PHYSICS IN MANY-SHEETED SPACE-TIME: PART I

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April 29, 2024

0.1 PREFACE

This book belongs to a series of online books summarizing the recent state Topological Geometrodynamics (TGD) and its applications. TGD can be regarded as a unified theory of fundamental interactions but is not the kind of unified theory as so called GUTs constructed by graduate students at seventies and eighties using detailed recipes for how to reduce everything to group theory. Nowadays this activity has been completely computerized and it probably takes only a few hours to print out the predictions of this kind of unified theory as an article in the desired format. TGD is something different and I am not ashamed to confess that I have devoted the last 37 years of my life to this enterprise and am still unable to write The Rules.

If I remember correctly, I got the basic idea of Topological Geometrodynamics (TGD) during autumn 1977, perhaps it was October. What I realized was that the representability of physical space-times as 4-dimensional surfaces of some higher-dimensional space-time obtained by replacing the points of Minkowski space with some very small compact internal space could resolve the conceptual difficulties of general relativity related to the definition of the notion of energy. This belief was too optimistic and only with the advent of what I call zero energy ontology the understanding of the notion of Poincare invariance has become satisfactory. This required also the understanding of the relationship to General Relativity.

It soon became clear that the approach leads to a generalization of the notion of space-time with particles being represented by space-time surfaces with finite size so that TGD could be also seen as a generalization of the string model. Much later it became clear that this generalization is consistent with conformal invariance only if space-time is 4-dimensional and the Minkowski space factor of imbedding space is 4-dimensional. During last year it became clear that 4-D Minkowski space and 4-D complex projective space CP_2 are completely unique in the sense that they allow twistor space with Kähler structure.

It took some time to discover that also the geometrization of also gauge interactions and elementary particle quantum numbers could be possible in this framework: it took two years to find the unique internal space (CP_2) providing this geometrization involving also the realization that family replication phenomenon for fermions has a natural topological explanation in TGD framework and that the symmetries of the standard model symmetries are much more profound than pragmatic TOE builders have believed them to be. If TGD is correct, main stream particle physics chose the wrong track leading to the recent deep crisis when people decided that quarks and leptons belong to same multiplet of the gauge group implying instability of proton.

There have been also longstanding problems.

- Gravitational energy is well-defined in cosmological models but is not conserved. Hence the conservation of the inertial energy does not seem to be consistent with the Equivalence Principle. Furthermore, the imbeddings of Robertson-Walker cosmologies turned out to be vacuum extremals with respect to the inertial energy. About 25 years was needed to realize that the sign of the inertial energy can be also negative and in cosmological scales the density of inertial energy vanishes: physically acceptable universes are creatable from vacuum. Eventually this led to the notion of zero energy ontology (ZEO) which deviates dramatically from the standard ontology being however consistent with the crossing symmetry of quantum field theories. In this framework the quantum numbers are assigned with zero energy states located at the boundaries of so called causal diamonds defined as intersections of future and past directed light-cones. The notion of energy-momentum becomes length scale dependent since one has a scale hierarchy for causal diamonds. This allows to understand the non-conservation of energy as apparent.

  Equivalence Principle as it is expressed by Einstein's equations follows from Poincare invariance once it is realized that GRT space-time is obtained from the many-sheeted space-time of TGD by lumping together the space-time sheets to a regionof Minkowski space and endowing it with an effective metric given as a sum of Minkowski metric and deviations of the metrices of space-time sheets from Minkowski metric. Similar description relates classical gauge potentials identified as components of induced spinor connection to Yang-Mills gauge potentials in GRT space-time. Various topological inhomogenities below resolution scale identified as particles are described using energy momentum tensor and gauge currents.

- From the beginning it was clear that the theory predicts the presence of long ranged classical electro-weak and color gauge fields and that these fields necessarily accompany classical electromagnetic fields.

  It took about 26 years to gain the maturity to admit the obvious: these fields are classical correlates for long range color and weak interactions assignable to dark matter. The only possible conclusion is that TGD physics is a fractal consisting of an entire hierarchy of fractal copies of standard model physics. Also the understanding of electro-weak massivation and screening of weak charges has been a long standing problem, and 32 years was needed to discover that what I call weak form of electric-magnetic duality gives a satisfactory solution of the problem and provides also surprisingly powerful insights to the mathematical structure of quantum TGD.

  The latest development was the realization that the well- definedness of electromagnetic charge as quantum number for the modes of the induced spinors field requires that the CP_2 projection of the region in which they are non-vanishing carries vanishing W boson field and is 2-D. This implies in the generic case their localization to 2-D surfaces: string world sheets and possibly also partonic 2-surfaces. This localization applies to all modes except covariantly constant right handed neutrino generating supersymmetry and mplies that string model in 4-D space-time is part of TGD. Localization is possible only for Kähler-Dirac assigned with Kähler action defining the dynamics of space-time surfaces. One must however leave open the question whether W field might vanish for the space-time of GRT if related to many-sheeted space-time in the proposed manner even when they do not vanish for space-time sheets.

I started the serious attempts to construct quantum TGD after my thesis around 1982. The original optimistic hope was that path integral formalism or canonical quantization might be enough to construct the quantum theory but the first discovery made already during first year of TGD was that these formalisms might be useless due to the extreme non-linearity and enormous vacuum degeneracy of the theory. This turned out to be the case.

- It took some years to discover that the only working approach is based on the generalization of Einstein's program. Quantum physics involves the geometrization of the infinite-dimensional "world of classical worlds" (WCW) identified as 3-dimensional surfaces. Still few years had to pass before I understood that general coordinate invariance leads to a more or less unique solution of the problem and in positive energyontology implies that space-time surfaces are analogous to Bohr orbits. This in positive energy ontology in which space-like 3-surface is basic object. It is not clear whether Bohr orbitology is necessary also in ZEO in which space-time surfaces connect space-like 3-surfaces at the light-like boundaries of causal diamond CD obtained as intersection of future and past directed light-cones (with CP_2 factor included). The reason is that the pair of 3-surfaces replaces the boundary conditions at single 3-surface involving also time derivatives. If one assumes Bohr orbitology then strong correlations between the 3-surfaces at the ends of CD follow. Still a couple of years and I discovered that quantum states of the Universe can be identified as classical spinor fields in WCW. Only quantum jump remains the genuinely quantal aspect of quantum physics.

- During these years TGD led to a rather profound generalization of the space-time concept. Quite general properties of the theory led to the notion of many-sheeted space-time with sheets representing physical subsystems of various sizes. At the beginning of 90s I became dimly aware of the importance of p-adic number fields and soon ended up with the idea that p-adic thermodynamics for a conformally invariant system allows to understand elementary particle massivation with amazingly few input assumptions. The attempts to understand p-adicity from basic principles led gradually to the vision about physics as a generalized number theory as an approach complementary to the physics as an infinite-dimensional spinor geometry of WCW approach. One of its elements was a generalization of the number concept obtained by fusing real numbers and various p-adic numbers along common rationals. The number theoretical trinity involves besides p-adic number fields also quaternions and octonions and the notion of infinite prime.

- TGD inspired theory of consciousness entered the scheme after 1995 as I started to write a book about consciousness. Gradually it became difficult to say where physics ends and consciousness theory begins since consciousness theory could be seen as a generalization of quantum measurement theory by identifying quantum jump as a moment of consciousness and by replacing the observer with the notion of self identified as a system which is conscious as long as it can avoid entanglement with environment. The somewhat cryptic statement "Everything is conscious and consciousness can be only lost" summarizes the basic philosophy neatly.

The idea about p-adic physics as physics of cognition and intentionality emerged also rather naturally and implies perhaps the most dramatic generalization of the space-time concept in which most points of p-adic space-time sheets are infinite in real sense and the projection to the real imbedding space consists of discrete set of points. One of the most fascinating outcomes was the observation that the entropy based on p-adic norm can be negative. This observation led to the vision that life can be regarded as something in the intersection of real and p-adic worlds. Negentropic entanglement has interpretation as a correlate for various positively colored aspects of conscious experience and means also the possibility of strongly correlated states stable under state function reduction and different from the conventional bound states and perhaps playing key role in the energy metabolism of living matter.

If one requires consistency of Negentropy Mazimization Pronciple with standard measurement theory, negentropic entanglement defined in terms of number theoretic negentropy is necessarily associated with a density matrix proportional to unit matrix and is maximal and is characterized by the dimension n of the unit matrix. Negentropy is positive and maximal for a p-adic unique prime dividing n.

- One of the latest threads in the evolution of ideas is not more than nine years old. Learning about the paper of Laurent Nottale about the possibility to identify planetary orbits as Bohr orbits with a gigantic value of gravitational Planck constant made once again possible to see the obvious. Dynamical quantized Planck constant is strongly suggested by quantum classical correspondence and the fact that space-time sheets identifiable as quantum coherence regions can have arbitrarily large sizes. Second motivation for the hierarchy of Planck constants comes from bio-electromagnetism suggesting that in living systems Planck constant could have large values making macroscopic quantum coherence possible. The interpretation of dark matter as a hierarchy of phases of ordinary matter characterized by the value of Planck constant is very natural.

  During summer 2010 several new insights about the mathematical structure and interpretation of TGD emerged. One of these insights was the realization that the postulated hierarchy of Planck constants might follow from the basic structure of quantum TGD. The point is that due to the extreme non-linearity of the classical action principle the correspondence between canonical momentum densities and time derivatives of the imbedding space coordinates is one-to-many and the natural description of the situation is in terms of local singular covering spaces of the imbedding space. One could speak about effective value of Planck constant $h_{eff} = n \times h$ coming as a multiple of minimal value of Planck constant. Quite recently it became clear that the non-determinism of Kähler action is indeed the fundamental justification for the hierarchy: the integer n can be also interpreted as the integer characterizing the dimension of unit matrix characterizing negentropic entanglement made possible by the many-sheeted character of the space-time surface.

  Due to conformal invariance acting as gauge symmetry the n degenerate space-time sheets must be replaced with conformal equivalence classes of space-time sheets and conformal transformations correspond to quantum critical deformations leaving the ends of space-time surfaces invariant. Conformal invariance would be broken: only the sub-algebra for which conformal weights are divisible by n act as gauge symmetries. Thus deep connections between conformal invariance related to quantum criticality, hierarchy of Planck constants, negentropic entanglement, effective p-adic topology, and non-determinism of Kähler action perhaps reflecting p-adic non-determinism emerges.

  The implications of the hierarchy of Planck constants are extremely far reaching so that the significance of the reduction of this hierarchy to the basic mathematical structure distinguishing between TGD and competing theories cannot be under-estimated.

From the point of view of particle physics the ultimate goal is of course a practical construction recipe for the S-matrix of the theory. I have myself regarded this dream as quite too ambitious taking into account how far reaching re-structuring and generalization of the basic mathematical structure of quantum physics is required. It has indeed turned out that the dream about explicit formula is unrealistic before one has understood what happens in quantum jump. Symmetries and general physical principles have turned out to be the proper guide line here. To give some impressions about what is required some highlights are in order.

- With the emergence of ZEO the notion of S-matrix was replaced with M-matrix defined between positive and negative energy parts of zero energy states. M-matrix can be interpreted as a complex square root of density matrix representable as a diagonal and positive square root of density matrix and unitary S-matrix so that quantum theory in ZEO can be said to define a square root of thermodynamics at least formally. M-matrices in turn bombine to form the rows of unitary U-matrix defined between zero energy states.

- A decisive step was the strengthening of the General Coordinate Invariance to the requirement that the formulations of the theory in terms of light-like 3-surfaces identified as 3-surfaces at which the induced metric of space-time surfaces changes its signature and in terms of space-like 3-surfaces are equivalent. This means effective 2-dimensionality in the sense that partonic 2-surfaces defined as intersections of these two kinds of surfaces plus 4-D tangent space data at partonic 2-surfaces code for the physics. Quantum classical correspondence requires the coding of the quantum numbers characterizing quantum states assigned to the partonic 2-surfaces to the geometry of space-time surface. This is achieved by adding to the modified Dirac action a measurement interaction term assigned with light-like 3-surfaces.

- The replacement of strings with light-like 3-surfaces equivalent to space-like 3-surfaces means enormous generalization of the super conformal symmetries of string models. A further generalization of these symmetries to non-local Yangian symmetries generalizing the recently discovered Yangian symmetry of $\mathcal{N} = 4$ supersymmetric Yang-Mills theories is highly suggestive. Here the replacement of point like particles with partonic 2-surfaces means the replacement of conformal symmetry of Minkowski space with infinite-dimensional super-conformal algebras. Yangian symmetry provides also a further refinement to the notion of conserved quantum numbers allowing to define them for bound states using non-local energy conserved currents.

- A further attractive idea is that quantum TGD reduces to almost topological quantum field theory. This is possible if the Kähler action for the preferred extremals defining WCW Kähler function reduces to a 3-D boundary term. This takes place if the conserved currents are so called Beltrami fields with the defining property that the coordinates associated with flow lines extend to single global coordinate variable. This ansatz together with the weak form of electric-magnetic duality reduces the Kähler action to Chern-Simons term with the condition that the 3-surfaces are extremals of Chern-Simons action subject to the constraint force defined by the weak form of electric magnetic duality. It is the latter constraint which prevents the trivialization of the theory to a topological quantum field theory. Also the identification of the Kähler function of WCW as Dirac determinant finds support as well as the description of the scattering amplitudes in terms of braids with interpretation in terms of finite measurement resolution coded to the basic structure of the solutions of field equations.

- In standard QFT Feynman diagrams provide the description of scattering amplitudes. The beauty of Feynman diagrams is that they realize unitarity automatically via the so called Cutkosky rules. In contrast to Feynman's original beliefs, Feynman diagrams and virtual particles are taken only as a convenient mathematical tool in quantum field theories. QFT approach is however plagued by UV and IR divergences and one must keep mind open for the possibility that a genuine progress might mean opening of the black box of the virtual particle.

  In TGD framework this generalization of Feynman diagrams indeed emerges unavoidably. Light-like 3-surfaces replace the lines of Feynman diagrams and vertices are replaced by 2-D partonic 2-surfaces. The approximate localization of the nodes of induced spinor fields to 2-D string world sheets (and possibly also to partonic 2-surfaces) implies a stringy formulation of the theory analogous to stringy variant of twistor formalism with string world sheets having interpretation as 2-braids. In TGD framework fermionic variant of twistor Grassmann formalism leads to a stringy variant of twistor diagrammatics in which basic fermions can be said to be on mass-shell but carry non-physical helicities in the internal lines. This suggests the generalization of the Yangian symmetry to infinite-dimensional super-conformal algebras.

What I have said above is strongly biased view about the recent situation in quantum TGD. This vision is single man's view and doomed to contain unrealistic elements as I know from experience. My dream is that young critical readers could take this vision seriously enough to try to demonstrate that some of its basic premises are wrong or to develop an alternative based on these or better premises. I must be however honest and tell that 32 years of TGD is a really vast bundle of thoughts and quite a challenge for anyone who is not able to cheat himself by taking the attitude of a blind believer or a light-hearted debunker trusting on the power of easy rhetoric tricks.

Karkkila, October 30, 2010, Finland

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ACKNOWLEDGEMENTS

Neither TGD nor these books would exist without the help and encouragement of many people. The friendship with Heikki and Raija Haila and their family and Kalevi and Ritva Tikkanen and their family have been kept me in contact with the everyday world and without this friendship I would not have survived through these lonely 45 lonely years most of which I have remained unemployed as a scientific dissident. I am happy that my children have understood my difficult position and like my friends have believed that what I am doing is something valuable although I have not received any official recognition for it.

During the last decade Tapio Tammi has helped me quite concretely by providing the necessary computer facilities and being one of the few persons in Finland with whom to discuss my work. Pertti Kärkkäinen is my old physicist friend and has provided continued economic support for a long time. I have also had stimulating discussions with Samuli Penttinen who has also helped to get through the economical situations in which there seemed to be no hope. The continual updating of fifteen online books means quite a heavy bureaucracy at the level of bits and without a systemization one ends up with endless copying and pasting and internal consistency is soon lost. Tommi Ullgren has provided both economic support and encouragement during years. Pekka Rapinoja has offered his help in this respect and I am especially grateful to him for my Python skills.

During the last five years I have had inspiring discussions with many people in Finland interested in TGD. We have had video discussions with Sini Kunnas and had podcast discussions with Marko Manninen related to the TGD based view of physics and consciousness. Marko has also helped in the practical issues related to computers and quite recently he has done a lot of testing of chatGPT helping me to get an overall view of what it is. The discussions in a Zoom group involving Marko Manninen, Tuomas Sorakivi and Rode Majakka have given me the valuable opportunity to clarify my thoughts.

The collaboration with Lian Sidorov was extremely fruitful and she also helped me to survive economically through the hardest years. The participation in CASYS conferences in Liege has been an important window to the academic world and I am grateful for Daniel Dubois and Peter Marcer for making this participation possible. The discussions and collaboration with Eduardo de Luna and Istvan Dienes stimulated the hope that the communication of new vision might not be a mission impossible after all. Also blog discussions have been very useful. During these years I have received innumerable email contacts from people around the world. I am grateful to Mark McWilliams, Paul Kirsch, Gary Ehlenberg, and Ulla Matfolk and many others for providing links to possibly interesting websites and articles. We have collaborated with Peter Gariaev and Reza Rastmanesh. These contacts have helped me to avoid the depressive feeling of being some kind of Don Quixote of Science and helped me to widen my views: I am grateful for all these people.

In the situation in which the conventional scientific communication channels are strictly closed it is important to have some loop hole through which the information about the work done can at least in principle leak to the public through the iron wall of academic censorship. Without any exaggeration I can say that without the world wide web I would not have survived as a scientist nor as an individual. Homepage and blog are however not enough since only the formally published result is a result in recent day science. Publishing is however impossible without direct support from power holders- even in archives like arXiv.org.

Situation changed as Andrew Adamatsky proposed the writing of a book about TGD when I had already gotten used to the thought that my work would not be published during my lifetime. The Prespacetime Journal and two other journals related to quantum biology and consciousness - all of them founded by Huping Hu - have provided this kind of loophole. In particular, Dainis Zeps, Phil Gibbs, and Arkadiusz Jadczyk deserve my gratitude for their kind help in the preparation of an article series about TGD catalyzing a considerable progress in the understanding of quantum TGD. Also the viXra archive founded by Phil Gibbs and its predecessor Archive Freedom have been of great help: Victor Christianto deserves special thanks for doing the hard work needed to run Archive Freedom. Also the Neuroquantology Journal founded by Sultan Tarlaci deserves a special mention for its publication policy.

And last but not least: there are people who experience as a fascinating intellectual challenge to spoil the practical working conditions of a person working with something which might be called unified theory: I am grateful for the people who have helped me to survive through the virus attacks, an activity which has taken roughly one month per year during the last half decade and given a strong hue of grey to my hair.

For a person approaching his 73th birthday it is somewhat easier to overcome the hard feelings due to the loss of academic human rights than for an inpatient youngster. Unfortunately the economic situation has become increasingly difficult during the twenty years after the economic depression in Finland which in practice meant that Finland ceased to be a constitutional state in the strong sense of the word. It became possible to depose people like me from society without fear about public reactions and the classification as dropout became a convenient tool of ridicule to circumvent the ethical issues. During the period when the right wing held political power this trend was steadily strengthening and the situation is the same as I am writing this. In this kind of situation the concrete help from individuals has been and will be of utmost importance. Against this background it becomes obvious that this kind of work is not possible without the support from outside and I apologize for not being able to mention all the people who have helped me during these years.

Karkkila, August 30, 2023, Finland

Matti Pitkänen

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Chapter 1

Introduction

1.1 Basic Ideas of Topological Geometrodynamics (TGD)

Standard model describes rather successfully both electroweak and strong interactions but sees them as totally separate and contains a large number of parameters which it is not able to predict. For about four decades ago unified theories known as Grand Unified Theories (GUTs) trying to understand electroweak interactions and strong interactions as aspects of the same fundamental gauge interaction assignable to a larger symmetry group emerged. Later superstring models trying to unify even gravitation and strong and weak interactions emerged. The shortcomings of both GUTs and superstring models are now well-known. If TGD - whose basic idea emerged towards the end of 1977 - would emerge now it would be seen as an attempt to solve the difficulties of these approaches to unification.

The basic physical picture behind the geometric vision of TGD corresponds to a fusion of two rather disparate approaches: namely TGD as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model. After 1995 number theoretic vision started to develop and was initiated by the success of mass calculations based on p-adic thermodynamics. Number theoretic vision involves all number fields and is complementary to the geometric vision: one can say that this duality is analogous to momentum-position duality of wave mechanics. TGD can be also regarded as topological quantum theory in a very general sense as already the attribute "Topological" in "TGD" makes clear. Space-time surfaces as minimal surfaces can be regarded as representatives of homology equivalence classes and p-adic topologies generalize the notion of local topology and apply to the description of correlates of cognition.

1.1.1 Geometric Vision Very Briefly

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [K1].

The basic vision and its relationship to existing theories is now rather well understood.

1. Space-times are representable as 4-surfaces in the 8-dimensional embedding space $H = M^4 \times CP_2$, where M^4 is 4-dimensional (4-D) Minkowski space and CP_2 is 4-D complex projective space (see Appendix).

2. Induction procedure (a standard procedure in fiber bundle theory, see Appendix) allows to geometrize various fields. Space-time metric characterizing gravitational fields corresponds to the induced metric obtained by projecting the metric tensor of H to the space-time surface. Electroweak gauge potentials are identified as projections of the components of CP_2 spinor connection to the space-time surface, and color gauge potentials as projections of CP_2 Killing vector fields representing color symmetries. Also spinor structure can be induced: induced spinor gamma matrices are projections of gamma matrices of H and induced spinor fields just H spinor fields restricted to space-time surface. Spinor connection is also projected. The interpretation is that distances are measured in embedding space metric and parallel translation using spinor connection of embedding space.

Twistor lift of TGD means that one can lift space-time surfaces in H to 6-D surfaces a analogs of twistor space of space-time surface in the Cartesian product of the twistor spaces of M^4 and CP_2, which are the only 4-manifolds allowing twistor space with Kähler structure [A35]. The twistor structure would be induced in some sense, and should coincide with that associated with the induced metric. Clearly, the 2-spheres defining the fibers of twistor spaces of M^4 and CP_2 must allow identification: this 2-sphere defines the S^2 fiber of the twistor space of the space-time surface. This poses a constraint on the embedding of the twistor space of space-time surfaces as sub-manifold in the Cartesian product of twistor spaces. The existence of Kähler structure allows to lift 4-D Kähler action to its 6-D counterparts and the 6-D counterpart of twistor space is obtained by its dimensional reduction so that one obtains a sphere bundle. This makes possible twistorialization for all space-time surfaces: in general relativity the general metric does not allow this.

3. A geometrization of quantum numbers is achieved. The isometry group of the geometry of CP_2 codes for the color gauge symmetries of strong interactions. Vierbein group codes for electroweak symmetries, and explains their breaking in terms of CP_2 geometry so that standard model gauge group results. There are also important deviations from the standard model: color quantum numbers are not spin-like but analogous to orbital angular momentum: this difference is expected to be seen only in CP_2 scale. In contrast to GUTs, quark and lepton numbers are separately conserved and family replication has a topological explanation in terms of topology of the partonic 2-surface carrying fermionic quantum numbers.

   M^4 and CP_2 are unique choices for many other reasons. For instance, they are the unique 4-D space-times allowing twistor space with Kähler structure. M^4 light-cone boundary allows a huge extension of 2-D conformal symmetries. M^4 and CP_2 allow quaternionic structures. Therefore standard model symmetries have number theoretic meaning.

4. Induced gauge potentials are expressible in terms of embedding space coordinates and their gradients and general coordinate invariance implies that there are only 4 field-like variables locally. Situation is thus extremely simple mathematically. The objection is that one loses linear superposition of fields. The resolution of the problem comes from the generalization of the concepts of particle and space-time.

   Space-time surfaces can be also particle like having thus finite size. In particular, space-time regions with Euclidian signature of the induced metric (temporal and spatial dimensions in the same role) emerge and have interpretation as lines of generalized Feynman diagrams. Particles in space-time can be identified as a topological inhomogeneities in background space-time surface which looks like the space-time of general relativity in long length scales.

   One ends up with a generalization of space-time surface to many-sheeted space-time with space-time sheets having extremely small distances of about 10^4 Planck lengths (CP_2 size). As one adds a particle to this kind of structure, it touches various space-time sheets and thus interacts with the associated classical fields. Their effects superpose linearly in good approximation and linear superposition of fields is replaced with that for their effects.

   This resolves the basic objection. It also leads to the understanding of how the space-time of general relativity and quantum field theories emerges from TGD space-time as effective space-time when the sheets of many-sheeted space-time are lumped together to form a region of Minkowski space with metric replaced with a metric identified as the sum of empty Minkowski metric and deviations of the metrics of sheets from empty Minkowski metric. Gauge potentials are identified as sums of the induced gauge potentials. TGD is therefore a microscopic theory from which the standard model and general relativity follow as a topological simplification, however forcing a dramatic increase of the number of fundamental field variables.

5. A further objection is that classical weak fields identified as induced gauge fields are long ranged and should cause large parity breaking effects due to weak interactions. These effects are indeed observed but only in living matter. The basic problem is that one has long ranged classical electroweak gauge fields. The resolution of the problem is that the quantum averages of induced weak and color gauge fields vanish due to the fact that color rotations affect both space-time surfaces and induced weak and color fields. Only the averages of

electromagnetic fields are nonvanishing. The correlations functions for weak fields are non-vanishing below Compton lengths of weak bosons. In living matter large values of effective Planck constant labelling phases of ordinary matter identified as dark matter make possible long ranged weak fields and color fields.

6. General coordinate invariance requires holography so that space-time surfaces are analogous to Bohr orbits for particles identified as 3-surfaces. Bohr orbit property would be naturally realized by a 4-D generalization of holomorphy of string world sheets and implies that the space-time surfaces are minimal surfaces apart from singularities. This holds true for any action as long as it is general coordinate invariant and constructible in terms of the induced geometry. String world sheets and light-like orbits of partonic 2-surfaces correspond to singularities at which the minimal surface property of the space-time surfaces realizing the preferred extremal property fails. Preferred extremals are not completely deterministic, which implies what I call zero energy ontology (ZEO) meaning that the Bohr orbits are the fundamental objects. This leads to a solution of the basic paradox of quantum measurement theory. Also the mathematically ill-defined path integral disappears and leaves only the well-defined functional integral over the Bohr orbits.

7. A string model-like picture emerges from TGD and one ends up with a rather concrete view about the topological counterpart of Feynman diagrammatics. The natural stringy action would be given by the string world sheet area, which is present only in the space-time regions with Minkowskian signature. Gravitational constant could be present as a fundamental constant in string action and the ratio $\hbar/G/R^2$ would be determined by quantum criticality conditions. The hierarchy of Planck constants $h_{eff}/h = n$ assigned to dark matter in TGD framework would allow to circumvent the objection that only objects of length of order Planck length are possible since string tension given by $T = 1/\hbar_{eff}G$ apart from numerical factor could be arbitrary small. This would make possible gravitational bound states as partonic 2-surfaces as structures connected by strings and solve the basic problem of superstring theories. This option allows the natural interpretation of M^4 type vacuum extremals with CP_2 projection, which is Lagrange manifold as good approximations for space-time sheets at macroscopic length scales. String area does not contribute to the Kähler function at all.

   Whether induced spinor fields associated with Kähler-Dirac action and de-localized inside the entire space-time surface should be allowed remains an open question: super-conformal symmetry strongly suggests their presence. A possible interpretation for the corresponding spinor modes could be in terms of dark matter, sparticles, and hierarchy of Planck constants.

It is perhaps useful to make clear what TGD is not and also what new TGD can give to physics.

1. TGD is *not* just General Relativity made concrete by using embeddings: the 4-surface property is absolutely essential for unifying standard model physics with gravitation and to circumvent the incurable conceptual problems of General Relativity. The many-sheeted space-time of TGD gives rise only at the macroscopic limit to GRT space-time as a slightly curved Minkowski space. TGD is *not* a Kaluza-Klein theory although color gauge potentials are analogous to gauge potentials in these theories.

   TGD space-time is 4-D and its dimension is due to completely unique conformal properties of light-cone boundary and 3-D light-like surfaces implying enormous extension of the ordinary conformal symmetries. Light-like 3-surfaces represent orbits of partonic 2-surfaces and carry fundamental fermions at 1-D boundaries of string world sheets. TGD is *not* obtained by performing Poincare gauging of space-time to introduce gravitation and is plagued by profound conceptual problems.

2. TGD is *not* a particular string model although string world sheets emerge in TGD very naturally as loci for spinor modes: their 2-dimensionality makes among other things possible quantum deformation of quantization known to be physically realized in condensed matter, and conjectured in TGD framework to be crucial for understanding the notion of finite measurement resolution. Hierarchy of objects of dimension up to 4 emerge from TGD: this obviously means analogy with branes of super-string models.

TGD is *not* one more item in the collection of string models of quantum gravitation relying on Planck length mystics. Dark matter becomes an essential element of quantum gravitation and quantum coherence in astrophysical scales is predicted just from the assumption that strings connecting partonic 2-surfaces are responsible for gravitational bound states.

TGD is *not* a particular string model although AdS/CFT duality of super-string models generalizes due to the huge extension of conformal symmetries and by the identification of WCW gamma matrices as Noether super-charges of super-symplectic algebra having a natural conformal structure.

3. TGD is *not* a gauge theory. In TGD framework the counterparts of also ordinary gauge symmetries are assigned to super-symplectic algebra (and its Yangian [A20] [B23, B20, B21]), which is a generalization of Kac-Moody algebras rather than gauge algebra and suffers a fractal hierarchy of symmetry breakings defining hierarchy of criticalities. TGD is *not* one more quantum field theory like structure based on path integral formalism: path integral is replaced with functional integral over 3-surfaces, and the notion of classical space-time becomes an exact part of the theory. Quantum theory becomes formally a purely classical theory of WCW spinor fields: only state function reduction is something genuinely quantal.

4. TGD view about spinor fields is *not* the standard one. Spinor fields appear at three levels. Spinor modes of the embedding space are analogs of spinor modes characterizing incoming and outgoing states in quantum field theories. Induced second quantized spinor fields at space-time level are analogs of stringy spinor fields. Their modes are localized by the well-definedness of electro-magnetic charge and by number theoretic arguments at string world sheets. Kähler-Dirac action is fixed by supersymmetry implying that ordinary gamma matrices are replaced by what I call Kähler-Dirac gamma matrices - this something new. WCW spinor fields, which are classical in the sense that they are not second quantized, serve as analogs of fields of string field theory and imply a geometrization of quantum theory.

5. TGD is in some sense an extremely conservative geometrization of entire quantum physics: *no* additional structures such as gauge fields as independent dynamical degrees of freedom are introduced: Kähler geometry and associated spinor structure are enough. "Topological" in TGD should not be understood as an attempt to reduce physics to torsion (see for instance [B18]) or something similar. Rather, TGD space-time is topologically non-trivial in all scales and even the visible structures of the everyday world represent non-trivial topology of space-time in the TGD Universe.

6. Twistor space - or rather, a generalization of twistor approach replacing masslessness in 4-D sense with masslessness in 8-D sense and thus allowing description of also massive particles - emerged originally as a technical tool, and its Kähler structure is possible only for $H = M^4 \times CP_2$. It however turned out that much more than a technical tool is in question. What is genuinely new is the infinite-dimensional character of the Kähler geometry making it highly unique, and its generalization to p-adic number fields to describe correlates of cognition. Also the hierarchy of Planck constants $h_{eff} = n \times h$ reduces to the quantum criticality of the TGD Universe and p-adic length scales and Zero Energy Ontology represent something genuinely new.

The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last 45 years to the realization of this dream and this has resulted in 26 online books about TGD and nine online books about TGD inspired theory of consciousness and of quantum biology.

A collection of 30 online books is now (August 2023) under preparation. The goal is to minimize overlap between the topics of the books and make the focus of a given book sharper.

1.1.2 Two Visions About TGD as Geometrization of Physics and Their Fusion

As already mentioned, TGD as a geometrization of physics can be interpreted both as a modification of general relativity and generalization of string models.

TGD as a Poincare Invariant Theory of Gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_\times CP_2$, where M^4 denotes Minkowski space and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [A30, A34, A25, A33].

The identification of the space-time as a sub-manifold [A31, A41] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity.

It soon however turned out that sub-manifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

The choice of H is unique from the condition that TGD has standard model symmetries. Also number theoretical vision selects $H = M^4 \times CP_2$ uniquely. M^4 and CP_2 are also unique spaces allowing twistor space with Kähler structure.

TGD as a Generalization of the Hadronic String Model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

This decay vertex does not however correspond to a direct generalization of trouser vertex of string models. Indeed, the important difference between TGD and string models is that the analogs of string world sheet diagrams do not describe particle decays but the propagation of particles via different routes. Particle reactions are described by generalized Feynman diagrams for which 3-D light-like surface describing particle propagating join along their ends at vertices. As 4-manifolds the space-time surfaces are therefore singular like Feynman diagrams as 1-manifolds.

Quite recently, it has turned out that fermionic strings inside space-time surfaces define an exact part of quantum TGD and that this is essential for understanding gravitation in long length scales. Also the analog of AdS/CFT duality emerges in that the Kähler metric can be defined either in terms of Kähler function identifiable as Kähler action assignable to Euclidian space-time regions or Kähler action + string action assignable to Minkowskian regions.

The recent view about construction of scattering amplitudes is very "stringy". By strong form of holography string world sheets and partonic 2-surfaces provide the data needed to construct scattering amplitudes. Space-time surfaces are however needed to realize quantum-classical correspondence necessary to understand the classical correlates of quantum measurement. There is a huge generalization of the duality symmetry of hadronic string models.

The proposal is that scattering amplitudes can be regarded as sequences of computational operations for the Yangian of super-symplectic algebra. Product and co-product define the basic vertices and realized geometrically as partonic 2-surfaces and algebraically as multiplication for the elements of Yangian identified as super-symplectic Noether charges assignable to strings. Any computational sequences connecting given collections of algebraic objects at the opposite boundaries of causal diamond (CD) produce identical scattering amplitudes.

Fusion of the Two Approaches via a Generalization of the Space-Time Concept

The problem is that the two approaches to TGD seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there could be "vapor phase" that is a "gas" of particle like 3-surfaces and string like objects (counterpart of the "baby universes" of GRT) and the non-conservation of energy in GRT corresponds to the transfer of energy between different sheets of the space-time and possible existence vapour phase.

. What one obtains is what I have christened as many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. ??** in the appendix of this book). One particular aspect is topological field quantization meaning that various classical fields assignable to a physical system correspond to space-time sheets representing the classical fields to that particular system. One can speak of the field body of a particular physical system. Field body consists of topological light rays, and electric and magnetic flux quanta. In Maxwell's theory the physical system does not possess this kind of field identity. The notion of the magnetic body is one of the key players in TGD inspired theory of consciousness and quantum biology. The existence of monopole flux tubes requiring no current as a source of the magnetic field makes it possible to understand the existence of magnetic fields in cosmological and astrophysical scales.

This picture became more detailed with the advent of zero energy ontology (ZEO). The basic notion of ZEO is causal diamond (CD) identified as the Cartesian product of CP_2 and of the intersection of future and past directed light-cones and having scale coming as an integer multiple of CP_2 size is fundamental. CDs form a fractal hierarchy and zero energy states decompose to products of positive and negative energy parts assignable to the opposite boundaries of CD defining the ends of the space-time surface. The counterpart of zero energy state in positive energy ontology is the pair of initial and final states of a physical event, say particle reaction.

At space-time level ZEO means that 3-surfaces are pairs of space-like 3-surfaces at the opposite light-like boundaries of CD. Since the extremals of Kähler action connect these, one can say that by holography the basic dynamical objects are the space-time surface connecting these 3-surfaces and identifiable as analogs of Bohr orbits. This changes totally the vision about notions like self-organization: self-organization by quantum jumps does not take for a 3-D system but for the entire 4-D field pattern associated with it.

General Coordinate Invariance (GCI) allows to identify the basic dynamical objects as space-like 3-surfaces at the ends of space-time surface at boundaries of CD: this means that space-time surface is analogous to Bohr orbit. An alternative identification of the lines of generalized Feynman diagrams is as light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian . Also the Euclidian 4-D regions can have a similar interpretation. The requirement that the two interpretations are equivalent, leads to a strong form of General Coordinate Invariance. The outcome is effective 2-dimensionality stating that the partonic 2-surfaces identified as intersections of the space-like ends of space-time surface and light-like wormhole throats are the fundamental objects. That only effective 2-dimensionality is in question is due to the effects caused by the failure of strict determinism of Kähler action. In finite length scale resolution these effects can be neglected below UV cutoff and above IR cutoff. One can also speak about a strong form of holography.

The understanding of the super symplectic invariance leads to the proposal that super symplectic algebra and other Kac-Moody type algebras labelled by non-negative multiples of basic conformal weights allow a hierarchy of symmetry breakings in which the analog of gauge symmetry breaks down to a genuine dynamical symmetry. This gives rise to fractal hierarchies of algebras and symmetry breakings. This breaking can occur also for ordinary conformal algebras if one restricts the conformal weights to be non-negative integers.

1.1.3 Basic Objections

Objections are the most powerful tool in theory building. The strongest objection against TGD is the observation that all classical gauge fields are expressible in terms of four embedding space coordinates only- essentially CP_2 coordinates. The linear superposition of classical gauge fields taking place independently for all gauge fields is lost. This would be a catastrophe without many-sheeted space-time. Instead of gauge fields, only the effects such as gauge forces are superposed. Particles topologically condense to several space-time sheets simultaneously and experience the sum of gauge forces. This transforms the weakness to extreme economy: in a typical unified theory the number of primary field variables is countered in hundreds if not thousands, now it is just four.

Second objection is that TGD space-time is quite too simple as compared to GRT space-time due to the embeddability to 8-D embedding space. One can also argue that Poincare invariant theory of gravitation cannot be consistent with General Relativity. The above interpretation makes it possible to understand the relationship to GRT space-time and how the Equivalence Principle (EP) follows from Poincare invariance of TGD. The interpretation of GRT space-time is as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrics of the space-time sheets from Minkowski metric. Poincare invariance strongly suggests classical EP for the GRT limit in long length scales at least. One can also consider other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with a very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of the standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Topological Field Quantization

Topological field quantization distinguishes between TGD based and more standard - say Maxwellian - notion of field. In Maxwell's fields created by separate systems superpose and one cannot tell which part of field comes from which system except theoretically. In TGD these fields correspond to different space-time sheets and only their effects on test particle superpose. Hence physical systems have well-defined field identifies - field bodies - in particular magnetic bodies.

The notion of magnetic body carrying dark matter with non-standard large value of Planck constant has become central concept in TGD inspired theory of consciousness and living matter, and by starting from various anomalies of biology one ends up to a rather detailed view about the role of magnetic body as intentional agent receiving sensory input from the biological body and controlling it using EEG and its various scaled up variants as a communication tool. Among other thins this leads to models for cell membrane, nerve pulse, and EEG.

1.1.4 Quantum TGD as Spinor Geometry of World of Classical Worlds

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and have been the following ones.

World of Classical Worlds

The notion of WCW reduces the interacting quantum theory to a theory of free WCW spinor fields.

1. Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude identified as WCW spinor in the configuration space CH ("world of classical worlds", WCW) consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included.

2. 4-D general coordinate invariance forces holography and replaces the ill-defined path integral over all space-time surfaces with a discrete sum over 4-D analogs of Bohr orbits for particles identified as 3-surfaces. Holography means that basic objects are these analogs of Bohr orbits. Since there is no quantization at the level of WCW, one has an analog of wave mechanics with point-like particles replaced with 4-D Bohr orbits.

3. One must geometrize WCW as the space of Bohr orbits. In an infinite-dimensional situation the existence of geometry requires maximal symmetries already in the case of loop spaces. Physics is unique from its mathematical existence.

   WCW is endowed with metric and spinor structure so that one can define various metric related differential operators, say Dirac operators, appearing in the field equations of the theory [1]

Identification of Kähler function

The evolution of these basic ideas has been rather slow but has gradually led to a rather beautiful vision. One of the key problems has been the definition of Kähler function. Kähler function is Kähler action for a preferred extremal assignable to a given 3-surface but what this preferred extremal is? The obvious first guess was as absolute minimum of Kähler action but could not be proven to be right or wrong. One big step in the progress was boosted by the idea that TGD should reduce to almost topological QFT in which braids would replace 3-surfaces in finite measurement resolution, which could be inherent property of the theory itself and imply discretization at partonic 2-surfaces with discrete points carrying fermion number.

It took long time to realize that there is no discretization in 4-D sense - this would lead to difficulties with basic symmetries. Rather, the discretization occurs for the parameters characterizing co-dimension 2 objects representing the information about space-time surface so that they belong to some algebraic extension of rationals. These 2-surfaces - string world sheets and partonic 2-surfaces - are genuine physical objects rather than a computational approximation. Physics itself approximates itself, one might say! This is of course nothing but strong form of holography.

1. TGD as almost topological QFT vision suggests that Kähler action for preferred extremals reduces to Chern-Simons term assigned with space-like 3-surfaces at the ends of space-time (recall the notion of causal diamond (CD)) and with the light-like 3-surfaces at which the signature of the induced metric changes from Minkowskian to Euclidian. Minkowskian and Euclidian regions would give at wormhole throats the same contribution apart from coefficients and in Minkowskian regions the $\sqrt{g_4}$ factorc coming from metric would be imaginary so that one would obtain sum of real term identifiable as Kähler function and imaginary term identifiable as the ordinary Minkowskian action giving rise to interference effects and stationary phase approximation central in both classical and quantum field theory.

   Imaginary contribution - the presence of which I realized only after 33 years of TGD - could also have topological interpretation as a Morse function. On physical side the emergence of Euclidian space-time regions is something completely new and leads to a dramatic modification of the ideas about black hole interior.

2. The way to achieve the reduction to Chern-Simons terms is simple. The vanishing of Coulomb contribution to Kähler action is required and is true for all known extremals if one makes a general ansatz about the form of classical conserved currents. The so called weak form of electric-magnetic duality defines a boundary condition reducing the resulting 3-D terms to Chern-Simons terms. In this way almost topological QFT results. But only “almost” since the Lagrange multiplier term forcing electric-magnetic duality implies that Chern-Simons action for preferred extremals depends on metric.

WCW spinor fields

Classical WCW spinor fields are analogous to Schrödinger amplitudes and the construction of WCW Kähler geometry reduces to the second quantization of free spinor fields of H.

[1]There are four kinds of Dirac operators in TGD. The geometrization of quantum theory requires Kähler metric definable either in terms of Kähler function identified as a the bosonic action for Euclidian space-time regions or as anti-commutators for WCW gamma matrices identified as conformal Noether super-charges associated with the second quantized modified Dirac action consisting of string world sheet term and possibly also modified Dirac action in Minkowskian space-time regions. These two possible definitions reflect a duality analogous to AdS/CFT duality.

1. The WCW metric is given by anticommutators of WCW gamma matrices which also have interpretation as supercharges assignable to the generators of WCW isometries and allowing expression as non-conserved Noether charges. Holography implies zero energy ontology (ZEO) meaning that zero energy states are superpositions of Bohr orbits connecting boundaries of causal diamond (CD). CDs form a fractal hierarchy and their space forming the spine of WCW is finite-dimensional and can be geometrized. The alternative interpretation is as a superposition of pairs of ordinary 3-D fermionic states assignable to the ends of the space-time surfaces.

2. There are several Dirac operators. WCW Dirac operator D_{WCW} appears in Super-symplectic gauge conditions analogous to Super Virasoro conditions. The algebraic variant of the H Dirac operator D_H appears in fermionic correlation functions: this is due to the fact that free fermions appearing as building bricks of WCW gamma matrices are modes of D_H. The modes of $_DH$ define the ground states of super-symplectic representations. There is also the modified Dirac operator D_{X^4} acting on the induced spinors at space-time surfaces and it is dictated by symmetry one the action fixing the space-time surfaces as Bohr orbits is fixed. D_H is needed since it determines the expressions of WCW gamma matrices as Noether charges assignable to 3-surfaces at the ends of WCW.

The role of modified Dirac action

1. By quantum classical correspondence, the construction of WCW spinor structure in sectors assignable to CDs reduces to the second quantization of the induced spinor fields of H. The basic action is so called modified Dirac action in which gamma matrices are replaced with the modified) gamma matrices defined as contractions of the canonical momentum currents of the bosonic action defining the space-time surfaces with the embedding space gamma matrices. In this way one achieves super-conformal symmetry and conservation of fermionic currents among other things and a consistent Dirac equation.

   Modified Dirac action is needed to define WCW gamma matrices as super charges assignable to WCW isometry generators identified as generators of symplectic transformations and by holography are needed only at the 3-surface at the boundaries of WCW. It is important to notice that the modified Dirac equation does not determine propagators since induced spinor fields are obtained from free second quantized spinor fields of H. This means enormous simplification and makes the theory calculable.

2. An important interpretational problem relates to the notion of the induced spinor connection. The presence of classical W boson fields is in conflict with the classical conservation of em charge since the coupling to classical W fields changes em charge.

   One way out of the problem is the fact that the quantum averages of weak and gluon fields vanish unlike the quantum average of the em field. This leads to a rather precise understanding of electroweak symmetry breaking as being due the fact that color symmetries rotate space-time surfaces and also affect the induced weak fields.

   One can also consider a stronger condition. If one requires that the spinor modes have well-defined em charge, one must assume that the modes in the generic situation are localized at 2-D surfaces - string world sheets or perhaps also partonic 2-surfaces - at which classical W boson fields vanish. Covariantly constant right handed neutrinos generating super-symmetries forms an exception. The vanishing of the Z^0 field is possible for Kähler-Dirac action and should hold true at least above weak length scales. This implies that the string model in 4-D space-time becomes part of TGD. Without these conditions classical weak fields can vanish above weak scale only for the GRT limit of TGD for which gauge potentials are sums over those for space-time sheets.

   The localization would simplify the mathematics enormously and one can solve exactly the Kähler-Dirac equation for the modes of the induced spinor field just like in super string models.

   At the light-like 3-surfaces the signature of the induced metric changes from Euclidian to Minkowskian so that $\sqrt{g_4}$ vanishes. One can pose the condition that the algebraic analog of the massless Dirac equation is satisfied by the modes of the modified-Dirac action assignable to the Chern-Simons-Kähler action.

1.1.5 Construction of scattering amplitudes

Reduction of particle reactions to space-time topology

Particle reactions are identified as topology changes [A38, A42, A47]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \to B + C$. Classically this corresponds to a path of WCW leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

During years this naïve and very rough vision has of course developed a lot and is not anymore quite equivalent with the original insight. In particular, the space-time correlates of Feynman graphs have emerged from theory as Euclidian space-time regions and the strong form of General Coordinate Invariance has led to a rather detailed and in many respects un-expected visions. This picture forces to give up the idea about smooth space-time surfaces and replace space-time surface with a generalization of Feynman diagram in which vertices represent the failure of manifold property. I have also introduced the word "world of classical worlds" (WCW) instead of rather formal "configuration space". I hope that "WCW" does not induce despair in the reader having tendency to think about the technicalities involved!

Construction of the counterparts of S-matrices

What does one mean with the counterpart of S-matrix in the TGD framework has been a long standing problem. The development of ZEO based quantum measurement theory has led to a rough overall view of the situation.

1. There are two kinds of state function reductions (SFRs). "Small" SFRs (SSFRs) following the TGD counterpart of a unitary time evolution defines a sequence of SFRs, which is analogous to a sequence of repeated quantum measurements associated with the Zeno effect. In wave mechanics nothing happens in these measurements. In quantum optics these measurements correspond to weak measurements. In TGD SSFR affects the zero energy state but leaves the 3-D state at the passive boundary of CD unaffected.

2. In TGD framework each SSFR is preceded by a counterpart of a unitary time evolution, which means dispersion in the space of CDs and unitary time evolution in fermionic degrees of freedom such that the passive boundary of CDs and 3-D states at it are unaffected but a superposition of CDs with varying active boundaries in the space of CDs is formed. In SSFR a localization in the space of CDs occurs such that the active is fixed. In a statistical sense the size of the CD increases and the increasing distance between the tips of the CD gives rise to the arrow of geometric time.

3. Also "big" SFRS (BSFRs) can occur and they correspond to ordinary SFRs. In BSFR the roles of the active and passive boundary are changed and this means that the arrow of time is changed. Big SFR occurs when the SSFR corresponds to a quantum measurement, which does not commute with the operators, which define the states at the passive boundary of CD as their eigenstates. This means a radical deviation from standard quantum measurement theory and has predictions in all scales.

4. One can assign the counterpart of S-matrix to the unitary time evolution between two subsequent SSFRs and also to the counterpart of S-matrix associated with BSFR. At least in the latter case the dimension of the state space can increase since at least BSFRs lead to the increase of the dimension of algebraic extension of rationals assignable to the space-time surface by $M^8 - H$ duality. Unitarity is therefore replaced with isometry.

5. I have also considered the possibility that unitary S-matrix could be replaced in the fermionic degrees of freedom with Kähler metric of the state space satisfying analogs of unitarity conditions but it seems that this is un-necessary and also too outlandish an idea.

The notion of M-matrix

1. The most ambitious dream is that zero energy states correspond to a complete solution basis for the Dirac operators associated with WCWs associated with the spaces of CDs with fixed passive boundary: this would define an S-matrix assignable to SFR. Also the analog of S-matrix for the localizations of the states to the active boundary assignable to the BSFR changing the state at the passive boundary of CD is needed.

2. If one allows entanglement between positive and energy parts of the zero energy state but assumes that the states at the passive boundary are fixed, one must introduce the counterpart of the density matrix, or rather its square root. This classical free field theory would dictate what I have called M-matrices defined between positive and negative energy parts of zero energy states which form orthonormal rows of what I call U-matrix as a matrix defined between zero energy states. A biven M-matrix in turn would decompose to a product of a hermitian square root of density matrix and unitary S-matrix.

3. M-matrix would define time-like entanglement coefficients between positive and negative energy parts of zero energy states (all net quantum numbers vanish for them) and can be regarded as a hermitian square root of density matrix multiplied by a unitary S-matrix. Quantum theory would be in a well-defined sense a square root of thermodynamics. The orthogonality and hermiticity of the M-matrices commuting with S-matrix means that they span infinite-dimensional Lie algebras acting as symmetries of the S-matrix. Therefore quantum TGD would reduce to group theory in a well-defined sense.

4. In fact the Lie algebra of Hermitian M-matrices extends to Kac-Moody type algebra obtained by multiplying hermitian square roots of density matrices with powers of the S-matrix. Also the analog of Yangian algebra involving only non-negative powers of S-matrix is possible and would correspond to a hierarchy of CDs with the temporal distances between tips coming as integer multiples of the CP_2 time.

   The M-matrices associated with CDs are obtained by a discrete scaling from the minimal CD and characterized by integer n are naturally proportional to a representation matrix of scaling: $S(n) = S^n$, where S is unitary S-matrix associated with the minimal CD [K60]. This conforms with the idea about unitary time evolution as exponent of Hamiltonian discretized to integer power of S and represented as scaling with respect to the logarithm of the proper time distance between the tips of CD.

5. I have also considered the notion of U-matrix. U-matrix elements between M-matrices for various CDs are proportional to the inner products $Tr[S^{-n_1} \circ H^i H^j \circ S^{n_2} \lambda]$, where λ represents unitarily the discrete Lorentz boost relating the moduli of the active boundary of CD and H^i form an orthonormal basis of Hermitian square roots of density matrices. $\circ$ tells that S acts at the active boundary of CD only. I have proposed a general representation for the U-matrix, reducing its construction to that of the S-matrix.

1.1.6 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space ("world of classical worlds", WCW), p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. The fourth thread deserves the name "TGD as a generalized number theory". It involves three separate threads: the fusion of real and various p-adic physics to a single coherent whole by requiring number theoretic universality discussed already, the formulation of quantum TGD in terms of complexified counterparts of classical number fields, and the notion of infinite prime. Note that one can identify subrings such as hyper-quaternions and hyper-octonions as sub-spaces of complexified classical number fields with Minkowskian signature of the metric defined by the complexified inner product.

The Threads in the Development of Quantum TGD

The development of TGD has involved several strongly interacting threads: physics as infinite-dimensional geometry; TGD as a generalized number theory, the hierarchy of Planck constants interpreted in terms of dark matter hierarchy, and TGD inspired theory of consciousness. In the following these threads are briefly described.

1. Quantum T(opological) G(eometro)D(ynamics) as a classical spinor geometry for infinite-dimensional WCW, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness and of quantum biology have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

2. The discussions with Tony Smith initiated a fourth thread which deserves the name "TGD as a generalized number theory". The basic observation was that classical number fields might allow a deeper formulation of quantum TGD. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and rather fruitful revision of the basic views about what the final form and physical content of quantum TGD might be. Together with the vision about the fusion of p-adic and real physics to a larger coherent structure these sub-threads fused to the "physics as generalized number theory" thread.

3. A further thread emerged from the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and strongly suggested by the failure of strict determinism for the fundamental variational principle. The identification of hierarchy of Planck constants labelling phases of dark matter would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

The chronology based identification of the threads is quite natural but not logical and it is much more logical to see p-adic physics, the ideas related to classical number fields, and infinite primes as sub-threads of a thread which might be called "physics as a generalized number theory". In the following I adopt this view. This reduces the number of threads to three corresponding to geometric, number theoretic and topological views of physics.

TGD forces the generalization of physics to a quantum theory of consciousness, and TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations.

Number theoretic vision very briefly

Number theoretic vision about quantum TGD involves notions like adelic physics, M^8-H duality and number theoretic universality. A short review of the basic ideas that have developed during years is in order.

1. The physical interpretation of M^8 is as an analog of momentum space and M^8-H duality is analogous to momentum-position duality of ordinary wave mechanics.

2. Adelic physics means that all classical number fields, all p-adic number fields and their extensions induced by extensions of rationals and defining adeles, and also finite number fields are basic mathematical building bricks of physics.

   The complexification of M^8, identified as complexified octonions, would provide a realization of this picture and M^8-H duality would map the algebraic physics in M^8 to the ordinary physics in $M^4\times CP_2$ described in terms of partial differential equations.

3. Negentropy Maximization Principle (NMP) states that the conscious information assignable with cognition representable measured in terms of p-adic negentropy increases in statistical sense.

   NMP is mathematically completely analogous to the second law of thermodynamics and number theoretic evolution as an unavoidable statistical increase of the dimension of the algebraic extension of rationals characterizing a given space-time region implies it. There is no paradox involved: the p-adic negentropy measures the conscious information assignable to the entanglement of two systems regarded as a conscious entity whereas ordinary entropy measures the lack of information about the quantums state of either entangled system.

4. Number theoretical universality requires that space-time surfaces or at least their M^8-H duals in M^8_c are defined for both reals and various p-adic number fields. This is true if they are defined by polynomials with integer coefficients as surfaces in M^8 obeying number theoretic holography realized as associativity of the normal space of 4-D surface using as holographic data 3-surfaces at mass shells identified in terms of roots of a polynomial. A physically motivated additional condition is that the coefficients of the polynomials are smaller than their degrees.

5. Galois confinement is a key piece of the number theoretic vision. It states that the momenta of physical states are algebraic integers in the extensions of rationals assignable to the space-time region considered. These numbers are in general complex and are not consistent with particle in box quantization. The proposal is that physical states satisfy Galois confinement being thus Galois singlets and having therefore total momenta, whose components are ordinary integers, when momentum unit defined by the scale of causal diamond (CD) is used.

6. The notion of p-adic prime was introduced in p-adic mass calculations that started the developments around 1995. p-Adic length scale hypothesis states that p-adic primes near powers of 2 have a special physical role (as possibly also the powers of other small primes such as $p=3$).

   The proposal is that p-adic primes correspond to ramified primes assignable to the extension and identified as divisors of the polynomial defined by the products of the root differences for the roots of the polynomial defining space-time space and having interpretation as values of, in general complex, virtual mass squared.

p-Adic TGD and fusion of real and p-adic physics to single coherent whole

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzles and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired "Universe as Computer" vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

In string model context one tries to reduces the physics to Planck scale. The price is the inability to say anything about physics in long length scales. In TGD p-adic physics takes care of this shortcoming by predicting the physics also in long length scales.

There were many interpretational and technical questions crying for a definite answer.

1. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *the* Physics? Should one perform p-adicization also at the level of the WCW? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations.
2. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question. Quite recently I however learned that the problem of making sense about p-adic integration has been for decades central problem in the frontier of mathematics and a lot of profound work has been done along same intuitive lines as I have proceeded in TGD framework. The basic idea is certainly the notion of algebraic continuation from the world of rationals belonging to the intersection of real world and various p-adic worlds.

Despite various uncertainties, the number of the applications of the poorly defined p-adic physics has grown steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

The key challenge is to fuse various p-adic physics and real physics to single larger structure. This has inspired a proposal for a generalization of the notion of number field by fusing real numbers and various p-adic number fields and their extensions along rationals and possible common algebraic numbers. This leads to a generalization of the notions of embedding space and space-time concept and one can speak about real and p-adic space-time sheets. One can talk about adelic space-time, embedding space, and WCW.

The corresponds of real 4-surfaces with the p-adic ones is induced by number theoretical discretization using points of 4-surfaces $Y^4 \subset M^8_c$ identifiable as 8-momenta, whose components are assumed to be algebraic integers in an extension of rationals defined by the extension of rationals associated with a polynomial P with integer coefficients smaller than the degree of P. These points define a cognitive representation, which is universal in the sense that it exists also in the algebraic extensions of p-adic numbers. The points of the cognitive representations associated with the mass shells with mass squared values identified as roots of P are enough since $M^8 - H$ duality can be used at both M^8 and H sides and also in the p-adic context. The mass shells are special in that they allow for Minkowski coordinates very large cognitive representations unlike the interiors of the 4-surfaces determined by holography by using the data defined by the 3-surfaces at the mass shells. The higher the dimension of the algebraic extension associated with P, the better the accuracy of the cognitive representation.

Adelization providing number theoretical universality reduces to algebraic continuation for the amplitudes from this intersection of reality and various p-adicities - analogous to a back of a book - to various number fields. There are no problems with symmetries but canonical identification is needed: various group invariant of the amplitude are mapped by canonical identification to various p-adic number fields. This is nothing but a generalization of the mapping of the p-adic mass squared to its real counterpart in p-adic mass calculations.

This leads to surprisingly detailed predictions and far reaching conjectures. For instance, the number theoretic generalization of entropy concept allows negentropic entanglement central for the applications to living matter (see **Fig.** `http://tgdtheory.fi/appfigures/cat.jpg` or **Fig. ??** in the appendix of this book). One can also understand how preferred p-adic primes could emerge as so called ramified primes of algebraic extension of rationals in question and characterizing string world sheets and partonic 2-surfaces. Preferred p-adic primes would be ramified primes for extensions for which the number of p-adic continuations of two-surfaces to space-time surfaces (imaginations) allowing also real continuation (realization of imagination) would be especially large. These ramifications would be winners in the fight for number theoretical survival. Also a generalization of p-adic length scale hypothesis emerges from NMP [K57].

The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to “mind stuff”, the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably a brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

Infinite primes

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory.

After the realization that infinite primes can be mapped to polynomials possibly representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of complexified quaternions and octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

The infinite primes at the first level of hierarchy, which represent analogs of bound states, can be mapped to irreducible polynomials, which in turn characterize the algebraic extensions of rationals defining a hierarchy of algebraic physics continuable to real and p-adic number fields. The products of infinite primes in turn define more general algebraic extensions of rationals. The interesting question concerns the physical interpretation of the higher levels in the hierarchy of infinite primes and integers mappable to polynomials of $n > 1$ variables.

1.1.7 An explicit formula for $M^8 - H$ duality

$M^8 - H$ duality is a generalization of momentum-position duality relating the number theoretic and geometric views of physics in TGD and, despite that it still involves poorly understood aspects, it has become a fundamental building block of TGD. One has 4-D surfaces $Y^4 \subset M^8_c$, where M^8_c is complexified M^8 having interpretation as an analog of complex momentum space and 4-D spacetime surfaces $X^4 \subset H = M^4 \times CP_2$. M^8_c, equivalently E^8_c, can be regarded as complexified octonions. M^8_c has a subspace M^4_c containing M^4.

Comment: One should be very cautious with the meaning of "complex". Complexified octonions involve a complex imaginary unit i commuting with the octonionic imaginary units I_k. i is assumed to also appear as an imaginary unit also in complex algebraic numbers defined by the roots of polynomials P defining holographic data in M^8_c.

In the following $M^8 - H$ duality and its twistor lift are discussed and an explicit formula for the dualities are deduced. Also possible variants of the duality are discussed.

Holography in H

$X^4 \subset H$ satisfies holography and is analogous to the Bohr orbit of a particle identified as a 3-surface. The proposal is that holography reduces to a 4-D generalization of holomorphy so that X^4 is a simultaneous zero of two functions of complex CP_2 coordinates and of what I have called Hamilton-Jacobi coordinates of M^4 with a generalized Kähler structure.

The simplest choice of the Hamilton-Jacobi coordinates is defined by the decomposition $M^4 = M^2 \times E^2$, where M^2 is endowed with hypercomplex structure defined by light-like coordinates (u, v), which are analogous to z and $\overline{z}$. Any analytic map $u \rightarrow f(u)$ defines a new set of light-like coordinates and corresponds to a solution of the massless d'Alembert equation in M^2. E^2 has some complex coordinates with imaginary unit defined by i.

The conjecture is that also more general Hamilton-Jacobi structures for which the tangent space decomposition is local are possible. Therefore one would have $M^4 = M^2(x) \times E^2(x)$. These would correspond to non-equivalent complex and Kähler structures of M^4 analogous to those possessed by 2-D Riemann surfaces and parametrized by moduli space.

Number theoretic holography in M^8_c

$Y^4 \subset M^8_c$ satisfies number theoretic holography defining dynamics, which should reduce to associativity in some sense. The Euclidian complexified normal space $N^4(y)$ at a given point y of Y^4 is required to be associative, i.e. quaternionic. Besides this, $N^4(i)$ contains a preferred complex Euclidian 2-D subspace $Y^2(y)$. Also the spaces $Y^2(x)$ define an integrable distribution. I have assumed that $Y^2(x)$ can depend on the point y of Y^4.

These assumptions imply that the normal space $N(y)$ of Y^4 can be parameterized by a point of $CP_2 = SU(3)/U(2)$. This distribution is always integrable unlike quaternionic tangent space distributions. $M^8 - H$ duality assigns to the normal space $N(y)$ a point of CP_2. M^4_c point y is mapped to a point $x \in M^4 \subset M^4 \times CP_2$ defined by the real part of its inversion (conformal transformation): this formula involves effective Planck constant for dimensional reasons.

The 3-D holographic data, which partially fixes 4-surfaces Y^4 is partially determined by a polynomial P with real integer coefficients smaller than the degree of P. The roots define mass squared values which are in general complex algebraic numbers and define complex analogs of mass shells in $M^4_c \subset M^8_c$, which are analogs of hyperbolic spaces H^3. The 3-surfaces at these mass shells define 3-D holographic data continued to a surface Y^4 by requiring that the normal space of Y^4 is associative, i.e. quaternionic. These 3-surfaces are not completely fixed but an interesting conjecture is that they correspond to fundamental domains of tessellations of H^3.

What does the complexity of the mass shells mean? The simplest interpretation is that the space-like M^4 coordinates (3-momentum components) are real whereas the time-like coordinate (energy) is complex and determined by the mass shell condition. One would have $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3 when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as a time coordinate. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to an equation of mass shell when $\sqrt{(Re(E)^2 - Im(E)^2)}$, expressed in terms of $Re(E)$, is taken as new energy coordinate $E_{eff} = \sqrt{(Re(E)^2 - Im(E)^2)}$. Is this deformation of H^3 in imaginary time direction equivalent with a region of the hyperbolic 3-space H^3?

One can look at the formula in more detail. Mass shell condition gives $Re^2(E) - Im(E)^2 - p^2 = Re(m^2)$ and $2Re(E)Im(E) = Im(m^2)$. The condition for the real parts gives H^3, when $\sqrt{Re^2(E) - Im(E)^2}$ is taken as an effective energy. The second condition allows to solve $Im(E)$ in terms of $Re(E)$ so that the first condition reduces to a dispersion relation for $Re(E)^2$.

$$Re(E)^2 = \frac{1}{2}(Re(m^2) - Im(m^2) + p^2)(1 \pm \sqrt{1 + \frac{2Im(m^2)^2}{(Re(m^2) - Im(m^2) + p^2)^2}} . \quad (1.1.1)$$

Only the positive root gives a non-tachyonic result for $Re(m^2) - Im(m^2) > 0$. For real roots with $Im(m^2) = 0$ and at the high momentum limit the formula coincides with the standard formula. For $Re(m^2) = Im(m^2)$ one obtains $Re(E)^2 \rightarrow Im(m^2)/\sqrt{2}$ at the low momentum limit $p^2 \rightarrow 0$. Energy does not depend on momentum at all: the situation resembles that for plasma waves.

Can one find an explicit formula for $M^8 - H$ duality?

The dream is an explicit formula for the $M^8 - H$ duality mapping $Y^4 \subset M^8_c$ to $X^4 \subset H$. This formula should be consistent with the assumption that the generalized holomorphy holds true for X^4.

The following proposal is a more detailed variant of the earlier proposal for which Y^4 is determined by a map g of $M^4_c \rightarrow SU(3)_c \subset G_{2,c}$, where $G_{2,c}$ is the complexified automorphism group of octonions and $SU(3)_c$ is interpreted as a complexified color group.

This map defines a trivial $SU(3)_c$ gauge field. The real part of g however defines a non-trivial real color gauge field by the non-linearity of the non-abelian gauge field with respect to the gauge potential. The quadratic terms involving the imaginary part of the gauge potential give an additional condition to the real part in the complex situation and cancel it. If only the real part of g contributes, this contribution would be absent and the gauge field is non-vanishing.

How could the automorphism $g(x) \subset SU(3) \subset G_2$ give rise to $M^8 - H$ duality?

1. The interpretation is that $g(y)$ at given point y of Y^4 relates the normal space at y to a fixed quaternionic/associative normal space at point y_0, which corresponds is fixed by some subgroup $U(2)_0 \subset SU(3)$. The automorphism property of g guarantees that the normal space is quaternionic/associative at y. This simplifies the construction dramatically.

2. The quaternionic normal sub-space (which has Euclidian signature) contains a complex sub-space which corresponds to a point of sphere $S^2 = SO(3)/O(2)$, where $SO(3)$ is the quaternionic automorphism group. The interpretation could be in terms of a selection of spin quantization axes. The local choice of the preferred complex plane would not be unique and is analogous to the possibility of having non-trivial Hamilton Jacobi structures in M^4 characterized by the choice of $M^2(x)$ and equivalently its normal subspace $E^2(x)$.

   These two structures are independent apart from dependencies forced by the number theoretic dynamics. Hamilton-Jacobi structure means a selection of the quantization axis of spin and energy by fixing a distribution of light-like tangent vectors of M^4 and the choice of the quaternionic normal sub-space fixes a choice of preferred quaternionic imaginary unit defining a quantization axis of the weak isospin.

3. The real part $Re(g(y))$ defines a point of $SU(3)$ and the bundle projection $SU(3) \rightarrow CP_2$ in turn defines a point of $CP_2 = SU(3)/U(2)$. Hence one can assign to g a point of CP_2 as $M^8 - H$ duality requires and deduce an explicit formula for the point. This means a realization of the dream.

4. The construction requires a fixing of a quaternionic normal space N_0 at y_0 containing a preferred complex subspace at a single point of Y^4 plus a selection of the function g. If M^4 coordinates are possible for Y^4, the first guess is that g as a function of complexified M^4 coordinates obeys generalized holomorphy with respect to complexified M^4 coordinates in the same sense and in the case of X^4. This might guarantee that the $M^8 - H$ image of Y^4 satisfies the generalized holomorphy.

5. Also space-time surfaces X^4 with M^4 projection having a dimension smaller than 4 are allowed. I have proposed that they might correspond to singular cases for the above formula: a kind of blow-up would be involved. One can also consider a more general definition of Y^4 allowing it to have a M^4 projection with dimension smaller than 4 (say cosmic strings). Could one have implicit equations for the surface Y^4 in terms of the complex coordinates of $SU(3)_c$ and M^4? Could this give for instance cosmic strings with a 2-D M^4 projection and CP_2 type extremals with 4-D CP_2 projection and 1-D light-like M^4 projection?

What could the number theoretic holography mean physically?

What could be physical meaning of the number theoretic holography? The condition that has been assumed is that the CP_2 coordinates at the mass shells of $M^4_c \subset M^8_c$ mapped to mass shells H^3 of $M^4 \subset M^4 \times CP_2$ are constant at the H^3. This is true if the $g(y)$ defines the same CP_2 point for a given component X^3_i of the 3-surface at a given mass shell. g is therefore fixed apart from a local $U(2)$ transformation leaving the CP_2 point invariant. A stronger condition would be that the CP_2 point is the same for each component of X^3_i and even at each mass shell but this condition seems to be unnecessarily strong.

Comment: One can o criticize this condition as too strong and one can consider giving up this condition. The motivation for this condition is that the number of algebraic points at the 3-surfaces associated with H^3 explodes since the coordinates associated with normal directions vanish. Kind of cognitive explosion would be in question.

$SU(3)$ corresponds to a subgroup of G_2 and one can wonder what the fixing of this subgroup could mean physically. G_2 is 14-D and the coset space $G_2/SU(3)$ is 6-D and a good guess is that it is just the 6-D twistor space $SU(3)/U(1) \times U(1)$ of CP_2: at least the isometries are the same.

The fixing of the $SU(3)$ subgroup means fixing of a CP_2 twistor. Physically this means the fixing of the quantization axis of color isospin and hypercharge.

Twistor lift of the holography

What is interesting is that by replacing $SU(3)$ with G_2, one obtains an explicit formula form the generalization of M^8-H duality to that for the twistorial lift of TGD!

One can also consider a twistorial generalization of the above proposal for the number theoretic holography by allowing local G_2 automorphisms interpreted as local choices of the color quantization axis. G_2 elements would be fixed apart from a local $SU(3)$ transformation at the components of 3-surfaces at mass shells. The choice of the color quantization axes for a connected 3-surface at a given mass shell would be the same everywhere. This choice is indeed very natural physically since 3-surface corresponds to a particle.

Is this proposal consistent with the boundary condition of the number theoretical holography mean in the case of 4-surfaces in M_c^8 and $M^4\times CP_2$?

1. The selection of $SU(3)\subset G_2$ for ordinary M^8-H duality means that the $G_{2,c}$ gauge field vanishes everywhere and the choice of color quantization axis is the same at all points of the 4-surface. The fixing of the CP_2 point to be constant at H^3 implies that the color gauge field at $H^3\subset M_c^8$ and its image $H^3\subset H$ vanish. One would have color confinement at the mass shells H_i^3, where the observations are made. Is this condition too strong?
2. The constancy of the G_2 element at mass shells makes sense physically and means a fixed color quantization axis. The selection of a fixed $SU(3)\subset G_2$ for entire space-time surface is in conflict with the non-constancy of G_2 element unless G_2 element differs at different points of 4-surface only by a multiplication of a local $SU(3)_0$ element, that is local $SU(3)$ transformation. This kind of variation of the G_2 element would mean a fixed color group but varying choice of color quantization axis.
3. Could one consider the possibility that the local $G_{2,c}$ element is free and defines the twistor lift of M^8-H duality as something more fundamental than the ordinary M^8-H duality based on $SU(3)_c$. This duality would make sense only at the mass shells so that only the spaces $H^3\times CP_2$ assignable to mass shells would make sense physically? In the interior CP_2 would be replaced with the twistor space $SU(3)/U(1)\times U(1)$. Color gauge fields would be non-vanishing at the mass shells but outside the mass shells one would have G_2 gauge fields.

   There is also a physical objection against the G_2 option. The 14-D Lie algebra representation of G_2 acts on the imaginary octonions which decompose with respect to the color group to $1\oplus 3\oplus \overline{3}$. The automorphism property requires that 1 can be transformed to 3 or $\overline{3}$ to themselves: this requires that the decomposition contains $3\oplus\overline{3}$. Furthermore, it must be possible to transform 3 and $\overline{3}$ to themselves, which requires the presence of 8. This leaves only the decomposition $8\oplus 3\oplus\overline{3}$. G_2 gluons would both color octet and triplets. In the TDG framework the only conceivable interpretation would be in terms of ordinary gluons and leptoquark-like gluons. This does not fit with the basic vision of TGD.

The choice of twistor as a selection of quantization axes should make sense also in the M^4 degrees of freedom. M^4 twistor corresponds to a choice of light-like direction at a given point of M^4. The spatial component of the light-like vector fixes the spin quantization axis. Its choice together with the light-likeness fixes the time direction and therefore the rest system and energy quantization axis. Light-like vector fixes also the choice of M^2 and of E^2 as its orthogonal complement. Therefore the fixing of M^4 twistor as a point of $SU(4)/SU(3)\times U(1)$ corresponds to a choice of the spin quantization axis and the time-like axis defining the rest system in which the energy is measured. This choice would naturally correspond to the Hamilton-Jacobi structure fixing the decompositions $M^2(x)\times E^2(x)$. At a given mass shell the choice of the quantization axis would be constant for a given X_i^3.

1.1.8 Hierarchy of Planck Constants and Dark Matter Hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that

Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

Dark Matter as Large $\hbar$ Phases

D. Da Rocha and Laurent Nottale [E25] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests that astrophysical systems are at some levels of the hierarchy of space-time sheets macroscopic quantum systems. The space-time sheets in question would carry dark matter.

Nottale's hypothesis would predict a gigantic value of h_{gr}. Equivalence Principle and the independence of gravitational Compton length on mass m implies however that one can restrict the values of mass m to masses of microscopic objects so that h_{gr} would be much smaller. Large h_{gr} could provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [K79].

It is natural to assign the values of Planck constants postulated by Nottale to the space-time sheets mediating gravitational interaction and identifiable as magnetic flux tubes (quanta) possibly carrying monopole flux and identifiable as remnants of cosmic string phase of primordial cosmology. The magnetic energy of these flux quanta would correspond to dark energy and magnetic tension would give rise to negative "pressure" forcing accelerate cosmological expansion. This leads to a rather detailed vision about the evolution of stars and galaxies identified as bubbles of ordinary and dark matter inside magnetic flux tubes identifiable as dark energy.

Certain experimental findings suggest the identification $h_{eff} = n\times = h_{gr}$. The large value of h_{gr} can be seen as a way to reduce the string tension of fermionic strings so that gravitational (in fact all!) bound states can be described in terms of strings connecting the partonic 2-surfaces defining particles (analogous to AdS/CFT description). The values $h_{eff}/h = n$ can be interpreted in terms of a hierarchy of breakings of super-conformal symmetry in which the super-conformal generators act as gauge symmetries only for a sub-algebras with conformal weights coming as multiples of n. Macroscopic quantum coherence in astrophysical scales is implied. If also Kähler-Dirac action is present, part of the interior degrees of freedom associated with the Kähler-Dirac part of conformal algebra become physical. A possible is that tfermionic oscillator operators generate super-symmetries and sparticles correspond almost by definition to dark matter with $h_{eff}/h = n > 1$. One implication would be that at least part if not all gravitons would be dark and be observed only through their decays to ordinary high frequency graviton ($E = hf_{high} = h_{eff}f_{low}$) of bunch of n low energy gravitons.

Hierarchy of Planck Constants from the Anomalies of Neuroscience and Biology

The quantal ELF effects of ELF em fields on vertebrate brain have been known since seventies. ELF em fields at frequencies identifiable as cyclotron frequencies in magnetic field whose intensity is about 2/5 times that of Earth for biologically important ions have physiological effects and affect also behavior. What is intriguing that the effects are found only in vertebrates (to my best knowledge). The energies for the photons of ELF em fields are extremely low - about 10^{-10} times lower than thermal energy at physiological temperatures- so that quantal effects are impossible in the framework of standard quantum theory. The values of Planck constant would be in these situations large but not gigantic.

This inspired the hypothesis that these photons correspond to so large a value of Planck constant that the energy of photons is above the thermal energy. The proposed interpretation was as dark photons and the general hypothesis was that dark matter corresponds to ordinary matter with non-standard value of Planck constant. If only particles with the same value of Planck constant can appear in the same vertex of Feynman diagram, the phases with different value of Planck constant are dark relative to each other. The phase transitions changing Planck constant can however make possible interactions between phases with different Planck constant but these interactions do not manifest themselves in particle physics. Also the interactions mediated by classical fields should be possible. Dark matter would not be so dark as we have used to believe.

The hypothesis $h_{eff} = h_{gr}$ - at least for microscopic particles - implies that cyclotron energies of charged particles do not depend on the mass of the particle and their spectrum is thus universal although corresponding frequencies depend on mass. In bio-applications this spectrum would correspond to the energy spectrum of bio-photons assumed to result from dark photons by h_{eff} reducing phase transition and the energies of bio-photons would be in visible and UV range associated with the excitations of bio-molecules.

Also the anomalies of biology (see for instance [K72, K73, K70]) support the view that dark matter might be a key player in living matter.

Dark Matter as a Source of Long Ranged Weak and Color Fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics.

The recent view about the solutions of Kähler- Dirac action assumes that the modes have a well-defined em charge and this implies that localization of the modes to 2-D surfaces (right-handed neutrino is an exception). Classical W boson fields vanish at these surfaces and also classical Z^0 field can vanish. The latter would guarantee the absence of large parity breaking effects above intermediate boson scale scaling like h_{eff}.

1.1.9 Twistors in TGD and connection with Veneziano duality

The twistorialization of TGD has two aspects. The attempt to generalize twistor Grassmannian approach emerged first. It was however followed by the realization that also the twistor lift of TGD at classical space-time level is needed. It turned out that the progress in the understanding of the classical twistor lift has been much faster - probably this is due to my rather limited technical QFT skills.

Twistor lift at space-time level

8-dimensional generalization of ordinary twistors is highly attractive approach to TGD [K94]. The reason is that M^4 and CP_2 are completely exceptional in the sense that they are the only 4-D manifolds allowing twistor space with Kähler structure [A35]. The twistor space of $M^4 \times CP_2$ is Cartesian product of those of M^4 and CP_2. The obvious idea is that space-time surfaces allowing twistor structure if they are orientable are representable as surfaces in H such that the properly induced twistor structure co-incides with the twistor structure defined by the induced metric.

In fact, it is enough to generalize the induction of spinor structure to that of twistor structure so that the induced twistor structure need not be identical with the ordinary twistor structure possibly assignable to the space-time surface. The induction procedure reduces to a dimensional reduction of 6-D Kähler action giving rise to 6-D surfaces having bundle structure with twistor sphere as fiber and space-time as base. The twistor sphere of this bundle is imbedded as sphere in the product of twistor spheres of twistor spaces of M^4 and CP_2.

This condition would define the dynamics, and the original conjecture was that this dynamics is equivalent with the identification of space-time surfaces as preferred extremals of Kähler action. The dynamics of space-time surfaces would be lifted to the dynamics of twistor spaces, which are sphere bundles over space-time surfaces. What is remarkable that the powerful machinery of complex analysis becomes available.

It however turned out that twistor lift of TGD is much more than a mere technical tool. First of all, the dimensionally reduction of 6-D Kähler action contained besides 4-D Kähler action also a volume term having interpretation in terms of cosmological constant. This need not bring anything new, since all known extremals of Kähler action with non-vanishing induced Kähler form are minimal surfaces. There is however a large number of embeddings of twistor sphere of space-time surface to the product of twistor spheres. Cosmological constant has spectrum and depends on length scale, and the proposal is that coupling constant evolution reduces to that for cosmological constant playing the role of cutoff length. That cosmological constant could transform from a mere nuisance to a key element of fundamental physics was something totally new and unexpected.

1. The twistor lift of TGD at space-time level forces to replace 4-D Kähler action with 6-D dimensionally reduced Kähler action for 6-D surface in the 12-D Cartesian product of 6-D twistor spaces of M^4 and CP_2. The 6-D surface has bundle structure with twistor sphere as fiber and space-time surface as base.

   Twistor structure is obtained by inducing the twistor structure of 12-D twistor space using dimensional reduction. The dimensionally reduced 6-D Kähler action is sum of 4-D Kähler action and volume term having interpretation in terms of a dynamical cosmological constant depending on the size scale of space-time surface (or of causal diamond CD in zero energy ontology (ZEO)) and determined by the representation of twistor sphere of space-time surface in the Cartesian product of the twistor spheres of M^4 and CP_2.

2. The preferred extremal property as a representation of quantum criticality would naturally correspond to minimal surface property meaning that the space-time surface is separately an extremal of both Kähler action and volume term almost everywhere so that there is no coupling between them. This is the case for all known extremals of Kähler action with non-vanishing induced Kähler form.

   Minimal surface property could however fail at 2-D string world sheets, their boundaries and perhaps also at partonic 2-surfaces. The failure is realized in minimal sense if the 3-surface has 1-D edges/folds (strings) and 4-surface 2-D edges/folds (string world sheets) at which some partial derivatives of the embedding space coordinates are discontinuous but canonical momentum densities for the entire action are continuous.

   There would be no flow of canonical momentum between interior and string world sheet and minimal surface equations would be satisfied for the string world sheet, whose 4-D counterpart in twistor bundle is determined by the analog of 4-D Kähler action. These conditions allow the transfer of canonical momenta between Kähler- and volume degrees of freedom at string world sheets. These no-flow conditions could hold true at least asymptotically (near the boundaries of CD).

   M^8-H duality suggests that string world sheets (partonic 2-surfaces) correspond to images of complex 2-sub-manifolds of M^8 (having tangent (normal) space which is complex 2-plane of octonionic M^8).

3. Cosmological constant would depend on p-adic length scales and one ends up to a concrete model for the evolution of cosmological constant as a function of p-adic length scale and other number theoretic parameters (such as Planck constant as the order of Galois group): this conforms with the earlier picture.

   Inflation is replaced with its TGD counterpart in which the thickening of cosmic strings to flux tubes leads to a transformation of Kähler magnetic energy to ordinary and dark matter. Since the increase of volume increases volume energy, this leads rapidly to energy minimum at some flux tube thickness. The reduction of cosmological constant by a phase transition however leads to a new expansion phase. These jerks would replace smooth cosmic expansion of GRT. The discrete coupling constant evolution predicted by the number theoretical vision could be understood as being induced by that of cosmological constant taking the role of cutoff parameter in QFT picture [L48].

Twistor lift at the level of scattering amplitudes and connection with Veneziano duality

The classical part of twistor lift of TGD is rather well-understood. Concerning the twistorialization at the level of scattering amplitudes the situation is much more difficult conceptually - I already mentioned my limited QFT skills.

1. From the classical picture described above it is clear that one should construct the 8-D twistorial counterpart of theory involving space-time surfaces, string world sheets and their boundaries, plus partonic 2-surfaces and that this should lead to concrete expressions for the scattering amplitudes.

   The light-like boundaries of string world sheets as carriers of fermion numbers would correspond to twistors as they appear in twistor Grassmann approach and define the analog for the massless sector of string theories. The attempts to understand twistorialization have been restricted to this sector.

2. The beautiful basic prediction would be that particles massless in 8-D sense can be massive in 4-D sense. Also the infrared cutoff problematic in twistor approach emerges naturally and reduces basically to the dynamical cosmological constant provided by classical twistor lift.

   One can assign 4-momentum both to the spinor harmonics of the embedding space representing ground states of super-conformal representations and to light-like boundaries of string world sheets at the orbits of partonic 2-surfaces. The two four-momenta should be identical by quantum classical correspondence: this could be seen as a concretization of Equivalence Principle. Also a connection with string model emerges.

3. As far as symmetries are considered, the picture looks rather clear. Ordinary twistor Grassmannian approach boils down to the construction of scattering amplitudes in terms of Yangian invariants for conformal group of M^4. Therefore a generalization of super-symplectic symmetries to their Yangian counterpart seems necessary. These symmetries would be gigantic but how to deduce their implications?

4. The notion of positive Grassmannian is central in the twistor approach to the scattering amplitudes in $calN = 4$ SUSYs. TGD provides a possible generalization and number theoretic interpretation of this notion. TGD generalizes the observation that scattering amplitudes in twistor Grassmann approach correspond to representations for permutations. Since 2-vertex is the only fermionic vertex in TGD, OZI rules for fermions generalizes, and scattering amplitudes are representations for braidings.

   Braid interpretation encourages the conjecture that non-planar diagrams can be reduced to ordinary ones by a procedure analogous to the construction of braid (knot) invariants by gradual un-braiding (un-knotting).

This is however not the only vision about a solution of non-planarity. Quantum criticality provides different view leading to a totally unexpected connection with string models, actually with the Veneziano duality, which was the starting point of dual resonance model in turn leading via dual resonance models to super string models.

1. Quantum criticality in TGD framework means that coupling constant evolution is discrete in the sense that coupling constants are piecewise constant functions of length scale replaced by dynamical cosmological constant. Loop corrections would vanish identically and the recursion formulas for the scattering amplitudes (allowing only planar diagrams) deduced in twistor Grassmann would involve no loop corrections. In particular, cuts would be replaced by sequences of poles mimicking them like sequences of point charge mimic line charges. In momentum discretization this picture follows automatically.

2. This would make sense in finite measurement resolution realized in number theoretical vision by number-theoretic discretization of the space-time surface (cognitive representation) as points with coordinates in the extension of rationals defining the adele [L28]. Similar discretization would take place for momenta. Loops would vanish at the level of discretization but what would happen at the possibly existing continuum limit: does the sequence of poles integrate to cuts? Or is representation as sum of resonances something much deeper?

3. Maybe it is! The basic idea of behind the original Veneziano amplitudes (see http://tinyurl.com/yyhwvbqb) was Veneziano duality. This 4-particle amplitude was generalized by Yoshiro Nambu, Holber-Beck Nielsen, and Leonard Susskind to N-particle amplitude (see http://tinyurl.com/yyvkx7as) based on string picture, and the resulting model was called dual resonance model. The model was forgotten as QCD emerged. Later came superstring models and led to M-theory. Now it has become clear that something went wrong, and it seems that one must return to the roots. Could the return to the roots mean a careful reconsideration of the dual resonance model?

4. Recall that Veneziano duality (1968) was deduced by assuming that scattering amplitude can be described as sum over s-channel resonances or t-channel Regge exchanges and Veneziano duality stated that hadronic scattering amplitudes have representation as sums over s- or t-channel resonance poles identified as excitations of strings. The sum over exchanges defined by t-channel resonances indeed reduces at larger values of s to Regge form.

   The resonances had zero width, which was not consistent with unitarity. Further, there were no counterparts for the *sum* of s-, t-, and u-channel diagrams with continuous cuts in the kinematical regions encountered in QFT approach. What puts bells ringing is the u-channel diagrams would be non-planar and non-planarity is the problem of twistor Grassmann approach.

5. Veneziano duality is true only for s- and t- channels but not been s- and u-channel. Stringy description makes t-channel and s-channel pictures equivalent. Could it be that in fundamental description u-channels diagrams cannot be distinguished from s-channel diagrams or t-channel diagrams? Could the stringy representation of the scattering diagrams make u-channel twist somehow trivial if handles of string world sheet representing stringy loops in turn representing the analog of non-planarity of Feynman diagrams are absent? The permutation of external momenta for tree diagram in absence of loops in planar representation would be a twist of π in the representation of planar diagram as string world sheet and would not change the topology of the string world sheet and would not involve non-trivial world sheet topology.

   For string world sheets loops would correspond to handles. The presence of handle would give an edge with a loop at the level of 3-surface (self energy correction in QFT). Handles are not allowed if the induced metric for the string world sheet has Minkowskian signature. If the stringy counterparts of loops are absent, also the loops in scattering amplitudes should be absent.

   This argument applies only inside the Minkowskian space-time regions. If string world sheets are present also in Euclidian regions, they might have handles and loop corrections could emerge in this manner. In TGD framework strings (string world sheets) are identified to 1-D edges/folds of 3-surface at which minimal surface property and topological QFT property fails (minimal surfaces as calibrations). Could the interpretation of edge/fold as discontinuity of some partial derivatives exclude loopy edges: perhaps the branching points would be too singular?

A reduction to a sum over s-channel resonances is what the vanishing of loops would suggest. Could the presence of string world sheets make possible the vanishing of continuous cuts even at the continuum limit so that continuum cuts would emerge only in the approximation as the density of resonances is high enough?

The replacement of continuous cut with a sum of *infinitely* narrow resonances is certainly an approximation. Could it be that the stringy representation as a sum of resonances with *finite* width is an essential aspect of quantum physics allowing to get rid of infinities necessarily accompanying loops? Consider now the arguments against this idea.

1. How to get rid of the problems with unitarity caused by the zero width of resonances? Could *finite* resonance widths make unitarity possible? Ordinary twistor Grassmannian approach predicts that the virtual momenta are light-like but complex: obviously, the imaginary part of the energy in rest frame would have interpretation as resonance with.

   In TGD framework this generalizes for 8-D momenta. By quantum-classical correspondence (QCC) the classical Noether charges are equal to the eigenvalues of the fermionic charges in Cartan algebrable (maximal set of mutually commuting observables) and classical TGD indeed predicts complex momenta (Kähler coupling strength is naturally complex). QCC thus supports this proposal.

2. Sum over resonances/exchanges picture is in conflict with QFT picture about scattering of particles. Could *finite* resonance widths due to the complex momenta give rise to the QFT type scattering amplitudes as one develops the amplitudes in Taylor series with respect to the resonance width? Unitarity condition indeed gives the first estimate for the resonance width.

   QFT amplitudes should emerge in an approximation obtained by replacing the discrete set of finite width resonances with a cut as the distance between poles is shorter than the resolution for mass squared.

   In superstring models string tension has single very large value and one cannot obtain QFT type behavior at low energies (for instance, scattering amplitudes in hadronic string model are concentrated in forward direction). TGD however predicts an entire hierarchy of p-adic length scales with varying string tension. The hierarchy of mass scales corresponding roughly to the lengths and thickness of magnetic flux tubes as thickened cosmic strings and characterized by the value of cosmological constant predicted by twistor lift of TGD. Could this give rise to continuous QCT type cuts at the limit when measurement resolution cannot distinguish between resonances?

   The dominating term in the sum over sums of resonances in t-channel gives near forward direction approximately the lowest mass resonance for strings with the smallest string tension. This gives the behavior $1/(t-m_{min}^2)$, where m_{min} corresponds to the longest mass scale involved (the largest space-time sheet involved), approximating the $1/t$-behavior of massless theories. This also brings in IR cutoff, the lack of which is a problem of gauge theories. This should give rise to continuous QFT type cuts at the limit when measurement resolution cannot distinguish between resonances.

1.2 Bird's Eye of View about the Topics of the Book

This book is mostly devoted to what might be called classical TGD.

1. In a well-defined sense classical TGD defined as the dynamics of space-time surfaces determining them as kind of generalized Bohr orbits can be regarded as an exact part of quantum theory and assuming quantum classical correspondence has served as an extremely valuable guideline in the attempts to interpret TGD, to form a view about what TGD really predicts, and to guess what the underlying quantum theory could be and how it deviates from standard quantum theory.

2. The notions of many-sheeted space-time, topological field quantization and the notion of field/magnetic body, follow from simple topological considerations. The observation that space-time sheets can have arbitrarily large sizes and their interpretation as quantum coherence regions forces to conclude that in TGD Universe macroscopic and macro-temporal quantum coherence are possible in arbitrarily long scales. Also long ranged classical color and electro-weak fields are an unavoidable prediction.

3. It took a considerable time to make the obvious conclusion: TGD Universe is fractal containing fractal copies of standard model physics at various space-time sheets and labeled by the collection of p-adic primes assignable to elementary particles and by the level of dark matter hierarchy characterized partially by the value of Planck constant labeling the pages of the book like structure formed by singular covering spaces of the embedding space $M^4 \times CP_2$ glued together along a four-dimensional back. Particles at different pages are dark relative to each other since purely local interactions defined in terms of the vertices of Feynman diagram involve only particles at the same page.

4. The new view about energy and time justified by the notion of zero energy ontology (ZEO) means that the sign of inertial energy depends on the time orientation of the space-time sheet and that negative energy space-time sheets serve as correlates for communications to the geometric past. This alone leads to profoundly new views about metabolism, long term memory, and realization of intentional action.

1.2.1 The Implications Deriving From The Topology Of Space-Time Surface And From The Properties Of Induced Gauge Fields

1. The general properties of Kähler action, in particular its vacuum degeneracy and failure of the classical determinism in the conventional sense, have rather far reaching implications. Space-time surfaces as a generalization of Bohr orbit provide not only a representation of quantum states but also sequences of quantum jumps and thus contents of consciousness. Vacuum degeneracy implies spin glass degeneracy in 4-D sense reflecting quantum criticality which is the fundamental characteristic of TGD Universe.
2. The detailed study of the simplest extremals of Kähler action interpreted as correlates for asymptotic self organization patterns provides additional insights [K11] . CP_2 type extremals representing elementary particles, cosmic strings, vacuum extremals, topological light rays ("massless extremal", ME), flux quanta of magnetic and electric fields represent the basic extremals. Pairs of wormhole throats identifiable as parton pairs define a completely new kind of particle carrying only color quantum numbers in ideal case and I have proposed their interpretation as quantum correlates for Boolean cognition. MEs and flux quanta of magnetic and electric fields are of special importance in living matter.

This general picture serves as a cornerstone of also TGD inspired view about cosmology and astrophysics. For obvious reasons the newest ideas developed during last year and still developing (in particular, the vision about dark matter) are not discussed in full depth yet.

1.2.2 Many-Sheeted Cosmology

The many-sheeted space-time concept, the new view about the relationship between inertial and gravitational four-momenta, the basic properties of the paired cosmic strings, the existence of the limiting temperature, the assumption about the existence of the phase dominated by cosmic strings, and quantum criticality imply a rather detailed picture of the cosmic evolution, which differs from that provided by the standard cosmology in several respects but has also strong resemblances with inflationary scenario.

Basic deviations from standard cosmology

The most important differences between TGD based and standard cosmology are following.

1. Many-sheetedness implies cosmologies inside cosmologies Russian doll like structure with a spectrum of Hubble constants.
2. TGD cosmology is also genuinely quantal: each quantum jump in principle recreates each sub-cosmology in 4-dimensional sense: this makes possible a genuine evolution in cosmological length scales so that the use of anthropic principle to explain why fundamental constants are tuned for life is not necessary.
3. The new view about energy means that inertial energy is negative for space-time sheets with negative time orientation and that the density of inertial energy vanishes in cosmological length scales. Therefore any cosmology is in principle creatable from vacuum and the problem of initial values of cosmology disappears. The density of matter near the initial moment is dominated by cosmic strings approaches to zero so that big bang is transformed to a silent whisper amplified to a relatively big bang.
4. Dark matter hierarchy with dynamical quantized Planck constant $h_{eff} = nh_0$, where h_0 is the minimum value of Planck constant ($h = 6h_0$ is the proposal) implies the presence of dark space-time sheets which differ from non-dark ones in that they define multiple coverings of M^4. Quantum coherence of dark matter in the length scale of space-time sheet involved implies that even in cosmological length scales Universe is more like a living organism than a thermal soup of particles.
5. Sub-critical and over-critical Robertson-Walker cosmologies are fixed completely from the imbeddability requirement apart from a single parameter characterizing the duration of the period after which transition to sub-critical cosmology necessarily occurs. The fluctuations of the microwave background reflect the quantum criticality of the critical period rather than

amplification of primordial fluctuations by exponential expansion. This and also the finite size of the space-time sheets predicts deviations from the standard cosmology.

This picture can be criticized since it is based on Kähler action. Twistor lift however predicts that action contains a small volume term proportional to dynamical cosmological constant. This means that Robertson-Walker cosmologies are not possible as preferred extremals except at the limit of vanishing cosmological contant corresponds to infinitely large space-time sheets and is also possible in principle.

Cosmic strings

Cosmic strings belong to the basic extremals of the Kähler action. The string tension of the cosmic strings is $T \simeq .2 \times 10^{-6}/G$ and slightly smaller than the string tension of the GUT strings and this makes them very interesting cosmologically. Concerning the understanding of cosmic strings a decisive breakthrough came through the identification of gravitational four-momentum as the difference of inertial momenta associated with matter and antimatter and the realization that the net inertial energy of the Universe vanishes. This forced to conclude cosmological constant in TGD Universe is non-vanishing. p-Adic length fractality predicts that Λ scales as $1/L^2(k)$ as a function of the p-adic scale characterizing the space-time sheet. The recent value of the cosmological constant comes out correctly. The gravitational energy density described by the cosmological constant is identifiable as that associated with topologically condensed cosmic strings and of magnetic flux tubes to which they are gradually transformed during cosmological evolution.

p-Adic fractality and simple quantitative observations lead to the hypothesis that pairs of cosmic strings are responsible for the evolution of astrophysical structures in a very wide length scale range. Large voids with size of order 10^8 light years can be seen as structures containing knotted and linked cosmic string pairs wound around the boundaries of the void. Galaxies correspond to same structure with smaller size and linked around the supra-galactic strings. This conforms with the finding that galaxies tend to be grouped along linear structures. Simple quantitative estimates show that even stars and planets could be seen as structures formed around cosmic strings of appropriate size. Thus Universe could be seen as fractal cosmic necklace consisting of cosmic strings linked like pearls around longer cosmic strings linked like...

1.2.3 Dark Matter And Quantization Of Gravitational Planck Constant

The notion of gravitational Planck constant having gigantic value is perhaps the most radical idea related to the astrophysical applications of TGD. D. Da Rocha and Laurent Nottale have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ $(\hbar = c = 1)$. v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale.

TGD predicts correctly the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n^2-fold: much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$-sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. An attractive solution of the matter antimatter asymmetry is based on the identification of also antimatter as dark matter.

1.2.4 The organization Of "Physics in Many-sheeted Space-time: Part I"

The book consists of 2 parts. In the 1st part the notion of many-sheeted space-time is discussed.

1. In the first two chapters the extremals of Kähler action are discussed. This discussion is somewhat out-of-date since twistor lift of TGD adds to Kähler action a volume term. All known extremals of Kähler action with non-vanishing induced Kähler form are however also minimal surfaces. Quantum criticality suggests that extremal has no dependence on Kähler coupling strength. This is achieve for minimal surface extremals of Kähler action. As a matter fact, minimal surface property would be lost at 2-D singular surfaces identifiable as string world sheets and partonic 2-surfaces.

2. Also two old chapters about possible implications of TGD for condensed matter physics written for at least about 15 years ago at least and updated only slightly. The phases of CP_2 complex coordinates could define phases of order parameters of macroscopic quantum phases so that the deviations of induced gauge field concept from the standard one could have direct experimental implications visible for instance in the properties of living matter and even in hydrodynamics. For instance, Z^0 magnetic gauge field could make itself visible in hydrodynamics and also Z^0 magnetic vortices could be involved with super-fluidity.

In the 2nd part the relationship between TGD and GRT is discussed. The discussion proceeds in the same order as time-line of TGD itself rather than starting from the final outcome.

1. The first two chapters are devoted TGD-GRT relationship and various gravitational anomalies.

2. The understanding of QFT-GRT limit of TGD required several decades. The key idea is that QFT limit means that the sheets of many-sheeted space-time are replaced with single region of M^4 made slightly curved so that the topology of the many-sheeted space-time is lost. The gauge potentials in this space are identified as sums of the induced gauge potentials associated with space-time sheets having same M^4 projection. Similar identification of gravitational field as deviation from flat Minkowski metric is performed. One can say that in many-sheeted space-time the fields of separate systems reside at different space-time sheets: systems have field identity - field body. A test particle touching the sheets carrying the fields however experiences the sum of their effects. At QFT limit sum of the effects is replaced with the sum of the fields. This distinction plays a key role in TGD inspired quantum biology.

3. Nottale's formula for gravitational Planck constant assignable to gravitational flux tubes plays a key role in TGD based view about quantum gravity and dark matter and its proper interpretation has been a long standing problem. Second problem concerns the origin of Newton's constant G.

   The original identification of Planck length was as CP_2 size R. It however turned out that R is about $10^{3.5}$ longer than Planck length. How does Planck length emerge from quantum criticality? Twistor lift and hierarchy of Planck constants led to a possible solution of the problem. One can indeed identify R as fundamental length and analog of Planck length. Newton's constant G would not be given as $G = R^2/\hbar$ but as $G = R^2/\hbar_{CP_2}$, where $\hbar_{M^4} = n_2\hbar_0$ is the number of sheets of space-time surface as covering of M^4. The value of n_{gr} in $h_{eff} = n_{gr}h_0$ appearing in Nottale's formmula is identified as $n = n_1n_2$, where n_1 is the number of sheets of space-time surface as covering of CP_2.

4. In chapter "Expanding Earth Model and Pre-Cambrian Evolution of Continents, Climate, and Life" the new view about space-time is applied in attempt to understand the mysteries associated with Cambrian Explosion and to the finding that the continents seem to fit nicely together if the radius of Earth is taken one half of its recent value. The explanation is in terms of rapid expansion of Earth radius induced by reduction of length scale dependent cosmological constant in turn inducing an expansion of the space-time sheet by factor of order 2.

1.3 Sources

The eight online books about TGD [K100, K95, K76, K64, K20, K61, K45, K82] and nine online books about TGD inspired theory of consciousness and quantum biology [K91, K14, K69, K13, K41, K51, K54, K81, K90] are warmly recommended for the reader willing to get overall view about what is involved.

My homepage (`http://tinyurl.com/ybv8dt4n`) contains a lot of material about TGD. In particular, a TGD glossary at `http://tinyurl.com/yd6jf3o7`).

I have published articles about TGD and its applications to consciousness and living matter in *Journal of Non-Locality* (`http://tinyurl.com/ycyrxj4o` founded by Lian Sidorov and in *Prespacetime Journal* (`http://tinyurl.com/ycvktjhn`), *Journal of Consciousness Research and Exploration* (`http://tinyurl.com/yba4f672`), and *DNA Decipher Journal* (`http://tinyurl.com/y9z52khg`), all of them founded by Huping Hu. One can find the list about the articles published at `http://tinyurl.com/ybv8dt4n`. I am grateful for these far-sighted people for providing a communication channel, whose importance one cannot overestimate.

1.4 The contents of the book

1.4.1 PART I: THE NOTION OF MANY-SHEETED SPACE-TIME

Extremals of the Kähler action

The physical interpretation of the Kähler function and the TGD based space-time concept are the basic themes of this book. The aim is to develop what might be called classical TGD at fundamental level. The strategy is simple: try to guess the general physical consequences of the geometry of the "world of classical worlds" (WCW) and of the TGD based gauge field concept and study the simplest extremals of Kähler action and try to abstract general truths from their properties.

The fundamental underlying assumptions are the following:

1. The notion of preferred extremals emerged during the period when I believed that positive energy ontology applies in TGD. In this framework the 4-surface associated with given 3-surface defined by Kähler function K as a preferred extremal of the Kähler action is identifiable as a classical space-time. Number theoretically preferred extremals would decompose to associative and co-associative regions. The reduction of the classical theory to the level of the Kähler-Dirac action implies that the preferred extremals are critical in the sense of allowing infinite number of deformations for which the second variation of Kähler action vanishes [**?**] It is not clear whether criticality and associativeity are consistent with each other. A further natural conjecture is that these critical deformations should act as conformal symmetries of light-like wormhole contacts at which the signature of the induced metric changes and preserve their light-likeness.

   Due to the preferred extremal property classical space-time can be also regarded as a generalized Bohr orbit - at least in positive energy ontology - so that the quantization of the various parameters associated with a typical extremal of the Kähler action is expected to take place in general. In TGD quantum states corresponds to quantum superpositions of these classical space-times so that this classical space-time is certainly not some kind of effective quantum average space-time.

2. In ZEO one can also consider the possibility that there is no selection of preferred extremal at all! The two space-like 3-surfaces at the ends of CD define the space-time surface connecting them apart from conformal symmetries acting as critical deformations. If 3-surface is identified as union of both space-like 3-surfaces and the light-like surfaces defining parton orbits connecting then, the conformal equivalence class of the preferred extremal is unique without any additional conditions! This conforms with the view about hierarchy of Planck constants requiring that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom and also with the idea that these surface together define analog for the Wilson loop. Actually all the discussions of this chapter are about extremals in general so that the attribute "preferred" is not relevant for them.

3. The bosonic vacuum functional of the theory is the exponent of the Kähler function $\Omega_B = exp(K)$. This assumption is the only assumption about the dynamics of the theory and is necessitated by the requirement of divergence cancellation in perturbative approach.
4. Renormalization group invariance and spin glass analogy. The value of the Kähler coupling strength is such that the vacuum functional $exp(K)$ is analogous to the exponent $exp(H/T)$ defining the partition function of a statistical system at critical temperature. This allows Kähler coupling strength to depend on zero modes of the configuration space metric and as already found there is very attractive hypothesis determining completely the dependence of the Kähler coupling strength on the zero modes based on p-adic considerations motivated by the spin glass analogy. Coupling constant evolution would be replaced by effective discrete evolution with respect to p-adic length scale and angle variable defined by the phases appearing in the algebraic extension of p-adic numbers in question.
5. In spin degrees of freedom the massless Dirac equation for the induced spinor fields with Kähler-Dirac action defines classical theory: this is in complete accordance with the proposed definition of the WCW spinor structure.

The geometrization of the classical gauge fields in terms of the induced gauge field concept is also important concerning the physical interpretation. Electro-weak gauge potentials correspond to the space-time projections of the spinor connection of CP_2, gluonic gauge potentials to the projections of the Killing vector fields of CP_2 and gravitational field to the induced metric. The topics to be discussed in this part of the book are summarized briefly in the following.

What the selection of preferred extremals of Kähler action might mean has remained a long standing problem and real progress occurred only quite recently (I am writing this towards the end of year 2003).

1. The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that all preferred extremals of Kähler action satisfy the condition. The vanishing of the Lorentz 4-force in turn implies local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. The condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.
2. The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Instanton current has a vanishing divergence for $D_{CP_2} < 4$, and Lorentz 4-force indeed vanishes. Four 4-dimensional projection the scalar function multiplying the instanton current can make it divergenceless. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.
3. By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that preferred extemals of Kähler action correspond to space-time sheets which at least asymptotically satisfy the generalized Beltrami conditions so that one can indeed assign to the final 3-surface a unique 4-surface apart from effects related to non-determinism. Preferred extremal property abstracted to purely algebraic generalized Beltrami conditions makes sense also in the p-adic context.

This chapter is mainly devoted to the study of the basic extremals of the Kähler action besides the detailed arguments supporting the view that the preferred extrema satisfy generalized Beltrami conditions at least asymptotically.

The newest results discussed in the last section about the weak form of electric-magnetic duality suggest strongly that Beltrami property is general and together with the weak form of electric-magnetic duality allows a reduction of quantum TGD to almost topological field theory with Kähler function allowing expression as a Chern-Simons term.

The surprising implication of the duality is that Kähler form of CP_2 must be replaced with that for $S^2 \times CP_2$ in order to obtain a WCW metric which is non-trivial in M^4 degrees of freedom. This modification implies much richer vacuum structure than the original Kähler action which is a good news as far as the description of classical gravitational fields in terms of small deformations of vacuum extremals with the four-momentum density of the topologically condensed matter given by Einstein's equations is considered. The breaking of Lorentz invariace from $SO(3,1)$ to $SO(3)$ is implied already by the geometry of CD but is extremely small for a given causal diamond (CD). Since a wave function over the Lorentz boosts and translates of CD is allowed, there is no actual breaking of Poincare invariance at the level of the basic theory. Beltrami property leads to a rather explicit construction of the general solution of field equations based on the hydrodynamic picture implying that single particle quantum numbers are conserved along flow lines defined by the instanton current. The construction generalizes also to the fermionic sector.

Identification of the preferred extremals of Kähler Action

Preferred extremal of Kähler action have remained one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what the attribute "preferred" really means. Symmetries give a clue to the problem. The conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [?]. Preferred extremal property should rely on this symmetry.

In Zero Energy Ontology (ZEO) preferred extremals are space-time surfaces connecting two space-like 3-surfaces at the ends of space-time surfaces at boundaries of causal diamond (CD). A natural looking condition is that the symplectic Noether charges associated with a sub-algebra of symplectic algebra with conformal weights n-multiples of the weights of the entire algebra vanish for preferred extremals. These conditions would be classical counterparts the the condition that super-symplectic sub-algebra annihilates the physical states. This would give a hierarchy of super-symplectic symmetry breakings and quantum criticalities having interpretation in terms of hierarchy of Planck constants $h_{eff} = n \times h$ identified as a hierarchy of dark matter. n could be interpreted as the number of space-time conformal gauge equivalence classes for space-time sheets connecting the 3-surfaces at the ends of space-time surface.

There are also many other proposals for what preferred extremal property could mean or imply. The weak form of electric-magnetic duality combined with the assumption that the contraction of the Kähler current with Kähler gauge potential vanishes for preferred extremals implies that Kähler action in Minkowskian space-time regions reduces to Chern-Simons terms at the light-like orbits of wormhole throats at which the signature of the induced metric changes its signature from Minkowskian to Euclidian. In regions with 4-D CP_2 projection (wormhole contacts) also a 3-D contribution not assignable to the boundary of the region might be possible. These conditions pose strong physically feasible conditions on extremals and might be true for preferred extremals too.

Number theoretic vision leads to a proposal that either the tangent space or normal space of given point of space-time surface is associative and thus quaternionic. Also the formulation in terms of quaternion holomorphy and quaternion-Kähler property is an attractive possibility. So called $M^8 - H$ duality is a variant of this vision and would mean that one can map associative/co-associative space-time surfaces from M^8 to H and also iterate this mapping from H to H to generate entire category of preferred extremals. The signature of M^4 is a general technical problem. For instance, the holomorphy in 2 complex variables could correspond to what I have called Hamilton-Jacobi property. Associativity/co-associativity of the tangent space makes sense also in Minkowskian signature.

In this chapter various views about preferred extremal property are discussed.

About Hydrodynamical and Thermodynamical Interpretations of TGD

This chapter is collected from the material related to the relationship between TGD and hydrodynamics one one hand and TGD and thermodynamics on the one hand. What I have called hydrodynamics ansatz is a proposal for what the preferred extremals of Kähler action might be. The basic vision behind the ansatz is the reduction of quantum TGD to almost topological QFT.

The basic condition is the vanishing of the contraction of the conserved Kähler current j with the induced Kähler gauge potential A implying the reduction of the Kähler action to 3-D contributions coming from the boundaries between space-time regions of Minkowskian and Euclidian signature.

Hydrodynamical interpretation demands that the flow parameters associated with the flow lines of isometry currents and Kähler current extend to global coordinates. Otherwise the flow line would resemble those for a gas of particles moving randomly. This leads to integrability conditions implying generalized Beltrami flow and Kähler action for the preferred extremals reduces to Chern-Simons action when the weak electro-weak duality is applied as boundary conditions. This allows also a definition of non-constant quantal order parameters depending on the spatial coordinates transversal to the flow lines.

Kiehn and others have studied Beltrami flows as integrable flows for which the flow lines define coordinate lines. In D=3 this requires that the rotor of the flow vector is parallel to the flow vector stating that Lorentz force vanishes. In D=4 the condition states that Lorentz 4-force vanishes so that also dissipation is absent. This kind of extremals are of special interest as asymptotic self-organization patterns: in fact all preferred extremals might satisfy these conditions. 3-D Beltrami flows are highly interesting topologically since the flow lines can get knotted. Their 4-D counterparts would have flow lines replaced with world sheets which can develop 2-knots. String world sheets carrying induced spinor fields are fundamental objects in TGD framework and they could indeed get knotted.

Kiehn has worked with both Betrami flows developed what he calls topological thermodynamics (TTD). This work is rather interesting from TGD point of view and the relationship between TTD and TGD is discussed in this chapter.

General View About Physics in Many-Sheeted Space-Time

This chapter, which is second part of a summary about the recent view about many-sheeted space-time, provides a summary of the developments in TGD that have occurred during last few years (the year I am writing this is 2007). The view is out-of-date in some respects. The most important steps of progress are following ones.

1. *Parton level formulation of quantum TGD*

The formulation of quantum TGD at partonic level identifying fundamental objects as light-like 3-surfaces having also interpretation as random light-like orbits of 2-D partons having arbitrarily large size. This picture reduces quantum TGD to an almost-topological quantum field theory and leads to a dramatic understanding of S-matrix. A generalization of Feynman diagrams emerges obtained by replacing lines of Feynman diagram with light-like 3-surfaces meeting along their ends at vertices. This picture is different from that of string models and means also a generalization of the view about space-time and 3-surface since these surfaces cannot be assumed to be a smooth manifold anymore.

The condition that the formulation in terms of light-like 3-surfaces is equivalent with that using pairs of space-like 3-surfaces at the ends of causal diamonds leads to strong from of holography stating that partonic 2-surfaces and their tangent space-data code for physics. It has turned out that fermionic string model in 4-D space-time emerges naturally from TGD. This is not yet taken into account in there considerations of the chapter.

2. *Zero energy ontology*

In zero energy ontology physical states are creatable from vacuum and have vanishing net quantum numbers, in particular energy. Zero energy states can be decomposed to positive and negative energy parts with definite geometro-temporal separation, call it T, and having interpretation in terms of initial and final states of particle reactions. Zero energy ontology is consistent with ordinary positive energy ontology at the limit when the time scale of the perception of observer is much shorter than T.

Zero energy ontology leads to the view about S-matrix as a characterizer of time-like entanglement associated with the zero energy state and a generalization of S-matrix to what might be called M-matrix emerges. M-matrix is complex square root of density matrix expressible as a product of real valued "modulus" and unitary matrix representing phase and can be seen as a matrix valued generalization of Schrödinger amplitude. Also thermodynamics becomes an inherent element of quantum theory in this approach. M-matrices in turn form orthogonal rows of U-matrix which is defined between zero energy states whereas S and M-matrices are defined by entanglement coefficients between positive and negative energy parts of zero energy states.

3. Fusion of real and p-adic physics to single one

The fusion of p-adic physics and real physics to single coherent whole requires generalization of the number concept obtained by gluing reals and various p-adic number fields along common algebraic numbers. This leads to a new vision about how cognition and intentionality make themselves visible in real physics via long range correlations realized via the effective p-adicity of real physics. The success of the p-adic length scale hypothesis and p-adic mass calculations suggest that cognition and intentionality are present already at elementary particle level. This picture leads naturally to an effective discretization of the real physics at the level of S-matrix and relying on the notion of number theoretic braid.

It has turned out that the notion of braid emerges naturally from the localization of spinor modes to 2-D surfaces in the generic case. Braids correspond to the orbits of the strings ends at given space-time sheet.

4. Dark matter hierarchy and hierarchy of Planck constants

Dark matter revolution with levels of the hierarchy labeled by values of Planck constant suggests a further generalization of the notion of embedding space and thus of space-time - at least as an effective mathematical tool. One can say that embedding space is a book like structure obtained by gluing together infinite number of copies of the embedding space like pages of a book: two copies characterized by singular discrete bundle structure are glued together along 4-dimensional set of common points. These points have physical interpretation in terms of quantum criticality. Particle states belonging to different sectors (pages of the book) can interact via field bodies representing space-time sheets which have parts belonging to two pages of this book.

It has turned out that the hierarchy of effective Planck constants $h_{eff} = n \times h$ follows from the quantum criticality implied by the non-determinism of Kähler action and that one can relate it to an infinite hierarchy of breakings of conformal symmetries acting on the orbits of light-like 3-surfaces leaving the space-like ends of space-time surface at boundaries of CD invariant. Hierarchy of conformal algebras corresponds to sub-algebras of conformal algebras with conformal weights coming as multiples of n.

5. Equivalence Principle and evolution of gravitational constant

The views about Equivalence Princple (EP) and GRT limit of TGD have changed quite a lot since 2007 and here the updated view is summarized. Before saying anything about evolution of gravitational constant one must understand whether it is a fundamental constant or prediction of quantum TGD. Also one should understand whether Equivalence Principle holds true and if so, in what sense. Also the identification of gravitational and inertial masses seems to be necessary.

At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Gravitational constant, cosmological constant, and various gauge couplings emerge as predictions. Planck length should be related to CP_2 size by a dimensionless numerical factor predicted by the theory. These constants need not be universal constants: cosmological constant is certainly very large for the Euclidian variant of GRT space-time. These constants could also depend on p-adic length scale. p-Adic coupling constant evolution suggests itself as a discretized variant of coupling constant evolution and p-adic scales would relate naturally to the size scales of causal diamonds: perhaps the integer n characterizing the multiple of CP_2 scale giving the distance between the tips of CD has p-adic prime p or its power as a divisor.

At the level of single space-time sheet and CD it is not possible to talk about coupling constant evolution since Kähler action and Kähler-Dirac action contain no coupling constants.

This description however gives rise to p-adic coupling constant evolution since the process of lumping together the sheets of the many-sheeted space-time gives a result which depends on the size scale of CD. If the non-deterministic dynamics of Kähler action for the maxima of Kähler function mimics p-adic non-determinism then one has hopes about p-adic coupling constant evolution. The p-adic prime and therefore also the length scale and coupling constants characterizing the dynamics for given CD would vary wildly as function of integer characterizing CD size scale. This could mean that the CDs whose size scales are related by multiplication of small integer are close to each other. They would be near to each other in logarithmic sense and logarithms indeed appear in running coupling constants. This "prediction" is of course subject to criticism.

6. Renormalization group equations for gauge couplings at space-time level

In classical TGD only Kähler coupling constant appears explicitly but does not affect the classical dynamics. Other gauge couplings do not appear at all in classical dynamics since the the definition of classical fields absorbs them as normalization constants. This suggests that the notion of continuous coupling constant evolution at space-time level is not needed in quantum TGD proper and emerges only at the QFT limit when space-time is replaced with general relativistic effective space-time.

For the known extremals of Kähler action gauge couplings are RG invariants inside single space-time sheet, which supports the view that discrete p-adic coupling constant evolution replacing the ordinary continuous coupling constant evolution emerges only when space-time sheets are lumped together to define GRT space-time. This evolution would have as parameters the p-adic length scale characterizing the causal diamond (CD) associated with particle and the phase factors characterizing the algebraic extension of p-adic numbers involved.

The p-adic prime and therefore also the length scale and coupling constants characterizing the dynamics for given CD would vary wildly as function of integer characterizing CD size scale. This could mean that the CDs whose size scales are related by multiplication of small integer are close to each other. They would be near to each other in logarithmic sense and logarithms indeed appear in running coupling constants. This "prediction" is of course subject to criticism.

7. Quantitative g for the values of coupling constants

All quantitative statements about coupling constants are bound to be guesswork as long as explicit formulas for M-matrix elements are lacking. p-Adic length scale hypothesis provides one guideline for the guesses. Second guideline is provided by number theoretical universality. Third guideline is general physical intuition. What is done can be however seen as exercises perhaps giving some familiarity with the basic notions.

The latest progress in the understanding of p-adic coupling constant evolution comes from a formula for Kähler coupling strength α_K in terms of Dirac determinant of the Kähler-Dirac operator associated with Kähler action.

The formula for α_K fixes its number theoretic anatomy and also that of other coupling strengths. The assumption that simple rationals (p-adicization) are involved can be combined with the input from p-adic mass calculations and with an old conjecture for the formula of gravitational constant allowing to express it in terms of CP_2 length scale and Kähler action of topologically condensed CP_2 type vacuum extremal. The prediction is that α_K is renormalization group invariant and equals to the value of fine structure constant at electron length scale characterized by M_{127}. Although Newton's constant is proportional to p-adic length scale squared it can be RG invariant thanks to exponential reduction due to the presence of the exponent of Kähler action associated with the two CP_2 type vacuum extremals representing the wormhole contacts associated with graviton. The number theoretic anatomy of R^2/G allows to consider two options. For the first one only M_{127} gravitons are possible number theoretically. For the second option gravitons corresponding to $p \simeq 2^k$ are possible.

A relationship between electromagnetic and color coupling constant evolutions based on the formula $1/\alpha_{em} + 1/\alpha_s = 1/\alpha_K$ is suggested by the induced gauge field concept, and would mean that the otherwise hard-to-calculate evolution of color coupling strength is fixed completely. The predicted value of α_s at intermediate boson length scale is correct.

In this chapter the above topics are discussed in detail. Also the possible role of so called super-symplectic gauge bosons in the understanding of non-perturbative phase of QCD and black-hole physics is discussed.

Hydrodynamics and CP_2 Geometry

The chapter is one of the earliest attempts to apply TGD to macroscopic physics and must be taken as such. The chapter begins with a brief summary of the basic notions related to many-sheeted space-time. A generalization of hydrodynamics to a p-adic hierarchy of hydrodynamics is considered and a mechanism of energy transfer between condensate levels is identified. It is suggested that TGD based generalization of Hawking-Bekenstein law holds even in macroscopic length scales and that hydrodynamical vortices behave in some aspects like elementary particles. TGD leads to a formulation of a general theory of phase transitions: the new element is the presence of several condensate levels.

It has much later become clear that the vision about elementary particles Euclidian space-time regions defining lines of generalized Feynman diagrams generalizes to macroscopic scales and that every macroscopic body should accompany such space-time sheet and thus in some aspects behave like elementary particle.

A topological model for the generation of the hydrodynamical turbulence is proposed. The basic idea is that hydrodynamical turbulence can be regarded as a spontaneous Kähler magnetization leading to the increase the value of Kähler function and therefore of the probability of the configuration. Kähler magnetization is achieved through the formation of a vortex cascade via the decay of the mother vortex by the emission of smaller daughter vortices. Vortices with various values of the fractal quantum number and with sizes related by a discrete scaling transformation appear in the cascade. The decay of the vortices takes place via the so called phase slippage process.

An encouraging result is the prediction for the size distribution of the vortices: the prediction is practically identical with that obtained from the model of Heisenberg but on rather different physical grounds. The model is rather insensitive to the p-adic scaling of vortices in the transition as long as it is smaller than $\lambda = 2^{-5}$. The model is also consistent with the assumption that the decay of a vortex to smaller vortices corresponds to a phase transition from a given level of dark matter hierarchy to a lower level so that the value of $\hbar$ is reduced by a factor $\lambda = v_0/n \simeq 2^{-11}/n$, $n = 1, 2, ...$ so that Compton length scales as well as sizes of vortices are reduced by this factor.

Macroscopic Quantum Phenomena and CP_2 Geometry

Topological field quantization is applied to a unified description of three macroscopic quantum phases: super conductors, super fluids and quantum Hall phase. The basic observation is that the formation of connections identified as join along boundaries bonds makes possible the formation of macroscopic quantum system from topological field quanta having size of the order of the coherence length ξ for ordinary phase. The presence of the connections makes possible supra flow and the presence of two levels of the topological condensate explains the two-fluid picture of super fluids. In standard physics, the order parameter is constant in the ground state. In TGD context, the non-simply connected topology of the 3-surface makes possible ground states with a covariantly constant order parameter characterized by the integers telling the change of the order parameter along closed homotopically nontrivial loops. Later an alternative identification of connections as Kähler magnetic flux tubes carrying magnetic monopole flux has emerged but does not change the general vision.

The role of the ordinary magnetic field in super conductivity is proposed to be taken by the Z^0 magnetic field in super fluidity and the mathematical descriptions of super conductors and super fluids become practically identical. The generalization of the quantization condition for the magnetic flux to a condition involving also a velocity circulation, plays a central role in the description of both phases and suggests a new description of the rotating super fluid and some new effects. A classical explanation for the fractional Quantum Hall effect in terms of the topological field quanta is proposed. Quantum Hall phase is very similar to the supra phases: an essential role is played by the generalized quantization condition and the hydrodynamic description of the Hall electrons. The role of Z^0 magnetic field is suggested by large parity breaking effects in biology.

The results obtained support the view that in condensed matter systems topological field quanta with size of the order of $\xi \simeq 10^{-8} - 10^{-7}$ meters are of special importance. This new length scale is expected to have also applications to less exotic phenomena of the condensed matter physics (the description of the conductors and di-electrics and ferromagnetism) and in hydrodynamics (the failure of the hydrodynamic approximation takes place at this length scale). These field quanta of course, correspond to only one condensate level and many length scales are expected to be present.

1.4.2 PART II: TGD AND GRT

The Relationship Between TGD and GRT

In this chapter the recent view about TGD as Poincare invariant theory of gravitation is discussed. Radically new views about ontology were necessary before it was possible to see what had been there all the time. Zero energy ontology states that all physical states have vanishing net quantum numbers. The hierarchy of dark matter identified as macroscopic quantum phases labeled by arbitrarily large values of Planck constant is second aspect of the new ontology.

1. Equivalence Principle and GRT limit of TGD

The views about Equivalence Princple (EP) and GRT limit of TGD have changed quite a lot since 2007 and here the updated view is summarized. Before saying anything about evolution of gravitational constant one must understand whether it is a fundamental constant or prediction of quantum TGD. Also one should understand whether Equivalence Principle holds true and if so, in what sense. Also the identification of gravitational and inertial masses seems to be necessary.

At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

2. The problem of cosmological constant

A further implication of dark matter hierarchy is that astrophysical systems correspond to stationary states analogous to atoms and do not participate to cosmic expansion in a continuous manner but via discrete quantum phase transitions in which gravitational Planck constant increases. By quantum criticality of these phase transitions critical cosmologies are excellent candidates for the modeling of these transitions. Imbeddable critical (and also over-critical) cosmologies are unique apart from a parameter determining their duration and represent accelerating cosmic expansion so that there is no need to introduce cosmological constant.

It indeed turns out possible to understand these critical phases in terms of quantum phase transition increasing the size of large modeled in terms of cosmic strings. A possible mechanism driving the strings to the boundaries of large voids could be repulsive interaction due to net charges of strings. Also repulsive gravitational acceleration could do this. In this framework cosmological constant like parameter does not characterize the density of dark energy but that of dark matter identifiable as quantum phases with large Planck constant.

A concrete interpretation for the dark matter is as Kähler magnetic energy of Kähler magnetic flux tubes, which are outcome of the expansion of primordial cosmic strings. Dark matter in turn corresponds to particles with non-standard value of Planck constant given by $h_{eff} = n\times$ residing at the Kähler magnetic flux tubes. The GRT limit of TGD allows a description of dark energy in terms of cosmological constant in Einstein's equations.

A further problem is that the naive estimate for the cosmological constant is predicted to be by a factor 10^{120} larger than its value deduced from the accelerated expansion of the Universe. In TGD framework the resolution of the problem comes naturally from the fact that large voids are quantum systems which follow the cosmic expansion only during the quantum critical phases.

p-Adic fractality predicting that cosmological constant is reduced by a power of 2 in phase transitions occurring at times $T(k) \propto 2^{k/2}$, which correspond to p-adic time scales. These phase transitions would naturally correspond to quantum phase transitions increasing the size of the large voids during which critical cosmology predicting accelerated expansion naturally applies. On the average $\Lambda(k)$ behaves as $1/a^2$, where a is the light-cone proper time. This predicts correctly the order of magnitude for observed value of Λ.

3. Topics of the chapter

The topics discussed in the chapter are following.

1. The basic principles of GRT (General Coordinate Invariance, Equivalence Principle, and Machian Principle) are discussed from TGD point of view.
2. The theory assuming that the most important solutio is applied to the vacuum extremal embeddings of Reissner-Nordström and Schwartschild metric.
3. A model for the final state of star indicates that Z^0 force, presumably created by dark matter, might have an important role in the dynamics of the compact objects. During year 2003, more than decade after the formulation of the model, the discovery of the connection between supernovas and gamma ray bursts provided strong support for the predicted axial magnetic and Z^0 magnetic flux tube structures predicted by the model for the final state of a rotating star. Two years later the interpretation of the predicted long range weak forces as being caused by dark matter emerged.

   The progress in understanding of hadronic mass calculations has led to the identification of what I call super-symplectic bosons and their super-counterparts as basic building blocks of hadrons. This notion leads also to a microscopic description of neutron stars and black-holes in terms of highly entangled string like objects in Hagedorn temperature and in very precise sense analogous to gigantic hadrons.
4. There is a brief summary about cosmic strings, which form a corner stone of TGD inspired cosmology.
5. The idea of entropic gravity is not consistent with what is already known about the quantal behavior of neutrons in the Earth's gravitational field. The discussion of entropic gravity in TGD framework however leads to fresh ideas about GRT limit of TGD and is therefore included.

TGD and Potential Anomalies of GRT

In this chapter the applications of TGD to various real or potential anomalies of GRT approach are discussed.

1. In the first section Allais effect as a possible evidence for large $\hbar$ dark gravitons is discussed.
2. TGD inspired model of gravimagnetism is studied. There are claims about strong gravimagnetism and these claims are considered in terms large $\hbar$ hypothesis.
3. The dependence of operationally defined light velocity on space-time sheet distinguishes between the sub-manifold gravity of TGD and the abstract manifold gravity GRT. Possible evidence for the effect is discussed. These effects are discussed in several sections. Also the time dilation effect caused by the warping of space-time sheet in absence of matter is considered.
4. There are also some considerations not strictly related to anomalies such as possible interpretations of Machian Principle in TGD framework.

About the Nottale's formula for h_{gr} and the possibility that Planck length l_P and CP_2 length R are identical?

Nottale's formula for the gravitational Planck constant $\hbar_{gr} = GMm/v_0$ involves parameter v_0 with dimensions of velocity. I have worked with the quantum interpretation of the formula but the physical origin of v_0 - or equivalently the dimensionless parameter $\beta_0 = v_0/c$ (to be used in the sequel) appearing in the formula has remained open hitherto. In this chapter a possible interpretation based on many-sheeted space-time concept, many-sheeted cosmology, and zero energy ontology (ZEO) is discussed. In ZEO the non-changing parts of zero energy states are assigned to the passive boundary of CD and β_0 should be assigned to it.

There are two measures for the size of the system. The M^4 size L_{M^4} is identifiable as the maximum of the radial M^4 distance from the tip of CD associated with the center of mass of the system along the light-like geodesic at the boundary of CD. System has also size L_{ind} defined defined in terms of the induced metric of the space-time surface, which is space-like at the boundary of CD. One has $L_{ind} < L_H$. The identification $\beta_0 = L_{M^4}/L_H$ does not allow the identification of $L_H = L_{M^4}$. L_H would however naturally corresponds to the size of the magnetic body of the system in turn identifiable as the size of CD.

One can deduce an estimate for β_0 by approximating the space-time surface as Robertson-Walker cosmology expected to be a good approximation near the passive light-like boundary of CD. The resulting formula is tested for planetary system and Earth. The dark matter assignable to Earth can be identified as the innermost part of inner core with volume, which is .01 per cent of the volume of Earth. Also the consistency of the Bohr quantization for dark and ordinary matter is discussed and leads to a number theoretical condition on the ratio of the ordinary and dark masses.

$\beta_0/4\pi$ is analogous to gravitational fine structure constant for $h_{eff} = h_{gr}$. Could one see it as fundamental coupling parameter appearing also in other interactions at quantum criticality in which ordinary perturbation series diverges? Remarkably, the value of G does not appear at all in the perturbative expansion in this region! Could G have several values? This suggests the generalization $G = l_P^2/\hbar \rightarrow G = R^2/\hbar_{eff}$ so that G would indeed have a spectrum and that Planck length l_P would be equal to CP_2 radius R so that only one fundamental length would be associated with twistorialization. Ordinary Newton's constant would be given by $G = R^2/h_{eff}$ with h_{eff}/h_0 having value in the range $10^7 - 10^8$.

The second topic of the chapter relates to the the fact that the measurements of G give differing results with differences between measurements larger than the measurement accuracy. This suggests that there might be some new physics involved. In TGD framework the hierarchy of Planck constants $h_{eff} = nh_0$, $h = 6h_0$ together with the condition that theory contains CP_2 size scale R as only fundamental length scale, suggest the possibility that Newtons constant is given by $G = R^2/\hbar_{eff}$, where R replaces Planck length ($l_P = \sqrt{\hbar G} \rightarrow l_P = R$) and $\hbar_{eff}/h$ is in the range $10^6 - 10^7$. The spectrum of Newton' constant is consistent with Newton's equations if the scaling of $\hbar_{eff}$ inducing scaling G is accompanied by opposite scaling of M^4 coordinates in $M^4 \times CP_2$: dark matter hierarchy would correspond to discrete hierarchy of scales given by breaking of scale invariance. In the special case $h_{eff} = h_{gr} = GMm/v0$ quantum critical dynamics as gravitational fine structure constant $(v_0/c)/4\pi$ as coupling constant and it has no dependence of the value of G or masses M and m.

In this chapter I consider a possible interpretation for the finding of a Chinese research group measuring two different values of G differing by 47 ppm in terms of varying h_{eff}. Also a model for fountain effect of superfluidity as de-localization of wave function and increase of the maximal height of vertical orbit due to the change of the gravitational acceleration g at surface of Earth induced by a change of h_{eff} due to super-fluidity is discussed. Also Podkletnov effect is considered. TGD inspired theory of consciousness allows to speculate about levitation experiences possibly induced by the modification of G_{eff} at the flux tubes for some part of the magnetic body accompanying biological body in TGD based quantum biology.

Expanding Earth Model and Pre-Cambrian Evolution of Continents, Climate, and Life

TGD inspired quantum cosmology predicts that astrophysical objects do not follow cosmic expansion except in jerk-wise quantum leaps increasing the gigantic value of the gravitational Planck constant h_{gr} characterizing space-time mediating gravitational interactions between two masses or gravitational self interactions. This assumption provides explanation for the apparent cosmological constant. As a matter fact, gigantic value of h_{gr}. By Equivalence principle and independence of gravitational acceleration on mass it is enough to assume that only microscopic systems have the gravitational flux tube contacts with central mass. In this case the value range of h_{gr} is consistent with the identification as $h_{eff} = n \times h$ introduced with motivations coming from biology and in TGD framework following from the non-determinism of Kähler action.

Also planets are predicted to expand in a stepwise manner allowing to imagine a new version of Expanding Earth theory originally postulated to explain the intriguing findings suggesting that continents have once formed a connected continent covering almost the entire surface of Earth but with radius which was one half of the recent one.

This leads also to a rather fascinating vision about biology. The mysterious Cambrian Explosion in which a large number of new species emerged suddenly (realized already Darwin as the strongest objection against his theory) could be understood if the life would have gone to underground lakes and seas formed during the expansion period as fractures were formed and the underground cavities expanded and were filled with water. This would have allowed the life to escape cosmic radiation, meteoric bombardment, and the extremely cold climate during Proterozoic period preceding the Cambrian Explosion and migrate back as highly developed life forms as the period of glaciations ended.

Before the Proterozoic era the radius of Earth would have been one half of its recent value and started to grow with gradually accelerating rate. This forces to rewrite the entire geological and climate history of Earth during the Proterozoic period.

1. The postulated physically implausible cyclic appearance of single connected super-continent containing all land mass can be given up and replaced with a single continent containing large inland seas. There is no need to postulate the existence of series of super-oceans whose ocean floor would have subduced totally so that no direct information about them would exist nowadays.

2. The dominating model for pre-Cambrian climate is so called Snowball Earth model inspired by the finding that signatures of glaciations have been found at regions of Earth, which should have been near Equator during the Proterozoic. Snowball model has several difficulties: in particular, there is a lot of evidence that a series of ordinary glaciations was in question. For $R/2$ option the regions located to Equator would have actually been near North Pole so that the glaciations would have indeed been ordinary glaciations proceeding from the poles. A killer prediction is the existence of non-glaciated regions at apparent southern latitudes around about 45 degrees and there is evidence for these indeed exists! The model makes also testable paleomagnetic killer predictions. In particular, during periods when the magnetic dipole in the direction of rotation axis the directions of the magnetic fields for $R/2$ model are predicted to be same at South Pole and apparent Equator and opposite for the standard option.

Part I

THE NOTION OF MANY-SHEETED SPACE-TIME

Chapter 2

Extremals of the Kähler Action

2.1 Introduction

In this chapter the classical field equations associated with the Kähler action are studied. The study of the extremals of the Kähler action has turned out to be extremely useful for the development of TGD. Towards the end of year 2003 quite dramatic progress occurred in the understanding of field equations and it seems that field equations might be in well-defined sense exactly solvable. The progress made during next five years led to a detailed understanding of quantum TGD at the fundamental parton level and this provides considerable additional insights concerning the interpretation of field equations.

2.1.1 About The Notion Of Preferred Extremal

The notion of preferred extremal has been central in classical TGD although the known solutions could be preferred or not: the main challenge has been to understand what "preferred" could mean.

In zero energy ontology (ZEO) one can also consider the releaving possibility that all extremals are preferred ones! The two space-like 3-surfaces at the ends of CD define the space-time surface connecting them apart from conformal symmetries acting as critical deformations. If 3-surface is identified as union of both space-like 3-surfaces and the light-like surfaces defining parton orbits connecting then, the conformal equivalence class of the preferred extremal is unique without any additional conditions! This conforms with the view about hierarchy of Planck constants requiring that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom and also with the idea that these surface together define analog for the Wilson loop. The non-determinism of Kähler action suggests that "preferred" could be obsolete in given length scale resolution.

Actually all the discussions of this chapter are about known extremals in general so that the attribute "preferred" is not relevant for them.

2.1.2 Beltrami Fields And Extremals

The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that preferred extremals satisfy the condition. The vanishing of the Lorentz 4-force in turn implies a local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. If Kähler action is defined by CP_2 Kähler form alone, the condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.

The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Instanton current has vanishing divergence for $D_{CP_2} < 4$, and Lorentz

4-force indeed vanishes. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.

Under additional conditions magnetic field reduces to what is known as Beltrami field. Beltrami fields are known to be extremely complex but highly organized structures. The natural conjecture is that topologically quantized many-sheeted magnetic and Z^0 magnetic Beltrami fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chirality selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

Field equations can be reduced to algebraic conditions stating that energy momentum tensor and second fundamental form have no common components (this occurs also for minimal surfaces in string models) and only the conditions stating that Kähler current vanishes, is light-like, or proportional to instanton current, remain and define the remaining field equations. The conditions guaranteeing topologization to instanton current can be solved explicitly. Solutions can be found also in the more general case when Kähler current is not proportional to instanton current. On basis of these findings there are strong reasons to believe that classical TGD is exactly solvable.

An important outcome is the notion of Hamilton-Jacobi structure meaning dual slicings of M^4 projection of preferred extremals to string world sheets and partonic 2-surfaces. The necessity of this slicing was discovered years later from number theoretic compactification and is now a key element of quantum TGD allowing to deduce Equivalence Principle in its stringy form from quantum TGD and formulate and understand quantum TGD in terms of Kähler-Dirac action assignable to Kähler action. The conservation of Noether charges associated with Kähler-Dirac action requires the vanishing of the second second variation of Kähler action for preferred extremals. Preferred extremals would thus define space-time representation for quantum criticality. Infinite-dimensional variant for the hierarchy of criticalities analogous to the hierarchy assigned to the extrema of potential function with levels labeled by the rank of the matrix defined by the second derivatives of the potential function in catastrophe theory would suggest itself.

A natural interpretation for deformations would be as conformal gauge symmetries due to the non-determinism of Kähler action. They would transform to each other preferred extremals having fixed 3-surfaces as ends at the boundaries of the causal diamond. They would preserve the value of Kähler action and those of conserved charges. The assumption is that there are n gauge equivalence classes of these surfaces and that n defines the value of the effective Planck constant $h_{eff} = n \times h$ in the effective GRT type description replacing many-sheeted space-time with single sheeted one.

2.1.3 In What Sense Field Equations Could Mimic Dissipative Dynamics?

By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. The nontrivial question is what this means in TGD framework.

1. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that preferred extremals of Kähler action correspond to space-time sheets which at least asymptotically satisfy generalized Beltrami conditions so that one can indeed assign to the final (rather than initial!) 3-surface a unique 4-surface apart from effects related to non-determinism. Preferred extremal property of Kähler action abstracted to purely algebraic generalized Beltrami conditions would make sense also in the p-adic context. The general solution ansatz discussed in the last section of the chapter assumes that all conserved isometry currents are proportional to instanton current so that various charges are conserved separately for all flow lines: this means esssentially the integrability of the theory. This ansatz is forced by the hypothesis that TGD reduces to almost topological QFT and this idea. The basic consequence is that dissipation is impossible classically.

2. A more radical view inspired by zero energy ontology is that the light-like 3-surfaces and corresponding space-time regions with Euclidian signature defining generalized Feynman diagrams provide a space-time representation of dissipative dynamics just as they provide this

representation in quantum field theory. Minkowskian regions would represent empty space so that the vanishing of Lorentz 4-force and absence of dissipation would be natural. This would mean very precise particle field duality and the topological pattern associated with the generalized Feynman diagram would represent dissipation. One could also interprete dissipation as transfer of energy between sheets of the many-sheeted space time and thus as an essentially topological phenomenon. This option seems to be the only viable one.

2.1.4 The Dimension Of CP_2 Projection As Classifier For The Fundamental Phases Of Matter

The dimension D_{CP_2} of CP_2 projection of the space-time sheet encountered already in p-adic mass calculations classifies the fundamental phases of matter. For $D_{CP_2} = 4$ empty space Maxwell equations hold true. The natural guess would be that this phase is chaotic and analogous to de-magnetized phase. $D_{CP_2} = 2$ phase is analogous to ferromagnetic phase: highly ordered and relatively simple. It seems however that preferred extremals can correspond only to small perturbations of these extremals resulting by topological condensation of CP_2 type vacuum extremals and through topological condensation to larger space-time sheets. $D_{CP_2} = 3$ is the analog of spin glass and liquid crystal phases, extremely complex but highly organized by the properties of the generalized Beltrami fields. This phase could be seen as the boundary between chaos and order and corresponds to life emerging in the interaction of magnetic bodies with bio-matter. It is possible only in a finite temperature interval (note however the p-adic hierarchy of critical temperatures) and characterized by chirality just like life.

The original proposal was that $D(CP_2) = 4$ phase is completely chaotic. This is not true if the reduction to almost topological QFT takes place. This phase must correspond to Maxwellian phase with a vanishing Kähler current as concluded already earlier. Various isometry currents are however proportional to the instanton current and conserved along the flow lines of the instanton current whose flow parameter extends to a global coordinate. Hence a completely chaotic phase is not in question even in this case.

2.1.5 Specific Extremals Of Kähler Action

The study of extremals of Kähler action represents more than decade old layer in the development of TGD.

1. The huge vacuum degeneracy is the most characteristic feature of Kähler action (any 4-surface having CP_2 projection which is Legendre sub-manifold is vacuum extremal, Legendre sub-manifolds of CP_2 are in general 2-dimensional). This vacuum degeneracy is behind the spin glass analogy and leads to the p-adic TGD. As found in the second part of the book, various particle like vacuum extremals also play an important role in the understanding of the quantum TGD.

2. The so called CP_2 type vacuum extremals have finite, negative action and are therefore an excellent candidate for real particles whereas vacuum extremals with vanishing Kähler action are candidates for the virtual particles. These extremals have one dimensional M^4 projection, which is light like curve but not necessarily geodesic and locally the metric of the extremal is that of CP_2: the quantization of this motion leads to Virasoro algebra. Space-times with topology $CP_2\#CP_2\#...CP_2$ are identified as the generalized Feynman diagrams with lines thickened to 4-manifolds of "thickness" of the order of CP_2 radius. The quantization of the random motion with light velocity associated with the CP_2 type extremals in fact led to the discovery of Super Virasoro invariance, which through the construction of the configuration space geometry, becomes a basic symmetry of quantum TGD.

3. There are also various non-vacuum extremals.

   (a) String like objects, with string tension of same order of magnitude as possessed by the cosmic strings of GUTs, have a crucial role in TGD inspired model for the galaxy formation and in the TGD based cosmology.

(b) The so called massless extremals describe non-linear plane waves propagating with the velocity of light such that the polarization is fixed in given point of the space-time surface. The purely TGD:eish feature is the light like Kähler current: in the ordinary Maxwell theory vacuum gauge currents are not possible. This current serves as a source of coherent photons, which might play an important role in the quantum model of bio-system as a macroscopic quantum system.

(c) In the so called Maxwell phase, ordinary Maxwell equations for the induced Kähler field would be satisfied in an excellent approximation. It is however far from clear whether this kind of extremals exist. Their non-existence would actually simplify the theory enormously since all extremals would have quantal character. The recent view indeed is that Maxwell phase makes sense only as as genuinely many-sheeted structure and solutions of Maxwell's equation appear only at the level of effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Gauge potentials in effective space-time are determined in the same way. Since the gauge potentials sum up, it is possible to understand how field configurations of Maxwell's theory emerge at this limit.

2.1.6 The Weak Form Of Electric-Magnetic Duality And Modification Of Kähler Action

The newest results discussed in the last section about the weak form of electric-magnetic duality suggest strongly that Beltrami property is general and together with the weak form of electric-magnetic duality allows a reduction of quantum TGD to almost topological field theory with Kähler function allowing expression as a Chern-Simons term.

Generalized Beltrami property leads to a rather explicit construction of the general solution of field equations based on the hydrodynamic picture implying that single particle quantum numbers are conserved along flow lines defined by the instanton current. The construction generalizes also to the fermionic sector and there are reasons to hope that TGD is completely integrable theory.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L8].

2.2 General Considerations

The solution families of field equations studied in this chapter were found already during eighties. The physical interpretation turned out to be the really tough problem. What is the principle selecting preferred extremals of Kähler action as analogs of Bohr orbits assigning to 3-surface X^3 a unique space-time surface $X^4(X^3)$? Does Equivalence Principle hold true and if so, in what sense? These have been the key questions. The realization that light-like 3-surfaces X_l^3 associated with the light-like wormhole throats at which the signature of the induced metric changes from Minkowskian to Euclidian led to the formulation of quantum TGD in terms of second quantized induced spinor fields at these surfaces. Together with the notion of number theoretical compactification this approach allowed to identify the conditions characterizing the preferred extremals. What is remarkable that these conditions are consistent with what is known about extremals.

Also a connection with string models emerges and partial understanding of the space-time realization of Equivalence Principle suggests itself. However, much more general argument allows to understand how GRT space-time appears from the many-sheeted space-time of TGD (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) as effective concept [K99]: this more general view is not in conflict with the much earlier proposal discussed below.

In this section the theoretical background behind field equations is briefly summarized. I will not repeat the discussion of previous two chapters [**?**, K40] summarizing the general vision about many-sheeted space-time, and consideration will be restricted to those aspects of vision leading to direct predictions about the properties of preferred extremals of Kähler action.

2.2.1 Number Theoretical Compactification And $M^8 - H$ Duality

The notion of hyper-quaternionic and octonionic manifold makes sense but it not plausible that $H = M^4 \times CP_2$ could be endowed with a hyper-octonionic manifold structure. Situation changes if H is replaced with hyper-octonionic M^8. Suppose that $X^4 \subset M^8$ consists of hyper-quaternionic and co-hyper-quaternionic regions. The basic observation is that the hyper-quaternionic sub-spaces of M^8 with a fixed hyper-complex structure (containing in their tangent space a fixed hyper-complex subspace M^2 or at least one of the light-like lines of M^2) are labeled by points of CP_2. Hence each hyper-quaternionic and co-hyper-quaternionic four-surface of M^8 defines a 4-surface of $M^4 \times CP_2$. One can loosely say that the number-theoretic analog of spontaneous compactification occurs: this of course has nothing to do with dynamics.

This picture was still too naïve and it became clear that not all known extremals of Kähler action contain fixed $M^2 \subset M^4$ or light-like line of M^2 in their tangent space.

1. The first option represents the minimal form of number theoretical compactification. M^8 is interpreted as the tangent space of H. Only the 4-D tangent spaces of light-like 3-surfaces X^3_l (wormhole throats or boundaries) are assumed to be hyper-quaternionic or co-hyper-quaternionic and contain fixed M^2 or its light-like line in their tangent space. Hyper-quaternionic regions would naturally correspond to space-time regions with Minkowskian signature of the induced metric and their co-counterparts to the regions for which the signature is Euclidian. What is of special importance is that this assumption solves the problem of identifying the boundary conditions fixing the preferred extremals of Kähler action since in the generic case the intersection of M^2 with the 3-D tangent space of X^3_l is 1-dimensional. The surfaces $X^4(X^3_l) \subset M^8$ would be hyper-quaternionic or co-hyper-quaternionic but would not allow a local mapping between the 4-surfaces of M^8 and H.

2. One can also consider a more local map of $X^4(X^3_l) \subset H$ to $X^4(X^3_l) \subset M^8$. The idea is to allow $M^2 \subset M^4 \subset M^8$ to vary from point to point so that $S^2 = SO(3)/SO(2)$ characterizes the local choice of M^2 in the interior of X^4. This leads to a quite nice view about strong geometric form of $M^8 - H$ duality in which M^8 is interpreted as tangent space of H and $X^4(X^3_l) \subset M^8$ has interpretation as tangent for a curve defined by light-like 3-surfaces at X^3_l and represented by $X^4(X^3_l) \subset H$. Space-time surfaces $X^4(X^3_l) \subset M^8$ consisting of hyper-quaternionic and co-hyper-quaternionic regions would naturally represent a preferred extremal of E^4 Kähler action. The value of the action would be same as CP_2 Kähler action. $M^8 - H$ duality would apply also at the induced spinor field and at the level of WCW . The possibility to assign $M^2(x) \subset M^4$ to each point of M^4 projection $P_{M^4}(X^4(X^3_l))$ is consistent with what is known about extremals of Kähler action with only one exception: CP_2 type vacuum extremals. In this case M^2 can be assigned to the normal space.

3. Strong form of $M^8 - H$ duality satisfies all the needed constraints if it represents Kähler isometry between $X^4(X^3_l) \subset M^8$ and $X^4(X^3_l) \subset H$. This implies that light-like 3-surface is mapped to light-like 3-surface and induced metrics and Kähler forms are identical so that also Kähler action and field equations are identical. The only differences appear at the level of induced spinor fields at the light-like boundaries since due to the fact that gauge potentials are not identical.

4. The map of $X^3_l \subset H \rightarrow X^3_l \subset M^8$ would be crucial for the realization of the number theoretical universality. $M^8 = M^4 \times E^4$ allows linear coordinates as those preferred coordinates in which the points of embedding space are rational/algebraic. Thus the point of $X^4 \subset H$ is algebraic if it is mapped to algebraic point of M^8 in number theoretic compactification. This of course restricts the symmetry groups to their rational/algebraic variants but this does not have practical meaning. Number theoretical compactification could thus be motivated by the number theoretical universality.

5. The possibility to use either M^8 or H picture might be extremely useful for calculational purposes. In particular, M^8 picture based on $SO(4)$ gluons rather than $SU(3)$ gluons could perturbative description of low energy hadron physics. The strong $SO(4)$ symmetry of low energy hadron physics can be indeed seen direct experimental support for the $M^8 - H$ duality.

Number theoretical compactification has quite deep implications for quantum TGD and is actually responsible for most of the progress in the understanding of the mathematical structure of quantum TGD. A very powerful prediction is that preferred extremals should allow slicings to either stringy world sheets or dual partonic 2-surfaces as well as slicing by light-like 3-surfaces. Both predictions are consistent with what is known about extremals.

1. If the distribution of planes $M^2(x)$ is integrable, it is possible to slice $X^4(X^3)$ to a union of 2-dimensional surfaces having interpretation as string world sheets and dual 2-dimensional copies of partonic surfaces X^2. This decomposition defining 2+2 Kaluza-Klein type structure could realize quantum gravitational holography and might allow to understand Equivalence Principle at space-time level in the sense that dimensional reduction defined by the integral of Kähler action over the 2-dimensional space labeling stringy world sheets gives rise to the analog of stringy action and one obtains string model like description of quantum TGD as dual for a description based on light-like partonic 3-surfaces. String tension is not however equal to the inverse of gravitational constant as one might naïvely expect but the connection is more delicate. As already mentioned, TGD-GRT connection and EP can be understood at general level only from very general arguments [K99].
2. Second implication is the slicing of $X^4(X_l^3)$ to light-like 3-surfaces Y_l^3 "parallel" to X_l^3. Also this slicing realizes quantum gravitational holography if one requires General Coordinate Invariance in the sense that the Dirac determinant differs for two 3-surfaces Y_l^3 in the slicing only by an exponent of a real part of a holomorphic function of WCW complex coordinates giving no contribution to the Kähler metric.
3. The square of the Dirac determinant would be equal to the modulus squared for the exponent of vacuum functional and would be formally defined as the product of conformal weights assignable to the modes of the Dirac operator at string world sheets at the ends of strings at partonic 2-surfaces defining the ends of Y_l^3. The detailed definition requires to specify what one means with the conformal weights assignable with the modes of the Kähler-Dirac operator.
4. The localization of the modes of Kähler-Dirac operator to 2-D surfaces (string world sheets and possibly partonic 2-surfaces) [K104] following from the condition that electromagnetic charges of the modes is well-defined is very strong restriction and reduces Dirac determinant to a product of Dirac determinants assignable with these 2-surfaces.

2.2.2 Preferred Extremal Property As Classical Correlate For Quantum Criticality, Holography, And Quantum Classical Correspondence

The Noether currents assignable to the Kähler-Dirac equation are conserved only if the first variation of the Kähler-Dirac operator D_K defined by Kähler action vanishes. This is equivalent with the vanishing of the second variation of Kähler action -at least for the variations corresponding to dynamical symmetries having interpretation as dynamical degrees of freedom which are below measurement resolution and therefore effectively gauge symmetries. The natural identification would be as conformal symmetries. The weaker condition would mean that the inner product defined by the integral of $D_\alpha \partial L_K/\partial h^k_\alpha \delta h^k$ over the space-time surface vanishes for the deformations defining dynamical symmetries but the field equations are not satisfied completely generally. The weaker condition would mean that the inner product defined by the integral of $D_\alpha \partial L_K/\partial h^k_\alpha \delta h^k$ over the space-time surface vanishes for the deformations defining dynamical symmetries but the field equations are not satisfied completely generally.

The vanishing of the second variation in interior of $X^4(X_l^3)$ is what corresponds exactly to quantum criticality so that the basic vision about quantum dynamics of quantum TGD would lead directly to a precise identification of the preferred extremals. Something which I should have noticed for more than decade ago!

