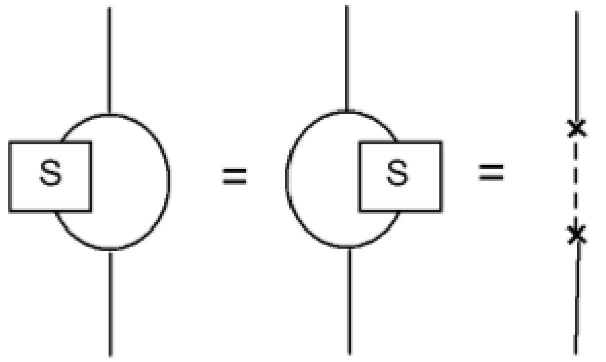

Figure 4: Graphical representation of antipode axiom $S\star id_H = id_H\star S = \eta\circ\epsilon$.

The notion of scalar product central for physical applications boils down to the notion of duality. Duality between Hopf algebras U and H means the existence of a morphism $x\rightarrow\Psi(x)$: $H\rightarrow U^\star$ defined by a bilinear form $\langle u,x\rangle = \Psi(x)(u)$ on $U\times H$, which is a bi-algebra morphism. This means that the conditions

$$\begin{aligned}
&\langle uv,x\rangle = \langle u\otimes v,\Delta(x)\rangle \ , \quad \langle u,xy\rangle = \langle\Delta(u),x\otimes y\rangle \ ,\\
&\langle 1,x\rangle = \epsilon(x) \ , \qquad\qquad\quad \langle u,1\rangle = \epsilon(u) \ ,\\
&\langle S(u),x\rangle = \langle u,S(x)\rangle
\end{aligned} \tag{A-2.15}$$

are satisfied. The first condition on multiplication and co-multiplication, when expressed graphically, states that the decay $x\rightarrow u\otimes v$ can be regarded as time reversal for the fusion of $u\otimes v\rightarrow x$. Second condition has analogous interpretation.

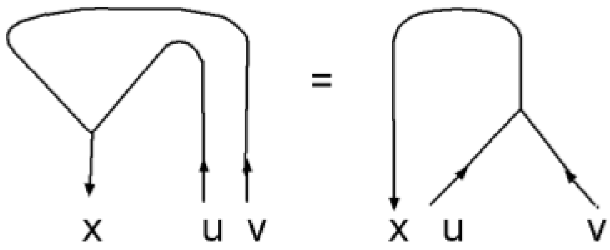

Figure 5: Graphical representation of the duality condition $\langle uv, x\rangle = \langle u \otimes v, \Delta(x)\rangle$.

Modules and comodules

Left and right algebra modules and algebra representations are defined in an obvious manner and satisfy associativity and unit axioms having diagrammatic representation similar to that for corresponding algebra axioms.

A left co-module corresponds a pair (V, Δ_V), where the co-action Δ_N: $V \to C \otimes V$ satisfies co-associativity axiom $(id_C \otimes \Delta_N) \circ \Delta_N = (\Delta \otimes id_N) \circ \Delta_N$ and co-unit axiom $(\epsilon \otimes id) \circ \Delta_N = id_N$. A right co-module is defined in an analogous manner. It is convenient to introduce Sweedlers's notation for Δ_N as $\Delta_N = \sum_{\{c\}} x_C \otimes x_N$.

One can define module and comodule morphisms and tensor product of modules and comodules in a rather obvious manner. The module N could be also algebra, call it A, in which case μ_A and η_A are assumed to act as H-comodule morphisms.

The standard example is quantum plane $A = M(2)_q$ is the free algebra generated variables x, y subject to relations $yx = qxy$ and having coefficients in k. The action of Δ_A reads as

$$\Delta_A \left(\begin{array}{c} x \\ y \end{array} \right) = \left(\begin{array}{cc} a & b \\ c & d \end{array} \right) \otimes \left(\begin{array}{c} x \\ y \end{array} \right) \quad .$$

Δ_A defines algebra morphism from A to $SL_e(2)_q \otimes A$: $\Delta_a(yx) = \Delta_A(y)\Delta_A(x) = q\Delta_A(x)\Delta_A(y) = \Delta(qxy)$.

Braided bi-algebras

$\Delta^{op} = \tau_{H,H} \circ \Delta$ defines the opposite co-algebra H^{op} of H. A braided bi-algebra $(H, \mu, \eta, \Delta, \epsilon)$ is called quasi-co-commutative (or quasi-triangular) if there exists an element R of algebra $H \otimes H$ such that for all $x \in H$ one has

$$\Delta^{op} = R\Delta R^{-1} \quad .$$

One can express R in the form

$$R = \sum_i s_i \otimes t_i \quad .$$

It is convenient to denote by R_{ij} the R matrix acting in i^{th} and j^{th} tensor factors of n^{th} tensor power of H. More precisely, R_{ij} can be defined as an operator acting in an n-fold tensor power of H by the formula $R_{ij} = y^{(1)} \otimes y^{(2)} \otimes ... \otimes y^{(p)}$, $p \leq n$, $y^{(k_i)} = s_i$ and $y^{(k_j)} = t_j$, $y^{(k)} = 1$ otherwise. For instance, one has $R_{13} = \sum_i s_i \otimes 1 \otimes t_i$.

With these prerequisites one can define a braided bi-algebra as a quasi-commutative bi-algebra $(H, \mu, \eta, \Delta, \epsilon, S, S^{-1}, R)$ as an algebra with a preferred element $R \in H \otimes H$ satisfying the two relations

$$\begin{aligned} (\Delta \otimes id_H)(R) &= R_{13}R_{23} \ , \\ (id_H \otimes \Delta)(R) &= R_{13}R_{12} \ . \end{aligned} \qquad \text{(A-2.16)}$$

Braided bi-algebras, known also as quasi-triangular bi-algebras, are central in the theory of quantum groups, R-matrices, and braid groups. By a direct calculations one can verify the following relations.

1. Yang-Baxter equations

$$R_{12}R_{13}R_{23} \;=\; R_{23}R_{13}R_{12} \;, \tag{A-2.17}$$

   and the relation

$$(\epsilon \otimes id_H)(R) \;=\; 1 \tag{A-2.18}$$

   hold true.

2. Since H has an invertible antipode S, one has

$$\begin{aligned}(S \otimes id_H)(R) &= R^{-1} = (id_H \otimes S^{-1})(R) \;, \\ (S \otimes S)(R) &= R \;.\end{aligned} \tag{A-2.19}$$

The graphical representation of the Yang-Baxter equation in terms of the relations of braid group generators is given in the figure **??**.

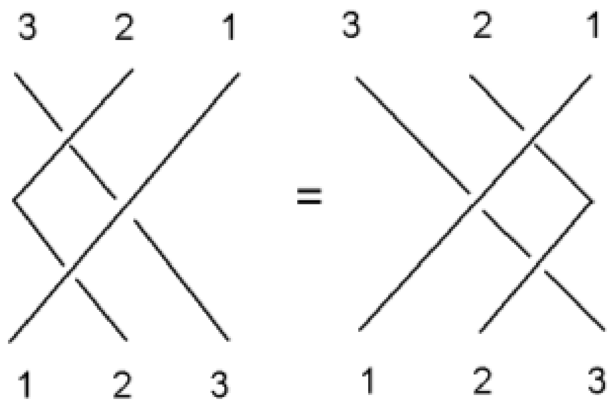

Figure 6: Graphical representation of Yang-Baxter equation $R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}$.

Ribbon algebras

Let H be a braided Hopf algebra with a universal matrix $R = \sum_i s_i \otimes t_i$ and set $u = \sum_i S(t_i)s_i$. It can be shown that u is invertible with the inverse $u^{-1} = \sum s_i S^2(t_i)$ and that $uS(u) = S(u)u$ is central element in H. Furthermore, one has $\epsilon(u) = 1$ and $\Delta(u) = (R_{21}R)^{-1}(u \otimes u)$, and the antipode is given for any $x \in H$ by $S^2(x) = uxu^{-1}$.

Ribbon algebra has besides $R \in H \otimes H$ also a second preferred element called θ. A braided Hopf algebra is called ribbon algebra if there exists a central element θ of H satisfying the relations

$$\Delta(\theta) = (R_{21}R)^{-1}(\theta \otimes \theta) \;\;,\;\;\; \epsilon(\theta) = 1 \;\;,\;\;\; S(\theta) = \theta \;\;. \tag{A-2.20}$$

It can be shown that θ^2 acts like $S(u)u$ on any finite-dimensional module [A29].

Drinfeld's quantum double

Drinfeld's quantum double construction allows to build a quasi-triangular Hopf algebra by starting from any Hopf algebra H and its dual $H^\star$, which exists in a finite-dimensional case always, and as a vector space is isomorphic with H. Besides duality normal ordering is second ingredient of the construction. Physically the generators of the algebra and its dual correspond to creation and annihilation operator type operators. Drinfeld's quantum double construction is represented in a very general manner in [A29]. A construction easier to understand by a physicist is discussed in [A51]. For this reason this representation is summarized here although the style differs from the representation of [A29] followed in the other parts of appendices.

Consider first what is known.

1. Duality means the existence of basis $\{e_a\}$ for H and $\{e^a\}$ for $H^\star$ and inner product (or evaluation as it is called in [A29]) $ev : H^\star \otimes H \to k$ defined as $ev(e^a e_b) \equiv \langle e^a, e_b\rangle = \delta^a_b$ and its inverse $\delta : k \to H^\star \otimes H$ defined by $\delta(1) = e^a e_a$. One can extend the inner product to an inner product in the tensor product $(H^\star \otimes H^\star) \otimes (H \otimes H)$ in an obvious manner.

2. The product (co-product) in H ($H^\star$) coincides with the co-product (product) in $H^\star$ (H) in the sense that one has

$$\begin{aligned} \langle e^c, e_a e_b\rangle &= m^c_{ab} = \langle \Delta(e^c), e_b \otimes e_a\rangle \ , \\ \langle e^a e^b, e_c\rangle &= \mu^{ab}_c = \langle e^a \otimes e^b, \Delta(e_c)\rangle \ , \end{aligned} \quad \text{(A-2.21)}$$

   These equations are quite general expressions for the duality expressed graphically in figure **??**.

3. The antipodes S for H and $H^\star$ can be represented as matrices

$$S_H(e_a) = S_a^{\ b} e_b \ , \quad S_{H^\star}(e^a) = (S^{-1})^a_{\ b} e^b \ . \quad \text{(A-2.22)}$$

The task is to construct algebra product μ and co-algebra product Δ, unit η and co-unit ϵ, antipode, and R-matrix R for for $H \otimes H^\star$. The natural basis for $H \otimes H^\star$ consists of $e_a \otimes e^b$.

1. Co-product Δ is simply the product of co-products

$$\Delta(e_a e^b) = \Delta(e_a)\Delta(e^b) = m^b_{vu}\mu^{cd}_a e_c e^u \otimes e_d e^v \ . \quad \text{(A-2.23)}$$

2. Product μ involves normal ordering prescription allowing to transform products $e^a e_b$ (elements of $H^\star \otimes H$) to combinations of basis elements $e_a e^b$ (elements of $H \otimes H^\star$. This map must be consistent with the requirement that co-product acts as an algebra morphism. Drinfeld's normal ordering prescription, or rather a map $c_{H^\star,H}$: $H^\star \otimes H \to H \otimes H^\star$ is given by

$$c_{H^\star,H}(e^a e_b) = R^{ac}_{bd} e_c e^d \ , \quad R^{ac}_{bd} = m^x_{kd} m^a_{xu} \mu^{vy}_b \mu^{ck}_y (S^{-1})^u_v e_c e^d \ . \quad \text{(A-2.24)}$$

   The details of the formula are far from being obvious: the axioms of tensor category with duality to be discussed later might allow to relate $R_{H^\star,H}$ to $R_{H,H}$ and this might help to understand the origin of the expression. Normal ordering map can be interpreted as braid operation exchanging H and $H^\star$ and the matrix defining the map could be regarded as R-matrix $R_{H\otimes H^\star}$.

3. The universal R-matrix is given by

$$R = (e_a \otimes id_{H^\star}) \otimes (id_H \otimes e^a) \ , \tag{A-2.25}$$

where the summation convention is applied. One can show that $R\Delta = \Delta^{op}R$ by a direct calculation.

4. The antipode $S_{H\otimes H^\star}$ follows from the product of antipodes for H and $H^\star$ using the fact that antipode is antihomomorphism using the normal ordering prescription

$$S_{H\otimes H^\star}(e_a e^b) = c_{H^\star,H}(S(e^b)S(e_a)) \ . \tag{A-2.26}$$

Quasi-Hopf algebras and Drinfeld associator

Braided Hopf algebras are quasi-commutative in the sense that one has $\Delta^{op} = R\Delta R^{-1}$. Also the strict co-associativity can be given up and this means that one has

$$(\Delta \otimes id)\Delta = \Phi(id \otimes \Delta)\Phi^{-1} \ , \tag{A-2.27}$$

where $\Phi \in H \otimes H \otimes H$ is known as Drinfeld's associator and appears in the of conformal fields theories. If the resulting structure satisfies also the so called Pentagon Axiom (to be discussed later, see Eq. A-2.36 and figure **??**), it is called quasi-Hopf algebra. Pentagon Axiom boils down to the condition

$$(id \otimes id \otimes \Delta)(\Phi)(\Delta \otimes id \otimes id)(\Phi) = (id \otimes \Phi)(id \otimes id \otimes \Delta)(\Phi)(\Phi \otimes id) \ . \tag{A-2.28}$$

The Yang-Baxter equation for quasi-Hopf algebra reads as

$$R_{12}\Phi_{312}R_{13}\Phi^{-1}_{1322}R_{23}\Phi_{123} = \Phi_{321}R_{23}\Phi^{-1}_{231}R_{13}\Phi_{213}R_{12}\Phi_{123} \ . \tag{A-2.29}$$

The left-hand side arises from a sequence of transformations

$$(12)3 \xrightarrow{\Phi_{123}} 1(23) \xrightarrow{R_{23}} 1(32) \xrightarrow{\Phi^{-1}_{132}} (31)2 \xrightarrow{R_{13}} 3(12) \xrightarrow{\Phi_{312}} 3(12) \xrightarrow{R_{12}} 3(21) \ .$$

The right-hand side arises from the sequence

$$(12)3 \xrightarrow{R_{12}} (21)3 \xrightarrow{\Phi_{213}} 2(13) \xrightarrow{R_{13}} 2(31) \xrightarrow{\Phi^{-1}_{231}} (23)1 \xrightarrow{R_{23}} (32)1 \xrightarrow{\Phi_{321}} 3(21) \ .$$

One can produce new quasi-Hopf algebras by gauge (or twist) transformations using invertible element $\Omega \in H \otimes H$ called twist operator

$$\begin{aligned} \Delta(a) &\rightarrow \Omega\Delta(a)\Omega^{-1} \ , \\ \Phi &\rightarrow \Omega_{23}(id \otimes \Delta)(\Omega)\Phi(\Delta \otimes id)(\Omega^{-1})\Omega^{-1}_{12} \ , \\ R &\rightarrow \Omega R \Omega^{-1} \ . \end{aligned} \tag{A-2.30}$$

Quasi-Hopf algebras appear in conformal field theories and correspond quantum universal enveloping algebras divided by their centralizer. Consider as an example the R-matrix R^{j_1,j_2} relating $j_1 \otimes j_2$ and $j_2 \otimes j_1$ representations $\Delta^{j_1,j_2}(a)$ and $\Delta^{j_2,j_1}(a)$ of the co-product Δ of $U(sl(2))_q$. $\Delta^{j,j}(a)$ commutes with R^{jj} for all elements of the quantum group. The action of $g_i = qR^{jj}$ acting in i^{th} and $(i+1)^{th}$ tensor factors extends to the representation $(V_j)^{\times n}$ in an obvious manner. From the Yang-Baxter equation it follows that the operators g_i define a representation of braid group B_n:

$$\begin{aligned} g_i g_{i+1} g_i &= g_{i+1} g_i g_{i+1} \ , \\ g_i g_j &= g_j g_i \ , \ \text{for} \ |j-k| \geq 2 \ . \end{aligned} \quad \text{(A-2.31)}$$

Under certain conditions the braid group generators generate the whole centralizer C_q^n for the representation of quantum group. For instance, this occurs for $j=1/2$. In this case the additional condition

$$g_i^2 = (q^2-1)g_i + q^2 \times 1 \ , \quad \text{(A-2.32)}$$

so that the centralizer is isomorphic with the Hecke algebra $H_n(q)$, which can be regarded as a q-deformation of permutation group S_n.

The result generalizes. In Wess-Zumino-Witten model based on group G the relevant algebraic structure is $U(G_q)/C^n(q)$. This is quasi-Hopf algebra and the so called Drinfeld associator characterizes the quasi-associativity.

A-2.3 Tensor Categories

Hopf algebras and related structures do not seem to be quite enough in order to formulate elegantly the construction of S-matrix in TGD framework. A more general structure known as a braided tensor category with left duality and twist operation making the category to a ribbon category is needed. The algebra product μ and co-product Δ must be generalized so that they appear as morphisms $\mu_{A\otimes B\rightarrow C}$ and $\Delta_{A\rightarrow B\otimes C}$: this gives hopes of describing 3-vertices algebraically. It is not clear whether one can assume single underlying algebra so that objects would correspond to different representations of this algebra or whether one allow even non-isomorphic algebras.

In the tensor category the tensor products of objects and corresponding morphisms belong to the category. In a braided category the objects $U\otimes V$ and $V\otimes U$ are related by a braiding morphism. The notion of braided tensor category appears naturally in topological and conformal quantum field theories and seems to be an appropriate tool also in TGD context. The basic category theoretical notions are discussed in [A29] and I have already earlier considered category theory as a possible tool in the construction of quantum TGD and TGD inspired theory of consciousness [K28].

In braided tensor categories one introduces the braiding morphism $c_{V,W}: V\otimes W\rightarrow W\otimes V$, which is closely related to R-matrix. In categories allowing duality arrows with both directions are allowed ad diagrams analogous to pair creation from vacuum are possible. In ribbon categories one introduces also the twist operation θ_V as a morphism of object and the Θ_W satisfies the axiom: $\theta_{V\otimes W} = (\theta_V\otimes\theta_W)c_{W,V}c_{V,W}$. One can also introduce morphisms with arbitrary number of incoming lines and outgoing lines and visualize them as boxes, coupons. Isotopy principle, originally related to link and knot diagrams provides a powerful tool allowing to interpret the basic axioms of ribbon categories in terms of isotopy invariance of the diagrams and to invent theorems by just isotoping.

Categories, functors, natural transformations

Categories [A29, A26, A54, A60] are roughly collections of objects A, B, C... and morphisms $f(A\rightarrow B)$ between objects A and B such that decomposition of two morphisms is always defined. Identity morphisms map objects to objects. Examples of categories are open sets of some topological spaces with continuous maps between them acting as morphisms, linear spaces with linear maps between them acting as morphisms, groups with group homomorphisms taking the role of morphisms. Practically any collection of mathematical structures can be regarded as a category. Morphisms can be very general: for instance, partial ordering $a\leq b$ can define a morphism $f(A\rightarrow B)$.

Functors between categories map objects to objects and morphisms to morphisms so that a product of morphisms is mapped to the product of the images and identity morphism is mapped to identity morphism. Functor $F : \mathcal{C} \rightarrow \mathcal{D}$ commutes also with the maps s and b assigning to a morphism $f : V \rightarrow W$ its source $s(f) = V$ and target $b(f) = W$.

A natural transformation between functors F and G from $\mathcal{C} \rightarrow \mathcal{C}'$ is a family of morphisms $\eta(V) : F(V) \rightarrow G(V)$ in $\mathcal{C}'$ indexed by objects V of $\mathcal{C}$ such that for any morphisms $f : V \rightarrow W$ in $\mathcal{C}$, the square

$$\begin{array}{ccc} F(V) & \overset{\eta(V)}{\longrightarrow} & G(V) \\ \downarrow F(f) & & \downarrow G(f) \\ F(W) & \overset{\eta(W)}{\longrightarrow} & G(W) \end{array} \tag{A-2.33}$$

commutes.

The functor $F : \mathcal{C} \rightarrow \mathcal{D}$ is said to be equivalence of categories if there exists a functor $G : \mathcal{D} \rightarrow \mathcal{C}$ such and natural isomorphisms

$$\eta : id_{\mathcal{D}} \rightarrow FG \text{ and } \theta : GF \rightarrow id_{\mathcal{C}}FG \ .$$

The notion of adjoint functor is a more general notion than equivalence of categories. In this case η and θ are natural transformations but not necessary natural isomorphisms in such a way that the composite maps

$$\begin{array}{ccccc} F(V) & \overset{\eta(F(V))}{\longrightarrow} & (FGF)(V) & \overset{F(\theta(V))}{\longrightarrow} & F(V) \\ & & & & \\ G(W) & \overset{G(\eta(W))}{\longrightarrow} & (GFG)(W) & \overset{\theta(G(W)))}{\longrightarrow} & G(W) \end{array} \tag{A-2.34}$$

are identify morphisms for all objects V in $\mathcal{C}$ and W in $\mathcal{D}$.

The product $C = AB$ for objects of categories is defined by the requirement that there exist projection morphisms π_A and π_B from C to A and B and that for any object D and pair of morphisms $f(D \rightarrow A)$ and $g(D \rightarrow B)$ there exist morphism $h(D \rightarrow C)$ such that one has $f = \pi_A h$ and $g = \pi_B h$. Graphically this corresponds to a square diagram in which pairs A, B and C, D correspond to the pairs formed by opposite vertices of the square and arrows DA and DB correspond to morphisms f and g, arrows CA and CB to the morphisms π_A and π_B and the arrow h to the diagonal DC. Examples of product categories are Cartesian products of topological spaces, linear spaces, differentiable manifolds, groups, etc. The tensor products of linear spaces and algebras provides an especially interesting example of product in the recent case. One can define also more advanced concepts such as limits and inverse limits. Also the notions of sheafs, presheafs, and topos are important.