For instance, the natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg` or **Fig. ??**in the appendix of this book).

The vanishing of second variations of preferred extremals -at least for deformations representing dynamical symmetries, suggests a generalization of catastrophe theory of Thom, where the rank of the matrix defined by the second derivatives of potential function defines a hierarchy of criticalities with the tip of bifurcation set of the catastrophe representing the complete vanishing of this matrix. In the recent case this theory would be generalized to infinite-dimensional context. There are three kind of variables now but quantum classical correspondence (holography) allows to reduce the types of variables to two.

1. The variations of $X^4(X^3_l)$ vanishing at the intersections of $X^4(X^3_l)$ with the light-like boundaries of causal diamonds CD would represent behavior variables. At least the vacuum extremals of Kähler action would represent extremals for which the second variation vanishes identically (the "tip" of the multi-furcation set).
2. The zero modes of Kähler function would define the control variables interpreted as classical degrees of freedom necessary in quantum measurement theory. By effective 2-dimensionality (or holography or quantum classical correspondence) meaning that the configuration space metric is determined by the data coming from partonic 2-surfaces X^2 at intersections of X^3_l with boundaries of CD, the interiors of 3-surfaces X^3 at the boundaries of CDs in rough sense correspond to zero modes so that there is indeed huge number of them. Also the variables characterizing 2-surface, which cannot be complexified and thus cannot contribute to the Kähler metric of WCW represent zero modes. Fixing the interior of the 3-surface would mean fixing of control variables. Extremum property would fix the 4-surface and behavior variables if boundary conditions are fixed to sufficient degree.
3. The complex variables characterizing X^2 would represent third kind of variables identified as quantum fluctuating degrees of freedom contributing to the WCW metric. Quantum classical correspondence requires 1-1 correspondence between zero modes and these variables. This would be essentially holography stating that the 2-D "causal boundary" X^2 of $X^3(X^2)$ codes for the interior. Preferred extremal property identified as criticality condition would realize the holography by fixing the values of zero modes once X^2 is known and give rise to the holographic correspondence $X^2 \rightarrow X^3(X^2)$. The values of behavior variables determined by extremization would fix then the space-time surface $X^4(X^3_l)$ as a preferred extremal.
4. Clearly, the presence of zero modes would be absolutely essential element of the picture. Quantum criticality, quantum classical correspondence, holography, and preferred extremal property would all represent more or less the same thing. One must of course be very cautious since the boundary conditions at X^3_l involve normal derivative and might bring in delicacies forcing to modify the simplest heuristic picture.

The basic question is whether number theoretic view about preferred extremals imply absolute minimization or something analogous to it.

1. The number theoretic conditions defining preferred extremals are purely algebraic and make sense also p-adically and this is enough since p-adic variants of field equations make sense although the notion of Kähler action does not make sense as integral. Despite this the identification of the vacuum functional as exponent of Kähler function as Dirac determinant allows to define the exponent of Kähler function as a p-adic number [K104].
2. The general objection against all extremization principles is that they do not make sense p-adically since p-adic numbers are not well-ordered.
3. These observations do not encourage the idea about equivalence of the two approaches. On the other hand, real and p-adic sectors are related by algebraic continuation and it could be quite enough if the equivalence were true in real context alone.

The finite-dimensional analogy allows to compare absolute minimization and criticality with each other.

1. Absolute minimization would select the branch of Thom's catastrophe surface with the smallest value of potential function for given values of control variables. In general this value would not correspond to criticality since absolute minimization says nothing about the values of control variables (zero modes).
2. Criticality forces the space-time surface to belong to the bifurcation set and thus fixes the values of control variables, that is the interior of 3-surface assignable to the partonic 2-surface, and realized holography. If the catastrophe has more than $N = 3$ sheets, several

preferred extremals are possible for given values of control variables fixing $X^3(X^2)$ unless one assumes that absolute minimization or some other criterion is applied in the bifurcation set. In this sense absolute minimization might make sense in the real context and if the selection is between finite number of alternatives is in question, it should be possible carry out the selection in number theoretically universal manner.

It must be emphasized that there are several proposals for what preferred extremal property could mean. For instance, one can consider the identification of space-time surface as quaternionic sub-manifold meaning that tangent space of space-time surface can be regarded as quaternionic sub-manifold of complexified octonions defining tangent space of embedding space. One manner to define "quaternionic sub-manifold" is by introducing octonionic representation of embedding space gamma matrices identified as tangent space vectors. It must be also assumed that the tangent space contains a preferred complex (commutative) sub-space at each point and defining an integrable distribution having identification as string world sheet (also slicing of space-time sheet by string world sheets can be considered). Associativity and commutativity would define the basic dynamical principle. A closely related approach is based on so called Hamilton-Jacobi structure [K11] defining also this kind of slicing and the approaches could be equivalent. A further approach is based on the identification of preferred extremal property as quantum criticality [K11].

2.2.3 Can One Determine Experimentally The Shape Of The Space-Time Surface?

The question "Can one determine experimentally the shape of the space-time surface?" does not relate directly to the topic of this chapter in technical sense, and the only excuse for its inclusion is the title of this section plus the fact that the general conceptual framework behind quantum TGD assumes an affirmative answer to this question. If physics were purely classical physics, operationalism in the strong sense of the word would require that one can experimentally determine the shape of the space-time as a surface of the embedding space with arbitrary accuracy by measuring suitable classical observables. In quantum physics situation is considerably more complex and quantum effects are both a blessing and a curse.

Measuring classically the shape of the space-time surface

Consider first the purely classical situation to see what is involved.

1. All classical gauge fields are expressible in terms of CP_2 coordinates and their space-time gradients so that the measurement of four field quantities with some finite resolution in some space-time volume could in principle give enough information to deduce the remaining field quantities. The requirement that space-time surface corresponds to an extremal of Kähler action gives a further strong consistency constraint and one can in principle test whether this constraint is satisfied. A highly over-determined system is in question.
2. The freedom to choose the space-time coordinates freely causes complications and it seems that one must be able to determine also the distances between the points at which the field quantities are determined. At purely classical Riemannian level this boils down to the measurement of the induced metric defining classical gravitational field. In macroscopic length scales one could base the approach to iterative procedure in which one starts from the assumption that the coordinates used are Minkowski coordinates and gravitational corrections are very weak.
3. The measurement of induced Kähler form in some space-time volume determines space-time surface only modulo canonical transformations of CP_2 and isometries of the embedding space. If one measures classical electromagnetic field, which is not canonical invariant in general case, with some precision, one can determine to what kind of surface space-time region corresponds apart from the action of the isometries of H.

Quantum measurement of the shape of the space-time surface

In practice the measurement of the shape of the space-time surface is necessarily a bootstrap procedure based on the model for space-time region and on the requirement of internal consistency.

Many-sheeted space-time and quantum phenomena produce considerable complications but also provide universal measurement standards.

Consider first how quantum effects could help to measure classical fields and distances.

1. The measurement of distances by measuring first induced metric at each point of space-time sheet is rather unpractical procedure. Many-sheeted space-time however comes in rescue here. p-Adic length scale hypothesis provides a hierarchy of natural length scales and one can use p-adic length and time scales as natural units of length and time: space-time sheets serve as meter sticks. For instance, length measurement reduces in principle to a finite number of operations using various space-time sheets with standardized lengths given by p-adic length scales. Also various transition frequencies and corresponding wavelengths provide universal time and length units. Atomic clock provides a standard example of this kind of time unit. A highly nontrivial implication is the possibility to deduce the composition of distant star from its spectral lines. Without p-adic length scale hypothesis the scales for the mass spectra of the elementary particles would be variable and atomic spectra would vary from point to point in TGD universe.

   Do the p-adic length scales correspond to the length units of the induced metric or of M^4_+ metric? If the topological condensation a meter stick space-time sheet at a larger space-time sheet does not stretch the meter stick but only bends it, the length topologically condensed meter stick in the induced metric equals to its original length measured using M^4_+ metric.

2. If superconducting order parameters are expressible in terms of the CP_2 coordinates (there is evidence for this, see the chapter "Macroscopic quantum phenomena and CP_2 geometry"), one might determine directly the CP_2 coordinates as functions of Minkowski coordinates and this would allow to estimate all classical fields directly and thus to deduce strong consistency constraints.

3. At quantum level only the fluxes of the classical fields through surface areas with some minimum size determined by the length scale resolution can be measured. In case of magnetic fields the quantization of the magnetic flux simplifies the situation dramatically. Topological field quantization quite generally modifies the measurement of continuous field variables to the measurement of fluxes. Interestingly, the construction of WCW geometry uses as WCW coordinates various electric and magnetic fluxes over 2-dimensional cross sections of 3-surface.

Quantum effects introduce also difficulties and restrictions.

1. Canonical transformations localized with respect to the boundary of the light cone or more general light like surfaces act as isometries of WCW and one can determine the space-time surface only modulo these isometries. Even more, only the values of the non-quantum fluctuating zero modes characterizing the shape and size of the space-time surface are measurable with arbitrary precision in quantum theory. At the level of conscious experience quantum fluctuating degrees of freedom correspond to sensory qualia like color having no classical geometric content.

2. Space-time surface is replaced by a new one in each quantum jump (or rather the superposition of perceptively equivalent space-time surfaces). Only in the approximation that the change of the space-time region in single quantum jump is negligible, the measurement of the shape of space-time surface makes sense. The physical criterion for this is that dissipation is negligible. The change of the space-time region in single quantum jump can indeed be negligible if the measurement is performed with a finite resolution.

3. Conscious experience of self is an average over quantum jumps defining moments of consciousness. In particular, only the average increment of the zero modes is experienced and this means that one cannot fix the space-time surface apart from canonical transformation affecting the zero modes. Again the notion of measurement resolution comes in rescue.

4. The possibility of coherent states of photons and gravitons brings in a further quantum complication since the effective classical em and gravitational fields are superpositions of classical field and the order parameter describing the coherent state. In principle the extremely strong constraints between the classical field quantities allow to measure both the order parameters of the coherent phases and classical fields.

Quantum holography and the shape of the space-time surface

If the Dirac determinant asssognable to the mass squared eigenvalue spectrum of the Kähler-Dirac operator $D_K(X^2)$ equals to the exponent of Kähler action of a preferred extremal, it is fair to say that a lot of information about the shape of the space-time surface is coded to physical observables, which eigenvalues indeed represent. Quantum gravitational holography due to the Bohr orbit like character of space-time surface reduces the amount of information needed. Only a finite number of eigenvalues is involved and the eigen modes are associated with the 3-D light-like wormhole throats rather than with the space-time surface itself. If the eigenvalues were known or could be measured with infinite accuracy, one could in principle fix the boundary conditions at X^3_l and solve field equations determining the preferred extremal of Kähler action.

What is of course needed is the complete knowledge of the light-like 3-surfaces X^3_l. Needless to say, in practice a complete knowledge of X^3_l is impossible since measurement resolution is finite. The notion number theoretic braid provides a precise realization for the finite measurement accuracy at space-time level. At the level of WCW spinors fields (world of classical worlds) just the fact that the number of eigenvalues is finite is correlate for the finite measurement accuracy. Furthermore, quantum states are actually quantum superpositions of 3-surfaces, which means that one can only speak about quantum average space-time surface for which the phase factors coding for the quantum numbers of elementary particles assigned to the strands of number theoretic braids are stationary so that correlation of classical gauge charges with quantum gauge charges is obtained.

2.3 The Vanishing Of Super-Conformal Charges As Gauge Conditions Selecting Preferred Extremals Of Kähler Action

Classical TGD [K11] involves several key questions waiting for clearcut answers.

1. The notion of preferred extremal emerges naturally in positive energy ontology, where Kähler metric assigns a unique (apart from gauge symmetries) preferred extremal to given 3-surface at M^4 time= constant section of embedding space $H = M^4 \times CP_2$. This would quantize the initial values of the time derivatives of embedding coordinates and this could correspond to the Bohr orbitology in quantum mechanics.

2. In zero energy ontology (ZEO) initial conditions are replaced by boundary conditions. One fixes only the 3-surfaces at the opposite boundaries of CD and in an ideal situation there would exist a unique space-time surface connecting them. One must however notice that the existence of light-like wormhole throat orbits at which the signature of the induced metric changes ($det(g_4) = 0$) its signature might change the situation. Does the attribute "preferred" become obsolete and does one lose the beautiful Bohr orbitology, which looks intuitively compelling and would realize quantum classical correspondence?

3. Intuitively it has become clear that the generalization of super-conformal symmetries by replacing 2-D manifold with metrically 2-D but topologically 3-D light-like boundary of causal diamond makes sense. Generalized super-conformal symmetries should apply also to the wormhole throat orbits which are also metrically 2-D and for which conformal symmetries respect $detg(g_4) = 0$ condition. Quantum classical correspondence demands that the generalized super-conforal invariance has a classical counterpart. How could this classical counterpart be realized?

4. Holography is one key aspect of TGD and mean that 3-surfaces dictate everything. In positive energy ontology the content of this statement would be rather obvious and reduce to Bohr orbitology but in ZEO situation is different. On the other hand, TGD strongly suggests strong form of holography based stating that partonic 2-surfaces (the ends of wormhole throat orbits at boundaries of CD) and tangent space data at them code for quantum physics of TGD. General coordinate invariance would be realized in strong sense: one could formulate the theory either in terms of space-like 3-surfaces at the ends of CD or in terms of light-like wormhole throat orbits. This would realize Bohr orbitology also in ZEO by reducing the

boundary conditions to those at partonic 2-surfaces. How to realize this explicitly at the level of field equations? This has been the challenge.

Answering questions is extremely useful activity. During last years my friend Hamed has posed continually questions related to the basic TGD. At this time Hamed asked about the derivation of field equations of TGD. In "simple" field theories involving some polynomial non-linearities the deduction of field equations is of course totally trivial process but in the extremely non-linear geometric framework of TGD situation is quite different.

While answering the questions I ended up with the following question. Could one assume that the variations at the light-like boundaries of CD vanish for all conformal variations, which are not isometries. For isometries the contributions from the ends of CD cancel each other automatically so that the corresponding variations need not vanish separately at boundaries of CD! This is extremely simple and profound fact. This would be nothing but the realisation of the analogs of conformal symmetries classically and give precise content for the notion of preferred external, Bohr orbitology, and strong form of holography. And the condition makes sense only in ZEO!

I attach below the answers to the questions of Hamed almost as such apart from slight editing and little additions, re-organization, and correction of typos.

2.3.1 Field Equations For Kähler Action

My friend Hamed made some questions relating to the derivation of field equations for the extremals of Kähler action, which led to the recent progress. I comment first these questions since they lead naturally to the basic new idea.

The addition of the volume term implied by the twistor lift of TGD [L83, L84], having interpretation in terms of cosmological constant, adds to the field equations only a term proportional to $D_\beta g^{alpha\beta}\partial_\beta h^k$. There are excellent reasons to believe that solutions of field equations representing preferred extremals as analogs of Bohr orbits are actually minimal surfaces except at singular surfaces of dimension $D>4$. One might speak of 4-D analogies of soap films spanned by frames.

The physical interpretation of the canonical momentum current

Hamed asked about the physical meaning of $T_k^n \equiv \partial L/\partial(\partial_n h^k)$ - normal components of canonical momentum labelled by the label k of embedding space coordinates - it is good to start from the physical meaning of a more general vector field

$$T_k^\alpha \equiv \frac{\partial L}{\partial(\partial_\alpha h^k)}$$

with both embedding space indices k and space-time indices α - canonical momentum currents. L refers to Kähler action.

1. One can start from the analogy with Newton's equations derived from action principle (Lagrangian). Now the analogs are the partial derivatives $\partial L/\partial(dx^k/dt)$. For a particle in potential one obtains just the momentum. Therefore the term canonical momentum current/density: one has kind of momentum current for each embedding space coordinate.
2. By contracting with Killing vector fields of the embedding space isometries j_A^k (Poincare and color) one indeed obtains conserved currents associated with isometries by Noether's theorem:

$$j^{A\alpha} = T_k^\alpha j^{Ak} \quad .$$

By field equations the divergences of these currents vanish and one obtains conserved charged-classical four-momentum and color charges:

$$D_\alpha T^{A\alpha} = 0 \quad .$$

The field equations are essentially hydrodynamical and replace Einstein's equations $T^{alpha\beta} = kG^{\alpha\beta}$. The conditions $D_\beta G^{\alpha\beta} = 0$, stating the vanishing of the covariant divergence of Einstein's tensor $G^{\alpha\beta}$, is the counterpart for the conservation of the isometry currents, but do not give rise to conserved charges.

3. The normal component of the conserved current must vanish at boundaries with one time-like direction if one has such:

$$T^{An} = 0.$$

Now one has wormhole throat orbits which are not genuine boundaries albeit analogous to them and one must be very careful. The quantity T_k^n determines the values of normal components of currents and must vanish at possible space-like boundaries.

Note that in TGD field equations reduce to the conservation of isometry currents as in hydrodynamics where basic equations are just conservation laws.

The basic steps in the derivation of field equations

First a general recipe for deriving field equations from Kähler action - or any action as a matter of fact.

1. At the first step one writes an expression of the variation of the Kähler action as sum of variations with respect to the induced metric g and induced Kähler form J. The partial derivatives in question are energy momentum tensor and contravariant Kähler form.
2. After this the variations of g and J are expressed in terms of variations of embedding space coordinates, which are the primary dynamical variables.
3. The integral defining the variation can be decomposed to a total divergence plus a term vanishing for extremals for all variations: this gives the field equations. Total divergence term gives a boundary term and it vanishes by boundary conditions if the boundaries in question have time-like direction.

   If the boundary is space-like, the situation is more delicate in TGD framework: this will be considered in the sequel. In TGD situation is also delicate also because the light-like 3-surfaces which are common boundaries of regions with Minkowskian or Euclidian signature of the induced metric are not ordinary topological boundaries. Therefore a careful treatment of both cases is required in order to not to miss important physics.

Expressing this summary more explicitly, the variation of the Kahler action with respect to the gradients of the embedding space coordinates reduces to the integral of

$$T_k^\alpha \partial_\alpha \delta h^k + \frac{\partial K}{\partial h^k}\delta h^k \quad .$$

The latter term comes only from the dependence of the embedding space metric and Kähler form on embedding space coordinates. One can use a simple trick. Assume that they do not depend at all on embedding space coordinates, derive field equations, and replaced partial derivatives by covariant derivatives at the end. Covariant derivative means covariance with respect to both space-time and embedding space vector indices for the tensorial quantities involved. The trick works because embedding space metric and Kähler form are covariantly constant quantities.

The integral of the first term $T_k^\alpha \partial_\alpha \delta h^k$ decomposes to two parts.

1. The first term, whose vanishing gives rise to field equations, is integral of

$$D_\alpha T_k^\alpha \delta h^k \quad .$$

2. The second term is integral of

$$\partial_\alpha (T_k^\alpha \delta h^k) \quad .$$

   This term reduces as a total divergence to a 3-D surface integral over the boundary of the region of fixed signature of the induced metric consisting of the ends of CD and wormhole throat orbits (boundary of region with fixed signature of induced metric). This term vanishes if the normal components T_k^n of canonical momentum currents vanishes at the boundary like region.

In the sequel the boundary terms are discussed explicitly and it will be found that their treatment indeed involves highly non-trivial physics.

Complex isometry charges and twistorialization

TGD space-time contains regions of both Minkowskian and Euclidian signature of metric. This has some highly non-trivial consequences.

1. Should one assume that $\sqrt{det(g_4)}$ is imaginary in Minkowskian and real in Euclidian region? For Kähler action this is sensible and Euclidian region would give a real negative contribution giving rise to exponent of Kähler function of WCW ("world of classical worlds") making the functional integral convergent. Minkowskian regions would give imaginary contribution to the exponent causing interference effects absolutely essential in quantum field theory. This contribution would correspond to Morse function for WCW .

   The implication would be that the classical four-momenta in Euclidian/Minkowskian regions are imaginary/real. What could the interpretation be? Should one accept as a fact that four-momenta are complex.

2. Twistor approach to TGD is now in quite good shape [K94]. $M^4 \times CP_2$ is the unique choice is one requires that the Cartesian factors allow twistor space with Kähler structure [A35] and classical TGD allows twistor formulation.

   In the recent formulation the fundamental fermions are assumed to propagate with light-like momenta along wormhole throats. At gauge theory limit particles must have massless or massive four-momenta. One can however also consider the possibility of complex massless momenta and in the standard twistor approach on mass shell massless particles appearing in graphs indeed have complex momenta. These complex momenta should by quantum classical correspondence correspond directly to classical complex momenta.

3. A funny question popping in mind is whether the massivation of particles could be such that the momenta remain massless in complex sense! The complex variant of light-likeness condition would be

   $$p_{re}^2 = p_{Im}^2 \ \ , \ \ p_{re} \cdot p_{Im} = 0 \ \ .$$

   Could one interpret p_{Im}^2 as the mass squared of the particle? Or could p_{Im}^2 code for the decay width of an unstable particle? This option does not look feasible.

4. The complex momenta could provide an elegant 4-D space-time level representation for the isometry quantum numbers at the level of embedding space. The ground states of the super-conformal representations have as building bricks the spinor harmonics of the embedding space which correspond to the analogs of massless particles in 8-D sense [K53]. Indeed, the condition giving mass squared eigenvalues for the spinor harmonics is just massless condition in $M^4 \times CP_2$.

   At the space-time level these conditions must be replaced by 4-D conditions and complex masslessness would be the elegant manner to realizes this. Also the massivation of massless states by p-adic thermodynamics could have similar description.

   This interpretation would also conform with $M^8 - M^4 \times CP_2$ duality [K102] at the level of momentum space.

2.3.2 Boundary Conditions At Boundaries Of CD

In positive energy ontology one would formulate boundary conditions as initial conditions by fixing both the 3-surface and associated canonical momentum densities at either end of CD (positions and momenta of particles in mechanics). This would bring asymmetry between boundaries of CD. In ZEO the basic boundary condition is that space-time surfaces have as their ends the members of pairs of surfaces at the ends of CD. Besides this one can have additional boundary conditions and the notion of preferred extremal suggests this.

Do boundary conditions realize quantum classical correspondence?

In TGD framework one must carefully consider the boundary conditions at the boundaries of CDs. What is clear that the time-like boundary contributions from the boundaries of CD to the variation must vanish.

1. This is true if the variations are assumed to vanish at the ends of CD. This might be however too strong a condition.
2. One cannot demand the vanishing of T^t_k (t refers to time coordinate as normal coordinate) since this would give only vacuum extremals. One could however require quantum classical correspondence for any Cartan sub-algebra of isometries whose elements define maximal set of isometry generators. The eigenvalues of quantal variants of isometry charge assignable to second quantized induced spinors at the ends of space-time surface are equal to the classical charges. Is this actually a formulation of Equivalence Principle, is not quite clear to me.

Do boundary conditions realize preferred extremal property as a choice of conformal gauge?

While writing this a completely new idea popped to my mind. What if one poses the vanishing of the boundary terms at boundaries of CDs as additional boundary conditions for *all* variations *except isometries* ? Of perhaps for all conformal variations (conformal in TGD sense)? This would *not* imply vanishing of isometry charges since the variations coming from the opposite ends of CD cancel each other! It soon became clear that this would allow to meet all the challenges listed in the beginning!

1. These conditions would realize Bohr orbitology also to ZEO approach and define what "preferred extremal" means.
2. The conditions would be very much like super-Virasoro conditions stating that the superconformal generators with non-vanishing conformal weight annihilate states or create zero norm states but no conditions are posed on generators with vanishing conformal weight (now isometries). One could indeed assume only deformations, which are local isometries assignable to the generalised conformal algebra of the $\delta M^4_+/-\times CP_2$. For arbitrary variations one would not require the vanishing. This could be the long sought for precise formulation of superconformal invariance at the level of classical field equations!

   It is enough co consider the weaker conditions that the conformal charges defined as integrals of corresponding Noether currents vanish. These conditions would be direct equivalents of quantal conditions.
3. The natural interpretation would be as a fixing of conformal gauge. This fixing would be motivated by the fact that WCW Kähler metric must possess isometries associated with the conformal algebra and can depend only on the tangent data at partonic 2-surfaces as became clear already for more than two decades ago. An alternative, non-practical option would be to allow all 3-surfaces at the ends of CD: this would lead to the problem of eliminating the analog of the volume of gauge group from the functional integral.
4. The conditions would also define precisely the notion of holography and its reduction to strong form of holography in which partonic 2-surfaces and their tangent space data code for the dynamics.

Needless to say, the modification of this approach could make sense also at partonic orbits.

2.3.3 Boundary Conditions At Parton Orbits

The contributions from the orbits of wormhole throats are singular since the contravariant form of the induced metric develops components which are infinite ($det(g_4)=0$). The contributions are real at Euclidian side of throat orbit and imaginary at the Minkowskian side so that they must be treated as independently.

Conformal gauge choice, preferred extremal property, hierarchy of Planck constants, and TGD as almost topological QFT

The generalization of the boundary conditions as a classical realization conformal gauge invariance is natural.

1. One can consider the possibility that under rather general conditions the normal components $T^n_k\sqrt{det(g_4)}$ approach to zero at partonic orbits since $det(g_4)$ is vanishing. Note however the appearance of contravariant appearing twice as index raising operator in Kähler action. If so, the vanishing of $T^n_k\sqrt{det(g_4)}$ need not fix completely the "boundary" conditions. In fact, I assign to the wormhole throat orbits conformal gauge symmetries so that just this is expected on physical grounds.

2. Generalized conformal invariance would suggest that the variations defined as integrals of $T^n_k\sqrt{det(g_4)}\delta h^k$ vanish in a non-trivial manner for the conformal algebra associated with the light-like wormhole throats with deformations respecting $det(g_4) = 0$ condition. Also the variations defined by infinitesimal isometries (zero conformal weight sector) should vanish since otherwise one would lose the conservation laws for isometry charges. The conditions for isometries might reduce to $T^n_k\sqrt{det(g_4)} \to 0$ at partonic orbits. Also now the interpretation would be in terms of fixing of conformal gauge.

3. Even $T^n_k\sqrt{g} = 0$ condition need not fix the partonic orbit completely. The Gribov ambiguity meaning that gauge conditions do not fix uniquely the gauge potential could have counterpart in TGD framework. It could be that there are several conformally non-equivalent space-time surfaces connecting 3-surfaces at the opposite ends of CD.

   If so, the boundary values at wormhole throats orbits could matter to some degree: very natural in boundary value problem thinking but new in initial value thinking. This would conform with the non-determinism of Kähler action implying criticality and the possibility that the 3-surfaces at the ends of CD are connected by several space-time surfaces which are physically non-equivalent.

4. The hierarchy of Planck [K35] constants assigned to dark matter, quantum criticality and even criticality indeed relies on the assumption that $h_{eff} = n \times h$ corresponds to n-fold coverings having n space-time sheets which coincide at the ends of CD and that conformal symmetries act on the sheets as gauge symmetries. One would have as Gribov copies n conformal equivalence classes of wormhole throat orbits and corresponding space-time surfaces. Depending on whether one fixes the conformal gauge one has n equivalence classes of space-time surfaces or just one representative from each conformal equivalent class.

5. There is also the question about the correspondence with the weak form of electric magnetic duality [K11]. This duality plus the condition that $j^\alpha A_\alpha = 0$ in the interior of space-time surface imply the reduction of Kähler action to Chern-Simons terms. This would suggest that the boundary variation of the Kähler action reduces to that for Chern-Simons action which is indeed well-defined for light-like 3-surfaces.

   If so, the gauge fixing would reduce to variational equations for Chern-Simons action! A weaker condition is that classical conformal charges vanish. This would give a nice connection to the vision about TGD as almost topological QFT. In TGD framework these conditions do not imply the vanishing of Kähler form at boundaries. The conditions are satisfied if the CP_2 projection of the partonic orbit is 2-D: the reason is that Chern-Simons term vanishes identically in this case.

Fractal hierarchy of conformal symmetry breakings

A further intuitively natural hypothesis is that there is a fractal hierarchy of breakings of conformal symmetry.

1. Only the generators of conformal sub-algebra with conformal weight multiple of n act as gauge symmetries. This would give infinite hierarchies of breakings of conformal symmetry interpreted in terms of criticality: in the hierarchy n_i divides n_{i+1}.

   Similar degeneracy would be associated with both the parton orbits and the space-like ends at CD boundaries and I have considered the possibility that the integer n appearing in h_{eff} has decomposition $n = n_1 n_2$ corresponding to the degeneracies associated with the two kinds of boundaries. Alternatively, one could have just $n = n_1 = n_2$ from the condition that the two conformal symmetries are 3-dimensional manifestations of single 4-D analog of conformal symmetry.

2. In the symmetry breaking $n_i \rightarrow n_{i+1}$ the conformal charges, which vanished earlier, would become non-vanishing. Could one require that they are conserved that is the contributions of the boundary terms at the ends of CD cancel each other? If so, one would have dynamical conformal symmetry.

What could the proper interpretation of the conformal hierarchies $n_i \rightarrow n_{i+1}$?

1. Could one interpret the hierarchy in terms of increasing measurement resolution? Conformal degrees of freedom below measurement resolution would be gauge degrees of freedom and the conformal hierarchies would correspond to an inclusion hierarchies for hyper-finite factors of type II_1 [K103]. If $h_{eff} = n \times h$ defines the conformal gauge sub-algebra, the improvement of the resolution would scale up the Compton scales and would quite concretely correspond to a zoom analogous to that done for Mandelbrot fractal to get new details visible. From the point of view of cognition the improving resolution would fit nicely with the recent view about h_{eff}/h as a kind of intelligence quotient.

   This interpretation might make sense for the symplectic algebra of $\delta M^4_\pm \times CP_2$ for which the light-like radial coordinate r_M of light-cone boundary takes the role of complex coordinate. The reason is that symplectic algebra acts as isometries.

2. Suppose that the Kähler action has vanishing variation under deformations defined by the broken conformal symmetries so that the corresponding conformal charges As a consequence, Kähler function would be critical with respect to the corresponding variations. The components of WCW Kähler metric expressible in terms of second derivatives of Kähler function can be however non-vanishing and have also components, which correspond to WCW coordinates associated with different partonic 2-surfaces. This conforms with the idea that conformal algebras extend to Yangian algebras generalizing the Yangian symmetry of $\mathcal{N} = 4$ symmetric gauge theories.

   In this kind of situation one could consider the interpretation in terms of criticality: the lower the criticality, the larger then value of h_{eff} and h and the higher the resolution.

3. n gives also the number of space-time sheets in the singular covering. Could the interpretation be in terms measurement resolution for counting the number of space-time sheets. Our recent quantum physics would only see single space-time sheet representing visible manner and dark matter would become visible only for $n > 1$.

As should have become clear, the derivation of field equations in TGD framework is not just an application of a formal recipe as in field theories and a lot of non-trivial physics is involved!

2.4 General View About Field Equations

In this section field equations are deduced and discussed in general level. The fact that the divergence of the energy momentum tensor, Lorentz 4-force, does not vanish in general, in principle makes possible the mimicry of even dissipation and of the second law. For asymptotic self organization patterns for which dissipation is absent the Lorentz 4-force must vanish. This condition is guaranteed if Kähler current is proportional to the instanton current in the case that CP_2 projection of the space-time sheet is smaller than four and vanishes otherwise. An attractive identification for the vanishing of Lorentz 4-force is as a condition equivalent with the selection of preferred extremal of Kähler action. This condition implies that covariant divergence of energy momentum tensor vanishes and in General Relativity context this leads to Einstein's equations. If preferred extremals correspond to absolute minima this principle would be essentially equivalent with the second law of thermodynamics. There are however could reasons to keep the identification of preferred extremely property open.

2.4.1 Field Equations

The requirement that Kähler action is stationary leads to the following field equations in the interior of the four-surface

$$D_\beta(T^{\alpha\beta}h^k_\alpha) - j^\alpha J^k{}_l\partial_\alpha h^l = 0 \ ,$$
$$T^{\alpha\beta} = J^{\nu\alpha}J_\nu{}^\beta - \frac{1}{4}g^{\alpha\beta}J^{\mu\nu}J_{\mu\nu} \ . \tag{2.4.1}$$

Here $T^{\alpha\beta}$ denotes the traceless canonical energy momentum tensor associated with the Kähler action. An equivalent form for the first equation is

$$T^{\alpha\beta}H^k_{\alpha\beta} - j^\alpha(J_\alpha{}^\beta h^k{}_\beta + J^k{}_l\partial_\alpha h^l) = 0 \ .$$
$$H^k_{\alpha\beta} = D_\beta\partial_\alpha h^k \ . \tag{2.4.2}$$

$H^k_{\alpha\beta}$ denotes the components of the

second fundamental form and $j^\alpha = D_\beta J^{\alpha\beta}$ is the gauge current associated with the Kähler field.

On the boundaries of X^4 and at wormhole throats the field equations are given by the expression

$$\frac{\partial L_K}{\partial_n h^k} = T^{n\beta}\partial_\beta h^k - J^{n\alpha}(J_\alpha{}^\beta\partial_\beta h^k + J^k_l)\partial_\alpha h^k) = 0 \ . \tag{2.4.3}$$

At wormhole throats problems are caused by the vanishing of metric determinant implying that contravariant metric is singular.

For M^4 coordinates boundary conditions are satisfied if one assumes

$$T^{n\beta} = 0 \tag{2.4.4}$$

stating that there is no flow of four-momentum through the boundary component or wormhole throat. This means that there is no energy exchange between Euclidian and Minkowskian regions so that Euclidian regions provide representations for particles as autonomous units. This is in accordance with the general picture [K40]. Note that momentum transfer with external world necessarily involves generalized Feynman diagrams also at classical level.

For CP_2 coordinates the boundary conditions are more delicate. The construction of WCW spinor structure [K104] led to the conditions

$$g_{ni} = 0 \ , \ J_{ni} = 0 \ . \tag{2.4.5}$$

$J^{ni} = 0$ does not and should not follow from this condition since contravariant metric is singular. It seems that limiting procedure is necessary in order to see what comes out.

The condition that Kähler electric charge defined as a gauge flux is non-vanishing would require that the quantity $J^{nr}\sqrt{g}$ is finite (here r refers to the light-like coordinate of X^3_l). Also $g^{nr}\sqrt{g_4}$ which is analogous to gravitational flux if n is interpreted as time coordinate could be non-vanishing. These conditions are consistent with the above condition if one has

$$\begin{array}{llll} J_{ni} = 0 \ , & g_{ni} = 0 \ , & J_{ir} = 0 \ , & g_{ir} = 0 \ , \\ J^{nk} = 0 \ \ k \neq r \ , & g^{nk} = 0 \ \ k \neq r \ , & J^{nr}\sqrt{g_4} \neq 0 \ , & g^{nr}\sqrt{g_4} \neq 0 \ . \end{array} \tag{2.4.6}$$

The interpretation of this conditions is rather transparent.

1. The first two conditions state that covariant form of the induced Kähler electric field is in direction normal to X^3_l and metric separate into direct sum of normal and tangential contributions. Fifth and sixth condition state the same in contravariant form for $k \neq n$.

2. Third and fourth condition state that the induced Kähler field at X^3_l is purely magnetic and that the metric of x^3_l reduces to a block diagonal form. The reduction to purely magnetic field is of obvious importance as far as the understanding of the generalized eigen modes of the Kähler-Dirac operator is considered [K104].
3. The last two conditions must be understood as a limit and $\neq$ means only the possibility of non-vanishing Kähler gauge flux or analog of gravitational flux through X^3_l.
4. The vision inspired by number theoretical compactification allows to identify r and n in terms of the light-like coordinates assignable to an integrable distribution of planes $M^2(x)$ assumed to be assignable to M^4 projection of $X^4(X^3_l)$. Later it will be found that Hamilton-Jacobi structure assignable to the extremals indeed means the existence of this kind of distribution meaning slicing of $X^4(X^3_l)$ both by string world sheets and dual partonic 2-surfaces as well as by light-like 3-surfaces Y^3_l.
5. The physical analogy for the situation is the surface of an ideal conductor. It would not be surprising that these conditions are satisfied by all induced gauge fields.

2.4.2 Topologization And Light-Likeness Of The Kähler Current As Alternative ways To Guarantee Vanishing Of Lorentz 4-Force

The general solution of 4-dimensional Einstein-Yang Mills equations in Euclidian 4-metric relies on self-duality of the gauge field, which topologizes gauge charge. This topologization can be achieved by a weaker condition, which can be regarded as a dynamical generalization of the Beltrami condition. An alternative manner to achieve vanishing of the Lorentz 4-force is light-likeness of the Kähler 4-current. This does not require topologization.

Topologization of the Kähler current for $D_{CP_2} = 3$: covariant formulation

The condition states that Kähler 4-current is proportional to the instanton current whose divergence is instanton density and vanishes when the dimension of CP_2 projection is smaller than four: $D_{CP_2} < 4$. For $D_{CP_2} = 2$ the instanton 4-current vanishes identically and topologization is equivalent with the vanishing of the Kähler current.

If the simplest vision about light-like 3-surfaces as basic dynamical objects is accepted $D_{CP_2} = 2$, corresponds to a non-physical situation and only the deformations of these surfaces - most naturally resulting by gluing of CP_2 type vacuum extremals on them - can represent preferred extremals of Kähler action. One can however speak about $D_{CP_2} = 2$ phase if 4-surfaces are obtained are obtained in this manner.

$$j^\alpha \equiv D_\beta J^{\alpha\beta} = \psi \times j^\alpha_I = \psi \times \epsilon^{\alpha\beta\gamma\delta} J_{\beta\gamma} A_\delta \ . \tag{2.4.7}$$

Here the function ψ is an arbitrary function $\psi(s^k)$ of CP_2 coordinates s^k regarded as functions of space-time coordinates. It is essential that ψ depends on the space-time coordinates through the CP_2 coordinates only. Hence the representation as an imbedded gauge field is crucial element of the solution ansatz.

The field equations state the vanishing of the divergence of the 4-current. This is trivially true for instanton current for $D_{CP_2} < 4$. Also the contraction of $\nabla\psi$ (depending on space-time coordinates through CP_2 coordinates only) with the instanton current is proportional to the winding number density and therefore vanishes for $D_{CP_2} < 4$.

The topologization of the Kähler current guarantees the vanishing of the Lorentz 4-force. Indeed, using the self-duality condition for the current, the expression for the Lorentz 4-force reduces to a term proportional to the instanton density:

$$\begin{aligned} j^\alpha J_{\alpha\beta} &= \psi \times j^\alpha_I J_{\alpha\beta} \\ &= \psi \times \epsilon^{\alpha\mu\nu\delta} J_{\mu\nu} A_\delta J_{\alpha\beta} \ . \end{aligned} \tag{2.4.8}$$

Since all vector quantities appearing in the contraction with the four-dimensional permutation tensor are proportional to the gradients of CP_2 coordinates, the expression is proportional to the instanton density, and thus winding number density, and vanishes for $D_{CP_2} < 4$.

Remarkably, the topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{k_l}\partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M^4_+ and CP_2 degrees of freedom to

$$T^{\alpha\beta}H^k_{\alpha\beta} = 0 \ . \tag{2.4.9}$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The earlier proposal that quaternion conformal invariance in a suitable sense might provide a general solution of the field equations could be seen as a generalization of the ordinary conformal invariance of string models. If the topologization of the Kähler current implying effective dimensional reduction in CP_2 degrees of freedom is consistent with quaternion conformal invariance, the quaternion conformal structures must differ for the different dimensions of CP_2 projection.

Topologization of the Kähler current for $D_{CP_2} = 3$: non-covariant formulation

In order to gain a concrete understanding about what is involved it is useful to repeat these arguments using the 3-dimensional notation. The components of the instanton 4-current read in three-dimensional notation as

$$\overline{j}_I = \overline{E} \times \overline{A} + \phi\overline{B} \ , \quad \rho_I = \overline{B}\cdot\overline{A} \ . \tag{2.4.10}$$

The self duality conditions for the current can be written explicitly using 3-dimensional notation and read

$$\begin{aligned} \nabla \times \overline{B} - \partial_t\overline{E} &= \overline{j} = \psi\overline{j}_I = \psi\left(\phi\overline{B} + \overline{E}\times\overline{A}\right) \ , \\ \nabla\cdot E &= \rho = \psi\rho_I \ . \end{aligned} \tag{2.4.11}$$

For a vanishing electric field the self-duality condition for Kähler current reduces to the Beltrami condition

$$\nabla \times \overline{B} = \alpha\overline{B} \ , \quad \alpha = \psi\phi \ . \tag{2.4.12}$$

The vanishing of the divergence of the magnetic field implies that α is constant along the field lines of the flow. When ϕ is constant and $\overline{A}$ is time independent, the condition reduces to the Beltrami condition with $\alpha = \phi = constant$, which allows an explicit solution [B11].

One can check also the vanishing of the Lorentz 4-force by using 3-dimensional notation. Lorentz 3-force can be written as

$$\rho_I\overline{E} + \overline{j}\times\overline{B} = \psi\overline{B}\cdot\overline{A}\overline{E} + \psi\left(\overline{E}\times\overline{A} + \phi\overline{B}\right)\times\overline{B} = 0 \ . \tag{2.4.13}$$

The fourth component of the Lorentz force reads as

$$\overline{j}\cdot\overline{E} = \psi\overline{B}\cdot\overline{E} + \psi\left(\overline{E}\times\overline{A} + \phi\overline{B}\right)\cdot\overline{E} = 0 \ . \tag{2.4.14}$$

The remaining conditions come from the induction law of Faraday and could be guaranteed by expressing $\overline{E}$ and $\overline{B}$ in terms of scalar and vector potentials.

The density of the Kähler electric charge of the vacuum is proportional to the the helicity density of the so called helicity charge $\rho = \psi\rho_I = \psi B\cdot A$. This charge is topological charge in the sense that it does not depend on the induced metric at all. Note the presence of arbitrary function ψ of CP_2 coordinates.

Further conditions on the functions appearing in the solution ansatz come from the 3 independent field equations for CP_2 coordinates. What is remarkable that the generalized self-duality condition for the Kähler current allows to understand the general features of the solution ansatz to very high degree without any detailed knowledge about the detailed solution. The question whether field equations allow solutions consistent with the self duality conditions of the current will be dealt later. The optimistic guess is that the field equations and topologization of the Kähler current relate to each other very intimately.

Vanishing or light likeness of the Kähler current guarantees vanishing of the Lorentz 4-force for $D_{CP_2} = 2$

For $D_{CP_2} = 2$ one can always take two CP_2 coordinates as space-time coordinates and from this it is clear that instanton current vanishes so that topologization gives a vanishing Kähler current. In particular, the Beltrami condition $\nabla \times \overline{B} = \alpha \overline{B}$ is not consistent with the topologization of the instanton current for $D_{CP_2} = 2$.

$D_{CP_2} = 2$ case can be treated in a coordinate invariant manner by using the two coordinates of CP_2 projection as space-time coordinates so that only a magnetic or electric field is present depending on whether the gauge current is time-like or space-like. Light-likeness of the gauge current provides a second manner to achieve the vanishing of the Lorentz force and is realized in case of massless extremals having $D_{CP_2} = 2$: this current is in the direction of propagation whereas magnetic and electric fields are orthogonal to it so that Beltrami conditions is certainly not satisfied.

Under what conditions topologization of Kähler current yields Beltrami conditions?

Topologization of the Kähler 4-current gives rise to magnetic Beltrami fields if either of the following conditions is satisfied.

1. The $\overline{E} \times \overline{A}$ term contributing besides $\phi \overline{B}$ term to the topological current vanishes. This requires that $\overline{E}$ and $\overline{A}$ are parallel to each other

$$\overline{E} = \nabla \Phi - \partial_t \overline{A} = \beta \overline{A} \tag{2.4.15}$$

   This condition is analogous to the Beltrami condition. Now only the 3-space has as its coordinates time coordinate and two spatial coordinatesand$\overline{B}$ is replaced with $\overline{A}$. Since E and B are orthogonal, this condition implies $\overline{B} \cdot \overline{A} = 0$ so that Kähler charge density is vanishing.

2. The vector $\overline{E} \times \overline{A}$ is parallel to $\overline{B}$.

$$\overline{E} \times \overline{A} = \beta \overline{B} \tag{2.4.16}$$

   The condition is consistent with the orthogonality of $\overline{E}$ and $\overline{B}$ but implies the orthogonality of $\overline{A}$ and $\overline{B}$ so that electric charge density vanishes

In both cases vector potential fails to define a contact structure since $B \cdot A$ vanishes (contact structures are discussed briefly below), and there exists a global coordinate along the field lines of $\overline{A}$ and the full contact structure is lost again. Note however that the Beltrami condition for magnetic field means that magnetic field defines a contact structure irrespective of whether $\overline{B} \cdot \overline{A}$ vanishes or not. The transition from the general case to Beltrami field would thus involve the replacement

$$(\overline{A}, \overline{B}) \rightarrow_{\nabla \times} (\overline{B}, \overline{j})$$

induced by the rotor.

One must of course take these considerations somewhat cautiously since the inner product depends on the induced 4-metric and it might be that induced metric could allow small vacuum charge density and make possible genuine contact structure.

Hydrodynamic analogy

The field equations of TGD are basically hydrodynamic equations stating the local conservation of the currents associated with the isometries of the embedding space. Therefore it is intriguing that Beltrami fields appear also as solutions of ideal magnetohydrodynamics equations and as steady solutions of non-viscous incompressible flow described by Euler equations [B36].

In hydrodynamics the role of the magnetic field is taken by the velocity field. This raises the idea that the incompressible flow could occur along the field lines of some natural vector field.

The considerations of the last section show that the instanton current defines a universal candidate as far as the general solution of the field equations is considered. All conserved currents defined by the isometry charges would be parallel to the instanton current: one can say each flow line of instanton current is a carrier of conserved quantum numbers. Perhaps even the flow lines of an incompressible hydrodynamic flow could in reasonable approximation correspond to those of instanton current.

The conservation laws are satisfied for each flow line separately and therefore it seems that one cannot have the analog of viscous hydrodynamic flow in this framework. One the other hand, quantum classical correspondence requires that also dissipative effects have space-time correlates. Does something go badly wrong?

The following argument suggests a way out of the problem. Dissipation is certaily due to the quantum jumps at scales below that associated with causal diamond (CD) associated with the observer and is thus assignable to sub-CDs. The quantum jumps for sub-CDs would eventually lead to a thermal ensemble of sub-CDs.

The usual description of dissipation in terms of viscocity and similar parameters emerges at the GRT-QFT limit of TGD replacing in long length scales the many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) with a piece of Minkowski space with effective metric defined by the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. This lumping of space-time sheets means that induced gauge fields and gravitational fields from various space-time sheet sum up and become random (by central limit theorems). Thus locally the dynamics is dissipation free for individual space-time sheets and dissipation emerges at the level of GRT space-time carrying effective metric and effective gauge fields.

The stability of generalized Beltrami fields

The stability of generalized Beltrami fields is of high interest since unstable points of space-time sheets are those around which macroscopic changes induced by quantum jumps are expected to be localized.

1. Contact forms and contact structures

The stability of Beltrami flows has been studied using the theory of contact forms in three-dimensional Riemann manifolds contact. Contact form is a one-form A (that is covariant vector field A_α) with the property $A \wedge dA \neq 0$. In the recent case the induced Kähler gauge potential A_α and corresponding induced Kähler form $J_{\alpha\beta}$ for any 3-sub-manifold of space-time surface define a contact form so that the vector field $A^\alpha = g^{\alpha\beta}A_\beta$ is not orthogonal with the magnetic field $B^\alpha = \epsilon^{\alpha\beta\delta}J_{\beta\gamma}$. This requires that magnetic field has a helical structure. Induced metric in turn defines the Riemann structure.

If the vector potential defines a contact form, the charge density associated with the topologized Kähler current must be non-vanishing. This can be seen as follows.

1. The requirement that the flow lines of a one-form X_μ defined by the vector field X^μ as its dual allows to define a global coordinate x varying along the flow lines implies that there is an integrating factor ϕ such that $\phi X = dx$ and therefore $d(\phi X) = 0$. This implies $dlog(\phi) \wedge X = -dX$. From this the necessary condition for the existence of the coordinate x is $X \wedge dX = 0$. In the three-dimensional case this gives $\overline{X} \cdot (\nabla \times \overline{X}) = 0$.
2. This condition is by definition not satisfied by the vector potential defining a contact form so that one cannot identify a global coordinate varying along the flow lines of the vector potential. The condition $\overline{B} \cdot \overline{A} \neq 0$ states that the charge density for the topologized Kähler current is non-vanishing. The condition that the field lines of the magnetic field allow a global coordinate requires $\overline{B} \cdot \nabla \times \overline{B} = 0$. The condition is not satisfied by Beltrami fields with $\alpha \neq 0$. Note that in this case magnetic field defines a contact structure.

Contact structure requires the existence of a vector ξ satisfying the condition $A(\xi) = 0$. The vector field ξ defines a plane field, which is orthogonal to the vector field A^α. Reeb field in turn is a vector field for which $A(X) = 1$ and $dA(X;) = 0$ hold true. The latter condition states the vanishing of the cross product $X \times B$ so that X is parallel to the Kähler magnetic field B^α and has unit projection in the direction of the vector field A^α. Any Beltrami field defines a Reeb field irrespective of the Riemannian structure.

2. Stability of the Beltrami flow and contact structures

Contact structures are used in the study of the topology and stability of the hydrodynamical flows [B25], and one might expect that the notion of contact structure and its proper generalization to the four-dimensional context could be useful in TGD framework also. An example giving some idea about the complexity of the flows defined by Beltrami fields is the Beltrami field in R^3 possessing closed orbits with all possible knot and link types simultaneously [B25] !

Beltrami flows associated with Euler equations are known to be unstable [B25]. Since the flow is volume preserving, the stationary points of the Beltrami flow are saddle points at which also vorticity vanishes and linear instabilities of Navier-Stokes equations can develop. From the point of view of biology it is interesting that the flow is stabilized by vorticity which implies also helical structures. The stationary points of the Beltrami flow correspond in TGD framework to points at which the induced Kähler magnetic field vanishes. They can be unstable by the vacuum degeneracy of Kähler action implying classical non-determinism. For generalized Beltrami fields velocity and vorticity (both divergence free) are replaced by Kähler current and instanton current.

More generally, the points at which the Kähler 4-current vanishes are expected to represent potential instabilities. The instanton current is linear in Kähler field and can vanish in a gauge invariant manner only if the induced Kähler field vanishes so that the instability would be due to the vacuum degeneracy also now. Note that the vanishing of the Kähler current allows also the generation of region with $D_{CP_2} = 4$. The instability of the points at which induce Kähler field vanish is manifested in quantum jumps replacing the generalized Beltrami field with a new one such that something new is generated around unstable points. Thus the regions in which induced Kähler field becomes weak are the most interesting ones. For example, unwinding of DNA could be initiated by an instability of this kind.

2.4.3 How To Satisfy Field Equations?

The topologization of the Kähler current guarantees also the vanishing of the term $j^\alpha J^{k_l}\partial_\alpha s^k$ in the field equations for CP_2 coordinates. This means that field equations reduce in both M^4_+ and CP_2 degrees of freedom to

$$T^{\alpha\beta}H^k_{\alpha\beta} = 0 . \qquad (2.4.17)$$

These equations differ from the equations of minimal surface only by the replacement of the metric tensor with energy momentum tensor. The following approach utilizes the properties of Hamilton Jacobi structures of M^4_+ introduced in the study of massless extremals and contact structures of CP_2 emerging naturally in the case of generalized Beltrami fields.

String model as a starting point

String model serves as a starting point.

1. In the case of Minkowskian minimal surfaces representing string orbit the field equations reduce to purely algebraic conditions in light cone coordinates (u, v) since the induced metric has only the component g_{uv}, whereas the second fundamental form has only diagonal components H^k_{uu} and H^k_{vv}.
2. For Euclidian minimal surfaces (u, v) is replaced by complex coordinates $(w, \overline{w})$ and field equations are satisfied because the metric has only the component $g^{w\overline{w}}$ and second fundamental form has only components of type H^k_{ww} and $H^{\overline{k}}_{\overline{w}\overline{w}}$. The mechanism should generalize to the recent case.

The general form of energy momentum tensor as a guideline for the choice of coordinates

Any 3-dimensional Riemann manifold allows always a orthogonal coordinate system for which the metric is diagonal. Any 4-dimensional Riemann manifold in turn allows a coordinate system for which 3-metric is diagonal and the only non-diagonal components of the metric are of form g^{ti}. This kind of coordinates might be natural also now. When $\overline{E}$ and $\overline{B}$ are orthogonal, energy momentum tensor has the form

$$T = \begin{pmatrix} \frac{E^2+B^2}{2} & 0 & 0 & EB \\ 0 & \frac{E^2+B^2}{2} & 0 & 0 \\ 0 & 0 & \frac{-E^2+B^2}{2} & 0 \\ EB & 0 & 0 & \frac{E^2-B^2}{2} \end{pmatrix} \tag{2.4.18}$$

in the tangent space basis defined by time direction and longitudinal direction $\overline{E}\times\overline{B}$, and transversal directions $\overline{E}$ and $\overline{B}$. Note that T is traceless.

The optimistic guess would be that the directions defined by these vectors integrate to three orthogonal coordinates of X^4 and together with time coordinate define a coordinate system containing only g^{ti} as non-diagonal components of the metric. This however requires that the fields in question allow an integrating factor and, as already found, this requires $\nabla \times X \cdot X = 0$ and this is not the case in general.

Physical intuition suggests however that X^4 coordinates allow a decomposition into longitudinal and transversal degrees freedom. This would mean the existence of a time coordinate t and longitudinal coordinate z the plane defined by time coordinate and vector $\overline{E}\times\overline{B}$ such that the coordinates $u = t-z$ and $v = t+z$ are light like coordinates so that the induced metric would have only the component g^{uv} whereas g^{vv} and g^{uu} would vanish in these coordinates. In the transversal space-time directions complex space-time coordinate coordinate w could be introduced. Metric could have also non-diagonal components besides the components $g^{w\overline{w}}$ and g^{uv}.

Hamilton Jacobi structures in M^4_+

Hamilton Jacobi structure in M^4_+ can understood as a generalized complex structure combing transversal complex structure and longitudinal hyper-complex structure so that notion of holomorphy and Kähler structure generalize.

1. Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct, not necessarily orthogonal, sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has an interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray. Assume that E^2 allows complex coordinates $w = E^1 + iE^2$ and $\overline{w} = E^1 - iE^2$. The simplest decomposition of this kind corresponds to the decomposition ($S^+ \equiv u = t+z, S^- \equiv v = t-z, w = x+iy, \overline{w} = x-iy$).

2. In accordance with this physical picture, S^+ and S^- define light-like curves which are normals to light-like surfaces and thus satisfy the equation:

$$(\nabla S_\pm)^2 = 0 \quad .$$

   The gradients of $S_\pm$ are obviously analogous to local light like velocity vectors $v = (1, \overline{v})$ and $\tilde{v} = (1, -\overline{v})$. These equations are also obtained in geometric optics from Hamilton Jacobi equation by replacing photon's four-velocity with the gradient ∇S: this is consistent with the interpretation of massless extremals as Bohr orbits of em field. $S_\pm = constant$ surfaces can be interpreted as expanding light fronts. The interpretation of $S_\pm$ as Hamilton Jacobi functions justifies the term Hamilton Jacobi structure.

   The simplest surfaces of this kind correspond to $t = z$ and $t = -z$ light fronts which are planes. They are dual to each other by hyper complex conjugation $u = t-z \rightarrow v = t+z$. One should somehow generalize this conjugation operation. The simplest candidate for the conjugation $S^+ \rightarrow S^-$ is as a conjugation induced by the conjugation for the arguments: $S^+(t-z, t+z, x, y) \rightarrow S^-(t-z, t+z, x, y) = S^+(t+z, t-z, x, -y)$ so that a dual pair is mapped to a dual pair. In transversal degrees of freedom complex conjugation would be involved.

3. The coordinates $(S_\pm, w, \overline{w})$ define local light cone coordinates with the line element having the form

$$\begin{aligned} ds^2 &= g_{+-}dS^+dS^- + g_{w\overline{w}}dwd\overline{w} \\ &+ g_{+w}dS^+dw + g_{+\overline{w}}dS^+d\overline{w} \\ &+ g_{-w}dS^-dw + g_{-\overline{w}}dS^-d\overline{w} \ . \end{aligned} \quad (2.4.19)$$

Conformal transformations of M^4_+ leave the general form of this decomposition invariant. Also the transformations which reduces to analytic transformations $w \rightarrow f(w)$ in transversal degrees of freedom and hyper-analytic transformations $S^+ \rightarrow f(S^+), S^- \rightarrow f(S^-)$ in longitudinal degrees of freedom preserve this structure.

4. The basic idea is that of generalized Kähler structure meaning that the notion of Kähler function generalizes so that the non-vanishing components of metric are expressible as

$$\begin{aligned} g_{w\overline{w}} &= \partial_w\partial_{\overline{w}}K \ , \quad g_{+-} = \partial_{S^+}\partial_{S^-}K \ , \\ g_{w\pm} &= \partial_w\partial_{S^\pm}K \ , \quad g_{\overline{w}\pm} = \partial_{\overline{w}}\partial_{S^\pm}K \ . \end{aligned} \quad (2.4.20)$$

for the components of the metric. The expression in terms of Kähler function is coordinate invariant for the same reason as in case of ordinary Kähler metric. In the standard light-cone coordinates the Kähler function is given by

$$K = w_0\overline{w}_0 + uv \ , \quad w_0 = x + iy \ , \quad u = t - z \ , \quad v = t + z \ . \quad (2.4.21)$$

The Christoffel symbols satisfy the conditions

$$\left\{ {k \atop w\ \overline{w}} \right\} = 0 \ , \quad \left\{ {k \atop +-} \right\} = 0 \ . \quad (2.4.22)$$

If energy momentum tensor has only the components $T^{w\overline{w}}$ and T^{+-}, field equations are satisfied in M^4_+ degrees of freedom.

5. The Hamilton Jacobi structures related by these transformations can be regarded as being equivalent. Since light-like 3- surface is, as the dynamical evolution defined by the light front, fixed by the 2-surface serving as the light source, these structures should be in one-one correspondence with 2-dimensional surfaces with two surfaces regarded as equivalent if they correspond to different time=constant snapshots of the same light front, or are related by a conformal transformation of M^4_+. Obviously there should be quite large number of them. Note that the generating two-dimensional surfaces relate also naturally to quaternion conformal invariance and corresponding Kac Moody invariance for which deformations defined by the M^4 coordinates as functions of the light-cone coordinates of the light front evolution define Kac Moody algebra, which thus seems to appear naturally also at the level of solutions of field equations.

The task is to find all possible local light cone coordinates defining one-parameter families 2-surfaces defined by the condition $S_i = constant$, $i = +\ or\ = -$, dual to each other and expanding with light velocity. The basic open questions are whether the generalized Kähler function indeed makes sense and whether the physical intuition about 2-surfaces as light sources parameterizing the set of all possible Hamilton Jacobi structures makes sense.

Hamilton Jacobi structure means the existence of foliations of the M^4 projection of X^4 by 2-D surfaces analogous to string word sheets labeled by w and the dual of this foliation defined by partonic 2-surfaces labeled by the values of S_i. Also the foliation by light-like 3-surfaces Y^3_l labeled by $S_\pm$ with $S_\mp$ serving as light-like coordinate for Y^3_l is implied. This is what number theoretic compactification and $M^8 - H$ duality predict when space-time surface corresponds to hyper-quaternionic surface of M^8 [K40, K88].

Contact structure and generalized Kähler structure of CP_2 projection

In the case of 3-dimensional CP_2 projection it is assumed that one can introduce complex coordinates $(\xi, \overline{\xi})$ and the third coordinate s. These coordinates would correspond to a contact structure in 3-dimensional CP_2 projection defining transversal symplectic and Kähler structures. In these coordinates the transversal parts of the induced CP_2 Kähler form and metric would contain only components of type $g_{w\overline{w}}$ and $J_{w\overline{w}}$. The transversal Kähler field $J_{w\overline{w}}$ would induce the Kähler magnetic field and the components J_{sw} and $J_{s\overline{w}}$ the Kähler electric field.

It must be emphasized that the non-integrability of the contact structure implies that J cannot be parallel to the tangent planes of $s = constant$ surfaces, s cannot be parallel to neither A nor the dual of J, and ξ cannot vary in the tangent plane defined by J. A further important conclusion is that for the solutions with 3-dimensional CP_2 projection topologized Kähler charge density is necessarily non-vanishing by $A \wedge J \neq 0$ whereas for the solutions with $D_{CP_2} = 2$ topologized Kähler current vanishes.

Also the CP_2 projection is assumed to possess a generalized Kähler structure in the sense that all components of the metric except s_{ss} are derivable from a Kähler function by formulas similar to M^4_+ case.

$$s_{w\overline{w}} = \partial_w \partial_{\overline{w}} K \quad , \quad s_{ws} = \partial_w \partial_s K \quad , \quad s_{\overline{w}s} = \partial_{\overline{w}} \partial_s K \quad . \tag{2.4.23}$$

Generalized Kähler property guarantees that the vanishing of the Christoffel symbols of CP_2 (rather than those of 3-dimensional projection), which are of type $\{^{\ k}_{\xi\ \overline{\xi}}\}$.

$$\{^{\ k}_{\xi\ \overline{\xi}}\} = 0 \ . \tag{2.4.24}$$

Here the coordinates of CP_2 have been chosen in such a manner that three of them correspond to the coordinates of the projection and fourth coordinate is constant at the projection. The upper index k refers also to the CP_2 coordinate, which is constant for the CP_2 projection. If energy momentum tensor has only components of type T^{+-} and $T^{w\overline{w}}$, field equations are satisfied even when if non-diagonal Christoffel symbols of CP_2 are present. The challenge is to discover solution ansatz, which guarantees this property of the energy momentum tensor.

A stronger variant of Kähler property would be that also s_{ss} vanishes so that the coordinate lines defined by s would define light like curves in CP_2. The topologization of the Kähler current however implies that CP_2 projection is a projection of a 3-surface with strong Kähler property. Using $(s, \xi, \overline{\xi}, S^-)$ as coordinates for the space-time surface defined by the ansatz $(w = w(\xi, s), S^+ = S^+(s))$ one finds that g_{ss} must be vanishing so that stronger variant of the Kähler property holds true for $S^- = constant$ 3-surfaces.

The topologization condition for the Kähler current can be solved completely generally in terms of the induced metric using $(\xi, \overline{\xi}, s)$ and some coordinate of M^4_+, call it x^4, as space-time coordinates. Topologization boils down to the conditions

$$\begin{aligned} \partial_\beta (J^{\alpha\beta} \sqrt{g}) &= 0 \text{ for } \alpha \in \{\xi, \overline{\xi}, s\} \ , \\ g^{4i} &\neq 0 \ . \end{aligned} \tag{2.4.25}$$

Thus 3-dimensional empty space Maxwell equations and the non-orthogonality of X^4 coordinate lines and the 3-surfaces defined by the lift of the CP_2 projection.

A solution ansatz yielding light-like current in $D_{CP_2} = 3$ case

The basic idea is that of generalized Kähler structure and solutions of field equations as maps or deformations of canonically imbedded M^4_+ respecting this structure and guaranteeing that the only non-vanishing components of the energy momentum tensor are $T^{\xi\xi}$ and T^{s-} in the coordinates $(\xi, \overline{\xi}, s, S^-)$.

1. The coordinates (w, S^+) are assumed to holomorphic functions of the CP_2 coordinates (s, ξ)

$$S^+ = S^+(s) \ , \quad w = w(\xi, s) \ . \tag{2.4.26}$$

Obviously S^+ could be replaced with S^-. The ansatz is completely symmetric with respect to the exchange of the roles of (s, w) and (S^+, ξ) since it maps longitudinal degrees of freedom to longitudinal ones and transverse degrees of freedom to transverse ones.

2. Field equations are satisfied if the only non-vanishing components of the energy momentum tensor are of type $T^{\xi\overline{\xi}}$ and T^{s-}. The reason is that the CP_2 Christoffel symbols for projection and projections of M^4_+ Christoffel symbols are vanishing for these lower index pairs.

3. By a straightforward calculation one can verify that the only manner to achieve the required structure of energy momentum tensor is to assume that the induced metric in the coordinates $(\xi, \overline{\xi}, s, S^-)$ has as non-vanishing components only $g_{\xi\overline{\xi}}$ and g_{s-}

$$g_{ss} = 0 \ , \quad g_{\xi s} = 0 \ , \quad g_{\overline{\xi} s} = 0 \ . \tag{2.4.27}$$

Obviously the space-time surface must factorize into an orthogonal product of longitudinal and transversal spaces.

4. The condition guaranteeing the product structure of the metric is

$$\begin{aligned} s_{ss} &= m_{+w}\partial_s w(\xi, s)\partial_s S^+(s) + m_{+\overline{w}}\partial_s \overline{w}(\xi, s)\partial_s S^+(s) \ , \\ s_{s\xi} &= m_{+w}\partial_\xi w(\xi)\partial_s S^+(s) \ , \\ s_{s\overline{\xi}} &= m_{+w}\partial_{\overline{\xi}} w(\overline{\xi})\partial_s S^+(s) \ . \end{aligned} \tag{2.4.28}$$

Thus the function of dynamics is to diagonalize the metric and provide it with strong Kähler property. Obviously the CP_2 projection corresponds to a light-like surface for all values of S^- so that space-time surface is foliated by light-like surfaces and the notion of generalized conformal invariance makes sense for the entire space-time surface rather than only for its boundary or elementary particle horizons.

5. The requirement that the Kähler current is proportional to the instanton current means that only the j^- component of the current is non-vanishing. This gives the following conditions

$$\begin{aligned} & j^\xi\sqrt{g} = \partial_\beta(J^{\xi\beta}\sqrt{g}) = 0 \ , \quad j^{\overline{\xi}}\sqrt{g} = \partial_\beta(J^{\overline{\xi}\beta}\sqrt{g}) = 0 \ , \\ & j^+\sqrt{g} = \partial_\beta(J^{+\beta}\sqrt{g}) = 0 \ . \end{aligned} \tag{2.4.29}$$

Since $J^{+\beta}$ vanishes, the condition

$$\sqrt{g}j^+ = \partial_\beta(J^{+\beta}\sqrt{g}) \ = \ 0 \tag{2.4.30}$$

is identically satisfied. Therefore the number of field equations reduces to three.

The physical interpretation of the solution ansatz deserves some comments.

1. The light-like character of the Kähler current brings in mind CP_2 type extremals for which CP_2 projection is light like. This suggests that the topological condensation of CP_2 type extremal occurs on $D_{CP_2} = 3$ helical space-time sheet representing zitterbewegung. In the case of many-body system light-likeness of the current does not require that particles are massless if particles of opposite charges can be present. Field tensor has the form $(J^{\xi\overline{\xi}}, J^{\xi-}, J^{\overline{\xi}-})$. Both helical magnetic field and electric field present as is clear when one replaces the coordinates (S^+, S^-) with time-like and space-like coordinate. Magnetic field dominates but the presence of electric field means that genuine Beltrami field is not in question.

2. Since the induced metric is product metric, 3-surface is metrically product of 2-dimensional surface X^2 and line or circle and obeys product topology. If preferred extremals correspond to asymptotic self-organization patterns, the appearance of the product topology and even metric is not so surprising. Thus the solutions can be classified by the genus of X^2. An interesting question is how closely the explanation of family replication phenomenon in terms of the topology of the boundary component of elementary particle like 3-surface relates to this. The heaviness and instability of particles which correspond to genera $g > 2$ (sphere with more than two handles) might have simple explanation as absence of (stable) $D_{CP_2} = 3$ solutions of field equations with genus $g > 2$.

3. The solution ansatz need not be the most general. Kähler current is light-like and already this is enough to reduce the field equations to the form involving only energy momentum tensor. One might hope of finding also solution ansätze for which Kähler current is time-like or space-like. Space-likeness of the Kähler current might be achieved if the complex coordinates $(\xi, \overline{\xi})$ and hyper-complex coordinates (S^+, S^-) change the role. For this solution ansatz electric field would dominate. Note that the possibility that Kähler current is always light-like cannot be excluded.

4. Suppose that CP_2 projection quite generally defines a foliation of the space-time surface by light-like 3-surfaces, as is suggested by the conformal invariance. If the induced metric has Minkowskian signature, the fourth coordinate x^4 and thus also Kähler current must be time-like or light-like so that magnetic field dominates. Already the requirement that the metric is non-degenerate implies $g_{s4} \neq 0$ so that the metric for the $\xi = constant$ 2-surfaces has a Minkowskian signature. Thus space-like Kähler current does not allow the lift of the CP_2 projection to be light-like.

Are solutions with time-like or space-like Kähler current possible in $D_{CP_2} = 3$ case?

As noticed in the section about number theoretical compactification, the flow of gauge currents along slices Y^3_l of $X^4(X^3_l)$ "parallel" to X^3_l requires only that gauge currents are parallel to Y^3_l and can thus space-like. The following ansatz gives good hopes for obtaining solutions with space-like and perhaps also time-like Kähler currents.

1. Assign to light-like coordinates coordinates (T, Z) by the formula $T = S^+ + S^-$ and $Z = S^+ - S^-$. Space-time coordinates are taken to be $(\xi, \overline{\xi}, s)$ and coordinate Z. The solution ansatz with time-like Kähler current results when the roles of T and Z are changed. It will however found that same solution ansatz can give rise to both space-like and time-like Kähler current.

2. The solution ansatz giving rise to a space-like Kähler current is defined by the equations

$$T = T(Z, s) \ , \quad w = w(\xi, s) \ . \tag{2.4.31}$$

   If T depends strongly on Z, the g_{ZZ} component of the induced metric becomes positive and Kähler current time-like.

3. The components of the induced metric are

$$\begin{aligned} g_{ZZ} &= m_{ZZ} + m_{TT}\partial_Z T \partial_s T \ , \quad & g_{Zs} &= m_{TT}\partial_Z T \partial_s T \ , \\ g_{ss} &= s_{ss} + m_{TT}\partial_s T \partial_s T \ , & g_{w\overline{w}} &= s_{w\overline{w}} + m_{w\overline{w}}\partial_\xi w \partial_{\overline{\xi}} \overline{w} \ , \\ g_{s\xi} &= s_{s\xi} \ , & g_{s\overline{\xi}} &= s_{s\overline{\xi}} \ . \end{aligned} \tag{2.4.32}$$

   Topologized Kähler current has only Z-component and 3-dimensional empty space Maxwell's equations guarantee the topologization.

In CP_2 degrees of freedom the contractions of the energy momentum tensor with Christoffel symbols vanish if T^{ss}, $T^{\xi s}$ and $T^{\xi\xi}$ vanish as required by internal consistency. This is guaranteed if the condition

$$J^{\xi s} \;=\; 0 \qquad (2.4.33)$$

holds true. Note however that $J^{\xi Z}$ is non-vanishing. Therefore only the components $T^{\xi\overline{\xi}}$ and $T^{Z\xi}$, $T^{Z\overline{\xi}}$ of energy momentum tensor are non-vanishing, and field equations reduce to the conditions

$$\begin{aligned} \partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g}) + \partial_Z(J^{\xi Z}\sqrt{g}) &= 0 \ , \\ \partial_{\xi}(J^{\overline{\xi}\xi}\sqrt{g}) + \partial_Z(J^{\overline{\xi} Z}\sqrt{g}) &= 0 \ . \end{aligned} \qquad (2.4.34)$$

In the special case that the induced metric does not depend on z-coordinate equations reduce to holomorphicity conditions. This is achieve if T depends linearly on Z: $T = aZ$.

The contractions with M^4_+ Christoffel symbols come from the non-vanishing of $T^{Z\xi}$ and vanish if the Hamilton Jacobi structure satisfies the conditions

$$\begin{aligned} &\{{}^{\ k}_{T\ w}\} = 0 \ , \quad \{{}^{\ k}_{T\ \overline{w}}\} = 0 \ , \\ &\{{}^{\ k}_{Z\ w}\} = 0 \ , \quad \{{}^{\ k}_{Z\ \overline{w}}\} = 0 \end{aligned} \qquad (2.4.35)$$

hold true. The conditions are equivalent with the conditions

$$\{{}^{\ k}_{\pm\ w}\} = 0 \ , \quad \{{}^{\ k}_{\pm\ \overline{w}}\} = 0 \ . \qquad (2.4.36)$$

These conditions possess solutions (standard light cone coordinates are the simplest example). Also the second derivatives of $T(s,Z)$ contribute to the second fundamental form but they do not give rise to non-vanishing contractions with the energy momentum tensor. The cautious conclusion is that also solutions with time-like or space-like Kähler current are possible.

$D_{CP_2} = 4$ case

The preceding discussion was for $D_{CP_2} = 3$ and one should generalize the discussion to $D_{CP_2} = 4$ case.

1. Hamilton Jacobi structure for M^4_+ is expected to be crucial also now.
2. One might hope that for $D_{CP_2} = 4$ the Kähler structure of CP_2 defines a foliation of CP_2 by 3-dimensional contact structures. This requires that there is a coordinate varying along the field lines of the normal vector field X defined as the dual of the three-form $A \wedge dA = A \wedge J$. By the previous considerations the condition for this reads as $dX = d(log\phi) \wedge X$ and implies $X \wedge dX = 0$. Using the self duality of the Kähler form one can express X as $X^k = J^{kl}A_l$. By a brief calculation one finds that $X \wedge dX \propto X$ holds true so that (somewhat disappointingly) a foliation of CP_2 by contact structures does not exist.

For $D_{CP_2} = 4$ case Kähler current vanishes and this case corresponds to what I have called earlier Maxwellian phase since empty space Maxwell's equations would be indeed satisfied, provided this phase exists at all. It however seems that Maxwell phase is probably realized differently.

1. Solution ansatz with a 3-dimensional M^4_+ projection

The basic idea is that the complex structure of CP_2 is preserved so that one can use complex coordinates (ξ^1, ξ^2) for CP_2 in which CP_2 Christoffel symbols and energy momentum tensor have automatically the desired properties. This is achieved the second light like coordinate, say v, is non-dynamical so that the induced metric does not receive any contribution from the longitudinal degrees of freedom. In this case one has

$$S^+ = S^+(\xi^1, \xi^2) \ , \quad w = w(\xi^1, \xi^2) \ , \quad S^- = constant \ . \qquad (2.4.37)$$

The induced metric does possesses only components of type $g_{i\overline{j}}$ if the conditions

$$g_{+w} = 0 \ , \quad g_{+\overline{w}} = 0 \ . \tag{2.4.38}$$

This guarantees that energy momentum tensor has only components of type $T^{i\bar{j}}$ in coordinates (ξ^1, ξ^2) and their contractions with the Christoffel symbols of CP_2 vanish identically. In M^4_+ degrees of freedom one must pose the conditions

$$\{^{\ k}_{w+}\} = 0 \ , \quad \{^{\ k}_{\overline{w}+}\} = 0 \ , \quad \{^{\ k}_{++}\} = 0 \ . \tag{2.4.39}$$

on Christoffel symbols. These conditions are satisfied if the the M^4_+ metric does not depend on S^+:

$$\partial_+ m_{kl} \ = \ 0 \ . \tag{2.4.40}$$

This means that m_{-w} and $m_{-\overline{w}}$ can be non-vanishing but like m_{+-} they cannot depend on S^+.

The second derivatives of S^+ appearing in the second fundamental form are also a source of trouble unless they vanish. Hence S^+ must be a linear function of the coordinates ξ^k:

$$S^+ \ = \ a_k \xi^k + \overline{a}_k \overline{\xi}^k \ . \tag{2.4.41}$$

Field equations are the counterparts of empty space Maxwell equations $j^\alpha = 0$ but with M^4_+ coordinates (u, w) appearing as dynamical variables and entering only through the induced metric. By holomorphy the field equations can be written as

$$\partial_j (J^{j\bar{i}} \sqrt{g}) = 0 \ , \quad \partial_{\bar{j}} (J^{\bar{j}i} \sqrt{g}) = 0 \ , \tag{2.4.42}$$

and can be interpreted as conditions stating the holomorphy of the contravariant Kähler form.

What is remarkable is that the M^4_+ projection of the solution is 3-dimensional light like surface and that the induced metric has Euclidian signature. Light front would become a concrete geometric object with one compactified dimension rather than being a mere conceptualization. One could see this as topological quantization for the notion of light front or of electromagnetic shock wave, or perhaps even as the realization of the particle aspect of gauge fields at classical level.

If the latter interpretation is correct, quantum classical correspondence would be realized very concretely. Wave and particle aspects would both be present. One could understand the interactions of charged particles with electromagnetic fields both in terms of absorption and emission of topological field quanta and in terms of the interaction with a classical field as particle topologically condenses at the photonic light front.

For CP_2 type extremals for which M^4_+ projection is a light like curve correspond to a special case of this solution ansatz: transversal M^4_+ coordinates are constant and S^+ is now arbitrary function of CP_2 coordinates. This is possible since M^4_+ projection is 1-dimensional.

2. Are solutions with a 4-dimensional M^4_+ projection possible?

The most natural solution ansatz is the one for which CP_2 complex structure is preserved so that energy momentum tensor has desired properties. For four-dimensional M^4_+ projection this ansatz does not seem to make promising since the contribution of the longitudinal degrees of freedom implies that the induced metric is not anymore of desired form since the components $g_{ij} = m_{+-}(\partial_{\xi^i} S^+ \partial_{\xi^j} S^- + m_{+-} \partial_{\xi^i} S^- \partial_{\xi^j} S^+)$ are non-vanishing.

1. The natural dynamical variables are still Minkowski coordinates $(w, \overline{w}, S^+, S^-)$ for some Hamilton Jacobi structure. Since the complex structure of CP_2 must be given up, CP_2 coordinates can be written as (ξ, s, r) to stress the fact that only "one half" of the Kähler structure of CP_2 is respected by the solution ansatz.

2. The solution ansatz has the same general form as in $D_{CP_2} = 3$ case and must be symmetric with respect to the exchange of M^4_+ and CP_2 coordinates. Transverse coordinates are mapped to transverse ones and longitudinal coordinates to longitudinal ones:

$$(S^+, S^-) = (S^+(s,r), S^-(s,r)) \quad , \quad w = w(\xi) \ . \tag{2.4.43}$$

This ansatz would describe ordinary Maxwell field in M^4_+ since the roles of M^4_+ coordinates and CP_2 coordinates are interchangeable.

It is however far from obvious whether there are any solutions with a 4-dimensional M^4_+ projection. That empty space Maxwell's equations would allow only the topologically quantized light fronts as its solutions would realize quantum classical correspondence very concretely.

The recent view conforms with this intuition. The Maxwell phase is certainly physical notion but would correspond effective fields experience by particle in many-sheeted space-time. Test particle topological condenses to all the space-time sheets with projection to a given region of Minkowski space and experiences essentially the sum of the effects caused by the induced gauge fields at different sheets. This applies also to gravitational fields interpreted as deviations from Minkowski metric.

The transition to GRT and QFT picture means the replacement of many-sheeted space-time with piece of Minkowski space with effective metric defined as the sum of Minkowski metric and deviations of the induced metrics of space-time sheets from Minkowski metric. Effective gauge potentials are sums of the induced gauge potentials. Hence the rather simple topologically quantized induced gauge fields associated with space-time sheets become the classical fields in the sense of Maxwell's theory and gauge theories.

$D_{CP_2} = 2$ case

Hamilton Jacobi structure for M^4_+ is assumed also for $D_{CP_2} = 2$, whereas the contact structure for CP_2 is in $D_{CP_2} = 2$ case replaced by the induced Kähler structure. Topologization yields vanishing Kähler current. Light-likeness provides a second manner to achieve vanishing Lorentz force but one cannot exclude the possibility of time- and space-like Kähler current.

1. Solutions with vanishing Kähler current

1. String like objects, which are products $X^2 \times Y^2 \subset M^4_+ \times CP_2$ of minimal surfaces Y^2 of M^4_+ with geodesic spheres S^2 of CP_2 and carry vanishing gauge current. String like objects allow considerable generalization from simple Cartesian products of $X^2 \times Y^2 \subset M^4 \times S^2$. Let $(w, \overline{w}, S^+, S^-)$ define the Hamilton Jacobi structure for M^4_+. $w = constant$ surfaces define minimal surfaces X^2 of M^4_+. Let ξ denote complex coordinate for a sub-manifold of CP_2 such that the embedding to CP_2 is holomorphic: $(\xi^1, \xi^2) = (f^1(\xi), f^2(\xi))$. The resulting surface $Y^2 \subset CP_2$ is a minimal surface and field equations reduce to the requirement that the Kähler current vanishes: $\partial_{\overline{\xi}}(J^{\xi\overline{\xi}}\sqrt{g_2}) = 0$. One-dimensional strings are deformed to 3-dimensional cylinders representing magnetic flux tubes. The oscillations of string correspond to waves moving along string with light velocity, and for more general solutions they become TGD counterparts of Alfven waves associated with magnetic flux tubes regarded as oscillations of magnetic flux lines behaving effectively like strings. It must be emphasized that Alfven waves are a phenomenological notion not really justified by the properties of Maxwell's equations.

2. Also electret type solutions with the role of the magnetic field taken by the electric field are possible. $(\xi, \overline{\xi}, u, v)$ would provide the natural coordinates and the solution ansatz would be of the form

$$(s, r) = (s(u,v), r(u,v)) \quad , \quad \xi = constant \ , \tag{2.4.44}$$

and corresponds to a vanishing Kähler current.

3. Both magnetic and electric fields are necessarily present only for the solutions carrying non-vanishing electric charge density (proportional to $\overline{B}\cdot\overline{A}$). Thus one can ask whether more general solutions carrying both magnetic and electric field are possible. As a matter fact, one must first answer the question what one really means with the magnetic field. By choosing the coordinates of 2-dimensional CP_2 projection as space-time coordinates one can define what one means with magnetic and electric field in a coordinate invariant manner. Since the CP_2 Kähler form for the CP_2 projection with $D_{CP_2}=2$ can be regarded as a pure Kähler magnetic field, the induced Kähler field is either magnetic field or electric field.

The form of the ansatz would be

$$(s,r) = (s,r)\,(u,v,w,\overline{w}) \ , \quad \xi = constant \ . \tag{2.4.45}$$

As a matter fact, CP_2 coordinates depend on two properly chosen M^4 coordinates only.

1. Solutions with light-like Kähler current

There are large classes of solutions of field equations with a light-like Kähler current and 2-dimensional CP_2 projection.

1. Massless extremals for which CP_2 coordinates are arbitrary functions of one transversal coordinate $e=f(w,\overline{w})$ defining local polarization direction and light like coordinate u of M^4_+ and carrying in the general case a light like current. In this case the holomorphy does not play any role.
2. The string like solutions thickened to magnetic flux tubes carrying TGD counterparts of Alfven waves generalize to solutions allowing also light-like Kähler current. Also now Kähler metric is allowed to develop a component between longitudinal and transversal degrees of freedom so that Kähler current develops a light-like component. The ansatz is of the form

$$\xi^i = f^i(\xi) \ , \quad w = w(\xi) \ , \quad S^- = s^- \ , \quad S^+ = s^+ + f(\xi,\overline{\xi}) \ .$$

Only the components $g_{+\xi}$ and $g_{+\overline{\xi}}$ of the induced metric receive contributions from the modification of the solution ansatz. The contravariant metric receives contributions to $g^{-\xi}$ and $g^{-\overline{\xi}}$ whereas $g^{+\xi}$ and $g^{+\overline{\xi}}$ remain zero. Since the partial derivatives $\partial_\xi\partial_+h^k$ and $\partial_{\overline{\xi}}\partial_+h^k$ and corresponding projections of Christoffel symbols vanish, field equations are satisfied. Kähler current develops a non-vanishing component j^-. Apart from the presence of the electric field, these solutions are highly analogous to Beltrami fields.

Could $D_{CP_2}=2\rightarrow 3$ transition occur in rotating magnetic systems?

I have studied the embeddings of simple cylindrical and helical magnetic fields in various applications of TGD to condensed matter systems, in particular in attempts to understand the strange findings about rotating magnetic systems [K96].

Let S^2 be the homologically non-trivial geodesic sphere of CP_2 with standard spherical coordinates $(U\equiv cos(\theta),\Phi)$ and let (t,ρ,ϕ,z) denote cylindrical coordinates for a cylindrical space-time sheet. The simplest possible space-time surfaces $X^4\subset M^4_+\times S^2$ carrying helical Kähler magnetic field depending on the radial cylindrical coordinate ρ, are given by:

$$\begin{array}{ll} U = U(\rho) \ , & \Phi = n\phi + kz \ , \\ J_{\rho\phi} = n\partial_\rho U \ , & J_{\rho z} = k\partial_\rho U \ . \end{array} \tag{2.4.46}$$

This helical field is not Beltrami field as one can easily find. A more general ansatz corresponding defined by

$$\Phi = \omega t + kz + n\phi$$

would in cylindrical coordinates give rise to both helical magnetic field and radial electric field depending on ρ only. This field can be obtained by simply replacing the vector potential with its rotated version and provides the natural first approximation for the fields associated with rotating magnetic systems.

A non-vanishing vacuum charge density is however generated when a constant magnetic field is put into rotation and is implied by the condition $\overline{E} = \overline{v} \times \overline{B}$ stating vanishing of the Lorentz force. This condition does not follow from the induction law of Faraday although Faraday observed this effect first. This is also clear from the fact that the sign of the charge density depends on the direction of rotation.

The non-vanishing charge density is not consistent with the vanishing of the Kähler 4-current and requires a 3-dimensional CP_2 projection and topologization of the Kähler current. Beltrami condition cannot hold true exactly for the rotating system. The conclusion is that rotation induces a phase transition $D_{CP_2} = 2 \rightarrow 3$. This could help to understand various strange effects related to the rotating magnetic systems [K96]. For instance, the increase of the dimension of CP_2 projection could generate join along boundaries contacts/flux tubes and wormhole contacts leading to the transfer of charge between different space-time sheets. The possibly resulting flow of gravitational flux to larger space-time sheets might help to explain the claimed antigravity effects.

2.4.4 $D_{CP_2} = 3$ Phase Allows Infinite Number Of Topological Charges Characterizing The Linking Of Magnetic Field Lines

When space-time sheet possesses a $D = 3$-dimensional CP_2 projection, one can assign to it a non-vanishing and conserved topological charge characterizing the linking of the magnetic field lines defined by Chern-Simons action density $A \wedge dA/4\pi$ for induced Kähler form. This charge can be seen as classical topological invariant of the linked structure formed by magnetic field lines.

The topological charge can also vanish for $D_{CP_2} = 3$ space-time sheets. In Darboux coordinates for which Kähler gauge potential reads as $A = P_k dQ^k$, the surfaces of this kind result if one has $Q^2 = f(Q^1)$ implying $A = f dQ^1$, $f = P_1 + P_2 \partial_{Q_1} Q^2$, which implies the condition $A \wedge dA = 0$. For these space-time sheets one can introduce Q^1 as a global coordinate along field lines of A and define the phase factor $exp(i \int A_\mu dx^\mu)$ as a wave function defined for the entire space-time sheet. This function could be interpreted as a phase of an order order parameter of super-conductor like state and there is a high temptation to assume that quantum coherence in this sense is lost for more general $D_{CP_2} = 3$ solutions.

Chern-Simons action is known as helicity in electrodynamics [B27]. Helicity indeed describes the linking of magnetic flux lines as is easy to see by interpreting magnetic field as incompressible fluid flow having A as vector potential: $B = \nabla \times A$. One can write A using the inverse of $\nabla\times$ as $A = (1/\nabla\times)B$. The inverse is non-local operator expressible as

$$\frac{1}{\nabla\times} B(r) = \int dV' \frac{(r-r')}{|r-r'|^3} \times B(r') \ ,$$

as a little calculation shows. This allows to write $\int A \cdot B$ as

$$\int dV A \cdot B = \int dV dV' B(r) \cdot \left(\frac{(r-r')}{|r-r'|^3} \times B(r') \right) \ ,$$

which is completely analogous to the Gauss formula for linking number when linked curves are replaced by a distribution of linked curves and an average is taken.

For $D_{CP_2} = 3$ field equations imply that Kähler current is proportional to the helicity current by a factor which depends on CP_2 coordinates, which implies that the current is automatically divergence free and defines a conserved charge for $D = 3$-dimensional CP_2 projection for which the instanton density vanishes identically. Kähler charge is not equal to the helicity defined by the inner product of magnetic field and vector potential but to a more general topological charge.

The number of conserved topological charges is infinite since the product of any function of CP_2 coordinates with the helicity current has vanishing divergence and defines a topological charge. A very natural function basis is provided by the scalar spherical harmonics of $SU(3)$ defining Hamiltonians of CP_2 canonical transformations and possessing well defined color quantum numbers. These functions define and infinite number of conserved charges which are also classical knot invariants in the sense that they are not affected at all when the 3-surface interpreted as a map from CP_2 projection to M^4_+ is deformed in M^4_+ degrees of freedom. Also canonical transformations induced by Hamiltonians in irreducible representations of color group affect these invariants via Poisson bracket action when the $U(1)$ gauge transformation induced by the canonical transformation corresponds to a single valued scalar function. These link invariants are additive in union whereas the quantum invariants defined by topological quantum field theories are multiplicative.

Also non-Abelian topological charges are well-defined. One can generalize the topological current associated with the Kähler form to a corresponding current associated with the induced electro-weak gauge fields whereas for classical color gauge fields the Chern-Simons form vanishes identically. Also in this case one can multiply the current by CP_2 color harmonics to obtain an infinite number of invariants in $D_{CP_2} = 3$ case. The only difference is that $A \wedge dA$ is replaced by $Tr(A \wedge (dA + 2A \wedge A/3))$.

There is a strong temptation to assume that these conserved charges characterize colored quantum states of the conformally invariant quantum theory as a functional of the light-like 3-surface defining boundary of space-time sheet or elementary particle horizon surrounding wormhole contacts. They would be TGD analogs of the states of the topological quantum field theory defined by Chern-Simons action as highest weight states associated with corresponding Wess-Zumino-Witten theory. These charges could be interpreted as topological counterparts of the isometry charges of WCW defined by the algebra of canonical transformations of CP_2.

The interpretation of these charges as contributions of light-like boundaries to WCW Hamiltonians would be natural. The dynamics of the induced second quantized spinor fields relates to that of Kähler action by a super-symmetry, so that it should define super-symmetric counterparts of these knot invariants. The anti-commutators of these super charges cannot however contribute to WCW Kähler metric so that topological zero modes are in question. These Hamiltonians and their super-charge counterparts would be responsible for the topological sector of quantum TGD.

2.4.5 Preferred Extremal Property And The Topologization/Light-Likeness Of Kähler Current?

The basic question is under what conditions the Kähler current is either topologized or light-like so that the Lorentz force vanishes. Does this hold for all preferred extremals of Kähler action? Or only asymptotically as suggested by the fact that generalized Beltrami fields can be interpreted as asymptotic self-organization patterns, when dissipation has become insignificant. Or does topologization take place in regions of space-time surface having Minkowskian signature of the induced metric? And what asymptotia actually means? Do absolute minima of Kähler action correspond to preferred extremals?

One can challenge the interpretation in terms of asymptotic self organization patterns assigned to the Minkowskian regions of space-time surface.

1. Zero energy ontology challenges the notion of approach to asymptotia in Minkowskian sense since the dynamics of light-like 3-surfaces is restricted inside finite volume $CD \subset M^4$ since the partonic 2-surfaces representing their ends are at the light-like boundaries of causal diamond in a given p-adic time scale.
2. One can argue that generic non-asymptotic field configurations have $D_{CP_2} = 4$, and would thus carry a vanishing Kähler four-current if Beltrami conditions were satisfied universally rather than only asymptotically. $j^\alpha = 0$ would obviously hold true also for the asymptotic configurations, in particular those with $D_{CP_2} < 4$ so that empty space Maxwell's field equations would be universally satisfied for asymptotic field configurations with $D_{CP_2} < 4$. The weak point of this argument is that it is 3-D light-like 3-surfaces rather than space-time surfaces which are the basic dynamical objects so that the generic and only possible case corresponds to $D_{CP_2} = 3$ for X^3_l. It is quite possible that preferred extremal property implies that $D_{CP_2} = 3$ holds true in the Minkowskian regions since these regions indeed represent empty space. Geometrically this would mean that the CP_2 projection does not change as the light-like coordinate labeling Y^3_l varies. This conforms nicely with the notion of quantum gravitational holography.
3. The failure of the generalized Beltrami conditions would mean that Kähler field is completely analogous to a dissipative Maxwell field for which also Lorentz force vanishes since $\overline{j} \cdot \overline{E}$ is non-vanishing (note that isometry currents are conserved although energy momentum tensor is not). Quantum classical correspondence states that classical space-time dynamics is by its

classical non-determinism able to mimic the non-deterministic sequence of quantum jumps at space-time level, in particular dissipation in various length scales defined by the hierarchy of space-time sheets. Classical fields would represent "symbolically" the average dynamics, in particular dissipation, in shorter length scales. For instance, vacuum 4-current would be a symbolic representation for the average of the currents consisting of elementary particles. This would seem to support the view that $D_{CP_2} = 4$ Minkowskian regions are present. The weak point of this argument is that there is fractal hierarchy of length scales represented by the hierarchy of causal diamonds (CDs) and that the resulting hierarchy of generalized Feynman graphs might be enough to represent dissipation classically.

4. One objection to the idea is that second law realized as an asymptotic vanishing of Lorentz-Kähler force implies that all space-like 3-surfaces approaching same asymptotic state have the same value of Kähler function assuming that the Kähler function assignable to space-like 3-surface is same for all space-like sections of $X^4(X_l^3)$ (assuming that one can realize general coordinate invariance also in this sense). This need not be the case. In any case, this need not be a problem since it would mean an additional symmetry extending general coordinate invariance. The exponent of Kähler function would be highly analogous to a partition function defined as an exponent of Hamiltonian with Kähler coupling strength playing the role of temperature.

It seems that asymptotic self-organization pattern need not be correct interpretation for non-dissipating regions, and the identification of light-like 3-surfaces as generalized Feynman diagrams encourages an alternative interpretation.

1. $M^8 - H$ duality states that also the H counterparts of co-hyper-hyperquaternionic surfaces of M^8 are preferred extremals of Kähler action. CP_2 type vacuum extremals represent the basic example of these and a plausible conjecture is that the regions of space-time with Euclidian signature of the induced metric represent this kind of regions. If this conjecture is correct, dissipation could be assigned with regions having Euclidian signature of the induced metric. This makes sense since dissipation has quantum description in terms of Feynman graphs and regions of Euclidian signature indeed correspond to generalized Feynman graphs. This argument would suggest that generalized Beltrami conditions or light-likeness hold true inside Minkowskian regions rather than only asymptotically.

2. One could of course play language games and argue that asymptotia is with respect to the Euclidian time coordinate inside generalized Feynman graps and is achieved exactly when the signature of the induced metric becomes Minkowskian. This is somewhat artificial attempt to save the notion of asymptotic self-organization pattern since the regions outside Feynman diagrams represent empty space providing a holographic representations for the matter at X_l^3 so that the vanishing of $j^\alpha F_{\alpha\beta}$ is very natural.

3. What is then the correct identification of asymptotic self-organization pattern. Could correspond to the negative energy part of the zero energy state at the upper light-like boundary δM^4_- of CD? Or in the case of phase conjugate state to the positive energy part of the state at δM^4_+? An identification consistent with the fractal structure of zero energy ontology and TGD inspired theory of consciousness is that the entire zero energy state reached by a sequence of quantum jumps represents asymptotic self-organization pattern represented by the asymptotic generalized Feynman diagram or their superposition. Biological systems represent basic examples about self-organization, and one cannot avoid the questions relating to the relationship between experience and geometric time. A detailed discussion of these points can be found in [K5].

Absolute minimization of Kähler action was the first guess for the criterion selecting preferred extremals. Absolute minimization in a strict sense of the word does not make sense in the p-adic context since p-adic numbers are not well-ordered, and one cannot even define the action integral as a p-adic number. The generalized Beltrami conditions and the boundary conditions defining the preferred extremals are however local and purely algebraic and make sense also p-adically. If absolute minimization reduces to these algebraic conditions, it would make sense.

2.4.6 Generalized Beltrami Fields And Biological Systems

The following arguments support the view that generalized Beltrami fields play a key role in living systems, and that $D_{CP_2} = 2$ corresponds to ordered phase, $D_{CP_2} = 3$ to spin glass phase and $D_{CP_2} = 4$ to chaos, with $D_{CP_2} = 3$ defining life as a phenomenon at the boundary between order and chaos. If the criteria suggested by the number theoretic compactification are accepted, it is not clear whether D_{CP_2} extremals can define preferred extremals of Kähler action. For instance, cosmic strings are not preferred extremals and the Y_l^3 associated with MEs allow only covariantly constant right handed neutrino eigenmode of $D_K(X^2)$. The topological condensation of CP_2 type vacuum extremals around $D_{CP_2} = 2$ type extremals is however expected to give preferred extremals and if the density of the condensate is low enough one can still speak about $D_{CP_2} = 2$ phase. A natural guess is also that the deformation of $D_{CP_2} = 2$ extremals transforms light-like gauge currents to space-like topological currents allowed by $D_{CP_2} = 3$ phase.

Why generalized Beltrami fields are important for living systems?

Chirality, complexity, and high level of organization make $D_{CP_2} = 3$ generalized Beltrami fields excellent candidates for the magnetic bodies of living systems.

1. Chirality selection is one of the basic signatures of living systems. Beltrami field is characterized by a chirality defined by the relative sign of the current and magnetic field, which means parity breaking. Chirality reduces to the sign of the function ψ appearing in the topologization condition and makes sense also for the generalized Beltrami fields.
2. Although Beltrami fields can be extremely complex, they are also extremely organized. The reason is that the function α is constant along flux lines so that flux lines must in the case of compact Riemann 3-manifold belong to 2-dimensional $\alpha = constant$ closed surfaces, in fact two-dimensional invariant tori [B36] .

For generalized Beltrami fields the function ψ is constant along the flow lines of the Kähler current. Space-time sheets with 3-dimensional CP_2 projection serve as an illustrative example. One can use the coordinates for the CP_2 projection as space-time coordinates so that one space-time coordinate disappears totally from consideration. Hence the situation reduces to a flow in a 3-dimensional sub-manifold of CP_2. One can distinguish between three types of flow lines corresponding to space-like, light-like and time-like topological current. The 2-dimensional $\psi = constant$ invariant manifolds are sub-manifolds of CP_2. Ordinary Beltrami fields are a special case of space-like flow with flow lines belonging to the 2-dimensional invariant tori of CP_2. Time-like and light-like situations are more complex since the flow lines need not be closed so that the 2-dimensional $\psi = constant$ surfaces can have boundaries.

For periodic self-organization patterns flow lines are closed and $\psi = constant$ surfaces of CP_2 must be invariant tori. The dynamics of the periodic flow is obtained from that of a steady flow by replacing one spatial coordinate with effectively periodic time coordinate. Therefore topological notions like helix structure, linking, and knotting have a dynamical meaning at the level of CP_2 projection. The periodic generalized Beltrami fields are highly organized also in the temporal domain despite the potentiality for extreme topological complexity.

For these reasons topologically quantized generalized Beltrami fields provide an excellent candidate for a generic model for the dynamics of biological self-organization patterns. A natural guess is that many-sheeted magnetic and Z^0 magnetic fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chiral selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

The intricate topological structures of DNA, RNA, and protein molecules are known to have a deep significance besides their chemical structure, and they could even define something analogous to the genetic code. Usually the topology and geometry of bio-molecules is believed to reduce to chemistry. TGD suggests that space-like generalized Beltrami fields serve as templates for the formation of bio-molecules and bio-structures in general. The dynamics of bio-systems would in turn utilize the time-like Beltrami fields as templates. There could even exist a mapping from the topology of magnetic flux tube structures serving as templates for bio-molecules to the templates of self-organized dynamics. The helical structures, knotting, and linking of bio-molecules would thus define a symbolic representation, and even coding for the dynamics of the bio-system analogous to written language.

$D_{CP_2} = 3$ systems as boundary between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos

The dimension of CP_2 projection is basic classifier for the asymptotic self-organization patterns.

1. *$D_{CP_2} = 4$ phase, dead matter, and chaos*

$D_{CP_2} = 4$ corresponds to the ordinary Maxwellian phase in which Kähler current and charge density vanish and there is no topologization of Kähler current. By its maximal dimension this phase would naturally correspond to disordered phase, ordinary "dead matter". If one assumes that Kähler charge corresponds to either em charge or Z^0 charge then the signature of this state of matter would be em neutrality or Z^0 neutrality.

2. *$D_{CP_2} = 2$ phase as ordered phase*

By the low dimension of CP_2 projection $D_{CP_2} = 2$ phase is the least stable phase possible only at cold space-time sheets. Kähler current is either vanishing or light-like, and Beltrami fields are not possible. This phase is highly ordered and much like a topological quantized version of ferro-magnet. In particular, it is possible to have a global coordinate varying along the field lines of the vector potential also now. The magnetic and Z^0 magnetic body of any system is a candidate for this kind of system. Z^0 field is indeed always present for vacuum extremals having $D_{CP_2} = 2$ and the vanishing of em field requires that $sin^2(\theta_W)$ (θ_W is Weinberg angle) vanishes.

3. *$D_{CP_2} = 3$ corresponds to living matter*

$D_{CP_2} = 3$ corresponds to highly organized phase characterized in the case of space-like Kähler current by complex helical structures necessarily accompanied by topologized Kähler charge density $\propto \overline{A}\cdot\overline{B} \neq 0$ and Kähler current $\overline{E}\times\overline{A}+\phi\overline{B}$. For time like Kähler currents the helical structures are replaced by periodic oscillation patterns for the state of the system. By the non-maximal dimension of CP_2 projection this phase must be unstable against too strong external perturbations and cannot survive at too high temperatures. Living matter is thus excellent candidate for this phase and it might be that the interaction of the magnetic body with living matter makes possible the transition from $D_{CP_2} = 2$ phase to the self-organizing $D_{CP_2} = 3$ phase.

Living matter which is indeed populated by helical structures providing examples of space-like Kähler current. Strongly charged lipid layers of cell membrane might provide example of time-like Kähler current. Cell membrane, micro-tubuli, DNA, and proteins are known to be electrically charged and Z^0 charge plays key role in TGD based model of catalysis discussed in [**?**]. For instance, denaturing of DNA destroying its helical structure could be interpreted as a transition leading from $D_{CP_2} = 3$ phase to $D_{CP_2} = 4$ phase. The prediction is that the denatured phase should be electromagnetically (or Z^0) neutral.

Beltrami fields result when Kähler charge density vanishes. For these configurations magnetic field and current density take the role of the vector potential and magnetic field as far as the contact structure is considered. For Beltrami fields there exist a global coordinate along the field lines of the vector potential but not along those of the magnetic field. As a consequence, the covariant consistency condition $(\partial_s - qeA_s)\Psi = 0$ frequently appearing in the physics of super conducting systems would make sense along the flow lines of the vector potential for the order parameter of Bose-Einstein condensate. If Beltrami phase is super-conducting, then the state of the system must change in the transition to a more general phase. It is impossible to assign slicing of 4-surface by 3-D surfaces labeled by a coordinate t varying along the flow lines. This means that one cannot speak about a continuous evolution of Schrödinger amplitude with t playing the role of time coordinate. One could perhaps say that the entire space-time sheet represents single quantum event which cannot be decomposed to evolution. This would conform with the assignment of macroscopic and macro-temporal quantum coherence with living matter.

The existence of these three phases brings in mind systems allowing chaotic de-magnetized phase above critical temperature T_c, spin glass phase at the critical point, and ferromagnetic phase below T_c. Similar analogy is provided by liquid phase, liquid crystal phase possible in the vicinity of the critical point for liquid to solid transition, and solid phase. Perhaps one could regard $D_{CP_2} = 3$

phase and life as a boundary region between $D_{CP_2} = 2$ order and $D_{CP_2} = 4$ chaos. This would naturally explain why life as it is known is possible in relatively narrow temperature interval.

Can one assign a continuous Schrödinger time evolution to light-like 3-surfaces?

Alain Connes wrote [A21] about factors of various types using as an example Schrödinger equation for various kinds of foliations of space-time to time=constant slices. If this kind of foliation does not exist, one cannot speak about time evolution of Schrödinger equation at all. Depending on the character of the foliation one can have factor of type I, II, or III. For instance, torus with slicing $dx = ady$ in flat coordinates, gives a factor of type I for rational values of a and factor of type II for irrational values of a.

1. 3-D foliations and type III factors

Connes mentioned 3-D foliations V which give rise to type III factors. Foliation property requires a slicing of V by a one-form v to which slices are orthogonal (this requires metric).

1. The foliation property requires that v multiplied by suitable scalar is gradient. This gives the integrability conditions $dv = w \wedge v$, $w = -d\psi/\psi = -dlog(\psi)$. Something proportional to $log(\psi)$ can be taken as a third coordinate varying along flow lines of v: the flow defines a continuous sequence of maps of 2-dimensional slice to itself.
2. If the so called Godbillon-Vey invariant defined as the integral of $dw \wedge w$ over V is non-vanishing, factor of type III is obtained using Schrödinger amplitudes for which the flow lines of foliation define the time evolution. The operators of the algebra in question are transversal operators acting on Schrödinger amplitudes at each slice. Essentially Schrödinger equation in 3-D space-time would be in question with factor of type III resulting from the exotic choice of the time coordinate defining the slicing.

2. What happens in case of light-like 3-surfaces?

In TGD light-like 3-surfaces are natural candidates for V and it is interesting to look what happens in this case. Light-likeness is of course a disturbing complication since orthogonality condition and thus contravariant metric is involved with the definition of the slicing. Light-likeness is not however involved with the basic conditions.

1. The one-form v defined by the induced Kähler gauge potential A defining also a braiding is a unique identification for v. If foliation exists, the braiding flow defines a continuous sequence of maps of partonic 2-surface to itself.
2. Physically this means the possibility of a super-conducting phase with order parameter satisfying covariant constancy equation $D\psi = (d/dt - ieA)\psi = 0$. This would describe a supra current flowing along flow lines of A.
3. If the integrability fails to be true, one *cannot* assign Schrödinger time evolution with the flow lines of v. One might perhaps say that 3-surface behaves like single quantum event not allowing slicing by a continuous Schrödinger time evolution.
4. The condition that the modes of the induced spinor field have well-defined em charge implies that CP_2 projection for the region of space-time in which induced spinor field is non-vanishing is 2-dimensional. In the generic case a localization to 2-surfaces - string world sheets and possibly partonic 2-surface. At light-like 3-surfaces this implies that modes are localized at 1-D curves so that the hydrodynamic picture is realized [K104].

3. Extremals of Kähler action

Some comments relating to the interpretation of the classification of the extremals of Kähler action by the dimension of their CP_2 projection are in order. It has been already found that the extremals can be classified according to the dimension D of the CP_2 projection of space-time sheet in the case that $A_a = 0$ holds true.

1. For $D_{CP_2} = 2$ integrability conditions for the vector potential can be satisfied for $A_a = 0$ so that one has generalized Beltrami flow and one can speak about Schrödinger time evolution associated with the flow lines of vector potential defined by covariant constancy condition

$D\psi = 0$ makes sense. Kähler current is vanishing or light-like. This phase is analogous to a super-conductor or a ferromagnetic phase. For non-vanishing A_a the Beltrami flow property is lost but the analogy with ferromagnetism makes sense still.

2. For $D_{CP_2} = 3$ foliations are lost. The phase is dominated by helical structures. This phase is analogous to spin glass phase around phase transition point from ferromagnetic to non-magnetized phase and expected to be important in living matter systems.

3. $D_{CP_2} = 4$ is analogous to a chaotic phase with vanishing Kähler current and to a phase without magnetization. The interpretation in terms of non-quantum coherent "dead" matter is suggestive.

An interesting question is whether the ordinary 8-D embedding space which defines one sector of the generalized embedding space could correspond to $A_a = 0$ phase. If so, then all states for this sector would be vacua with respect to M^4 quantum numbers. M^4-trivial zero energy states in this sector could be transformed to non-trivial zero energy states by a leakage to other sectors.

2.4.7 About Small Perturbations Of Field Equations

The study of small perturbations of the known solutions of field equations is a standard manner to get information about the properties of the solutions, their stability in particular. Fourier expansion is the standard manner to do the perturbation theory. In the recent case an appropriate modification of this ansatz might make sense if the solution in question is representable as a map $M^4_+ \rightarrow CP_2$, and the perturbations are rapidly varying when compared to the components of the induced metric and Kähler form so that one can make adiabatic approximation and approximate them as being effectively constant. Presumably also restrictions on directions of wave 4-vectors $k_\mu = (\omega, \overline{k}))$ are necessary so that the direction of wave vector adapts to the slowly varying background as in ray optics. Also Hamilton Jacobi structure is expected to modify the most straightforward approach. The four CP_2 coordinates are the dynamical variables so that the situation is relatively simple.

A completely different approach is inspired by the physical picture. In this approach one glues CP_2 type vacuum extremal to a known extremal and tries to deduce the behavior of the deformed extremal in the vicinity of wormhole throat by posing the general conditions on the slicing by light-like 3-surfaces Y^3_l. This approach is not followed now.

Generalized plane waves

Individual plane waves are geometrically very special since they represent a deformation of the space-time surface depending on single coordinate only. Despite this one might hope that plane waves or their appropriate modifications allowing to algebraize the treatment of small perturbations could give useful information also now.

1. Lorentz invariance plus the translational invariance due to the assumption that the induced metric and Kähler form are approximately constant encourage to think that the coordinates reduce Minkowski coordinates locally with the orientation of the local Minkowski frame depending slowly on space-time position. Hamilton Jacobi $(S^+, S^-, w, \overline{w})$ are a good candidate for this kind of coordinates. The properties of the Hamilton Jacobi structure and of the solution ansatz suggest that excitations are generalized plane waves in longitudinal degrees of freedom only so that four-momentum would be replaced by the longitudinal momentum. In transverse degrees of freedom one might expect that holomorphic plane-waves $exp(ik_T w)$, where k_T is transverse momentum, make algebraization possible.

   For time-like longitudinal momenta one can choose the local M^4 coordinates in such a way that longitudinal momentum reduces to $(\omega_0, 0)$, where ω_0 plays the role of rest mass and is analogous to the plasma frequency serving as an infrared cutoff for plasma waves. In these coordinates the simplest candidates for excitations with time-like momentum would be of form $\Delta s^k = \epsilon a^k exp(i\omega_0 u)$, where s^k are some real coordinates for CP_2, a^k are Fourier coefficients, and time-like coordinate is defined as $u = S^+ + S^-$. The excitations moving with light velocity correspond to $\omega_0 = 0$, and one must treat this case separately using plane wave $exp(i\omega S^\pm)$, where ω has continuum of values.

2. It is possible that only some preferred CP_2 coordinates are excited in longitudinal degrees of freedom. For $D_{CP_2} = 3$ ansatz the simplest option is that the complex CP_2 coordinate ξ depends analytically on w and the longitudinal CP_2 coordinate s obeys the plane wave ansatz. $\xi(w) = a \times exp(ik_T w)$, where k_T is transverse momentum allows the algebraization of the solution ansatz also in the transversal degrees of freedom so that a dispersion relation results. For imaginary values of k_T and ω the equations are real.

2. General form for the second variation of the field equations

For time-like four-momentum the second variation of field equations contains three kinds of terms. There are terms quadratic in ω_0 and coming from the second derivatives of the deformation, terms proportional to $i\omega_0$ coming from the variation with respect to the derivatives of CP_2 coordinates, and terms which do not depend on ω_0 and come from the variations of metric and Kähler form with respect to the CP_2 coordinates.

In standard perturbation theory the terms proportional to $i\omega_0$ would have interpretation as analogs of dissipative terms. This forces to assume that ω_0 is complex: note that in purely imaginary ω_0 the equations are real. The basic assumption is that Kähler action is able to mimic dissipation despite the fact that energy and momentum are conserved quantities. The vanishing of the Lorentz force has an interpretation as the vanishing of the dissipative effects. This would suggest that the terms proportional to $i\omega_0$ vanish for the perturbations of the solution preserving the non-dissipative character of the asymptotic solutions. This might quite well result from the vanishing of the contractions with the deformation of the energy momentum tensor with the second fundamental form and of energy momentum tensor with the deformation of the second fundamental form coming from first derivatives.

Physical intuition would suggest that dissipation-less propagation is possible only along special directions. Thus the vanishing of the linear terms should occur only for special directions of the longitudinal momentum vector, say for light-like four-momenta in the direction of coordinate lines of S^+ or S^-. Quite generally, the sub-space of allowed four-momenta is expected to depend on position since the components of metric and Kähler form are slowly varying. This dependence is completely analogous with that appearing in the Hamilton Jacobi (ray-optics) approach to the approximate treatment of wave equations and makes sense if the phase of the plane wave varies rapidly as compared to the variation of CP_2 coordinates for the unperturbed solution.

Complex values of ω_0 are also possible, and would allow to deduce important information about the rate at which small deviations from asymptotia vanish as well as about instabilities of the asymptotic solutions. In particular, for imaginary values of ω_0 one obtains completely well-defined solution ansatz representing exponentially decaying or increasing perturbation.

High energy limit

One can gain valuable information by studying the perturbations at the limit of very large four-momentum. At this limit the terms which are quadratic in the components of momentum dominate and come from the second derivatives of the CP_2 coordinates appearing in the second fundamental form. The resulting equations reduce for all CP_2 coordinates to the same condition

$$T^{\alpha\beta} k_\alpha k_\beta = 0 \ .$$

This condition is generalization of masslessness condition with metric replaced by the energy momentum tensor, which means that light velocity is replaced by an effective light velocity. In fact, energy momentum tensor effectively replaces metric also in the modified Dirac equation whose form is dictated by super symmetry. Light-like four momentum is a rather general solution to the condition and corresponds to $\omega_0 = 0$ case.

Reduction of the dispersion relation to the graph of swallowtail catastrophe

Also the general structure of the equations for small perturbations allows to deduce highly non-trivial conclusions about the character of perturbations.

1. The equations for four CP_2 coordinates are simultaneously satisfied if the determinant associated with the equations vanishes. This condition defines a 3-dimensional surface in the

4-dimensional space defined by ω_0 and coordinates of 3-space playing the role of slowly varying control parameters. 4×4 determinant results and corresponds to a polynomial which is of order $d=8$ in ω_0. If the determinant is real, the polynomial can depend on ω_0^2 only so that a fourth order polynomial in $w=\omega_0^2$ results.

2. Only complex roots are possible in the case that the terms linear in $i\omega_0$ are non-vanishing. One might hope that the linear term vanishes for certain choices of the direction of slowly varying four-momentum vector $k^\mu(x)$ at least. For purely imaginary values of ω_0 the equations determinant are real always. Hence catastrophe theoretic description applies in this case at least, and the so called swallow tail [A29] with three control parameters applies to the situation.

3. The general form of the vanishing determinant is

$$D(w,a,b,c) = w^4 - ew^3 - cw^2 - bw - a \ .$$

The transition from the oscillatory to purely dissipative case changes only the sign of w. By the shift $w=\hat{w}+e/4$ the determinant reduces to the canonical form

$$D(\hat{w},a,b,c) = \hat{w}^4 - c\hat{w}^2 - b\hat{w} - a$$

of the swallowtail catastrophe. This catastrophe has three control variables, which basically correspond to the spatial 3-coordinates on which the induced metric and Kähler form depend. The variation of these coefficients at the space-time sheet of course covers only a finite region of the parameter space of the swallowtail catastrophe. The number of real roots for $w=\omega_0^2$ is four, two, or none since complex roots appear in complex conjugate pairs for a real polynomial. The general shape of the region of 3-space is that for a portion of swallow tail catastrophe (see **Fig. 2.1**).

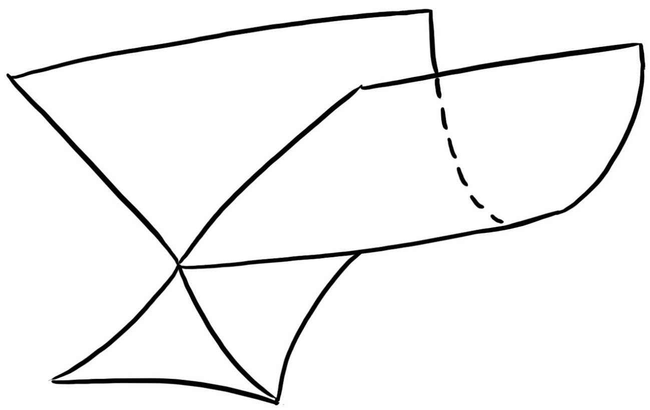

Figure 2.1: The projection of the bifurcation set of the swallowtail catastrophe to the 3-dimensional space of control variables. The potential function has four extrema in the interior of the swallowtail bounded by the triangles, no extrema in the valley above the swallowtail, and 2 extrema elsewhere.

4. The dispersion relation for the "rest mass" ω_0 (decay rate for the imaginary value of ω_0) has at most four real branches, which conforms with the fact that there are four dynamical variables. In real case ω_0 is analogous to plasma frequency acting as an infrared cutoff for the frequencies of plasma excitations. To get some grasp on the situation notice that for $a=0$ the swallowtail reduces to $\hat{w}=0$ and

$$\hat{w}^3 - c\hat{w} - b = 0 \ ,$$

which represents the cusp catastrophe easy to illustrate in 3-dimensional space. Cusp (see **Fig. 6.1**) in turn reduces for $b = 0$ to $\hat{w} = 0$ and fold catastrophe $\hat{w} = \pm\sqrt{c}$. Thus the catastrophe surface becomes 4-sheeted for $c \geq 0$ for sufficiently small values of the parameters a and b. The possibility of negative values of $\hat{w}$ in principle allows $\omega^2 = \hat{w} + e/4 < 0$ solutions identifiable as exponentially decaying or amplified perturbations. At the high frequency limit the 4 branches degenerate to a single branch $T^{\alpha\beta}k_\alpha k_\beta = 0$, which as a special case gives light-like four-momenta corresponding to $\omega_0 = 0$ and the origin of the swallowtail catastrophe.

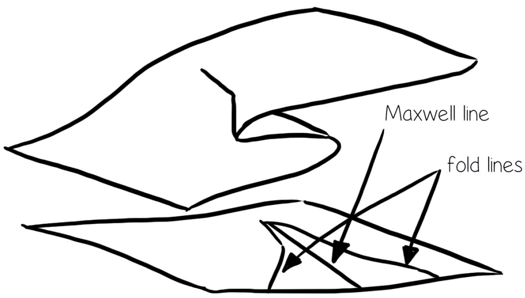

Figure 2.2: Cusp catastrophe. Vertical direction corresponds to the behavior variable and orthogonal directions to control variables.

5. It is quite possible that the imaginary terms proportional to $i\omega_0$ cannot be neglected in the time-like case. The interpretation would be as dissipative effects. If these effects are not too large, an approximate description in terms of butterfly catastrophe makes still sense. Note however that the second variation contains besides gravitational terms potentially large dissipative terms coming from the variation of the induced Kähler form and from the variation of CP_2 Christoffel symbols.
6. Additional complications are encountered at the points, where the induced Kähler field vanishes since the second variation vanishes identically at these points. By the arguments represented earlier, these points quite generally represent instabilities.

2.5 Vacuum Extremals

Vacuum extremals come as two basic types: CP_2 type vacuum extremals for which the induced Kähler field and Kähler action are non-vanishing and the extremals for which the induced Kähler field vanishes. The deformations of both extremals are expected to be of fundamental importance in TGD universe. Vacuum extremals are not gravitational vacua and they are indeed fundamental in TGD inspired cosmology.

2.5.1 CP_2 Type Extremals

CP_2 type vacuum extremals

These extremals correspond to various isometric embeddings of CP_2 to $M^4_+ \times CP_2$. One can also drill holes to CP_2. Using the coordinates of CP_2 as coordinates for X^4 the embedding is given by the formula

$$\begin{aligned} m^k &= m^k(u) \ , \\ m_{kl}\dot{m}^k\dot{m}^l &= 0 \ , \end{aligned} \quad (2.5.1)$$

where $u(s^k)$ is an arbitrary function of CP_2 coordinates. The latter condition tells that the curve representing the projection of X^4 to M^4 is light like curve. One can choose the functions $m^i, i = 1,2,3$ freely and solve m^0 from the condition expressing light likeness so that the number of this kind of extremals is very large.

The induced metric and Kähler field are just those of CP_2 and energy momentum tensor $T^{\alpha\beta}$ vanishes identically by the self duality of the Kähler form of CP_2. Also the canonical current $j^\alpha = D_\beta J^{\alpha\beta}$ associated with the Kähler form vanishes identically. Therefore the field equations in the interior of X^4 are satisfied. The field equations are also satisfied on the boundary components of CP_2 type extremal because the non-vanishing boundary term is, besides the normal component of Kähler electric field, also proportional to the projection operator to the normal space and vanishes identically since the induced metric and Kähler form are identical with the metric and Kähler form of CP_2.

As a special case one obtains solutions for which M^4 projection is light like geodesic. The projection of $m^0 = constant$ surfaces to CP_2 is $u = constant$ 3-sub-manifold of CP_2. Geometrically these solutions correspond to a propagation of a massless particle. In a more general case the interpretation as an orbit of a massless particle is not the only possibility. For example, one can imagine a situation, where the center of mass of the particle is at rest and motion occurs along a circle at say (m^1, m^2) plane. The interpretation as a massive particle is natural. Amusingly, there is nice analogy with the classical theory of Dirac electron: massive Dirac fermion moves also with the velocity of light (zitterbewegung). The quantization of this random motion with light velocity leads to Virasoro conditions and this led to a breakthrough in the understanding of the p-adic QFT limit of TGD. Furthermore, it has turned out that Super Virasoro invariance is a general symmetry of WCW geometry and quantum TGD and appears both at the level of embedding space and space-time surfaces.

The action for all extremals is same and given by the Kähler action for the embedding of CP_2. The value of the action is given by

$$S = -\frac{\pi}{8\alpha_K} . \tag{2.5.2}$$

To derive this expression we have used the result that the value of Lagrangian is constant: $L = 4/R^4$, the volume of CP_2 is $V(CP_2) = \pi^2 R^4/2$ and the definition of the Kähler coupling strength $k_1 = 1/16\pi\alpha_K$ (by definition, πR is the length of CP_2 geodesics). Four-momentum vanishes for these extremals so that they can be regarded as vacuum extremals. The value of the action is negative so that these vacuum extremals are indeed favored by the minimization of the Kähler action.

The absolute minimization of Kähler action was the original suggestion for what preferred extremal property could mean, and suggested that ordinary vacuums with vanishing Kähler action density are unstable against the generation of CP_2 type extremals. The same conclusion however follows also from the mere vacuum degeneracy of Käbler action. There are even reasons to expect that CP_2 type extremals are for TGD what black holes are for GRT. This identification seems reasonable: the 4-D lines of generalized Feynman graphs [K38] would be regions with Euclidian signature of induced metric and identifiable as deformations of CP_2 type vacuum extremals, and even TGD counterparts of blackholes would be analogous to lines of Feynman diagrams. Their M^4 projection would be of course arbitrarily of macroscopic size. The nice generalization of the area law for the entropy of black hole [K39] supports this view.

In accordance with the basic ideas of TGD topologically condensed vacuum extremals should somehow correspond to massive particles. The properties of the CP_2 type vacuum extremals are in accordance with this interpretation. Although these objects move with a velocity of light, the motion can be transformed to a mere zitterbewegung so that the center of mass motion is trivial. Even the generation of the rest mass could might be understood classically as a consequence of the minimization of action. Long range Kähler fields generate negative action for the topologically condensed vacuum extremal (momentum zero massless particle) and Kähler field energy in turn is identifiable as the rest mass of the topologically condensed particle.

An interesting feature of these objects is that they can be regarded as gravitational instantons [A33]. A further interesting feature of CP_2 type extremals is that they carry nontrivial classical color charges. The possible relationship of this feature to color confinement raises interesting questions. Could one model classically the formation of the color singlets to take place through the emission of "colorons": states with zero momentum but non-vanishing color? Could these peculiar states reflect the infrared properties of the color interactions?

Are CP_2 type non-vacuum extremals possible?

The isometric embeddings of CP_2 are all vacuum extremals so that these extremals as such cannot correspond to physical particles. One obtains however non-vacuum extremals as deformations of these solutions. There are several types of deformations leading to non-vacuum solutions. In order to describe some of them, recall the expressions of metric and Kähler form of CP_2 in the coordinates (r, Θ, Ψ, Φ) [A30] are given by

$$\begin{aligned} \frac{ds^2}{R^2} &= \frac{dr^2}{(1+r^2)^2} + \frac{r}{2(1+r^2))^2}(d\Psi + cos(\Theta)d\Phi)^2 \\ &+ \frac{r^2}{(4(1+r^2)}(d\Theta^2 + sin^2\Theta d\Phi^2) \ , \\ J &= \frac{r}{(1+r^2)}dr \wedge (d\Psi + cos(\Theta)d\Phi) \\ &- \frac{r^2}{(2(1+r^2)}sin(\Theta)d\Theta \wedge d\Phi \ . \end{aligned} \quad (2.5.3)$$

The scaling of the line element is defined so that πR is the length of the CP_2 geodesic line. Note that Φ and Ψ appear as "cyclic" coordinates in metric and Kähler form: this feature plays important role in the solution ansatze to be described.

Let $M^4 = M^2 \times E^2$ denote the decomposition of M^4 to a product of 2-dimensional Minkowski space and 2-dimensional Euclidian plane. This decomposition corresponds physically to the decomposition of momentum degrees of freedom for massless particle: E^2 corresponds to polarization degrees of freedom.

There are several types of non-vacuum extremals.

"Virtual particle" extremals: the mass spectrum is continuous (also Euclidian momenta are allowed) but these extremals reduce to vacuum extremals in the massless limit.

2. Massless extremals.

Consider first an example of virtual particle extremal. The simplest extremal of this type is obtained in the following form

$$m^k = a^k\Psi + b^k\Phi \ . \quad (2.5.4)$$

Here a^k and b^k are some constant quantities. Field equations are equivalent to the conditions expressing four-momentum conservation and are identically satisfied the reason being that induced metric and Kähler form do not depend on the coordinates Ψ and Φ.

Extremal describes 3-surface, which moves with constant velocity in M^4. Four-momentum of the solution can be both space and time like. In the massless limit solution however reduces to a vacuum extremal. Therefore the interpretation as an off mass shell massless particle seems appropriate.

Massless extremals are obtained from the following solution ansatz.

$$\begin{aligned} m^0 &= m^3 = a\Psi + b\Phi \ , \\ (m^1, m^2) &= (m^1(r,\Theta), m^2(r,\Theta)) \ . \end{aligned} \quad (2.5.5)$$

Only E^2 degrees of freedom contribute to the induced metric and the line element is obtained from

$$ds^2 = ds^2_{CP_2} - (dm^1)^2 - (dm^2)^2 \ . \quad (2.5.6)$$

Field equations reduce to conservation condition for the componenents of four-momentum in E^2 plane. By their cyclicity the coordinates Ψ and Φ disappear from field equations and one obtains essentially current conservation condition for two-dimensional field theory defined in space spanned by the coordinates r and Θ.

$$\begin{aligned} (J_a^i)_{,i} &= 0 \ , \\ J_a^i &= T^{ij} f^a_{,j} \sqrt{g} \ . \end{aligned} \tag{2.5.7}$$

Here the index i and a refer to r and Θ and to E^2 coordinates m^1 and m^2 respectively. T^{ij} denotes the canonical energy momentum tensor associated with Kähler action. One can express the components of T^{ij} in terms of induced metric and CP_2 metric in the following form

$$T^{ij} = (-g^{ik}g^{jl} + g^{ij}g^{kl}/2)s_{kl} \ . \tag{2.5.8}$$

This expression holds true for all components of the energy momentum tensor.

Since field equations are essentially two-dimensional conservation conditions they imply that components of momentum currents can be regarded as vector fields of some canonical transformations

$$J_a^i = \varepsilon^{ij} H^a_{,j} \ , \tag{2.5.9}$$

where ε^{ij} denotes two-dimensional constant symplectic form. An open problem is whether one could solve field equations exactly and whether there exists some nonlinear superposition principle for the solutions of these equations. Solutions are massless since transversal momentum densities vanish identically.

Consider as a special case the solution obtained by assuming that one E^2 coordinate is constant and second coordinate is function $f(r)$ of the variable r only. Field equations reduce to the following form

$$f_{,r} = \pm \frac{k}{(1+r^2)^{1/3}} \sqrt{r^2 - k^2(1+r^2)^{4/3}} \ . \tag{2.5.10}$$

The solution is well defined only for sufficiently small values of the parameter k appearing as integration constant and becomes ill defined at two singular values of the variable r. Boundary conditions are identically satisfied at the singular values of r since the radial component of induced metric diverges at these values of r. The result leads to suspect that the generation of boundary components dynamically is a general phenomenon so that all non-vacuum solutions have boundary components in accordance with basic ideas of TGD.

$CP_2\#CP_2\#...\#CP_2$: s as generalized Feynman graphs

There are reasons to believe that point like particles might be identified as CP_2 type extremals in TGD approach. Also the geometric counterparts of the massless on mass shell particles and virtual particles have been identified. It is natural to extend this idea to the level of particle interactions: the lines of Feynman diagrams of quantum field theory are thickened to four-manifolds, which are in a good approximation CP_2 type vacuum extremals. This would mean that generalized Feynman graphs are essentially connected sums of CP_2: s (see **Fig. 2.3**): $X^4 = CP_2\#CP_2....\#CP_2$).

Unfortunately, this picture seems to be oversimplified. First, it is questionable whether the cross sections for the scattering of CP_2 type extremals have anything to do with the cross sections associated with the standard gauge interactions. A naïve geometric argument suggests that the cross section should reflect the geometric size of the scattered objects and therefore be of the order of CP_2 radius for topologically non-condensed CP_2 type extremals. The observed cross sections would result at the first level of condensation, where particles are effectively replaced by surfaces with size of order Compton length. Secondly, the $h_{vac} = -D$ rule, considered in the previous chapter, suggests that only real particles correspond to the CP_2 type extremals whereas virtual

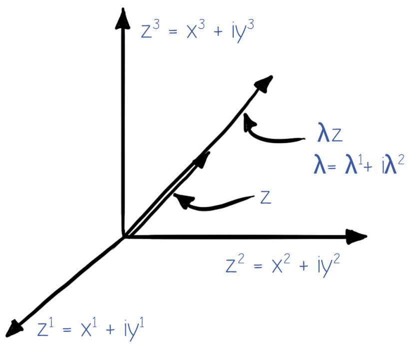

Figure 2.3: Topological sum of CP_2: s as Feynman graph with lines thickened to four-manifolds

particles in general correspond to the vacuum extremals with a vanishing Kähler action. The reason is that the negative exponent of the Kähler action reduces the contribution of the CP_2 type extremals to the functional integral very effectively. Therefore the exchanges of CP_2 type extremals are suppressed by the negative exponent of the Kähler action very effectively so that geometric scattering cross section is obtained.

2.5.2 Vacuum Extremals With Vanishing Kähler Field

Vacuum extremals correspond to 4-surfaces with vanishing Kähler field and therefore to gauge field zero configurations of gauge field theory. These surfaces have CP_2 projection, which is Legendre manifold. The condition expressing Legendre manifold property is obtained in the following manner. Kähler potential of CP_2 can be expressed in terms of the canonical coordinates (P_i, Q_i) for CP_2 as

$$A = \sum_k P_k dQ^k \ . \tag{2.5.11}$$

The conditions

$$P_k = \partial_{Q^k} f(Q^i) \ , \tag{2.5.12}$$

where $f(Q^i)$ is arbitrary function of its arguments, guarantee that Kähler potential is pure gauge. It is clear that canonical transformations, which act as local $U(1)$ gauge transformations, transform different vacuum configurations to each other so that vacuum degeneracy is enormous. Also M^4_+ diffeomorphisms act as the dynamical symmetries of the vacuum extremals. Some sub-group of these symmetries extends to the isometry group of the WCW in the proposed construction of the configuration space metric. The vacuum degeneracy is still enhanced by the fact that the topology of the four-surface is practically free.

Vacuum extremals are certainly not absolute minima of the action. For the induced metric having Minkowski signature the generation of Kähler electric fields lowers the action. For Euclidian signature both electric and magnetic fields tend to reduce the action. Therefore the generation of Euclidian regions of space-time is expected to occur. CP_2 type extremals, identifiable as real (as contrast to virtual) elementary particles, can be indeed regarded as these Euclidian regions.

Particle like vacuum extremals can be classified roughly by the number of the compactified dimensions D having size given by CP_2 length. Thus one has $D_{CP_2} = 3$ for CP_2 type extremals, $D_{CP_2} = 2$ for string like objects, $D_{CP_2} = 1$ for membranes and $D_{CP_2} = 0$ for pieces of M^4. As already mentioned, the rule $h_{vac} = -D$ relating the vacuum weight of the Super Virasoro representation to the number of compactified dimensions of the vacuum extremal is very suggestive. $D < 3$ vacuum extremals would correspond in this picture to virtual particles, whose contribution to the generalized Feynman diagram is not suppressed by the exponential of Kähler action unlike that associated with the virtual CP_2 type lines.

M^4 type vacuum extremals (representable as maps $M^4_+ \rightarrow CP_2$ by definition) are also expected to be natural idealizations of the space-time at long length scales obtained by smoothing out small scale topological inhomogenuities (particles) and therefore they should correspond to space-time of GRT in a reasonable approximation.

In both cases the vanishing of Kähler action per volume in long length scales makes vacuum extremals excellent idealizations for the smoothed out space-time surface. Robertson-Walker cosmologies provide a good example in this respect. As a matter fact the smoothed out space-time is not a mere fictive concept since larger space-time sheets realize it as a essential part of the Universe.

Several absolute minima could be possible and the non-determinism of the vacuum extremals is not expected to be reduced completely. The remaining degeneracy could be even infinite. A good example is provided by the vacuum extremals representable as maps $M^4_+ \rightarrow D^1$, where D^1 is one-dimensional curve of CP_2. This degeneracy could be interpreted as a space-time correlate for the non-determinism of quantum jumps with maximal deterministic regions representing quantum states in a sequence of quantum jumps.

2.6 Non-Vacuum Extremals

2.6.1 Cosmic Strings

Cosmic strings are extremals of type $X^2 \times S^2$, where X^2 is minimal surface in M^4_+ (analogous to the orbit of a bosonic string) and S^2 is the homologically non-trivial geodesic sphere of CP_2. The action of these extremals is positive and thus absolute minima are certainly not in question. One can however consider the possibility that these extremals are building blocks of the absolute minimum space-time surfaces since the absolute minimization of the Kähler action is global rather than a local principle. A more general approach gives up absolute minimization as definition of preferred extremal property and there are indeed several proposals for what preferred extremal property could mean. Cosmic strings can contain also Kähler charged matter in the form of small holes containing elementary particle quantum numbers on their boundaries and the negative Kähler electric action for a topologically condensed cosmic string could cancel the Kähler magnetic action.

The string tension of the cosmic strings is given by

$$T = \frac{1}{8\alpha_K R^2} \simeq .2210^{-6}\frac{1}{G} \ , \tag{2.6.1}$$

where $\alpha_K \simeq \alpha_{em}$ has been used to get the numerical estimate. The string tension is of the same order of magnitude as the string tension of the cosmic strings of GUTs and this leads to the model of the galaxy formation providing a solution to the dark matter puzzle as well as to a model for large voids as caused by the presence of a strongly Kähler charged cosmic string. Cosmic strings play also fundamental role in the TGD inspired very early cosmology.

2.6.2 Massless Extremals

Massless extremals (or topological light rays) are characterized by massless wave vector p and polarization vector ε orthogonal to this wave vector. Using the coordinates of M^4 as coordinates for X^4 the solution is given as

$$\begin{array}{ll} s^k = f^k(u,v) \ , & \\ u = p\cdot m \ , & v = \varepsilon \cdot m \ , \\ p\cdot\varepsilon = 0 \ , & p^2 = 0 \ . \end{array}$$

CP_2 coordinates are arbitrary functions of $p\cdot m$ and $\varepsilon\cdot m$. Clearly these solutions correspond to plane wave solutions of gauge field theories. It is important to notice however that linear super position doesn't hold as it holds in Maxwell phase. Gauge current is proportional to wave vector and its divergence vanishes as a consequence. Also cylindrically symmetric solutions for which the transverse coordinate is replaced with the radial coordinate $\rho = \sqrt{m_1^2 + m_2^2}$ are possible. In fact, v can be *any* function of the coordinates m^1, m^2 transversal to the light like vector p.

Boundary conditions on the boundaries of the massless extremal are satisfied provided the normal component of the energy momentum tensor vanishes. Since energy momentum tensor is of the form $T^{\alpha\beta} \propto p^\alpha p^\beta$ the conditions $T^{n\beta} = 0$ are satisfied if the M^4 projection of the boundary is given by the equations of form

$$\begin{array}{c} H(p \cdot m, \varepsilon \cdot m, \varepsilon_1 \cdot m) = 0 \ , \\ \varepsilon \cdot p = 0 \ , \qquad \varepsilon_1 \cdot p = 0 \ , \quad \varepsilon \cdot \varepsilon_1 = 0 \ . \end{array} \tag{2.6.2}$$

where H is arbitrary function of its arguments. Recall that for M^4 type extremals the boundary conditions are also satisfied if Kähler field vanishes identically on the boundary.

The following argument suggests that there are not very many ways to satisfy boundary conditions in case of M^4 type extremals. The boundary conditions, when applied to M^4 coordinates imply the vanishing of the normal component of energy momentum tensor. Using coordinates, where energy momentum tensor is diagonal, the requirement boils down to the condition that at least one of the eigen values of $T^{\alpha\beta}$ vanishes so that the determinant $det(T^{\alpha\beta})$ must vanish on the boundary: this condition defines 3-dimensional surface in X^4. In addition, the normal of this surface must have same direction as the eigen vector associated with the vanishing eigen value: this means that three additional conditions must be satisfied and this is in general true in single point only. The boundary conditions in CP_2 coordinates are satisfied provided that the conditions

$$J^{n\beta} J^k_{\ l} \partial_\beta s^l = 0$$

are satisfied. The identical vanishing of the normal components of Kähler electric and magnetic fields on the boundary of massless extremal property provides a way to satisfy all boundary conditions but it is not clear whether there are any other ways to satisfy them.

The characteristic feature of the massless extremals is that in general the Kähler gauge current is non-vanishing. In ordinary Maxwell electrodynamcis this is not possible. This means that these extremals are accompanied by vacuum current, which contains in general case both weak and electromagnetic terms as well as color part.

A possible interpretation of the solution is as the exterior space-time to a topologically condensed particle with vanishing mass described by massless CP_2 type extremal, say photon or neutrino. In general the surfaces in question have boundaries since the coordinates s^k are bounded this is in accordance with the general ideas about topological condensation. The fact that massless plane wave is associated with CP_2 type extremal combines neatly the wave and particle aspects at geometrical level.

The fractal hierarchy of space-time sheets implies that massless extremals should interesting also in long length scales. The presence of a light like electromagnetic vacuum current implies the generation of coherent photons and also coherent gravitons are generated since the Einstein tensor is also non-vanishing and light like (proportional to $k^\alpha k^\beta$). Massless extremals play an important role in the TGD based model of bio-system as a macroscopic quantum system. The possibility of vacuum currents is what makes possible the generation of the highly desired coherent photon states.

2.6.3 Does GRT really allow gravitational radiation: could cosmological constant save the situation?

In Facebook discussion Nils Grebäck mentioned Weyl tensor and I learned something that I should have noticed long time ago. Wikipedia article (see `http://tinyurl.com/y7fsnzk8`) lists the basic properties of Weyl tensor as the traceless part of curvature tensor, call it R. Weyl tensor C is vanishing for conformally flat space-times. In dimensions D=2,3 Weyl tensor vanishes identically so that they are always conformally flat: this obviously makes the dimension $D = 3$ for space very special. Interestingly, one can have non-flat space-times with nonvanishing Weyl tensor but the vanishing Schouten/Ricci/Einstein tensor and thus also with vanishing energy momentum tensor.

The rest of curvature tensor R can be expressed in terms of so called Kulkarni-Nomizu product $P \cdot g$ of Schouten tensor P and metric tensor g: $R = C + P \cdot g$, which can be also transformed to a definition of Weyl tensor using the definition of curvature tensor in terms of Christoffel symbols as the fundamental definition. Kulkarni-Nomizu product $\cdot$ is defined as tensor product of two 2-tensors with symmetrization with respect to first and second index pairs plus antisymmetrization with respect to second and fourth indices.

Schouten tensor P is expressible as a combination of Ricci tensor Ric defined by the trace of R with respect to the first two indices and metric tensor g multiplied by curvature scalar s (rather than R in order to use index free notation without confusion with the curvature tensor). The expression reads as

$$P = \frac{1}{D-2}\left[Ric - \frac{s}{2(D-1)}g\right] \quad .$$

Note that the coefficients of Ric and g differ from those for Einstein tensor. Ricci tensor and Einstein tensor are proportional to energy momentum tensor by Einstein equations relate to the part.

Weyl tensor is assigned with gravitational radiation in GRT. What I see as a serious interpretational problem is that by Einstein's equations gravitational radiation would carry no energy and momentum in absence of matter. One could argue that there are no free gravitons in GRT if this interpretation is adopted! This could be seen as a further argument against GRT besides the problems with the notions of energy and momentum: I had not realized this earlier.

Interestingly, in TGD framework so called massless extremals (MEs) [K11, K6, K65] are four-surfaces, which are extremals of Kähler action, have Weyl tensor equal to curvature tensor and therefore would have interpretation in terms of gravitons. Now these extremals are however non-vacuum extremals.

1. Massless extremals correspond to graphs of possibly multi-valued maps from M^4 to CP_2. CP_2 coordinates are arbitrary functions of variables $u = k \cot m$ and $w = \epsilon \cdot m$. k is light-like wave vector and ϵ space-like polarization vector orthogonal to k so that the interpretation in terms of massless particle with polarization is possible. ME describes in the most general case a wave packet preserving its shape and propagating with maximal signal velocity along a kind of tube analogous to wave guide so that they are ideal for precisely targeted communications and central in TGD inspired quantum biology. MEs do not have Maxwellian counterparts. For instance, MEs can carry light-like gauge currents parallel to them: this is not possible in Maxwell's theory.

2. I have discussed a generalization of this solution ansatz so that the directions defined by light-like vector k and polarization vector ϵ orthogonal to it are not constant anymore but define a slicing of M^4 by orthogonal curved surfaces (analogs of string world sheets and space-like surfaces orthogonal to them). MEs in their simplest form at least are minimal surfaces and actually extremals of practically any general coordinate invariance action principle. For instance, this is the case if the volume term suggested by the twistor lift of Kähler action [K38] and identifiable in terms of cosmological constant is added to Kähler action.

3. MEs carry non-trivial induced gauge fields and gravitational fields identified in terms of the induced metric. I have identified them as correlates for particles, which correspond to pairs of wormhole contacts between two space-times such that at least one of them is ME. MEs would accompany to both gravitational radiation and other forms or radiation classically and serve as their correlates. For massless extremals the metric tensor is of form

$$g = m + a\epsilon \otimes \epsilon + bk \otimes k + c(\epsilon \otimes kv + k \otimes \epsilon) \quad ,$$

where m is the metric of empty Minkowski space. The curvature tensor is necessarily quadrilinear in polarization vector ϵ and light-like wave vector k (light-like ifor both M^4 and ME metric) and from the general expression of Weyl tensor C in terms of R and g it is equal to curvature tensor: $C = R$.

Hence the interpretation as graviton solution conforms with the GRT interpretation. Now however the energy momentum tensor for the induced Kähler form is non-vanishing and bilinear in velocity vector k and the interpretational problem is avoided.

What is interesting that also at GRT limit cosmological constant saves gravitons from reducing to vacuum solutions. The deviation of the energy density given by cosmological term from that for Minkowski metric is identifiable as gravitonic energy density. The mysterious cosmological constant would be necessary for making gravitons non-vacuum solutions. The value of graviton amplitude would be determined by the continuity conditions for Einstein's equations with cosmological term. The p-adic evolution of cosmological term predicted by TGD is however difficult to understand in GRT framework.

2.6.4 Gravitational memory effect and quantum criticality of TGD

Gary Ehlenberg sent an interesting post about the gravitational memory effect (see this and this).

Classical gravitational waves would leave a memory of its propagation to the metric of space-time affecting distances between mass points. The computations are done by treating Einstein's theory as a field theory in the background defined by the energy momentum tensor of matter and calculations are carried out only in the lowest non-trivial order.

There are two kinds of effects: the linear memory effect occurs for instance when a planet moves along non-closed hyperbolic orbit around a star and involves only the energy momentum tensor of the system. The non-linear memory effect also involves the energy momentum tensor of gravitational radiation as a source added to the energy momentum tensor of matter.

The effect is accumulative and involves integration over the history of the matter source over the entire past. The reason why the memory effect is non-vanishing is basically that the source of the gravitational radiation is quadratic in metric. In Maxwellian electrodynamics the source does not have this property.

I have never thought of the memory effect. The formula used to estimate the effect is however highly interesting.

1. In the formula for the non-linear memory effect, that is for the action of d'Alembert operator acting on the radiation contribution to the metric, the source term is obtained by adding to the energy momentum tensor of the matter, the energy momentum tensor of the gravitational radiation.
2. This formula can be iterated and if the limit as a fixed point exists, the energy momentum tensor of the gravitational radiation produced by the total energy momentum tensor, including also the radiative contribution, should vanish. This brings in mind fractals and criticality.

   One of the basic facts about iteration for polynomials is that it need not always converge. Limit cycles typically emerge. In more complex situations also objects known as strange attractors can appear. Does the same problem occur now, when the situation is much much more complex?
3. What is interesting is that gravitational wave solutions indeed have vanishing energy momentum tensors. This is problematic if one considers them as radiation in empty space. In the presence of matter, this might be true only for very special background metrics as a sum of matter part and radiation part: just these gravitationally critical fixed point metrics.

   Could the fixed point property of these metrics (matter plus gravitational radiation) be used to gain information of the total metric as sum of matter and gravitational parts?
4. As a matter of fact, all solutions of non-linear field theories are constructed by similar iteration and the radiative contribution in a given order is determined by the contribution in lower orders.

   Under what conditions can one assume convergence of the perturbation series, that is fixed point property? Are limit cycles and chaotic attractors, and only a specialist knows what, unavoidable? Could this fixed point property have some physical relevance? Could the fixed points correspond in quantum field theory context to fixed points of the renormalization group and lead to quantization of coupling constants?

Does the fixed point property have a TGD counterpart?

1. In the TGD, framework Einstein's equations are expected only at the QFT limit at which space-time sheets are replaced with a single region of M^4 carrying gauge fields and gravitational fields, which are sums of the induced fields associated with space-time sheets. What happens at the level of the basic TGD?

What is intriguing, is that quantum criticality is the basic principle of TGD and fixes discrete coupling constant evolution: could the quantum criticality realize itself also as gravitational criticality in the above sense? And even the idea that perturbation series can converge only at critical points and becomes actually trivial?

2. What does the year 2023 version of classical TGD say about the situation? In TGD, space-time surfaces obey almost deterministic holography required by general coordinate invariance [L91, L78]. Holography follows from the general coordinate invariance and implies that path integral trivializes to sum over the analogs of Bohr orbits of particles represented as 3-D surfaces. This states quantum criticality and fixed point property: radiative contributions vanish. This also implies a number theoretic view of coupling constant evolution based on number theoretic vision about physics.

   $M^8 - H$ duality [L64, L65, L90] implies that the space-time regions defined by Bohr orbits are extremely simple and form an evolutionary hierarchy characterized by extensions of rationals associated with polynomials characterizing the counterparts of space-time surfaces as 4-surfaces in complexified M^8 mapped to space-time surfaces in $M^4 \times CP_2$ by $M^8 - H$ duality. There is also universality: the Bohr orbits in H are minimal surfaces [L82], which satisfy a 4-D generalization of 2-D holomorphy and are independent of the action principle as long as it is general coordinate invariant and constructible in terms of the induced geometry. The only dependence on coupling constants comes from singularities at which minimal surface property fails. Also classical conserved quantities depend on coupling constants.

3. The so called "massless extremals" (MEs) represent radiation with very special properties such as precisely targeted propagation with light velocity, absence of dispersion of wave packed, and restricted linear superposition for massless modes propagating in the direction of ME. They are analogous to laser beams, Bohr orbits for radiation fields. The gauge currents associated with MEs are light-like and Lorentz 4-force vanishes.

4. Could the Einstein tensor of ME vanish? The energy momentum tensor expressed in terms of Einstein tensor involves a dimensional parameter and measures the breaking of scale invariance. MEs are conformally invariant objects: does this imply the vanishing of the Einstein tensor? Note however that the energy momentum tensor assignable to the induced gauge fields is non-vanishing: however, its scale covariance is an inherent property of gauge fields so that it need not vanish.

2.6.5 Generalization Of The Solution Ansatz Defining Massless Extremals (MEs)

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

The solution ansatz for MEs has developed gradually to an increasingly general form and the following formulation is the most general one achieved hitherto. Rather remarkably, it rather closely resembles the solution ansatz for the CP_2 type extremals and has direct interpretation in terms of geometric optics. Equally remarkable is that the latest generalization based on the introduction of the local light cone coordinates was inspired by quantum holography principle.

Local light cone coordinates

The solution involves a decomposition of M^4_+ tangent space localizing the decomposition of Minkowski space to an orthogonal direct sum $M^2 \oplus E^2$ defined by light-like wave vector and polarization vector orthogonal to it. This decomposition defines what might be called local light cone coordinates.

1. Denote by m^i the linear Minkowski coordinates of M^4. Let (S^+, S^-, E^1, E^2) denote local coordinates of M^4_+ defining a *local* decomposition of the tangent space M^4 of M^4_+ into a direct *orthogonal* sum $M^4 = M^2 \oplus E^2$ of spaces M^2 and E^2. This decomposition has interpretation in terms of the longitudinal and transversal degrees of freedom defined by local light-like four-velocities $v_\pm = \nabla S_\pm$ and polarization vectors $\epsilon_i = \nabla E^i$ assignable to light ray.

2. With these assumptions the coordinates $(S_\pm, E^i)$ define local light cone coordinates with the metric element having the form

$$ds^2 = 2g_{+-}dS^+dS^- + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \ . \tag{2.6.3}$$

If complex coordinates are used in transversal degrees of freedom one has $g_{11} = g_{22}$.

3. This family of light cone coordinates is not the most general family since longitudinal and transversal spaces are orthogonal. One can also consider light-cone coordinates for which one non-diagonal component, say m_{1+}, is non-vanishing if the solution ansatz is such that longitudinal and transversal spaces are orthogonal for the induced metric.

A conformally invariant family of local light cone coordinates

The simplest solutions to the equations defining local light cone coordinates are of form $S_\pm = k\cdot m$ giving as a special case $S_\pm = m^0 \pm m^3$. For more general solutions of from

$$S_\pm = m^0 \pm f(m^1, m^2, m^3) \ , \ (\nabla_3 f)^2 = 1 \ ,$$

where f is an otherwise arbitrary function, this relationship reads as

$$S^+ + S^- = 2m^0 \ .$$

This condition defines a natural rest frame. One can integrate f from its initial data at some two-dimensional $f = constant$ surface and solution describes curvilinear light rays emanating from this surface and orthogonal to it. The flow velocity field $\overline{v} = \nabla f$ is irrotational so that closed flow lines are not possible in a connected region of space and the condition $\overline{v}^2 = 1$ excludes also closed flow line configuration with singularity at origin such as $v = 1/\rho$ rotational flow around axis.

One can identify E^2 as a local tangent space spanned by polarization vectors and orthogonal to the flow lines of the velocity field $\overline{v} = \nabla f(m^1, m^2, m^3)$. Since the metric tensor of any 3-dimensional space allows always diagonalization in suitable coordinates, one can always find coordinates (E^1, E^2) such that (f, E^1, E^2) form orthogonal coordinates for $m^0 = constant$ hyperplane. Obviously one can select the coordinates E^1 and E^2 in infinitely many ways.

Closer inspection of the conditions defining local light cone coordinates

Whether the conformal transforms of the local light cone coordinates $\{S_\pm = m^0 \pm f(m^1, m^2, m^3), E^i\}$ define the only possible compositions $M^2 \oplus E^2$ with the required properties, remains an open question. The best that one might hope is that any function S^+ defining a family of light-like curves defines a local decomposition $M^4 = M^2 \oplus E^2$ with required properties.

1. Suppose that S^+ and S^- define light-like vector fields which are not orthogonal (proportional to each other). Suppose that the polarization vector fields $\epsilon_i = \nabla E^i$ tangential to local E^2 satisfy the conditions $\epsilon_i \cdot \nabla S^+ = 0$. One can formally integrate the functions E^i from these condition since the initial values of E^i are given at $m^0 = constant$ slice.

2. The solution to the condition $\nabla S_+ \cdot \epsilon_i = 0$ is determined only modulo the replacement

$$\epsilon_i \rightarrow \hat{\epsilon}_i = \epsilon_i + k\nabla S_+ \ ,$$

where k is any function. With the choice

$$k = -\frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-}$$

one can satisfy also the condition $\hat{\epsilon}_i \cdot \nabla S^- = 0$.

3. The requirement that also $\hat{\epsilon}_i$ is gradient is satisfied if the integrability condition

$$k = k(S^+)$$

is satisfiedin this case $\hat{\epsilon}_i$ is obtained by a gauge transformation from ϵ_i. The integrability condition can be regarded as an additional, and obviously very strong, condition for S^- once S^+ and E^i are known.

4. The problem boils down to that of finding local momentum and polarization directions defined by the functions S^+, S^- and E^1 and E^2 satisfying the orthogonality and integrability conditions

$$(\nabla S^+)^2 = (\nabla S^-)^2 = 0 \ , \quad \nabla S^+ \cdot \nabla S^- \neq 0 \ ,$$

$$\nabla S^+ \cdot \nabla E^i = 0 \ , \qquad \frac{\nabla E^i \cdot \nabla S^-}{\nabla S^+ \cdot \nabla S^-} = k_i(S^+) \ .$$

The number of integrability conditions is 3+3 (all derivatives of k_i except the one with respect to S^+ vanish): thus it seems that there are not much hopes of finding a solution unless some discrete symmetry relating S^+ and S^- eliminates the integrability conditions altogether.

A generalization of the spatial reflection $f \rightarrow -f$ working for the separable Hamilton Jacobi function $S_\pm = m^0 \pm f$ ansatz could relate S^+ and S^- to each other and trivialize the integrability conditions. The symmetry transformation of M^4_+ must perform the permutation $S^+ \leftrightarrow S^-$, preserve the light-likeness property, map E^2 to E^2, and multiply the inner products between M^2 and E^2 vectors by a mere conformal factor. This encourages the conjecture that all solutions are obtained by conformal transformations from the solutions $S_\pm = m^0 \pm f$.

General solution ansatz for MEs for given choice of local light cone coordinates

Consider now the general solution ansatz assuming that a local wave-vector-polarization decomposition of M^4_+ tangent space has been found.

1. Let $E(S^+, E^1, E^2)$ be an arbitrary function of its arguments: the gradient ∇E defines at each point of E^2 an S^+-dependent (and thus time dependent) polarization direction orthogonal to the direction of local wave vector defined by ∇S^+. Polarization vector depends on E^2 position only.

2. Quite a general family of MEs corresponds to the solution family of the field equations having the general form

$$s^k = f^k(S^+, E) \ ,$$

where s^k denotes CP_2 coordinates and f^k is an arbitrary function of S^+ and E. The solution represents a wave propagating with light velocity and having definite S^+ dependent polarization in the direction of ∇E. By replacing S^+ with S^- one obtains a dual solution. Field equations are satisfied because energy momentum tensor and Kähler current are light-like so that all tensor contractions involved with the field equations vanish: the orthogonality of M^2 and E^2 is essential for the light-likeness of energy momentum tensor and Kähler current.

3. The simplest solutions of the form $S_\pm = m^0 \pm m^3$, $(E^1, E^2) = (m^1, m^2)$ and correspond to a cylindrical MEs representing waves propagating in the direction of the cylinder axis with light velocity and having polarization which depends on point (E^1, E^2) and S^+ (and thus time). For these solutions four-momentum is light-like: for more general solutions this cannot be the case. Polarization is in general case time dependent so that both linearly and circularly polarized waves are possible. If m^3 varies in a finite range of length L, then "free" solution represents geometrically a cylinder of length L moving with a light velocity. Of course, ends could be also anchored to the emitting or absorbing space-time surfaces.

4. For the general solution the cylinder is replaced by a three-dimensional family of light like curves and in this case the rectilinear motion of the ends of the cylinder is replaced with a curvilinear motion with light velocity unless the ends are anchored to emitting/absorbing space-time surfaces. The non-rotational character of the velocity flow suggests that the freely moving particle like 3-surface defined by ME cannot remain in a infinite spatial volume. The most general ansatz for MEs should be useful in the intermediate and nearby regions of a radiating object whereas in the far away region radiation solution is excepted to decompose to cylindrical ray like MEs for which the function $f(m^1, m^2, m^2)$ is a linear function of m^i.

5. One can try to generalize the solution ansatz further by allowing the metric of M^4_+ to have components of type g_{i+} or g_{i-} in the light cone coordinates used. The vanishing of T^{11}, T^{+1}, and T^{--} is achieved if $g_{i\pm}=0$ holds true for the induced metric. For $s^k = s^k(S^+,E^1)$ ansatz neither $g_{2\pm}$ nor g_{1-} is affected by the embedding so that these components of the metric must vanish for the Hamilton Jacobi structure:

$$ds^2 = 2g_{+-}dS^+dS^- + 2g_{1+}dE^1dS^+ + g_{11}(dE^1)^2 + g_{22}(dE^2)^2 \ . \tag{2.6.4}$$

$g_{1+}=0$ can be achieved by an additional condition

$$m_{1+} = s_{kl}\partial_1 s^k \partial_+ s^k \ . \tag{2.6.5}$$

The diagonalization of the metric seems to be a general aspect of preferred extremals. The absence of metric correlations between space-time degrees of freedom for asymptotic self-organization patterns is somewhat analogous to the minimization of non-bound entanglement in the final state of the quantum jump.

Are the boundaries of space-time sheets quite generally light like surfaces with Hamilton Jacobi structure?

Quantum holography principle naturally generalizes to an approximate principle expected to hold true also in non-cosmological length and time scales.

1. The most general ansatz for topological light rays or massless extremals (MEs) inspired by the quantum holographic thinking relies on the introduction of the notion of local light cone coordinates S_+, S_-, E_1, E_2. The gradients ∇S_+ and ∇S_- define two light like directions just like Hamilton Jacobi functions define the direction of propagation of wave in geometric optics. The two polarization vector fields ∇E_1 and ∇E_2 are orthogonal to the direction of propagation defined by either S_+ or S_-. Since also E_1 and E_2 can be chosen to be orthogonal, the metric of M^4_+ can be written locally as $ds^2 = g_{+-}dS_+dS_- + g_{11}dE_1^2 + g_{22}dE_2^2$. In the earlier ansatz S_+ and S_- where restricted to the variables $k\cdot m$ and $\tilde{k}\cdot m$, where k and $\tilde{k}$ correspond to light like momentum and its mirror image and m denotes linear M^4 coordinates: these MEs describe cylindrical structures with constant direction of wave propagation expected to be most important in regions faraway from the source of radiation.
2. Boundary conditions are satisfied if the 3-dimensional boundaries of MEs have one light like direction (S_+ or S_- is constant). This means that the boundary of ME has metric dimension $d=2$ and is characterized by an infinite-dimensional super-symplectic and super-conformal symmetries just like the boundary of the embedding space $M^4_+ \times CP_2$: The boundaries are like moments for mini big bangs (in TGD based fractal cosmology big bang is replaced with a silent whisper amplified to not necessarily so big bang).
3. These observations inspire the conjecture that boundary conditions for M^4 like space-time sheets fixed by the absolute minimization of Kähler action quite generally require that space-time boundaries correspond to light like 3-surfaces with metric dimension equal to $d=2$. This does not yet imply that light like surfaces of embedding space would take the role of the light cone boundary: these light like surface could be seen only as a special case of causal determinants analogous to event horizons.

2.6.6 Maxwell Phase

"Maxwell phase" corresponds to small deformations of the M^4 type vacuum extremals. Since energy momentum tensor is quadratic in Kähler field the term proportional to the contraction of the energy momentum tensor with second fundamental form drops from field equations and one obtains in lowest order the following field equations

$$j^\alpha J^k_{\ l} s^l_{,\alpha} = 0 \ . \tag{2.6.6}$$

These equations are satisfied if Maxwell's equations

$$j^{\alpha} = 0 \tag{2.6.7}$$

hold true. Massless extremals and Maxwell phase clearly exclude each other and it seems that they must corresponds to different space-time sheets.

The explicit construction of these extremals reduces to the task of finding an embedding for an arbitrary free Maxwell field to H. One can also allow source terms corresponding to the presence of the point like charges: these should correspond to the regions of the space-time, where the flat space-time approximation of the space-time fails. The regions where the approximation defining the Maxwell phase fails might correspond to a topologically condensed CP_2 type extremals, for example. As a consequence, Kähler field is superposition of radiation type Kähler field and of Coulomb term. A second possibility is the generation of "hole" with similar Coulombic Kähler field.

An important property of the Maxwell phase (also of massless extremals) is its approximate canonical invariance. Canonical transformations do not spoil the extremal property of the four-surface in the approximation used, since it corresponds to a mere $U(1)$ gauge transformation. This implies the counterpart of the vacuum degeneracy, that is, the existence of an enormous number of four-surfaces with very nearly the same action. Also there is an approximate $Diff(M^4_+)$ invariance.

The canonical degeneracy has some very interesting consequences concerning the understanding of the electro-weak symmetry breaking and color confinement. Kähler field is canonical invariant and satisfies Maxwells equations. This is in accordance with the identification of Kähler field as $U(1)$ part of the electro-weak gauge field. Electromagnetic gauge field is a superposition of Kähler field and Z^0 field$\gamma = 3J - sin^2(\theta_W)Z^0/2$ so that also electromagnetic gauge field is long ranged assuming that Z^0 and W^+ fields are short ranged. These fields are not canonical invariants and their behavior seems to be essentially random, which implies short range correlations and the consequent massivation.

There is an objection against this argument. For the known $D < 4$ solutions of field equations weak fields are not random at all. These situations could represent asymptotic configurations assignable to space-time sheets. This conforms with the interpretation that weak gauge fields are essentially massless within the asymptotic space-time sheets representing weak bosons. Gauge fields are however transferred between space-time sheets through # contacts modellable as pieces of CP_2 type extremals having $D_{CP_2} = 4$. In contrast to Kähler and color gauge fluxes, weak gauge fluxes are not conserved in the Euclidian time evolution between the 3-D causal horizons separating the Euclidian # contact from space-time sheets with Minkowskian signature. This non-conservation implying the loss of coherence in the transfer of fields between space-time sheets is a plausible mechanism for the loss of correlations and massivation of the weak gauge fields.

Classical gluon fields are proportional to Kähler field and to the Hamiltonians associated with the color isometry generators.

$$g^A_{\alpha\beta} = kH^A J_{\alpha\beta} \ . \tag{2.6.8}$$

This implies that the direction of gluon fields in color algebra is random. One can always perform a canonical transformation, which reduces to a global color rotation in some arbitrary small region of space-time and reduces to identity outside this region. The proportionality of a gluon field to Kähler form implies that there is a classical long range correlation in X^4 degrees of freedom: in this sense classical gluon fields differ from massive electro-weak fields in Maxwell phase.

2.6.7 Stationary, Spherically Symmetric Extremals

The stationary, spherically symmetric extremals of the Kähler action imbeddable in $M^4 \times S^2$, where S^2 is geodesic sphere, are the simplest extremals, which one can study as models for the space-time surrounding a topologically condensed particle, say CP_2 type vacuum extremal. In the region near the particle the spherical symmetry is an unrealistic assumption since it excludes the presence of magnetic fields needed to cancel the total Kähler action. The stationarity is also unrealistic assumption since zitterbewegung seems to provide a necessary mechanism for generating Kähler magnetic field and for satisfying boundary conditions. Also the imbeddability to $M^4 \times S^2$ implies unrealistic relationship between Z^0 and photon charges.

According to the general wisdom, the generation of a Kähler electric field must take place in order to minimize the action and it indeed turns out that the extremal is characterized by essentially $1/r^2$ Kähler electric field. The necessary presence of a hole or of a topologically condensed object is also demonstrated: it is impossible to find extremals well defined in the region surrounding the origin. It is impossible to satisfy boundary conditions at a hole: this is in accordance with the idea that Euclidian region corresponding to a CP_2 type extremal performing zitterbewegung is generated. In case of CP_2 type extremal radius is of the order of the Compton length of the particle and in case of a "hole" of the order of Planck length. The value of the vacuum frequency ω is of order of particle mass whereas for macroscopic vacuum extremals it must be of the order of $1/R$. This does not lead to a contradiction if the concept of a many-sheeted space-time is accepted.

The Poincare energy of the exterior region is considerably smaller than the gravitational mass; this conforms with the interpretation that gravitational mass is sum of absolute values of positive and negative inertial masses associated with matter and negative energy antimatter. It is quite possible that classical considerations cannot provide much understanding concerning the inertial masses of topologically condensed particles. Electro-weak gauge forces are considerably weaker than the gravitational force at large distances, when the value of the frequency parameter ω is of order $1/R$. Both these desirable properties fail to be true if CP_2 radius is of order Planck length as believed earlier.

In light of the general ideas about topological condensation it is clear that in planetary length scales these kind of extremals cannot provide a realistic description of space-time. Indeed, spherically symmetric extremals predict a wrong rate for the precession of the perihelion of Mercury. Scwhartschild and Reissner-Nordström metric do this and indeed allow embedding as vacuum extremals for which the inertial masses of positive energy matter and negative energy antimatter sum up to zero.

This does not yet resolve the interpretational challenge due to the unavoidable long range color and weak gauge fields. A dark matter hierarchy giving rise to a hierarchy of color and electro-weak physics characterized by increasing values of weak and confinement scales explains these fields. # contacts involve a pair of causal horizons at which the Euclidian metric signature of # contact transforms to Minkowskian one. These causal horizons have interpretation as partons so that # contact can be regarded as a bound state of partons bound together by a gravitational instanton (CP_2 type extremal). # contacts provide basic example of dark matter creating long ranged weak fields.

An important result is the correlation between the sign of the vacuum frequency ω and that of the Kähler charge, which is of opposite sign for fermions and anti-fermions. This suggests an explanation for matter-antimatter asymmetry. Matter and antimatter condense stably on disjoint regions of the space-time surface at different space-time sheets. Stable antimatter could correspond to negative time orientation and negative energy. This leads to a model for the primordial generation of matter as spontaneous generation of zero energy # contacts between space-time sheets of opposite time orientations. If CP conjugation is not exact symmetry, # contacts and their CP conjugates are created with slightly different rates and this gives rise to CP asymmetry at each of the two space-time sheets involved. After the splitting of # contacts and subsequent annihilation of particles and antiparticles at each space-time sheet, the two space-time sheets contain only positive energy matter and negative energy antimatter.

General solution ansatz

The general form of the solution ansatz is obtained by assuming that the space-time surface in question is a sub-manifold of $M^4 \times S^2$, where S^2 is the homologically non-trivial geodesic sphere of CP_2. S^2 is most conveniently realized as $r = \infty$ surface of CP_2, for which all values of the coordinate Ψ correspond to same point of CP_2 so that one can use Θ and Φ as the coordinates of S^2.

The solution ansatz is given by the expression

$$\begin{aligned} cos(\Theta) &= u(r) \ , \\ \Phi &= \omega t \ , \\ m^0 &= \lambda t \ , \\ r_M &= r \ , \quad \theta_M = \theta \ , \quad \phi_M = \phi \ . \end{aligned} \tag{2.6.9}$$

The induced metric is given by the expression

$$ds^2 = \left[\lambda^2 - \frac{R^2}{4}\omega^2(1-u^2)\right] dt^2 - (1+\frac{R^2}{4}\theta_{,r}^2)dr^2 - r^2 d\Omega^2 \ . \tag{2.6.10}$$

The value of the parameter λ is fixed by the condition $g_{tt}(\infty) = 1$:

$$\lambda^2 - \frac{R^2}{4}\omega^2(1-u(\infty)^2) = 1 \ . \tag{2.6.11}$$

From the condition $e^0 \wedge e^3 = 0$ the non-vanishing components of the induced Kähler field are given by the expression

$$J_{tr} = \frac{\omega}{4}u_{,r} \ . \tag{2.6.12}$$

Geodesic sphere property implies that Z^0 and photon fields are proportional to Kähler field:

$$\begin{aligned} \gamma &= (3-p/2)J \ , \\ Z^0 &= J \ . \end{aligned} \tag{2.6.13}$$

From this formula one obtains the expressions

$$\begin{aligned} Q_{em} &= \frac{(3-p/2)}{4\pi\alpha_{em}}Q_K \ , \quad Q_Z = \frac{1}{4\pi\alpha_Z}Q \ , \\ Q &\equiv \frac{J_{tr}4\pi r^2}{\sqrt{-g_{rr}g_{tt}}} \ . \end{aligned} \tag{2.6.14}$$

for the electromagnetic and Z^0 charges of the solution using e and g_Z as unit.

Field equations can be written as conditions for energy momentum conservation (two equations is in principle all what is needed in the case of geodesic sphere). Energy conservation holds identically true and conservation of momentum, say, in z-direction gives the equation

$$(T^{rr}z_{,r})_{,r} + (T^{\theta\theta}z_{,\theta})_{,\theta} = 0 \ . \tag{2.6.15}$$

Using the explicit expressions for the components of the energy momentum tensor

$$\begin{aligned} T^{rr} &= g^{rr}L/2 \ , \\ T^{\theta\theta} &= -g^{\theta\theta}L/2 \ , \\ L &= g^{tt}g^{rr}(J_{tr})^2\sqrt{g}/2 \ , \end{aligned} \tag{2.6.16}$$

and the following notations

$$\begin{aligned} A &= g^{tt}g^{rr}r^2\sqrt{-g_{tt}g_{rr}} \ , \\ X &\equiv (J_{tr})^2 \ , \end{aligned} \tag{2.6.17}$$

the field equations reduce to the following form

$$(g^{rr}AX)_{,r} - \frac{2AX}{r} = 0 \ . \quad (2.6.18)$$

In the approximation $g^{rr} = 1$ this equation can be readily integrated to give $AX = C/r^2$. Integrating Eq. (5.6.7), one obtains integral equation for X

$$J_{tr} = \frac{q}{r_c}(|g_{rr}|^3 g_{tt})^{1/4} exp(\int_{r_c}^{r} dr \frac{g_{rr}}{r})\frac{1}{r} \ , \quad (2.6.19)$$

where q is integration constant, which is related to the charge parameter of the long range Kähler electric field associated with the solution. r_c denotes the critical radius at which the solution ceases to be well defined.

The inspection of this formula shows that J_{tr} behaves essentially as $1/r^2$ Coulomb field. This behavior doesn't depend on the detailed properties of the solution ansatz (for example the imbeddability to $M^4 \times S^2$): stationarity and spherical symmetry is what matters only. The compactness of CP_2 means that stationary, spherically symmetric solution is not possible in the region containing origin. This is in concordance with the idea that either a hole surrounds the origin or there is a topologically condensed CP_2 type extremal performing zitterbewegung near the origin and making the solution non-stationary and breaking spherical symmetry.

Second integration gives the following integral equation for CP_2 coordinate $u = cos(\Theta)$

$$u(r) = u_0 + \frac{4q}{\omega}\int_{r_c}^{r}(-g_{rr}^3 g_{tt})^{1/4}\frac{1}{r}exp(\int_{r_c}^{r} dr \frac{g_{rr}}{r}) \ . \quad (2.6.20)$$

Here u_0 denotes the value of the coordinate u at $r = r_0$.

The form of the field equation suggests a natural iterative procedure for the numerical construction of the solution for large values of r.

$$u_n(r) = T_{n-1} \ , \quad (2.6.21)$$

where T_{n-1} is evaluated using the induced metric associated with u_{n-1}. The physical content of the approximation procedure is clear: estimate the gravitational effects using lower order solution since these are expected to be small.

A more convenient manner to solve u is based on Taylor expansion around the point $V \equiv 1/r = 0$. The coefficients appearing in the power series expansion $u = \sum_n u_n A^n V^n \ : \ A = q/\omega$ can be solved by calculating successive derivatives of the integral equation for u.

The lowest order solution is simply

$$u_0 = u_\infty \ , \quad (2.6.22)$$

and the corresponding metric is flat metric. In the first order one obtains for $u(r)$ the expression

$$u = u_\infty - \frac{4q}{\omega r} \ , \quad (2.6.23)$$

which expresses the fact that Kähler field behaves essentially as $1/r^2$ Coulomb field. The behavior of u as a function of r is identical with that obtained for the embedding of the Reissner-Nordström solution.

To study the properties of the solution we fix the signs of the parameters in the following manner:

$$u_\infty < 0 \ , \quad q < 0 \ , \quad \omega > 0 \quad (2.6.24)$$

(reasons become clear later).

Concerning the behavior of the solution one can consider two different cases.

1. The condition $g_{tt} > 0$ hold true for all values of Θ. In this case u decreases and the rate of decrease gets faster for small values of r. This means that in the lowest order the solution becomes certainly ill defined at a critical radius $r = r_c$ given by the the condition $u = 1$: the reason is that u cannot get values large than one. The expression of the critical radius is given by

$$\begin{aligned} r_c &\geq \frac{4q}{(|u_\infty| + 1)\omega} \\ &= \frac{4\alpha Q_{em}}{(3 - p/2)} \frac{1}{(|u_\infty| + 1)\omega} \quad . \end{aligned} \tag{2.6.25}$$

The presence of the critical radius for the actual solution is also a necessity as the inspection of the expression for J_{tr} shows: $\partial_r \theta$ grows near the origin without bound and $u = 1$ is reached at some finite value of r. Boundary conditions require that the quantity $X = T^{rr}\sqrt{g}$ vanishes at critical radius (no momentum flows through the boundary). Substituting the expression of J_{tr} from the field equation to T^{rr} the expression for X reduces to a form, from which it is clear that X cannot vanish. The cautious conclusion is that boundary conditions cannot be satisfied and the underlying reason is probably the stationarity and spherical symmetry of the solution. Physical intuition suggests that that CP_2 type extremal performing zitterbewegung is needed to satisfy the boundary conditions.

2. g_{tt} vanishes for some value of Θ. In this case the radial derivative of u together with g_{tt} can become zero for some value of $r = r_c$. Boundary conditions can be satisfied only provided $r_c = 0$. Thus it seems that for the values of ω satisfying the condition $\omega^2 = \frac{4\lambda^2}{R^2 sin^2(\Theta_0)}$ it might be possible to find a globally defined solution. The study of differential equation for u however shows that the ansatz doesn't work. The conclusion is that although the boundary is generated it is not possible to satisfy boundary conditions.

A direct calculation of the coefficients u_n from power series expansion gives the following third order polynomial approximation for u $(V = 1/r)$

$$\begin{aligned} u &= \sum_n u_n A^n V^n \quad , \\ u_0 &= u_\infty (< 0) \quad , \quad u_1 = 1 \quad , \\ u_2 &= K|u_\infty| \quad , \quad u_3 = K(1 + 4K|u_\infty|) \quad , \\ A &\equiv \frac{4q}{\omega} \quad , \quad K \equiv \omega^2 \frac{R^2}{4} \quad . \end{aligned} \tag{2.6.26}$$

The coefficients u_2 and u_3 are indeed positive which means that the value of the critical radius gets larger at least in these orders.

Solution contains three parameters: Kähler electric flux $Q = 4\pi q$, parameter ωR and parameter u_∞. The latter parameters can be regarded as parameters describing the properties of a flat vacuum extremal (lowest order solution) to which particle like solution is glued and are analogous to the parameters describing symmetry broken vacuum in gauge theories.

Solution is not a realistic model for topological condensation

The solution does not provide realistic model for topological condensation although it gives indirect support for some essential assumptions of TGD based description of Higgs mechanism.

1. When the value of ω is of the order of CP_2 mass the solution could be interpreted as the "exterior metric" of a "hole".
   i) The radius of the hole is of the order of CP_2 length and its mass is of the order of CP_2 mass.
   ii) Kähler electric field is generated and charge renormalization takes place classically at CP_2 length scales as is clear from the expression of $Q(r)$: $Q(r) \propto (\frac{-g_{rr}}{g_{tt}})^{1/4}$ and charge increases

at short distances.
iii) The existence of the critical radius is unavoidable but boundary conditions cannot be satisfied. The failure to satisfy boundary conditions might be related to stationarity or to the absence of magnetic field. The motion of the boundary component with velocity of light might be the only manner to satisfy boundary conditions. Second possibility is the breaking of spherical symmetry by the generation of a static magnetic field.
iv) The absence of the Kähler magnetic field implies that the Kähler action has an infinite magnitude and the probability of the configuration is zero. A more realistic solution ansatz would break spherical symmetry containing dipole type magnetic field in the nearby region of the hole. The motion of the boundary with a velocity of light could serves as an alternative mechanism for the generation of magnetic field. The third possibility, supported by physical intuition, is that one must give up "hole" type extremal totally.

2. For sufficiently large values of r and for small values of ω (of the order of elementary particle mass scale), the solution might provide an approximate description for the region surrounding elementary particle. Although it is not possible to satisfy boundary conditions the order of magnitude estimate for the size of critical radius ($r_c \simeq \alpha/\omega$) should hold true for more realistic solutions, too. The order of magnitude for the critical radius is smaller than Compton length or larger if the vacuum parameter ω is larger than the mass of the particle. In macroscopic length scales the value of ω is of order $1/R$. This does not lead to a contradiction if the many-sheeted space-time concept is accepted so that $\omega < m$ corresponds to elementary particle space-time sheet. An unrealistic feature of the solution is that the relationship between Z^0 and em charges is not correct: Z^0 charge should be very small in these length scales.

Exterior solution cannot be identified as a counterpart of Schwartshild solution

The first thing, which comes into mind is to ask whether one might identify exterior solution as the TGD counterpart of the Schwartshild solution. The identification of gravitational mass as absolute value of inertial mass which is negative for antimatter implies that vacuum extremals are vacua only with respect to the inertial four-momentum and have a non-vanishing gravitational four-momentum. Hence, in the approximation that the net density of inertial mass vanishes, vacuum extremals provide the proper manner to model matter, and the identification of the ansatz for a spherically symmetric extremal as the counterpart of Scwhartschild metric is certainly not possible. It is however useful to show explicitly that the identification is indeed unrealistic. The solution is consistent with Equivalence Principle but the electro-weak gauge forces are considerably weaker than gravitational forces. A wrong perihelion shift is also predicted so that the identification as an exterior metric of macroscopic objects is out of question.

1. Is Equivalence Principle respected?

The following calculation demonstrates that Equivalence Principle might not be satisfied for the solution ansatz (which need not actually define a preferred extremal!).

The gravitational mass of the solution is determined from the asymptotic behavior of g_{tt} and is given by

$$M_{gr} = \frac{R^2}{G}\omega q u_\infty \ , \qquad (2.6.27)$$

and is proportional to the Kähler charge q of the solution.

One can estimate the gravitational mass density also by applying Newtonian approximation to the time component of the metric $g_{tt} = 1 - 2\Phi_{gr}$. One obtains Φ_{gr} corresponds in the lowest order approximation to a solution of Einstein's equations with the source consisting of a mass point at origin and the energy density of the Kähler electric field. The effective value of gravitational constant is however $G_{eg} = 8R^2\alpha_K$. Thus the only sensible interpretation is that the density of Kähler (inertial) energy is only a fraction $G/G_{eq} \equiv \epsilon \simeq .22 \times 10^{-6}$ of the density of gravitational mass. Hence the densities of positive energy matter and negative energy antimatter cancel each other in a good approximation.

The work with cosmic strings lead to a possible interpretation of the solution as a space-time sheet containing topologically condensed magnetic flux tube idealizable as a point. The negative

Kähler electric action must cancel the positive Kähler magnetic action. The resulting structure in turn can condense to a vacuum extremal and Schwartshild metric is a good approximation for the metric.

One can estimate the contribution of the exterior region ($r > r_c$) to the inertial mass of the system and Equivalence principle requires this to be a fraction of order ϵ about the gravitational mass unless the region $r < r_c$ contains negative inertial mass density, which is of course quite possible. Approximating the metric with a flat metric and using first order approximation for $u(r)$ the energy reduces just to the standard Coulomb energy of charged sphere with radius r_c

$$\begin{aligned} M_I(ext) &= \frac{1}{32\pi\alpha_K}\int_{r>r_c} E^2\sqrt{g}d^3x \\ &\simeq \frac{\lambda q^2}{2\alpha_K r_c} \ , \\ \lambda &= \sqrt{1+\frac{R^2}{4}\omega^2(1-u_\infty^2)} \ (>1) \ . \end{aligned} \quad (2.6.28)$$

Approximating the metric with flat metric the contribution of the region $r > r_c$ to the energy of the solution is given by

$$M_I(ext) = \frac{1}{8\alpha_K}\lambda q\omega(1+|u_\infty|) \ . \quad (2.6.29)$$

The contribution is proportional to Kähler charge as expected. The ratio of external inertial and gravitational masses is given by the expression

$$\begin{aligned} \frac{M_I(ext)}{M_{gr}} &= \frac{G}{4R^2\alpha_K}x \ , \\ x &= \frac{(1+|u_\infty|)}{|u_\infty|} > 1 \ . \end{aligned} \quad (2.6.30)$$

In the approximation used the ratio of external inertial and gravitational masses is of order 10^{-6} for $R \sim 10^4\sqrt{G}$ implied by the p-adic length scale hypothesis and for $x \sim 1$. The result conforms with the above discussed interpretation.

The result forces to challenge the underlying implicit assumptions behind the calculation.

1. Many-sheeted space-time means that single space-time sheet need not be a good approximation for astrophysical systems. The GRT limit of TGD can be interpreted as obtained by lumping many-sheeted space-time time to Minkowski space with effective metric defined as sum M^4 metric and sum of deviations from M^4 metric for various space-time sheets involved [K99]. This effective metric should correspond to that of General Relativity and Einstein's equations reflect the underlying Poincare invariance. Gravitational and cosmological constants follow as predictions and EP is satisfied.
2. The systems considered need not be preferred extremals of Kähler action so that one cannot take the results obtained too seriously. For vacuum extremals one does not encounter this problem at all and it could be that vacuum extremals with induced metric identified as GRT metric are a good approximation in astrophysical systems. This requires that single-sheetedness is a good approximation. TGD based single-sheeted models for astrophysical and cosmological systems rely on this assumption.

2. Z^0 and electromagnetic forces are much weaker than gravitational force

The extremal in question carries Kähler charge and therefore also Z^0 and electromagnetic charge. This implies long range gauge interactions, which ought to be weaker than gravitational interaction in the astrophysical scales. This is indeed the case as the following argument shows.

Expressing the Kähler charge using Planck mass as unit and using the relationships between gauge fields one obtains a direct measure for the strength of the Z^0 force as compared with the strength of gravitational force.

$$Q_Z \equiv \varepsilon_Z M_{gr}\sqrt{G} \ . \tag{2.6.31}$$

The value of the parameter ε_Z should be smaller than one. A transparent form for this condition is obtained, when one writes $\Phi = \omega t = \Omega m^0 : \Omega = \lambda\omega$:

$$\varepsilon_Z = \frac{\alpha_K}{\alpha_Z}\frac{1}{\pi(1+|u_\infty|)\Omega R}\sqrt{\frac{G}{R}} \ . \tag{2.6.32}$$

The order of magnitude is determined by the values of the parameters $\sqrt{\frac{G}{R^2}} \sim 10^{-4}$ and ΩR. Global Minkowskian signature of the induced metric implies the condition $\Omega R < 2$ for the allowed values of the parameter ΩR. In macroscopic length scales one has $\Omega R \sim 1$ so that Z^0 force is by a factor of order 10^{-4} weaker than gravitational force. In elementary particle length scales with $\omega \sim m$ situation is completely different as expected.

3. The shift of the perihelion is predicted incorrectly

The g_{rr} component of Reissner-Nordström and TGD metrics are given by the expressions

$$g_{rr} = -\frac{1}{(1-\frac{2GM}{r})} \ , \tag{2.6.33}$$

and

$$g_{rr} \simeq 1 - \frac{\frac{Rq}{\omega^2}}{\left[1-(u_\infty - \frac{4q}{\omega r})^2\right]r^4} \ , \tag{2.6.34}$$

respectively. For reasonable values of q, ω and u_∞ the this terms is extremely small as compared with $1/r$ term so that these expressions differ by $1/r$ term.

The absence of the $1/r$ term from g_{rr}-component of the metric predicts that the shift of the perihelion for elliptic plane orbits is about $2/3$ times that predicted by GRT so that the identification as a metric associated with objects of a planetary scale leads to an experimental contradiction. Reissner-Nordström solutions are obtained as vacuum extremals so that standard predictions of GRT are obtained for the planetary motion.

One might hope that the generalization of the form of the spherically symmetric ansatz by introducing the same modification as needed for the embedding of Reissner-Nordstrm̈ metric might help. The modification would read as

$$\begin{aligned} cos(\Theta) &= u(r) \ , \\ \Phi &= \omega t + f(r) \ , \\ m^0 &= \lambda t + h(r) \ , \\ r_M &= r \ , \quad \theta_M = \theta \ , \quad \phi_M = \phi \ . \end{aligned} \tag{2.6.35}$$

The vanishing of the g_{tr} component of the metric gives the condition

$$\lambda\partial_r h - \frac{R^2}{4}sin^2(\Theta)\omega\partial_r f = 0 \ . \tag{2.6.36}$$

The expression for the radial component of the metric transforms to

$$g_{rr} \simeq \partial_r h^2 - 1 - \frac{R^2}{4}(\partial_r\Theta)^2 - \frac{R^2}{4}sin^2(\Theta)\partial_r f^2 \ , \tag{2.6.37}$$

Essentially the same perihelion shift as for Schwartschild metric is obtained if g_{rr} approaches asymptotically to its expression for Schwartschild metric. This is guaranteed if the following conditions hold true:

$$f(r)_{r\rightarrow\infty} \rightarrow \omega r \ , \ \Lambda^2 - 1 = \frac{R^2\omega^2}{4} sin^2(\Theta_\infty) \ll \frac{2GM}{\langle r\rangle} \ . \quad (2.6.38)$$

In the second equation $\langle r\rangle$ corresponds to the average radius of the planetary orbit.

The field equations for this ansatz can be written as conditions for energy momentum and color charge conservation. Two equations are enough to determine the functions $\Theta(r)$ and $f(r)$. The equation for momentum conservation is same as before. Second field equation corresponds to the conserved isometry current associated with the color isometry $\Phi \rightarrow \Phi + \epsilon$ and gives equation for f.

$$[T^{rr} f_{,r} s_{\Phi\Phi}\sqrt{g}]_{,r} = 0 \ . \quad (2.6.39)$$

The conservation laws associated with other infinitesimal $SU(2)$ rotations of S^2_I should be satisfied identically. This equation can be readily integrated to give

$$T^{rr} f_{,r} s_{\Phi\Phi}\sqrt{g_{tt}g_{rr}} = \frac{C}{r^2} \ . \quad (2.6.40)$$

Unfortunately, the result is inconsistent with the $1/r^4$ behavior of T^{rr} and $f \rightarrow \omega r$ implies by correct red shift.

It seems that the only possible way out of the difficulty is to replace spherical symmetry with a symmetry with respect to the rotations around z-axis. The simplest modification of the solution ansatz is as follows:

$$m^0 = \lambda t + h(\rho) \ , \quad \Phi = \omega t + k\rho \quad .$$

Thanks to the linear dependence of Φ on ρ, the conservation laws for momentum and color isospin reduce to the same condition. The ansatz induces a small breaking of spherical symmetry by adding to $g_{\rho\rho}$ the term

$$(\partial_\rho h)^2 - \frac{R^2}{4} sin^2(\Theta)k^2 \ .$$

One might hope that in the plane $\theta = \pi/2$, where $r = \rho$ holds true, the ansatz could behave like Schwartschild metric if the conditions discussed above are posed (including the condition $k = \omega$). The breaking of the spherical symmetry in the planetary system would be coded already to the gravitational field of Sun.

Also the study of the embeddings of Reissner-Nordström metric as vacuum extremals and the investigation of spherically symmetric (inertial) vacuum extremals for which gravitational four-momentum is conserved [K99] leads to the conclusion that the loss of spherical symmetry due to rotation is inevitable characteristic of realistic solutions.

2.6.8 Maxwell Hydrodynamics As A Toy Model For TGD

The field equations of TGD are extremely non-linear and all known solutions have been discovered by symmetry arguments. Chern-Simons term plays essential role also in the construction of solutions of field equations and at partonic level defines braiding for light-like partonic 3-surfaces expected to play key role in the construction of S-matrix. The inspiration for this section came from Terence Tao's blog posting *2006 ICM: Etienne Ghys, "Knots and dynamics"* [A45] giving an elegant summary about amazing mathematical results related to knots, links, braids and hydrodynamical flows in dimension $D = 3$. Posting tells about really amazing mathematical results related to knots.

Chern-Simons term as helicity invariant

Tao mentions helicity as an invariant of fluid flow. Chern-Simons action defined by the induced Kähler gauge potential for light-like 3-surfaces has interpretation as helicity when Kähler gauge potential is identified as fluid velocity. This flow can be continued to the interior of space-time sheet. Also the dual of the induced Kähler form defines a flow at the light-like partonic surfaces but not in the interior of space-time sheet. The lines of this flow can be interpreted as magnetic field lines. This flow is incompressible and represents a conserved charge (Kähler magnetic flux).

The question is which of these flows should define number theoretical braids. Perhaps both of them can appear in the definition of S-matrix and correspond to different kinds of partonic matter (electric/magnetic charges, quarks/leptons?, ...). Second kind of matter could not flow in the interior of space-time sheet. Or could interpretation in terms of electric magnetic duality make sense?

Helicity is not gauge invariant and this is as it must be in TGD framework since CP_2 symplectic transformations induce $U(1)$ gauge transformation, which deforms space-time surface an modifies induced metric as well as classical electroweak fields defined by induced spinor connection. Gauge degeneracy is transformed to spin glass degeneracy.

Maxwell hydrodynamics

In TGD Maxwell's equations are replaced with field equations which express conservation laws and are thus hydrodynamical in character. With this background the idea that the analogy between gauge theory and hydrodynamics might be applied also in the reverse direction is natural. Hence one might ask what kind of relativistic hydrodynamics results if assumes that the action principle is Maxwell action for the four-velocity u^α with the constraint term saying that light velocity is maximal signal velocity.

1. For massive particles the length of four-velocity equals to 1: $u^\alpha u_\alpha = 1$. In massless case one has $u^\alpha u_\alpha = 0$. Geometrically this means that one has sigma model with target space which is 3-D Lobatschevski space or at light-cone boundary. This condition means the addition of constraint term

$$\lambda(u^\alpha u_\alpha - \epsilon) \tag{2.6.41}$$

   to the Maxwell action. $\epsilon = 1/0$ holds for massive/massless flow. In the following the notation of electrodynamics is used to make easier the comparison with electrodynamics.

2. The constraint term destroys gauge invariance by allowing to express A^0 in terms of A^i but in general the constraint is not equivalent to a choice of gauge in electrodynamics since the solutions to the field equations with constraint term are not solutions of field equations without it. One obtains field equations for an effectively massive em field with Lagrange multiplier λ having interpretation as photon mass depending on space-time point:

$$\begin{aligned} j^\alpha &= \partial_\beta F^{\alpha\beta} = \lambda A^\alpha \ , \\ A^\alpha &\equiv u^\alpha \ , \ F^{\alpha\beta} = \partial^\beta A^\alpha - \partial^\alpha A^\beta \ . \end{aligned} \tag{2.6.42}$$

3. In electrodynamic context the natural interpretation would be in terms of spontaneous massivation of photon and seems to occur for both values of ϵ. The analog of em current given by λA^α is in general non-vanishing and conserved. This conservation law is quite strong additional constraint on the hydrodynamics. What is interesting is that breaking of gauge invariance does not lead to a loss of charge conservation.

4. One can solve λ by contracting the equations with A_α to obtain

$$\lambda = j^\alpha A_\alpha$$

   for $\epsilon = 1$. For $\epsilon = 0$ one obtains

$$j^\alpha A_\alpha = 0$$

stating that the field does not dissipate energy: λ can be however non-vanishing unless field equations imply $j^\alpha = 0$. One can say that for $\epsilon = 0$ spontaneous massivation can occur. For $\epsilon = 1$ massivation is present from the beginning and dissipation rate determines photon mass: a natural interpretation for $\epsilon = 1$ would be in terms of thermal massivation of photon. Non-tachyonicity fixes the sign of the dissipation term so that the thermodynamical arrow of time is fixed by causality.

5. For $\epsilon = 0$ massless plane wave solutions are possible and one has

$$\partial_\alpha \partial_\beta A^\beta = \lambda A_\alpha \ .$$

$\lambda = 0$ is obtained in Lorentz gauge which is consistent with the condition $\epsilon = 0$. Also superpositions of plane waves with same polarization and direction of propagation are solutions of field equations: these solutions represent dispersionless precisely targeted pulses. For superpositions of plane waves λ with 4-momenta, which are not all parallel λ is non-vanishing so that non-linear self interactions due to the constraint can be said to induce massivation. In asymptotic states for which gauge symmetry is not broken one expects a decomposition of solutions to regions of space-time carrying this kind of pulses, which brings in mind final states of particle reactions containing free photons with fixed polarizations.

6. Gradient flows satisfying the conditions

$$A_\alpha = \partial_\alpha \Phi \ , \ A^\alpha A_\alpha = \epsilon \tag{2.6.43}$$

give rise to identically vanishing hydrodynamical gauge fields and $\lambda = 0$ holds true. These solutions are vacua since energy momentum tensor vanishes identically. There is huge number of this kind of solutions and spin glass degeneracy suggests itself. Small deformations of these vacuum flows are expected to give rise to non-vacuum flows.

7. The counterparts of charged solutions are of special interest. For $\epsilon = 0$ the solution $(u^0, u^r) = (Q/r)(1,1)$ is a solution of field equations outside origin and corresponds to electric field of a point charge Q. In fact, for $\epsilon = 0$ any ansatz $(u^0, u^r) = f(r)(1,1)$ satisfies field equations for a suitable choice of $\lambda(r)$ since the ratio of equations associate with j^0 and j^r gives an equation which is trivially satisfied. For $\epsilon = 1$ the ansatz $(u^0, u^r) = (cosh(u), sinh(u))$ expressing solution in terms of hyperbolic angle linearizes the field equation obtained by dividing the equations for j^0 and j^r to eliminate λ. The resulting equation is

$$\partial_r^2 u + \frac{2\partial_r u}{r} = 0$$

for ordinary Coulomb potential and one obtains $(u^0, u^r) = (cosh(u_0 + k/r), sinh(u_0 + k/r))$. The charge of the solution at the limit $r \to \infty$ approaches to the value $Q = sinh(u_0)k$ and diverges at the limit $r \to 0$. The charge increases exponentially as a function of $1/r$ near origin rather than logarithmically as in QED and the interpretation in terms of thermal screening suggests itself. Hyperbolic ansatz might simplify considerably the field equations also in the general case.

Similarities with TGD

There are strong similarities with TGD which suggests that the proposed model might provide a toy model for the dynamics defined by Kähler action.

1. Also in TGD field equations are essentially hydrodynamical equations stating the conservation of various isometry charges. Gauge invariance is broken for the induced Kähler field although Kähler charge is conserved. There is huge vacuum degeneracy corresponding to vanishing of induced Kähler field and the interpretation is in terms of spin glass degeneracy.

2. Also in TGD dissipation rate vanishes for the known solutions of field equations and a possible interpretation is as space-time correlates for asymptotic non-dissipating self organization patterns.

3. In TGD framework massless extremals represent the analogs for superpositions of plane waves with fixed polarization and propagation direction and representing targeted and dispersionless propagation of signal. Gauge currents are light-like and non-vanishing for these solutions. The decomposition of space-time surface to space-time sheets representing particles is much more general counterpart for the asymptotic solutions of Maxwell hydrodynamics with vanishing λ.

4. In TGD framework one can consider the possibility that the four-velocity assignable to a macroscopic quantum phase is proportional to the induced Kähler gauge potential. In this kind of situation one could speak of a quantal variant of Maxwell hydrodynamics, at least for light-like partonic 3-surfaces. For instance, the condition

$$D^\alpha D_\alpha \Psi = 0 \ , \ D_\alpha \Psi = (\partial_\alpha - iq_K A_\alpha)\Psi$$

for the order parameter of the quantum phase corresponds at classical level to the condition $p^\alpha = q_K Q^\alpha + l^\alpha$, where q_K is Kähler charge of fermion and l^α is a light-like vector field naturally assignable to the partonic boundary component. This gives $u^\alpha = (q_K Q^\alpha + l^\alpha)/m$, $m^2 = p^\alpha p_\alpha$, which is somewhat more general condition. The expressibility of u^α in terms of the vector fields provided by the induced geometry is very natural.

The value ϵ depends on space-time region and it would seem that also $\epsilon = -1$ is possible meaning tachyonicity and breaking of causality. Kähler gauge potential could however have a time-like pure gauge component in M^4 possibly saving the situation. The construction of quantum TGD at parton level indeed forces to assume that Kähler gauge potential has Lorentz invariant M^4 component $A_a = constant$ in the direction of the light-cone proper time coordinate axis a. Note that the decomposition of WCW to sectors consisting of space-time sheets inside future or past light-cone of M^4 is an essential element of the construction of WCW geometry and does not imply breaking of Poincare invariance. Without this component $u_\alpha u^\alpha$ could certainly be negative. The contribution of M^4 component could prevent this for preferred extremals.

If TGD is taken seriously, these similarities force to ask whether Maxwell hydrodynamics might be interpreted as a nonlinear variant of electrodynamics. Probably not: in TGD em field is proportional to the induced Kähler form only in special cases and is in general non-vanishing also for vacuum extremals.

Chapter 3

Identification of the Preferred extremals of Kähler Action

3.1 Introduction

Preferred extremal of Kähler action have remained one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what the attribute "preferred" really means. Symmetries give a clue to the problem. The conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [K94]. Preferred extremal property should rely on this symmetry.

In Zero Energy Ontology (ZEO) preferred extremals are space-time surfaces connecting two space-like 3-surfaces at the ends of space-time surfaces at boundaries of causal diamond (CD). A natural looking condition is that the symplectic Noether charges associated with a sub-algebra of symplectic algebra with conformal weights n-multiples of the weights of the entire algebra vanish for preferred extremals. These conditions would be classical counterparts the condition that super-symplectic sub-algebra annihilates the physical states. This would give a hierarchy of super-symplectic symmetry breakings and quantum criticalities having interpretation in terms of hierarchy of Planck constants $h_{eff} = n \times h$ identified as a hierarchy of dark matter. n could be interpreted as the number of space-time conformal gauge equivalence classes for space-time sheets connecting the 3-surfaces at the ends of space-time surface.

There are also many other proposals for what preferred extremal property could mean or imply. The weak form of electric-magnetic duality combined with the assumption that the contraction of the Kähler current with Kähler gauge potential vanishes for preferred extremals implies that Kähler action in Minkowskian space-time regions reduces to Chern-Simons terms at the light-like orbits of wormhole throats at which the signature of the induced metric changes its signature from Minkowskian to Euclidian. In regions with 4-D CP_2 projection (wormhole contacts) also a 3-D contribution not assignable to the boundary of the region might be possible. These conditions pose strong physically feasible conditions on extremals and might be true for preferred extremals too.

Number theoretic vision leads to a proposal that either the tangent space or normal space of given point of space-time surface is associative and thus quaternionic. Also the formulation in terms of quaternion holomorphy and quaternion-Kähler property is an attractive possibility. So called $M^8 - H$ duality is a variant of this vision and would mean that one can map associative/co-associative space-time surfaces from M^8 to H and also iterate this mapping from H to H to generate entire category of preferred extremals. The signature of M^4 is a general technical problem. For instance, the holomorphy in 2 complex variables could correspond to what I have called Hamilton-Jacobi property. Associativity/co-associativity of the tangent space makes sense also in Minkowskian signature.

In this chapter various views about preferred extremal property are discussed.

3.1.1 Preferred Extremals As Critical Extremals

The study of the Kähler-Dirac equation leads to a detailed view about criticality. Quantum criticality [D13] fixes the values of Kähler coupling strength as the analog of critical temperature. Quantum criticality implies that second variation of Kähler action vanishes for critical deformations and the existence of conserved current except in the case of Cartan algebra of isometries. Quantum criticality allows to fix the values of couplings appearing in the measurement interaction by using the condition $K \rightarrow K + f + \overline{f}$. p-Adic coupling constant evolution can be understood also and corresponds to scale hierarchy for the sizes of causal diamonds (CDs).

The discovery that the hierarchy of Planck constants realized in terms of singular covering spaces of $CD \times CP_2$ can be understood in terms of the extremely non-linear dynamics of Kähler action implying 1-to-many correspondence between canonical momentum densities and time derivatives of the embedding space coordinates led to a further very concrete understanding of the criticality at space-time level and its relationship to zero energy ontology [K46].

Criticality is accompanied by conformal invariance and this leads to the proposal that critical deformations correspond to Kac-Moody type conformal algebra respecting the light-likeness of the partonic orbits and acting trivially at partonic 2-surfaces. Sub-algebras of conformal algebras with conformal weights divisible by integer n would act as gauge symmetries and these algebras would form an inclusion hierarchy defining hierarchy of symmetry breakings. n would also characterize the value of Planck constant $h_{eff} = n \times h$ assignable to various phases of dark matter.

3.1.2 Construction Of Preferred Extremals

There has been considerable progress in the understanding of both preferred extremals and Kähler-Dirac equation.

1. For preferred extremals the generalization of conformal invariance to 4-D situation is very attractive idea and leads to concrete conditions formally similar to those encountered in string model [K11]. In particular, Einstein's equations with cosmological constant would solve consistency conditions and field equations would reduce to a purely algebraic statements analogous to those appearing in equations for minimal surfaces if one assumes that space-time surface has Hermitian structure or its Minkowskian variant Hamilton-Jacobi structure (Appendix).
2. The older approach based on basic heuristics for massless equations, on effective 3-dimensionality, weak form of electric magnetic duality, and Beltrami flows is also promising. An alternative approach is inspired by number theoretical considerations and identifies space-time surfaces as associative or co-associative sub-manifolds of octonionic embedding space [K88].

The basic step of progress was the realization that the known extremals of Kähler action - certainly limiting cases of more general extremals - can be deformed to more general extremals having interpretation as preferred extremals.

1. The generalization boils down to the condition that field equations reduce to the condition that the traces $Tr(TH^k)$ for the product of energy momentum tensor and second fundamental form vanish. In string models energy momentum tensor corresponds to metric and one obtains minimal surface equations. The equations reduce to purely algebraic conditions stating that T and H^k have no common components. Complex structure of string world sheet makes this possible.

   Stringy conditions for metric stating $g_{zz} = g_{\overline{z}\overline{z}} = 0$ generalize. The condition that field equations reduce to $Tr(TH^k) = 0$ requires that the terms involving Kähler gauge current in field equations vanish. This is achieved if Einstein's equations hold true (one can consider also more general way to satisfy the conditions). The conditions guaranteeing the vanishing of the trace in turn boil down to the existence of Hermitian structure in the case of Euclidian signature and to the existence of its analog - Hamilton-Jacobi structure - for Minkowskian signature (Appendix). These conditions state that certain components of the induced metric vanish in complex coordinates or Hamilton-Jacobi coordinates.
2. In string model the replacement of the embedding space coordinate variables with quantized ones allows to interpret the conditions on metric as Virasoro conditions. In the recent case a

generalization of classical Virasoro conditions to four-dimensional ones would be in question. An interesting question is whether quantization of these conditions could make sense also in TGD framework at least as a useful trick to deduce information about quantum states in WCW degrees of freedom.

3. The interpretation of the extended algebra as Yangian [A20] [B20] suggested previously [K94] to act as a generalization of conformal algebra in TGD Universe is attractive. There is also the conjecture that preferred extremals could be interpreted as quaternionic of co-quaternionic 4-surface of the octonionic embedding space with octonionic representation of the gamma matrices defining the notion of tangent space quanternionicity.

3.2 Weak Form Electric-Magnetic Duality And Its Implications

The notion of electric-magnetic duality [B5] was proposed first by Olive and Montonen and is central in $\mathcal{N}=4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K25] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.

2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2,-1,-1)$ and could be proportional to color hyper charge.

3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.

4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.

5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges

are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current. Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

3.2.1 Could A Weak Form Of Electric-Magnetic Duality Hold True?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the embedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.
2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.
3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.
4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} . \quad (3.2.1)$$

A more general form of this duality is suggested by the considerations of [K46] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B2] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} \ . \quad (3.2.2)$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1+K)J_{12} \ , \quad (3.2.3)$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L1] , [L1] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \quad (3.2.4)$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \quad (3.2.5)$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ \ p = sin^2(\theta_W) \ . \end{aligned} \quad (3.2.6)$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi}\times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \quad (3.2.7)$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the Kähler-Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K68] supports this interpretation.

3. The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \rightarrow \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.

4. The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z \ . \tag{3.2.8}$$

This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} \ . \tag{3.2.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \rightarrow 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{12} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \tag{3.2.10}$$

Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L1]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K75]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.
2. GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [K99]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.
3. Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.
4. GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

3.2.2 Magnetic Confinement, The Short Range Of Weak Forces, And Color Confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L \overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L \nu_R$. $\nu_L \overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately

to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.

2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hypercharge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$

has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D8] .

Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [**?**] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_{\pm}$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies ZEO. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

2. The addition of the particles $X^{\pm}$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the

vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

3. How should one describe the bound state formed by the fermion and $X^{\pm}$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K57] . If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.
4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K58] .

3.2.3 Could Quantum TGD Reduce To Almost Topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the Kähler-Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j^{\alpha}_K A_{\alpha}$ plus and integral of the boundary term $J^{n\beta}A_{\beta}\sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta}A_{\beta}\sqrt{g_4}$ over the ends of the 3-surface.
2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K\epsilon^{n\beta\gamma\delta}J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K\epsilon^{0\beta\gamma\delta}J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $h \rightarrow n\times h$ would effectively describe this. Boundary conditions would however give $1/n$ factor so that $\hbar$ would disappear from the Kähler function! It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j^{α}_K either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [K11]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.

2. The original naïve conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_\alpha (J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma} J_{\beta\ gamma})\sqrt{g_4} d^3x \ . \tag{3.2.11}$$

   The (1,1) part of second variation contributing to M^4 metric comes from this term.

3. This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta} K(J_{\gamma\delta} + \epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.

4. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^\alpha \partial_\alpha \phi = -j^\alpha A_\alpha \ . \tag{3.2.12}$$

   This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^\alpha/dt = j_K^\alpha$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j_\beta^K \partial_\gamma j^K_{delta} \ = \ 0 \ . \tag{3.2.13}$$

   j_K is a four-dimensional counterpart of Beltrami field [B11] and could be called generalized Beltrami field.

   The integrability conditions follow also from the construction of the extremals of Kähler action [K11] . The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^\alpha \partial_\alpha \phi = \partial_\alpha j^\alpha \phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j_K^\alpha \phi$ and $j_I^\alpha \phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

5. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations

$A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^\alpha \partial_\alpha \phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_\alpha(j^\alpha \phi) = 0 . \tag{3.2.14}$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_\phi^e = \int j^0 \phi \sqrt{g_4} d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q_\phi^m = \sum \int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

6. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the Kähler-Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential couples to the Kähler-Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.

7. A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q_ϕ^m and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

8. Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to Kähler-Dirac action as boundary term.

   Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

   One can assign to partonic orbits Chern-Simons Dirac action and now the condition would be that the action of C-S-D operator equals to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator. Twistor Grassmann approach suggests that also the virtual fermions reduce effectively to massless on-shell states but have non-physical helicity.

To sum up, one could understand the basic properties of WCW metric in this framework. Effective 2-dimensionality would result from the existence of an infinite number of conserved charges in two different time directions (genuine conservation laws plus gauge fixing). The infinite-dimensional symmetric space for given values of zero modes corresponds to the Cartesian product of the WCWs associated with the partonic 2-surfaces at both ends of CD and the generalized Chern-Simons term decomposes into a sum of terms from the ends giving single particle Kähler functions and to the terms from light-like wormhole throats giving interaction term between positive and negative energy parts of the state. Hence Kähler function could be calculated without any knowledge about the interior of the space-time sheets and TGD would reduce to almost topological QFT as speculated earlier. Needless to say this would have immense boost to the program of constructing WCW Kähler geometry.

3.3 An attempt to understand preferred extremals of Kähler action

Preferred extremal of Kähler action is one of the basic poorly defined notions of TGD. There are pressing motivations for understanding what "preferred" really means. For instance, the conformal invariance of string models naturally generalizes to 4-D invariance defined by quantum Yangian of quantum affine algebra (Kac-Moody type algebra) characterized by two complex coordinates and therefore explaining naturally the effective 2-dimensionality [K27]. The problem is however how to assign a complex coordinate with the string world sheet having Minkowskian signature of metric. One can hope that the understanding of preferred extremals could allow to identify two preferred complex coordinates whose existence is also suggested by number theoretical vision giving preferred role for the rational points of partonic 2-surfaces in preferred coordinates. The best one could hope is a general solution of field equations in accordance with the hints that TGD is integrable quantum theory.

3.3.1 What "preferred" could mean?

The first question is what preferred extremal could mean.

1. In positive energy ontology preferred extremal would be a space-time surface assignable to given 3-surface and unique in the ideal situation: since one cannot pose conditions to the normal derivatives of embedding space coordinates at 3-surface, there is infinity of extremals. Some additional conditions are required and space-time surface would be analogous to Bohr orbit: hence the attribute "preferred". The problem would be to understand what "preferred" could mean. The non-determinism of Kähler action however destroyed this dream in its original form and led to zero energy ontology (ZEO).

2. In ZEO one considers extremals as space-time surfaces connecting two space-like 3-surfaces at the boundaries. One might hope that these 4-surfaces are unique. The non-determinism of Kähler action suggests that this is not the case. At least there is conformal invariance respecting the light-likeness of the 3-D parton orbits at which the signature of the induced metric changes: the conformal transformations would leave the space-like 3-D ends or at least partonic 2-surfaces invariant. This non-determinism would correspond to quantum criticality.

3. Effective 2-dimensionality follows from strong form of general coordinate invariance (GCI) stating that light-like partonic orbits and space-like 3-surfaces at the ends of space-time surface are equivalent physically: partonic 2-surfaces and their 4-D tangent space data would determine everything. One can however worry about how effective 2-dimensionality relates to the fact that the modes of the induced spinor field are localized at string world sheets and partonic 2-surface. Are the tangent space data equivalent with the data characterizing string world sheets as surfaces carrying vanishing electroweak fields?

   There is however a problem: the hierarchy of Planck constants (dark matter) requires that the conformal equivalence classes of light-like surfaces must be counted as physical degrees of freedom so that either space-like or light-like surfaces do not seem to be quite enough.

   Should one then include also the light-like partonic orbits to the what one calls 3-surface? The resulting connected 3-surfaces would define analogs of Wilson loops. Could the conformal equivalence class of the preferred extremal be unique without any additional conditions? If so, one could get rid of the attribute "preferred". The fractal character of the many-sheeted space-time however suggests that one can have this kind of uniqueness only in given length scale resolution and that "radiative corrections" due to the non-determinism are always present.

These considerations show that the notion of preferred extremal is still far from being precisely defined and it is not even clear whether the attribute "preferred" is needed. If not then the question is what are the extremals of Kähler action.

3.3.2 What is known about extremals?

A lot is is known about properties of extremals and just by trying to integrate all this understanding, one might gain new visions. The problem is that all these arguments are heuristic and rely heavily on physical intuition. The following considerations relate to the space-time regions having Minkowskian signature of the induced metric. The attempt to generalize the construction also to Euclidian regions could be very rewarding. Only a humble attempt to combine various ideas to a more coherent picture is in question.

The core observations and visions are following.

1. Hamilton-Jacobi coordinates for M^4 (discussed in this chapter) define natural preferred coordinates for Minkowskian space-time sheet and might allow to identify string world sheets for X^4 as those for M^4. Hamilton-Jacobi coordinates consist of light-like coordinate m and its dual defining local 2-plane $M^2 \subset M^4$ and complex transversal complex coordinates $(w, \overline{w})$ for a plane E_x^2 orthogonal to M_x^2 at each point of M^4. Clearly, hyper-complex analyticity and complex analyticity are in question.
2. Space-time sheets allow a slicing by string world sheets (partonic 2-surfaces) labelled by partonic 2-surfaces (string world sheets).
3. The quaternionic planes of octonion space containing preferred hyper-complex plane are labelled by CP_2, which might be called CP_2^{mod} [K88]. The identification $CP_2 = CP_2^{mod}$ motivates the notion of $M^8 - -M^4 \times CP_2$ duality [K24]. It also inspires a concrete solution ansatz assuming the equivalence of two different identifications of the quaternionic tangent space of the space-time sheet and implying that string world sheets can be regarded as strings in the 6-D coset space $G_2/SU(3)$. The group G_2 of octonion automorphisms has already earlier appeared in TGD framework.
4. The duality between partonic 2-surfaces and string world sheets in turn suggests that the $CP_2 = CP_2^{mod}$ conditions reduce to string model for partonic 2-surfaces in $CP_2 = SU(3)/U(2)$. String model in both cases could mean just hypercomplex/complex analyticity for the coordinates of the coset space as functions of hyper-complex/complex coordinate of string world sheet/partonic 2-surface.

The considerations of this section lead to a revival of an old very ambitious and very romantic number theoretic idea.

1. To begin with express octonions in the form $o = q_1 + Iq_2$, where q_i is quaternion and I is an octonionic imaginary unit in the complement of fixed a quaternionic sub-space of octonions. Map preferred coordinates of $H = M^4 \times CP_2$ to octonionic coordinate, form an arbitrary octonion analytic function having expansion with real Taylor or Laurent coefficients to avoid problems due to non-commutativity and non-associativity. Map the outcome to a point of H to get a map $H \rightarrow H$. This procedure is nothing but a generalization of Wick rotation to get an 8-D generalization of analytic map.
2. Identify the preferred extremals of Kähler action as surfaces obtained by requiring the vanishing of the imaginary part of an octonion analytic function. Partonic 2-surfaces and string world sheets would correspond to commutative sub-manifolds of the space-time surface and of embedding space and would emerge naturally. The ends of braid strands at partonic 2-surface would naturally correspond to the poles of the octonion analytic functions. This would mean a huge generalization of conformal invariance of string models to octonionic conformal invariance and an exact solution of the field equations of TGD and presumably of quantum TGD itself.

3.3.3 Basic ideas about preferred extremals

The slicing of the space-time sheet by partonic 2-surfaces and string world sheets

The basic vision is that space-time sheets are sliced by partonic 2-surfaces and string world sheets. The challenge is to formulate this more precisely at the level of the preferred extremals of Kähler action.

1. Almost topological QFT property means that the Kähler action reduces to Chern-Simons terms assignable to 3-surfaces. This is guaranteed by the vanishing of the Coulomb term in the action density implied automatically if conserved Kähler current is proportional to the instanton current with proportionality coefficient some scalar function.

2. The field equations reduce to the conservation of isometry currents. An attractive ansatz is that the flow lines of these currents define global coordinates. This means that these currents are Beltrami flows [B11] so that corresponding 1-forms J satisfy the condition $J \wedge dJ = 0$. These conditions are satisfied if

$$J = \Phi \nabla \Psi$$

hold true for conserved currents. From this one obtains that Ψ defines global coordinate varying along flow lines of J.

3. A possible interpretation is in terms of local polarization and momentum directions defined by the scalar functions involved and natural additional conditions are that the gradients of Ψ and Φ are orthogonal:

$$\nabla \Phi \cdot \nabla \Psi = 0 \ ,$$

and that the Ψ satisfies massless d'Alembert equation

$$\nabla^2 \Psi = 0$$

as a consequence of current conservation. If Ψ defines a light-like vector field - in other words

$$\nabla \Psi \cdot \nabla \Psi = 0 \ ,$$

the light-like dual of Φ -call it Φ_c- defines a light-like like coordinate and Φ and Φ_c defines a light-like plane at each point of space-time sheet.

If also Φ satisfies d'Alembert equation

$$\nabla^2 \Phi = 0 \ ,$$

also the current

$$K = \Psi \nabla \Phi$$

is conserved and its flow lines define a global coordinate in the polarization plane orthogonal to time-lik plane defined by local light-like momentum direction.

If Φ allows a continuation to an analytic function of the transversal complex coordinate, one obtains a coordinatization of space-time surface by Ψ and its dual (defining hyper-complex coordinate) and $w, \overline{w}$. Complex analyticity and its hyper-complex variant would allow to provide space-time surface with four coordinates very much analogous with Hamilton-Jacobi coordinates of M^4.

This would mean a decomposition of the tangent space of space-time surface to orthogonal planes defined by light-like momentum and plane orthogonal to it. If the flow lines of J defined Beltrami flow it seems that the distribution of momentum planes is integrable.

4. General arguments suggest that the space-time sheets allow a slicing by string world sheets parametrized by partonic 2-surfaces or vice versa. This would mean a intimate connection with the mathematics of string models. The two complex coordinates assignable to the Yangian of affine algebra would naturally relate to string world sheets and partonic 2-surfaces and the highly non-trivial challenge is to identify them appropriately.

Hamilton-Jacobi coordinates for M^4

The earlier attempts to construct preferred extremals [K11] led to the realization that so called Hamilton-Jacobi coordinates (m, w) for M^4 define its slicing by string world sheets parametrized by partonic 2-surfaces. m would be pair of light-like conjugate coordinates associated with an integrable distribution of planes M^2 and w would define a complex coordinate for the integrable distribution of 2-planes E^2 orthogonal to M^2. There is a great temptation to assume that these coordinates define preferred coordinates for M^4.

1. The slicing is very much analogous to that for space-time sheets and the natural question is how these slicings relate. What is of special interest is that the momentum plane M^2 can be defined by massless momentum. The scaling of this vector does not matter so that these planes are labelled by points z of sphere S^2 telling the direction of the line $M^2 \cap E^3$, when one assigns rest frame and therefore S^2 with the preferred time coordinate defined by the line connecting the tips of CD. This direction vector can be mapped to a twistor consisting of a spinor and its conjugate. The complex scalings of the twistor $(u, \overline{u}) \to \lambda u, \overline{u}/\lambda)$ define the same plane. Projective twistor like entities defining CP_1 having only one complex component instead of three are in question. This complex number defines with certain prerequisites a local coordinate for space-time sheet and together with the complex coordinate of E^2 could serve as a pair of complex coordinates (z, w) for space-time sheet. This brings strongly in mind the two complex coordinates appearing in the expansion of the generators of quantum Yangian of quantum affine algebra [K27].
2. The coordinate Ψ appearing in Beltrami flow defines the light-like vector field defining M^2 distribution. Its hyper-complex conjugate would define Ψ_c and conjugate light-like direction. An attractive possibility is that Φ allows analytic continuation to a holomorphic function of w. In this manner one would have four coordinates for M^4 also for space-time sheet.
3. The general vision is that at each point of space-time surface one can decompose the tangent space to $M^2(x) \subset M^4 = M^2_x \times E^2_x$ representing momentum plane and polarization plane $E^2 \subset E^2_x \times T(CP_2)$. The moduli space of planes $E^2 \subset E^6$ is 8-dimensional and parametrized by $SO(6)/SO(2) \times SO(4)$ for a given E^2_x. How can one achieve this selection and what conditions it must satisfy? Certainly the choice must be integrable but this is not the only condition.

Space-time surfaces as associative/co-associative surfaces

The idea that number theory determines classical dynamics in terms of associativity condition means that space-time surfaces are in some sense quaternionic surfaces of an octonionic space-time. It took several trials before the recent form of this hypothesis was achieved.

1. Octonionic structure is defined in terms of the octonionic representaton of gamma matrices of the embedding space existing only in dimension $D = 8$ since octonion units are in one-one correspondence with tangent vectors of the tangent space. Octonionic real unit corresponds to a preferred time axes (and rest frame) identified naturally as that connecting the tips of CD. What modified gamma matrices mean depends on variational principle for space-time surface. For volume action one would obtain induced gamma matrices. For Kähler action one obtains something different. In particular, the modified gamma matrices do not define vector basis identical with tangent vector basis of space-time surface.
2. Quaternionicity means that the modified gamma matrices defined as contractions of gamma matrices of H with canonical momentum densities for Kähler action span quaternionic sub-space of the octonionic tangent space [K104, K77]. A further condition is that each quaternionic space defined in this manner contains a preferred hyper-complex subspace of octonions.
3. The sub-space defined by the modified gamma matrices does not co-incide with the tangent space of space-time surface in general so that the interpretation of this condition is far from obvious. The canonical momentum densities need not define four independent vectors at given point. For instance, for massless extremals these densities are proportional to light-like vector so that the situation is degenerate and the space in question reduces to 2-D hyper-complex sub-space since light-like vector defines plane M^2.

The obvious questions are following.

1. Does the analog of tangent space defined by the octonionic modified gammas contain the local tangent space $M^2 \subset M^4$ for preferred extremals? For massless extremals [K11] this condition would be true. The orthogonal decomposition $T(X^4) = M^2 \oplus_\perp E^2$ can be defined at each point if this is true. For massless extremals also the functions Ψ and Φ can be identified.

2. One should answer also the following delicate question. Can M^2 really depend on point x of space-time? CP_2 as a moduli space of quaternionic planes emerges naturally if M^2 is *same* everywhere. It however seems that one should allow an integrable distribution of M^2_x such that M^2_x is same for all points of a given partonic 2-surface.

   How could one speak about fixed CP_2 (the embedding space) at the entire space-time sheet even when M^2_x varies?

   (a) Note first that G_2 (see `http://tinyurl.com/y9rrs7un`) defines the Lie group of octonionic automorphisms and G_2 action is needed to change the preferred hyper-octonionic sub-space. Various $SU(3)$ subgroups of G_2 are related by G_2 automorphism. Clearly, one must assign to each point of a string world sheet in the slicing parameterizing the partonic 2-surfaces an element of G_2. One would have Minkowskian string model with G_2 as a target space. As a matter fact, this string model is defined in the target space $G_2/SU(3)$ having dimension $D = 6$ since $SU(3)$ automorphisms leave given $SU(3)$ invariant.

   (b) This would allow to identify at each point of the string world sheet standard quaternionic basis - say in terms of complexified basis vectors consisting of two hyper-complex units and octonionic unit q_1 with "color isospin" $I_3 = 1/2$ and "color hypercharge" $Y = -1/3$ and its conjugate $\overline{q}_1$ with opposite color isospin and hypercharge.

   (c) The CP_2 point assigned with the quaternionic basis would correspond to the $SU(3)$ rotation needed to rotate the standard basis to this basis and would actually correspond to the first row of $SU(3)$ rotation matrix. Hyper-complex analyticity is the basic property of the solutions of the field equations representing Minkowskian string world sheets. Also now the same assumption is highly natural. In the case of string models in Minkowski space, the reduction of the induced metric to standard form implies Virasoro conditions and similar conditions are expected also now. There is no need to introduce action principle -just the hyper-complex analycitity is enough-since Kähler action already defines it.

3. The WZW model (see `http://tinyurl.com/ydxcvfhv`) inspired approach to the situation would be following. The parameterization corresponds to a map $g : X^2 \to G_2$ for which g defines a flat G_2 connection at string world sheet. WZW type action would give rise to this kind of situation. The transition $G_2 \to G_2/SU(3)$ would require that one gauges $SU(3)$ degrees of freedom by bringing in $SU(3)$ connection. Similar procedure for $CP_2 = SU(3)/U(2)$ would bring in $SU(3)$ valued chiral field and $U(2)$ gauge field. Instead of introducing these connections one can simply introduce $G_2/SU(3)$ and $SU(3)/U(2)$ valued chiral fields. What this observation suggests that this ansatz indeed predicts gluons and electroweak gauge bosons assignable to string like objects so that the mathematical picture would be consistent with physical intuition.

The two interpretations of CP_2

An old observation very relevant for what I have called $M^8 - H$ duality [K24] is that the moduli space of quaternionic sub-spaces of octonionic space (identifiable as M^8) containing preferred hyper-complex plane is CP_2. Or equivalently, the space of two planes whose addition extends hyper-complex plane to some quaternionic subspace can be parametrized by CP_2. This CP_2 can be called it CP_2^{mod} to avoid confusion. In the recent case this would mean that the space $E^2(x) \subset E^2_x \times T(CP_2)$ is represented by a point of CP_2^{mod}. On the other hand, the embedding of space-time surface to H defines a point of "real" CP_2. This gives two different CP_2s.

1. The highly suggestive idea is that the identification $CP_2^{mod} = CP_2$ (apart from isometry) is crucial for the construction of preferred extremals. Indeed, the projection of the space-time point to CP_2 would fix the local polarization plane completely. This condition for $E^2(x)$ would be purely local and depend on the values of CP_2 coordinates only. Second condition

for $E^2(x)$ would involve the gradients of embedding space coordinates including those of CP_2 coordinates.

2. The conditions that the planes M^2_x form an integrable distribution at space-like level and that M^2_x is determined by the modified gamma matrices. The integrability of this distribution for M^4 could imply the integrability for X^2. X^4 would differ from M^4 only by a deformation in degrees of freedom transversal to the string world sheets defined by the distribution of M^2s.

   Does this mean that one can begin from vacuum extremal with constant values of CP_2 coordinates and makes them non-constant but allows to depend only on transversal degrees of freedom? This condition is too strong even for simplest massless extremals for which CP_2 coordinates depend on transversal coordinates defined by $\epsilon \cdot m$ and $\epsilon \cdot k$. One could however allow dependence of CP_2 coordinates on light-like M^4 coordinate since the modification of the induced metric is light-like so that light-like coordinate remains light-like coordinate in this modification of the metric.

   Therefore, if one generalizes directly what is known about massless extremals, the most general dependence of CP_2 points on the light-like coordinates assignable to the distribution of M^2_x would be dependence on either of the light-like coordinates of Hamilton-Jacobi coordinates but not both.

3.3.4 What could be the construction recipe for the preferred extremals assuming $CP_2 = CP_2^{mod}$ identification?

The crucial condition is that the planes $E^2(x)$ determined by the point of $CP_2 = CP_2^{mod}$ identification and by the tangent space of $E^2_x \times CP_2$ are same. The challenge is to transform this condition to an explicit form. $CP_2 = CP_2^{mod}$ identification should be general coordinate invariant. This requires that also the representation of E^2 as (e^2, e^3) plane is general coordinate invariant suggesting that the use of preferred CP_2 coordinates - presumably complex Eguchi-Hanson coordinates - could make life easy. Preferred coordinates are also suggested by number theoretical vision. A careful consideration of the situation would be required.

The modified gamma matrices define a quaternionic sub-space analogous to tangent space of X^4 but not in general identical with the tangent space: this would be the case only if the action were 4-volume. I will use the notation $T^m_x(X^4)$ about the modified tangent space and call the vectors of $T^m_x(X^4)$ modified tangent vectors. I hope that this would not cause confusion.

$CP_2 = CP_2^{mod}$ condition

Quaternionic property of the counterpart of $T^m_x(X^4)$ allows an explicit formulation using the tangent vectors of $T^m_x(X^4)$.

1. The unit vector pair (e_2, e_3) should correspond to a unique tangent vector of H defined by the coordinate differentials dh^k in some natural coordinates used. Complex Eguchi-Hanson coordinates [L1] are a natural candidate for CP_2 and require complexified octonionic imaginary units. If octonionic units correspond to the tangent vector basis of H uniquely, this is possible.

2. The pair (e_2, e_3) as also its complexification $(q_1 = e_2 + ie_3, \overline{q}_1 = e_2 - ie_3)$ is expressible as a linear combination of octonionic units $I_2, ...I_7$ should be mapped to a point of $CP_2^{mod} = CP_2$ in canonical manner. This mapping is what should be expressed explicitly. One should express given (e_2, e_3) in terms of $SU(3)$ rotation applied to a standard vector. After that one should define the corresponding CP_2 point by the bundle projection $SU(3) \rightarrow CP_2$.

3. The tangent vector pair

$$(\partial_w h^k, \partial_{\overline{w}} h^k)$$

   defines second representation of the tangent space of $E^2(x)$. This pair should be equivalent with the pair $(q_1, \overline{q}_1)$. Here one must be however very cautious with the choice of coordinates. If the choice of w is unique apart from constant the gradients should be unique. One can use also real coordinates (x, y) instead of $(w = x + iy, \overline{w} = x - iy)$ and the pair (e_2, e_3). One can project the tangent vector pair to the standard vielbein basis which must correspond to the octonionic basis

$$(\partial_x h^k, \partial_y h^k) \rightarrow (\partial_x h^k e_k^A e_A, \partial_y h^k e_k^A) e_A) \leftrightarrow (e_2, e_3) \ ,$$

where the e_A denote the octonion units in 1-1 correspondence with vielbein vectors. This expression can be compared to the expression of (e_2, e_3) derived from the knowledge of CP_2 projection.

Formulation of quaternionicity condition in terms of octonionic structure constants

One can consider also a formulation of the quaternionic tangent planes in terms of (e_2, e_3) expressed in terms of octonionic units deducible from the condition that unit vectors obey quaternionic algebra. The expressions for octonionic (see `http://tinyurl.com/5m5lqr`) *resp.* quaternionic (see `http://tinyurl.com/3rr79p9`) structure constants can be found at [A11] *resp.* [A13].

1. The ansatz is

$$\begin{aligned} \{E_k\} &= \{1, I_1, E_2, E_3\} \ , \\ E_2 &= E_{2k} e^k \equiv \sum_{k=2}^{7} E_{2k} e^k \ , \ E_3 = E_{3k} e^k \equiv \sum_{k=2}^{7} E_{3k} e^k \ , \\ |E_2| &= 1 \ , \ |E_3| = 1 \ . \end{aligned} \quad (3.3.1)$$

2. The multiplication table for octonionic units expressible in terms of octonionic triangle (see `http://tinyurl.com/5m5lqr`) [A11] gives

$$f^{1kl} E_{2k} = E_{3l} \ , \ f^{1kl} E_{3k} = -E_{2l} \ , \ f^{klr} E_{2k} E_{3l} = \delta_1^r \ . \quad (3.3.2)$$

   Here the indices are raised by unit metric so that there is no difference between lower and upper indices. Summation convention is assumed. Also the contribution of the real unit is present in the structure constants of third equation but this contribution must vanish.

3. The conditions are linear and quadratic in the coefficients E_{2k} and E_{3k} and are expected to allow an explicit solution. The first two conditions define homogenous equations which must allow solution. The coefficient matrix acting on (E_2, E_3) is of the form

$$\begin{pmatrix} f_1 & 1 \\ -1 & f_1 \end{pmatrix} \ ,$$

   where 1 denotes unit matrix. The vanishing of the determinant of this matrix should be due to the highly symmetric properties of the structure constants. In fact the equations can be written as eigen conditions

$$f_1 \circ (E_2 \pm i E_3) = \mp i (E_2 \pm i E_3) \ ,$$

   and one can say that the structure constants are eigenstates of the hermitian operator defined by I_1 analogous to color hyper charge. Both values of color hyper charged are obtained.

Explicit expression for the $CP_2 = CP_2^{mod}$ conditions

The symmetry under $SU(3)$ allows to construct the solutions of the above equations directly.

1. One can introduce complexified basis of octonion units transforming like $(1, 1, 3, \overline{3})$ under SU(3). Note the analogy of triplet with color triplet of quarks. One can write complexified basis as $(1, e_1, (q_1, q_2, q_3), (\overline{q}_1 \overline{q}_2, \overline{q}_3))$. The expressions for complexified basis elements are

$$(q_1, q_2, q_3) = \frac{1}{\sqrt{2}} (e_2 + ie_3, e_4 + ie_5, e_6 + ie_7) \ .$$

These options can be seen to be possible by studying octonionic triangle in which all lines containing 3 units defined associative triple: any pair of octonion units at this kind of line can be used to form pair of complexified unit and its conjugate. In the tangent space of $M^4 \times CP_2$ the basis vectors q_1, and q_2 are mixtures of E^2_x and CP_2 tangent vectors. q_3 involves only CP_2 tangent vectors and there is a temptation to interpret it as the analog of the quark having no color isospin.

2. The quaternionic basis is real and must transform like $(1, 1, q_1, \overline{q}_1)$, where q_1 is any quark in the triplet and $\overline{q}_1$ its conjugate in antitriplet. Having fixed some basis one can perform SU(3) rotations to get a new basis. The action of the rotation is by 3×3 special unitary matrix. The over all phases of its rows do not matter since they induce only a rotation in (e_2, e_3) plane not affecting the plane itself. The action of $SU(3)$ on q_1 is simply the action of its first row on (q_1, q_2, q_3) triplet:

$$\begin{aligned} q_1 &\rightarrow (Uq)_1 = U_{11}q_1 + U_{12}q_2 + U_{13}q_3 \equiv z_1q_1 + z_2q_2 + z_3q_3 \\ &= z_1(e_2 + ie_3) + z_2(e_4 + ie_5) + z_3(e_6 + ie_7) \ . \end{aligned} \tag{3.3.3}$$

The triplets (z_1, z_2, z_3) defining a complex unit vector and point of S^5. Since overall phase does not matter a point of CP_2 is in question. The new real octonion units are given by the formulas

$$\begin{aligned} e_2 &\rightarrow Re(z_1)e_2 + Re(z_2)e_4 + Re(z_3)e_6 - Im(z_1)e_3 - Im(z_2)e_5 - Im(z_3)e_7 \ , \\ e_3 &\rightarrow Im(z_1)e_2 + Im(z_2)e_4 + Im(z_3)e_6 + Re(z_1)e_3 + Re(z_2)e_5 + Re(z_3)e_7 \ . \end{aligned} \tag{3.3.4}$$

For instance the CP_2 coordinates corresponding to the coordinate patch (z_1, z_2, z_3) with $z_3 \neq 0$ are obtained as $(\xi_1, \xi_2) = (z_1/z_3, z_2/z_3)$.

Using these expressions the equations expressing the conjecture $CP_2 = CP_2^{mod}$ equivalence can be expressed explicitly as first order differential equations. The conditions state the equivalence

$$(e_2, e_3) \leftrightarrow (\partial_x h^k e_k^A e_A, \partial_y h^k e_k^A e_A) \ , \tag{3.3.5}$$

where e_A denote octonion units. The comparison of two pairs of vectors requires normalization of the tangent vectors on the right hand side to unit vectors so that one takes unit vector in the direction of the tangent vector. After this the vectors can be equated. This allows to expresses the contractions of the partial derivatives with vielbein vectors with the 6 components of e_2 and e_3. Each condition gives 6+6 first order partial differential equations which are non-linear by the presence of the overal normalization factor for the right hand side. The equations are invariant under scalings of (x, y). The very special form of these equations suggests that some symmetry is involved.

It must be emphasized that these equations make sense only in preferred coordinates: ordinary Minkowski coordinates and Hamilton-Jacobi coordinates for M^4 and Eguchi-Hanson complex coordinates in which $SU(2) \times U(1)$ is represented linearly for CP_2. These coordinates are preferred because they carry deep physical meaning.

Does TGD boil down to two string models?

It is good to look what have we obtained. Besides Hamilton-Jacobi conditions, and $CP_2 = CP_2^{mod}$ conditions one has what one might call string model with 6-dimensional $G_2/SU(3)$ as targent space. The orbit of string in $G_2/SU(3)$ allows to deduce the G_2 rotation identifiable as a point of $G_2/SU(3)$ defining what one means with standard quaternionic plane at given point of string world sheet. The hypothesis is that hyper-complex analyticity solves these equations.

The conjectured electric-magnetic duality implies duality between string world sheet and partonic 2-surfaces central for the proposed mathematical applications of TGD [K47, K48, K86, K56]. This duality suggests that the solutions to the $CP_2 = CP_2^{mod}$ conditions could reduce to holomorphy with respect to the coordinate w for partonic 2-surface plus the analogs of Virasoro conditions. The dependence on light-like coordinate would appear as a parametric dependence.

If this were the case, TGD would reduce at least partially to what might be regarded as dual string models in $G_2/SU(3)$ and $SU(3)/U(2)$ and also to string model in M^4 and X^4! In the previous arguments one ends up to string models in moduli spaces of string world sheets and partonic 2-surfaces. TGD seems to yield an inflation of string models! This not actually surprising since the slicing of space-time sheets by string world sheets and partonic 2-surfaces implies automatically various kinds of maps having interpretation in terms of string orbits.

3.4 In What Sense TGD Could Be An Integrable Theory?

During years evidence supporting the idea that TGD could be an integrable theory in some sense has accumulated. The challenge is to show that various ideas about what integrability means form pieces of a bigger coherent picture. Of course, some of the ideas are doomed to be only partially correct or simply wrong. Since it is not possible to know beforehand what ideas are wrong and what are right the situation is very much like in experimental physics and it is easy to claim (and has been and will be claimed) that all this argumentation is useless speculation. This is the price that must be paid for real thinking.

Integrable theories allow to solve nonlinear classical dynamics in terms of scattering data for a linear system. In TGD framework this translates to quantum classical correspondence. The solutions of Kähler-Dirac equation define the scattering data. This data should define a real analytic function whose octonionic extension defines the space-time surface as a surface for which its imaginary part in the representation as bi-quaternion vanishes. There are excellent hopes about this thanks to the reduction of the Kähler-Dirac equation to geometric optics.

In the following I will first discuss briefly what integrability means in (quantum) field theories, list some bits of evidence for integrability in TGD framework, discuss once again the question whether the different pieces of evidence are consistent with other and what one really means with various notions. An an outcome I represent what I regard as a more coherent view about integrability of TGD. The notion of octonion analyticity developed in the previous section is essential for the for what follows.

3.4.1 What Integrable Theories Are?

The following is an attempt to get some bird's eye of view about the landscape of integrable theories.

Examples of integrable theories

Integrable theories are typically non-linear 1+1-dimensional (quantum) field theories. Solitons and various other particle like structures are the characteristic phenomenon in these theories. Scattering matrix is trivial in the sense that the particles go through each other in the scattering and suffer only a phase change. In particular, momenta are conserved. Korteveg- de Vries equation (see `http://tinyurl.com/3cyt8hk`) [B3] was motivated by the attempt to explain the experimentally discovered shallow water wave preserving its shape and moving with a constant velocity. Sine-Gordon equation (see `http://tinyurl.com/yafl243x`) [B8] describes geometrically constant curvature surfaces and defines a Lorentz invariant non-linear field theory in 1+1-dimensional space-time, which can be applied to Josephson junctions (in TGD inspired quantum biology it is encountered in the model of nerve pulse [K75]). Non- linear Schrödinger equation (see `http://tinyurl.com/y88efbo7`) [B6] having applications to optics and water waves represents a further example. All these equations have various variants.

From TGD point of view conformal field theories represent an especially interesting example of integrable theories. (Super-)conformal invariance is the basic underlying symmetry and by its infinite-dimensional character implies infinite number of conserved quantities. The construction of the theory reduces to the construction of the representations of (super-)conformal algebra. One can solve 2-point functions exactly and characterize them in terms of (possibly anomalous) scaling dimensions of conformal fields involved and the coefficients appearing in 3-point functions can be solved in terms of fusion rules leading to an associative algebra for conformal fields. The basic applications are to 2-dimensional critical thermodynamical systems whose scaling invariance generalizes to conformal invariance. String models represent second application in which a collection of super-conformal field theories associated with various genera of 2-surface is needed to describe loop corrections to the scattering amplitudes. Also moduli spaces of conformal equivalence classes become important.

Topological quantum field theories (see http://tinyurl.com/lsvx7g3) are also examples of integrable theories. Because of its independence on the metric Chern-Simons action (see http://tinyurl.com/ydgsqm2c) is in 3-D case the unique action defining a topological quantum field theory. The calculations of knot invariants (for TGD approach see [K47]), topological invariants of 3-manifolds and 4-manifolds, and topological quantum computation (see http://tinyurl.com/dkpo4y) (for a model of DNA as topological quantum computer see [K2]) represent applications of this approach. TGD as almost topological QFT means that the Kähler action for preferred extremals reduces to a surface term by the vanishing of Coulomb term in action and by the weak form of electric-magnetic duality reduces to Chern-Simons action. Both Euclidian and Minkowskian regions give this kind of contribution.

$\mathcal{N} = 4$ SYM is the a four-dimensional and very nearly realistic candidate for an integral quantum field theory. The observation that twistor amplitudes allow also a dual of the 4-D conformal symmetry motivates the extension of this symmetry to its infinite-dimensional Yangian variant [A20]. Also the enormous progress in the construction of scattering amplitudes suggests integrability. In TGD framework Yangian symmetry would emerge naturally by extending the symplectic variant of Kac-Moody algebra from light-cone boundary to the interior of causal diamond and the Kac-Moody algebra from light-like 3-surface representing wormhole throats at which the signature of the induced metric changes to the space-time interior [K94].

About mathematical methods

The mathematical methods used in integrable theories are rather refined and have contributed to the development of the modern mathematical physics. Mention only quantum groups, conformal algebras, and Yangian algebras.

The basic element of integrability is the possibility to transform the non-linear classical problem for which the interaction is characterized by a potential function or its analog to a linear scattering problem depending on time. For instance, for the ordinary Schrödinger function one can solve potential once single solution of the equation is known. This does not work in practice. One can however gather information about the asymptotic states in scattering to deduce the potential. One cannot do without information about bound state energies too.

In TGD framework asymptotic states correspond to partonic 2-surfaces at the two light-like boundaries of CD (more precisely: the largest CD involved and defining the IR resolution for momenta). From the scattering data coding information about scattering for various values of energy of the incoming particle one deduced the potential function or its analog.

1. The basic tool is inverse scattering transform known as Gelfand-Marchenko-Levitan (GML) transform (see http://tinyurl.com/y9f7ybln) described in simple terms in [B9].

   (a) In 1+1 dimensional case the S-matrix characterizing scattering is very simple since the only thing that can take place in scattering is reflection or transmission. Therefore the S-matrix elements describe either of these processes and by unitarity the sum of corresponding probabilities equals to 1. The particle can arrive to the potential either from left or right and is characterized by a momentum. The transmission coefficient can have a pole meaning complex (imaginary in the simplest case) wave vector serving as a signal for the formation of a bound state or resonance. The scattering data are represented by the reflection and transmission coefficients as function of time.

   (b) One can deduce an integral equation for a propagator like function $K(t,x)$ describing how delta pulse moving with light velocity is scattered from the potential and is expressible in terms of time integral over scattering data with contributions from both scattering states and bound states. The derivation of GML transform [B9] uses time reversal and time translational invariance and causality defined in terms of light velocity. After some tricks one obtains the integral equation as well as an expression for the time independent potential as $V(x) = K(x,x)$. The argument can be generalized to more complex problems to deduce the GML transform.

2. The so called Lax pair (see http://tinyurl.com/yc93nw53) is one manner to describe integrable systems [B4]. Lax pair consists of two operators L and M. One studies what might be identified as "energy" eigenstates satisfying $L(x,t)\Psi = \lambda\Psi$. λ does not depend on time and one can say that the dynamics is associated with x coordinate whereas as t is time coordinate parametrizing different variants of eigenvalue problem with the same spectrum for L. The operator $M(t)$ does not depend on x at all and the independence of λ on time implies the condition

$$\partial_t L = [L, M] \ .$$

   This equation is analogous to a quantum mechanical evolution equation for an operator induced by time dependent "Hamiltonian" M and gives the non-linear classical evolution equation when the commutator on the right hand side is a multiplicative operator (so that it does not involve differential operators acting on the coordinate x). Non-linear classical dynamics for the time dependent potential emerges as an integrability condition.

   One could say that $M(t)$ introduces the time evolution of $L(t,x)$ as an automorphism which depends on time and therefore does not affect the spectrum. One has $L(t,x) = U(t)L(0,x)U^{-1}(t)$ with $dU(t)/dt = M(t)U(t)$. The time evolution of the analog of the quantum state is given by a similar equation.

3. A more refined view about Lax pair is based on the observation that the above equation can be generalized so that M depends also on x. The generalization of the basic equation for $M(x,t)$ reads as

$$\partial_t L - \partial_x M - [L, M] = 0 \ .$$

   The condition has interpretation as a vanishing of the curvature of a gauge potential having components $A_x = L, A_t = M$. This generalization allows a beautiful geometric formulation of the integrability conditions and extends the applicability of the inverse scattering transform. The monodromy of the flat connection becomes important in this approach. Flat connections in moduli spaces are indeed important in topological quantum field theories and in conformal field theories.

4. There is also a connection with the so called Riemann-Hilbert problem (see http://tinyurl.com/ybay4qjg) [A15]. The monodromies of the flat connection define monodromy group and Riemann-Hilbert problem concerns the existence of linear differential equations having a given monodromy group. Monodromy group emerges in the analytic continuation of an analytic function and the action of the element of the monodromy group tells what happens for the resulting many-valued analytic function as one turns around a singularity once ("mono-"). The linear equations obviously relate to the linear scattering problem. The flat connection (M,L) in turn defines the monodromy group. What is needed is that the functions involved are analytic functions of (t,x) replaced with a complex or hyper-complex variable. Again Wick rotation is involved. Similar approach generalizes also to higher dimensional moduli spaces with complex structures.

   In TGD framework the effective 2-dimensionality raises the hope that this kind of mathematical apparatus could be used. An interesting possibility is that finite measurement resolution could be realized in terms of a gauge group or Kac-Moody type group represented by trivial gauge potential defining a monodromy group for n-point functions. Monodromy invariance would hold for the full n-point functions constructed in terms of analytic n-point functions and their conjugates. The ends of braid strands are natural candidates for the singularities around which monodromies are defined.

3.4.2 Why TGD Could Be Integrable Theory In Some Sense?

There are many indications that TGD could be an integrable theory in some sense. The challenge is to see which ideas are consistent with each other and to build a coherent picture where everything finds its own place.

1. 2-dimensionality or at least effective 2-dimensionality seems to be a prerequisite for integrability. Effective 2-dimensionality is suggested by the strong form of General Coordinate Invariance implying also holography and generalized conformal invariance predicting infinite number of conservation laws. The dual roles of partonic 2-surfaces and string world sheets supports a four-dimensional generalization of conformal invariance. Twistor considerations [K94, K8] indeed suggest that Yangian invariance and Kac-Moody invariances combine to a 4-D analog of conformal invariance induced by 2-dimensional one by algebraic continuation.
2. Octonionic representation of embedding space Clifford algebra and the identification of the space-time surfaces as quaternionic space-time surfaces would define a number theoretically natural generalization of conformal invariance. The reason for using gamma matrix representation is that vector field representation for octonionic units does not exist. The problem concerns the precise meaning of the octonionic representation of gamma matrices.

   Space-time surfaces could be quaternionic also in the sense that conformal invariance is analytically continued from string curve to 8-D space by octonion real-analyticity. The question is whether the Clifford algebra based notion of tangent space quaternionicity is equivalent with octonionic real-analyticity based notion of quaternionicity.

   The notions of co-associativity and co-quaternionicity make also sense and one must consider seriously the possibility that associativity-co-associativity dichotomy corresponds to Minkowskian-Euclidian dichotomy.
3. Field equations define hydrodynamic Beltrami flows satisfying integrability conditions of form $J \wedge dJ = 0$.
   (a) One can assign local momentum and polarization directions to the preferred extremals and this gives a decomposition of Minkowskian space-time regions to massless quanta analogous to the 1+1-dimensional decomposition to solitons. The linear superposition of modes with 4-momenta with different directions possible for free Maxwell action does not look plausible for the preferred extremals of Kähler action. This rather quantal and solitonic character is in accordance with the quantum classical correspondence giving very concrete connection between quantal and classical particle pictures. For 4-D volume action one does not obtain this kind of decomposition. In 2-D case volume action gives superposition of solutions with different polarization directions so that the situation is nearer to that for free Maxwell action and is not like soliton decomposition.
   (b) Beltrami property in strong sense allows to identify 4 preferred coordinates for the space-time surface in terms of corresponding Beltrami flows. This is possible also in Euclidian regions using two complex coordinates instead of hyper-complex coordinate and complex coordinate. The assumption that isometry currents are parallel to the same light-like Beltrami flow implies hydrodynamic character of the field equations in the sense that one can say that each flow line is analogous to particle carrying some quantum numbers. This property is not true for all extremals (say cosmic strings).
   (c) The tangent bundle theoretic view about integrability is that one can find a Lie algebra of vector fields in some manifold spanning the tangent space of a lower-dimensional manifolds and is expressed in terms of Frobenius theorem (see `http://tinyurl.com/of6vfz5`) [A3]. The gradients of scalar functions defining Beltrami flows appearing in the ansatz for preferred exremals would define these vector fields and the slicing. Partonic 2-surfaces would correspond to two complex conjugate vector fields (local polarization direction) and string world sheets to light-like vector field and its dual (light-like momentum directions). This slicing generalizes to the Euclidian regions.
4. Infinite number of conservation laws is the signature of integrability. Classical field equations follow from the condition that the vector field defined by Kähler-Dirac gamma matrices has vanishing divergence and can be identified an integrability condition for the Kähler-Dirac equation guaranteeing also the conservation of super currents so that one obtains an infinite number of conserved charges.
5. Quantum criticality is a further signal of integrability. 2-D conformal field theories describe critical systems so that the natural guess is that quantum criticality in TGD framework relates to the generalization of conformal invariance and to integrability. Quantum criticality implies that Kähler coupling strength is analogous to critical temperature. This condition

does affects classical field equations only via boundary conditions expressed as weak form of electric magnetic duality at the wormhole throats at which the signature of the metric changes.

For finite-dimensional systems the vanishing of the determinant of the matrix defined by the second derivatives of potential is similar signature and applies in catastrophe theory. Therefore the existence of vanishing second variations of Kähler action should characterize criticality and define a property of preferred extremals. The vanishing of second variations indeed leads to an infinite number of conserved currents [K11] following the conditions that the deformation of Kähler-Dirac gamma matrix is also divergenceless and that the Kähler-Dirac equation associated with it is satisfied.

3.4.3 Could TGD Be An Integrable Theory?

Consider first the abstraction of integrability in TGD framework. Quantum classical correspondence could be seen as a correspondence between linear quantum dynamics and non-linear classical dynamics. Integrability would realize this correspondence. In integrable models such as Sine-Gordon equation particle interactions are described by potential in 1+1 dimensions. This too primitive for the purposes of TGD. The vertices of generalized Feynman diagrams take care of this. At lines one has free particle dynamics so that the situation could be much simpler than in integrable models if one restricts the considerations to the lines or Minkowskian space-time regions surrounding them.

The non-linear dynamics for the space-time sheets representing incoming lines of generalized Feynman diagram should be obtainable from the linear dynamics for the induced spinor fields defined by Kähler-Dirac operator. There are two options.

1. Strong form of the quantum classical correspondence states that each solution for the linear dynamics of spinor fields corresponds to space-time sheet. This is analogous to solving the potential function in terms of a single solution of Schrödinger equation. Coupling of space-time geometry to quantum numbers via measurement interaction term is a proposal for realizing this option. It is however the quantum numbers of positive/negative energy parts of zero energy state which would be visible in the classical dynamics rather than those of induced spinor field modes.
2. Only overall dynamics characterized by scattering data- the counterpart of S-matrix for the Kähler-Dirac operator- is mapped to the geometry of the space-time sheet. This is much more abstract realization of quantum classical correspondence.
3. Can these two approaches be equivalent? This might be the case since quantum numbers of the state are not those of the modes of induced spinor fields.

What the scattering data could be for the induced spinor field satisfying Kähler-Dirac equation?

1. If the solution of field equation has hydrodynamic character, the solutions of the Kähler-Dirac equation can be localized to light-like Beltrami flow lines of hydrodynamic flow. These correspond to basic solutions and the general solution is a superposition of these. There is no dispersion and the dynamics is that of geometric optics at the basic level. This means geometric optics like character of the spinor dynamics.

   Solutions of the Kähler-Dirac equation are completely analogous to the pulse solutions defining the fundamental solution for the wave equation in the argument leading from wave equation with external time independent potential to Marchenko-Gelfand-Levitan equation allowing to identify potential in terms of scattering data. There is however no potential present now since the interactions are described by the vertices of Feynman diagram where the particle lines meet. Note that particle like regions are Euclidian and that this picture applies only to the Minkowskian exteriors of particles.
2. Partonic 2-surfaces at the ends of the line of generalized Feynman diagram are connected by flow lines. Partonic 2-surfaces at which the signature of the induced metric changes are in a special position. Only the imaginary part of the bi-quaternionic value of the octonion valued map is non-vanishing at these surfaces which can be said to be co-complex 2-surfaces. By

geometric optics behavior the scattering data correspond to a diffeomorphism mapping initial partonic 2-surface to the final one in some preferred complex coordinates common to both ends of the line.

3. What could be these preferred coordinates? Complex coordinates for S^2 at light-cone boundary define natural complex coordinates for the partonic 2-surface. With these coordinates the diffeomorphism defining scattering data is diffeomorphism of S^2. Suppose that this map is real analytic so that maps "real axis" of S^2 to itself. This map would be same as the map defining the octonionic real analyticity as algebraic extension of the complex real analytic map. By octonionic analyticity one can make large number of alternative choices for the coordinates of partonic 2-surface.

4. There can be non-uniqueness due to the possibility of $G_2/SU(3)$ valued map characterizing the local octonionic units. The proposal is that the choice of octonionic imaginary units can depend on the point of string like orbit: this would give string model in $G_2/SU(3)$. Conformal invariance for this string model would imply analyticity and helps considerably but would not probably fix the situation completely since the element of the coset space would constant at the partonic 2-surfaces at the ends of CD. One can of course ask whether the $G_2/SU(3)$ element could be constant for each propagator line and would change only at the 2-D vertices?

This would be the inverse scattering problem formulated in the spirit of TGD. There could be also dependence of space-time surface on quantum numbers of quantum states but not on individual solution for the induced spinor field since the scattering data of this solution would be purely geometric.

3.5 Do Geometric Invariants Of Preferred Extremals Define Topological Invariants Of Space-time Surface And Code For Quantumphysics?

The recent progress in the understanding of preferred extremals [K11] led to a reduction of the field equations to conditions stating for Euclidian signature the existence of Kähler metric. The resulting conditions are a direct generalization of corresponding conditions emerging for the string world sheet and stating that the 2-metric has only non-diagonal components in complex/hypercomplex coordinates. Also energy momentum of Kähler action and has this characteristic (1, 1) tensor structure. In Minkowskian signature one obtains the analog of 4-D complex structure combining hyper-complex structure and 2-D complex structure.

The construction lead also to the understanding of how Einstein's equations with cosmological term follow as a consistency condition guaranteeing that the covariant divergence of the Maxwell's energy momentum tensor assignable to Kähler action vanishes. This gives $T = kG + \Lambda g$. By taking trace a further condition follows from the vanishing trace of T:

$$R = \frac{4\Lambda}{k} . \tag{3.5.1}$$

That any preferred extremal should have a constant Ricci scalar proportional to cosmological constant is very strong prediction. Note that the accelerating expansion of the Universe would support positive value of Λ. Note however that both Λ and $k \propto 1/G$ are both parameters characterizing one particular preferred extremal. One could of course argue that the dynamics allowing only constant curvature space-times is too simple. The point is however that particle can topologically condense on several space-time sheets meaning effective superposition of various classical fields defined by induced metric and spinor connection.

The following considerations demonstrate that preferred extremals can be seen as canonical representatives for the constant curvature manifolds playing central role in Thurston's geometrization theorem (see `http://tinyurl.com/y8bbzlnr`) [A18] known also as hyperbolization theorem implying that geometric invariants of space-time surfaces transform to topological invariants. The generalization of the notion of Ricci flow to Maxwell flow in the space of metrics and further to

Kähler flow for preferred extremals in turn gives a rather detailed vision about how preferred extremals organize to one-parameter orbits. It is quite possible that Kähler flow is actually discrete. The natural interpretation is in terms of dissipation and self organization.

Quantum classical correspondence suggests that this line of thought could be continued even further: could the geometric invariants of the preferred extremals could code not only for space-time topology but also for quantum physics? How to calculate the correlation functions and coupling constant evolution has remained a basic unresolved challenge of quantum TGD. Could the correlation functions be reduced to statistical geometric invariants of preferred extemals? The latest (means the end of 2012) and perhaps the most powerful idea hitherto about coupling constant evolution is quantum classical correspondence in statistical sense stating that the statistical properties of a preferred extremal in quantum superposition of them are same as those of the zero energy state in question. This principle would be quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This principle would allow to deduce correlation functions and S-matrix from the statistical properties of single preferred extremal alone using classical intuition. Also coupling constant evolution would be coded by the statistical properties of the representative preferred extremal.

3.5.1 Preferred Extremals Of Kähler Action As Manifolds With Constant Ricci Scalar Whose Geometric Invariants Are Topological-Invariants

An old conjecture inspired by the preferred extremal property is that the geometric invariants of space-time surface serve as topological invariants. The reduction of Kähler action to 3-D Chern-Simons terms (see `http://tinyurl.com/ybp86sho`) [K11] gives support for this conjecture as a classical counterpart for the view about TGD as almost topological QFT. The following arguments give a more precise content to this conjecture in terms of existing mathematics.

1. It is not possible to represent the scaling of the induced metric as a deformation of the space-time surface preserving the preferred extremal property since the scale of CP_2 breaks scale invariance. Therefore the curvature scalar cannot be chosen to be equal to one numerically. Therefore also the parameter $R = 4\Lambda/k$ and also Λ and k separately characterize the equivalence class of preferred extremals as is also physically clear.

   Also the volume of the space-time sheet closed inside causal diamond CD remains constant along the orbits of the flow and thus characterizes the space-time surface. Λ and even $k \propto 1/G$ can indeed depend on space-time sheet and p-adic length scale hypothesis suggests a discrete spectrum for Λ/k expressible in terms of p-adic length scales: $\Lambda/k \propto 1/L_p^2$ with $p \simeq 2^k$ favored by p-adic length scale hypothesis. During cosmic evolution the p-adic length scale would increase gradually. This would resolve the problem posed by cosmological constant in GRT based theories.

2. One could also see the preferred extremals as 4-D counterparts of constant curvature 3-manifolds in the topology of 3-manifolds. An interesting possibility raised by the observed negative value of Λ is that most 4-surfaces are constant negative curvature 4-manifolds. By a general theorem coset spaces (see `http://tinyurl.com/y8d3udpr`) H^4/Γ, where $H^4 = SO(1,4)/SO(4)$ is hyperboloid of M^5 and Γ a torsion free discrete subgroup of $SO(1,4)$ [A7]. It is not clear to me, whether the constant value of Ricci scalar implies constant sectional curvatures and therefore hyperbolic space property. It could happen that the space of spaces with constant Ricci curvature contain a hyperbolic manifold as an especially symmetric representative. In any case, the geometric invariants of hyperbolic metric are topological invariants.

   By Mostow rigidity theorem (see `http://tinyurl.com/yacbu8sk`) [A10] finite-volume hyperbolic manifold is unique for $D > 2$ and determined by the fundamental group of the manifold. Since the orbits under the Kähler flow preserve the curvature scalar the manifolds at the orbit must represent different embeddings of one and hyperbolic 4-manifold. In 2-D case the moduli space for hyperbolic metric for a given genus $g > 0$ is defined by Teichmueller parameters and has dimension $6(g-1)$. Obviously the exceptional character of $D = 2$ case relates to conformal invariance. Note that the moduli space in question (see `http://tinyurl.com/ybowqm5v`) plays a key role in p-adic mass calculations [K22].

In the recent case Mostow rigidity theorem could hold true for the Euclidian regions and maybe generalize also to Minkowskian regions. If so then both "topological" and "geometro" in "Topological GeometroDynamics" would be fully justified. The fact that geometric invariants become topological invariants also conforms with "TGD as almost topological QFT" and allows the notion of scale to find its place in topology. Also the dream about exact solvability of the theory would be realized in rather convincing manner.

These conjectures are the main result independent of whether the generalization of the Ricci flow discussed in the sequel exists as a continuous flow or possibly discrete sequence of iterates in the space of preferred extremals of Kähler action. My sincere hope is that the reader could grasp how far reaching these result really are.

3.5.2 Is There A Connection Between Preferred Extremals And AdS$_4$/CFT Correspondence?

The preferred extremals satisfy Einstein Maxwell equations with a cosmological constant and have negative scalar curvature for negative value of Λ. 4-D space-times with hyperbolic metric provide canonical representation for a large class of four-manifolds and an interesting question is whether these spaces are obtained as preferred extremals and/or vacuum extremals.

4-D hyperbolic space with Minkowski signature is locally isometric with AdS$_4$. This suggests at connection with AdS$_4$/CFT correspondence of M-theory. The boundary of AdS would be now replaced with 3-D light-like orbit of partonic 2-surface at which the signature of the induced metric changes. The metric 2-dimensionality of the light-like surface makes possible generalization of 2-D conformal invariance with the light-like coordinate taking the role of complex coordinate at light-like boundary. AdS could represent a special case of a more general family of space-time surfaces with constant Ricci scalar satistying Einstein-Maxwell equations and generalizing the AdS$_4$/CFT correspondence. There is however a strong objection from cosmology: the accelerated expansion of the Universe requires positive value of Λ and favors De Sitter Space dS_4 instead of AdS_4.

These observations provide motivations for finding whether AdS$_4$ and/or dS_4 allows an embedding as a vacuum extremal to $M^4\times S^2\subset M^4\times CP_2$, where S^2 is a homologically trivial geodesic sphere of CP_2. It is easy to guess the general form of the embedding by writing the line elements of, M^4, S^2, and AdS$_4$.

1. The line element of M^4 in spherical Minkowski coordinates (m, r_M, θ, ϕ) reads as

$$ds^2 = dm^2 - dr_M^2 - r_M^2 d\Omega^2 \ . \tag{3.5.2}$$

2. Also the line element of S^2 is familiar:

$$ds^2 = -R^2(d\Theta^2 + sin^2(\theta)d\Phi^2) \ . \tag{3.5.3}$$

3. By visiting in Wikipedia (see `http://tinyurl.com/y9hw95ql`) one learns that in spherical coordinate the line element of AdS$_4$/dS_4 is given by

$$\begin{aligned} ds^2 &= A(r)dt^2 - \frac{1}{A(r)}dr^2 - r^2 d\Omega^2 \ , \\ A(r) &= 1 + \epsilon y^2 \ , \ y = \frac{r}{r_0} \ , \\ \epsilon &= 1 \ \text{for} \ AdS_4 \ , \ \epsilon = -1 \ \text{for} \ dS_4 \ . \end{aligned} \tag{3.5.4}$$

4. From these formulas it is easy to see that the ansatz is of the same general form as for the embedding of Schwartschild-Nordstöm metric:

$$\begin{aligned} &m = \Lambda t + h(y) \ , \quad r_M = r \ , \\ &\Theta = s(y) \ , \qquad \Phi = \omega(t + f(y)) \ . \end{aligned} \tag{3.5.5}$$

The non-trivial conditions on the components of the induced metric are given by

$$\begin{aligned}
g_{tt} &= \Lambda^2 - x^2 sin^2(\Theta) = A(r) \ , \\
g_{tr} &= \frac{1}{r_0}\left[\Lambda\frac{dh}{dy} - x^2 sin^2(\theta)\frac{df}{dr}\right] = 0 \ , \\
g_{rr} &= \frac{1}{r_0^2}\left[(\frac{dh}{dy})^2 - 1 - x^2 sin^2(\theta)(\frac{df}{dy})^2 - R^2(\frac{d\Theta}{dy})^2\right] = -\frac{1}{A(r)} \ , \\
x &= R\omega \ .
\end{aligned} \quad (3.5.6)$$

By some simple algebraic manipulations one can derive expressions for $sin(\Theta)$, df/dr and dh/dr.

1. For $\Theta(r)$ the equation for g_{tt} gives the expression

$$\begin{aligned}
sin(\Theta) &= \pm\frac{P^{1/2}}{x} \ , \\
P &= \Lambda^2 - A = \Lambda^2 - 1 - \epsilon y^2 \ .
\end{aligned} \quad (3.5.7)$$

The condition $0 \leq sin^2(\Theta) \leq 1$ gives the conditions

$$\begin{array}{rcll}
(\Lambda^2 - x^2 - 1)^{1/2} \leq & y & \leq (\Lambda^2 - 1)^{1/2} & \text{for } \epsilon = 1 \ (AdS_4) \ , \\
(-\Lambda^2 + 1)^{1/2} \leq & y & \leq (x^2 + 1 - \Lambda^2)^{1/2} & \text{for } \epsilon = -1 \ (dS_4) \ .
\end{array} \quad (3.5.8)$$

Only a spherical shell is possible in both cases. The model for the final state of star considered in [K99] predicted similar layer layer like structure and inspired the proposal that stars quite generally have an onion-like structure with radii of various shells characterize by p-adic length scale hypothesis and thus coming in some powers of $\sqrt{2}$. This brings in mind also Titius-Bode law.

2. From the vanishing of g_{tr} one obtains

$$\frac{dh}{dy} = \frac{P}{\Lambda}\frac{df}{dy} \ . \quad (3.5.9)$$

3. The condition for g_{rr} gives

$$(\frac{df}{dy})^2 = \frac{r_0^2}{AP}[A^{-1} - R^2(\frac{d\Theta}{dy})^2] \ . \quad (3.5.10)$$

Clearly, the right-hand side is positive if $P \geq 0$ holds true and $Rd\Theta/dy$ is small. One can express $d\Theta/dy$ using chain rule as

$$(\frac{d\Theta}{dy})^2 = \frac{x^2y^2}{P(P-x^2)} \ . \quad (3.5.11)$$

One obtains

$$(\frac{df}{dy})^2 = \Lambda r_0^2\frac{y^2}{AP}\left[\frac{1}{1+y^2} - x^2(\frac{R}{r_0})^2\frac{1}{P(P-x^2)}\right] \ . \quad (3.5.12)$$

The right hand side of this equation is non-negative for certain range of parameters and variable y. Note that for $r_0 \gg R$ the second term on the right hand side can be neglected. In this case it is easy to integrate $f(y)$.

The conclusion is that both AdS_4 and dS^4 allow a local embedding as a vacuum extremal. Whether also an embedding as a non-vacuum preferred extremal to $M^4 \times S^2$, S^2 a homologically non-trivial geodesic sphere is possible, is an interesting question.

3.5.3 Generalizing Ricci Flow To Maxwell Flow For 4-Geometries And Kähler Flow For Space-Time Surfaces

The notion of Ricci flow has played a key part in the geometrization of topological invariants of Riemann manifolds. I certainly did not have this in mind when I choose to call my unification attempt "Topological Geometrodynamics" but this title strongly suggests that a suitable generalization of Ricci flow could play a key role in the understanding of also TGD.

Ricci flow and Maxwell flow for 4-geometries

The observation about constancy of 4-D curvature scalar for preferred extremals inspires a generalization of the well-known volume preserving Ricci flow (see `http://tinyurl.com/2cwlzh9l`) [A14] introduced by Richard Hamilton. Ricci flow is defined in the space of Riemann metrics as

$$\frac{dg_{\alpha\beta}}{dt} = -2R_{\alpha\beta} + 2\frac{R_{avg}}{D}g_{\alpha\beta} \ . \tag{3.5.13}$$

Here R_{avg} denotes the average of the scalar curvature, and D is the dimension of the Riemann manifold. The flow is volume preserving in average sense as one easily checks ($\langle g^{\alpha\beta}dg_{\alpha\beta}/dt\rangle = 0$). The volume preserving property of this flow allows to intuitively understand that the volume of a 3-manifold in the asymptotic metric defined by the Ricci flow is topological invariant. The fixed points of the flow serve as canonical representatives for the topological equivalence classes of 3-manifolds. These 3-manifolds (for instance hyperbolic 3-manifolds with constant sectional curvatures) are highly symmetric. This is easy to understand since the flow is dissipative and destroys all details from the metric.

What happens in the recent case? The first thing to do is to consider what might be called Maxwell flow in the space of all 4-D Riemann manifolds allowing Maxwell field.

1. First of all, the vanishing of the trace of Maxwell's energy momentum tensor codes for the volume preserving character of the flow defined as

$$\frac{dg_{\alpha\beta}}{dt} = T_{\alpha\beta} \ . \tag{3.5.14}$$

Taking covariant divergence on both sides and assuming that d/dt and D_α commute, one obtains that $T^{\alpha\beta}$ is divergenceless.

This is true if one assumes Einstein's equations with cosmological term. This gives

$$\frac{dg_{\alpha\beta}}{dt} = kG_{\alpha\beta} + \Lambda g_{\alpha\beta} = kR_{\alpha\beta} + (-\frac{kR}{2} + \Lambda)g_{\alpha\beta} \ . \tag{3.5.15}$$

The trace of this equation gives that the curvature scalar is constant. Note that the value of the Kähler coupling strength plays a highly non-trivial role in these equations and it is quite possible that solutions exist only for some critical values of α_K. Quantum criticality should fix the allow value triplets (G, Λ, α_K) apart from overall scaling

$$(G, \Lambda, \alpha_K) \rightarrow (xG, \Lambda/x, x\alpha_K \ .$$

Fixing the value of G fixes the values remaining parameters at critical points. The rescaling of the parameter t induces a scaling by x.

2. By taking trace one obtains the already mentioned condition fixing the curvature to be constant, and one can write

$$\frac{dg_{\alpha\beta}}{dt} = kR_{\alpha\beta} - \Lambda g_{\alpha\beta} \ . \tag{3.5.16}$$

Note that in the recent case $R_{avg} = R$ holds true since curvature scalar is constant. The fixed points of the flow would be Einstein manifolds (see `http://tinyurl.com/ybrnakuu`) [A2, A36] satisfying

$$R_{\alpha\beta} = \frac{\Lambda}{k} g_{\alpha\beta} \quad (3.5.17)$$

.

3. It is by no means obvious that continuous flow is possible. The condition that Einstein-Maxwell equations are satisfied might pick up from a completely general Maxwell flow a discrete subset as solutions of Einstein-Maxwell equations with a cosmological term. If so, one could assign to this subset a sequence of values t_n of the flow parameter t.

4. I do not know whether 3-dimensionality is somehow absolutely essential for getting the topological classification of closed 3-manifolds using Ricci flow. This ignorance allows me to pose some innocent questions. Could one have a canonical representation of 4-geometries as spaces with constant Ricci scalar? Could one select one particular Einstein space in the class four-metrics and could the ratio Λ/k represent topological invariant if one normalizes metric or curvature scalar suitably. In the 3-dimensional case curvature scalar is normalized to unity. In the recent case this normalization would give $k = 4\Lambda$ in turn giving $R_{\alpha\beta} = g_{\alpha\beta}/4$. Does this mean that there is only single fixed point in local sense, analogous to black hole toward which all geometries are driven by the Maxwell flow? Does this imply that only the 4-volume of the original space would serve as a topological invariant?

Maxwell flow for space-time surfaces

One can consider Maxwell flow for space-time surfaces too. In this case Kähler flow would be the appropriate term and provides families of preferred extremals. Since space-time surfaces inside CD are the basic physical objects are in TGD framework, a possible interpretation of these families would be as flows describing physical dissipation as a four-dimensional phenomenon polishing details from the space-time surface interpreted as an analog of Bohr orbit.

1. The flow is now induced by a vector field $j^k(x,t)$ of the space-time surface having values in the tangent bundle of embedding space $M^4 \times CP_2$. In the most general case one has Kähler flow without the Einstein equations. This flow would be defined in the space of all space-time surfaces or possibly in the space of all extremals. The flow equations reduce to

$$h_{kl} D_\alpha j^k(x,t) D_\beta h^l = \frac{1}{2} T_{\alpha\beta} \ . \quad (3.5.18)$$

The left hand side is the projection of the covariant gradient $D_\alpha j^k(x,t)$ of the flow vector field $j^k(x,t)$ to the tangent space of the space-time surface. D_{alpha} is covariant derivative taking into account that j^k is embedding space vector field. For a fixed point space-time surface this projection must vanish assuming that this space-time surface reachable. A good guess for the asymptotia is that the divergence of Maxwell energy momentum tensor vanishes and that Einstein's equations with cosmological constant are well-defined.

Asymptotes corresponds to vacuum extremals. In Euclidian regions CP_2 type vacuum extremals and in Minkowskian regions to any space-time surface in any 6-D sub-manifold $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2 having therefore vanishing induced Kähler form. Symplectic transformations of CP_2 combined with diffeomorphisms of M^4 give new Lagrangian manifolds. One would expect that vacuum extremals are approached but never reached at second extreme for the flow.

If one assumes Einstein's equations with a cosmological term, allowed vacuum extremals must be Einstein manifolds. For CP_2 type vacuum extremals this is the case. It is quite possible that these fixed points do not actually exist in Minkowskian sector, and could be replaced with more complex asymptotic behavior such as limit, chaos, or strange attractor.

2. The flow could be also restricted to the space of preferred extremals. Assuming that Einstein Maxwell equations indeed hold true, the flow equations reduce to

$$h_{kl}D_\alpha j^k(x,t)\partial_\beta h^l = \frac{1}{2}(kR_{\alpha\beta} - \Lambda g_{\alpha\beta}) \ . \qquad (3.5.19)$$

   Preferred extremals would correspond to a fixed sub-manifold of the general flow in the space of all 4-surfaces.

3. One can also consider a situation in which $j^k(x,t)$ is replaced with $j^k(h,t)$ defining a flow in the entire embedding space. This assumption is probably too restrictive. In this case the equations reduce to

$$(D_r j_l(x,t) + D_l j_r)\partial_\alpha h^r \partial_\beta h^l = kR_{\alpha\beta} - \Lambda g_{\alpha\beta} \ . \qquad (3.5.20)$$

   Here D_r denotes covariant derivative. Asymptotia is achieved if the tensor $D_k j_l + D_k j_l$ becomes orthogonal to the space-time surface. Note for that Killing vector fields of H the left hand side vanishes identically. Killing vector fields are indeed symmetries of also asymptotic states.

It must be made clear that the existence of a continuous flow in the space of preferred extremals might be too strong a condition. Already the restriction of the general Maxwell flow in the space of metrics to solutions of Einstein-Maxwell equations with cosmological term might lead to discretization, and the assumption about reprentability as 4-surface in $M^4 \times CP_2$ would give a further condition reducing the number of solutions. On the other hand, one might consiser a possibility of a continuous flow in the space of constant Ricci scalar metrics with a fixed 4-volume and having hyperbolic spaces as the most symmetric representative.

Dissipation, self organization, transition to chaos, and coupling constant evolution

A beautiful connection with concepts like dissipation, self-organization, transition to chaos, and coupling constant evolution suggests itself.

1. It is not at all clear whether the vacuum extremal limits of the preferred extremals can correspond to Einstein spaces except in special cases such as CP_2 type vacuum extremals isometric with CP_2. The imbeddability condition however defines a constraint force which might well force asymptotically more complex situations such as limit cycles and strange attractors. In ordinary dissipative dynamics an external energy feed is essential prerequisite for this kind of non-trivial self-organization patterns.

   In the recent case the external energy feed could be replaced by the constraint forces due to the imbeddability condition. It is not too difficult to imagine that the flow (if it exists!) could define something analogous to a transition to chaos taking place in a stepwise manner for critical values of the parameter t. Alternatively, these discrete values could correspond to those values of t for which the preferred extremal property holds true for a general Maxwell flow in the space of 4-metrics. Therefore the preferred extremals of Kähler action could emerge as one-parameter (possibly discrete) families describing dissipation and self-organization at the level of space-time dynamics.

2. For instance, one can consider the possibility that in some situations Einstein's equations split into two mutually consistent equations of which only the first one is independent

$$\begin{aligned} xJ^\alpha_{\ \nu}J^{\nu\beta} &= R^{\alpha\beta} \ , \\ L_K &= xJ^\alpha_{\ \nu}J^{\nu\beta} = 4\Lambda \ , \\ x &= \frac{1}{16\pi\alpha_K} \ . \end{aligned} \qquad (3.5.21)$$

   Note that the first equation indeed gives the second one by tracing. This happens for CP_2 type vacuum extremals.

Kähler action density would reduce to cosmological constant which should have a continuous spectrum if this happens always. A more plausible alternative is that this holds true only asymptotically. In this case the flow equation could not lead arbitrary near to vacuum extremal, and one can think of situation in which $L_K = 4\Lambda$ defines an analog of limiting cycle or perhaps even strange attractor. In any case, the assumption would allow to deduce the asymptotic value of the action density which is of utmost importance from calculational point of view: action would be simply $S_K = 4\Lambda V_4$ and one could also say that one has minimal surface with Λ taking the role of string tension.

3. One of the key ideas of TGD is quantum criticality implying that Kähler coupling strength is analogous to critical temperature. Second key idea is that p-adic coupling constant evolution represents discretized version of continuous coupling constant evolution so that each p-adic prime would correspond a fixed point of ordinary coupling constant evolution in the sense that the 4-volume characterized by the p-adic length scale remains constant. The invariance of the geometric and thus geometric parameters of hyperbolic 4-manifold under the Kähler flow would conform with the interpretation as a flow preserving scale assignable to a given p-adic prime. The continuous evolution in question (if possible at all!) might correspond to a fixed p-adic prime. Also the hierarchy of Planck constants relates to this picture naturally. Planck constant $\hbar_{eff} = n\hbar$ corresponds to a multi-furcation generating n-sheeted structure and certainly affecting the fundamental group.

4. One can of course question the assumption that a continuous flow exists. The property of being a solution of Einstein-Maxwell equations, imbeddability property, and preferred extremal property might allow allow only discrete sequences of space-time surfaces perhaps interpretable as orbit of an iterated map leading gradually to a fractal limit. This kind of discrete sequence might be also be selected as preferred extremals from the orbit of Maxwell flow without assuming Einstein-Maxwell equations. Perhaps the discrete p-adic coupling constant evolution could be seen in this manner and be regarded as an iteration so that the connection with fractality would become obvious too.

Does a 4-D counterpart of thermodynamics make sense?

The interpretation of the Kähler flow in terms of dissipation, the constancy of R, and almost constancy of L_K suggest an interpretation in terms of 4-D variant of thermodynamics natural in zero energy ontology (ZEO), where physical states are analogs for pairs of initial and final states of quantum event are quantum superpositions of classical time evolutions. Quantum theory becomes a "square root" of thermodynamics so that 4-D analog of thermodynamics might even replace ordinary thermodynamics as a fundamental description. If so this 4-D thermodynamics should be qualitatively consistent with the ordinary 3-D thermodynamics.

1. The first naïve guess would be the interpretation of the action density L_K as an analog of energy density $e = E/V_3$ and that of R as the analog to entropy density $s = S/V_3$. The asymptotic states would be analogs of thermodynamical equilibria having constant values of L_K and R.

2. Apart from an overall sign factor ϵ to be discussed, the analog of the first law $de = Tds - pdV/V$ would be

$$dL_K = kdR + \Lambda\frac{dV_4}{V_4} \ .$$

One would have the correspondences $S \rightarrow \epsilon RV_4$, $e \rightarrow \epsilon L_K$ and $k \rightarrow T$, $p \rightarrow -\Lambda$. $k \propto 1/G$ indeed appears formally in the role of temperature in Einstein's action defining a formal partition function via its exponent. The analog of second law would state the increase of the magnitude of ϵRV_4 during the Kähler flow.

3. One must be very careful with the signs and discuss Euclidian and Minkowskian regions separately. Concerning purely thermodynamic aspects at the level of vacuum functional Euclidian regions are those which matter.

   (a) For CP_2 type vacuum extremals $L_K \propto E^2 + B^2$, $R = \Lambda/k$, and Λ are positive. In thermodynamical analogy for $\epsilon = 1$ this would mean that pressure is negative.

(b) In Minkowskian regions the value of $R = \Lambda/k$ is negative for $\Lambda < 0$ suggested by the large abundance of 4-manifolds allowing hyperbolic metric and also by cosmological considerations. The asymptotic formula $L_K = 4\Lambda$ considered above suggests that also Kähler action is negative in Minkowskian regions for magnetic flux tubes dominating in TGD inspired cosmology: the reason is that the magnetic contribution to the action density $L_K \propto E^2 - B^2$ dominates.

Consider now in more detail the 4-D thermodynamics interpretation in Euclidian and Minkowskian regions assuming that the evolution by quantum jumps has Kähler flow as a space-time correlate.

1. In Euclidian regions the choice $\epsilon = 1$ seems to be more reasonable one. In Euclidian regions $-\Lambda$ as the analog of pressure would be negative, and asymptotically (that is for CP_2 type vacuum extremals) its value would be proportional to $\Lambda \propto 1/GR^2$, where R denotes CP_2 radius defined by the length of its geodesic circle.

   A possible interpretation for negative pressure is in terms of string tension effectively inducing negative pressure (note that the solutions of the Kähler-Dirac equation indeed assign a string to the wormhole contact). The analog of the second law would require the increase of RV_4 in quantum jumps. The magnitudes of L_K, R, V_4 and Λ would be reduced and approach their asymptotic values. In particular, V_4 would approach asymptotically the volume of CP_2.

2. In Minkowskian regions Kähler action contributes to the vacuum functional a phase factor analogous to an imaginary exponent of action serving in the role of Morse function so that thermodynamics interpretation can be questioned. Despite this one can check whether thermodynamic interpretation can be considered. The choice $\epsilon = -1$ seems to be the correct choice now. $-\Lambda$ would be analogous to a negative pressure whose gradually decreases. In 3-D thermodynamics it is natural to assign negative pressure to the magnetic flux tube like structures as their effective string tension defined by the density of magnetic energy per unit length. $-R \geq 0$ would entropy and $-L_K \geq 0$ would be the analog of energy density.

   $R = \Lambda/k$ and the reduction of Λ during cosmic evolution by quantum jumps suggests that the larger the volume of CD and thus of (at least) Minkowskian space-time sheet the smaller the negative value of Λ.

   Assume the recent view about state function reduction explaining how the arrow of geometric time is induced by the quantum jump sequence defining experienced time [K5]. According to this view zero energy states are quantum superpositions over CDs of various size scales but with common tip, which can correspond to either the upper or lower light-like boundary of CD. The sequence of quantum jumps the gradual increase of the average size of CD in the quantum superposition and therefore that of average value of V_4. On the other hand, a gradual decrease of both $-L_K$ and $-R$ looks physically very natural. If Kähler flow describes the effect of dissipation by quantum jumps in ZEO then the space-time surfaces would gradually approach nearly vacuum extremals with constant value of entropy density $-R$ but gradually increasing 4-volume so that the analog of second law stating the increase of $-RV_4$ would hold true.

3. The interpretation of $-R > 0$ as negentropy density assignable to entanglement is also possible and is consistent with the interpretation in terms of second law. This interpretation would only change the sign factor ϵ in the proposed formula. Otherwise the above arguments would remain as such.

3.5.4 Could Correlation Functions, S-Matrix, And Coupling Constant Evolution Be Coded The Statistical Properties Of Preferred Extremals?

How to calculate the correlation functions and coupling constant evolution has remained a basic unresolved challenge. Generalized Feynman diagrams provide a powerful vision which however does not help in practical calculations. Some big idea has been lacking.

Quantum classical correspondence states that all aspects of quantum states should have correlates in the geometry of preferred extremals. In particular, various elementary particle propagators should have a representation as properties of preferred extremals. This would allow to realize the old dream about being able to say something interesting about coupling constant evolution although it is not yet possible to calculate the M-matrices and U-matrix. The general structure of U-matrix is however understood [K60]. Hitherto everything that has been said about coupling constant evolution has been rather speculative arguments except for the general vision that it reduces to a discrete evolution defined by p-adic length scales. General first principle definitions are however much more valuable than ad hoc guesses even if the latter give rise to explicit formulas.

In quantum TGD and also at its QFT limit various correlation functions in given quantum state should code for its properties. By quantum classical correspondence these correlation functions should have counterparts in the geometry of preferred extremals. Even more: these classical counterparts for a given preferred extremal ought to be identical with the quantum correlation functions for the superposition of preferred extremals. This correspondence could be called quantum ergodicity by its analogy with ordinary ergodicity stating that the member of ensemble becomes representative of ensemble.

This principle would be a quantum generalization of ergodic theorem stating that the time evolution of a single member of ensemble represents the ensemble statistically. This symmetry principle analogous to holography might allow to fix S-matrix uniquely even in the case that the hermitian square root of the density matrix appearing in the M-matrix would lead to a breaking of quantum ergodicity as also 4-D spin glass degeneracy suggests.

This principle would allow to deduce correlation functions from the statistical properties of single preferred extremal alone using just classical intuition. Also coupling constant evolution would be coded by the statistical properties of preferred extremals. Quantum ergodicity would mean an enormous simplification since one could avoid the horrible conceptual complexities involved with the functional integrals over WCW .

This might of course be too optimistic guess. If a sub-algebra of symplectic algebra acts as gauge symmmetries of the preferred extremals in the sense that corresponding Noether charges vanish, it can quite well be that correlations functions correspond to averages for extremals belonging to single conformal equivalence class.

1. The marvellous implication of quantum ergodicity would be that one could calculate everything solely classically using the classical intuition - the only intuition that we have. Quantum ergodicity would also solve the paradox raised by the quantum classical correspondence for momentum eigenstates. Any preferred extremal in their superposition defining momentum eigenstate should code for the momentum characterizing the superposition itself. This is indeed possible if every extremal in the superposition codes the momentum to the properties of classical correlation functions which are identical for all of them.

2. The only manner to possibly achieve quantum ergodicity is in terms of the statistical properties of the preferred extremals. It should be possible to generalize the ergodic theorem stating that the properties of statistical ensemble are represented by single space-time evolution in the ensemble of time evolutions. Quantum superposition of classical worlds would effectively reduce to single classical world as far as classical correlation functions are considered. The notion of finite measurement resolution suggests that one must state this more precisely by adding that classical correlation functions are calculated in a given UV and IR resolutions meaning UV cutoff defined by the smallest CD and IR cutoff defined by the largest CD present.

3. The skeptic inside me immediately argues that TGD Universe is 4-D spin glass so that this quantum ergodic theorem must be broken. In the case of the ordinary spin classes one has not only statistical average for a fixed Hamiltonian but a statistical average over Hamiltonians. There is a probability distribution over the coupling parameters appearing in the Hamiltonian. Maybe the quantum counterpart of this is needed to predict the physically measurable correlation functions.

   Could this average be an ordinary classical statistical average over quantum states with different classical correlation functions? This kind of average is indeed taken in density matrix formalism. Or could it be that the square root of thermodynamics defined by ZEO actually gives automatically rise to this average? The eigenvalues of the "hermitian square root" of the density matrix would code for components of the state characterized by different classical correlation functions. One could assign these contributions to different "phases".

4. Quantum classical correspondence in statistical sense would be very much like holography (now individual classical state represents the entire quantum state). Quantum ergodicity would pose a rather strong constraint on quantum states. This symmetry principle could actually fix the spectrum of zero energy states to a high degree and fix therefore the M-matrices given by the product of hermitian square root of density matrix and unitary S-matrix and unitary U-matrix constructible as inner products of M-matrices associated with CDs with various size scales [K60].

5. In TGD inspired theory of consciousness the counterpart of quantum ergodicity is the postulate that the space-time geometry provides a symbolic representation for the quantum states and also for the contents of consciousness assignable to quantum jumps between quantum states. Quantum ergodicity would realize this strongly self-referential looking condition. The positive and negative energy parts of zero energy state would be analogous to the initial and final states of quantum jump and the classical correlation functions would code for the contents of consciousness like written formulas code for the thoughts of mathematician and provide a sensory feedback.

How classical correlation functions should be defined?

1. General Coordinate Invariance and Lorentz invariance are the basic constraints on the definition. These are achieved for the space-time regions with Minkowskian signature and 4-D M^4 projection if linear Minkowski coordinates are used. This is equivalent with the contraction of the indices of tensor fields with the space-time projections of M^4 Killing vector fields representing translations. Accepting ths generalization, there is no need to restrict oneself to 4-D M^4 projection and one can also consider also Euclidian regions identifiable as lines of generalized Feynman diagrams.

   Quantum ergodicity very probably however forces to restrict the consideration to Minkowskian and Euclidian space-time regions and various phases associated with them. Also CP_2 Killing vector fields can be projected to space-time surface and give a representation for classical gluon fields. These in turn can be contracted with M^4 Killing vectors giving rise to gluon fields as analogs of graviton fields but with second polarization index replaced with color index.

2. The standard definition for the correlation functions associated with classical time evolution is the appropriate starting point. The correlation function $G_{XY}(\tau)$ for two dynamical variables $X(t)$ and $Y(t)$ is defined as the average $G_{XY}(\tau) = \int_T X(t)Y(t+\tau)dt/T$ over an interval of length T, and one can also consider the limit $T \to \infty$. In the recent case one would replace τ with the difference $m_1 - m_2 = m$ of M^4 coordinates of two points at the preferred extremal and integrate over the points of the extremal to get the average. The finite time interval T is replaced with the volume of causal diamond in a given length scale. Zero energy state with given quantum numbers for positive and negative energy parts of the state defines the initial and final states between which the fields appearing in the correlation functions are defined.

3. What correlation functions should be considered? Certainly one could calculate correlation functions for the induced spinor connection given electro-weak propagators and correlation functions for CP_2 Killing vector fields giving correlation functions for gluon fields using the description in terms of Killing vector fields. If one can uniquely separate from the Fourier transform uniquely a term of form $Z/(p^2 - m^2)$ by its momentum dependence, the coefficient Z can be identified as coupling constant squared for the corresponding gauge potential component and one can in principle deduce coupling constant evolution purely classically. One can imagine of calculating spinorial propagators for string world sheets in the same manner. Note that also the dependence on color quantum numbers would be present so that in principle all that is needed could be calculated for a single preferred extremal without the need to construct QFT limit and to introduce color quantum numbers of fermions as spin like quantum numbers (color quantum numbers corresponds to CP_2 partial wave for the tip of the CD assigned with the particle).

Many detailed speculations about coupling constant evolution to be discussed in the sections below must be taken as innovative guesses doomed to have the eventual fate of guesses. The notion of quantum ergodicity could however be one of the really deep ideas about coupling constant evolution comparable to the notion of p-adic coupling constant evolution. Quantum Ergodicity (briefly

QE) would also state something extremely non-trivial also about the construction of correlation functions and S-matrix. Because this principle is so new, the rest of the chapter does not yet contain any applications of QE. This should not lead the reader to under-estimate the potential power of QE.

3.6 About Deformations Of Known Extremals Of Kähler Action

I have done a considerable amount of speculative guesswork to identify what I have used to call preferred extremals of Kähler action. The difficulty is that the mathematical problem at hand is extremely non-linear and that I do not know about existing mathematical literature relevant to the situation. One must proceed by trying to guess the general constraints on the preferred extremals which look physically and mathematically plausible. The hope is that this net of constraints could eventually chrystallize to Eureka! Certainly the recent speculative picture involves also wrong guesses. The need to find explicit ansatz for the deformations of known extremals based on some common principles has become pressing. The following considerations represent an attempt to combine the existing information to achieve this.

3.6.1 What Might Be The Common Features Of The Deformations Of Known Extremals

The dream is to discover the deformations of all known extremals by guessing what is common to all of them. One might hope that the following list summarizes at least some common features.

Effective three-dimensionality at the level of action

1. Holography realized as effective 3-dimensionality also at the level of action requires that it reduces to 3-dimensional effective boundary terms. This is achieved if the contraction $j^\alpha A_\alpha$ vanishes. This is true if j^α vanishes or is light-like, or if it is proportional to instanton current in which case current conservation requires that CP_2 projection of the space-time surface is 3-dimensional. The first two options for j have a realization for known extremals. The status of the third option - proportionality to instanton current - has remained unclear.
2. As I started to work again with the problem, I realized that instanton current could be replaced with a more general current $j = *B \wedge J$ or concretely: $j^\alpha = \epsilon^{\alpha\beta\gamma\delta} B_\beta J_{\gamma\delta}$, where B is vector field and CP_2 projection is 3-dimensional, which it must be in any case. The contractions of j appearing in field equations vanish automatically with this ansatz.
3. Almost topological QFT property in turn requires the reduction of effective boundary terms to Chern-Simons terms: this is achieved by boundary conditions expressing weak form of electric magnetic duality. If one generalizes the weak form of electric-magnetic duality to $J = \Phi * J$ one has $B = d\Phi$ and j has a vanishing divergence for 3-D CP_2 projection. This is clearly a more general solution ansatz than the one based on proportionality of j with instanton current and would reduce the field equations in concise notation to $Tr(TH^k) = 0$.
4. Any of the alternative properties of the Kähler current implies that the field equations reduce to $Tr(TH^k) = 0$, where T and H^k are shorthands for Maxwellian energy momentum tensor and second fundamental form and the product of tensors is obvious generalization of matrix product involving index contraction.

Could Einstein's equations emerge dynamically?

For j^α satisfying one of the three conditions, the field equations have the same form as the equations for minimal surfaces except that the metric g is replaced with Maxwell energy momentum tensor T.

1. This raises the question about dynamical generation of small cosmological constant Λ: $T = \Lambda g$ would reduce equations to those for minimal surfaces. For $T = \Lambda g$ Kähler-Dirac gamma matrices would reduce to induced gamma matrices and the Kähler-Dirac operator would be

proportional to ordinary Dirac operator defined by the induced gamma matrices. One can also consider weak form for $T = \Lambda g$ obtained by restricting the consideration to a sub-space of tangent space so that space-time surface is only "partially" minimal surface but this option is not so elegant although necessary for other than CP_2 type vacuum extremals.

2. What is remarkable is that $T = \Lambda g$ implies that the divergence of T which in the general case equals to $j^{\beta} J^{\alpha}_{\beta}$ vanishes. This is guaranteed by one of the conditions for the Kähler current. Since also Einstein tensor has a vanishing divergence, one can ask whether the condition to $T = \kappa G + \Lambda g$ could the general condition. This would give Einstein's equations with cosmological term besides the generalization of the minimal surface equations. GRT would emerge dynamically from the non-linear Maxwell's theory although in slightly different sense as conjectured [K99] ! Note that the expression for G involves also second derivatives of the embedding space coordinates so that actually a partial differential equation is in question. If field equations reduce to purely algebraic ones, as the basic conjecture states, it is possible to have $Tr(GH^k) = 0$ and $Tr(gH^k) = 0$ separately so that also minimal surface equations would hold true.

   What is amusing that the first guess for the action of TGD was curvature scalar. It gave analogs of Einstein's equations as a definition of conserved four-momentum currents. The recent proposal would give the analog of ordinary Einstein equations as a dynamical constraint relating Maxwellian energy momentum tensor to Einstein tensor and metric.

3. Minimal surface property is physically extremely nice since field equations can be interpreted as a non-linear generalization of massless wave equation: something very natural for non-linear variant of Maxwell action. The theory would be also very "stringy" although the fundamental action would not be space-time volume. This can however hold true only for Euclidian signature. Note that for CP_2 type vacuum extremals Einstein tensor is proportional to metric so that for them the two options are equivalent. For their small deformations situation changes and it might happen that the presence of G is necessary. The GRT limit of TGD discussed in [K99] [L5] indeed suggests that CP_2 type solutions satisfy Einstein's equations with large cosmological constant and that the small observed value of the cosmological constant is due to averaging and small volume fraction of regions of Euclidian signature (lines of generalized Feynman diagrams).

4. For massless extremals and their deformations $T = \Lambda g$ cannot hold true. The reason is that for massless extremals energy momentum tensor has component T^{vv} which actually quite essential for field equations since one has $H^k_{vv} = 0$. Hence for massless extremals and their deformations $T = \Lambda g$ cannot hold true if the induced metric has Hamilton-Jacobi structure meaning that g^{uu} and g^{vv} vanish. A more general relationship of form $T = \kappa G + \Lambda G$ can however be consistent with non-vanishing T^{vv} but require that deformation has at most 3-D CP_2 projection (CP_2 coordinates do not depend on v).

5. The non-determinism of vacuum extremals suggest for their non-vacuum deformations a conflict with the conservation laws. In, also massless extremals are characterized by a non-determinism with respect to the light-like coordinate but like-likeness saves the situation. This suggests that the transformation of a properly chosen time coordinate of vacuum extremal to a light-like coordinate in the induced metric combined with Einstein's equations in the induced metric of the deformation could allow to handle the non-determinism.

Are complex structure of CP_2 and Hamilton-Jacobi structure of M^4 respected by the deformations?

The complex structure of CP_2 and Hamilton-Jacobi structure of M^4 could be central for the understanding of the preferred extremal property algebraically.

1. There are reasons to believe that the Hermitian structure of the induced metric ((1, 1) structure in complex coordinates) for the deformations of CP_2 type vacuum extremals could be crucial property of the preferred extremals. Also the presence of light-like direction is also an essential elements and 3-dimensionality of M^4 projection could be essential. Hence a good guess is that allowed deformations of CP_2 type vacuum extremals are such that (2, 0) and (0, 2) components the induced metric and/or of the energy momentum tensor vanish. This gives rise to the conditions implying Virasoro conditions in string models in quantization:

$$g_{\xi^i\xi^j} = 0 \ , \quad g_{\overline{\xi}^i\overline{\xi}^j} = 0 \ , \quad i,j = 1,2 \ . \tag{3.6.1}$$

Holomorphisms of CP_2 preserve the complex structure and Virasoro conditions are expected to generalize to 4-dimensional conditions involving two complex coordinates. This means that the generators have two integer valued indices but otherwise obey an algebra very similar to the Virasoro algebra. Also the super-conformal variant of this algebra is expected to make sense.

These Virasoro conditions apply in the coordinate space for CP_2 type vacuum extremals. One expects similar conditions hold true also in field space, that is for M^4 coordinates.

2. The integrable decomposition $M^4(m) = M^2(m)+E^2(m)$ of M^4 tangent space to longitudinal and transversal parts (non-physical and physical polarizations) - Hamilton-Jacobi structure- could be a very general property of preferred extremals and very natural since non-linear Maxwellian electrodynamics is in question. This decomposition led rather early to the introduction of the analog of complex structure in terms of what I called Hamilton-Jacobi coordinates $(u, v, w, \overline{w})$ for M^4. (u, v) defines a pair of light-like coordinates for the local longitudinal space $M^2(m)$ and $(w, \overline{w})$ complex coordinates for $E^2(m)$. The metric would not contain any cross terms between $M^2(m)$ and $E^2(m)$: $g_{uw} = g_{vw} = g_{u\overline{w}} = g_{v\overline{w}} = 0$.

   A good guess is that the deformations of massless extremals respect this structure. This condition gives rise to the analog of the constraints leading to Virasoro conditions stating the vanishing of the non-allowed components of the induced metric. $g_{uu} = g_{vv} = g_{ww} = g_{\overline{w}\overline{w}} = g_{uw} = g_{vw} = g_{u\overline{w}} = g_{v\overline{w}} = 0$. Again the generators of the algebra would involve two integers and the structure is that of Virasoro algebra and also generalization to super algebra is expected to make sense. The moduli space of Hamilton-Jacobi structures would be part of the moduli space of the preferred extremals and analogous to the space of all possible choices of complex coordinates. The analogs of infinitesimal holomorphic transformations would preserve the modular parameters and give rise to a 4-dimensional Minkowskian analog of Virasoro algebra. The conformal algebra acting on CP_2 coordinates acts in field degrees of freedom for Minkowskian signature.

Field equations as purely algebraic conditions

If the proposed picture is correct, field equations would reduce basically to purely algebraically conditions stating that the Maxwellian energy momentum tensor has no common index pairs with the second fundamental form. For the deformations of CP_2 type vacuum extremals T is a complex tensor of type (1, 1) and second fundamental form H^k a tensor of type (2, 0) and (0, 2) so that $Tr(TH^k) =$ is true. This requires that second light-like coordinate of M^4 is constant so that the M^4 projection is 3-dimensional. For Minkowskian signature of the induced metric Hamilton-Jacobi structure replaces conformal structure. Here the dependence of CP_2 coordinates on second light-like coordinate of $M^2(m)$ only plays a fundamental role. Note that now T^{vv} is non-vanishing (and light-like). This picture generalizes to the deformations of cosmic strings and even to the case of vacuum extremals.

3.6.2 What Small Deformations Of CP_2 Type Vacuum Extremals Could Be?

I was led to these arguments when I tried find preferred extremals of Kähler action, which would have 4-D CP_2 and M^4 projections - the Maxwell phase analogous to the solutions of Maxwell's equations that I conjectured long time ago. It however turned out that the dimensions of the projections can be $(D_{M^4} \leq 3, D_{CP_2} = 4)$ or $(D_{M^4} = 4, D_{CP_2} \leq 3)$. What happens is essentially breakdown of linear superposition so that locally one can have superposition of modes which have 4-D wave vectors in the same direction. This is actually very much like quantization of radiation field to photons now represented as separate space-time sheets and one can say that Maxwellian superposition corresponds to union of separate photonic space-time sheets in TGD.

Approximate linear superposition of fields is fundamental in standard physics framework and is replaced in TGD with a linear superposition of effects of classical fields on a test particle topologically condensed simultaneously to several space-time sheets. One can say that linear superposition is replaced with a disjoint union of space-time sheets. In the following I shall restrict the consideration to the deformations of CP_2 type vacuum extremals.

Solution ansatz

I proceed by the following arguments to the ansatz.

1. Effective 3-dimensionality for action (holography) requires that action decomposes to vanishing $j^\alpha A_\alpha$ term + total divergence giving 3-D "boundary" terms. The first term certainly vanishes (giving effective 3-dimensionality) for

$$D_\beta J^{\alpha\beta} = j^\alpha = 0 \ .$$

   Empty space Maxwell equations, something extremely natural. Also for the proposed GRT limit these equations are true.

2. How to obtain empty space Maxwell equations $j^\alpha = 0$? The answer is simple: assume self duality or its slight modification:

$$J = *J$$

   holding for CP_2 type vacuum extremals or a more general condition

$$J = k * J \ ,$$

   In the simplest situation k is some constant not far from unity. * is Hodge dual involving 4-D permutation symbol. $k = constant$ requires that the determinant of the induced metric is apart from constant equal to that of CP_2 metric. It does not require that the induced metric is proportional to the CP_2 metric, which is not possible since M^4 contribution to metric has Minkowskian signature and cannot be therefore proportional to CP_2 metric.

   One can consider also a more general situation in which k is scalar function as a generalization of the weak electric-magnetic duality. In this case the Kähler current is non-vanishing but divergenceless. This also guarantees the reduction to $Tr(TH^k) = 0$. In this case however the proportionality of the metric determinant to that for CP_2 metric is not needed. This solution ansatz becomes therefore more general.

3. Field equations reduce with these assumptions to equations differing from minimal surfaces equations only in that metric g is replaced by Maxwellian energy momentum tensor T. Schematically:

$$Tr(TH^k) = 0 \ ,$$

   where T is the Maxwellian energy momentum tensor and H^k is the second fundamental form - asymmetric 2-tensor defined by covariant derivative of gradients of embedding space coordinates.

How to satisfy the condition $Tr(TH^k) = 0$?

It would be nice to have minimal surface equations since they are the non-linear generalization of massless wave equations. It would be also nice to have the vanishing of the terms involving Kähler current in field equations as a consequence of this condition. Indeed, $T = \kappa G + \Lambda g$ implies this. In the case of CP_2 vacuum extremals one cannot distinguish between these options since CP_2 itself is constant curvature space with $G \propto g$. Furthermore, if G and g have similar tensor structure the algebraic field equations for G and g are satisfied separately so that one obtains minimal surface property also now. In the following minimal surface option is considered.

1. The first opton is achieved if one has

$$T = \Lambda g \ .$$

Maxwell energy momentum tensor would be proportional to the metric! One would have dynamically generated cosmological constant! This begins to look really interesting since it appeared also at the proposed GRT limit of TGD [L5] (see `http://tinyurl.com/hzkldnb`). Note that here also non-constant value of Λ can be considered and would correspond to a situation in which k is scalar function: in this case the the determinant condition can be dropped and one obtains just the minimal surface equations.

2. Very schematically and forgetting indices and being sloppy with signs, the expression for T reads as

$$T = JJ - g/4Tr(JJ) \ .$$

Note that the product of tensors is obtained by generalizing matrix product. This should be proportional to metric.

Self duality implies that $Tr(JJ)$ is just the instanton density and does not depend on metric and is constant.

For CP_2 type vacuum extremals one obtains

$$T = -g + g = 0 \ .$$

Cosmological constant would vanish in this case.

3. Could it happen that for deformations a small value of cosmological constant is generated? The condition would reduce to

$$JJ = (\Lambda - 1)g \ .$$

Λ must relate to the value of parameter k appearing in the generalized self-duality condition. For the most general ansatz Λ would not be constant anymore.

This would generalize the defining condition for Kähler form

$$JJ = -g \ \ (i^2 = -1 \ \ geometrically)$$

stating that the square of Kähler form is the negative of metric. The only modification would be that index raising is carried out by using the induced metric containing also M^4 contribution rather than CP_2 metric.

4. Explicitly:

$$J_{\alpha\mu}J^{\mu}_{\ \beta} = (\Lambda - 1)g_{\alpha\beta} \ .$$

Cosmological constant would measure the breaking of Kähler structure. By writing $g = s + m$ and defining index raising of tensors using CP_2 metric and their product accordingly, this condition can be also written as

$$Jm = (\Lambda - 1)mJ \ .$$

If the parameter k is constant, the determinant of the induced metric must be proportional to the CP_2 metric. If k is scalar function, this condition can be dropped. Cosmological constant would not be constant anymore but the dependence on k would drop out from the field equations and one would hope of obtaining minimal surface equations also now. It however seems that the dimension of M^4 projection cannot be four. For 4-D M^4 projection the contribution of the M^2 part of the M^4 metric gives a non-holomorphic contribution to CP_2 metric and this spoils the field equations.

For $T = \kappa G + \Lambda g$ option the value of the cosmological constant is large - just as it is for the proposed GRT limit of TGD [K99] [L5]. The interpretation in this case is that the average value of cosmological constant is small since the portion of space-time volume containing generalized Feynman diagrams is very small.

More detailed ansatz for the deformations of CP_2 type vacuum extremals

One can develop the ansatz to a more detailed form. The most obvious guess is that the induced metric is apart from constant conformal factor the metric of CP_2. This would guarantee self-duality apart from constant factor and $j^\alpha = 0$. Metric would be in complex CP_2 coordinates tensor of type (1, 1) whereas CP_2 Riemann connection would have only purely holomorphic or anti-holomorphic indices. Therefore CP_2 contributions in $Tr(TH^k)$ would vanish identically. M^4 degrees of freedom however bring in difficulty. The M^4 contribution to the induced metric should be proportional to CP_2 metric and this is impossible due to the different signatures. The M^4 contribution to the induced metric breaks its Kähler property but would preserve Hermitian structure.

A more realistic guess based on the attempt to construct deformations of CP_2 type vacuum extremals is following.