Tensor categories

Let $\mathcal{C}$ be a category. Tensor product $\otimes$ is a functor from $\mathcal{C} \times \mathcal{C}$ to $\mathcal{C}$ if

1. there is an object $V \otimes W$ associated with any pair (V, W) of objects of $\mathcal{C}$

2. there is an morphism $f \otimes g$ associated with any pair (f, g) of morphisms of $\mathcal{C}$ such that $s(f \otimes g) = s(f) \otimes s(g)$ and $b(f \otimes g) = b(f) \otimes b(g)$,

3. if f' and g' are morphisms such that $s(f') = b(f)$ and $s(g') = b(g)$ then $(f' \otimes g') \circ (f \otimes g) = (f' \circ f) \otimes (g' \circ g)$,

4. $id_{V \otimes W} = id_{W \otimes V}$.

   Any functor with these properties is called tensor product. The tensor product of vector spaces provides the most familiar example of a tensor product functor.

   In figure **??** the general rules for graphical representations of morphisms are given.

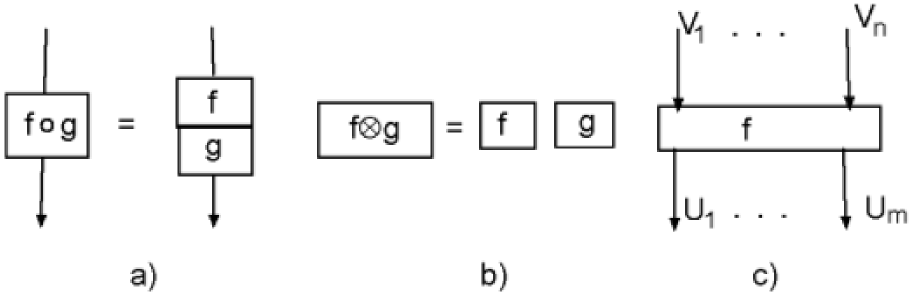

Figure 7: The graphical representation of morphisms. a) $g \circ f$: $V \rightarrow W$, b) $f \otimes g$, c) f : $U_1 \otimes ... \otimes U_m \rightarrow V_1 \otimes .... \otimes V_n$.

An associativity constraint for the tensor product is a natural isomorphism

$$a : \otimes(\otimes \times id) \rightarrow \otimes(id \times \otimes) \ .$$

On basis of general definition of natural isomorphisms (see Eq. A-2.33) one can conclude that for any triple (U, V, W) of objects of $\mathcal{C}$ there exists an isomorphism

$$\begin{array}{ccc} (U \otimes V) \otimes W & \overset{a_{U,V,W}}{\rightarrow} & U \otimes (V \otimes W) \\ \downarrow (f \otimes g) \otimes h & & \downarrow f \otimes (g \otimes h) \\ (U' \otimes V') \otimes W' & \overset{a_{U',V\ prime,W'}}{\longrightarrow} & U' \otimes (V' \otimes W') \end{array} \quad \text{(A-2.35)}$$

Associativity constraints satisfies Pentagon Axiom [A29] if the following diagrams commutes.

$$\begin{array}{ccc} U \otimes (V \otimes W) \otimes X & \underset{a_{U,V,W} \otimes id_X}{\longleftarrow} & ((U \otimes V) \otimes W)) \otimes X \\ & & \downarrow a_{U \otimes V,W,X} \\ \downarrow a_{U,V \otimes W,X} & & (U \otimes V) \otimes (W \otimes X) \\ & & \downarrow a_{U,V,W \otimes X} \\ U \otimes ((V \otimes W) \otimes X) & \overset{id_U \otimes a_{V,W,X}}{\longrightarrow} & U \otimes (V \otimes (W \otimes X)) \end{array} \quad \text{(A-2.36)}$$

Pentagon axiom has been already mentioned while discussing the definition of quasi-Hopf algebras. In figure **??** are graphical illustrations of associativity morphism $a(U, V, W)$, Triangle Axiom, and Pentagon Axiom are given.

Assume that an object I is fixed in the category. A left unit constraint with respect to I is a natural isomorphism

$$l : \otimes(I \times id) \rightarrow id$$

By Eq. A-2.33 this means that for any object V of C there exists an isomorphism

$$l_V : I \otimes V \rightarrow V \quad \text{(A-2.37)}$$

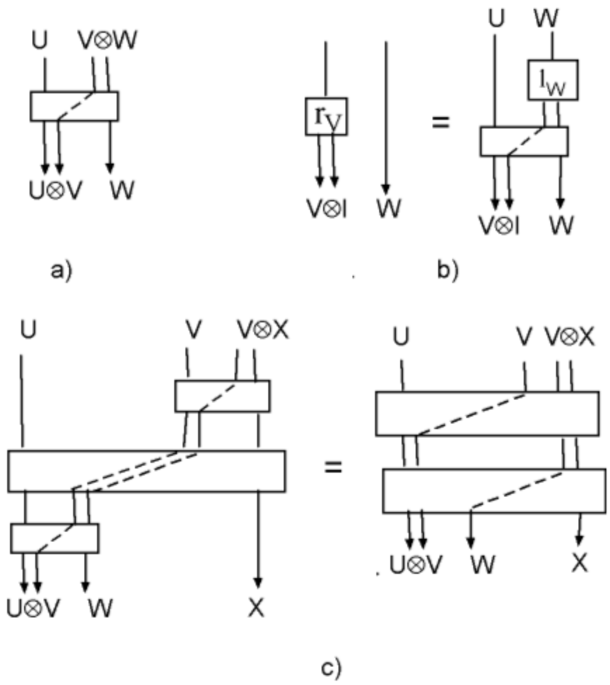

Figure 8: Graphical representations of a) the associativity isomorphism $a_{U,V,W}$, b) Triangle Axiom, c) Pentagon Axiom.

such that

$$\begin{array}{ccc} I \otimes V & \xrightarrow{l_V} & V \\ \downarrow id_I \otimes f & & \downarrow f \\ I \otimes V' & \xrightarrow{l_{V'}} & V' \end{array} \tag{A-2.38}$$

The right unit constraint $r : \otimes(id \times I) \to id$ can be defined in a completely analogous manner.

Given an associativity constraint a, and left and right unit constraints l, r with respect to an object I, one can say that the Triangle Axiom is satisfies if the triangle

$$\begin{array}{ccc} (V \otimes I) \otimes W & \xrightarrow{a_{V,I,W}} & V \otimes (I \otimes W) \\ & r_V \otimes id_W \searrow \quad \swarrow id_W \otimes l_W & \\ & V \otimes W & \end{array} \tag{A-2.39}$$

commutes (see figure **??**).

These ingredients lead allow to define tensor category $(\mathcal{C}, I, a, l, r)$ as a category $\mathcal{C}$ which is equipped with a tensor product $\otimes : \mathcal{C} \times \mathcal{C} \to \mathcal{C}$ satisfying associativity constraint a, left unit constraint l and right unit constraint r with respect to I, such that Pentagon Axiom and Triangle Axiom are satisfied.

The definition of a tensor functor $F : \mathcal{C} \to \mathcal{D}$ involves also additional isomorphisms. $\phi_0 : I \to F(I)$ satisfies commutative diagrams involving right and left unit constraints l and r. The family of isomorphisms

$$\phi_2(U, V) : F(U) \otimes F(V) \to F(U \otimes V)$$

satisfies a commutative diagram stating that ϕ_2 commutes with associativity constraints. The interested reader can consult [A29] for details. One can also define the notions of natural tensor transformation, natural tensor isomorphism, and tensor equivalence between tensor categories by applying the general category theoretical tools.

Keeping track of associativity isomorphisms is obviously a rather heavy burden. Fortunately, it can be shown that one can assign to a tensor category $\mathcal{C}$ a strictly associative (or briefly, strict) tensor category which is tensor equivalent of $\mathcal{C}$.

Braided tensor categories

Braided tensor categories satisfy also commutativity constraint c besides associativity constraint a. Denote by $\tau : \mathcal{C} \times \mathcal{C} \to \mathcal{C} \times \mathcal{C}$ the flip functor defined by $\tau(V, W) = (W, V)$. Commutativity constraint is a natural isomorphism

$$c : \otimes \to \otimes\tau \quad .$$

This means that for any pair (V, W) of objects there exists isomorphism

$$c_{V,W} : V \otimes W \to W \otimes V$$

such that the square

$$\begin{array}{ccc} V \otimes W & \xrightarrow{c_{V,W}} & W \otimes V \\ \downarrow f \otimes g & & \downarrow g \otimes f \\ V' \otimes W' & \xrightarrow{c_{V',W'}} & W' \otimes V' \end{array} \qquad \text{(A-2.40)}$$

commutes.

The commutativity constraint satisfies Hexagon Axiom if the two hexagonal diagrams

(H1)

$$\begin{array}{ccc} U \otimes (V \otimes W) & \xrightarrow{c_{U,V\otimes W}} & (V \otimes W) \otimes U \\ \uparrow a_{U,V,W} & & \downarrow a_{V,W,U} \\ (U \otimes V) \otimes W & & V \otimes (W \otimes U) \\ \downarrow c_{U,V} \otimes id_W & & \uparrow id_V \otimes c_{U,W} \\ (V \otimes U) \otimes W & \xrightarrow{a_{V,U,W}} & V \otimes (U \otimes W) \end{array} \qquad \text{(A-2.41)}$$

and (H2)

$$\begin{array}{ccc} (U \otimes V) \otimes W & \xrightarrow{c_{U\otimes V,W}} & W \otimes (U \otimes V) \\ \uparrow a^{-1}_{U,V,W} & & \downarrow a^{-1}_{W,U,V} \\ U \otimes (V \otimes W) & & (W \otimes U) \otimes V \\ \downarrow id_U \otimes c_{V,W} & & \uparrow c_{U,W} \otimes id_V \\ U \otimes (W \otimes V) & \xrightarrow{a^{-1}_{U,W,V}} & (U \otimes W) \otimes V \end{array} \qquad \text{(A-2.42)}$$

commute.

The braiding operation $c_{V,W}$ and the association operation $a(U,V,W)$, and pentagon and hexagon axioms are illustrated in the figure **??** below.

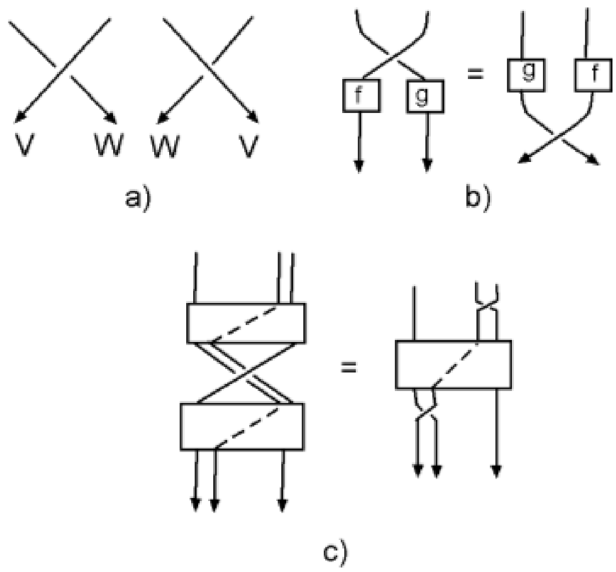

Figure 9: Graphical representations a) of the braiding morphism $c_{V,W}$ and its inverse $c^{-1}_{V,W}$, b) of naturality of $c_{V,W}$, c) of First Hexagon Axiom.

Duality and tensor categories

The notion of a dual of the finite-dimensional vector space as a space of linear maps from V to field k can lifted to a concept applying for tensor category. A strict (strictly associative) tensor category $(\mathcal{C},\otimes,I)$ with unit object I is said to possess left duality if for each object V of $\mathcal{C}$ there exists an object $V^\star$ and morphisms

$$b_V : I \rightarrow V\otimes V^\star \ \text{ and } \ d_V : V^\star\otimes V \rightarrow I$$

such that

$$(id\otimes d_V)(b_V\otimes id_V) = id_V \ \ and \ \ (d_V\otimes id_{V^\star})(id_{V^\star}\otimes b_V) = id_{V^\star} \ \ . \tag{A-2.43}$$

One can define the transpose of f in terms of b_V and d_V. The idea how this is achieved is obvious from **Fig. ??**.

$$f^\star = (d_V\otimes id_{U^\star})(id_{V^\star}\otimes f\otimes id_{U^\star})(id_{V^\star}\otimes b_U) \ \ . \tag{A-2.44}$$

Also the braiding operation $c_{V^\star,W}$ can be expressed in terms of $c^{-1}_{V,W}$, b_V and d_V by using the isotopy of **Fig. 10**:

$$c_{V^\star,W} = (d_V\otimes id_{W\otimes V^\star})(id_{V^\star}\otimes c^{-1}_{V,W}\otimes id_{V^\star})(id_{V^\star\otimes W}\otimes b_V) \ \ . \tag{A-2.45}$$

Drinfeld quantum double can be regarded as a tensor product of Hopf algebra and its dual and in this case one can introduce morphisms $ev_H : H\otimes H^\star\rightarrow k$ defined as $e^i\otimes e_j\rightarrow \delta^i_j$ defining inner product and its inverse $\delta : k\rightarrow H\otimes H$ defined as $1\rightarrow e^ie_i$, where summation over i is understood. For categories these morphisms are generalized to morphism d_V from objects V of category to unit object I and b_V from I to object of category. The elements of H and $H^\star$ are described as strands with opposite directions, whereas d_V and b_V correspond to annihilation and creation of strand–anti-strand pair as show in **Fig.** 10.

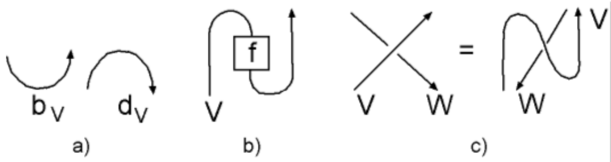

Figure 10: Graphical representations a) of the morphisms b_V and d_V, b) of the transpose $f^\star$, c) of braiding operation $c_{V^\star,W}$ expressed in terms of $c_{V,W}$.

Ribbon categories

According to the definition of [A29] ribbon category is a strict braided tensor category $(\mathcal{C}, \otimes, I)$ with a left duality with a family of natural morphisms $\theta_V : V \rightarrow V$ indexed by the objects V of $\mathcal{C}$ satisfying the conditions

$$\begin{aligned} \theta_{V\otimes W} &= \theta_V \otimes \theta_W c_{W,V} c_{V,W} \ , \\ \theta_{V^\star} &= (\theta_V)^\star \end{aligned} \qquad \text{(A-2.46)}$$

for all objects V, W of $\mathcal{C}$. The naturality of twist means for for any morphisms $f : V \rightarrow W$ one has $\theta_W f = f\theta_V$. The graphical representation for the axioms and is in **Fig. 11**.

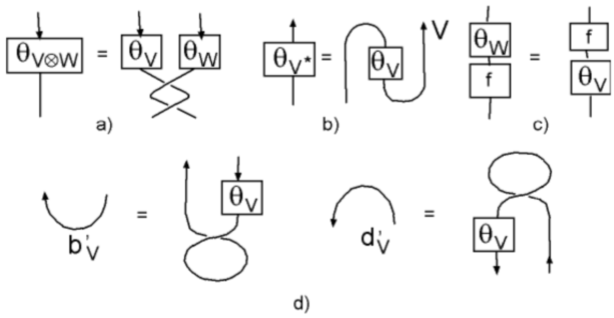

Figure 11: Graphical representations a) of $\theta_{V\otimes W} = \theta_V \otimes \theta_W c_{W,V} c_{V,W}$, b) of $\theta_{V^\star} = (\theta_V)^\star$, c) of $\theta_W f = f\theta_V$, d) of right duality for a ribbon category.

The existence of the twist operation provides $\mathcal{C}$ with right duality necessary in order to define trace (see **Fig. ??**).

$$\begin{aligned} d'_V &= (id_{V^\star} \otimes \theta_V) c_{V,V^\star} b_V \ , \\ b'_V &= d_V c_{V,V^\star} (\theta_V \otimes id_{V^\star}) \ . \end{aligned} \qquad \text{(A-2.47)}$$

One can define quantum trace for any endomorphisms f of ribbon category:

$$tr_q(f) = d'_V (f \otimes id_{V^\star}) b_V = d_V c_{V,V^\star} (\theta_V f \otimes id_{V^\star}) b_V \ . \qquad \text{(A-2.48)}$$

Again the graphical representation is the best manner to understand the definition, see **Fig. ??**. Quantum trace has the basic properties of trace: $tr_q(fg) = tr_q(gf)$, $tr_q(f \otimes g) = tr_q(f) tr_q(g)$, $tr_q(f) = tr_q(f^\star)$. The proof of these properties is easiest using isotropy principle.

The quantum dimension of an object V of ribbon category can be defined as the quantum trace for the identity morphism of V: $dim_q(V) = tr_q(id_V) = d'_V b_V$. Quantum dimension is

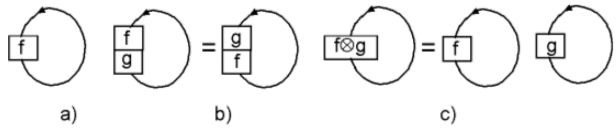

Figure 12: Graphical representations of a) $tr_q(f)$, b) of $tr_q(fg) = tr_q(gf)$, c) of $tr(f \otimes g) = tr(f)tr(g)$.

represented as a vacuum bubble. Quantum dimension satisfies the conditions $dim_q(V \otimes W) = dim_q(V)dim_q(W)$ and $dim_q(V) = dim_q(V^\star)$.

A more general definition of ribbon category inspired by the considerations of [A51] is obtained by allowing the generalization of morphisms μ and Δ so that they become morphisms $\mu_{A\otimes B\rightarrow C}$ and $\Delta_{C\rightarrow A\otimes B}$ of ribbon category. Graphically the general morphism with arbitrary number of incoming outgoing strands can be represented as a box or "coupon". An important special case of ribbon categories consists of modules over braided Hopf algebras allowing ribbon algebra structure.

A-3 Axiomatic Approach To S-Matrix Based On The Notion Of Quantum Category

This section can be regarded as an attempt of a physicists with some good intuitions and intentions but rather poor algebraic skills to formulate basic axioms about S-matrix in terms of what might be called quantum category. The basic result is an interpretation for the equivalence of loop diagrams with tree diagrams as a consequence of basic algebra and co-algebra axioms generalized to the level of tensor category. The notion of quantum category emerges naturally as a generalization of ribbon category, when algebra product and co-algebra product are interpreted as morphisms between different objects of the ribbon category.

The general picture suggest that the operations Δ and μ generalized to algebra homomorphisms $A \rightarrow B\otimes C$ and $A\otimes B \rightarrow C$ in a tensor category whose objects are either representations of an algebra or even algebras might provide an appropriate mathematical tool for saying something interesting about S-matrix in TGD Universe. These algebras need not necessarily be bi-algebras. In the following it is demonstrated that the equivalence of loop diagrams to tree diagrams follows from suitably generalized bi-algebra axioms. Also the interpretation of various morphisms involved with Hopf algebra structure is discussed.

A-3.1 Δ And μ And The Axioms Eliminating Loops

The first task is to find a physical interpretation for the basic algebraic operations and how the basic algebra axioms might allow to eliminate loops. The physical interpretation of morphisms Δ and μ as algebra or category morphisms has been already discussed. As already found, the condition that Δ (μ) acts as an algebra (co-algebra) morphism leads to a condition stating that a box graph for 2-particle scattering is equivalent with tree graph. It is interesting to identify the corresponding conditions in the case of self energy loops and vertex corrections.

The condition

$$\mu_{B\otimes C\rightarrow A} \circ \Delta_{A\rightarrow B\otimes C} = K \times id_A \ , \tag{A-3.1}$$

where K is a numerical factor, is a natural additional condition stating that a line with a self energy loop is equivalent with a line without the loop. The condition is illustrate in figure **??**. For the co-commutative tensor algebra $T(V)$ of vector space with $\Delta(x) = 1\otimes x + x\otimes 1$ one would have $K = 2$ for the generators of $T(V)$. For a product of n generators one has $K = 2^n$.

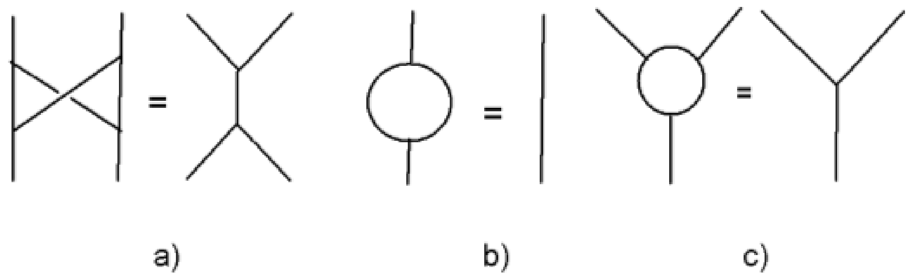

Figure 13: Graphical representations for the conditions a) $(id \otimes \mu \otimes id)(\Delta \otimes \Delta) = \Delta \circ \mu$, b) $\mu_{B\otimes C\to A} \circ \Delta_{A\to B\otimes C} = K \times id_A$, and c) $(\mu \otimes id) \circ (\Delta \otimes id) \circ \Delta = K \times \Delta$.

The condition $\Delta_{A\to B\otimes C} \circ \mu_{B\otimes C\to A} = K \times id_A$ cannot hold true since multiplication is not an irreversible process. If this were the case one could reduce tree diagrams to collections of free propagator lines.

In quantum field theories also vertex corrections are a source of divergences. The requirement that the graph representing a vertex correction is equivalent with a simple tree graph representing a decay gives an additional algebraic condition. For bi-algebras the condition would read

$$(\mu \otimes id) \circ (\Delta \otimes id) \circ \Delta = K\Delta \ , \tag{A-3.2}$$

where K is a simple multiplicative factor. In fact, for the co-commutative tensor algebra $T(V)$ of vector space the left hand side would be $3 \times \Delta(x)$ giving $K = 3$ for generators $T(V)$. The condition is illustrated in figure ??.

Using the standard formulas of appendix for quantum groups one finds that in the case of $U_q(sl(2))$ the condition $\mu \circ \Delta(X) = K_X X$, K_X constant, is not true in general. Rather, one has $\mu \circ \Delta(X) = XK_X(q^{H/2} + q^{-H/2}, q^{1/2}, q^{-1/2})$. The action on the vacuum state is however proportional to that of X, being given by $K_X(2, 1, 1)X$. The function K_X for a given X can be deduced from $\mu \circ \Delta(X_\pm) = q^{H/2}X_\pm + X_\pm q^{-H/2} = X_\pm(q^{\pm 1/2} + q^{H/2} + q^{-H/2})$. The eigen states of Cartan algebra generators are expected to be eigen states of $\mu \circ \Delta$ also in the case of a general quantum group. $\mu \circ \Delta$ is analogous to a single particle operator like kinetic energy and its action on multi-particle state is a sum over all tensor factors with $\mu \circ \Delta$ applied to each of them. For eigen states of $\mu \circ \Delta$ the projective equivalence of loop diagrams with tree diagrams would make sense.

Since self energy loops, vertex corrections, and box diagrams represent the basic divergences of renormalizable quantum field theories, these axioms raise the hope that the basic infinities of quantum field theories could be eliminated by the basic axioms for the morphisms of quantum category.

There are also morphisms related to the topology changes in which the 3-surface remains connected. For instance, processes in which the number of boundary components can change could be of special relevance if the family replication phenomenon reduces to the boundary topology. Also 3-topology can change. The experience with topological quantum field theories [A41], stimulates the hope that the braid group representations of the topological invariants of 3-topology might be of help in the construction of S-matrix.

The equivalence of loop diagrams with tree diagrams must have algebraic formulation using the language of standard quantum field theory. In the third section it was indeed found that thanks to the presence of the emission of vacuons, the equivalence of loop diagrams with tree diagrams corresponds to the vanishing of loop corrections in the standard quantum field theory framework. Furthermore, the non-cocommutative Hopf algebra of Feynman diagrams discussed in [A33] becomes co-commutative when the loop corrections vanish so that TGD program indeed has an elegant algebraic formulation also in the standard framework.

A-3.2 The Physical Interpretation Of Non-Trivial Braiding And Quasi-Associativity

The exchange of the tensor factors by braiding could also correspond to a physically non-trivial but unitary operation as it indeed does in anyon physics [D38, D33]. What would differentiate between elementary particles and anyons would be the non-triviality of the super-canonical and Super Kac-Moody conformal central extensions which have the same origin (addition of a multiplication by a multiple of the Hamiltonian of a canonical transformation to the action of isometry generator). The proposed interpretation of braiding acting in the complex plane in which the conformal weights of the elements of the super-canonical algebra represent punctures justifies the non-triviality. Hexagon Axioms would state that two generalized Feynman diagrams involving exchanges, dissociations and re-associations are equivalent.

An interesting question is whether the association $(A, B) \rightarrow (A \otimes B)$ could be interpreted as a formation of bound state entanglement between A and B. A possible space-time correlate for association is topological condensation of A and B to the same space-time sheet. Association would be trivial if all particles are at same space-time sheet X^4 but non-trivial if some subset of particles condense at an intermediate space-time sheet Y^4 condensing in turn at X^4.

Be as it may, association isomorphisms $a_{A,B,C}$ would state that the state space obtained by binding A with bound bound states $(B \otimes C)$ is unitarily related with the state space obtained by binding $(A \otimes B)$ bound states with C. With this interpretation Pentagon axiom would state that two generalized Feynman diagrams depicted in figure **??** leading from initial to final to final state by dissociation and re-association are equivalent.

A-3.3 Generalizing The Notion Of Bi-Algebra Structures At The Level Of WCW

WCW of 3-surfaces decomposes into sectors corresponding to different 3-topologies. Also other signatures might be involved and I have proposed that the sectors are characterized by the collection of p-adic primes labelling space-time sheets of the 3-surface and that a given space-time surface could be characterized by an infinite prime or integer. The general problem is to continue various geometric structures from a given sector A of WCW to other sector B.

An especially interesting special case corresponds to a continuation from 1-particle sector to two-particle sector or vice versa and corresponds to TGD variant of 3-vertex. All these continuations involve the embedding of a structure associated with the sector A to a structure associated with sector B. For the continuation from 1-particle sector to 2-particle sector the map is analogous to co-algebra homomorphism Δ. For the reverse continuation it is analogous to the algebra product μ. Now however one does not have maps $\Delta : A \rightarrow A \otimes A$ and $\mu : A \otimes A \rightarrow A$ but $\Delta : A \rightarrow B \otimes C$ and $\mu : B \otimes C \rightarrow A$ unless the algebras are isomorphic. $\mu \circ \Delta = id$ should hold true as an additional condition but $\Delta \circ \mu = id$ cannot hold true since product maps many pairs to the same element.

Continuation of the WCW spinor structure

The basic example of a structure to be continued is configuration space spinor structure. WCW spinor fields in different sectors should be related to each other. The isometry generators and gamma matrices of WCW span a super-canonical algebra. The continuation requires that the super algebra basis of different sectors are related. Also vacua must be related. Isometry generators correspond to bosonic generators of the super-canonical algebra. There is also a natural extension of the super-canonical algebra defined by the Poisson structure of WCW .

This view suggests that in the first approximation one could see the construction of S-matrix as following process.

1. Incoming/outgoing states correspond to positive/negative energy states localized to the sectors of WCW with fixed 3-topologies.

2. In order to construct an S-matrix matrix element between two states localized in sectors A and B, one must continue the state localized in A to B or vice versa and calculate overlap. The continuation involves a sequence of morphisms mapping various structures between sectors. In particular, topological transformations describing particle decay and fusion are possible

so that the analogs of product μ and co-product Δ are involved. The construction of three-manifold topological invariants [A41] in topological quantum field theories provides concrete ideas about how to proceed.

3. The S-matrix element describing a particular transition can be expressed as any path leading from the sector A to B or vice versa. There is a huge symmetry very much analogous to the independence of the final result of the analytic continuation on the path chosen since generalized Feynman graphs allow all moves changing intermediate topologies so that initial and final 3-topologies are same. Generalized conformal invariance probably also poses restrictions on possible paths of continuation. In the path integral approach one would have simply sum over all these equivalent paths and thus encounter the fundamental difficulties related to the infinite-dimensional integration.

4. Quantum classical correspondence suggests that the continuation operation has a space-time correlate. That is, the preferred extremal of Kähler action going through the initial and final 3-sheets defines a sequence of transitions changing the topology of 3-sheet. The localization to a particular sector of course selects particular preferred extremal. There are two possible interpretations. Either the continuation from A is not possible to all possible sectors but only to those with 3-topologies appearing in X^4, or the preferred extremal represents some kind of minimal continuation involving minimum amount of calculational labor.

5. Quantum classical correspondence and the possibility to represent the rows of S-matrix as zero energy quantum states suggests that the paths for continuation can be also represented at the space-time level, perhaps in terms of braided flux tubes connecting two light like 3-surfaces representing the initial and final states of particle reaction. Since light like 3-surfaces are metrically two-dimensional and allow conformal invariance, this suggests a connection with braid diagrams in the sense that it should be possible to regard the paths connecting sectors of WCW consisting of unions of disjoint 3-surfaces (corresponding interacting 4-surfaces are connected) as generalized braids for which also decay and fusion for the strands of braid are possible. Quantum algebra structure and effective metric 2-dimensionality of the light like 3-surfaces suggests different braidings for flux tubes connecting boundaries of 3-surfaces define non-equivalent 3-surfaces.

Co-multiplication and second quantized induced spinor fields

At the microscopic level the construction of S-matrix reduces to understanding what happens for the classical spinor fields in a vertex, which corresponds to an incoming 3-surface A decaying to two outgoing 3-surfaces B and C. At the classical level incoming spinor field A develops into a spinor fields B and C expressible as linear combinations of appropriate spinor basis. At quantum level one must understand how the Fock space defined by the incoming spinor fields of A is mapped to the tensor product of Fock spaces of B and C. The idea about the possible importance of co-algebras came with the realization that this mapping is obviously is very much like a co-product. Co-algebras and bi-algebras possessing both algebra and co-algebra structure indeed suggest a general approach giving hopes of understanding how Feynman diagrammatics generalizes to TGD framework.

The first guess is that fermionic oscillator operators are mapped by the embedding Δ to a superposition of operators $a^{\dagger}_{Bn}\otimes Id_C$ and $Id_B\otimes a^{\dagger}_{Cn}$ with obvious formulas for Hermitian conjugates. Δ induces the mapping of higher Fock states and the construction of S-matrix should reduce to the construction of this map.

Δ is analogous to the definition for co-product operation although there is also an obvious difference due to the fact that Δ imbeds algebra A to $B \otimes C$ rather than to $A \otimes A$. Only in the case that the algebras are isomorphic, the situation reduces to that for Hopf algebras. Category theoretical approach however allows to consider a more general situation in which Δ is a morphism in the category of Fock algebras associated with 3-surfaces.

Δ preserves fermion number and should respect Fock algebra structure, in particular commute with the anti-commutation relations of fermionic oscillator operators. The basis of fermionic oscillator operators would naturally correspond to fermionic super-canonical generators in turn defining WCW gamma matrices.

Since any leg can be regarded as incoming leg, strong consistency conditions result on the coefficients in the expression

$$\Delta(a^{\dagger}_{An}) = C(A,B)_n{}^m a^{\dagger}_{Bm} \otimes Id_C + C(A,C)_n{}^m Id_B \otimes a^{\dagger}_{Cm} \quad \text{(A-3.3)}$$

by forming the cyclic permutations in A,B,C. This option corresponds to the co-commutative situation and quantum group structure. If identity matrices are replaced with something more general, co-product becomes non-cocommutative.

A-3.4 Ribbon Category As A Fundamental Structure?

There exists a generalization of the braided tensor category inspired by the axiomatic approach to topological quantum field theories which seems to almost catch the proposed mathematical requirements. This category is also called ribbon [A3] [A92] but in more general sense than it is defined in [A29].

One adds to the tangle diagrams (braid diagrams with both directions of strands and possibility of strand–anti-strand annihilation) also "coupons", which are boxes representing morphisms with arbitrary numbers of incoming and outgoing strands. As a special case 3-particle vertices are obtained. The strands correspond to representations of a fixed Hopf algebra H.

In the recent case it would seem safest to postulate that strands correspond to algebras, which can be different because of the potential dependence of the details of Fock algebra on 3-topology and other properties of 3-surface. For instance, WCW metric defined by anti-commutators of the gamma matrices is degenerate for vacuum extremals so that the infinite Clifford algebra is definitely "smaller" than for surfaces with $D \geq 3$-dimensional CP_2 projection.

One might feel that the full ribbon algebra is an un-necessary luxury since only 3-particle vertices are needed since higher vertices describing decays of 3-surfaces can be decomposed to 3-vertices in the generic case. On the other hand, many-sheeted space-time and p-adic fractality suggest that coupons with arbitrary number of incoming and outgoing strands are needed in order to obtain the p-adic hierarchy of length scale dependent theories.

The situation would be the same as in the effective quantum field theories involving arbitrarily high vertices and would require what might be called universal algebra allowing n-ary multiplications and co-multiplications rather than only binary ones. Also strands within strands hierarchy is strongly suggestive and would require a fractal generalization of the ribbon algebra. Note that associativity and commutativity conditions for morphisms which more than three incoming and outgoing lines would force to generalize the notion of R-matrix and would bring in conditions stating that more complex loop diagrams are equivalent with tree diagrams.

A-3.5 Minimal Models And TGD

Quaternion conformal invariance with non-vanishing c and k for anyons is highly attractive option and minimal super-conformal field theories attractive candidate since they describe critical systems and TGD Universe is indeed a quantum critical system.

Rational conformal field theories and TGD

The highest weight representations of Virasoro algebra are known as Verma modules containing besides the ground state with conformal weight Δ the states generated by Virasoro generators L_n, $n \geq 0$. For some values of Δ Verma module contains states with conformal weight $\Delta + l$ annihilated by Virasoro generators L_n, $n \geq 1$. In this case the number of primary fields is reduced since Virasoro algebra acts as a gauge algebra. The conformal weights Δ of the Verma modules allowing null states are given by the Kac formula

$$\begin{aligned} \Delta_{mm'} &= \Delta_0 + \frac{1}{4}(\alpha_+ m + \alpha_- m')^2 \ , \ m,m' \in \{1,2...\} \ , && \text{(A-3.4)} \\ \Delta_0 &= \frac{1}{24}(c-1) \ , \\ \alpha_\pm &= \frac{\sqrt{1-c} \pm \sqrt{25-c}}{\sqrt{24}} \ . && \text{(A-3.5)} \end{aligned}$$

The descendants $\prod_{n\geq 1} L_n^{k_n}|\Delta\rangle$ annihilated by L_n, $n > 0$, have conformal weights at level $l = \sum_n nk_n = mm'$.

In the general case the operator products of primary fields satisfying these conditions form an algebra spanned by infinitely many primary fields. The situation changes if the central charge c satisfies the condition

$$c = 1 - \frac{6(p'-p)^2}{pp'} \ , \quad \text{(A-3.6)}$$

where p and p' are mutually prime positive integers satisfying $p < p'$. In this case the Kac weights are rational

$$\Delta_{m,m'} = \frac{(mp' - m'p)^2 - (p'-p)^2)}{4pp'} \ , \ 0 < m < p \ , \ 0 < m' < p' \ . \quad \text{(A-3.7)}$$

Obviously, the number of primary fields is finite. This option does not seem to be realistic in TGD framework were super-conformal invariance is realized.

For $N = 1$ super-conformal invariance the unitary representations have central extension and conformal weights given by

$$\begin{aligned} c &= \frac{3}{2}(1 - \frac{8}{m(m+2)}) \ , \\ \Delta_{p,q}(NS) &= \frac{\left[(m+2)p) - mq)\right]^2 - 4}{8m(m+2)} \ , \ 0 \leq p \leq m \ , \ 1 \leq q \leq m+2 \ . \end{aligned} \quad \text{(A-3.8)}$$

For Ramond representations the conformal weights are

$$\Delta_{p,q}(R) = \Delta(NS) + \frac{1}{16} \ . \quad \text{(A-3.9)}$$

The states with vanishing conformal weights correspond to light elementary particles and the states with $p = q$ have vanishing conformal weight in NS sector. Also this option is non-realistic since in TGD framework super-generators carry fermion number so that G cannot be a Hermitian operator.

$N = 2$ super-conformal algebra is the most interesting one from TGD point of view since it involves also a bosonic $U(1)$ charge identifiable as fermion number and $G^{\pm}(z)$ indeed carry $U(1)$ charge[1]. Hence one has $N = 2$ super-conformal algebra is generated by the energy momentum tensor $T(z)$, $U(1)$ current $J(z)$, and super generators $G^{\pm}(z)$. $U(1)$ current would correspond to fermion number and super generators would involve contraction of covariantly constant neutrino spinor with second quantized induced spinor field. The further facts that $N = 2$ algebra is associated naturally with Kähler geometry, that the partition functions associated with $N = 2$ super-conformal representations are modular invariant, and that $N = 2$ algebra defines so called

[1]I realized that TGD super-conformal algebra corresponds to $N = 2$ algebra while writing this and proposed it earlier as a generalization of super-conformal algebra!

chiral ring defining a topological quantum field theory [A51], lend further support for the belief that $N = 2$ super-conformal algebra acts in super-canonical degrees of freedom.

The values of c and conformal weights for $N = 2$ super-conformal field theories are given by

$$\begin{aligned} c &= \frac{3k}{k+2} \ , \\ \Delta_{l,m}(NS) &= \frac{l(l+2)-m^2}{4(k+2)} \ , \ l = 0,1,...,k \ , \\ q_m &= \frac{m}{k+2} \ , \ m = -l,-l+2,....,l-2,l \ . \end{aligned} \tag{A-3.10}$$

q_m is the fractional value of the $U(1)$ charge, which would now correspond to a fractional fermion number. For $k = 1$ one would have $q = 0, 1/3, -1/3$, which brings in mind anyons. $\Delta_{l=0,m=0} = 0$ state would correspond to a massless state with a vanishing fermion number. Note that $SU(2)_k$ Wess-Zumino model has the same value of c but different conformal weights. More information about conformal algebras can be found from the appendix of [A51].

For Ramond representation $L_0 - c/24$ or equivalently G_0 must annihilate the massless states. This occurs for $\Delta = c/24$ giving the condition $k = 2\left[l(l+2) - m^2\right]$ (note that k must be even and that $(k,l,m) = (4,1,1)$ is the simplest non-trivial solution to the condition). Note the appearance of a fractional vacuum fermion number $q_{vac} = \pm c/12 = \pm k/4(k+2)$. I have proposed that NS and Ramond algebras could combine to a larger algebra containing also lepto-quark type generators.

Quaternion conformal invariance [K126] encourages to consider the possibility of super-symmetrizing also spin and electro-weak spin of fermions. In this case the conformal algebra would extend to a direct sum of Ramond and NS $N = 8$ algebras associated with quarks and leptons. This algebra in turn extends to a larger algebra if lepto-quark generators acting as half odd-integer Virasoro generators are allowed. The algebra would contain spin and electro-weak spin as fermionic indices. Poincare and color Kac-Moody generators would act as symplectically extended isometry generators on WCW Hamiltonians expressible in terms of Hamiltonians of $X_l^3 \times CP_2$. Electro-weak and color Kac-Moody currents have conformal weight $h = 1$ whereas T and G have conformal weights $h = 2$ and $h = 3/2$.

The experience with $N = 4$ super-conformal invariance suggests that the extended algebra requires the inclusion of also second quantized induced spinor fields with $h = 1/2$ and their super-partners with $h = 0$ and realized as fermion-anti-fermion bilinears. Since G and Ψ are labelled by 2×4 spinor indices, super-partners would correspond to $2\times(3+1) = 8$ massless electro-weak gauge boson states with polarization included. Their inclusion would make the theory highly predictive since induced spinor and electro-weak fields are the fundamental fields in TGD.

In TGD framework both quark and lepton numbers correspond to NS and Ramond type representations, which in conformal field theories can be assigned to the topologies of complex plane and cylinder. This would suggest that a given 3-surface allows either NS or Ramond representation and is either leptonic or quark like but one must be very cautious with this kind of conclusion. Interestingly, NS and Ramond type representations allow a symmetry acting as a spectral flow in the indices of the generators and transforming NS and Ramond type representations continuously to each other [A51]. The flow acts as

$$\begin{aligned} L_n &\to L_n + \alpha J_n + \frac{c}{6}\alpha^2 \delta_{n,0} \\ J_n &\to J_n + \frac{c}{3}\alpha\delta_{n,0} \ , \\ G_n^{\pm} &\to G_{n\pm\alpha}^{\pm} \ . \end{aligned} \tag{A-3.11}$$

The choice $\alpha = \pm 1/2$ transforms NS representation to Ramond representation. The idea that leptons could be transformed to quarks in a continuous manner does not sound attractive in TGD framework. Note that the action of Super Kac-Moody Virasoro algebra in the space of super-canonical conformal weights can be interpreted as a spectral flow.

Co-product for Super Kac-Moody and Super Virasoro algebras

By the previous considerations the quantized conformal weights z_1, z_2, z_3 of super-canonical generators defining punctures of 2-surface should correspond to line punctures of 3-surface. One cannot avoid the thought that these line punctures should meet at single point so that three-vertex would have also quantum field theoretical interpretation.

Each point z_k corresponds to its own Virasoro algebra $V_k = \{L_n^{z_k)}\}$ and Kac-Moody algebra $J_k = \{J_n^{z_k)}\}$ defined by Laurent series of $T(z)$ and $J(z)$ at z_k. Also super-generators are involved. To minimize notational labor denote by $X_n^{z_k)}$, $k = 1, 2, 3$ the generators in question.

The co-algebra product for Super-Virasoro and Super-Kac-Moody involves in the case of fusion $A_1 \otimes A_2 \to A_3$ a co-algebra product assigning to the generators $X_n^{z_3)}$ direct sum of generators of $X_k^{z_1)}$ and $X_l^{z_2)}$. The most straightforward approach is to express the generators $X_n^{z_3)}$ in terms of generators $X_k^{z_1)}$ and $X_l^{z_2)}$. This is achieved by using the expressions for generators as residy integrals of energy momentum tensor and Kac Moody currents. For Virasoro generators this is carried out explicitly in [A51]. The resulting co-product conserves the value of central extension whereas for the naïve co-product this would not be the case. Obviously, the geometric co-product does not conserve conformal weight.

A-4 Some Examples Of Bi-Algebras And Quantum Groups

The appendix summarizes briefly the simplest bi- and Hopf algebras and some basic constructions related to quantum groups.

A-4.1 Hecke Algebra And Temperley-Lieb Algebra

Braid group is accompanied by several algebras. For Hecke algebra, which is particular case of braid algebra, one has

$$\begin{aligned} e_{n+1}e_ne_{n+1} &= e_ne_{n+1}e_n \ , \\ e_n^2 &= (t-1)e_n + t \ . \end{aligned} \tag{A-4.1}$$

The algebra reduces to that for symmetric group for $t = 1$.

Hecke algebra can be regarded as a discrete analog of Kac Moody algebra or loop algebra with G replaced by S_n. This suggests a connection with Kac-Moody algebras and embedding of Galois groups to Kac-Moody group. $t = p^n$ corresponds to a finite field. Fractal dimension $t = \mathcal{M} : \mathcal{N}$ relates naturally to braid group representations: fractal dimension of quantum quaternions might be appropriate interpretation. t=1 gives symmetric group. Infinite braid group could be seen as a quantum variant of Galois group for algebraic closure of rationals.