1. Physical intuition suggests that M^4 coordinates can be chosen so that one has integrable decomposition to longitudinal degrees of freedom parametrized by two light-like coordinates u and v and to transversal polarization degrees of freedom parametrized by complex coordinate w and its conjugate. M^4 metric would reduce in these coordinates to a direct sum of longitudinal and transverse parts. I have called these coordinates Hamilton-Jacobi coordinates.
2. w would be holomorphic function of CP_2 coordinates and therefore satisfy the analog of massless wave equation. This would give hopes about rather general solution ansatz. u and v cannot be holomorphic functions of CP_2 coordinates. Unless wither u or v is constant, the induced metric would receive contributions of type (2, 0) and (0, 2) coming from u and v which would break Kähler structure and complex structure. These contributions would give no-vanishing contribution to all minimal surface equations. Therefore either u or v is constant: the coordinate line for non-constant coordinate -say u- would be analogous to the M^4 projection of CP_2 type vacuum extremal.
3. With these assumptions the induced metric would remain $(1,1)$ tensor and one might hope that $Tr(TH^k)$ contractions vanishes for all variables except u because the there are no common index pairs (this if non-vanishing Christoffel symbols for H involve only holomorphic or anti-holomorphic indices in CP_2 coordinates). For u one would obtain massless wave equation expressing the minimal surface property.
4. If the value of k is constant the determinant of the induced metric must be proportional to the determinant of CP_2 metric. The induced metric would contain only the contribution from the transversal degrees of freedom besides CP_2 contribution. Minkowski contribution has however rank 2 as CP_2 tensor and cannot be proportional to CP_2 metric. It is however enough that its determinant is proportional to the determinant of CP_2 metric with constant proportionality coefficient. This condition gives an additional non-linear condition to the solution. One would have wave equation for u (also w and its conjugate satisfy massless wave equation) and determinant condition as an additional condition.

   The determinant condition reduces by the linearity of determinant with respect to its rows to sum of conditions involved 0, 1, 2 rows replaced by the transversal M^4 contribution to metric given if M^4 metric decomposes to direct sum of longitudinal and transversal parts. Derivatives with respect to derivative with respect to particular CP_2 complex coordinate appear linearly in this expression they can depend on u via the dependence of transversal metric components on u. The challenge is to show that this equation has (or does not have) non-trivial solutions.
5. If the value of k is scalar function the situation changes and one has only the minimal surface equations and Virasoro conditions.

What makes the ansatz attractive is that special solutions of Maxwell empty space equations are in question, equations reduces to non-linear generalizations of Euclidian massless wave equations, and possibly space-time dependent cosmological constant pops up dynamically. These properties are true also for the GRT limit of TGD [L5] (see `http://tinyurl.com/hzkldnb`).

3.6.3 Hamilton-Jacobi Conditions In Minkowskian Signature

The maximally optimistic guess is that the basic properties of the deformations of CP_2 type vacuum extremals generalize to the deformations of other known extremals such as massless extremals, vacuum extremals with 2-D CP_2 projection which is Lagrangian manifold, and cosmic strings characterized by Minkowskian signature of the induced metric. These properties would be following.

1. The recomposition of M^4 tangent space to longitudinal and transversal parts giving Hamilton-Jacobi structure. The longitudinal part has hypercomplex structure but the second light-like coordinate is constant: this plays a crucial role in guaranteeing the vanishing of contractions in $Tr(TH^k)$. It is the algebraic properties of g and T which are crucial. T can however have light-like component T^{vv}. For the deformations of CP_2 type vacuum extremals $(1,1)$ structure is enough and is guaranteed if second light-like coordinate of M^4 is constant whereas w is holomorphic function of CP_2 coordinates.
2. What could happen in the case of massless extremals? Now one has 2-D CP_2 projection in the initial situation and CP_2 coordinates depend on light-like coordinate u and single real transversal coordinate. The generalization would be obvious: dependence on single light-like coordinate u and holomorphic dependence on w for complex CP_2 coordinates. The constraint is $T = \Lambda g$ cannot hold true since T^{vv} is non-vanishing (and light-like). This property restricted to transversal degrees of freedom could reduce the field equations to minimal surface equations in transversal degrees of freedom. The transversal part of energy momentum tensor would be proportional to metric and hence covariantly constant. Gauge current would remain light-like but would not be given by $j = *d\phi \wedge J$. $T = \kappa G + \Lambda g$ seems to define the attractive option.

It therefore seems that the essential ingredient could be the condition

$$T = \kappa G + \lambda g \ ,$$

which has structure (1, 1) in both $M^2(m)$ and $E^2(m)$ degrees of freedom apart from the presence of T^{vv} component with deformations having no dependence on v. If the second fundamental form has (2, 0)+(0, 2) structure, the minimal surface equations are satisfied provided Kähler current satisfies on of the proposed three conditions and if G and g have similar tensor structure.

One can actually pose the conditions of metric as complete analogs of stringy constraints leading to Virasoro conditions in quantization to give

$$g_{uu} = 0 \ , \quad g_{vv} = 0 \ , \quad g_{ww} = 0 \ , \quad g_{\overline{w}\overline{w}} = 0 \ . \tag{3.6.2}$$

This brings in mind the generalization of Virasoro algebra to four-dimensional algebra for which an identification in terms of non-local Yangian symmetry [A20] [B23, B20, B21] has been proposed [K94]. The number of conditions is four and the same as the number of independent field equations. One can consider similar conditions also for the energy momentum tensor T but allowing non-vanishing component T^{vv} if deformations has no v-dependence. This would solve the field equations if the gauge current vanishes or is light-like. On this case the number of equations is 8. First order differential equations are in question and they can be also interpreted as conditions fixing the coordinates used since there is infinite number of ways to choose the Hamilton-Jacobi coordinates.

One can can try to apply the physical intuition about general solutions of field equations in the linear case by writing the solution as a superposition of left and right propagating solutions:

$$\xi^k = f_+^k(u,w) + f_+^k(v,w) \ . \tag{3.6.3}$$

This could guarantee that second fundamental form is of form (2, 0)+(0, 2) in both M^2 and E^2 part of the tangent space and these terms if $Tr(TH^k)$ vanish identically. The remaining terms involve contractions of T^{uw}, $T^{u\overline{w}}$ and T^{vw}, $T^{v\overline{w}}$ with second fundamental form. Also these terms should sum up to zero or vanish separately. Second fundamental form has components coming from f_+^k and f_-^k

Second fundamental form H^k has as basic building bricks terms $\hat{H}^k$ given by

$$\hat{H}^k_{\alpha\beta} \quad = \quad \partial_\alpha\partial_\beta h^k + \binom{k}{l\ m} \partial_\alpha h^l \partial_\beta h^m \ . \tag{3.6.4}$$

For the proposed ansatz the first terms give vanishing contribution to H^k_{uv}. The terms containing Christoffel symbols however give a non-vanishing contribution and one can allow only f^k_+ or f^k_- as in the case of massless extremals. This reduces the dimension of CP_2 projection to $D = 3$.

What about the condition for Kähler current? Kähler form has components of type $J_{w\overline{w}}$ whose contravariant counterpart gives rise to space-like current component. J_{uw} and $J_{u\overline{w}}$ give rise to light-like currents components. The condition would state that the $J^{w\overline{w}}$ is covariantly constant. Solutions would be characterized by a constant Kähler magnetic field. Also electric field is represent. The interpretation both radiation and magnetic flux tube makes sense.

3.6.4 Deformations Of Cosmic Strings

In the physical applications it has been assumed that the thickening of cosmic strings to Kähler magnetic flux tubes takes place. One indeed expects that the proposed construction generalizes also to the case of cosmic strings having the decomposition $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, where X^2 is minimal surface and Y^2 a complex homologically non-trivial sub-manifold of CP_2. Now the starting point structure is Hamilton-Jacobi structure for $M^2_m \times Y^2$ defining the coordinate space.

1. The deformation should increase the dimension of either CP_2 or M^4 projection or both. How this thickening could take place? What comes in mind that the string orbits X^2 can be interpreted as a distribution of longitudinal spaces $M^2(x)$ so that for the deformation w coordinate becomes a holomorphic function of the natural Y^2 complex coordinate so that M^4 projection becomes 4-D but CP_2 projection remains 2-D. The new contribution to the X^2 part of the induced metric is vanishing and the contribution to the Y^2 part is of type $(1,1)$ and the ansatz $T = \kappa G + \Lambda g$ might be needed as a generalization of the minimal surface equations The ratio of κ and G would be determined from the form of the Maxwellian energy momentum tensor and be fixed at the limit of undeformed cosmic strong to $T = (ag(Y^2) - bg(Y^2)$. The value of cosmological constant is now large, and overall consistency suggests that $T = \kappa G + \Lambda g$ is the correct option also for the CP_2 type vacuum extremals.

2. One could also imagine that remaining CP_2 coordinates could depend on the complex coordinate of Y^2 so that also CP_2 projection would become 4-dimensional. The induced metric would receive holomorphic contributions in Y^2 part. As a matter fact, this option is already implied by the assumption that Y^2 is a complex surface of CP_2.

3.6.5 Deformations Of Vacuum Extremals?

What about the deformations of vacuum extremals representable as maps from M^4 to CP_2?

1. The basic challenge is the non-determinism of the vacuum extremals. One should perform the deformation so that conservation laws are satisfied. For massless extremals there is also non-determinism but it is associated with the light-like coordinate so that there are no problems with the conservation laws. This would suggest that a properly chosen time coordinate consistent with Hamilton-Jacobi decomposition becomes light-like coordinate in the induced metric. This poses a conditions on the induced metric.

2. Physical intuition suggests that one cannot require $T = \Lambda g$ since this would mean that the rank of T is maximal whereas the original situation corresponds to the vanishing of T. For small deformations rank two for T looks more natural and one could think that T is proportional to a projection of metric to a 2-D subspace. The vision about the long length scale limit of TGD is that Einstein's equations are satisfied and this would suggest $T = kG$ or $T = \kappa G + \Lambda g$. The rank of T could be smaller than four for this ansatz and this conditions binds together the values of κ and G.

3. These extremals have CP_2 projection which in the generic case is 2-D Lagrangian sub-manifold Y^2. Again one could assume Hamilton-Jacobi coordinates for X^4. For CP_2 one could assume Darboux coordinates (P_i, Q_i), $i = 1, 2$, in which one has $A = P_i dQ^i$, and that $Y^2 \subset CP_2$ corresponds to $Q_i = constant$. In principle P_i would depend on arbitrary manner on M^4 coordinates. It might be more convenient to use as coordinates (u, v) for M^2 and (P_1, P_2) for Y^2. This covers also the situation when M^4 projection is not 4-D. By its 2-dimensionality

Y^2 allows always a complex structure defined by its induced metric: this complex structure is not consistent with the complex structure of CP_2 (Y^2 is not complex sub-manifold).

Using Hamilton-Jacobi coordinates the pre-image of a given point of Y^2 is a 2-dimensional sub-manifold X^2 of X^4 and defines also 2-D sub-manifold of M^4. The following picture suggests itself. The projection of X^2 to M^4 can be seen for a suitable choice of Hamilton-Jacobi coordinates as an analog of Lagrangian sub-manifold in M^4 that is as surface for which v and $Im(w)$ vary and u and $Re(w)$ are constant. X^2 would be obtained by allowing u and $Re(w)$ to vary: as a matter fact, (P_1, P_2) and $(u, Re(w))$ would be related to each other. The induced metric should be consistent with this picture. This would requires $g_{uRe(w)} = 0$.

For the deformations Q_1 and Q_2 would become non-constant and they should depend on the second light-like coordinate v only so that only g_{uu} and g_{uw} and $gu\overline{w}$ $g_{w,w}$ and $g_{\overline{w},\overline{w}}$ receive contributions which vanish. This would give rise to the analogs of Virasoro conditions guaranteeing that T is a tensor of form $(1,1)$ in both M^2 and E^2 indices and that there are no cross components in the induced metric. A more general formulation states that energy momentum tensor satisfies these conditions. The conditions on T might be equivalent with the conditions for g and G separately.

4. Einstein's equations provide an attractive manner to achieve the vanishing of effective 3-dimensionality of the action. Einstein equations would be second order differential equations and the idea that a deformation of vacuum extremal is in question suggests that the dynamics associated with them is in directions transversal to Y^2 so that only the deformation is dictated partially by Einstein's equations.

5. Lagrangian manifolds do not involve complex structure in any obvious manner. One could however ask whether the deformations could involve complex structure in a natural manner in CP_2 degrees of freedom so that the vanishing of g_{ww} would be guaranteed by holomorphy of CP_2 complex coordinate as function of w.

   One should get the complex structure in some natural manner: in other words, the complex structure should relate to the geometry of CP_2 somehow. The complex coordinate defined by say $z = P_1 + iQ^1$ for the deformation suggests itself. This would suggest that at the limit when one puts $Q_1 = 0$ one obtains $P_1 = P_1(Re(w))$ for the vacuum extremals and the deformation could be seen as an analytic continuation of real function to region of complex plane. This is in spirit with the algebraic approach. The vanishing of Kähler current requires that the Kähler magnetic field is covariantly constant: $D_z J^{z\overline{z}} = 0$ and $D_{\overline{z}} J^{z\overline{z}} = 0$.

6. One could consider the possibility that the resulting 3-D sub-manifold of CP_2 can be regarded as contact manifold with induced Kähler form non-vanishing in 2-D section with natural complex coordinates. The third coordinate variable- call it s- of the contact manifold and second coordinate of its transversal section would depend on time space-time coordinates for vacuum extremals. The coordinate associated with the transversal section would be continued to a complex coordinate which is holomorphic function of w and u.

7. The resulting thickened magnetic flux tubes could be seen as another representation of Kähler magnetic flux tubes: at this time as deformations of vacuum flux tubes rather than cosmic strings. For this ansatz it is however difficult to imagine deformations carrying Kähler electric field.

3.6.6 About The Interpretation Of The Generalized Conformal Algebras

The long-standing challenge has been finding of the direct connection between the super-conformal symmetries assumed in the construction of the geometry of the "world of classical worlds" (WCW) and possible conformal symmetries of field equations. 4-dimensionality and Minkowskian signature have been the basic problems. The recent construction provides new insights to this problem.

1. In the case of string models the quantization of the Fourier coefficients of coordinate variables of the target space gives rise to Kac-Moody type algebra and Virasoro algebra generators are quadratic in these. Also now Kac-Moody type algebra is expected. If one were to perform a quantization of the coefficients in Laurents series for complex CP_2 coordinates, one would obtain interpretation in terms of $su(3) = u(2) + t$ decomposition, where t corresponds to CP_3:

the oscillator operators would correspond to generators in t and their commutator would give generators in $u(2)$. SU(3)/SU(2) coset representation for Kac-Moody algebra would be in question. Kac-Moody algebra would be associated with the generators in both M^4 and CP_2 degrees of freedom. This kind of Kac-Moody algebra appears in quantum TGD.

2. The constraints on induced metric imply a very close resemblance with string models and a generalization of Virasoro algebra emerges. An interesting question is how the two algebras acting on coordinate and field degrees of freedom relate to the super-conformal algebras defined by the symplectic group of $\delta M^4_+ \times CP_2$ acting on space-like 3-surfaces at boundaries of CD and to the Kac-Moody algebras acting on light-like 3-surfaces. It has been conjectured that these algebras allow a continuation to the interior of space-time surface made possible by its slicing by 2-surfaces parametrized by 2-surfaces. The proposed construction indeed provides this kind of slicings in both M^4 and CP_2 factor.

3. In the recent case, the algebras defined by the Fourier coefficients of field variables would be Kac-Moody algebras. Virasoro algebra acting on preferred coordinates would be expressed in terms of the Kac-Moody algebra in the standard Sugawara construction applied in string models. The algebra acting on field space would be analogous to the conformal algebra assignable to the symplectic algebra so that also symplectic algebra is present. Stringy pragmatist could imagine quantization of symplectic algebra by replacing CP_2 coordinates in the expressions of Hamiltonians with oscillator operators. This description would be counterpart for the construction of spinor harmonics in WCW and might provide some useful insights.

4. For given type of space-time surface either CP_2 or M^4 corresponds to Kac-Moody algebra but not both. From the point of view of quantum TGD it looks as that something were missing. An analogous problem was encountered at GRT limit of TGD [L5]. When Euclidian space-time regions are allowed Einstein-Maxwell action is able to mimic standard model with a surprising accuracy but there is a problem: one obtains either color charges or M^4 charges but not both. Perhaps it is not enough to consider either CP_2 type vacuum extremal or its exterior but both to describe particle: this would give the direct product of the Minkowskian and Euclidian algebras acting on tensor product. This does not however seem to be consistent with the idea that the two descriptions are duality related (the analog of T-duality).

3.7 About TGD counterparts of classical field configurations in Maxwell's theory

Classical physics is an exact part of TGD so that the study of extremals of dimensionally reduces 6-D Kähler action can provide a lot of intuition about quantum TGD and see how quantum-classical correspondence is realized. In the following I will try to develop further understanding about TGD counterparts of the simplest field configurations in Maxwell's theory.

In the sequel CP_2 type extremals will be considered from the point of view of quantum criticality and the view about string world sheets, their lightlike boundaries as carriers of fermion number, and the ends as point like particles as singularities acting as sources for minimal surfaces satisfying non-linear generalization of d'Alembert equation.

I will also discuss the delicacies associated with M^4 Kähler structure and its connection with what I call Hamilton-Jacobi structure and with M^8 approach based on classical number fields. I will argue that the breaking of CP symmetry associated with M^4 Kähler structure is small without any additional assumptions: this is in contrast with the earlier view.

The difference between TGD and Maxwell's theory and consider the TGD counterparts of simple em field configurations will be also discussed. Topological field quantization provides a geometric view about formation of atoms as bound states based on flux tubes as correlates for binding, and allows to identify space-time correlates for second quantization. These considerations force to take seriously the possibility that preferred extremals besides being minimal surfaces also possess generalized holomorphy reducing field equations to purely algebraic conditions and that minimal surfaces without this property are not preferred extremals. If so, at microscopic level only CP_2 type extremals, massless extremals, and string like objects and their deformations would exist as preferred extremals and serve as building bricks for the counterparts of Maxwellian field configurations and the counterparts of Maxwellian field configurations such as Coulomb potential would emerge only at the QFT limit.

3.7.1 About differences between Maxwell's ED and TGD

TGD differs from Maxwell's theory in several important aspects.

1. The TGD counterparts of classical electroweak gauge potentials are induced from component of spinor connection of CP_2. Classical color gauge potentials corresponds to the projections of Killing vector fields of color isometries.
2. Also M^4 has Kähler potential, which is induced to space-time surface and gives rise to an additional $U(1)$ force. The couplings of M^4 gauge potential to quarks and leptons are of same sign whereas the couplings of CP_2 Kähler potential to B and L are of opposite sign so that the contributions to 6-D Kähler action reduce to separate terms without interference term.

   Coupling to induced M^4 Kähler potential implies CP breaking. This could explain the small CP breaking in hadronic systems and also matter antimatter asymmetry in which there are opposite matter-antimatter asymmetries inside cosmic strings and their exteriors respectively. A priori it is however not obvious that the CP breaking is small.
3. General coordinate invariance implies that there are only 4 local field like degrees of freedom so that for extremals with 4-D M^4 projection corresponding to GRT space-time both metric, electroweak and color gauge potentials can be expressed in terms four CP_2 coordinates and their gradients. Preferred extremal property realized as minimal surface condition means that field equations are satisfied separately for the 4-D Kähler and volume action reduces the degrees of freedom further.

   If the CP_2 part of Kähler form is non-vanishing, minimal surface conditions can be guaranteed by a generalization of holomorphy realizing quantum criticality (satisfied by known extremals). One can say that there is no dependence on coupling parameters. If CP_2 part of Kähler form vanishes identically, the minimal surface condition need not be guaranteed by holomorphy. It is not at all clear whether quantum criticality and preferred extremal property allow this kind of extremals.
4. Supersymplectic symmetries act as isometries of "world of classical worlds" (WCW). In a well-defined sense supersymplectic symmetry generalizes 2-D conformal invariance to 4-D context. The key observation here is that light-like 3-surfaces are metrically 2-D and therefore allow extended conformal invariance.

   Preferred extremal property realizing quantum criticality boils down to a condition that sub-algebra of SSA and its commutator with SSA annihilate physical states and that corresponding Noether charges vanish. These conditions could be equivalent with minimal surface property. This implies that the set of possible field patterns is extremely restricted and one might talk about "archetypal" field patterns analogous to partial waves or plane waves in Maxwell's theory.
5. Linear superposition of the archetypal field patterns is not possible. TGD however implies the notion of many-sheeted space-time and each sheet can carry its own field pattern. A test particle which is space-time surface itself touches all these sheets and experiences the sum of the effects caused by fields at various sheets. Effects are superposed rather than fields and this is enough. This means weakening of the superposition principle of Maxwell's theory and the linear superposition of fields at same space-time sheet is replaced with set theoretic union of space-time sheets carrying the field patterns whose effects superpose.

   This observation is also essential in the construction of QFT limit of TGD. The gauge potentials in standard model and gravitational field in general relativity are superpositions of those associated with space-time sheets idealized with slightly curved piece of Minkowski space M^4.
6. An important implication is that each system has field identity - field body or magnetic body (MB). In Maxwell's theory superposition of fields coming from different sources leads to a loss of information since one does not anymore now which part of field came from particular source. In TGD this information loss does not happen and this is essential for TGD inspired quantum biology.

Remark: An interesting algebraic analog is the notion of co-algebra. Co-product is analogous to reversal of product AB= C in the sense that it assigns to C and a linear combination of products $\sum A_i \otimes B_i$ such that $A_i B_i = C$. Quantum groups and co-algebras are indeed important in TGD and it might be that there is a relationship. In TGD inspired quantum biology magnetic body plays a key role as an intentional agent receiving sensory data from biological body and using it as motor instrument.

7. I have already earlier considered a space-time correlate for second quantization in terms of sheets of covering for $h_{eff} = nh_0$. In [L49] it is proposed that n factorizes as $n = n_1 n_2$ such that n_1 (n_2) is the number sheets for space-time surface as covering of CP_2 (M^4). One could have quantum mechanical linear superposition of space-time sheets, each with a particular field pattern. This kind state would correspond to single particle state created by quantum field in QFT limit. For instance, one could have spherical harmonic for orientations of magnetic flux tube or electric flux tube.

   One could also have superposition of configurations containing several space-time sheets simultaneously as analogs of many-boson states. Many-sheeted space-time would correspond to this kind many-boson states. Second quantization in quantum field theory (QFT) could be seen as an algebraic description of many-sheetedness having no obvious classical correlate in classical QFT.

8. Flux tubes should be somehow different for gravitational fields, em fields, and also weak and color gauge fields. The value of $n = n_1 n_2$ [L49] for gravitational flux tubes is very large by Nottale formula $\hbar_{eff} = \hbar_{gr} = GMm/v_0$. The value of n_2 for gravitational flux tubes is $n_2 \sim 10^7$ if one accepts the formula $G = R^2/n_2\hbar$. For em fields much smaller values of n and therefore of n_2 are suggestive. There the value of n measuring in adelic physics algebraic complexity and evolutionary level would distinguish between gravitational and em flux tubes.

   Large value of n would mean quantum coherence in long scales. For gravitation this makes sense since screening is absent unlike for gauge interactions. Note that the large value of $h_{eff} = h_{gr}$ implies that $\alpha_{em} = e^2/4\pi\hbar_{eff}$ is extremely small for gravitational flux tubes so that they would indeed be gravitational in an excellent approximation.

   n would be the dimension of extension of rationals involved and n_2 would be the number space-time sheets as covering of M^4. If this picture is correct, gravitation would correspond to much larger algebraic complexity and much larger value of Planck constant. This conforms with the intuition that gravitation plays essential role in the quantum physics of living matter.

   There are also other number theoretic characteristics such as ramified primes of the extension identifiable as preferred p-adic primes in turn characterizing elementary particle. Also flux tubes mediating weak and strong interactions should allow characterization in terms of number theoretic parameters. There are arguments that in atomic physics one has $h = 6h_0$. Since the quantum coherence scale of hadrons is smaller than atomic scale, one can ask whether one could have $h_{eff} < h$.

3.7.2 CP_2 type extremals as ultimate sources of fields and singularities

CP_2 type extremals have Euclidian signature of induced metric and therefore represent the most radical deviation from Maxwell's ED, gauge theories, and GRT. CP_2 type extremal with light-like geodesic as M^4 projection represents a model for wormhole contact. The light-like orbit of partonic 2-surface correspond to boundary between wormhole contact and Minkowskian region and is associated with both throats of wormhole contact. The throats of wormhole contact can carry part of a boundary of string world sheet connecting the partonic orbits associated with different particles. These light-like lines can carry fermion number and would correspond to lines of TGD counterparts of twistor diagrams.

These world lines would correspond to singularities for the minimal surface equations analogous to sources of massless vector fields carrying charge [L48, L55]. These singularities would serve as ultimate sources of classical em fields. Various currents would consist of wormhole throat pairs representing elementary particle and carrying charges at the partonic orbits. Two-sheetedness is essential and could be interpreted in terms of a double covering formed by space-time sheet glued along their common boundary. This necessary since space-time sheet has a finite size being not continuable beyond certain minimal size as preferred extremal since some of the real coordinates would become complex.

Quantum criticality for CP_2 type extremals

TGD predicts a hierarchy of quantum criticalities. The increase in criticality means that some space-time sheets for space-time surface regarded as a covering with sheets related by Galois group of extension of rationals degenerate to single sheet. The action of Galois group would reduce to that for its subgroup.

This is analogous to the degeneration of some roots of polynomial to single root and in M^8 representation space-time sheets are indeed quite concretely roots of octonionic polymomial defined by vanishing of real or imaginary part in the decomposition $o = q_1 + iq_2$ of octonion to a sum quaternionic real and imaginary parts.

The hierarchy of criticalities is closely related to the hierarchy of Planck constants $h_{eff}/h_0 = n = n_1n_2$, where n_1 corresponds to number of sheets as covering over CP_2 and n_2 as covering over M^4. One can also consider special cases in which M^4 projection has dimension $D < 4$. The proposal is that n corresponds to the dimension of Galois group for extension of rationals defining the level of dark matter hierarchy. If n is prime, one has either $n_1 = 1$ or $n_2 = 1$.

It seems that the range of n_2 is rather limited since the expression for Newton's constant as $G = R^2/n_2\hbar$ varies in rather narrow range. If the covering has symmetries assignable to some discrete subgroup of $SU(3)$ acting as isometries of CP_2 this could be understood. The increase of criticality could mean that n_1 or n_2 or both are reduced.

What is the position of CP_2 type extremals in the hierarchies of Planck constants and quantum criticalities?

1. Consider first n_2. CP_2 type extremal have 1-D geodesic line as M^4 projection. The light-like geodesic as 1-D structure could be interpreted as covering for which two geodesic lines along the orbits of opposite throats of wormhole contact form a kind of time loop. In this case one would have $n_2 = 2$ and one could have $n = 2p$, p prime.

   In this sense CP_2 type extremal or at least its core would be maximally critical. Deformations replacing the light-like geodesic as projection with higher-D region of M^4 presumably reduce criticality and one has $n_2 > 2$ is obtained. Whether this is possible inside wormhole contact is not clear. One can imagine that as one approaches partonic 2-surface, the criticality and degeneration increase in CP_2 degrees of freedom step by step and reach maximum in its core. This would be like realization of Thom's catastrophe involving parts with various degrees of criticalities.

   At the flux tubes mediating gravitational interaction $n_2 \sim 10^7$ would hold true in the exterior of associated CP_2 type extremals. This would suggests that CP_2 type extremals have maximal criticality in M^4 degrees of freedom and M^4 covering reduces to 2-fold covering for wormhole contacts.

2. What about criticality as n_1-fold covering of CP_2. This covering corresponds to a situation in which CP_2 coordinates as field in M^4 have given values of CP_2 coordinates n_1 times. A lattice like structure formed by n_1 wormhole contacts is suggestive. n_1 can be arbitrary large in principle and the gravitational Planck constant $h_{gr}/h_0 = n_1n_2$ would correspond to this situation. Singularities would now correspond to a degeneration of some wormhole contacts to single wormhole contact and could have interpretation in terms of fusion of particles to single particle. One might perhaps interpret elementary particle reaction vertices as catastrophes.

Wormhole contacts can be regarded as CP_2 type extremals having two holes corresponding to the 3-D orbits of wormhole contacts. Mathematician would probably speak of a blow up. CP_2 type extremals is glued to surrounding Minkowskian space-time sheets at the 3-D boundaries of these holes. At the orbit of partonic 2-surface the induced 4-metric degenerates to 3-D metric and 4-D tangent space becomes metrically 3-D. Light-likeness of the M^4 projection would correspond to this. For CP_2 type extremal 3 space-like M^4 directions of Minkowskian region would transmute to CP_2 directions at the light-like geodesic and time direction would become light-like. This is like graph of function for which tangent becomes vertical. For deformations of CP_2 type extremals this process could take place in several steps, one dimension in given step. This process could take place inside CP_2 or outside it depending on which order the transmutation of dimensions takes place.

3.7.3 Delicacies associated with M^4 Kähler structure

Twistor lift forces to assume that also M^4 possesses the analog of Kähler form, and Minkowskian signature does not prevent this [K18]. M^4 Kähler structure breaks CP symmetry and provides a very attractive manner to break CP symmetry and explain generation of matter antimatter symmetry and CP breaking in hadron physics. The CP breaking is very small characterized by a dimensionless number of order 10^{-9} identifiable as photon/baryon ratio. Can one understand the smallness of CP breaking in TGD framework?

Hamilton-Jacobi structure

Hamilton-Jacobi structure [K43] can be seen as a generalization of complex structure and involves a local but integrable selection of subspaces of various dimension for the tangent space of M^4. Integrability means that the selected subspaces are tangent spaces of a sub-manifold of M^4. M^8-H duality allows to interpret this selection as being induced by a global selection of a hierarchy of real, complex, and quaternionic subspaces associated with octonionic structure mapped to M^4 in such a way that this global selection becomes local at the level of H.

1. The 4-D analog of conformal invariance is due to very special conformal properties of light-like 3-surfaces and light-cone boundary of M^4. This raises hopes about construction of general solution families by utilizing the generalized form of conformal invariance. Massless extremals (MEs) in fact define extremely general solution family of this kind and involve light-like direction vector k and polarization vector ϵ orthogonal to it defining decomposition $M^4 = M^2 \times E^2$. I have proposed that this decomposition generalizes to local but integrable decomposition so that the distributions for M^2 and E^2 integrate to string world sheets and partonic 2-surfaces.
2. One can have decomposition $M^4 = M^2 \times E^2$ such that one has Minkowskian analog of conformal symmetry in M^2. This decomposition is defined by the vectors k and ϵ. An unproven conjecture is that these vectors can depend on point and the proposed Hamilton-Jacobi structure would mean a *local* decomposition of tangent space of M^4 , which is integrable meaning that local M^2s integrate to string world sheet in M^4 and local E^2s integrate to closed 2-surface as special case corresponds to partonic 2-surface. Generalizing the terminology, one could talk about family of partonic surfaces. These decompositions could define families of exremals.

   An integrable decomposition of M^4 to string world sheets and partonic 2-surfaces would characterize the preferred extremals with 4-D M^4 projection. Integrable distribution would mean assignment of partonic 2-surface to each point of string world sheet and vice versa.
3. M^4 Kähler form defines unique decomposition $M^2 \times E^2$. This is however not consistent Lorentz invariance. To cure this problem one must allow moduli space for M^4 Kähler forms such that one can assign to each Hamilton-Jacobi structure M^4 Kähler form defining the corresponding integrable surfaces in terms of light-like vector and polarization vector whose directions depend on point of M^4.

   This looks strange since the very idea is that the embedding space if unique. However, this local decomposition could be secondary being associated only with $H = M^4 \times CP_2$ and emerge in $M^8 - H$ duality mapping of space-time surfaces $X^4 \subset M^8$ to surfaces in $M^4 \times CP_2$. There is a moduli space for octonion structures in M^8 defined as a choice of preferred time axis M^1 (rest system), preferred M^2 defining hypercomplex place and preferred direction (light-like vector), and quaternionic plane $M^2 \times E^2$ (also polarization direction is included). Lorentz boosts mixing the real and imaginary octonion coordinates and changing the direction of time axis give rise to octonion structures not equivalent with the original one.

   Thus the choice $M^1 \subset M^2 \subset M^4 = M^2 \times E^2 \subset M^8$ is involved with the definition of octonion structure and quaternionion structure. The image of this decomposition under M^8-H duality mapping quaternionic tangent space of $X^4 \subset M^8$ containing M^1 and M^2 as sub-spaces would be such that the image of $M^1 \subset M^2 \subset M^2 \times E^2$ depends on point of $M^4 \subset H$ in integrable manner so that Hamilton-Jacobi structure in H is obtained.

Also CP_2 allows the analog of Hamilton-Jacobi structure as a local decomposition integrating to a family of geodesic spheres S^2_I as analog of partonic 2-surfaces with complex structure and having at each point as a fiber different S^2_I - these spheres necessary intersect at single point. This decomposition could correspond to the 4-D complex structure of CP_2 and complex coordinates of CP_2 would serve as coordinates for the two geodesic spheres.

Could one imagine decompositions in which fiber is 2-D Lagrangian manifold - say S^2_{II} - with vanishing induced Kähler form and not possessing induced complex structure? S^2_{II} does not have complex structure as induced complex structure and is therefore analogous to M^2. S^2_{II} coordinates would be functions of string world sheet coordinates (in special as analytic in hypercomplex sense and describing wave propagating with light-velocity). S^2_I coordinates would be analytic functions of complex coordinates of partonic 2-surface.

CP breaking and M^4 Kähler structure

The CP breaking induce by M^4 Kähler structure should be small. Is this automatically true or must one make some assumptions to achieve this.

Could one guarantee this by brute force by assuming M^4 and CP_2 parts of Kähler action to have different normalizations. The proposal for the length scale evolution of cosmological constant however relies on almost cancellation M^4 induced Kähler forms of M^4 and CP_2 parts due to the fact that the induced forms differ from each other by a rotation of the twistor sphere S^2. The S^2 part $M^4 \times S^2$ Kähler for can have opposite with respect to $T(CP_2) = SU(3)/U(1) \times U(1)$ Kähler so that for trivial rotation the forms cancel completely. If the normalizations of Kähler actions differ this cannot happen at the level of 4-D Kähler action.

To make progress, it is useful to look at the situation more concretely.

1. Kähler action is dimensionless. The square of Kähler form is metric so that $J_{kl}J^{kl}$ is dimensionless. One must include to the 4-D Kähler action a dimensional factor $1/L^4$ to make it dimensionless. The natural choice for L is as the radius R of CP_2 geodesic sphere to radius of twistor spheres for M^4 and CP_2. Note however that there is numerical constant involved and if it is changed there must be a compensating change of Kähler coupling strength. Therefore M^4 contribution to action is proportional to the volume of M^4 region using R^4 as unit. This contribution is very large for macroscopic regions of M^4 unless self-duality of M^4 Kähler form would not cause cancellation ($E^2 - B^2 = 0$).

2. What about energy density? The naïve expectation based on Maxwell's theory is that the energy density assignable to M^4 Kähler form is by self-duality proportional to $E^2 + B^2 = 2E^2$ and non-vanishing. By naïve order of magnitude estimate using Maxwellian formula for the energy of this kind extremal is proportional to Vol_3/R^4 and very large. Does this exclude these extremals or should one assume that they have very small volume? For macroscopic lengths of one should assume extremely thin MEs with thickness smaller than R. Could one have 2-fold covering formed by gluing to copies of very thin MEs together along their boundaries. This does not look feasible.

   Luckily, the Maxwellian intuition fails in TGD framework. The Noether currents associated in presence of M^4 Kähler action involve also a term coming from the variation of the induced M^4 Kähler form. This term guarantees that canonical momentum currents as H-vector fields are orthogonal to the space-time surface. In the case of CP_2 type extremals this causes the cancellation of the canonical momentum currents associated with Kähler action and corresponding contributions to conserved charges. The complete symmetry between M^4 and CP_2 and also physical intuition demanding that canonically imbedded M^4 os vacuum require that cancellation takes place also for M^4 part so that only the term corresponding to cosmological constant remains.

M^4 Kähler form and CP breaking for various kinds of extremals

I have considered already earlier the proposal that CP breaking is due to M^4 Kähler form [K18]. CP breaking is very small and the proposal inspired by the Cartesian product structure of the embedding space and its twistor bundle and also by the similar decomposition of $T(M^4) = M^4 \times S^2$ was that the coefficient of M^4 part of Kähler action can be chosen to be much smaller than the coefficient of CP_2 part. The proposed mechanism giving rise to p-adic length scale evolution of cosmological constant however requires that the coefficients of are identical. Luckily, the CP breaking term is automatically very small as the following arguments based on the examination of various kinds of extremals demonstrate.

1. For CP_2 type extremals with light-like M^4 geodesics as M^4 projection the induced M^4 Kähler form vanishes so that there is no CP breaking. For small deformations CP_2 type extremals thickening the M^4 projection the induced M^4 Kähler form is non-vanishing. An attractive hypothesis is that the small CP breaking parameter quantifies the order of magnitude of the induced M^4 Kähler form. This picture could allow to understand CP breaking of hadrons.
2. Canonically imbedded M^4 is a minimal surface. A small breaking of CP symmetry is generated in small deformations of M^4. In particular, for massless extremals (MEs) having 4-D M^4 projection the action associated with M^4 part of Kähler action vanishes at the M^4 limit when the local polarization vector characterizing ME approaches zero. The small CP breaking is characterized by the size of the polarization vector ϵ giving a contribution to the induced metric. This conforms with the perturbative CP breaking.
3. String like objects of type $X^4 = X^2 \times Y^2 \subset M^4 \times CP_2$, where X^2 is minimal surface and Y^2 is 2-surface in CP_2. The M^4 projection contains only electric part but no magnetic part. The M^4 part of action is proportional to the volume Y^2 and therefore very small. This in turn guarantees smallness of CP breaking effects.
   (a) If Y^2 is homologically non-trivial (magnetic flux tube carries monopole flux), CP_2 part of action is large since action density is proportional $1/\sqrt{det(g_2)}$ for Y^2 and therefore large. The thickening of the flux tube however reduces the value of the action by flux conservation as discussed already earlier.
   M^4 and CP_2 contributions to the actions are of opposite sign but M^4 contribution os however very small as compared to CP_2 contribution. One can look the situation in $M^2 \times S^2$ coordinates. The transverse deformation would correspond to the dependence of E^2 coordinates on S^2 coordinates. The induced Kähler form would give a contribution to the S^2 part of induced Kähler form whose size would characterize CP breaking.
   (b) Y^2 can be also homologically trivial. In particular, for $Y^2 = S^2_{II}$ the CP_2 contribution to the total Kähler action vanishes and only the small M^4 contribution proportional to the area of Y^2 remains.

3.7.4 About TGD counterparts for the simplest classical field patterns

What could be the TGD counterparts of typical configurations of classical fields? Since minimal surface equation is a nonlinear generalization of massless field equations, one can hope that the simplest solutions of Maxwell's equations have TGD analogs. The strong non-linearity poses a strong constraint, which can be solved if the extremal allows generalization of holomorphic structure so that field equations are trivially true since they involve in complex coordinates a contraction of tensors of type (1,1) with tensors of type (2,0) or (0,2). It is not clear whether minimal surface property reducing to holomorphy is equivalent with preferred extremal property.

Can one have the basic field patterns such as multipoles as structures with 4-D M^4 projection or could it be that flux tube picture based on spherical harmonics for the orientation of flux tube is all that one can have? Same question can be made for radiation fields having MEs as archetypal representatives in TGD framework. What about the possible consistency problems produced by M^4 Kähler form breaking Lorentz invariance?

I have considered these questions already earlier. The following approach is just making questions and guesses possibly helping to develop general ideas about the correspondence.

1. In QFT approach one expresses fields as superpositions of partial waves, which are indeed very simple field patterns and the coefficients in the superposition become oscillator operators. What could be the analogs of partial waves in TGD? Simultaneous extremals of Kähler action and volume strongly suggest themselves as carriers of field archetypes but the non-linearity of field equations does not support the idea that partial waves could be realized at classical level as extremals with 4-D M^4 projection. A more plausible option is that they correspond

to spherical harmonics for the orientation of flux tube carrying say electric flux. Could the flux tubes of various kinds serve as building of all classical fields?

2. String-like objects $X^2 \times Y^2 \subset M^4 \times CP_2$, where string world sheet X^2 is minimal surface and Y^2 is sub-manifold of CP_2 and their deformations in M^4 degrees of freedom transversal to X^2 and depending on the coordinates Y^2 are certainly good candidates for archetypal field configurations.

   Y^2 can be homologically trivial and could correspond to Lagrangian sub-manifold. Y^2 can also carry homology charge n identifiable as Kähler magnetic charge and correspond to complex sub-manifold of CP_2 with complex structure induced from that of CP_2.

The simplest option corresponds to geodesic sphere $Y^2 = S^2$. There are two geodesic spheres in CP_2 and they correspond to simplest string like objects.

1. S^2_I has Kähler magnetic charge of one unit and the cosmic and its deformations carry monopole flux. These field configurations are not possible in Maxwell's electrodynamics and the proposal is that they appear in all length scales. The model for the formation of galaxies solving also the problem of galactic dark matter relies on long cosmic strings. They are proposed to appear also in biology.

2. S^2_{II} is homologically trivial so that magnetic flux over it vanishes although magnetic field is non-vanishing. Note that although the Kähler magnetic field is vanishing, the electromagnetic ordinary magnetic field is non-vanishing because em field is a combination of Kähler form and component of CP_2 curvature form with vanishing weak isospin. The total flux of ordinary magnetic field over S^2_{II} vanishes whereas electric flux can be non-vanishing.

Coulomb fields

By the vanishing of magnetic flux flux tubes for S^2_{II} cannot represent ordinary magnetic field. They can however serve as radial flux tubes carrying electromagnetic flux. Magnetic flux tubes indeed allow time dependent deformations for which the phase angles of CP_2 coordinates depend linearly of M^4 time coordinate. This would give rise to an archetypal flux tube representation of the electric field created by point charge. Also gravitational flux tubes should correspond to this kind flux tubes emanating radially from the source.

Charge quantization suggests that these flux tubes carry unit charge. In the case of charged elementary particle there would be only single flux tubes but there would be wave function for its orientation having no angular dependence. In principle, this wave function can any spherical harmonic.

Does the orientation angle dependence of flux distribution have any counterpart in Maxwell's theory. One would have the analog of $1/r$ Coulomb potential with the modulus squared of spherical harmonic Y_{lm} modulating it. Could one consider the possibility that in atoms the spherical harmonics for excited states correspond to this kind of distribution for the electric flux coming from nucleus. The probability amplitude for electrons touching the flux tube would inherit this distribution.

For many particle system with large em charge there would be large number of radial flux tubes and the approximation of electric field with Coulomb field becomes natural. In the case of atoms this limit is achieved for large enough nuclear charges. This does not exclude the possibility of having space-time surfaces carrying Coulomb potential in Maxwellian sense: in this case however the field equations cannot solved by holomorphy and quantum criticality might exclude these configurations.

What about gravitation? The notion of gravitational Planck constant requires that Planck mass replaced in TGD framework by CP_2 mass defining the unit of gravitational flux - $h_{gr}0GMm/v_0$ cannot be smaller than h_0. What happens in systems possessing mass smaller than CP_2 mass? Are gravitational flux tubes absent. Is gravitational interaction absent in this kind of systems or is its description analogous to string model description meaning that $h_{gr} = h_0$ for masses smaller than CP_2 mass?

Magnetic fields

As such S^2_{II} flux tubes cannot serve as counterparts of ordinary magnetic fields. The flux tubes have now boundary and the current at boundary creates the magnetic field inside the tube. This would mean cutting of a disk D^2 from S^2_{II} so that the net magnetic flux becomes non-vanishing.

The assumption has been that genuine boundaries are not possible since conservation laws very probably prevent them (the normal components of canonical momentum currents should vanish at boundaries but this is not possible). This requires that this flux tube must be glued along the boundary of $D^2 \times D^1$ to surrounding space-time surface X^4, which has a similar hole. At the boundary of this hole the space-time surface must turn to the direction of CP_2 meaning that the dimension of M^4 projection is reduced to $D = 2$. Algebraic geometer would talk about blow-up.

Ordinary multipole magnetic field could correspond to spherical harmonic for the orientation of this kind flux tubes. They could also carry electric flux but the em charge could be fractionized. These flux tubes might relate to anyons carrying fractional em charge. Also the fractional charges of quarks could classically correspond to flux tubes mediating both color magnetic field and em flux. The spherical harmonic in question corresponds to that associated with electron in atoms.

Magnetic and electric fields associated with straight current wire

Magnetic and electric fields associated with straight current wire need not allow representation as archetypes since they are obviously macroscopic entities.

1. Is the magnetic field associated with straight current wire representable in terms of extremal with 4-D M^4 projection. The magnetic field lines rotate around the current and it is does not seem natural to model it the field in terms of flux tubes. Forget the presence of M^4 Kähler form. One can imbed this kind of magnetic field as a surface with 4-D M^4 projection and possessing cylindrical symmetry. Line current would correspond to a source of the magnetic field and could be realized as a flux tube carrying em current and topologically condensed to the space-time sheet in question.

   The embedding however fails at certain critical radius and the assumption is that no boundaries are allowed by conservation laws. Should one glue the structure to the surrounding space-time surface at this radius. In Maxwell's theory one would have surface current in direction opposite to the source cancelling the magnetic field outside. Could this current have interpretation as a return current?

   One can also imagine glueing its copy to it along the boundary at critical radius. It would seem that the magnetic fields must have same direction at the boundary and therefore also in interior.

2. What about current ring? Separation of variables is essential for the simplest embeddings implying a reduction of partial different equations to differential equation. There is rather small number of coordinates system in E^3 in which Laplacian allows separation of variables. The metric is diagonal in these coordinates. One example is toroidal coordinates assignable with a current ring having toroidal geometry. This would allow a construction of minimal surface solution in some finite volume. Minimal surface property would *not* reduce to complex analyticity for these extremals and they would be naturally associated $M^4 \times S^2_{II}$.

Remark: This kind of extremals are not holomorphic and could be excluded by quantum criticality and preferred extremal property. GRT space-time would be idealization making sense only at the QFT limit of TGD.

Time dependent fields

What about time dependent fields such as the field created by oscillating dipole and radiation fields? One can imagine quantal and classical option.

1. The simplest possibility is reduction to quantum description at single particle level. The dipole current corresponds to a wave function for the source particle system consisting of systems with opposite total charge.

Spherical harmonics representing multipoles would induce wave function for the orientations of MEs (topological light ray) carrying radial wave. This is certainly the most natural options as far radiation field at large distances from sources is considered. One can also have second quantization in the proposed sense giving rise to multi-photon states and one can also define coherent states.

One should also understand time dependent fields near sources having also non-radiative part. This requires a model for source such as oscillating dipole. The simplest possibility is that in the case of dipole there are charges of opposite sign with oscillating distance creating Coulomb fields represented in the proposed manner. It is however not obvious that preferred extremals of this kind exist.

2. One can consider also classical description. The model of elementary particle as consisting of two wormhole contacts, whose throats effectively serve as end of monopole flux tubes at the two sheets involved suggests a possible model. If the wormhole contacts carry opposite em charges realized in terms of fermion and antifermions an oscillating dipole could correspond to flux tube whose length oscillates. This means generation of radiation and for elementary particles this would suggest instability against decay. One can however consider excitation which decay to ground states - say for hadrons. For scaled up variants of this structure this would not mean instability although energy is lost and the system must end up to non-oscillating state.

   One possibility is that there are two charges at different space-time sheets connected by wormhole contacts and oscillating by their mutual interaction in harmonic oscillator state. Ground state would be stable and have not dipole moment.

Effectively 2-D systems

In classical electrodynamics effectively 2-D systems are very special in that they allow conformal invariance assignable to 2-D Laplacian.

1. Since minimal surface equation is generalization of massless d'Alambertian and since field equations are trivially true for analytic solutions, one can hope that the basic solutions of 4-D d'Alembertian generalize in TGD framework. This would conform with the universality of quantum criticality meaning that coupling parameters disappear from field equations. Conformal invariance or its generalization would mean huge variety of field patterns. This suggests that effectively 2-D systems serve as basic building bricks of more complex field configurations. Flux tubes of various kinds would represent basic examples of this kind of surfaces. Also the magnetic end electric fields associated with straight current wire would serve as an example.

2. Are there preferred extremals analogous to the solutions of field equations of general relativity in faraway regions, where they become simple and might allow an analog in TGD framework? If our mathematical models reflect the preferred extremals as archetypal structures, this could be the case.

   Forget for a moment the technicalities related to M^4 Kähler form. One can construct a spherically symmetric ansatz in $M^4 \times S^2_{II}$ as a minimal surface for which Φ depends linearly on time t and u is function of r. The ansatz reduces to a highly non-linear differential equation for u. In this case hyper-complex analyticity is obviously not satisfied. This ansatz could give the analog of Schwartschild metric giving also the electric field of point charge appearing as source of the non-linear variant of d'Alembertian. It is however far from clear whether this kind extremals is allowed as preferred extremals.

   Under which conditions spherically symmetric ansatz is consistent with M^4 Kähler form? Obviously, the M^4 Kähler form must be spherically symmetric as also the Hamilton-Jacobi structure it. Suppose local Hamilton-Jacobi structures for which M^2s integrate to t,r coordinate planes and E^2s integrate to (θ,ϕ) sphere are allowed and that M^4 Kähler form defines this decomposition. In this case there are hopes that consistency conditions can be satisfied. Note however that M^4 Kähler form defines in this case orthogonal magnetic and electric monopole fields defining an analog of instanton. Can one really allow this or should one exclude the time line with $r=0$?

Similar M^4 Kähler structure can be associated with cylindrical coordinates and other separable coordinates system. M^4 Kähler structure would define Hamilton-Jacobi structure.

3.8 Minimal surfaces and TGD

The twistor lift of TGD [K94, K78, L50] meant a revolution in the understanding of TGD and led to a new view about what preferred extremal property means physically and why it is needed.

1. The construction of twistor lift of TGD replaces space-time surfaces with 6-D surfaces but requires that they are dynamically effectively 4-D as the analogs of twistor space having the structure of S^2 bundle with space-time surface as the base. This requires dimensional reduction making S^2 fiber of the twistor space non-dynamical.

   One can say that twistor structure is induced from that for 12-D product of the geometric 6-D twistor spaces of M^4 and CP_2. The condition that 6-D Kähler action exists requires that the twistor spaces of M^4 and CP_2 have Kähler structure. This condition allows only $H = M^4 \times CP_2$ [A35]. The condition that one obtains standard model symmetries leads to the same conclusion.

2. The dimensionally reduced Kähler action decomposes to a sum of 4-D Kähler action and volume term. The interaction is as analog of Maxwell action plus action of point-like particle replaced with 3-D surface. The coefficient of the volume term has an interpretation as cosmological constant having a discrete spectrum [L55]. The natural proposal it that it depends on p-adic length scale approaching zero in long length scales. This solves the cosmological constant problem.

3. I had actually known for decades that all non-vacuum extremals of 4-D Kähler action are minimal surfaces thus minimizing the space-time volume in the induced metric. This is because the field equations for Kähler action for known non-vacuum extremals were reduced essentially to algebraic conditions realizing holomorphy. Also so called CP_2 type vacuum extremals of 4-D Kähler action are minimal surfaces. This finding conforms with the fact that in $M^8 - H$ duality [L23] one has regard field equations as purely algebraic conditions at M^8 side of the duality.

   This inspired the proposal that preferred extremal property of space-time surface is realized by requiring that space-time surfaces as base spaces of these 6-D twistor spaces are quite generally minimal surfaces, and therefore represent a non-linear geometrization for the notion of massless field in accordance with conformal invariance forced by quantum criticality.

   Also a more general proposal that space-time contains regions inside which there is an exchange of canonical momenta between Kähler action and volume term was considered. Minimal surface regions would correspond to incoming particles and non-minimal ones to interaction regions.

   Later this proposal was simplified by requiring that interaction regions are 2-D string world sheets as singularities: this implied that string world sheets required by general considerations [K104] indeed emerge from 4-D action. This could happen also at the 1-D boundaries of string world sheets at 3-D light-like boundaries between Minkowskian and Euclidian regions behaving like ordinary point-like particles and carrying fermion number, and in the most general case also at these 3-D light-like 3-surfaces.

3.8.1 Space-time surfaces as singular minimal surfaces

From the physics point this is not surprising since minimal surface equations are the geometric analog for massless field equations.

1. The boundary value problem in TGD is analogous to that defining soap films spanned by frames: space-time surface is thus like a 4-D soap film. Space-time surface has 3-D ends at the opposite boundaries of causal diamond of M^4 with points replaced with CP_2: I call this 8-D object just causal diamond (CD). Geometrically CD brings in mind big-bang followed by big crunch.

These 3-D ends are like the frame of a soap film. This and the Minkowskian signature guarantees the existence of minimal surface extremals. Otherwise one would expect that the non-compactness does not allow minimal surfaces as non-self-intersecting surfaces.

2. Space-time is a 4-surface in 8-D $H = M^4 \times CP_2$ and is a minimal surface, which can have 2-D or 1-D singularities identifiable as string world sheets having 1-D singularities as light-like orbits - they could be geodesics of space-time surface.

   Remark: I considered in [L41] the possibility that the minimal surface property could fail only at the reaction vertices associated with partonic 2-surfaces defining the ends of string world sheet boundaries. This condition however seems to be too strong. It is essential that the singular surface defines a sub-manifold giving deltafunction like contribution to the action density and that one can assign conserved quantities to this surface. This requires that the singular contributions to energy momentum tensor and canonical momentum currents as spacetime vectors are parallel to the singular surface. Singular points do not satisfy this condition.

   String boundaries represent orbits of fundamental point-like fermions located at 3-D light-like surfaces which represent orbits of partonic 2-surfaces. String world sheets are minimal surfaces and correspond to stringy objects associated with say hadrons. There are also degrees of freedom associated with space-time interior. One have objects of various dimension which all are minimal surfaces. Modified Dirac equation extends the field equations to supersymmetric system and assigns fermionic degrees of freedom to these minimal surfaces of varying dimension.

   From the physics point of view, the singular surfaces are analogous to carriers of currents acting as point- and string-like sources of massless field equations.

3. Geometrically string world sheets are analogous to folds of paper sheet. Space-time surfaces are extremals of an action which is sum of volume term having interpretation in terms of cosmological constant and what I call Kähler action - analogous to Maxwell action. Outside singularities one has minimal surfaces stationary with respect to variations of both volume term and Kähler action - note the analogy with free massless field. At singularities there is an exchange of conserved quantities between volume and Kähler degrees of freedom analogous to the interaction of charged particle with electromagnetic field. One can see TGD as a generalization of a dynamics of point-like particle coupled to Maxwell field by making particle 3-D surface.

4. The condition that the exchange of conserved charges such as four-momentum is restricted to lower-D surfaces realizes preferred extremal property as a consequence of quantum criticality demanding a universal dynamics independent of coupling parameters [L55]. Indeed, outside the singularities the minimal surfaces dynamics has no explicit dependence on coupling constants provided local minimal surface property guarantees also the local stationarity of Kähler action.

   Preferred extremal property has also other formulations. What is essential is the generalization of super-conformal symmetry playing key role in super string models and in the theory of 2-D critical systems so that field equations reduce to purely algebraic conditions just like for analytic functions in 2-D space providing solutions of Laplace equations.

5. TGD provides a large number of specific examples about closed minimal surfaces [K6]. Cosmic strings are objects, which are Cartesian products of minimal surfaces (string world sheets) in M^4 and of complex algebraic curves (2-D surfaces). Both are minimal surfaces and extremize also Kähler action. These algebraic surfaces are non-contractible and characterized by homology charge having interpretation as Kähler magnetic charge. These surfaces are genuine minima just like the geodesics at torus.

   CP_2 contains two kinds of geodesic spheres, which are trivially minimal surfaces. The reason is that the second fundamental form defining as its trace the analogs of external curvatures in the normal space of the surfaces vanishes identically. The geodesic sphere of the first kind is non-contractible minimal surface and absolute minimum. Geodesic spheres of second kind is contractible and one has Minimax type situation.

   These geodesic spheres are analogous to 2-planes in flat 3-space with vanishing external curvatures. For a generic minimal surface in 3-space the principal curvatures are non-vanishing and sum up to zero. This implies that minimal surfaces look locally like saddles. For 2-plane the curvatures vanish identically so that saddle is not formed.

3.8.2 Kähler action as Morse function in the space of minimal 4-surfaces

It was found that surface volume could define a Morse function in the space of surfaces. What about the situation in TGD, where volume is replaced with action which is sum of volume term and Kähler action [L50, L48, L55]?

Morse function interpretation could appear in two ways. The first possibility is that the action defines an analog of Morse function in the space of 4-surfaces connecting given 3-surfaces at the boundaries of CD. Could it be that there is large number of preferred extremals connecting given 3-surfaces at the boundaries of CD? This would serve as analogy for the existence of infinite number of closed surfaces in the case of compact embedding space. The fact that preferred extremals extremize almost everywhere two different actions suggests that this is not the case but one must consider also this option.

1. The simplest realization of general coordinate invariance would allow only single preferred extremal but I have considered also the option for which one has several preferred extremals. In this case one encounters problem with the definition of Kähler function which would become many-valued unless one is ready to replace 3-surfaces with its covering so that each preferred extremal associated with the given 3-surface gives rise to its own 3-surface in the covering space. Note that analogy with the definition of covering space of say circle by replacing points with the set of homologically equivalence classes of closed paths at given point (rotating arbitrary number of times around circle).
2. Number theoretic vision [K102, K38] suggests that these possibly existing different preferred extremals are analogous to same algebraic computation but performed in different ways or theorem proved in different ways. There is always the shortest manner to do the computation and an attractive idea is that the physical predictions of TGD do not depend on what preferred extremal is chosen.
3. An interesting question is whether the "drum theorem" could generalize to TGD framework. If there exists infinite series of preferred extremals which are singular minimal surfaces, the volume of space-time surface for surfaces in the series would depend only on the volume of the CD containing it. The analogy with the high frequencies and drum suggests that the surfaces in the series have more and more local details. In number theoretic vision this would correspond to emergence of more and more un-necessary pieces to the computation. One cannot exclude the possibility that these details are analogs for what is called loop corrections in quantum field theory.
4. If the action defines Morse action, the preferred extremals give information about its topology. Note however that the requirement that one has extremum of both volume term and Kähler action almost everywhere is an extremely strong additional condition and corresponds physically to quantum criticality.

   Remark: The original assumption was that the space-time surface decomposes to critical regions which are minimal surfaces locally and to non-critical regions inside which there is flow of canonical momentum currents between volume and Kähler degrees of freedom. The stronger hypothesis is that this flow occurs at 2-D and 1-D surfaces only.

3.8.3 Kähler function as Morse function in the space of 3-surfaces

The notion of Morse function can make sense also in the space of 3-surfaces - the world of classical worlds which in zero energy ontology consists of pairs of 3-surfaces at opposite boundaries of CD connected by preferred extremal of Kähler action [K25, K77, L50, L48]. Kähler action for the preferred extremal is assumed to define Kähler function defining Kähler metric of WCW via its second derivates $\partial_K \partial_{\overline{L}} K$. Could Kähler function define a Morse function?

1. First of all, Morse function must be a genuine function. For general Kähler metric this is not the case. Rather, Kähler function K is a section in a $U(1)$ bundle consisting of patches transforming by real part of a complex gradient as one moves between the patches of the

bundle. A good example is CP_2, which has non-trivial topology, and which decomposes to 3 coordinate patches such that Kähler functions in overlapping patches are related bythe analog of U(1) gauge transformation.

Kähler action for preferred extremal associated with given 3-surface is however uniquely defined unless one includes Chern-Simons term which changes in U(1) gauge transformation for Kähler gauge potential of CP_2.

2. What could one conclude about the topology of WCW if the action for preferred extremal defines a Morse function as a functional of 3-surface? This function cannot have saddle points: in a region of WCW around saddle point the WCW metric depending on the second derivatives of Morse function would not be positive definite, and this is excluded by the positivity of Hilbert space inner product defined by the Kähler metric essential for the unitarity of the theory. This would suggest that the space of 3-surfaces has very simple topology if Kähler function.

   This is too hasty conclusion! WCW metric is expected to depend also on zero modes, which do not contribute to the WCW line element. What suggests itself is bundle structure. Zero modes define the base space and dynamical degrees of freedom contributing to WCW line element as fiber. The space of zero modes can be topologically complex.

There is a fascinating open problem related to the metric of WCW.

1. The conjecture is that WCW metric possess the symplectic symmetries of $\Delta M^4_+ \times CP_2$ as isometries. In infinite dimensional case the existence of Riemann/Kähler geometry is not at all obvious as the work of Dan Freed demonstrated in the case of loops spaces [A27], and the maximal group of isometries would guarantee the existence of WCW Kähler geometry. Geometry would be determined by symmetries alone and all points of the space would be metrically equivalent. WCW would be an infinite-dimensional analog of symmetric space.

2. Isometry group property does not require that symplectic symmetries leave Kähler action, and even less volume term for preferred extremal, invariant. Just the opposite: if the action would remain invariant, Kähler function and Kähler metric would be trivial!

3. The condition for the existence of symplectic isometries must fix the ratio of the coefficients of Kähler action and volume term highly uniquely. The physical interpretation is in terms of quantum criticality realized mathematically in terms of the symplectic symmetry serving as analog of ordinary conformal symmetry characterizing 2-D critical systems. Note that at classical level quantum criticality realized as minimal surface property says nothing outside singular surfaces since the field equations in this regions are algebraic. At singularities the situation changes. Note also that the minimal surface property is a geometric analog of masslessness which in turn is a correlate of criticality.

4. Twistor lift of TGD [**?**]eads to a proposal for the spectra of Kähler coupling strength and cosmological constant allowed by quantum criticality [L48]. What is surprising that cosmological constant identified as the coefficient of the volume term takes the role of cutoff mass in coupling constant evolution in TGD framework. Coupling constant evolution discretizes in accordance with quantum criticality which must give rise to infinite-D group of WCW isometries. There is also a connection with number theoretic vision in which coupling constant evolution has interpretation in terms of extensions of rationals [K102, L28, L23].

3.8.4 Kähler calibrations: an idea before its time?

While updating book introductions I was surprised to find that I had talked about so called calibrations of sub-manifolds as something potentially important for TGD and later forgotten the whole idea! A closer examination however demonstrated that I had ended up with the analog of this notion completely independently later as the idea that preferred extremals are minimal surfaces apart form 2-D singular surfaces, where there would be exchange of Noether charges between Kähler and volume degrees of freedom.

1. The original idea that I forgot too soon was that the notion of calibration (see `http://tinyurl.com/y3lyead3`) generalizes and could be relevant for TGD. A calibration in Riemann

manifold M means the existence of a k-form ϕ in M such that for any orientable k-D sub-manifold the integral of ϕ over M equals to its k-volume in the induced metric. One can say that metric k-volume reduces to homological k-volume.

Calibrated k-manifolds are minimal surfaces in their homology class, in other words their volume is minimal. Kähler calibration is induced by the k^{th} power of Kähler form and defines calibrated sub-manifold of real dimension $2k$. Calibrated sub-manifolds are in this case precisely the complex sub-manifolds. In the case of CP_2 they would be complex curves (2-surfaces) as has become clear.

2. By the Minkowskian signature of M^4 metric, the generalization of calibrated sub-manifold so that it would apply in $M^4 \times CP_2$ is non-trivial. Twistor lift of TGD however forces to introduce the generalization of Kähler form in M^4 (responsible for CP breaking and matter antimatter asymmetry) and calibrated manifolds in this case would be naturally analogs of string world sheets and partonic 2-surfaces as minimal surfaces. Cosmic strings are Cartesian products of string world sheets and complex curves of CP_2. Calibrated manifolds, which do not reduce to Cartesian products of string world sheets and complex surfaces of CP_2 should also exist and are minimal surfaces.

   One can also have 2-D calibrated surfaces and they could correspond to string world sheets and partonic 2-surfaces which also play key role in TGD. Even discrete points assignable to partonic 2-surfaces and representing fundamental fermions play a key role and would trivially correspond to calibrated surfaces.

3. Much later I ended up with the identification of preferred extremals as minimal surfaces by totally different route without realizing the possible connection with the generalized calibrations. Twistor lift and the notion of quantum criticality led to the proposal that preferred extremals for the twistor lift of Kähler action containing also volume term are minimal surfaces. Preferred extremals would be separately minimal surfaces and extrema of Kähler action and generalization of complex structure to what I called Hamilton-Jacobi structure would be an essential element. Quantum criticality outside singular surfaces would be realized as decoupling of the two parts of the action. May be all preferred extremals be regarded as calibrated in generalized sense.

   If so, the dynamics of preferred extremals would define a homology theory in the sense that each homology class would contain single preferred extremal. TGD would define a generalized topological quantum field theory with conserved Noether charges (in particular rest energy) serving as generalized topological invariants having extremum in the set of topologically equivalent 3-surfaces.

   It is interesting to recall that the original proposal for the preferred extremals as absolute minima of Kähler action has transformed during years to a proposal that they are absolute minima of volume action within given homology class and having fixed ends at the boundaries of CD.

4. The experience with CP_2 would suggest that the Kähler structure of M^4 defining the counterpart of form ϕ is unique. There is however infinite number of different closed self-dual Kähler forms of M^4 defining what I have called Hamilton-Jacobi structures. These forms can have subgroups of Poincare group as symmetries. For instance, magnetic flux tubes correspond to given cylindrically symmetry Kähler form. The problem disappears as one realizes that Kähler structures characterize families of preferred extremals rather than M^4 itself.

If the notion of calibration indeed generalizes, one ends up with the same outcome - preferred extremals as minimal surfaces with 2-D string world sheets and partonic 2-surfaces as singularities - from many different directions.

1. Quantum criticality requires that dynamics does not depend on coupling parameters so that extremals must be separately extremals of both volume term and Kähler action and therefore minimal surfaces for which these degrees of freedom decouple except at singular 2-surfaces, where the necessary transfer of Noether charges between two degrees of freedom takes place at these. One ends up with string picture but strings alone are of course not enough. For instance, the dynamical string tension is determined by the dynamics for the twistor lift.

2. Almost topological QFT picture implies the same outcome: topological QFT property fails only at the string world sheets.

3. Discrete coupling constant evolution, vanishing of loop corrections, and number theoretical condition that scattering amplitudes make sense also in p-adic number fields, requires a representation of scattering amplitudes as sum over resonances realized in terms of string world sheets.

4. In the standard QFT picture about scattering incoming states are solutions of free massless field equations and interaction regions the fields have currents as sources. This picture is realized by the twistor lift of TGD in which the volume action corresponds to geodesic length and Kähler action to Maxwell action and coupling corresponds to a transfer of Noether charges between volume and Kähler degrees of freedom. Massless modes are represented by minimal surfaces arriving inside causal diamond (CD) and minimal surface property fails in the scattering region consisting of string world sheets.

5. Twistor lift forces M^4 to have generalize Kähler form and this in turn strongly suggests a generalization of the notion of calibration. At physics side the implication is the understanding of CP breaking and matter anti-matter asymmetry.

6. $M^8 - H$ duality requires that the dynamics of space-time surfaces in H is equivalent with the algebraic dynamics in M^8. The effective reduction to almost topological dynamics implied by the minimal surface property implies this. String world sheets (partonic 2-surfaces) in H would be images of complex (co-complex sub-manifolds) of $X^4 \subset M^8$ in H. This should allows to understand why the partial derivatives of embedding space coordinates can be discontinuous at these edges/folds but there is no flow between interior and singular surface implying that string world sheets are minimal surfaces (so that one has conformal invariance).

The analogy with foams in 3-D space deserves to be noticed.

1. Foams can be modelled as 2-D minimal surfaces with edges meeting at vertices. TGD space-time could be seen as a dynamically generated foam in 4-D many-sheeted space-time consisting of 2-D minimal surfaces such that also the 4-D complement is a minimal surface. The counterparts for vertices would be light-like curves at light like orbits of partonic 2-surfaces from which several string world sheets can emanate.

2. Can one imagine something more analogous to the usual 3-D foam? Could the light-like orbits of partonic 2-surfaces define an analog of ordinary foam? Could also partonic 2-surfaces have edges consisting of 2-D minimal surfaces joined along edges representing strings connecting fermions inside partonic 2-surface?

   For years ago I proposed what I called as symplectic QFT (SQFT) as an analog of conformal QFT and as part of quantum TGD [K19]. SQFT would have symplectic transformations as symmetries, and provide a description for the symplectic dynamics of partonic 2-surfaces. SQFT involves an analog of triangulation at partonic 2-surfaces and Kähler magnetic fluxes associated with them serve as observables. The problem was how to fix this kind of network. Partonic foam could serve as a concrete physical realization for the symplectic network and have fundamental fermions at vertices. The edges at partonic 2-surfaces would be space-like geodesics. The outcome would be a calibration involving objects of all dimensions $0 \leq D \leq 4$ - a physical analog of homology theory.

3.9 Are space-time boundaries possible in the TGD framework?

One of the key ideas of TGD from the very beginning was that the space-time surface has boundaries and we see them directly as boundaries of physical objects.

It however turned out that it is not at all clear whether the boundary conditions stating that no isometry currents flow out of the boundary, can be satisfied. Therefore the cautious conclusion was that perhaps the boundaries are only apparent. For instance, the space-time regions correspond to maps $M^4 \rightarrow CP_2$, which are many-valued and have as turning points, which have 3-D projections to M^4. The boundary surfaces between regions with Minkowskian and Euclidean signatures of the induced metric seem to be unavoidable, at least those assignable to deformations of CP_2 type extremals assignable to wormhole contacts.

There are good reasons to expect that the possible boundaries are light-like and possibly also satisfy the $det(g_4) = 0$ condition and I have considered the boundary conditions but have not been able to make definite conclusions about how they could be realized.

1. The action principle defining space-times as 4-surfaces in $H = M^4 \times CP_2$ as preferred extremals contains a 4-D volume term and the Kähler action plus possible boundary term if boundaries are possible at all. This action would give rise to a boundary term representing a normal flow of isometry currents through the boundary. These currents should vanish.

2. There could also be a 3-D boundary part in the action but if the boundary is light-like, it cannot depend on the induced metric. The Chern-Simons term for the Kähler action is the natural choice. Twistor lift suggests that it is present also in M^4 degrees of freedom. Topological field theories utilizing Chern-Simons type actions are standard in condensed matter physics, in particular in the description of anyonic systems, so that the proposal is not so radical as one might think. One might even argue that in anyonic systems, the fundamental dynamics of the space-time surface is not masked by the information loss caused by the approximations leading to the field theory limit of TGD.

   Boundary conditions would state that the normal components of the isometry currents are equal to the divergences of Chern-Simons currents and in this way guarantee conservation laws. In CP_2 degrees of freedom the conditions would be for color currents and in M^4 degrees of freedom for 4-momentum currents.

3. This picture would conform with the general view of TGD. In zero energy ontology (ZEO) [L61, L76] phase transitions would be induced by macroscopic quantum jumps at the level of the magnetic body (MB) of the system. In ZEO, they would have as geometric correlates classical deterministic time evolutions of space-time surface leading from the initial to the final state [L52]. The findings of Minev et al provide [L52] lend support for this picture.

3.9.1 Light-like 3-surfaces from $det(g_4) = 0$ condition

How the light-like 3- surfaces could be realized?

1. A very general condition considered already earlier is the condition $det(g_4) = 0$ at the light-like 4-surface. This condition means that the tangent space of X^4 becomes metrically 3-D and the tangent space of X^3 becomes metrically 2-D. In the local light-like coordinates, $(u, v, W, \overline{W})$ $guv = g_{vu}$) would vanish (g_{uu} and g_{vv} vanish by definition.

   Could $det(g_4) = 0$ and $det(g_3) = 0$ condition implied by it allow a universal solution of the boundary conditions? Could the vanishing of these dimensional quantities be enough for the extended conformal invariance?

2. 3-surfaces with $det(g_4) = 0$ could represent boundaries between space-time regions with Minkowskian and Euclidean signatures or genuine boundaries of Minkowskian regions.

   A highly attractive option is that what we identify the boundaries of physical objects are indeed genuine space-time boundaries so that we would directly see the space-time topology. This was the original vision. Later I became cautious with this interpretation since it seemed difficult to realize, or rather to understand, the boundary conditions.

   The proposal that the outer boundaries of different phases and even molecules make sense and correspond to 3-D membrane like entities [L82], served as a partial inspiration for this article but this proposal is not equivalent with the proposal that light-like boundaries defining genuine space-time boundaries can carry isometry charges and fermions.

3. How does this relate to $M^8 - H$ duality [L64, L65]? At the level of rational polynomials P determined 4-surfaces at the level of M^8 as their "roots" and the roots are mass shells. The points of M^4 have interpretation as momenta and would have values, which are algebraic integers in the extension of rationals defined by P.

   Nothing prevents from posing the additional condition that the region of $H^3 \subset M^4 \subset M^8$ is finite and has a boundary. For instance, fundamental regions of tessellations defining hyperbolic manifolds (one of them appears in the model of the genetic code [L74]) could be considered. $M^8 - H$ duality would give rise to holography associating to these 3-surfaces space-time surfaces in H as minimal surfaces with singularities as 4-D analogies to soap films with frames.

The generalization of the Fermi torus and its boundary (usually called Fermi sphere) as the counterpart of unit cell for a condensed matter cubic lattice to a fundamental region of a tessellation of hyperbolic space H^3 acting is discussed is discussed in [L84]. The number of tessellations is infinite and the properties of the hyperbolic manifolds of the "unit cells" are fascinating. For instance, their volumes define topological invariants and hyperbolic volumes for knot complements serve as knot invariants.

This picture resonates with an old guiding vision about TGD as an almost topological quantum field theory (QFT) [K46, K6, K105], which I have even regarded as a third strand in the 3-braid formed by the basic ideas of TGD based on geometry-number theory-topology trinity.

1. Kähler Chern-Simons form, also identifiable as a boundary term to which the instanton density of Kähler form reduces, defines an analog of topological QFT.
2. In the recent case the metric is however present via boundary conditions and in the dynamics in the interior of the space-time surface. However, the preferred extremal property essential for geometry-number theory duality transforms geometric invariants to topological invariants. Minimal surface property means that the dynamics of volume and Kähler action decouple outside the singularities, where minimal surface property fails. Coupling constants are present in the dynamics only at these lower-D singularities defining the analogs of frames of a 4-D soap film.

   Singularities also include string worlds sheets and partonic 2-surfaces. Partonic two-surfaces play the role of topological vertices and string world sheets couple partonic 2-orbits to a network. It is indeed known that the volume of a minimal surface can be regarded as a homological invariant.
3. If the 3-surfaces assignable to the mass shells H^3 define unit cells of hyperbolic tessellations and therefore hyperbolic manifolds, they also define topological invariants. Whether also string world sheets could define topological invariants is an interesting question.

3.9.2 Can one allow macroscopic Euclidean space-time regions

Euclidean space-time regions are not allowed in General Relativity. Can one allow them in TGD?

1. CP_2 extremals with a Euclidean induced metric and serving as correlates of elementary particles are basic pieces of TGD vision. The quantum numbers of fundamental fermions would reside at the light-like orbit of 2-D wormhole throat forming a boundary between Minkowskian space-time sheet and Euclidean wormhole contact- parton as I have called it. More precisely, fermionic quantum numbers would flow at the 1-D ends of 2-D string world sheets connecting the orbits of partonic 2-surfaces. The signature of the 4-metric would change at it.
2. It is difficult to invent any mathematical reason for excluding even macroscopic surfaces with Euclidean signature or even deformations of CP_2 type extremals with a macroscopic size. The simplest deformation of Minkowski space is to a flat Euclidean space as a warping of the canonical embedding $M^4 \subset M^4 \times S^1$ changing its signature.
3. I have wondered whether space-time sheets with an Euclidean signature could give rise to black-hole like entities. One possibility is that the TGD variants of blackhole-like objects have a space-time sheet which has, besides the counterpart of the ordinary horizon, an additional inner horizon at which the signature changes to the Euclidean one. This could take place already at Schwarzschild radius if g_{rr} component of the metric does not change its sign.

3.9.3 But are the normal components of isometry currents finite?

Whether this scenario works depends on whether the normal components for the isometry currents are finite.

1. $det(g_4) = 0$ condition gives boundaries of Euclidean and Minkowskian regions as 3-D light-like minimal surfaces. There would be no scales in accordance with generalized conformal invariance. g_{uv} in light-cone coordinates for M^2 vanishes and implies the vanishing of $det(g_4)$ and light-likeness of the 3-surface.

   What is important is that the formation of these regions would be unavoidable and they would be stable against perturbations.

2. $g^{uv}\sqrt{|g_4|}$ is finite if $det(g_4) = 0$ condition is satisfied, otherwise it diverges. The terms $g^{ui}\partial_i h^k\sqrt{|g_4|}$ must be finite. $g^{ui} = cof(g_{iu})/det(g_4)$ is finite since $g_{uv}g_{vu}$ in the cofactor cancels it from the determinant in the expression of g^{ui}. The presence of $\sqrt{|g_4|}$ implies that the these contributions to the boundary conditions vanish. Therefore only the condition boundary condition for g^{uv} remains.

3. If also Kähler action is present, the conditions are modified by replacing $T^{uk} = g^{u\alpha}\partial_\alpha h^k\sqrt{|g_4|}$ with a more general expression containing also the contribution of Kähler action. I have discussed the details of the variational problem in [K11, K6].

   The Kähler contribution involves the analogy of Maxwell's energy momentum tensor, which comes from the variation of the induced metric and involves sum of terms proportional to $J_{\alpha\mu}J^{beta}_{\mu}$ and $g^{\alpha\beta}J^{\mu\nu}J_{\mu\nu}$.

   In the first term, the dangerous index raisings by g^{uv} appear 3 times. The most dangerous term is given by $J^{uv}J_v^v\sqrt{|g|} = g^{u\mu}g^{v\nu}J_{\alpha\beta}g^{vu}J_{vu}\sqrt{|g|}$. The divergent part is $g^{uv}g^{vu}J_{uv}g^{vu}J_{vu}\sqrt{|g|}$. The diverging g^{uv} appears 3 times and $J_{uv} = 0$ condition eliminates two of these. $g^{vu}\sqrt{|g|}$ is finite by $\sqrt{|g|} = 0$ condition. $J_{uv} = 0$ guarantees also the finiteness of the most dangerous part in $g^{\alpha\beta}J^{\mu\nu}J_{\mu\nu}\sqrt{|g|}$.

   There is also an additional term coming from the variation of the induced Kähler form. This to the normal component of the isometry current is proportional to the quantity $J^{n\alpha}J_l^k\partial_\beta h^l\sqrt{|g|}$. Also now, the most singular term in $J^{u\beta} = g^{u\mu}g^{\beta\nu}J_{\mu\nu}$ corresponds to J^{uv} giving $g^{uv}g^{vu}J^{uv}\sqrt{|g|}$. This term is finite by $J_{uv} = 0$ condition.

   Therefore the boundary conditions are well-defined but only because $det(g_4) = 0$ condition is assumed.

4. Twistor lift strongly suggests that the assignment of the analogy of Kähler action also to M^4 and also this would contribute. All terms are finite if $det(g_4) = 0$ condition is satisfied.

5. The isometry currents in the normal direction must be equal to the divergences of the corresponding currents assignable to the Chern-Simons action at the boundary so that the flow of isometry charges to the boundary would go to the Chern-Simons isometry charges at the boundary.

   If the Chern-Simons term is absent, one expects that the boundary condition reduces to $\partial_v h^k = 0$. This would make X^3 2-dimensional so that Chern-Simons term is necessary. Note that light-likeness does not force the M^4 projection to be light-like so that the expansion of X^2 need not take with light-velocity. If CP_2 complex coordinates are holomorphic functions of W depending also on $U = v$ as a parameter, extended conformal invariance is obtained.

3.9.4 $det(g_4) = 0$ condition as a realization of quantum criticality

Quantum criticality is the basic dynamical principle of quantum TGD. What led to its discovery was the question "How to make TGD unique?". TGD has a single coupling constant, Kähler couplings strength, which is analogous to a critical temperature. The idea was obvious: require quantum criticality. This predicts a spectrum of critical values for the Kähler coupling strength. Quantum criticality would make the TGD Universe maximally complex. Concerning living matter, quantum critical dynamics is ideal since it makes the system maximally sensitive and maximallt reactive.

Concerning the realization of quantum criticality, it became gradually clear that the conformal invariance accompanying 2-D criticality, must be generalized. This led to the proposal that super symplectic symmetries, extended isometries and conformal symmetries of the metrically 2-D boundary of lightcone of M^4, and the extension of the Kac-Moody symmetries associated with the light-like boundaries of deformed CP_2 type extremals should act as symmetries of TGD extending the conformal symmetries of 2-D conformal symmetries. These huge infinite-D symmetries are also required by the existence of the Kähler geometry of WCW [K46, K25, K77] [L78, L88].

However, the question whether light-like boundaries of 3-surfaces with scale larger than CP_2 are possible, remained an open question. On the basis of preceding arguments, the answer seems to be affirmative and one can ask for the implications.

1. At M^8 level, the concrete realization of holography would involve two ingredients. The intersections of the space-time surface with the mass shells H^3 with mass squared value determined as the roots of polynomials P and the tlight-like 3-surfaces as $det(g_4) = 0$ surfaces as boundaries (genuine or between Minkowskian and Euclidean regions) associated by $M^8 - H$ duality to 4-surface of M^8 having associative normal space, which contains commutative 2-D subspace at each point. This would make possible both holography and $M^8 - H$ duality.

   Note that the identification of the algebraic geometric characteristics of the counterpart of $det(g_4) = 0$ surface at the level of H remains still open.

   Since holography determines the dynamics in the interior of the space-time surface from the boundary conditions, the classical dynamics can be said to be critical also in the interior.

2. Quantum criticality means ability to self-organize. Number theoretical evolution allows us to identify evolution as an increase of the algebraic complexity. The increase of the degree n of polynomial P serves as a measure for this. $n = h_{eff}/h_0$ also serves as a measure for the scale of quantum coherence, and dark matter as phases of matter would be characterized by the value of n.

3. The 3-D boundaries would be places where quantum criticality prevails. Therefore they would be ideal seats for the development of life. The proposal that the phase boundaries between water and ice serve as seats for the evolution of prebiotic life, is discussed from the point of TGD based view of quantum gravitation involving huge value of gravitational Planck constant $\hbar_{eff} = \hbar_{gr} = GMm/v_0$ making possible quantum coherence in astrophysical scales [L85]. Density fluctuations would play an essential role, and this would mean that the volume enclosed by the 2-D M^4 projection of the space-time boundary would fluctuate. Note that these fluctuations are possible also at the level of the field body and magnetic body.

4. It has been said that boundaries, where the nervous system is located, distinguishes living systems from inanimate ones. One might even say that holography based on $det(g_4) = 0$ condition realizes nervous systems in a universal manner.

5. I have considered several variants for the holography in the TGD framework, in particular strong form of holography (SH). SH would mean that either the light-like 3-surfaces or the 3-surfaces at the ends of the causal diamond (CD) determine the space-time surface so that the 2-D intersections of the 3-D ends of the space-time surface with its light-like boundaries would determine the physics.

   This condition is perhaps too strong but a fascinating, weaker, possibility is that the internal consistency requires that the intersections of the 3-surface with the mass shells H^3 are identifiable as fundamental domains for the coset spaces $SO(1,3)/\Gamma$ defining tessellations of H^3 and hyperbolic manifolds. This would conform nicely with the TGD inspired model of genetic code [L74].

Chapter 4

About Hydrodynamical and Thermodynamical Interpretations of TGD

This chapter is collected from the material related to the relationship between TGD and hydrodynamics one one hand and TGD and thermodynamics on the one hand.

What I have called hydrodynamics ansatz is a proposal for what the preferred extremals of Kähler action might be. The basic vision behind the ansatz is the reduction of quantum TGD to almost topological QFT. The basic condition is the vanishing of the contraction of the conserved Kähler current j with the induced Kähler gauge potential A implying the reduction of the Kähler action to 3-D contributions coming from the boundaries between space-time regions of Minkowskian and Euclidian signature.

The latter ones do not appear in standard physics but in TGD they serve as 4-D space-time correlates for lines of generalized Feynman graphs: the concrete identification is as wormhole contacts connecting 2 space-time sheets and carrying magnetic flux. Wormhole contacts appear necessarily as pairs due to the presence of the magnetic monopole flux and elementary particles correspond to this kind of pairs. Weak form of electric-magnetic duality reduces the 3-D contributions to Chern-Simons terms. The interior of wormhole contact can in principle contain additional 3-D contribution besides the "boundary" contribution.

Hydrodynamical interpretation demands that the flow parameters associated with the flow lines of isometry currents and Kähler current extend to global coordinates. Otherwise the flow line would resemble those for a gas of particles moving randomly. This leads to integrability conditions implying generalized Beltrami flow and Kähler action for the preferred extremals reduces to Chern-Simons action when weak electro-weak duality is applied as boundary conditions. This allows also a definition of non-constant quantal order parameters depending on the spatial coordinates transversal to the flow lines.

The strongest form of the hydrodynamical interpretation requires that all conserved currents are parallel to Kähler current. In the more general case one would have several hydrodynamic flows. Also the braidings (several of them for the most general ansatz) assigned with the light-like 3-surfaces are naturally defined by the flow lines of conserved currents. The independent behavior of particles at different flow lines can be seen as a realization of the complete integrability of the theory. In free quantum field theories on mass shell Fourier components are in a similar role but the geometric interpretation in terms of flow is of course lacking. This picture should generalize also to the solution of the Kähler-Dirac equation.

The localization of the induced spinor fields to string world sheets with massless Dirac action as 1-D boundary term forcing the fermion lines to be embedding space geodesics makes this picture very concrete. One could even say that the light-like orbits of partonic 2-surfaces containing possibly several parallel fermions define discrete bundles of flow lines.

TGD relies on Zero Energy Ontology (ZEO). In ZEO quantum theory can be at least formally seen as a "square root" of thermodynamics. The rows of unitary U-matrix between zero energy states are identified as M-matrices. M-matrix defining entanglement coefficients between positive and negative energy parts of zero energy state is identified as a square root of density matrix reducing to a product of real and positive square root of the density matrix and unitary S-matrix.

Kiehn [B40] and others have studied Beltrami flows [B11] as integrable flows for which the flow lines define coordinate lines. In D=3 this requires that the rotor of the flow vector is parallel to the flow vector stating that Lorentz force vanishes. In D=4 the condition states that Lorentz 4-force vanishes so that also dissipation is absent. This kind of extremals are of special interest as asymptotic self-organization patterns and in fact all preferred extremals might satisfy these conditions. 3-D Beltrami flows are highly interesting topologically since the flow lines can get knotted. Their 4-D counterparts would have flow lines replaced with world sheets which can develop 2-knots. String world sheets carrying induced spinor fields are fundamental objects in TGD framework and they could indeed get knotted.

Kiehn has worked with both Betrami flows developed what he calls topological thermodynamics (TTD) [B43]. This work is rather interesting from TGD point of view and the relationship between TTD and TGD is discussed in this chapter.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L8].

4.1 Hydrodynamic Interpretation Of Extremals

4.1.1 Possible Role Of Beltrami Flows And Symplectic Invariance In The Description Of Gauge And Gravitational Interactions

One of the most recent observations made by people working with twistors is the finding of Monteiro and O'Connell described in the preprint "The Kinematic Algebra From the Self-Dual Sector" (see `http://tinyurl.com/ya2oa8na`) [B35]. The claim is that one can obtain supergravity amplitudes by replacing the color factors with kinematic factors which obey formally 2-D symplectic algebra defined by the plane defined by light-like momentum direction and complexified variable in the plane defined by polarizations. One could say that momentum and polarization dependent kinematic factors are in exactly the same role as the factors coming from Yang-Mills couplings. Unfortunately, the symplectic algebra looks rather formal object since the first coordinate is light-like coordinate and second coordinate complex transverse coordinate. It could make sense only in the complexification of Minkowski space.

In any case, this would suggest that the gravitational gauge group (to be distinguished from diffeomorphisms) is symplectic group of some kind having enormous representative power as we know from the fact that the symmetries of practically any physical system are realized in terms of symplectic transformations. According to the authors of [B35] one can identify the Lie algebra of symplectic group of sphere with that of $SU(N)$ at large N limit in suitable basis. What makes this interesting is that at large N limit non-planar diagrams which are the problem of twistor Grassmann approach vanish: this is old result of t'Hooft, which initiated the developments leading to AdS/CFT correspondence.

The symplectic group of $\delta M^4_\pm \times CP_2$ is the isometry algebra of WCW and I have proposed that the effective replacement of gauge group with this group implies the vanishing of non-planar diagrams [K94]. The extension of SYM to a theory of also gravitation in TGD framework could make Yangian symmetry exact, resolve the infrared divergences, and the problems caused by non-planar diagrams. It would also imply stringy picture in finite measurement resolution. Also the construction of the non-commutative homology and cohomology in TGD framework led to the lifting of Galois group algebras to their braided variants realized as symplectic flows [K56] and to the conjecture that in finite measurement resolution the cohomology obtained in this manner represents WCW ("world of classical worlds") spinor fields (or at least something very essential about them).

It is however difficult to understand how one could generalize the symplectic structure so that also symplectic transformations involving light-like coordinate and complex coordinate of the partonic 2-surface would make sense in some sense. In fact, a more natural interpretation for the kinematic algebra would in terms of volume preserving flows which are also Beltrami flows [B11, B28]. This gives a connection with quantum TGD since Beltrami flows define a basic dynamical symmetry for the preferred extremals of Kähler action which might be called Maxwellian phase.

1. Classical TGD is defined by Kähler action which is the analog of Maxwell action with Maxwell field expressed as the projection of CP_2 Kähler form. The field equations are extremely non-linear and only the second topological half of Maxwell equations is satisfied. The remaining equations state conservation laws for various isometry currents. Actually much more general conservation laws are obtained.
2. As a special case one obtains solutions analogous to those for Maxwell equations but there are also other objects such as CP_2 type vacuum extremals providing correlates for elementary particles and string like objects: for these solutions it does not make sense to speak about QFT in Minkowski space-time. For the Maxwell like solutions linear superposition is lost but a superposition holds true for solutions with the same local direction of polarization and massless four-momentum. This is a very quantal outcome (in accordance with quantum classical correspondence) since also in quantum measurement one obtains final state with fixed polarization and momentum. So called massless extremals (topological light rays) analogous to wave guides containing laser beam and its phase conjugate are solutions of this kind. The solutions are very interesting since no dispersion occurs so that wave packet preserves its form and the radiation is precisely targeted.
3. Maxwellian preferred extremals decompose in Minkowskian space-time regions to regions that can be regarded as classical space-time correlates for massless particles. Massless particles are characterized by polarization direction and light-like momentum direction. Now these directions can depend on position and are characterized by gradients of two scalar functions Φ and Ψ. Φ defines light-like momentum direction and the square of the gradient of Φ in Minkowski metric must vanish. Ψ defines polarization direction and its gradient is orthogonal to the gradient of Φ since polarization is orthogonal to momentum.
4. The flow has the additional property that the coordinate associated with the flow lines integrates to a global coordinate. Beltrami flow is the term used by mathematicians. Beltrami property means that the condition $j\wedge dj=0$ is satisfied. In other words, tjhe current is in the plane defined by its exterior derivative. The above representation obviously guarantees this. Beltrami property allows to assign order parameter to the flow depending only the parameter varying along flow line.

   This is essential for the hydrodynamical interpretation of the preferred extremals which relies on the idea that varies conservation laws hold along flow lines. For instance, super-conducting phase requires this kind of flow and velocity along flow line is gradient of the order parameter. The breakdown of super-conductivity would mean topologically the loss of the Beltrami flow property. One might say that the space-time sheets in TGD Universe represent analogs of supra flow and this property is spoiled only by the finite size of the sheets. This strongly suggests that the space-time sheets correspond to perfect fluid flows with very low viscosity to entropy ratio and one application is to the observed perfect flow behavior of quark gluon plasma.
5. The current $J=\Phi\nabla\Psi$ has vanishing divergence if besides the orthogonality of the gradients the functions Ψ and Φ satisfy massless d'Alembert equation. This is natural for massless field modes and when these functions represent constant wave vector and polarization also d'Alembert equations are satisfied. One can actually add to $\nabla\Psi$ a gradient of an arbitrary function of Φ this corresponds to U(1) gauge invariance and the addition to the polarization vector a vector parallel to light-like four-momentum. One can replace Φ by any function of Φ so that one has Abelian Lie algebra analogous to $U(1)$ gauge algebra restricted to functions depending on Φ only.

The general Beltrami flow gives as a special case the kinetic flow associated by Monteiro and O'Connell with plane waves. For ordinary plane wave with constant direction of momentum vector and polarization vector one could take $\Phi=cos(\phi)$, $\phi=k\cdot m$ and $\Psi=\epsilon\cdot m$. This would give a real flow. The kinematical factor in SYM diagrams corresponds to a complexified flow $\Phi=exp(i\phi)$ and $\Psi=\phi+w$, where w is complex coordinate for polarization plane or more naturally, complexificaton of the coordinate in polarization direction. The flow is not unique since gauge invariance allows to modify ϕ term. The complexified flow is volume preserving only in the formal algebraic sense and satisfies the analog of Beltrami condition only in Dolbeault cohomology where d is identified as complex exterior derivative ($df = df/dz\ dz$ for holomorphic functions). In ordinary cohomology it fails. This formal complex flow of course does not define a real diffeomorphism at space-time level: one should replace Minkowski space with its complexification to get a genuine flow.

The finding of Monteiro and O'Connel encourages to think that the proposed more general Abelian algebra pops up also in non-Abelian YM theories. Discretization by braids would actually select single polarization and momentum direction. If the volume preserving Beltrami flows characterize the basic building bricks of radiation solutions of both general relativity and YM theories, it would not be surprising if the kinematic Lie algebra generators would appear in the vertices of YM theory and replace color factors in the transition from YM theory to general relativity. In TGD framework the construction of vertices at partonic two-surfaces would define local kinematic factors as effectively constant ones.

4.1.2 A General Solution Ansatz Based On Almost Topological QFT Property

The basic vision behind the ansatz is the reduction of quantum TGD to almost topological QFT. This requires that the flow parameters associated with the flow lines of isometry currents and Kähler current extend to global coordinates. This leads to integrability conditions implying generalized Beltrami flow and Kähler action for the preferred extremals reduces to Chern-Simons action when weak electro-weak duality is applied as boundary conditions. The strongest form of the hydrodynamical interpretation requires that all conserved currents are parallel to Kähler current. In the more general case one would have several hydrodynamic flows. Also the braidings (several of them for the most general ansatz) assigned with the light-like 3-surfaces are naturally defined by the flow lines of conserved currents. The independent behavior of particles at different flow lines can be seen as a realization of the complete integrability of the theory. In free quantum field theories on mass shell Fourier components are in a similar role but the geometric interpretation in terms of flow is of course lacking. This picture should generalize also to the solution of the Kähler-Dirac equation.

Basic field equations

Consider first the equations at general level.

1. The breaking of the Poincare symmetry due to the presence of monopole field occurs and leads to the isometry group $T\times SO(3)\times SU(3)$ corresponding to time translations, rotations, and color group. The Cartan algebra is four-dimensional and field equations reduce to the conservation laws of energy E, angular momentum J, color isospin I_3, and color hypercharge Y.

2. Quite generally, one can write the field equations as conservation laws for I, J, I_3, and Y.

$$D_\alpha\left[D_\beta(J^{\alpha\beta}H_A) - j_K^\alpha H^A + T^{\alpha\beta}j_A^l h_{kl}\partial_\beta h^l\right] = 0 \ . \quad (4.1.1)$$

The first term gives a contraction of the symmetric Ricci tensor with antisymmetric Kähler form and vanishes so that one has

$$D_\alpha\left[j_K^\alpha H^A - T^{\alpha\beta}j_A^k h_{kl}\partial_\beta h^l\right] = 0 \ . \quad (4.1.2)$$

For energy one has $H_A = 1$ and energy current associated with the flow lines is proportional to the Kähler current. Its divergence vanishes identically.

3. One can express the divergence of the term involving energy momentum tensor as as sum of terms involving $j_K^\alpha J_{\alpha\beta}$ and contraction of second fundamental form with energy momentum tensor so that one obtains

$$j_K^\alpha D_\alpha H^A = j_K^\alpha J_\alpha{}^\beta j_\beta^A + T^{\alpha\beta}H_{\alpha\beta}^k j_k^A \ . \quad (4.1.3)$$

Hydrodynamical solution ansatz

The characteristic feature of the solution ansatz would be the reduction of the dynamics to hydrodynamics analogous to that for a continuous distribution of particles initially at the end of X^3 of the light-like 3-surface moving along flow lines defined by currents j_A satisfying the integrability condition $j_A \wedge dj_A = 0$. Field theory would reduce effectively to particle mechanics along flow lines with conserved charges defined by various isometry currents. The strongest condition is that all isometry currents j_A and also Kähler current j_K are proportional to the same current j. The more general option corresponds to multi-hydrodynamics.

Conserved currents are analogous to hydrodynamical currents in the sense that the flow parameter along flow lines extends to a global space-time coordinate. The conserved current is proportional to the gradient $\nabla\Phi$ of the coordinate varying along the flow lines: $J = \Psi\nabla\Phi$ and by a proper choice of Ψ one can allow to have conservation. The initial values of Ψ and Φ can be selected freely along the flow lines beginning from either the end of the space-time surface or from wormhole throats.

If one requires hydrodynamics also for Chern-Simons action (effective 2-dimensionality is required for preferred extremals), the initial values of scalar functions can be chosen freely only at the partonic 2-surfaces. The freedom to chose the initial values of the charges conserved along flow lines at the partonic 2-surfaces means the existence of an infinite number of conserved charges so that the theory would be integrable and even in two different coordinate directions. The basic difference as compared to ordinary conservation laws is that the conserved currents are parallel and their flow parameter extends to a global coordinate.

1. The most general assumption is that the conserved isometry currents

$$J_A^\alpha = j_K^\alpha H^A - T^{\alpha\beta} j_A^k h_{kl}\partial_\beta h^l \tag{4.1.4}$$

and Kähler current are integrable in the sense that $J_A \wedge J_A = 0$ and $j_K \wedge j_K = 0$ hold true. One could imagine the possibility that the currents are not parallel.

2. The integrability condition $dJ_A \wedge J_A = 0$ is satisfied if one one has

$$J_A = \Psi_A d\Phi_A \ . \tag{4.1.5}$$

The conservation of J_A gives

$$d*(\Psi_A d\Phi_A) = 0 \ . \tag{4.1.6}$$

This would mean separate hydrodynamics for each of the currents involved. In principle there is not need to assume any further conditions and one can imagine infinite basis of scalar function pairs (Ψ_A, Φ_A) since criticality implies infinite number deformations implying conserved Noether currents.

3. The conservation condition reduces to d'Alembert equation in the induced metric if one assumes that $\nabla\Psi_A$ is orthogonal with every $d\Phi_A$.

$$d*d\Phi_A = 0 \ , \ d\Psi_A \cdot d\Phi_A = 0 \ . \tag{4.1.7}$$

Taking $x = \Phi_A$ as a coordinate the orthogonality condition states $g^{xj}\partial_j\Psi_A = 0$ and in the general case one cannot solve the condition by simply assuming that Ψ_A depends on the coordinates transversal to Φ_A only. These conditions bring in mind $p\cdot p = 0$ and $p \cdot e$ condition for massless modes of Maxwell field having fixed momentum and polarization. $d\Phi_A$ would correspond to p and $d\Psi_A$ to polarization. The condition that each isometry current corresponds its own pair (Ψ_A, Φ_A) would mean that each isometry current corresponds to independent light-like momentum and polarization. Ordinary free quantum field theory would support this view whereas hydrodynamics and QFT limit of TGD would support single flow.

These are the most general hydrodynamical conditions that one can assume. One can consider also more restricted scenarios.

1. The strongest ansatz is inspired by the hydrodynamical picture in which all conserved isometry charges flow along same flow lines so that one would have

$$J_A = \Psi_A d\Phi \ . \quad (4.1.8)$$

In this case same Φ would satisfy simultaneously the d'Alembert type equations.

$$d*d\Phi = 0 \ , \ d\Psi_A \cdot d\Phi = 0. \quad (4.1.9)$$

This would mean that the massless modes associated with isometry currents move in parallel manner but can have different polarizations. The spinor modes associated with light-light like 3-surfaces carry parallel four-momenta, which suggest that this option is correct. This allows a very general family of solutions and one can have a complete 3-dimensional basis of functions Ψ_A with gradient orthogonal to $d\Phi$.

2. Isometry invariance under $T \times SO(3) \times SU(3)$ allows to consider the possibility that one has

$$J_A = k_A \Psi_A d\Phi_{G(A)} \ , \ d*(d\Phi_G(A)) = 0 \ , \ d\Psi_A \cdot d\Phi_G(A)) = 0 \ . \quad (4.1.10)$$

where $G(A)$ is T for energy current, $SO(3)$ for angular momentum currents and $SU(3)$ for color currents. Energy would thus flow along its own flux lines, angular momentum along its own flow lines, and color quantum numbers along their own flow lines. For instance, color currents would differ from each other only by a numerical constant. The replacement of Ψ_A with $\Psi_{G(A)}$ would be too strong a condition since Killing vector fields are not related by a constant factor.

To sum up, the most general option is that each conserved current J_A defines its own integrable flow lines defined by the scalar function pair (Ψ_A, Φ_A). A complete basis of scalar functions satisfying the d'Alembert type equation guaranteeing current conservation could be imagined with restrictions coming from the effective 2-dimensionality reducing the scalar function basis effectively to the partonic 2-surface. The diametrically opposite option corresponds to the basis obtained by assuming that only single Φ is involved.

The proposed solution ansatz can be compared to the earlier ansatz [K46] stating that Kähler current is topologized in the sense that for $D(CP_2) = 3$ it is proportional to the identically conserved instanton current (so that 4-D Lorentz force vanishes) and vanishes for $D(CP_2) = 4$ (Maxwell phase). This hypothesis requires that instanton current is Beltrami field for $D(CP_2) = 3$. In the recent case the assumption that also instanton current satisfies the Beltrami hypothesis in strong sense (single function Φ) generalizes the topologization hypothesis for $D(CP_2) = 3$. As a matter fact, the topologization hypothesis applies to isometry currents also for $D(CP_2) = 4$ although instanton current is not conserved anymore.

Can one require the extremal property in the case of Chern-Simons action?

Effective 2-dimensionality is achieved if the ends and wormhole throats are extremals of Chern-Simons action. The strongest condition would be that space-time surfaces allow orthogonal slicings by 3-surfaces which are extremals of Chern-Simons action.

Also in this case one can require that the flow parameter associated with the flow lines of the isometry currents extends to a global coordinate. Kähler magnetic field $B = *J$ defines a conserved current so that all conserved currents would flow along the field lines of B and one would have 3-D Beltrami flow. Note that in magnetohydrodynamics the standard assumption is that currents flow along the field lines of the magnetic field.

For wormhole throats light-likeness causes some complications since the induced metric is degenerate and the contravariant metric must be restricted to the complement of the light-like direction. This means that d'Alembert equation reduces to 2-dimensional Laplace equation. For space-like 3-surfaces one obtains the counterpart of Laplace equation with partonic 2-surfaces serving as sources. The interpretation in terms of analogs of Coulomb potentials created by 2-D charge distributions would be natural.

Almost perfect fluids seems to be abundant in Nature. For instance, QCD plasma was originally thought to behave like gas and therefore have a rather high viscosity to entropy density ratio $x = \eta/s$. Already RHIC found that it however behaves like almost perfect fluid with x near to the minimum predicted by AdS/CFT. The findings from LHC gave additional conform the discovery [C7]. Also Fermi gas is predicted on basis of experimental observations to have at low temperatures a low viscosity roughly 5-6 times the minimal value [D6] . In the following the argument that the preferred extremals of Kähler action are perfect fluids apart from the symmetry breaking to space-time sheets is developed. The argument requires some basic formulas summarized first.

The detailed definition of the viscous part of the stress energy tensor linear in velocity (oddness in velocity relates directly to second law) can be found in [D2] .

1. The symmetric part of the gradient of velocity gives the viscous part of the stress-energy tensor as a tensor linear in velocity. Velocity gradient decomposes to a term traceless tensor term and a term reducing to scalar.

$$\partial_i v_j + \partial_j v_i = \frac{2}{3}\partial_k v^k g_{ij} + (\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v^k g_{ij}) \ . \tag{4.1.11}$$

The viscous contribution to stress tensor is given in terms of this decomposition as

$$\sigma_{visc;ij} = \zeta \partial_k v^k g_{ij} + \eta(\partial_i v_j + \partial_j v_i - \frac{2}{3}\partial_k v^k g_{ij}) \ . \tag{4.1.12}$$

From $dF^i = T^{ij}S_j$ it is clear that bulk viscosity ζ gives to energy momentum tensor a pressure like contribution having interpretation in terms of friction opposing. Shear viscosity η corresponds to the traceless part of the velocity gradient often called just viscosity. This contribution to the stress tensor is non-diagonal and corresponds to momentum transfer in directions not parallel to momentum and makes the flow rotational. This term is essential for the thermal conduction and thermal conductivity vanishes for ideal fluids.

2. The 3-D total stress tensor can be written as

$$\sigma_{ij} = \rho v_i v_j - p g_{ij} + \sigma_{visc;ij} \ . \tag{4.1.13}$$

The generalization to a 4-D relativistic situation is simple. One just adds terms corresponding to energy density and energy flow to obtain

$$T^{\alpha\beta} = (\rho - p) u^\alpha u^\beta + p g^{\alpha\beta} - \sigma^{\alpha\beta}_{visc} \ . \tag{4.1.14}$$

Here u^α denotes the local four-velocity satisfying $u^\alpha u_\alpha = 1$. The sign factors relate to the concentrations in the definition of Minkowski metric ($(1, -1, -1, -1)$).

3. If the flow is such that the flow parameters associated with the flow lines integrate to a global flow parameter one can identify new time coordinate t as this flow parameter. This means a transition to a coordinate system in which fluid is at rest everywhere (comoving coordinates in cosmology) so that energy momentum tensor reduces to a diagonal term plus viscous term.

$$T^{\alpha\beta} = (\rho - p) g^{tt} \delta^\alpha_t \delta^\beta_t + p g^{\alpha\beta} - \sigma^{\alpha\beta}_{visc} \ . \tag{4.1.15}$$

In this case the vanishing of the viscous term means that one has perfect fluid in strong sense.

The existence of a global flow parameter means that one has

$$v_i = \Psi\partial_i\Phi \ . \quad (4.1.16)$$

Ψ and Φ depend on space-time point. The proportionality to a gradient of scalar Φ implies that Φ can be taken as a global time coordinate. If this condition is not satisfied, the perfect fluid property makes sense only locally.

AdS/CFT correspondence allows to deduce a lower limit for the coefficient of shear viscosity as

$$x = \frac{\eta}{s} \geq \frac{\hbar}{4\pi} \ . \quad (4.1.17)$$

This formula holds true in units in which one has $k_B = 1$ so that temperature has unit of energy.

What makes this interesting from TGD view is that in TGD framework perfect fluid property in appropriately generalized sense indeed characterizes locally the preferred extremals of Kähler action defining space-time surface.

1. Kähler action is Maxwell action with U(1) gauge field replaced with the projection of CP_2 Kähler form so that the four CP_2 coordinates become the dynamical variables at QFT limit. This means enormous reduction in the number of degrees of freedom as compared to the ordinary unifications. The field equations for Kähler action define the dynamics of space-time surfaces and this dynamics reduces to conservation laws for the currents assignable to isometries. This means that the system has a hydrodynamic interpretation. This is a considerable difference to ordinary Maxwell equations. Notice however that the "topological" half of Maxwell's equations (Faraday's induction law and the statement that no non-topological magnetic are possible) is satisfied.

2. Even more, the resulting hydrodynamical system allows an interpretation in terms of a perfect fluid. The general ansatz for the preferred extremals of field equations assumes that various conserved currents are proportional to a vector field characterized by so called Beltrami property. The coefficient of proportionality depends on space-time point and the conserved current in question. Beltrami fields by definition is a vector field such that the time parameters assignable to its flow lines integrate to single global coordinate. This is highly non-trivial and one of the implications is almost topological QFT property due to the fact that Kähler action reduces to a boundary term assignable to wormhole throats which are light-like 3-surfaces at the boundaries of regions of space-time with Euclidian and Minkowskian signatures. The Euclidian regions (or wormhole throats, depends on one's tastes) define what I identify as generalized Feynman diagrams.

   Beltrami property means that if the time coordinate for a space-time sheet is chosen to be this global flow parameter, all conserved currents have only time component. In TGD framework energy momentum tensor is replaced with a collection of conserved currents assignable to various isometries and the analog of energy momentum tensor complex constructed in this manner has no counterparts of non-diagonal components. Hence the preferred extremals allow an interpretation in terms of perfect fluid without any viscosity.

This argument justifies the expectation that TGD Universe is characterized by the presence of low-viscosity fluids. Real fluids of course have a non-vanishing albeit small value of x. What causes the failure of the exact perfect fluid property?

1. Many-sheetedness of the space-time is the underlying reason. Space-time surface decomposes into finite-sized space-time sheets containing topologically condensed smaller space-time sheets containing.... Only within given sheet perfect fluid property holds true and fails at wormhole contacts and because the sheet has a finite size. As a consequence, the global flow parameter exists only in given length and time scale. At embedding space level and in zero energy ontology the phrasing of the same would be in terms of hierarchy of causal diamonds (CDs).

2. The so called eddy viscosity is caused by eddies (vortices) of the flow. The space-time sheets glued to a larger one are indeed analogous to eddies so that the reduction of viscosity to eddy viscosity could make sense quite generally. Also the phase slippage phenomenon of super-conductivity meaning that the total phase increment of the super-conducting order parameter is reduced by a multiple of 2π in phase slippage so that the average velocity proportional to the increment of the phase along the channel divided by the length of the channel is reduced by a quantized amount.

   The standard arrangement for measuring viscosity involves a lipid layer flowing along plane. The velocity of flow with respect to the surface increases from $v = 0$ at the lower boundary to v_{upper} at the upper boundary of the layer: this situation can be regarded as outcome of the dissipation process and prevails as long as energy is fed into the system. The reduction of the velocity in direction orthogonal to the layer means that the flow becomes rotational during dissipation leading to this stationary situation.

   This suggests that the elementary building block of dissipation process corresponds to a generation of vortex identifiable as cylindrical space-time sheets parallel to the plane of the flow and orthogonal to the velocity of flow and carrying quantized angular momentum. One expects that vortices have a spectrum labelled by quantum numbers like energy and angular momentum so that dissipation takes in discrete steps by the generation of vortices which transfer the energy and angular momentum to environment and in this manner generate the velocity gradient.

3. The quantization of the parameter x is suggestive in this framework. If entropy density and viscosity are both proportional to the density n of the eddies, the value of x would equal to the ratio of the quanta of entropy and kinematic viscosity η/n for single eddy if all eddies are identical. The quantum would be $\hbar/4\pi$ in the units used and the suggestive interpretation is in terms of the quantization of angular momentum. One of course expects a spectrum of eddies so that this simple prediction should hold true only at temperatures for which the excitation energies of vortices are above the thermal energy. The increase of the temperature would suggest that gradually more and more vortices come into play and that the ratio increases in a stepwise manner bringing in mind quantum Hall effect. In TGD Universe the value of h_{eff} can be large in some situations so that the quantal character of dissipation could become visible even macroscopically. Whether this a situation with large h_{eff} is encountered even in the case of QCD plasma is an interesting question.

The following poor man's argument tries to make the idea about quantization a little bit more concrete.

1. The vortices transfer momentum parallel to the plane from the flow. Therefore they must have momentum parallel to the flow given by the total cm momentum of the vortex. Before continuing some notations are needed. Let the densities of vortices and absorbed vortices be n and n_{abs} respectively. Denote by $v_{\|}$ *resp.* $v_{\perp}$ the components of cm momenta parallel to the main flow *resp.* perpendicular to the plane boundary plane. Let m be the mass of the vortex. Denote by S are parallel to the boundary plane.

2. The flow of momentum component parallel to the main flow due to the absorbed at S is

   $$n_{abs} m v_{\|} v_{\perp} S \quad . \tag{4.1.18}$$

   This momentum flow must be equal to the viscous force

   $$F_{visc} = \eta \frac{v_{\|}}{d} \times S \quad . \tag{4.1.19}$$

   From this one obtains

$$\eta = n_{abs} m v_{\perp} d \ . \tag{4.1.20}$$

If the entropy density is due to the vortices, it equals apart from possible numerical factors to

$$s = n$$

so that one has

$$\frac{\eta}{s} = m v_{\perp} d \ . \tag{4.1.21}$$

This quantity should have lower bound $x = \hbar/4\pi$ and perhaps even quantized in multiples of x, Angular momentum quantization suggests strongly itself as origin of the quantization.

3. Local momentum conservation requires that the comoving vortices are created in pairs with opposite momenta and thus propagating with opposite velocities $v_{\perp}$. Only one half of vortices is absorbed so that one has $n_{abs} = n/2$. Vortex has quantized angular momentum associated with its internal rotation. Angular momentum is generated to the flow since the vortices flowing downwards are absorbed at the boundary surface.

   Suppose that the distance of their center of mass lines parallel to plane is $D = \epsilon d$, ϵ a numerical constant not too far from unity. The vortices of the pair moving in opposite direction have same angular momentum $mv \ \ D/2$ relative to their center of mass line between them. Angular momentum conservation requires that the sum these relative angular momenta cancels the sum of the angular momenta associated with the vortices themselves. Quantization for the total angular momentum for the pair of vortices gives

$$\frac{\eta}{s} = \frac{n\hbar}{\epsilon} \tag{4.1.22}$$

   Quantization condition would give

$$\epsilon = 4\pi \ . \tag{4.1.23}$$

   One should understand why $D = 4\pi d$ - four times the circumference for the largest circle contained by the boundary layer- should define the minimal distance between the vortices of the pair. This distance is larger than the distance d for maximally sized vortices of radius $d/2$ just touching. This distance obviously increases as the thickness of the boundary layer increases suggesting that also the radius of the vortices scales like d.

4. One cannot of course take this detailed model too literally. What is however remarkable that quantization of angular momentum and dissipation mechanism based on vortices identified as space-time sheets indeed could explain why the lower bound for the ratio η/s is so small.

4.1.3 Hydrodynamic Picture In Fermionic Sector

Super-symmetry inspires the conjecture that the hydrodynamical picture applies also to the solutions of the Kähler-Dirac equation. This would mean that the solutions of Dirac equation can be localized to lower-dimensional surface or even flow lines.

Basic objection

The obvious objection against the localization to sub-manifolds is that it is not consistent with uncertainty principle in transversal degrees of freedom. More concretely, the assumption that the mode is localized to a lower-dimensional surface of X^4 implies that the action of the transversal part of Dirac operator in question acts on delta function and gives something singular.

The situation changes if the Dirac operator in question has vanishing transversal part at the lower-dimensional surface. This is not possible for the Dirac operator defined by the induced metric but is quite possible in the case of Kähler-Dirac operator. For instance, in the case of massless extremals Kähler-Dirac gamma matrices are non-vanishing in single direction only and the solution modes could be one-dimensional. For more general preferred extremals such as cosmic strings this is not the case.

In fact, there is a strong physical argument in favor of the localization of spinor modes at 2-D string world sheets so that hydrodynamical picture would result but with flow lines replaced with fermionic string world sheets.

1. Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.
2. The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action and requires that the part of the Kähler-Dirac operator transversal to 2-surface vanishes.
3. This localization does not hold for cosmic string solutions which however have 2-D CP_2 projection which should have vanishing weak fields so that 4-D spinor modes with well-defined em charge are possible.
4. A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

4-dimensional Kähler-Dirac equation and hydrodynamical picture

In following consideration is restricted to preferred extremals for which one has decomposition to regions characterized by local light-like vector and polarization direction. In this case one has good hopes that the modes can be restricted to 1-D light-like geodesics.

Consider first the solutions of of the induced spinor field in the interior of space-time surface.

1. The local inner products of the modes of the induced spinor fields define conserved currents

$$\begin{aligned} D_\alpha J^\alpha_{mn} &= 0 \ , \\ J^\alpha_{mn} &= \overline{u}_m \hat{\Gamma}^\alpha u_n \ , \\ \hat{\Gamma}^\alpha &= \frac{\partial L_K}{\partial(\partial_\alpha h^k)}\Gamma_k \ . \end{aligned} \tag{4.1.24}$$

The conjecture is that the flow parameters of also these currents extend to a global coordinate so that one would have in the completely general case the condition

$$\begin{aligned} J^{\alpha}_{mn} &= \Phi_{mn} d\Psi_{mn} \ , \\ d*(d\Phi_{mn}) &= 0 \ , \ \nabla\Psi_{mn}\cdot\Phi_{mn} = 0 \ . \end{aligned} \tag{4.1.25}$$

The condition $\Phi_{mn} = \Phi$ would mean that the massless modes propagate in parallel manner and along the flow lines of Kähler current. The conservation condition along the flow line implies tht the current component J_{mn} is constant along it. Everything would reduce to initial values at the ends of the space-time sheet boundaries of CD and 3-D Kähler-Dirac equation would reduce everything to initial values at partonic 2-surfaces.

2. One might hope that the conservation of these super currents for all modes is equivalent with the Kähler-Dirac equation. The modes u_n appearing in Ψ in quantized theory would be kind of "square roots" of the basis Φ_{mn} and the challenge would be to deduce the modes from the conservation laws.

3. The quantization of the induced spinor field in 4-D sense would be fixed by those at 3-D space-like ends by the fact that the oscillator operators are carried along the flow lines as such so that the anti-commutator of the induced spinor field at the opposite ends of the flow lines at the light-like boundaries of CD is in principle fixed by the anti-commutations at the either end. The anti-commutations at 3-D surfaces cannot be fixed freely since one has 3-D Chern-Simons flow reducing the anti-commutations to those at partonic 2-surfaces.

The following argument suggests that induced spinor fields are in a suitable gauge simply constant along the flow lines of the Kähler current just as massless spinor modes are constant along the geodesic in the direction of momentum.

1. The Kähler-Dirac gamma matrices are of form $T^{\alpha}_k\Gamma^k$, $T^{\alpha}_k = \partial L_K/\partial(\partial_\alpha h^k)$. The H-vectors T^{α}_k can be expressed as linear combinations of a subset of Killing vector fields j^k_A spanning the tangent space of H. For CP_2 the natural choice are the 4 Lie-algebra generators in the complement of $U(2)$ sub-algebra. For CD one can used generator time translation and three generators of rotation group SO(3). The completeness of the basis defined by the subset of Killing vector fields gives completeness relation $h^k_l = j^{Ak}j_{Ak}$. This implies $T^{\alpha k} = T^{\alpha k}j^A_k j^k_A = T^{\alpha A}j^k_A$. One can defined gamma matrices Γ_A as $\Gamma_k j^k_A$ to get $T^{\alpha}_k\Gamma^k = T^{\alpha A}\Gamma_A$.

2. This together with the condition that all isometry currents are proportional to the Kähler current (or if this vanishes to same conserved current- say energy current) satisfying Beltrami flow property implies that one can reduce the Kähler-Dirac equation to an ordinary differential equation along flow lines. The quantities T^{tA} are constant along the flow lines and one obtains

$$T^{tA}j_A D_t\Psi = 0 \ . \tag{4.1.26}$$

By choosing the gauge suitably the spinors are just constant along flow lines so that the spinor basis reduces by effective 2-dimensionality to a complete spinor basis at partonic 2-surfaces.

4.2 Does Thermodynamics Have A Representation At The Level Of Space-Time Geometry?

R. Kiehn has proposed what he calls Topological Thermodynamics (TTD) (see `http://tinyurl.com/y9z3jtre`) [B43] as a new formulation of thermodynamics. The basic vision is that thermodynamical equations could be translated to differential geometric statements using the notions of differential forms and Pfaffian system (see `http://tinyurl.com/6ks8jkc`) [A12]. That TTD differs from TGD by a single letter is not enough to ask whether some relationship between them might exist. Quantum TGD can however in a well-defined sense be regarded as a square root of thermodynamics in zero energy ontology (ZEO) and this leads leads to ask seriously whether TTD might help to understand TGD at deeper level. The thermodynamical interpretation of space-time dynamics would obviously generalize black hole thermodynamics to TGD framework and already earlier some concrete proposals have been made in this direction.

One can raise several questions. Could the preferred extremals of Kähler action code for the square root of thermodynamics? Could induced Kähler gauge potential and Kähler form (essentially Maxwell field) have formal thermodynamic interpretation? The vacuum degeneracy of Kähler action implies 4-D spin glass degeneracy and strongly suggests the failure of strict determinism for the dynamics of Kähler action for non-vacuum extremals too. Could thermodynamical irreversibility and preferred arrow of time allow to characterize the notion of preferred extremal more sharply?

It indeed turns out that one can translate Kiehn's notions to TGD framework rather straightforwardly.

1. Kiehn's work 1- form corresponds to induced Kähler gauge potential implying that the vanishing of instanton density for Kähler form becomes a criterion of reversibility and irreversibility is localized on the (4-D) "lines" of generalized Feyman diagrams, which correspond to space-like signature of the induced metric. The localization of heat production to generalized Feynman diagrams conforms nicely with the kinetic equations of thermodynamics based on reaction rates deduced from quantum mechanics. It also conforms with Kiehn's vision that dissipation involves topology change.
2. Heat produced in a given generalized Feynman diagram is just the integral of instanton density and the condition that the arrow of geometric time has definite sign classically fixes the sign of produced heat to be positive. In this picture the preferred extremals of Kähler action would allow a trinity of interpretations as non-linear Maxwellian dynamics, thermodynamics, and integrable hydrodynamics.
3. The 4-D spin glass degeneracy of TGD breaking of ergodicity suggests that the notion of global thermal equilibrium is too naïve. The hierarchies of Planck constants and of p-adic length scales suggests a hierarchical structure based on CDs withing CDs at embedding space level and space-time sheets topologically condensed at larger space-time sheets at space-time level. The arrow of geometric time for quantum states could vary for sub-CDs and would have thermodynamical space-time correlates realized in terms of distributions of arrows of geometric time for sub-CDs, sub-sub-CDs, etc...

The hydrodynamical character of classical field equations of TGD means that field equations reduce to local conservation laws for isometry currents and Kähler gauge current. This requires the extension of Kiehn's formalism to include besides forms and exterior derivative also induced metric, index raising operation transforming 1-forms to vector fields, duality operation transforming k- forms to n-k forms, and divergence which vanishes for conserved currents.

4.2.1 Motivations And Background

It is good to begin by discussing the motivations for the geometrization of thermodynamics and by introducing the existing mathematical framework identifying space-time surfaces as preferred extremals of Kähler action.

ZEO and the need for the space-time correlates for square root of thermodynamics

Quantum classical correspondence is basic guiding principle of quantum TGD. In ZEO TGD can be regarded as a complex square root of thermodynamics so that the thermodynamics should have correlates at the level of the geometry of space-time.

1. Zero energy states consist of pairs of positive and negative energy states assignable to opposite boundaries of a causal diamond (CD). There is entire hierarchy of CDs characterized by their scale coming as an integer multiple of a basic scale (also their Poincare transforms are allowed).
2. In ZEO zero energy states are automatically time-irreversible in the sense that either end of the causal diamond (CD) corresponds to a state consisting of single particle states with well-defined quantum numbers. In other words, this end of CD carries a prepared state. The other end corresponds to a superposition of states which can have even different particle numbers: this is the case in particle physics experiment typically. State function reduction reduces the

second end of CD to a prepared state. This process repeats itself. This suggests that the arrow of time or rather, its geometric counterpart which we experience, alternates. This need not however be the case if quantum classical correspondence holds true.

3. To illustrate what I have in mind consider a path towel, which has been been folded forth and back. Assume that the direction in which folding is carried is time direction. Suppose that the inhabitant of bath towel Universe is like the habitant of the famous Flatland and therefore not able to detect the folding of the towel. If the classical dynamics of towel is time irreversible (time corresponds to the direction in which the folding takes place), the inhabitant sees ever lasting irreversible time evolution with single arrow of geometric time identified as time coordinate for the towel: no changes in the arrow of geometric time. If the inhabitant is able to make measurements about 3-D space the situation he or she might be able to see that his time evolution takes place forth and back with respect to the time coordinate of higher-dimensional embedding space.

4. One might understand the arrow of time - albeit differently as in normal view about the situation - if classical time evolution for the preferred extremals of Kähler action defines a geometric correlate for quantum irreversibility of zero energy states. There are of course other space-time sheets and other CDs present an it might be possible to detect the alternation of the arrow of geometric time at embedding space level by making measurements giving information about their geometric arrows of time [K5].

By quantum classical correspondence one expects that the geometric arrow of time - irreversibility - for zero energy states should have classical counterparts at the level of the dynamics of preferred extremals of Kähler action. What could be this counterpart? Thermodynamical evolution by quantum jumps does not obey ordinary variational principle that would make it deterministic: Negentropy Maximization Principle (NMP) [K57] for state function reductions of system is analogous to Second Law for an ensemble of copies of system and actually implies it. Could one mimic irreversibility by single classical evolution defined by a preferred extremal? Note that the dynamics of preferred extremals is not actually strictly deterministic in the ordinary sense of the word: the reason is the enormous vacuum degeneracy implying 4-D spin glass degeneracy. This makes it possible to mimic not only quantum states but also sequences of quantum jumps by piece-wise deterministic evolution.

Preferred extremals of Kähler action

In Quantum TGD the basic arena of quantum dynamics is "world of classical worlds" (WCW [K95]), see `http://tinyurl.com/ycqyk49f`). Purely classical spinor fields in this infinite-dimensional space define quantum states of the Universe. General Coordinate Invariance (GCI) implies that classical worlds can be regarded as either 3-surfaces or 4-D space-time surfaces analogous to Bohr orbits. Strong form of GCI implies in ZEO strong form of holography in the sense that the points of WCW effectively correspond to collections of partonic 2-surfaces belonging to both ends of causal diamonds (CDs) plus their 4-D tangent space-time data.

Kähler geometry reduces to the notion of Kähler function [K46] and by quantum classical correspondence a good guess is that Kähler function corresponds to so called Kähler action for Euclidian space-time regions. Minkowskian space-time regions give a purely imaginary to Kähler action (square root of metric determinant is imaginary) and this contribution plays the role of Morse function for WCW . Stationary phase approximation imples that in first the approximation the extremals of the Kähler *function* (to be distinguished from preferred extremals of Kähler *action* !) select one particular 3-surface and corresponding classical space-time surface (Bohr orbit) as that defining "classical physics".

GCI implies holography and holography suggests that action reduces to 3-D terms. This is true if one has $j^\mu A_\mu = 0$ in the interior of space-time. If one assumes so called weak form of electric-magnetic duality [K104] at the real and effective boundaries of space-time surface (3-D surfaces at the ends of CDs and the light-like 3-surfaces at which the signature of induced 4-metric changes so that 4-metric is degenerate), one obtains a reduction of Kähler action to Chern-Simons terms at the boundaries. TGD reduces to almost topological QFT. "Almost" means that the induced metric does not disappear completely from the theory since it appears in the conditions expressing weak form of electric magnetic duality and in the condition $j^\mu A_\mu = 0$.

The strong form of holography implies effective 2-dimensionality and this suggests the reduction of Chern-Simons terms to 2-dimensional areas of string world sheets and possible of partonic 2-surfaces. This would mean almost reduction to string theory like theory with string tension becoming a dynamic quantity.

Under additional rather general conditions the contributions from Minkowskian and Euclidian regions of space-time surface are apart from the value of coefficient identical at light-like 3-surfaces. At space-like 3-surfaces at the ends of space-time surface they need not be identical.

Quantum classical correspondence suggests that space-time surfaces provide a representation for the square root of thermodynamics and therefore also for thermodynamics. In general relativity black hole thermodynamics suggests the same. This idea is not new in TGD framework. For instance, Hawking-Bekenstein formula (see `http://tinyurl.com/3ytg9m6`) for blackbody entropy [B1] allows a p-adic generalization (see `http://tinyurl.com/y9e22rr6`) in terms of area of partonic 2-surfaces [K63]. The challenge is to deduce precise form of this correspondence and here Kiehn's topological thermodynamics might help in this task.

4.2.2 Kiehn's Topological Thermodynamics (TTD)

The basic in the work of Kiehn is that thermodynamics allows a topological formulation in terms of differential geometry.

1. Kiehn introduces also the notions of Pfaff system (see `http://tinyurl.com/6ks8jkc`) and Pfaff dimension as the number of non-vanishing forms in the sequence for given 1-form such as W or Q: W, dW, $W \wedge dW$, $dW \wedge dW$. Pfaff dimension $D \leq 4$ tells that one can describe W as sum $W = \sum W_k dx^k$ of gradients of D variables. $D = 4$ corresponds to open system, $D = 3$ to a closed system and $W \wedge dW \neq 0$ defines what can be regarded as a chirality. For $D = 2$ chirality vanishes no spontaneous parity breaking.
2. Kiehn's king idea that Pfaffian systems provide a universal description of thermodynamical reversibility. Kiehn introduces heat 1- form Q. System is thermodynamically reversible if Q is integrable. In other words, the condition $Q \wedge dQ = 0$ holds true which implies that one can write $Q = TdS$: Q allows an integrable factor T and is expressible in terms of the gradient of entropy. $Q = TdS$ condition implies that Q correspond to a global flow defined by the coordinate lines of S. This in turn implies that it is possible define phase factors depending on S along the flow line: this relates to macroscopic quantum coherence for macroscopic quantum phases.
3. The first law expressing the work 1-form W as $W = Q - dU = TdS - dU$ for reversible processes. This gives $dW \wedge dW = 0$. The condition $dW \wedge dW \neq 0$ therefore characterizes irreversible processes.
4. Symplectic transformations are natural in Kiehn's framework but not absolutely essential.

Reader is encouraged to get familiar with Kiehn's examples [B43] about the description of various simple thermodynamical systems in this conceptual framework. Kiehn has also worked with the differential topology of electrodynamics and discussed concepts like integrable flows known as Beltrami flows. These flows generalized to TGD framework and are in key role in the construction of proposals for preferred extremals of Kähler action: the basic idea would be that various conserved isometry currents define Beltrami flows so that theit flow lines can be associated with coordinate lines.

4.2.3 Attempt To Identify TTD In TGD Framework

Let us now try to identify TTD or its complex square root in TGD framework.

The role of symplectic transformations

Symplectic transformations are important in Kiehn's approach although they are not a necessary ingredient of it and actually impossible to realize in Minkowski space-time.

1. Symplectic symmetries of WCW induced by synmplectic symmetries of CP_2 and light-like boundary of CD are important also in TGD framework [K25] and define the isometries of

WCW . As a matter fact, symplectic group parameterizes the quantum fluctuating degrees of freedom and zero modes defining classical variables are symplectic invariants. One cannot assign to entire space-time surfaces symplectic structure although this is possible for partonic 2-surfaces.

2. The symplectic transformations of CP_2 act on the Kähler gauge potential as $U(1)$ gauge transformations formally but modify the shape of the space-time surface. These symplectic transformations are symmetries of Kähler action only in the vacuum sector which as such does not belong to WCW whereas small deformations of vacua belong. Therefore genuine gauge symmetries are not in question. One can of course formally assign to Kähler gauge potential a separate $U(1)$ gauge invariance.

3. Vacuum extremals with at most 2-D CP_2 projection (Lagrangian sub-manifold) form an infinite-dimensional space. Both M^4 diffeomorphisms and symplectic transformations of CP_2 produce new vacuum extremals, whose small deformations are expected to correspond preferred extremals. This gives rise to 4-D spin glass degeneracy (see `http://tinyurl.com/y9e22rr6`) [K63] to be distinguished from 4-D gauge degeneracy.

Identification of basic 1-forms of TTD in TGD framework

Consider nex the identification of the basic variables which are forms of various degrees in TTD.

1. Kähler gauge potential is analogous to work 1-form W. In classical electrodynamics vector potential indeed has this interpretation. $dW \wedge dW$ is replaced with $J \wedge J$ defining instanton density ($E_K \cdot B_K$ in physicist's notation) for Kähler form and its non-vanishing - or equivalently 4-dimensionality of CP_2 projection of space-time surface - would be the signature of irreversibility. $dJ = 0$ holds true only locally and one can have magnetic monopoles since CP_2 has non-trivial homology. Therefore the non-trivial topology of CP_2 implying that the counterpart of W is not globablly defined, brings in non-trivial new element to Kiehn's theory.

2. Chirality $C-S = A \wedge J$ is essentially Chern-Simons 3-form and in ordinary QFT non-vanishing of $C-S$- if present in action - means parity breaking in ordinary quantum field theories. Now one must be very cautious since parity is a symmetry of the embedding space rather than that of space-time sheet.

3. Pfaff dimension equals to the dimension of CP_2 projection and has been used to classify existing preferred extremals. I have called the extremals with 4-D CP_2 projection chaotic and so called CP_2 vacuum extremals with 4-D CP_2 projection correspond to such extremals. Massless extremals or topological light rays correspond to $D = 2$ as do also cosmic strings. In Euclidian regions preferred extremals with $D = 4$ are are possible but not in Minkowskian regions if one accepts effective 3-dimensionality. Here one must keep mind open.

Irreversibility identified as a non-vanishing of the instanton density $J \wedge J$ has a purely geometrical and topological description in TGD Universe if one accepts effective 3-dimensionality.

1. The effective 3-dimensionality for space-time sheets (holography implied by general coordinate invariance) implies that Kähler action reduces to Chern-Simons terms so that the Pfaff dimension is at most $D = 3$ for Minkowskian regions of space-time surface so that they are are thermodynamically reversible.

2. For Euclidian regions (say deformations of CP_2 type vacuum extremals) representing orbits of elementary particles and lines of generalized Feynman diagrams $D = 4$ is possible. Therefore Euclidian space-like regions of space-time would be solely responsible for the irreversibility. This is quite strong conclusion but conforms with the standard quantum view about thermodynamics according to which various particle reaction rates deduced from quantum theory appear in kinetic equations giving rise to irreversible dynamics at the level of ensembles. The presence of Morse function coming from Minkowskian regions is natural since square root of thermodynamics is in question. Morse function is analogous to the action in QFTs whereas Kähler function is analogous to Hamiltonian in thermodynamics. Also this conforms with the square root of TTD interpretation.

Instanton current, instanton density, and irreversibility

Classical TGD has the structure of hydrodynamics in the sense that field equations are conservation laws for isometry currents and Kähler current. These are vector fields although induced metric allows to transform them to forms. This aspect should be visible also in thermodynamic interpretation and forces to add to the Kiehn's formulation involving only forms and exterior derivative also induced metric transforming 1-forms to vector fields, the duality mapping 4-k forms and k-forms to each other, and divergence operation.

It was already found that irreversibility and dissipation corresponds locally to a non-vanishing instanton density $J \wedge J$. This form can be regarded as exterior derivative of Chern-Simons 3-form or equivalently as divergence of instanton current.

1. The dual of C-S 3-form given by $*(A \wedge J)$ defines what I have called instanton current. This current is not conserved in general and the interpretation as a heat current would be natural. The exterior derivative of C-S gives instanton density $J \wedge J$. Equivalently, the divergence of instanton currengives the dual of $J \wedge J$ and the integral of instanton density gives the analog of instanton number analogous to the heat generated in a given space-time volume. Note that in Minkowskian regions one can multiply instanton current with a function of CP_2 coordinates without losing closedness property so that infinite number similar conserved currents is possible.

   The heat 3-form is expressible in terms of Chern-Simons 3-form and for preferred extremals it would be proportional to the weight sum of Kähler actions from Minkowskian and Euclidian regions (coeffecients are purely imaginary and real in these two regions). Instead of single real quantity one would have complex quantity characterizing irreversibility. Complexity would conform with the idea that quantum TGD is complex square root of thermodynamics.

2. The integral of heat 3-form over effective boundaries associated with a given space-time region define the net heat flow from that region. Only the regions defining the lines of generalized Feynman diagrams give rise to non-vanishing heat fluxes. Second law states that one has $\Delta Q \geq 0$. Generalized second law means at the level of quantum classical correspondence would mean that depending on the arrow of geometric time for zero energy state ΔQ is defined as difference between upper and lower or lower and upper boundaries of CD. This condition applied to CD and sub-CD: s would generalize the conditions familiar from hydrodynamics (stating for instance that for shock waves the branch of bifurcation for which the entropy increases is selected). Note that the field equations of TGD are hydrodynamical in the sense that they express conservation of various isometry currents. The naïve picture about irreversibility is that classical dynamics generates CP_2 type vacuum extremals so that the number of outgoing lines of generalized Feynman diagram is higher than that of incoming ones. Therefore that the number of space-like 3-surfaces giving rise to Chern-Simons contribution is larger at the end of CD corresponding to the final (negative energy) state.

3. A more precise characterization of the irreversible states involves several non-trivial questions.

   (a) By the failure of strict classical determinism the condition that for a given CD the number outgoing lines is not smaller than incoming lines need not provide a unique manner to fix the preferred extremal when partonic 2-surfaces at the ends are fixed. Could the arrow of geometric time depend on sub-CD as the model for living matter suggests (recall also phase conjugate light rays)?
   In ordinary quantum mechanical approach to kinetic equations also the reactions, which decrease entropy are allowed but their weight is smaller in thermal equilibrium. Could this fact be described as a probability distribution for the arrow of time associated for the sub-CDs, sub-sub-CDs, etc... ? Space-time correlates for quantal thermodynamics would be probability distributions for space-time sheets and hierarchy of sub-CDs.

   (b) 4-D spin glass degeneracy suggests breaking of ergodic hypothesis: could this mean that one does not have thermodynamical equilibrium but very large number of spin glass states caused by the frustation for which induced Kähler form provides a representation? Could these states correspond to a varying arrow of geometric time for sub-CDs? Or could different deformed vacuum extremals correspond to different space-time sheets in thermal equilibrium with different thermal parameters.

Also Kähler current and isometry currents are needed

The conservation Kähler current and of isometry currents imply the hydrodynamical character of TGD.

1. The conserved Kähler current j_K is defined as 3-form $j_K = *(d*J)$, where $d*J$ is closed 3-form and defines the counterpart of d^*dW. Field equations for preferred extremals require $*j_K \wedge A = 0$ satsfied if one Kähler current is proportional to instanton current: $*j_K \propto A \wedge J$. As a consequence Kähler action reduces to 3-dimensional Chern-Simons terms (classical holography) and Minkowskian space-time regions have at most 3-D CP_2 projection (Pfaff dimension $D \leq 3$) so that one has $J \wedge J = 0$ and reversibility. This condition holds true for preferred extremals representing macroscopically the propagation of massless quanta but not Euclidian regions representing quanta themselves and identifiable as basic building bricks of wormhole contacts between Minkowskian space-time sheets.
2. A more general proposal is that all conserved currents transformed to 1-forms using the induced metric (classical gravitation comes into play!) are integrable: in other words, on has $j \wedge dj = 0$ for both isometry currents and Kähler current. This would mean that they are analogous to heat 1-forms in the reversible case and therefore have a representation analogous to $Q = TdS$, $W = PdV$, μdN and the coordinate along flowline defines the analog of S, V, or N (note however that dS, dV, dN would more naturally correspond to 3-forms than 1-forms, see below) A stronger form corresponds to the analog of hydrodynamics for one particle species: all one-forms are proportional (by scalar function) to single 1-form which is $A \wedge J$ (all quantum number flows are parallel to each other).

Questions

There are several questions to be answered.

1. In Darboux coordinates in which one has $A = P_1 dQ^1 + P_2 dQ_2$. The identification of A as counterpart for $W = PdV - \mu dN$ comes first in mind. For thermodynamical equilibria one would have $TdS = dU + W$ translating to $TdS = dU + A$ so that Q for reversible processes would be apart from $U(1)$ gauge transformation equal to the Kähler gauge potential. Symplectic transformations of CP_2 generate $U(1)$ gauge transformations and dU might have interpretation in terms of energy flow induced by this kind of transformation. Recall however that symplectic transformations are not symmetries of space-time surfaces but only of the WCW metric and act on partonic 2-surfaces and their tangent space data as such.
2. Does the conserved Kähler current j_K have any thermodynamical interpretation? Clearly the counterparts of conserved (and also non-conserved quantities) in Kiehn's formulation would be 3-forms with vanishing curl $d(*j_K) = 0$ in conserved case. Therefore it seems impossible to reduce them to 1-forms unless one introduces divergence besides exterior derivative as a basic differential operation.

   The hypothesis that the flow lines of these 1-forms associated with j_K vector field are integrable implies that they are gradients apart from the presence of integrating factor. Reduction to a gradient ($j = dU$) means that U satisfies massless d'Alembert equation $d*dU = 0$. Note that local polarization and light-like momentum are gradients of scalar functions which satisfy massless d'Alembert equation for the Minkowskian space-time regions representing propagating of massless quanta.
3. In genuinely 3-dimensional context S, V, N are integrals of 3-forms over 3-surfaces for some current defining 3-form. This is in conflict with Kiehn's description where they are 0-forms. One can imagine three cures and first two ones look
   (a) The integrability of the flows allows to see them as superposition of independent 1-dimensional flows. This picture would make it natural to regard the TGD counterparts of S, V, N as 0-forms rather than 2-forms. This would also allow to deduce $J \wedge J = 0$ as a reversibility condition using Kiehn's argument.
   (b) Unless one requires integrable flows, one must consider the replacement of $Q = TdS$ *resp.* $W = PdV$ *resp.* μdN $Q = TdS$ *resp.* $W = PdV$ *resp.* μdN where W, Q, dS,dV, anddN with 3-forms. So that S, V, N would be 2-forms and the 3-integrals of dS, dV, dN over

3-surfaces would reduce to integrals over partonic 2-surfaces, which is of course highly non-trivial but physically natural implication of the effective 2-dimensionality. First law should now read as $*W = T*dS - *dU$ and would give $d*W = dT \wedge *dS + Td*dS + d*dU$. If S and U as 2-forms satisfy massless d'Alembert equation, one obtains $d*W = dT \wedge *dS$ giving $d*W \wedge d*W = 0$ as the reversibility condition. If one replaces $W \leftrightarrow A$ correspondence with $*W \leftrightarrow A$ correspondence, one obtains the vanishing of instanton density as a condition for reversibility. For the preferred extremals having interpretation as massless modes the massless d'Alembert equations are satisfied and it might that this option makes sense and be equivalent with the first option.

(c) In accordance with the idea that finite measurement resolution is realized at the level of Kähler-Dirac equation, its solutions at light-like 3-surfaces reduces to solutions restricted to lines connecting partonic 2-surfaces. Could one regard W, Q, dS, dV, and dN as singular one-forms restricted to these lines? The vanishing of instanton density would be obtained as a condition for reversibility only at the braid strands, and one could keep the original view of Kiehn. Note however that the instanton density could be non-vanishing elsewhere unless one develops a separate argument for its vanishing. For instance, the condition that isometries of embedding space say translations produce braid ends points for which instanton density also vanishes for the reversible situation might be enough.

To sum up, it seems that TTD allows to develop considerable insights about how classical space-time surfaces could code for classical thermodynamics. An essential ingredient seems to be the reduction of the hydrodynamical flows for isometry currents to what might be called perfect flows decomposing to 1-dimensional flows with conservation laws holding true for individual flow lines. An interesting challenge is to find expressions for total heat in terms of temperature and entropy. Blackhole-elementary particle analogy suggest the reduction as well as effective 2-dimensionality suggest the reduction of the integrals of Chern-Simons terms defining total heat flux to two 2-D volume integrals over string world sheets and/or partonic 2-surfaces and this would be quite near to Hawking-Bekenstein formula.

4.3 Robert Kiehn's Ideas About Falaco Solitons And Generation Of Turbulent Wake From TGD Perspective

I have been reading two highly interesting articles by Robert Kiehn. The first article has the title "Hydrodynamics wakes and minimal surfaces with fractal boundaries" (see `http://tinyurl.com/y8emhmt7`) [B41]. Second article is titled "Instability patterns, wakes and topological limit sets" (see `http://tinyurl.com/y8v4e3xr`) [B42]. There are very many contacts on TGD inspired vision and its open interpretational problems.

The notion of Falaco soliton has surprisingly close resemblance with Kähler magnetic flux tubes defining fundamental structures in TGD Universe. Fermionic strings are also fundamental structures of TGD accompanying magnetic flux tubes and this supports the vision that these string like objects could allow reduction of various condensed matter phenomena such as sound waves -usually regarded as emergent phenomena allowing only highly phenomenological description - to the fundamental microscopic level in TGD framework. This can be seen as the basic outcome of this article.

Kiehn proposed a new description for the generation of various instability patterns of hydrodynamics flows (Kelvin-Helmholtz and Rayleigh-Taylor instabilities) in terms of hyperbolic dynamics so that a connection with wave phenomena like interference and diffraction would emerge. The role of characteristic surfaces as surfaces of tangential and also normal discontinuities is central for the approach. In TGD framework the characteristic surfaces have as analogs light-like wormhole throats at which the signature of the induced 4-metric changes and these surfaces indeed define boundaries of two phases and of material objects in general. This inspires a more detailed comparison of Kiehn's approach with TGD.

4.3.1 Falaco Solitons And TGD

In the first article [B41] Kiehn tells about his basic motivations. The first motivating observations were related to so called Falaco solitons. Second observation was related to the so called mushroom pattern associated with Rayleigh–Taylor instability (see http://tinyurl.com/brypvgm) or fingering instability [B7], which appears in very many contexts, the most familiar being perhaps the mushroom shaped cloud created by a nuclear explosion. The idea was that both structures whose stability is not easy to understand in standard hydrodynamics, could have topological description.

Falaco solitons are very fascinating objects. Kiehn describes in detail the formation and properties in [B41]: anyone possessing swimming pool can repeat these elegant and simple experiments. The vortex string connecting the end singularities - dimpled indentations at the surface of water - is the basic notion. Kiehn asks whether there migh be a deeper connection with a model of mesons in which strings connecting quark and antiquark appear. The formation of spiral structures around the end gaps in the initial formative states of Falaco soliton is emphasized and compared to the structure of spiral galaxies. The suggestion is that galaxies could appear as pairs connected by strings.

Kähler magnetic tubes carrying monopole flux are central in TGD and have several interesting resemblances with Falaco solutions.

1. In TGD framework so called cosmic strings fundamental primordial objects (see http://tinyurl.com/yblk638z). They have 2-D Minkowski space projection and 2-D CP_2 projection so that one can say that there is no space-time in ordinary sense present during the primordial phase. During cosmic evolution their time= constant M^4 projection gradually thickens from ideal string to a magnetic flux tube. Among other things this explains the presence of magnetic fields in all cosmic scale not easy to understand in standard view. The decay of cosmic strings generates visible and dark matter much in the same manner as the decay of inflaton field does in inflationary scenario. One however avoids the many problems of inflationary scenario.

   Cosmic strings would contain ordinary matter and dark matter around them like necklace contains pearls along it. Cosmic strings carry Kähler magnetic monopole flux which stabilizes them. The magnetic field energy explains dark energy. Magnetic tension explains the negative "pressure" explaining accelerated expansion. The linear distribution of field energy along cosmic strings gives rise to logarithmic gravitational potential, which explains the constant velocity spectrum of distant stars around galaxy and therefore galactic dark matter.

2. Magnetic flux tubes form a fractal structure and the notion of Falaco soliton has also an analogy in TGD based description of elementary particles. In TGD framework the ends caps of vortices correspond to pairs of wormhole throats connected by short wormhole contact and there is a magnetic flux tube carrying monopole flux at both space-time sheets.

   So called Kähler-Dirac equation assigns with this flux tube 1-D closed string and to it string world sheets, which might be 2-D minimal surface of space-time surface [K104]. Rather surprisingly, string model in 4-D space-time emerges naturally in TGD framework and has also very special properties due to the knotting of strings as 1-knots and knotting of string world sheets as 2-knots. Braiding and linking of strings is also involved and make dimension D=4 for space-time completely unique.

   Both elementary particles and hadron like state are describable in terms of these string like objects. Wormhole throats are the basic building brick of particles which are in the simplest situation two-sheeted structure with wormhole contact structures connecting the sheets and giving rise to one or more closed flux tubes accompanied by closed strings.

4.3.2 Stringy Description Of Condensed Matter Physics And Chemistry?

What is important that magnetic flux tubes and associated string world sheets can also connect wormhole throats associated with different elementary particles in the sense that their boundaries are along light-like wormhole throats assignable to different elementary particles. These string worlds sheets therefore mediate interactions between elementary particles.

1. What these interactions are? Could *string world sheets could provide a microscopic first principle description of condensed matter phenomena* - in particular of sound waves and

various waves analogs of sound waves usually regarded as emergent phenomena requiring phenomenological models of condensed matter?

The hypothesis that this is the case would allow to test basic assumptions of quantum TGD at the level of condensed matter physics. String model in 4-D space-time could describe concrete experimental everyday reality rather than esoteric Planck length scale physics! The phenomena of condensed matter physics often thought to be high level emergent phenomena would have first principle microscopic description at the level of space-time geometry.

2. The idea about stringy reductionism extends also to chemistry. One of the poorly understanding basic notions of molecular chemistry is the formation of valence bond as pairing of two valence electrons belonging to different atoms. Could this pairing correspond to a formation of a closed Kähler magnetic flux tube with two wormhole contacts carrying quantum numbers of electron? Could also Cooper pairs be regarded as this kind of structure with long connecting pair of flux tubes between electron carrying wormhole contacts as has been suggested already earlier?

3. The proposal indeed is that TGD inspired biochemistry and neuroscience indeed has magnetic flux tubes and flux sheets as a key element. For instance, the notion of magnetic body plays a key role in TGD inspired view about EEG and magnetic flux tubes represent braid strands in the model for DNA-cell membrane system as topological quantum computer [K2].

One can argue that this is not a totally new idea: basically one particular variant of holography[1] is in question and follows in TGD framework from general coordinate invariance alone: the geometry of world of classical worlds must assign to a given 3-surface a unique space-time surface.

1. The fashionable manner to realize holography is by replacing 4-D space-time with 10-D one. String world sheets in 10-D space-time $AdS_5 - S_5$ connecting the points of 4+5-D boundary of $AdS_5 - S_5$ are hoped to provide a dual description of even condensed matter phenomena in the case that the system is described by a theory enjoying conformal invariance in 4-D sense.

2. In TGD framework holography is much more concrete: 3-D light-like 3-surfaces (giving rise to generalized conformal invariance by their metric 2-dimensionality) are enough. One has actually a strong form of holography stating that 2-D partonic 2-surfaces plus their 4-D tangent space data are enough. Partonic 2-surfaces define the ends of light-like 3-surfaces at the ends of space-time surface at the light-like 7-D boundaries of causal diamonds. 10-D space is replaced with the familiar 4-D space-time and 4+5-D boundary with end 2-D ends of 3-D light-like wormhole orbits (plus 4-D tangent space data). These partonic 2-surfaces are highly analogous to the 2-D sections of your characteristic surfaces.

Consider now how sound waves as and various oscillations of this kind could be understood in terms of string word sheets. String world sheets have both geometric and fermionic degrees of freedom.

1. A good first guess is that string world sheet is minimal surface in space-time - this does not mean minimal surface property in embedding space and the non-vanishing second fundamental form- in particular its CP_2 part should have physical meaning - maybe the parameter that would be called Higgs vacuum expectation in QFT limit of TGD could relate to it.

2. Another possibility that I have proposed is that a minimal surface of embedding space (not the minimal surface is geometric analog for a solution of massless wave equation) but in the effective metric defined by the anti-commutators of modified gamma matrices defined by the canonical momentum densities of Kähler action is in question: in this case one might even dream about the possibility that the analog of light-velocity defined by the effective metric has interpretation as sound velocity.

For string world sheets as minimal surfaces of X^4 (the first option) oscillations would propagate with light-velocity but as one adds massive particle momenta at wormhole throats defining their ends the situation changes due to the additional inertia making impossible propagation with light-velocity. Consideration of the situation for ordinary non-relativistic condensed matter string

[1] The equivalent of holography emerged from the construction of the Kähler geometry of "world of classical worlds" as an implication of general coordinate invariance around 1990, about five years before it was introduced by t'Hooft and Susskind.

with masses at ends as a simple example, the velocity of propagation is in the first naïve estimate just square root of the ratio of the magnetic energy of string portion to its total energy which also concludes the mass at its ends. Kähler magnetic energy is given by string tension which has a spectrum determined by p-adic length scale hypothesis so that one ends up with a rough quantitative picture and coil understand the dependence of the sound velocity on temperature.

In TGD framework massless quanta moving in different directions correspond to different space-time sheets: linear superposition for fields is replaced with a set theoretic union and effects superpose instead of fields. This would hold true also for sound waves which would always be restricted at stringy world sheets: superposition can make sense only for wave moving in exactly the same direction. This of course conforms with the properties of phonons so that Bohr orbitology would be realized for sound waves and ordinary description of sound waves would be only an approximation. The fundamental difference between light and sound defining fundamental qualia would be the dimension of the quanta as geometric structures.

4.3.3 New Manner To Understand The Generation Of Turbulent Wake

Kiehn proposes a new manner to understand the generation of turbulent wake (see `http://tinyurl.com/y8v4e3xr`) [B42]. The dynamics generating it would be that of hyperbolic wave equation rather than diffusive parabolic or elliptic dynamics. The decay of the turbulence would however obey the diffusive parabolic dynamics. Therefore sound velocity and supersonic velocities would be involved with the generation of the turbulence.

Kiehn considers Landau's nonlinear model for a scalar potential of velocity in the case of 2-D compressible isentropic fluid as an example. The wave equation is given by

$$(c^2-\Phi_x^2)\Phi_{xx}+(c^2-\Phi_y^2)\Phi_{yy}-2\Phi_x\Phi_u\Phi_{xy} \quad = \quad 0 \ . \tag{4.3.1}$$

Here c denotes sound velocity and velocity is given by $v=\nabla\Phi$. 3-D generalization is obvious. This partial differential equation for the velocity potential is quasi-linear equation of the form

$$A\Phi_{\eta\eta}+2B\Phi_{\eta\xi}+C\Phi_{\xi\xi} \quad = \quad 0 \ . \tag{4.3.2}$$

The characteristic surfaces contain imbedded curves which are given by solutions to ordinary differentia equations

$$\frac{d\eta}{d\xi} \quad = \quad \frac{B\pm(B^2AC)^{1/2}}{C} \ . \tag{4.3.3}$$

Real solutions are possible when the argument of the square root is positive. This is true when the local velocity exceeds the local characteristic speed c. These characteristic lines combine to form characteristic surfaces.