Temperley-Lieb algebra assignable with Jones inclusions of hyper-finite factors of type II_1 with $\mathcal{M} : \mathcal{N} < 4$ is given by the relations

$$\begin{aligned} e_{n+1}e_nen+1 &= e_{n+1} \\ e_ne_{n+1}e_n &= e_n \ , \\ e_n^2 &= te_n \ , \ \ , t = -\sqrt{\mathcal{M}:\mathcal{N}} = -2cos(\pi/n) \ \ , n = 3,4,... \end{aligned} \tag{A-4.2}$$

The conditions involving three generators differ from those for braid group algebra since e_n are now proportional to projection operators. An alternative form of this algebra is given by

$$\begin{aligned} e_{n+1}e_nen+1 &= te_{n+1} \\ e_ne_{n+1}e_n &= te_n \ , \\ e_n^2 &= e_n = e_n^* \ , \ \ , t = -\sqrt{\mathcal{M}:\mathcal{N}} = -2cos(\pi/n) \ \ , n = 3,4,... \end{aligned} \tag{A-4.3}$$

This representation reduces to that for Temperley-Lieb algebra with obvious normalization of projection operators. These algebras are somewhat analogous to function fields but the value of coordinate is fixed to some particular values. An analogous discretization for function fields corresponds to a formation of number theoretical braids.

A-4.2 Simplest Bi-Algebras

Let $k(x_1,..,x_n)$ denote the free algebra of polynomials in variables x_i with coefficients in field k. x_i can be regarded as points of a set. The algebra $Hom(k(x_1,...,x_n),A)$ of algebra homomorphisms $k(x_1,...,x_n)\rightarrow A$ can be identified as A^n since by the homomorphism property the images $f(x_i)$ of the generators $x_1,...x_n$ determined the homomorphism completely. Any commutative algebra A can be identified as the $Hom(k[x],A)$ with a particular homomorphism corresponding to a line in A determined uniquely by an element of A.

The matrix algebra $M(2)$ can be defined as the polynomial algebra $k(a,b,c,d)$. Matrix multiplication can be represented universally as an algebra morphism Δ from from $M_2=k(a,b,c,d)$ to $M_2^{\otimes 2}=k(a',a'',b',b'',c',c'',d',d'')$ to $k(a,b,c,d)$ in matrix form as

$$\Delta\left(\begin{array}{cc} a & b \\ c & d \end{array}\right)=\left(\begin{array}{cc} a' & b' \\ c' & d' \end{array}\right)\left(\begin{array}{cc} a'' & b'' \\ c'' & d'' \end{array}\right) \ .$$

This morphism induces algebra multiplication in the matrix algebra $M_2(A)$ for any commutative algebra A.

$M(2)$, $GL_e(2)$ and $SL_e(2)$ provide standard examples about bi-algebras. $SL_e(2)$ can be defined as a commutative algebra by dividing free polynomial algebra $k(a,b,c,d)$ spanned by the generators a,b,c,d by the ideal $det-1=ad-bc-1=0$ expressing that the determinant of the matrix is one. In the matrix representation μ and η are defined in obvious manner and μ gives powers of the matrix

$$A=\left(\begin{array}{cc} a & b \\ c & d \end{array}\right) \ .$$

Δ, counit ϵ, and antipode S can be written in case of $SL_e(2)$ as

$$\left(\begin{array}{cc} \Delta(a) & \Delta(b) \\ \Delta(c) & \Delta(d) \end{array}\right)=\left(\begin{array}{cc} a & b \\ c & d \end{array}\right)\otimes\left(\begin{array}{cc} a & b \\ c & d \end{array}\right) \ ,$$

$$\left(\begin{array}{cc} \epsilon(a) & \epsilon(b) \\ \epsilon(c) & \epsilon(d) \end{array}\right)=\left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right) \ .$$

$$S\left(\begin{array}{cc} a & b \\ c & d \end{array}\right)=(ad-bc)^{-1}\left(\begin{array}{cc} d & -b \\ -c & a \end{array}\right) \ .$$

Note that matrix representation is only an economical manner to summarize the action of Δ on the generators a,b,c,d of the algebra. For instance, one has $\Delta(a)=a\rightarrow a\otimes a+b\otimes c$. The resulting algebra is both commutative and co-commutative.

$SL_e(2)_q$ can be defined as a Hopf algebra by dividing the free algebra generated by elements a,b,c,d by the relations

$$\begin{array}{cc} ba=qab \ , & db=qbd \ , \\ ca=qac \ , & dc=QCD \ , \\ bc=cb \ , & ad-da=(q^{-1}-1)bc \ , \end{array}$$

and the relation

$$det_q=ad-q^{-1}bc=1$$

stating that the quantum determinant of $SL_e(2)_q$ matrix is one.

μ,η,Δ,ϵ are defined as in the case of $SL_e(2)$. Antipode S is defined by

$$S\left(\begin{array}{cc} a & b \\ c & d \end{array}\right)=det_q^{-1}\left(\begin{array}{cc} d & -qb \\ -q^{-1}c & a \end{array}\right) \ .$$

The relations above guarantee that it defines quantum inverse of A. For q an n^{th} root of unity, $S^{2n} = id$ holds true which signals that these parameter values are somehow exceptional. This result is completely general.

Given an algebra, the R point of $SL_q(2)$ is defined as a four-tuple (A, B, C, D) in R^4 satisfying the relations defining the point of $SL_q(2)$. One can say that R-points provide representations of the universal quantum algebra $SL_q(2)$.

A-4.3 Quantum Group $U_Q(Sl(2))$

Quantum group $U_q(sl(2))$ or rather, quantum enveloping algebra of $sl(2)$, can be constructed by applying Drinfeld's quantum double construction (to avoid confusion note that the quantum Hopf algebra associated with $SL_e(2)$ is the quantum analog of a commutative algebra generated by powers of a 2×2 matrix of unit determinant).

The commutation relations of $sl(2)$ read as

$$[X_+, X_-] = H \ , \quad [H, X_\pm] = \pm 2X_\pm \ . \tag{A-4.4}$$

$U_q(sl(2))$ allows co-algebra structure given by

$$\begin{aligned} &\Delta(J) = J \otimes 1 + 1 \otimes J \ , \quad S(J) = -J \ , \quad \epsilon(J) = 0 \ , \quad J = X_\pm, H \ , \\ &S(1) = 1 \ , \qquad\qquad\qquad \epsilon(1) = 1 \ . \end{aligned} \tag{A-4.5}$$

The enveloping algebras of Borel algebras $U(B_\pm)$ generated by $\{1, X_+, H\}$ $\{1, X_-, hH\}$ define the Hopf algebra H and its dual $H^\star$ in Drinfeld's construction. h could be called Planck's constant vanishes at the classical limit. Note that $H^\star$ reduces to $\{1, X_-\}$ at this limit. Quantum deformation parameter q is given by $exp(2h)$. The duality map $\star : H \to H^\star$ reads as

$$\begin{aligned} &a \to a^\star \ , \quad ab = (ab)^\star = b^\star a^\star \ , \\ &1 \to 1 \ , \quad H \to H^\star = hH \ , \quad X_+ \to (X_+)^\star = hX_- \ . \end{aligned} \tag{A-4.6}$$

The commutation relations of $U_q(sl(2)$ read as

$$[X_+, X_-] = \frac{q^H - q^{-H}}{q - q^{-1}} \ , \quad [H, X_\pm] = \pm 2X_\pm \ . \tag{A-4.7}$$

Co-product Δ, antipode S, and co-unit ϵ differ from those $U(sl(2))$ only in the case of $X_\pm$:

$$\begin{aligned} &\Delta(X_\pm) = X_\pm \otimes q^{H/2} + q^{-H/2} \otimes X_\pm \ , \\ &S(X_\pm) = -q^{\pm 1} X_\pm \ . \end{aligned} \tag{A-4.8}$$

When q is not a root of unity, the universal R-matrix is given by

$$R = q^{\frac{H \otimes H}{2}} \sum_{n=0}^{\infty} \frac{(1-q^{-2})^n}{[n]_q!} q^{\frac{n(1-n)}{2}} q^{\frac{nH}{2}} X_+^n \otimes q^{-\frac{nH}{2}} X_-^n \ . \tag{A-4.9}$$

When q is m: th root of unity the q-factorial $[n]_q!$ vanishes for $n \geq m$ and the expansion does not make sense.

For q not a root of unity the representation theory of quantum groups is essentially the same as of ordinary groups. When q is m^{th} root of unity, the situation changes. For $l = m = 2n$ n^{th} powers of generators span together with the Casimir operator a sub-algebra commuting with the whole algebra providing additional numbers characterizing the representations. For $l = m = 2n+1$ same happens for m^{th} powers of Lie-algebra generators. The generic representations are not fully reducible anymore. In the case of $U_q(sl(2))$ irreducibility occurs for spins $n < l$ only. Under certain conditions on q it is possible to decouple the higher representations from the theory. Physically the reduction of the number of representations to a finite number means a symmetry analogous to a gauge symmetry. The phenomenon resembles the occurrence of null vectors in the case of Virasoro and Kac Moody representations and there indeed is a deep connection between quantum groups and Kac-Moody algebras [A51].

One can wonder what is the precise relationship between $U_q(sl(2)$ and $SL_q(2)$ which both are quantum groups using loose terminology. The relationship is duality. This means the existence of a morphism $x \rightarrow \Psi(x)\ M_q(2) \rightarrow U_q^\star$ defined by a bilinear form $\langle u, x\rangle = \Psi(x)(u)$ on $U_q \times M_q(2)$, which is bi-algebra morphism. This means that the conditions

$$\langle uv, x\rangle = \langle u\otimes v, \Delta(x)\rangle \ , \quad \langle u, xy\rangle = \langle \Delta(u), x\otimes y\rangle \ ,$$
$$\langle 1, x\rangle = \epsilon(x) \ , \qquad\qquad \langle u, 1\rangle = \epsilon(u)$$

are satisfied. It is enough to find $\Psi(x)$ for the generators $x = A, B, C, D$ of $M_q(2)$ and show that the duality conditions are satisfied. The representation

$$\rho(E) = \left(\begin{array}{cc} 0 & 1 \\ 0 & 0 \end{array}\right) \ , \quad \rho(F) = \left(\begin{array}{cc} 0 & 0 \\ 1 & 0 \end{array}\right) \ , \quad \rho(K = q^H) = \left(\begin{array}{cc} q & 0 \\ 0 & q^{-1} \end{array}\right) \ ,$$

extended to a representation

$$\rho(u) = \left(\begin{array}{cc} A(u) & B(u) \\ C(u) & D(u) \end{array}\right)$$

of arbitrary element u of $U_q(sl(2)$ defines for elements in $U_q^\star$. It is easy to guess that $A(u), B(u), C(u), D(u)$, which can be regarded as elements of $U_q^\star$, can be regarded also as R points that is images of the generators a, b, c, d of $SL_q(2)$ under an algebra morphism $SL_q(2) \rightarrow U_q^\star$.

A-4.4 General Semisimple Quantum Group

The Drinfeld's construction of quantum groups applies to arbitrary semi-simple Lie algebra and is discussed in detail in [A51]. The construction relies on the use of Cartan matrix.

Quite generally, Cartan matrix $A = \{a_{ij}\}$ is $n\times n$ matrix satisfying the following conditions:

1. A is indecomposable, that is does not reduce to a direct sum of matrices.

2. $a_{ij} \leq 0$ holds true for $i < j$.

3. $a_{ij} = 0$ is equivalent with $a_{ij} = 0$.

A can be normalized so that the diagonal components satisfy $a_{ii} = 2$.

The generators e_i, f_i, k_i satisfying the commutations relations

$$\begin{array}{ll} k_ik_j = k_jk_i \ , & k_ie_j = q_i^{a_{ij}}e_jk_i \ , \\ k_if_j = q_i^{-a_{ij}}e_jk_i \ , & e_if_j - f_je_i = \delta_{ij}\frac{k_i - k_i^{-1}}{q_i - q_i^{-1}} \ , \end{array} \qquad \text{(A-4.10)}$$

and so called Serre relations

$$\begin{array}{l} \sum_{l=0}^{1-a_{ij}}(-1)^l \left[\begin{array}{c} 1-a_{ij} \\ l \end{array}\right]_{q_i} e_i^{1-a_{ij}-l}e_je_i^l = 0, \ \ i\neq j \ , \\ \sum_{l=0}^{1-a_{ij}}(-1)^l \left[\begin{array}{c} 1-a_{ij} \\ l \end{array}\right]_{q_i} f_i^{1-a_{ij}-l}f_jf_i^l = 0 \ , \ \ i\neq j \ . \end{array} \qquad \text{(A-4.11)}$$

Here $q_i = q^{D_i}$ where one has $D_ia_{ij} = a_{ij}D_i$. $D_i = 1$ is the simplest choice in this case.

Comultiplication is given by

$$\begin{aligned} \Delta(k_i) &= k_i \otimes k_i \ , & \text{(A-4.12)}\\ \Delta(e_i) &= e_i \otimes k_i + 1 \otimes e_i \ , & \text{(A-4.13)}\\ \Delta(f_i) &= f_i \otimes 1 + k_i^{-1} \otimes 1 \ . & \text{(A-4.14)}\\ & & \text{(A-4.15)} \end{aligned}$$

The action of antipode S is defined as

$$S(e_i) = -e_i k_i^{-1} \ , \quad S(f_i) = -k_i f_i \ , \quad S(k_i) = -k_i^{-1} \ . \tag{A-4.16}$$

A-4.5 Quantum Affine Algebras

The construction of Drinfeld and Jimbo generalizes also to the case of untwisted affine Lie algebras, which are in one-one correspondence with semisimple Lie algebras. The representations of quantum deformed affine algebras define corresponding deformations of Kac-Moody algebras. In the following only the basic formulas are summarized and the reader not familiar with the formalism can consult a more detailed treatment can be found in [A51].

1. *Affine algebras*

The Cartan matrix A is said to be of affine type if the conditions $det(A) = 0$ and $a_{ij}a_{ji} \geq 4$ (no summation) hold true. There always exists a diagonal matrix D such that $B = DA$ is symmetric and defines symmetric bilinear degenerate metric on the affine Lie algebra.

The Dynkin diagrams of affine algebra of rank l have $l+1$ vertices (so that Cartan matrix has one null eigenvector). The diagrams of semisimple Lie-algebras are sub-diagrams of affine algebras. From the $(l+1) \times (l+1)$ Cartan matrix of an untwisted affine algebra $\hat{A}$ one can recover the $l \times l$ Cartan matrix of A by dropping away 0: th row and column.

For instance, the algebra A_1^1, which is affine counterpart of $SL_e(2)$, has Cartan matrix a_{ij}

$$A = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$$

with a vanishing determinant.

Quite generally, in untwisted case quantum algebra $U_q(\hat{G}_l)$ as $3(l+1)$ generators e_i, f_i, k_i $(i = 0, 1, .., l)$ satisfying the relations of Eq. A-4.11 for Cartan matrix of $\mathcal{G}^{(1)}$. Affine quantum group is obtained by adding to $U_q(\hat{\mathcal{G}}_l)$ a derivation d satisfying the relations

$$[d, e_i] = \delta_{i0} e_i \ , \quad [d, f_i] = \delta_{i0} f_i, \quad [d, k_i] = 0 \ . \tag{A-4.17}$$

with comultiplication $\Delta(d) = d \otimes 1 + 1 \otimes d$.

2. *Kac Moody algebras*

The undeformed extension $\hat{\mathcal{G}}_l$ associated with the affine Cartan matrix $\mathcal{G}_l^{(1)}$ is the Kac Moody algebra associated with the group G obtained as the central extension of the corresponding loop algebra. The loop algebra is defined as

$$L_e(\mathcal{G}) = \mathcal{G} \otimes C\left[t, t^{-1}\right] \ , \tag{A-4.18}$$

where $C\left[t, t^{-1}\right]$ is the algebra of Laurent polynomials with complex coefficients. The Lie bracket is

$$[x \times P, y \otimes Q] = [x, y] \otimes PQ \ . \tag{A-4.19}$$

The non-degenerate bilinear symmetric form $(,)$ in $\mathcal{G}_l$ induces corresponding form in $L_e(\mathcal{G}_l)$ as $(x \otimes P, y \otimes Q) = (x, y)PQ$.

A two-cocycle on $L_e(\mathcal{G}_l)$ is defined as

$$\Psi(a,b) = Res(\frac{da}{dt}, b) \ , \qquad \text{(A-4.20)}$$

where the residue of a Laurent is defined as $Res(\sum_n a_n t^n) = a_{-1}$. The two-cocycle satisfies the conditions

$$\begin{aligned} \Psi(a,b) &= -\Psi(b,a) \ , \\ \Psi([a,b],c) + \Psi([b,c],a) + \Psi([c,a],b) &= 0 \ . \end{aligned} \qquad \text{(A-4.21)}$$

The two-cocycle defines the central extension of loop algebra $L_e(\mathcal{G}_l)$ to Kac Moody algebra $L_e(\mathcal{G}_l)\otimes Cc$, where c is a new central element commuting with the loop algebra. The new bracket is defined as $[,]+\Psi(,)c$. The algebra $\tilde{L}(\mathcal{G}_l)$ is defined by adding the derivation d which acts as td/dt measuring the conformal weight.

The standard basis for Kac Moody algebra and corresponding commutation relations are given by

$$\begin{aligned} J^x_n &= x \otimes t^n \ , \\ [J^x_n, J^y_m] &= J^{[x,y]}_{n+m} + n\delta_{m+n,0}c \ . \end{aligned} \qquad \text{(A-4.22)}$$

The finite dimensional irreducible representations of G defined representations of Kac Moody algebra with a vanishing central extension $c = 0$. The highest weight representations are characterized by highest weight vector $|v\rangle$ such that

$$\begin{aligned} J^x_n|v\rangle &= 0, \ \ n > 0 \ , \\ c|v\rangle &= k|v\rangle \ . \end{aligned} \qquad \text{(A-4.23)}$$

3. Quantum affine algebras

Drinfeld has constructed the quantum affine extension $U_q(\mathcal{G}_l)$ using quantum double construction. The construction of generators uses almost the same basic formulas as the construction of semi-simple algebras. The construction involves the automorphism $D_t : U_q(\tilde{\mathcal{G}}_l) \otimes C\left[t, t^{-1}\right] \rightarrow U_q(\tilde{\mathcal{G}}_l) \otimes C\left[t, t^{-1}\right]$ given by

$$\begin{array}{ll} D_t(e_i) = t^{\delta_{i0}}e_i \ , & D_t(f_i) = t^{\delta_{i0}}f_i \ , \\ D_t(k_i) = k_i & D_t(d) = d \ , \end{array} \qquad \text{(A-4.24)}$$

and the co-product

$$\Delta_t(a) = (D_t \otimes 1)\Delta(a) \ , \quad \Delta^{op}_t(a) = (D_t \otimes 1)\Delta^{op}(a) \ , \qquad \text{(A-4.25)}$$

where the $\Delta(a)$ is the co-product defined by the same general formula as applying in the case of semi-simple Lie algebras. The universal R-matrix is given by

$$\mathcal{R}(t) = (D_t \otimes 1)\mathcal{R} \ , \qquad \text{(A-4.26)}$$

and satisfies the equations

$$\begin{aligned} &\mathcal{R}(t)\Delta_t(a) = \Delta^{op}_t(a)\mathcal{R} \ , \\ &(\Delta_z \otimes id)\mathcal{R}(u) = \mathcal{R}_{13}(zu)\mathcal{R}_{23}(u) \ , \\ &(id \otimes \Delta_u)\mathcal{R}(zu) = \mathcal{R}_{13}(z)\mathcal{R}_{12}(zu) \ , \\ &\mathcal{R}_{12}(t)\mathcal{R}_{13}(tw)\mathcal{R}_{23}(w) = \mathcal{R}_{23}(w)\mathcal{R}_{13}(tw)\mathcal{R}_{12}(t) \ . \end{aligned} \qquad \text{(A-4.27)}$$

The infinite-dimensional representations of affine algebra give representations of Kac-Moody algebra when one restricts the consideration to generations e_i, f_i, k_i, $i > 0$.

A-5 Introduction

Originally this appendix was meant to be a purely technical summary of basic facts but in its recent form it tries to briefly summarize those basic visions about TGD which I dare to regarded stabilized. I have added illustrations making it easier to build mental images about what is involved and represented briefly the key arguments. This chapter is hoped to help the reader to get fast grasp about the concepts of TGD.

The basic properties of embedding space and related spaces are discussed and the relationship of CP_2 to the standard model is summarized. The basic vision is simple: the geometry of the embedding space $H = M^4 \times CP_2$ geometrizes standard model symmetries and quantum numbers. The assumption that space-time surfaces are basic objects, brings in dynamics as dynamics of 3-D surfaces based on the induced geometry. Second quantization of free spinor fields of H induces quantization at the level of H, which means a dramatic simplification.

The notions of induction of metric and spinor connection, and of spinor structure are discussed. Many-sheeted space-time and related notions such as topological field quantization and the relationship many-sheeted space-time to that of GRT space-time are discussed as well as the recent view about induced spinor fields and the emergence of fermionic strings. Also the relationship to string models is discussed briefly.

Various topics related to p-adic numbers are summarized with a brief definition of p-adic manifold and the idea about generalization of the number concept by gluing real and p-adic number fields to a larger book like structure analogous to adele [L50, L49]. In the recent view of quantum TGD [L133], both notions reduce to physics as number theory vision, which relies on $M^8 - H$ duality [L100, L101] and is complementary to the physics as geometry vision.

Zero energy ontology (ZEO) [L91] [K130] has become a central part of quantum TGD and leads to a TGD inspired theory of consciousness as a generalization of quantum measurement theory having quantum biology as an application. Also these aspects of TGD are briefly discussed.