Velocity field would be compressible ($\nabla\cdot v\neq 0$) but irrotational ($\nabla\times v=0$) in this approach whereas in standard approach velocity field would be incompressible ($\nabla\cdot v=0$) but irrotational ($\nabla\times v\neq 0$). There would be two phases in which these two different options would be realized and at the boundary the dynamics would be both in-compressible and irrotational and these boundaries would correspond to characteristic surfaces which are minimal surfaces which evolve with time somehow. The presence of scalar function satisfying Laplace equation ($\nabla^2\Phi=0$) would serve as a signature of this.

The emergence of this hyperbolic dynamics would explain the sharpness and long-lived character of the singular structures. Kiehn also proposes that the formation of wake could have analogies with diffraction and interference - basic aspects of wave motion. This picture does not conform with standard view which assumes diffusive parabolic or elliptic dynamics as the origin of the wake turbulence.

Characteristic surfaces and light-like wormhole throat orbits

Characteristic surface is key notion in Kiehn's approach and he suggests that the creation of wakes relies on hyperbolic dynamics (see http://tinyurl.com/y8v4e3xr) in restricted regions [B42]. If I have understood correctly, the boundaries of vortices created in the process could be seen as this kind of characteristic surfaces: some physical quantities would have tangential discontinuities at them since a boundary between different phases (fluid and air) would be in question.

Another situation corresponds to a shock wave in which case there is a flow of matter through the characteristic surface. Also boundary patterns associated with Kelvin-Helmholtz instability (see http://tinyurl.com/p92zx) (formation of waves due to wind and their breaking) and Rayleigh-Taylor instability (see http://tinyurl.com/brypvgm) (the formation of mushroom like fingers of heavier substance resting above lighter one).

The proposal of Kiehn is that the characteristic minimal surfaces have the following general form:

$$\begin{array}{ll} u=\frac{d\eta}{ds}=A(\rho)\times sin(Q(s))\ , & v=\frac{d\eta}{ds}=-A(\rho)\times cos(Q(s))\ , \\ w=F(u,v)=Q(u/v=s) & Q(s)=arctan(s)\ . \end{array} \tag{4.3.4}$$

If $F(u,v)$ satisfies the equation

$$(1+F_v^2)F_{uu}+(1+F_u^2)F_{vv}-2F_uF_vF_{uv} = 0\ . \tag{4.3.5}$$

This expresses the vanishing of the trace of the second fundamental form, actually the component corresponding to the coordinate w. The minimal surface in question is known as right helicoid.

In TGD framework light-like 3-surfaces defined by wormhole throats are the counterparts of characteristic surfaces.

1. By their light-likeness the light-like wormhole throats are analogous to characteristic surfaces (In TGD context light-velocity of course replace local sound velocity). Since the signature of the metric changes at wormhole throats, the 4-D tangent space reduces to 3-D in metric sense at them so that they indeed are singular in a unique sense. Gravitational effects imply that they need not look expanding in Minkowski coordinates. The light-velocity in the induced metric is in general smaller than maximal signal velocity in Minkowski space and can be arbitrarily small.

2. In TGD framework light-like 3-surfaces would be naturally associated with phase boundaries defining boundaries of physical objects. They would be light-like metrically degenerate 3-surfaces in space-time along which the space-time sheet assignable to fluid flow meets the space-time sheet assignable to say air. The generation of wake turbulence would in TGD framework mean the decay of a large 3-surface representing a laminar flow to sheet of separate cylindrical 3-surfaces representing vortex sheet. Also the amalgamation of vortices can be considered as a reverse process.

3. Interesting question related to the time evolution of these 2-D boundaries. In TGD framework it should give rise to 3-D light-like surface. The simulations for the evolution of Kelvin-Helmholtz instability and Rayleigh-Taylor mushroom pattern (see http://tinyurl.com/brypvgm) in Wikipedia and its seems that at the initial stages there is period of growth bringing in mind expanding light-front: the velocity of expansion is not its value in Minkowski space but corresponds to that assignable to the induced metric and can be much smaller. Recall also that in TGD framework gravitational effects are large near the singularity so that growth is not with the light-velocity in vacuum.

The proposal of Kiehn that very special minimal surfaces (right helicoids) are in question would in TGD framework correspond to a light-like 3-surfaces representing light-like orbits of these minimal surfaces presumably expanding at least in the beginning of the time evolution.

Minkowskian hydrodynamics/Maxwellian dynamics as hyperbolic dynamics and Euclidian hydrodynamics as elliptic dynamics

In Kiehn's proposal both the hyperbolic wave dynamics (about which Maxwell's equations provide a simple linear example) and diffusive elliptic or parabolic dynamics are present. In TGD framework both aspects are present at the level of field equations and correspond to the hyperbolic dynamics in Minkowskian space-time regions and elliptic dynamics in Euclidian space-time regions.

The dynamics of preferred extremals can be seen in two ways. Either as hydrodynamics or as Maxwellian dynamics with Bohr rules expressing the decomposition of the field to quanta-magnetic flux quanta or massless radiation quanta.

1. Maxwellian hydrodynamics involves a considerable restriction: superposition of modes moving in different directions is not allowed: one has just left-movers or right-movers in given direction, not both. Preferred extremals are "Bohr orbit like" and resemble outcomes of state function reduction measuring polarization and wave vector. The linear superposition of fields is replaced with the superposition of effects. The test particle topologically condenses to several space-time sheets simultaneously and experiences the sum of the forces of classical fields associated with the space-time sheets. Therefore one avoids the worst objection against TGD that I have been able to invent. Only four primary field like variables would replace the multitude of primary fields encountered in a typical unification. Besides this one has second quantized induced spinor fields.

2. Field equations are hydrodynamical in the sense that the field equations state classical conservation laws of four-momentum and color charges. In fermionic sector conservation of electromagnetic charge (in quantum sense so that different charge states for spinor mode do not mix) requires the localization of solutions to 2-D string world sheets for all states except right-handed neutrino. This leads to 2-D conformal invariance. A possible identification of string world sheet is as 2-D minimal surface of space-time (rather than that of embedding space).

   What is remarkable that in Minkowskian space-time regions most preferred extremals (magnetic flux tube structures define an exception to this) are locally analogous to the modes of massless field with polarization direction and light-like momentum direction which in the general case can depend on position so that one has curvilinear light-like curve as analog of light-ray. The curvilinear light-like orbits results when two parallel preferred extremals with constant light-like direction form bound states via the formation of magnetically charged wormhole contact structures identifiable as elementary particles. Total momentum is conserved and is time-like for this kind of states, and the hypothesis is that the values of mass squared are given by p-adic thermodynamics. The conservation of Kähler current holds true as also its integrability in the sense of Frobenius giving $j = \Psi\nabla\Phi$. Besides this massless wave equations hold true for both Ψ and Φ. This looks like 4-D generalization of your equations at the characteristic defined by phase boundary.

3. In Euclidian regions one has naturally elliptic "hydrodynamics". Euclidian regions correspond for 4-D CP_2 projection to the 4-D "lines" of generalized Feynman diagrams. Their M^4 projections can be arbitrary large and the proposal is that the space-time sheet characterizing the macroscopic objects is actually Euclidian. In $AdS_5 - S^5$correspondence the corresponding idea is that macroscopic object is described as a blackhole in 10-D space. Now blackhole interiors have Euclidian signature as lines of generalized Feynman diagrams and blackhole interior does not differ from the interior of any system in any dramatical manner. Whether the Euclidian and Minkowskian dynamics are dual of each other or whether both are necessary is an open question.

Chapter 5

General View About Physics in Many-Sheeted Space-Time

5.1 Introduction

In previous chapter "General View About Physics in Many-Sheeted Space-Time" the notion of many-sheeted space-time concept and the understanding of coupling constant evolution at space-time level were discussed without reference to the newest developments in quantum TGD. In this chapter this picture is completed by a summary of the new rather dramatic developments in TGD that have occurred during last few years (the year I am writing this is 2007). The most important steps of progress are following ones.

5.1.1 Parton Level Formulation Of Quantum TGD

The formulation of quantum TGD at partonic level identifying fundamental objects as light-like 3-surfaces having also interpretation as random light-like orbits of 2-D partons having arbitrarily large size. This picture reduces quantum TGD to an almost-topological quantum field theory and leads to a dramatic understanding of S-matrix. A generalization of Feynman diagrams emerges obtained by replacing lines of Feynman diagram with light-like 3-surfaces meeting along their ends at vertices. This picture is different from that of string models and means also a generalization of the view about space-time and 3-surface since these surfaces cannot be assumed to be a smooth manifold anymore.

Extended super-conformal invariance involving the fusion of ordinary Super-Kac Moody symmetries and so called super-symplectic invariance generalizing the Kac-Moody algebra by replacing the Lie algebra of finite-dimensional Lie group with that for symplectic transformations of $\delta M^4_\pm \times CP_2$ plays a key role in this framework. The help of professionals in this branch of mathematics would be badly needed in order to develop a detailed understanding about the predicted particle spectrum.

5.1.2 Zero Energy Ontology

The notion of zero energy ontology emerged implicitly in cosmological context from the observation that the imbeddings of Robertson-Walker metrics are always vacuum extremals. In fact, practically all solutions of Einstein's equations have this property very naturally. The explicit formulation emerged with the progress in the formulation of quantum TGD. In zero energy ontology physical states are creatable from vacuum and have vanishing net quantum numbers, in particular energy. Zero energy states can be decomposed to positive and negative energy parts with definite geometro-temporal separation, call it T, and having interpretation in terms of initial and final states of particle reactions. Zero energy ontology is consistent with ordinary positive energy ontology at the limit when the time scale of the perception of observer is much shorter than T. One of the implications is a new view about fermions and bosons allowing to understand Higgs mechanism among other things.

Zero energy ontology leads to the view about S-matrix as a characterizer of time-like entanglement associated with the zero energy state and a generalization of S-matrix to what might be called M-matrix emerges. M-matrix is complex square root of density matrix expressible as a product of real valued "modulus" and unitary matrix representing phase and can be seen as a matrix valued generalization of Schrödinger amplitude. Also thermodynamics becomes an inherent element of quantum theory in this approach.

5.1.3 Fusion Of Real And P-Adic Physics To Single One

The fusion of p-adic physics and real physics to single coherent whole requires generalization of the number concept obtained by gluing reals and various p-adic number fields along common algebraic numbers. This leads to a completely new vision about how cognition make themselves visible in real physics via long range correlations realized via the effective p-adicity of real physics. The success of p-adic length scale hypothesis and p-adic mass calculations suggest that cognition and intentionality are present already at elementary particle level. This picture leads naturally to an effective discretization of the real physics at the level of S-matrix and relying on the notion of umber theoretic braid.

5.1.4 Dark Matter Hierarchy And Hierarchy Of Planck Constants

The idea about hierarchy of Planck constants relying on generalization of the embedding space was inspired both by empirical input (Bohr quantization of planetary orbits and anomalies of biology) and by the mathematics of hyper-finite factors of type II_1 combined with the quantum classical correspondence. Consider first the mathematical structure in question.

1. The Clifford algebra of World of Classical Worlds (WCW) creating many fermion states is a standard example of an algebra expressible as a direct integral of copies of von Neumann algebras known as hyper-finite factor of type II_1 (HFFs). The inclusions of HFFs relate very intimately to the notion of finite measurement resolution. There is a canonical hierarchy of Jones inclusions [A1] labeled by finite subgroups of SU(2) [A46]. Quantum classical correspondence suggests that these inclusions have space-time correlates [K103, K35] and the generalization of embedding space would provide these correlates.

2. The space $CD\times CP_2$, where $CD \subset M^4$ is so called causal diamond identified as the intersection of future and past directed light-cones defines the basic geometric structure in zero energy ontology. The positive (negative) energy part of the zero energy state is located to the lower (upper) light-like boundaries of $CD \times CP_2$ and has interpretation as the initial (final) state of the physical event in standard positive energy ontology. p-Adic length scale hypothesis follows if one assumes that the temporal distance between the tips of CD comes as an octave of fundamental time scale defined by the size of CP_2. The "world of classical worlds" (WCW) is union of sub-WCWs associated with spaces $CD\times CP_2$ with different locations in $M^4\times CP_2$.

3. One can say that causal diamond CD and the space CP_2 appearing as factors in $CD \times CP_2$ forms the basic geometric structure in zero energy ontology, is replaced with a book like structure obtained by gluing together infinite number of singular coverings and factor spaces of CD *resp.* CP_2 together. The copies are glued together along a common "back" $M^2 \subset M^2$ of the book in the case of CD. In the case of CP_2 the most general option allows two backs corresponding to the two non-isometric geodesic spheres S^2_i, $i = I, II$, represented as sub-manifolds $\xi^1 = \overline{\xi}^2$ and $\xi^1 = \xi^2$ in complex coordinates transforming linearly under $U(2) \subset SU(3)$. Color rotations in CP_2 produce different choices of this pair.

4. The selection of geodesic spheres S^2 and M^2 is an imbedding space correlate for the fixing of quantization axes and means symmetry breaking at the level of embedding space geometry. WCW is union over all possible choices of CD and pairs of geodesic spheres so that at the level no symmetry breaking takes place. The points of M^2 and S^2 have a physical interpretation in terms of quantum criticality with respect to the phase transition changing Planck constant (leakage to another page of the book through the back of the book).

5. The pages of the singular coverings are characterized by finite subgroups G_a and G_b of $SU(2)$ and these groups act in covering or leave the points of factor space invariant. The pages

are labeled by Planck constants $\hbar(CD) = n_a\hbar_0$ and $\hbar(CP_2) = n_b\hbar_0$, where n_a and n_b are integers characterizing the orders of maximal cyclic subgroups of G_a and G_b. For singular factor spaces one has $\hbar(CD) = \hbar_0/n_a$ and $\hbar(CP_2) = \hbar_0/n_b$. The observed Planck constant corresponds to $\hbar = (\hbar(CD)/\hbar(CP_2)) \times \hbar_0$. What is also important is that $(\hbar/\hbar_0)^2$ appears as a scaling factor of M^4 covariant metric so that Kähler action via its dependence on induced metric codes for radiative corrections coming in powers of ordinary Planck constant: therefore quantum criticality and vanishing of radiative corrections to functional integral over WCW does not mean vanishing of radiative corrections.

The interpretation in terms of dark matter comes as follows.

1. Large values of $\hbar$ make possible macroscopic quantum phase since all quantum scales are scaled upwards by $\hbar/\hbar_0$. Anyonic and charge fractionization effects allow to "measure" $\hbar(CD)$ and $\hbar(CP_2)$ rather than only their ratio. $\hbar(CD) = \hbar(CP_2) = \hbar_0$ corresponds to what might be called standard physics without any anyonic effects and visible matter is identified as this phase.
2. Particle states belonging to different pages of the book can interact via classical fields and by exchanging particles, such as photons, which leak between the pages of the book. This leakage means a scaling of frequency and wavelength in such a way that energy and momentum of photon are conserved. Direct interactions in which particles from different pages appear in the same vertex of generalized Feynman diagram are impossible. This seems to be enough to explain what is known about dark matter. This picture differs in many respects from more conventional models of dark matter making much stronger assumptions and has far reaching implications for quantum biology, which also provides support for this view about dark matter.

5.1.5 Equivalence Principle And Evolution Of Coupling Constants

The views about Equivalence Princple (EP) and GRT limit of TGD have changed quite a lot since 2007 and here the updated view is summarized. Before saying anything about evolution of gravitational constant one must understand whether it is a fundamental constant or prediction of quantum TGD. Also one should understand whether Equivalence Principle holds true and if so, in what sense. Also the identification of gravitational and inertial masses seems to be necessary.

At classical level EP follows from the interpretation of GRT space-time as effective space-time obtained by replacing many-sheeted space-time with Minkowski space with effective metric determined as a sum of Minkowski metric and sum over the deviations of the induced metrices of space-time sheets from Minkowski metric. Poincare invariance suggests strongly classical EP for the GRT limit in long length scales at least. One can consider also other kinds of limits such as the analog of GRT limit for Euclidian space-time regions assignable to elementary particles. In this case deformations of CP_2 metric define a natural starting point and CP_2 indeed defines a gravitational instanton with very large cosmological constant in Einstein-Maxwell theory. Also gauge potentials of standard model correspond classically to superpositions of induced gauge potentials over space-time sheets.

Gravitational constant, cosmological constant, and various gauge couplings emerge as predictions. Planck length should be related to CP_2 size by a dimensionless numerical factor predicted by the theory. These constants need not be universal constants: cosmological constant is certainly very large for the Euclidian variant of GRT space-time. These constants could also depend on p-adic length scale. p-Adic coupling constant evolution suggests itself as a discretized variant of coupling constant evolution and p-adic scales would relate naturally to the size scales of causal diamonds: perhaps the integer n characterizing the multiple of CP_2 scale giving the distance between the tips of CD has p-adic prime p or its power as a divisor.

At the level of single space-time sheet and CD it is not possible to talk about coupling constant evolution since Kähler action and Kähler-Dirac action contain no coupling constants.

This description however gives rise to p-adic coupling constant evolution since the process of lumping together the sheets of the many-sheeted space-time gives a result which depends on the size scale of CD. If the non-deterministic dynamics of Kähler action for the maxima of Kähler function mimics p-adic non-determinism then one has hopes about p-adic coupling constant evolution. The p-adic prime and therefore also the length scale and coupling constants characterizing the dynamics for given CD would vary wildly as function of integer characterizing CD size scale. This could mean that the CDs whose size scales are related by multiplication of small integer are close to each other. They would be near to each other in logarithmic sense and logarithms indeed appear in running coupling constants. This "prediction" is of course subject to criticism.

All this is a work in progress and there are many uncertainties involved. Despite this it seems that it is good to sum up the recent view in order to make easier to refer to the new developments in the existing chapters.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L8].

5.2 The New Developments In Quantum TGD

This section summarizes the developments in quantum TGD which have taken place during last few years.

5.2.1 Reduction Of Quantum TGD To Parton Level

It took surprisingly long time before the realization that quantum TGD can be reduced to parton level in the sense that fundamental objects are light-like 3-surfaces (of arbitrary size). This identification follows from 4-D general coordinate invariance. Light-likeness in turn implies effective 2-dimensionality of the fermionic dynamics. 4-D space-time sheets are identified as preferred extrema of Kähler action. A stronger form of holography is that Kähler-Dirac action and Chern-Simons action for light-like partonic 3-surfaces defined the Kähler action as a logarithm of the fermionic determinant.

Magic properties of 3-D light-like surfaces and generalization of super-conformal symmetries

The very special conformal properties of both boundary $\delta M^4_\pm$ of 4-D light-cone and of light-like partonic 3-surfaces X^3 imply a generalization and extension of the super-conformal symmetries of super-string models to 3-D context [K25, K24]. Both the Virasoro algebras associated with the light-like coordinate r and to the complex coordinate z transversal to it define super-conformal algebras so that the structure of conformal symmetries is much richer than in string models.

1. The symplectic transformations of $\delta M^4_\pm \times CP_2$ give rise to an infinite-dimensional symplectic/symplectic algebra having naturally a structure of Kac-Moody type algebra with respect to the light-like coordinate of $\delta M^4_\pm = S^2 \times R_+$ and with finite-dimensional Lie group G replaced with the symplectic group. The conformal transformations of S^2 localized with respect to the light like coordinate act as conformal symmetries analogous to those of string models. The super-symplectic algebra, call it SC, made local with respect to partonic 2-surface can be regarded as a Kac-Moody algebra associated with an infinite-dimensional Lie algebra.
2. The light-likeness of partonic 3-surfaces is respected by conformal transformations of H made local with respect to the partonic 3-surface and gives to a generalization of bosonic Kac-Moody algebra, call it KM, Also now the longitudinal and transversal Virasoro algebras emerge. The commutator $[KM, SC]$ annihilates physical states.
3. Fermionic Kac-Moody algebras act as algebras of left and right handed spinor rotations in M^4 and CP_2 degrees of freedom. Also the Kähler-Dirac operator allows super-conformal symmetries as gauge symmetries of its generalized eigen modes.

Quantum TGD as almost topological quantum field theory at parton level

The original belief was that the light-like character of basic dynamical objects X^3_l at which the signature of the induced metric changes implies that Chern-Simons action for the induced Kähler gauge potential of CP_2 determines the classical dynamics of partonic 3-surfaces [K104]. This turned out to be a wrong guess: Kähler action and corresponding Kähler-Dirac action is enough.

1. Number theoretical compactification and the properties of known extremals of Kähler action suggests strongly the slicing of space-time surface by 3-D light-like surfaces Y_l^3 parallel to X_l^3. The surfaces Y_l^3 behave as independent dynamical units in the sense that conserved currents flow along them so that quantum holography is realized. Number theoretic compactification allows also dual slicings of $X^4(X_l^3)$ by string world sheets Y^2 and partonic 2-surfaces X^2.
2. The Kähler-Dirac action obtained as the super-symmetric counterpart Kähler action fixes the dynamics of the second quantized free fermionic fields in terms of which WCW gamma matrices and WCW spinor s can be constructed. The essential difference to the ordinary massless Dirac action is that induced gamma matrices are replaced by the contractions of the symplectic momentum densities Kähler action with embedding space gamma matrices. Therefore the effective metric defined by the Kähler-Dirac gamma matrices replaces ordinary gamma matrices and the corresponding effective metric can be non-singular even when induced metric is degenerate. Effective 3-dimensionality means that the modes of the induced spinor field are constant with respect to the light-like coordinate labeling the slices Y_l^3.
3. Kähler-Dirac action is consistent with the symmetries of Kähler action provided its first variation with respect to H coordinates vanishes - or equivalently- the second variation of Kähler action varies. This would realize quantum criticality at space-time level. The second variation vanishes only for those deformations which correspond to conserved currents.

   The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg` or **Fig. ??** in the appendix of this book).
4. Kähler-Dirac operator decomposes as $D_K = D_K(Y^2) + D_K(X^2)$ and its zero modes for effectively 3-D solutions can be chosen to be generalized eigenmodes of $D_K(X^2)$. The product of the generalized eigenvalues of $D_K(X^2)$ defines the exponent of Kähler function conjectured to reduce to Kähler action for the preferred extremal.

Fermionic statistics is geometrized in terms of spinor geometry of WCW since gamma matrices are linear combinations of fermionic oscillator operators identifiable also as super-symplectic generators [K104]. Only the light-likeness property involving the notion of induced metric breaks the topological QFT property of the theory so that the theory is as close to a physically trivial theory as it can be.

The resulting generalization of $N = 4$ super-conformal symmetry [A40] involves super-symplectic algebra (SC)and super Kac-Moody algebra (SKM) [K24] There are considerable differences as compared to string models. Super generators carry fermion number, no sparticles are predicted (at least super Poincare invariance is not obtained), SKM algebra and corresponding Virasoro algebra associated with light-like coordinates of X^3 and $\delta M^4_\pm$ do not annihilate physical states which justifies p-adic thermodynamics used in p-adic mass calculations, four-momentum does not appear in Virasoro generators so that there are no problems with Lorentz invariance, and mass squared is p-adic thermal expectation of conformal weight.

5.2.2 Quantum Measurement Theory With Finite Measurement Resolution

Infinite-dimensional Clifford algebra of CH can be regarded as a canonical example of a von Neumann algebra known as a hyper-finite factor of type II_1 [A37, A46] (shortly HFF) characterized by the defining condition that the trace of infinite-dimensional unit matrix equals to unity: $Tr(Id) = 1$. In TGD framework the most obvious implication is the absence of fermionic normal ordering infinities whereas the absence of bosonic divergences is guaranteed by the basic properties of WCW Kähler geometry, in particular the non-locality of the Kähler function as a functional of 3-surface.

The special properties of this algebra, which are very closely related to braid and knot invariants [A32, A28], quantum groups [A46], non-commutative geometry [A26], spin chains, integrable models [B37], topological quantum field theories [A44], conformal field theories, and at the level of concrete physics to anyons [D11], generate several new insights and ideas about the structure of quantum TGD.

Jones inclusions $\mathcal{N} \subset \mathcal{M}$ [A1, A46] of these algebras lead to quantum measurement theory with a finite measurement resolution characterized by $\mathcal{N}$ [K103, K35]. Quantum Clifford algebra $\mathcal{M}/\mathcal{N}$ interpreted as $\mathcal{N}$-module creates physical states modulo measurement resolution. Complex rays of the state space resulting in the ordinary state function reduction are replaced by $\mathcal{N}$-rays and the notions of unitarity, hermiticity, and eigenvalue generalize [K23, K35].

The notion of entanglement generalizes so that entanglement coefficients are N-valued. Generalized eigenvalues are in turn N-valued hermitian operators. S- and U-matrices become N valued and probabilities are obtained from N-valued probabilities as traces.

Non-commutative physics would be interpreted in terms of a finite measurement resolution rather than something emerging below Planck length scale. An important implication is that a finite measurement sequence can never completely reduce quantum entanglement so that entire universe would necessarily be an organic whole. Topologically condensed space-time sheets could be seen as correlates for sub-factors which correspond to degrees of freedom below measurement resolution. Topological condensation in turn corresponds to the inclusion $\mathcal{N} \subset \mathcal{M}$. This is however not the only possible interpretation.

5.2.3 Hierarchy Of Planck Constants

The idea about hierarchy of Planck constants relying on generalization of the embedding space was inspired both by empirical input (Bohr quantization of planetary orbits) and by the mathematics of hyper-finite factors of type II_1 combined with the quantum classical correspondence.

The generalization of embedding space concept and hierarchy of Planck constants

Quantum classical correspondence suggests that Jones inclusions [A1] have space-time correlates [K103, K35]. There is a canonical hierarchy of Jones inclusions labeled by finite subgroups of SU(2) [A46] This leads to a generalization of the embedding space obtained by gluing an infinite number of copies of H regarded as singular bundles over $H/G_a \times G_b$, where $G_a \times G_b$ is a subgroup of $SU(2) \times SU(2) \subset SL(2,C) \times SU(3)$. Gluing occurs along a factor for which the group is same. The generalized embedding space has clearly a book like structure with pages of books intersecting along 4-D sub-manifold $M^2 \times S^2$, S^2 a geodesic sphere of CP_2 characterizing the choice of quantization axes. Entire configuration space is union over “books” corresponding to various choices of this sub-manifold.

The groups in question define in a natural manner the direction of quantization axes for for various isometry charges and this hierarchy seems to be an essential element of quantum measurement theory. Ordinary Planck constant, as opposed to Planck constants $\hbar_a = n_a \hbar_0$ and $\hbar_b = n_b \hbar_0$ appearing in the commutation relations of symmetry algebras assignable to M^4 and CP_2, is naturally quantized as $\hbar = (n_a/n_b)\hbar_0$, where n_i is the order of maximal cyclic subgroup of G_i. The hierarchy of Planck constants is interpreted in terms of dark matter hierarchy [K35]. What is also important is that $(n_a/n_b)^2$ appear as a scaling factor of M^4 metric so that Kähler action via its dependence on induced metric codes for radiative corrections coming in powers of ordinary Planck constant: therefore quantum criticality and vanishing of radiative corrections to functional integral over WCW does not mean vanishing of radiative corrections.

G_a would correspond directly to the observed symmetries of visible matter induced by the underlying dark matter [K35]. For instance, in living matter molecules with 5- and 6-cycles could directly reflect the fact that free electron pairs associated with these cycles correspond to $n_a = 5$ and $n_a = 6$ dark matter possibly responsible for anomalous conductivity of DNA [K35, K15] and recently reported strange properties of graphene [D9]. Also the tetrahedral and icosahedral symmetries of water molecule clusters could have similar interpretation [K33], [D14].

A further fascinating possibility is that the observed indications for Bohr orbit quantization of planetary orbits [E25] could have interpretation in terms of gigantic Planck constant for underlying dark matter [K79] so that macroscopic and -temporal quantum coherence would be possible in astrophysical length scales manifesting itself in many ways: say as preferred directions of quantization axis (perhaps related to the CMB anomaly) or as anomalously low dissipation rates.

Since the gravitational Planck constant is proportional to the product of the gravitational masses of interacting systems, it must be assigned to the field body of the two systems and characterizes the interaction between systems rather than systems themselves. This observation applies quite generally and each field body of the system (em, weak, color, gravitational) is characterized by its own Planck constant.

In the gravitational case the order of G_a is gigantic and at least GM_1m/v_0, $v_0 = 2^{-11}$ the favored value. The natural interpretation is as a discrete rotational symmetry of the gravitational field body of the system having both gravimagnetic and gravi-electric parts. The subgroups of G_a for which order is a divisor of the order of G_a define broken symmetries at the lower levels of dark matter hierarchy, in particular symmetries of visible matter.

The number theoretically simple ruler-and-compass integers having as factors only first powers of Fermat primes and power of 2 would define a physically preferred sub-hierarchy of quantum criticality for which subsequent levels would correspond to powers of 2: a connection with p-adic length scale hypothesis suggests itself. Ruler and compass hypothesis implies that besides p-adic length scales also their 3- and 5- multiples should be important. Note that in the structure of chromosomes p-adic length scale $L(151) \simeq 10$ characterizes beads-on-string structure of DNA whereas the length scale $3L(151)$ appears in the coiling of this structure.

Implications of dark matter hierarchy

The basic implication of dark matter hierarchy is hierarchy of macroscopic quantum coherent systems covering all length scales. The presence of this hierarchy is visible as exact discrete symmetries of field bodies reflecting at the level of visible matter as broken symmetries. In case of gravitational interaction these symmetries are highest and also the scale of quantum coherence is astrophysical. Together with ruler-and-compass hypothesis and p-adic length scale hypothesis this leads to very powerful predictions and p-adic length scale hypothesis might reduce to the ruler-and-compass hypothesis.

At the level of condensed matter one application is nuclear string model explaining also the selection rules of cold fusion and predicting that dark copy of weak physics with atomic scale defining the range of weak interaction is involved. Note that cold fusion has recently gained considerable support. High T_c super-conductivity is second application of dark matter hierarchy.

The 5- and 6-fold symmetries of the sugar backbone of DNA suggest that corresponding cyclic groups or cyclic groups having these groups as factors are symmetries of dark matter part of DNA presumably consisting of what is called as free electron pairs assignable to 5- and 6-cycles. The model allows to understand the observed high conductivity of DNA not consistent with the insulator property of DNA at the level of visible matter.

Dark matter and bio-control

The hierarchy of dark matters provides rather concrete realization for the vision about living matter as quantum critical system. This vision will be discussed in more detail later.

The large Planck constants characterize various field bodies of physical system. This gives justification to the notion of (magnetic) field body which plays key role in TGD inspired model of living matter serving as intentional agent controlling the behavior of field body. For instance, the model of EEG relies and of bio-control relies on this notion. The large value of the Planck constant is absolutely essential since for a given low frequency it allows to have gauge boson energy above thermal threshold. Large value of Planck constant is essential for time mirror mechanism which is behind the models of metabolism, long term memory, and intentional action.

The huge values of gravitational Planck constant supports the vision of Penrose [J5] about the special role of quantum gravitation in living matter. In TGD framework the proposal of Penrose and Hameroff for the emergence of consciousness known as Orch-Or (Orchestrated Objective Reduction [J6]) is however too restricted since it gives a very special role to micro-tubules.

A reasonable guess - based on the hypothesis that transition to dark matter phase occurs when perturbation theory for standard value of Planck constant fails - is that $GMm > 1$ is the criterion for the transition to dark phase for the gravitational field body characterizing the interaction between the two masses so that Planck mass becomes the critical mass for this transition. For the density of water this means size scale of.1 mm, the size of large neuron.

5.2.4 Zero Energy Ontology

Zero energy ontology has roots in TGD inspired cosmology [K80]. The problem has been that the embeddings of Robertson-Walker cosmologies have vanishing densities of Poincare momenta identified as inertial momenta whereas gravitational energy density is non-vanishing. This led to the conclusion that one must allow space-time sheets with both time orientations such that the signs of Poincare energies are different for them and total density of inertial energy vanishes. Gravitational momenta can be identified as difference of the Poincare momenta and need not be conserved.

Construction of S-matrix and zero energy ontology

The construction of S-matrix allows to formulate this picture more sharply. Zero energy states have positive and negative energy parts located in geometric past and future and S-matrix can be identified as time-like entanglement coefficients between these states. Positive energy ontology is a good approximation in time scales shorter than the temporal distance between positive and negative energy states. This picture leads also to a generalization of Feynman graphs obtained by gluing light-like partonic 3-surfaces together along their ends at vertices. These Feynman cobordisms become a basic element of quantum TGD having interpretation as almost topological QFT and category theoretical formulation of quantum TGD emerges.

Elementary particles and zero energy ontology

At the level of elementary particles zero energy ontology means that fermionic quantum numbers are located at the light-like throats of wormhole contacts connecting CP_2 type extremals with Euclidian signature of induced metric to space-time sheets with Minkowskian signature of induced metric. Gauge bosons in turn correspond to pieces of CP_2 type extremals connecting positive and negative energy space-time sheets with fermion and anti-fermion quantum numbers at the throats of the wormhole contact. Depending on the sign of net energy one has ordinary boson or its phase conjugate. Gravitons correspond to pairs of fermion or gauge boson pair with particle and antiparticle connected by flux tube. This string picture emerges automatically if one assumes that the fermions of the conformal field theory associated with partonic 3-surface are free. It is also possible to have gauge bosons corresponding to single wormhole throat: these particles correspond to bosonic generators of super-symplectic algebra and excitations which correspond to genuine WCW degrees of freedom so that description in terms of quantum field theory in fixed background space-time need not work.

5.2.5 U- And S-Matrices

In quite early stage physical arguments led to the conclusion that the universal U-matrix associated with quantum jump must be distinguished from the S-matrix characterizing the rates of particle reactions. The notion of zero energy ontology was however needed before it became possible to characterize the difference between these matrices in a more precise manner.

Some distinctions between U- and S-matrices

The distinctions between U- and S-matrices discussed in more detail in [K60] have become rather clear.

1. U-matrix is the universal unitary matrix assignable to quantum jump between zero energy states whereas S-matrix can be identified assigned with the square root of density matrix expressible as its hermitian square root multiplied with a unitary S-matrix, which is universal. M-matrices from in ZEO an orthonormal basis of hermitian matrices so that the choice of the density matrix is not arbitrary. M-matrix defines time-like entanglement coefficients between positive and negative energy parts of the zero energy state. S-matrix characterizes zero energy states and actually codes the physics unless one is interested in consciousness.
2. State function reduction for S-matrix elements reduces the entanglement between positive and negative energy parts of a given zero energy state and is completely analogous to ordinary

quantum measurement reducing entanglement between systems having space-like separation. It can take place at either boundary of CD and the sequence of repeated state function reduction at passive boundary give rise to self as conscious entity which dies and re-incarnates when the first reduction to the opposite boundary of CD takes place.

3. U-matrix is unitary as also S-matrix. For HFFs of type II_1 M-matrix can be taken to be S-matrix since the trace of the unit matrix equals to one. In the most general case S-matrix can be regarded as a "square" root of the density matrix assignable to time like entanglement: this hypothesis would unify the notions of S-matrix and density matrix and one could regard quantum states as matrix analogs of Schrö;dinger amplitudes expressible as products of its modulus (square root of probability density replaced with square root of density matrix) and phase (possibly universal unitary S-matrix). Thermal S-matrices define an important special case and thermodynamics becomes an integral part of quantum theory in zero energy ontology.

What can one say about the general structure of U-, M-, and S-matrices?

In Zero Energy Ontology (ZEO) S-matrix must be replaced with the triplet U-matrix, M-matrix, and S-matrix. U-matrix realizes unitary time evolution in the space for zero energy states realized geometrically as dispersion in the moduli space of causal diamonds (CDs) leaving second boundary (passive boundary) of CD and states at it fixed.

This process can be seen as the TGD counterpart of repeated state function reductions leaving the states at passive boundary unaffected and affecting only the member of state pair at active boundary (Zeno effect). In TGD inspired theory of consciousness self corresponds to the sequence these state function reductions. M-matrix describes the entanglement between positive and negative energy parts of zero energy states and is expressible as a hermitian square root H of density matrix multiplied by a unitary matrix S, which corresponds to ordinary S-matrix, which is universal and depends only the size scale n of CD through the formula $S(n) = S^n$. M-matrices and H-matrices defined by hermitian square roots of density matrices form an orthonormal basis at given CD and H-matrices would naturally correspond to the generators of super-symplectic algebra.

The first state function reduction to the opposite boundary corresponds to what happens in quantum physics experiments. The relationship between U- and S-matrices has remained poorly understood. In [K60] this relationship is analyzed by starting from basic principles. One ends up to formulas allowing to understand the architecture of U-matrix and to reduce its construction to that for S-matrix having interpretation as exponential of the generator L_{-1} of the Virasoro algebra associated with the super-symplectic algebra.

The first state function reduction to the opposite boundary corresponds to what happens in quantum physics experiments. The relationship between U- and S-matrices has remained poorly understood.

The original view about the relationship was a purely formal guess: M-matrices would define the orthonormal rows of U-matrix. This guess is not correct physically and one must consider in detail what U-matrix really means.

1. First about the geometry of CD [K60]. The boundaries of CD will be called passive and active: passive boundary correspond to the boundary at which repeated state function reductions take place and give rise to a sequence of unitary time evolutions U followed by localization in the moduli of CD each. Active boundary corresponds to the boundary for which U induces delocalization and modifies the states at it.

   The moduli space for the CDs consists of a discrete subgroup of scalings for the size of CD characterized by the proper time distance between the tips and the sub-group of Lorentz boosts leaving passive boundary and its tip invariant and acting on the active boundary only. This group is assumed to be represented unitarily by matrices Λ forming the same group for all values of n.

   The proper time distance between the tips of CDs is quantized as integer multiples of the minimal distance defined by CP_2 time: $T = nT_0$. Also in quantum jump in which the size scale n of CD increases the increase corresponds to integer multiple of T_0. Using the logarithm of proper time, one can interpret this in terms of a scaling parametrized by an integer. The possibility to interpret proper time translation as a scaling is essential for having a manifest Lorentz invariance: the ordinary definition of S-matrix introduces preferred rest system.

2. The physical interpretation would be roughly as follows. M-matrix for a given CD codes for the physics as we usually understand it. M-matrix is product of square root of density matrix and S-matrix depending on the size scale of CD and is the analog of thermal S-matrix. State function at the opposite boundary of CD corresponds to what happens in the state function reduction in particle physics experiments. The repeated state function reductions at same boundary of CD correspond to TGD version of Zeno effect crucial for understanding consciousness. Unitary U-matrix describes the time evolution zero energy states due to the increase of the size scale of CD (at least in statistical sense). This process is dispersion in the moduli space of CDs: all possible scalings are allowed and localization in the space of moduli of CD localizes the active boundary of CD after each unitary evolution.

From this picture one ends up to general formulas [K60] allowing to understand the architecture of U-matrix and to reduce its construction to that for S-matrix having interpretation as exponential of the generator L_1 of the Virasoro algebra associated with the super-symplectic algebra.

Number theoretic universality and S-matrix

The fact that zero energy states are created by p-adic to-real transitions and must be number theoretically universal suggests strongly that the data about partonic 2-surfaces contributing to S-matrix elements come from the intersection of real partonic 2-surface and its p-adic counterpart satisfying same algebraic equations. The intersection consists of algebraic points and contains as subset number theoretic braids central for the proposed construction of S-matrix.

The question is whether also states for which S-matrix receives data from non-algebraic points should be allowed or whether the data can come even from continuous string like structures at partonic 2-surfaces as standard conformal field theory picture would suggest. If also S-matrix is algebraic, one can wonder whether there is any difference between p-adic and real physics at all. The latter option would mean that intentional action is followed by a unitarity process U analogous to a dispersion of completely localized particle implied by Schröndinger equation.

The algebraic universality of S-matrix could mean that S-matrix is obtained as an algebraic continuation of an S-matrix in algebraic extension of rationals by replacing incoming momenta and other continuous quantum numbers with real ones. Similar continuation should make sense in p-adic sector. S-matrix and U-matrix in a given algebraic extension of rationals or p-adics are not in general diagonalizable. Thus number theory would allows to avoid the paradoxical conclusion that S-matrix is always diagonal in a suitable basis.

Later progress in understanding of quantum TGD allows to refine and simplify this view dramatically. In adelic vision real and p-adic numbers are aspects of existence in all length scales and mean that cognition is present at all levels rather than emerging. Intentions have interpretation in terms of state function reductions in ZEO and there is no need to identify p-adic space-time sheets as their correlates.

5.2.6 Number Theoretic Ideas

p-Adic physics emerged roughly at the same time via p-adic mass calculations. The interpretation of p-adic physics as physics of cognition emerged.

Cognition would be present already at elementary particle level and p-adic fractality would be the experimental signature of it making itself visible in elementary particle mass spectrum among other things. The success of p-adic mass calculations provides strong support for the hypothesis.

This led gradually to the vision about physics as generalized number theory. It involves three separate aspects.

1. The p-adic approach led eventually to the program of fusing real physics and various p-adic physics to a single coherent whole by generalizing the number concept by gluing reals and various p-adics to a larger structure along common rationals and algebraics (see **Fig.** `http://tgdtheory.fi/appfigures/book.jpg` or **Fig. ??** in the appendix of this book).

This inspired the notion of algebraic universality stating that for instance S-matrix should result by algebraic continuation from rational or at most algebraic valued S-matrix.

The notion of number theoretic braid belonging to the algebraic intersection of real and p-adic partonic 2-surface obeying same algebraic equations emerged also and gives a further connection with topological QFT: s. The perturbation theoretic definition of S-matrix is definitely excluded in this approach and TGD indeed leads to the understanding of coupling constant evolution at the level of "free" theory as a discrete p-adic coupling constant evolution so that radiative corrections are not needed for this purpose.

2. Also the classical number fields relate closely to TGD and the vision is that embedding space M^4xCP_2 emerges from the physics based on hyper-octonionic 8-space with associativity as the fundamental dynamical principle both at classical and quantum level. Hyper-octonion space M^8 with space-time surface identified as hyper-quanternionic sub-manifolds or their duals and M^4xCP_2 would provide in this framework dual ways to describe physics and this duality would provide TGD counterpart for compactification.

3. The construction of infinite primes is analogous to repeated second quantization of super-symmetric arithmetic quantum field theory. This notion implies a further generalization of real and p-adic numbers allowing space-time points to have infinitely complex number theoretic structure not visible at the level of real physics. The idea is that space-time points define the Platonia able to represent in its structure arbitrarily complex mathematical structures and that space-time points could be seen as evolving structures becoming quantum jump by quantum jump increasingly complex number theoretically. Even the world of classical worlds (light-like 3-surfaces) and quantum states of Universe might be represented in terms of the number theoretic anatomy of space-time points (number theoretic Brahman=Atman and algebraic holography).

S-matrix as a functor and the groupoid structure formed by S-matrices

In zero energy ontology S-matrix can be seen as a functor from the category of Feynman cobordisms to the category of operators. S-matrix can be identified as a "square root" of the positive energy density matrix $S = \rho_+^{1/2} S_0$, where S_0 is a unitary matrix and ρ_+ is the density matrix for positive energy part of the zero energy state. Obviously one has $SS^\dagger = \rho_+$. $S^\dagger S = \rho_-$ gives the density matrix for negative energy part of zero energy state. Clearly, S-matrix can be seen as matrix valued generalization of Schrödinger amplitude. Note that the "indices" of the S-matrices correspond to WCW spinor s (fermions and their bound states giving rise to gauge bosons and gravitons) and to WCW degrees of freedom (world of classical worlds). For hyper-finite factor of II_1 it is not strictly speaking possible to speak about indices since the matrix elements are traces of the S-matrix multiplied by projection operators to infinite-dimensional subspaces from right and left.

The functor property of S-matrices implies that they form a multiplicative structure analogous but not identical to groupoid [A5]. Recall that groupoid has associative product and there exist always right and left inverses and identity in the sense that ff^{-1} and $f^{-1}f$ are always defined but not identical and one has $fgg^{-1} = f$ and $f^{-1}fg = g$.

The reason for the groupoid like property is that S-matrix is a map between state spaces associated with initial and final sets of partonic surfaces and these state spaces are different so that inverse must be replaced with right and left inverse. The defining conditions for groupoid are replaced with more general ones. Also now associativity holds but the role of inverse is taken by hermitian conjugate. Thus one has the conditions $fgg^\dagger = f\rho_{g,+}$ and $f^\dagger fg = \rho_{f,-}g$, and the conditions $ff^\dagger = \rho_+$ and $f^\dagger f = \rho_-$ are satisfied. Here $\rho_\pm$ is density matrix associated with positive/negative energy parts of zero energy state. If the inverses of the density matrices exist, groupoid axioms hold true since $f_L^{-1} = f^\dagger \rho_{f,+}^{-1}$ satisfies $ff_L^{-1} = Id_+$ and $f_R^{-1} = \rho_{f,-}^{-1} f^\dagger$ satisfies $f_R^{-1} f = Id_-$.

There are good reasons to believe that also tensor product of its appropriate generalization to the analog of co-product makes sense with non-triviality characterizing the interaction between the systems of the tensor product. If so, the S-matrices would form very beautiful mathematical structure bringing in mind the corresponding structures for 2-tangles and N-tangles. Knowing how incredibly powerful the group like structures have been in physics, one has good reasons to hope that groupoid like structure might help to deduce a lot of information about the quantum dynamics of TGD.

A word about nomenclature is in order. S has strong associations to unitarity and it might be appropriate to replace S with some other letter. The interpretation of S-matrix as a generalized Schrödinger amplitude would suggest Ψ-matrix. Since the interaction with Kea's M-theory blog (with M denoting Monad or Motif in this context) was crucial for the realization of the the connection with density matrix, also M-matrix might work. S-matrix as a functor from the category of Feynman cobordisms in turn suggests C or F. Or could just Matrix denoted by M in formulas be enough?

Number theoretic braids

The notion of number theoretic braid has gradually evolved to a fundamental notion in quantum TGD and both number theoretical universality (p-adicization), TGD as almost-TQFT, and the notion of finite measurement resolution lead to this notion. The decisive proof of the notion came from the observation that the special properties of Kähler action imply this concept. In the quantization of induced spinor fields the number of fermionic oscillators is finite so that anti-commutation relations can hold true only for a finite point set defining the points of the number theoretic braid. The natural identification of the number theoretic braid is as the intersection of M^4 (CP_2) projection of X_l^3 with the back M^2 of M^4 book (back S_i^2, $i = I, II$, of CP_2 book) so that the points of braid would be always quantum critical. Both homologically trivial ($i = I$) and non-trivial geodesic sphere ($i = II$) can be considered in the case of CP_2 so that there would be three possibly equivalent braidings defining kind of holy trinity.

The notion of number theoretic braid is especially interesting from the point of view of quantum biology. Generalized Feynman diagrams obtained by gluing light-like partonic 3-surfaces (whose sizes can be arbitrarily large) along their ends and define what might be called Feynman cobordisms. The first expectation was that number theoretic braids replicate in the vertices identifiable as partonic 2-surfaces at which the incoming and outgoing lines of generalized Feynman diagram meet. This would be nice but is not the case since by the lacking anti-commutatitivity of the incoming and outgoing oscillator operators the lines need not meet in this manner. This suggested an attractive information theoretic interpretation of generalized Feynman diagrams. Incoming and outgoing "lines" would give rise to topological quantum computations characterized by corresponding M-matrices, vertices would represent the replication of number theoretic braids analogous to DNA replication, and internal lines would be analogous to quantum communications. One could of generalize this simple view about computation by allowing creation of new strands instead of mere replication.

Number theoretic braids are associated with light-like 3-surfaces and can be said to have both dynamical and static characteristics. Partonic 2-surfaces as sub-manifolds of space-like 3-surface can also become linked and knotted and would naturally define space-like counterparts of tangles. Number theoretic braids could define dynamical topological quantum computation like operations whereas partonic 2-surfaces associated with say RNA could define as their space-like counterparts tangles and in the special case braids analogous to printed quantum programs so that there is duality between space-like and light-like braids [K2]. In terms of dance metaphor the dynamical braiding defined by the light like braid points interpreted as dancers has as a dual space-like braiding resulting as the threads connecting the feet of the dancers get tangled. An interesting question is how light-like and space-like braidings are transformed to each other: could this process correspond to a conscious reading like process and how closely DNA relates to language so that reading and writing would be fundamental processes appearing in all scales.

It came as a pleasant surprise that the idea about duality of space-like and light-like braidings inspired by DNA as topological quantum computer [J4] [K2] is realized at the level of basic quantum TGD [K104]. The dual slicings of $X^4(X_l^3)$ to string world sheets Y^2 and partonic 2-surfaces X^2 generalize the original picture in the sense that one can speak either about partons or string world sheets as basic objects. The strings connecting points of braid strands in X_l^3 would define space-like braidings whereas time like braidings are associated with X_l^3. The light-like braiding at X_l^3 induces the space-like braiding of strings connecting the points of the strands to the strands of other braids.

Dark matter hierarchy and hierarchy of quantum critical systems in modular degrees of freedom

Dark matter hierarchy corresponds to a hierarchy of conformal symmetries Z_n of partonic 2-surfaces with genus $g \geq 1$ such that factors of n define subgroups of conformal symmetries of Z_n. By the decomposition $Z_n = \prod_{p|n} Z_p$, where $p|n$ tells that p divides n, this hierarchy corresponds to an hierarchy of increasingly quantum critical systems in modular degrees of freedom. For a given prime p one has a sub-hierarchy Z_p, $Z_{p^2} = Z_p \times Z_p$, etc... such that the moduli at n+1: th level are contained by n: th level. In the similar manner the moduli of Z_n are sub-moduli for each prime factor of n. This mapping of integers to quantum critical systems conforms nicely with the general vision that biological evolution corresponds to the increase of quantum criticality as Planck constant increases. This hierarchy would also define a hierarchy of conscious entities and could relate directly to mathematical cognition.

The group of conformal symmetries could be also non-commutative discrete group having Z_n as a subgroup. This inspires a very short-lived conjecture that only the discrete subgroups of SU(2) allowed by Jones inclusions are possible as conformal symmetries of Riemann surfaces having $g \geq 1$. Besides Z_n one could have tetrahedral and icosahedral groups plus cyclic group Z_{2n} with reflection added but not Z_{2n+1} nor the symmetry group of cube. The conjecture is wrong. Consider the orbit of the subgroup of rotational group on standard sphere of E^3, put a handle at one of the orbits such that it is invariant under rotations around the axis going through the point, and apply the elements of subgroup. You obtain a Riemann surface having the subgroup as its isometries. Hence all discrete subgroups of SU(2) can act even as isometries for some value of g.

The number theoretically simple ruler-and-compass integers having as factors only first powers of Fermat primes and power of 2 would define a physically preferred sub-hierarchy of quantum criticality for which subsequent levels would correspond to powers of 2: a connection with p-adic length scale hypothesis suggests itself.

Spherical topology is exceptional since in this case the space of conformal moduli is trivial and conformal symmetries correspond to the entire $SL(2,C)$. This would suggest that only the fermions of lowest generation corresponding to the spherical topology are maximally quantum critical. This brings in mind Jones inclusions for which the defining subgroup equals to $SU(2)$ and Jones index equals to $\mathcal{M}/\mathcal{N} = 4$. In this case all discrete subgroups of $SU(2)$ label the inclusions. These inclusions would correspond to fiber space $CP_2 \rightarrow CP_2/U(2)$ consisting of geodesic spheres of CP_2. In this case the discrete subgroup might correspond to a selection of a subgroup of $SU(2) \subset SU(3)$ acting non-trivially on the geodesic sphere. Cosmic strings $X^2 \times Y^2 \subset M^4 \times CP_2$ having geodesic spheres of CP_2 as their ends could correspond to this phase dominating the very early cosmology.

5.3 Identification Of Elementary Particles And The Role Of Higgs In Particle Massivation

The development of the recent view about the identification of elementary particles and particle massivation has taken fifteen years since the discovery of p-adic thermodynamics around 1993. p-Adic thermodynamics worked excellently from the beginning for fermions. Only the understanding of gauge boson masses turned out to be problematic and group theoretical arguments led to the proposal that Higgs boson should be present and give the dominating contribution to the masses of gauge bosons whereas the contribution to fermion masses should be small and even negligible. The detailed understanding of quantum TGD at partonic level eventually led to the realization that the coupling to Higgs is not needed after all. The deviation Δh of the ground state conformal weight from negative integer has interpretation as effective Higgs contribution since Higgs vacuum expectation is naturally proportional to Δh but the coupling to Higgs does not cause massivation. In the following I summarize the basic identification of elementary particles and massivation. A more detailed discussion can be found in [?].

5.3.1 Identification Of Elementary Particles

The developments in the formulation of quantum TGD which have taken place during the period 2005-2007 [K24, K23] suggest dramatic simplifications of the general picture discussed in the earlier version of this chapter. p-Adic mass calculations [K53, K62, K58] leave a lot of freedom concerning the detailed identification of elementary particles.

Elementary fermions and bosons

The basic open question is whether the *theory is on some sense free at parton level* as suggested by the recent view about the construction of S-matrix (actually its generalization M-matrix) and by the almost topological QFT property of quantum TGD at parton level [K23]. If partonic 2-surfaces at elementary particle level carry only free many-fermion states, no bi-local composites of second quantized induced spinor field would be needed in the construction of the quantum states and this would simplify the theory enormously.

If this is the case, the basic conclusion would be that light-like 3-surfaces - in particular the ones at which the signature of induced metric changes from Minkowskian to Euclidian - are carriers of fermionic quantum numbers. These regions are associated naturally with CP_2 type vacuum extremals identifiable as correlates for elementary fermions if only fermion number ± 1 is allowed for the stable states. The question however arises about the identification of elementary bosons.

Wormhole contacts with two light-like wormhole throats carrying fermion and anti-fermion quantum numbers are the first thing that comes in mind. The wormhole contact connects two space-time sheets with induced metric having Minkowski signature. Wormhole contact itself has an Euclidian metric signature so that there are two wormhole throats which are light-like 3-surfaces and would carry fermion and anti-fermion number. In this case a delicate question is whether the space-time sheets connected by wormhole contacts have opposite time orientations or not. If this the case the two fermions would correspond to positive and negative energy particles.

I considered first the identification of only Higgs as a wormhole contact but there is no reason why this identification should not apply also to gauge bosons (certainly not to graviton). This identification would imply quite a dramatic simplification since the theory would be free at single parton level and the only stable parton states would be fermions and anti-fermions.

This picture allows to understand the difference between fermions and gauge bosons and Higgs particle. For fermions topological explanation of family replication predicts three fermionic generations [K22] corresponding to handle numbers $g = 0, 1, 2$ for the partonic 2-surface. In the case of gauge bosons and Higgs this replication is not visible. This could be due to the fact that gauge bosons form singlet and octet representation of the dynamical $SU(3)$ group associated with the handle number $g = 0, 1, 2$ since bosons correspond to pairs of handles. If octet representation is heavy the experimental absence of family replication for bosons can be understood.

Graviton and other stringy states

Fermion and anti-fermion can give rise to only single unit of spin since it is impossible to assign angular momentum with the relative motion of wormhole throats. Hence the identification of graviton as single wormhole contact is not possible. The only conclusion is that graviton must be a superposition of fermion-anti-fermion pairs and boson-anti-boson pairs with coefficients determined by the coupling of the parton to graviton. Graviton-graviton pairs might emerge in higher orders. Fermion and anti-fermion would reside at the same space-time sheet and would have a non-vanishing relative angular momentum. Also bosons could have non-vanishing relative angular momentum and Higgs bosons must indeed possess it.

Gravitons are stable if the throats of wormhole contacts carry non-vanishing gauge fluxes so that the throats of wormhole contacts are connected by flux tubes carrying the gauge flux. The mechanism producing gravitons would the splitting of partonic 2-surfaces via the basic vertex. A connection with string picture emerges with the counterpart of string identified as the flux tube connecting the wormhole throats. Gravitational constant would relate directly to the value of the string tension.

The development of the understanding of gravitational coupling has had many twists and it is perhaps to summarize the basic misunderstandings.

1. CP_2 length scale R, which is roughly $10^{3.5}$ times larger than Planck length $l_P = \sqrt{\hbar G}$, defines a fundamental length scale in TGD. The challenge is to predict the value of Planck length $\sqrt{\hbar G}$. The outcome was an identification of a formula for $R^2/\hbar G$ predicting that the magnitude of Kähler coupling strength α_K is near to fine structure constant in electron length scale (for ordinary value of Planck constant should be added here).
2. The emergence of the parton level formulation of TGD finally demonstrated that G actually appears in the fundamental parton level formulation of TGD as a fundamental constant characterizing the M^4 part of CP_2 Kähler gauge potential [K104, K68]. This part is pure gauge in the sense of standard gauge theory but necessary to guarantee that the theory does not reduce to topological QFT. Quantum criticality requires that G remains invariant under p-adic coupling constant evolution and is therefore predictable in principle at least.
3. The TGD view about coupling constant evolution [L48] predicts the proportionality $G \propto L_p^2$, where L_p is p-adic length scale. Together with input from p-adic mass calculations one ends up to two conclusions. The correct conclusion was that Kähler coupling strength is equal to the fine structure constant in the p-adic length scale associated with Mersenne prime $p = M_{127} = 2^{127} - 1$ assignable to electron [L48]. I have considered also the possibility that α_K would be equal to electro-weak $U(1)$ coupling in this scale.
4. The additional - wrong- conclusion was that gravitons must always correspond to the p-adic prime M_{127} since G would otherwise vary as function of p-adic length scale. As a matter fact, the question was for years whether it is G or g_K^2 which remains invariant under p-adic coupling constant evolution. I found both options unsatisfactory until I realized that RG invariance is possible for both g_K^2 and G! The point is that the exponent of the Kähler action associated with the piece of CP_2 type vacuum extremal assignable with the elementary particle is exponentially sensitive to the volume of this piece and logarithmic dependence on the volume fraction is enough to compensate the $L_p^2 \propto p$ proportionality of G and thus guarantee the constancy of G.

The explanation for the small value of the gravitational coupling strength serves as a test for the proposed picture. The exchange of ordinary gauge boson involves the exchange of single CP_2 type extremal giving the exponent of Kähler action compensated by state normalization. In the case of graviton exchange two wormhole contacts are exchanged and this gives second power for the exponent of Kähler action which is not compensated. It would be this additional exponent that would give rise to the huge reduction of gravitational coupling strength from the naïve estimate $G \sim L_p^2$.

Gravitons are obviously not the only stringy states. For instance, one obtains spin 1 states when the ends of string correspond to gauge boson and Higgs. Also non-vanishing electro-weak and color quantum numbers are possible and stringy states couple to elementary partons via standard couplings in this case. TGD based model for nuclei as nuclear strings having length of order $L(127)$ [K84] suggests that the strings with light M_{127} quark and anti-quark at their ends identifiable as companions of the ordinary graviton are responsible for the strong nuclear force instead of exchanges of ordinary mesons or color van der Waals forces.

Also the TGD based model of high T_c super-conductivity involves stringy states connecting the space-time sheets associated with the electrons of the exotic Cooper pair [K15, K16]. Thus stringy states would play a key role in nuclear and condensed matter physics, which means a profound departure from stringy wisdom, and breakdown of the standard reductionistic picture.

Spectrum of non-stringy states

The 1-throat character of fermions is consistent with the generation-genus correspondence. The 2-throat character of bosons predicts that bosons are characterized by the genera (g_1, g_2) of the wormhole throats. Note that the interpretation of fundamental fermions as wormhole contacts with second throat identified as a Fock vacuum is excluded.

The general bosonic wave-function would be expressible as a matrix M_{g_1,g_2} and ordinary gauge bosons would correspond to a diagonal matrix $M_{g_1,g_2} = \delta_{g_1,g_2}$ as required by the absence of neutral flavor changing currents (say gluons transforming quark genera to each other). 8 new gauge bosons are predicted if one allows all 3×3 matrices with complex entries orthonormalized with respect to trace meaning additional dynamical $SU(3)$ symmetry. Ordinary gauge bosons would be

$SU(3)$ singlets in this sense. The existing bounds on flavor changing neutral currents give bounds on the masses of the boson octet. The 2-throat character of bosons should relate to the low value $T = 1/n \ll 1$ for the p-adic temperature of gauge bosons as contrasted to $T = 1$ for fermions.

If one forgets the complications due to the stringy states (including graviton), the spectrum of elementary fermions and bosons is amazingly simple and almost reduces to the spectrum of standard model. In the fermionic sector one would have fermions of standard model. By simple counting leptonic wormhole throat could carry $2^3 = 8$ states corresponding to 2 polarization states, 2 charge states, and sign of lepton number giving 8+8=16 states altogether. Taking into account phase conjugates gives 16+16=32 states.

In the non-stringy boson sector one would have bound states of fermions and phase conjugate fermions. Since only two polarization states are allowed for massless states, one obtains $(2+1) \times (3+1) = 12$ states plus phase conjugates giving 12+12=24 states. The addition of color singlet states for quarks gives 48 gauge bosons with vanishing fermion number and color quantum numbers. Besides 12 electro-weak bosons and their 12 phase conjugates there are 12 exotic bosons and their 12 phase conjugates. For the exotic bosons the couplings to quarks and leptons are determined by the orthogonality of the coupling matrices of ordinary and boson states. For exotic counterparts of W bosons and Higgs the sign of the coupling to quarks is opposite. For photon and Z^0 also the relative magnitudes of the couplings to quarks must change. Altogether this makes 48+16+16=80 states. Gluons would result as color octet states. Family replication would extend each elementary boson state into $SU(3)$ octet and singlet and elementary fermion states into $SU(3)$ triplets.

What about light-like boundaries and macroscopic wormhole contacts?

Light-like boundaries of the space-time sheet as also wormhole throats can have macroscopic size and can carry free many-fermion states but not elementary bosons. Number theoretic braids and anyons might be assignable to these structures. Deformations of cosmic strings to magnetic flux tubes with a light-like outer boundary are especially interesting in this respect.

If the ends of a string like object move with light velocity as implied by the usual stringy boundary conditions they indeed define light-like 3-surfaces. Many-fermion states could be assigned at the ends of string. One could also connect in pairwise manner the ends of two time-like strings having opposite time orientation using two space-like strings so that the analog of boson state consisting of two wormhole contacts and analogous to graviton would result. "Wormhole throats" could have arbitrarily long distance in M^4.

Wormhole contacts can be regarded as slightly deformed CP_2 type extremals only if the size of M^4 projection is not larger than CP_2 size. The natural question is whether one can construct macroscopic wormhole contacts at all.

1. The throats of wormhole contacts cannot belong to vacuum extremals. One might however hope that small deformations of macrosopic vacuum extremals could yield non-vacuum wormhole contacts of macroscopic size.

2. A large class of macroscopic wormhole contacts which are vacuum extremals consists of surfaces of form $X_1^2 \times X_2^2 \subset (M^1 \times Y^2) \times E^3$, where Y^2 is Lagrangian manifold of CP_2 (induced Kähler form vanishes) and $M^4 = M^1 \times E^3$ represents decomposition of M^1 to time-like and space-like sub-spaces. X_2^2 is a stationary surface of E^3. Both $X_1^2 \subset M^1 \times CP_2$ and X_2^2 have an Euclidian signature of metric except at light-like boundaries $X_a^1 \times X_2^2$ and $X_b^1 \times X_2^2$ defined by ends of X_1^2 defining the throats of the wormhole contact.

3. This kind of vacuum extremals could define an extremely general class of macroscopic wormhole contacts as their deformations. These wormhole contacts describe an interaction of wormhole throats regarded as closed strings as is clear from the fact that X^2 can be visualized as an analog of closed string world sheet X_1^2 in $M^1 \times Y^2$ describing a reaction leading from a state with a given number of incoming closed strings to a state with a given number of outgoing closed strings which correspond to wormhole throats at the two space-time sheets involved.

If one accepts the hierarchy of Planck constants [K35] leading to the generalization of the notion of embedding space, the identification of anyonic phases in terms of macroscopic light-like surfaces emerges naturally. In this kind of states large fermion numbers are possible. Dark matter would correspond to this kind of phases and "partonic" 2-surfaces could have even astrophysical size. Also black holes can be identified as dark matter at light-like 3-surfaces analogous to black hole horizons and possessing gigantic value of Planck constant [K68].

5.3.2 New View About The Role Of Higgs Boson In Massivation

The proposed identifications challenge the standard model view about particle massivation.

1. The standard model inspired interpretation would be that Higgs vacuum expectation associated with the coherent state of neutral Higgs wormhole contacts generates gauge boson mass. The TGD counterpart of Higgs would be however not H-scalar but complex CP_2 tangent vector. There are no covariantly constant vector fields in CP_2 so that the idea about Higgs vacuum expectation is not mathematically feasible. This led to the original exaggerated conclusion that TGD does not allow Higgs: it is however only Higgs vacuum expectation which does not look plausible. Fermionic mass would be solely due to p-adic thermodynamics. Also in the case gauge boson masses one encounters a problem: the natural guess for the p-adic prime as M_{89} represents too small gauge boson masses, and it is very difficult to understand Weinberg angle, which is essentially group theoretical notion.
2. The Kähler-Dirac equation plus well-definedness of em charge requires that the spinor modes are restricted to stringy curves connecting the throats of two wormhole contacts associated with the elementary particles and carrying monopole fluxes. One can say that the wormhole throats are connected by flux tube behaving like string. The obvious idea is that the flux tube gives additional contribution to the mass squared, which can be interpreted as a contribution to the conformal weight of the ground state. If the string tension is proportional to gauge coupling strength for W and Z and to the counterpart of Higgs self coupling λ for Higgs one can explain the mass ratios of gauge bosons.
3. Besides the thermodynamical contribution to the particle mass there would be a small contribution from the ground state conformal weight unless this weight is not negative integer. Gauge boson mass would correspond to the ground state conformal weight present in both fermionic and bosonic states and in the case of gauge bosons this contribution would dominate due to the small value of p-adic temperature. For fermions p-adic thermodynamics for super Virasoro algebra would give the dominating contribution to the mass.
4. The remaining problem is to understand how the negative value of the ground state conformal weight emerges. This negative conformal weight compensated by the action of Super Virasoro generators is necessary for the success of p-adic mass calculations. The intuitive expectation is that the solution of this problem must relate to the Euclidian signature of the regions representing lines of generalized Feynman diagrams.
   (a) Kähler-Dirac action gives for the solutions of Dirac action a boundary term which is essentially contraction of the normal component of the vector defined by Kähler-Dirac gamma matrices. In absence of measurement interaction terms the boundary condition for K-D equation states $\Gamma^n\Psi = 0$ at the stringy curves at the space-like ends of space-time surface. Γ^n must be lightlike and the assumption is that the spinor modes are generalized eigenmodes of Γ^n: $\Gamma^n\Psi = p^k\gamma_k\Psi = 0$ where p^k is constant light-like four-momentum. This conforms with the idea that all fermions are massless and massive states of super-conformal representations emerges as bound states of fermions at wormhole throats. Elementary particles would correspond to pair of wormholes with magnetic flux flowing between the throats at the two space-time sheets involved. Massivation would be many-sheeted phenomenon. The string like objects would have string tension explaining the masses of weak bosons at microscopic level.
   Very naïvely, $\Gamma^n\Psi = 0$ is possible only in the regions of space-like 3-surface which belong to Minkowskian space-time regions. Since Kähler-Dirac gamma matrices are in question it can however happen that the effective metric of string world sheet defined by Γ^α is degenerate. If CP_2 projection is 4-D as it is for CP_2 type extremals, one however ecpects that Γ^α is not degenerate inside wormhole contacts, and one can even question the localization of the spinor modes to 2-D string world sheets in these regions. The TGD based variant of stringy diagrammatics would indeed involve massless fermionic propagators only in the Minkowskian regions. The interaction of fermions at opposite throats

of wormhole contacts would be described by stringy propagator $1/L_0$ or its non-local generalization to the product $(1/G)(1) \times (1/G)^\dagger(2)$ with supergenerators $G(i)$ assigned with the opposite wormhole throats.

(b) One can add to the Kähler action measurement interaction term fixing the space-time surfaces to have conserved classical identical to their quantum counterparts belonging to Cartan algebra of symmetries. This can be achieved by adding Lagrange multiplier terms. These terms contribute to the Kähler-Dirac action a term at space-like ends of 3-surface and this term modifies the TGD counterpart of massless Dirac equation. The original generalized massless generalized eigenvalue spectrum associated with $p^k\gamma_k\Psi = 0$ of Γ^n is modified to massive spectrum given by the condition

$$\Gamma^n\Psi = -\sum_i \lambda_i \Gamma^\alpha_{Q_i} D_\alpha \Psi = p^k\gamma_k\Psi \ ,$$

where Q_i refers to i: th conserved charge. Fermions are not massless anymore. This description is certainly over-simplified since several wormhole throats are involved. It is also only a formal description for the values of quantum numbers Q_i. One might say that $(\Gamma^n)^2$ serves as the analog of Higgs field vacuum expectation defined at the string curve.

(c) It is not clear whether the tachyonic value of mass squared for ground state of superconformal representations can emerge from this kind of description. This might be possible inside wormhole contacts which have Euclidian signature of induced metric and define the lines of generalized Feynman diagrams.

5.3.3 General Mass Formulas

In the following general view about p-adic mass formulas and related problems is discussed.

Mass squared as a thermal expectation of super Kac-Moody conformal weight

The general view about particle massivation is based on the generalized coset construction allowing to understand the p-adic thermal contribution to mass squared as a thermal expectation value of the conformal weight for super Kac-Moody Virasoro algebra ($SKMV$) or equivalently super-symplectic Virasoro algebra (SSV). Conformal invariance holds true only for the generators of the differences of $SKMV$ and SSV generators. In the case of SSV and $SKMV$ only the generators L_n, $n > 0$, annihilate the physical states. Obviously the actions of super-symplectic Virasoro (SSV) generators and Super Kac-Moody Virasoro generators on physical states are identical. The interpretation is in terms of Equivalence Principle. p-Adic mass expectation value is same irrespective of whether it is calculated for the excitations created by SSV or $KKMV$ generators and p-adic mass calculations are consisted with super-conformal invariance.

1. Super-Kac Moody conformal weights must be negative for elementary fermions and this can be understood if the ground state conformal weight corresponds to the square of the imaginary eigenvalue of the modified Dirac operator having dimensions of mass. If the value of ground state conformal weight is not negative integer, a contribution to mass squared analogous to Higgs expectation is obtained.
2. Massless state is thermalized with respect to SKMV (or SSV) with thermal excitations created by generators L_n, $n > 0$.

Under what conditions conformal weight is additive

The question whether four- momentum or conformal weight is additive in p-adic mass calculations becomes acute in hadronic mass calculations. Only the detailed understanding of quantum TGD at partonic level allowed to understand the situation. One can consider three options.

1. Conformal weight and thus mass squared is additive only inside the regions of X^3_l, which correspond to non-vanishing of induced Kähler magnetic field since these behave effectively as separate 3-surfaces as far as eigenmodes of the Kähler-Dirac operator are considered. The spectrum of the ground state conformal weights is indeed different for these regions in the general case. The four-momenta associated with different regions would be additive. This

makes sense since the tangent space of $X^4(X_l^3)$ contains at each point of X_l^3 a subspace $M^2(x)) \subset M^4$ defining the plane of non-physical polarizations and the natural interpretation is that four-momentum is in this plane. Hence the problem of original mass calculations forcing to assign all partonic four-momenta to a fixed plane M^2 is avoided.

2. If assigns independent translational degrees of freedom only to disjoint partonic 2-surfaces, a separate mass formula for each X_i^2 would result and four-momenta would be additive:

$$M_i^2 = \sum_i L_{0i}(SKM) \ . \tag{5.3.1}$$

Here $L_{0i}(SKM)$ contains a CP_2 cm term giving the CP_2 contribution to the mass squared known once the spinorial partial waves associated with super generators used to construct the state are known. Also vacuum conformal weight is included.

3. At the other extreme one has the option is based on the assignment of the mass squared with the total cm. This option looked the only reasonable one for 15 years ago. This would give

$$M^2 = (\sum_i p_i)^2 = \sum_i M_i^2 + 2\sum_{i\neq j} p_i \cdot p_j = -\sum_i L_{0i}(SKM) \ . \tag{5.3.2}$$

The additivity of mass squared is strong condition and p-adic mass calculations for hadrons suggest that it holds true for quarks of low lying hadrons. For this option the decomposition of the net four momentum to a sum of individual momenta can be regarded as subjective unless there is a way to measure the individual masses.

Mass formula for bound states of partons

The coefficient of proportionality between mass squared and conformal weight can be deduced from the observation that the mass squared values for CP_2 Dirac operator correspond to definite values of conformal weight in p-adic mass calculations. It is indeed possible to assign to partonic 2-surface X^2 CP_2 partial waves correlating strongly with the net electro-weak quantum numbers of the parton so that the assignment of ground state conformal weight to CP_2 partial waves makes sense. In the case of M^4 degrees of freedom it is not possible to talk about momentum eigen states since translations take parton out of δH_+ so that momentum must be assigned with the tip of the light-cone containing the particle.

The additivity of conformal weight means additivity of mass squared at parton level and this has been indeed used in p-adic mass calculations. This implies the conditions

$$(\sum_i p_i)^2 = \sum_i m_i^2 \tag{5.3.3}$$

The assumption $p_i^2 = m_i^2$ makes sense only for massless partons moving collinearly. In the QCD based model of hadrons only longitudinal momenta and transverse momentum squared are used as labels of parton states, which would suggest that one has

$$\begin{aligned} p_{i,||}^2 &= m_i^2 \ , \\ -\sum_i p_{i,\perp}^2 + 2\sum_{i,j} p_i \cdot p_j &= 0 \ . \end{aligned} \tag{5.3.4}$$

The masses would be reduced in bound states: $m_i^2 \rightarrow m_i^2 - (p_T^2)_i$. This could explain why massive quarks can behave as nearly massless quarks inside hadrons.

5.4 Super-Symplectic Degrees Of Freedom

5.4.1 What Could Happen In The Transition To Non-Perturbative QCD?

What happens mathematically in the transition to non-perturbative QCD has remained more or less a mystery. The number theoretical considerations of [K3] inspired the idea that Planck constant is dynamical and has a spectrum given as $\hbar(n) = n\hbar_0$, where n characterizes the quantum phase $q = exp(i2\pi/n)$ associated with Jones inclusion. The strange finding that the orbits of planets seem to obey Bohr quantization rules with a gigantic value of Planck constant inspired the hypothesis that the increase of Planck constant provides a unique mechanism allowing strongly interacting system to stay in perturbative phase [K79, K35]. The resulting model allows to understand dark matter as a macroscopic quantum phase in astrophysical length and time scales, and strongly suggest a connection with dark matter and biology.

The phase transition increasing Planck constant could provide a model for the transition to confining phase in QCD. When combined with the recent ideas about value spectrum of Kähler coupling strength one ends up with a rather explicit model about non-perturbative aspects of hadron physics already successfully applied in hadron mass calculations [K62]. Mersenne primes seem to define the p-adic length scales of gauge bosons and of hadronic space-time sheets. The quantization of Planck constant provides additional insight to p-adic length scales hypothesis and to the preferred role of Mersenne primes.

Super-symplectic gluons and non-perturbative aspects of hadron physics

According to the model of hadron masses [K62], in the case of light pseudo-scalar mesons the contribution of quark masses to the mass squared of meson dominates whereas spin 1 mesons contain a large contribution identified as color interaction conformal weight (color magnetic spin-spin interaction conformal weight and color Coulombic conformal weight). This conformal weight cannot however correspond to the ordinary color interactions alone and is negative for pseudo-scalars and compensated by some unknown contribution in the case of pion in order to avoid tachyonic mass. Quite generally this realizes the idea about light pseudo-scalar mesons as Goldstone bosons. Analogous mass formulas hold for baryons but in this case the additional contribution which dominates.

The unknown contribution can be assigned to the $k = 107$ hadronic space-time sheet and must correspond to the non-perturbative aspects of QCD and the failure of the quantum field theory approach at low energies. In TGD the failure of QFT picture corresponds to the presence of WCW degrees of freedom ("world of classical worlds") in which super-symplectic algebra acts. The failure of the approximation assuming single fixed background space-time is in question.

The purely bosonic generators carry color and spin quantum numbers: spin has however the character of orbital angular momentum. The only electro-weak quantum numbers of super-generators are those of right-handed neutrino. If the super-generators degrees carry the quark spin at high energies, a solution of proton spin puzzle emerges.

The presence of these degrees of freedom means that there are two contributions to color interaction energies corresponding to the ordinary gluon exchanges and exchanges of super-symplectic gluons. It turns out the model assuming same topological mixing of super-symplectic bosons identical to that experienced by U type quarks leads to excellent understanding of hadron masses assuming that hadron spin correlates with the super-symplectic particle content of the hadronic space-time sheet.

According to the argument already discussed, at the hadronic $k = 107$ space electro-weak interactions would be absent and classical $U(1)$ action should vanish. This is guaranteed if $\alpha_{U(1)}$ diverges. This would give

$$\alpha_s = \alpha_K = \frac{1}{4} \quad .$$

This would give also a quantitative articulation for the statement that strong interactions are charge independent.

This α_s would correspond to the interaction via super-symplectic colored gluons and would lead to the failure of perturbation theory. By the general criterion stating that the failure of perturbation theory leads to a phase transition increasing the value of Planck constant one expects that the value of $\hbar$ increases [K35]. The value leaving the value of α_K invariant would be $\hbar \rightarrow 26\hbar$ and would mean that p-adic length scale L_{107} is replaced with length scale $26L_{107} = 46$ fm, the size of large nucleus so that also the basic length scale nuclear physics would be implicitly coded into the structure of hadrons.

Why Mersenne primes should label a fractal hierarchy of physics?

There are motivations for the working hypothesis stating that there is fractal hierarchy of copies of standard model physics, and that Mersenne primes label both hadronic space-time sheets and gauge bosons. The reason for this is not yet well understood and I have considered several speculative explanations.

1. First picture

The first thing to come in mind is that Mersenne primes correspond to fixed points of the discrete p-adic coupling constant evolution, most naturally to the maxima of the color coupling constant strength. This would mean that gluons are emitted with higher probability than in other p-adic length scales.

There is however an objection against this idea. If one accepts the new vision about non-perturbative aspects of QCD, it would seem that super-symplectic bosons or the interaction between super-symplectic bosons and quarks for some reason favors Mersenne primes. However, if color coupling strength corresponds to $\alpha_K = \alpha_s = 1/4$ scaled down by the increase of the Planck constant, the evolution of super-symplectic color coupling strength does not seem to play any role. What becomes large should be a geometric "form factor", when the boson in the vertex corresponds to Mersenne prime rather than "bare" coupling.

The resolution of the problem could be that boson emission vertices $g(p_1, p_2, p_3)$ are functions of p-adic primes labeling the particles of the vertices so that actually three p-adic length scales are involved instead of single length scale as in the ordinary coupling constant evolution. Hence one can imagine that the interaction between particles corresponding to primes near powers of 2 and Mersenne primes is especially strong and analogous to a resonant interaction. The geometric resonance due to the fact that the length scales involved are related by a fractal scaling by a power of 2 would make the form factors $F(p_1 \simeq 2^{k_1}, p_2 \simeq 2^{k_2}, M_n)$ large. The selection of primes near powers of two and Mersenne bosons would be analogous to evolutionary selection of a population consisting of species able to interact strongly.

Since $k = 113$ quarks are possible for $k = 107$ hadron physics, it seems that quarks can have flux tubes directed to M_n space-times with $n < k$. This suggests that neighboring Mersenne primes compete for flux tubes of quarks. For instance, when the p-adic length scale characterizing quark of M_{107} hadron physics begins to approach M_{89} quarks tend to feed their gauge flux to M_{89} space-time sheet and M_{89} hadron physics takes over and color coupling strength begins to increase. This would be the space-time correlate for the loss of asymptotic freedom.

2. Second picture

Preferred values of Planck constants could play a key role in the selection of Mersenne primes. Ruler-and-compass hypothesis predicts that Planck constants, which correspond to ratios of ruler and compass integers proportional to a product of distinct Fermat primes (four of them are known) and any power of two are favored. As a special case one obtains ruler and compass integers. As a consequence, p-adic length scales have satellites obtained by multiplying them with ruler-and-compass integers, and entire fractal hierarchy of power-of-two multiples of a given p-adic length scale results.

Mersenne length scales would be special since their satellites would form a subset of satellites of shorter Mersenne length scales. The copies of standard model physics associated with Mersenne primes would define a kind of resonating subset of physics since corresponding wavelengths and frequencies would coincide. This would also explain why fermions labeled by primes near power of two couple strongly with Mersenne primes.

5.4.2 Super-Symplectic Bosons As A Particular Kind Of Dark Matter

Super-symplectic bosons

TGD predicts also exotic bosons which are analogous to fermion in the sense that they correspond to single wormhole throat associated with CP_2 type vacuum extremal whereas ordinary gauge bosons corresponds to a pair of wormhole contacts assignable to wormhole contact connecting positive and negative energy space-time sheets. These bosons have super-conformal partners with quantum numbers of right handed neutrino and thus having no electro-weak couplings. The bosons are created by the purely bosonic part of super-symplectic algebra [K25, K104], whose generators belong to the representations of the color group and 3-D rotation group but have vanishing electro-weak quantum numbers. Their spin is analogous to orbital angular momentum whereas the spin of ordinary gauge bosons reduces to fermionic spin. Recall that super-symplectic algebra is crucial for the construction of WCW Kähler geometry. If one assumes that super-symplectic gluons suffer topological mixing identical with that suffered by say U type quarks, the conformal weights would be (5, 6, 58) for the three lowest generations. The application of super-symplectic bosons in TGD based model of hadron masses is discussed in [K62] and here only a brief summary is given.

As explained in [K62], the assignment of these bosons to hadronic space-time sheet is an attractive idea.

1. Quarks explain only a small fraction of the baryon mass and that there is an additional contribution which in a good approximation does not depend on baryon. This contribution should correspond to the non-perturbative aspects of QCD. A possible identification of this contribution is in terms of super-symplectic gluons. Baryonic space-time sheet with $k = 107$ would contain a many-particle state of super-symplectic gluons with net conformal weight of 16 units. This leads to a model of baryons masses in which masses are predicted with an accuracy better than 1 per cent.
2. Hadronic string model provides a phenomenological description of non-perturbative aspects of QCD and a connection with the hadronic string model indeed emerges. Hadronic string tension is predicted correctly from the additivity of mass squared for $J = 2$ bound states of super-symplectic quanta. If the topological mixing for super-symplectic bosons is equal to that for U type quarks then a 3-particle state formed by 2 super-symplectic quanta from the first generation and 1 quantum from the second generation would define baryonic ground state with 16 units of conformal weight. A very precise prediction for hadron masses results by assuming that the spin of hadron correlates with its super-symplectic particle content.
3. Also the baryonic spin puzzle caused by the fact that quarks give only a small contribution to the spin of baryons, could find a natural solution since these bosons could give to the spin of baryon an angular momentum like contribution having nothing to do with the angular momentum of quarks.
4. Super-symplectic bosons suggest a solution to several other anomalies related to hadron physics. The events observed for a couple of years ago in RHIC [C11] suggest a creation of a black-hole like state in the collision of heavy nuclei and inspire the notion of color glass condensate of gluons, whose natural identification in TGD framework would be in terms of a fusion of hadronic space-time sheets containing super-symplectic matter materialized also from the collision energy. In the collision, valence quarks connected together by color bonds to form separate units would evaporate from their hadronic space-time sheets in the collision, and would define TGD counterpart of Pomeron, which experienced a reincarnation for few years ago [C14]. The strange features of the events related to the collisions of high energy cosmic rays with hadrons of atmosphere (the particles in question are hadron like but the penetration length is anomalously long and the rate for the production of hadrons increases as one approaches surface of Earth) could be also understood in terms of the same general mechanism.

Topological evaporation, quark gluon plasma and Pomeron

Topological evaporation of elementary particles means nothing if CP_2 type vacuum extremal evaporates so that one must assume that it is quark space-time sheet or join along boundaries block of quark space-time sheets which evaporates. Second new element is the identification of valence quarks as dark matter in the sense of having large $\hbar$: $\hbar_s \simeq (n/v_0)\hbar$, $v_0 \simeq 2^{-11}$, $n = 1$ so that Compton length is scaled by the same factor. Quark gluon plasma would correspond to a phase with ordinary value $\hbar$ and possibly also sea partons can be regarded as this kind of phase. Color bonds between partons are possible also in this phase.

Concerning the evaporation there are two options.

1. The space-time sheets of sea partons are condensed at much larger space-time sheets defined by the space-time sheets of valence quarks connected by color bonds. Topological evaporation of the parton sea would correspond to the splitting of # contacts connecting sea partons space-time sheets to valence quark space-time sheets.
2. Sea partons condensed at a larger space-time sheet which in turn condenses at the space-time sheet of valence quarks. In this case topological evaporation occurs for the entire sea parton space-time sheet.

One can consider two possible scenarios for topological evaporation of quarks and gluons.

1. Color gauge charge is not identified as gauge flux and single secondarily condensed quark space-time sheet can suffer topological evaporation. In this case quark gluon plasma could be identified as vapor phase state for quarks and gluons.
2. Color gauge charge is identified as gauge flux and only join along boundaries blocks formed from quarks can evaporate. Join along boundaries contacts are naturally identified as color flux tubes between quarks. These tubes need not be static. Quark gluon plasma corresponds to condensed state in which the flux tubes between quark like 3-surfaces are broken. The evaporation of single quark is possible but as a consequence a compensating color charge develops on the interior of the outer boundary of the evaporated quark and the process probably can be interpreted as an emission of meson from hadron. The production of hadrons in hadron collision could be interpreted as a topological evaporation process for sea and valence quarks.

The problematic feature of scenario 1) is the understanding of color confinement In scenario 2) color confinement of the vapor phase particles is an automatic consequence of the assumption that color charge corresponds to gauge flux classically (gauge field is $H^A J_{\alpha\beta}$, H^A being the Hamiltonian of the color isometry. This does not however exclude the possibility that hadron might feed part of its color isospin or hypercharge gauge flux to surrounding condensate. The concept of anomalous hypercharge introduced in earlier work as proportional to electromagnetic charge indeed suggests this kind of possibility. It should be noticed that for the vacuum extremals of Kähler action induced Kähler field and thus also color fields vanish identically.

The alternatives a) and b) have an additional nice feature that they lead to elegant description for the mysterious concept of Pomeron originally introduced to describe hadronic diffractive scattering as the exchange of Pomeron Regge trajectory [C18]. No hadrons belonging to Pomeron trajectory were however found and via the advent of QCD Pomeron was almost forgotten. Pomeron has recently experienced reincarnation [C14, C13, C16]. In Hera [C14] $e-p$ collisions, in which proton scatters essentially elastically whereas jets in the direction of incoming virtual photon emitted by electron are observed. These events can be understood by assuming that proton emits color singlet particle carrying a small fraction of proton's momentum. This particle in turn collides with the virtual photon (antiproton) whereas proton scatters essentially elastically.

The identification of the color singlet particle as Pomeron looks natural since Pomeron emission describes nicely the diffractive scattering of hadrons. Analogous hard diffractive scattering events in pX diffractive scattering with $X = \bar{p}$ [C13] or $X = p$ [C16] have also been observed. What happens is that proton scatters essentially elastically and the emitted Pomeron collides with X and suffers hard scattering so that large rapidity gap jets in the direction of X are observed. These results suggest that Pomeron is real and consists of ordinary partons.

The TGD identification of Pomeron is as sea partons in vapor phase. In TGD inspired phenomenology events involving Pomeron correspond to pX collisions, where incoming X collides with proton, when sea quarks have suffered coherent simultaneous (by color confinement) evaporation into vapor phase. System X sees only the sea left behind in the evaporation and scatters from it whereas dark valence quarks continue without noticing X and condense later to form quasi-elastically scattered proton. If X suffers hard scattering from the sea, the peculiar hard diffractive scattering events are observed. The fraction of these events is equal to the fraction f of time spent by sea quarks in vapor phase.

Dimensional arguments suggest a rough order of magnitude estimate for $f \sim \alpha_K \sim 1/137 \sim 10^{-2}$ for f. The fraction of the peculiar deep inelastic scattering events at Hera is about 5 percent, which suggest that f is about 6.8 times larger and of same order of magnitude as QCD α_s The time spent in condensate is by dimensional arguments of the order of the p-adic length scale $L(M_{107})$, not far from proton Compton length. Time dilation effects at high collision energies guarantee that valence quarks indeed stay in vapor phase during the collision. The identification of Pomeron as sea explains also why Pomeron Regge trajectory does not correspond to actual on mass shell particles.

The existing detailed knowledge about the properties of sea structure functions provides a stringent test for the TGD based scenario. According to [C13] Pomeron structure function seems to consist of soft ($(1-x)^5$), hard ($(1-x)$) and super-hard component (delta function like component at $x=1$). The peculiar super hard component finds explanation in TGD based picture. The structure function $q_P(x,z)$ of parton in Pomeron contains the longitudinal momentum fraction z of the Pomeron as a parameter and $q_P(x,z)$ is obtained by scaling from the sea structure function $q(x)$ for proton $q_P(x,z) = q(zx)$. The value of structure function at $x=1$ is non-vanishing: $q_P(x=1,z) = q(z)$ and this explains the necessity to introduce super hard delta function component in the fit of [C13].

Simulating big bang in laboratory

An important steps in the development of ideas were stimulated by the findings made during period 2002-2005 in Relativist Heavy Ion Collider (RHIC) in Brookhaven compared with the finding of America and for full reason.

1. The first was finding of longitudinal Lorentz invariance at single particle level suggesting a collective behavior. This was around 2002.
2. The collective behavior which was later interpreted in terms of color glass condensate meaning the presence of a blob of liquid like phase decaying later to quark gluon plasma since it was found that the density of what was expected to be quark gluon plasma was about ten times higher than expected.
3. The last finding is that this object seems to absorb partons like black hole and behaves like evaporating black hole.

In my personal Theory Universe the history went as follows.

1. I proposed 2002 a model for Gold-Gold collision as a mini big bang identified as a scaled down variant of TGD inspired cosmology. This makes sense because in TGD based critical cosmology the initial state has vanishing mass per comoving volume instead of being infinite as in radiation dominated cosmology. Any phase transition involving a generation of a new space-time sheet might proceed in this universal manner.
2. Cosmic string soup in the primordial stage is replaced by a tangle of color flux tubes containing the color glass condensate. Flux tubes correspond to flow lines of incompressible liquid flow and non-perturbative macroscopic quantum phase with a very large $\hbar$ is in question. Gravitational constant is replaced by strong gravitational constant defined by the relevant p-adic length scale squared since color flux tubes are analogs of hadronic strings. Presumably L_p, $p = M_107 = 2^107 - 1$, is the p-adic length scale since Mersenne prime M_{107} labels the space-time sheet at which partons feed their color gauge fluxes. Temperature during this phase could correspond to Hagedorn temperature for strings and is determined by string tension. Density would be maximal.
3. Next phase is critical phase in which the notion of space-time in ordinary sense makes sense and 3-space is flat since there is no length scale in critical system (so that curvature vanishes). During this critical phase phase a transition to quark gluon plasma occurs. The duration of this phase fixes all relevant parameters such as temperature (which is the analog of Hagedorn temperature corresponding since critical density is maximal density of gravitational mass in TGD Universe).

4. The next phase is radiation dominated quark gluon plasma phase and then follows hadronization to matter dominated phase provided cosmological picture still applies.

Since black hole formation and evaporation is very much like formation big crunch followed by big bang, the picture is more or less equivalent with the picture in which black hole like object consisting of string like objects (mass is determined by string length just as it is determined by the radius for black holes) is formed and then evaporates. Black hole temperature corresponds to Hagedorn temperature and to the duration of critical period of the mini cosmology.

Are ordinary black-holes replaced with super-symplectic black-holes in TGD Universe?

Some variants of super string model predict the production of small black-holes at LHC. I have never taken this idea seriously but in a well-defined sense TGD predicts black-holes associated with super-symplectic gravitons with strong gravitational constant defined by the hadronic string tension. The proposal is that super-symplectic black-holes have been already seen in Hera, RHIC, and the strange cosmic ray events.

Baryonic super-symplectic black-holes of the ordinary M_{107} hadron physics would have mass 934.2 MeV, very near to proton mass. The mass of their M_{89} counterparts would be 512 times higher, about 478 GeV if quark massses scale also by this factor. This need not be the case: if one has $k = 113 \rightarrow 103$ instead of 105 one has 434 GeV mass. "Ionization energy" for Pomeron, the structure formed by valence quarks connected by color bonds separating from the space-time sheet of super-symplectic black-hole in the production process, corresponds to the total quark mass and is about 170 MeV for ordinary proton and 87 GeV for M_{89} proton. This kind of picture about black-hole formation expected to occur in LHC differs from the stringy picture since a fusion of the hadronic mini black-holes to a larger black-hole is in question.

An interesting question is whether the ultrahigh energy cosmic rays having energies larger than the GZK cut-off of 5×10^{10} GeV are baryons, which have lost their valence quarks in a collision with hadron and therefore have no interactions with the microwave background so that they are able to propagate through long distances.

In neutron stars the hadronic space-time sheets could form a gigantic super-symplectic black-hole and ordinary black-holes would be naturally replaced with super-symplectic black-holes in TGD framework (only a small part of black-hole interior metric is representable as an induced metric). This obviously means a profound difference between TGD and string models.

1. Hawking-Bekenstein black-hole entropy would be replaced with its p-adic counterpart given by

$$S_p = (\frac{M}{m(CP_2)})^2 \times log(p) \ , \tag{5.4.1}$$

where $m(CP_2)$ is CP_2 mass, which is roughly 10^{-4} times Planck mass. M is the contribution of p-adic thermodynamics to the mass. This contribution is extremely small for gauge bosons but for fermions and super-symplectic particles it gives the entire mass.

2. If p-adic length scale hypothesis $p \simeq 2^k$ holds true, one obtains

$$S_p = klog(2) \times (\frac{M}{m(CP_2)})^2, \tag{5.4.2}$$

$m(CP_2) = \hbar/R$, R the "radius" of CP_2, corresponds to the standard value of $\hbar_0$ for all values of $\hbar$.

3. Hawking-Bekenstein area law gives in the case of Schwartschild black-hole

$$S = \frac{A}{4G} \times \hbar = \pi G M^2 \times \hbar \ . \tag{5.4.3}$$

For the p-adic variant of the law Planck mass is replaced with CP_2 mass and $klog(2) \simeq log(p)$ appears as an additional factor. Area law is obtained in the case of elementary particles if k is prime and wormhole throats have M^4 radius given by p-adic length scale $L_k = \sqrt{k}R$ which is exponentially smaller than L_p. For macroscopic super-symplectic black-holes modified area law results if the radius of the large wormhole throat equals to Schwartschild radius. Schwartschild radius is indeed natural: in [K99] I have shown that a simple deformation of the Schwartschild exterior metric to a metric representing rotating star transforms Schwartschild horizon to a light-like 3-surface at which the signature of the induced metric is transformed from Minkowskian to Euclidian.

4. The formula for the gravitational Planck constant appearing in the Bohr quantization of planetary orbits and characterizing the gravitational field body mediating gravitational interaction between masses M and m [K79] reads as

$$\hbar_{gr} = \frac{GMm}{v_0}\hbar_0 \ .$$

$v_0 = 2^{-11}$ is the preferred value of v_0. One could argue that the value of gravitational Planck constant is such that the Compton length $\hbar_{gr}/M$ of the black-hole equals to its Schwartshild radius. This would give

$$\hbar_{gr} = \frac{GM^2}{v_0}\hbar_0 \ , \quad v_0 = 1/2 \ . \tag{5.4.4}$$

The requirement that $\hbar_{gr}$ is a ratio of ruler-and-compass integers expressible as a product of distinct Fermat primes (only four of them are known) and power of 2 would quantize the mass spectrum of black hole [K79]. Even without this constraint M^2 is integer valued using p-adic mass squared unit and if p-adic length scale hypothesis holds true this unit is in an excellent approximation power of two.

5. The gravitational collapse of a star would correspond to a process in which the initial value of v_0 , say $v_0 = 2^{-11}$, increases in a stepwise manner to some value $v_0 \leq 1/2$. For a supernova with solar mass with radius of 9 km the final value of v_0 would be $v_0 = 1/6$. The star could have an onion like structure with largest values of v_0 at the core as suggested by the model of planetary system. Powers of two would be favored values of v_0. If the formula holds true also for Sun one obtains $1/v_0 = 3 \times 17 \times 2^{13}$ with 10 per cent error.

6. Black-hole evaporation could be seen as means for the super-symplectic black-hole to get rid of its electro-weak charges and fermion numbers (except right handed neutrino number) as the antiparticles of the emitted particles annihilate with the particles inside super-symplectic black-hole. This kind of minimally interacting state is a natural final state of star. Ideal super-symplectic black-hole would have only angular momentum and right handed neutrino number.

7. In TGD light-like partonic 3-surfaces are the fundamental objects and space-time interior defines only the classical correlates of quantum physics. The space-time sheet containing the highly entangled cosmic string might be separated from environment by a wormhole contact with size of black-hole horizon.

   This looks the most plausible option but one can of course ask whether the large partonic 3-surface defining the horizon of the black-hole actually contains all super-symplectic particles so that super-symplectic black-hole would be single gigantic super-symplectic parton. The interior of super-symplectic black-hole would be a space-like region of space-time, perhaps resulting as a large deformation of CP_2 type vacuum extremal. Black-hole sized wormhole contact would define a gauge boson like variant of the black-hole connecting two space-time sheets and getting its mass through Higgs mechanism. A good guess is that these states are extremely light.

5.5 Number Theoretic Compactification And $M^8 - H$ Duality

This section summarizes the basic vision about number theoretic compactification reducing the classical dynamics to associativity or co-associativity. Originally $M^8 - H$ duality was introduced as a number theoretic explanation for $H = M^4 \times CP_2$. Much later it turned out that the completely exceptional twistorial properties of M^4 and CP_2 are enough to justify $X^4 \subset H$ hypothesis. Skeptic could therefore criticize the introduction of M^8 (actually its complexification) as an un-necessary mathematical complication producing only unproven conjectures and bundle of new statements to be formulated precisely. However, if quaternionicity can be realized in terms of M^8_c using O_c-real analytic functions and if quaternionicity is equivalent with preferred extremal property, a huge simplification results and one can say that field equations are exactly solvable.

One can question the feasibility of $M^8 - H$ duality if the dynamics is purely number theoretic at the level of M^8 and determined by Kähler action at the level of H. Situation becomes more democratic if Kähler action defines the dynamics in both M^8 and H: this might mean that associativity could imply field equations for preferred extremals or vice versa or there might be equivalence between two. This means the introduction Kähler structure at the level of M^8, and motivates also the coupling of Kähler gauge potential to M^8 spinors characterized by Kähler charge or em charge. One could call this form of duality strong form of $M^8 - H$ duality.

The strong form $M^8 - H$ duality boils down to the assumption that space-time surfaces can be regarded either as 4-surfaces of H or as surfaces of M^8 or even M^8_c composed of associative and co-associative regions identifiable as regions of space-time possessing Minkowskian *resp.* Euclidian signature of the induced metric. They have the same induced metric and Kähler form and WCW associated with H should be essentially the same as that associated with M^8. Associativity corresponds to hyper-quaterniocity at the level of tangent space and co-associativity to co-hyper-quaternionicity - that is associativity/hyper-quaternionicity of the normal space. Both are needed to cope with known extremals. Since in Minkowskian context precise language would force to introduce clumsy terms like hyper-quaternionicity and co-hyper-quaternionicity, it is better to speak just about associativity or co-associativity.

Remark: The original assumption was that space-times could be regarded as surfaces in M^8 rather than in its complexification M^8_c identifiable as complexified octonions. This assumption is un-necessarily strong and if one assumes that octonion-real analytic functions characterize these surfaces M^8_c must be assumed.

For the octonionic spinor fields the octonionic analogs of electroweak couplings reduce to mere Kḧler or electromagnetic coupling and the solutions reduce to those for spinor d'Alembertian in 4-D harmonic potential breaking $SO(4)$ symmetry. Due to the enhanced symmetry of harmonic oscillator, one expects that partial waves are classified by $SU(4)$ and by reduction to $SU(3) \times U(1)$ by em charge and color quantum numbers just as for CP_2 - at least formally.

Harmonic oscillator potential defined by self-dual em field splits M^8 to $M^4 \times E^4$ and implies Gaussian localization of the spinor modes near origin so that E^4 effectively compactifies. The The resulting physics brings strongly in mind low energy physics, where only electromagnetic interaction is visible directly, and one cannot avoid associations with low energy hadron physics. These are some of the reasons for considering $M^8 - H$ duality as something more than a mere mathematical curiosity.

Remark: The Minkowskian signatures of M^8 and M^4 produce technical nuisance. One could overcome them by Wick rotation, which is however somewhat questionable trick. $M^8_c = O_c$ provides the proper formulation.

1. The proper formulation is in terms of complexified octonions and quaternions involving the introduction of commuting imaginary unit j.
2. Hyper-quaternions/octonions define as subspace of complexified quaternions/octonions spanned by real unit and jI_k, where I_k are quaternionic units. These spaces are obviously not closed under multiplication. One can however however define the notion of associativity for the subspace of M^8 by requiring that the products and sums of the tangent space vectors generate complexified quaternions.
3. Ordinary quaternions Q are expressible as $q = q_0 + q^k I_k$. Hyper-quaternions are expressible

as $q = q_0 + jq^kI_k$ and form a subspace of complexified quaternions $Q_c = Q \oplus jQ$. Similar formula applies to octonions and their hyper counterparts which can be regarded as subspaces of complexified octonions $O \oplus jO$. Tangent space vectors of H correspond hyper-quaternions $q_H = q_0 + jq^kI_k + jiq_2$ defining a subspace of doubly complexified quaternions: note the appearance of two imaginary units.

The recent definitions of associativity and M^8 duality has evolved slowly from in-accurate characterizations and there are still open questions.

1. Kähler form for M^8 non-trivial only in $E^4 \subset M^8$ implies unique decomposition $M^8 = M^4 \times E^4$ needed to define $M^8 - H$ duality uniquely. This applies also to M^8_c. This forces to introduce also Kähler action, induced metric and induced Kähler form. Could strong form of duality meant that the space-time surfaces in M^8 and H have same induced metric and induced Kähler form? Could the WCW s associated with M^8 and H be identical with this assumption so that duality would provide different interpretations for the same physics?
2. One can formulate associativity in M^8 (or M^8_c) by introducing octonionic structure in tangent spaces or in terms of the octonionic representation for the induced gamma matrices. Does the notion have counterpart at the level of H as one might expect if Kähler action is involved in both cases? The analog of this formulation in H might be as quaternionic "reality" since tangent space of H corresponds to complexified quaternions: I have however found no acceptable definition for this notion.

   The earlier formulation is in terms of octonionic flat space gamma matrices replacing the ordinary gamma matrices so that the formulation reduces to that in M^8 tangent space. This formulation is enough to define what associativity means although one can protest. Somehow H is already complex quaternionic and thus associative. Perhaps this just what is needed since dynamics has two levels: *embedding space level* and *space-time level*. One must have embedding space spinor harmonics assignable to the ground states of super-conformal representations and quaternionicity and octonionicity of H tangent space would make sense at the level of space-time surfaces.
3. Whether the associativity using induced gamma matrices works is not clear for massless extremals (MEs) and vacuum extremals with the dimension of CP_2 projection not larger than 2.
4. What makes this notion of associativity so fascinating is that it would allow to iterate duality as a sequence $M^8 \rightarrow H \rightarrow H...$ by mapping the space-time surface to $M^4 \times CP_2$ by the same recipe as in case of M^8. This brings in mind the functional composition of O_c-real analytic functions (O_c denotes complexified octonions: complexification is forced by Minkowskian signature) suggested to produced associative or co-associative surfaces. The associative (co-associative) surfaces in M^8 would correspond to loci for vanishing of imaginary (real) part of octonion-real-analytic function.

It might be possible to define associativity in H also in terms of Kähler-Dirac gamma matrices defined by Kähler action (certainly not M^8).

1. All known extremals are associative or co-associative in H in this sense. This would also give direct correlation with the variational principle. For the known preferred extremals this variant is successful partially because the Kähler-Dirac gamma matrices need not span the entire tangent space. The space spanned by the Kähler-Dirac gammas is not necessarily tangent space. For instance for CP_2 type vacuum extremals the Kähler-Dirac gamma matrices are CP_2 gamma matrices plus an additional light-like component from M^4 gamma matrices.

   If the space spanned by Kähler-Dirac gammas has dimension D smaller than 3 co-associativity is automatic. If the dimension of this space is $D = 3$ it can happen that the triplet of gammas spans by multiplication entire octonionic algebra. For $D = 4$ the situation is of course non-trivial.
2. For Kähler-Dirac gamma matrices the notion of co-associativity can produce problems since Kähler-Dirac gamma matrices do not in general span the tangent space. What does co-associativity mean now? Should one replace normal space with orthogonal complement of the space spanned by Kähler-Dirac gamma matrices? Co-associativity option must be considered

for $D = 4$ only. CP_2 type vacuum extremals provide a good example. In this case the Kähler-Dirac gamma matrices reduce to sums of ordinary CP_2 gamma matrices and ligt-like M^4 contribution. The orthogonal complement for the Kähler-Dirac gamma matrices consists of dual light-like gamma matrix and two gammas orthogonal to it: this space is subspace of M^4 and trivially associative.

5.5.1 Basic Idea Behind $M^8 - M^4 \times CP_2$ Duality

If four-surfaces $X^4 \subset M^8$ under some conditions define 4-surfaces in $M^4 \times CP_2$ indirectly, the spontaneous compactification of super string models would correspond in TGD to two different ways to interpret the space-time surface. This correspondence could be called number theoretical compactification or $M^8 - H$ duality.

The hard mathematical facts behind the notion of number theoretical compactification are following.

1. One must assume that M^8 has unique decomposition $M^8 = M^4 \times E^4$. This decomposition generalizes also to the case of M^8_c. This would be most naturally due to Kähler structure in E^4 defined by a self-dual Kähler form defining parallel constant electric and magnetic fields in Euclidian sense. Besides Kähler form there is vector field coupling to sigma matrix representing the analog of strong isospin: the corresponding octonionic sigma matrix however is imaginary unit times gamma matrix - say ie_1 in M^4 - defining a preferred plane M^2 in M^4. Here it is essential that the gamma matrices of E^4 defined in terms of octonion units commute to gamma matrices in M^4. What is involved becomes clear from the Fano triangle illustrating octonionic multiplication table.
2. The space of hyper-complex structures of the hyper-octonion space - they correspond to the choices of plane $M^2 \subset M^8$ - is parameterized by 6-sphere $S^6 = G^2/SU(3)$. The subgroup $SU(3)$ of the full automorphism group G_2 respects the a priori selected complex structure and thus leaves invariant one octonionic imaginary unit, call it e_1. Fixed complex structure therefore corresponds to a point of S^6.
3. Quaternionic sub-algebras of M^8 (and M^8_c) are parametrized by $G_2/U(2)$. The quaternionic sub-algebras of octonions with fixed complex structure (that is complex sub-space defined by real and preferred imaginary unit and parametrized by a point of S^6) are parameterized by $SU(3)/U(2) = CP_2$ just as the complex planes of quaternion space are parameterized by $CP_1 = S^2$. Same applies to hyper-quaternionic sub-spaces of hyper-octonions. $SU(3)$ would thus have an interpretation as the isometry group of CP_2, as the automorphism sub-group of octonions, and as color group. Thus the space of quaternionic structures can be parametrized by the 10-dimensional space $G_2/U(2)$ decomposing as $S^6 \times CP_2$ locally.
4. The basic result behind number theoretic compactification and $M^8 - H$ duality is that associative sub-spaces $M^4 \subset M^8$ containing a fixed commutative sub-space $M^2 \subset M^8$ are parameterized by CP_2. The choices of a fixed hyper-quaternionic basis $1, e_1, e_2, e_3$ with a fixed complex sub-space (choice of e_1) are labeled by $U(2) \subset SU(3)$. The choice of e_2 and e_3 amounts to fixing $e_2 \pm \sqrt{-1}e_3$, which selects the $U(2) = SU(2) \times U(1)$ subgroup of $SU(3)$. $U(1)$ leaves 1 invariant and induced a phase multiplication of e_1 and $e_2 \pm e_3$. $SU(2)$ induces rotations of the spinor having e_2 and e_3 components. Hence all possible completions of $1, e_1$ by adding e_2, e_3 doublet are labeled by $SU(3)/U(2) = CP_2$.

Consider now the formulation of $M^8 - H$ duality.

1. The idea of the standard formulation is that associative manifold $X^4 \subset M^8$ has at its each point associative tangent plane. That is X^4 corresponds to an integrable distribution of $M^2(x) \subset M^8$ parametrized 4-D coordinate x that is map $x \to S^6$ such that the 4-D tangent plane is hyper-quaternionic for each x.
2. Since the Kähler structure of M^8 implies unique decomposition $M^8 = M^4 \times E^4$, this surface in turn defines a surface in $M^4 \times CP_2$ obtained by assigning to the point of 4-surface point $(m, s) \in H = M^4 \times CP_2$: $m \in M^4$ is obtained as *projection* $M^8 \to M^4$ (this is modification to the earlier definition) and $s \in CP_2$ parametrizes the quaternionic tangent plane as point of CP_2. Here the local decomposition $G_2/U(2) = S^6 \times CP_2$ is essential for achieving uniqueness.

3. One could also map the associative surface in M^8 to surface in 10-dimensional $S^6 \times CP_2$. In this case the metric of the image surface cannot have Minkowskian signature and one cannot assume that the induced metrics are identical. It is not known whether S^6 allows genuine complex structure and Kähler structure which is essential for TGD formulation.
4. Does duality imply the analog of associativity for $X^4 \subset H$? The tangent space of H can be seen as a sub-space of doubly complexified quaternions. Could one think that quaternionic sub-space is replaced with sub-space analogous to that spanned by real parts of complexified quaternions? The attempts to define this notion do not however look promising. One can however define associativity and co-associativity for the tangent space M^8 of H using octonionization and can formulate it also terms of induced gamma matrices.
5. The associativity defined in terms of induced gamma matrices in both in M^8 and H has the interesting feature that one can assign to the associative surface in H a new associative surface in H by assigning to each point of the space-time surface its M^4 projection and point of CP_2 characterizing its associative tangent space or co-associative normal space. It seems that one continue this series ad infinitum and generate new solutions of field equations! This brings in mind iteration which is standard manner to generate fractals as limiting sets. This certainly makes the heart of mathematician beat.
6. Kähler structure in $E^4 \subset M^8$ guarantees natural $M^4 \times E^4$ decomposition. Does associativity imply preferred extremal property or vice versa, or are the two notions equivalent or only consistent with each other for preferred extremals?

A couple of comments are in order.

1. This definition generalizes to the case of M_c^8: all that matters is that tangent space-is is complexified quaternionic and there is a unique identification $M^4 \subset M_c^8$: this allows to assign the point of 4-surfaces a point of $M^4 \times CP_2$. The generalization is needed if one wants to formulate the hypothesis about O_c real-analyticity as a way to build quaternionic space-time surfaces properly.
2. This definition differs from the first proposal for years ago stating that each point of X^4 contains a *fixed* $M^2 \subset M^4$ rather than $M_2(x) \subset M^8$ and also from the proposal assuming integrable distribution of $M^2(x) \subset M^4$. The older proposals are not consistent with the properties of massless extremals and string like objects for which the counterpart of M^2 depends on space-time point and is not restricted to M^4. The earlier definition $M^2(x) \subset M^4$ was problematic in the co-associative case since for the Euclidian signature is is not clear what the counterpart of $M^2(x)$ could be.
3. The new definition is consistent with the existence of Hamilton-Jacobi structure meaning slicing of space-time surface by string world sheets and partonic 2-surfaces with points of partonic 2-surfaces labeling the string world sheets [K11]. This structure has been proposed to characterize preferred extremals in Minkowskian space-time regions at least.
4. Co-associative Euclidian 4-surfaces, say CP_2 type vacuum extremal do not contain integrable distribution of $M^2(x)$. It is normal space which contains $M^2(x)$. Does this have some physical meaning? Or does the surface defined by $M^2(x)$ have Euclidian analog?

   A possible identification of the analog would be as string world sheet at which W boson field is pure gauge so that the modes of the modified Dirac operator [K104] restricted to the string world sheet have well-defined em charge. This condition appears in the construction of solutions of Kähler-Dirac operator.

   For octonionic spinor structure the W coupling is however absent so that the condition does not make sense in M^8. The number theoretic condition would be as commutative or co-commutative surface for which imaginary units in tangent space transform to real and imaginary unit by a multiplication with a fixed imaginary unit! One can also formulate co-associativity as a condition that tangent space becomes associative by a multiplication with a fixed imaginary unit.

   There is also another justification for the distribution of Euclidian tangent planes. The idea about associativity as a fundamental dynamical principle can be strengthened to the statement that space-time surface allows slicing by hyper-complex or complex 2-surfaces, which are commutative or co-commutative inside space-time surface. The physical interpretation would be as Minkowskian or Euclidian string world sheets carrying spinor modes. This would give a connection with string model and also with the conjecture about the general structure of preferred extremals.

5. Minimalist could argue that the minimal definition requires octonionic structure and associativity *only* in M^8. There is no need to introduce the counterpart of Kähler action in M^8 since the dynamics would be based on associativity or co-associativity alone. The objection is that one must assumes the decomposition $M^8 = M^4 \times E^4$ without any justification.

   The map of space-time surfaces to those of $H = M^4 \times CP_2$ implies that the space-time surfaces in H are in well-defined sense quaternionic. As a matter of fact, the standard spinor structure of H can be regarded as quaternionic in the sense that gamma matrices are essentially tensor products of quaternionic gamma matrices and reduce in matrix representation for quaternions to ordinary gamma matrices. Therefore the idea that one should introduce octonionic gamma matrices in H is questionable. If all goes as in dreams, the mere associativity or co-associativity would code for the preferred extremal property of Kähler action in H. One could at least hope that associativity/co-associativity in H is consistent with the preferred extremal property.

6. One can also consider a variant of associativity based on modified gamma matrices - but only in H. This notion does not make sense in M^8 since the very existence of quaternionic tangent plane makes it possible to define $M^8 - H$ duality map. The associativity for modified gamma matrices is however consistent with what is known about extremals of Kähler action. The associativity based on induced gamma matrices would correspond to the use of the space-time volume as action. Note however that gamma matrices are *not* necessary in the definition.

5.5.2 Hyper-Octonionic Pauli "Matrices" And The Definition Of Associativity

Octonionic Pauli matrices suggest an interesting possibility to define precisely what associativity means at the level of M^8 using gamma matrices (for background see [K94, K8]).

1. According to the standard definition space-time surface $X^4 \subset M^8$ is associative if the tangent space at each point of X^4 in $X^4 \subset M^8$ picture is associative. The definition can be given also in terms of octonionic gamma matrices whose definition is completely straightforward.

2. Could/should one define the analog of associativity at the level of H? One can identify the tangent space of H as M^8 and can define octonionic structure in the tangent space and this allows to define associativity locally. One can replace gamma matrices with their octonionic variants and formulate associativity in terms of them locally and this should be enough.

   Skeptic however reminds M^4 allows hyper-quaternionic structure and CP_2 quaternionic structure so that complexified quaternionic structure would look more natural for H. The tangent space would decompose as $M^8 = HQ + ijQ$, weher j is commuting imaginary unit and HQ is spanned by real unit and by units iI_k, where i second commutating imaginary unit and I_k denotes quaternionic imaginary units. There is no need to make anything associative.

   There is however far from obvious that octonionic spinor structure can be (or need to be!) defined globally. The lift of the CP_2 spinor connection to its octonionic variant has questionable features: in particular vanishing of the charged part and reduction of neutral part to photon. Therefore is is unclear whether associativity condition makes sense for $X^4 \subset M^4 \times CP_2$. What makes it so fascinating is that it would allow to iterate duality as a sequences $M^8 \rightarrow H \rightarrow H$.... This brings in mind the functional composition of octonion real-analytic functions suggested to produced associative or co-associative surfaces.

I have not been able to settle the situation. What seems the working option is associativity in both M^8 and H and Kähler-Dirac gamma matrices defined by appropriate Kähler action and correlation between associativity and preferred extremal property.

5.5.3 Are Kähler And Spinor Structures Necessary In M^8?

If one introduces M^8 as dual of H, one cannot avoid the idea that hyper-quaternionic surfaces obtained as images of the preferred extremals of Kähler action in H are also extremals of M^8

Kähler action with same value of Kähler action defining Kähler function. As found, this leads to the conclusion that the $M^8 - H$ duality is Kähler isometry. Coupling of spinors to Kähler potential is the next step and this in turn leads to the introduction of spinor structure so that quantum TGD in H should have full M^8 dual.

Are also the 4-surfaces in M^8 preferred extremals of Kähler action?

It would be a mathematical miracle if associative and co-associative surfaces in M^8 would be in 1-1 correspondence with preferred extremals of Kähler action. This motivates the question whether Kähler action make sense also in M^8. This does not exclude the possibility that associativity implies or is equivalent with the preferred extremal property.

One expects a close correspondence between preferred extremals: also now vacuum degeneracy is obtained, one obtains massless extremals, string like objects, and counterparts of CP_2 type vacuum extremals. All known extremals would be associative or co-associative if modified gamma matrices define the notion (possible only in the case of H).

The strongest form of duality would be that the space-time surfaces in M^8 and H have same induced metric same induced Kähler form. The basic difference would be that the spinor connection for surfaces in M^8 would be however neutral and have no left handed components and only em gauge potential. A possible interpretation is that M^8 picture defines a theory in the phase in which electroweak symmetry breaking has happened and only photon belongs to the spectrum.

The question is whether one can define WCW also for M^8. Certainly it should be equivalent with WCW for H: otherwise an inflation of poorly defined notions follows. Certainly the general formulation of the WCW geometry generalizes from H to M^8. Since the matrix elements of symplectic super-Hamiltonians defining WCW gamma matrices are well defined as matrix elements involve spinor modes with Gaussian harmonic oscillator behavior, the non-compactness of E^4 does not pose any technical problems.

Spinor connection of M^8

There are strong physical constraints on M^8 dual and they could kill the hypothesis. The basic constraint to the spinor structure of M^8 is that it reproduces basic facts about electro-weak interactions. This includes neutral electro-weak couplings to quarks and leptons identified as different H-chiralities and parity breaking.

1. By the flatness of the metric of E^4 its spinor connection is trivial. E^4 however allows full S^2 of covariantly constant Kähler forms so that one can accommodate free independent Abelian gauge fields assuming that the independent gauge fields are orthogonal to each other when interpreted as realizations of quaternionic imaginary units. This is possible but perhaps a more natural option is the introduction of just single Kähler form as in the case of CP_2.
2. One should be able to distinguish between quarks and leptons also in M^8, which suggests that one introduce spinor structure and Kähler structure in E^4. The Kähler structure of E^4 is unique apart form $SO(3)$ rotation since all three quaternionic imaginary units and the unit vectors formed from them allow a representation as an antisymmetric tensor. Hence one must select one preferred Kähler structure, that is fix a point of S^2 representing the selected imaginary unit. It is natural to assume different couplings of the Kähler gauge potential to spinor chiralities representing quarks and leptons: these couplings can be assumed to be same as in case of H.
3. Electro-weak gauge potential has vectorial and axial parts. Em part is vectorial involving coupling to Kähler form and Z^0 contains both axial and vector parts. The naïve replacement of sigma matrices appearing in the coupling of electroweak gauge fields takes the left handed parts of these fields to zero so that only neutral part remains. Further, gauge fields correspond to curvature of CP_2 which vanishes for E^4 so that only Kähler form form remains. Kähler form couples to 3L and q so that the basic asymmetry between leptons and quarks remains. The resulting field could be seen as analog of photon.
4. The absence of weak parts of classical electro-weak gauge fields would conform with the standard thinking that classical weak fields are not important in long scales. A further prediction is that this distinction becomes visible only in situations, where H picture is

necessary. This is the case at high energies, where the description of quarks in terms of $SU(3)$ color is convenient whereas $SO(4)$ QCD would require large number of E^4 partial waves. At low energies large number of $SU(3)$ color partial waves are needed and the convenient description would be in terms of $SO(4)$ QCD. Proton spin crisis might relate to this.

Dirac equation for leptons and quarks in M^8

Kähler gauge potential would also couple to octonionic spinors and explain the distinction between quarks and leptons.

1. The complexified octonions representing H spinors decompose to $1+1+3+\overline{3}$ under SU(3) representing color automorphisms but the interpretation in terms of QCD color does not make sense. Rather, the triplet and single combine to two weak isospin doublets and quarks and leptons corresponds to "spin" states of octonion valued 2-spinor. The conservation of quark and lepton numbers follows from the absence of coupling between these states.
2. One could modify the coupling so that coupling is on electric charge by coupling it to electromagnetic charge which as a combination of unit matrix and sigma matrix is proportional to $1+kI_1$, where I_1 is octonionic imaginary unit in $M^2 \subset M^4$. The complexified octonionic units can be chosen to be eigenstates of Q_{em} so that Laplace equation reduces to ordinary scalar Laplacian with coupling to self-dual em field.
3. One expects harmonic oscillator like behavior for the modes of the Dirac operator of M^8 since the gauge potential is linear in E^4 coordinates. One possibility is Cartesian coordinates is $A(A_x, A_y, A_z, A_t) = k(-y, x, t, -z)$. Thhe coupling would make E^4 effectively a compact space.
4. The square of Dirac operator gives potential term proportional to $r^2 = x^2 + y^2 + z^2 + t^2$ so that the spectrum of 4-D harmonic oscillator operator and $SO(4)$ harmonics localized near origin are expected. For harmonic oscillator the symmetry enhances to $SU(4)$.

   If one replaces Kähler coupling with em charge symmetry breaking of $SO(4)$ to vectorial $SO(3)$ is expected since the coupling is proportional to $1 + ike_1$ defining electromagnetic charge. Since the basis of complexified quaternions can be chosen to be eigenstates of e_1 under multiplication, octonionic spinors are eigenstates of em charge and one obtains two color singles $1 \pm e_1$ and color triplet and antitriplet. The color triplets cannot be however interpreted in terms of quark color.

   Harmonic oscillator potential is expected to enhance $SO(3)$ to SU(3). This suggests the reduction of the symmetry to $SU(3) \times U(1)$ corresponding to color symmetry and em charge so that one would have same basic quantum numbers as tof CP_2 harmonics. An interesting question is how the spectrum and mass squared eigenvalues of harmonics differ from those for CP_2.
5. In the square of Dirac equation $J^{kl}\Sigma_{kl}$ term distinguishes between different em charges ($\Sigma_k l$ reduces by self duality and by special properties of octonionic sigma matrices to a term proportional to iI_1 and complexified octonionic units can be chosen to be its eigenstates with eigen value ± 1. The vacuum mass squared analogous to the vacuum energy of harmonic oscillator is also present and this contribution are expected to cancel themselves for neutrinos so that they are massless whereas charged leptons and quarks are massive. It remains to be checked that quarks and leptons can be classified to triality $T = \pm 1$ and $t = 0$ representations of dynamical $SU(3)$ respectively.

What about the analog of Kähler Dirac equation

Only the octonionic structure in $T(M^8)$ is needed to formulate quaternionicity of space-time surfaces: the reduction to O_c-real-analyticity would be extremely nice but not necessary (O_c denotes complexified octonions needed to cope with Minkowskian signature). Most importantly, there might be no need to introduce Kähler action (and Kähler form) in M^8. Even the octonionic representation of gamma matrices is un-necessary. Neither there is any absolute need to define octonionic Dirac equation and octonionic Kähler Dirac equation nor octonionic analog of its solutions nor the octonionic variants of embedding space harmonics.

It would be of course nice if the general formulas for solutions of the Kähler Dirac equation in H could have counterparts for octonionic spinors satisfying quaternionicity condition. One can indeed wonder whether the restriction of the modes of induced spinor field to string world sheets defined by integrable distributions of hyper-complex spaces $M^2(x)$ could be interpretated in terms of commutativity of fermionic physics in M^8. $M^8 - H$ correspondence could map the octonionic spinor fields at string world sheets to their quaternionic counterparts in H. The fact that only holomorphy is involved with the definition of modes could make this map possible.

5.5.4 How Could One Solve Associativity/Co-Associativity Conditions?

The natural question is whether and how one could solve the associativity/-co-associativity conditions explicitly. One can imagine two approaches besides $M^8 \to H \to H...$ iteration generating new solutions from existing ones.

Could octonion-real analyticity be equivalent with associativity/co-associativity?

Analytic functions provide solutions to 2-D Laplace equations and one might hope that also the field equations could be solved in terms of octonion-real-analyticity at the level of M^8 perhaps also at the level of H. Signature however causes problems - at least technical. Also the compactness of CP_2 causes technical difficulties but they need not be insurmountable.

For E^8 the tangent space would be genuinely octonionic and one can define the notion octonion-real analytic map as a generalization of real-analytic function of complex variables (the coefficients of Laurent series are real to guarantee associativity of the series). The argument is complexified octonion in $O \oplus iO$ forming an algebra but not a field. The norm square is Minkowskian as difference of two Euclidian octonionic norms: $N(o_1 + io_2) = N(o_1) - N(o_2)$ and vanishes at 15-D light cone boundary. Obviously, differential calculus is possible outside the light-cone boundary. Rational analytic functions have however poles at the light-cone boundary. One can wonder whether the poles at M^4 light-cone boundary, which is subset of 15-D light-cone boundary could have physical significance and relevant for the role of causal diamonds in ZEO.

The candidates for associative surfaces defined by O_c-real-analytic functions (I use O_c for complexified octonions) have Minkowskian signature of metric and are 4-surfaces at which the projection of $f(o_1 + io_2)$ to $Im(O_1)$, $iIm(O_2)$, and $iRe(Q_2) \oplus Im(Q_1)$ vanish so that only the projection to hyper-quaternionic Minkowskian sub-space $M^4 = Re(Q_1) + iIm(Q_2)$ with signature $(1, -1, -, 1-, 1)$ is non-vanishing. The inverse image need not belong to M^8 and in general it belongs to M_c^8 but this is not a problem: all that is needed that the tangent space of inverse image is complexified quaternionic. If this is the case then $M^8 - H$ duality maps the tangent space of the inverse image to CP_2 point and image itself defines the point of M^4 so that a point of H is obtained. Co-associative surfaces would be surfaces for which the projections of image to $Re(O_1)$, $iRe(O_2)$, and to $Im(O_1)$ vanish so that only the projection to $iIm(O_2)$ with signature $(-1, -1, -1, -1)$ is non-vanishing.

The inverse images as 4-D sub-manifolds of M_c^8 (not M^8!) are excellent candidates for associative and co-associative 4-surfaces since $M^8 - H$ duality assignes to them a 4-surface in $M^4 \times CP_2$ if the tangent space at given point is complexified quaternionic. This is true if one believes on the analytic continuation of the intuition from complex analysis (the image of real axes under the map defined by O_c-real-analytic function is real axes in the new coordinates defined by the map: the intuition results by replacing "real" by "complexified quaternionic"). The possibility to solve field equations in this manner would be of enormous significance since besides basic arithmetic operations also the functional decomposition of O_c-real-analytic functions produces similar functions. One could speak of the algebra of space-time surfaces.

What is remarkable that the complexified octonion real analytic functions are obtained by analytic continuation from single real valued function of real argument. The real functions form naturally a hierarchy of polynomials (maybe also rational functions) and number theoretic vision suggests that there coefficients are rationals or algebraic numbers. Already for rational coefficients hierarchy of algebraic extensions of rationals results as one solves the vanishing conditions. There is a temptation to regard this hierarchy coding for space-time sheets as an analog of DNA.

Note that in the recent formulation there is no need to pose separately the condition about integrable distribution of $M^2(x) \subset M^4$.

Quaternionicity condition for space-time surfaces

Quaternionicity actually has a surprisingly simple formulation at the level of space-time surfaces. The following discussion applies to both M^8 and H with minor modifications if one accepts that also H can allow octonionic tangent space structure, which does not require gamma matrices.

1. Quaternionicity is equivalent with associativity guaranteed by the vanishing of the associator $A(a, b, c) = a(bc) - (ab)c$ for any triplet of imaginary tangent vectors in the tangent space of the space-time surface. The condition must hold true for purely imaginary combinations of tangent vectors.

2. If one is able to choose the coordinates in such a way that one of the tangent vectors corresponds to real unit (in the embedding map embedding space M^4 coordinate depends only on the time coordinate of space-time surface), the condition reduces to the vanishing of the octonionic product of remaining three induced gamma matrices interpreted as octonionic gamma matrices. This condition looks very simple - perhaps too simple!- since it involves only first derivatives of the embedding space vectors.

   One can of course whether quaternionicity conditions replace field equations or only select preferred extremals. In the latter case, one should be able to prove that quaternionicity conditions are consistent with the field equations.

3. Field equations would reduce to tri-linear equations in in the gradients of embedding space coordinates (rather than involving embedding space coordinates quadratically). Sum of analogs of 3×3 determinants deriving from $a \times (b \times b)$ for different octonion units is involved.

4. Written explicitly field equations give in terms of vielbein projections e^A_α, vielbein vectors e^A_k, coordinate gradients $\partial_\alpha h^k$ and octonionic structure constants f_{ABC} the following conditions stating that the projections of the octonionic associator tensor to the space-time surface vanishes:

$$\begin{aligned} e^A_\alpha e^B_\beta e^C_\gamma A^E_{ABC} &= 0 \ , \\ A^E_{ABC} &= f_{AD}{}^E f_{BC}{}^D - f_{AB}{}^D f_{DC}{}^E \ , \\ e^A_\alpha &= \partial_\alpha h^k e^A_k \ , \\ \Gamma_k &= e^A_k \gamma_A \ . \end{aligned} \tag{5.5.1}$$

   The very naïve idea would be that the field equations are indeed integrable in the sense that they reduce to these tri-linear equations. Tri-linearity in derivatives is highly non-trivial outcome simplifying the situation further. These equations can be formulated as the as purely algebraic equations written above plus integrability conditions

$$F^A_{\alpha\beta} = D_\alpha e^A_\beta - D_\beta e^A_\alpha = 0 \ . \tag{5.5.2}$$

   One could say that vielbein projections define an analog of a trivial gauge potential. Note however that the covariant derivative is defined by spinor connection rather than this effective gauge potential which reduces to that in SU(2). Similar formulation holds true for field equations and one should be able to see whether the field equations formulated in terms of derivatives of vielbein projections commute with the associatitivity conditions.

5. The quaternionicity conditions can be formulated as vanishing of generalization of Cayley's hyperdeterminant for "hypermatrix" a_{ijk} with 2-valued indiced (see `http://tinyurl.com/ya7h3n9z`). Now one has 8 hyper-matrices with 3 8-valued indices associated with the vanishing $A^E_{BCD} x^B y^C z^D = 0$ of trilinear forms defined by the associators. The conditions say somethig only about the octonioni structure constants and since octonionic space allow quaternionic sub-spaces these conditions must be satisfied.

The inspection of the Fano triangle [A39] (see **Fig. 5.1**) expressing the multiplication table for octonionic imaginary units reveals that give any two imaginary octonion units e_1 and e_2 their product e_1e_2 (or equivalently commutator) is imaginary octonion unit (2 times octonion unit) and the three units span together with real unit quaternionic sub-algebra. There it seems that one can generate local quaternionic sub-space from two imaginary units plus real unit. This generalizes to the vielbein components of tangent vectors of space-time surface and one can build the solutions to the quaternionicity conditions from vielbein projections e_1, e_2, their product $e_3 = k(x)e_1e_2$ and real fourth "time-like" vielbein component which must be expressible as a combination of real unit and imaginary units:

$$e_0 = a \times 1 + b^i e_i$$

For static solutions this condition is trivial. Here summation over i is understood in the latter term. Besides these conditions one has integrability conditions and field equations for Kähler action. This formulation suggests that quaternionicity is additional - perhaps defining - property of preferred extremals.

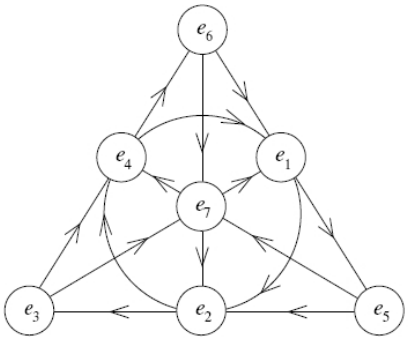

Figure 5.1: Octonionic triangle: the six lines and one circle containing three vertices define the seven associative triplets for which the multiplication rules of the ordinary quaternion imaginary units hold true. The arrow defines the orientation for each associative triplet. Note that the product for the units of each associative triplets equals to real unit apart from sign factor.

5.5.5 Quaternionicity At The Level Of Embedding Space Quantum Numbers

From the multiplication table of octonions as illustrated by Fano triangle [A39] one finds that all edges of the triangle, the middle circle and the three the lines connecting vertices to the midpoints of opposite side define triplets of quaternionic units. This means that by taking real unit and any imaginary unit in quaternionic M^4 algebra spanning $M^2 \subset M^4$ and two imaginary units in the complement representing CP_2 tangent space one obtains quaternionic algebra. This suggests an explanation for the preferred M^2 contained in tangent space of space-time surface (the M^2: s could form an integrable distribution). Four-momentum restricted to M^2 and I_3 and Y interpreted as tangent vectors in CP_2 tangent space defined quaterionic sub-algebra. This could give content for the idea that quantum numbers are quaternionic.

I have indeed proposed that the four-momentum belongs to M^2. If $M^2(x)$ form a distribution as the proposal for the preferred extremals suggests this could reflect momentum exchanges between different points of the space-time surface such that total momentum is conserved or momentum exchange between two sheets connected by wormhole contacts.

5.5.6 Questions

In following some questions related to $M^8 - H$ duality are represented.

Could associativity condition be formulated using modified gamma matrices?

Skeptic can criticize the minimal form of $M^8 - H$ duality involving no Kähler action in M^8 is unrealistic. Why just Kähler action? What makes it so special? The only defense that I can imagine is that Kähler action is in many respects unique choice.

An alternative approach would replace induced gamma matrices with the modified ones to get the correlation In the case of M^8 this option cannot work. One cannot exclude it for H.

1. For Kähler action the Kähler-Dirac gamma matrices $\Gamma^\alpha = \frac{\partial L_K}{\partial h^k_\alpha}\Gamma^k$, $\Gamma_k = e^A_k\gamma_A$, assign to a given point of X^4 a 4-D space which need not be tangent space anymore or even its sub-space. The reason is that canonical momentum current contains besides the gravitational contribution coming from the induced metric also the "Maxwell contribution" from the induced Kähler form not parallel to space-time surface. In the case of M^8 the duality map to H is therefore lost.
2. The space spanned by the Kähler-Dirac gamma matrices need not be 4-dimensional. For vacuum extremals with at most 2-D CP_2 projection Kähler-Dirac gamma matrices vanish identically. For massless extremals they span 1- D light-like subspace. For CP_2 vacuum extremals the modified gamma matrices reduces to ordinary gamma matrices for CP_2 and the situation reduces to the quaternionicity of CP_2. Also for string like objects the conditions are satisfied since the gamma matrices define associative sub-space as tangent space of $M^2 \times S^2 \subset M^4 \times CP_2$. It seems that associativity is satisfied by all known extremals. Hence Kähler-Dirac gamma matrices are flexible enough to realize associativity in H.
3. Kähler-Dirac gamma matrices in Dirac equation are required by super conformal symmetry for the extremals of action and they also guarantee that vacuum extremals defined by surfaces in $M^4 \times Y^2$, Y^2 a Lagrange sub-manifold of CP_2, are trivially hyper-quaternionic surfaces. The modified definition of associativity in H does not affect in any manner $M^8 - H$ duality necessarily based on induced gamma matrices in M^8 allowing purely number theoretic interpretation of standard model symmetries. One can however argue that the most natural definition of associativity is in terms of induced gamma matrices in both M^8 and H.

Remark: A side comment not strictly related to associativity is in order. The anti-commutators of the Kähler-Dirac gamma matrices define an effective Riemann metric and one can assign to it the counterparts of Riemann connection, curvature tensor, geodesic line, volume, etc... One would have two different metrics associated with the space-time surface. Only if the action defining space-time surface is identified as the volume in the ordinary metric, these metrics are equivalent. The index raising for the effective metric could be defined also by the induced metric and it is not clear whether one can define Riemann connection also in this case. Could this effective metric have concrete physical significance and play a deeper role in quantum TGD? For instance, AdS-CFT duality leads to ask whether interactions be coded in terms of the gravitation associated with the effective metric.

Now skeptic can ask why should one demand $M^8 - H$ correspondence if one in any case is forced to introduced Kähler also at the level of M^8? Does $M^8 - H$ correspondence help to construct preferred extremals or does it only bring in a long list of conjectures? I can repeat the questions of the skeptic.

Minkowskian-Euclidian $\leftrightarrow$ associative–co-associative?

The 8-dimensionality of M^8 allows to consider both associativity of the tangent space and associativity of the normal space- let us call this co-associativity of tangent space- as alternative options. Both options are needed as has been already found. Since space-time surface decomposes into regions whose induced metric possesses either Minkowskian or Euclidian signature, there is a strong temptation to propose that Minkowskian regions correspond to associative and Euclidian regions to co-associative regions so that space-time itself would provide both the description and its dual.

The proposed interpretation of conjectured associative-co-associative duality relates in an interesting manner to p-adic length scale hypothesis selecting the primes $p \simeq 2^k$, k positive integer as preferred p-adic length scales. $L_p \propto \sqrt{p}$ corresponds to the p-adic length scale defining the size of the space-time sheet at which elementary particle represented as CP_2 type extremal is topologically condensed and is of order Compton length. $L_k \propto \sqrt{k}$ represents the p-adic length scale of the wormhole contacts associated with the CP_2 type extremal and CP_2 size is the natural length unit now. Obviously the quantitative formulation for associative-co-associative duality would be in terms $p \to k$ duality.

Can $M^8 - H$ duality be useful?

Skeptic could of course argue that $M^8 - H$ duality generates only an inflation of unproven conjectures. This might be the case. In the following I will however try to defend the conjecture. One can however find good motivations for $M^8 - H$ duality: both theoretical and physical.

1. If $M^8 - H$ duality makes sense for induced gamma matrices also in H, one obtains infinite sequence if dualities allowing to construct preferred extremals iteratively. This might relate to octonionic real-analyticity and composition of octonion-real-analytic functions.
2. $M^8 - H$ duality could provide much simpler description of preferred extremals of Kähler action as hyper-quaternionic surfaces. Unfortunately, it is not clear whether one should introduce the counterpart of Kähler action in M^8 and the coupling of M^8 spinors to Kähler form. Note that the Kähler form in E^4 would be self dual and have constant components: essentially parallel electric and magnetic field of same constant magnitude.
3. $M^8 - H$ duality provides insights to low energy physics, in particular low energy hadron physics. M^8 description might work when H-description fails. For instance, perturbative QCD which corresponds to H-description fails at low energies whereas M^8 description might become perturbative description at this limit. Strong $SO(4) = SU(2)_L \times SU(2)_R$ invariance is the basic symmetry of the phenomenological low energy hadron models based on conserved vector current hypothesis (CVC) and partially conserved axial current hypothesis (PCAC). Strong $SO(4) = SU(2)_L \times SU(2)_R$ relates closely also to electro-weak gauge group $SU(2)_L \times U(1)$ and this connection is not well understood in QCD description. $M^8 - H$ duality could provide this connection. Strong $SO(4)$ symmetry would emerge as a low energy dual of the color symmetry. Orbital $SO(4)$ would correspond to strong $SU(2)_L \times SU(2)_R$ and by flatness of E^4 spin like $SO(4)$ would correspond to electro-weak group $SU(2)_L \times U(1)_R \subset SO(4)$. Note that the inclusion of coupling to Kähler gauge potential is necessary to achieve respectable spinor structure in CP_2. One could say that the orbital angular momentum in $SO(4)$ corresponds to strong isospin and spin part of angular momentum to the weak isospin.

   This argument does not seem to be consistent with $SU(3) \times U(1) \subset SU(4)$ symmetry for Mx Dirac equation. One can however argue that $SU(4)$ symmetry combines $SO(4)$ multiplets together. Furthermore, $SO(4)$ represents the isometries leaving Kähler form invariant.

$M^8 - H$ duality in low energy physics and low energy hadron physics

$M^8 - H$ can be applied to gain a view about color confinement. The basic idea would be that $SO(4)$ and $SU(3)$ provide provide dual descriptions of quarks using E^4 and CP_2 partial waves and low energy hadron physics corresponds to a situation in which M^8 picture provides the perturbative approach whereas H picture works at high energies.

A possible interpretation is that the space-time surfaces vary so slowly in CP_2 degrees of freedom that can approximate CP_2 with a small region of its tangent space E^4. One could also say that color interactions mask completely electroweak interactions so that the spinor connection of CP_2 can be neglected and one has effectively E^4. The basic prediction is that $SO(4)$ should appear as dynamical symmetry group of low energy hadron physics and this is indeed the case.

Consider color confinement at the long length scale limit in terms of $M^8 - H$ duality.

1. At high energy limit only lowest color triplet color partial waves for quarks dominate so that QCD description becomes appropriate whereas very higher color partial waves for quarks and gluons are expected to appear at the confinement limit. Since WCW degrees of freedom begin to dominate, color confinement limit transcends the descriptive power of QCD.
2. The success of $SO(4)$ sigma model in the description of low lying hadrons would directly relate to the fact that this group labels also the E^4 Hamiltonians in M^8 picture. Strong $SO(4)$ quantum numbers can be identified as orbital counterparts of right and left handed

electro-weak isospin coinciding with strong isospin for lowest quarks. In sigma model pion and sigma boson form the components of E^4 valued vector field or equivalently collection of four E^4 Hamiltonians corresponding to spherical E^4 coordinates. Pion corresponds to S^3 valued unit vector field with charge states of pion identifiable as three Hamiltonians defined by the coordinate components. Sigma is mapped to the Hamiltonian defined by the E^4 radial coordinate. Excited mesons corresponding to more complex Hamiltonians are predicted.

3. The generalization of sigma model would assign to quarks E^4 partial waves belonging to the representations of $SO(4)$. The model would involve also 6 $SO(4)$ gluons and their $SO(4)$ partial waves. At the low energy limit only lowest representations would be important whereas at higher energies higher partial waves would be excited and the description based on CP_2 partial waves would become more appropriate.

4. The low energy quark model would rely on quarks moving $SO(4)$ color partial waves. Left *resp.* right handed quarks could correspond to $SU(2)_L$ *resp.* $SU(2)_R$ triplets so that spin statistics problem would be solved in the same manner as in the standard quark model.

5. Family replication phenomenon is described in TGD framework the same manner in both cases so that quantum numbers like strangeness and charm are not fundamental. Indeed, p-adic mass calculations allowing fractally scaled up versions of various quarks allow to replace Gell-Mann mass formula with highly successful predictions for hadron masses [K62].

To my opinion these observations are intriguing enough to motivate a concrete attempt to construct low energy hadron physics in terms of $SO(4)$ gauge theory.

5.5.7 Summary

The overall conclusion is that the most convincing scenario relies on the associativity/co-associativity of space-time surfaces define by induced gamma matrices and applying both for M^8 and H. The fact that the duality can be continued to an iterated sequence of duality maps $M^8 \rightarrow H \rightarrow H...$ is what makes the proposal so fascinating and suggests connection with fractality.

The introduction of Kähler action and coupling of spinors to Kähler gauge potentials is highly natural. One can also consider the idea that the space-time surfaces in M^8 and H have same induced metric and Kähler form: for iterated duality map this would mean that the steps in the map produce space-time surfaces which identical metric and Kähler form so that the sequence might stop. M^8_H duality might provide two descriptions of same underlying dynamics: M^8 description would apply in long length scales and H description in short length scales.

5.6 Weak Form Electric-Magnetic Duality And Its Implications

The notion of electric-magnetic duality [B5] was proposed first by Olive and Montonen and is central in $\mathcal{N} = 4$ supersymmetric gauge theories. It states that magnetic monopoles and ordinary particles are two different phases of theory and that the description in terms of monopoles can be applied at the limit when the running gauge coupling constant becomes very large and perturbation theory fails to converge. The notion of electric-magnetic self-duality is more natural since for CP_2 geometry Kähler form is self-dual and Kähler magnetic monopoles are also Kähler electric monopoles and Kähler coupling strength is by quantum criticality renormalization group invariant rather than running coupling constant. The notion of electric-magnetic (self-)duality emerged already two decades ago in the attempts to formulate the Kähler geometric of world of classical worlds. Quite recently a considerable step of progress took place in the understanding of this notion [K25] . What seems to be essential is that one adopts a weaker form of the self-duality applying at partonic 2-surfaces. What this means will be discussed in the sequel.

Every new idea must be of course taken with a grain of salt but the good sign is that this concept leads to precise predictions. The point is that elementary particles do not generate monopole fields in macroscopic length scales: at least when one considers visible matter. The first question is whether elementary particles could have vanishing magnetic charges: this turns out to be impossible. The next question is how the screening of the magnetic charges could take place and leads to an identification of the physical particles as string like objects identified as pairs magnetic charged wormhole throats connected by magnetic flux tubes.

1. The first implication is a new view about electro-weak massivation reducing it to weak confinement in TGD framework. The second end of the string contains particle having electroweak isospin neutralizing that of elementary fermion and the size scale of the string is electro-weak scale would be in question. Hence the screening of electro-weak force takes place via weak confinement realized in terms of magnetic confinement.
2. This picture generalizes to the case of color confinement. Also quarks correspond to pairs of magnetic monopoles but the charges need not vanish now. Rather, valence quarks would be connected by flux tubes of length of order hadron size such that magnetic charges sum up to zero. For instance, for baryonic valence quarks these charges could be $(2, -1, -1)$ and could be proportional to color hyper charge.
3. The highly non-trivial prediction making more precise the earlier stringy vision is that elementary particles are string like objects: this could become manifest at LHC energies.
4. The weak form electric-magnetic duality together with Beltrami flow property of Kähler leads to the reduction of Kähler action to Chern-Simons action so that TGD reduces to almost topological QFT and that Kähler function is explicitly calculable. This has enormous impact concerning practical calculability of the theory.
5. One ends up also to a general solution ansatz for field equations from the condition that the theory reduces to almost topological QFT. The solution ansatz is inspired by the idea that all isometry currents are proportional to Kähler current which is integrable in the sense that the flow parameter associated with its flow lines defines a global coordinate. The proposed solution ansatz would describe a hydrodynamical flow with the property that isometry charges are conserved along the flow lines (Beltrami flow). A general ansatz satisfying the integrability conditions is found.

   The strongest form of the solution ansatz states that various classical and quantum currents flow along flow lines of the Beltrami flow defined by Kähler current (Kähler magnetic field associated with Chern-Simons action). Intuitively this picture is attractive. A more general ansatz would allow several Beltrami flows meaning multi-hydrodynamics. The integrability conditions boil down to two scalar functions: the first one satisfies massless d'Alembert equation in the induced metric and the gradients of the scalar functions are orthogonal. The interpretation in terms of momentum and polarization directions is natural.

5.6.1 Could A Weak Form Of Electric-Magnetic Duality Hold True?

Holography means that the initial data at the partonic 2-surfaces should fix the WCW metric. A weak form of this condition allows only the partonic 2-surfaces defined by the wormhole throats at which the signature of the induced metric changes. A stronger condition allows all partonic 2-surfaces in the slicing of space-time sheet to partonic 2-surfaces and string world sheets. Number theoretical vision suggests that hyper-quaternionicity *resp.* co-hyperquaternionicity constraint could be enough to fix the initial values of time derivatives of the embedding space coordinates in the space-time regions with Minkowskian *resp.* Euclidian signature of the induced metric. This is a condition on modified gamma matrices and hyper-quaternionicity states that they span a hyper-quaternionic sub-space.

Definition of the weak form of electric-magnetic duality

One can also consider alternative conditions possibly equivalent with this condition. The argument goes as follows.

1. The expression of the matrix elements of the metric and Kähler form of WCW in terms of the Kähler fluxes weighted by Hamiltonians of $\delta M^4_\pm$ at the partonic 2-surface X^2 looks very attractive. These expressions however carry no information about the 4-D tangent space of the partonic 2-surfaces so that the theory would reduce to a genuinely 2-dimensional theory, which cannot hold true. One would like to code to the WCW metric also information about the electric part of the induced Kähler form assignable to the complement of the tangent space of $X^2 \subset X^4$.

2. Electric-magnetic duality of the theory looks a highly attractive symmetry. The trivial manner to get electric magnetic duality at the level of the full theory would be via the identification of the flux Hamiltonians as sums of of the magnetic and electric fluxes. The presence of the induced metric is however troublesome since the presence of the induced metric means that the simple transformation properties of flux Hamiltonians under symplectic transformations -in particular color rotations- are lost.

3. A less trivial formulation of electric-magnetic duality would be as an initial condition which eliminates the induced metric from the electric flux. In the Euclidian version of 4-D YM theory this duality allows to solve field equations exactly in terms of instantons. This approach involves also quaternions. These arguments suggest that the duality in some form might work. The full electric magnetic duality is certainly too strong and implies that space-time surface at the partonic 2-surface corresponds to piece of CP_2 type vacuum extremal and can hold only in the deep interior of the region with Euclidian signature. In the region surrounding wormhole throat at both sides the condition must be replaced with a weaker condition.

4. To formulate a weaker form of the condition let us introduce coordinates (x^0, x^3, x^1, x^2) such (x^1, x^2) define coordinates for the partonic 2-surface and (x^0, x^3) define coordinates labeling partonic 2-surfaces in the slicing of the space-time surface by partonic 2-surfaces and string world sheets making sense in the regions of space-time sheet with Minkowskian signature. The assumption about the slicing allows to preserve general coordinate invariance. The weakest condition is that the generalized Kähler electric fluxes are apart from constant proportional to Kähler magnetic fluxes. This requires the condition

$$J^{03}\sqrt{g_4} = KJ_{12} \ . \tag{5.6.1}$$

A more general form of this duality is suggested by the considerations of [K46] reducing the hierarchy of Planck constants to basic quantum TGD and also reducing Kähler function for preferred extremals to Chern-Simons terms [B2] at the boundaries of CD and at light-like wormhole throats. This form is following

$$J^{n\beta}\sqrt{g_4} = K\epsilon \times \epsilon^{n\beta\gamma\delta}J_{\gamma\delta}\sqrt{g_4} \ . \tag{5.6.2}$$

Here the index n refers to a normal coordinate for the space-like 3-surface at either boundary of CD or for light-like wormhole throat. ϵ is a sign factor which is opposite for the two ends of CD. It could be also opposite of opposite at the opposite sides of the wormhole throat. Note that the dependence on induced metric disappears at the right hand side and this condition eliminates the potentials singularity due to the reduction of the rank of the induced metric at wormhole throat.

5. Information about the tangent space of the space-time surface can be coded to the WCW metric with loosing the nice transformation properties of the magnetic flux Hamiltonians if Kähler electric fluxes or sum of magnetic flux and electric flux satisfying this condition are used and K is symplectic invariant. Using the sum

$$J_e + J_m = (1+K)J_{12} \ , \tag{5.6.3}$$

where J denotes the Kähler magnetic flux, , makes it possible to have a non-trivial WCW metric even for $K = 0$, which could correspond to the ends of a cosmic string like solution carrying only Kähler magnetic fields. This condition suggests that it can depend only on Kähler magnetic flux and other symplectic invariants. Whether local symplectic coordinate invariants are possible at all is far from obvious, If the slicing itself is symplectic invariant then K could be a non-constant function of X^2 depending on string world sheet coordinates. The light-like radial coordinate of the light-cone boundary indeed defines a symplectically invariant slicing and this slicing could be shifted along the time axis defined by the tips of CD.

Electric-magnetic duality physically

What could the weak duality condition mean physically? For instance, what constraints are obtained if one assumes that the quantization of electro-weak charges reduces to this condition at classical level?

1. The first thing to notice is that the flux of J over the partonic 2-surface is analogous to magnetic flux

$$Q_m = \frac{e}{\hbar}\oint BdS = n \ .$$

n is non-vanishing only if the surface is homologically non-trivial and gives the homology charge of the partonic 2-surface.

2. The expressions of classical electromagnetic and Z^0 fields in terms of Kähler form [L1] , [L1] read as

$$\begin{aligned} \gamma &= \frac{eF_{em}}{\hbar} = 3J - sin^2(\theta_W)R_{03} \ , \\ Z^0 &= \frac{g_Z F_Z}{\hbar} = 2R_{03} \ . \end{aligned} \quad (5.6.4)$$

Here R_{03} is one of the components of the curvature tensor in vielbein representation and F_{em} and F_Z correspond to the standard field tensors. From this expression one can deduce

$$J = \frac{e}{3\hbar}F_{em} + sin^2(\theta_W)\frac{g_Z}{6\hbar}F_Z \ . \quad (5.6.5)$$

3. The weak duality condition when integrated over X^2 implies

$$\begin{aligned} \frac{e^2}{3\hbar}Q_{em} + \frac{g_Z^2 p}{6}Q_{Z,V} &= K\oint J = Kn \ , \\ Q_{Z,V} &= \frac{I_V^3}{2} - Q_{em} \ , \ p = sin^2(\theta_W) \ . \end{aligned} \quad (5.6.6)$$

Here the vectorial part of the Z^0 charge rather than as full Z^0 charge $Q_Z = I_L^3 + sin^2(\theta_W)Q_{em}$ appears. The reason is that only the vectorial isospin is same for left and right handed components of fermion which are in general mixed for the massive states.

The coefficients are dimensionless and expressible in terms of the gauge coupling strengths and using $\hbar = r\hbar_0$ one can write

$$\begin{aligned} \alpha_{em}Q_{em} &+ p\frac{\alpha_Z}{2}Q_{Z,V} = \frac{3}{4\pi} \times rnK \ , \\ \alpha_{em} &= \frac{e^2}{4\pi\hbar_0} \ , \ \alpha_Z = \frac{g_Z^2}{4\pi\hbar_0} = \frac{\alpha_{em}}{p(1-p)} \ . \end{aligned} \quad (5.6.7)$$

4. There is a great temptation to assume that the values of Q_{em} and Q_Z correspond to their quantized values and therefore depend on the quantum state assigned to the partonic 2-surface. The linear coupling of the Kähler-Dirac operator to conserved charges implies correlation between the geometry of space-time sheet and quantum numbers assigned to the partonic 2-surface. The assumption of standard quantized values for Q_{em} and Q_Z would be also seen as the identification of the fine structure constants α_{em} and α_Z. This however requires weak isospin invariance.