A-6 Embedding space $M^4 \times CP_2$

Space-times are regarded as 4-surfaces in $H = M^4 \times CP_2$ the Cartesian product of empty Minkowski space - the space-time of special relativity - and compact 4-D space CP_2 with size scale of order 10^4 Planck lengths. One can say that embedding space is obtained by replacing each point m of empty Minkowski space with 4-D tiny CP_2. The space-time of general relativity is replaced by a 4-D surface in H which has very complex topology. The notion of many-sheeted space-time gives an idea about what is involved.

Fig. 1. Embedding space $H = M^4 \times CP_2$ as Cartesian product of Minkowski space M^4 and complex projective space CP_2. `http://tgdtheory.fi/appfigures/Hoo.jpg`

Denote by M^4_+ and M^4_- the future and past directed lightcones of M^4. Denote their intersection, which is not unique, by CD. In zero energy ontology (ZEO) [L91, L119] [K130] causal diamond (CD) is defined as cartesian product $CD \times CP_2$. Often I use CD to refer just to $CD \times CP_2$ since CP_2 factor is relevant from the point of view of ZEO.

Fig. 2. Future and past light-cones M^4_+ and M^4_-. Causal diamonds (CD) are defined as their intersections. `http://tgdtheory.fi/appfigures/futurepast.jpg`

Fig. 3. Causal diamond (CD) is highly analogous to Penrose diagram but simpler. `http://tgdtheory.fi/appfigures/penrose.jpg`

A rather recent discovery was that CP_2 is the only compact 4-manifold with Euclidian signature of metric allowing twistor space with Kähler structure. M^4 is in turn is the only 4-D space with Minkowskian signature of metric allowing twistor space with Kähler structure [A57] so that $H = M^4 \times CP_2$ is twistorially unique.

One can loosely say that quantum states in a given sector of "world of classical worlds" (WCW) are superpositions of space-time surfaces inside CDs and that positive and negative energy parts of zero energy states are localized and past and future boundaries of CDs. CDs form a hierarchy. One can have CDs within CDs and CDs can also overlap. The size of CD is characterized by the proper time distance between its two tips. One can perform both translations and also Lorentz boosts of CD leaving either boundary invariant. Therefore one can assign to CDs a moduli space and speak about wave function in this moduli space.

In number theoretic approach it is natural to restrict the allowed Lorentz boosts to some discrete subgroup of Lorentz group and also the distances between the tips of CDs to multiples of CP_2 radius defined by the length of its geodesic. Therefore the moduli space of CDs discretizes. The quantization of cosmic recession velocities for which there are indications, could relate to this quantization.

A-6.1 Basic facts about CP_2

CP_2 as a four-manifold is very special. The following arguments demonstrate that it codes for the symmetries of standard models via its isometries and holonomies.

CP_2 as a manifold

CP_2, the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space C^3 under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) \ . \tag{A-6.1}$$

Here λ is any non-zero complex number. Note that CP_2 can be also regarded as the coset space $SU(3)/U(2)$. The pair z^i/z^j for fixed j and $z^i \neq 0$ defines a complex coordinate chart for CP_2. As j runs from 1 to 3 one obtains an atlas of three coordinate charts covering CP_2, the charts being holomorphically related to each other (e.g. CP_2 is a complex manifold). The points $z^3 \neq 0$ form a subset of CP_2 homoeomorphic to R^4 and the points with $z^3 = 0$ a set homeomorphic to S^2. Therefore CP_2 is obtained by "adding the 2-sphere at infinity to R^4".

Besides the standard complex coordinates $\xi^i = z^i/z^3$, $i = 1, 2$ the coordinates of Eguchi and Freund [A43] will be used and their relation to the complex coordinates is given by

$$\begin{aligned} \xi^1 &= z + it \ , \\ \xi^2 &= x + iy \ . \end{aligned} \tag{A-6.2}$$

These are related to the "spherical coordinates" via the equations

$$\begin{aligned} \xi^1 &= rexp(i\frac{(\Psi + \Phi)}{2})cos(\frac{\Theta}{2}) \ , \\ \xi^2 &= rexp(i\frac{(\Psi - \Phi)}{2})sin(\frac{\Theta}{2}) \ . \end{aligned} \tag{A-6.3}$$

The ranges of the variables r, Θ, Φ, Ψ are $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively.

Considered as a real four-manifold CP_2 is compact and simply connected, with Euler number Euler number 3, Pontryagin number 3 and second $b = 1$.

Fig. 4. CP_2 as manifold. `http://tgdtheory.fi/appfigures/cp2.jpg`

Metric and Kähler structure of CP_2

In order to obtain a natural metric for CP_2, observe that CP_2 can be thought of as a set of the orbits of the isometries $z^i \rightarrow exp(i\alpha)z^i$ on the sphere S^5: $\sum z^i\bar{z}^i = R^2$. The metric of CP_2 is obtained by projecting the metric of S^5 orthogonally to the orbits of the isometries. Therefore the distance between the points of CP_2 is that between the representative orbits on S^5.

The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}} d\xi^a d\bar{\xi}^b \ , \tag{A-6.4}$$

where the Hermitian, in fact Kähler metric $g_{a\bar{b}}$ is defined by

$$g_{a\bar{b}} = R^2 \partial_a \partial_{\bar{b}} K \ , \tag{A-6.5}$$

where the function K, Kähler function, is defined as

$$\begin{aligned} K &= log(F) \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-6.6}$$

The Kähler function for S^2 has the same form. It gives the S^2 metric $dzd\bar{z}/(1+r^2)^2$ related to its standard form in spherical coordinates by the coordinate transformation $(r,\phi) = (tan(\theta/2),\phi)$.

The representation of the CP_2 metric is deducible from S^5 metric is obtained by putting the angle coordinate of a geodesic sphere constant in it and is given

$$\frac{ds^2}{R^2} = \frac{(dr^2 + r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2 + \sigma_2^2)}{F} \ , \tag{A-6.7}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{A-6.8}$$

R denotes the radius of the geodesic circle of CP_2. The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2 \sum_A e_k^A e_l^A \ , \tag{A-6.9}$$

are given by

$$\begin{aligned} e^0 &= \frac{dr}{F} \ , \quad e^1 = \frac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 &= \frac{r\sigma_2}{\sqrt{F}} \ , \quad e^3 = \frac{r\sigma_3}{F} \ . \end{aligned} \tag{A-6.10}$$

The explicit representations of vierbein vectors are given by

$$\begin{aligned} e^0 &= \frac{dr}{F} \ , \quad e^1 = \frac{r(sin\Theta cos\Psi d\Phi + sin\Psi d\Theta)}{2\sqrt{F}} \ , \\ e^2 &= \frac{r(sin\Theta sin\Psi d\Phi - cos\Psi d\Theta)}{2\sqrt{F}} \ , \quad e^3 = \frac{r(d\Psi + cos\Theta d\Phi)}{2F} \ . \end{aligned} \tag{A-6.11}$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2) \ . \tag{A-6.12}$$

From this expression one finds that at coordinate infinity $r=\infty$ line element reduces to $\frac{r^2}{4F}(d\Theta^2+sin^2\Theta d\Phi^2)$ of S^2 meaning that 3-sphere degenerates metrically to 2-sphere and one can say that CP_2 is obtained by adding to R^4 a 2-sphere at infinity.

The vierbein connection satisfying the defining relation

$$de^A = -V^A_B \wedge e^B \ , \tag{A-6.13}$$

is given by

$$\begin{array}{llllll} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r-\frac{1}{r})e^3 \ , & V_{12} & = & (2r+\frac{1}{r})e^3 \ . \end{array} \tag{A-6.14}$$

The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{llllll} R_{01} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , & R_{23} & = & e^0\wedge e^1 - e^2\wedge e^3 \ , \\ R_{02} & = & e^0\wedge e^2 - e^3\wedge e^1 \ , & R_{31} & = & -e^0\wedge e^2 + e^3\wedge e^1 \ , \\ R_{03} & = & 4e^0\wedge e^3 + 2e^1\wedge e^2 \ , & R_{12} & = & 2e^0\wedge e^3 + 4e^1\wedge e^2 \ . \end{array} \tag{A-6.15}$$

Metric defines a real, covariantly constant, and therefore closed 2-form J

$$J = -is_{a\bar{b}}d\xi^a d\bar{\xi}^b \ , \tag{A-6.16}$$

the so called Kähler form. Kähler form J defines in CP_2 a symplectic structure because it satisfies the condition

$$J^k_{\ r}J^{rl} = -s^{kl} \ . \tag{A-6.17}$$

The condition states that J and g give representations of real unit and imaginary units related by the formula $i^2=-1$.

Kähler form is expressible locally in terms of Kähler gauge potential

$$J = dB \ , \tag{A-6.18}$$

where B is the so called Kähler potential, which is not defined globally since J describes homological magnetic monopole.

$dJ=ddB=0$ gives the topological half of Maxwell equations (vanishing of magnetic charges and Faraday's induction law) and self-duality $*J=J$ reduces the remaining equations to $dJ=0$. Hence the Kähler form can be regarded as a curvature form of a $U(1)$ gauge potential B carrying a magnetic charge of unit $1/2g$ (g denotes the gauge coupling).

The magnetic flux of J through a 2-surface in CP_2 is proportional to its homology equivalence class, which is integer valued. The explicit representations of J and B are given by

$$\begin{array}{lll} B & = & 2re^3 \ , \\ J & = & 2(e^0\wedge e^3 + e^1\wedge e^2) = \frac{r}{F^2}dr\wedge(d\Psi + cos\Theta d\Phi) + \frac{r^2}{2F}sin\Theta d\Theta\wedge d\Phi \ . \end{array} \tag{A-6.19}$$

The vierbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type (1, 1).

Useful coordinates for CP_2 are the so called canonical (or symplectic or Darboux) coordinates in which the Kähler potential and Kähler form have very simple expressions

$$
\begin{aligned}
B &= \sum_{k=1,2} P_k dQ_k \ , \\
J &= \sum_{k=1,2} dP_k \wedge dQ_k \ .
\end{aligned} \tag{A-6.20}
$$

The relationship of the canonical coordinates to the "spherical" coordinates is given by the equations

$$
\begin{aligned}
P_1 &= -\frac{1}{1+r^2} \ , \\
P_2 &= -\frac{r^2 cos\Theta}{2(1+r^2)} \ , \\
Q_1 &= \Psi \ , \\
Q_2 &= \Phi \ .
\end{aligned} \tag{A-6.21}
$$

Spinors In CP_2

CP_2 doesn't allow spinor structure in the conventional sense [A31]. However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of CP_2 play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space M. The parallel propagation around a closed curve with a base point x leads to a rotated vierbein at x: $e^A = R^A_B e^B$ and one can associate to each closed path an element of $SO(4)$.

Consider now a one-parameter family of closed curves $\gamma(v) : v \in (0,1)$ with the same base point x and $\gamma(0)$ and $\gamma(1)$ trivial paths. Clearly these paths define a sphere S^2 in M and the element $R^A_B(v)$ defines a closed path in $SO(4)$. When the sphere S^2 is contractible to a point e.g., homologically trivial, the path in $SO(4)$ is also contractible to a point and therefore represents a trivial element of the homotopy group $\Pi_1(SO(4)) = Z_2$.

For a homologically nontrivial 2-surface S^2 the associated path in $SO(4)$ can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group Spin(4) (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallel propagate also spinors and by the above construction associate a closed path of Spin(4) to the surface S^2. Now, however this path corresponds to a lift of the corresponding $SO(4)$ path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the non-allowed -1-factor associated with the parallel transport of the spinor around the sphere S^2 by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating -1-factor. For a $U(1)$ gauge potential this factor is given by the exponential $exp(i2\Phi)$, where Φ is the magnetic flux through the surface. This factor has the value -1 provided the $U(1)$ potential carries half odd multiple of Dirac charge $1/2g$. In case of CP_2 the required gauge potential is half odd multiple of the Kähler potential B defined previously. In the case of $M^4 \times CP_2$ one can in addition couple the spinor components with different chiralities independently to an odd multiple of $B/2$.

Geodesic sub-manifolds of CP_2

Geodesic sub-manifolds are defined as sub-manifolds having common geodesic lines with the embedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors h^k_α (understood as vectors of H) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to H and X^4.

In [A90] a general characterization of the geodesic sub-manifolds for an arbitrary symmetric space G/H is given. Geodesic sub-manifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra g of the group G. The Lie triple system t is defined as a subspace of g characterized by the closedness property with respect to double commutation

$$[X,[Y,Z]] \in t \ \text{ for } \ X,Y,Z \in t \ . \tag{A-6.22}$$

$SU(3)$ allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that $SU(3)$ allows two nonequivalent $SU(2)$ algebras corresponding to subgroups $SO(3)$ (orthogonal 3×3 matrices) and the usual isospin group $SU(2)$. By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic sub-manifold of CP_2.

Standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S^2_I \ : \ \xi^1 = \bar{\xi}^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S^2_{II} \ : \ \xi^1 = \xi^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Phi = 0) \ .$$

The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for S^2_I. S^2_{II} is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

A-6.2 CP_2 geometry and Standard Model symmetries

Identification of the electro-weak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B36] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \tag{A-6.23}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \otimes \gamma_5$, $1 \otimes \gamma_5$ and $\gamma_5 \otimes 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4)$ having as its covering group $SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{A-6.24}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$
\begin{array}{rclcrcl}
V_{01} & = & -\frac{e^1}{r} & , & V_{23} & = & \frac{e^1}{r} \ , \\
V_{02} & = & -\frac{e^2}{r} & , & V_{31} & = & \frac{e^2}{r} \ , \\
V_{03} & = & (r-\frac{1}{r})e^3 & , & V_{12} & = & (2r+\frac{1}{r})e^3 \ ,
\end{array}
\quad (A\text{-}6.25)
$$

and

$$B = 2re^3 \ , \quad (A\text{-}6.26)$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \quad (A\text{-}6.27)$$

where one have defined

$$
\begin{array}{rcl}
I_L^1 & = & \frac{(\Sigma_{01}-\Sigma_{23})}{2} \ , \\
I_L^2 & = & \frac{(\Sigma_{02}-\Sigma_{13})}{2} \ .
\end{array}
\quad (A\text{-}6.28)
$$

A_{ch} is clearly left handed so that one can perform the identification of the gauge potential as

$$W^{\pm} = \frac{2(e^1 \pm ie^2)}{r} \ , \quad (A\text{-}6.29)$$

where $W^{\pm}$ denotes the charged intermediate vector boson.

The covariantly constant curvature tensor is given by

$$
\begin{array}{l}
R_{01} = -R_{23} = e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\
R_{02} = -R_{31} = e^0 \wedge e^2 - e^3 \wedge e^1 \ , \\
R_{03} = 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ , \\
R_{12} = 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ .
\end{array}
\quad (A\text{-}6.30)
$$

The charged part of the curvature tensor is left handed.

This is to be compared with the Weyl tensor, which defines a representation of quaternionic imaginary units.

$$
\begin{array}{l}
W_{03} = W_{12} \equiv 2I_3 = 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\
W_{01} = W_{23} \equiv I_1 = -e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\
W_{02} = W_{31} \equiv I_2 = -e^0 \wedge e^2 - e^3 \wedge e^1 \ .
\end{array}
\quad (A\text{-}6.31)
$$

The charged part of the Weyl tensor is right-handed and that the relative sign of the two terms in the curvature tensor and Weyl tensor are opposite.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$
\begin{array}{rcl}
X & = & re^3 \ , \\
Y & = & \frac{e^3}{r} \ ,
\end{array}
\quad (A\text{-}6.32)
$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$\begin{aligned} \bar{\gamma} &= aX + bY \ , \\ \bar{Z}^0 &= cX + dY \ , \end{aligned} \quad \text{(A-6.33)}$$

where the normalization condition

$$ad - bc = 1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$\begin{aligned} A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\ &+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ . \end{aligned} \quad \text{(A-6.34)}$$

Identifying Σ_{12} and $\Sigma_{03} = 1 \times \gamma_5\Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \ . \quad \text{(A-6.35)}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \quad \text{(A-6.36)}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$\begin{aligned} Q_{em} &= \Sigma^{12} + \frac{(n_+1_+ + n_-1_-)}{6} \ , \\ I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ . \end{aligned} \quad \text{(A-6.37)}$$

The fields γ and Z^0 are defined via the relations

$$\begin{aligned} \gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX + bY) \ , \\ Z^0 &= 4(a+b)\bar{Z}^0 = 4(X - Y) \ . \end{aligned} \quad \text{(A-6.38)}$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} \ , \quad \text{(A-6.39)}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of the Weinberg angle is a dynamical problem. The original approach was based on the assumption that it makes sense to talk about electroweak action defined at fundamental level and introduce a symmetry breaking by adding an additional term proportional to Kähler action. The recent view is that Kähler action plus volume term defines the fundamental action.

The Weinberg angle is completely fixed if one requires that the electroweak action contains no cross term of type γZ^0. This leads to a definite value for the Weinberg angle.

One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle. As a matter fact, color gauge action identifying color gauge field as proportional to $H^A J_{\alpha\beta}$ is proportional to Kähler action. A possible interpretation would be as a sum of electroweak and color gauge interactions.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+ 1_+ + n_- 1_-) \ , \tag{A-6.40}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \end{aligned} \tag{A-6.41}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-6.42}$$

Evaluating the expressions above, one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{12} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{A-6.43}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \tag{A-6.44}$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned} L_{ew} &= L_{sym} + fJ^{\alpha\beta}J_{\alpha\beta} \ , \\ L_{sym} &= \frac{1}{4g^2}Tr(F^{\alpha\beta}F_{\alpha\beta}) \ , \end{aligned} \tag{A-6.45}$$

where the trace is taken in spinor representation, in terms of γ and Z^0 one obtains for the coefficient X of the γZ^0 cross term (this coefficient must vanish) the expression

$$\begin{aligned} X &= -\frac{K}{2g^2} + \frac{fp}{18} \ , \\ K &= Tr\left[Q_{em}(I_L^3 - sin^2\theta_W Q_{em})\right] \ , \end{aligned} \tag{A-6.46}$$

This parameter can be calculated by substituting the values of quark and lepton charges and weak isospins.

In the general case the value of the coefficient K is given by

$$K = \sum_i \left[-\frac{(18 + 2n_i^2)sin^2\theta_W}{9}\right] \ , \tag{A-6.47}$$

where the sum is over the spinor chiralities, which appear as elementary fermions and n_i is the integer describing the coupling of the spinor field to the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9\sum_i 1}{(fg^2 + 2\sum_i(18+n_i^2))} . \quad \text{(A-6.48)}$$

In the scenario where both leptons and quarks are elementary fermions the value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{9}{(\frac{fg^2}{2} + 28)} . \quad \text{(A-6.49)}$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is not far from the typical value 9/24 of GUTs at high energies [B3]. The experimental value at the scale length scale of the electron can be deduced from the ratio of W and Z boson masses as $sin^2\theta_W = 1 - (m_W/m_Z)^2 \simeq .22290$. This ratio and also the weak boson masses depend on the length scale.

If one interprets the additional term proportional to J as color action, one could perhaps interpret the value of Weinberg angle as expressing a connection between strong and weak coupling constant evolution. The limit $f \rightarrow 0$ should correspond to an infinite value of color coupling strength and at this limit one would have $sin^2\theta_W = \frac{9}{28}$ for $f/g^2 \rightarrow 0$. This does not make sense since the Weinberg angle is in the standard model much smaller in QCD scale Λ corresponding roughly to pion mass scale. The Weinberg angle is in principle predicted by the p-adic coupling constant evolution fixed by the number theoretical vision of TGD.

One could however have a sum of electroweak action, correction terms changing the value of Weinberg angle, and color action and coupling constant evolution could be understood in terms of the coupling parameters involved.

Electroweak symmetry breaking

One of the hardest challenges in the development of the TGD based view of weak symmetry breaking was the fact that classical field equations allow space-time surfaces with finite but arbitrarily large size. For a fixed space-time surface, the induced gauge fields, including classical weak fields, are long ranged. On the other hand, the large mass for weak bosons would require a short correlation length. How can one understand this together with the fact that a photon has a long correlation length?

In zero energy ontology quantum states are superpositions of space-time surfaces as analogs of almost unique Bohr orbits of particles identified as 3-D surfaces. For some reason the superposition should be such that the quantum averages of weak gauge boson fields vanish below the weak scale whereas the quantum average of electromagnetic fields is non-vanishing.

This is indeed the case.

1. The supersymplectic symmetries form isometries of the world of classical worlds (WCW) and they act in CP_2 degrees of freedom as symplectic transformations leaving the CP_2 symplectic form J invariant and therefore also its contribution to the electromagnetic field since this part is the same for all space-time surfaces in the superposition of space-time surfaces as a representation of supersymplectic isometry group (as a special case a representation of color group).

2. In TGD, color and electroweak symmetries acting as holonomies are not independent and for the $SU(2)_L$ part of induced spinor connection the symplectic transformations induces $SU(2)_L \times U(1)_R$ gauge transformation. This suggests that the quantum expectations of the induced weak fields over the space-time surfaces vanish above the quantum coherence scale. The averages of W and of the left handed part of Z^0 should therefore vanish.

3. $\langle Z^0 \rangle$ should vanish. For $U(1)_R$ part of Z^0, the action of gauge transformation is trivial in gauge theory. Now however the space-time surface changes under symplectic transformations

and this could make the average of the right-handed part of Z^0 vanishing. The vanishing of the average of the axial part of the Z^0 is suggested by the partially conserved axial current hypothesis.

One can formulate this picture quantitatively.

1. The electromagnetic field [L137] contains, besides the induced Kähler form, also the induced curvature form R_{12}, which couples vectorially. Conserved vector current hypothesis suggests that the average of R_{12} is non-vanishing. One can express the neutral part of the induced gauge field in terms of induced spinor curvature and Kähler form J as

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) = J + 2e^0 \wedge e^3 \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) = 3J - 2e^0 \wedge e^3 \ , \end{aligned} \tag{A-6.50}$$

2. The induced fields γ and Z^0 (photon and Z- boson) can be expressed as

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{12} \ , \\ Z^0 &= 2R_{03} = 2(J + 2e^0 \wedge e^3) \end{aligned} \tag{A-6.51}$$

$$per. \tag{A-6.52}$$

The condition $\langle Z^0 \rangle = 0$ gives $2\langle e^0 \wedge e^3 \rangle = -2J$ and this in turn gives $\langle R_{12} \rangle = 4J$. The average over γ would be

$$\langle \gamma \rangle = (3 - 4sin^2\theta_W)J \ .$$

For $sin^2\theta_W = 3/4 \ langle\gamma\rangle$ would vanish.

The quantum averages of classical weak fields quite generally vanish. What about correlation functions?

1. One expects that the correlators of classical weak fields as color invariants, and perhaps even symplectic invariants, are non-vanishing below the Compton length since in this kind of situation the points in the correlation function belong to the same 3-surface representing particle, such as hadron.

2. The intuitive picture is that in longer length scales one has disjoint 3-surfaces with a size scale of Compton length. If the states associated with two disjoint 3-surfaces are separately color invariant there are no correlations in color degrees of freedom and correlators reduce to the products of expectations of classical weak fields and vanish. This could also hold when the 3-surfaces are connected by flux tube bonds.

   Below the Compton length weak bosons would thus behave as correlated massless fields. The Compton lengths of weak bosons are proportional to the value of effective Planck constant h_{eff} and in living systems the Compton lengths are proposed to be even of the order of cell size. This would explain the mysterious chiral selection in living systems requiring large parity violation.

3. What about the averages and correlators of color gauge fields? Classical color gauge fields are proportional to the products of Hamiltonians of color isometries induced Kähler form and the expectations of color Hamiltonians give vanishing average above Compton length and therefore vanishing average. Correlators are non-vanishing below the hadron scale. Gluons do not propagate in long scales for the same reason as weak bosons. This is implied by color confinement, which has also classical description in the sense that 3-surfaces have necessarily a finite size.

   A large value of h_{eff} allows colored states even in biological scales below the Compton length since in this kind of situation the points in the correlation function belong to the same 3-surface representing particle, such as dark hadron.

Discrete symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:

1. Symmetries must be realized as purely geometric transformations.

2. Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [B9] .

The action of the reflection P on spinors of is given by

$$\Psi \rightarrow P\Psi = \gamma^0 \otimes \gamma^0 \Psi \ . \tag{A-6.53}$$

in the representation of the gamma matrices for which γ^0 is diagonal. It should be noticed that W and Z^0 bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of P.

The guess that a complex conjugation in CP_2 is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$\begin{aligned} m^k &\rightarrow T(M^k) \ , \\ \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \gamma^1\gamma^3 \otimes 1\Psi \ . \end{aligned} \tag{A-6.54}$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in CP_2:

$$\begin{aligned} \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \Psi^\dagger\gamma^2\gamma^0 \otimes 1 \ . \end{aligned} \tag{A-6.55}$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

A-7 Induction procedure and many-sheeted space-time

Since the classical gauge fields are closely related in TGD framework, it is not possible to have space-time sheets carrying only single kind of gauge field. For instance, em fields are accompanied by Z^0 fields for extremals of Kähler action.

Classical em fields are always accompanied by Z^0 field and some components of color gauge field. For extremals having homologically non-trivial sphere as a CP_2 projection em and Z^0 fields are the only non-vanishing electroweak gauge fields. For homologically trivial sphere only W fields are non-vanishing. Color rotations does not affect the situation.

For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has U(1) holonomy by 2-dimensionality of the CP_2 projection. Color gauge field has $U(1)$ holonomy for all space-time surfaces and quantum classical correspondence suggest a weak form of color confinement meaning that physical states correspond to color neutral members of color multiplets.

A-7.1 Induction procedure for gauge fields and spinor connection

Induction procedure for gauge potentials and spinor structure is a standard procedure of bundle theory. If one has embedding of some manifold to the base space of a bundle, the bundle structure can be induced so that it has as a base space the imbedded manifold, whose points have as fiber the fiber if embedding space at their image points. In the recent case the embedding of space-time surface to embedding space defines the induction procedure. The induced gauge potentials and gauge fields are projections of the spinor connection of the embedding space to the space-time surface (see http://tgdtheory.fi/appfigures/induct.jpg).

Induction procedure makes sense also for the spinor fields of embedding space and one obtains geometrization of both electroweak gauge potentials and of spinors. The new element is induction of gamma matrices which gives their projections at space-time surface.

As a matter fact, the induced gamma matrices cannot appear in the counterpart of massless Dirac equation. To achieve super-symmetry, Dirac action must be replaced with Kähler-Dirac action for which gamma matrices are contractions of the canonical momentum currents of Kähler action with embedding space gamma matrices. Induced gamma matrices in Dirac action would correspond to 4-volume as action.

Fig. 9. Induction of spinor connection and metric as projection to the space-time surface. http://tgdtheory.fi/appfigures/induct.jpg.

A-7.2 Induced gauge fields for space-times for which CP_2 projection is a geodesic sphere

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional CP_2 projection, only vacuum extremals and space-time surfaces for which CP_2 projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing W fields and homologically non-trivial sphere to non-vanishing W fields but vanishing γ and Z^0. This can be verified by explicit examples.

$r = \infty$ surface gives rise to a homologically non-trivial geodesic sphere for which e_0 and e_3 vanish imply the vanishing of W field. For space-time sheets for which CP_2 projection is $r = \infty$ homologically non-trivial geodesic sphere of CP_2 one has

$$\gamma = (\frac{3}{4} - \frac{sin^2(\theta_W)}{2})Z^0 \simeq \frac{5Z^0}{8} \quad .$$

The induced W fields vanish in this case and they vanish also for all geodesic sphere obtained by $SU(3)$ rotation.

$Im(\xi^1) = Im(\xi^2) = 0$ corresponds to homologically trivial geodesic sphere. A more general representative is obtained by using for the phase angles of standard complex CP_2 coordinates constant values. In this case e^1 and e^3 vanish so that the induced em, Z^0, and Kähler fields vanish but induced W fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D CP_2 projection color rotations and weak symmetries commute.

A-7.3 Many-sheeted space-time

TGD space-time is many-sheeted: in other words, there are in general several space-sheets which have projection to the same M^4 region. Second manner to say this is that CP_2 coordinates are many-valued functions of M^4 coordinates. The original physical interpretation of many-sheeted space-time time was not correct: it was assumed that single sheet corresponds to GRT space-time and this obviously leads to difficulties since the induced gauge fields are expressible in terms of only four embedding space coordinates.

Fig. 10. Illustration of many-sheeted space-time of TGD. http://tgdtheory.fi/appfigures/manysheeted.jpg

Superposition of effects instead of superposition of fields

The first objection against TGD is that superposition is not possible for induced gauge fields and induced metric. The resolution of the problem is that it is effects which need to superpose, not the fields.

Test particle topologically condenses simultaneously to all space-time sheets having a projection to same region of M^4 (that is touches them). The superposition of effects of fields at various space-time sheets replaces the superposition of fields.This is crucial for the understanding also how GRT space-time relates to TGD space-time, which is also in the appendix of this book).

Wormhole contacts

Wormhole contacts are key element of many-sheeted space-time. One does not expect them to be stable unless there is non-trivial Kähler magnetic flux flowing through then so that the throats look like Kähler magnetic monopoles.

Fig. 11. Wormhole contact. `http://tgdtheory.fi/appfigures/wormholecontact.jpg`

Since the flow lines of Kähler magnetic field must be closed this requires the presence of another wormhole contact so that one obtains closed monopole flux tube decomposing to two Minkowskian pieces at the two space-time sheets involved and two wormhole contacts with Euclidian signature of the induced metric. These objects are identified as space-time correlates of elementary particles and are clearly analogous to string like objects.

The relationship between the many-sheeted space-time of TGD and of GRT space-time

The space-time of general relativity is single-sheeted and there is no need to regard it as surface in H although the assumption about representability as vacuum extremal gives very powerful constraints in cosmology and astrophysics and might make sense in simple situations.

The space-time of GRT can be regarded as a long length scale approximation obtained by lumping together the sheets of the many-sheeted space-time to a region of M^4 and providing it with an effective metric obtained as sum of M^4 metric and deviations of the induced metrics of various space-time sheets from M^4 metric. Also induced gauge potentials sum up in the similar manner so that also the gauge fields of gauge theories would not be fundamental fields.

Fig. 12. The superposition of fields is replaced with the superposition of their effects in many-sheeted space-time. `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg`

Space-time surfaces of TGD are considerably simpler objects that the space-times of general relativity and relate to GRT space-time like elementary particles to systems of condensed matter physics. Same can be said about fields since all fields are expressible in terms of embedding space coordinates and their gradients, and general coordinate invariance means that the number of bosonic field degrees is reduced locally to 4. TGD space-time can be said to be a microscopic description whereas GRT space-time a macroscopic description. In TGD complexity of space-time topology replaces the complexity due to large number of fields in quantum field theory.

Topological field quantization and the notion of magnetic body

Topological field quantization also TGD from Maxwell's theory. TGD predicts topological light rays ("massless extremals (MEs)") as space-time sheets carrying waves or arbitrary shape propagating with maximal signal velocity in single direction only and analogous to laser beams and carrying light-like gauge currents in the generi case. There are also magnetic flux quanta and electric flux quanta. The deformations of cosmic strings with 2-D string orbit as M^4 projection gives rise to magnetic flux tubes carrying monopole flux made possible by CP_2 topology allowing homological Kähler magnetic monopoles.

Fig. 13. Topological quantization for magnetic fields replaces magnetic fields with bundles of them defining flux tubes as topological field quanta. `http://tgdtheory.fi/appfigures/field.jpg`

The imbeddability condition for say magnetic field means that the region containing constant magnetic field splits into flux quanta, say tubes and sheets carrying constant magnetic field. Unless one assumes a separate boundary term in Kähler action, boundaries in the usual sense are forbidden except as ends of space-time surfaces at the boundaries of causal diamonds. One obtains typically pairs of sheets glued together along their boundaries giving rise to flux tubes with closed cross section possibly carrying monopole flux.

These kind of flux tubes might make possible magnetic fields in cosmic scales already during primordial period of cosmology since no currents are needed to generate these magnetic fields: cosmic string would be indeed this kind of objects and would dominated during the primordial period. Even superconductors and maybe even ferromagnets could involve this kind of monopole flux tubes.

A-7.4 Embedding space spinors and induced spinors

One can geometrize also fermionic degrees of freedom by inducing the spinor structure of $M^4 \times CP_2$.

CP_2 does not allow spinor structure in the ordinary sense but one can couple the opposite H-chiralities of H-spinors to an $n = 1$ ($n = 3$) integer multiple of Kähler gauge potential to obtain a respectable modified spinor structure. The em charges of resulting spinors are fractional (integer valued) and the interpretation as quarks (leptons) makes sense since the couplings to the induced spinor connection having interpretation in terms electro-weak gauge potential are identical to those assumed in standard model.

The notion of quark color differs from that of standard model.

1. Spinors do not couple to color gauge potential although the identification of color gauge potential as projection of $SU(3)$ Killing vector fields is possible. This coupling must emerge only at the effective gauge theory limit of TGD.

2. Spinor harmonics of embedding space correspond to triality $t = 1$ ($t = 0$) partial waves. The detailed correspondence between color and electroweak quantum numbers is however not correct as such and the interpretation of spinor harmonics of embedding space is as representations for ground states of super-conformal representations. The wormhole pairs associated with physical quarks and leptons must carry also neutrino pair to neutralize weak quantum numbers above the length scale of flux tube (weak scale or Compton length). The total color quantum numbers or these states must be those of standard model. For instance, the color quantum numbers of fundamental left-hand neutrino and lepton can compensate each other for the physical lepton. For fundamental quark-lepton pair they could sum up to those of physical quark.

The well-definedness of em charge is crucial condition.

1. Although the embedding space spinor connection carries W gauge potentials one can say that the embedding space spinor modes have well-defined em charge. One expects that this is true for induced spinor fields inside wormhole contacts with 4-D CP_2 projection and Euclidian signature of the induced metric.

2. The situation is not the same for the modes of induced spinor fields inside Minkowskian region and one must require that the CP_2 projection of the regions carrying induced spinor field is such that the induced W fields and above weak scale also the induced Z^0 fields vanish in order to avoid large parity breaking effects. This condition forces the CP_2 projection to be 2-dimensional. For a generic Minkowskian space-time region this is achieved only if the spinor modes are localized at 2-D surfaces of space-time surface - string world sheets and possibly also partonic 2-surfaces.

3. Also the Kähler-Dirac gamma matrices appearing in the modified Dirac equation must vanish in the directions normal to the 2-D surface in order that Kähler-Dirac equation can be satisfied. This does not seem plausible for space-time regions with 4-D CP_2 projection.

4. One can thus say that strings emerge from TGD in Minkowskian space-time regions. In particular, elementary particles are accompanied by a pair of fermionic strings at the opposite space-time sheets and connecting wormhole contacts. Quite generally, fundamental fermions would propagate at the boundaries of string world sheets as massless particles and wormhole contacts would define the stringy vertices of generalized Feynman diagrams. One obtains geometrized diagrammatics, which brings looks like a combination of stringy and Feynman diagrammatics.

5. This is what happens in the the generic situation. Cosmic strings could serve as examples about surfaces with 2-D CP_2 projection and carrying only em fields and allowing delocalization of spinor modes to the entire space-time surfaces.

A-7.5 About induced gauge fields

In the following the induced gauge fields are studied for general space-time surface without assuming the preferred extremal property (Bohr orbit property). Therefore the following arguments are somewhat obsolete in their generality.

Space-times with vanishing em, Z^0, or Kähler fields

The following considerations apply to a more general situation in which the homologically trivial geodesic sphere and extremal property are not assumed. It must be emphasized that this case is possible in TGD framework only for a vanishing Kähler field.

Using spherical coordinates (r, Θ, Ψ, Φ) for CP_2, the expression of Kähler form reads as

$$
\begin{aligned}
J &= \frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta) d\Phi) + \frac{r^2}{2F} sin(\Theta) d\Theta \wedge d\Phi \ , \\
F &= 1 + r^2 \ .
\end{aligned}
\tag{A-7.1}
$$

The general expression of electromagnetic field reads as

$$
\begin{aligned}
F_{em} &= (3+2p)\frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta) d\Phi) + (3+p)\frac{r^2}{2F} sin(\Theta) d\Theta \wedge d\Phi \ , \\
p &= sin^2(\Theta_W) \ ,
\end{aligned}
\tag{A-7.2}
$$

where Θ_W denotes Weinberg angle.

1. The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$
\begin{aligned}
\Psi &= k\Phi \ , \\
(3+2p)\frac{1}{r^2 F}(d(r^2)/d\Theta)(k + cos(\Theta)) &+ (3+p) sin(\Theta) = 0 \ ,
\end{aligned}
\tag{A-7.3}
$$

hold true. The conditions imply that CP_2 projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$
\begin{aligned}
r &= \sqrt{\frac{X}{1-X}} \ , \\
X &= D\left[|\frac{k+u}{C}| \right]^{\epsilon} \ , \\
u &\equiv cos(\Theta) \ , \ C = k + cos(\Theta_0) \ , \ D = \frac{r_0^2}{1+r_0^2} \ , \ \epsilon = \frac{3+p}{3+2p} \ ,
\end{aligned}
\tag{A-7.4}
$$

where C and D are integration constants. $0 \leq X \leq 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \leq 1$ and $r = \infty$ to $X = 1$ giving $|u + k| = [(1 + r_0^2)/r_0^2)]^{(3+2p)/(3+p)}$ achieved only for

$$
sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\frac{3+2p}{3+p}} \leq k + 1 \ ,
$$

where $sign(x)$ denotes the sign of x.

The expressions for Kähler form and Z^0 field are given by

$$J = -\frac{p}{3+2p} X du \wedge d\Phi \ ,$$
$$Z^0 = -\frac{6}{p} J \ . \qquad \text{(A-7.5)}$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range Z^0 vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

2. The vanishing of Z^0 fields is achieved by the replacement of the parameter ϵ with $\epsilon = 1/2$ as becomes clear by considering the condition stating that Z^0 field vanishes identically. Also the relationship $F_{em} = 3J = -\frac{3}{4}\frac{r^2}{F} du \wedge d\Phi$ is useful.

3. The vanishing Kähler field corresponds to $\epsilon = 1, p = 0$ in the formula for em neutral space-times. In this case classical em and Z^0 fields are proportional to each other:

$$Z^0 = 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u} du \wedge d\Phi = (k+u) du \wedge d\Phi \ ,$$
$$r = \sqrt{\frac{X}{1-X}} \ , \quad X = D|k+u| \ ,$$
$$\gamma = -\frac{p}{2} Z^0 \ . \qquad \text{(A-7.6)}$$

For a vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible.

The effective form of CP_2 metric for surfaces with 2-dimensional CP_2 projection

The effective form of the CP_2 metric for a space-time having vanishing em,Z^0, or Kähler field is of practical value in the case of vacuum extremals and is given by

$$ds^2_{eff} = (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta} d\Theta^2 + s^{eff}_{\Phi\Phi} d\Phi^2] \ ,$$
$$s^{eff}_{\Theta\Theta} = X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ ,$$
$$s^{eff}_{\Phi\Phi} = X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ , \qquad \text{(A-7.7)}$$

and is useful in the construction of vacuum embedding of, say Schwartchild metric.

Topological quantum numbers

Space-times for which either em, Z^0, or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers (ω_1 and ω_2) are frequency type parameters, two (k_1 and k_2) are wave vector like quantum numbers, two of the quantum numbers (n_1 and n_2) are integers. The parameters ω_i and n_i will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of CP_2 coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$\begin{aligned} \Psi &= \omega_2 m^0 + k_2 m^3 + n_2\phi + \text{Fourier expansion} \ , \\ \Phi &= \omega_1 m^0 + k_1 m^3 + n_1\phi + \text{Fourier expansion} \ . \end{aligned} \quad \text{(A-7.8)}$$

m^0,m^3 and ϕ denote the coordinate variables of the cylindrical M^4 coordinates) so that one has $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$. The regions of the space-time surface with given values of the vacuum parameters ω_i,k_i and n_i and m and C are bounded by the surfaces at which space-time surface becomes ill-defined, say by $r > 0$ or $r < \infty$ surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters r_0 and Θ_0. At $r = \infty$ surfaces n_2,ω_2 and m can change since all values of Ψ correspond to the same point of CP_2: at $r = 0$ surfaces also n_1 and ω_1 can change since all values of Φ correspond to same point of CP_2, too. If $r = 0$ or $r = \infty$ is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global embedding for, say a constant magnetic field. Although global embedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate u in general possesses discontinuous derivative at $r = 0$ and $r = \infty$ surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteeing the absence of edges are satisfied.

For instance, the vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 \ , \quad \text{(A-7.9)}$$

is satisfied. In particular, the ratio ω_2/ω_1 is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter n_1 and n_2 (ω_1 and ω_2) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

A-8 The relationship of TGD to QFT and string models

The recent view of the relationship of TGD to QFT and string models has developed slowly during years and it seems that in a certain sense TGD means a return to roots: instead of QFT like description involving path integral one would have wave mechanics for 3-surfaces.

A-8.1 TGD as a generalization of wave mechanism obtained by replacing point-like particles with 3-surfaces

The first vision of TGD was as a generalization of quantum field theory (string models) obtained by replacing pointlike particles (strings) as fundamental objects with 3-surfaces.

The later work has revealed that TGD could be seen as a generalization of the wave mechanism based on the replacement of a point-like particle with 3-D surface. This is due to holography implied by general coordinate invariance. The definition of the metric of the "world of classical worlds" (WCW) must assign a unique or at least almost unique space-time surface to a given 3-surface. This 4-surface is analogous to Bohr orbit so that also Bohr orbitology becomes an exact part of quantum physics. The failure of strict determinism forces to replace 3-surfaces with 4-surfaces and this leads to zero energy ontology (ZEO) in which quantum states are superpositions of space-time surfaces [K57, K36, K95] [L122, L133].

Fig. 5. TGD replaces point-like particles with 3-surfaces. `http://tgdtheory.fi/appfigures/particletgd.jpg`

A-8.2 Extension of superconformal invariance

The fact that light-like 3-surfaces are effectively metrically 2-dimensional and thus possess generalization of 2-dimensional conformal symmetries with light-like radial coordinate defining the analog of second complex coordinate suggests that this generalization could work and extend the super-conformal symmetries to their 4-D analogs.

The boundary $\delta M^4_+ = S^2 \times R_+$- of 4-D light-cone M^4_+ is also metrically 2-dimensional and allows extended conformal invariance. Also the group of isometries of light-cone boundary and of light-like 3-surfaces is infinite-dimensional since the conformal scalings of S^2 can be compensated by S^2-local scaling of the light-like radial coordinate of R_+. These simple facts mean that 4-dimensional Minkowski space and 4-dimensional space-time surfaces are in a completely unique position as far as symmetries are considered.

In fact, this leads to a generalization of the Kac-Moody type symmetries of string models. $\delta M^4_+ \times CP_2$ allows huge supersymplectic symmetries for which the radial light-like coordinate of δM^4_+ plays the role of complex string coordinate in string models. These symmetries are assumed to act as isometries of WCW.

A-8.3 String-like objects and strings

String like objects obtained as deformations of cosmic strings $X^2 \times Y^2$, where X^2 is minimal surface in M^4 and Y^2 a holomorphic surface of CP_2 are fundamental extremals of Kähler action having string world sheet as M^4 projections. Cosmic strings dominate the primordial cosmology of the TGD Universe and the inflationary period corresponds to the transition to radiation dominated cosmology for which space-time sheets with 4-D M^4 projection dominate.

Also genuine string-like objects emerge from TGD. The conditions that the em charge of modes of induces spinor fields is well-defined requires in the generic case the localization of the modes at 2-D surfaces -string world sheets and possibly also partonic 2-surfaces. This in Minkowskian space-time regions.