The value of K from classical quantization of Kähler electric charge

The value of K can be deduced by requiring classical quantization of Kähler electric charge.

1. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K would give the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant which should invariant under coupling constant evolution by quantum criticality. Within experimental uncertainties one has $\alpha_K = g_K^2/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.
2. The quantization of Planck constants makes the condition highly non-trivial. The most general quantization of r is as rationals but there are good arguments favoring the quantization as integers corresponding to the allowance of only singular coverings of CD andn CP_2. The point is that in this case a given value of Planck constant corresponds to a finite number pages of the "Big Book". The quantization of the Planck constant implies a further quantization of K and would suggest that K scales as $1/r$ unless the spectrum of values of Q_{em} and Q_Z allowed by the quantization condition scales as r. This is quite possible and the interpretation would be that each of the r sheets of the covering carries (possibly same) elementary charge. Kind of discrete variant of a full Fermi sphere would be in question. The interpretation in terms of anyonic phases [K68] supports this interpretation.
3. The identification of J as a counterpart of $eB/\hbar$ means that Kähler action and thus also Kähler function is proportional to $1/\alpha_K$ and therefore to $\hbar$. This implies that for large values of $\hbar$ Kähler coupling strength $g_K^2/4\pi$ becomes very small and large fluctuations are suppressed in the functional integral. The basic motivation for introducing the hierarchy of Planck constants was indeed that the scaling $\alpha \to \alpha/r$ allows to achieve the convergence of perturbation theory: Nature itself would solve the problems of the theoretician. This of course does not mean that the physical states would remain as such and the replacement of single particles with anyonic states in order to satisfy the condition for K would realize this concretely.
4. The condition $K = g_K^2/\hbar$ implies that the Kähler magnetic charge is always accompanied by Kähler electric charge. A more general condition would read as

$$K = n \times \frac{g_K^2}{\hbar}, n \in Z \ . \tag{5.6.8}$$

   This would apply in the case of cosmic strings and would allow vanishing Kähler charge possible when the partonic 2-surface has opposite fermion and anti-fermion numbers (for both leptons and quarks) so that Kähler electric charge should vanish. For instance, for neutrinos the vanishing of electric charge strongly suggests $n = 0$ besides the condition that abelian Z^0 flux contributing to em charge vanishes.

It took a year to realize that this value of K is natural at the Minkowskian side of the wormhole throat. At the Euclidian side much more natural condition is

$$K = \frac{1}{hbar} \ . \tag{5.6.9}$$

In fact, the self-duality of CP_2 Kähler form favours this boundary condition at the Euclidian side of the wormhole throat. Also the fact that one cannot distinguish between electric and magnetic charges in Euclidian region since all charges are magnetic can be used to argue in favor of this form. The same constraint arises from the condition that the action for CP_2 type vacuum extremal has the value required by the argument leading to a prediction for gravitational constant in terms of the square of CP_2 radius and α_K the effective replacement $g_K^2 \to 1$ would spoil the argument.

The boundary condition $J_E = J_B$ for the electric and magnetic parts of Kählwer form at the Euclidian side of the wormhole throat inspires the question whether all Euclidian regions could be self-dual so that the density of Kähler action would be just the instanton density. Self-duality follows if the deformation of the metric induced by the deformation of the canonically imbedded

CP_2 is such that in CP_2 coordinates for the Euclidian region the tensor $(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\mu\beta})/\sqrt{g}$ remains invariant. This is certainly the case for CP_2 type vacuum extremals since by the light-likeness of M^4 projection the metric remains invariant. Also conformal scalings of the induced metric would satisfy this condition. Conformal scaling is not consistent with the degeneracy of the 4-metric at the wormhole.

Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{12} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \quad (5.6.10)$$

   Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L1]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K75]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.

2. GRT limit of TGD with space-time surfaces replaced with abstract 4-geometries would naturally correspond to Einstein-Maxwell theory with cosmological constant which is non-vanishing only in Euclidian regions of space-time so that both Reissner-Nordström metric and CP_2 are allowed as simplest possible solutions of field equations [K99]. The extremely small value of the observed cosmological constant needed in GRT type cosmology could be equal to the large cosmological constant associated with CP_2 metric multiplied with the 3-volume fraction of Euclidian regions.

3. Also at GRT limit quantum theory would reduce to almost topological QFT since Einstein-Maxwell action reduces to 3-D term by field equations implying the vanishing of the Maxwell current and of the curvature scalar in Minkowskian regions and curvature scalar + cosmological constant term in Euclidian regions. The weak form of electric-magnetic duality would guarantee also now the preferred extremal property and prevent the reduction to a mere topological QFT.

4. GRT limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

The weak form of electric-magnetic duality has surprisingly strong implications for the basic view about quantum TGD as following considerations show.

5.6.2 Magnetic Confinement, The Short Range Of Weak Forces, And Color Confinement

The weak form of electric-magnetic duality has surprisingly strong implications if one combines it with some very general empirical facts such as the non-existence of magnetic monopole fields in macroscopic length scales.

How can one avoid macroscopic magnetic monopole fields?

Monopole fields are experimentally absent in length scales above order weak boson length scale and one should have a mechanism neutralizing the monopole charge. How electroweak interactions become short ranged in TGD framework is still a poorly understood problem. What suggests itself is the neutralization of the weak isospin above the intermediate gauge boson Compton length by neutral Higgs bosons. Could the two neutralization mechanisms be combined to single one?

1. In the case of fermions and their super partners the opposite magnetic monopole would be a wormhole throat. If the magnetically charged wormhole contact is electromagnetically neutral but has vectorial weak isospin neutralizing the weak vectorial isospin of the fermion only the electromagnetic charge of the fermion is visible on longer length scales. The distance of this wormhole throat from the fermionic one should be of the order weak boson Compton length. An interpretation as a bound state of fermion and a wormhole throat state with the quantum numbers of a neutral Higgs boson would therefore make sense. The neutralizing throat would have quantum numbers of $X_{-1/2} = \nu_L\overline{\nu}_R$ or $X_{1/2} = \overline{\nu}_L\nu_R$. $\nu_L\overline{\nu}_R$ would not be neutral Higgs boson (which should correspond to a wormhole contact) but a super-partner of left-handed neutrino obtained by adding a right handed neutrino. This mechanism would apply separately to the fermionic and anti-fermionic throats of the gauge bosons and corresponding space-time sheets and leave only electromagnetic interaction as a long ranged interaction.
2. One can of course wonder what is the situation situation for the bosonic wormhole throats feeding gauge fluxes between space-time sheets. It would seem that these wormhole throats must always appear as pairs such that for the second member of the pair monopole charges and I_V^3 cancel each other at both space-time sheets involved so that one obtains at both space-time sheets magnetic dipoles of size of weak boson Compton length. The proposed magnetic character of fundamental particles should become visible at TeV energies so that LHC might have surprises in store!

Well-definedness of electromagnetic charge implies stringiness

Well-definedness of electromagnetic charged at string world sheets carrying spinor modes is very natural constraint and not trivially satisfied because classical W boson fields are present. As a matter fact, all weak fields should be effectively absent above weak scale. How this is possible classical weak fields identified as induced gauge fields are certainly present.

The condition that em charge is well defined for spinor modes implies that the space-time region in which spinor mode is non-vanishing has 2-D CP_2 projection such that the induced W boson fields are vanishing. The vanishing of classical Z^0 field can be poses as additional condition - at least in scales above weak scale. In the generic case this requires that the spinor mode is restricted to 2-D surface: string world sheet or possibly also partonic 2-surface. This implies that TGD reduces to string model in fermionic sector. Even for preferred extremals with 2-D projecting the modes are expected to allow restriction to 2-surfaces. This localization is possible only for Kähler-Dirac action.

A word of warning is however in order. The GRT limit or rather limit of TGD as Einstein Yang-Mills theory replaces the sheets of many-sheeted space-time with Minkowski space with effective metric obtained by summing to Minkowski metric the deviations of the induced metrics of space-time sheets from Minkowski metric. For gauge potentials a similar identification applies. YM-Einstein equations coupled with matter and with non-vanishing cosmological constant are expected on basis of Poincare invariance. One cannot exclude the possibility that the sums of weak gauge potentials from different space-time sheet tend to vanish above weak scale and that well-definedness of em charge at classical level follows from the effective absence of classical weak gauge fields.

Magnetic confinement and color confinement

Magnetic confinement generalizes also to the case of color interactions. One can consider also the situation in which the magnetic charges of quarks (more generally, of color excited leptons and quarks) do not vanish and they form color and magnetic singles in the hadronic length scale. This would mean that magnetic charges of the state $q_{\pm 1/2} - X_{\mp 1/2}$ representing the physical quark would not vanish and magnetic confinement would accompany also color confinement. This would explain why free quarks are not observed. To how degree then quark confinement corresponds to magnetic confinement is an interesting question.

For quark and antiquark of meson the magnetic charges of quark and antiquark would be opposite and meson would correspond to a Kähler magnetic flux so that a stringy view about meson emerges. For valence quarks of baryon the vanishing of the net magnetic charge takes place provided that the magnetic net charges are $(\pm 2, \mp 1, \mp 1)$. This brings in mind the spectrum of color hyper charges coming as $(\pm 2, \mp 1, \mp 1)/3$ and one can indeed ask whether color hypercharge correlates with the Kähler magnetic charge. The geometric picture would be three strings connected to single vertex. Amusingly, the idea that color hypercharge could be proportional to color hyper charge popped up during the first year of TGD when I had not yet discovered CP_2 and believed on $M^4 \times S^2$.

p-Adic length scale hypothesis and hierarchy of Planck constants defining a hierarchy of dark variants of particles suggest the existence of scaled up copies of QCD type physics and weak physics. For p-adically scaled up variants the mass scales would be scaled by a power of $\sqrt{2}$ in the most general case. The dark variants of the particle would have the same mass as the original one. In particular, Mersenne primes $M_k = 2^k - 1$ and Gaussian Mersennes $M_{G,k} = (1+i)^k - 1$ has been proposed to define zoomed copies of these physics. At the level of magnetic confinement this would mean hierarchy of length scales for the magnetic confinement.

One particular proposal is that the Mersenne prime M_{89} should define a scaled up variant of the ordinary hadron physics with mass scaled up roughly by a factor $2^{(107-89)/2} = 512$. The size scale of color confinement for this physics would be same as the weal length scale. It would look more natural that the weak confinement for the quarks of M_{89} physics takes place in some shorter scale and M_{61} is the first Mersenne prime to be considered. The mass scale of M_{61} weak bosons would be by a factor $2^{(89-61)/2} = 2^{14}$ higher and about 1.6×10^4 TeV. M_{89} quarks would have virtually no weak interactions but would possess color interactions with weak confinement length scale reflecting themselves as new kind of jets at collisions above TeV energies.

In the biologically especially important length scale range 10 nm -2500 nm there are as many as four scaled up electron Compton lengths $L_e(k) = \sqrt{5}L(k)$: they are associated with Gaussian Mersennes $M_{G,k}$, $k = 151, 157, 163, 167$. This would suggest that the existence of scaled up scales of magnetic-, weak- and color confinement. An especially interesting possibly testable prediction is the existence of magnetic monopole pairs with the size scale in this range. There are recent claims about experimental evidence for magnetic monopole pairs [D8] .

Magnetic confinement and stringy picture in TGD sense

The connection between magnetic confinement and weak confinement is rather natural if one recalls that electric-magnetic duality in super-symmetric quantum field theories means that the descriptions in terms of particles and monopoles are in some sense dual descriptions. Fermions would be replaced by string like objects defined by the magnetic flux tubes and bosons as pairs of wormhole contacts would correspond to pairs of the flux tubes. Therefore the sharp distinction between gravitons and physical particles would disappear.

The reason why gravitons are necessarily stringy objects formed by a pair of wormhole contacts is that one cannot construct spin two objects using only single fermion states at wormhole throats. Of course, also super partners of these states with higher spin obtained by adding fermions and anti-fermions at the wormhole throat but these do not give rise to graviton like states [?] . The upper and lower wormhole throat pairs would be quantum superpositions of fermion anti-fermion pairs with sum over all fermions. The reason is that otherwise one cannot realize graviton emission in terms of joining of the ends of light-like 3-surfaces together. Also now magnetic monopole charges are necessary but now there is no need to assign the entities $X_{\pm}$ with gravitons.

Graviton string is characterized by some p-adic length scale and one can argue that below this length scale the charges of the fermions become visible. Mersenne hypothesis suggests that some Mersenne prime is in question. One proposal is that gravitonic size scale is given by electronic Mersenne prime M_{127}. It is however difficult to test whether graviton has a structure visible below this length scale.

What happens to the generalized Feynman diagrams is an interesting question. It is not at all clear how closely they relate to ordinary Feynman diagrams. All depends on what one is ready to assume about what happens in the vertices. One could of course hope that zero energy ontology could allow some very simple description allowing perhaps to get rid of the problematic aspects of Feynman diagrams.

1. Consider first the recent view about generalized Feynman diagrams which relies ZEO. A highly attractive assumption is that the particles appearing at wormhole throats are on mass shell particles. For incoming and outgoing elementary bosons and their super partners they would be positive it resp. negative energy states with parallel on mass shell momenta. For virtual bosons they the wormhole throats would have opposite sign of energy and the sum of on mass shell states would give virtual net momenta. This would make possible twistor description of virtual particles allowing only massless particles (in 4-D sense usually and in 8-D sense in TGD framework). The notion of virtual fermion makes sense only if one assumes in the interaction region a topological condensation creating another wormhole throat having no fermionic quantum numbers.

2. The addition of the particles $X^{\pm}$ replaces generalized Feynman diagrams with the analogs of stringy diagrams with lines replaced by pairs of lines corresponding to fermion and $X_{\pm 1/2}$. The members of these pairs would correspond to 3-D light-like surfaces glued together at the vertices of generalized Feynman diagrams. The analog of 3-vertex would not be splitting of the string to form shorter strings but the replication of the entire string to form two strings with same length or fusion of two strings to single string along all their points rather than along ends to form a longer string. It is not clear whether the duality symmetry of stringy diagrams can hold true for the TGD variants of stringy diagrams.

3. How should one describe the bound state formed by the fermion and $X^{\pm}$? Should one describe the state as superposition of non-parallel on mass shell states so that the composite state would be automatically massive? The description as superposition of on mass shell states does not conform with the idea that bound state formation requires binding energy. In TGD framework the notion of negentropic entanglement has been suggested to make possible the analogs of bound states consisting of on mass shell states so that the binding energy is zero [K57] . If this kind of states are in question the description of virtual states in terms of on mass shell states is not lost. Of course, one cannot exclude the possibility that there is infinite number of this kind of states serving as analogs for the excitations of string like object.

4. What happens to the states formed by fermions and $X_{\pm 1/2}$ in the internal lines of the Feynman diagram? Twistor philosophy suggests that only the higher on mass shell excitations are possible. If this picture is correct, the situation would not change in an essential manner from the earlier one.

The highly non-trivial prediction of the magnetic confinement is that elementary particles should have stringy character in electro-weak length scales and could behaving to become manifest at LHC energies. This adds one further item to the list of non-trivial predictions of TGD about physics at LHC energies [K58] .

5.6.3 Could Quantum TGD Reduce To Almost Topological QFT?

There seems to be a profound connection with the earlier unrealistic proposal that TGD reduces to almost topological quantum theory in the sense that the counterpart of Chern-Simons action assigned with the wormhole throats somehow dictates the dynamics. This proposal can be formulated also for the Kähler-Dirac action action. I gave up this proposal but the following argument shows that Kähler action with weak form of electric-magnetic duality effectively reduces to Chern-Simons action plus Coulomb term.

1. Kähler action density can be written as a 4-dimensional integral of the Coulomb term $j^{\alpha}_K A_{\alpha}$ plus and integral of the boundary term $J^{n\beta}A_{\beta}\sqrt{g_4}$ over the wormhole throats and of the quantity $J^{0\beta}A_{\beta}\sqrt{g_4}$ over the ends of the 3-surface.
2. If the self-duality conditions generalize to $J^{n\beta} = 4\pi\alpha_K\epsilon^{n\beta\gamma\delta}J_{\gamma\delta}$ at throats and to $J^{0\beta} = 4\pi\alpha_K\epsilon^{0\beta\gamma\delta}J_{\gamma\delta}$ at the ends, the Kähler function reduces to the counterpart of Chern-Simons action evaluated at the ends and throats. It would have same value for each branch and the replacement $h \rightarrow n \times h$ would effectively describe this. Boundary conditions would however give $1/n$ factor so that $\hbar$ would disappear from the Kähler function! It is somewhat surprising that Kähler action gives Chern-Simons action in the vacuum sector defined as sector for which Kähler current is light-like or vanishes.

Holography encourages to ask whether also the Coulomb interaction terms could vanish. This kind of dimensional reduction would mean an enormous simplification since TGD would reduce to an almost topological QFT. The attribute "almost" would come from the fact that one has non-vanishing classical Noether charges defined by Kähler action and non-trivial quantum dynamics in M^4 degrees of freedom. One could also assign to space-time surfaces conserved four-momenta which is not possible in topological QFTs. For this reason the conditions guaranteeing the vanishing of Coulomb interaction term deserve a detailed analysis.

1. For the known extremals j^{α}_K either vanishes or is light-like ("massless extremals" for which weak self-duality condition does not make sense [K11]) so that the Coulomb term vanishes identically in the gauge used. The addition of a gradient to A induces terms located at the ends and wormhole throats of the space-time surface but this term must be cancelled by the other boundary terms by gauge invariance of Kähler action. This implies that the M^4 part of WCW metric vanishes in this case. Therefore massless extremals as such are not physically realistic: wormhole throats representing particles are needed.
2. The original naïve conclusion was that since Chern-Simons action depends on CP_2 coordinates only, its variation with respect to Minkowski coordinates must vanish so that the WCW metric would be trivial in M^4 degrees of freedom. This conclusion is in conflict with quantum classical correspondence and was indeed too hasty. The point is that the allowed variations of Kähler function must respect the weak electro-magnetic duality which relates Kähler electric field depending on the induced 4-metric at 3-surface to the Kähler magnetic field. Therefore the dependence on M^4 coordinates creeps via a Lagrange multiplier term

$$\int \Lambda_{\alpha}(J^{n\alpha} - K\epsilon^{n\alpha\beta\gamma}J_{\beta\ gamma})\sqrt{g_4}d^3x \quad . \tag{5.6.11}$$

The (1,1) part of second variation contributing to M^4 metric comes from this term.
3. This erratic conclusion about the vanishing of M^4 part WCW metric raised the question about how to achieve a non-trivial metric in M^4 degrees of freedom. The proposal was a modification of the weak form of electric-magnetic duality. Besides CP_2 Kähler form there would be the Kähler form assignable to the light-cone boundary reducing to that for $r_M = constant$ sphere - call it J^1. The generalization of the weak form of self-duality would be $J^{n\beta} = \epsilon^{n\beta\gamma\delta}K(J_{\gamma\delta}+\epsilon J^1_{\gamma\delta})$. This form implies that the boundary term gives a non-trivial contribution to the M^4 part of the WCW metric even without the constraint from electric-magnetic duality. Kähler charge is not affected unless the partonic 2-surface contains the tip of CD in its interior. In this case the value of Kähler charge is shifted by a topological contribution. Whether this term can survive depends on whether the resulting vacuum extremals are consistent with the basic facts about classical gravitation.

4. The Coulombic interaction term is not invariant under gauge transformations. The good news is that this might allow to find a gauge in which the Coulomb term vanishes. The vanishing condition fixing the gauge transformation ϕ is

$$j_K^{\alpha}\partial_{\alpha}\phi = -j^{\alpha}A_{\alpha} \ . \tag{5.6.12}$$

This differential equation can be reduced to an ordinary differential equation along the flow lines j_K by using $dx^{\alpha}/dt = j_K^{\alpha}$. Global solution is obtained only if one can combine the flow parameter t with three other coordinates- say those at the either end of CD to form space-time coordinates. The condition is that the parameter defining the coordinate differential is proportional to the covariant form of Kähler current: $dt = \phi j_K$. This condition in turn implies $d^2t = d(\phi j_K) = d(\phi j_K) = d\phi \wedge j_K + \phi dj_K = 0$ implying $j_K \wedge dj_K = 0$ or more concretely,

$$\epsilon^{\alpha\beta\gamma\delta} j^K_{\beta}\partial_{\gamma} j^K_{delta} = 0 \ . \tag{5.6.13}$$

j_K is a four-dimensional counterpart of Beltrami field [B11] and could be called generalized Beltrami field.

The integrability conditions follow also from the construction of the extremals of Kähler action [K11] . The conjecture was that for the extremals the 4-dimensional Lorentz force vanishes (no dissipation): this requires $j_K \wedge J = 0$. One manner to guarantee this is the topologization of the Kähler current meaning that it is proportional to the instanton current: $j_K = \phi j_I$, where $j_I = {}^*(J \wedge A)$ is the instanton current, which is not conserved for 4-D CP_2 projection. The conservation of j_K implies the condition $j_I^{\alpha}\partial_{\alpha}\phi = \partial_{\alpha}j^{\alpha}\phi$ and from this ϕ can be integrated if the integrability condition $j_I \wedge dj_I = 0$ holds true implying the same condition for j_K. By introducing at least 3 or CP_2 coordinates as space-time coordinates, one finds that the contravariant form of j_I is purely topological so that the integrability condition fixes the dependence on M^4 coordinates and this selection is coded into the scalar function ϕ. These functions define families of conserved currents $j_K^{\alpha}\phi$ and $j_I^{\alpha}\phi$ and could be also interpreted as conserved currents associated with the critical deformations of the space-time surface.

5. There are gauge transformations respecting the vanishing of the Coulomb term. The vanishing condition for the Coulomb term is gauge invariant only under the gauge transformations $A \rightarrow A + \nabla\phi$ for which the scalar function the integral $\int j_K^{\alpha}\partial_{\alpha}\phi$ reduces to a total divergence a giving an integral over various 3-surfaces at the ends of CD and at throats vanishes. This is satisfied if the allowed gauge transformations define conserved currents

$$D_{\alpha}(j^{\alpha}\phi) = 0 \ . \tag{5.6.14}$$

As a consequence Coulomb term reduces to a difference of the conserved charges $Q_{\phi}^e = \int j^0\phi\sqrt{g_4}d^3x$ at the ends of the CD vanishing identically. The change of the Chern-Simons type term is trivial if the total weighted Kähler magnetic flux $Q_{\phi}^m = \sum\int J\phi dA$ over wormhole throats is conserved. The existence of an infinite number of conserved weighted magnetic fluxes is in accordance with the electric-magnetic duality. How these fluxes relate to the flux Hamiltonians central for WCW geometry is not quite clear.

6. The gauge transformations respecting the reduction to almost topological QFT should have some special physical meaning. The measurement interaction term in the Kähler-Dirac interaction corresponds to a critical deformation of the space-time sheet and is realized as an addition of a gauge part to the Kähler gauge potential of CP_2. It would be natural to identify this gauge transformation giving rise to a conserved charge so that the conserved charges would provide a representation for the charges associated with the infinitesimal critical deformations not affecting Kähler action. The gauge transformed Kähler gauge potential couples to the Kähler-Dirac equation and its effect could be visible in the value of Kähler function and therefore also in the properties of the preferred extremal. The effect on WCW metric would however vanish since K would transform only by an addition of a real part of a holomorphic function.

7. A first guess for the explicit realization of the quantum classical correspondence between quantum numbers and space-time geometry is that the deformation of the preferred extremal due to the addition of the measurement interaction term is induced by a $U(1)$ gauge transformation induced by a transformation of $\delta CD \times CP_2$ generating the gauge transformation represented by ϕ. This interpretation makes sense if the fluxes defined by Q_{ϕ}^{m} and corresponding Hamiltonians affect only zero modes rather than quantum fluctuating degrees of freedom.

8. Later a simpler proposal assuming Kähler action with Chern-Simons term at partonic orbits and Kähler-Dirac action with Chern-Simons Dirac term at partonic orbits emerged. Measurement interaction terms would correspond to Lagrange multiplier terms at the ends of space-time surface fixing the values of classical conserved charges to their quantum values. Super-symmetry requires the assignment of this kind of term also to Kähler-Dirac action as boundary term.

   Kähler-Dirac equation gives rise to a boundary condition at space-like ends of the space-time surface stating that the action of the Kähler-Dirac gamma matrix in normal direction annihilates the spinor modes. The normal vector would be light-like and the value of the incoming on mass shell four-momentum would be coded to the geometry of the space-time surface and string world sheet.

   One can assign to partonic orbits Chern-Simons Dirac action and now the condition would be that the action of C-S-D operator equals to that of massless M^4 Dirac operator. C-S-D Dirac action would give rise to massless Dirac propagator. Twistor Grassmann approach suggests that also the virtual fermions reduce effectively to massless on-shell states but have non-physical helicity.

5.7 How To Define Generalized Feynman Diagrams?

S-matrix codes to a high degree the predictions of quantum theories. The longstanding challenge of TGD has been to construct or at least demonstrate the mathematical existence of S-matrix- or actually M-matrix which generalizes this notion in ZEO (ZEO) [K76] . This work has led to the notion of generalized Feynman diagram and the challenge is to give a precise mathematical meaning for this object. The attempt to understand the counterpart of twistors in TGD framework [K94] has inspired several key ideas in this respect but it turned out that twistors themselves need not be absolutely necessary in TGD framework.

1. The notion of generalized Feyman diagram defined by replacing lines of ordinary Feynman diagram with light-like 3-surfaces (elementary particle sized wormhole contacts with throats carrying quantum numbers) and vertices identified as their 2-D ends - I call them partonic 2-surfaces is central. Speaking somewhat loosely, generalized Feynman diagrams (plus background space-time sheets) define the “world of classical worlds” (WCW). These diagrams involve the analogs of stringy diagrams but the interpretation is different: the analogs of stringy loop diagrams have interpretation in terms of particle propagating via two different routes simultaneously (as in the classical double slit experiment) rather than as a decay of particle to two particles. For stringy diagrams the counterparts of vertices are singular as manifolds whereas the entire diagrams are smooth. For generalized Feynman diagrams vertices are smooth but entire diagrams represent singular manifolds just like ordinary Feynman diagrams do. String like objects however emerge in TGD and even ordinary elementary particles are predicted to be magnetic flux tubes of length of order weak gauge boson Compton length with monopoles at their ends as shown in accompanying article. This stringy character should become visible at LHC energies.

2. ZEO (ZEO) and causal diamonds (intersections of future and past directed light-cones) define second key ingredient. The crucial observation is that in ZEO it is possible to identify off mass shell particles as pairs of on mass shell fermions at throats of wormhole contact since both positive and negative signs of energy are possible and one obtains also space-like total momenta for wormhole contact behaving as a boson. The localization of fermions to string world sheets and the fact that super-conformal generator G carries fermion number combined with twistorial consideration support the view that the propagators at fermionic lines are of

form $(1/G)ip^k\gamma_k(1/G^\dagger + h.c.$ and thus hermitian. In strong models $1/G$ would serve as a propagator and this requires Majorana condition fixing the dimension of the target space to 10 or 11.

3. A powerful constraint is number theoretic universality requiring the existence of Feynman amplitudes in all number fields when one allows suitable algebraic extensions: roots of unity are certainly required in order to realize p-adic counterparts of plane waves. Also embedding space, partonic 2-surfaces and WCW must exist in all number fields and their extensions. These constraints are enormously powerful and the attempts to realize this vision have dominated quantum TGD for last two decades.

4. Representation of 8-D gamma matrices in terms of octonionic units and 2-D sigma matrices is a further important element as far as twistors are considered [K94] . Kähler-Dirac gamma matrices at space-time surfaces are quaternionic/associative and allow a genuine matrix representation. As a matter fact, TGD and WCW could be formulated as study of associative local sub-algebras of the local Clifford algebra of 8-D embedding space parameterized by quaternionic space-time surfaces.

5. A central conjecture has been that associative (co-associative) 4-surfaces correspond to preferred extremals of Kähler action [K104]. It took long time to realize that in ZEO the notion of preferred extremal might be un-necessary! The reason is that 3-surfaces are now pairs of 3-surfaces at boundaries of causal diamonds and for deterministic dynamics the space-time surface connecting them is expected to be more or less unique. Now the action principle is non-deterministic but the non-determinism would give rise to additional discrete dynamical degrees of freedom naturally assignable to the hierarchy of Planck constants $h_{eff} = n \times h$, n the number of space-time surface with same fixed ends at boundaries of CD and with same values of Kähler action and of conserved quantities. One must be however cautions: this leaves the possibility that there is a gauge symmetry present so that the n sheets correspond to gauge equivalence classes of sheets. Conformal invariance is associated with criticality and is expected to be present also now.

   One can of course also ask whether one can assume that the pairs of 3-surfaces at the ends of CD are totally un-correlated. If this assumption is not made then preferred extremal property would make sense also in ZEO and imply additional correlation between the members of these pairs. This kind of correlations would correspond to the Bohr orbit property, which is very attractive space-time correlate for quantum states. This kind of correlates are also expected as space-time counterpart for the correlations between initial and final state in quantum dynamics.

6. A further conjecture has been that preferred extremals are in some sense critical (second variation of Kähler action could vanish for infinite number of deformations defining a super-conformal algebra). The non-determinism of Kähler action implies this property for $n > 0$ in $h_{eff} = nh$. If the criticality is present, it could correspond to conformal gauge invariance defined by sub-algebras of conformal algebra with conformal weights coming as multiples of n and isomorphic to the conformal algebra itself.

7. As far as twistors are considered, the first key element is the reduction of the octonionic twistor structure to quaternionic one at space-time surfaces and giving effectively 4-D spinor and twistor structure for quaternionic surfaces.

Quite recently quite a dramatic progress took place in this approach [K104, K94] .

1. The progress was stimulated by the simple observation that on mass shell property puts enormously strong kinematic restrictions on the loop integrations. With mild restrictions on the number of parallel fermion lines appearing in vertices (there can be several since fermionic oscillator operator algebra defining SUSY algebra generates the parton states)- all loops are manifestly finite and if particles has always mass -say small p-adic thermal mass also in case of massless particles and due to IR cutoff due to the presence largest CD- the number of diagrams is finite. Unitarity reduces to Cutkosky rules [B44] automatically satisfied as in the case of ordinary Feynman diagrams.

2. Ironically, twistors which stimulated all these development do not seem to be absolutely necessary in this approach although they are of course possible. Situation changes if one does

not assume small p-adically thermal mass due to the presence of massless particles and one must sum infinite number of diagrams. Here a potential problem is whether the infinite sum respects the algebraic extension in question.

This is about fermionic and momentum space aspects of Feynman diagrams but not yet about the functional (not path-) integral over small deformations of the partonic 2-surfaces. The basic challenges are following.

1. One should perform the functional integral over WCW degrees of freedom for fixed values of on mass shell momenta appearing in the internal lines. After this one must perform integral or summation over loop momenta. Note that the order is important since the space-time surface assigned to the line carries information about the quantum numbers associated with the line by quantum classical correspondence realized in terms of Kähler-Dirac operator.
2. One must define the functional integral also in the p-adic context. p-Adic Fourier analysis relying on algebraic continuation raises hopes in this respect. p-Adicity suggests strongly that the loop momenta are discretized and ZEO predicts this kind of discretization naturally.

It indeed seems that the functional integrals over WCW could be carried out at general level both in real and p-adic context. This is due to the symmetric space property (maximal number of isometries) of WCW required by the mere mathematical existence of Kähler geometry [K46] in infinite-dimensional context already in the case of much simpler loop spaces [A27] .

1. The p-adic generalization of Fourier analysis allows to algebraize integration- the horrible looking technical challenge of p-adic physics- for symmetric spaces for functions allowing the analog of discrete Fourier decomposition. Symmetric space property is indeed essential also for the existence of Kähler geometry for infinite-D spaces as was learned already from the case of loop spaces. Plane waves and exponential functions expressible as roots of unity and powers of p multiplied by the direct analogs of corresponding exponent functions are the basic building bricks and key functions in harmonic analysis in symmetric spaces. The physically unavoidable finite measurement resolution corresponds to algebraically unavoidable finite algebraic dimension of algebraic extension of p-adics (at least some roots of unity are needed). The cutoff in roots of unity is very reminiscent to that occurring for the representations of quantum groups and is certainly very closely related to these as also to the inclusions of hyper-finite factors of type II_1 defining the finite measurement resolution.
2. WCW geometrization reduces to that for a single line of the generalized Feynman diagram defining the basic building brick for WCW. Kähler function decomposes to a sum of "kinetic" terms associated with its ends and interaction term associated with the line itself. p-Adicization boils down to the condition that Kähler function, matrix elements of Kähler form, WCW Hamiltonians and their super counterparts, are rational functions of complex WCW coordinates just as they are for those symmetric spaces that I know of. This would allow a continuation to p-adic context.

In the following this vision about generalized Feynman diagrams is discussed in more detail.

5.7.1 Questions

The goal is a proposal for how to perform the integral over WCW for generalized Feynman digrams and the best manner to proceed to this goal is by making questions.

What does finite measurement resolution mean?

The first question is what finite measurement resolution means.

1. One expects that the algebraic continuation makes sense only for a finite measurement resolution in which case one obtains only finite sums of what one might hope to be algebraic functions. The finiteness of the algebraic extension would be in fact equivalent with the finite measurement resolution.
2. Finite measurement resolution means a discretization in terms of number theoretic braids. p-Adicization condition suggests that one must allow only the number theoretic braids. For

these the ends of braid at boundary of CD are algebraic points of the embedding space. This would be true at least in the intersection of real and p-adic worlds.

3. The question is whether one can localize the points of the braid. The necessity to use momentum eigenstates to achieve quantum classical correspondence in the Kähler-Dirac action [K104] suggests however a de-localization of braid points, that is wave function in space of braid points. In real context one could allow all possible choices for braid points but in p-adic context only algebraic points are possible if one wants to replace integrals with sums. This implies finite measurement resolution analogous to that in lattice. This is also the only possibility in the intersection of real and p-adic worlds.

   A non-trivial prediction giving a strong correlation between the geometry of the partonic 2-surface and quantum numbers is that the total number $n_F + n_{\overline{F}}$ of fermions and anti-fermions is bounded above by the number n_{alg} of algebraic points for a given partonic 2-surface: $n_F + n_{\overline{F}} \leq n_{alg}$. Outside the intersection of real and p-adic worlds the problematic aspect of this definition is that small deformations of the partonic 2-surface can radically change the number of algebraic points unless one assumes that the finite measurement resolution means restriction of WCW to a sub-space of algebraic partonic surfaces.

4. Braids defining propagator lines for fundamental fermions (to be distinguished from observer particles) emerges naturally. Braid strands correspond to the boundaries of string world sheets at which the modes of induced spinor fields are localized from the condition that em charge is well-defined: induced W field and above weak scale also Z^0 field vanish at them.

   In order to obtain non-trivial fermion propagator one must add to Dirac action 1-D Dirac action in induced metric with the boundaries of string world sheets at the light-like parton orbits. Its bosonic counterpart is line-length in induced metric. Field equations imply that the boundaries are light-like geodesics and fermion has light-like 8-momentum. This suggests strongly a connection with quantum field theory and an 8-D generalization of twistor Grassmannian approach. By field equations the bosonic part of this action does not contribute to the Kähler action. The light-like 8-momenta p^k have same M^4 and CP_2 mass squared and latter correspond to the the eigenvalues of the CP_2 spinor d'Alembertian by quantum-classical correspondence.

5. One has also discretization of the relative position of the second tip of CD at the hyperboloid isometric with mass shell. Only the number of braid points and their momenta would matter, not their positions.

6. The quantum numbers characterizing positive and negative energy parts of zero energy states couple directly to space-time geometry via the measurement interaction terms in Kähler action expressing the equality of classical conserved charges in Cartan algebra with their quantal counterparts for space-time surfaces in quantum superposition. This makes sense if classical charges parametrize zero modes. The localization in zero modes in state function reduction would be the WCW counterpart of state function collapse.

How to define integration in WCW degrees of freedom?

The basic question is how to define the integration over WCW degrees of freedom.

1. What comes mind first is Gaussian perturbation theory around the maxima of Kähler function. Gaussian and metric determinants cancel each other and only algebraic expressions remain. Finiteness is not a problem since the Kähler function is non-local functional of 3-surface so that no local interaction vertices are present. One should however assume the vanishing of loops required also by algebraic universality and this assumption look unrealistic when one considers more general functional integrals than that of vacuum functional since free field theory is not in question. The construction of the inverse of the WCW metric defining the propagator is also a very difficult challenge. Duistermaat-Hecke theorem states that something like this known as localization might be possible and one can also argue that something analogous to localization results from a generalization of mean value theorem.

2. Symmetric space property is more promising since it might reduce the integrations to group theory using the generalization of Fourier analysis for group representations so that there

would be no need for perturbation theory in the proposed sense. In finite measurement resolution the symmetric spaces involved would be finite-dimensional. Symmetric space structure of WCW could also allow to define p-adic integration in terms of p-adic Fourier analysis for symmetric spaces. Essentially algebraic continuation of the integration from the real case would be in question with additional constraints coming from the fact that only phase factors corresponding to finite algebraic extensions of rationals are used. Cutoff would emerge automatically from the cutoff for the dimension of the algebraic extension.

How to define generalized Feynman diagrams?

Integration in symmetric spaces could serve as a model at the level of WCW and allow both the understanding of WCW integration and p-adicization as algebraic continuation. In order to get a more realistic view about the problem one must define more precisely what the calculation of the generalized Feynman diagrams means.

1. WCW integration must be carried out separately for all values of the momenta associated with the internal lines. The reason is that the spectrum of eigenvalues λ_i of the Kähler-Dirac operator D depends on the momentum of line and momentum conservation in vertices translates to a correlation of the spectra of D at internal lines.
2. For tree diagrams algebraic continuation to the p-adic context if the expression involves only the replacement of the generalized eigenvalues of D as functions of momenta with their p-adic counterparts besides vertices. If these functions are algebraically universal and expressible in terms of harmonics of symmetric space , there should be no problems.
3. If loops are involved, one must integrate/sum over loop momenta. In p-adic context difficulties are encountered if the spectrum of the momenta is continuous. The integration over on mass shell loop momenta is analogous to the integration over sub-CDs, which suggests that internal line corresponds to a $sub-CD$ in which it is at rest. There are excellent reasons to believe that the moduli space for the positions of the upper tip is a discrete subset of hyperboloid of future light-cone. If this is the case, the loop integration indeed reduces to a sum over discrete positions of the tip. p-Adizication would thus give a further good reason why for ZEO.
4. Propagator is expressible in terms of the inverse of generalized eigenvalue and there is a sum over these for each propagator line. At vertices one has products of WCW harmonics assignable to the incoming lines. The product must have vanishing quantum numbers associated with the phase angle variables of WCW. Non-trivial quantum numbers of the WCW harmonic correspond to WCW quantum numbers assignable to excitations of ordinary elementary particles. WCW harmonics are products of functions depending on the "radial" coordinates and phase factors and the integral over the angles leaves the product of the first ones analogous to Legendre polynomials $P_{l,m}$, These functions are expected to be rational functions or at least algebraic functions involving only square roots.
5. In ordinary QFT incoming and outgoing lines correspond to propagator poles. In the recent case this would mean that incoming stringy lines at the ends of CD correspond to fermions satisfying the stringy mass formula serving as a generalization of masslessness condition.

5.7.2 Generalized Feynman Diagrams At Fermionic And Momentum SpaceLevel

Negative energy ontology has already led to the idea of interpreting the virtual particles as pairs of positive and negative energy wormhole throats. Hitherto I have taken it as granted that ordinary Feynman diagrammatics generalizes more or less as such. It is however far from clear what really happens in the verties of the generalized Feynman diagrams. The safest approach relies on the requirement that unitarity realized in terms of Cutkosky rules in ordinary Feynman diagrammatics allows a generalization. This requires loop diagrams. In particular, photon-photon scattering can take place only via a fermionic square loop so that it seems that loops must be present at least in the topological sense.

One must be however ready for the possibility that something unexpectedly simple might emerge. For instance, the vision about algebraic physics allows naturally only finite sums for diagrams and does not favor infinite perturbative expansions. Hence the true believer on algebraic physics might dream about finite number of diagrams for a given reaction type. For simplicity generalized Feynman diagrams without the complications brought by the magnetic confinement since by the previous arguments the generalization need not bring in anything essentially new.

The basic idea of duality in early hadronic models was that the lines of the dual diagram representing particles are only re-arranged in the vertices. This however does not allow to get rid of off mass shell momenta. ZEO encourages to consider a stronger form of this principle in the sense that the virtual momenta of particles could correspond to pairs of on mass shell momenta of particles. If also interacting fermions are pairs of positive and negative energy throats in the interaction region the idea about reducing the construction of Feynman diagrams to some kind of lego rules might work.

Virtual particles as pairs of on mass shell particles in ZEO

The first thing is to try to define more precisely what generalized Feynman diagrams are. The direct generalization of Feynman diagrams implies that both wormhole throats and wormhole contacts join at vertices.

1. A simple intuitive picture about what happens is provided by diagrams obtained by replacing the points of Feynman diagrams (wormhole contacts) with short lines and imagining that the throats correspond to the ends of the line. At vertices where the lines meet the incoming on mass shell quantum numbers would sum up to zero. This approach leads to a straightforward generalization of Feynman diagrams with virtual particles replaced with pairs of on mass shell throat states of type $++$, $--$, and $+-$. Incoming lines correspond to $++$ type lines and outgoing ones to $--$ type lines. The first two line pairs allow only time like net momenta whereas $+-$ line pairs allow also space-like virtual momenta. The sign assigned to a given throat is dictated by the sign of the on mass shell momentum on the line. The condition that Cutkosky rules generalize as such requires $++$ and $--$ type virtual lines since the cut of the diagram in Cutkosky rules corresponds to on mass shell outgoing or incoming states and must therefore correspond to $++$ or $--$ type lines.

2. The basic difference as compared to the ordinary Feynman diagrammatics is that loop integrals are integrals over mass shell momenta and that all throats carry on mass shell momenta. In each vertex of the loop mass incoming on mass shell momenta must sum up to on mass shell momentum. These constraints improve the behavior of loop integrals dramatically and give excellent hopes about finiteness. It does not however seem that only a finite number of diagrams contribute to the scattering amplitude besides tree diagrams. The point is that if a the reactions $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$,, where N_i denote particle numbers, are possible in a common kinematical region for N_2-particle states then also the diagrams $N_1 \rightarrow N_2 \rightarrow N_2 \rightarrow N_3$ are possible. The virtual states N_2 include all all states in the intersection of kinematically allow regions for $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_3$. Hence the dream about finite number possible diagrams is not fulfilled if one allows massless particles. If all particles are massive then the particle number N_2 for given N_1 is limited from above and the dream is realized.

3. For instance, loops are not possible in the massless case or are highly singular (bringing in mind twistor diagrams) since the conservation laws at vertices imply that the momenta are parallel. In the massive case and allowing mass spectrum the situation is not so simple. As a first example one can consider a loop with three vertices and thus three internal lines. Three on mass shell conditions are present so that the four-momentum can vary in 1-D subspace only. For a loop involving four vertices there are four internal lines and four mass shell conditions so that loop integrals would reduce to discrete sums. Loops involving more than four vertices are expected to be impossible.

4. The proposed replacement of the elementary fermions with bound states of elementary fermions and monopoles $X_\pm$ brings in the analog of stringy diagrammatics. The 2-particle wave functions in the momentum degrees of freedom of fermion and $X_\pm$ might allow more flexibility and allow more loops. Note however that there are excellent hopes about the finiteness of the theory also in this case.

Loop integrals are manifestly finite

One can make also more detailed observations about loops.

1. The simplest situation is obtained if only 3-vertices are allowed. In this case conservation of momentum however allows only collinear momenta although the signs of energy need not be the same. Particle creation and annihilation is possible and momentum exchange is possible but is always light-like in the massless case. The scattering matrices of supersymmetric YM theories would suggest something less trivial and this raises the question whether something is missing. Magnetic monopoles are an essential element of also these theories as also massivation and symmetry breaking and this encourages to think that the formation of massive states as fermion $X_{\pm}$ pairs is needed. Of course, in TGD framework one has also high mass excitations of the massless states making the scattering matrix non-trivial.
2. In YM theories on mass shell lines would be singular. In TGD framework this is not the case since the propagator is defined as the inverse of the 3-D dimensional reduction of the Kähler-Dirac operator D containing also coupling to four-momentum (this is required by quantum classical correspondence and guarantees stringy propagators),

$$\begin{aligned} D &= i\hat{\Gamma}^{\alpha}p_{\alpha}+\hat{\Gamma}^{\alpha}D_{\alpha} \ , \\ p_{\alpha} &= p_k\partial_{\alpha}h^k \ . \end{aligned} \tag{5.7.1}$$

   The propagator does not diverge for on mass shell massless momenta and the propagator lines are well-defined. This is of course of essential importance also in general case. Only for the incoming lines one can consider the possibility that 3-D Dirac operator annihilates the induced spinor fields. All lines correspond to generalized eigenstates of the propagator in the sense that one has $D_3\Psi=\lambda\gamma\Psi$, where γ is Kähler-Dirac gamma matrix in the direction of the stringy coordinate emanating from light-like surface and D_3 is the 3-dimensional dimensional reduction of the 4-D Kähler-Dirac operator. The eigenvalue λ is analogous to energy. Note that the eigenvalue spectrum depends on 4-momentum as a parameter.
3. Massless incoming momenta can decay to massless momenta with both signs of energy. The integration measure $d^2k/2E$ reduces to dx/x where $x\geq 0$ is the scaling factor of massless momentum. Only light-like momentum exchanges are however possible and scattering matrix is essentially trivial. The loop integrals are finite apart from the possible delicacies related to poles since the loop integrands for given massless wormhole contact are proportional to dx/x^3 for large values of x.
4. Irrespective of whether the particles are massless or not, the divergences are obtained only if one allows too high vertices as self energy loops for which the number of momentum degrees of freedom is $3N-4$ for N-vertex. The construction of SUSY limit of TGD in [?] led to the conclusion that the parallelly propagating N fermions for given wormhole throat correspond to a product of N fermion propagators with same four-momentum so that for fermions and ordinary bosons one has the standard behavior but for $N>2$ non-standard so that these excitations are not seen as ordinary particles. Higher vertices are finite only if the total number N_F of fermions propagating in the loop satisfies $N_F>3N-4$. For instance, a 4-vertex from which $N=2$ states emanate is finite.

Taking into account magnetic confinement

What has been said above is not quite enough. The weak form of electric-magnetic duality [B5] leads to the picture about elementary particles as pairs of magnetic monopoles inspiring the notions of weak confinement based on magnetic monopole force. Also color confinement would have magnetic counterpart. This means that elementary particles would behave like string like objects in weak boson length scale. Therefore one must also consider the stringy case with wormhole throats replaced with fermion-$X_{\pm}$ pairs ($X_{\pm}$ is electromagnetically neutral and $\pm$ refers to the sign of the weak isospin opposite to that of fermion) and their super partners.

1. The simplest assumption in the stringy case is that fermion-$X_{\pm}$ pairs behave as coherent objects, that is scatter elastically. In more general case only their higher excitations identifiable in terms of stringy degrees of freedom would be created in vertices. The massivation of

these states makes possible non-collinear vertices. An open question is how the massivation fermion-$X_\pm$ pairs relates to the existing TGD based description of massivation in terms of Higgs mechanism and Kähler-Dirac operator.

2. Mass renormalization could come from self energy loops with negative energy lines as also vertex normalization. By very general arguments supersymmetry implies the cancellation of the self energy loops but would allow non-trivial vertex renormalization [?] .
3. If only 3-vertices are allowed, the loops containing only positive energy lines are possible if on mass shell fermion-$X_\pm$ pair (or its superpartner) can decay to a pair of positive energy pair particles of same kind. Whether this is possible depends on the masses involved. For ordinary particles these decays are not kinematically possible below intermediate boson mass scale (the decays $F_1 \to F_2 + \gamma$ are forbidden kinematically or by the absence of flavor changing neutral currents whereas intermediate gauge bosons can decay to on mass shell fermion-anti-fermion pair).
4. The introduction of IR cutoff for 3-momentum in the rest system associated with the largest CD (causal diamond) looks natural as scale parameter of coupling constant evolution and p-adic length scale hypothesis favors the inverse of the size scale of CD coming in powers of two. This parameter would define the momentum resolution as a discrete parameter of the p-adic coupling constant evolution. This scale does not have any counterpart in standard physics. For electron, d quark, and u quark the proper time distance between the tips of CD corresponds to frequency of 10 Hz, 1280 Hz, and 160 Hz: all these frequencies define fundamental bio-rhythms [K32] .

These considerations have left completely untouched one important aspect of generalized Feynman diagrams: the necessity to perform a functional integral over the deformations of the partonic 2-surfaces at the ends of the lines- that is integration over WCW. Number theoretical universality requires that WCW and these integrals make sense also p-adically and in the following these aspects of generalized Feynman diagrams are discussed.

5.7.3 Harmonic Analysis In WCW As a way To Calculate WCWFunctional Integrals

Previous examples suggest that symmetric space property, Kähler and symplectic structure and the use of symplectic coordinates consisting of canonically conjugate pairs of phase angles and corresponding "radial" coordinates are essential for WCW integration and p-adicization. Kähler function, the components of the metric, and therefore also metric determinant and Kähler function depend on the "radial" coordinates only and the possible generalization involves the identification the counterparts of the "radial" coordinates in the case of WCW.

Conditions guaranteeing the reduction to harmonic analysis

The basic idea is that harmonic analysis in symmetric space allows to calculate the functional integral over WCW.

1. Each propagator line corresponds to a symmetric space defined as a coset space G/H of the symplectic group and Kac-Moody group and one might hope that the proposed p-adicization works for it- at least when one considers the hierarchy of measurement resolutions forced by the finiteness of algebraic extensions. This coset space is as a manifold Cartesian product $(G/H) \times (G/H)$ of symmetric spaces G/H associated with ends of the line. Kähler metric contains also an interaction term between the factors of the Cartesian product so that Kähler function can be said to reduce to a sum of "kinetic" terms and interaction term.
2. Effective 2-dimensionality and ZEO allow to treat the ends of the propagator line independently. This means an enormous simplification. Each line contributes besides propagator a piece to the exponent of Kähler action identifiable as interaction term in action and depending on the propagator momentum. This contribution should be expressible in terms of generalized spherical harmonics. Essentially a sum over the products of pairs of harmonics associated with the ends of the line multiplied by coefficients analogous to $1/(p^2 - m^2)$ in the case of the ordinary propagator would be in question. The optimal situation is that the pairs

are harmonics and their conjugates appear so that one has invariance under G analogous to momentum conservation for the lines of ordinary Feynman diagrams.

3. Momentum conservation correlates the eigenvalue spectra of the Kähler-Dirac operator D at propagator lines [K104] . G-invariance at vertex dictates the vertex as the singlet part of the product of WCW harmonics associated with the vertex and one sums over the harmonics for each internal line. p-Adicization means only the algebraic continuation to real formulas to p-adic context.

4. The exponent of Kähler function depends on both ends of the line and this means that the geometries at the ends are correlated in the sense that Kähler form contains interaction terms between the line ends. It is however not quite clear whether it contains separate "kinetic" or self interaction terms assignable to the line ends. For Kähler function the kinetic and interaction terms should have the following general expressions as functions of complex WCW coordinates:

$$\begin{aligned} K_{kin,i} &= \sum_n f_{i,n}(Z_i)\overline{f_{i,n}(Z_i)} + c.c \ , \\ K_{int} &= \sum_n g_{1,n}(Z_1)\overline{g_{2,n}(Z_2)} + c.c \ , i = 1,2 \ . \end{aligned} \quad (5.7.2)$$

Here $K_{kin,i}$ define "kinetic" terms and K_{int} defines interaction term. One would have what might be called holomorphic factorization suggesting a connection with conformal field theories.

Symmetric space property -that is isometry invariance- suggests that one has

$$f_{i,n} = f_{2,n} \equiv f_n \ , \ g_{1,n} = g_{2,n} \equiv g_n \quad (5.7.3)$$

such that the products are invariant under the group H appearing in G/H and therefore have opposite H quantum numbers. The exponent of Kähler function does not factorize although the terms in its Taylor expansion factorize to products whose factors are products of holomorphic and antiholomorphic functions.

5. If one assumes that the exponent of Kähler function reduces to a product of eigenvalues of the Kähler-Dirac operator eigenvalues must have the decomposition

$$\lambda_k = \prod_{i=1,2} exp\left[\sum_n c_{k,n} g_n(Z_i)\overline{g_n(Z_i)} + c.c\right] \times exp\left[\sum_n d_{k,n} g_n(Z_1)\overline{g_n(Z_2)} + c.c\right] \quad (5.7.4)$$

Hence also the eigenvalues coming from the Dirac propagators have also expansion in terms of G/H harmonics so that in principle WCW integration would reduce to Fourier analysis in symmetric space.

Generalization of WCW Hamiltonians

This picture requires a generalization of the view about configuration space Hamiltonians since also the interaction term between the ends of the line is present not taken into account in the previous approach.

1. The proposed representation of WCW Hamiltonians as flux Hamiltonians [K25, K104]

$$\begin{aligned} Q(H_A) &= \int H_A(1+K)Jd^2x \ , \\ J &= \epsilon^{\alpha\beta}J_{\alpha\beta} \ , \ J^{03}\sqrt{g_4} = KJ_{12} \ . \end{aligned} \quad (5.7.5)$$

works for the kinetic terms only since J cannot be the same at the ends of the line. The formula defining K assumes weak form of self-duality (^{03} refers to the coordinates in the complement of X^2 tangent plane in the 4-D tangent plane). K is assumed to be symplectic invariant and constant for given X^2. The condition that the flux of $F^{03} = (\hbar/g_K)J^{03}$ defining the counterpart of Kähler electric field equals to the Kähler charge g_K gives the condition $K = g_K^2/\hbar$, where g_K is Kähler coupling constant. Within experimental uncertainties one has $\alpha_K = g_K^/4\pi\hbar_0 = \alpha_{em} \simeq 1/137$, where α_{em} is finite structure constant in electron length scale and $\hbar_0$ is the standard value of Planck constant.

The assumption that Poisson bracket of WCW Hamiltonians reduces to the level of embedding space - in other words $\{Q(H_A), Q(H_B\} = Q(\{H_A, H_B\})$ - can be justified. One starts from the representation in terms of say flux Hamiltonians $Q(H_A)$ and *defines* $J_{A,B}$ as $J_{A,B} \equiv Q(\{H_A, H_B\})$. One has $\partial H_A/\partial t_B = \{H_B, H_A\}$, where t_B is the parameter associated with the exponentiation of H_B. The inverse J^{AB} of $J_{A,B} = \partial H_B/\partial t_A$ is expressible as $J^{A,B} = \partial t_A/\partial H_B$. From these formulas one can deduce by using chain rule that the bracket $\{Q(H_A), Q(H_B\} = \partial t_C Q(H_A) J^{CD} \partial t_D Q(H_B)$ of flux Hamiltonians equals to the flux Hamiltonian $Q(\{H_A, H_B\})$.

2. One should be able to assign to WCW Hamiltonians also a part corresponding to the interaction term. The symplectic conjugation associated with the interaction term permutes the WCW coordinates assignable to the ends of the line. One should reduce this apparently nonlocal symplectic conjugation (if one thinks the ends of line as separate objects) to a non-local symplectic conjugation for $\delta CD \times CP_2$ by identifying the points of lower and upper end of CD related by time reflection and assuming that conjugation corresponds to time reflection. Formally this gives a well defined generalization of the local Poisson brackets between time reflected points at the boundaries of CD. The connection of Hermitian conjugation and time reflection in quantum field theories is is in accordance with this picture.

3. The only manner to proceed is to assign to the flux Hamiltonian also a part obtained by the replacement of the flux integral over X^2 with an integral over the projection of X^2 to a sphere S^2 assignable to the light-cone boundary or to a geodesic sphere of CP_2, which come as two varieties corresponding to homologically trivial and non-trivial spheres. The projection is defined as by the geodesic line orthogonal to S^2 and going through the point of X^2. The hierarchy of Planck constants assigns to CD a preferred geodesic sphere of CP_2 as well as a unique sphere S^2 as a sphere for which the radial coordinate r_M or the light-cone boundary defined uniquely is constant: this radial coordinate corresponds to spherical coordinate in the rest system defined by the time-like vector connecting the tips of CD. Either spheres or possibly both of them could be relevant.

Recall that also the construction of number theoretic braids and symplectic QFT [K23] led to the proposal that braid diagrams and symplectic triangulations could be defined in terms of projections of braid strands to one of these spheres. One could also consider a weakening for the condition that the points of the number theoretic braid are algebraic by requiring only that the S^2 coordinates of the projection are algebraic and that these coordinates correspond to the discretization of S^2 in terms of the phase angles associated with θ and ϕ.

This gives for the corresponding contribution of the WCW Hamiltonian the expression

$$Q(H_A)_{int} = \int_{S^2_\pm} H_A X \delta^2(s_+, s_-) d^2 s_\pm = \int_{P(X^2_+)\cap P(X^2_-)} \frac{\partial(s^1, s^2)}{\partial(x^1_\pm, x^2_\pm)} d^2 x_\pm \quad . \tag{5.7.6}$$

Here the Poisson brackets between ends of the line using the rules involve delta function $\delta^2(s_+, s_-)$ at S^2 and the resulting Hamiltonians can be expressed as a similar integral of $H_{[A,B]}$ over the upper or lower end since the integral is over the intersection of S^2 projections. The expression must vanish when the induced Kähler form vanishes for either end. This is achieved by identifying the scalar X in the following manner:

$$\begin{aligned} X &= J_+^{kl} J^-_{kl} \ , \\ J_\pm^{kl} &= (1 + K_\pm) \partial_\alpha s^k \partial_\beta s^l J_\pm^{\alpha\beta} \ . \end{aligned} \tag{5.7.7}$$

The tensors are lifts of the induced Kähler form of $X^2_\pm$ to S^2 (not CP_2).

4. One could of course ask why these Hamiltonians could not contribute also to the kinetic terms and why the brackets with flux Hamiltonians should vanish. This relate to how one *defines* the Kähler form. It was shown above that in case of flux Hamiltonians the definition of Kähler form as brackets gives the basic formula $\{Q(H_A), Q(H_B)\} = Q(\{H_A, H_B\}$ and same should hold true now. In the recent case $J_{A,B}$ would contain an interaction term defined in terms of flux Hamiltonians and the previous argument should go through also now by identifying Hamiltonians as sums of two contributions and by introducing the doubling of the coordinates t_A.
5. The quantization of the Kähler-Dirac operator must be reconsidered. It would seem that one must add to the super-Hamiltonian completely analogous term obtained by replacing $(1+K)J$ with $X\partial(s^1, s^2)/\partial(x^1_\pm, x^2_\pm)$. Besides the anti-commutation relations defining correct anti-commutators to flux Hamiltonians, one should pose anti-commutation relations consistent with the anti-commutation relations of super Hamiltonians. In these anti-commutation relations $(1+K)J\delta^2(x,y)$ would be replaced with $X\delta^2(s^+, s^-)$. This would guarantee that the oscillator operators at the ends of the line are not independent and that the resulting Hamiltonian reduces to integral over either end for $H_{[A,B]}$.
6. In the case of CP_2 the Hamiltonians generating isometries are rational functions. This should hold true also now so that p-adic variants of Hamiltonians as functions in WCW would make sense. This in turn would imply that the components of the WCW Kähler form are rational functions. Also the exponentiation of Hamiltonians make sense p-adically if one allows the exponents of group parameters to be functions $Exp_p(t)$.

Does the expansion in terms of partial harmonics converge?

The individual terms in the partial wave expansion seem to be finite but it is not at all clear whether the expansion in powers of K actually converges.

1. In the proposed scenario one performs the expansion of the vacuum functional $exp(K)$ in powers of K and therefore in negative powers of α_K. In principle an infinite number of terms can be present. This is analogous to the perturbative expansion based on using magnetic monopoles as basic objects whereas the expansion using the contravariant Kähler metric as a propagator would be in positive powers of α_K and analogous to the expansion in terms of magnetically bound states of wormhole throats with vanishing net value of magnetic charge. At this moment one can only suggest various approaches to how one could understand the situation.
2. Weak form of self-duality and magnetic confinement could change the situation. Performing the perturbation around magnetic flux tubes together with the assumed slicing of the space-time sheet by stringy world sheets and partonic 2-surfaces could mean that the perturbation corresponds to the action assignable to the electric part of Kähler form proportional to α_K by the weak self-duality. Hence by $K = 4\pi\alpha_K$ relating Kähler electric field to Kähler magnetic field the expansion would come in powers of a term containing sum of terms proportional to α_K^0 and α_K. This would leave to the scattering amplitudes the exponents of Kähler function at the maximum of Kähler function so that the non-analytic dependence on α_K would not disappear.

A further reason to be worried about is that the expansion containing infinite number of terms proportional to α_K^0 could fail to converge.

1. This could be also seen as a reason for why magnetic singlets are unavoidable except perhaps for $\hbar < \hbar_0$. By the holomorphic factorization the powers of the interaction part of Kähler action in powers of $1/\alpha_K$ would naturally correspond to increasing and opposite net values of the quantum numbers assignable to the WCW phase coordinates at the ends of the propagator line. The magnetic bound states could have similar expansion in powers of α_K as pairs of states with arbitrarily high but opposite values of quantum numbers. In the functional integral these quantum numbers would compensate each other. The functional integral would leave only an expansion containing powers of α_K starting from some finite possibly negative (unless one assumes the weak form of self-duality) power. Various gauge coupling strengths are expected to be proportional to α_K and these expansions should reduce to those in powers of α_K.

2. Since the number of terms in the fermionic propagator expansion is finite, one might hope on basis of super-symmetry that the same is true in the case of the functional integral expansion. By the holomorphic factorization the expansion in powers of K means the appearance of terms with increasingly higher quantum numbers. Quantum number conservation at vertices would leave only a finite number of terms to tree diagrams. In the case of loop diagrams pairs of particles with opposite and arbitrarily high values of quantum numbers could be generated at the vertex and magnetic confinement might be necessary to guarantee the convergence. Also super-symmetry could imply cancellations in loops.

Could one do without flux Hamiltonians?

The fact that the Kähler functions associated with the propagator lines can be regarded as interaction terms inspires the question whether the Kähler function could contain only the interaction terms so that Kähler form and Kähler metric would have components only between the ends of the lines.

1. The basic objection is that flux Hamiltonians too beautiful objects to be left without any role in the theory. One could also argue that the WCW metric would not be positive definite if only the non-diagonal interaction term is present. The simplest example is Hermitian 2×2-matrix with vanishing diagonal for which eigenvalues are real but of opposite sign.
2. One could of course argue that the expansions of $exp(K)$ and λ_k give in the general powers $(f_n\overline{f_n})^m$ analogous to diverging tadpole diagrams of quantum field theories due to local interaction vertices. These terms do not produce divergences now but the possibility that the exponential series of this kind of terms could diverge cannot be excluded. The absence of the kinetic terms would allow to get rid of these terms and might be argued to be the symmetric space counterpart for the vanishing of loops in WCW integral.
3. In ZEO this idea does not look completely non-sensical since physical states are pairs of positive and negative energy states. Note also that in quantum theory only creation operators are used to create positive energy states. The manifest non-locality of the interaction terms and absence of the counterparts of kinetic terms would provide a trivial manner to get rid of infinities due to the presence of local interactions. The safest option is however to keep both terms.

Summary

The discussion suggests that one must treat the entire Feynman graph as single geometric object with Kähler geometry in which the symmetric space is defined as product of what could be regarded as analogs of symmetric spaces with interaction terms of the metric coming from the propagator lines. The exponent of Kähler function would be the product of exponents associated with all lines and contributions to lines depend on quantum numbers (momentum and color quantum numbers) propagating in line via the coupling to the Kähler-Dirac operator. The conformal factorization would allow the reduction of integrations to Fourier analysis in symmetric space. What is of decisive importance is that the entire Feynman diagrammatics at WCW level would reduce to the construction of WCW geometry for a single propagator line as a function of quantum numbers propagating on the line.

Chapter 6

Hydrodynamics and CP_2 Geometry

6.1 Introduction

The understanding of the turbulence is a longstanding problem in hydrodynamics [B38, B34]. This problem is acute also in astrophysics [E43]. where the proper understanding of the turbulence associated with the astrophysical systems, such as the mass accretion in a binary star, is lacking. A generally accepted point of view is that Navier-Stokes equations provide a correct description of the hydrodynamics and that the problems are of purely technical nature, being analogous to the difficulties encountered in the understanding of the color confinement.

6.1.1 Basic Ideas And Concepts

TGD approach to the description of the fundamental interactions suggests a fresh approach to the basic problems of the hydrodynamics. The new physical ideas are the following ones.

The notion of topological condensate

The concept of topological condensate: the criticality of the Kähler function and topological arguments suggest that 3-space has many-sheeted, fractal like, hierarchical structure consisting of 3-surfaces with boundary, topological field quanta, condensed on larger topological field quanta. The n:th level of the topological condensate is characterized by a length scale $L(n)$ giving lower bound for the size of the topological quanta at this level.

Various gauge fluxes and gravitational flux associated with a given topological field quantum flow to the lower condensate level via # contacts near the boundaries of the topological field quanta, whose microscopic description in terms of partons is discussed in [**?**]. The outer surfaces of the macroscopic bodies are identified as the boundaries of the topological field quanta condensed in the background 3-space.

Topological field quanta are characterized by certain vacuum quantum numbers and the space-time in the astrophysical length scales corresponds to the large vacuum quantum number limit of TGD. In the present situation hydrodynamic vortex provides a good candidate for a topological field quantum condensed on the background 3-space and at a given level vortices must have size not smaller than the length scale $L(n)$. Actually this picture of the space-time requires the generalization of the ordinary hydrodynamics to a hierarchy of hydrodynamics, one for each condensate level and also the modelling of the energy transfer between various condensate levels. In this chapter only the modifications of the hydrodynamics associated with a given condensate level are considered.

The join along boundaries bond makes it possible to glue topological field quanta together to form a larger coherent quantum systems from simpler basic units. Since dissipation corresponds to a loss of the quantum coherence, the formation of the join along boundaries bonds should play a key role in the understanding of the dissipation, in particular hydrodynamic dissipation.

A concrete topological description for the dissipation is following. The basic mechanism of the dissipation at condensate level n are the inelastic collisions of the condensed topological field quanta involving the formation and splitting of the join along boundaries bonds and leading to the transfer of the kinetic energy to the kinetic energy of the topological field quanta at higher condensate level n_1 with $L(n_1) < L(n)$. Eventually the kinetic energy of the flow ends up to the atomic condensate levels, where the collisions of atoms take care of the dissipation. The modelling of this mechanism requires a model for the coupling between hydrodynamics associated with two different condensate levels.

It has much later become clear that the vision about elementary particles Euclidian space-time regions defining lines of generalized Feynman diagrams generalizes to macroscopic scales and that every macroscopic body should accompany such space-time sheet and thus in some aspects behave like elementary particle.

Long range color and electroweak gauge fields created by dark matter

TGD predicts classical long ranged color and weak forces, in particular Z^0 force. The study of the imbeddings for various metrics [K99] suggests strongly that at long length scales matter is accompanied by long range electro-weak gauge fields. For vacuum extremals em field can vanish while Z^0 field is non-vanishing: this requires that Weinberg angle satisfies $sin^2(\theta_W) = 0$ in this phase. In the astrophysical length scales Z^0 charge is proportional to the gravitational mass of the system, when Planck mass is used as unit: $Q_Z = \epsilon_1 m/m_{Pl}$, where ϵ_1 is numerical factor smaller.

Also long ranged classical W fields are possible as well as classical long ranged color fields. The proper interpretation is in terms of scaled down hierarchy of weak and color physics assignable to a hierarchy of dark matters coupling to ordinary matter only via gravitation directly. These physics manifest themselves already in nuclear physics [K84] and condensed matter physics [K33]. in particular in the physics of living matter. The appearance of classical Z^0 fields in the bio-systems could explain chirality selection in the living matter.

TGD based model for atomic nuclei predicts that nucleons are connected by color bonds connecting exotic quarks with mass of order MeV. These quarks couple to light variants of weak bosons with Compton length of order atomic radius so that the range of these exotic weak forces would be about atomic radius. These color bonds can have also net em and weak charges so that nucleus develops an anomalous weak charge. More generally, a hierarchy of scaled up variants of weak and color physics is predicted and the range 10 nm-2.5 μm containing the electron Compton lengths $L_e(k) = \sqrt{5}L(k)$ associated with four Gaussian Mersennes is especially interesting in this respect.

As a consequence, the dark matter part of condensed matter system serves as a source of Z^0 electric and magnetic fields. These fields are vacuum screened above the relevant weak length scale L_w. This means that the space-time sheets of weak bosons are of size L_w and weak gauge fluxes are not conserved in # contacts to larger space-time sheets. The outcome is randomness and loss of coherence in length scales longer than L_w.

In particular, moving matter at given dark space-time sheet creates Z^0 magnetic field

$$\nabla \times B_Z \simeq g_Z N\beta \ . \tag{6.1.1}$$

where N is the density of weak isospin of dark matter using neutrino isospin as a unit. This formula makes sense below the appropriate weak length scale determined by the mass of dark weak bosons in question. Above this length scale vacuum screening occurs. Z^0 electric field satisfies also the appropriate source equation.

Although the Z^0 fields as such are extremely weak, the topological obstructions caused by the CP_2 topology for the imbeddings of the Z^0 magnetic fields are nontrivial. CP_2 topology generates structures: the hydrodynamical flow decomposes into what could be called flux quanta of the Z^0 magnetic field. It will be later found that under rather natural assumptions the sizes of the flux quanta are indeed of the same order of magnitude as the sizes of the typical structures associated with the hydrodynamic flow. In particular, for large systems typically encountered in astrophysics, the geometry of CP_2 is bound to become important.

Around 2004 the idea about hierarchy of Planck constants explaining dark matter emerged. Since weak scale is proportional to h_{eff}, the prediction is that it could be even macroscopic for large enough value of h_{eff}. Around 2012 the realization that the modes of induced spinor field are localized to 2-D surfaces in generic case. Induced W fields and possibly also Z^0 fields vanish at these surfaces so that strong parity breaking effects are not present.

6.1.2 Z^0 Magnetic Fields And Hydrodynamics

In [K84] long ranged color and weak forces associated with the color bonds between nucleons inside atomic nuclei are proposed as an explanation for the basic properties of the ordinary liquid phase and for the anomalous characteristics of liquid water. The mathematical similarity between incompressible hydrodynamical flow and Maxwell equations for magnetic field forces to ask whether Z^0 magnetic fields created by the dark matter component of condensed matter system might provide deeper insights into the physics of hydrodynamical flow. The general study of solutions of field equations [K11] indeed leads to very general mathematical insights in this respect providing a classification of asymptotic flow patterns in terms of the dimension of CP_2 projection varying in the range $2 \leq D \leq 4$.

Z^0 magnetic fields and transition to turbulence

The concept of the Z^0 magnetic field suggests a new approach to the problem of understanding how the transition to turbulence takes place. The transition to a turbulence might be understood simply as a spontaneous Z^0 magnetization. Flow decomposes into eddies carrying a Z^0 magnetic field in the direction of the rotation axis of the eddy. Due to the viscosity, the size of the eddy grows until its size becomes critical. Vortices dissipate their energy and angular momentum by the emission of daughter vortices: the emission is a generalization of the process known as a phase slippage in super fluidity [D15]. This mechanism suggests fractal like structure for the development of the hydrodynamic turbulence. In fact, it will be found CP_2 geometry implies naturally fractal like structures [A23] and the model for the turbulence relies heavily on the assumption that the sizes of the daughter eddies are related to the size of the mother eddy by a discrete scaling transformation.

Turbulence and Z^0 magnetization

TGD suggests a first principle explanation for the occurrence of a spontaneous Z^0 (and Kähler) magnetization and therefore of turbulence. The probability of the configuration is proportional to the exponent of the Kähler function. Kähler function corresponds to the absolute minimum of the Kähler action and Kähler magnetic (electric) fields give a positive (negative) contribution to the Kähler action so that a transition to a configuration containing Kähler magnetic fields can take place provided the configuration is energetically possible and corresponds to the minimum of Kähler action.

It turns out that for a certain critical values of the flow parameters, Kähler magnetization takes place and implies the generation of the eddies and turbulence. The mechanism leading to the increase of the Kähler action is however not the generation of magnetic Kähler action but the decrease of the magnitude of the Kähler electric contribution as is understandable from the fact that Kähler magnetic fields of the flow are in general by a factor β (β is the typical flow velocity) weaker than the Kähler electric fields. The decrease of the Kähler electric contribution follows from the fact that the Kähler electric field of the vortex becomes small near the core of the vortex. It should be noticed that a similar explanation might apply to other types of phase transitions, say spontaneous magnetization.

6.1.3 Topics Of The Chapter

The topics of the chapter are following.

1. The chapter begins with an updated review of the basic aspects of the many-sheeted space-time concept.
2. Hydrodynamical and thermodynamical hierarchies associated with the p-adic length scale hierarchy are considered. A generalization of hydrodynamics to a p-adic hierarchy of hydrodynamics is performed and a mechanism of energy transfer between condensate levels is identified. Mary Selvam has found a fascinating connection between the distribution of

primes and the distribution of vortex radii in turbulent flow in atmosphere. These observations provide new insights into p-adic length scale hypothesis and suggest that TGD based generalization of Hawking-Bekenstein law holds even in macroscopic length scales and that hydrodynamical vortices behave in some aspects like elementary particles.

3. General ideas about the description of phase transitions in terms of configuration space geometry (configuration space understood as the space of 3-surfaces, the "world of classical worlds") are considered. The new element is the presence of several condensate levels.

4. Some simple cylindrically symmetric flows are studied and it is shown that the sizes of the flux structures are of a correct order of magnitude under rather natural assumptions about the vacuum parameters characterizing electrovac neutral space-time.

5. A detailed model for the generation of turbulence as a spontaneous Kähler (implying both em and Z^0 magnetization) magnetization in the case of the channel flow is discussed.

An encouraging result is the prediction for the size distribution of the vortices: the prediction is practically identical with that obtained from the model of Heisenberg but on rather different physical grounds. The model is rather insensitive to the p-adic scaling of vortices in the transition as long as it is smaller than $\lambda = 2^{-5}$. The model is also consistent with the assumption that the decay of a vortex to smaller vortices corresponds to a phase transition from a given level of dark matter hierarchy to a lower level so that the value of $\hbar$ is reduced by a factor $\lambda = v_0/n \simeq 2^{-11}/n$, $n = 1, 2, ...$ so that Compton length scales as well as sizes of vortices are reduced by this factor.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. There are concept maps about topics related to the contents of the chapter prepared using CMAP realized as html files. Links to all CMAP files can be found at `http://tgdtheory.fi/cmaphtml.html` [L7]. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L8].

6.2 Many-Sheeted Space-Time Concept

In this section the basic phenomenology related to the many-sheeted space-time concept (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) is introduced. In [**?**] a more refined and more up-to-date review of these notions relying on number theoretic vision can be found. The vision about the role of dark matter in condensed matter and living matter is summarized in [K33].

6.2.1 Basic Concepts Related To Topological Condensation And Evaporation

The most up-to-date discussion of the notions such as topological condensation and evaporation, gauge charges, transfer of gauge field between different space-time sheets, ... can be found in [**?**].

CP_2 type vacuum extremals

CP_2 type extremals behave like elementary particles (in particular, light-likeness of M^4 projection gives rise to Virasoro conditions). CP_2 type vacuum extremals have however vanishing four-momentum although they carry classical color charges. This raises the question how they can gain elementary particle quantum numbers.

In topological condensation of CP_2 type vacuum extremal a light-like causal horizon is created. Number theoretical considerations strongly suggest that the horizon carries elementary particle numbers and can be identified as a parton. The quantum numbers or parton would serve as sources of the classical gauge fields created by the causal horizon.

In topological evaporation CP_2 type vacuum extremal carrying only classical color charges is created. This would suggest that the scattering of CP_2 type vacuum extremals defines a topological quantum field theory resulting as a limit of quantum gravitation (CP_2 is gravitational instanton) and that CP_2 type extremals define the counterparts of vacuum lines appearing in the formulation of generalized Feynman diagrams.

contacts as parton pairs

The earlier view about # contacts as passive mediators of classical gauge and gravitational fluxes is not quite correct. The basic modification is due to the fact that one can assign parton or parton pair to the # contact so that it becomes a particle like entity. This means that an entire p-adic hierarchy of new physics is predicted.

1. Formally # contact can be constructed by drilling small spherical holes S^2 in the 3-surfaces involved and connecting the spherical boundaries by a tube $S^2 \times D^1$. For instance, CP_2 type extremal can be glued to space-time sheet with Minkowskian signature or space-time sheets with Minkowskian signature can be connected by # contact having Euclidian signature of the induced metric. Also more general contacts are possible since S^2 can be replaced with a 2-surface of arbitrary genus and family replication phenomenon can be interpreted in terms of the genus.

   The # contact connecting two space-time sheets with Minkowskian signature of metric is accompanied by two "elementary particle horizons", which are light-like 3-surfaces at which the induced 4-metric becomes degenerate. Since these surfaces are causal horizons, it is not clear whether # contacts can mediate classical gauge interactions. If there is an electric gauge flux associated with elementary particle horizon it tends to be either infinite by the degeneracy of the induced metric. It is not clear whether boundary conditions allow to have finite gauge fluxes of electric type. A similar difficulty is encountered when one tries to assign gravitational flux to the # contact: in this case even the existence of flux in non-singular case is far from obvious. Hence the naïve extrapolation of Newtonian picture might not be quite correct.
2. Number theoretical considerations suggests that the two light-like horizons associated with # contacts connecting space-time sheets act as dynamical units analogous to shock waves or light fronts carrying quantum numbers so that the identification as partons is natural. Quantum holography would suggest itself in the sense that the quantum numbers associated with causal horizons would determine the long range fields inside space-time sheets involved.
3. # contacts can be modeled in terms of CP_2 type extremals topologically condensed simultaneously to the two space-time sheets involved. The topological condensation of CP_2 type extremal creates only single parton and this encourages the interpretation as elementary particle. The gauge currents for CP_2 type vacuum extremals have a vanishing covariant divergence so that there are no conserved charges besides Kähler charge. Hence electro-weak gauge charges are not conserved classically in the region between causal horizons whereas color gauge charges are. This could explain the vacuum screening of electro-weak charges at space-time level. This is required since for the known solutions of field equations other than CP_2 type extremals vacuum screening does not occur.
4. In the special case space-time sheets have opposite time orientations and the causal horizons carry opposite quantum numbers (with four-momentum included) the # contact would serve the passive role of flux mediator and one could assign to the contact generalized gauge fluxes as quantum numbers associated with the causal horizons. This is the case if the contact is created from vacuum in topological condensation so that the quantum numbers associated with the horizons define naturally generalized gauge fluxes. Kind of generalized quantum dipoles living in two space-times simultaneously would be in question. # contacts in the ground state for space-time sheets with opposite time orientation can be also seen as zero energy parton-antiparton pairs bound together by a piece of CP_2 type extremal.
5. When space-time sheets have same time orientation, the two-parton state associated with the # contact has non-vanishing energy and it is not clear whether it can be stable.

$\#_B$ contacts as bound parton pairs

Besides # contacts also flux tubes (JABs, $\#_B$ contacts) are possible. They can connect outer boundaries of space-time sheets or the boundaries of small holes associated with the interiors of two space-time sheets which can have Minkowskian signature of metric and can mediate classical gauge fluxes and are excellent candidates for mediators of gauge interactions between space-time sheet glued to a larger space-time sheet by topological sum contacts and join along boundaries contacts. The size scale of the causal horizons associated with parton pairs can be arbitrary whereas the size scale of # contacts is given by CP_2 radius.

The existence of the holes for real space-time surfaces is a natural consequence of the induced gauge field concept: for sufficiently strong gauge fields the imbeddability of gauge field as an induced gauge field fails and hole in space-time appears as a consequence. The holes connected by $\#_B$ contacts obey field equations, and a good guess is that they are light-like 3-surfaces and carry parton quantum numbers. This would mean that both # and $\#_B$ contacts allow a fundamental description in terms of pair of partons.

Magnetic flux tubes provide a representative example of $\#_B$ contact. Instead of $\#_B$ contact also more descriptive terms such as join along boundaries bond (JAB), color bond, and magnetic flux tube are used. $\#_B$ contacts serve also as a space-time correlate for bound state formation and one can even consider the possibility that entanglement might have braiding of bonds defined by # contacts as a space-time correlate [K3].

The formation of join along boundaries bonds/flux tubes could become important at the quantum limit, when the thermal de Broglie wave length $\lambda_{th} = \frac{2\pi}{\sqrt{2Tm}}$ (roughly the minimal size for the p-adic 3-surface at which particle with thermal momentum $p = \sqrt{2Tm}$ can condense) is of same order of magnitude as average separation between particles. A tempting identification for the formation of the flux tubes is as Bose-Einstein condensation taking place at same temperature range.

For solids join along boundaries/flux tube binding energy E_{join} can be, at least partially, regarded as the reduction of kinetic energy resulting from the elimination of translational degrees of freedom in the join along boundaries bond. Also the de-localization energy of particles, say conduction electrons contributes to E_{join} (de-localization is made possible by the formation of bridges between p-adic blocks).

Topological condensation and evaporation

Topological condensation corresponds to a formation of # or $\#_B$ contacts between space-time sheets. Topological evaporation means the splitting of # or $\#_B$ contacts. In the case of elementary particles the process changes almost nothing since the causal horizon carrying parton quantum numbers does not disappear. The evaporated CP_2 type vacuum extremal having interpretation as a gravitational instanton can carry only color quantum numbers.

As # contact splits partons are created at the two space-time sheets involved. This process can obviously generate from vacuum space-time sheets carrying particles with opposite signs of energies and other quantum numbers. Positive energy matter and negative energy anti-matter could be thus created by the formation of # contacts with zero net quantum numbers which then split to produce pair of positive and negative energy particles at different space-time sheets having opposite time orientations. This mechanism would allow a creation of positive energy matter and negative energy antimatter with an automatic separation of matter and antimatter at space-time sheets having different time orientation. This might resolve elegantly the puzzle posed by matter-antimatter asymmetry.

The creation of # contact leads to an appearance of radial gauge field in condensate and this seems to be impossible at the limit of infinitely large space-time sheet since it involves a radical instantaneous change in field line topology. The finite size of the space-time sheet can however resolve the difficulty.

If all quantum numbers of elementary particle are expressible as gauge fluxes, the quantum numbers of topologically evaporated particles should vanish. In the case of color quantum numbers and Poincare quantum numbers there is no obvious reason why this should be the case. Despite this the cancellation of the interior quantum numbers by those at boundaries or light-like causal determinants could occur and would conform with the effective 2-dimensionality stating that quantum states are characterized by partonic boundary states associated with causal determinants. This could be also seen as a holographic duality of interior and boundary degrees of freedom [K83].

6.2.2 Can One Regard # *Resp.* $\#_B$ Contacts As Particles *Resp.* String Like Objects?

#-contacts have obvious particle like aspects identifiable as either partons or parton pairs. $\#_B$ contacts in turn behave like string like objects. Using the terminology of M-theory, $\#_B$ contacts connecting the boundaries of space-time sheets could be also seen as string like objects connecting two branes. Again the ends holes at the ends of $\#_B$ contacts carry well defined gauge charges.

contacts as particles and $\#_B$ contacts as string like objects?

The fact that # contacts correspond to parton pairs raises the hope that it is possible to apply p-adic thermodynamics to calculate the masses of # contact and perhaps even the masses of the partons. If this the case, one has an order of magnitude estimate for the first order contribution to the mass of the parton as $m \sim 1/L(p_i)$, $i = 1, 2$. It can of course happen that the first order contribution vanishes: in this case an additional factor $1/\sqrt{p_i}$ appears in the estimate and makes the mass extremely small.

For # contacts connecting space-time sheets with opposite time orientations the vanishing of the net four-momentum requires $p_1 = p_2$. According to the number theoretic considerations below it is possible to assign several p-adic primes to a given space-time sheet and the largest among them, call it p_{max}, determines the p-adic mass scale. The milder condition is that p_{max} is same for the two space-time sheets.

There are some motivations for the working hypothesis that # contacts and the ends of $\#_B$ contacts feeding the gauge fluxes to the lower condensate levels or vice versa tend to be located near the boundaries of space-time sheets. For gauge charges which are not screened by vacuum charges (em and color charges) the embedding of the gauge fields created by the interior gauge charges becomes impossible near the boundaries and the only possible manner to satisfy boundary conditions is that gauge fluxes flow to the larger space-time sheet and space-time surface becomes a vacuum extremal of the Kähler action near the boundary.

For gauge bosons the density of boundary $\#_B$ contacts should be very small in length scales, where matter is essentially neutral. For gravitational $\#_B$ contacts the situation is different. One might well argue that there is some upper bound for the gravitational flux associated with single # or $\#_B$ contact (or equivalently the gravitational mass associated with causal horizon) given by Planck mass or CP_2 mass so that the number of gravitational contacts is proportional to the mass of the system.

The TGD based explanation for Podkletnov effect [H6] is based on the assumption that magnetically charged # contacts are carries of gravitational flux equal to Planck mass and predicts effect with correct order of magnitude. The model generalizes also to the case of $\#_B$ contacts. The lower bound for the gravitational flux quantum must be rather small: the mass $1/L(p)$ determined by the p-adic prime associated with the larger space-time sheet is a first guess for the unit of flux.

Could # and $\#_B$ contacts form Bose-Einstein condensates?

The description as # contact as a parton pair suggests that it is possible to assign to # contacts inertial mass, say of order $1/L(p)$, they should be describable using d'Alembert type equation for a scalar field. # contacts couple dynamically to the geometry of the space-time since the induced metric defines the d'Alembertian. There is a mass gap and hence # contacts could form a Bose-Einstein (BE) condensate to the ground state. If # contacts are located near the boundary of the space-time surface, the d'Alembert equation would be 3-dimensional. One can also ask whether # contacts define a particular form of dark matter having only gravitational interactions with the ordinary matter.

Also the probability amplitudes for the positions of the ends of $\#_B$ contacts located at the boundary of the space-time sheet could be described using an order parameter satisfying d'Alembert equation with some mass parameter and whether the notion of Bose-Einstein condensate makes sense also now. The model for atomic nucleus assigns to the ends of the $\#_B$ contact realized as a color magnetic flux tube quark and anti-quark with mass scale given by $k = 127$ (MeV scale) [K84].

This inspires the question whether # and $\#_B$ contacts could be essential for understanding bio-systems as macroscopic quantum systems [K17]. The BE condensate associated with the # contacts behaves in many respects like super conductor: for instance, the concept of Josephson junction generalizes. As a matter fact, it seems that $\#_B$ contacts, join along boundaries, or magnetic flux tubes could indeed be a key element of not only living matter but even nuclear matter and condensed matter in TGD Universe. One application of the concept is the TGD based explanation [K106] of Comorosan1 effect [I15, I7] in terms of # contact Josephson currents appearing at molecular level.

The transfer of fields between space-time sheets and # and $\#_B$ contacts

The penetration of the external electric and magnetic fields from external world to subsystem (from larger space-time sheet to a smaller one) and vice versa must take place via the creation and re-arrangement of the # and $\#_B$ contacts and also by the generation of # and $\#_B$ contact currents. The unique coupling of the wormhole BE condensate to the geometry of the boundary of the space-time sheet together with the classical electromagnetic interaction between wormholes and electrons implies coupling between electrons and the shape and size of the 3-surface. This coupling might make it possible to understand how bio-systems are able to control their size and shape.

Exotic effects related to the many-sheeted space-time

The hopping of electrons (most probably unpaired valence electrons) from the atomic space-time sheet to non-atomic space-time sheets might be energetically favorable under some circumstances and would lead to the formation of "exotic atoms" and effective electronic alchemy since the chemical properties of the atom are presumably determined by the electronic properties of the atomic space-time sheet [K31]. The "exotic" electrons on non-atomic space-time sheets provide an ideal mechanism for energy and charge transfer since dissipative effects are small and even the temperature at these space-time sheets might be much smaller than the temperature at the atomic space-time sheet. In this respect bio-systems are especially interesting.

The interaction of the exotic electrons with the wormhole BE condensate takes place via the classical electromagnetic interaction generating excitations of the # contact BE condensate. The mechanism is completely analogous to the ordinary mechanism of super conductivity in which electromagnetic interaction of electrons with nuclei excites phonons. Since the gap energy is of order $1/L(p)$ characterizing the size of the p-adic space-time sheet, one can consider the possibility of high temperature super conductivity.

One can even consider the possibility that the presence of electrons on "wrong" space-time sheets makes if favorable for some atomic nuclei to feed their electromagnetic charges to non-atomic space-time sheets. This would in principle make possible Trojan horse mechanism of cold nuclear fusion since two nuclei feeding their electromagnetic gauge fluxes on different space-time sheets do not see the Coulomb wall [K84].

Also ions can drop to larger space-time sheets. In [K15, K16] a model of ionic high T_c super conductivity explaining certain peculiar effects of the em radiation on living matter is considered. These effects actually provide support for the view that living systems are macroscopic quantum systems.

6.2.3 Number Theoretical Considerations

Number theoretical considerations allow to develop more quantitative vision about the how p-adic length scale hypothesis relates to the ideas just described.

How to define the notion of elementary particle?

p-Adic length scale hierarchy forces to reconsider carefully also the notion of elementary particle. p-Adic mass calculations led to the idea that particle can be characterized uniquely by single p-adic prime characterizing its mass squared. It however turned out that the situation is probably not so simple.

The work with modelling dark matter suggests that particle could be characterized by a collection of p-adic primes to which one can assign weak, color, em, gravitational interactions, and possibly also other interactions. It would also seem that only the space-time sheets containing common primes in this collection can interact. This leads to the notions of relative and partial darkness. An entire hierarchy of weak and color physics such that weak bosons and gluons of given physics are characterized by a given p-adic prime p and also the fermions of this physics contain space-time sheet characterized by same p-adic prime, say M_{89} as in case of weak interactions. In this picture the decay widths of weak bosons do not pose limitations on the number of light particles if weak interactions for them are characterized by p-adic prime $p \neq M_{89}$. Same applies to color interactions.

The p-adic prime characterizing the mass of the particle would perhaps correspond to the largest p-adic prime associated with the particle. Graviton which corresponds to infinitely long ranged interactions, could correspond to the same p-adic prime or collection of them common to all particles. This might apply also to photons. Infinite range might mean that the flux tubes mediating these interactions can be arbitrarily long but their transversal sizes are characterized by the p-adic length scale in question.

The natural question is what this collection of p-adic primes characterizing particle means? The hint about the correct answer comes from the number theoretical vision, which suggests that at fundamental level the branching of boundary components to two or more components, completely analogous to the branching of line in Feynman diagram, defines vertices [K23, K86].

1. If space-time sheets correspond holographically to multi-p p-adic topology such that largest p determines the mass scale, the description of particle reactions in terms of branchings indeed makes sense. This picture allows also to understand the existence of different scaled up copies of QCD and weak physics. Multi-p p-adicity could number theoretically correspond to q-adic topology for $q = m/n$ a rational number consistent with p-adic topologies associated with prime factors of m and n ($1/p$-adic topology is homeomorphic with p-adic topology).

2. One could also imagine that different p-adic primes in the collection correspond to different space-time sheets condensed at a larger space-time sheet or boundary components of a given space-time sheet. If the boundary topologies for gauge bosons are completely mixed, as the model of hadrons forces to conclude, this picture is consistent with the topological explanation of the family replication phenomenon and the fact that only charged weak currents involve mixing of quark families. The problem is how to understand the existence of different copies of say QCD. The second difficult question is why the branching leads always to an emission of gauge boson characterized by a particular p-adic prime, say M_{89}, if this p-adic prime does not somehow characterize also the particle itself.

What effective p-adic topology really means?

The need to characterize elementary particle p-adically leads to the question what p-adic effective topology really means. p-Adic mass calculations leave actually a lot of room concerning the answer to this question.

1. The naïvest option is that each space-time sheet corresponds to single p-adic prime. A more general possibility is that the boundary components of space-time sheet correspond to different p-adic primes. This view is not favored by the view that each particle corresponds to a collection of p-adic primes each characterizing one particular interaction that the particle in question participates.

2. A more abstract possibility is that a given space-time sheet or boundary component can correspond to several p-adic primes. Indeed, a power series in powers of given integer n gives rise to a well-defined power series with respect to all prime factors of n and effective multi-p-adicity could emerge at the level of field equations in this manner.

One could say that space-time sheet or boundary component corresponds to several p-adic primes through its effective p-adic topology in a hologram like manner. This option is the most flexible one as far as physical interpretation is considered. It is also supported by the number theoretical considerations predicting the value of gravitational coupling constant [K86].

An attractive hypothesis is that only space-time sheets characterized by integers n_i having common prime factors can be connected by join along boundaries bonds and can interact by particle exchanges and that each prime p in the decomposition corresponds to a particular interaction mediated by an elementary boson characterized by this prime.

Do infinite primes code for q-adic effective space-time topologies?

Besides the hierarchy of space-time sheets, TGD predicts, or at least suggests, several hierarchies such as the hierarchy of infinite primes [K86], hierarchy of Jones inclusions [K103], hierarchy of dark matters with increasing values of $\hbar$ [K33, K31], the hierarchy of extensions of given p-adic number field, and the hierarchy of selves and quantum jumps with increasing duration with respect to geometric time. There are good reasons to expect that these hierarchies are closely related.

1. Some facts about infinite primes

The hierarchy of infinite primes can be interpreted in terms of an infinite hierarchy of second quantized super-symmetric arithmetic quantum field theories allowing a generalization to quaternionic or perhaps even octonionic context [K86]. Infinite primes, integers, and rationals have decomposition to primes of lower level.

Infinite prime has fermionic and bosonic parts having no common primes. Fermionic part is finite and corresponds to an integer containing and bosonic part is an integer multiplying the product of all primes with fermionic prime divided away. The infinite prime at the first level of hierarchy corresponds in a well defined sense a rational number $q = m/n$ defined by bosonic and fermionic integers m and n having no common prime factors.

2. Do infinite primes code for effective q-adic space-time topologies?

The most obvious question concerns the space-time interpretation of this rational number. Also the question arises about the possible relation with the integers characterizing space-time sheets having interpretation in terms of multi-p-adicity. On can assign to any rational number $q = m/n$ so called q-adic topology. This topology is not consistent with number field property like p-adic topologies. Hence the rational number q assignable to infinite prime could correspond to an effective q-adic topology.

If this interpretation is correct, arithmetic fermion and boson numbers could be coded into effective q-adic topology of the space-time sheets characterizing the non-determinism of Kähler action in the relevant length scale range. For instance, the power series of $q > 1$ in positive powers with integer coefficients in the range $[0, q)$ define q-adically converging series, which also converges with respect to the prime factors of m and can be regarded as a p-adic power series. The power series of q in negative powers define in similar converging series with respect to the prime factors of n.

I have proposed earlier that the integers defining infinite rationals and thus also the integers m and n characterizing finite rational could correspond at space-time level to particles with positive *resp.* negative time orientation with positive *resp.* negative energies. Phase conjugate laser beams would represent one example of negative energy states. With this interpretation super-symmetry exchanging the roles of m and n and thus the role of fermionic and bosonic lower level primes would correspond to a time reversal.

1. The first interpretation is that there is single q-adic space-time sheet and that positive and negative energy states correspond to primes associated with m and n respectively. Positive (negative) energy space-time sheets would thus correspond to p-adicity ($1/p$-adicity) for the field modes describing the states.

2. Second interpretation is that particle (in extremely general sense that entire universe can be regarded as a particle) corresponds to a pair of positive and negative energy space-time sheets labelled by m and n characterizing the p-adic topologies consistent with $m-$ and n-adicities. This looks natural since Universe has necessary vanishing net quantum numbers. Unless one allows the non-uniqueness due to $m/n = mr/nr$, positive and negative energy space-time sheets can be connected only by # contacts so that positive and negative energy space-time sheets cannot interact via the formation of $\#_B$ contacts and would be therefore dark matter with respect to each other.

Positive energy particles and negative energy antiparticles would also have different mass scales. If the rate for the creation of # contacts and their CP conjugates are slightly different, say due to the presence of electric components of gauge fields, matter antimatter asymmetry could be generated primordially.

k	127	131	137	139	149
L_p/m	$2.04E-12$	$8.19E-12$	$6.53E-11$	$1.31E-10$	$4.18E-9$
k	151	157	163	167	173
L_p/m	$8.33E-9$	$6.69E-8$	$5.34E-7$	$2.13E-6$	$1.71E-5$
k	179	181	191	193	
L_p/m	$1.37E-4$	$2.74E-4$	$8.85E-3$	$1.75E-2$	

Table 6.1: p-Adic length scales $L_p = 2^{k-127}L_{127}$, $p \simeq 2^k$, $L_{127} \equiv \frac{\pi\sqrt{5+Y}}{m_e}$, $Y = .0317$, k prime, possibly relevant to condensed matter physics.

These interpretations generalize to higher levels of the hierarchy. There is a homomorphism from infinite rationals to finite rationals. One can assign to a product of infinite primes the product of the corresponding rationals at the lower level and to a sum of products of infinite primes the sum of the corresponding rationals at the lower level and continue the process until one ends up with a finite rational. Same applies to infinite rationals. The resulting rational $q = m/n$ is finite and defines q-adic effective topology, which is consistent with all the effective p-adic topologies corresponding to the primes appearing in factorizations of m and n. This homomorphism is of course not 1-1.

If this picture is correct, effective p-adic topologies would appear at all levels but would be dictated by the infinite-p p-adic topology which itself could refine infinite-P p-adic topology [K86] coding information too subtle to be caught by ordinary physical measurements.

Obviously, one could assign to each elementary particle infinite prime, integer, or even rational to this a rational number $q = m/n$. q would associate with the particle q-adic topology consistent with a collection of p-adic topologies corresponding to the prime factors of m and n and characterizing the interactions that the particle can participate directly. In a very precise sense particles would represent both infinite and finite numbers.

Under what conditions space-time sheets can be connected by $\#_B$ contact?

Assume that particles are characterized by a p-adic prime determining it mass scale plus p-adic primes characterizing the gauge bosons to which they couple and assume that $\#_B$ contacts mediate gauge interactions. The question is what kind of space-time sheets can be connected by $\#_B$ contacts.

1. The first working hypothesis that comes in mind is that the p-adic primes associated with the two space-time sheets connected by $\#_B$ contact must be identical. This would require that particle is many-sheeted structure with no other than gravitational interactions between various sheets. The problem of the multi-sheeted option is that the characterization of events like electron-positron annihilation to a weak boson looks rather clumsy.
2. If the notion of multi-p p-adicity is accepted, space-time sheets are characterized by integers and the largest prime dividing the integer might characterize the mass of the particle. In this case a common prime factor p for the integers characterizing the two space-time sheets could be enough for the possibility of $\#_B$ contact and this contact would be characterized by this prime. If no common prime factors exist, only $\#$ contacts could connect the space-time sheets. This option conforms with the number theoretical vision. This option would predict that the transition to large $\hbar$ phase occurs simultaneously for all interactions.

6.2.4 Physically Interesting P-Adic Length Scales In Condensed Matter Systems

Table 6.1 lists the p-adic length scales L_p. p near prime power of 2, which might be interesting as far as condensed matter is considered. It must be emphasized that the definition of length scale is bound to contain some unknown numerical factor and numbers should not be taken too literally.

Notice that the length scales $L(137)$ and $L(139)$ are quite near to the typical atomic length scale and this suggests that the lattice structures of solid state physics might be understood in terms of structures formed by gluing together p-adic cubes with size $L(137)$ by join along boundaries bonds/flux tubes.

6.3 Hydrodynamical And Thermodynamical Hierarchies

The existence of p-adic length scale hierarchy suggests a new approach to hydrodynamics. There is hydrodynamic flow associated with each condensate level h. The particles at level are condensate blocks of the previous level having typically size $L_{upper}(k)$ larger than $L(k)$ and hydrodynamic approximation fails at this length scale. It will be found that the phenomenon of sono-luminescence can be interpreted as evidence for the hydrodynamical hierarchy. The masses of these particles are just the masses of condensate blocks. The energy dissipation at given level takes place via the collisions of condensate blocks and one can get an order of magnitude estimate for the viscosity $\nu(k)$ and other transport coefficients at level k using kinetic gas theory for condensate blocks.

There must exist also energy transfer mechanism transporting energy and angular momentum to higher condensate levels and eventually to atomic condensation level and this mechanism should be work at length scales $L < L_{upper}(k)$, at which hydrodynamic approximation fails at level k. The mechanism to be proposed is completely analogous with the penetration of magnetic fields into super conductor and should be possible in sufficiently long length scales: the convective zone of Sun provides a possible realization of the mechanism. The hierarchy means quite rich possibilities for flows: the fluid need not be in same phase at all levels, the temperatures (temperature distributions) at different levels need not be identical. The character of the flow need not be same at different levels (turbulent/ non-turbulent, rotational/irrotational, etc).

6.3.1 Dissipation By The Collisions Of Condensate Blocks

Collisions of condensate blocks at level k provide one possible dissipation mechanism and just as in molecular case the mechanism can be characterized viscosity coefficient. One can generalize kinetic gas theory estimate for the kinetic viscosity at level k in straightforward manner.

$$\begin{aligned}
\nu(k) &= \lambda(k)\beta \ , \\
\lambda(k) &= \frac{1}{N(block)\sigma(k)} \ , \\
\sigma(k) &\sim 4\pi L_{upper}^2(k) \ , \\
N(block) &\sim \frac{1}{L_{upper}(k)^3} \ , \\
\beta &\sim \beta_{th} \sim \sqrt{\frac{T(k)}{M(block)}} \ , \\
M(block) &\sim N(nucleus) M L_{upper}^3(k) \ ,
\end{aligned} \tag{6.3.1}$$

where the average velocity β is replaced with thermal velocity to obtain order of magnitude estimate. More explicitly,

$$\begin{aligned}
\nu(k) &= \sqrt{\frac{L(139)}{L_{upper}(k)}}\nu(139) \ , \\
\nu(139) &= \frac{1}{4\pi N(nucleus,139)}\sqrt{\frac{T}{M}}L(139) \ .
\end{aligned} \tag{6.3.2}$$

The order of magnitude $\nu(139)$ is roughly the same as the order of magnitude for ordinary viscosity at room temperatures determined by the size of the atom. From formula it is clear that $\nu(k)$

scales as $\sqrt{1/L(k)}$. This means that the importance of the collisions of the condensate blocks as dissipation mechanism decreases rapidly in long p-adic length scales. This does not necessarily mean the absence of dissipation since mechanisms of energy transfer between condensate levels must exist. Reynolds number criterion implies that the flow is in sufficiently long p-adic length scales always turbulent.

The collisions of the condensate block need not be elastic and the collision at level k in general involves simultaneous collisions at levels $k_1 < k$ up to atomic condensate levels so that it leads to energy dissipation at all condensate levels $k_1 \geq k$. An interesting challenge is the description of shock waves in this picture. A shock wave at level k corresponds to "traffic jam" in shock front involving the collisions of the condensate blocks at level k. This in turn is expected to lead to shock waves propagating inside condensate blocks at level $k_{prev} < k$ and so on. Shock wave hierarchy ends up to the atomic condensate level $k = 131$.

6.3.2 Energy Transfer Between Different Condensate Levels In Turbulent Flow

The model for the generation of hydrodynamic turbulence is based on the idea that hydrodynamic vortices correspond to topological field quanta, that is cylindrical 3-surfaces with finite radius carrying Kähler electric and magnetic fields. The completely new feature is the presence of ordinary or Z^0 magnetic fields determining the size of the hydrodynamic vortices. Even the Reynolds number criterion could be formulated in terms of these fields. The naïve expectation would be that the vortices could be characterized as either em or Z^0 vortices. This is actually not the case since induced gauge field concept implies that em fields are accompanied by Z^0 fields and vice versa for extremals of Kähler action. The study of the embeddings for Kähler electric and magnetic fields led to the conclusion that vorticities are specified by two frequency type parameters ω_i and by two integers n_i related to the space-time dependence of the phases of the two complex CP_2 coordinates plus and integer m: the vortices with different value of fractal quantum number m were related by a power of a discrete scaling transformation to each other. The decay of vortices to smaller vortices leading to a cascade was suggested to be the basic mechanism for the generation of turbulence. The model led to estimates for Reynolds number for the transition to turbulence in channel flow and for the exponent Δ appearing in the Fourier transform $T(k) \propto k^{\Delta}$ of the kinetic energy density of the flow. In recent context the model for the decay of vortices can be regarded as a kinetic model for the vortices of level k appearing as particles at the level k_{next}.

p-Adic picture of condensed matter suggests a considerable generalization of this model. Of course, a lot of work is needed to construct a detailed quantitative model but some general features of the model are evident.

1. The proposed cascade mechanism as such works at single condensate level for vortices having size larger than $L_{upper}(k)$: below this length scale the hydrodynamic approximation fails. The lower bound for the vortex size was assumed to be some scale not much above atomic size so that description might apply as such at condensate level $k = k_Z$.
2. The idea already due to Kolmogorov [B33] is that the generation of turbulence involves the interaction between many length scales: in turbulent situation constant power ϵ is fed to the system of size l and the rate of the energy flow between any subsequent levels in the hierarchy of length scales is constant and dissipation becomes important at the highest levels of the hierarchy, which correspond to the shortest length scales $L_0 \sim l/Re^{3/4}$ related by to the length scale of the entire flow. This idea leads directly to important dimensional estimates making possible to deduce the form of the velocity correlation function in length scales at which dissipation is not important. It is perhaps worth of recalling that the turbulence model gives slightly different value for the exponent Δ associated with the energy density.

This interaction between different scales corresponds to the decay of vortices to smaller vortices with scaled down values of the vorticity and critical radius: this picture probably still applies at single condensate level down to the vortex radii of order $L_{upper}(k)$, where the hydrodynamical approximation fails. If the size of the block is much larger than the size λ_0 of eddies important for energy dissipation (having Reynolds number of order one) collisions of the condensate blocks at level k cannot take care of energy dissipation. Using the standard order of magnitude estimate for λ_0 [B33] the criterion for dissipation via collisions to be possible reads as

$$L_{upper}(k) < \lambda_0(k) ,$$
$$\lambda_0(k) = \frac{l}{Re^{3/4}(k)} = (\frac{L(139)}{L(k)})^{3/8}\lambda_0(139) . \quad (6.3.3)$$

$\lambda_0(139)$ is roughly of same order of magnitude as the estimate based on molecular viscosity and it is clear that in long p-adic length scales the condition cannot be met. One has $\lambda_0(k) \sim 2^{-(k-139)/2}10^{-3}l$ (assuming for definiteness $R \sim 10^4$ in turbulent flow) and L is bound to be smaller than $L_{upper}(k)$ unless l is very large as compared with $L(k)$. Since constant energy dissipation is taking place there must exist some mechanism of energy and angular momentum transfer between condensate levels and this mechanism is expected to be at work below the length scales below, at which hydrodynamic approximation works.

The structure of the topological condensate suggests much more general realization for the idea about interacting length scales: besides vortices related by powers of discrete scaling transformation also different levels k of topological condensate correspond to the levels of the hierarchy. The external source of energy and angular momentum is at some level $k >> 131$ (a concrete example is provided by channel flow) and the flow of energy occurs first from large to smaller eddies at level k in accordance with the standard picture and continues to the higher level k_{prev} via some energy transfer mechanism and repeats itself at level k_{prev}.

If condensate has hierarchical structure the flow occurs in good approximation only between two subsequent condensate levels. The previous work suggests that the mechanism is based on generation of vortices at level k_{cr} and that ordinary and Z^0 magnetic fields might play key role in the mechanism. The length scale $L(k_Z)$ means clearly a borderline in the generation of turbulence. For levels with $k > k_Z$ the electro-weak gauge fields are of Z^0 and em type and there is no motion in atomic length scales. At level $k = k_Z$ the motion is transferred to atomic level since nuclei feed their Z^0 charges directly at $k = k_Z$ level. At levels $k > k_Z$ ordinary magnetic vortices should take the role of Z^0 and em vortices. $k = k_Z$ level is special in the sense that the entire fluid motion at length scales $k > k_Z$ is seen in the flow pattern of Z^0 # throats at this level. It should be also noticed that p-adic quantized version of hydrodynamics (whatever it might mean!) is in principle involved at level $k = k_Z$.

p-Adic TGD suggests a detailed mechanisms for the flow of energy, angular momentum and magnetic flux from level k to level k_{prev}.

1. In the simplified description there are two kind of lumps of rotational energy at level k. The rigid body rotation of the condensate blocks of level k_{prev} condensed on level k and the vortices formed by the condensate blocks, each block rotating according to the law $\beta(\rho) = K/\rho$, where K is vorticity (essentially the total angular momentum) and ρ the distance from the vortex axis. The basic energy transfer process must take place at the level of single condensate block of size not very much larger than $L(k)$ produced as the end result of the cascade process. The block is in rigid body rotation and the destruction of the super fluidity by rigid body rotation of the vessel containing super fluid suggests the mechanism. When the approximately constant magnetic field created by vortex motion at level k is sufficiently strong at the position of the block it penetrates to the level k_{prev}.

2. According to the previous proposal this mechanism is following. For $\Omega < \Omega_{crit}$ the Z^0 and/or em magnetic fields created by the rotation of the # throats on the boundary of the block at level k are those of an extended magnetic dipole: inside the vortex the field lines run in the direction of vortex. For $\Omega = \Omega_{cr}$ something very peculiar happens: the magnetic field created by the rotational flow penetrates to the higher condensate level via # contacts formed at the upper and lower end of the vortex, which behave as magnetic dipoles at levels k and k_{prev}. This means that the magnetic flux runs from the level k to k_{prev} and vice versa at the opposite ends of the vortex and the conservation of magnetic flux implies that average magnetic fluxes are identical on the two levels. The field inside the vortex cylinder disappears at level k and only the field lines of the return flux outside the vortex are preserved. Since magnetic flux and angular momentum are closely related this requires that the rotating block is set in rigid body motion with angular momentum opposite to the angular momentum of the entire block

in vortex motion. There blocks in vortex would rotate in opposite direction as compared to the vortex and angular momentum is indeed transferred from level k to k_{prev}.

3. The analogy with super conduticity/super fluidity suggests that the process cannot take place for too small value of magnetic field/rotational velocity at level k. Since the vorticity can be written as $K = \beta\rho$ the condition $K > K_{cr}$ is analogous (but not equivalent) with the Reynolds number criterion $ud > Re_{cr}\nu$. The criterion $K > K_{cr}$ translates into the condition $B > B_{cr}$. The physical content of the condition is probably the following. In the absence of the vortices liquid at level k_{prev} tends to form large join along boundaries/flux tube blocks: for dense liquids only very few large join along boundaries/flux tube block are present whereas for the gases there are only few flux tubes present. The formation of vortices splits flux tubes at the boundaries of the vortices and some energy $E_{join}(k)$ must be taken from the flow to split single flux tubes if present.

4. The criterion for the penetration of magnetic field must be local in the sense that only the energetics of a single join along boundaries bond/flux tube is involved. A natural guess is that the magnetic energy contained in the volume of the bond is larger than the binding energy of the bond: $E_B > E(join)$. Since B is proportional to the vorticity K, the criterion gives critical vorticity K_{cr}. The dependence of $E_B \propto b/L^3(k)$ with b integer implies that the dependence of E_B and $E(join)$ on $L(k)$ is same and K_{cr} does not depend on condensate level. In this case $K_{cr} < K_{Re} \equiv ud = Re \cdot \nu$ holds true unless b is very large integer of order 10^{39} and criterion is identically satisfied for turbulent flow. If b is rational number with small denominator, one has effectively $E_{join} = b/L(k)$ for the real counterpart of the energy and one obtains $K_{cr} \propto L(k)$, which is probably the correct alternative. In sufficiently long length scales (perhaps all physically interesting length scales) one has $K_{cr} > K_{Re} \equiv ud = Re \cdot \nu$, which implies a lower bound for the size of the vortices of the turbulent flow in the range $K_{Re} < K < K_{cr}$. This means that for liquids the energy transfer mechanism comes into play for very large Reynolds numbers only and should manifest itself in long (perhaps astrophysical) length scales only. For gases the situation is different since the criterion makes sense only provided the density of the join along boundaries bonds is large (incompressible flow) and in ordinary gas flow the criterion is not needed.

5. The disappearance of the vortices at the highest condensation levels can be regarded as resulting from the annihilation of magnetic monopoles associated with the upper and lower ends of the vortices. One possibility is self destruction, when the mopoles and upper and lower ends annihilate. Second possibility is the annihilation of two different vortices. At lower level the process implies the recombination of the magnetic field lines at positions of monopoles.

6. A possible astrophysical example of the proposed energy transfer process is provided by the convective zone of the Sun, where the presence of the magneticized vortex like structures of all sizes is directly visible. Only observational limitations set lower bound for the radii of the vortices. The ends of the magnetic dipoles are visible and also the recombination of field lines of magnetic fields (this can be regarded as annihilation of magnetic monopoles!) occurs frequently [E40].

7. The assumption that the flow consists of vortices carrying almost constant magnetic fields, is not necessary. What is important is the behavior of the magnetic field created by the main flow in the region of single condensate block participating in the flow. If the magnetic field does not vary much in the region of the block, the penetration can take place via the same mechanism into the block. A possible test for the proposed scenario is the flow in the external magnetic field at level $k < k_Z$: for some critical value of field (probably rather high) the flow should become turbulent. One can also consider creating external Z^0 magnetic fields in the interior of, say rotating cylinder, and finding whether they affect the properties of the non-turbulent flow inside the cylinder.

6.3.3 The Magnetic Fields Associated With Vortex And Rigid Body Flows

The magnetic field associated with vortex flow $\beta = K/\rho$ (ρ is the distance from the axis of vortex) is given by

$$
\begin{aligned}
B_C &= A_C K ln(\frac{\rho}{\rho_0}),\ C = em, Z \ , \\
A_C &= \frac{g_C q_C}{\sqrt{\epsilon_C(k)}} n(nucleus) \ , \\
Q_Z &= (A-Z)Q_Z(n) \quad Q_{em} = Z \ ,
\end{aligned} \tag{6.3.4}
$$

where ρ_0 is some finite radius at which the flow ceases to be vortex flow and is expected to change to rigid body flow (single condensate block rotates as rigid body). ϵ_C will be assumed to satisfy the simple scaling law $\epsilon_C(k) \propto L(k)^6$. The field is in good approximation constant in region of vortex so that critical field condition leading to the penetration of the field to higher level occurs almost simultaneously in vortex but proceeds from boundary to interior.

The magnetic field associated with rigid body flow $\beta = \Omega\rho$ is given by

$$
B_C = A_C \Omega \frac{\rho^2}{2} \ , \tag{6.3.5}
$$

where the parameter A_C defined in previous formula. At critical value of vortex magnetic field condensate blocks rotating in vortex flow like rigid bodies begin to rotate counterclockwise with regard to the vortex flow and the angular rotation velocity is such that

i) the magnetic fluxes created by rigid body flow and vortex flow cancel each other or

ii) angular momentum in the region of condensate block is transferred to higher condensate level.

Denoting the radius of a rigidly rotating block in the vortex flow by ρ_{rig} and by ρ_1 the distance of the block from the axis of vortex flow one obtains for the value of the angular velocity parameter Ω

$$
\begin{aligned}
\Omega &\simeq \frac{2K}{\rho_{rig}^2} X \ , \\
X &= ln(\rho_1/rho_0) \ .
\end{aligned} \tag{6.3.6}
$$

An almost identical condition

$$
\Omega \simeq \frac{2K}{\rho_{rig}^2} \ , \tag{6.3.7}
$$

is obtained if one requires that entire angular momentum of the rigidly rotating block in vortex flow is transferred to higher condensate level so that the two models are equivalent with logarithmic accuracy.

6.3.4 Criticality Condition

Consider next the criticality condition for vortex magnetic field or equivalently vorticity $K \sim ud$ to derive the analog of Reynolds number criterion $ud > Re \cdot \nu$ for single vortex. The condition states that magnetic field energy in the volume of join along boundaries bond/flux tubes is larger than flux tubeing energy E_{join}.

$$
E_B(bond) > E_{join} \ , \tag{6.3.8}
$$

to derive more quantitative criterion one must make some additional assumptions. The volume of flux tube at level k is assumed to be of order $L^3(k)$ since bond should consist of few p-adic cubes glued together along their walls. E_{join} is of form $bp^{3/2}$ p-adically. If b is integer the real counterpart of the energy behaves as $1/L(k)^3$ and if b is rational number with small denominator the real counterpart of energy behaves as $a/L(k)$, $a<1$.

The following argument suggests that b must be a genuine rational number. The radius ρ_{cr} of the condensate block determined from the imbeddability requirement of the magnetic field as induced gauge field must be equal to the radius $L_{upper}(k) \propto L(k)$ of the block determined by the stability against topological evaporation. This is possible only provided $\rho_{cr} \propto L(k)$ holds true. It will be later found that the dependence of ρ_{cr} on p-adic length scales is as follows

$$\rho_{cr} \propto \frac{\epsilon_C^{1/4}}{K^{1/2}} \propto \frac{L(k)^{3/2}}{b^{1/2}} \ . \tag{6.3.9}$$

For integer b this gives $\rho_{cr} \propto L(k)^{3/2}$ so that the critical radius is larger than $L_{upper}(k)$ at large length scales. If b is rational number one indeed has $\rho_{cr} \propto L(k)$ and ρ_{cr}. In this case both K_{cr} and ρ_{cr} are proportional to $L(k)$ as suggested by fractality.

If Z^0 magnetic fields dominate at levels $k>k_Z$ levels the condition reduces for $E_{join} = b/L(k)$ to the form

$$\begin{aligned} K &> K_{cr}(k) \ , \\ K_{cr}(Z,k) &= K_Z L(k) \ , \\ K_Z &= b^{1/2}2^{-41}\frac{\sqrt{\epsilon_Z(k_Z)}}{g_Z(A-Z)}B \ , \\ B &= \frac{\sqrt{2}}{N(nucleus,139)}\frac{1}{ln(\frac{\rho_1}{\rho_0})} \ , \end{aligned} \tag{6.3.10}$$

which gives $K_{cr}(k) \sim \sqrt{b}\cdot 5\cdot 10^{-2}L(k)$ for $\epsilon_Z(k_Z) \sim 10^{24}$. At condensate levels $k<k_Z$, where ordinary magnetic fields are in question, the condition reads

$$\begin{aligned} K_{cr}(em,k) &= K_{em}L(k) \ , \\ K_{em} &= b^{1/2}2^{12}\frac{\sqrt{\epsilon_Z(131)}}{Ze}BL(k) \ . \end{aligned} \tag{6.3.11}$$

B is given by the previous formula. This gives $K_{cr}(k) \sim \sqrt{b}10^4L(k)$ $(b<1)$. The value of $K_{Re} = Re\cdot\nu$ is of order 10^{-10} m for typical values $Re = 10^4$ and $\nu \sim 10^{-14}$ m so that K_{cr} is always larger than K_{Re} unless b is very small. This means that below the length scale $L(k_Z)$ the proposed energy transfer mechanism comes into play at very large Reynolds numbers of order

$$Re \sim \frac{K_{cr}(em,k)}{\nu} \sim 10^5 b^{1/2}\frac{L(k)}{L(107)} \ , \tag{6.3.12}$$

whereas for gas phase the situation is different. When $L_{upper}(k)$ is much larger than the size $L_0 \sim l/Re^{3/4}$ for dissipative eddies with $Re \sim 1$ and $K<K_{cr}$ so that the collisions of the join along boundaries blocks nor the proposed energy transfer mechanism cannot take care of the dissipation and some other mechanisms of dissipation must be active: one possibility is heating leading to the splitting of the flux tubes.

The assumption that the mechanism is at work in the convective zone of Sun gives information on the value value of the parameter b. Assuming $\beta \sim 10^{-5}$ and $L_{upper}(k) \sim 10^7$ m one obtains from $K \sim L_{upper}\beta \sim 10^2$ m. The criterion gives $b^{1/2}L(k) \leq 2\cdot 10^2$ m. An estimate for b is obtained using the relation $L_{upper}(k) \leq AL(k)$, $A \sim 10^2$: for $L_{upper} \sim 10^2L(k)$ one obtains $b \sim 4\cdot 10^{-6}$. $L(k)$ and therefore b can be estimated if one has some idea about the value of B_Z: this together with estimate for K gives grasp on the value of $\epsilon_Z(k)$ and scaling law gives estimate for $L(k)$.

The condition implies that typical angular velocities Ω for rigid body rotation behave as $\Omega(k) \propto 1/L(k)$ and that average rotation velocities $\beta(k)$ are identical for all condensate levels. This implies that the frequency spectrum associated with the flow is superposition of form

$$F_{tot}(\omega) = \sum_{k\ prime} a_k F(\omega \frac{L(k_0)}{L(k)}) \ , \tag{6.3.13}$$

and the general form of the spectrum in principle provides a test for p-adic length scale hypothesis. $\beta(k) = constant$ suggests that spatial correlation function for velocity is constant and its Fourier spectrum corresponds to white noise spectrum.

For completeness it is useful to give the values of K_{cr} also for the $E_{join} = bL_0^2/L^3(k)$ ($L_0 \sim 10^4\sqrt{G}$ being the fundamental p-adic length scale) case.

$$K_{cr}(C) = k_C L_0 \ . \tag{6.3.14}$$

The only difference with respect to previous formulas is the replacement $L(k) \rightarrow L_0$. For small values of b the condition is automatically satisfied for reasonable values of K and the sizes of vortices should have no lower bound above atomic length scales: this is not in accordance with the estimate $\lambda_0 \sim l/Re^{3/4}$ of Kolmogorov theory.

6.3.5 Sono-Luminescence, Z^0 Plasma Waves, And Hydrodynamic Hierarchy

Sono-luminescence [D5], [D5] is a peculiar phenomenon, which might provide an application for the hydrodynamical hierarchy. The radiation pressure of a resonant sound field in a liquid can trap a small gas bubble at a velocity node. At a sufficiently high sound intensity the pulsations of the bubble are large enough to prevent its contents from dissolving in the surrounding liquid. For an air bubble in water, a still further increase in intensity causes the phenomenon of sono-luminescence above certain threshold for the sound intensity. What happens is that the minimum and maximum radii of the bubble decrease at the threshold and picosecond flash of broad band light extending well into ultraviolet is emitted. Rather remarkably, the emitted frequencies are emitted simultaneously during very short time shorter than 50 picoseconds, which suggests that the mechanism involves formation of coherent states of photons. The transition is very sensitive to external parameters such as temperature and sound field amplitude.

A plausible explanation for the sono-luminescence is in terms of the heating caused by shock waves launched from the boundary of the adiabatically contracting bubble [D5], [D5] . The temperature jump across a strong shock is proportional to the square of Mach number and increases with decreasing bubble radius. After the reflection from the minimum radius $R_s(min)$ the outgoing shock moves into the gas previously heated by the incoming shock and the increase of the temperature after focusing is approximately given by $T/T_0 = M^4$, where M is Mach number at focusing and $T_0 \sim 300\ K$ is the temperature of the ambient liquid. The observed spectrum of sono-luminescence is explained as a brehmstrahlung radiation emitted by plasma at minimum temperature $T \sim 10^5\ K$. There is a fascinating possibility that sono-luminescence relates directly to the classical Z^0 force: this point is discussed in [K84].

Even standard model reproduces nicely the time development of the bubble and sono-luminescence spectrum and explains sensitivity to the external parameters [D5], [D5]. The problem is to understand how the length scales are generated and explain the jump-wise transition to sono-luminescence and the decrease of the bubble radius at sono-luminescence: ordinary hydrodynamics predicts continuous increase of the bubble radius. The length scales are the ambient radius R_0 (radius of the bubble, when gas is in pressure of 1 atm) and the minimum radius $R_s(min)$ of the shock wave determining the temperature reached in shock wave heating. Zero radius is certainly not reached since shock front is susceptible to instabilities.

Since p-adic length scale hypothesis introduces a hierarchy of hydrodynamics with each hydrodynamics characterized by a p-adic cutoff length scale there are good hopes of achieving a better understanding of these length scales in TGD. The change in bubble size in turn could be understood as a change in the "primary" condensation level of the bubble.

1. The bubble of air is characterized by its primary condensation level k. The minimum size of the bubble at level k must be larger than the p-adic length scale $L(k)$. This suggests that the transition to photo-luminescence corresponds to the change in the primary condensation level of the air bubble. In the absence of photo-luminescence the level can be assumed to be $k = 163$ with $L(163) \sim .76\ \mu m$ in accordance with the fact that the minimum bubble radius is above $L(163)$. After the transition the primary condensation level of the air bubble is $k = 157$ with $L(157) \sim .07\ \mu m$. In the transition the minimum radius of the bubble decreases below $L(163)$ but should not decrease below $L(157)$: this hypothesis is consistent with the experimental data [D5] , [D5].
2. The particles of hydrodynamics at level k have minimum size $L(k_{prev})$. For $k = 163$ one has $k_{prev} = 157$ and for $k = 157$ $k_{prev} = 151$ with $L(151) \sim 11.8\ nm$. It is natural to assume that the minimum size of the particle at level k gives also the minimum radius for the spherical shock wave since hydrodynamic approximation fails below this length scale. This means that the minimum radius of the shock wave decreases from $R_s(min, 163) = L(157)$ to $R_s(min, 157) = L(151)$ in the transition to sono-luminescence. The resulting minimum radius is 11 nm and much smaller than the radius .1 $\mu\ m$ needed to explain the observed radiation if it is emitted by plasma.

A quantitative estimate goes along lines described in [D5], [D5].

1. The radius of the spherical shock is given by

$$R_s = At^\alpha \ , \tag{6.3.15}$$

where t is the time to the moment of focusing and α depends on the equation of state (for water one has $\alpha \sim .7$).
2. The collapse rate of the adiabatically compressing bubble obeys

$$\frac{dR}{dt} = c_0(\frac{2}{3\gamma}\frac{\rho_0}{\rho}(\frac{R_m}{R_0})^3)^{1/2} \ , \tag{6.3.16}$$

where c_0 is the sound velocity in gas, γ is the heat capacity ratio and ρ_0/ρ is the ratio of densities of the ambient gas and the liquid.
3. Assuming that the shock is moving with velocity c_0 of sound in gas, when the radius of the bubble is equal to the ambient radius R_0 one obtains from previous equations for the Mach number M and for the radius of the shock wave

$$\begin{aligned} M &= \frac{\frac{dR_s}{dt}}{c_0} = (t_0/t)^{\alpha-1} \ , \\ R_s &= R_0(t/t_0)^\alpha \ , \\ t_0 &= \frac{\alpha R_0}{c_0} \ . \end{aligned} \tag{6.3.17}$$

where t_0 is the time that elapses between the moment, when the bubble radius is R_0 and the instant, when the shock would focus to zero radius in the ideal case. For $R_0 = L(167)$ (order of magnitude is this) and for $R_s(min) = L(151)$ one obtains $R_0/R_s(min) = 256$ and $M \simeq 10.8$ at the minimum shock radius.
4. The increase of the temperature immediately after the focusing is approximately given by

$$\frac{T}{T_0} \simeq M^4 = (\frac{R_0}{R_s})^{\frac{4(1-\alpha)}{\alpha}} \simeq 1.3 \cdot 10^4 \ . \tag{6.3.18}$$

For $T_0 = 300\ K$ this gives $T \simeq 4 \cdot 10^6\ K$: the temperature is far below the temperature needed for fusion.

In principle the further increase of the temperature can lead to further transitions. The next transition would correspond to the transition $k = 157 \rightarrow k = 151$ with the minimum size of particle changing as $L(k_{prev}) \rightarrow L(149)$. The next transition corresponds to the transition to $k = 149$ and $L(k_{prev}) \rightarrow L(141)$. The values of the temperatures reached depend on the ratio of the ambient size R_0 of the bubble and the minimum radius of the shock wave. The fact that R_0 is expected to be of the order of $L(k_{next})$ suggests that the temperatures achieved are not sufficiently high for nuclear fusion to take place.

6.3.6 P-Adic Length Scale Hypothesis, Hydrodynamic Turbulence, And Distribution Of Primes

The work of Indian meteorologists Mary Selvam [H1] related to the turbulent atmospheric flows provides additional very interesting insight to p-adic length scale hypothesis and suggests that n-ary p-adic length scales corresponding to very large values of n are realized in hydrodynamical turbulence, and that hydrodynamical vortices could be regarded as elementary particle like objects on the space-time sheets at which they are condensed topologically.

1. The distribution of vortex sizes is same as distribution of primes

Selvam studies the distribution for the ratio $z = R/r$ of large vortex radius R to smallest vortex radius r, and finds that this distribution is the same as the distribution of primes in region of rather small primes. This could be understood if vortex radii are prime multiples of r

$$R = kr \ , \ k \ \text{prime} \ ,$$

and if each prime appears with the same probability. This assumption can be actually loosened: one can also interpret r as the p-adic length scale associated with minimum size vortex interpreted as space-time sheet. Selvam also argues that vortex dynamics has quantal features and that vortices could in some aspects be regarded as quantum objects.

2. p-Adic length scale hypothesis from elementary particle blackhole analogy

One can try to understand results on basis of the p-adic length scale hypothesis $p \simeq 2^{k^m}$, k prime, m positive integer.

1. At quantum level p-Adic length scale hypothesis follows from the generalization of Hawking-Bekenstein law for the radius of elementary particle horizon defined as the surface at which the Euclidian signature of the induced metric of the space-time sheet containing topologically condensed particle changes to Minkowskian signature of the metric in regions faraway from particle. Ordinary elementary particles corresponds to CP_2 type extremals condensed on larger space-time sheet with size of order $L_p = \sqrt{p}l$, $l \simeq 10^4$ Planck lengths. Generalized Hawking-Bekenstein law implies that the p-adic entropy of elementary particle characterized by p-adic prime p is proportional to the surface area of the elementary particle horizon. Since entropy is proportional to $log(p)$, the radius r of the elementary particle horizon satisfies $r^2 \propto log(p)$.

2. The idea is to require that the radius of the elementary particle horizon itself is m-ary p-adic length scale. For $p \simeq 2^{k^m}$ this is indeed the case if generalized Hawking-Bekenstein law holds and one has

$$r = \sqrt{k^m} \times l \ , \ k \ \text{prime} \ .$$

   For $m = 2$ one has

$$r = kl \ .$$

   This is the same law as holds true for the vortex radii except that l corresponds to Planck length scale rather than macroscopic size of the minimal vortex. Therefore a generalization replacing l with the size of the minimal vortex is needed.

3. Does generalization of Hawking-Bekenstein hold true also for vortices regarded as elementary particles?

One must be able to generalize the notion of elementary particle by allowing also larger space-time surfaces than CP_2 type extremals as models of particle and to assume that the metric of the space-time sheet at which particle is condensed has Euclidian metric signature inside the particle region, now inside the region covered by vortex.

1. A more general situation allowed by the p-adic length scale hypothesis corresponds to vortices topologically condensed at space-time sheets with size of order of n-ary p-adic length scale

$$L_p(n) = p^{n/2}L_p \ , \quad p \simeq 2^{k^m} \ .$$

In this case generalized Hawking-Bekenstein law implies that the radius of the elementary particle horizon is given by

$$r = k^m \times L \ , \quad L = \frac{n}{2} \times l \ .$$

$m = 2$ applies in the situation studied by Mary Selvam. Also the values of k can be small in this case. What is important is that the fundamental p-adic length scale l has been effectively replaced by $L = nl/2$. This is in accordance with the idea of fractality.

2. The requirement that r is also now p-adic length scale would imply that the length scale $k^m \times \frac{n}{2} \times l$ is p-adic length scale. This does not make sense except possibly as an approximation. p-Adic length scale hypothesis however suggests that the new fundamental length scale L itself is some n-ary p-adic length scale. The simplest possibility is that $n/2$ is large prime p_1 so that one has

$$n = 2p_1 \ , \quad r = p_1 l \ .$$

$L = p_1 l$ and clearly corresponds to the secondary p-adic length scale associated with p_1 satisfying itself p-adic length scale hypothesis $p_1 \simeq 2^{k_1^{m_1}}$. This assumption provides the scenario with strong predictive power since the number of the secondary p-adic length scales is not very high.

3. Does atmospheric turbulence provide a fractally scaled version of elementary particle physics?

In the length scale range between .1 meters and Earth circumference the following p-adic primes $p_1 = n/2$ are possible:

$$\begin{array}{l} p_1 \simeq 2^{k_1^{m^1}} \ , \\ k_1^{m^1} = 101, 103, 107, 109, 113, 11^2 = 121, 5^3 = 125, 127 \ . \end{array}$$

There would be only 8 minimal vortex sizes in this length scale range, which is very strong and testable prediction. What is fascinating is that these secondary length scales correspond to the p-adic primes associated with quarks, atomic nuclei, and leptons so that the physics of vortices in atmosphere might in some sense be regarded as a fractal copy of elementary particle and nuclear physics! Note that the length scale $L(n,k)$ giving the size of the space-time sheet at which vortex is condensed, is given by

$$L(n,k^2) \simeq 2^{2^{k_1-1}\times k^2} \ ,$$

and is completely super-astronomical already for small values of k.

4. Does the space-time region at which vortex is condensed have Euclidian metric signature?

What this model implies is that the induced metric at the space-time sheet at which vortex is condensed, should have Euclidian signature inside radius r. TGD indeed allows huge number of vacuum extremals with Euclidian signature: signature becomes Euclidian if the dependence of the CP_2 coordinates on M^4_+ coordinates is too fast. The simplest situation is encountered when the angle coordinate ϕ associated with CP_2 geodesic circle satisfies the condition $\phi = \omega t$, $\omega \geq 1/R$, where $2\pi R$ is the length of the CP_2 geodesic circle and t is Minkowski time coordinate. From this it is clear that time gradients must be typically larger than $1/R$, where R is CP_2 size, for Euclidization to happen. Also criticality of the preferred extremals of Kähler action (there exists infinite number of deformations with a vanishing second variation identifiable in terms of conformal symmetries) is consistent with the formation of Euclidian regions. Thus field equations support the idea that space-time sheets can contain Euclidian regions of even macroscopic size. Inside the region covered by the vortex light would not propagate at all and Euclidian regions would be in some respects analogous to black holes. Vortex space-time sheets itself would obey good old Minkowskian physics.

5. Connection with dark matter hierarchy

The remarks above were written much before the realization that TGD "predicts" a dark matter hierarchy with the values of Planck constant $\hbar(n) = \lambda^n \hbar(1)$, $\lambda = n/v_0 \simeq n \times 2^{11}$, $n = 1, 2, ...$ λ is predicted to be integer and also sub-harmonics could be allowed. This means that also the scaled up variants of the p-adic length scale hierarchy appears. For the preferred value of $\lambda \simeq 2^{11}$ precise predictions of preferred time and length scales corresponding to small values of p-adic primes follow. In particular, the TGD based interpretation [K79, K31] of Nottale's proposal [E25] for the quantization of planetary orbits in terms of a gigantic value of gravitational Planck constant means that huge scalings are possible so that quantum effects are present in astrophysical and even cosmological length scales. The proposed picture might be consistent with this view since also $\hbar(1)$ is predicted to have a discrete spectrum varying by a factor 2.

6.3.7 Thermodynamical Hierarchy

p-Adic TGD suggests the replacement of the ordinary thermodynamic description of the condensed matter with a hierarchy of p-adic thermodynamics, one for each p-adic level. Above the p-adic length scale $L(k)$ this thermodynamics is ordinary real thermodynamics. Below the length scale $L(k)$ p-adic thermodynamics is probably needed (assuming that thermodynamic description makes sense at all).

The general formulation might look like follows.

1. There is thermodynamics associated with each p-adic level of the condensate (in analogy with p-adic conformal field theory limit of TGD). The order parameters for ordinary condensed matter are particle densities at each level of the condensate. Besides this block densities describing the density of $p_1 < p_2$-adic blocks of matter at level $p_2 > p_1$ are present. Join along boundaries bonds/flux tubes give rise to bound state formation and corresponding densities can also be present. In spin systems also block densities for spin are present and can be identified as densities for magnetic domains with preferred sizes given by the p-adic cutoff length scales $L(k)$ given by prime powers of two.
2. The basic variational principle is the absolute minimization of free energy subject to certain constraints such as the constraint fixing total pressure: absolute minimization would be in accordance with the absolute minimization of Kähler action (only one candidate for the characterization of preferred extremals) and implies the so called Maxwell rule for phase transitions. Free energy contains three parts: the "ordinary" free energy F_o, TGD based contribution to free energy and constraint term

$$F = F_{TGD} + F_o + F_{constraint} \ . \tag{6.3.19}$$

The "ordinary" free energy F_o at level p is sum of single particle free energies for p_1-adic blocks with $p_1 < p$ and the block-block interaction energies plus higher order interaction energies

$$F_o = \sum_i F_i + \sum_{ij} F_{ij} + ... \ . \tag{6.3.20}$$

The index $p_1 \simeq 2^k$, k prime, labelling different p_1-blocks is included in the index i. Ordinary thermodynamics suggests general forms for these terms. By fractality the various parameters appearing in free energies associated with different p-adic levels should be related by simple scaling laws. For instance, van der Waals type form should be appropriate for the free energy associated with a given block density of fluid at a given level of condensate. Also the general form for the block-block interaction terms can be guessed on general grounds.

The free energy has the general form

$$F_{TGD} = \sum_i N_i(-E_{cond} - E_{join}) + \sum_{ij} E^{ij}_{int} + F_{gr} \ . \tag{6.3.21}$$

The energy decomposes into a sum of the condensation energies $E_{cond} = \frac{b(k)}{L(k)}$ and join along boundaries/flux tube binding energies E_{join} for blocks and of Kähler interaction energy and gravitational binding energy. According to the previous arguments, gravitational binding energy becomes important only in length scales $L(k) > \frac{1}{T}$. Depending on whether the condensate level is of electromagnetic or Z^0 type Kähler interaction energy corresponds either to electromagnetic or Z^0 Coulomb energy. Also magnetic interaction energies are possible. The general order of magnitude estimate for Kähler interaction energy is obtained if one accepts the previously proposed general picture of the electromagnetically neutral topological condensate.

One can understand these terms as coming from the Boltzmann weight $exp(E_{cond} + E_{join} + E_{gr} - E_K)$ appearing in the partition function associated with p: th level of the condensate. Kähler interaction energy is actually thermal average of the Kähler interaction energy and contains small temperature dependence. Due to its smallness it seems however safe to neglect this dependence. There is also a second reason for separating the ordinary contributions and those present only in TGD framework. Ordinary free energy is related to short range interactions and is not sensitive to the finite size of the p-adic surface whereas Kähler interaction energy corresponds to long range interaction and depends strongly on the size of the p-adic surface.

Besides these terms also Lagrange multiplier terms, such as a term

$$F_{const} = \lambda(p_{ext} - \frac{\partial F}{\partial V}) \ . \tag{6.3.22}$$

fixing the pressure to the external pressure at the highest level of the condensate, are present. The condensation level at which the constraint term appears corresponds naturally to the length scale $L(k) \sim \frac{1}{T}$ determined by the temperature: above this length scales gravitational interaction dominates. At the lower levels of the condensate this kind of pressure term is not present and the minimization of free energy fixes completely the various densities at these levels of the condensate. The important consequence is that the density of say, fluid, at short length scales should be fixed completely by the minimization conditions and should not depend on the external pressure at all. The external pressure changes the density of blocks but the not the density inside blocks. An exception is provided by solid phases, for which join along boundaries implies the formation of lattice so that only single block density is present for an ideal solid.

At high temperatures and in long length scales Kähler interaction energy and condensation energy are completely negligible in general. At low temperatures and short length scales as well as in critical systems the situation is different. The formation of supra phases and also of ordinary solids by join along boundaries operation provide examples of the situation, where the Kähler energy probably must be taken into account.

6.4 WCW Geometry And Phase Transitions

The definition of the WCW Kähler geometry has beautiful catastrophe theoretic interpretation. As a matter fact, catastrophe theory enters at two levels. First, Kähler function $K(X^3)$ is defined as the preferred extremal of Kähler action and associates a unique space-time surface $X^4(X^3)$ to a given 3-surface X^3. It can quite well happen that the preferred extremal of the Kähler action as a function of the varied parameters changes in discontinuous manner. Secondly, "quantum average effective space-times" correspond to the preferred extremals $X^4(X^3_{max})$ associated with the maxima of Kähler function as function of 3-surface and has so called zero modes as external control parameters and also now catastrophes are possible.

6.4.1 Basic Ideas Of The Catastrophe Theory

To understand the connection consider first the definition of the ordinary catastrophe theory [A29]. In catastrophe theory one considers the extrema of a potential function depending on dynamical variables x as function of external parameters c. The basic space decomposes locally into cartesian product $E = C \times X$ of control variables c, appearing as parameters in the potential function $V(c, x)$ and of state variables x appearing as dynamical variables. Equilibrium states of the system correspond to the extrema of the potential $V(x, c)$ with respect to the variables x and in the absence of symmetries they form a sub-manifold of M with dimension equal to that of the parameter space C. In some regions of C there are several extrema of potential function and the extremum value of x as a function of c is multi-valued. These regions of $C \times X$ are referred to as catastrophes. The simplest example is cusp catastrophe (see **Fig. 6.1**) with two control parameters and one state variable.

In catastrophe regions the actual equilibrium state must be selected by some additional physical requirement. If system obeys flow dynamics defined by first order differential equations the catastrophic jumps take place along the folds of the cusp catastrophe (delay rule). On the other hand, the Maxwell rule obeyed by the thermodynamic phase transitions, states that the equilibrium state corresponds to the absolute minimum of the potential function and the state of the system changes in discontinuous manner along the Maxwell line in the middle between the folds of the cusp (see **Fig. 6.1**). As far as discontinuous behavior is considered fold catastrophe is the basic catastrophe: all catastrophes contain folds as there "satellites" and one aim of the catastrophe theory is to derive all possible ways for the stable organization of folds into higher catastrophes. The fundamental result of the catastrophe theory is that for dimensions d of C smaller than 5 there are only 7 basic catastrophes and polynomial potential functions provide a canonical representation for the catastrophes: fold catastrophe corresponds to a third order polynomial (in the fold the two real roots become a pair of complex conjugate roots), cusp to fourth order polynomial, etc.. The development of the fold catastrophe means that the minimum of a potential function decomposes to two minima so that previous minimum becomes local maximum.

6.4.2 WCW Geometry And Catastrophe Theory

Consider now how catastrophe theory emerges from the definition of the Kähler function. The most obvious identification for the parameter space C would be as the space of all 3-surfaces in $H = M^4_+ \times CP_2$. In order to get rid of the difficulties related to $Diff^4$ invariance one must however restrict the consideration to 3-surfaces belonging to H_a: the set of 3-surfaces of $M^4_+ \times CP_2$ with constant M^4_+ proper time coordinate. The counterpart of the total space $E = C \times X$ can be identified as the space of the solutions of the Euler Lagrange equations associated with Kähler action (one could consider all 4-surfaces but this is not necessary) and decomposes only locally into Cartesian product. Intuitively the space X corresponds to the time derivatives for the variables specifying the space X and in Hamiltonian formalism to the canonical momenta. If the initial value problem is well defined, the values of C and X coordinates specify the extremum uniquely. In TGD this is not in general true as the extremely large vacuum degeneracy of the Kähler action strongly suggests.

Potential function corresponds to the Kähler action restricted to the solution space of the Euler Lagrange equations. Catastrophe surface corresponds to the four-surfaces found by extremizing Kähler action with respect to the variables of X (time derivatives of coordinates of C specifying X^3 in H_a) keeping the variables of C specifying 3-surface X^3 fixed. Extremization with respect to time derivatives implies a phenomenon analogous to the Bohr quantization since canonical momenta cannot be chosen freely as in the ordinary initial value problems of the classical physics. When catastrophe occurs there are several extremizing 4-surfaces going through the given 3-surface: otherwise one obtains just the sought for preferred extremal.

The requirement that Kähler function (Kähler action in Euclidian space-time regions) corresponds to absolute minimum is just Maxwell's rule in infinite dimensional context and implies that phase transition type catastrophic quantum jumps are typical for TGD Universe. Cusp catastrophe provides a simple concretization of the situation (see **Fig. 6.1**) The set M ("Maxwell set")of the critical 3-surfaces corresponds to the Maxwell line of the cusp catastrophe and forms codimension one set in configuration space. For 3-surfaces near to the Maxwell set M small one parameter deformation in the direction normal to it can induce large deformation of the 4-surface associated with it. This implies initial value sensitivity with respect to the coordinate X_n associated with the normal direction. Kähler function itself is continuous on Maxwell surface and mathematical consistency requires that also Kähler metric is continuous on Maxwell surface. A good example of a catastrophic jump is provided by a topology changing quantum jump (3-surface decays to two disjoint 3-surfaces) identifiable as 3-particle vertex.

The present situation differs from the ordinary catastrophe theory in several respects.

1. The parameter space C is infinite dimensional so that there seems to be no hope of having finite classification for catastrophes in TGD Universe. Of course, all lower dimensional catastrophes are expected to be present in TGD, too.
2. Kähler action possesses vacuum degeneracy and one cannot exclude the possibility that the absolute minima of the Kähler action are degenerate: this implies further modifications to the standard picture of catastrophe theory.
3. Maxwell rule follows as a theorem in Quantum TGD whereas in ordinary catastrophe theory delay rule (jumps takes place along the folds) follows as a theorem. The latter implies that the description of phase transitions is not possible using the catastrophe theory associated with flows. These observations suggests that classical dynamics (for instance the classical dynamics associated with Kähler action) obeys delay rule whereas quantum dynamics obeys Maxwell rule and that the phenomena of super cooling and super heating are related to classical dynamics and ordinary phase transitions are induced by quantum fluctuations.

The existence of the catastrophes is implied by the vacuum degeneracy of the Kähler action. For example, for pieces of Minkowski space in $M^4_+ \times CP_2$ the second variation of the Kähler action vanishes identically and only the fourth variation is non-vanishing: these 4-surfaces correspond to the tip of the cusp catastrophe (see **Fig. 6.1**). There are also space-time surfaces for which second variation is non-vanishing for special deformations only and a hierarchy of subsets in the space of extremal 4-surfaces with decreasing degeneracy of the second variation defines the boundaries of the projection of the catastrophe surface to the space of 3-surfaces. By p-adic fractality there are good reasons to expect that there are catastrophes in all length scales so that the increase in p-adic resolution leads to emergence of new smaller catastrophes on a given portion of the catastrophe surface.

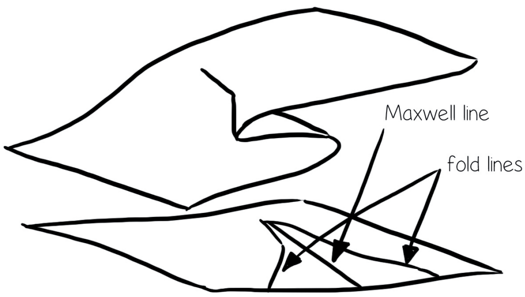

Figure 6.1: Cusp catastrophe

6.4.3 Quantum TGD And Catastrophe Theory

Catastrophes appear also in a second manner in TGD. As explained in the second part of the book, WCW allows an infinite number of zero modes. Zero modes characterize the size and shape of the 3-surface but do not appear in the line element of the configuration space metric. In good approximation WCW functional integrals associated with the S-matrix elements can in principle be calculated using perturbation theory around the maxima of the Kähler function and one can define "quantum average effective space-times" as the space-time surfaces $X^4(X^3_{max})$ associated with the maxima. Since the vacuum functional of the theory is the exponent of the Kähler function, the ill-defined Gaussian and metric determinants cancel each other and what remains is an integral over the zero modes. In general, for given values of the zero modes there are several maxima of the Kähler function and zero modes are in the role of the control parameters whereas the coordinates fixing the maximum of Kähler function for given values of the zero modes are in the role of the state variables. Also now infinite-dimensional catastrophe theory is in question.

The values of the vacuum functional at the Maxwell line of the cusp catastrophe same at the two sheets of the catastrophe but when one moves away from the Maxwell line, the second sheet begins to dominate due to the exponential dependence of the vacuum functional on Kähler function. One can also consider quantum jumps associated with the catastrophes: if the states represented by the points of the catastrophe surface are quantum entangled with the states of the external world or measurement apparatus E, one has, in the case of a cusp catastrophe, entanglement of the two sheets of the catastrophe with the states of E.

According to the strong form of Negentropy Maximization Principle, the quantum jumps selecting and of the sheets can occur when the quantum entanglement/entanglement entropy is large, actually largest in the set of all possible quantum subsystems. This is indeed the case at the Maxwell line, where the values of the Kähler function defining the entanglement probabilities at two sheets are identical so that entanglement entropy is maximized. Hence the region near the Maxwell line is predicted to be the region, where macroscopic phase transition like quantum jumps can occur and it is an intriguing possibility that thermal phase transitions basically correspond to this kind of quantum jumps. Strong form of NMP actually suggests that large number of nearly degenerate maxima must be involved so that the entanglement entropy becomes large.

6.4.4 TGD Based Description Of Phase Transitions

The above described mathematical structure should somehow reflect its presence also in the quantum description of the ordinary condensed matter phase transitions. Quantum criticality means that quantum states in TGD Universe are analogous to the states of a critical system and long range quantum correlations are predicted in all length scales. In principle, all quantum states are predicted to be critical in some time and length scale. The appearance of the join along boundaries-/flux tube condensates provides a concrete realization for quantum criticality. Spin glass analogy is in turn related to the enormous vacuum degeneracy of the Kähler action. This means the appearance of infinite number of zero modes of the Kähler function, which characterize the size and shape of the 3-surface as well as the classical induced Kähler field and play the role of universal control parameters in the catastrophe theory. Zero modes are the quantum counterparts of macroscopic state variables, to which thermodynamical variables should reduce at quantum level, and clearly they have no counterpart in the ordinary quantum field theories.

The strong form of Negentropy Maximization Principle states that the quantum jump in a given quantum state is performed by a subsystem for which the quantum jump to an eigenstate of the density matrix gives maximum negentropy gain. There are good arguments suggesting that the second law of thermodynamics follows from the strong form of Negentropy Maximization Principle [K57].

1. State function reductions increase the negentropy of the subsystem in ensemble but only the subsystem for which negentropy gain is maximal, can make the quantum jump and reduce its entanglement entropy. In order to get the possibility to make quantum jump (and be conscious according to TGD inspired theory of consciousness), the subsystem must be able to generate entanglement entropy very effectively: therefore strong NMP favors the generation of entanglement entropy and, rather paradoxically, implies both evolution and the second law of thermodynamics as different sides of the same coin.

2. The maximum for the real counterpart of the p-adic entropy is proportional to $ln(p)$ and this implies that cosmological evolution leading to the emergence of larger p-adic length scales in the topological condensate favors also the increase of the entanglement entropy.

   Hence, *if* one can indeed identify thermal entropy as an entanglement entropy, there are good hopes that second law of thermodynamics follows as a consequence.

This picture leads to a straightforward generalization of Haken's non-equilibrium thermodynamics description of the self-organizing systems [B30] with configuration zero modes appearing in the role of the order parameters and the negative of the Kähler function playing the role of the potential function. The classical dynamics given by Langevin and Focker-Planck equations is replaced with the nondeterministic dynamics defined by quantum jumps. Quantum jump can be regarded as a basic step of self-organization.

As a special case, quantum description of the thermodynamical phase transitions should emerge. Quantum entanglement of the almost degenerate configurations near Maxwell line would be the purely quantal element of the quantum theory of phase transitions. The absolute minimization of the thermodynamical free energy and Maxwell rule would basically follow from the assumption that phase transition is induced by a quantum jump selecting between various maxima of the Kähler function and from the maximization of the Kähler function plus strong form of Negentropy Maximization Principle. The super cooling and super heating effects could be interpreted as produced by classical dynamics defined by the absolute minimization of the Kähler action for which the delay rule holds true. It must be however emphasized that absolute minimization of Kähler action is only one candidate identification of preferred extremals of Kähler action.

6.5 Embeddings Of The Cylindrically Symmetric Flows

In order to find orders of magnitude for the critical radii, the embeddings of some simple cylindrically symmetric flows will be considered. It is more convenient to consider Z^0 field instead of the Kähler field: these fields are proportional to each other for electrovac space-times: $J = pZ^0/6$ ($p = sin^2(\theta_W)$).

6.5.1 The General Form Of The Embedding Of The Cylindrically Symmetric Rotational Flow

In the following the flows at condensate levels $n \geq n_Z$ will be considered so that Z^0 fields are expected to dominate over the electromagnetic fields. Since the neutrinos screening the nuclear Z^0 charge are not expected to participate in the flow, only the Z^0 charge coming from level $n-1$ contributes to the spatial components of the Z^0 gauge current density at the level n and the time like component of the current density is therefore much smaller than the spatial components. This motivates the study of the field configurations for which Z^0 electric field is negligibly small as compared to Z^0 magnetic field.

1. Theof em and Z^0 fields for vacuum extremals is given by $\gamma/Z^0 = -p/2$, $p = sin^2(\theta_W)$. Vanishing of the electromagnetic field is achieved for $p = 0$. It is indeed possible that Weinberg angle vanishes for vacuum extremals. The CP_2 projection of the embedding is two-dimensional, which implies the orthogonality of the magnetic and electric fields belonging to the same condensate level. On basis of the results of appendix Z^0 and em fields for vacuum extremals are given by

$$\begin{aligned} Z^0 &= (k+u)du \wedge d\Phi \ , \\ \gamma &= -\frac{p}{2}Z^0 \ . \end{aligned} \tag{6.5.1}$$

Here $u = cos(\Theta)$ and Φ corresponds to spherical coordinates.