Fig. 6. Well-definedness of em charge forces the localization of induced spinor modes to 2-D surfaces in generic situations in Minkowskian regions of space-time surface. `http://tgdtheory.fi/appfigures/fermistring.jpg`

A-8.4 TGD view of elementary particles

The TGD based view about elementary particles has two key aspects.

1. The space-time correlates of elementary particles are identified as pairs of wormhole contacts with Euclidean signature of metric and having 4-D CP_2 projection. Their throats behave effectively as Kähler magnetic monopoles so that wormhole throats must be connected by Kähler magnetic flux tubes with monopole flux so that closed flux tubes are obtained.

2. At the level of H Fermion number is carried by the modes of the induced spinor field. In space-time regions with Minkowski signature the modes are localized at string world sheets connecting the wormhole contacts.

Fig. 7. TGD view about elementary particles. a) Particle orbit corresponds to a 4-D generalization of a world line or b) with its light-like 3-D boundary (holography). c) Particle world lines have Euclidean signature of the induced metric. d) They can be identified as wormhole contacts. e) The throats of wormhole contacts carry effective Kähler magnetic charges so that wormhole contacts must appear as pairs in order to obtain closed flux tubes. f) Wormhole contacts are accompanied by fermionic strings connecting the throats at the same sheet: the strings do not extend inside the wormhole contacts. `http://tgdtheory.fi/appfigures/elparticletgd.jpg`

Particle interactions involve both stringy and QFT aspects.

1. The boundaries of string world sheets correspond to fundamental fermions. This gives rise to massless propagator lines in generalized Feynman diagrammatics. One can speak of “long” string connecting wormhole contacts and having a hadronic string as a physical counterpart.

Long strings should be distinguished from wormhole contacts which due to their super-conformal invariance behave like "short" strings with length scale given by CP_2 size, which is 10^4 times longer than Planck scale characterizing strings in string models.

2. Wormhole contact defines basic stringy interaction vertex for fermion-fermion scattering. The propagator is essentially the inverse of the superconformal scaling generator L_0. Wormhole contacts containing fermion and antifermion at its opposite throats behave like virtual bosons so that one has BFF type vertices typically.

3. In topological sense one has 3-vertices serving as generalizations of 3-vertices of Feynman diagrams. In these vertices 4-D "lines" of generalized Feynman diagrams meet along their 3-D ends. One obtains also the analogs of stringy diagrams but stringy vertices do not have the usual interpretation in terms of particle decays but in terms of propagation of particles along two different routes.

Fig. 8. a) TGD analogs of Feynman and string diagrammatics at the level of space-time topology. b) The 4-D analogs of both string diagrams and QFT diagrams appear but the interpretation of the analogs stringy diagrams is different. `http://tgdtheory.fi/appfigures/tgdgraphs.jpg`

A-9 About the selection of the action defining the Kähler function of the "world of classical worlds" (WCW)

The proposal is that space-time surfaces correspond to preferred extremals of some action principle, being analogous to Bohr orbits, so that they are almost deterministic. The action for the preferred extremal would define the Kähler function of WCW [K57, K95].

How unique is the choice of the action defining WCW Kähler metric? The problem is that twistor lift strongly suggests the identification of the preferred extremals as 4-D surfaces having 4-D generalization of complex structure and that a large number of general coordinate invariant actions constructible in terms of the induced geometry have the same preferred extremals.

A-9.1 Could twistor lift fix the choice of the action uniquely?

The twistor lift of TGD [K98] [L122, L126, L127] generalizes the notion of induction to the level of twistor fields and leads to a proposal that the action is obtained by dimensional reduction of the action having as its preferred extremals the counterpart of twistor space of the space-time surface identified as 6-D surface in the product $T(M^4) \times T(CP_2)$ twistor spaces of $T(M^4)$ and $T(CP_2)$ of M^4 and CP_2. Only M^4 and CP_2 allow a twistor space with Kähler structure [A57] so that TGD would be unique. Dimensional reduction is forced by the condition that the 6-surface has S^2-bundle structure characterizing twistor spaces and the base space would be the space-time surface.

1. Dimensional reduction of 6-D Kähler action implies that at the space-time level the fundamental action can be identified as the sum of Kähler action and volume term (cosmological constant). Other choices of the action do not look natural in this picture although they would have the same preferred extremals.

2. Preferred extremals are proposed to correspond to minimal surfaces with singularities such that they are also extremals of 4-D Kähler action outside the singularities. The physical analogue are soap films spanned by frames and one can localize the violation of the strict determinism and of strict holography to the frames.

3. The preferred extremal property is realized as the holomorphicity characterizing string world sheets, which generalizes to the 4-D situation. This in turn implies that the preferred extremals are the same for any general coordinate invariant action defined on the induced gauge fields and induced metric apart from possible extremals with vanishing CP_2 Kähler action.

   For instance, 4-D Kähler action and Weyl action as the sum of the tensor squares of the components of the Weyl tensor of CP_2 representing quaternionic imaginary units constructed from the Weyl tensor of CP_2 as an analog of gauge field would have the same preferred extremals and only the definition of Kähler function and therefore Kähler metric of WCW would change. One can even consider the possibility that the volume term in the 4-D action could be assigned to the tensor square of the induced metric representing a quaternionic or octonionic real unit.

Action principle does not seem to be unique. On the other hand, the WCW Kähler form and metric should be unique since its existence requires maximal isometries.

Unique action is not the only way to achieve this. One cannot exclude the possibility that the Kähler gauge potential of WCW in the complex coordinates of WCW differs only by a complex gradient of a holomorphic function for different actions so that they would give the same Kähler form for WCW. This gradient is induced by a symplectic transformation of WCW inducing a U(1) gauge transformation. The Kähler metric is the same if the symplectic transformation is an isometry.

Symplectic transformations of WCW could give rise to inequivalent representations of the theory in terms of action at space-time level. Maybe the length scale dependent coupling parameters of an effective action could be interpreted in terms of a choice of WCW Kähler function, which maximally simplifies the computations at a given scale.

1. The 6-D analogues of electroweak action and color action reducing to Kähler action in 4-D case exist. The 6-D analog of Weyl action based on the tensor representation of quaternionic imaginary units does not however exist. One could however consider the possibility that only the base space of twistor space $T(M^4)$ and $T(CP_2)$ have quaternionic structure.

2. Kähler action has a huge vacuum degeneracy, which clearly distinguishes it from other actions. The presence of the volume term removes this degeneracy. However, for minimal surfaces having CP_2 projections, which are Lagrangian manifolds and therefore have a vanishing induced Kähler form, would be preferred extremals according to the proposed definition. For these 4-surfaces, the existence of the generalized complex structure is dubious.

   For the electroweak action, the terms corresponding to charged weak bosons eliminate these extremals and one could argue that electroweak action or its sum with the analogue of color action, also proportional Kähler action, defines the more plausible choice. Interestingly, also the neutral part of electroweak action is proportional to Kähler action.

Twistor lift strongly suggests that also M^4 has the analog of Kähler structure. M^8 must be complexified by adding a commuting imaginary unit i. In the E^8 subspace, the Kähler structure of E^4 is defined in the standard sense and it is proposed that this generalizes to M^4 allowing also generalization of the quaternionic structure. M^4 Kähler structure violates Lorentz invariance but could be realized at the level of moduli space of these structures.

The minimal possibility is that the M^4 Kähler form vanishes: one can have a different representation of the Kähler gauge potential for it obtained as generalization of symplectic transformations acting non-trivially in M^4. The recent picture about the second quantization of spinors of $M^4 \times CP_2$ assumes however non-trivial Kähler structure in M^4.

A-9.2 Two paradoxes

TGD view leads to two apparent paradoxes.

1. If the preferred extremals satisfy 4-D generalization of holomorphicity, a very large set of actions gives rise to the same preferred extremals unless there are some additional conditions restricting the number of preferred extremals for a given action.

2. WCW metric has an infinite number of zero modes, which appear as parameters of the metric but do not contribute to the line element. The induced Kähler form depends on these degrees of freedom. The existence of the Kähler metric requires maximal isometries, which suggests that the Kähler metric is uniquely fixed apart from a conformal scaling factor Ω depending on zero modes. This cannot be true: galaxy and elementary particle cannot correspond to the same Kähler metric.

Number theoretical vision and the hierarchy of inclusions of HFFs associated with supersymplectic algebra actings as isometries of WcW provide equivalent realizations of the measurement resolution. This solves these paradoxes and predicts that WCW decomposes into sectors for which Kähler metrics of WCW differ in a natural way.

The hierarchy subalgebras of supersymplectic algebra implies the decomposition of WCW into sectors with different actions

Supersymplectic algebra of $\delta M^4_+ \times CP_2$ is assumed to act as isometries of WCW [L133]. There are also other important algebras but these will not be discussed now.

1. The symplectic algebra A of $\delta M^4_+ \times CP_2$ has the structure of a conformal algebra in the sense that the radial conformal weights with non-negative real part, which is half integer, label the elements of the algebra have an interpretation as conformal weights.

   The super symplectic algebra A has an infinite hierarchy of sub-algebras [L133] such that the conformal weights of sub-algebras $A_{n(SS)}$ are integer multiples of the conformal weights of the entire algebra. The superconformal gauge conditions are weakened. Only the subalgebra $A_{n(SS)}$ and the commutator $[A_{n(SS)}, A]$ annihilate the physical states. Also the corresponding classical Noether charges vanish for allowed space-time surfaces.

   This weakening makes sense also for ordinary superconformal algebras and associated Kac-Moody algebras. This hierarchy can be interpreted as a hierarchy symmetry breakings, meaning that sub-algebra $A_{n(SS)}$ acts as genuine dynamical symmetries rather than mere gauge symmetries. It is natural to assume that the super-symplectic algebra A does not affect the coupling parameters of the action.

2. The generators of A correspond to the dynamical quantum degrees of freedom and leave the induced Kähler form invariant. They affect the induced space-time metric but this effect is gravitational and very small for Einsteinian space-time surfaces with 4-D M^4 projection.

   The number of dynamical degrees of freedom increases with $n(SS)$. Therefore WCW decomposes into sectors labelled by $n(SS)$ with different numbers of dynamical degrees of freedom so that their Kähler metrics cannot be equivalent and cannot be related by a symplectic isometry. They can correspond to different actions.

Number theoretic vision implies the decomposition of WCW into sectors with different actions

The number theoretical vision leads to the same conclusion as the hierarchy of HFFs. The number theoretic vision of TGD based on M^8-H duality [L133] predicts a hierarchy with levels labelled by the degrees $n(P)$ of rational polynomials P and corresponding extensions of rationals characterized by Galois groups and by ramified primes defining p-adic length scales.

These sequences allow us to imagine several discrete coupling constant evolutions realized at the level H in terms of action whose coupling parameters depend on the number theoretic parameters.

1. Coupling constant evolution with respect to $n(P)$

The first coupling constant evolution would be with respect to $n(P)$.

1. The coupling constants characterizing action could depend on the degree $n(P)$ of the polynomial defining the space-time region by $M^8 - H$ duality. The complexity of the space-time surface would increase with $n(P)$ and new degrees of freedom would emerge as the number of the rational coefficients of P.

2. This coupling constant evolution could naturally correspond to that assignable to the inclusion hierarchy of hyperfinite factors of type II_1 (HFFs). I have indeed proposed [L133] that the degree $n(P)$ equals to the number $n(braid)$ of braids assignable to HFF for which super symplectic algebra subalgebra $A_{n(SS)}$ with radial conformal weights coming as $n(SS)$-multiples of those of entire algebra A. One would have $n(P) = n(braid) = n(SS)$. The

number of dynamical degrees of freedom increases with n which just as it increases with $n(P)$ and $n(SS)$.

3. The actions related to different values of $n(P) = n(braid) = n(SS)$ cannot define the same Kähler metric since the number of allowed space-time surfaces depends on $n(SS)$.

   WCW could decompose to sub-WCWs corresponding to different actions, a kind of theory space. These theories would not be equivalent. A possible interpretation would be as a hierarchy of effective field theories.

4. Hierarchies of composite polynomials define sequences of polynomials with increasing values of $n(P)$ such that the order of a polynomial at a given level is divided by those at the lower levels. The proposal is that the inclusion sequences of extensions are realized at quantum level as inclusion hierarchies of hyperfinite factors of type II_1.

   A given inclusion hierarchy corresponds to a sequence $n(SS)_i$ such that $n(SS)_i$ divides $n(SS)_{i+1}$. Therefore the degree of the composite polynomials increases very rapidly. The values of $n(SS)_i$ can be chosen to be primes and these primes correspond to the degrees of so called prime polynomials [L129] so that the decompositions correspond to prime factorizations of integers. The "densest" sequence of this kind would come in powers of 2 as $n(SS)_i = 2^i$. The corresponding p-adic length scales (assignable to maximal ramified primes for given $n(SS)_i$) are expected to increase roughly exponentially, say as 2^{r2^i}. $r = 1/2$ would give a subset of scales $2^{r/2}$ allowed by the p-adic length scale hypothesis. These transitions would be very rare.

   A theory corresponding to a given composite polynomial would contain as sub-theories the theories corresponding to lower polynomial composites. The evolution with respect to $n(SS)$ would correspond to a sequence of phase transitions in which the action genuinely changes. For instance, color confinement could be seen as an example of this phase transition.

5. A subset of p-adic primes allowed by the p-adic length scale hypothesis $p \simeq 2^k$ defining the proposed p-adic length scale hierarchy could relate to n_S changing phase transition. TGD suggests a hierarchy of hadron physics corresponding to a scale hierarchy defined by Mersenne primes and their Gaussian counterparts [K73, K74]). Each of them would be characterized by a confinement phase transition in which n_S and therefore also the action changes.

2. Coupling constant evolutions with respect to ramified primes for a given value of $n(P)$

For a given value of $n(P)$, one could have coupling constant sub-evolutions with respect to the set of ramified primes of P and dimensions $n = h_{eff}/h_0$ of algebraic extensions. The action would only change by U(1) gauge transformation induced by a symplectic isometry of WCW. Coupling parameters could change but the actions would be equivalent.

The choice of the action in an optimal manner in a given scale could be seen as a choice of the most appropriate effective field theory in which radiative corrections would be taken into account. One can interpret the possibility to use a single choice of coupling parameters in terms of quantum criticality.

The range of the p-adic length scales labelled by ramified primes and effective Planck constants h_{eff}/h_0 is finite for a given value of $n(SS)$.

The first coupling constant evolution of this kind corresponds to ramified primes defining p-adic length scales for given $n(SS)$.

1. Ramified primes are factors of the discriminant $D(P)$ of P, which is expressible as a product of non-vanishing root differents and reduces to a polynomial of the n coefficients of P. Ramified primes define p-adic length scales assignable to the particles in the amplitudes scattering amplitudes defined by zero energy states.

   P would represent the space-time surface defining an interaction region in $N-$-particle scattering. The N ramified primes dividing $D(P)$ would characterize the p-adic length scales assignable to these particles. If $D(P)$ reduces to a single ramified prime, one has elementary particle [L129], and the forward scattering amplitude corresponds to the propagator.

This would give rise to a multi-scale p-adic length scale evolution of the amplitudes analogous to the ordinary continuous coupling constant evolution of n-point scattering amplitudes with respect to momentum scales of the particles. This kind of evolutions extend also to evolutions with respect to $n(SS)$.

2. According to [L129], physical constraints require that $n(P)$ and the maximum size of the ramified prime of P correlate.

   A given rational polynomial of degree $n(P)$ can be always transformed to a polynomial with integer coefficients. If the integer coefficients are smaller than $n(P)$, there is an upper bound for the ramified primes. This assumption also implies that finite fields become fundamental number fields in number theoretical vision [L129].

3. p-Adic length scale hypothesis [L134] in its basic form states that there exist preferred primes $p \simeq 2^k$ near some powers of 2. A more general hypothesis states that also primes near some powers of 3 possibly also other small primes are preferred physically. The challenge is to understand the origin of these preferred scales.

   For polynomials P with a given degree $n(P)$ for which discriminant $D(P)$ is prime, there exists a maximal ramified prime. Numerical calculations suggest that the upper bound depends exponentially on $n(P)$.

   Could these maximal ramified primes satisfy the p-adic length scale hypothesis or its generalization? The maximal prime defines a fixed point of coupling constant evolution in accordance with the earlier proposal. For instance, could one think that one has $p \simeq 2^k$, $k = n(SS)$? Each p-adic prime would correspond to a p-adic coupling constant sub-evolution representable in terms of symplectic isometries.

Also the dimension n of the algebraic extension associated with P, which is identified in terms of effective Planck constant $h_{eff}/h_0 = n$ labelling different phases of the ordinary matter behaving like dark matter, could give rise to coupling constant evolution for given $n(SS)$. The range of allowed values of n is finite. Note however that several polynomials of a given degree can correspond to the same dimension of extension.

Number theoretic discretization of WCW and maxima of WCW Kähler function

Number theoretic approach involves a unique discretization of space-time surface and also of WCW. The question is how the points of the discretized WCW correspond to the preferred extremals.

1. The exponents of Kähler function for the maxima of Kähler function, which correspond to the universal preferred extremals, appear in the scattering amplitudes. The number theoretical approach involves a unique discretization of space-time surfaces defining the WCW coordinates of the space-time surface regarded as a point of WCW.

   In [L133] it is assumed that these WCW points appearing in the number theoretical discretization correspond to the maxima of the Kähler function. The maxima would depend on the action and would differ for ghd maxima associated with different actions unless they are not related by symplectic WCW isometry.

2. The symplectic transformations of WCW acting as isometries are assumed to be induced by the symplectic transformations of $\delta M^4_+ \times CP_2$ [K57, K36]. As isometries they would naturally permute the maxima with each other.

A-10 Number theoretic vision of TGD

Physics as number theory vision is complementary to the physics as geometry vision and has developed gradually since 1993. Langlands program is the counterpart of this vision in mathematics [L131].

The notion of p-adic number fields emerged with the motivation coming from the observation that elementary particle mass scales and mass ratios could be understood in terms of the so-called p-adic length scale hypothesis [K77, K66, K33]. The fusion of the various p-adic physics leads to what I call adelic physics [L50, L49]. Later the hypothesis about hierarchy of Planck constants labelling phases of ordinary matter behaving like dark matter emerged [K38, K39, K40, K40].

Eventually this led to that the values of effective Planck constant could be identified as the dimension of an algebraic extension of rationals assignable to polynomials with rational coefficients. This led to the number theoretic vision in which so-called M^8-H duality [L100, L101] plays a key role. M^8 (actually a complexification of real M^8) is analogous to momentum space so that the duality generalizes momentum position duality for point-like particles. M^8 has an interpretation as complexified octonions.

The dynamics of 4-surfaces in M^8 is coded by polynomials with rational coefficients, whose roots define mass shells H^3 of $M^4\subset M^8$. It has turned out that the polynomials satisfy stringent additional conditions and one can speak of number theoretic holography [L129, L131]. Also the ordinary 3→ 4 holography is needed to assign 4-surfaces with these 3-D mass shells. The number theoretic dynamics is based on the condition that the normal space of the 4-surface in M^8 is associative (quaternionic) and contains a commutative complex sub-space. This makes it possible to assign to this surface space-time surface in $H=M^4\times CP_2$.

At the level of H the space-time surfaces are by holography preferred extremals and are assumed to be determined by the twistor lift of TGD [K98] giving rise to an action which is sum of the Kähler action and volume term. The preferred extremals would be minimal surfaces analogous to soap films spanned by frames. Outside frames they would be simultaneous extremals of the Kähler action, which requires a generalization of the holomorphy characterizing string world sheets.

In the following only p-adic numbers and hierarchy of Planck constants will be discussed.

A-10.1 p-Adic numbers and TGD

p-Adic number fields

p-Adic numbers (p is prime: 2, 3, 5, ...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [A27]. p-Adic numbers are representable as power expansion of the prime number p of form

$$x = \sum_{k\geq k_0} x(k)p^k,\ x(k)=0,....,p-1 \ . \quad \text{(A-10.1)}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} \ . \quad \text{(A-10.2)}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) \ , \quad \text{(A-10.3)}$$

where $\varepsilon(x)=k+....$ with $0<k<p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

The distance function $d(x,y)=|x-y|_p$ defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x,z) \leq max\{d(x,y),d(y,z)\} \ . \quad \text{(A-10.4)}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x,y) \quad \leq \quad D \ . \tag{A-10.5}$$

This division of the metric space into classes has following properties:

1. Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.

2. Distances of points x and y inside single class are smaller than distances between different classes.

3. Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [B29]. The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

Canonical correspondence between p-adic and real numbers

The basic challenge encountered by p-adic physicist is how to map the predictions of the p-adic physics to real numbers. p-Adic probabilities provide a basic example in this respect. Identification via common rationals and canonical identification and its variants have turned out to play a key role in this respect.

1. Basic form of the canonical identification

There exists a natural continuous map $I : R_p \rightarrow R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \rightarrow x = \sum_{k<N} y_k p^{-k} \ , \\ y_k &\in \{0,1,..,p-1\} \ . \end{aligned} \tag{A-10.6}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also decimal expansion is not unique (1 = 0.999...) for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^{N} x_k p^{-k} \ , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \ . \end{aligned} \tag{A-10.7}$$

The p-adic images associated with these expansions are different

$$\begin{aligned} y_1 &= \sum_{k=N_0}^{N} x_k p^k \ , \\ y_2 &= \sum_{k=N_0}^{N-1} x_k p^k + (x_N - 1)p^N + (p-1)p^{N+1} \sum_{k=0,..} p^k \\ &= y_1 + (x_N - 1)p^N - p^{N+1} \ , \end{aligned} \quad \text{(A-10.8)}$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite pinary digits. The finite pinary digit expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

2. The topology induced by canonical identification

The topology induced by the canonical identification in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see **Fig. A-10.1**) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

Fig. 14. The real norm induced by canonical identification from 2-adic norm. `http://tgdtheory.fi/appfigures/norm.png`

The linear structure of the p-adic numbers induces a corresponding structure in the set of the non-negative real numbers and p-adic linearity in general differs from the ordinary concept of linearity. For example, p-adic sum is equal to real sum only provided the summands have no common pinary digits. Furthermore, the condition $x +_p y < max\{x, y\}$ holds in general for the p-adic sum of the real numbers. p-Adic multiplication is equivalent with the ordinary multiplication only provided that either of the members of the product is power of p. Moreover one has $x \times_p y < x \times y$ in general. The p-Adic negative -1_p associated with p-adic unit 1 is given by $(-1)_p = \sum_k (p-1)p^k$ and defines p-adic negative for each real number x. An interesting possibility is that p-adic linearity might replace the ordinary linearity in some strongly nonlinear systems so these systems would look simple in the p-adic topology.