2. Z^0 charge density of matter is assumed to serve as a source of Z^0 fields and in the idealization that matter consists of identical nuclei (A, Z) one can write the charge density as

$$\rho_Z = -K_Z N_n \ , \ K_Z = \frac{g_Z^2}{4\sqrt{\epsilon_Z}}\frac{N}{A} \ . \tag{6.5.2}$$

Here N_n is the density of nucleons and $N/\sqrt{\epsilon_Z}$ is the weak isospin per nucleus using neutrino isospin as a unit. ϵ_Z depends on the p-adic length scale involved and p-adic fractality suggests the scaling

$$\frac{N}{\sqrt{\epsilon_Z}} \propto N_0 \times (\frac{L(k_0)}{L(k)})^3 = N_0 \times 2^{-3(k-k_0)/2}$$

as a function of $p \simeq 2^k$. Prime values of k are favored and $k = 113, 151, 157, 163, 167$ corresponding to Mersenne primes are especially interesting.

The general situation corresponds to a flow for which the matter rotates around the z-axis with velocity $\beta(\rho)$ and creates Z^0 magnetic field in the z-direction. The Z^0 magnetic field associated with the flow at n: th condensate level is given by

$$B^Z = K_Z N_n \int \beta(\rho) d\rho \ . \tag{6.5.3}$$

The spatial dependence of the Z^0 electric field is same as that of B^Z and this means that Z^0 charge density serving as the source of E^Z cannot be constant: a possible resolution of the problem is that the screening neutrinos at level n arrange themselves so that Z^0 charge density is not constant although the nucleon density is.

Using coordinates $(r, u = cos(\Theta), \Psi, \Phi)$ for CP_2, the cylindrically symmetric electromagnetically neutral embedding of this flow is obtained in the form

$$\begin{aligned} u &= u(\rho) \ , \\ \Psi &= \omega_2 m^0 + n_2\phi \ , \quad \Phi = \omega_1 m^0 + n_1\phi \ , \end{aligned} \tag{6.5.4}$$

where the relationship between the variables r and Θ is fixed by the vacuum extremal property (see Appendix of the book). The value of the parameter k is given by $k = \omega_2/\omega_1 = n_2/n_1$.

From the general expression for the Z^0 field in the vacuum extremal space-time one obtains the following differential equation for u:

$$\begin{aligned} B^Z &= (k+u)n_1 \frac{\partial_\rho u}{\rho} \ , \\ &= K_Z N_n \int \beta(\rho) d\rho \ , \end{aligned} \tag{6.5.5}$$

which gives the relationship between u and ρ in the following form

$$\int (k+u) du = \frac{K_Z N_n}{n_1} \int d\rho\rho \int d\rho \beta(\rho) \ . \tag{6.5.6}$$

Assuming that $u = -1$ corresponds to the z-axis and the boundary of topological field quantum to $u = 1$, one obtains an expression for the critical radius:

$$\begin{aligned} \int_0^{\rho_{cr}} d\rho\rho \int \beta(\rho) d\rho &= -\frac{n_1}{K_Z N_n} \times 2k \ , \\ K_Z &= \frac{g_Z^2}{4\sqrt{\epsilon_Z}} \frac{N}{A} \end{aligned} \tag{6.5.7}$$

An attractive possibility is that the structures associated with the ordinary hydrodynamic flow might be understood as consequences of CP_2 geometry. It will be found that the order of magnitude estimates give quantitative support for this guess.

One obtains also a quantization of Z^0 magnetic flux as

$$\int B_Z da \quad = \quad 2\pi n_1 \int (k+u)du = 4\pi k n_1 \ , \tag{6.5.8}$$

What is nice is that the quantization condition eliminates the dependence of the critical radius on the poorly known vacuum quantum numbers totally. The least one can hope is that the condition fixes orders of magnitude correctly.

p-Adic length scale hypothesis suggests a simple scaling for the flow velocities guaranteeing that ρ_{cr} scales as $L(k)$. $K_Z \propto L(k)^{-3}$ scaling, which follows from the assumption that the number of dark Z^0 charges per p-adic volume does not depend on p, implies the scaling

$$\int \beta_k(\rho) d\rho \propto L(k)^{-3}$$

achieved for

$$\beta_k(\rho) \propto (\frac{\rho}{L(k)})^k L(k) \ .$$

The decay of a structure characterized by the p-adic length scale $L(k)$ to smaller structures with smaller values of k could provide a general mechanism for generating fractal structures [A23]. The model of turbulence favors the scaling $2^k = 2^5$ for the vortices in the hierarchy. This scaling could also correspond to the Mersenne prime $M_5 = 2^5 - 1 = 31$.

CP_2 topology is bound to become important for large scale flows. The central ill understood problem in the astrophysics is the understanding of the turbulence and the dissipation of the angular momentum [E43]. From the foregoing it is clear that TGD approach might provide understanding concerning several astrophysical problems [E43]. An interesting test for the ideas is the possible existence of the nested fractal structures related by discrete scale transformations.

6.5.2 Orders Of Magnitude For Some Vacuum Parameters

The space-time associated with the flow is characterized by several parameters. Besides the parameters ω_i and n_i there are integer valued parameter m and the parameter u_0. In the following estimates for the general orders of magnitude for some of these parameters will be derived.

An estimate for the parameter ϵ_Z

The requirement that gravitational interaction is stronger than Z^0 force in long length scales implies $\epsilon_Z(n \to \infty) \geq 10^{36}$. At the condensate level $n = n_Z$ at which elementary particles feed their Z^0 charges the estimate

$$\epsilon_Z \sim 10^{20} \ ,$$

holds true by the argument related to the dissipation in super fluid flow, to be developed later. For the Z^0 magnetic field at level n the $\epsilon_Z(n-1)$, rather than $\epsilon_Z(n)$, appears in the expression of B_Z (assuming that dark neutrinos do not participate in the flow) so that at level $n = n_Z$ strong B_Z fields are possible ($\epsilon_Z = 1$).

An estimate for the quantum number n_1

An essentially similar estimate have been already carried out in the previous chapter. The requirement that angular momentum density is of correct order of magnitude, gives an estimate for the value of the parameter n_1. The expression of the conserved gravitational angular momentum current in the z-direction is given by

$$J^\alpha \quad = \quad T^{\alpha\beta}_{gr} \partial_\beta m^k m_{kl} j^l \ , \tag{6.5.9}$$

where j^k denotes the vector field associated with the infinitesimal rotation and $T^{\alpha\beta}_{gr}$ denotes gravitational energy momentum tensor defined by Einstein's equations. For the angular momentum density one obtains in the cylindrical M^4 coordinates for X^4 the expression

$$J^t = T^{t\phi}_{gr}\rho^2 \ . \tag{6.5.10}$$

The leading order contribution to the angular momentum density comes from the non-vanishing of the metric component

$$\begin{aligned} g_{t\phi} &= s^{eff}_{\Phi\Phi}\omega_1 n_1 = -\frac{R^2}{4}X\times\left[(1-X)(k+u)^2+1-u^2\right]\omega_1 n_1 \ , \\ X &= D|k+u| \ , \ D=\frac{r_0^2}{1+r_0^2}\times\frac{1}{k+u_0} \ , \ r_0=r(u_0) \ , \end{aligned} \tag{6.5.11}$$

and one obtains the order of magnitude estimate

$$J^t \simeq -T^{tt}_{gr}g_{t\phi}\rho^2 \simeq \rho_m\frac{R^2}{4}\omega_1 n_1 \ . \tag{6.5.12}$$

In order to obtain a correct order of magnitude for the angular momentum density associated with the rotational flow one must have

$$\frac{R^2}{4}\omega_1 n_1 \simeq \rho\beta(\rho) \ , \tag{6.5.13}$$

which implies

$$n_1 \simeq \frac{L}{R}\beta \ , \tag{6.5.14}$$

where L and β are the typical scale and velocity associated with the flow. It is clear that n_1 is an enormous number: essentially the size of the rotational flow measured using CP_2 length as a unit.

Estimate for ω_2, n_2 and m

The values of the parameters ω_2 and n_2 and m remain free and the attractive possibility is that the value of the parameter n_2 is small, perhaps of the order of one. If this the case then the value of the parameter ω_2 is also small

$$\frac{\omega_2}{\omega_1} = \frac{n_2}{n_1} \simeq \frac{R}{L}\frac{n_2}{\beta} \ . \tag{6.5.15}$$

The first guess is that at microscopic scales the order of magnitude for ω_2 corresponds to the p-adic lengths scale of dark matter particles in question and ω_1 is of order CP_2 mass as the embeddings of Schwartchild metric as a vacuum extremal suggest [K99]. $\omega_2 \sim m_e$ gives $n_2 \sim 10^{-19}(L/R)\beta$. For $L = 0.1$ meters and $\beta \simeq 10^{-8}$ one would have $n_2 \sim 10^6$.

6.5.3 Critical Radii For Some Special Flows

In order to get concrete picture of the situation it is useful to calculate the critical radius for some special flows.

Vortex flow

The velocity field is irrotational except on the z-axis and velocity and Z^0 magnetic fields are given by

$$\begin{aligned} \beta &= \frac{K}{\rho} \ , \\ B^Z &= K_Z N_n K ln(\frac{\rho}{\rho_0}) \ . \end{aligned} \tag{6.5.16}$$

Assuming that $r = 0$ on the z-axis, one obtains for the critical radius the equation

$$\rho_{cr}^2 (ln(\frac{\rho_{cr}}{\rho_0}) - \frac{1}{2}) = -\frac{2n_1 k}{K_Z N_n K} \ . \tag{6.5.17}$$

To a logarithmic accuracy, this gives the order of magnitude estimate

$$\rho_{cr} = 2\sqrt{\frac{n_1 k}{K_Z N_n K}} \ . \tag{6.5.18}$$

The size of the critical radius decreases as the vorticity K increases.

Rigid body flow

Velocity field and Z^0 magnetic field are given by

$$\begin{aligned} \beta &= \Omega\rho \ , \\ B_Z &= K_Z N \Omega \frac{\rho^2}{2} \ . \end{aligned} \tag{6.5.19}$$

The value of the critical radius is given by the condition

$$\rho_{cr} = (\frac{16 k n_1}{K_Z N_n \Omega})^{1/4} \ , \tag{6.5.20}$$

for the quantized Z^0 magnetic flux.

6.6 Transition To The Turbulence In Channel Flow

In sequel a general model for the transition to turbulent flow is proposed. In order to see whether the proposed scenario has anything to do with the reality it is useful to look whether one can understand the generation of a turbulence in some simple situation, which is chosen to be channel flow. The consideration is restricted to the length scales $L > \xi$ so that Z^0 magnetization should play key role in the generation of turbulence if the proposed general model is correct.

6.6.1 Transition To The Turbulence

In the following a general model for the transition to turbulent flow below length scale identifiable as a weak length scale characterizing dark weak bosons L_w associated with the largest vortices. Similar model might apply also in the case of magnetic fields. In the following the phrase "Kähler field" refers either to the ordinary electromagnetic field or Z^0 field or possibly to their linear combination depending on the context.

1. The probability of the configuration is proportional to the exponent of the Kähler function so that the most probable configurations correspond to a large value of the Kähler action. Kähler action can be increased by making either the magnitude of the Kähler electric part smaller or the magnitude of the Kähler magnetic part larger. The first mechanism is expected to be at work at non-relativistic velocities since the ratio of the Kähler magnetic and electric contributions to the Kähler action is expected to be of the order of β^2, where β is typical flow velocity. The transition to configuration with larger Kähler action is expected to take place provided it is energetically possible and is consistent with the minimization of the Kähler action.

2. Spontaneous Kähler magnetization provides the means to generate a positive action. The Kähler action of the Kähler magnetized space-time domain should be larger than that associated with the same domain without magnetization. It turns out that the Kähler electric action associated to a vortex region moving with the fluid has smaller magnitude than that associated with the same volume of the original flow: the reason is that Kähler electric field associated with the vortex is small near the core of the vortex. CP_2 geometry implies that the stable domains of the Kähler magnetization have some finite critical size. Kähler magnetized domains correspond to vortices and due to the viscosity, vortices grow until their achieve a critical size.

3. Vortex must get somehow rid of its angular momentum and kinetic energy and the topological quantum numbers n_1 and n_2 must become zero. One candidate for the region, where new vortices are produced is the region near the critical radius, where the velocity gradients are large so that the viscosity plays important role. The vortices created in this region cannot however lead to a decrease of n_1 and n_2. The process leading to a decrease of n_1 and n_2 is a generalization of the process known as phase slippage in super fluidity [D12]. Daughter vortices are created at the core of the mother vortex and they propagate under the action of Magnus and friction forces to the boundary of the mother vortex and carry away the quantum numbers n_1 and n_2 of the mother vortex gradually.

   For the flow $\beta = K/\rho$, which is irrotational outside the symmetry axis, which actually corresponds to a cylindrical hole of finite radius r, this hypothesis makes sense since the variation of velocity is large in normal direction in the core and dissipation rate therefore largest near the boundary of the hole. The radius r defines a natural lower bound for the sizes of vortices involved.

4. The transition to turbulence involves the generation vortices of various sizes related by scale transformations. That this is the case is suggested by the following argument. It is an empirical fact that the size of the daughter vortices is smaller than the size of mother vortex (this assumption forms the basis of Kolmogorov and Heisenberg theories of turbulence [B34]). The conservation laws of energy and angular momentum however imply that daughter vorticities cannot be larger than mother vorticity. The critical radii of the mother and daughter vortices are related by the scale transformation $\rho_{cr} \rightarrow \lambda\rho_{cr}$. λ is expected to be a negative power of 2 and it turns out that $\lambda < 2^{-5}$ is consistent with the Heisenberg's model for the generation of turbulence. In fact, a distribution $\lambda(k) = 2^{-k}$, $k \geq 5$, for vortex sizes might be allowed.

   The hypothesis that vortex decay corresponds to a decay of higher levels in the dark matter hierarchy by de-coherence such that $\hbar$ is reduced by could a factor $\lambda = v_0/n \simeq 2^{-11}/n$, $n = 1, 2, ...$, is consistent with the proposal. The decay would correspond to a decay of Bose-Einstein condensates of corresponding weak bosons to those at the lower level of darkness and thus having Compton lengths reduced by λ.

5. The transition to turbulence can be understood as a a fractal like process. In the case of the channel flow, the walls serve as sources of the mother vortices with large critical radii. These vortices in turn decay to smaller vortices. At a given condensate level the process stops, when the size of the daughter vortices is so small that the hydrodynamics approximation fails so that the radius of the smallest vortices is of same order of magnitude as the length scale $L(n)$ giving the size of smallest structures at the condensate level in question. A necessary condition for the process to occur is that the total Kähler action generated is positive. The criterion for the process to occur is that the total Kähler action associated with the cascade is positive.

It should be emphasized that the decomposition of the space-time into above described regions is very general phenomenon characteristic for TGD. It happens for a general space-time with vanishing electromagnetic fields and also for more general space-time surfaces: the condition in question might state the vanishing of the Kähler field or electromagnetic field or the proportionality of the Kähler field and electromagnetic field. This suggests rather unexpected support for the basic assumptions of TGD: many of the fractal structures encountered in Nature might be direct manifestations of CP_2 geometry!

6.6.2 Definition Of The Model

The transition to turbulence is cascade process.

1. Mother vortices having initial radius ρ_0 are created at the walls of the channel, where the velocity gradients are large and viscosity plays important role. Let ξ is the length scale above which hydrodynamic approximation works. ξ should be of the order of atomic length scale $a = 10^{-10}$ m.
   In the rest frame of the vortex the velocity field is given by
   $$\beta(\rho) = \frac{K}{\rho} . \tag{6.6.1}$$
   The sign of the vorticity is such that the formation of the vortices tends to make velocity zero at the walls of the channel.
2. Mother vortices move across the channel under the combined action of the Magnus force $F = K \times v$ and friction force and reach a critical size. Mother vortices dissipate their energy and angular momentum by the emission of daughter vortices by the phase slippage process. The critical radius of the daughter vortices is by a factor λ smaller than the critical radius of the mother vortex. The value of λ remains a parameter to be fitted.
3. The process repeats itself until the size of the daughter vortices is of the order of ξ and hydrodynamic approximation fails.

6.6.3 Estimates For The Parameters

Consider now a more quantitative definition of the process.

1. The order of magnitude estimates for the parameters $k(0) \equiv k$ and $\rho_{cr}(0) \equiv \rho_1$ are obtained in the following manner.
   i) ρ_1 should be smaller than the width of the channel for obvious geometric reasons:
   $$\rho_1 \leq d . \tag{6.6.2}$$
   This same estimate follows from the requirement that the configuration with a vortex possesses larger Kähler action than the configuration without any vortex as will be found later.
   ii) An upper bound for the vorticity K is obtained by requiring that the flow velocity at $\rho \simeq \xi$ is not larger than the thermal velocity (sound velocity could be taken as an alternative lower bound: orders of magnitude are same): $K/\xi \leq \beta_{th}$, which gives
   $$K \leq K_{max} \simeq \xi\beta_{th} . \tag{6.6.3}$$
2. A natural requirement is that the rotation velocity of the vortices at the critical radius is of the same order of magnitude as the velocity of the main flow
   $$\frac{K}{\rho_1} \simeq \beta . \tag{6.6.4}$$

This condition guarantees that the angular momentum of the vortex is of the same order of magnitude as the angular momentum for the main flow in the vortex region. Substituting this constraint and the upper bound for k to the condition $\rho_1 \leq d$ one obtains Reynolds number type criterion

$$\frac{\beta d}{\nu} \geq R_{cr} \equiv \frac{\beta_{th}\xi}{\nu} , \tag{6.6.5}$$

when the vorticity K is maximal ($K \simeq \xi$ in units $c = 1$).

3. A kinetic theory estimate for the order of magnitude estimate of the gas viscosity gives a correct order of magnitude in case of the small viscosity liquids, too and is given by

$$\nu \simeq \frac{\beta_{th}}{N\sigma} , \tag{6.6.6}$$

where N is the density of nucleons in the liquid. Typically one has $N \simeq 10^{30}/Am^3$ (A is atomic mass number) and $\sigma \sim a^2$, $a = 10^{-10}$ m is atomic cross section: $\sigma \simeq 10^{-20}$ m^2 holds true for liquids at room temperature.

4. Using the order of magnitude estimate for the kinematic viscosity ν one obtains

$$R_{cr} = \frac{\beta_{th}\xi}{\nu} \simeq N\sigma\xi \simeq Na^3\xi \sim \frac{10^4}{A} \times \frac{\xi}{a} . \tag{6.6.7}$$

For $\xi \sim a$ the value of R_{cr} is of the correct order of magnitude since the fully developed turbulence sets in at Reynolds numbers of this order of magnitude. In case of water more careful estimate using the actual value of the kinematic viscosity and thermal velocity in room temperature gives $R_{cr} \simeq 1200 - 12000$.

According to this criterion turbulence can develop also for smaller Reynolds numbers by vortices with $K \leq d \leq \xi\beta_{th}/\beta$ (as it does) but not all vorticities allowed by the velocity condition are possible. For the critical Reynolds number means that all possible vorticities allowed by $\beta(\xi) \leq \beta_{th}$ are allowed and for larger Reynolds numbers the upper limit for the size of vortices: $\rho_{cr} \leq \xi\beta_{th}/\beta \leq d$ is strictly smaller than the width of the channel.

The critical Reynolds number follows from the geometric condition $\rho_{cr} \leq d$ in the case of a channel flow. It will be later found that the same condition follows also from the requirement that the generation of the vortex increases the Kähler action so that same kind of condition is expected also in case of, say, the flow between two rotating disks.

6.6.4 Kähler Fields Associated With The Cascade Process

In the following a simple model for the Kähler electric and magnetic fields associated with the main flow and vortices will be constructed. The following simplifying assumptions about the flow are made:
i) The flow takes place in a channel of height h, width d and length L.
ii) The flow velocity β is constant throughout the channel.
iii) The main flow has a constant density $\rho_m \equiv Nm_p$, possesses kinematic viscosity ν and thermal velocity β_{th}.

Consider first the Kähler electric and magnetic fields associated with the main flow and a vortex assumed to have its axis in the z-direction.

1. When the fluid is at rest, it creates Kähler electric field, which near the symmetry axis of the flow is cylindrically symmetric for long and wide channel is in the z-direction and given by

$$\begin{aligned} E^K_\rho &= K_Z N_n \frac{\rho}{2} , \\ K_Z &= \epsilon_1 10^{-19} . \end{aligned} \tag{6.6.8}$$

Near the walls $x = d$ and $x = 0$ and far from the corners, the Kähler electric field is to a good approximation orthogonal to the wall and given by the expression

$$E_x^K = K_Z N_n (x - \frac{d}{2}) \ . \qquad (6.6.9)$$

Same applies on the walls $z = 0$ and $z = h$. The small effects caused by the density gradients on the Kähler electric field, are neglected.

2. The Kähler magnetic field near the axis of the symmetry has circles as its flow lines and the magnitude of the field is given by

$$B_\phi^K = K_Z N_n \beta \frac{\rho^2}{2} \ . \qquad (6.6.10)$$

Near the walls $x = d$ and $x = 0$ and sufficiently far from the corners the Kähler magnetic field is given by the expression

$$B_z^K = K_Z N_n \beta (x - \frac{d}{2}) \ . \qquad (6.6.11)$$

3. The Kähler magnetic field created by the locally irrotational vortex vortex is given by the expression

$$B_z^K = K_Z N_n K ln(\frac{\rho}{\rho_0}) \ . \qquad (6.6.12)$$

4. The Kähler electric field created by the vortex can be estimated by assuming the simplest possible embedding with vanishing electromagnetic fields ($\Psi = \omega_2 m^0 + n_2 \phi$ and $\Phi = \omega_1 m^0 + n_1 \phi$). The relationship between Kähler electric field and Kähler magnetic field is given by

$$\begin{aligned} E_\rho^K &= \frac{\omega_1}{n_1} B_z^K \rho \\ &= \frac{\omega_1}{n_1} K_Z N_n K ln(\frac{\rho}{\rho_0}) \simeq K_Z N_n ln(\frac{\rho}{\rho_0})\rho \ , \end{aligned} \qquad (6.6.13)$$

and apart from the logarithmic factor behaves like the field created by a constant charge density. The last estimate is obtained using the previous order of magnitude estimate for the size of the integer n_1: $n_1 \simeq K/\sqrt{G}$. From this relationship one obtains an estimate for S_B^2/S_E^2:
$S_B^2/S_E^2 \simeq K^2/\rho_{cr}^2 \ll 1$.

6.6.5 Order Of Magnitude Estimate For The Change Of The Kähler Action And Reynolds Criterion

In the following a rough order of magnitude estimate for the various contributions to Kähler action and numerical criteria for the transition to the turbulence are derived. The estimates are based on the following assumptions.

1. The Kähler fields associated with the moving vortices are obtained by Lorentz boosts leaving the Kähler action of the vortex invariant.
2. Kähler magnetic contributions to the Kähler action are neglected so that the increase of the Kähler action must result from the decrease of the magnitude of the Kähler electric part of the action. This is indeed expected to take place since the Kähler electric field of the vortex is small near the vortex core.

3. The Kähler action resulting from the interaction of the main flow and vortex is neglected. For the Kähler electric part of the action this assumption is well founded by the symmetry considerations. The Kähler electric field of the vortex is radially symmetric and in the region, where this field has a considerable magnitude, the Kähler field of the main flow is constant to a good approximation so that the integral $\int E_{vortex} \cdot E_{flow} d^4x$ vanishes to a good approximation. The corresponding magnetic interaction term can be neglected by its smallness.

As a consequence the change in the Kähler action is simply the change in the Kähler electric contribution to Kähler action, when the Kähler electric field of the main flow is replaced with the Kähler electric field of the vortex inside the space-time volume occupied by the vortex and the condition for the generation of turbulence reads as

$$\delta S_E^K = S_E^K(vortex) - S_E^K(flow) \geq 0 \ . \tag{6.6.14}$$

For the vortex of $n:th$ generation $S_E^K(n)$ has order of magnitude given by

$$\begin{aligned} S_E^K(n) &= \frac{1}{16\pi\alpha_K} \int E_n \cdot E_n d^4x \\ &\propto K_Z^2 N_n^2 (\rho_{cr}^4(n)) h \frac{\pi}{4} \tau(n) \ , \end{aligned} \tag{6.6.15}$$

where $\tau(n)$ is the average lifetime of the n: th generation vortex. The value of $S_E^K(flow)$ near the wall has order of magnitude given by the expression

$$\begin{aligned} S_E^K(flow) &= \frac{1}{16\pi\alpha_K} \int E_{flow} \cdot E_{flow} d^4x \\ &\propto K_Z^2 N_n^2 d^2 (\rho_{cr}^2(n)) h \frac{\pi}{4} \tau(n) \end{aligned} \tag{6.6.16}$$

to a logarithmic accuracy. From the condition $S^K(vortex) \geq S^K(flow)$ one obtains to the same logarithmic accuracy

$$\rho_{cr}(n) \leq d \ , \tag{6.6.17}$$

which is identical to the condition obtained by a purely geometric argument. The condition is satisfied for all vortices in the cascade if it is satisfied for the initiating vortex.

Some comments on the condition is in order.

1. The condition poses an upper bound for the vorticities of the mother vortices: $K \leq \beta d$ in addition to the bound $K\xi \leq \beta_{th}$ and implies for the vortices with the maximal vorticity the condition $\beta d/\nu \geq 2/\beta_s$ as found already earlier. This means that full turbulence becomes possible at critical Reynolds number. Partially developed turbulence is possible for smaller Reynolds numbers, too. The vortices with the largest vorticity increase Kähler action most effectively and this suggests that the ordinary dissipation for a non-turbulent flow corresponds to the formation of small mother vortices.

2. Also flows without turbulence are possible since the condition states only that the most probable flows are turbulent. This is indeed what has been observed in the case of real flows: by appropriate experimental arrangements one can hinder the development of the turbulence up to rather high Reynolds numbers.

3. The critical Reynolds number derived from the requirement of large Kähler function has a correct order of magnitude for laboratory scale flows: $R_{cr} \sim \frac{10^4}{A} \times \frac{\xi}{a}$ ($R_{cr} \sim 10^4/A$ at room temperature).

4. The result is insensitive to the details of the cascade model since the first vortex serves as the bottle neck of the cascade.

6.6.6 Phase Slippage As A Mechanism For The Decay Of Vortices

Phase slippage in TGD context

Vortices must somehow dissipate their energy and angular momentum. Since angular momentum is proportional to the integer n_1 this means that some mechanism for reducing the value of n_1 must exist. This kind of mechanism is indeed known in the context of super fluidity and known as phase slippage [D12]. In case of the channel flow phase slippage means that the order parameter χ, which is completely analogous to the angle variables Ψ and Φ, develops in the following manner.

The original linear behavior $\chi = kx$, where x is the coordinate in the direction of flow is gradually deformed to a behavior for which χ changes by a multiple of 2π at single point $x = x_0$ and behaves otherwise linearly (see **Fig. 6.2**). Since χ and $\chi + n2\pi$ correspond to the same physical situation the result means that one replace the graph of χ with graph without the jump. This process implies dissipation: the value of the momentum like quantum number k has decreased by a discrete amount. Physically the phase slippage corresponds to the propagation of a vortex across the channel although this is not quite obvious: the quantized vorticity of the vortex is n/M so that vorticity is conserved in the process (see **Fig. ??**).

In the present context the phase slippage process has a nice geometric interpretation. A pair of $r = \infty$ and $r = 0$ surfaces is generated in the process. Ψ (Φ) can change discontinuously on the these surfaces and Ψ (Φ) indeed changes by a multiple of 4π (2π) and a phase slippage is generated. In present case it is quite obvious that this process corresponds to a propagation of a vortex across the channel.

The process can be generalized to provide a dissipation mechanism for the vortices. Daughter vortex is generated on the core of the decaying vortex and moves under the action of Magnus and friction forces in radial direction and finally leaves mother vortex. The quantum numbers n_1 and n_2 associated with the process are conserved.

$$n_k(mother, i) = n_k(mother, f) + n_k(daughter) \ , \quad k = 1, 2 \ . \tag{6.6.18}$$

If one assumes that K and n_1 are proportional to each other as they should be by the semiclassical argument, the critical radius of the mother vortex doesn't change in the process. If this process repeats itself sufficiently many times n_2 and n_1 become zero gradually resulting in a complete dissipation for the energy and angular momentum of the original vortex.

A model for the emission of the daughter vortices

A natural manner to model the emission of daughter vortices is as a stochastic process. Vortices are characterized by the quantum label $\Lambda = (n_1, n_2, \omega_1, \omega_2, m)$ and phase slippage corresponds to the emission process

$$\Lambda_1 \rightarrow \Lambda_2 + \Lambda_3 \ , \tag{6.6.19}$$

characterized by the decay rates

$$\Gamma(\Lambda_1 \rightarrow \Lambda_2 + \Lambda_3) \ . \tag{6.6.20}$$

Also the reverse process is possible but there are good reasons to assume that the fusion of the two vortices is a rather rare process.

It is straightforward to write general kinetic equations for the distribution of vortices as a function of Λ and in particular, as a function of the critical radius: this in turn leads to the distribution of the kinetic energy of the vortex as function of the the size of the vortex predicted also in the Heisenberg model for turbulence [B38, B34]. In order to get grasp of the situation it is however useful to make some simplifying assumptions about the decay of the vortices.

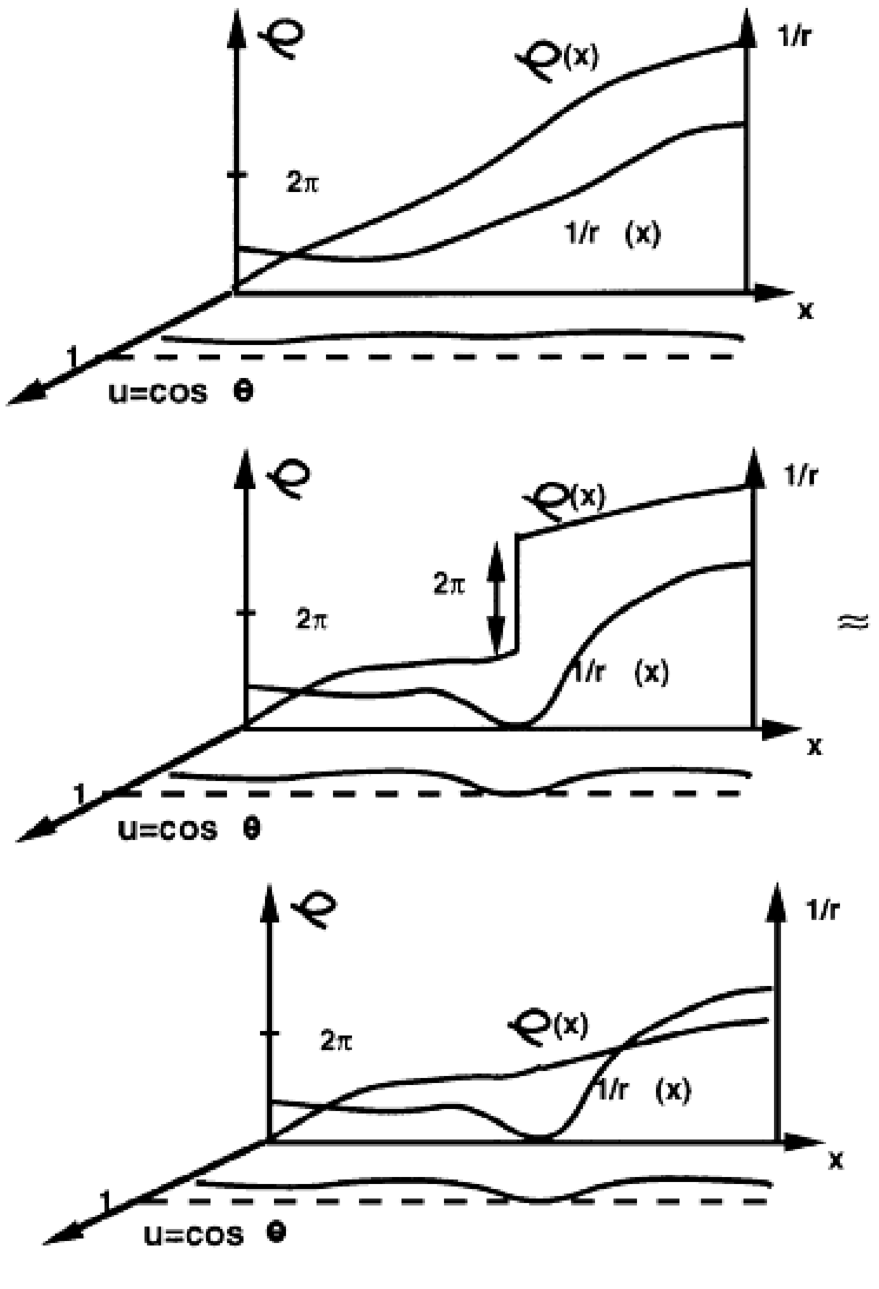

Figure 6.2: Phase slippage process and CP_2 geometry

1. Vortex growth is a rapid process as compared to the motion of vortex between the core and the boundary of the mother vortex. This implies that the integer m associated with the daughter vortex must be smaller than the integer associated with the mother vortex. For simplicity it is assumed

$$m(daughter) \; = \; m(mother) - 1 \; . \tag{6.6.21}$$

2. The ratio $n_1/n_2 = \omega_1/\omega_2$ remains constant in the decay process if possible: this implies that the change in the functional relationship between CP_2 coordinates u and r is minimized. Since the ratio n_1/k is constant by a semiclassical argument implying that angular momentum is proportional to n_1, the conservation law

$$\frac{n_i}{k} \; = \; constant \; , \tag{6.6.22}$$

holding true for all vortices of the cascade is suggestive.

3. The conservation law implies that the critical radius, vorticity and and n_i of the daughter vortex are given by

$$\begin{aligned} \rho_{crit}(daughter) &= \lambda \rho_{crit}(mother) \; , \\ k(daughter) &= \lambda k(mother) \; , \\ n_i(daughter) &\simeq \lambda n_i(mother) \; , \\ \lambda &= 2^{-x} \; . \end{aligned} \tag{6.6.23}$$

The value of x is expected to be integer and will be fixed by the comparison with experiment. The assumption

$$k(daughter) = \lambda k(mother) \; ,$$

makes sense if one gives up the assumption that magnetic flux is quantized irrespective of the value of n_1 as is clear by looking at the expression Eq. (6.5.18) for the critical radius for the vortex flow. One can however allow the increase of n (n_1 is multiple of n rather than arbitrary integer):

$$n(daughter) = \frac{n(mother)}{\lambda^2} \; ,$$

as is clear from the formula for the critical radius to achieve the quantization of magnetic flux. If magnetic flux quantization is assumed with the parameter $n = 1$ (n_1 integer) one must have

$$k(daughter) = \frac{k(mother)}{\lambda} \; ,$$

in order to get the critical radius correctly. The increase of k might be forced by the angular momentum conservation: if daughter vortices are created on the boundary of the mother vortex (as implied by the geometric picture) in the layer of a thickness $\rho_{crit}(daughter)$, the requirement that the angular momentum of the daughter vortices is of same order of magnitude as that of mother vortex, implies the desired formula. One must however remember that this argument need not make sense since flow equilibrium rather than decay of single vortex is in question. Also the increase of the average rotation velocity in small length scales looks un-physical feature. In any case, there are two possible scenarios:

a) Quantized magnetic flux and $n_i/k = constant$: ,
$k(daughter) = \lambda k(mother)$,
$n(daughter) = \frac{n(mother)}{\lambda^2}$, (6.6.24)

Quantized magnetic flux and $n = 1$:
$k(daughter) = \frac{k(mother)}{\lambda}$,

and the scenario 1) looks more attractive.

For the mother vortex the corresponding quantities are after the decay given by

$$\begin{aligned} \rho_{crit}(mother,f) &= \rho_{crit}(mother,i) \ , \\ k(mother,f) &= k(mother,i)(1-\lambda) \ , \\ n_i(mother,f) &\simeq n_i(mother,i)(1-\lambda) \ . \end{aligned} \qquad (6.6.25)$$

The process stops, when the condition $n_2(mother,f) = n_2(1-\lambda)^{N_f} \leq \lambda$ (n_2 refers to the mother vortex created at the wall) is satisfied, which gives the estimate

$$N_f(n_2) \simeq \frac{(ln(n_2)+ln(\lambda))}{|ln(1-\lambda))|} \ , \qquad (6.6.26)$$

for the total number of the daughter generations with $m(daughter) = m(mother) - 1$ born in the dissipation of the mother vortex by the emission of the daughter vortices.

The distribution of the vortices as a function of the critical radius

Consider now the evaluation of the distribution for the number $N(\rho)$ of the vortices as function of the critical radius ρ.

1. The number of the daughters in the k: th generation having is given by

$$\begin{aligned} N_d(k) &\simeq \prod_{i=0}^{i=k} N_f(i) \ , \\ N_f(i) &= \frac{(ln(n_2)+(i+1)ln(\lambda))}{|ln(1-\lambda))|} \ . \end{aligned} \qquad (6.6.27)$$

2. The size distribution is obtained by expressing the number k of generations in terms of the critical radius

$$k = -\frac{ln(\rho_m/\rho))}{ln(\lambda)} \ . \qquad (6.6.28)$$

Here ρ_m denotes the initial value of the vortex radius created at the wall of the channel. Assuming that the size distribution $N(\rho_m)$ for the mother vortices emitted at the wall is known, one obtains the following expression for the size distribution of vortices

$$\begin{aligned} N(\rho) &= \int N(\rho|\rho_m)N(\rho_m)d\rho_m \ , \\ N(\rho|\rho_m) &= \prod_{i=0}^{k} N_f(i) \ , \\ N_f(i) &= \frac{(ln(n_2)+(i+1)ln(\lambda)}{|ln(1-\lambda)|} \ . \end{aligned} \qquad (6.6.29)$$

An approximate expression of $N(\rho/\rho_m)$ holding true for small values of ρ is given by

$$
\begin{aligned}
N(\rho|\rho_m) &\simeq D(\frac{\rho_m}{\rho})^{\alpha+1/ln(\lambda)} \ , \\
D &= B^{-\frac{B}{ln(\lambda)}}A^{\frac{A}{ln(\lambda)}} \ , \\
A &= ln(n_2)+ln(\lambda) \ , \\
B &= A+ln(\frac{\rho}{\rho_m}) \ , \\
\alpha &= -\frac{ln(-\frac{1}{ln(1-\lambda)})}{ln(\lambda)}) \simeq 1 \ .
\end{aligned} \tag{6.6.30}
$$

D is a slowly varying logarithmic factor so that $N(\rho_m|\rho)$ behaves as the power $\rho^{1+\frac{1}{ln(\lambda)}}$ for all values of ρ_m. This implies that for small radii the general form of the size distribution is universal

$$
N(\rho) \simeq C(\frac{\rho_m}{\rho})^{\alpha+\frac{1}{ln(\lambda)}} \ , \tag{6.6.31}
$$

where C is some constant, which is determined once the rate of the energy dissipation is known.

The distribution of the kinetic energy of vortex per mass density ρ_m as a function of the vortex radius ρ can be evaluated using the formula

$$
\frac{T(\rho)}{\rho_m} = \pi\int_0^{\rho}\beta^2(\rho)\rho d\rho \ . \tag{6.6.32}
$$

1. For $\beta = K/\rho$ one obtains at the limit of the small radii

$$
T(\rho) = C\pi K^2 ln(\frac{\rho}{\rho_0})(\frac{\rho}{\rho_0})^{\alpha+\frac{1}{ln(\lambda)}} \ . \tag{6.6.33}
$$

The leading order behavior of the Fourier transform of the energy function defined as $\hat{T}(p) \equiv \int exp(ip\rho)T(\rho)d\rho$ is for small values of the wave vector given by

$$
\begin{aligned}
\hat{T}(p) &\simeq p^{\Delta} \ , \\
\Delta &= -1-\alpha-\frac{1}{ln(\lambda)}) \ .
\end{aligned} \tag{6.6.34}
$$

2. For $\beta = \Omega\rho$ one obtains

$$
\begin{aligned}
T(\rho) &= C\pi\Omega^2(\frac{\rho}{\rho_0})^{4+\alpha+\frac{1}{ln(\lambda)}} \ , \\
\Delta &= -4-\alpha-\frac{1}{ln(\lambda)} \ .
\end{aligned} \tag{6.6.35}
$$

In the Heisenberg model for the turbulence [B38, B34] a similar form is obtained and the exponent is in that case equal to $\Delta = -5/3$ and experimentally verified in some cases. It should also be noticed that according to [B38] the assumptions implying $\Delta = -5/3$ in the Heisenberg model are not strictly true for the small values of the vortex radii. On basis of this result it seems that the values of $\Delta(TGD) = -4-\alpha+\frac{1}{ln(\lambda)}$ are un-physical in the case of the rigid body flow.

Only the flow $\beta = K/\rho$ predicting constant Z^0 magnetic field apart from logarithmic corrections predicts physically acceptable values of Δ. For $\lambda = 2^5$ one would have $\Delta(TGD) = -1-\alpha-\frac{1}{ln(\lambda)} \simeq -1.709$ to be compared with $-5/3 = -1.667$ of the Heisenberg model. The deviation from the prediction of Heisenber model is 2.5 per cent. The prediction does not depend strongly on the value of of the $\lambda = 2^{-x}$ and at the limit $x = \infty$ one has $\Delta = -2$. Hence a statistical distribution for the p-adic scalings involved with the decay does not affect dramatically the prediction.

The general vision about dark matter hierarchy characterized by the values of Planck constant given by $\hbar(n) = \lambda^{-n}\hbar(1)$, $\lambda = v_0/n \simeq 2^{-11}/n$, n integer, encourages to consider the possibility that the scaling is associated with a transition $\hbar(n) \rightarrow \hbar(n-1)$ to a lower level in the dark matter hierarchy accompanied by the reduction of Compton lengths and Compton times by factor λ. The decay to smaller vortices would correspond to a reduction of quantum coherence via a decay of dark weak bosons to lower level dark weak bosons. For $n = 1$ one has $\Delta = -1.869$. For $n = 3$ one would have $\Delta = -1.885$.

Chapter 7

Macroscopic Quantum Phenomena and CP_2 Geometry

7.1 Introduction

Super conductivity, super fluidity and quantum Hall effect are examples of macroscopic quantum phenomena and it is instructive to apply the TGD inspired topological ideas about the formation of the macroscopic quantum systems to these phenomena. This chapter is is written for about 15 years ago and I hope that the reader does not forget that much has occurred in TGD since then.

For instance, Z^0 magnetic fields are suggested to be important for understanding super fluidity without precise characterization of there origin. About 15 years after writing the first version of this chapter, it became clear that the source of the long ranged Z^0 fields, as well as other weak fields and color gauge fields predicted by the classical theory could be dark matter at various space-time sheets. Also a precise number theoretic characterization of dark matter, or actually infinite hierarchy of dark matters, emerged. Already earlier it had become clear that the theory predicts a fractal hierarchy of scaled down copies of electro-weak and color physics. I have not added any discussion of the origin of Z^0 classical gauge fields here. This kind of discussion can be found in [**?**, K84, K33].

Around 2012 it became clear that the condition that the em charge of the modes of the induced spinor field is well-defined forces in the generic case the localization of the nodes to 2-D surfaces carrying vanishing W fields and above weak scale also vanishing Z^0 fields. This resolves the problems caused by the strong breaking of parity symmetry.

In the first section the general ideas of the TGD inspired description of supra phases are described. The aim is to make clear the close similarity between super conductivity and super fluidity by treating these phenomena in parallel. What makes possible the unified description is the hypothesis that the role of the ordinary magnetic field in the super conductivity is taken by the Z^0 magnetic field in the super fluid phase.

In the second, more technical section, certain simple imbeddings of Kähler electric and magnetic fields created by matter and relevant to the applications of the theory, are studied.

In the third section a TGD inspired phenomenological description of Quantum Hall effect is proposed. A more refined view about Quantum Hall effect developed about 15 years later can be found in [K3] . In the last section the TGD inspired description of less exotic condensed matter phenomena (conductors, di-electrics and magnetism) using TGD based concepts will be discussed briefly.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L8].

7.2 General Theory

RGE invariance predicts that 3-space should have fractal like structure consisting of topological field quanta of all possible sizes glued on each other by the topological sum operation. The join along boundaries bond provides a tool for constructing larger quantum systems from the smaller ones. Since dissipation corresponds to a loss of the quantum coherence, flux tube should provide a key to a topological description of the dissipation. The generation of the long range classical Z^0 fields is a phenomenon characteristic for TGD, and is expected to be important in the small vacuum quantum number limit of TGD at the condensate levels $n \geq n_Z$ $L(n) \geq \xi \sim 10^{-6}$ m. For supra phases the correlation lengths are such that classical Z^0 force should not have any role in their description.

The mathematical similarities between super conductors and super liquids however suggest that Z^0 magnetic field might play same role in the description of dissipation of super fluids as ordinary magnetic field in the description of the super conductors. The many sheeted structure of the topological condensate and length scale hierarchy remains rather implicit in the following considerations and the most relevant condensation levels are "atomic condensation level" at which electrons and nuclei are condensed and the level n_Z at which nuclei feed their Z^0 charges.

7.2.1 Identification Of The Topological Field Quanta

Both super conductors and Super fluids are characterized by the coherence length ξ. This length tells the size of the largest possible coherent quantum subsystem in the ordinary phase and becomes infinite, when the transition to the supra phase takes place. Below the critical point the value of ξ is finite, but there is macroscopic quantum coherence since the order parameter develops vacuum expectation value. Since topological field quanta correspond in TGD framework to coherent quantum systems a natural assumption is that the relevant topological field quanta have size of the order of ξ. ξ is typically of the order of $\xi \simeq 10^{-8} - 10^{-7}$ meters for super conductors, for super fluid He^4 ξ is of the order of atomic length scale and for He^3 ξ is of the order of 10^{-8} meters. This suggests that also ordinary matter behaves like supra phase in the length scales shorter than ξ. Of course, the corresponding time scale is rather short for the typical velocities of the supra flows.

In accordance with RGE hypothesis, it is assumed that topological field quanta of size ξ have suffered topological condensation in the background 3-space and topological field quanta in turn contain matter as topologically condensed 3-surfaces having size of atomic length scale

The size of the topological field quantum is determined by the vacuum quantum numbers associated with it. Since the size of the topological field quantum is rather small, the values of the vacuum quantum ω_1, ω_2 must be small. A first principle explanation for the finite size of the topological field quantum is the maximization of Kähler function. The contribution of the Kähler electric field to the Kähler action is smaller in magnitude if topological field quantization takes place: the reason is that Kähler electric field necessarily vanishes at some point(s) inside the topological field quantum.

In the simplest model for a topological field quantum matter serves as a source of Kähler field, which in present case is purely electromagnetic field and possible due to the incomplete screening of the nuclear electromagnetic charge by electrons. The critical radius associated with the embedding of the Kähler electric field gives the size of the topological field quantum, which should be of the order of ξ. The simplest model for the field quantum is as a spherical region. The join along boundaries/flux tube condensate of the topological field quanta serves as a model for the ordinary phase.

The sizes of the field quanta are exponentially sensitive to the value of the fractal quantum number m, which is small in present case. The order of magnitude for ω_1 is not much larger than proton mass: the estimates give $\omega_1 = (10^{2.5} - 10^3) m_p$ (m_p is proton mass).

In the astrophysical length scales and possibly also in the background 3-space surrounding topological field quanta in question the value of ω_1 is of the order of $m_0 \sim 1/R \sim 10^{-4} m_{Pl}$, where R is CP_2 radius.

Inside each topological field quantum one must perform a choice of the quantization axis and in the ordinary phase these choices are not correlated in accordance with the idea that quantum coherence is lost. In supra phase the presence of the flux tubes implies that same choice of the quantization axis must be performed in the whole phase and the global choice of the quantization axis is analogous to that taking place in the quantum measurement.

7.2.2 Formation Of The Supra Phase

Supra phase corresponds to lattice like structure of the topological field quanta of size ξ joined together by the join along boundaries bonds/flux tubes. In the lowest order approximation one can regard this lattice as a network formed by straight cylinders glued together by bonds. In supraphase the quantum numbers n_1 associated with the composite field quanta must vanish identically since otherwise the coordinate Φ is discontinuous somewhere on the bond joining the neighbouring field quanta and the field quantum in question separates from the supra phase. Exception is formed by the direction of the quantization axis, where bonds survive.

Two-fluid picture topologically

Supra flow is made possible by the bonds between the neighbouring topological field quanta and there is no essential difference between super conductors and super fluids in this respect. In case of the super conductors the topological field quanta form a rigid lattice but in case of super fluids topological field quanta are able to move. This freedom implies the two-fluid picture of the super fluidity as the following argument shows.

1. Normal liquid corresponds to the topological field quanta (of size ξ), which flow in the background 3-space. Since the bonds are absent in the ordinary phase, the matter condensed on the topological field quanta follows the flow of the topological field quanta so that topological field quanta can be regarded as effective fluid particles and their mass density is that of the liquid: $\rho_n = \rho$.
2. In supra phase the presence of the bonds make possible the flow of the topologically condensed matter and if the bonds are stable the condensed matter flows completely freely: $\rho_s = \rho$. This means that topological field quanta itself lose totally their inertia so that $\rho_n = 0$. Although the flow of the topological field quanta is possible it does not correspond to the flow of an inertial mass. This is certainly the situation at sufficiently low temperatures.
3. For temperatures slightly below T_c the situation is known to be intermediate between these two situations and two-fluid hydrodynamics [D15] is a good phenomenological description of the situation. One can consider two alternative explanations for this state of affairs. The first explanation is that the fluid is a mixture of the normal and super fluid components not only in critical temperature but also little below it so that one can speak about two fluids with average densities satisfying the condition $\rho_n + \rho_s = \rho$. The second alternative is that for the temperatures close to T_c the bonds are not completely stable and condensed matter doesn't flow completely freely so that topological field quanta do not lose their inertia totally.
4. One can understand also the frictionless supra flow in this picture. For example, in the frictionless supra flow in a channel, the topological field quanta are at rest with respect to the walls of the channel and only the matter condensed on the field quanta flows.

It should be emphasized that in TGD framework it is not possible to apply two-fluid picture to the description of the electrons in Super conductors since the particles of the "normal fluid" correspond to topological field quanta rather than electrons or atoms.

Ground states for the supra phases

In the ground state of the super conductor, the order parameter is covariantly constant with respect to the covariant derivative defined by the electromagnetic gauge potential. Covariant constancy indeed makes sense since, in the absence of the magnetic fields, the gauge potential is pure gauge in the spatial degrees of freedom. In the standard physics context the first homotopy group of the 3-space is trivial and gauge potential can always be gauge transformed away so that the order parameter is just constant in the ground state. In TGD context, the first homotopy of 3-surface is nontrivial and very complicated for a join along boundaries/flux tube condensate formed from the topological field quanta glued by the flux tubes. This implies that there is rich structure of different covariantly constant ground states, which look macroscopically identical since the splitting of single flux tube is not expected to affect the macroscopic properties of the system.

The induced gauge potential is in the case of the super conductors just the electromagnetic gauge potential. Assuming that Z^0 gauge fields are absent, one obtains the proportionality of the electromagnetic and Kähler gauge potentials:

$$A_{em} = 3A_K = 3P^k dQ_k \ . \tag{7.2.1}$$

Here P_k and Q_k are canonical coordinates for CP_2. An especially natural choice for the canonical coordinates is the one for which Q_k, $k = 1,2$ correspond to the phase angles Ψ and Φ associated with the complex CP_2 coordinates for which the action of $U(2)$ rotations is linear.

In case of the supra fluids Z^0 gauge potential if electromagnetic neutrality holds true and again the gauge potential is proportional to Kähler potential

$$\begin{aligned} A_Z &= \frac{6}{p}A_K = \frac{6}{p}P^k dQ_k \ , \\ p &\equiv sin^2(\theta_W) \ . \end{aligned} \tag{7.2.2}$$

If the ground state has vanishing gauge field the induced Kähler field must vanish and one has vacuum extremal of the Kähler action satisfying

$$P_k = \partial_k f(Q_i) \ , \tag{7.2.3}$$

where f is arbitrary function of the coordinates Q_i. In case that Q_i correspond to the angle coordinates Ψ and Φ of CP_2 one can write $f(Q_i)$ as a sum of a zero mode part and Fourier expansion

$$f = m\Psi + n\Phi + \sum_{kl} c_{kl} exp(ik\Psi + il\Phi) \ . \tag{7.2.4}$$

The covariant constancy condition for an order parameter possessing em (Z^0) charge Q_{em} (Q_Z) reads as

$$\begin{aligned} (\partial_\mu + ia\partial_\mu f)\psi &= 0 \ , \\ a_{em} &= 3Q_{em} \ , \\ a_Z &= \frac{6Q_Z}{p} \ . \end{aligned} \tag{7.2.5}$$

The solution of the condition is

$$\begin{aligned} \psi &= exp(iS)\psi_0 \ , \\ S_{em} &= -3Q_{em}f \ , \\ S_Z &= -\frac{6Q_Z}{p}f \ . \end{aligned} \tag{7.2.6}$$

in the two cases respectively.

The phase increments around the closed homotopically nontrivial loops clearly characterize the ground state of the supra phase. In the electromagnetic case the change of the phase of ψ around a closed loop equals to

$$\Delta S_{em} = 3Q_{em}(m\Delta\Psi + n\Delta\Psi) \ , \tag{7.2.7}$$

and is clearly a multiple of 2π (also for quarks!) since m and n appearing in the expansion of f are in general integers. For supra fluids one has

$$\Delta S_Z = \frac{6Q_Z}{p}(m\Delta\Psi + n\Delta\Phi) \ , \tag{7.2.8}$$

The values of Q_Z for proton and neutron are $Q_Z(neutron) = -1/4$ and $Q_Z(proton) = 1/4 - p$ so that one has for an order parameter describing the supra flow of nuclei (A, Z)

$$\Delta S_Z = 6(\frac{((2Z-A)}{4p} - Z))(m\Delta\Psi + n\Delta\Phi) \ , \tag{7.2.9}$$

The increment is *not* integer multiple of 2π without additional conditions on the value of the Weinberg angle. If p is rational number of form $p = r/s$, s must divide m and n. For instance, for $sin^2(\theta_W) = 1/4$ the vectorial couplings of the electron and proton to Z^0 field vanish and the average Z^0 charge of neutron is $Q_Z(n) = -1/4 = p$ so that one has in general $Q_Z(nucleus) = -(A-Z)/4$ and the increment of S_Z is automatically multiple of 2π for all choices of m and n:

$$\Delta S_Z = -6(A-Z)(m\Delta\Psi + n\Delta\Phi) \ . \tag{7.2.10}$$

Also for $p = 3/8$ the condition is identically satisfied.

For more complicated supra phases (Super liquid He^3) the order parameter possesses several components but also now a similar situation results. It is tempting to assume that for more general states the phase factor of ψ is power of S. If this is the case then supra phases are exceptional in the sense that CP_2 angle coordinates appear as physical observables rather than only the gauge fields (proportional to the gradients of CP_2 coordinates) as in the ordinary ordinary phase. What is clear is that the information about the homotopy of the state is coded into the phase of the order parameter. This state of affairs is especially interesting as far the applications to the BE condensate of the charged # throats possibly having an important role in bio-systems, are considered.

Binding energies and critical temperatures

What makes the supra flow possible are the bonds. Cooper pair also stabilize the bonds in case of the super conductors and ^3He super fluid. This becomes clear from the fact that the electrons of the Cooper pair have an average distance, which is considerably larger than ξ (about 10^{-6} meters in super conductors [D15]) so that the splitting of the bonds destroys Cooper pairs. Energy is however needed to destroy Cooper pairs and this implies stability. If the energy associated with the bonds were negligible with the binding energy associated with the Cooper pairs the phase transition leading to super conducting phase would be a first order transition involving non-vanishing latent heat. This is however not the case [D15]. This means that the binding energy of the Cooper pairs doesn't leave super conductor and probably goes to the energy associated with the bonds. Therefore the stabilization mechanism relies on the difficulty of transferring the bond energy to the Cooper pairs.

A rough estimate for the binding energy for the Cooper pair provides a test for the proposed ideas. In the ordinary phase conduction electrons tend to be confined inside the topological field quanta so that by Uncertainty Principle they possess kinetic energy of the order of

$$T \simeq \frac{1}{2m_e\xi^2} \ . \tag{7.2.11}$$

In the super conducting phase conduction electrons are not localized inside single field quantum so that the average kinetic energy is smaller and the order of magnitude estimate

$$\Delta E \simeq \frac{1}{4m_e\xi^2} \ , \tag{7.2.12}$$

for the binding energy of the Cooper pair is obtained. For $\xi \simeq 10^{-7}$ meters one obtains $\Delta E \simeq 10^{-4} eV$, which corresponds to the temperature of $T_c \simeq 0.25\ K$. The order of magnitude is correct.

For a high temperature super conductors with $T_c \simeq 100\ K$, the estimate gives $\xi \simeq 10^{-9}$ meters. High temperature super conductors have layered structure. In case of $YBa_2CU_2O_7$ the coherence length is $\xi_c = 1.5-3$ Angströms in the direction orthogonal to the layers and $\xi_{ab} = 14\pm2$ Angströms in the direction of the layers [D4]. The supra current is known to be confined inside the layers so that ξ_{ab} should determine the critical temperature: the orders of magnitude are consistent with the formula correlating Δ and ξ in the example considered and also more generally, since the transversal coherence lengths are known to be by an order of magnitude smaller than for the ordinary super conductors.

For the binding energy of the super fluid particles one obtains a completely analogous estimate (m_e is replaced with the mass of He^3 or He^4 nucleus) and correct order of magnitude estimates are obtained for both He^3 and He^4 having widely different values of ξ (ξ is about 10^{-8} meters and few Angströms for ^4He and ^3He respectively). From the binding energies one can estimate the critical temperatures ($T_c \simeq \Delta E$) and correct order of magnitude estimates are obtained.

The presence of super fluid phase in neutron stars has been suggested [D15]: Cooper pairs correspond to paired neutrons. The size of the field quantum is of the order of $\xi = 10^{-15} - 10^{-14}$ meters (this estimate is derived in the second section). For the critical temperature one obtains: $T_c \simeq 1/4m_n\xi^2 = 10^{11} - 10^{13}$ K.

In BCS theory ΔE is expressed in the following form [D15]

$$\begin{aligned} \Delta &= 2\omega_D exp(-\frac{2}{N(0)V}) \ , \\ \omega_D &= \frac{c_s 6^{1/3}\pi}{N^{1/3}} \ . \end{aligned} \tag{7.2.13}$$

Here ω_D is Debye frequency, $N(0)$ is the density of states on the surface of the Fermi sphere and $V(0)$ characterizes the strength of the attractive force between the electrons of the Cooper pair. N is the number density of atoms and c_s is the velocity sound. The proportionality to ω_D implies isotope effect: $\Delta \propto 1/A^\alpha$, where α is typically of the order of $\alpha \simeq 1/2$, which has been verified experimentally [D15]. Assuming that both formulas are correct one gets a relationship between the vacuum quantum numbers ω_1 and ω_2 since ξ corresponds to the radius of the topological field quantum and is expressible in terms of the vacuum quantum numbers.

7.2.3 Generalized Quantization Conditions

In the standard formulation of the quantum description of Super conductivity one starts from Schrödinger amplitude ψ_s for supra phase. The expression for the matrix element of the electric current is given by

$$\begin{aligned} \bar{j}_e &= -i\frac{e}{2m}(\bar{\psi}_s \bar{D}\psi_s - c.c.) \ , \\ \bar{D} &= \nabla + iqe\bar{A} \ . \end{aligned} \tag{7.2.14}$$

Here q denotes the charge of the superconducting charge carrier in units of e. $q = -2$ for the super conductors encountered in laboratory. One can write ψ_s in the form $\psi_s = \sqrt{n_S} exp(iS)$.

Since n_s is in a good approximation constant in supra phase the expressions for the electric current and velocity operator can be written as

$$\begin{aligned} \bar{j}_e &= -\frac{e}{m} n_s(\nabla + qe\bar{A}) \ , \\ \bar{v}_s &= \frac{1}{m}(\nabla S + qe\bar{A}) \ . \end{aligned} \tag{7.2.15}$$

Since S is single valued, one obtains by integrating over a closed curve a formula relating the magnetic flux and velocity circulation for the carriers of the super current to each other.

$$\oint \bar{v}\cdot d\bar{l} - \frac{qe}{m}\oint \bar{A}\cdot d\bar{l} = \frac{n2\pi}{m} . \tag{7.2.16}$$

If the velocity field vanishes in the curve in question, one obtains the standard quantization of the magnetic flux.

By taking a curl of the formula for $\bar{v}_s$ and using Maxwell's equations one gets the standard formula

$$\begin{aligned} \nabla^2 \bar{B} &= \frac{\bar{B}}{\lambda^2} , \\ \lambda^2 &= \frac{2m}{n_s q^2 e^2} . \end{aligned} \tag{7.2.17}$$

Here λ is the peneration length for the magnetic field in the super conductor.

TGD predicts that vacuum Z^0 field can become long ranged at small vacuum quantum number limit of TGD and super fluidity might correspond to this kind of situation. If this is indeed the case then the previous formulas for the super conductors generalize in an obvious manner to the case of Super fluids

$$\oint \bar{v}\cdot d\bar{l} - \frac{Q_Z g_Z}{M}\oint \bar{A}_Z\cdot d\bar{l} = \frac{n2\pi}{M} . \tag{7.2.18}$$

Here M is the mass of the super fluid particle (He^4 or the Cooper pair formed by two He^3 atoms), g_Z is the gauge coupling of the Z^0 gauge interaction ($g_Z^2 = e^2/sin\theta_W cos\theta_W$) and Q_Z is Z^0 charge of the super fluid particle. Q_Z is defined as the expectation value over the spin degrees of freedom

$$\begin{aligned} Q_Z &= \langle I_L^3 - pQ_{em}\rangle , \\ p &= sin^2(\theta_W) . \end{aligned} \tag{7.2.19}$$

The values of Q_Z for quarks and electron at rest are

$$Q_Z(u) = \frac{1}{4} - \frac{2p}{3} , \quad Q_Z(d) = -\frac{1}{4} + \frac{p}{3} , \quad Q_Z(e) = -\frac{1}{4} + p . \tag{7.2.20}$$

From these one obtains the values of Q_Z for proton and neutron: $Q_Z(p) = 1/4 - p$ and $Q_Z(n) = -1/4$ respectively. The values of Q_Z for He^4 and He^3 are

$$Q_Z(^4He) = -\frac{1}{2} , \quad Q_Z(^3He) = -\frac{1}{4} . \tag{7.2.21}$$

If the magnetic flux associated with Z^0 magnetic field vanishes one obtains the standard formula for the quantization of the velocity circulation of the super fluid. The expression for the penetration depth of the Z^0 magnetic field reads as

$$\lambda^2 = \frac{2M}{N_s Q_Z^2 g_Z^2} . \tag{7.2.22}$$

The order of magnitude of λ is of the order of $10^{-5} - 10^{-6}$ meters in accordance with the basic assumption $\xi \sim 10^{-6}$ meters for the scale at which classical Z^0 force becomes important. In this formula N_s is the entire super fluid density (essentially Z^0 charge density) and the formula makes sense at the condensation level at which the nuclei feed their Z^0 charges. At the higher condensate levels n, one must replace the density with the actual density of Z^0 charge $N_s \rightarrow N_s/\sqrt{\epsilon_Z(n)}$ (due to the neutrino screening$\epsilon_Z(n)$ is rather large number).

It will found that this generalization implies considerable differences between TGD based and standard descriptions of the super fluidity. For example, the counterpart of the magnetic flux quantum is predicted and is a good candidate for the elementary excitation leading to the dissipative super fluid flow at critical velocity considerably smaller than that associated with the known elementary excitations.

7.2.4 Dissipation In Super Fluids: Critical Velocities

Dissipation, or equivalently the loss of the quantum coherence results, when the lifetimes of the bonds connecting neighbouring field quanta are short and the joining and the splitting of the bonds provides the needed dissipation mechanism. One mechanism leading to a loss of the quantum coherence is thermal noise: the critical temperature has been already evaluated. In case of super conductors (super fluids) also external magnetic (Z^0 magnetic) fields lead to a loss of the quantum coherence: the values of the critical magnetic fields can be evaluated for the super conductors of type II and super fluids from the quantization condition. At a high enough flow velocity, the generation of the elementary excitations of the supra phase leads to dissipation. The estimates for the orders of magnitude for the critical velocities for the setup of the dissipation will be derived and are correct in both cases.

Critical velocity for super fluids

The so called Principle of Super Fluidity provides an explanation for the critical velocity of the Super fluid [D15]. The application of the energy and momentum conservation to the emission of elementary excitation of energy ϵ and momentum p by flow implies the condition $v \geq \epsilon/p$ and therefore the critical velocity is given by the formula

$$v_L = Min\{\frac{\epsilon}{p}\} . \tag{7.2.23}$$

In case of the super conductors the formula gives $v_L = \Delta(T)/k_F$ (Δ is the energy gap associated with the Cooper pair and k_F is Fermi momentum): the order of magnitude is correct. In case of Super fluids the critical velocities deduced from the roton and phonon spectrum (239 m/s and 58 m/s respectively) are several orders of magnitude larger than the velocities $v_{cr} \simeq 6 \cdot 10^{-3}$ meters), where the dissipation is known to set up. Velocity vortex predicts a critical velocity, which is too large by an order of magnitude. The hitherto unsolved problem is to identify the excitations giving rise to the dissipation in the supra flow.

The TGD based candidate for the excitation is Z^0 magnetic flux quantum. Z^0 magnetic flux quantum can appear at the condensate level with $L(n) \geq 10^{-6}$ meters to which nuclei feed their Z^0 charges so that the super fluid flow (typically rotating vessel) must have size scale much larger than this length scale. Both hydrodynamic and magnetic excitations are vortex like structures and in order to estimate orders of magnitude they can be idealized as straight vortices with a cylindrical symmetry, possessing Z^0 magnetic field in the direction of the vortex and rotational velocity field (to be studied in detail in the next section).

A general order of magnitude estimate for the critical velocity is obtained by assuming that at velocities higher than the critical velocity the kinetic energy of the supra phase goes to the energy of the excitation in question. The criticality criterion states that $dE_K(R)/dl$, the kinetic energy of the supra flow per unit length of the vortex of radius R and $dE_{ex}(R)/dl$, the energy of the excitation per unit length of the vortex, are identical:

$$\frac{dE_K(R)}{dl} = \frac{dE_{ex}(R)}{dl} .$$

This implies for the critical velocity the expression

$$v_{cr} = \sqrt{\frac{2}{NM\pi R^2}}\sqrt{\frac{dE_{ex}(R)}{dl}} \ . \tag{7.2.24}$$

Let us consider now in more detail the magnetic and hydrodynamic vortices.

a) Z^0 magnetic flux quantum

For the Z^0 magnetic flux quantum it is natural to assume that the core of the vortex corresponds to $n_1 \neq 0$ excitation since the requirement that no magnetic field is present implies $n_2/n_1 = \omega_2/\omega_1$ so that both n_2 and n_1 must be non-vanishing. A reasonable idealization for the vortex core is as a cylinder of radius ξ. Inside the vortex core the order parameter of the supra phase is constant so that the condition

$$\oint \bar{v}\cdot d\bar{l} - \frac{Q_Z g_Z}{M_4}\oint \bar{A}_Z \cdot d\bar{l} = 0 \ , \tag{7.2.25}$$

holding true for the ground states described by covariantly constant order parameter, is appropriate. The general quantization condition allows $n \neq 0$ but this implies singular velocity in the core of the vortex so that it will be dropped from consideration.

Since $B_Z = B_Z^0$ is constant, one can solve $\bar{v}$

$$v = \frac{g_Z Q_Z B_Z^0}{2M_4}\rho \ . \tag{7.2.26}$$

The core rotates like a rigid body and the rotation frequency is just the rotation frequency of Z^0 charged particle in Z^0 magnetic field. $\nabla^2 B_Z = 0 \neq B_Z/\lambda^2$ so that the matter inside the vortex core is not in supra phase.

Outside the vortex core the conditions

$$\begin{aligned} \oint \bar{v}\cdot d\bar{l} - \frac{Q_Z g_Z}{M_4}\int B_Z da &= \frac{n2\pi}{M_4} \ , \\ \nabla^2 B_Z &= \frac{B_Z}{\lambda^2} \ . \end{aligned} \tag{7.2.27}$$

are satisfied.

Both Z^0 magnetic and velocity fields decay exponentially. At large distances one obtains flux quantization and the constant value of B_Z inside the vortex core is fixed by the flux quantization condition:

$$B_Z^0 = [-2\int_{\xi}^{\infty} B_Z \rho d\rho + \frac{2n}{g_Z Q_Z}]\frac{1}{\xi^2} \ . \tag{7.2.28}$$

For order of magnitude purposes one can use the approximation

$$B_Z^0 \simeq \frac{2n}{q_Z Q_Z \lambda^2} \ . \tag{7.2.29}$$

Since the magnitude of B_Z^0 is quantized in integer multiples, all values of n are possible.

There are two contributions to the energy density of the flux quantum. The energy E_B of the Z^0 magnetic field and the kinetic energy T_{rot} of the rotating super fluid particles. The latter contribution is negligible ($T_{rot}/E_B \simeq (\xi/\lambda)^2$) so that it is enough to consider the magnetic energy density. Since B_Z is largest in the core of the vortex the most conservative form for the criterion is obtained by requiring that the kinetic energy density $T_K = N_s M_4 v^2/2$ of the super fluid flow equals to the Z^0 magnetic energy density $E_B = B_Z^2/2$ inside the core. This condition gives the following expression for the critical velocity

$$v_{cr}(magn) = \frac{B_Z^0}{\sqrt{NM_4}} \simeq g_z Q_Z \sqrt{\frac{N_s}{M_4^3}} \ . \qquad (7.2.30)$$

Substituting the typical value of N_s: $N_s \simeq 10^{28.5}/m^3$ one finds $v_{cr} \simeq 10^{-3}$ m/s. The value of the critical velocity is indeed known to be few millimeters in second [B19], [D15] !

b) Hydrodynamic vortices

The velocity field of the vortex behaves as k/ρ, where $k = n2\pi/M$ is the quantized vorticity. The kinetic energy of the vortex is of the order of $M_4 k^2 ln(\lambda/\xi)/2$ so that one obtains for the critical velocity the expression

$$v_{cr}(hydro) \simeq \sqrt{2} ln(\lambda/\xi) v_{cr}(magn) \ . \qquad (7.2.31)$$

Substituting the numerical values of the parameters, one finds that the numerical factor is of the order of ten so that hydrodynamic critical velocity is too large by an order of magnitude [B19], [D15].

Critical velocities for the super conductors

To derive the critical velocities for the super conductors of type II one can apply considerations formally identical with the previous ones. The structure of the magnetized vortices is similar to that of Z^0 magnetized vortices and at the critical velocity the kinetic energy density of the super conducting phase must be identical to the magnetic energy density of $n_1 = 1$ excitation:

$$\frac{n_s m_e \beta_c^2}{2} = \frac{B_c^2}{2} \ . \qquad (7.2.32)$$

Using the expression for the number density of the super conducting electrons $n_s = \frac{m_e}{e^2\lambda^2}$ one gets

$$\beta_c = \frac{B_c \lambda e}{m_e} \ . \qquad (7.2.33)$$

Using the estimate for B_c one obtains $\beta_c \simeq \frac{\sqrt{4\pi}}{m_e\lambda}$ for the super conductors of type II. The order of magnitude obtained, typically 10^2 m/s, is correct [D15]. For super conducting elements of type I β_c is considerably smaller since both the critical field and λ are smaller: the order of magnitude is few meters per second and considerably smaller than the critical velocity v_L obtained from the Landau criterion.

7.2.5 Meissner Effect

Meissner effect is one of the basic effects of super conductivity and it is of interest to find the TGD based description of the effect and how Meissner effect generalizes to the super fluid phase.

Meissner effect in superconductors

Meissner effect differs for the super conductors of type I and II. For super conductors of type I, the external field penetrates the whole super conductor if it has strength larger than the critical strength B_c. For super conductors of type II the external magnetic field begins to penetrate after having reached certain critical value B_{c_1} and total penetration takes place at considerably larger value of B_{c_2}. The penetration takes place as flux quanta

$$\int B \cdot da = \frac{m\pi}{e} , \tag{7.2.34}$$

where m is integer. This condition follows from the general quantization conditions provided the velocity of the super conducting charge carriers vanishes for large distances from the core of the magnetic flux quantum.

The TGD inspired model for the Meissner effect is based on the following observations.

1. The study of the simple models for the topological field quanta to be carried out later shows that in the supra phase topological field quanta have vanishing magnetic vacuum quantum numbers (n_1, n_2) and that there is a nontrivial magnetic field associated with $(n_1, n_2) \neq (0, 0)$ excitations of the topological field quanta. Magnetic field is in the direction of the quantization axis and is approximately constant for a cylindrically symmetric field quantum. The flux of this magnetic field is also quantized by purely topological reasons.

   For $(n_1 = 0, n_2 \neq 0)$ magnetic field is also non-vanishing and this field doesn't cut the bonds between the field quanta so that one could in principle construct a magnetic field in super conductor using these excitations. If, however, the condition

$$k \equiv \frac{\omega_2}{\omega_1} \ll 1 , \tag{7.2.35}$$

   holds true, then the flux associated with $(n_1 \neq 0, n_2 = 0)$ is much smaller than for $n_1 = 0$ excitations and it is energetically more favorable to excite $n_1 \neq 0$ excitations so that super conductivity is lost. The study of the simple models for field quanta shows that the assumption that ω_1 has same value for all supra phases, implies this condition.

2. The flux of the critical magnetic field is typically of the order of 10^{-2} Tesla and the flux of B_{c_2} over the field quantum of radius $\xi \simeq 10^{-7}$ m is considerably smaller than the quantized value of the magnetic flux for the super conducting elements (mostly of type I).

3. Since λ is much smaller than ξ for super conductors of type I, the magnetic flux associated with the magnetic vortex is smaller than the quantized magnetic flux, which together with the quantization condition implies that the velocity associated with the vortex cannot approach zero in large distances so that the kinetic energy of the vortex is large and this kind of excitation is not energetically favorable in case of the super conductors of type I. Rather, the magnetic field penetrates as $n_1 \neq 0$ excitation into each topological field quantum separately and as a result the bonds between field quanta are destroyed in the directions transversal to the magnetic field and supra phase is destroyed. For the super conductors of type II λ is large as compared to the radius of the vortex core and magnetic field can penetrate in the form of the flux quanta.

These observations suggest the following description of the Meissner effect.

a) Meissner effect for the super conductor of type I

Magnetic field penetrates into super conductors of type I as topologically nontrivial ($n_1 = 1 \neq 0$) excitations of the individual field quanta (see **Fig. ??**). The critical magnetic field is just that associated with $n_1 = 1$ excitation and the penetration of the magnetic field tends to destroy the bonds between the neighbouring field quanta since Φ becomes necessarily discontinuous on the bond. The bonds in the direction of $\bar{B}$ form an exception and might well survive. A structure consisting of topologically condensed cylinder like structures (see **Fig. ??**) results. That super conductivity disappears totally is suggested by the observation that $\Lambda = 0$ inside these structures and by the fact electrons rotate in the magnetic field.

The quantization of the magnetic flux takes place in case of Super conductors of type I, too, but the unit is now defined by B_c and smaller than the standard unit. The requirement that the magnetic field associated with the $n_1 = 1$ field quantum equals to B_c gives condition on the vacuum parameters of type I super conductor.

It would be nice if one could estimate the value of the critical magnetic field or equivalently, the value of the magnetic field associated with the $n_1 = 1$ excitation. The prediction is possible provided one can estimate the values of the vacuum quantum numbers associated with the embedding of Kähler electric field of matter: in the next section this kind of estimate is carried out.

b) Meissner effect for the super conductor of type II

Magnetic field penetrates into super conductors of type II as approximately cylindrical field quanta. The core of the cylinder corresponds to a topological field quantum of radius of order ξ, which has suffered topologically nontrivial ($n_1 \neq 0$) excitation. Since the flux associated with $n = 1$ quantum is considerably maller than that required by the quantization of magnetic flux, an exponentially damped magnetic field is created in the surrounding field quanta. This field corresponds to a topologically trivial deformation ($n_1 = 0!$) in the dependence of Φ on the M^4 coordinates and therefore the bonds connecting nearby neighbours are not destroyed and this region corresponds to a supra phase. The quantized magnetic flux is essentially given by the region surrounding the core.

The value of the critical magnetic field B_{c_1} can be estimated by noticing that the external magnetic field decomposes into field quanta with the property that the total flux of field quanta is same as that associated with the external field. This gives

$$B = n_v \frac{\pi}{e} \ , \tag{7.2.36}$$

where n_v is the number of flux quanta per unit area. As an estimate for n_v one can take $n_v \simeq 1/\pi\lambda^2$, so that one obtains the estimate

$$B_{c_1} \simeq \frac{1}{e\lambda^2} \ . \tag{7.2.37}$$

The order of magnitude is about 10^{-1} Tesla for $\lambda \simeq 10^{-7}$ m: for Nb, which is the only superconducting element of type II the order of magnitude for critical magnetic field is indeed this [D15]. The value of the magnetic field associated with $n = 1$ excitation cannot be very much larger than this field. It is natural to identify B_{c_2} as the magnetic field associated with $n_1 = 1$ excitation and so that the previous estimate combined with the estimate for B_1 gives $B_{c_2} \simeq 2B_{c_1}$.

Notice that the proposed model explains why ferromagnetic materials cannot be super conducting provided one can assume that the condition $k \ll 1$ holds true generally (ω_1 depends only weakly on material).

Meissner effect for super fluids

TGD predicts that Meissner effect is possible for super fluids, too and that super fluids are completely analogous to super conductors of type II. The magnetic vortices in the super fluid correspond to the quanta of the Z^0 magnetic flux.

The critical value of B_Z cannot be obtained directly from the experiment. The critical value of B_Z can be estimated by generalizing the formula of B_c for super conductors of type II and the formula for the penetration length λ

$$\begin{aligned} B_c^Z &\simeq \frac{1}{Q_Z g_Z \lambda^2} \ , \\ \lambda^2 &= \frac{2M}{Q_Z^2 g_Z^2 N_s} \ , \end{aligned} \tag{7.2.38}$$

where M is the mass of the super fluid particle and g_Z is Z^0 coupling constant and N_s the number density of the super fluid particles.

Superfluid should prohibit the penetration of Z^0 magnetic field created by some external source by creating surface flow. The obvious question is whether one can imagine any experimental tests for the prediction. To get grasp of the situation one can consider the following simple experimental arrangement.

A cylinder containing super fluid is surrounded by a rotating cylinder (see **Fig. ??**). The rotation of the outer cylinder creates Kähler magnetic and therefore also Z^0 magnetic field. Meissner effect implies that a surface flow is generated on the boundary of the super fluid vessel possessing direction opposite to that of rotation. A related effect would be the penetration of the Z^0 magnetic field in the form of vortices creating visible hydrodynamic vortices in the liquid. Unfortunately, the Z^0 field in question is extremely weak (for ordinary vacuum quantum numbers) so that the surface flow needed to cancel the Z^0 magnetic field is very small and might imply that the effect is not observable. Also the penetration of the field in the form of vortices is very improbable since penetration takes place only above some critical field strength, which is quite large.

Consider next a simple quantitive model for the situation. The constant axial Kähler magnetic field created by the rotating outer cylinder is given by the expression

$$\begin{aligned} B_{out}^K &= \epsilon_1(out) N_{out} \Omega_{out} S_{out} \ , \\ S_{out} &= \pi(R_1^2 - R_0^2) \ , \end{aligned} \tag{7.2.39}$$

where S_{out} denotes the cross-sectional area of the outer cylinder with the inner radius R_0 and outer radius R_1 and rotating with the angular velocity Ω_{out}.

The constant axial magnetic field created by a surface current of thickness λ rotating around the superfluid cylinder of radius R is given by

$$\begin{aligned} B_{in}^K &= \epsilon_1(in) N_s \Omega_{in} Sin \ , \\ S_{in} &= \pi(R^2 - (R-\lambda)^2) \ , \end{aligned} \tag{7.2.40}$$

where N_s denotes the density of the super fluid particles and Ω_{in} is the rotation velocity of the super fluid flow.

These fields must cancel each other inside the super fluid so that a condition for the ratio of rotation frequencies results

$$\begin{aligned} \frac{\Omega_{in}}{\Omega_{out}} &= \frac{\epsilon_1(out)}{\epsilon_1(in)} \frac{N_{out}}{N_s} \frac{S_{out}}{S_{in}} \\ &\simeq \frac{\epsilon_1(out)}{\epsilon_1(in)} \frac{N_{out}}{N_s} \frac{R_1^2}{2R\lambda} \ , \end{aligned} \tag{7.2.41}$$

where the assumption $R_1 \gg R_0$ is made. An order of magnitude estimate for Ω_{in} is obtained using magnitudes $R_1 = 1\ m$, $R = 10^{-3}\ m$, $\lambda \simeq 10^{-5}\ m$ and $\Omega_{out} \simeq 10^3/s$. The Z^0 current fed from the "previous" condensate level serves as source of Z^0 magnetic field at level n since neutrinos do not participate in the flow. The estimate for the ratio of parameters $\epsilon_1(n_Z)$ is obtained as follows: at nuclear condensate level one has $\epsilon_1 \sim 10^{19} g_Z$ (no screening) and at the condensate level n_Z one has $\sqrt{\epsilon_Z(n_Z)} \sim 10^{10} - 10^{11}$ from the estimate to be carried out in next subsection, which gives $\epsilon_1(out) \in 10^8 g_Z - 10^9 g_Z$. This gives

$$\Omega_{in} \sim 10^{-10}\frac{N_0}{N_s}\Omega_{out} \geq \frac{1}{200\ minutes} \ , \tag{7.2.42}$$

for $\epsilon_Z = 10^{11}$. Whether the existence or non-existence of this kind of effect could be determined experimentally remains an open question.

Rotating super fluid

In the two fluid theory the condition that super fluid flow is irrotational ($\nabla \times \bar{v} = 0$) seems to exclude the rigid body rotation of the super fluid. On the average super fluid phase is however known to rotate like rigid body [B19] , [D15] and the problem is to explain this result.

a) Hydrodynamic vortices

The generally accepted resolution of the difficulty [D15] is that super fluid flow decomposes into hydrodynamic vortices, with the property that the flow is irrotational inside the vortices except in the core of the vortex where super fluid density vanishes: this is achieved if the velocity is given by $v = k/\rho$. The requirement that super fluid wave function is single valued, implies the quantization of the circulation for the vortex

$$\oint \bar{v}\cdot d\bar{r} = \frac{n2\pi}{M} \ ,$$

implying the condition $k = n/M$. Vortices in turn form a regular array, which rotates like a rigid body. The average vorticity per surface area is given by $n_v k$, where k must be same as the vorticity of the rigid body rotation: this gives for the density of vortices the expression

$$n_v(hydro) = \frac{2\Omega}{2\pi k} = \frac{\Omega M_4}{\pi} \ . \tag{7.2.43}$$

The vortex core, where super fluid density vanishes according to the conventional theory, should have radius $\rho_0 \simeq 10^{-10}\ m$. Although the vortices as such are not visible there is indirect experimental evidence for the existence of the vortex like structures, in particular for the existence of vortex cores [B19] , [D15] possessing inner core radius of order $10^{-10}\ m$.

The generation of the vortices should begin at some critical angular velocity Ω (the circulation of the rigid body flow being of the order of the quantum of circulation at this value of Ω: this kind of effect has indeed been observed: the critical velocity is however smaller than the predicted one [D15].

One can wonder what happens at the rotation velocities smaller than the critical one. Does super fluid flow like a rigid body or does it rotate at all? There is some experimental evidence supporting the view that super fluid does not rotate for sufficiently low rotation velocities so that the behavior is analogous to Meissner effect with Ω playing the role of the magnetic field.

b) Z^0 magnetic vortices

Consider now an alternative TGD inspired description of the situation. The problem is clearly created by the velocity circulation condition, which implies that supra flow is irrotational almost everywhere. In TGD approach the quantization condition however contains also the contribution of the Z^0 magnetic flux besides the velocity circulation so that there is no reason to require that velocity field has vanishing curl anymore! Assuming that super fluid flows as rigid body one can adjust B_Z so that the quantization condition is satisfied.

$$B_Z = \frac{2\Omega M_4}{g_Z Q_Z} \ . \tag{7.2.44}$$

The resulting field is rather weak as compared to the critical B_Z. Ω must be of the order of $10^7/s$ (ten orders of magnitude larger than the critical rotation velocity for the formation of vortices!) to guarantee that B_Z is equal to the critical B_Z. This suggests that B_Z vortices cannot appear at rotation velocities studied and that the generation of the velocity vortices is the correct solution of the problem.

There are also other counter arguments. First, since the required field is much smaller than the critical field it seems impossible to imbed this magnetic field into super phase (one should excite some topological field quanta to $n_1 \neq 0$ state). Secondly, the generation of the subcritical magnetic field is excluded by the Meissner effect. Thirdly, $\nabla^2 B_Z = 0 \neq B_Z/\lambda^2$ so that super phase would be destroyed if constant B_Z is generated. On the other hand, the solution has the nice feature that the rigid body rotation of the super fluid could be regarded as a direct experimental evidence for the existence of macroscopic Z^0 field.

One manner to escape these problems is to argue that B_Z is constant in average sense only and that the actual field is consists of a network of Z^0 magnetic flux quanta in rigid body motion. The requirement that the total flux over the cross section of the container is same as the flux of constant field gives for the density of magnetic flux quanta per unit area the expression

$$n_v(magn) = \frac{\Omega M_4}{\pi} . \tag{7.2.45}$$

The density is identical with that obtained for hydrodynamical vortices! This observation suggests the solution to the discrepancy and a more detailed mechanism for the destruction of superfluidity. Super fluidity is destroyed, when Z^0 magnetic field (created by rotating Z^0 charge density) at condensation level $n_1 > n_Z$ ($L(n_Z) \sim 10^{-6}\ m$) penetrates to the level n_Z in form of flux quanta with strength B_c^Z. The conservation of magnetic flux explains why the average field strength at the level n_Z is identical with the penetrating field strength at the level n_1. Since Z^0 charge current of the previous level serves as source of Z^0 magnetic at level n one obtains as a byproduct an estimate for the value of $\epsilon_Z(n_Z)$ from the formula 7.2.38 for the critical Z^0 magnetic field strength giving $\sqrt{\epsilon_Z(n_Z)} \sim 10^{10}$ and $\epsilon_1 \sim 10^9 g_Z$ so that neutrino screening of Z^0 charge at level n_Z is rather effective.

Which of the mechanisms is correct or are both mechanisms at work? In order to answer this question one should verify experimentally whether the vortices observed in a rotating super fluid are really velocity vortices or Z^0 magnetic vortices or both. Since the critical velocity for the Z^0 magnetic vortices is smaller than for the hydrodynamical vortices, one might argue that at critical angular velocity Z^0 magnetic vortices appear and hydrodynamic vortices appear for larger angular velocities. Some indirect support for the TGD based scenario indeed exists. The study of the rotating ^3He has demonstrated that the angular velocity Ω and ordinary magnetic field B play very similar physical role in the texture of the rotating ^3He and that the texture of ^3He is rather sensitive to both these parameters. In TGD picture one can replace Ω and B by B_Z and B and a rich structure of the quantized excitations is predicted.

7.2.6 Phase Slippage

The so called phase slippage [D12] provides a mechanism for the dissipation in the case o superfluids. Also this phenomenon has natural interpretation in terms of the flux quantization. The conventional description of the phase slippage is in terms of angle like order parameter χ. For linear flow the order parameter behaves linearly as a function of the coordinate x in the direction of the flow

$$\chi(x) = kx , \tag{7.2.46}$$

where k can be interpreted as the momentum of the super fluid particle.

In the phase slippage the graph of $\chi(x)$ as a function of x is deformed so that χ jumps by an integer multiple of 2π at some point x_0 and stays linear for $x \leq x_0$ and for $x \geq x_0$. The value of k must however decrease for $x \geq x_0$ and this means that the momentum of the super fluid particle decreases and dissipation occurs. Since the discontinuity is multiple of 2π the graph can be replaced with a new one without any discontinuity and smoothed out so that the graph of χ is linear with new value of the momentum k. The change in the momentum k is quantized:

$$\Delta k = n\frac{2\pi}{L} , \qquad (7.2.47)$$

where L is the length of the channel. The process corresponds physically to the propagation of the vortex generated at the wall of the channel across the channel under the action of Magnus and friction forces and the integer n associated with the vortex ($\chi = n\phi$) equals to the integer associated with the Δk.

The process has obvious geometric interpretation in TGD approach. The angles Ψ and Φ are the counterparts of the angle like order parameter χ and phase slippage corresponds to the propagation of a vortex ($r = 0$ at the axis of the vortex and $r = \infty$ at the surface of the vortex) through the channel. In general the vortex is characterized by two integers n_1 and n_2. It has been already shown that the ordinary hydrodynamical dissipation and generation of turbulence might be understood in terms of the phase slippage process: the only difference with respect to the super fluidity is that the integers n_i and frequencies ω_i are much larger now: ordinary hydrodynamical system is obtained from the super fluid in the limit of the large quantum numbers.

7.3 Models For The Topological Field Quanta

In the sequel simple models for the electromagnetic and Z^0 gauge fields created by condensed matter are studied. The aim is to get some grasp on the physically reasonable values of the vacuum parameters appearing in the embedding by using as experimental input the values of coherence length ξ and critical magnetic fields. Two kinds of embeddings are studied.

1. Spherically symmetric, electrovac embedding of Z^0 condensate levels $n \geq n_Z$) or ordinary electric field (condensate levels $n < n_Z$) created by matter serves as a simple model for the topological field quanta in the ordinary condensed phase.

2. Cylindrically symmetric field quantum serves as an idealization for the linear structures obtained by glueing spherically symmetric topological field quanta together using joing along boundaries operation and is interesting as a model for the core of various vortex like structures. Several embeddings of this kind are constructed.
   i) An embedding of cylindrically symmetric em/Z_0 electric field for matter at rest is constructed assuming that matter density serves as the source of em/Z_0 electric field.
   ii) By applying a boost in the direction of cylinder axis an embedding of the em/Z_0 magnetic field associated with say super fluid flow is obtained.
   iii) Allowing non-vanishing quantum numbers n_i an embedding of a constant Z^0/m magnetic magnetic field in the direction of the cylinder axis is obtained. The requirement that the magnetic flux of this field is quantized in the standard manner, poses and additional condition on the vacuum parameters. One can construct ordinary magnetic fields in the length scales $n \geq n_Z$ as deformations of Z^0 electric field configuration. As a consequence of the construction procedure, the critical radius of all these embeddings depends on the properties of the matter only.

The dependence of the critical radii on the vacuum quantum numbers is studied and estimates for the vacuum numbers of topological field quanta are deduced Ordinary phase with $\omega_1 \sim m_0 \sim 10^{-4} m_{Pl}$ is shown to correspond to the large quantum number limit in the sense that the critical radii are macroscopic and therefore also magnetic flux m as well as the quantum numbers ω_i and n_i are very large. The embedding of the magnetic field is obtained non-perturbatively in the sense that the change Δn_i needed to generate the magnetic field satisfies the condition $\Delta n_i / n_i \gg 1$.

Supra phases correspond to the small quantum number limit and to Z^0 neutral space-times: using ξ and B_c as inputs, it is found that the parameter ω_1 is of the order of $10^{2.5} - 10^3$ proton masses. The assumption that ω_1 is same for all super conductors implies $\omega_2/\omega_1 \ll 1$, which condition in turn is necessary condition for Meissner effect to take place. The value of the fractal quantum number m is assumed to be zero for ^4He and -2 for the other supra phases.

the non-vanishing value of m affects radically the value of ϵ_1 so that estimates have considerable uncertainties.

7.3.1 The Kähler Field Created By A Constant Mass Density

In the following the em/Z^0 electric field created by an Z^0/em neutral, constant mass distribution assuming that mass distribution serves as a source of pure em/Z^0 field proportional to Kähler field, are studied. Although the mass distribution itself is homogeneous, Kähler electric field necessarily breaks translational symmetry. Concerning the applications in mind, the breaking of the translational symmetry to the spherical or cylindrical symmetry is the most natural one and will therefore be considered in the sequel. Also the embedding of a spherically (cylindrically) symmetric Kähler electric field can break spherical (cylindrical symmetry) since several gauge potentials are possible by gauge invariance and different gauges are related by the canonical transformations of CP_2 and correspond to different four-surfaces: it is assumed however that embedding is spherically (cylindrically) symmetric, too. What makes the cylindrically symmetric field configuration so interesting is that one can construct several physically interesting field configurations from it by modifying the values of the vacuum quantum numbers so that electrovac conditions cease to hold true.

To begin with, recall the conditions guaranteeing the vanishing of either Z^0 or electromagnetic gauge fields

$$\begin{aligned} r &= tan(X) \ , \quad \Psi = k\Phi \ , \\ X &= \frac{ln(|(u+k)/C|)\epsilon}{2} \ . \end{aligned} \tag{7.3.1}$$

One must chose the branch of arcus tangent in the expression of X in terms of r and this implies the condition $m\pi \leq X \leq (2m+1)\pi/2$, where m is an integer fixing the branch of the arcustangent and will be referred to as euantum number. The following remarks are useful for what follows:

1. The vanishing of the Z^0 field is achieved for

$$\epsilon = \epsilon(em) = \frac{1}{2} \ ,$$

and the vanishing of the electromagnetic field is achieved for

$$\epsilon = \epsilon(Z) = \frac{(3+p)}{(3+2p)} \ ,$$

($p = sin^2(\theta_W) \simeq 0.234$).

2. The CP_2 projection of the embedding is two-dimensional, which implies the orthogonality of the magnetic and electric fields belonging to same condensate level. Z^0/em field is proportional to induced Kähler form for the embeddings in question

$$\begin{aligned} \gamma &= k_{em}J = a_{em}sin^2X du\wedge d\Phi \ , \\ k_{em} &= 3 \ , \quad a_{em} = -\frac{3}{4} \ , \\ Z^0 &= k_Z J = a_Z sin^2X du\wedge d\Phi \ , \\ k_Z &= \frac{6}{p} \ , \quad a_Z = -\frac{3}{3+p} \ . \end{aligned} \tag{7.3.2}$$

One consequence of $F_{em} = 3J$ is that the # throats feeding magnetic flux to/from a purely electromagnetic condensate level behave on given space-time sheet as magnetic monopoles with magnetic charge quantized in multiples of the magnetic charge associated with the ordinary Dirac monopole: what is peculiar is that the magnetic charge is divisible by 3. As quantum effects are considered the # throats behave as extremely tiny magnetic dipoles.

3. Electromagnetic/Z^0 charge density of matter is assumed to serve as source of em/Z^0 fields and in the idealization that matter consists of identical nuclei (A, Z) one can write the charge density as

$$\begin{aligned}
\rho_{em} &= \frac{e^2}{\sqrt{\epsilon_{em}}}\frac{Z}{A}N = K_{em}N \ , \\
\rho_Z &= -\frac{g_Z^2}{4\sqrt{\epsilon_Z}}\frac{A-Z}{A}N = K_Z N \ ,
\end{aligned} \tag{7.3.3}$$

where N is the density of the nucleons. It has been assumed that only neutrons contribute to the nuclear Z^0 charge.

The formulas associated with the spherically and cylindrically symmetric embeddings differ from each other by numerical factors only and the cylindrically symmetric case will be considered first. Assuming cylindrical symmetry em/Z^0 electric field is radial and its magnitude is given by

$$\begin{aligned}
|E_\rho^{em}| &= \delta K_{em}\frac{N\rho}{2} \ , \\
|E_\rho^Z| &= \delta K_Z \frac{N\rho}{2} \ , \\
\delta &= 1 \ ,
\end{aligned} \tag{7.3.4}$$

The numerical factor δ is introduced in order to generalize the results to spherically symmetric case easily.

Cylindrically symmetric embedding of the em/Z^0 electric field is obtained through the ansatz

$$\begin{aligned}
\Phi &= \omega_1 t \ , \quad \Psi = \omega_2 t \ , \quad u = u(\rho) \ , \\
k &= \frac{\omega_2}{\omega_1} \ .
\end{aligned} \tag{7.3.5}$$

One can define $\omega_1 = m_p\sqrt{(\epsilon_i)}x$, where x is numerical factor not very far from unity in astrophysical scales. The dependence of u on ρ is fixed from the imbeddability condition for the appropriate electric field

$$\begin{aligned}
\frac{a_i}{k_i}sin^2X\partial_\rho u\omega_1 &= \delta K_i N\frac{\rho}{2} \ , \\
i &= em, Z^0 \ .
\end{aligned} \tag{7.3.6}$$

From this expression one can integrate u as a function of ρ

$$\int_{u_0}^{u} sin^2(X(u))du = \delta\frac{K_i k_i}{a_i\omega_1}N\rho^2 \ . \tag{7.3.7}$$

This equation determines the value of the critical radius of the embedding as a function of u_0, the value of u at $r=\infty$ surface provided $u=0$ at $r=0$ surface. Performing the integral, one obtains the condition

$$\begin{aligned}
\rho_{cr} &= \sqrt{\frac{2a_i\omega_1}{\delta K_i k_i N}}\sqrt{2(u_0+k)}exp(-m\pi/\epsilon(i))X(\epsilon(i)) \ , \\
X(\epsilon) &= \sqrt{\frac{(2+\epsilon^2)exp(\pi/\epsilon)+\epsilon^2)}{(1+\epsilon^2)}} \ , \\
i &= em, Z^0 \ .
\end{aligned} \tag{7.3.8}$$

Here u_0 is the value of $u = cos(\Theta)$ at the axis of the vortex ($k = \omega_2/\omega_1$) and various parameters with index i are defined in the previous formulas.

The general orders of magnitude become clear, when one writes the formula in a numerical form by using the density $N_0 = 10^{30}/m^3$ is a reference density of atomic nuclei.

1. In electromagnetic case one obtains

$$\begin{aligned} \rho_{cr} &\simeq X \cdot 3.7 \cdot 10^{-6} \text{ meters} , \\ X &= \sqrt{(u_0+k)}\sqrt{\epsilon_{em} x}\sqrt{\frac{A}{Z}\frac{N_0}{N}}\frac{1}{\sqrt{\delta}}10^{-2.7288m} , \\ \omega_1 &= \sqrt{\epsilon_{em}} x m(proton) . \end{aligned} \tag{7.3.9}$$

   The critical radius for spherically symmetric embedding is obtained by replacing $\delta = 1$ with $\delta = 2/3$.

2. In Z^0 case one obtains

$$\begin{aligned} \rho_{cr} &\simeq X \cdot 7.75 \cdot 10^{-7} \text{ meters} , \\ X &= \sqrt{(u_0+k)}\sqrt{\epsilon_Z x}\sqrt{\frac{A}{(A-Z)}\frac{N_0}{N}}\frac{1}{\sqrt{\delta}}10^{-1.46m} , \\ \omega_1 &= \sqrt{\epsilon_{em}} x m(proton) . \end{aligned} \tag{7.3.10}$$

   for $p = sin^2(\theta_W) = 1/4$.

The previous formulas contain still unknown parameters ($u_0 + k, x$) but order of magnitude estimates are possible for the critical radius since the value of $u_0 \leq 1$ is not expected to be anomalously small.

For the em neutral space-time there are two especially interesting special cases.

1. For $\sqrt{\epsilon_Z} \sim 10^{18}$ (so that Z^0 force is of the same order of magnitude as gravitational force) and for $m = 0$ critical radius is about 10^{11} m, which is roughly the size of the solar system.

2. For $\sqrt{\epsilon_Z} \sim 10^{10}$ (level n_Z) and for $m = 0$ one has $\rho_{cr} \sim 10^3$ m in typical condensed matter densities.
   For Z^0 neutral space-time expected to be important in sub-cellular length scales $m = 0$, $x = 1$ and $\epsilon_{em} = 1$ (no charge screening by electrons) the critical radius is about 10^{-6} meters. If one assumes $\omega_1 = \epsilon_{em} m_e x$ (replacing $m(proton)$ by m_e) with $x \sim 1$ one obtains critical radius of order $10^{-8} - 10^{-7}$ meters, which is of same order of magnitude as characteristic length parameters for super conductors. Same is achieved by assuming $m = -1$ instead of $m = 0$.

Critical radius depends exponentially on the value of the integer m and the embeddings with different values of m are related by a discrete scale transformation $\rho_{cr} \rightarrow exp(-m\pi/\epsilon)\rho_{cr}$: the "fundamental" change of scale is given $exp(\pi/\epsilon) \simeq 28.9$ in the electromagnetically neutral case (note the dependence on $sin^2(\theta_W)$) and by 535.5 in the Z^0 neutral case. Of course, it is not at all obvious whether the scaled up surfaces are structurally stable.

Using the BCS expression and TGD based estimate for the binding energy of the Cooper pairs, one obtains the formula

$$\rho_{cr} \simeq \frac{1}{\sqrt{m_e}\Delta} exp(\frac{1}{N(0)V}) , \tag{7.3.11}$$

which gives relationship between vacuum parameters and parameters of BCS model [D15].

7.3.2 The Embedding Of A Constant Magnetic Field

The embedding of constant em/Z^0 magnetic field is obtained from the corresponding electric field associated with the constant mass density assuming that Ψ and Φ depend also on the angle ϕ

$$\begin{aligned} \Phi &= \omega_1 t + n_1\phi \ , \quad \Psi = \omega_2 t + n_2\phi \ , \quad u = u(\rho) \ , \\ k &= \frac{\omega_2}{\omega_1} = \frac{n_2}{n_1} \ . \end{aligned} \tag{7.3.12}$$

The condition $n_2/n_1 = k$ guarantees electromagnetic neutrality. Magnetic fields are in the direction of the z-axis and their magnitudes are given by the expression

$$\begin{aligned} |B_i| &= |\frac{n_1}{\omega_1}\frac{E_i}{\rho}| = \frac{n_1 N}{\omega_1}\delta\frac{K_i}{2} \ , \\ i &= em, Z^0 \ . \end{aligned} \tag{7.3.13}$$

and are indeed constant.

The magnetic flux associated with the topological field quantum is in the electromagnetic case given by

$$\Phi = \int B_{em} da = -n_1\frac{3}{4}(u_0+k)exp(-4m\pi)\frac{(9exp(2\pi)+1)}{5}\pi \ . \tag{7.3.14}$$

The quantitization of the magnetic flux gives a condition for the parameters u_0 and k. The requirement that the flux is quantized in multiples of the elementary flux quantum irrespective of the value of n_1 implies the condition

$$\begin{aligned} \frac{3}{4}(u_0+k)exp(-4m\pi)\frac{(9exp(2\pi)+1)}{5} &= \frac{1}{n} \ , \\ n &= 1 \ . \end{aligned} \tag{7.3.15}$$

The more general condition $n > 1$ corresponds to the assumption that n_1 is multiple of of n.

Applying this condition to the expression for the critical radius, one has

$$\begin{aligned} \rho_{cr} &= \sqrt{\frac{A}{Z}}\sqrt{\epsilon_{em}x}\sqrt{\frac{2}{e^2}\frac{m(proton)}{N}}\frac{1}{\sqrt{n}} \\ &\sim \sqrt{\frac{A}{Z}}\sqrt{\epsilon_{em}x}\frac{1}{\sqrt{n}}\cdot 1.6\cdot 10^{-7} \ \text{meters} \ , \\ B^{em} &= \frac{2n_1}{\rho_{cr}^2} = 2n_1 n\frac{Z}{A}\frac{e^2}{2\epsilon_{em}x}\frac{N}{m(proton)} \ . \end{aligned} \tag{7.3.16}$$

The requirement that the radius of the flux quantum is of order $10^{-8} - 10^{-7}$ meters (magnetic penetration length for the super conductor) gives in $n = 1$ case the estimate $\sqrt{\epsilon_{em}x} \sim 1$ at the condensation level in question. Since $\epsilon_{em} \geq 1$ holds true this means that $x < 1$ must hold true. An alternative possibility is that $n > 1$ holds true instead of $n = 1$. The third possibility is that the imbeddability condition gives only an upper bound for the critical radius and that stability conditions give additional constraints. An additional restriction for the values of the free parameters comes from the requirement that the critical magnetic field ought to be of the order of $B_{cr} \simeq 10^{-2}$ $Tesla$ for the super conductors of type I and larger for the super conductors of type II. The critical magnetic field obviously corresponds to the smallest possible magnetic field allowed by the flux quantization and this estimate does not give anything new at order of magnitude level.

The quantization of the Z^0 magnetic flux gives

$$
\begin{aligned}
a_Z(u_0+k)exp(-2m\pi/\epsilon(Z))C(\epsilon(Z)) &= \frac{1}{n} \ , \\
n &= 1 \ ,
\end{aligned}
\tag{7.3.17}
$$

and reduces the expression for the critical radius and magnetic field to the form

$$
\begin{aligned}
\rho_{cr} &= \sqrt{\frac{A}{(A-Z)}}\sqrt{\epsilon_Z x}\sqrt{\frac{8}{g_Z^2}\frac{m(proton)}{N}} \ , \\
B^Z &= \frac{2n_1}{\rho_{cr}^2} \\
&= n_1 n\frac{(A-Z)}{A}\frac{g_Z^2}{4\epsilon_Z x}\frac{N}{m(proton)} \ ,
\end{aligned}
\tag{7.3.18}
$$

completely analogous to the expressions deduced in the electromagnetic case.

In $n > n_Z$ case Z^0 magnetic fields are expected to dominate over the Z^0 electric fields: the reason is that the screening neutrinos probably do not contribute to the Z^0 gauge current density acting as the source of Z^0 magnetic field but contribute to Z^0 charge density causing a very effective screening. This means that the the source of Z^0 magnetic field at level n corresponds to the Z^0 charge density (and ϵ_Z) associated with level $n-1$. In particular, at level n_Z there is no screening for Z^0 magnetic field. For $n > n_Z$ one can generate approximately constant ordinary magnetic fields by giving up the condition $n_2/n_1 = \omega_2/\omega_1$. The expression for the magnetic field strength is given by

$$
\begin{aligned}
|B^{em}| &= \frac{(3+p)(3+2p)}{6}B^Z \\
&= 2n_1\frac{(3+p)(3+2p)}{6}\frac{(A-Z)}{A}\frac{g_Z^2}{8\epsilon_Z x}\frac{N}{m(proton)} \ , \\
p &= sin^2(\theta_W) \ ,
\end{aligned}
\tag{7.3.19}
$$

where the quantization condition for Z^0 flux is used (the least one can hope is that one might fix the orders of magnitudes correctly for free parameters). At the level $n = n_Z$ one can generate fields of order one Tesla (Tesla corresponds roughly to $N/m(proton)$) at small quantum number limit ($\epsilon_Z(n_Z-1) = 1$). At the next level the field of one Tesla requires $n_1 \sim 10^{20}$ for $\epsilon_Z x \sim 10^{20}$ so that large quantum number limit is in question.

7.3.3 Magnetic Fields Associated With Constant Velocity Flows

One can construct a simple candidate for the Kähler magnetic field associated with a fluid flow with a constant velocity by boosting the cylindrically symmetric Kähler electric field in the direction of the cylinder axis:

$$
\begin{aligned}
\Phi &= \omega_1 t + k_1 z \ , \quad \Psi = \omega_2 t + k_2 z \ , \quad u = u(\rho) \ , \\
k_i &= \omega_i \beta \ .
\end{aligned}
\tag{7.3.20}
$$

The field lines are circles around the z-axis and the strength of the Kähler magnetic and Z^0 magnetic fields are given by

$$
\begin{aligned}
|B_K| &= |\frac{k_1}{\omega_1}E_K| \ , \\
B_Z &= \frac{3}{sin^2(\theta_W)}B^K \ .
\end{aligned}
\tag{7.3.21}
$$

Super fluid flow is a natural application for this mechanism for generating magnetic field. In this case the cylindrical symmetry of the Kähler electric field is indeed very natural. Note that the although the flux tubes are in the direction of the flow the critical radius doesn't depend on the flow velocity.

In order to obtain non-vanishing magnetic field (associated, say, with super conducting current) one must give up the condition that the field is obtained by a boost. For example, one can assume that $k_1 \neq \omega_1\beta$. An interesting possibility is that the magnetic field associated with the super conducting current is obtained in this manner. It should be noticed that one can obtain also helical magnetic fields by performing boost to a configuration with non-vanishing magnetic field.

7.4 Quantum Hall Effect From Topological Field Quantization

The concept of the topological field quantum and the ideas about the formation of macroscopic quantum systems and about the topological description of the dissipation provide a classical TGD based description of Quantum Hall effect very similar to that found for supra phases. This approach was proposed for long time ago and later I have proposed an approach based on the hierarchy of Planck constants assumed to represent phases of dark matter [K68].

7.4.1 The Effect

Consider first briefly the effect. The effect is observed two-dimensional systems consisting of a conducting slab in a strong magnetic field perpendicular to the slab. When potential difference V is applied in the y-direction of the slab (see **Fig. 7.1**), the Lorentz force induces a transversal current. The current is proportional to the electric field associated with the potential:

$$j_x = \sigma_{xy}E_y \ , \tag{7.4.1}$$

where σ_{xy} is the transversal conductivity.

Two kinds of effects have been observed at low temperatures ($T \simeq 1\ K$) and using strong magnetic fields $B \simeq 10\ T$.

1. In integer quantum Hall effect σ_{xy} is quantized in units of the fine structure constant

$$\sigma_{xy} = n \times 2\alpha \ , \tag{7.4.2}$$

   where n is integer (see **Fig. ??**).

2. In the fractional quantum Hall effect σ_{xy} is quantized in fractional units

$$\sigma_{xy} = \frac{n}{m} \times 2\alpha \ , \tag{7.4.3}$$

   where the integer m is fixed. Several values of m have been found to be possible.

7.4.2 The Model

One can understand Quantum Hall effect in TGD framework using the following arguments.

Conduction electrons as a mesoscopic quantum system

Assume that in the Quantum Hall phase conduction electrons form a mesoscopic quantum system, which means that topological field quanta with size of the order of $\xi \simeq 10^{-8} - 10^{-7}$ meters are glued together by flux tube to form a lattice like structure. The bonds must be stable since otherwise their splitting and rejoining causes an additional dissipation contributing to the transversal conductivity and Quantum Hall effect is lost. The estimate for the critical temperature $T_c \simeq 1/2m_e\xi^2$ used for the supra phases applies also now and correctly gives $T_c \simeq 1\ K$.

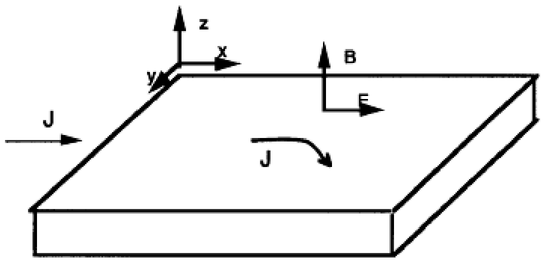

Figure 7.1: Quantum Hall effect

How to avoid the splitting of the joing along boundaries bonds in a strong magnetic field

Since a strong magnetic field (of the order of few Tesla) is present, individual topological field quanta are excited to $(n_1, n_2) \neq 0$ states. There are *two* possible ways to avoid the breaking of the bonds between the neighbouring topological field quanta:

1. The condition $n_1 = 0$ is satisfied for all topological field quanta. $n_1 = 0$ field quanta are favored if the condition

$$k \equiv \omega_2/\omega_1 \gg 1 \ , \tag{7.4.4}$$

is satisfied so that $n_2 = 0$ quanta have much larger magnetic flux than $n_1 = 0$ field quanta. If $k \gg 1$ condition is satisfied, the magnetic field inside the flux quantum can change in discrete, but sufficiently small, steps, when external magnetic field is varied. For the values of the vacuum quantum numbers encountered for the supra phases, the value of n_2 ought to be rather large, of the order of $10 - 100$ in Quantum Hall phase. A possible problem of this scenario is that the flux associated with the $n_1 = 1$ quantum is of same order as the flux of the external magnetic field: why this excitation is not generated?

The $k \ll 1$ condition encountered in the case of supra phases leads to difficulties. The magnetic field associated with $n_1 = 0$ excitations is large and of the order of the external magnetic field if same values for vacuum quantum numbers are assumed as for the supra phases so that external could excite these excitations. The problem is that the magnetic field associated with $n_1 \neq 0$ excitations is much smaller and its is difficult to understand why the variation of the external magnetic field does not not excite them (with the consequence that Quantum Hall phase disappears).

2. The condition $u = cos(\Theta) = \pm 1$ is satisfied on the $r = \infty$ boundaries of the field quanta. In this case both n_1 and n_2 can vary freely. For the magnetic fields used and for the values of parameters found for supra phases n_1 should be of the order of $n_1 = 1$ and n_2 can have much larger values. This makes possible the variation of the magnetic flux inside the field quantum in discrete steps, the step being however reasonably small. Thus it seems that this alternative is the physical one.

Quantization conditions

Assume that the quantization conditions

$$\int \bar{B}_{em} \cdot d\bar{a} - m \oint \bar{v} \cdot d\bar{l} = \frac{m \times 2\pi}{qe} \ , \tag{7.4.5}$$

encountered in the case of the supra phases are satisfied in Quantum Hall phase, too. Since the magnetic flux inside the topological field quanta is quantized in multiples of certain basic unit associated with n_1, which is much smaller than the standard flux quantum, the velocity field must adjust itself inside each flux quantum so that the quantization condition is satisfied. This is achieved if the velocity field is a super position of two terms

$$\bar{v} = \bar{v}_0 + \bar{v}_{rot} \ , \tag{7.4.6}$$

where $\bar{v}_0$ is essentially constant velocity field associated to the Hall current and $\bar{v}_{rot}$ is a local velocity field inside the topological field quantum, whose function is to cancel the failure of the magnetic field to satisfy the standard flux quantization condition

$$m_e \oint \bar{v}_{rot} \cdot d\bar{l} = -\frac{m2\pi}{qe} + \int \bar{B}_{em} \cdot d\bar{a} \equiv -\int \Delta\bar{B} \cdot d\bar{a} \ . \tag{7.4.7}$$

Here $\bar{v}_{rot}$ corresponds to a rigid body rotation in the constant magnetic field $\Delta\bar{B}$, which is the difference between the actual field and the field for which magnetic flux is quantized in standard units. Obviously, the external magnetic field must be so strong that the flux through a topological field quantum is of the order of the field quantum: otherwise unrealistically large local velocities are needed to guarantee quantization condition (or m would be equal to zero).

Carriers of the Hall current as an incompressible 2-dimensional liquid

Assume that the carriers of the Hall current behave like an incompressible, two-dimensional liquid (this assumption is made in the competing models, too [D3]). Assume also that the Euler equations are satisfied and write them into the following form

$$n_e m_e \frac{\partial \bar{v}}{\partial t} = -\nabla p - n_e m_e \nabla(\frac{v^2}{2}) + n_e m_e \bar{v} \times (\nabla \times \bar{v}) + n_e q e(\bar{E} + \bar{v} \times \bar{B}) \ . \tag{7.4.8}$$

Here n_e is the number of Hall current carriers per unit area orthogonal to the direction of magnetic field and m_e is the mass of the current carrier (electron).

Stationary state

The stationary situation for which the velocity can be decomposed in the manner already described is characterized by the conditions

$$\begin{aligned} \bar{v} &= \bar{v}_0 + \bar{v}_{rot} \ , \\ \frac{\partial \bar{v}}{\partial t} &= 0 \ , \\ \nabla[p + n_e m_e(\frac{v^2}{2} - \frac{v_{rot}^2}{2})] + X &= 0 \ , \\ X &\equiv n_e q e \bar{v}_{rot} \times \bar{B} \ . \end{aligned} \tag{7.4.9}$$

The remaining equation leads to the formula for transversal conductivity

$$n_e m_e \bar{v}_0 \times (\nabla \times \bar{v}_{rot}) + q e n_e \bar{E} + q e n_e \bar{v}_0 \times \bar{B} = 0 \ . \tag{7.4.10}$$

Before deriving the expression for the transversal conductivity it is useful to verify that the solution ansatz works. One can subsitute to the quantity $X \equiv \bar{v}_{rot} \times \bar{B}$ the expression of $\bar{v}_{rot}$ obtained from quantization condition (rigid body rotation) and one finds that this term is also expressible as a gradient: $X = a\nabla(B^2\rho^2)$, where a is some numerical constant. This implies that second condition reduces to a condition of form

$$p + n_e m_e(\frac{v^2}{2} - \frac{v_{rot}^2}{2}) + n_e qeaB^2\rho^2 \quad = \quad p_0 = \text{constant} \ . \tag{7.4.11}$$

This condition is a local condition referring to the properties of the flow inside the topological field quanta and is not essential for Quantum Hall effect.

Hall current

In order to obtain expression for the Hall current one can integrate the third condition involving Lorentz force over a transversal (orthogonal to B) surface area associated with one or more topological field quanta. One obtains the following expression for the Hall current $\bar{j}_H = qen_e\bar{v}_0$

$$\begin{aligned} j_H &= \sigma_{xy}E \ , \\ \sigma_{xy} &= -e\frac{\int n_e da}{(\int \bar{B}\cdot d\bar{a} - m_e \oint \bar{v}\cdot d\bar{l})} \ . \end{aligned} \tag{7.4.12}$$

Same expression can be obtained also directly form the Euler equations under much milder assumptions by integrating the x- component of the equations over the surface area. All the terms in Euler equation and not appearing in the formula for the Hall current (∇v^2, ∇v_{rot}^2, ∇p, $\bar{v}_{rot}\times\bar{B}$) give vanishing contribution to the integral over the field quantum provided they correspond to the variations of the physical quantities, whose average vanishes in length scales larger than the size of the topological field quantum.

One can write this formula in a form exhibiting fractional Quantum Hall effect by noticing that the integral $\oint n_e da$ is just n_{free}, the number of the carriers of Hall current inside the topological field quantum (or the several of them) and is quantized! The general quantization condition in turn implies that the denominator is integer multiple of $2\pi/qe$. What one obtains is the following formula for the transversal conductivity

$$\sigma_{xy} \quad = \quad -\frac{n_{free}e^2}{m2\pi} \ . \tag{7.4.13}$$

One obtains integer quantum Hall effect for $m = 1$ and fractional quantum Hall effect for $m \geq 1$.

Comments

Some comments concerning the proposed scenario are in order.

1. For a macroscopic quantum system consisting of a very large number of the topological field quanta n_{free} and m are so large that the value of the conductivity is practically continuous without any further assumptions. If one however assumes that the values of m and the number of the free charge carriers are same for all topological field quanta then it is possible to realize the situation, where n_{free}/m can be written as a ratio of small integers.

2. All integer values for m (in accordance with the experimental facts!) are possible (not only odd integers as in case of the anyon super conductivity in its simplest version [D11]). m corresponds to the angular momentum of an electron rotating around the flux tube in accordance with the Laughlin's proposal for the state functions of charge carriers [D11]. Since $m = 1$ angular momentum is expected to be most probable in low temperatures and for low magnetic fields, fractional quantum Hall effect is expected to be more rare phenomenon than integer Hall effect.

3. When magnetic field is kept as constant and potential V is varied the number of the free charge carriers inside the flux quantum changes in discrete steps at some critical values of the potential so that plateaus of σ_{xy} result. When magnetic field is varied compensating, velocity fields inside the field quanta are generated in order to preserve the quantization condition. When magnetic field is suitable, a change in the vacuum quantum number n_2 and possibly n_1 takes place and the rotational velocity field $\bar{v}_{rot}$ goes to zero. This doesn't lead to a change of the transversal conductivity in general. When total magnetic flux becomes sufficiently near to its quantized value also the integer m characterizing the flux quantum can change so that the fractional number characterizing quantum Hall effect changes. This kind of a transition can be regarded as a phase transition taking place in the whole specimen.
4. The proposed explanation differs from the more standard explanations in some respects.
   (a) The concept of fractional filling fractions follows from the quantization conditions and from the concept of the topological field quantum.
   (b) No reference is made to fractional statistics or to fractional electric charges.
   (c) The situation $m = 0$ is particularly interesting physically. In this case the transversal Hall conductivity is formally infinite. The only reasonable solution of the Euler equation in this case seems to be that for which the velocity in the transversal direction vanishes so that Hall effect and magnetic field is effectively absent (!) and classically (probably not quantum mechanically) there is a continuous acceleration in the direction of the electric field. Clearly the slab behaves as a super conductor apart from the presence of $\bar{v}_{rot}$ term in velocity.
   (d) The standard models for the fractional Quantum Hall effect predict also super conductivity together with the breaking of CP invariance. In present case the presence of the classical Z^0 electric vacuum fields suggests small parity breaking. This effect takes however place in ordinary supra phases, too and possibly in all condensed matter systems.

7.5 TGD And Condensed Matter

In previous sections we have applied TGD to a rather exotic condensed matter phenomena. Quite contrary to the original expectations it has turned out that TGD might have applications to less exotic condensed matter phenomena, too. In fact, it seems that TGD might be applied to reformulate the description of conductors, di-electrics, and magnetism using topological concepts.

7.5.1 Electronic Conductivity And Topological Field Quantization

The standard Drude model for conductors [B29] starts from the equilibrium condition $dv/dt = v/\tau - eE/m_e = 0$ to derive the expression for the conductivity of a metal as $\sigma = Ne^2\tau/m_e$. τ is interpreted as the average time between two collisions and is obtained from the estimate $\tau \simeq a/v_{th}$, where a is the distance between atoms and v_{th} is thermal velocity. The estimate is by a factor $10^2 - 10^3$ too small at low temperatures and approaches the observed conductivity at high temperatures only. A correct order of magnitude estimate is obtained if a is replaced with the size $\xi \simeq 10^{-8} - 10^{-7}$ meters of topological field quantum in accordance with the idea that ordinary metal behaves as a super conductor at length scales smaller than ξ. The decrease of the conductivity at higher temperatures can be understood, too: the joing along boundaries bonds between atoms become more and more unstable as the temperature is increased.

7.5.2 Dielectrics And Topological Field Quantization

Why do electrons then move freely in length scales smaller than ξ? This can be understood by introducing a TGD based description of a dielectric to be discussed in more detail later. The point is that there are two condensation levels present. This means that the electric flux D (electric displacement) associated with a test charge divides into two parts. First part P (polarization) flows at the first level of the condensate (in particular along the bonds joining topological field quanta of atomic size). Second part E (electric field flows at the background space-time, which corresponds to a larger space-time sheet. Since total electric flux is conserved, the fractions of electric flux sum up to one: $1/\varepsilon_1 + 1/\varepsilon_2 = 1$ ($D = E + P$), where the fractions are defined in terms of the dielectric constants ε_1 and ε_2 associated with the two levels of condensation. For an ideal conductor all electric flux runs to the larger space-time sheet and there are no electric fields at the first level of the condensate: electrons move freely! For and ideal di-electric all electric flux flows at the first level of condensation and strong electric fields are associated with the join along boundaries bonds/flux tubes.

7.5.3 Magnetism And Topological Field Quantization

Same kind of argumentation should work in case of magnetism, too. The magnetic flux H created by a test current can be decomposed to two parts. The first part M (magnetization) flows through the first level of the condensate and second part B (magnetic field) flows through the larger space-time sheet. Again one can associate susceptibilities μ_1 and μ_2 ($\mu_1 + \mu_2 = 1$) to both levels of the condensate to describe the properties of a simple magnetic substance.

The mechanism underlying spontaneous magnetization is not very well understood [B19, B29], [D16] and an interesting question is whether the magnetic domains in the spontaneous magnetization could be understood using TGD based concepts. The quantization of the field strength for a flux quantum implies that macroscopic magnetization results if the magnetic fields of $n_1 \geq 0$ excitations associated with these flux quanta are oriented in parallel. From the known values of the magnetic fields in ferromagnets and from the sizes of the magnetized domains it is possible to estimate the values of $\omega_1$andthe fractal quantum number m. The typical values of the magnetic fields are of the order of 10^{-2} Tesla and stable domains of magnetization are known to have size of the order of 10^{-8} meters. The fact that the orders of magnitude are same as for super conductors suggests that the sizes of the topological field quanta do not depend strongly on the properties of the condensed matter system.

In the phenomenological theory of the ferromagnetism the so called Weiss molecular field appears [B29]. If this field is present then the magnetic moments of individual electrons are oriented parallel and magnetization is essentially the density of the magnetic moments per volume: $M \propto N_e \mu_e$. The problem is that this field is very large, having magnitude of the order of 10^3 Tesla, which is about 10^5 times large than the actual magnetic field!

Standard explanation is that this field is only an effective field giving a short hand description of essentially quantum level phenomena (so called exchange interaction between electrons, which favors parallel spins for the electrons of the neighboring atoms). A possible TGD based classical explanation is that there is indeed magnetic field of this strength present. This field is present at the "zeroth" level of condensation that is inside the field quanta having atomic size (which are glued together by the flux tubes). Again the field strength is quantized and flux quantum is related by the scaling factor $(\xi_1/\xi_0)^2 \simeq 10^4 - 10^6$ to the magnetic field quantum at the first condensation level. The order of magnitude is indeed correct!

Part II

TGD AND GRT

Chapter 8

The Relationship Between TGD and GRT

8.1 Introduction

In this chapter the recent view about TGD as Poincare invariant theory of gravitation is discussed. It must be admitted that the development of the proper interpretation of the theory has been rather slow and involved rather weird twists motivated by conformist attitudes. Typically these attempts have brought into theory ad hoc identifications of say gravitational four-momentum although theory itself has from very beginning provided completely general formulas.

Perhaps the real problem has been that radically new views about ontology were necessary before it was possible to see what had been there all the time. Zero energy ontology states that all physical states have vanishing net quantum numbers. The hierarchy of dark matter identified as macroscopic quantum phases labeled by arbitrarily large values of Planck constant is second aspect of the new ontology.

8.1.1 Does Equivalence Principle Hold True In TGD Universe?

The motivation for TGD as a Poincare invariant theory of gravitation was that the notion of four-momentum is poorly defined in curved space-time since corresponding Noether currents do not exist. There however seems to be a fundamental obstacle against the existence of a Poincare invariant theory of gravitation related to the notions of inertial and gravitational energy.

1. The conservation laws of inertial energy and momentum assigned to the fundamental action would be exact in this kind of a theory. Gravitational four-momentum can be assigned to the curvature scalar as Noether currents and is thus completely well-defined unlike in GRT. Equivalence Principle requires that inertial and gravitational four-momenta are identical. This is satisfied if curvature scalar defines the fundamental action principle crucial for the definition of quantum TGD. Curvature scalar as a fundamental action is however non-physical and had to be replaced with so called Kähler action.
2. One can question Equivalence Principle because the conservation of gravitational four-momentum seems to fail in cosmological scales. Zero Energy Ontology however implies that four-momentum is length scale dependent notion so that the problem disappers.
3. For the extremals of Kähler action the Noether currents associated with curvature scalar are well-defined but non-conserved. Also for vacuum extremals satisfying Einstein's equations gravitational four-momentum fails to be conserved and non-conservation becomes large for small values of cosmic time. This looks fine but the problem is whether the possible failure of Equivalence Principle is so serious that it leads to conflict with experimental facts.

TGD view about quantum gravity led first to a technical understanding of what the equality of inertial and gravitational four-momenta means. This equality is something ill-definable in GRT context whether the conservation laws for four-momentum is purely local stating the vanishing of the covariant divergences of energy momentum-tensor.

The path leading to the recent view about GRT limit of TGD and EP at classical level has been long and tortuous. The earlier attempts to understand the relationship between TGD and GRT have been in terms of solutions of Einstein's equations imbeddable to $M^4 \times CP_2$. It turned out however that the problems related to TGD-GRT relationship and EP have been basically pseudo-problems due to the too restricted vision about what TGD limit of TGD could be. One must introduce GRT space-time as a fictive notion naturally obtained from Minkowski space by replacing its metric with effective metric describing gravitation: this is also the spirit of perturbation theoretic approach to quantum gravity. GRT emerges from TGD as a simplified concept replacing many-sheeted space-time.

1. The replacement of superposition of fields with superposition of their effects on test particle simultaneously topologically condensed to the space-time sheet means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets. This resolves also the objections due to the lacking superposition fields at given space-time sheet and strong correlations between different induced quantities (metric and spinor connection).
2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric in standard coordinates for the space-time sheets. One could replace flat metric of M^4 with effective metric as sum of flat metric and deviations associated with various space-time sheets "above" the M^4 point. This effective metric of M^4 regarded as independent space would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD. Also standard model gauge potentials can be defined as effective fields in the same way and one expects that classical electroweak fields vanish in the length scales above weak scale.

   The fact that Maxwell's electrodynamics, gauge theories and GRT work so well suggests that many-sheetedness is really present and only in special situations becomes manifest. Example of this kind of situation is represented by the propagation of light signal along different space-time sheets so that it spends different times on the travel. Encouragingly, two neutrino bursts followed by photon burst arrived from SN1987 supernova.

This resolves also the worries related to Equivalence Principle. TGD can be seen as a "microscopic" theory behind TGD and the understanding of the microscopic elements becomes the main focus of theoretical and hopefully also experimental work some day.

8.1.2 Zero Energy Ontology

In zero energy ontology one replaces positive energy states with zero energy states with positive and negative energy parts of the state at the boundaries of future and past direct light-cones forming a causal diamond. All conserved quantum numbers of the positive and negative energy states are of opposite sign so that these states can be created from vacuum. "Any physical state is creatable from vacuum" becomes thus a basic principle of quantum TGD and together with the notion of quantum jump resolves several philosophical problems (What was the initial state of universe?, What are the values of conserved quantities for Universe, Is theory building completely useless if only single solution of field equations is realized?).

At the level of elementary particle physics positive and negative energy parts of zero energy state are interpreted as initial and final states of a particle reaction so that quantum states become physical events. Equivalence Principle would hold true in the sense that the classical gravitational four-momentum of the vacuum extremal whose small deformations appear as the argument of configuration space spinor field is equal to the positive energy of the positive energy part of the zero energy quantum state.

The vacuum extremals are absolutely essential for the TGD based view about long length scale limit about gravitation but involve the assumption that solutions of Einstein's equations allowing imbedding as vacuum extremal are in physically preferred role. Already the Kähler action defined by CP_2 Kähler form J allows enormous vacuum degeneracy: any four-surface having Lagrangian sub-manifold of CP_2 as its CP_2 projection is a vacuum extremal. The dimension of these sub-manifolds is at most two. Robertson-Walker cosmologies correspond to vacua with respect to inertial energy and in fact with respect to all quantum numbers. They are not vacua with respect to gravitational charges defined as Noether charges associated with the curvature scalar. Also more general imbeddings of Einstein's equations are typically vacuum extremals with respect to Noether charges assignable to Kähler action since otherwise one ends up with conflict between imbeddability and dynamics. This suggests that physical states have vanishing net quantum numbers quite generally. The construction of quantum theory [K40, K24] indeed leads naturally to zero energy ontology stating that everything is creatable from vacuum.

Zero energy states decompose into positive and negative energy parts having identification as initial and final states of particle reaction in time scales of perception longer than the geometro-temporal separation T of positive and negative energy parts of the state. If the time scale of perception is smaller than T, the usual positive energy ontology applies.

In zero energy ontology inertial four-momentum is a quantity depending on the temporal time scale T used and in time scales longer than T the contribution of zero energy states with parameter $T_1 < T$ to four-momentum vanishes. This scale dependence alone implies that it does not make sense to speak about conservation of inertial four-momentum in cosmological scales. Hence it would be in principle possible to identify inertial and gravitational four-momenta and achieve strong form of Equivalence Principle. It however seems that this is not the correct approach to follow.

The concept of negative potential energy is completely standard notion in physics. Perhaps so standard that physicists have begun to regard it as understood. The precise physical origin of the negative potential energy is however complete mystery, and one is forced to take the potential energy as a purely phenomenological concept deriving from quantum theory as an effective description.

In TGD framework topological field quantization leads to the hypothesis that quantum concepts should have geometric counterparts. Also potential energy should have precise correlate at the level of description based on topological field quanta. This could be the case. As already explained, ZEO allows space-time sheets to have both positive and negative time orientations. This in turn implies that also the sign of energy can be also negative. This suggests that the generation of negative energy space-time sheets representing virtual gravitons together with energy conservation makes possible the generation of huge gravitationally induced kinetic energies and gravitational collapse. In this process inertial energy would be conserved since instead, of positive energy gravitons, the inertial energy would go to the energy of matter.

This picture has a direct correlate in quantum field theory where the exchange negative energy virtual bosons gives rise to the interaction potential. The gravitational red-shift of microwave background photons is the strongest support for the non-conservation of energy in General Relativity. In TGD it could have concrete explanation in terms of absorption of negative energy virtual gravitons by photons leading to gradual reduction of their energies. This explanation is consistent with the classical geometry based explanation of the red-shift based on the stretching of electromagnetic wave lengths. This explanation is also consistent with the intuition based on Feynman diagram description of gravitational acceleration in terms of graviton exchanges.

8.1.3 Dark Matter Hierarchy And Hierarchy Of Planck Constants

The idea about hierarchy of Planck constants relying on generalization of the embedding space was inspired both by empirical input (Bohr quantization of planetary orbits) and by the mathematics of hyper-finite factors of type II_1 combined with the quantum classical correspondence.

Quantum classical correspondence suggests that Jones inclusions [A1] have space-time correlates [K103, K35]. There is a symplectic hierarchy of Jones inclusions labeled by finite subgroups of SU(2) [A46] This leads to a generalization of the embedding space obtained by gluing an infinite number of copies of H regarded as singular bundles over $H/G_a \times G_b$, where $G_a \times G_b$ is a subgroup of $SU(2) \times SU(2) \subset SL(2,C) \times SU(3)$. Gluing occurs along a factor for which the group is same. The generalized embedding space has clearly a book like structure with pages of books intersecting along 4-D sub-manifold $M^2 \times S^2$, S^2 a geodesic sphere of CP_2 characterizing the choice of quantization axies. Entire configuration space is union over "books" corresponding to various choices of this sub-manifold.

The groups in question define in a natural way the direction of quantization axes for for various isometry charges and this hierarchy seems to be an essential element of quantum measurement theory. Ordinary Planck constant, as opposed to Planck constants $\hbar_a = n_a \hbar_0$ and $\hbar_b = n_b \hbar_0$

appearing in the commutation relations of symmetry algebras assignable to M^4 and CP_2, is naturally quantized as $\hbar = (n_a/n_b)\hbar_0$, where n_i is the order of maximal cyclic subgroup of G_i. The hierarchy of Planck constants is interpreted in terms of dark matter hierarchy [K35]. What is also important is that $(n_a/n_b)^2$ appear as a scaling factor of M^4 metric so that Kähler action via its dependence on induced metric codes for radiative corrections coming in powers of ordinary Planck constant: therefore quantum criticality and vanishing of radiative corrections to functional integral over WCW does not mean vanishing of radiative corrections.

G_a would correspond directly to the observed symmetries of visible matter induced by the underlying dark matter [K35]. For instance, in living matter molecules with 5- and 6-cycles could directly reflect the fact that free electron pairs associated with these cycles correspond to $n_a = 5$ and $n_a = 6$ dark matter possibly responsible for anomalous conductivity of DNA [K35, K15] and recently reported strange properties of graphene [D9]. Also the tetrahedral and icosahedral symmetries of water molecule clusters could have similar interpretation [K33]. [D14].

A further fascinating possibility is that the observed indications for Bohr orbit quantization of planetary orbits [E25] could have interpretation in terms of gigantic Planck constant for underlying dark matter [K79] so that macroscopic and -temporal quantum coherence would be possible in astrophysical length scales manifesting itself in many way: say as preferred directions of quantization axis (perhaps related to the CMB anomaly) or as anomalously low dissipation rates.

Since the gravitational Planck constant is proportional to the product of the gravitational masses of interacting systems, it must be assigned to the field body of the two systems and characterizes the interaction between systems rather than systems themselves. This observation applies quite generally and each field body of the system (em, weak, color, gravitational) is characterized by its own Planck constant.

In the gravitational case the order of G_a is gigantic and at least GM_1m/v_0, $v_0 = 2^{-11}$ the favored value. The natural interpretation is as a discrete rotational symmetry of the gravitational field body of the system having both gravimagnetic and gravi-electric parts. The subgroups of G_a for which order is a divisor of the order of G_a define broken symmetries at the lower levels of dark matter hierarchy, in particular symmetries of visible matter.

The number theoretically simple ruler-and-compass integers having as factors only first powers of Fermat primes and power of 2 would define a physically preferred sub-hierarchy of quantum criticality for which subsequent levels would correspond to powers of 2: a connection with p-adic length scale hypothesis suggests itself. Ruler and compass hypothesis implies that besides p-adic length scales also their 3- and 5- multiples should be important. Note that in the structure of chromosomes p-adic length scale $L(151) \simeq 10$ characterizes beads-on-string structure of DNA whereas the length scale $3L(151)$ appears in the coiling of this structure.

It has turned that there are good hopes of reducing the hierarchy of Planck constants to the basic TGD [K11]. By the extreme non-linearity of the Kähler action the correspondence between the time derivatives of the embedding space coordinates and canonical momentum densities is many-to-one. This leads naturally to the introduction of covering spaces of $CD \times CP_2$, which are singular in the sense that the sheets of the covering co-incide at the ends of CD and at wormhole throats. One can say that quantum criticality means also the instability of the 3-surfaces defined by throats and ends against the decay to several space-time sheets and consequence charge fractionization. The interpretation is as an instability caused by too strong density of mass and making perturbative description possible since the matter density at various branches is reduced. The situation can be described mathematically either by using effectively only single sheet but an integer multiple of Planck constant or many-sheeted covering and ordinary value of Planck constant. In [K35] the argument that this indeed leads to hierarchy of Planck constants including charge fractionization is developed in detail. The restriction to singular coverings is consistent with the experimental constraints and means that only integer valued Planck constants are possible. A given value of Planck constant corresponds only to a finite number of the pages of the Big Book and that the evolution by quantum jumps is analogous to a diffusion at half-line and tends to increase the value of Planck constant.

8.1.4 The Problem Of Cosmological Constant

A further implication of dark matter hierarchy is that astrophysical systems correspond to stationary states analogous to atoms and do not participate to cosmic expansion in a continuous way but via discrete quantum phase transitions in which gravitational Planck constant increases. By quantum criticality of these phase transitions critical cosmologies are excellent candidates for the modelling of these transitions. Imbeddable critical cosmologies are unique apart from a parameter determining their duration and represent accelerating cosmic expansion so that there is no need to introduce cosmological constant.

It indeed turns out possible to understand these critical phases in terms of quantum phase transition increasing the size of large modeled in terms of "big" cosmic strings with negative gravitational mass whose repulsive gravitation drives "galactic" cosmic strings with positive gravitational mass to the boundaries of the void. In this framework cosmological constant like parameter does not characterize the density of dark energy but that of dark matter identifiable as quantum phases with large Planck constant.

A further problem is that the naive estimate for the cosmological constant is predicted to be by a factor 10^{120} larger than its value deduced from the accelerated expansion of the Universe. In TGD framework the resolution of the problem comes naturally from the fact that large voids are quantum systems which follow the cosmic expansion only during the quantum critical phases.

p-Adic fractality predicting that cosmological constant is reduced by a power of 2 in phase transitions occurring at times $T(k) \propto 2^{k/2}$, which correspond to p-adic time scales. These phase transitions would naturally correspond to quantum phase transitions increasing the size of the large voids during which critical cosmology predicting accelerated expansion naturally applies. On the average $\Lambda(k)$ behaves as $1/a^2$, where a is the light-cone proper time. This predicts correctly the order of magnitude for observed value of Λ.

8.1.5 Topics Of The Chapter

The notion of many-sheeted space-time has been extensively discussed in the previous chapters [?, K40] and is therefore left out from this chapter. The topics included in this chapter are following.

The first three sections are devoted to the general theoretical picture.

1. There is a discussion of General Coordinate Invariance, Equivalence Principle, and Machian Principle in TGD context with a special emphasis on the views about the relationship of inertial and gravitational masses, the zero energy ontology, and dark matter hierarchy. A special emphasis is given to the notion of four-momentum and the latest (year 2013) vision about the situation is discussed.
2. The vacuum extremal imbeddings of Reissner-Nordström and Schwartschild metric are studied. The interpretational problems involved were responsible for much of the tension which eventually led to the recent understanding of Equivalence Principle in TGD framework.

The remaining sections are devoted to examples about applications.

1. A simple model for the final state of a star is proposed. The model indicates that Z^0 force, presumably created by dark matter, might have an important role in the dynamics of the compact objects. During year 2003, more than decade after the formulation of the model, the discovery of the connection between supernovas and gamma ray bursts [E26] provided strong support for the predicted axial magnetic and Z^0 magnetic flux tube structures predicted by the model for the final state of a rotating star. Two years later the interpretation of the predicted long range weak forces as being caused by dark matter emerged.

   The progress in the understanding of hadronic mass calculations [K62] has led to the identification of so called super-symplectic bosons and their super-counterparts as basic building blocks of hadrons. This notion suggests also a microscopic description of neutron stars and black-holes in terms of highly entangled string like objects in Hagedorn temperature and in very precise sense analogous to gigantic hadrons.
2. The idea of entropic gravity is not consistent with what is already known about the quantal behavior of neutrons in the Earth's gravitational field. The discussion of entropic gravity

in TGD framework however leads to fresh ideas about GRT limit of TGD and is therefore included.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L8].

8.2 Basic Principles Of General Relativity From TGD Point Of View

General Coordinate Invariance, Equivalence Principle are corner stones of general relativity and one expects that they hold true also in TGD some sense. The earlier attempts to understand the relationship between TGD and GRT have been in terms of solutions of Einstein's equations imbeddable to $M^4 \times CP_2$ instead of introducing GRT space-time as a fictive notion naturally emerging from TGD as a simplified concept replacing many-sheeted space-time. This resolves also the worries related to Equivalence Principle. TGD can be seen as a "microscopic" theory behind TGD and the understanding of the microscopic elements becomes the main focus of theoretical and hopefully also experimental work some day.

Objections against TGD have turned out to be the best route to the correct interpretation of the theory. A very general objection against TGD relies on the notion of induced gauge fields and metric implying extremely strong constraints between classical gauge fields for preferred extremals. These constraints cannot hold true for gauge fields in the usual sense. Also linear superposition is lost. The solution of the problem comes from simple observation: it is not fields which superpose but their effects on test particle topologically condensed to space-time sheets carrying the classical fields. Superposition is replaced with set theoretic union. This leads also naturally to explicit identification of the effective metric and gauge potentials defined in M^4 and defining GRT limit of TGD.

Finite length scale resolution is central notion in TGD and implies that the topological inhomogenities (space-time sheets and other topological inhomogenities) are treated as point-like objects and described in terms of energy momentum tensor of matter and various currents coupling to effective YM fields and effective metric important in length scales above the resolution scale. Einstein's equations with coupling to gauge fields and matter relate these currents to the Einstein tensor and metric tensor of the effective metric of M^4. The topological inhomogenities below cutoff scale serve determine the curvature of the effective metric.

The original proposal, which I called smoothed out space-time, took into account the topological inhomogenities but neglected many-sheetedness in length scales above resolution scale. I also identified the effective metric can be identified as induced metric: this is very strong assumption although the properties of vacuum extremals support this identification at least in some important special cases.

The attempts to understand Kähler-Dirac (or Kähler-Dirac-) action has provided very strong boost to the understanding of the basic problems related to GRT-TGD relationship, understanding of EP means at quantum level in TGD, and how the properties of induced electroweak gauge potentials can be consistent with what is known about electroweak interactions: for instance, if is far from clear how em charge can be well-defined for the modes of the induced spinor field and how the effective absence of weak bosons above weak scale is realized at classical level for Kähler-Dirac action.

8.2.1 General Coordinate Invariance

General Coordinate Invariance plays in the formulation of quantum TGD even deeper role than in that of GRT. Since the fundamental objects are 3-D surfaces, the construction of the geometry of the configuration space of 3-surfaces (the world of classical worlds, WCW) requires that the definition of the geometry assigns to a given 3-surface X^3 a unique space-time surface $X^4(X^3)$. This space-time surface is completely analogous to Bohr orbit, which means a completely unexpected connection with quantum theory.

General Coordinate Invariance is analogous to gauge symmetry and requires gauge fixing. The definition assigning $X^4(X^3)$ to given X^3 must be such that the outcome is same for all 4-diffeomorphs of X^3. This condition is highly non-trivial since $X^4(X^3) = X^4(Y^3)$ must hold true if X^3 and Y^3 are 4-diffeomorphs. One manner to satisfy this condition is by assuming quantum holography and weakened form of General Coordinate Invariance: there exists physically preferred 3-surfaces X^3 defining $X^4(X^3)$, and the 4-diffeomorphs Y^3 of X^3 at $X^4(X^3)$ provide classical holograms of X^3: $X^4(Y^3) = X^4(X^3)$ is trivially true. Zero energy ontology allows to realize this form of General Coordinate Invariance.

1. In ZEO WCW decomposes into a union of sub-WCWs associated with causal diamonds $CD \times CP_2$ (CD denotes the intersection of future and past directed light-cones of M^4), and the intersections of space-time surface with the light-light boundaries of $CD \times CP_2$ are excellent candidates for preferred space-like 3-surfaces X^3. The 3-surfaces at $\delta CD \times CP_2$ are indeed physically special since they carry the quantum numbers of positive and negative energy parts of the zero energy state.
2. Preferred 3-surfaces could be also identified as light-like 3-surfaces X_l^3 at which the Euclidian signature of the induced space-time metric changes to Minkowskian. Also light-like boundaries of X^4 can be considered. These 3-surfaces are assumed to carry elementary particle quantum numbers and their intersections with the space-like 3-surfaces X^3 are 2-dimensional partonic surfaces so that effective 2-dimensionality consistent with the conformal symmetries of X_l^3 results if the identifications of 3-surfaces are physically equivalent. Light-like 3-surfaces are identified as generalized Feynman diagrams and due to the presence of 2-D partonic 2-surfaces representing vertices fail to be 3-manifolds. Generalized Feynman diagrams could be also identified as Euclidian regions of space-time surface.
3. General Coordinate Invariance in minimal form requires that the slicing of $X^4(X_l^3)$ by light light 3-surfaces Y_l^3 "parallel" to X_l^3 predicted by number theoretic compactification gives rise to quantum holography in the sense that the data associated with any Y_l^3 allows an equivalent formulation of quantum TGD. This poses a strong condition on the spectra of the Kähler-Dirac operator at Y_l^3 and thus to the preferred extremals of Kähler action since the WCW Kähler functions defined by various choices of Y_l^3 can differ only by a sum of a holomorphic function and its conjugate [K104, K24] .

8.2.2 The Basic Objection Against TGD

The basic objection against TGD is that induced metrics for space-time surfaces in $M^4 \times CP_2$ form an extremely limited set in the space of all space-time metrics appearing in the path integral formulation of General Relativity. Even special metrics like the metric of a rotating black hole fail to be imbeddable as an induced metric. For instance, one can argue that TGD cannot reproduce the post-Newtonian approximation to General Relativity since it involves linear superposition of gravitational fields of massive objects. As a matter fact, Holger B. Nielsen- one of the very few colleagues who has shown interest in my work - made this objection for at least two decades ago in some conference and I remember vividly the discussion in which I tried to defend TGD with my poor English.

The objection generalizes also to induced gauge fields expressible solely in terms of CP_2 coordinates and their gradients. This argument is not so strong as one might think first since in standard model only classical electromagnetic field plays an important role.

1. Any electromagnetic gauge potential has in principle a local embedding in some region. Preferred extremal property poses strong additional constraints and the linear superposition of massless modes possible in Maxwell's electrodynamics is not possible.
2. There are also global constraints leading to topological quantization playing a central role in the interpretation of TGD and leads to the notions of field body and magnetic body having non-trivial application even in non-perturbative hadron physics. For a very large class of preferred extremals space-time sheets decompose into regions having interpretation as geometric counterparts for massless quanta characterized by local polarization and momentum directions. Therefore it seems that TGD space-time is very quantal. Is it possible to obtain from TGD what we have used to call classical physics at all?

The imbeddability constraint has actually highly desirable implications in cosmology. The enormously tight constraints from imbeddability imply that imbeddable Robertson-Walker cosmologies with infinite duration are sub-critical so that the most pressing problem of General Relativity disappears. Critical and over-critical cosmologies are unique apart from a parameter characterizing their duration and critical cosmology replaces both inflationary cosmology and cosmology characterized by accelerating expansion. In inflationary theories the situation is just the opposite of this: one ends up with fine tuning of inflaton potential in order to obtain recent day cosmology.

Despite these and many other nice implications of the induced field concept and of sub-manifold gravity the basic question remains. Is the imbeddability condition too strong physically? What about linear superposition of fields which is exact for Maxwell's electrodynamics in vacuum and a good approximation central also in gauge theories. Can one obtain linear superposition in some sense?

1. Linear superposition for small deformations of gauge fields makes sense also in TGD but for space-time sheets the field variables would be the deformations of CP_2 coordinates which are scalar fields. One could use preferred complex coordinates determined about SU(3) rotation to do perturbation theory but the idea about perturbations of metric and gauge fields would be lost. This does not look promising. Could linear superposition for fields be replaced with something more general but physically equivalent?
2. This is indeed possible. The basic observation is utterly simple: what we know is that the *effects* of gauge fields superpose. The assumption that fields superpose is un-necessary! This is a highly non-trivial lesson in what operationalism means for theoreticians tending to take these kind of considerations as mere "philosophy".
3. The hypothesis is that the superposition of effects of gauge fields occurs when the M^4 projections of space-time sheets carrying gauge and gravitational fields intersect so that the sheets are extremely near to each other and can touch each other (CP_2 size is the relevant scale).

A more detailed formulation goes as follows.

1. One can introduce common M^4 coordinates for the space-time sheets. A test particle (or real particle) is identifiable as a wormhole contact and is therefore point-like in excellent approximation. In the intersection region for M^4 projections of space-time sheets the particle forms topological sum contacts with all the space-time sheets for which M^4 projections intersect.
2. The test particle experiences the sum of various gauge potentials of space-time sheets involved. For Maxwellian gauge fields linear superposition is obtained. For non-Abelian gauge fields gauge fields contain interaction terms between gauge potentials associated with different space-time sheets. Also the quantum generalization is obvious. The sum of the fields induces quantum transitions for states of individual space time sheets in some sense stationary in their internal gauge potentials.
3. The linear superposition applies also in the case of gravitation. The induced metric for each space-time sheet can be expressed as a sum of Minkowski metric and CP_2 part having interpretation as gravitational field. The natural hypothesis that in the above kind of situation the effective metric is sum of Minkowski metric with the sum of the CP_2 contributions from various sheets. The effective metric for the system is well-defined and one can calculate a curvature tensor for it among other things and it contains naturally the interaction terms between different space-time sheets. At the Newtonian limit one obtains linear superposition of gravitational potentials. One can also postulate that test particles moving along geodesics in the effective metric. These geodesics are not geodesics in the metrics of the space-time sheets.
4. This picture makes it possible to interpret classical physics as the physics based on effective gauge and gravitational fields and applying in the regions where there are many space-time sheets which M^4 intersections are non-empty. The loss of quantum coherence would be due to the effective superposition of very many modes having random phases.

The effective superposition of the CP_2 parts of the induced metrics gives rise to an effective metric which is not in general imbeddable to $M^4 \times CP_2$. Therefore many-sheeted space-time makes possible a rather wide repertoire of 4-metrics realized as effective metrics as one might have expected and the basic objection can be circumvented In asymptotic regions where one can expect single sheetedness, only a rather narrow repertoire of "archetypal" field patterns of gauge fields and gravitational fields defined by topological field quanta is possible.

The skeptic can argue that this still need not make possible the embedding of a rotating black hole metric as induced metric in any physically natural manner. This might be the case but need of course not be a catastrophe. We do not really know whether rotating blackhole metric is realized in Nature. I have indeed proposed that TGD predicts new physics [K99]. Unfortunately, gravity probe B could not check whether this new physics is there since it was located at equator where the new effects vanish.

8.2.3 How GRT And Equivalence Principle Emerge From TGD?

The question how TGD relates to General Relativity Theory (GRT) has been a rich source of problems during last 37 years. In the light of after-wisdom the problems have been due to my too limited perspective. I have tried to understand GRT limit in the TGD framework instead of introducing GRT space-time as a fictive notion naturally emerging from TGD as a simplified concept replacing many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. ??** in the appendix of this book) . This resolves also the worries related to Equivalence Principle.

TGD itself gains the status of "microscopic" theory of gravity and the experimental challenges relate to how make the microscopy of gravitation experimentally visible. This involves questions such as "How to make the presence of Euclidian space-time regions visible?",

> How to reveal many-sheeted character of space-time, topological field quantization, and the presence of magnetic flux tubes?,"How to reveal quantum gravity as understood in TGD involving in an essential manner gravitational Planck constant h_{gr} identifiable as h_{eff} inspired by anomalies of bio-electromagnetism?

[K73].

More technical questions relate to the Kähler-Dirac action, in particular to how conservation laws are realized. During all these years several questions have been lurking at the boarder of conscious and sub-conscious. How can one guarantee that em charge is well-defined for the spinor modes when classical W fields are present? How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induced vielbein rotation of weak fields? The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string picture a part of TGD.

TGD and GRT

Concerning GRT limit the basic questions are the following ones.

1. Is it really possible to obtain a realistic theory of gravitation if general space-time metric is replaced with induced metric depending on 8 embedding space coordinates (actually only 4 by general coordinate invariance?
2. What happens to Einstein equations?
3. What about breaking of Poincare invariance, which seems to be real in cosmological scales? Can TGD cope with it?
4. What about Equivalence Principle (EP)
5. Can one predict the value of gravitational constant?
6. What about TGD counterpart of blackhole, which certainly represents the boundary of realm in which GRT applies?

Consider first possible answers to the first three questions.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences

sum of the effects caused by the classical fields at the space-time sheets (see **Fig.** `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg` or **??** in the appendix of this book).