These results suggest that canonical identification is involved with some deeper mathematical structure. The following inequalities hold true:

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |x|_p |y|_R \leq (xy)_R &\leq x_R y_R \ , \end{aligned} \quad \text{(A-10.9)}$$

where $|x|_p$ denotes p-adic norm. These inequalities can be generalized to the case of $(R_p)^n$ (a linear vector space over the p-adic numbers).

$$\begin{aligned} (x+y)_R &\leq x_R + y_R \ , \\ |\lambda|_p |y|_R \leq (\lambda y)_R &\leq \lambda_R y_R \ , \end{aligned} \quad \text{(A-10.10)}$$

where the norm of the vector $x \in T_p^n$ is defined in some manner. The case of Euclidian space suggests the definition

$$(x_R)^2 = (\sum_n x_n^2)_R \ . \qquad \text{(A-10.11)}$$

These inequalities resemble those satisfied by the vector norm. The only difference is the failure of linearity in the sense that the norm of a scaled vector is not obtained by scaling the norm of the original vector. Ordinary situation prevails only if the scaling corresponds to a power of p.

These observations suggests that the concept of a normed space or Banach space might have a generalization and physically the generalization might apply to the description of some non-linear systems. The nonlinearity would be concentrated in the nonlinear behavior of the norm under scaling.

3. Modified form of the canonical identification

The original form of the canonical identification is continuous but does not respect symmetries even approximately. This led to a search of variants which would do better in this respect. The modification of the canonical identification applying to rationals only and given by

$$I_Q(q = p^k \times \frac{r}{s}) = p^k \times \frac{I(r)}{I(s)} \qquad \text{(A-10.12)}$$

is uniquely defined for rationals, maps rationals to rationals, has also a symmetry under exchange of target and domain. This map reduces to a direct identification of rationals for $0 \leq r < p$ and $0 \leq s < p$. It has turned out that it is this map which most naturally appears in the applications. The map is obviously continuous locally since p-adically small modifications of r and s mean small modifications of the real counterparts.

Canonical identification is in a key role in the successful predictions of the elementary particle masses. The predictions for the light elementary particle masses are within extreme accuracy same for I and I_Q but I_Q is theoretically preferred since the real probabilities obtained from p-adic ones by I_Q sum up to one in p-adic thermodynamics.

4. Generalization of number concept and notion of embedding space

TGD forces an extension of number concept: roughly a fusion of reals and various p-adic number fields along common rationals is in question. This induces a similar fusion of real and p-adic embedding spaces. Since finite p-adic numbers correspond always to non-negative reals n-dimensional space R^n must be covered by 2^n copies of the p-adic variant R^n_p of R^n each of which projects to a copy of R^n_+ (four quadrants in the case of plane). The common points of p-adic and real embedding spaces are rational points and most p-adic points are at real infinity.

Real numbers and various algebraic extensions of p-adic number fields are thus glued together along common rationals and also numbers in algebraic extension of rationals whose number belong to the algebraic extension of p-adic numbers. This gives rise to a book like structure with rationals and various algebraic extensions of rationals taking the role of the back of the book. Note that Neper number is exceptional in the sense that it is algebraic number in p-adic number field Q_p satisfying $e^p \ mod \ p = 1$.

Fig. 15. Various number fields combine to form a book like structure. `http://tgdtheory.fi/appfigures/book.jpg`

For a given p-adic space-time sheet most points are literally infinite as real points and the projection to the real embedding space consists of a discrete set of rational points: the interpretation in terms of the unavoidable discreteness of the physical representations of cognition is natural. Purely local p-adic physics implies real p-adic fractality and thus long range correlations for the real space-time surfaces having enough common points with this projection.

p-Adic fractality means that M^4 projections for the rational points of space-time surface X^4 are related by a direct identification whereas CP_2 coordinates of X^4 at these points are related by I, I_Q or some of its variants implying long range correlates for CP_2 coordinates. Since only a discrete set of points are related in this manner, both real and p-adic field equations can be satisfied and there are no problems with symmetries. p-Adic effective topology is expected to be a good approximation only within some length scale range which means infrared and UV cutoffs. Also multi-p-fractality is possible.

The notion of p-adic manifold

The notion of p-adic manifold is needed in order to fuse real physics and various p-adic physics to a larger structure which suggests that real and p-adic number fields should be glued together along common rationals bringing in mind adeles. The notion is problematic because p-adic topology is totally disconnected implying that p-adic balls are either disjoint or nested so that ordinary definition of manifold using p-adic chart maps fails. A cure is suggested to be based on chart maps from p-adics to reals rather than to p-adics (see the appendix of the book)

The chart maps are interpreted as cognitive maps, "thought bubbles".

Fig. 16. The basic idea between p-adic manifold. `http://tgdtheory.fi/appfigures/padmanifold.jpg`

There are some problems.

1. Canonical identification does not respect symmetries since it does not commute with second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map arithmetic operations which requires pinary cutoff below which chart map takes rationals to rationals so that commutativity with arithmetics and symmetries is achieved in finite resolution: above the cutoff canonical identification is used

2. Canonical identification is continuous but does not map smooth p-adic surfaces to smooth real surfaces requiring second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map requiring completion of the image to smooth preferred extremal of Kähler action so that chart map is not unique in accordance with finite measurement resolution

3. Canonical identification violates general coordinate invariance of chart map: (cognition-induced symmetry breaking) minimized if p-adic manifold structure is induced from that for p-adic embedding space with chart maps to real embedding space and assuming preferred coordinates made possible by isometries of embedding space: one however obtains several in-equivalent p-adic manifold structures depending on the choice of coordinates: these cognitive representations are not equivalent.

A-10.2 Hierarchy of Planck constants and dark matter hierarchy

Hierarchy of Planck constants was motivated by the "impossible" quantal effects of ELF em fields on vertebrate cyclotron energies $E = hf = \hbar \times eB/m$ are above thermal energy is possible only if $\hbar$ has value much larger than its standard value. Also Nottale's finding that planetary orbits migh be understood as Bohr orbits for a gigantic gravitational Planck constant.

Hierachy of Planck constant would mean that the values of Planck constant come as integer multiples of ordinary Planck constant: $h_{eff} = n \times h$. The particles at magnetic flux tubes characterized by h_{eff} would correspond to dark matter which would be invisible in the sense that only particle with same value of h_{eff} appear in the same vertex of Feynman diagram.

Hierarchy of Planck constants would be due to the non-determism of the Kähler action predicting huge vacuum degeneracy allowing all space-time surfaces which are sub-manfolds of any $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2. For agiven Y^2 one obtains new manifolds Y^2 by applying symplectic transformations of CP_2.

Non-determinism would mean that the 3-surface at the ends of causal diamond (CD) can be connected by several space-time surfaces carrying same conserved Kähler charges and having same values of Kähler action. Conformal symmetries defined by Kac-Moody algebra associated with the embedding space isometries could act as gauge transformations and respect the light-likeness property of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian (Minkowskianb space-time region transforms to wormhole contact say). The number of conformal equivalence classes of these surfaces could be finite number n and define discrete physical degree of freedom and one would have $h_{eff} = n \times h$. This degeneracy would mean "second quantization" for the sheets of n-furcation: not only one but several sheets can be realized.

This relates also to quantum criticality postulated to be the basic characteristics of the dynamics of quantum TGD. Quantum criticalities would correspond to an infinite fractal hierarchy of broken conformal symmetries defined by sub-algebras of conformal algebra with conformal weights coming as integer multiples of n. This leads also to connections with quantum criticality and hierarchy of broken conformal symmetries, p-adicity, and negentropic entanglement which by consistency with standard quantum measurement theory would be described in terms of density matrix proportional $n \times n$ identity matrix and being due to unitary entanglement coefficients (typical for quantum computing systems).

Formally the situation could be described by regarding space-time surfaces as surfaces in singular n-fold singular coverings of embedding space. A stronger assumption would be that they are expressible as as products of n_1 -fold covering of M^4 and n_2-fold covering of CP_2 meaning analogy with multi-sheeted Riemann surfaces and that M^4 coordinates are n_1-valued functions and CP_2 coordinates n_2 -valued functions of space-time coordinates for $n = n_1 \times n_2$. These singular coverings of embedding space form a book like structure with singularities of the coverings localizable at the boundaries of causal diamonds defining the back of the book like structure.

Fig. 17. Hierarchy of Planck constants. `http://tgdtheory.fi/appfigures/planckhierarchy.jpg`

A-10.3 $M^8 - H$ duality as it is towards the end of 2021

The view of $M^8 - H$ duality (see Appendix **??**) has changed considerably towards the end 2021 [L122] after the realization that this duality is the TGD counterpart of momentum position duality of wave mechanics, which is lost in QFTs. Therefore M^8 and also space-time surface is analogous to momentum space. This forced us to give up the original simple identification of the points $M^4 \subset M^4 \times E^4 = M^8$ and of $M^4 \times CP_2$ so that it respects Uncertainty Principle (UP).

The first improved guess for the duality map was the replacement with the inversion $p^k \rightarrow m^k = \hbar_{eff} p^k/p^2$ conforming in spirit with UP but turned out to be too naive.

The improved form [L122] of the $M^8 - H$ duality map takes mass shells $p^2 = m^2$ of $M^4 \subset M^8$ to cds with size $L(m) = \hbar_{eff}/m$ with a common center. The slicing by mass shells is mapped to a Russian doll like slicing by cds. Therefore would be no CDs in M^8 contrary to what I believed first.

Quantum classical correspondence (QCC) inspires the proposal that the point $p^k \in M^8$ is mapped to a geodesic line corresponding to momentum p^k starting from the common center of cds. Its intersection with the opposite boundary of cd with size $L(m)$ defines the image point. This is not yet quite enough to satisfy UP but the additional details [L122] are not needed in the sequel.

The 6-D brane-like special solutions in M^8 are of special interest in the TGD inspired theory of consciousness. They have an M^4 projection which is $E = E_n$ 3-ball. Here E_n is a root of the real polynomial P defining $X^4 \subset M^8_c$ (M^8 is complexified to M^8_c) as a "root" of its octonionic continuation [L100, L101]. E_n has an interpretation as energy, which can be complex. The original interpretation was as moment of time. For this interpretation, $M^8 - H$ duality would be a linear identification and these hyper planes would be mapped to hyperplanes in $M^4 \subset H$. This motivated the term "very special moment in the life of self" for the image of the $E = E_n$ section of $X^4 \subset M^8$ [L83]. This notion does not make sense at the level M^8 anymore.

The modified $M^8 - H$ duality forces us to modify the original interpretation [L122]. The point $(E_n, p = 0)$ is mapped $(t_n = \hbar_{eff}/E_n, 0)$. The momenta (E_n, p) in $E = E_n$ plane are mapped to the boundary of cd and correspond to a continuous time interval at the boundary of CD: "very special moment" becomes a "very special time interval".

The quantum state however corresponds to a set of points corresponding to quark momenta, which belong to a cognitive representation and are therefore algebraic integers in the extension determined by the polynomial. These active points in E_n are mapped to a discrete set at the boundary of cd(m). A "very special moment" is replaced with a sequence of "very special moments".

So called Galois confinement [L114] forces the total momenta for bound states of quarks and antiquarks to be rational integers invariant under Galois group of extension of rationals determined by the polynomial P [L122]. These states correspond to states at boundaries of sub-CDs so that one obtains a hierarchy. Galois confinement provides a universal number theoretic mechanism for the formation of bound states.

A-11 Zero energy ontology (ZEO)

ZEO is implied by the holography forced in the TGD framework by general coordinate invariance.

A-11.1 Basic motivations and ideas of ZEO

The following gives a brief summary of ZEO [L91] [K130].

1. In ZEO quantum states are not 3-dimensional but superpositions of 4-dimensional deterministic time evolutions connecting ordinary initial 3-dimensional states. By holography they are equivalent to pairs of ordinary 3-D states identified as initial and final states of time evolution. One can say that in the TGD framework general coordinate invariance implies holography and the slight failure of its determinism in turn forces ZEO.

   Quantum jumps replace this state with a new one: a superposition of deterministic time evolutions is replaced with a new superposition. Classical determinism of individual time evolution is not violated and this solves the basic paradox of quantum measurement theory. There are two kinds of quantum jumps: ordinary ("big") state function reductions (BSFRs) changing the arrow of time and "small" state function reductions (SSFRs) (weak measurements) preserving it and giving rise to the analog of Zeno effect [L91].

2. To avoid getting totally confused it is good to emphasize some aspects of ZEO.

   (a) ZEO does not mean that physical states in the usual 3-D sense as snapshots of time evolution would have zero energy state pairs defining zero energy states as initial and final states have same conserved quantities such as energy. Conservation implies that one can adopt the conventions that the values of conserved quantities are opposite for these states so that their sum vanishes: one can think that incoming and outgoing particles come from geometric past and future is the picture used in quantum field theories.

   (b) ZEO means two times: subjective time as sequence of quantum jumps and geometric time as space-time coordinate. These times are identifiable but are strongly correlated.

3. In BSFRs the arrow of time is changed and the time evolution in the final state occurs backwards with respect to the time of the external observer. BSFRs can occur in all scales since TGD predicts a hierarchy of effective Planck constants with arbitrarily large values. There is empirical support for BSFRs.

   (a) The findings of Minev et al [L77] in atomic scale can be explained by the same mechanism [L77]. In BSFR a final zero energy state as a superposition of classical deterministic time evolutions emerges and for an observer with a standard arrow of time looks like a superposition of deterministic smooth time evolutions leading to the final state. Interestingly, once this evolution has started, it cannot be stopped unless one changes the stimulus signal inducing the evolution in which case the process does not lead to anywhere: the interpretation would be that BSFR back to the initial state occurs!

   (b) Libets' experiments about active aspects of consciousness [J5] can be understood. Subject person raises his finger and neural activity starts before the conscious decision to do so. In the physicalistic framework it is thought to lead to raising of the finger. The problem with the explanation is that the activity beginning .5 seconds earlier seems to be dissipation with a reversed arrow of time: from chaotic and disordered to ordered at around .15 seconds. ZEO explanation is that macroscopic quantum jump occurred and generated a signal proceeding backwards in time and generated neural activity and dissipated to randomness.

   (c) Earthquakes involve a strange anomaly: they are preceded by ELF radiation. One would expect that they generate ELF radiation. The identification as BSFR would explain the anomaly [L82]. In biology the reversal of the arrow of time would occur routinely and be a central element of biological self-organization, in particular self-organized quantum criticality (see [L86, L139].

A-11.2 Some implications of ZEO

ZEO has profound implications for understanding self-organization and self-organized quantum criticality in terms of dissipation with non-standard arrow of time looking like generation of structures [L86, L139]. ZEO could also allow understanding of what planned actions - like realizing the experiment under consideration - could be.

1. Second law in the standard sense does not favor - perhaps even not allow - realization of planned actions. ZEO forces a generalization of thermodynamics: dissipation with a non-standard arrow of time for a subsystem would look like self-organization and planned action and its realization.

   Could most if not all planned action be like this - induced by BSFR in the geometric future and only apparently planned? There would be however the experience of planning and realizing induced by the signals from geometric future by a higher level in the hierarchy of conscious entities predicted by TGD! In long time scales we would be realizing our fates or wishes of higher level conscious entities rather than agents with completely free will.

2. The notion of magnetic body (MB) serving as a boss of ordinary matter would be central. MB carries dark matter as $h_{eff} = nh_0$ phases of ordinary matter with n serving as a measure for algebraic complexity of extension of rationals as its dimension and defining a kind of universal IQ. There is a hierarchy of these phases and MBs labelled by extension of rationals and the value of n.

   MBs would form a hierarchy of bosses - a realization for master slave hierarchy. Ordinary matter would be at the bottom and its coherent behavior would be induced from quantum coherence at higher levels. BSFR for higher level MB would give rise to what looks like planned actions and experienced as planned action at the lower levels of hierarchy. One could speak of planned actions inducing a cascade of planned actions in shorter time scales and eventually proceeding to atomic level.

A-12 Some notions relevant to TGD inspired consciousness and quantum biology

Below some notions relevant to TGD inspired theory of consciousness and quantum biology.

A-12.1 The notion of magnetic body

Topological field quantization inspires the notion of field body about which magnetic body is especially important example and plays key role in TGD inspired quantum biology and consciousness theory. This is a crucial departure fromt the Maxwellian view. Magnetic body brings in third level to the description of living system as a system interacting strongly with environment. Magnetic body would serve as an intentional agent using biological body as a motor instrument and sensory receptor. EEG would communicated the information from biological body to magnetic body and Libet's findings from time delays of consciousness support this view.

The following pictures illustrate the notion of magnetic body and its dynamics relevant for quantum biology in TGD Universe.

Fig. 18. Magnetic body associated with dipole field. `http://tgdtheory.fi/appfigures/fluxquant.jpg`

Fig. 19. Illustration of the reconnection by magnetic flux loops. `http://tgdtheory.fi/appfigures/reconnect1.jpg`

Fig. 20. Illustration of the reconnection by flux tubes connecting pairs of molecules. `http://tgdtheory.fi/appfigures/reconect2.jpg`

Fig. 21. Flux tube dynamics. a) Reconnection making possible magnetic body to "recognize" the presence of another magnetic body, b) braiding, knotting and linking of flux tubes making possible topological quantum computation, c) contraction of flux tube in phase transition reducing the value of h_{eff} allowing two molecules to find each other in dense molecular soup. http://tgdtheory.fi/appfigures/fluxtubedynamics.jpg

A-12.2 Number theoretic entropy and negentropic entanglement

TGD inspired theory of consciousness relies heavily p-Adic norm allows an to define the notion of Shannon entropy for rational probabilities (and even those in algebraic extension of rationals) by replacing the argument of logarithm of probability with its p-adic norm. The resulting entropy can be negative and the interpretation is that number theoretic entanglement entropy defined by this formula for the p-adic prime minimizing its value serves as a measure for conscious information. This negentropy characterizes two-particle system and has nothing to do with the formal negative negentropy assignable to thermodynamic entropy characterizing single particle. Negentropy Maximization Principle (NMP) implies that number theoretic negentropy increases during evolution by quantum jumps. The condition that NMP is consistent with the standard quantum measurement theory requires that negentropic entanglement has a density matrix proportional to unit matrix so that in 2-particle case the entanglement matrix is unitary.

Fig. 22. Schrödinger cat is neither dead or alive. For negentropic entanglement this state would be stable. http://tgdtheory.fi/appfigures/cat.jpg

A-12.3 Life as something residing in the intersection of reality and p-adicities

In TGD inspired theory of consciousness p-adic space-time sheets correspond to space-time correlates for thoughts and intentions. The intersections of real and p-adic preferred extremals consist of points whose coordinates are rational or belong to some extension of rational numbers in preferred embedding space coordinates. They would correspond to the intersection of reality and various p-adicities representing the "mind stuff" of Descartes. There is temptation to assign life to the intersection of realities and p-adicities. The discretization of the chart map assigning to real space-time surface its p-adic counterpart would reflect finite cognitive resolution.

At the level of "world of classical worlds" (WCW) the intersection of reality and various p-adicities would correspond to space-time surfaces (or possibly partonic 2-surfaces) representable in terms of rational functions with polynomial coefficients with are rational or belong to algebraic extension of rationals.

The quantum jump replacing real space-time sheet with p-adic one (vice versa) would correspond to a buildup of cognitive representation (realization of intentional action).

Fig. 23. The quantum jump replacing real space-time surface with corresponding p-adic manifold can be interpreted as formation of though, cognitive representation. Its reversal would correspond to a transformation of intention to action. http://tgdtheory.fi/appfigures/padictoreal.jpg

A-12.4 Sharing of mental images

The 3-surfaces serving as correlates for sub-selves can topologically condense to disjoint large space-time sheets representing selves. These 3-surfaces can also have flux tube connections and this makes possible entanglement of sub-selves, which unentangled in the resolution defined by the size of sub-selves. The interpretation for this negentropic entanglement would be in terms of sharing of mental images. This would mean that contents of consciousness are not completely private as assumed in neuroscience.

Fig. 24. Sharing of mental images by entanglement of subselves made possible by flux tube connections between topologically condensed space-time sheets associated with mental images. http://tgdtheory.fi/appfigures/sharing.jpg

A-12.5 Time mirror mechanism

Zero energy ontology (ZEO) is crucial part of both TGD and TGD inspired consciousness and leads to the understanding of the relationship between geometric time and experience time and how the arrow of psychological time emerges. One of the basic predictions is the possibiity of negative energy signals propagating backwards in geometric time and having the property that entropy basically associated with subjective time grows in reversed direction of geometric time. Negative energy signals inspire time mirror mechanism (see **Fig.** http://tgdtheory.fi/appfigures/timemirror.jpg or **Fig.** 24 in the appendix of this book) providing mechanisms of both memory recall, realization of intentational action initiating action already in geometric past, and remote metabolism. What happens that negative energy signal travels to past and is reflected as positive energy signal and returns to the sender. This process works also in the reverse time direction.

Fig. 25. Zero energy ontology allows time mirror mechanism as a mechanism of memory recall. Essentially "seeing" in time direction is in question. http://tgdtheory.fi/appfigures/timemirror.jpg

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