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric in standard coordinates for the space-time sheets. One could replace flat metric of M^4 with effective metric as sum of metric and deviations associated with various space-time sheets "above" the M^4 point. This effective metric of M^4 regarded as independent space would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD. Also standard model gauge potentials can be defined as effective fields in the same manner and one expects that classical electroweak fields vanish in the length scales above weak scale.

3. This picture brings in mind the old intuitive notion of smoothed out quantum average space-time thought to be realized as surface in $M^4 \times CP_2$ rather than in terms of averages metric and gauge potentials in M^4. The problem of this approach was that it was not possible to imagine any quantitative recipe for the averaging and this was essentially dur to the sub-manifold assumption.

4. One could generalize this picture and consider effective metrics for CP_2 and $M^2 \times CP_2$ corresponding to CP_2 type vacuum extremals describing elementary particles and cosmic strings respectively.

5. Einstein's equations could hold true for the effective metric. The vanishing of the covariant divergence of energy momentum tensor would be a remnant of Poincare invariance actually still present in the sense of Zero Energy Ontology (ZEO) but having realization as global conservation laws.

6. The breaking of Poincare invariance at the level of effective metric could have interpretation as effective breaking due to zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

The following considerations are about answers to the fourth and fifth questions.

1. EP at classical level would hold true in local sense if Einstein's equations hold true for the effective metric. Underlying Poincare invariance suggests local covariant conservation laws.

2. The value of gravitational constant is in principle a prediction of theory containing only radius as fundamental scale and Kähler coupling strength as only coupling constant analogous to critical temperature. In GRT inspired quantum theory of gravitation Planck length scale given by $L_P = \sqrt{\hbar_{eff} \times G}$ is the fundamental length scale. In TGD size R defines it and it is independent of h_{eff}. The prediction for gravitational constant is prediction for the TGD counterpart of L_P: $L_P^2 = R^2/n$, n dimensionless constant. The prediction for G would be $G = R^2/(n \times \hbar_{eff})$ or $G = R^2/(n \times \hbar_{eff,min})$. The latter option is the natural one.

Interesting questions relate to the fate of blackholes in TGD framework.

1. Blackhole metric as such is quite possible as effective metric since there is no need to imbed it to embedding space. One could however argue that blackhole metric is so simple that it must be realizable as single-sheeted space-time surface. This is indeed possible above some radius which can be smaller than Schwarschild radius. This is due to the compactness of CP_2 . A general result is that the embedding carriers non-vanishing gauge charge say em charge. This need not have physical significance if the metric of GRT corresponds to the effective metric obtained by the proposed recipe.

2. TGD forces to challenge the standard view about black holes. For instance, could it be that blackhole interior corresponds microscopically to Euclidian space time regions? For these CP_2 endowed with effective metric would be appropriate GRT type description. Reissner-Nordström metric with cosmological constant indeed allows CP_2 as solution [K99]. M^4 region and CP_2 region would be joined along boundaries at which determinant of four-metric vanishes. If the radial component of R-N metric is required to be finite, one indeed obtains metric with vanishing determinant at horizon and it is natural to assume that the metric inside is Euclidian. Similar picture would applied to the cosmic strings as spaces $M^2 \times S^2$ with effective metric.

3. Could holography hold true in the sense that blackhole horizon is replaced with a partonic 2-surface with astrophysical size and having light-like orbit as also black-hole horizon has.
4. The notion of gravitational Planck constant $\hbar_{gr} = GMm/v_0$, where v_0 is typical rotation velocity in the system consisting of masses M and m, has been one of the speculative aspects of TGD. h_{gr} would be assigned with "gravitational" magnetic flux tube connecting the systems in question and it has turned out that the identification $h_{gr} = h_{eff}$ makes sense in particle length scales. The gravitational Compton length is universal and given $\lambda_{gr} = GM/v_0$. This strongly suggests that quantum gravity becomes important already above Schwarschild radius $r_S = 2GMm$. The critical velocity at which gravitational Compton length becomes smaller than r_S is $v_0/c = 1/\sqrt{2}$. All astrophysical objects would be genuinely quantal objects in TGD Universe point and blackholes would lose their unique role. An experimental support for these findings comes from experiments of Tajmar *et al* [E37, E57] [K73].

For few ago entropic gravity [B12, B46] was a buzzword in blogs. The idea was that gravity would have a purely thermodynamical origin. I have commented the notion of entropic gravity from the point of view of TGD earlier [K99].

The basic objection is standard QM against the entropic gravity is that gravitational interaction of neutrons with Earth's gravitational field is describable by Schrödinger equation and this does not fit with thermodynamical description.

Although the idea as such does not look promising TGD indeed suggests that the correlates for thermodynamical quantities at space-time level make sense in ZEO leading to the view that quantum TGD is square root of thermodynamics.

Th interesting question is whether temperature has space-time correlate.

1. In Zero Energy Ontology quantum theory can be seen as a square root of thermodynamics formally and this raises the question whether ordinary temperature could parametrize wave functions having interpretation as square roots of thermal distributions in ZEO. The quantum model for cell membrane [K32] having the usual thermodynamical model as limit gives support for this idea. If this were the case, temperature would have by quantum classical correspondence direct space-time correlate.
2. A less radical view is that temperature can be assigned with the effective space-time metric only. The effective metric associated with M^4 defining GRT limit of TGD is defined statistically in terms of metric of many-sheeted space-time and would naturally contain in its geometry thermodynamical parameters. The averaging over the WCW spinors fields involving integral over 3-surfaces is also involved.

Equivalence Principle

Equivalence Principle has several interpretations.

1. The global form form of Equivalence Principle (EP) realized in Newtonian gravity states that inertial mass = gravitational mass (mass is replaces with four-momentum in the possible relativistic generalization). This form does not make sense in general relativity since four-momentum is not well-defined: this problem is the starting point TGD.
2. The local form of EP can be expressed in terms of Einstein's equations. Local covariant conservation law does not imply global conservation law since energy momentum tensor is indeed tensor. One can try to define gravitational mass as something making sense in special cases. The basic problem is that there is no unique identification of empty space Minkowski coordinates. Gravitational mass could be identified as a a parameter appearing in asymptotic expression of solutions of Einstein's equations.

In TGD framework EP need not be problem of principle.

1. In TGD gravitational interaction couples to inertial four-momentum, which is well-defined as classical Noether charge associated with Kähler action. The very close analogy of TGD with string models suggest the same.
2. Only if one assumes that gravitational and inertial exist separately and are forced to be identical, one ends up with potential problems in TGD. This procedure might have sound physical basis in TGD but one should identify it in convincing manner.

3. In cosmology mass is not conserved, which in positive energy ontology would suggests breaking of Poincare invariance. In Zero Energy Ontology (ZEO) this is not the case. The conserved four-momentum assignable to either positive or negative energy part of the states in the basis of zero energy states depends on the scale of causal diamond (CD). Note that in ZEO zero energy states can be also superpositions of states with different four-momenta and even fermion numbers as in case of coherent state formed by Cooper pairs.

Consider now EP in quantum TGD.

1. Inertial momentum is defined as Noether charge for Kähler action.
2. One can assign to Kähler-Dirac action quantal four-momentum (I will use "Kähler-Dirac" instead of "modified" used in earlier work) [K104]. Its conservation is however not at all all trivial since embedding space coordinates appear in KD action like external fields. It however seems that at least for the modes localized at string world sheets the four-momentum conservation could be guaranteed by an assumption motivated by holomorphy [K104]. The assumption states that the variation of holomorphic/antiholomorphic Kähler-Dirac gamma matrices induced by isometry is superposition of K-D gamma matrices of same type.
3. Quantum Classical Correspondence (QCC) suggests that the eigenvalues of quantal four-momentum are equal to those of Kähler four-momentum. If this is the case, QCC would imply EP and force conservation of antal four-momenta even if the assumption about variations of gamma matrices fails! This could be realized in terms of Lagrange multiplier terms added to Kähler action and localized at the ends of CD and analogous to constraint terms in ordinary thermodynamics.
4. QCC generalizes to Cartan sub-algebra of symmetries and would give a correlation between geometry of space-time sheet and conserved quantum numbers. One can consider even stronger form of QCC stating that classical correlation functions at space-time surface are same as the quantal once.

The understanding of EP at classical level has been a long standing head-ache in TGD framework. What seems to be the eventual solution looks disappointingly trivial in the sense that its discovery requires only some common sense.

The trivial but important observation is that the GRT limit of TGD does *not* require that the space-times of GRT limit are imbeddable to the embedding space $M^4 \times CP_2$. The most elegant understanding of EP at classical level relies on following argument suggesting how GRT space-time emerges from TGD as an effective notion.

1. Particle experiences the sum of the effects caused by gravitational forces. The linear superposition for gravitational fields is replaced with the sum of effects describable in terms of effective metric in GRT framework. Hence it is natural to identify the metric of the effective space-time as the sum of M^4 metric and the deviations of various space-time sheets to which particle has topological sum contacts. This metric is defined for the M^4 serving as coordinate space and is not in general expressible as induced metric.
2. Underlying Poincare invariance is not lost but global conservation laws are lost for the effective space-time. A natural assumption is that global energy-momentum conservation translates to the vanishing of covariant divergence of energy momentum tensor.
3. By standard argument this implies Einstein's equations with cosmological constant Λ: this at least in statistical sense. Λ would parametrize the presence of topologically condensed magnetic flux tubes. Both gravitational constant and cosmological constant would come out as predictions.

This picture is in principle all that is needed. TGD is in this framework a "microscopic" theory of gravitation and GRT describes statistically the many-sheetedness in terms of single sheeted space-time identified as M^4 as manifold. All notions related to many-sheeted space-time - such as cosmic strings, magnetic flux tubes, generalized Feynman diagrams representing deviations from GRT. The theoretical and experimental challenge is discover what these deviations are and how to make them experimentally visible.

One can of course ask whether EP or something akin to it could be realized for preferred extremals of Kähler action.

1. In cosmological and astrophysical models vacuum extremals play a key role. Could small deformations of them provide realistic enough models for astrophysical and cosmological scales in statistical sense?

2. Could preferred extremals satisfy something akin to Einstein's equations? Maybe! The mere condition that the covariant divergence of energy momentum tensor for Kähler action vanishes, is satisfied if Einsteins equations with cosmological terms are satisfied. One can however consider also argue that this condition can be satisfied also in other ways. For instance, four-momentum currents associated with them be given by Einstein's equations involving several cosmological "constants". The vanishing of covariant divergence would however give a justification for why energy momentum tensor is locally conserved for the effective metric and thus gives rise to Einstein's equations.

EP as quantum classical correspondence

Quite recently I returned to an old question concerning the meaning of Equivalence Principle (EP) in TGD framework.

Heretic would of course ask whether the question about whether EP is true or not is a pseudo problem due to uncritical assumption there really are two different four-momenta which must be identified. If even the identification of these two different momenta is difficult, the pondering of this kind of problem might be waste of time.

At operational level EP means that the scattering amplitudes mediated by graviton exchange are proportional to the product of four-momenta of particles and that the proportionality constant does not depend on any other parameters characterizing particle (except spin). The are excellent reasons to expect that the stringy picture for interactions predicts this.

1. The old idea is that EP reduces to the coset construction for Super Virasoro algebra using the algebras associated with G and H. The four-momenta assignable to these algebras would be identical from the condition that the differences of the generators annihilate physical states and identifiable as inertial and gravitational momenta. The objection is that for the preferred 3-surface H by definition acts trivially so that time-like translations leading out from the boundary of CD cannot be contained by H unlike G. Hence four-momentum is not associated with the Super-Virasoro representations assignable to H and the idea about assigning EP to coset representations does not look promising.

2. Another possibility is that EP corresponds to quantum classical correspondence (QCC) stating that the classical momentum assignable to Kähler action is identical with gravitational momentum assignable to Super Virasoro representations. This view might be equivalent with coset space view. This forced to reconsider the questions about the precise identification of the Kac-Moody algebra and about how to obtain the magic five tensor factors required by p-adic mass calculations [K99].

   A more precise formulation for EP as QCC comes from the observation that one indeed obtains two four-momenta in TGD approach. The classical four-momentum assignable to the Kähler action and that assignable to the Kähler-Dirac action. This four-momentum is an operator and QCC would state that given eigenvalue of this operator must be equal to the value of classical four-momentum for the space-time surfaces assignable to the zero energy state in question. In this form EP would be highly non-trivial. It would be justified by the Abelian character of four-momentum so that all momentum components are well-defined also quantum mechanically. One can also consider the splitting of four-momentum to longitudinal and transversal parts as done in the parton model for hadrons: this kind of splitting would be very natural at the boundary of CD. The objection is that this correspondence is nothing more than QCC.

3. A further possibility is that duality of light-like 3-surfaces and space-like 3-surfaces holds true. This is the case if the action of symplectic algebra can be defined at light-like 3-surfaces or even for the entire space-time surfaces. This could be achieved by parallel translation of light-cone boundary providing slicing of CD. The four-momenta associated with the two representations of super-symplectic algebra would be naturally identical and the interpretation would be in terms of EP.

8.2.4 The Recent View About Kähler-Dirac Action

The understanding of Kähler-Dirac action and equation have provided very strong boost to the understanding of the basic problems related to GRT-TGD relationship, understanding of howe EP means at quantum level in TGD, and how the properties of induced electroweak gauge potentials can be consistent with what is known about electroweak interactions.

The understanding of Kähler Dirac action has been second long term project. How can one guarantee that em charge is well-defined for the spinor modes when classical W fields are present? How to avoid large parity breaking effects due to classical Z^0 fields? How to avoid the problems due to the fact that color rotations induced vielbein rotation of weak fields? The common answer to these questions is restriction of the modes of induced spinor field to 2-D string world sheets (and possibly also partonic 2-surfaces) such that the induced weak fields vanish. This makes string picture a part of TGD.

Kähler-Dirac action

8.2.5 Kähler-Dirac Action

Kähler-Dirac equation

8.2.6 Kähler-Dirac Equation In The Interior Of Space-Time Surface

The solution of K-D equation at string world sheets is very much analogous to that in string models and holomorphy (actually, its Minkowskian counterpart) plays a key role. Note however the K-D gamma matrices might not necessarily define effective metric with Minkowskian signature even for string world sheets. Second point to notice is that one can consider also solutions restricted to partonic 2-surfaces. Physical intuition suggests that they are very important because wormhole throats carry particle quantum numbers and because wormhole contacts mediat the interaction between space-time sheets. Whether partonic 2-surfaces are somehow dual to string world sheets remains an open question.

1. Conformal invariance/its Minkowskian variant based on hyper-complex numbers realized at string world sheets suggests a general solution of Kähler-Dirac equation. The solution ansatz is essentially similar to that in string models.
2. Second half of complexified Kähler-Dirac gamma matrices annihilates the spinors which are either holomorphic or anti-holomorphic functions of complex (hyper-complex) coordinate.
3. What about possible modes delocalized into entire 4-D space-time sheet possible if there are preferred extremals for which induced gauge field has only em part. What suggests itself is global slicing by string world sheets and obtain the solutions as integrals over localized modes over the slices.

The understanding of symmetries (isometries of embedding space) of K-D equation has turned out to be highly non-trivial challenge. The problem is that embedding space coordinates appear in the role of external fields in K-D equation. One cannot require the vanishing of the variations of the K-D action with respect to the embedding space-time coordinates since the action itself is second quantized object. Is it possible to have conservation laws associated with the embedding space isometries?

1. Quantum classical correspondence (QCC) suggests the conserved Noether charges for Kähler action are equal to the eigenvalues of the Noether charges for Kähler-Dirac action. The quantal charge conservation would be forced by hand. This condition would realize also Equivalence Principle.
2. Second possibility is that the current following from the vanishing of second variation of Kähler action and the modification of Kähler gamma matrices defined by the deformation are linear combinations of holomorphic or anti-holomorphic gammas just like the gamma matrix itself so that K-D remains true. Conformal symmetry would therefore play a fundamental role. Isometry currents would be conserved although variations with respect to embedding space coordinates would not vanish in general.

3. The natural expectation is that the number of critical deformations is infinite and corresponds to conformal symmetries naturally assignable to criticality. The number n of conformal equivalence classes of the deformations can be finite and n would naturally relate to the hierarchy of Planck constants $h_{eff} = n \times h$ (see **Fig. ??** also in the Appendix).

8.2.7 Boundary Terms For Kähler-Dirac Action

Weak form of E-M duality implies the reduction of Kähler action to Chern-Simons terms for preferred extremals satisfying $j \cdot A = 0$ (contraction of Kähler current and Kähler gauge potential vanishes). One obtains Chern-Simons terms at space-like 3-surfaces at the ends of space-time surface at boundaries of causal diamond and at light-like 3-surfaces defined by parton orbits having vanishing determinant of induced 4-metric. The naïve guess that consistency requires Kähler-Dirac-Chern Simons equation at partonic orbits. This need not however be correct and therefore it is best to carefully consider what one wants.

What one wants?

It is could to make first clear what one really wants.

1. What one wants is generalized Feynman diagrams demanding massless Dirac propagators at the boundaries of string world sheets interpreted as fermionic lines of generalized Feynman diagrams. This gives hopes that twistor Grassmannian approach emerges at QFT limit. This boils down to the condition

$$\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$$

   at the space-like ends of space-time surface. This condition makes sense also at partonic orbits although they are not boundaries in the usual sense of the word. Here however delicacies since g_4 vanishes at them. The localization of induced spinor fields to string world sheets implies that fermionic propagation takes place along their boundaries and one obtains the braid picture.

   The general idea is that the space-time geometry near the fermion line would *define* the four-momentum propagating along the line and quantum classical correspondence would be realized. The integral over four-momenta would be included to the functional integral over 3-surfaces.

   The basic condition is that $\sqrt{g_4}\Gamma^n$ is constant at the boundaries of string world sheets and depends only on the piece of this boundary representing fermion line rather than on its point. Otherwise the propagator does not exist as a global notion. Constancy allows to write $\sqrt{g_4}\Gamma^n\Psi = p^k\gamma_k\Psi$ since only M^4 gamma matrices are constant.

2. If p^k is light-like one can assume massless Dirac equation and restriction of the induced spinor field inside the Euclidian regions defining the line of generalized Feynman diagram. The interpretation would be as on mass-shell massless fermion. If p^k is not light-like, this is not possible and induced spinor field is delocalized outside the Euclidian portions of the line of generalized Feynman diagram: interactions would be basically due to the dispersion of induced spinor fields to Minkowskian regions. The interpretation would be as a virtual particle. The challenge is to find whether this interpretation makes sense and whether it is possible to articulate this idea mathematically. The alternative assumption is that also virtual particles can localized inside Euclidian regions.

3. One can wonder what the spectrum of p_k could be. If the identification as virtual momenta is correct, continuous mass spectrum suggests itself. For the incoming lines of generalized Feynman diagram one expects light-like momenta so that Γ^n should be light-like. This assumption is consistent with super-conformal invariance since physical states would correspond to bound states of massless fermions, whose four-momenta need not be parallel. Stringy mass spectrum would be outcome of super-conformal invariance and 2-sheetedness forced by boundary conditions for Kähler action would be essential for massivation. Note however that the string curves along the space-like ends of space-time surface are also internal lines and expected to carry virtual momentum: classical picture suggests that p^k tends to be space-like.

Chern-Simons Dirac action from mathematical consistency

A further natural condition is that the possible boundary term is well-defined. At partonic orbits the boundary term of Kähler-Dirac action need not be well-defined since $\sqrt{g_4}\Gamma^n$ becomes singular. This leaves only Chern-Simons Dirac action

$$\overline{\Psi}\Gamma^{\alpha}_{C-S}D_{\alpha}\Psi$$

under consideration at both sides of the partonic orbits and one can consider continuity of C-S-D action as the boundary condition. Here Γ^{α}_{C-S} denotes the C-S-D gamma matrix, which does not depend on the induced metric and is non-vanishing and well-defined. This picture conforms also with the view about TGD as almost topological QFT.

One could restrict Chern-Simons-Dirac action to partonic orbits since they are special in the sense that they are not genuine boundaries. Also Kähler action would naturally contain Chern-Simons term.

One can require that the action of Chern-Simons Dirac operator is equal to multiplication with $ip^k\gamma_k$ so that massless Dirac propagator is the outcome. Since Chern-Simons term involves only CP_2 gamma matrices this would define the analog of Dirac equation at the level of embedding space. I have proposed this equation already earlier and introduction this it as generalized eigenvalue equation having pseudomomenta p^k as its solutions.

If space-like ends of space-time surface involve no Chern-Simons term, one obtains the boundary condition

$$\sqrt{g_4}\Gamma^n\Psi = 0 \tag{8.2.1}$$

at them. Ψ would behave like massless mode locally. The condition $\sqrt{g_4}\Gamma^n\Psi = \gamma^k p_k\Psi = 0$ would state that incoming fermion is massless mode globally. If Chern-Simons term is present one obtains also Chern-Simons term in this condition but also now fermion would be massless in global sense. The physical interpretation would be as incoming massless fermions.

8.2.8 About The Notion Of Four-Momentum In TGD Framework

The starting point of TGD was the energy problem of General Relativity [K99]. The solution of the problem was proposed in terms of sub-manifold gravity and based on the lifting of the isometries of space-time surface to those of $M^4 \times CP_2$ in which space-times are realized as 4-surfaces so that Poincare transformations act on space-time surface as an 4-D analog of rigid body rather than moving points at space-time surface. It however turned out that the situation is not at all so simple.

There are several conceptual hurdles and I have considered several solutions for them. The basic source of problems has been Equivalence Principle (EP): what does EP mean in TGD framework [K99]? A related problem has been the interpretation of gravitational and inertial masses, or more generally the corresponding 4-momenta. In General Relativity based cosmology gravitational mass is not conserved and this seems to be in conflict with the conservation of Noether charges. The resolution is in terms of ZEO (ZEO), which however forces to modify slightly the original view about the action of Poincare transformations.

A further problem has been quantum classical correspondence (QCC): are quantal four-momenta associated with super conformal representations and classical four-momenta associated as Noether charges with Kähler action for preferred extremals identical? Could inertial-gravitational duality - that is EP - be actually equivalent with QCC? Or are EP and QCC independent dualities. A powerful experimental input comes p-adic mass calculations [K61] giving excellent predictions provided the number of tensor factors of super-Virasoro representations is five, and this input together with Occam's razor strongly favors QCC=EP identification.

There is also the question about classical realization of EP and more generally, TGD-GRT correspondence.

Twistor Grassmannian approach has meant a technical revolution in quantum field theory (for attempts to understand and generalize the approach in TGD framework see [K94]. This approach seems to be extremely well suited to TGD and I have considered a generalization of this approach from $\mathcal{N} = 4$ SUSY to TGD framework by replacing point like particles with string world sheets in TGD sense and super-conformal algebra with its TGD version: the fundamental objects are now massless fermions which can be regarded as on mass shell particles also in internal lines (but with unphysical helicity). The approach solves old problems related to the realization of stringy amplitudes in TGD framework, and avoids some problems of twistorial QFT (IR divergences and the problems due to non-planar diagrams). The Yangian [A20] [B23, B20, B21] variant of 4-D conformal symmetry is crucial for the approach in $\mathcal{N} = 4$ SUSY, and implies the recently introduced notion of amplituhedron [B14]. A Yangian generalization of various super-conformal algebras seems more or less a "must" in TGD framework. As a consequence, four-momentum is expected to have characteristic multilocal contributions identifiable as multipart on contributions now and possibly relevant for the understanding of bound states such as hadrons.

Scale dependent notion of four-momentum in zero energy ontology

Quite generally, General Relativity does not allow to identify four-momentum as Noether charges but in GRT based cosmology one can speak of non-conserved mass [K80], which seems to be in conflict with the conservation of four-momentum in TGD framework. The solution of the problem comes in terms of ZEO (ZEO) [K5, K101], which transforms four-momentum to a scale dependent notion: to each causal diamond (CD) one can assign four-momentum assigned with say positive energy part of the quantum state defined as a quantum superposition of 4-surfaces inside CD.

ZEO is necessary also for the fusion of real and various p-adic physics to single coherent whole. ZEO also allows maximal "free will" in quantum jump since every zero energy state can be created from vacuum and at the same time allows consistency with the conservation laws. ZEO has rather dramatic implications: in particular the arrow of thermodynamical time is predicted to vary so that second law must be generalized. This has especially important implications in living matter, where this kind of variation is observed.

More precisely, this superposition corresponds to a spinor field in the "world of classical worlds" (WCW) [K101]: its components - WCW spinors - correspond to elements of fermionic Fock basis for a given 4-surface - or by holography implied by general coordinate invariance (GCI) - for 3-surface having components at both ends of CD. Strong form of GGI implies strong form of holography (SH) so that partonic 2-surfaces at the ends of space-time surface plus their 4-D tangent space data are enough to fix the quantum state. The classical dynamics in the interior is necessary for the translation of the outcomes of quantum measurements to the language of physics based on classical fields, which in turn is reduced to sub-manifold geometry in the extension of the geometrization program of physics provided by TGD.

Holography is very much reminiscent of QCC suggesting trinity: GCI-holography-QCC. Strong form of holography has strongly stringy flavor: string world sheets connecting the wormhole throats appearing as basic building bricks of particles emerge from the dynamics of induced spinor fields if one requires that the fermionic mode carries well-defined electromagnetic charge [K104].

Are the classical and quantal four-momenta identical?

One key question concerns the classical and quantum counterparts of four-momentum. In TGD framework classical theory is an exact part of quantum theory. Classical four-momentum corresponds to Noether charge for preferred extremals of Kähler action. Quantal four-momentum in turn is assigned with the quantum superposition of space-time sheets assigned with CD - actually WCW spinor field analogous to ordinary spinor field carrying fermionic degrees of freedom as analogs of spin. Quantal four-momentum emerges just as it does in super string models - that is as a parameter associated with the representations of super-conformal algebras. The precise action of translations in the representation remains poorly specified. Note that quantal four-momentum does not emerge as Noether charge: at at least it is not at all obvious that this could be the case.

Are these classical and quantal four-momenta identical as QCC would suggest? If so, the Noether four-momentum should be same for all space-time surfaces in the superposition. QCC suggests that also the classical correlation functions for various general coordinate invariant local quantities are same as corresponding quantal correlation functions and thus same for all 4-surfaces in quantum superposition - this at least in the measurement resolution used. This would be an extremely powerful constraint on the quantum states and to a high extend could determined the U-, M-, and S-matrices.

QCC seems to be more or less equivalent with SH stating that in some respects the descriptions based on classical physics defined by Kähler action in the interior of space-time surface and the quantal description in terms of quantum states assignable to the intersections of space-like 3-surfaces at the boundaries of CD and light-like 3-surfaces at which the signature of induced metric changes. SH means effective 2-dimensionality since the four-dimensional tangent space data at partonic 2-surfaces matters. SH could be interpreted as Kac-Mody and symplectic symmetries meaning that apart from central extension they act almost like gauge symmetries in the interiors of space-like 3-surfaces at the ends of CD and in the interiors of light-like 3-surfaces representing orbits of partonic 2-surfaces. Gauge conditions are replaced with Super Virasoro conditions. The word "almost" is of course extremely important.

What Equivalence Principle (EP) means in quantum TGD?

EP states the equivalence of gravitational and inertial masses in Newtonian theory. A possible generalization would be equivalence of gravitational and inertial four-momenta. In GRT this correspondence cannot be realized in mathematically rigorous manner since these notions are poorly defined and EP reduces to a purely local statement in terms of Einstein's equations.

What about TGD? What could EP mean in TGD framework?

1. Is EP realized at both quantum and space-time level? This option requires the identification of inertial and gravitational four-momenta at both quantum and classical level. It is now clear that at classical level EP follows from very simple assumption that GRT space-time is obtained by lumping together the space-time sheets of the many-sheeted space-time and by the identification the effective metric as sum of M^4 metric and deviations of the induced metrics of space-time sheets from M^2 metric: the deviations indeed define the gravitational field defined by multiply topologically condensed test particle. Similar description applies to gauge fields. EP as expressed by Einstein's equations would follow from Poincare invariance at microscopic level defined by TGD space-time. The effective fields have as sources the energy momentum tensor and YM currents defined by topological inhomogenities smaller than the resolution scale.

2. QCC would require the identification of quantal and classical counterparts of both gravitational and inertial four-momenta. This would give three independent equivalences, say $P_{I,class} = P_{I,quant}$, $P_{gr,class} = P_{gr,quant}$, $P_{gr,class} = P_{I,quant}$, which imply the remaining ones.

   Consider the condition $P_{gr,class} = P_{I,class}$. At classical level the condition that the standard energy momentum tensor associated with Kähler action has a vanishing divergence is guaranteed if Einstein's equations with cosmological term are satisfied. If preferred extremals satisfy this condition they are constant curvature spaces for non-vanishing cosmological constant.It must be emphasized that field equations are extremely non-linear and one must also consider preferred extremals (which could be identified in terms of space-time regions having so called Hamilton-Jacobi structure): hence these proposals are guesses motivated by what is known about exact solutions of field equations.

   Consider next $P_{gr,class} = P_{I,class}$. At quantum level I have proposed coset representations for the pair of super conformal algebras g and $h \subset g$ which correspond to the coset space decomposition of a given sector of WCW with constant values of zero modes. The coset construction would state that the differences of super-Virasoro generators associated with g resp. h annhilate physical states.

   The identification of the algebras g and h is not straightforward. The algebra g could be formed by the direct sum of super-symplectic and super Kac-Moody algebras and its sub-algebra h for which the generators vanish at partonic 2-surface considered. This would correspond to the idea about WCW as a coset space G/H of corresponding groups (consider as a model $CP_2 = SU(3)/U(2)$ with $U(2)$ leaving preferred point invariant). The sub-algebra h in question includes or equals to the algebra of Kac-Moody generators vanishing at the partonic 2-surface. A natural choice for the preferred WCW point would be as maximum of Kähler function in Euclidian regions: positive definiteness of Kähler function allows only single maximum for fixed values of zero modes). Coset construction states that differences of super Virasoro generators associated with g and h annihilate physical states. This implies that corresponding four-momenta are identical that is Equivalence Principle.

The objection against the identification h in the decomposition $g = t + h$ of the symplectic algebra as Kac-Moody algebra is that this does not make sense mathematically. The strong form of holography implied by strong form of General Coordinate Invariance however implies that the action of Kac-Moody algebra for the maxima of Kähler function induces unique action of sub-algebra of symplectic algebra so that the identification makes sense after all [K25].

3. Does EP reduce to one aspect of QCC? This would require that classical Noether four-momentum identified as inertial momentum equals to the quantal four-momentum assignable to the states of super-conformal representations and identifiable as gravitational four-momentum. There would be only one independent condition: $P_{class} \equiv P_{I,class} = P_{gr,quant} \equiv P_{quant}$.

   Holography realized as AdS/CFT correspondence states the equivalence of descriptions in terms of gravitation realized in terms of strings in 10-D space-time and gauge fields at the boundary of AdS. What is disturbing is that this picture is not completely equivalent with the proposed one. In this case the super-conformal algebra would be direct sum of super-symplectic and super Kac-Moody parts.

Which of the options looks more plausible? The success of p-adic mass calculations [K61] have motivated the use of them as a guideline in attempts to understand TGD. The basic outcome was that elementary particle spectrum can be understood if Super Virasoro algebra has five tensor factors. Can one decide the fate of the two approaches to EP using this number as an input?

This is not the case. For both options the number of tensor factors is five as required. Four tensor factors come from Super Kac-Moody and correspond to translational Kac-Moody type degrees of freedom in M^4, to color degrees of freedom and to electroweak degrees of freedom ($SU(2) \times U(1)$). One tensor factor comes from the symplectic degrees of freedom in $\Delta CD \times CP_2$ (note that Hamiltonians include also products of δCD and CP_2 Hamiltonians so that one does not have direct sum!).

The reduction of EP to the coset structure of WCW sectors would be extremely beautiful property. But also the reduction of EP to QCC looks very nice and deep, and it seems that the coset option is definitely wrong: the reason is that for H in G/H decomposition the four-momentum vanishes.

TGD-GRT correspondence and Equivalence Principle

One should also understand how General Relativity and EP emerge at classical level. The understanding comes from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig. ??** in the Appendix).

2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.

3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Kähler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in ZEO (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

How translations are represented at the level of WCW?

The four-momentum components appearing in the formulas of super conformal generators correspond to infinitesimal translations. In TGD framework one must be able to identify these infinitesimal translations precisely. As a matter of fact, finite measurement resolution implies that it is probably too much to assume infinitesimal translations. Rather, finite exponentials of translation generators are involved and translations are discretized. This does not have practical signficance since for optimal resolution the discretization step is about CP_2 length scale.

Where and how do these translations act at the level of WCW? ZEO provides a possible answer to this question.

1. Discrete Lorentz transformations and time translations act in the space of CDs: inertial four-momentum

Quantum state corresponds also to wave function in moduli space of CDs. The moduli space is obtained from given CD by making all boosts for its non-fixed boundary: boosts correspond to a discrete subgroup of Lorentz group and define a lattice-like structure at the hyperboloid for which proper time distance from the second tip of CD is fixed to $T_n = n \times T(CP_2)$. The quantization of cosmic redshift for which there is evidence, could relate to this lattice generalizing ordinary 3-D lattices from Euclidian to hyperbolic space by replacing translations with boosts (velocities).

The additional degree of freedom comes from the fact that the integer $n > 0$ obtains all positive values. One has wave functions in the moduli space defined as a pile of these lattices defined at the hyperboloid with constant value of $T(CP_2)$: one can say that the points of this pile of lattices correspond to Lorentz boosts and scalings of CDs defining sub-WCW:s.

The interpretation in terms of group which is product of the group of shifts $T_n(CP_2) \rightarrow T_{n+m}(CP_2)$ and discrete Lorentz boosts is natural. This group has same Cartesian product structure as Galilean group of Newtonian mechanics. This would give a discrete rest energy and by Lorentz boosts discrete set of four-momenta giving a contribution to the four-momentum appearing in the super-conformal representation.

What is important that each state function reduction would mean localisation of either boundary of CD (that is its tip). This localization is analogous to the localization of particle in position measurement in E^3 but now discrete Lorentz boosts and discrete translations $T_n --> T_{n+m}$ replace translations. Since the second end of CD is necessary del-ocalized in moduli space, one has kind of flip-flop: localization at second end implies de-localization at the second end. Could the localization of the second end (tip) of CD in moduli space correspond to our experience that momentum and position can be measured simultaneously? This apparent classicality would be an illusion made possible by ZEO.

The flip-flop character of state function reduction process implies also the alternation of the direction of the thermodynamical time: the asymmetry between the two ends of CDs would induce the quantum arrow of time. This picture also allows to understand what the experience growth of geometric time means in terms of CDs.

2. The action of translations at space-time sheets

The action of embedding space translations on space-time surfaces possibly becoming trivial at partonic 2-surfaces or reducing to action at δCD induces action on space-time sheet which becomes ordinary translation far enough from end end of space-time surface. The four-momentum in question is very naturally that associated with Kähler action and would therefore correspond to inertial momentum for $P_{I,class} = P_{quant,gr}$ option. Indeed, one cannot assign quantal four-momentum to Kähler action as an operator since canonical quantization badly fails. In finite measurement infinitesimal translations are replaced with their exponentials for $P_{I,class} = P_{quant,gr}$ option.

What looks like a problem is that ordinary translations in the general case lead out from given CD near its boundaries. In the interior one expects that the translation acts like ordinary translation. The Lie-algebra structure of Poincare algebra including sums of translation generators with positive coefficient for time translation is preserved if only time-like superpositions if generators are allowed also the commutators of time-like translation generators with boost generators give time like translations. This defines a Lie-algebraic formulation for the arrow of geometric time. The action of time translation on preferred etxremal would be ordinary translation plus continuation of the translated preferred extremal backwards in time to the boundary of CD. The transversal space-like translations could be made Kac-Moody algebra by multiplying them with functions which vanish at δCD.

A possible interpretation would be that $P_{quant,gr}$ corresponds to the momentum assignable to the moduli degrees of freedom and $P_{cl,I}$ to that assignable to the time like translations. $P_{quant,gr} = P_{cl,I}$ would code for QCC. Geometrically quantum classical correspondence would state that time-like translation shift both the interior of space-time surface and second boundary of CD to the geometric future/past while keeping the second boundary of space-time surface and CD fixed.

Yangian and four-momentum

Yangian symmetry implies the marvellous results of twistor Grassmannian approach to $\mathcal{N} = 4$ SUSY culminating in the notion of amplituhedron which promises to give a nice projective geometry interpretation for the scattering amplitudes [B14]. Yangian symmetry is a multilocal generalization of ordinary symmetry based on the notion of co-product and implies that Lie algebra generates receive also multilocal contributions. I have discussed these topics from slightly different point of view in [K94], where also references to the work of pioneers can be found.

1. Yangian symmetry

The notion equivalent to that of Yangian was originally introduced by Faddeev and his group in the study of integrable systems. Yangians are Hopf algebras which can be assigned with Lie algebras as the deformations of their universal enveloping algebras. The elegant but rather cryptic looking definition is in terms of the modification of the relations for generating elements [K94] . Besides ordinary product in the enveloping algebra there is co-product Δ which maps the elements of the enveloping algebra to its tensor product with itself. One can visualize product and co-product is in terms of particle reactions. Particle annihilation is analogous to annihilation of two particle so single one and co-product is analogous to the decay of particle to two. Δ allows to construct higher generators of the algebra.

Lie-algebra can mean here ordinary finite-dimensional simple Lie algebra, Kac-Moody algebra or Virasoro algebra. In the case of SUSY it means conformal algebra of M^4- or rather its super counterpart. Witten, Nappi and Dolan have described the notion of Yangian for super-conformal algebra in very elegant andconcrete manner in the article *Yangian Symmetry in D=4 superconformal Yang-Mills theory* [B20] . Also Yangians for gauge groups are discussed.

In the general case Yangian resembles Kac-Moody algebra with discrete index n replaced with a continuous one. Discrete index poses conditions on the Lie group and its representation (adjoint representation in the case of $\mathcal{N} = 4$ SUSY). One of the conditions conditions is that the tensor product $R \otimes R^*$ for representations involved contains adjoint representation only once. This condition is non-trivial. For $SU(n)$ these conditions are satisfied for any representation. In the case of $SU(2)$ the basic branching rule for the tensor product of representations implies that the condition is satisfied for the product of any representations.

Yangian algebra with a discrete basis is in many respects analogous to Kac-Moody algebra. Now however the generators are labelled by non-negative integers labeling the light-like incoming and outgoing momenta of scattering amplitude whereas in in the case of Kac-Moody algebra also negative values are allowed. Note that only the generators with non-negative conformal weight appear in the construction of states of Kac-Moody and Virasoro representations so that the extension to Yangian makes sense.

The generating elements are labelled by the generators of ordinary conformal transformations acting in M^4 and their duals acting in momentum space. These two sets of elements can be labelled by conformal weights $n = 0$ and $n = 1$ and and their mutual commutation relations are same as for Kac-Moody algebra. The commutators of $n = 1$ generators with themselves are however something different for a non-vanishing deformation parameter h. Serre's relations characterize the difference and involve the deformation parameter h. Under repeated commutations the generating elements generate infinite-dimensional symmetric algebra, the Yangian. For $h = 0$ one obtains just one half of the Virasoro algebra or Kac-Moody algebra. The generators with $n > 0$ are $n + 1$-local in the sense that they involve $n + 1$-forms of local generators assignable to the ordered set of incoming particles of the scattering amplitude. This non-locality generalizes the notion of local symmetry and is claimed to be powerful enough to fix the scattering amplitudes completely.

2. How to generalize Yangian symmetry in TGD framework?

As far as concrete calculations are considered, it is not much to say. It is however possible to keep discussion at general level and still say something interesting (as I hope!). The key question is whether it could be possible to generalize the proposed Yangian symmetry and geometric picture behind it to TGD framework.

1. The first thing to notice is that the Yangian symmetry of $\mathcal{N} = 4$ SUSY in question is quite too limited since it allows only single representation of the gauge group and requires massless particles. One must allow all representations and massive particles so that the representation of symmetry algebra must involve states with different masses, in principle arbitrary spin and arbitrary internal quantum numbers. The candidates are obvious: Kac-Moody algebras [A9] and Virasoro algebras [A16] and their super counterparts. Yangians indeed exist for arbitrary super Lie algebras. In TGD framework conformal algebra of Minkowski space reduces to Poincare algebra and its extension to Kac-Moody allows to have also massive states.
2. The formal generalization looks surprisingly straightforward at the formal level. In ZEO one replaces point like particles with partonic two-surfaces appearing at the ends of light-like orbits of wormhole throats located to the future and past light-like boundaries of causal diamond ($CD \times CP_2$ or briefly CD). Here CD is defined as the intersection of future and past directed light-cones. The polygon with light-like momenta is naturally replaced with a polygon with more general momenta in ZEO and having partonic surfaces as its vertices. Non-point-likeness forces to replace the finite-dimensional super Lie-algebra with infinite-dimensional Kac-Moody algebras and corresponding super-Virasoro algebras assignable to partonic 2-surfaces.
3. This description replaces disjoint holomorphic surfaces in twistor space with partonic 2-surfaces at the boundaries of $CD \times CP_2$ so that there seems to be a close analogy with Cachazo-Svrcek-Witten picture. These surfaces are connected by either light-like orbits of partonic 2-surface or space-like 3-surfaces at the ends of CD so that one indeed obtains the analog of polygon.

What does this then mean concretely (if this word can be used in this kind of context)?

1. At least it means that ordinary Super Kac-Moody and Super Virasoro algebras associated with isometries of $M^4 \times CP_2$ annihilating the scattering amplitudes must be extended to a co-algebras with a non-trivial deformation parameter. Kac-Moody group is thus the product of Poincare and color groups. This algebra acts as deformations of the light-like 3-surfaces representing the light-like orbits of particles which are extremals of Chern-Simon action with the constraint that weak form of electric-magnetic duality holds true. I know so little about the mathematical side that I cannot tell whether the condition that the product of the representations of Super-Kac-Moody and Super-Virasoro algebras contains adjoint representation only once, holds true in this case. In any case, it would allow all representations of finite-dimensional Lie group in vertices whereas $\mathcal{N} = 4$ SUSY would allow only the adjoint.
2. Besides this ordinary kind of Kac-Moody algebra there is the analog of Super-Kac-Moody algebra associated with the light-cone boundary which is metrically 3-dimensional. The finite-dimensional Lie group is in this case replaced with infinite-dimensional group of symplectomorphisms of $\delta M^4_{+/-}$ made local with respect to the internal coordinates of the partonic 2-surface. This picture also justifies p-adic thermodynamics applied to either symplectic or isometry Super-Virasoro and giving thermal contribution to the vacuum conformal and thus to mass squared.
3. The construction of TGD leads also to other super-conformal algebras and the natural guess is that the Yangians of all these algebras annihilate the scattering amplitudes.
4. Obviously, already the starting point symmetries look formidable but they still act on single partonic surface only. The discrete Yangian associated with this algebra associated with the closed polygon defined by the incoming momenta and the negatives of the outgoing momenta acts in multi-local manner on scattering amplitudes. It might make sense to speak about polygons defined also by other conserved quantum numbers so that one would have generalized light-like curves in the sense that state are massless in 8-D sense.

3.Could Yangian symmetry provide a new view about conserved quantum numbers?

The Yangian algebra has some properties which suggest a new kind of description for bound states. The Cartan algebra generators of $n = 0$ and $n = 1$ levels of Yangian algebra commute. Since the co-product Δ maps $n = 0$ generators to $n = 1$ generators and these in turn to generators with high value of n, it seems that they commute also with $n \geq 1$ generators. This applies to four-momentum, color isospin and color hyper charge, and also to the Virasoro generator L_0 acting on Kac-Moody algebra of isometries and defining mass squared operator.

Could one identify total four momentum and Cartan algebra quantum numbers as sum of contributions from various levels? If so, the four momentum and mass squared would involve besides the local term assignable to wormhole throats also n-local contributions. The interpretation in terms of n-parton bound states would be extremely attractive. n-local contribution would involve interaction energy. For instance, string like object would correspond to $n = 1$ level and give $n = 2$-local contribution to the momentum. For baryonic valence quarks one would have 3-local contribution corresponding to $n = 2$ level. The Yangian view about quantum numbers could give a rigorous formulation for the idea that massive particles are bound states of massless particles.

8.3 Embedding Of The Reissner-NordströOm Metric

The recent view about how GRT relates to TGD differs considerably from that during period 1980-1990 when the calculations related to the embeddings of the basic solutions of Einstein's equations were carried out. At that time I believed that the physically most relevant space-times of GRT would allow embedding as sub-manifolds of $M^4 \times CP_2$. This is certainly not true for all of them: consider only rotating blackholes. Year 2014 - 37 years from discovery of TGD - was the year when I finally realized that the problems related to this relationship were mostly pseudo-problems. As described in previous section, the space-time of GRT emerges elegantly as statistical concept from quantum TGD and the notion of many-sheeted space-time. Space-time sheets represent the microscopic level of gravity not seen in GRT and preferred extremal property makes them extremely simple objects. The experimental challenge is to reveal the presence of these simpled building bricks.

One can of course consider seriously the possibility that the solutions of Einstein's equations imbeddable as vacuum extremals might be physically special. This justifies the discussions of this section as also the section devoted to the final state of the star.

In the following the embedding of electromagnetically neutral Reissner-Nordström metric to $M^4_+ \times CP_2$ will be studied. The embedding generalizes to an embedding of any spherically symmetric metric. The embeddings as vacuum extremals reduce to embeddings into 6-dimensional $M^4 \times Y^2$, Y^2 Lagrange manifold (vanishing induced Kähler form). Any vacuum extremal defines a solution of Einstein's equations if energy momentum tensor is defined by Einstein's equations. Non-vacuum embeddings of Reissner-Nordström solutions would correspond to homologically non-trivial geodesic sphere of CP_2, and it is implausible that non-vacuum embeddings could be extremals. Whether the embeddings of the metrics believed to describe rotating objects in GRT Universe are possible at all, is not known but it might well be that the dimension of the embedding space is too low to allow them. This would mean that the predictions of TGD concerning gravi-magnetism can differ from those of GRT.

8.3.1 Two Basic Types Of Embeddings

One can construct a large number of embeddings for Reissner-Nordström metric. These embeddings need not be extremals of Kähler action except when they are represent vacua.

1. X^4 could be a sub-manifold of $M^4 \times S^2_i$, $i = I, II$, where S^2_i is one of the geodesic spheres of CP_2. For $i = II$ the embeddings are vacuum extremals but this is not the case for $i = I$. The properties of these embeddings are essentially those associated with the spherically symmetric stationary extremals of the Kähler action. Long range electromagnetic and Z^0 fields assignable to dark matter [?] are present but the corresponding forces are by a factor 10^{-4} weaker than gravitational force, when the parameter ωR is of order one.

2. The vacuum extremals of the Kähler action are the physically most interesting candidates for the embeddings of solutions of Einstein's equations. For these embeddings electro-weak fields

are in general non-vanishing. Em neutrality is possible to achieve only for $p = sin^2(\theta_W) = 0$. Long ranged W^+ and W^- fields can be present and they induce a small mixing between charged dark lepton and corresponding neutrino spinors.

8.3.2 The Condition Guaranteeing The Vanishing Of Em, Z^0, Or Kähler Fields

In order to obtain embedding with vanishing em, Z^0, or Kähler field, one must pose the condition guaranteeing the vanishing of corresponding field (see the Appendix of the book). For extremals of Kähler action em Z^0 fields are always simultaneously present unless Weinberg angle vanishes. In practice only the condition guaranteeing vanishing of Kähler field is thus interesting.

Using coordinates $(r, u = cos(\Theta), \Psi, \Phi)$ for CP_2 the surfaces in question can be expressed as

$$\begin{aligned} r &= \sqrt{\frac{X}{1-X}} \ , \\ X &= D|k+u|^{\epsilon} \ , \\ u &\equiv cos(\Theta) \ , \ D = \frac{r_0^2}{1+r_0^2} \times \frac{1}{C} \ , \quad C = |k + cos(\Theta_0)|^{\epsilon} \ . \end{aligned} \tag{8.3.1}$$

Here C and D are integration constants. The value of the parameter ϵ characterizes which field vanishes:

$$\begin{aligned} &a) \ \epsilon = \tfrac{3+p}{3+2p} \ , \quad b) \ \epsilon = \tfrac{1}{2} \ , \quad c) \ \epsilon = 1 \ , \\ &p = sin^2(\Theta_W) \ . \end{aligned} \tag{8.3.2}$$

Here a/b/c corresponds to the vanishing of em/Z^0/Kähler field.

$0 \leq X \leq 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \leq 1$ and $r = \infty$ to $X = 1$ giving $|u + k| = [(1 + r_0^2)/r_0^2)]^{\epsilon}$ achieved only for

$$sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\epsilon} \leq k+1 \ ,$$

where $sign(x)$ denotes the sign of x.

These embeddings obviously possess a 2-dimensional CP_2 projection. The generation of long range vacuum weak and color electric fields is a purely TGD based phenomenon related to the fact that gauge fields are not primary dynamical variables.

For future purposes it is convenient to list the explicit expressions of relevant gauge field when em or Kähler field vanishes.

1. Using coordinates $(u = cos(\Theta), \Phi)$ the expressions for the Kähler form and Z^0 field for space-time surfaces with vanishing em field read as

$$\begin{aligned} J &= -\frac{p}{3+2p} X du \wedge d\Phi \ , \quad X = D|k+u|^{\frac{3+p}{3+2p}} \\ Z^0 &= -\frac{6}{p} J \ . \end{aligned} \tag{8.3.3}$$

2. For vacuum extremals ($\epsilon = 1$) classical em and Z^0 fields are proportional to each other:

$$\begin{aligned} Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u} du \wedge d\Phi = (k+u) du \wedge d\Phi \ , \\ r &= \sqrt{\frac{X}{1-X}} \ , \ X = D|k+u| \ , \\ \gamma &= -\frac{p}{2} Z^0 \ . \end{aligned} \tag{8.3.4}$$

For a vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible. The only reasonable physical interpretation seems to be in terms of a hierarchy of electro-weak physics with arbitrarily light weak boson mass scales.

The effective form of the CP_2 metric is given by

$$\begin{aligned}
ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta}d\Theta^2 + s^{eff}_{\Phi\Phi}d\Phi^2] \ , \\
s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\
s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ . \qquad (8.3.5)
\end{aligned}$$

This expression is useful in the construction of electromagnetically neutral embedding of, say Schwartchild metric. For $k \neq 1$ $u = \pm 1$ corresponds in general to circle rather than single point as is clear from the fact that $s^{eff}_{\Phi\Phi}$ is non-vanishing at $u = \pm 1$ so that u and Φ parameterize a piece of cylinder.

8.3.3 Embedding Of Reissner-Nordström Metric

The embedding of R-N metric to be discussed generalizes with minor modifications to an embedding of a spherically symmetric star model characterized by a mass density $\rho(r_M)$ and pressure $p(r_M)$ since the corresponding line element can be written in the form $ds^2 = A(r_M)dt^2 - B(r_M)dr_M^2 - r_M^2 d\Omega^2$ [E54]. For vacuum extremal a solution of field equations results.

Denote the coordinates of M^4_+ by (m^0, r_M, θ, ϕ) and those of X^4 by (t, r_M, θ, ϕ). The expression for Reissner-Nordström metric reads as

$$\begin{aligned}
ds^2 &= Adt^2 - Bdr_M^2 - r_M^2 d\Omega^2 \ , \\
A &= 1 - \frac{a}{r_M} - \frac{b}{r_M^2} \ , \quad B = \frac{1}{A} \ , \\
a &= 2GM \ , \quad b = G\pi q^2 \ . \qquad (8.3.6)
\end{aligned}$$

The embedding is given by the expression

$$\begin{aligned}
\Phi &= \omega_1 t + f(r_M) \ , \\
\Psi &= k\Phi = \omega_2 t + kf(r_M) \ , \\
m^0 &= \lambda t + h(r_M) \ , \\
\lambda &= \sqrt{1 + \frac{R^2\omega_1^2}{4}s^{eff}_{\Phi\Phi}(\infty)} \ , \quad k = \frac{\omega_2}{\omega_1} \ . \qquad (8.3.7)
\end{aligned}$$

The components of s^{eff} are given by Eq. 8.3.5 and general form of embedding by Eqs. 8.3.1 and 8.3.2.

The functions $f(r_M)$ and $h(r_M)$ are determined by the condition

$$\lambda\partial_{r_M} h = \frac{R^2}{4}s^{eff}_{\Phi\Phi}\omega_1\partial_{r_M} f \qquad (8.3.8)$$

resulting from the requirement $g_{tr_M} = 0$ and from the expression for $g_{r_M r_M} = -B$:

$$\begin{aligned}
h &= \int dr_M\sqrt{Y} \ , \quad Y = \frac{Y_1}{Y_2} \ , \\
Y_1 &= -B + 1 + \frac{R^2}{4}s^{eff}_{\Theta\Theta}\frac{(\partial_{r_M}u)^2}{(1-u^2)} \ , \\
Y_2 &= 1 - \frac{4\lambda^2}{R^2\omega_1^2}\frac{s^{eff}_{\Theta\Theta}}{s^{eff}_{\Phi\Phi}} \ . \qquad (8.3.9)
\end{aligned}$$

The condition $Y>0$ at the limit $r\rightarrow\infty$ gives non-trivial conditions. Y_1 is positive at large values of r_M and this gives

$$Y_1=-B+1+s^{eff}_{\Theta\Theta}\frac{(\partial_{r_M}u)^2}{(1-u^2)}\geq 0$$

for the allowed values of r_M. Y_1 can change sign at some critical radius above Schwartshild radius $r_S=2GM$ since B becomes infinite at r_S: this can be avoided only provided one has $u\rightarrow 1$ at $r_M\rightarrow r_S$. Y_2 must preserve its sign and this is possible if the value of $R\omega_1$ is sufficiently large. Below $r=r_S$ Y_1 has positive and also Y_2 can be positive down to some critical radius. At $r=r_S$ Y_1 has infinite discontinuity in case that Y_1 approaches finite value from above and CP_2 coordinates are continuous. It is easy to see that square root singularity of Θ as a function of r_M-r_S is in question so that the function h is continuous so that the solution is well-defined.

The dependence of $u\equiv cos(\Theta)$ on radial coordinate r_M is determined by the expression for $g_{tt}=A$ giving the condition

$$A=\lambda^2-\frac{R^2\omega_1^2}{4}s^{eff}_{\Phi\Phi}\omega_1^2 \ . \tag{8.3.10}$$

The asymptotic behavior of the coordinate $u=cos(\Theta)$ is of form

$$u \simeq u_{\infty}+\frac{K}{r_M} \ , \tag{8.3.11}$$

u_{∞} is fixed by the condition $A(\infty)=1$:

$$\begin{aligned} \lambda^2 &- \frac{R^2\omega_1^2}{4}s^{eff}_{\Phi\Phi}(\infty)=1 \ , \\ s^{eff}_{\Phi\Phi} &= X\times\left[(1-X)(k+u)^2+1-u^2\right] \ , \ X=D|k+u|^{\epsilon} \ . \end{aligned} \tag{8.3.12}$$

The value of K is given by

$$K = \frac{8GM}{R^2\omega_1^2}\left[\frac{\partial s^{eff}_{\Phi\Phi}}{\partial u}(\infty)\right]^{-1} \ . \tag{8.3.13}$$

The values of K and u_{∞} depend on parameters $\lambda, R\omega_1, k, D$.

For definiteness one can assume that the value of u at infinity is non-negative:

$$u_{\infty} \geq 0 \ . \tag{8.3.14}$$

There are two different solution types depending on the sign of the parameter K.

1. For $K<0$ u decreases and approaches to $u_{min}\geq 0$ as r_M decreases.
2. For $K>0$ u increases and approaches to $u_{max}\leq 1$. The requirement that the solution can be continued below Schwartshild radius allows only this option. Below Schwartshild radius u must transform to a solution of type a).

Imbeddability breaks for a critical value of the radial coordinate

The imbeddability breaks for some critical value of the coordinate r_M. The extremal value of u and the radius r_c below which the embedding fails corresponds to the maximum possible value of $s^{eff}_{\Phi\Phi}$. This value corresponds either to $u = 0,1$ or to a vanishing derivative of $s^{eff}_{\Phi\Phi}$

$$\frac{\partial s^{eff}_{\Phi\Phi}}{\partial u} = 0 \ . \tag{8.3.15}$$

For $\epsilon = 1$ corresponding to vacuum extremals s_{eff} is a fourth order polynomial as a function of u depending on external parameters. One has

$$s^{eff}_{\Phi\Phi} = D|k+u| \times \left[(1-D|k+u|)(k+u)^2 + 1 - u^2\right] \ . \tag{8.3.16}$$

s_{eff} becomes negative for very large values of u. Hence a restriction of the standard form of the dual of the cusp catastrophe to the range $u \in (0,1)$ results. Depending on the values of external parameters there are either 2 maxima or single maximum. For $k = 1$ the positive extremum correspond to $u = 1/|D|$.

In the case of the Schwartshild metric this gives for the critical radius the expression

$$\begin{aligned} r_c &= \frac{r_S}{\delta} \ , \\ \delta &= 1 - \lambda^2 + \frac{R^2\omega_1^2}{4} s^{eff}_{\Phi\Phi}(max) \ , \\ r_S &= 2GM \ . \end{aligned} \tag{8.3.17}$$

The existing evidence for black hole like objects suggests that it would be better to have $\delta >> 1$ in order to get embeddings of the Schwartschild metric containing also horizon and part of the interior region. A sufficiently large value of $R\omega_1$ indeed allows to have arbitrarily small value of r_c. There the experimental evidence for the existence of black hole like objects leads to no problems.

The vacuum extremal embeddings of Schwartschild metric possess electro-weak charges

The vacuum embeddings of Reissner-Nodrström and Scwartshild metric necessarily possess some non-vanishing electro-weak charges. Consider first vacuum extremals. Z^0 electric field Z^0_{tr} is proportional to ω_1

$$Z^0_{tr_M} = \omega_1 (k+u) \partial_{r_M} u \ . \tag{8.3.18}$$

The gauge flux through a sphere with radius r_M depends on r_M so that Z^0 vacuum charge density is necessarily present.

The condition $\theta \propto \sqrt{r - r_S}$ allowing to continue the embedding below $r_M < r_S$ implies that gauge fluxes, which are proportional to $sin(\Theta)\partial_{r_M}\Theta$, are finite at $r = r_S$ so that the renormalizations of gauge couplings remain finite at least down to Schwartschild radius.

At large distances the gauge flux approaches to

$$\begin{aligned} Q_Z(\infty) &= \frac{1}{g_Z} \int_{r_M \to \infty} Z^0_{tr_M} r_M^2 d\Omega \ , \\ &= \frac{4\pi}{g_Z} \omega_1 (k + u_\infty) K = \frac{4\pi}{g_Z} (k + u_\infty) \frac{8GM}{R^2\omega_1} \left[\frac{\partial s^{eff}_{\Phi\Phi}}{\partial u}(\infty)\right]^{-1} \end{aligned} \tag{8.3.19}$$

at the limit $r_M \to \infty$. Z^0 charge is proportional to the gravitational mass. The gauge flux grows at small distances in accordance with the general wisdom about the coupling constant evolution of $U(1)$ gauge field.

The requirement that Z^0 force is weaker than gravitational force expressed as the condition

$$\frac{Q_Z^2}{GM^2} \ll 1$$

implies

$$\frac{32\pi}{R\omega_1 g_Z}(k+u_\infty)\left[\frac{\partial s^{eff}_{\Phi\Phi}}{\partial u}(\infty)\right]^{-1} \ll \frac{R}{\sqrt{G}} \quad . \tag{8.3.20}$$

It seems that a sufficiently large value of $R\omega_1$ allows arbitrarily small values for both the Z^0 charge and the critical radius r_c. In the earliest scenario, which was based on the assumption that CP_2 radius is of order Planck length the situation was different. It is clear that the larger radius of CP_2 makes it possible to avoid too strong classical electro-weak forces.

The non-extremal embedding to S^2_I studied in detail here is Kähler charged and therefore also Z^0 charged since the condition $Z^0 = 6J/p$ holds true by electromagnetic neutrality. The value of the Kähler charge for non-vacuum embedding depends on the distance from the origin

$$\begin{aligned} Q_K(r_M) &= \frac{1}{g_K}\int_{r_M=const} J_{tr_M} r_M^2 d\Omega \ , \\ J_{tr_M} &= -\frac{p}{2(3+p)}\omega_1 |k+u|^{\frac{3+p}{3+2p}}\partial_{r_M} u \ , \end{aligned} \tag{8.3.21}$$

The expression for the charge differs only in minor details from that for Z^0 charge for vacuum extremals. Essentially similar conclusions about the behavior of the gauge charges hold true also in the case of vacuum extremals and the expressions differ only by the value of the parameter ϵ characterizing whether em, Z^0, of Kähler field vanishes.

Equivalence Principle and critical radius

When one considers Equivalence Principle (EP), one must keep in mind that the Kähler charged embeddings of Reissner Nodström and Scwartschild metrics are *not* extremals of Kähler action. Second thing that one must forget is that the most feasible realization of EP at quantum level seems is based on the identification of classical Noether charges in Cartan algebra with the eigenvalues of their quantum counterparts assignable to Kähler-Dirac action. At classical level EP follows at GRT limit obtained by lumping many-sheeted space-time to M^4 with effective metric satisfying Einstein's equations as a reflection of the underlying Poincare invariance [K99]. With this backgrounds the problems due to EP reduce to pseudo-problems due to too narrow interpretation caused by the attempt to identify GRT space-time with single space-time sheet.

1. Equivalence Principle and embeddings as vacuum extremals

In the case of vacuum extremals the naïve interpretation is that net inertial energy density of the space-time outside the topologically condensed space-time sheet representing charged system is vanishing but the density of gravitational energy is non-vanishing and non-conserved in general. The gravitational mass of the topologically condensed space-time sheet however consists of both inertial and purely gravitational contribution. For RN solution it is natural to interpret the gravitational mass as the gravitational energy of the classical gauge fields. For genuine RN case the densities of inertial color gauge charges vanish but those for gravitational color gauge charges in $SO(3) \subset SU(3)$ are in general non-vanishing. Schwartschild metric possesses necessarily a vacuum densities of some electro-weak gauge charges but the contribution to the gravitational energy momentum tensor vanishes.

EP obviously fails if applied to vacuum extremal. The proper interpretation is that for vacuum extremals induced metric can be identified with the effective metric of GRT space-time obtained by lumping the sheets of many-sheeted space-time to Minkowski space with effective metric. EP is true for GRT space-time satisfying Einstein equations reflecting Poincare invariance and also the vanishing of divergence of the energy momentum tensor for the proposed general ansatz to solutions of field equations.

2. Equivalence Principle and embeddings as non-vacuum extremals

One can consider Equivalence Principle in the case of Kähler charged embeddings only if one believes that the embedding is in a reasonable approximation an extremal. Equivalence Principle requires that the Kähler mass of the solution should be smaller than its gravitational mass. This does not pose any conditions on the critical radius since the density of Kähler charge can change sign inside the critical radius (meaning that antimatter dominates inside the critical radius). Thus no constraints results.

The strongest form of Equivalence Principle in TGD context (rather than applying at GRT limit as reflection of Poincare invariance) would require that the Kähler mass of the solution equals to its gravitational mass. It is difficult to see how this could be implied by any deep principle. This requirement poses a lower limit to the critical radius since the Kähler energy outside the critical radius should be smaller than the gravitational mass of the system. In the lowest order approximation this energy is given by the expression

$$\begin{aligned} \frac{E_K}{M} &= \frac{1}{8\pi\alpha_K M}\int_{r_M\geq r_c}\lambda E_K^2 dV \\ &= \frac{\lambda Q_K^2}{GM^2}\frac{r_S}{r_c} \ . \end{aligned} \tag{8.3.22}$$

The requirement that electro-weak interactions are much weak than gravitational interaction imply the condition $Q_K^2/GM^2 \ll 1$ so that the ratio can be equal to 1 as Equivalence Principle requires only if $r_S/r_c \gg 1$ holds true.

Gravitational energy is not conserved for vacuum embedding of Reissner-Nordström metric

The inertial energy associated with Kähler action inside a ball of given radius is not conserved for Reissner-Nordström metric imbedded as a non-vacuum extremal since extremal of Kähler action is not in question. This follows from the dependence $m^0 = \lambda t + h(r_M)$ implying that energy current has a radial component and from the non-vanishing of $T^{r_M r_M}$. The non-conservation is not due to the outflow of energy but due to the fact that in the case of Kähler charged embedding field equations are not satisfied. The basic reason is that the contraction of the energy momentum tensor with the second fundamental form is non-vanishing.

For vacuum extremals it is gravitational energy which fails to be conserved. For instance, for the embedding of Reissner-Nordström this happens. Only at the limit of Schwartshild metric gravitational energy is conserved. The vacuum extremals which are extremals of Einstein-Hilbert action for the induced metric conserve gravitational four momenta and color charges and are excellent candidates for models of the asymptotic state of star.

The simplest interpretation for the non-conservation of gravitational energy without losing Equivalence Principle is in terms of zero energy ontology. In zero energy ontology the extremals of curvature scalar have interpretation in terms of infinitely long time scale associated with the causal diamond.

The non-stationarity of the vacuum extremal embedding ($m^0 = \lambda t + h(r_M)$) of R-N metric leads to the following expression for the rate of the change of gravitational energy per time inside a sphere of radius r

$$\begin{aligned} \frac{dE_{vap}/dt}{E(r_M)} &= \frac{dE(r_M)/dt}{E(r_M)} + X \ , \\ X &= \frac{\int T^{r_M r_M}\partial_{r_M} m^0\sqrt{g}d\Omega}{E(r_M)} \ , \\ E(r_M) &= \int T^{tt}\partial_0 m^0\sqrt{g}dV \ . \end{aligned} \tag{8.3.23}$$

The latter term depending on $T^{r_M r_M}$ takes into account the flow of gravitational energy through boundaries of the sphere and is in general non-vanishing for Reissner-Nordström metric.

Since the proposed solution ansatz works also in the more general case of a stationary spherically symmetric star model, characterized by the pressure $p(r_M)$ and the energy density $\rho(r_M)$, one can write a general order of magnitude estimate for the gravitational energy transfer associated with the boundary of the sphere approximating $h(r_M)$ with the corresponding function for the Schwartshild metric for large values of r_M as

$$X \simeq -\partial_{r_M} h(r_M)\frac{4\pi p r_M^2}{M} . \quad (8.3.24)$$

The explicit expression for $\partial_{r_M} h(r_M)$ is given by

$$\begin{aligned} \partial_{r_M} h(r_M) &\simeq \frac{1}{\lambda}\sqrt{\frac{Y_1}{Y_2}} , \\ Y_1 &= -B + 1 + \frac{R^2}{4} s^{eff}_{\Theta\Theta}\frac{(\partial_{r_M} u)^2}{(1-u^2)} , \\ 1 - \frac{4\lambda^2}{R^2\omega_1^2}\frac{s^{eff}_{\Theta\Theta}}{s^{eff}_{\Phi\Phi}} & . \end{aligned} \quad (8.3.25)$$

Here B is determined by the Einstein equations defining the star model and can be approximated with its value for Schwartshild metric.

At the surface of the Sun ($r_M \simeq 6\cdot 10^8\ m$, particle density $n \simeq 10^{21}/m^3$, $T \simeq 0.5\ eV$, $M \simeq 10^{57} m_p$ and pressure $p \simeq nT$) the order of magnitude of this term is about $X/E \simeq 2\pi K 10^{-13}/year$. For $K \sim 1$ (obtained if the radius of CP_2 is of order Planck length) the loss would be of the same order of magnitude as the inertial energy loss associated with the solar wind: $K \sim 1/k$, $10^4 < k << 10^8$, however implies that the loss is roughly four orders of magnitudes smaller.

It should be noticed that in the case of matter dominated cosmology shows that the rate for the reduction of the gravitational energy is of the order of $(dE/da)/E \simeq 1/a \simeq 10^{-11}/year$, which is of the same order as the fusion energy production of Sun. Thus it would seem that the rate for the change of gravitational energy in cosmological length scales is same as that for the inertial energy in the solar length scale.

8.4 A Model For The Final State Of The Star

Also this chapter was written decades ago and reflects different views about how TGD space-time relates to GRT space-time.

1. The assumption that single preferred extremal describes the final state might be over-simplifying since many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book) having GRT space-time as a model is the correct description, at least according to the recent view.
2. At the time of writing the role of classical electroweak gauge fields was poorly understood although above weak scale it was clear that somehow their effects should be small. In living matter parity breaking effects are large and it is quite possible that classical Z^0 forces is present. This is allowed also by the recent view about Kähler-Dirac equation stating that spinor modes are restricted to 2-D string world sheets carrying vanishing W fields (to guarantee well-defined em charge) and possibly also vanishing Z^0 fields to guarantee vectorial couplings and experimental absence of Z^0 coupling in long length scales. An interesting possibility is that string world sheets can also carry Z^0 field below weak scale which for dark phase with large Planck constant is scaled up.

The best one can hope, is that the general features of the model as preferred extremal could be reflected also in the properties of GRT space-time.

As found, the energy production by fusion inside stars is of the same order of magnitude as the rate of change for the gravitational energy associated with the recent matter dominated cosmology. Since no energy is produced in the final state of the star, the stationary solutions provide a natural model for the final state of the star.

Besides stationarity, there is also a second new element, namely color and electro-weak long ranged forces coupling to the dark matter. For instance, for Kähler charged extremals one necessary has classical Z^0 force even when classical em force can vanish. For Schwartshild solution this force becomes very strong at small values of the radial distance. Therefore the presence of the Z^0 force, and presumably also other classical electro-weak forces, *might* play crucial role in the dynamics of the compact objects. The most plausible physical interpretation would be in terms of dark matter.

The topics to be discussed in the following are:

1. Spherically symmetric stationary model for the final state of the star. It is found that the model cannot be completely realistic since the stationarity assumption fails at the origin and at the surface of the star.
2. Generalization of the model to what could be called dynamo model in order to achieve stationarity.
3. The possible consequences of long range weak and color forces associated with dark matter, in particular the Z^0 force, concerning the dynamics of the compact objects.

The original discussion was based on a different view about energy and motivated the study of Kähler charged solutions with the stationarity property. These 4-surfaces are *not* extremals of the Kähler action. The replacement of the stationary solutions with vacuum extremals requires however only the replacement of the geodesic sphere S^2_I with S^2_{II} implying that both em and Z^0 fields are unavoidably present (or even $W^{\pm}$ fields, depending on vacuum extremal). A serious limitation of the model is that it is single-sheeted. Indeed, the fact that the rotation axis and magnetic axis of super novae are different can be seen as a signal of many-sheeted-ness: the dominantly em and Z^0 fields would reside at different space-time sheets and would correspond to ordinary and dark matter. Of course, entire hierarchy of space-time sheets are expected to be present.

8.4.1 Spherically Symmetric Model

The simplest model for the final state of the star that one can imagine is obtained by assuming time translation invariance plus spherical symmetry and imbeddability to $M^4 \times S^2_i$, where S^2_i, $i = I, II$ is the geodesic sphere of CP_2. For the homologically non-trivial sphere S^2_I the solution is *not* an extremal whereas S^2_{II} gives an extremal with a vanishing density of inertial energy. In the original discussion cosmological constant was assumed to vanish. There are excellent reasons to assume that this constant is so small that it does not have any appreciable effects in the scale of the star and can thus be neglected. The nice feature of this kind of model is that symmetry assumptions plus stationarity requirement fix almost completely the model: no assumptions about the equation of state for the matter inside the star are needed.

The solution ansatz giving rise to vacuum extremal corresponds to a surface $X^4 \subset M^4_+ \times S^2_{II}$, where S^2_{II} is the homologically trivial geodesic sphere of CP_2. The solution ansatz has the same general form as the embedding of spherically symmetric metric.

$$\begin{aligned} m^0 &= \lambda t + h(r) \ , \\ \Theta &= \Theta(r) \ , \\ \Phi &= \omega t + k(r) \ . \end{aligned} \tag{8.4.1}$$

The requirement that g_{tr} vanishes, implies a relationship between the functions $h(r)$ and $k(r)$. One might think that the simplest model is obtained, when the functions $h(r)$ and $k(r)$ vanish identically. One doesn't however obtain physically acceptable solutions in this manner: this is seen by expressing the g_{rr} component of the metric in terms of the mass function

$$-g_{rr} = 1 + \frac{R^2}{4}(\partial_r \Theta)^2 = \frac{1}{1 - \frac{2GM(r)}{r}} \ .$$

At the radii of order star radius (larger than Schwartshild radius $r_S = 2GM$) the gradient of Θ must be of the order of 1/R and this is inconsistent with the finite range of possible values for Θ.

As already shown the field equations $G^{\alpha\beta}D_\beta\partial_\alpha h^k = 0$ are obtained by varying the integral of the curvature scalar over the space time surface. Field equations reduce to conservation conditions for suitably chosen conserved current: for instance the relevant components of the gravitational 4-momentum and gravitational color currents and express the conservation of gravitational four-momentum current and corresponding color currents.

The expression for the induced metric is given by

$$\begin{aligned} ds^2 &= Bdt^2 - Adr^2 - r^2 d\Omega^2 \ , \\ B &= \lambda^2 - \frac{R^2\omega^2}{4} sin^2\Theta \ , \\ A &= 1 + \frac{R^2}{4}(\partial_r\Theta)^2 + \frac{R^2}{4} sin^2\Theta(\partial_r k)^2 - (\partial_r h)^2 \ . \end{aligned} \tag{8.4.2}$$

The vanishing of the g_{tr} component of the metric implies the condition

$$\lambda\partial_r h - \frac{R^2}{4}\sin^2\Theta\omega\partial_r k = 0 \ . \tag{8.4.3}$$

The expressions for the components of Einstein tensor for spherically symmetric stationary metric are given by

$$\begin{aligned} G^{rr} &= \frac{1}{A^2}(-\frac{\partial_r B}{Br} + \frac{(A-1)}{r^2}) \ , \\ G^{\theta\theta} &= \frac{1}{r^2}[-\frac{\partial_r^2 B}{2BA} + \frac{1}{2Ar}(\frac{\partial_r A}{A} - \frac{\partial_r B}{B}) \\ &+ \frac{\partial_r B}{4AB}(\frac{\partial_r A}{A} + \frac{\partial_r B}{B}] \ , \\ G^{tt} &= \frac{1}{AB}(-\frac{\partial_r A}{Ar} + \frac{(1-A)}{r^2}) \ . \end{aligned} \tag{8.4.4}$$

A solution of the field equations with one-dimensional CP_2 projection and vanishing gauge fields is obtained by specifying the solution ansatz in the following manner

$$\begin{aligned} \Theta &= \frac{\pi}{2} \ , \\ h(r) &= hr \ , \\ k(r) &= kr \ . \end{aligned} \tag{8.4.5}$$

The requirement that g_{rt} vanishes gives the condition

$$h\lambda = R^2\omega k/4 \ .$$

The functions A and B are in this case just constants. Since A differs from unity, the resulting metric is however non-flat and the non-vanishing components of the Einstein tensor are given by the expressions

$$\begin{aligned} G^{tt} &= \frac{(1-A)}{ABr^2} \ , \\ G^{rr} &= -\frac{(1-A)}{A^2r^2} \ . \end{aligned} \tag{8.4.6}$$

Field equations can be written as conservation conditions, say for the components of gravitational 4-momentum and the conserved "gravitational" color charges associated with the symmetry $\Phi \rightarrow \Phi + \varepsilon$. Quite generally, "gravitational" isometry currents have only time and radial components and radial component represent radial flow to or from the origin. Since the time component is time independent, the field equations state that radial flow is constant so that radial component of the current must behave as $1/r^2$. This is guaranteed provided the condition

$$\partial_r(G^{rr}\sqrt{g}) = 0 \quad (8.4.7)$$

holds true: this is indeed the case since G^{rr} is proportional to $1/r^2$.

The radial flow of gravitational energy is non-vanishing due to $m^0 = \lambda tt + h(r)$ behavior and given by the expression

$$J^r = \frac{G^{rr}h}{16\pi G} \ .$$

The conservation condition for J^r fails to be satisfied at origin, which acts as a source or a sink for the gravitational energy. Conservation law fails also at the surface of the star.

One can consider several interpretations.

1. Gravitational mass could be genuinely non-conserved at these locations. Dark particles *resp.* antiparticles with positive *resp.* negative energy would be created in the center of star such that the net density of inertial energy remains zero. Positive and negative energy particles would flow along their own space-time sheets to the outer surface and annihilate there so that there would be no net growth of the gravitational mass. The simplest possibility is that # contacts, which correspond to bound states of parton and negative energy antiparton [**?**], split to give rise to particles of opposite inertial energy. At the outer surface # contacts fuse together again.

2. Second option assumes the conservation of gravitational four-momentum. At the surface the non-conservation could result from the flow of the gravitational 4-momentum to a larger space-time sheet via flux tubes. No net flow of inertial energy would be involved since positive and negative energy flows must cancel each other. For example, for a physically acceptable solution the gravitational energy might flow radially from or towards the z-axis, flow to say north pole at the surface of the object and return back along z-axis. Gravitational energy could also flow at origin to a second space-time sheet and return back at the surface of the star.

The (gravitational) mass function of the solution is given by the expression

$$M(r) = \frac{\lambda}{16\pi G}\frac{(A-1)}{AB}\sqrt{AB}r \ . \quad (8.4.8)$$

Mass is positive for Minkowskian metric with $B > 0$ and Euclidian metric but negative for the interior black hole metric ($A < 0, B < 0$). Mass is proportional to the radius of the star and in order to obtain an object with about Schwartshild radius one must assume that the parameter k is of the order of $1/R$.

Concerning the physical interpretation of the $\Theta = \pi/2$ solution following remarks are in order:

1. Various gauge fields vanish since CP_2 projection is actually a geodesic circle. The interpretation is that various gauge charges vanish. Note that one-dimensional CP_2 projection conforms with the similar property of Robertson Walker cosmologies.

2. Both gravitational, color and weak forces vanish inside the star and the motion along radial geodesics takes place with constant velocity. This is consistent with the radial flow of gravitational energy.

3. Solution ansatz allows generalizations. For example, the following modification is stationary with respect to energy: $m^0 = \lambda t$, $\Phi = \omega_1 t + k_1 r$, $\Psi = \omega_2 t + k_2 r$, $u = constant < \infty$, $\Theta = \pi/2$. By choosing the values of the parameters suitably all the field equations are satisfied but stationarity is not achieved.

4. The solution should allow gluing to the Schwartshild metric at $\Theta = \pi/2$. As found, for the embedding of Schwartshild metric the $\Theta = 0$ correspond to the Schwartchild radius so that $\Theta = \pi/2$ would most naturally correspond to $r_M < r_S$. Since radial gauge fluxes are non-vanishing and finite at Schwartshild radius, they must be non-vanishing $\Theta = \pi/2$ surface too, so that the star would carry surface charges and behave somewhat like a conducting sphere.

8.4.2 Dynamo Model

The previous considerations have shown that the spherically symmetric solution is probably not physically realistic as such and it seems also clear that spherical symmetry must be given up and be replaced with a symmetry with respect to rotations around z-axis in order to obtain more realistic solutions. Since realistic stars rotate and have strong magnetic fields it is natural to ask whether rotation and magnetic fields might provide remedy for the pathological features of the solution. The rotation of the gauge charged matter (in "gravitational" sense) indeed creates classical gauge magnetic fields, which become very strong near the surface of the star, where the condition $\Theta \simeq \pi/2$ holds. If matter is approximately gauge neutral in the interior, the gauge fields should vanish to a very good approximation in the interior and the previous solution should be a good approximation to the actual situation. The rotating star could therefore be regarded as a rotating electro-weak conductor. Both Z^0 and em fields are present for a vanishing Kähler field and the ratio of field strengths is $\gamma/Z^0 = -sin^2(\theta_W)/2 \simeq -1/8$ (see Appendix) so that Z^0 field dominates.

The generation of strong em and Z^0 electric and magnetic fields suggests a mechanism guaranteeing the stability of the solution: star behaves like a dynamo. For solutions with a 2-dimensional CP_2 projection em and Z^0 electric and magnetic fields are automatically orthogonal. For $\Theta \simeq \pi/2$ they are very strong and dominate over gravitation and centrifugal force. Therefore the stability of the surface region naturally results from the cancelation of the electric and magnetic em and Z^0 forces ($\bar{E} + \bar{v} \times \bar{B} = 0$), which takes place, when the velocity field of the matter is suitably chosen. This condition is completely analogous to the vanishing of Kähler Lorentz 4-force which seems to be a general property of the solutions of field equations [K11] and there are reasons to hope non-vacuum extremals describing rotating star can be found.

Conditions for the vanishing of the induced Kähler field

Although the situation becomes too complicated in order to allow the finding of exact solutions describing rotating star, one can identify some general properties of the solution ansatz describing the rotating configuration with Kähler electric and magnetic fields. In order to study the general properties of the solution ansatz in its most general form the explicit expressions for the line element and Kähler form of CP_2 given by the expression

$$\begin{aligned} ds^2 &= \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + cos(\Theta)d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + sin^2(\Theta)d\Phi^2) \ , \\ J &= \frac{r}{2F^2}dr \wedge (d\Psi + cos(\Theta)d\Phi) + \frac{r^2}{2F}sin\Theta d\Theta \wedge d\Phi \ , \\ F &= 1 + r^2 \ , \end{aligned} \tag{8.4.9}$$

0
are needed.

The vanishing of Kähler field is can be guaranteed by the conditions (not the most general ones, symplectic transformations generate new solutions)

$$\begin{aligned} \Phi &= q\Psi \ , \\ \frac{dr}{d\theta} &= -qrF\frac{sin(\theta)}{1 + qcos(\theta)} \ . \end{aligned} \tag{8.4.10}$$

Note that this ansatz excludes the case $qcos(\Theta) = -1$ for which only $W^{\pm}$ fields are non-vanishing. For this ansatz the expressions for em and Z^0 fields (see Appendix for general formulas) are

$$\gamma = -sin^2(\Theta_W)R_{03} \ , \qquad Z^0 = 2R_{03} \ ,$$
$$R_{03} = -qr^2Fsin(\theta)d\Theta\wedge d\Psi \ . \tag{8.4.11}$$

Here R_{03} denotes a component of spinor curvature.

Topological quantum numbers

The crucial point is that the expansions for the angle coordinates Φ and Ψ using spherical coordinates contain linear terms in t, r and ϕ

$$\begin{aligned} \Phi &= n_1\phi + \omega_1 t + k_1 \ , \\ \Psi &= n_2\phi + \omega_2 t + k_2 \ . \end{aligned} \tag{8.4.12}$$

The functions k_1 and k_2 corresponds to Fourier expansion in terms of the plane waves $exp(in\phi)$ with coefficients depending on the coordinates (t, r, θ).

The terms depending linearly on ϕ imply a nontrivial topological structure for the gauge fields not present for the ordinary Maxwell fields. What happens is that space-time divides into regions, which correspond to different values of the topological quantum numbers (n_1, n_2). In the boundaries of these regions the values of the coordinates u and Θ must be such that different values of Φ and Ψ correspond to same point of CP_2. From the expression of the line element one finds that for Ψ the point $u = 0$ and the sphere $u = \infty$ corresponds to these kinds of points. For Φ the surfaces $u = 0$ and $u = \infty$, $\Theta = 0$ correspond to these kinds of surfaces. The form of Φ and Ψ implies that both electric and magnetic gauge fields are nontrivial and rather closely related as is clear from the expression for the Kähler form. Therefore the non-triviality of the winding numbers n_1 and n_2 is what seems to be the crucial, purely TGD based feature of rotating gauge field structures.

Stationary, axially symmetric ansatz with a non-vanishing Kähler field

To make the discussion more concrete, let us assume that the induced metric is invariant with respect to rotations around z-axis and time translations. This is achieved if CP_2 coordinates (apart from linear dependence on ϕ) depend on the coordinates r_M and θ only.

$$\begin{aligned} r &= r(r_M, \theta) \\ \Theta &= \Theta(r_M, \theta) \ , \\ k_i &= k_i(r_M, \theta) \ , \quad i = 1, 2 \ . \end{aligned} \tag{8.4.13}$$

This kind of ansatz is clearly consistent: field equations reduce to four equations since second fundamental form is orthogonal to the four-surface and there are four free functions of r and θ: one has effectively two dimensional field theory. Since the general solution ansatz for field equations relies on the vanishing of the Lorentz Kähler force central for the dynamo mechanism, it is of interest to study the general properties of the solution ansatz with a non-vanishing Kähler field. This ansatz can give as special cases space-time sheets carrying Z^0 and em fields with magnetic fields having different rotation axis.

In order to further simplify the discussion let us assume that X^4 corresponds to a submanifold of $M^4 \times S^2_I$. For instance, the ansatz

$$\begin{aligned} r &= \infty \ , \\ \Theta &= \Theta(r_M, \theta) \ , \\ \Phi &= n\phi + \omega t + k(r_M, \theta) \ . \end{aligned} \tag{8.4.14}$$

is consistent with this assumption. A simpler ansatz is obtained by assuming $k(r_M, \theta) = 0$. This ansatz has the following properties.

1. Induced Kähler (and Z^0-) electric and magnetic fields are automatically orthogonal since CP_2 projection is two-dimensional. In fact, the orthogonality holds to an excellent approximation also for the values of u different but near to $u = \infty$ since the resulting additional components of the Kähler field are extremely small. Kähler electric and magnetic fields are given by

$$\begin{aligned} E_{r_M} &= J_{r_M t} = -\partial_{r_M} cos(\Theta)\omega/2 \ , \\ E_\theta &= -\partial_\theta cos(\Theta)\omega/2 \ , \\ B_\theta &= -\partial_{r_M} cos(\Theta) n/2 \ , \\ B_{r_M} &= -\partial_\theta cos(\Theta) n/2 \ . \end{aligned} \tag{8.4.15}$$

   The field strengths are related by

$$\begin{aligned} E &= vB \ , \\ v &= \frac{\omega}{n}\sqrt{-\frac{g_{\phi\phi}}{g_{tt}}} \simeq \frac{\omega}{n}\rho \ , \end{aligned} \tag{8.4.16}$$

   where ρ denotes radial distance from the rotation axis. v can be interpreted as a velocity type parameter. The requirement that $v < 1$ gives a lower bound for the value of n: $n > \omega r_0$, where r_0 denotes the radius of the star: the condition implies that n must be larger than the mass of the star using Planck mass as unit. Somewhat counter intuitively, small rotation velocities seem to correspond to large values of n.

2. Kähler electric and magnetic fields indeed provide a possible mechanism guaranteeing the stability of the star at the surface, where Z^0 forces dominate over gravitation and centrifugal force. Star behaves like a dynamo: matter rotates with a velocity guaranteeing the vanishing of the Z^0 force. It should be noticed that no upper bound for the rotation velocity except that resulting from causality is obtained ($\Omega < 1/r_0$). Therefore this mechanism might explain the observed very large rotation velocities (for instance in Super Nova SN1987A), which are hard to understand in GRT based models [E55].

3. The ansatz indeed describes a rotating object. First, the dynamo mechanism for the stability necessitates the presence of rotation and determines rotation velocity also. Secondly, the presence of Kähler magnetic field can be understood as being created by the rotation of gauge charges. Thirdly, the $g_{t\phi}$ component of the induced metric and therefore the angular momentum density $J_z^t \propto G^{t\phi} r^2 sin^2\theta$ is non-vanishing. A rough order of magnitude estimate for the angular momentum gives $J \simeq M\sqrt{G}n$. In order to obtain angular momentum of order $MR \simeq GM^2$ the order of magnitude for the parameter n must be $n \simeq M\sqrt{G}$ or the mass of the star using Planck mass as unit or: notice that also the Kähler charge of the star is of the same order of magnitude.

4. The gluing of the solution to Schwartshild solution realized as a vacuum extremal is possible at a surface $\Theta = 0$, which corresponds to Scwartschild radius, since at this surface different values of Φ correspond to same point of CP_2. The gluing condition gives additional constraint $u = \infty$ at $r_M = r_S$.

5. The experience with the radially symmetric solution ansatz suggests that Θ is very nearly constant $\Theta \simeq \pi/2$ in the interior and varies considerably only at the surface of the star where Θ must go to zero in order to allow gluing to Schwartshild metric at $r = r_S$. A possible picture is therefore the following. On z-axis there is a Z^0 charged vortex creating radial Z^0 electric field and Z^0 magnetic field in the direction of the vortex. In order to obtain cyclic energy flow matter velocity near the surface of the star must have besides the rotational component a component in θ direction (Z^0 force vanishes in this direction).

6. An interesting possibility is that the vortex actually corresponds to a Kähler charged cosmic string which has gradually lost its enormous inertial mass by a generating pairs of positive and negative energy particles, such that positive energy particles have left the string and participated in the formation of the star. The weakening of the magnetic field would have

forced a gradual thickening of the cosmic string to an ordinary magnetic flux tube. This "stars as pearls in necklace" picture would be consistent with the idea that cosmic strings serve as seeds of galaxy and star formation. Both negative and positive energy strings should be present in order to guarantee vanishing of net inertial energy and one can wonder whether the axis of Z^0 and em magnetic fields correspond to these two kinds of strings.

Does Sun have a solid surface?

The model for the asymptotic state of star predicts that mass at given space-time sheet is concentrated in a spherical shell so that star would have a multi-sheeted onion-like structure. This brings in mind the model for the formation of planetary systems in which spherical layers of quantum coherent dark matter serve as templates for the formation of visible matter which eventually condensed to planets [K79, K31]. It would not be surprising if also younger stars and also planets would possess similar structure. This picture is in conflict with the simplest model of Sun as a gas sphere.

Around 2005 new satellites had begun to provide information about what lurks beneath the photosphere. The pictures produced by Lockheed Martin's Trace Satellite and YOHKOH, TRACE and SOHO satellite programs are publicly available in the web. SERTS program for the spectral analysis suggest a new picture challenging the simple gas sphere picture [E47]. The visual inspection of the pictures combined with spectral analysis has led Michael Moshina to suggests that Sun has a solid, conductive spherical surface layer consisting of calcium ferrite. The article of [E47] [E47] provides impressive pictures, which in my humble non-specialist opinion support this view. Of course, I have not worked personally with the analysis of these pictures so that I do not have the competence to decide how compelling the conclusions of Moshina are. In any case, I think that his web article [E47] deserves a summary.

Before SERTS people were familiar with hydrogen, helium, and calcium emissions from Sun. The careful analysis of SERTS spectrum however suggest the presence of a layer or layers containing ferrite and other heavy metals. Besides ferrite SERTS found silicon, magnesium, manganese, chromium, aluminum, and neon in solar emissions. Also elevated levels of sulphur and nickel were observed during more active cycles of Sun. In the gas sphere model these elements are expected to be present only in minor amounts. As many as 57 different types of emissions from 10 different kinds of elements had to be considered to construct a picture about the surface of the Sun.

Moshina has visually analyzed the pictures constructed from the surface of Sun using light at wave lengths corresponding to three lines of ferrite ions (171, 195, 284 Angstroms). On basis of his analysis he concludes that the spectrum originates from rigid and fixed surface structures, which can survive for days. A further analysis shows that these rigid structure rotate uniformly.

The existence of rigid structures idealizable as spherical shells in the first approximation would conform with the model for the final state of star extrapolated to a qualitative picture about the structure younger stars.

8.4.3 Z^0 Force And Dynamics Of Compact Objects

The fact that long ranged color fields and weak fields, in particular Z^0 electric fields, could become strong under certain conditions and in fact dominate over gravitation might have interesting consequences in the physics of compact objects. Besides the dynamo mechanism guaranteeing the stability of the compact object the following ideas come immediately into mind.

1. In GRT based models Super Nova explosion is explained in terms of the pressure of the collapsed matter. Numerical simulations however fail to produce the explosion [E55] and it might even be that GRT based models in fact predict the collapse to black hole. Z^0 and em electric fields created by dark matter plus the existence of particles with Z^0 charge, which are suggested by TGD based model of nucleus and condensed matter to be present already in ordinary condensed matter, might provide a natural mechanism preventing the formation of the black hole (also excluded by the failure of complete imbeddability). When matter collapses to a sufficiently small volume the value of Θ approaches $\pi/2$ in the surface region and very strong repulsive radial Z^0 force is generated and could indeed lead to the explosion. Very light exotic variants of Higgs bosons identified as wormhole contacts having left handed weak charge provides a possible mechanism generating Z^0 charge.

2. The strong Z^0 fields at the surface of the star might provide energy source and acceleration mechanism for very high energy cosmic rays and a mechanism producing very high energy X-rays. These rays would be dark matter particles but could transform to ordinary matter by the mechanism discussed in [K33, K31]. For instance, one can imagine the ejection of a particle beam from the surface of a compact object: particles in dark matter phase gain very high energies in the Z^0 electric field and emit brehmstrahlung in the direction of their motion: most intense emission appears in the region very near the surface of the star, where the Z^0 electric field is strongest. This kind of mechanism might provide alternative explanation for the pulsars. In standard explanations the emission takes place in the direction of the magnetic axis, which does not coincide with the rotation axis. In present case the emission point could be anywhere on the surface of the star and magnetic and rotation axes might well coincide as they do in the simplest model. What one has to do is to invent a mechanism creating the surface instability pushing the matter from the surface of the star to the Kähler electric field.
3. The topological character of the magnetic structures might have applications also in the physics of the ordinary stars. It is known that solar magnetic fields correspond to definite isolated structures [E18]. Since electromagnetic fields must be accompanied by Kähler fields it is tempting to assume that these structures indeed correspond to the structures predicted by TGD. At the surface of the Sun the value of Θ near $\pi/2$ are possible and therefore Z^0 force can be very strong inside the magnetic structures.

8.4.4 Correlation Between Gamma Ray Bursts And Supernovae And Dynamo Model ForThe Final State Of The Star

The correlation between gamma ray bursts and supernovae is certainly the cosmological discovery of the year 2003 [E26, E30].

1. The first indications for supernova gamma ray burst connection came 1998 when a supernova was seen few days after the gamma ray burst in the same region of sky. In this case the intensity of the burst was however by four orders of magnitude weaker than for the typical gamma ray bursts so that the idea about the correlation was not taken seriously. On 29 March, observers recorded a burst christened as GRB030329. On 6 April, theorists at the Technion Institute of Technology in Israel and CERN in Geneva predicted that there would be signs of a supernova in the visible light and infrared spectra on 8 April [E26]. On cue, two days later, observers picked up the telltale spectrum of a type Ic supernova in the same region of sky, triggered as the collapsing star lost hydrogen from its surface. It has now become clear that a large class of gamma ray bursts correlate with supernovae of type Ib and Ic [E17], and that they could thus be powered by the mere core collapse leading to supernova. Recall that supernovae of type II involve hydrogen lines unlike those of type I. Supernovae of type Ib shows Helium lines, and Ic shows neither hydrogen nor helium but intermediate mass elements instead. Supernovae of type Ib and Ic are thought to result as core collapse of massive stars.
2. One of the most enigmatic findings were the "mystery spots" accompanying supernova SN1987A at a distance of few light weeks at the symmetry axis at opposite sides of the supernova [E32]. Their luminosity was nearly 5 per cent of the maximal one. SN1987A was also accompanied by an expanding axi-symmetric remnant surrounded by three concentric rings.
3. The latest finding [E21] is that the radiation associated with the gamma ray bursts is maximally polarized. The polarization degree is the incredible 80 ± 20 per cent, which tells that it must be generated in an extremely strong magnetic field rather than in a simple explosion. The magnetic field must have a strong component parallel to the eye sight direction.

According to the updated model discussed in detail in [K26], cosmic strings transform in topological condensation to magnetic flux tubes about which they represent a limiting case. Primordial magnetic flux tubes forming ferro-magnet like structures become seeds for gravitational condensation leading to the formation of stars and galaxies. The TGD based model for the asymptotic state of a rotating star as dynamo leads to the identification of the predicted magnetic flux tube at the rotation axis of the star as Z^0 magnetic flux tube of primordial origin and assignable to dark matter. Besides Z^0 magnetic flux tube structure also magnetic flux tube structure exists at different space-time sheet but is in general not parallel to the Z^0 magnetic structure. This structure cannot have primordial origin (the magnetic field of star can even flip its polarity).

The flow of matter along Z^0 magnetic (rotation) axis generates synchrotron radiation, which escapes as a precisely targeted beam along magnetic axis and leaves the star. The identification is as the rotating light beam associated with ordinary neutron stars. During the core collapse leading to the supernova this beam becomes gamma ray burst. The mechanism is very much analogous to the squeezing of the tooth paste from the tube.

TGD based models of nuclei [K84] and condensed matter [K33] suggests that the nuclei of dense condensed matter develop anomalous color and weak charges coupling to dark weak bosons having Compton length L_w of order atomic size. Also lighter copies of weak bosons can be important in living matter. This weak charge is vacuum screened above L_w and by dark particles below it. Dark neutrinos, which according to TGD based explanation of tritium beta decay anomaly [K84] should have the same mass scale as ordinary neutrinos, are good candidates for screening dark particles. The Z^0 charge unbalance caused by the ejection of screening dark neutrinos hinders the gravitational collapse. The strong radial compression amplifies the tooth paste effect in this kind of situation so that there are hopes to understand the observed incredibly high polarization of 80 ± 20 per cent [E21].

8.4.5 Z^0 Force And Super Nova Explosion

The mechanism behind Super Nova explosion is not completely understood. The general picture is roughly the following.

1. The formation of iron means the end of the nuclear processes. The inner parts of the star contract and the degeneracy pressure of the non-relativistic electrons ($E_F \propto \rho^{2/3}$) increases and compensates the gravitational force. The equilibrium state is not stable. When the mass of the iron core approaches Chandrasekhar mass $1.4 M_{Sun}$ electrons become relativistic. The milder dependence of the electron Fermi energy on density $E_F \propto \rho^{1/3}$ at the relativistic limit leads to the loss of stability. The high Fermi energy of the electrons allows also the reactions $p + e^- \rightarrow n + \nu_e$ implying decrease of the electronic pressure and neutronization of nuclear matter in the core. Gravitational collapse starts.
2. Collapse stops, when the density of the core reaches the density of the nuclear matter. The degeneracy pressure of the neutrons stops contraction, a shock wave is created and the shock wave and neutrino radiation blow the outer regions of the star away so that Super Nova explosion results.

The problem of this scenario is that numerical simulations do not lead to a strong enough Super Nova explosion and the star tends to collapse into a black hole. A repulsive long ranged Z^0 force predicted by TGD based model of atomic nuclei [K84] generating an additional pressure provides a possible mechanism hindering the collapse and leading to the explosion.

1. The TGD based model for nuclei [K84] and condensed matter [K33] suggests that the nuclei of dense condensed matter develop anomalous color and weak charges coupling to dark weak bosons having Compton length L_w of order atomic size. Weak charge is due to the charged color bonds between nucleons: for instance, tetraneutron can be understood as an alpha particle containing two negatively charged color bonds [K84]. This weak charge is vacuum screened above L_w and by dark particles below it. The charged bonds could exist and also generated between nucleons of different nuclei during collapse.

   Dark neutrinos, which according to the TGD based explanation of tritium beta decay anomaly [K84] should have the same mass scale as ordinary neutrinos, are good candidates for screening dark weak force partially below length scale L_w. In equilibrium color force compensates the partially screened Z^0 force in the bonds. For the ordinary condensed matter densities vacuum screening effectively eliminates the force between neighboring nuclei, and the force makes it visible only via low compressibility. The gravitational collapse could be hindered by the strong additional pressure created by the repulsive L_w-ranged weak interaction between nucleons becoming manifest in the resulting dense phase.
2. In the initial state Z^0 charge is screened by dark neutrinos below L_w so that the repulsive Z^0 force is weaker than gravitational force and attractive color force associated with the

bonds. Neutronization reactions $p+e^- \rightarrow n+\nu_e$ trigger the collapse. During collapse density increases so that dark neutrinos are not able to screen the anomalous Z^0 charge density. The dark neutrino radiation escaping from the star can also reduce the Z^0 screening. The resulting repulsive weak force implies a rapid increase of pressure with increasing density and thus a very low compressibility as it is proposed to imply also in the case of ordinary condensed matter [K33]. The repulsive weak force thus stops the collapse to black-hole.

3. The study of the spherically symmetric star models as 4-surfaces imbedded in $M^4_+ \times CP_2$ shows that the extreme nonlinearity of Kähler action implies that Z^0 force dominates over gravitation near the surface of the star.

8.4.6 Microscopic Description Of Black-Holes In TGD Universe

In TGD framework the embedding of the metric for the interior of Schwartshild black-hole fails below some critical radius. This strongly suggests that only the exterior metric of black-hole makes sense in TGD framework and that TGD must provide a microscopic description of black-holes. Somewhat unexpectedly, I ended up with this description from a model of hadrons.

Super-symplectic algebra is a generalization of Kac-Moody algebra obtained by replacing the finite-dimensional group G with the group of symplectic transformations of $\delta M^4_\pm \times CP_2$. This algebra defines the group of isometries for the "world of classical worlds" and together with the Kac-Moody algebra assignable to the deformations of light-like 3-surfaces representing orbits of 2-D partonic surfaces it defines the mathematical backbone of quantum TGD as almost topological QFT.

From the point of view of experimentalist the basic question is how these super-symplectic degrees of freedom reflect themselves in existing physics and the pleasant surprise was that super-symplectic bosons explain what might be called the missing hadronic mass and spin. The point is that quarks explain only about 170 MeV of proton mass. Also the spin puzzle of proton is known for years. Also precise mass formulas for hadrons emerge.

Super-symplectic degrees of freedom represent dark matter in electro-weak sense and highly entangled hadronic strings in Hagedorn temperature are very much analogous to black-holes. This indeed generalizes to a microscopic model for black-holes created when hadronic strings fuse together in high density.

Super-symplectic bosons

TGD predicts also exotic bosons which are analogous to fermion in the sense that they correspond to single wormhole throat associated with CP_2 type vacuum extremal whereas ordinary gauge bosons corresponds to a pair of wormhole contacts assignable to wormhole contact connecting positive and negative energy space-time sheets. These bosons have super-conformal partners with quantum numbers of right handed neutrino and thus having no electro-weak couplings. The bosons are created by the purely bosonic part of super-symplectic algebra [K25, K104], whose generators belong to the representations of the color group and 3-D rotation group but have vanishing electro-weak quantum numbers. Their spin is analogous to orbital angular momentum whereas the spin of ordinary gauge bosons reduces to fermionic spin. Recall that super-symplectic algebra is crucial for the construction of WCW Kähler geometry. If one assumes that super-symplectic gluons suffer topological mixing identical with that suffered by say U type quarks, the conformal weights would be (5, 6, 58) for the three lowest generations. The application of super-symplectic bosons in TGD based model of hadron masses is discussed in [K62] and here only a brief summary is given.

As explained in [K62], the assignment of these bosons to hadronic space-time sheet is an attractive idea.

1. Quarks explain only a small fraction of the baryon mass and that there is an additional contribution which in a good approximation does not depend on baryon. This contribution should correspond to the non-perturbative aspects of QCD. A possible identification of this contribution is in terms of super-symplectic gluons. Baryonic space-time sheet with $k = 107$ would contain a many-particle state of super-symplectic gluons with net conformal weight of 16 units. This leads to a model of baryons masses in which masses are predicted with an accuracy better than 1 per cent.

2. Hadronic string model provides a phenomenological description of non-perturbative aspects of QCD and a connection with the hadronic string model indeed emerges. Hadronic string tension is predicted correctly from the additivity of mass squared for $J = 2$ bound states of super-symplectic quanta. If the topological mixing for super-symplectic bosons is equal to that for U type quarks then a 3-particle state formed by 2 super-symplectic quanta from the first generation and 1 quantum from the second generation would define baryonic ground state with 16 units of conformal weight. A very precise prediction for hadron masses results by assuming that the spin of hadron correlates with its super-symplectic particle content.
3. Also the baryonic spin puzzle caused by the fact that quarks give only a small contribution to the spin of baryons, could find a natural solution since these bosons could give to the spin of baryon an angular momentum like contribution having nothing to do with the angular momentum of quarks.
4. Super-symplectic bosons suggest a solution to several other anomalies related to hadron physics. The events observed for a couple of years ago in RHIC [C11] suggest a creation of a black-hole like state in the collision of heavy nuclei and inspire the notion of color glass condensate of gluons, whose natural identification in TGD framework would be in terms of a fusion of hadronic space-time sheets containing super-symplectic matter materialized also from the collision energy. In the collision, valence quarks connected together by color bonds to form separate units would evaporate from their hadronic space-time sheets in the collision, and would define TGD counterpart of Pomeron, which experienced a reincarnation for few years ago [C14]. The strange features of the events related to the collisions of high energy cosmic rays with hadrons of atmosphere (the particles in question are hadron like but the penetration length is anomalously long and the rate for the production of hadrons increases as one approaches surface of Earth) could be also understood in terms of the same general mechanism.

Are ordinary black-holes replaced with super-symplectic black-holes in TGD Universe?

Some variants of super string model predict the production of small black-holes at LHC. I have never taken this idea seriously but in a well-defined sense TGD predicts black-hole like states associated with super-symplectic gravitons with strong gravitational constant defined by the hadronic string tension. The proposal is that super-symplectic black-holes have been already seen in Hera, RHIC, and the strange cosmic ray events.

Baryonic super-symplectic black-holes of the ordinary M_{107} hadron physics would have mass 934.2 MeV, very near to proton mass. The mass of their M_{89} counterparts would be 512 times higher, about 478 GeV. "Ionization energy" for Pomeron, the structure formed by valence quarks connected by color bonds separating from the space-time sheet of super-symplectic black-hole in the production process, corresponds to the total quark mass and is about 170 MeV for ordinary proton and 87 GeV for M_{89} proton. This kind of picture about black-hole formation expected to occur in LHC differs from the stringy picture since a fusion of the hadronic mini black-holes to a larger black-hole is in question.

An interesting question is whether the ultrahigh energy cosmic rays having energies larger than the GZK cut-off of 5×10^{10} GeV are baryons, which have lost their valence quarks in a collision with hadron and therefore have no interactions with the microwave background so that they are able to propagate through long distances.

In neutron stars the hadronic space-time sheets could form a gigantic super-symplectic black-hole and ordinary black-holes would be naturally replaced with super-symplectic black-holes in TGD framework (only a small part of black-hole interior metric is representable as an induced metric). This obviously means a profound difference between TGD and string models.

1. Hawking-Bekenstein black-hole entropy would be replaced with its p-adic counterpart given by

$$S_p = (\frac{M}{m(CP_2)})^2 \times log(p) \ , \qquad (8.4.17)$$

where $m(CP_2)$ is CP_2 mass, which is roughly 10^{-4} times Planck mass. M is the contribution of p-adic thermodynamics to the mass. This contribution is extremely small for gauge bosons but for fermions and super-symplectic particles it gives the entire mass.

2. If p-adic length scale hypothesis $p \simeq 2^k$ holds true, one obtains

$$S_p = klog(2) \times (\frac{M}{m(CP_2)})^2 , \quad (8.4.18)$$

$m(CP_2) = \hbar/R$, R the "radius" of CP_2, corresponds to the standard value of $\hbar_0$ for all values of $\hbar$.

3. Hawking-Bekenstein area law gives in the case of Schwartschild black-hole

$$S = \frac{A}{4G} \times \hbar = \pi G M^2 \times \hbar \ . \quad (8.4.19)$$

For the p-adic variant of the law Planck mass is replaced with CP_2 mass and $klog(2) \simeq log(p)$ appears as an additional factor. Area law is obtained in the case of elementary particles if k is prime and wormhole throats have M^4 radius given by p-adic length scale $L_k = \sqrt{k}R$ which is exponentially smaller than L_p. For macroscopic super-symplectic black-holes modified area law results if the radius of the large wormhole throat equals to Schwartschild radius. Schwartschild radius is indeed natural: a simple deformation of the Schwartschild exterior metric to a metric representing rotating star transforms Schwartschild horizon to a light-like 3-surface at which the signature of the induced metric is transformed from Minkowskian to Euclidian.

4. The formula for the gravitational Planck constant appearing in the Bohr quantization of planetary orbits and characterizing the gravitational field body mediating gravitational interaction between masses M and m [K79] reads as

$$\hbar_{gr} = \frac{GMm}{v_0}\hbar_0 \ .$$

$v_0 = 2^{-11}$ is the preferred value of v_0. One could argue that the value of gravitational Planck constant is such that the Compton length $\hbar_{gr}/M$ of the black-hole equals to its Schwartshild radius. This would give

$$\hbar_{gr} = \frac{GM^2}{v_0}\hbar_0 \ , \quad v_0 = 1/2 \ . \quad (8.4.20)$$

The requirement that $\hbar_{gr}$ is a ratio of ruler-and-compass integers expressible as a product of distinct Fermat primes (only four of them are known) and power of 2 would quantize the mass spectrum of black hole [K79]. Even without this constraint M^2 is integer valued using p-adic mass squared unit and if p-adic length scale hypothesis holds true this unit is in an excellent approximation power of two.

5. The gravitational collapse of a star would correspond to a process in which the initial value of v_0, say $v_0 = 2^{-11}$, increases in a stepwise manner to some value $v_0 \leq 1/2$. For a supernova with solar mass with radius of 9 km the final value of v_0 would be $v_0 = 1/6$. The star could have an onion like structure with largest values of v_0 at the core as suggested by the model of planetary system. Powers of two would be favored values of v_0. If the formula holds true also for Sun one obtains $1/v_0 = 3 \times 17 \times 2^{13}$ with 10 per cent error.

6. Black-hole evaporation could be seen as means for the super-symplectic black-hole to get rid of its electro-weak charges and fermion numbers (except right handed neutrino number) as the antiparticles of the emitted particles annihilate with the particles inside super-symplectic black-hole. This kind of minimally interacting state is a natural final state of star. Ideal super-symplectic black-hole would have only angular momentum and right handed neutrino number.

7. In TGD light-like partonic 3-surfaces are the fundamental objects and space-time interior defines only the classical correlates of quantum physics. The space-time sheet containing the highly entangled cosmic string might be separated from environment by a wormhole contact with size of black-hole horizon.

This looks the most plausible option but one can of course ask whether the large partonic 3-surface defining the horizon of the black-hole actually contains all super-symplectic particles so that super-symplectic black-hole would be single gigantic super-symplectic parton. The interior of super-symplectic black-hole would be a space-like region of space-time, perhaps resulting as a large deformation of CP_2 type vacuum extremal. Black-hole sized wormhole contact would define a gauge boson like variant of the black-hole connecting two space-time sheets and getting its mass through Higgs mechanism. A good guess is that these states are extremely light.

Anyonic view about blackholes

A new element to the model of black hole comes from the vision that black hole horizon as a light-like 3-surface corresponds to a light-like orbit of light-like partonic 2-surface. This allows two kinds of black holes. Fermion like black hole would correspond to a deformed CP_2 type extremal which Euclidian signature of metric and topologically condensed at a space-time sheet with a Minkowskian signature. Boson like black hole would correspond to a wormhole contact connecting two space-time sheets with Minkowskian signature. Wormhole contact would be a piece deformed CP_2 type extremal possessing two light-like throats defining two black hole horizons very near to each other. It does not seem absolutely necessary to assume that the interior metric of the black-hole is realized in another space-time sheet with Minkowskian signature.

Second new element relates to the value of Planck constant. For $\hbar_{gr} = 4GM^2$ the Planck length $L_P(\hbar) = \sqrt{\hbar G}$ equals to Schwartschild radius and Planck mass equals to $M_P(\hbar) = \sqrt{\hbar/G} = 2M$. If the mass of the system is below the ordinary Planck mass: $M \leq m_P(\hbar_0)/2 = \sqrt{\hbar_0/4G}$, gravitational Planck constant is smaller than the ordinary Planck constant.

Black hole surface contains ultra dense matter so that perturbation theory is not expected to converge for the standard value of Planck constant but do so for gravitational Planck constant. If the phase transition increasing Planck constant is a friendly gesture of Nature making perturbation theory convergent, one expects that only the black holes for which Planck constant is such that $GM^2/4\pi\hbar < 1$ holds true are formed. Black hole entropy -being proportional to $1/\hbar$- is of order unity so that TGD black holes are not very entropic. $\hbar = GM^2/v_0$, $v_0 = 1/4$, would hold true for an ideal black hole with Planck length $(\hbar G)^{1/2}$ equal to Schwartshild radius $2GM$. Since black hole entropy is inversely proportional to $\hbar$, this would predict black hole entropy to be of order single bit. This of course looks totally non-sensible if one believes in standard thermodynamics. For the star with mass equal to 10^{40} Planck masses the entropy associated with the initial state of the star would be roughly the number of atoms in star equal to about 10^{60}. Black hole entropy proportional to $GM^2/\hbar$ would be of order 10^{80} provided the standard value of $\hbar$ is used as unit. This stimulates some questions.

1. Does second law pose an upper bound on the value of $\hbar$ of dark black hole from the requirement that black hole has at least the entropy of the initial state. The maximum value of $\hbar$ would be given by the ratio of black hole entropy to the entropy of the initial state and about 10^{20} in the example consider to be compared with $GM^2/v_0 \sim 10^{80}$.

2. Or should one generalize thermodynamics in a way suggested by zero energy ontology by making explicit distinction between subjective time (sequence of quantum jumps) and geometric time? The arrow of geometric time would correlate with that of subjective time. One can argue that the geometric time has opposite direction for the positive and negative energy parts of the zero energy state interpreted in standard ontology as initial and final states of quantum event. If second law would hold true with respect to subjective time, the formation of ideal dark black hole would destroy entropy only from the point of view of observer with standard arrow of geometric time. The behavior of phase conjugate laser light would be a more mundane example. Do self assembly processes serve as example of non-standard arrow of geometric time in biological systems? In fact, zero energy state is geometrically analogous to a big bang followed by big crunch. One can however criticize the basic assumption as ad

hoc guess. One should really understand the the arrow of geometric time. This is discussed in detail in [L2].

If the partonic 2-surface surrounds the tip of causal diamond CD, the matter at its surface is in anyonic state with fractional charges. Anyonic black hole can be seen as single gigantic elementary particle stabilized by fractional quantum numbers of the constituents preventing them from escaping from the system and transforming to ordinary visible matter. A huge number of different black holes are possible for given value of $\hbar$ since there is infinite variety of pairs (n_a, n_b) of integers giving rise to same value of $\hbar$.

One can imagine that the partonic surface is not exact sphere except for ideal black holes but contains large number of magnetic flux tubes giving rise to handles. Also a pair of spheres with different radii can be considered with surfaces of spheres connected by braided flux tubes. The braiding of these handles can represent information and one can even consider the possibility that black hole can act as a topological quantum computer. There would be no sharp difference between the dark parts of black holes and those of ordinary stars. Only the volume containing the complex flux tube structures associated with the orbits of planets and various objects around star would become very small for black hole so that the black hole might code for the topological information of the matter collapsed into it.

8.5 TGD Based Model For Cosmic Strings

The model for cosmic strings has forced to question all cherished assumptions including positive energy ontology, Equivalence Principle, and positivity of gravitational energy.

8.5.1 Zero Energy Ontology And Cosmic Strings

There are two kinds of cosmic strings: free and topological condensed ones.

1. Free cosmic strings need not be as such preferred extremals of Kähler action. This statement of course depends on what one means with “preferred extremal”. The original belief was that preferred extremal corresponds to an absolute minima of Kähler action and does not look a promising identification anymore except perhaps in Euclidian regions where Kähler action is positive definite. There are several other proposals based on some abstract geometric property. One proposal assumes the existence of so called Hamilton-Jacobi structure [K99]. A proposal inspired by number theoretc vision assumes quaternionic structure in some sense as a characterized of preferred extremal [K88]. Since the magnetic field of cosmic string corresponds to CP_2 degrees of freedom with Euclidian signature electric duals do not probably exist.

   Also criticality could be the defining property of preferred extremals. In the new formulation preferred extremals correspond to quantum criticality identified as the vanishing of the second variation of Kähler action at least for the deformations defining symmetries of Kähler action [K104, K40]. The symmetries very probably correspond to conformal symmetries acting as or almost as gauge symmetries. The number of conformal equivalence classes of space-time sheets with same Kähler action and conserved charges is expected to be finite and correspond to n in $h_{eff} = n \times h$ defining the hierarchy of Planck constants labelling phases of dark matter (see **Fig.** `http://tgdtheory.fi/appfigures/planckhierarchy.jpg` or **Fig. ??** in the appendix of this book).

2. In long enough length and time scales Kähler action per volume must vanish so that the idealization of cosmology as a vacuum extremal becomes possible and there must be some mechanism compensating the positive action of the free cosmic strings. The mechanism need not be local.

The original picture was based on dynamical cancellation mechanism involving generation of strong Kähler electric fields in the topological condensation such that the Kähler electric action compensates the Kähler magnetic action. Maybe this picture is the most realistic considered hitherto.

One possible cancellation mechanism relies on zero energy ontology. If the sign of the Kähler action is assumed to depend on time orientation - this assumption is vulnerable to criticism - t would be opposite for positive and negative energy space-time sheets and the actions associated with them would cancel if the field configurations are identical. Hence zero energy states would naturally have small Kähler action. Obviously this mechanism is non-local and looks rather questionable.

In this framework zero energy states correspond to cosmologies leading from big bang to big crunch separated by some time interval T of geometric time. Quantum jumps can gradually increase the value T and TGD inspired theory of consciousness suggests that the increase of T might relate to the shift for the contents of conscious experience towards geometric future. In particular, what is usually regarded as cosmology could have started from zero energy state with a small value of T.

8.5.2 Topological Condensation Of Cosmic Strings

1. Exterior metrics of topologically condensed $g > 1$ strings

If the sign of the gravitational string tension is negative, the simple embedding of the metric existing for positive string tension ceases to exist. There exists however a different embedding for which angle excess is in a good approximation same as for the flat solution. The solution is not flat anymore and this implies outwards radial gravitational acceleration. The embedding of the exterior metric also fails beyond a critical radius. This is not the only possible exterior metric: also non-flat exterior metric are possible and look actually more plausible and also this metric implies radial outwards acceleration as one might indeed expect. What remains to be shown that these metrics do not only yield small angle defect but are also consistent with Newtonian intuitions as the constant velocity spectrum for distant stars around galaxies suggests.

The natural interpretation would be as a mechanism generating large void around a central cosmic string having $g > 1$ and negative string tension and containing at its boundary $g = 1$ positive energy cosmic strings with string tension equal to Kähler string tension. Since angle surplus instead of angle deficit is predicted for $g > 1$ strings, lense effect transforms in this case to angle divergence and one avoids the basic objection against big cosmic strings. The emergence of preferred axes defined by $g > 1$ strings in the scale of large void could relate to the anomalies observed in Cosmic Microwave Background.

Negative gravitational energy of $g > 1$ cosmic strings could be regarded as that part of gravitational energy which causes the accelerated cosmic expansion by driving galactic strings to the boundaries of large voids which then induces phase transition increasing the size of the voids. This kind of acceleration is encountered already at the level of Newton's equations when some of the gravitational masses are negative.

2. Exterior metrics of topologically condensed $g = 1$ strings

One cannot assume that the exterior metric of the galactic $g = 1$ strings is the one predicted by assuming $G = 0$ in the exterior region. This would mean that metric decomposes as $g = g_2(X^2)+g_2(Y^2)$. $g(X^2)$ would be flat as also $g_2(Y^2)$ expect at the position of string. The resulting angle defect due to the replacement of plane Y^2 with cone would be large and give rise to lense effect of same magnitude as in the case of GUT cosmic strings. This lensing has not been observed.

The constant velocity spectrum for distant stars of galaxies and the fact that galaxies are organized along strings suggests that these string generate in a good approximation Newtonian potential. This potential predicts constant velocity spectrum with a correct value velocity.

In the stationary situation one expects that the exterior metric of galactic string corresponds to a small deformation of vacuum extremal of Kähler action which is also extremal of the curvature scalar in the induced metric. This allows a solution ansatz which conforms with Newtonian intuitions and for which metric decomposes as $g = g_1 + g_3$, where g_1 corresponds to axis in the direction of string and g_3 remaining $1 + 2$ directions.

8.5.3 Dark Energy Is Replaced With Dark Matter In TGD Framework

The first thing that comes in mind is that negative gravitational energy could be the TGD counterpart for the positive dark vacuum energy known to dominate over the mass density in cosmological length scales and believed to cause the accelerated cosmic expansion. This argument is wrong.

1. The gigantic value of gravitational Planck constant implies that dark matter makes TGD Universe a macroscopic quantum system even in cosmological length scales. Astrophysical systems become stationary quantum systems which participate in cosmic expansion only via quantum phase transitions increasing the value of gravitational Planck constant. Critical cosmologies, which are determined apart from a single parameter in TGD Universe, are natural during all quantum phase transitions, in particular the phase transition periods increasing the size of large voids and having interpretation in terms of an increase of gravitational Planck constant. Cosmic expansion is predicted to be accelerating during these periods. The mere criticality requires that besides ordinary matter there is a contribution $\Omega_\Lambda \simeq .74$ to the mass density besides visible matter and dark matter.

2. The essential characteristic of dark energy is its negative pressure. Negative gravitational energy could effectively create this negative pressure during phase transitions increasing the size of large voids. Since negative gravitational mass would be basically responsible for the accelerated expansion, one can assume that dark energy is actually dark matter.

3. Note that the pressure is negative during critical period. This is however interpreted as a correlate for the expansion caused by the phase transition increasing Planck constant rather than being due to positive cosmological constant or quintessence with negative pressure.

8.5.4 The Values For The TGD Counterpart Of Cosmological Constant

One can introduce a parameter characterizing the contribution of dark mass to the mass density during critical periods and call it cosmological constant recalling however that the contribution does not correspond to negative pressure now. The value of this parameter is same as in the standard cosmology from mere criticality assumption.

What is new that p-adic fractality predicts that Λ scales as $1/L^2(k)$ as a function of the p-adic scale characterizing the space-time sheet implying a series of phase transitions reducing Λ. The order of magnitude for the recent value of the cosmological constant comes out correctly. The gravitational energy density assignable to the cosmological constant is identifiable as that associated with topologically condensed cosmic strings and magnetic flux tubes to which they are gradually transformed during cosmological evolution.

The naïve expectation would be the density of cosmic strings would behave as $1/a^2$ as function of M^4_+ proper time. The vision about dark matter as a phase characterized by gigantic Planck constant however implies that large voids do not expand in continuous manner during cosmic evolution but in discrete quantum jumps increasing the value of the gravitational Planck constant and thus increasing the size of the large void as a quantum state. Since the set of preferred values of Planck constant is closed under multiplication by powers of 2, p-adic length scales L_p, $p \simeq 2^k$ form a preferred set of sizes scales for the large voids.

Classically one can understand the occurrence of the phase transitions increasing the size of the void as resulting when the galactic strings end up to the boundary of the large void in the repulsive gravitational field of the big string.

8.5.5 Matter-Antimatter Asymmetry And Cosmic Strings

Despite huge amount of work done during last decades (during the GUT era the problem was regarded as being solved!) matter-antimatter asymmetry remains still an unresolved problem of cosmology. A possible resolution of the problem is matter-antimatter asymmetry in the sense that cosmic strings contain antimatter and their exteriors matter. The challenge would be to understand the mechanism generating this asymmetry. The vanishing of the net gauge charges of cosmic string allows this symmetry since electro-weak charges of quarks and leptons can cancel each other.

The challenge is to identify the mechanism inducing the CP breaking necessary for the matter-antimatter asymmetry. Quite a small CP breaking inside cosmic strings would be enough.

1. The key observation is that vacuum extremals as such are not physically acceptable: small deformations of vacuum extremals to non-vacua are required. This applies also to cosmic strings since as such they do note present preferred extremals. The reason is that the preferred

extremals involve necessary regions with Euclidian signature providing four-dimensional representations of generalized Feynman diagrams with particle quantum numbers at the light-like 3-surfaces at which the induced metric is degenerate.

2. The simplest deformation of vacuum extremals and cosmic strings would be induced by the topological condensation of CP_2 type vacuum extremals representing fermions. The topological condensation at larger space-time surface in turn creates bosons as wormhole contacts.
3. This process induces a Kähler electric fields and could induce a small Kähler electric charge inside cosmic string. This in turn would induce CP breaking inside cosmic string inducing matter antimatter asymmetry by the minimization of the ground state energy. Conservation of Kähler charge in turn would induce asymmetry outside cosmic string and the annihilation of matter and antimatter would then lead to a situation in which there is only matter.
4. Either galactic cosmic strings or big cosmic strings (in the sense of having large string tension) at the centers of galactic voids or both could generate the asymmetry and in the recent scenario big strings are not necessary. One might argue that the photon to baryon ratio $r \sim 10^{-9}$ characterizing matter asymmetry quantitatively must be expressible in terms of some fundamental constant possibly characterizing cosmic strings. The ratio $\epsilon = G/\hbar R^2 \simeq 4 \times 10^{-8}$ is certainly a fundamental constant in TGD Universe. By replacing R with $2\pi R$ would give $\epsilon = G/(2\pi R)^2 \simeq 1.0 \times 10^{-9}$. It would not be surprising if this parameter would determine the value of r.

The model can be criticized.

1. The model suggest only a mechanism and one can argue that the Kähler electric fields created by topological condensates could be random and would not generate any Kähler electric charge. Also the sign of the asymmetry could depend on cosmic string. A CP breaking at the fundamental level might be necessary to fix the sign of the breaking locally.
2. The model is not the only one that one can imagine. It is only required that antimatter is somewhere else. Antimatter could reside also at other p-adic space-time sheets and at the dark space-time sheets with different values of Planck constant.

The needed CP breaking is indeed predicted by the fundamental formulation of quantum TGD in terms of the Kähler-Dirac action associated with Kähler action and its generalization allowing include instanton term as imaginary part of Kähler action inducing CP breaking [K104, K68] .

1. The key idea in the formulation of quantum TGD in terms of modified Dirac equation associated with Kähler action is that the Dirac determinant defined by the generalized eigenvalues assignable to the Dirac operator D_K equals to the vacuum functional defined as the exponent of Kähler function in turn identifiable as Kähler action for a preferred extremal, whose proper identification becomes a challenge. In ZEO (ZEO) 3-surfaces are pairs of space-like 3-surfaces assignable to the boundaries of causal diamond (CD) and for deterministic action principle this suggests that the extremals are unique. In presence of non-determinism the situation changes.
2. The huge vacuum degeneracy of Kähler action suggests that for given pair of 3-surfaces at the boundaries of CD there is a continuum of extremals with the same Kähler action and conserved charges obtained from each other by conformal transformations acting as gauge symmetries and respecting the light-likeness of wormhole throats (as well as the vanishing of the determinant of space-time metric at them). The interpretation is in terms of quantum criticality with the hierarchy of symmetries defining a hierarchy of criticalities analogous to the hierarchy defined by the rank of the matrix defined by the second derivatives of potential function in Thom's catastrophe theory.
3. The number of gauge equivalence classes is expected to be finite integer n and the proposal is that it corresponds to the value of the effective Planck constant $h_{eff} = n \times h$ so that a connection with dark matter hierarchy labelled by values of n emerges [K35].
4. This representation generalizes - at least formally. One could add an imaginary instanton term to the Kähler function and corresponding Kähler-Dirac operator D_K so that the generalized eigenvalues assignable to D_K become complex. The generalized eigenvalues correspond to

the square roots of the eigenvalues of the operator $DD^\dagger = (p^k\gamma_k + \Gamma^n)(p^k\gamma_k + \Gamma^n)^\dagger$ acting at the boundaries of string world sheets carrying fermion modes and it seems that only space-like 3-surfaces contribute. Γ^n is the normal component of the vector defined by Kähler-Dirac gamma matrices. One can define Dirac determinant formally as the product of the eigenvalues of $DD^\dagger$.

The conjecture is that the resulting Dirac determinant equals to the exponent of Kähler action and imaginary instanton term for the preferred extremal. The instanton term does not contribute to the WCW metric but could provide a first principle description for CP breaking and anyonic effects. It also predicts the dependence of these effects on the page of the book like structure defined by the generalized embedding space realizing the dark matter hierarchy with levels labeled by the value of Planck constant.

5. In the case of cosmic strings CP breaking could be especially significant and force the generation of Kähler electric charge. Instanton term is proportional to $1/h_{eff}$ so that CP breaking would be small for the gigantic values of h_{eff} characterizing dark matter. For small values of h_{eff} the breaking is large provided that the topological condensation is able to make the CP_2 projection of cosmic string four-dimensional so that the instanton contribution to the complexified Kähler action is non-vanishing and large enough. Since instanton contribution as a local divergence reduces to the contributions assignable to the light-like 3-surfaces X_l^3 representing topologically condensed particles, CP breaking is large if the density of topologically condensed fermions and wormhole contacts generated by the condensation of cosmic strings is high enough.

8.6 Entropic Gravity In TGD Framework

Entropic gravity (EG) introduced by Verline [B12] has stimulated a great interest. One of the most interesting reactions is the commentary of Sabine Hossenfelder [B46]. The article of Kobakhidze [B10] relies on experiments supporting the existence of Schrödinger amplitudes of neutron in the gravitational field of Earth develops an argument suggesting that EG hypothesis in the form in which it excludes gravitons is wrong. The following arguments represent TGD inspired view about what entropic gravity (see `http://tinyurl.com/y8utv7s`) (EG) could be if one throws out the unnecessary assumptions such as the emerging dimensions and absence of gravitons. Before continuing I want to express my gratitude to Prof. Masud Chaichian for the stimulus which led to a re-evaluation and reformulation of EG hypothesis. I want also to represent my thanks to Archil Kobakhidze for his clarifications concerning his argument against Verlinde's entropic gravity.

1. If one does not believe in TGD, one could start from the idea that stochastic quantization (see `http://tinyurl.com/y9rcnbzj`) or something analogous to it might imply something analogous to entropic gravity (EG). What is required is the replacement of the path integral with functional integral. More precisely, one has functional integral in which the real contribution to Kähler action of the preferred extremal from Euclidian regions of the space-time surface to the exponent represents Kähler function and the imaginary contribution from Minkowskian regions serves as a Morse function so that the counterpart of Morse theory in WCW is obtained on stationary phase approximation in accordance with the vision about TGD as almost topological QFT [K56]. The exponent of Kähler function is the new element making the functional integral well-defined and the presence of phase factor gives rise to the interference effects characteristic for quantum field theories although one does not integrate over all space-time surfaces. In zero energy ontology one has however pairs of 3-surfaces at the opposite light-like boundaries of CD so that something very much analogous to path integral is obtained.

2. Holography requires that everything reduces to the level of 3-metrics and more generally, to the level of 3-D field configurations. Something like this happens if one can approximate the functional integral with the integral over small deformations for the minima of the action. This happens in precise sense in completely integral quantum field theories.

   The basic vision behind quantum TGD is that this approximation is much nearer to reality than the original theory. In other words, holography is realized in the sense that to a given 3-surface the metric of WCW assigns a unique space-time and this space-time serves as the analog of Bohr orbit and allows to realize 4-D general coordinate invariance in the space of 3-surfaces so that classical theory becomes an exact part of quantum theory. This point of view will be adopted in the following also in the framework of general relativity where one considers abstract 4-geometries instead of 4-surfaces: functional integral should be over 3-geometries with the definition of Kähler metric assigning to 3-geometry a unique 4-geometry.

3. A powerful constraint is that the functional integral is free of divergences. Both 4-D path integral and stochastic quantization for gravitation fail in this respect due to the local divergences (in super-gravity situation might be different). The TGD inspired approach reducing quantum TGD to almost topological QFT with Chern-Simons term and a constraint term depending on metric associated with preferred 3-surfaces allows to circumvent this difficulty. This picture will applied to the quantization of GRT and one could see the resulting theory as a guess for what GRT limit of TGD could be. The first guess that Kähler function corresponds to Einstein-Maxwell action for this kind of preferred extremal turns out to be correct. An essential and radically new element of TGD is the possibility of space-time regions with Euclidian signature of the induced metric replacing the interiors of blackholes: this element will be assumed also now. The conditions that CP_2 represents and extremal of EYM action requires cosmological constant in Euclidian regions determined by the constant curvature of CP_2 and one can ask whether the average value of cosmological constant over 3-space could correspond to the cosmological constant explaining accelerating cosmic expansion.

4. Before going to a more precise formulation it is better to discuss how the phenomenology of EG with gravitons and without the fuzzy assumption about the emergence of space-time could be understood in TGD framework. This article is kind of continuation to the earlier article published in http://www.scribd.com/doc/45928480/PSTJ-V1-10-More-Possible-Games-in- Town-ContinuedPrespace-Time Journal [?], where the proposal that Quantum TGD as a hermitian square root of thermodynamics might imply something analogous to entropic gravity since S-matrix is replaced with the analog of thermal S-matrix. The article of Hossenfelder (see `http://tinyurl.com/ondxowl`) [B46] has been of great help. Entropic gravity is generalized in TGD framework so that all interactions are entropic: the reason is that in zero energy ontology (ZEO) the $S-$matrix is replaced with $M-$matrix defining a square root of thermodynamics in a well defined sense.

8.6.1 The Phenomenology Of EG In TGD Framework

In TGD framework one can consider a modification of EG allowing gravitons. In this framework thermodynamics is assigned with the virtual gravitons (and also real) flowing along the flux tubes mediating gravitational interaction. The entropy proportional to the length of flux tube corresponds to the entropy assigned with the holographic screen and temperature is the temperature of gravitons decreasing with distance just like the temperature of the radiation from Sun decreases as $1/r^2$: this is due to the absence of gravitonic heat sources in empty space.

TGD based view about EG leads also to new views. The basic objection against EG is that it applies also to electromagnetic interactions and leads to negative temperatures. In zero energy ontology the resolution of the problem could be that matter and antimatter correspond to opposite arrow sof geometric time and therefore different causal diamonds and space-time sheets: this could explain also the apparent absence of antimatter.

EG with gravitons and without emergence of space-time

The following arguments explain how the basic formulas of EG follow from TGD framework assuming that virtual gravitons reside at flux tubes connecting interacting systems.

1. The argument originally to Kobakhidze (see `http://tinyurl.com/y93oypyo`) [B10] suggests that EG in the strong sense predicting the absence of gravitons is inconsistent with experimental facts. The argument does not mention gravitons but relies on the experimental fact that neutron bound states in Earth's gravitational field exist. Chaichian *et al* [B16] however claim the argument contains an error because the formula (8) of [?]or the density matrix of neutron plus screen reading as

$$\rho_S(z+\Delta z) = \rho_N(z+\Delta z)\times \rho_{S/N}(z)$$

gives constant density matrix for screen when one removes neutron and this is certainly not true. According to Kobakhidze (private communication) the theory of Verlinde implies that the removal of neutron effectively removes the screen from $z+\Delta z$ to z. I leave it for the reader to decide what is the truth. Second challengable assumption of Kobakhidze used before equation (10) of [B10] is the additivity of the entropies of the screen and neutron: the interaction with the screen implies interaction entropy and the question is whether it can be neglected.

2. According to Chaichian *et al* [B16] that there exist transitions between the excited states suggest that the emission of gravitons must be involved (one can of course consider also electromagnetic transitions). This assumption is not testable since the rate of graviton induced transitions is extremely low. This result together with the vision about quantum theory as a square root of thermodynamics suggests that one must consider a modification of EG such that it allows gravitons and try to assign entropy and temperature to some real systems.

3. Suppose that one takes EG formulas seriously but accepts the existence of gravitons. EG should be understandable in terms of the classical space-time correlates of gravitational interaction assignable to virtual gravitons with space-like momenta. Could virtual gravitons mediating the gravitational flux through a hologram surface be responsible for the gravitational entropy?

   Could one assign entropy to the gravitons inside flux tube like structures from the source and traversing the holographic screen and carrying virtual gravitons with wave length much shorter than the distance to source so that quantum coherence for gravitons is lost? If the density of entropy per unit length of the flux tube is constant, gravitational entropy is proportional to the length of flux tube from the source to the constant potential surface so that $S \propto \Phi_{gr}A$ hypothesis would follow as a consequence.

4. Why the temperature of graviton carrying flux tubes should be reduced as $1/r^2$ with distance in the case of a spherically symmetric source? Could the masses serve as heat sources creating thermal ensemble of gravitons? The virtual gravitons emitted at the source would be at certain temperature just as ordinary photons created in Sun. The gravitons flowing along the flux tubes would cool- maybe by the expansion of the transversal cross section of the flux tube- and the condition that heat is not created or absorbed in the empty space would imply $1/r^2$ behavior. The flux tubes carrying virtual gravitons would serve as counterparts of long strings in holographic argument. In TGD the string like objects indeed appear quite concretely.

5. By using reduced mass, gravitational temperature and entropy become symmetric as functions of the masses of two objects. This assumption makes sense only in many-sheeted space-time for which each pair of systems is characterized by its own flux tubes (space-time sheets) mediating the gravitational interaction. Also the notion of gravitational Planck constant proportional to GMm makes sense only if it characterizes the flux tubes.

6. Unless the special nature of gravitational force as inertial force distinguishes gravitation from other interactions representing genuine forces, EG argument applies also in electrodynamics. The temperature in this case is proportional to the projection of the electric field which is in the direction of the normal of constant potential surface and has wrong sign for the second sign of the charge. Could the negative temperature implying instability relating somehow to the matter antimatter asymmetry? Antimatter and antimatter could not appear in same space-time region because either of them would give rise to negative temperature for flux tubes carrying virtual photons. In TGD framework similar outcome results also from totally different arguments and states that matter and antimatter should reside at different space-time sheets. Antimatter could be also dark in TGD sense. This point will be discussed in detail below and will be related to the generation of thermodynamical arrow of time which would be different for particles and antiparticles. In this case the reduced mass must be replaced with reduced charge $Q_1Q_2/(Q_+Q_2)$ to achieve symmetry.

7. Could one say that in the GRANIT experiment (see `http://tinyurl.com/ybg4kmgu`) [C15] giving support for the description of the neutron in Earth's gravitational field using Schrödinger

equation the entropy of neutron plus screen is just the entropy associated with the Coulomb potential of Earth and neutron obtainable as $S(r) \propto (\phi_{gr,Earth} + \Phi_{gr,neutron})A$? The gravitational potential appearing in the Schrödinger equation would be expressible essentially as the entropy per surface area and -as already noticed- this could be a mere accident having nothing to do with the real nature of gravitational force.

The assignment of entropy with the lines of generalized Feynman graphs is consistent with the replacement of S-matrix with $M-$matrix identified as a product of S-matrix and a Hermitian square root of density matrix commuting with $S-$matrix. These Hermitian square roots commute with $S-$matrix and generate infinite-D symmetry algebra of $S-$matrix defining a generalization of Yangian [A20] [B23, B20, B21] in ZEO since they are multi-local with respect to the partonic 2-surfaces located at the two light-like boundaries of CD. This algebra generated by zero energy states generalizes the twistorial Yangian and allowing CDs with integer multiples of basic scale one obtains a generalization of Kac-Moody algebra in which the non-commutative phase S^n generalizes the commutative phase factor $exp(in\phi)$ of Kac-Moody algebra. Also vacuum functional can be interpreted as a complex square root of density matrix for ground states with Minkowskian part of Kähler action defining the phase and the exponent of Euclidian part defining the modulus.

Could gravity reduce to entropic force in long length scales?

The pessimistic view is that the possibility to regard gravitation as an entropic force is purely accidental and follows from the fact that gravitational potential happens to represent the density of gravitonic entropy per surface area and gravitonic temperature happens to be proportional to the normal component of the gravitational acceleration. On the other hand, one can develop an argument in which the absorption l of virtual gravitons with wavelength must shorter than the distance between the two systems is analogous to radiation pressure and describable in terms of entropic gravity.

The proposal that both virtual and real gravitons are characterized by temperature and entropy is questionable in standard quantum theory. It however makes sense in ZEO in which $S-$matrix is replaced with $M-$matrix identifiable as a Hermitian square root of density matrix so that thermodynamics emerges even at the level of virtual particles. That it does so conforms with the fact that the basic building blocks of virtual particles are on mass shell massless particles. Allowing negative energies one can have also space-like net values of virtual momenta and virtual particles differ from incoming ones only in that the bound state conditions for masses is given up. The resulting powerful constraints on virtual momenta allowing to avoid both UV and IR divergences and justify twistorial description for both on mass shell particles and virtual particles.

Flux tube picture for gravitational interaction

Consider now the emission of gravitational radiation and its absorpion allowing also virtual gravitons. In the picture about flux tubes as space-time sheets carrying gravitons between two objects there are two cases as I have discussed earlier but without realizing that these cases could correspond to to non-entropic and entropic gravitation respectively.

Remark: The flux tube picture emerged from the attempt to understand why the gravitational Planck constant introduced by Nottale (see `http://tinyurl.com/ya6f3s4l`) [E25] and taken seriously by me as characteristics of dark gravitons is proportional to the masses of Sun and planet: the explanation is that $\hbar_{gr}$ is associated with flux tubes connecting these objects. It follows also from fractal string picture with string like objects identified as flux tubes.

In the minimal formulation the hierarchy of Planck constants [K35] coming as integer multiples of ordinary Planck constant and assigned to dark matter can be understood as an effective hierarchy due to the possibility of many-sheeted classical solutions of field equations with identical canonical momentum densities at various sheets implied by the huge vacuum degeneracy of Kähler action.

1. When the wavelength of gravitons is longer than that of flux tube, the graviton serves as a string connecting the systems (say ends of long bar, of the receiving system and source-not in practice) together and induces at classical level coherent oscillations of the relative distance.

In the detection of gravitational waves this kind arrangement should appear and typically appears. For instance, for millisecond pulsar the graviton wavelength is about 10^5 meters. This would represent quantum realm in which entropic gravity does not apply. Classical description however works in accordance with quantum classical correspondence.

Remark: If one is ready to take seriously the idea about large gravitational Planck constant, the wavelengths would be very long and one would be practically always in this realm.

2. When the wave length of gravitons is shorter than flux tube, the graviton beam losses its coherence and is characterized by temperature and entropy and generates on the receiver something analogous to gravitational radiation pressure induced by virtual particles (this pressure is however negative for gravitation!). This would generate entropic force with definite direction since the momentum of virtual gravitons is of the same sign.
   (a) This would suggest that gravitational waves with wavelengths shorter than the size of the detector should not be detectable via standard empirical arrangements.
   (b) A stronger condition would be that gravitational waves with wave lengths shorter than the distance between source and receiver cannot be detected: this would effectively conform with EG and predict that gravitational waves ill not be detected. This should have no practical consequences since even in the case of neutrons of GRANIT experiment the wavelength seems to be of order 10^5 meters from the peV energy scale of the bound states in Earth's gravitational field.
3. Entropic gravity is not in conflict with the geometrization of gravitational interaction since also thermodynamics should have space-time correlates by quantum classical correspondence. In accordance with stringy vision about short range gravitation, gravitational interaction in non-entropic realm is mediated by flux tubes connecting the masses involved and acting like strings.

The identification of the temperature and entropy

One can look at the situation also at more quantitative level. The natural guess for the temperature parameter would be as Unruh temperature

$$T_{gr} = \frac{\hbar}{2\pi}a \ , \tag{8.6.1}$$

where a is the projection of the gravitational acceleration along the normal of the gravitational potential = constant surface. In the Newtonian limit it would be acceleration associated with the relative coordinates and correspond to the reduced mass and equal to $a = G(m_1 + m_2)/r^2$.

One could identify T_{gr} also as the magnitude of gravitational acceleration. In this case the definition would involved only be purely local. This is in accordance with the character of temperature as intensive property.

The general relativistic objection against the generalization is that gravitation is not a genuine force: only a genuine acceleration due to other interactions than gravity should contribute to the Unruh temperature so that gravitonic Unruh temperature should vanish. On the other hand, any genuine force should give rise to an acceleration. The sign of the temperature parameter would be different for attractive and repulsive forces so that negative temperatures would become possible. Also the lack of general coordinate invariance is a heavy objection against the formula.

1. Gravitonic temperature in TGD Universe

In TGD Universe the situation is different. In this case the definition of temperature as magnitude of local acceleration is more natural.

1. Space-time surface is sub-manifold of the embedding space and one can talk about acceleration of a point like particle in embedding space $M^4 \times CP_2$. This acceleration corresponds to the trace of the second fundamental form for the embedding and is completely well-defined and general coordinate invariant quantity and vanishes for the geodesics of the embedding space. Since acceleration is a purely geometric quantity this temperature would be same for flux sheets irrespective of whether they mediate gravitational or some other interactions so that all kinds of virtual particles would be characterized by this same temperature.

2. One could even generalize T_{gr} to a purely local position dependent parameter by identifying it as the magnitude of second fundamental form at given point of space-time surface. This would mean that the temperature in question would have purely geometric correlate. This temperature would be alwas non-negative. This purely local definition would also save from possible inconsistencies in the definition of temperature resulting from the assumption that its sign depends on whether the interaction is repulsive or attractive.

3. The trace of the second fundamental form -call it H- and thus T_{gr} vanishes for minimal surfaces. Examples of minimal surfaces are string-like objects [**?**, **?**] massless extremals [K53, K65], and CP_2 type vacuum extremals [K26] with M^4 projection which is light-like geodesic [K11]. Vacuum extremals with at most 2-D Lagrangian CP_2 projection has a non-vanishing H and this is true also for their deformations defining the counterpart of GRT space-time. Also the deformations of cosmic strings with 2-D M^4 projection to magnetic flux tubes with 4-D M^4 projection are expected to be non-minimal surfaces. Same applies to the deformations of CP_2 vacuum extremals near the region where the signature of the induced metric changes. The predictedcosmic string dominated phase of primordial cosmology [K80] would correspond to the vanishing gravitonic temperature. Also generic CP_2 type vacuum extremals have non-vanishing H.

4. Massless extremals define an excellent macroscopic space-time correlate for gravitons. The massivation of gravitons is however strongly suggested by simple considerations encouraged by twistorial picture and wormhole throats connecting parallel MEs define the basic building bricks of gravitons and would bring in non-vanishing geometric temperature, (extremely small but non-vanishing) graviton mass, and gravitonic entropy.

   (a) The M^4 projection of CP_2 type vacuum extremal is random light-like curve rather than geodesic of M^4 (this gives rise to Virasoro conditions [K11]). The mass scale defined by the second fundamental form describing acceleration is non-vanishing. I have indeed assigned this scale as well as the mixing of M^4 and CP_2 gamma matrices inducing mixing of M^4 chiralities to massivation. The original proposal was that the trace of second fundamental form could be identifiable as classical counterpart of Higgs field. One can speak of light-like randomness above a given length scale defined by the inverse of the length of the acceleration vector.

   (b) This suggests a connection with p-adic mass calculations: the p-adic mass scale m_p is proportional to the acceleration and thus could be given by the geometric temperature: $m_p = nR^{-1}p^{-1/2} \sim \hbar H = \hbar a$, where $R \sim 10^4 L_{Pl}$ is CP_2 radius, and n some numerical constant of order unity. This would determine the mass scale of the particle and relate it to the momentum exchange along corresponding CP_2 type vacuum extremal. Local graviton mass scale at the flux tubes mediating gravitational interaction would be essentially the geometric temperature.

   (c) Interestingly, for photons at the flux tubes mediating Coulomb interactions in hydrogen atom this mass scale would be of order $\hbar a \sim e^2\hbar/m_p n^4 a_0^2 \sim 10^{-5}/n^4$ eV, which is of same order of magnitude as Lamb shift (see `http://tinyurl.com/ycp2ch8b`), which corresponds to 10^{-6} eV energy scale for $n = 2$ level of hydrogen atom. Hence it might be possible to kill the hypothesis rather easily.

   (d) Note that momentum exchange is space-like for Coulomb interaction and the trace H^k of second fundamental form would be space-like vector. It seems that one define mass scale as $H = \sqrt{-H^k H_k}$ to get a real quantity.

   (e) This picture is in line with the view that also the bosons usually regarded as massless possess a small mass serving as an IR cufoff. This vision is inspired by zero energy ontology and twistorial considerations [K94, K8]. The prediction that Higgs is completely eaten by gauge bosons in massivation is a prediction perhaps testable at LHC already during year 2011.

Remark: In MOND (see `http://tinyurl.com/qt875`) theory of dark matter (see `http://tinyurl.com/qt875`) a critical value of acceleration is introduced. I do not believe personally to MOND and TGD explains galactic rotation curves without any modification of Newtonian dynamics in terms of dark matter assignable to cosmic strings containing galaxies like around it like pearls in necklace. In TGD framework the critical acceleration would be the acceleration above which the gravitational acceleration caused by the dark matter associated with the cosmic strings traversing along galactic plane orthogonally and behaving as $1/\rho$ overcomes the acceleration caused by the galactic matter and behaving as $1/\rho^2$. Could this critical acceleration correspond to a critical temperature T_{gr}- presumably determined by an appropriate p-adic length scale and coming as a power $2^{-k/2}$ by p-adic length scale hypothesis? Could critical value of H perhaps characterize also a critical magnitude for the deformation from minimal surface extremal? The critical acceleration in Milgrom's model is about 1.2×10^{-10} m/s^2 and corresponds to a time scale of 10^{12} years, which is of the order of the age of the Universe.

The formula contains Planck constant and the obvious question of the inhabitant of TGD Universe is whether the Planck constant can be identified with the ordinary Planck constant or with the *effective* Planck constant coming as integer multiple of it [?]

1. For the ordinary value of $\hbar$ the gravitational Unruh temperature is extremely small. To make things more concrete one can express the Unruh temperature in gravitational case in terms of Schwartschild radius $r_S = 2GMm$ at Newtonian limit. This gives

$$T_{gr} = \frac{\hbar}{4\pi r_S}\frac{M+m}{M}(\frac{r_S}{r})^2 \ . \quad (8.6.2)$$

   Even at Schwartschild radius the temperature corresponds to Compton length of order $4\pi r_S$ for $m << M$.

2. Suppose that Planck constant is gravitational Planck constant $\hbar_{gr} = GMm/v_0$, where $v_0 \simeq 2^{-11}$ holds true for inner planets in solar system [?] This would give

$$T_{gr} = \frac{m}{8\pi v_0}\frac{M+m}{M}(\frac{r_S}{r})^2 \ .$$

   The value is gigantic so that one must assume that the temperature parameter corresponds to the minimum value of Planck constant. This conforms with the identification of the p-adic mass scale in terms of the geometric temperature.

2. Gravitonic entropy in TGD Universe

A good guess for the value of gravitational entropy (gravitonic entropy associated with the flux tube mediating gravitational interaction) comes from the observation that it should be proportional to the flux tube length.The relationship $dE = TdS$ suggests $S \propto \phi_{gr}/T_{gr}$ as the first guess in Newtonian limit. A better guess would be

$$S_{gr} = -\frac{V_{gr}}{T_{gr}} = \frac{M+m}{M}\frac{r}{\hbar m}, \ , \quad (8.6.3)$$

The replacement $M \rightarrow M + m$ appearing in the Newtonian equations of motion for the reduced mass has been performed to obtain symmetry with respect to the exchange of the masses.

The entropy would depend on the interaction mediated by the space-time sheet in question which suggests that the generalization is

$$S = -\frac{V(r)}{T_{gr}} \ . \quad (8.6.4)$$

Here $V(r)$ is the potential energy of the interaction. The sign of S depends on whether the interaction is attractive or repulsive and also on the sign of the temperature. For a repulsive interaction the entropy would be negative so that the state would be thermodynamically unstable in ordinary thermodynamics.

The integration of $dE = TdS$ in the case of Coulomb potential gives $E = V(r) - V(0)$ for both options. If the charge density near origin is constant, one has $V(r) \propto r^2$ in this region implying $V(0) = 0$ so that one obtains Coulombic interaction energy $E = V(r)$. Hence thermodynamical interpretation makes sense formally.

The challenge is to generalize the formula of entropy in Lorentz invariant and general coordinate invariant manner. Basically the challenge is to express the interaction energy in this manner. Entropy characterizes the entire flux tube and is therefore a non-local quantity. This justifies the use of interaction energy in the formula. In principle the dynamics defined by the extremals of Kähler action predicts the dependence of the interaction energy on Minkowskian length of the flux tube, which is well-defined in TGD Universe. Entropy should be also a scalar. This is achieved since the rest frame is fixed uniquely by the time direction defined by the time-like line connecting the tips of CD: the interaction energy in rest frame of CD defines a scalar. Note that the sign of entropy correlates with the sign of interaction energy so that the repulsive situation would be thermodynamically unstable and this indeed suggests that antimatter should have opposite arrow of time.

The sign of entropy for a Coulomb type interaction potential is always positive for the identification of T_{gr} as the normal component of gravitational acceleration whereas T_{gr} can be negative. If T_{gr} corresponds to the magnitude of the acceleration, entropy is negative for repulsive Coulomb interaction.

Negative temperatures/entropies for virtual bosons and the spontaneous generation of the arrow of time in ZEO

Negative entropies/temperatures are especially interesting from the point of ZEO in which causal diamonds (CDs) containing positive and negative energy states at their future and past light-like boundaries. Note that the term CD is used somewhat loosely about Cartesian product of CD and CP_2. Note also that CD is highly analogous to Penrose diagram and defines causal unit in quantum TGD.

There is also a fractal hierarchy CDs within CDs and the minimum number theoretically motivated assumption is that the scales of CDs come as integer multiples of CP_2 scale. Poincare transforms of CDs with respect to another tip are allowed and the position of the second tip with respect to the first one is quantized for number theoretic reasons and corresponds to a lattice like structure in the proper time constant hyperboloid of M^4_+. This has some highly non-trivial cosmological implications such as quantization of cosmic redshifts for which there is empirical evidence.

Both definitions lead to very similar predictions.

1. The identification of temperature T_{gr} as a scalar defined by the length of second fundamental form is favored in TGD framework. Entropy is defined in terms of interaction energy by the formula $S = -V/T_{gr}$. This definition can be defended in TGD Universe by Poincare invariance and general coordinate invariance. In this case temperature is always non-negative and entropy is positive for attractive interactions but negative for repulsive interactions. Therefore systems consisting mostly from matter or antimatter and having repulsive electromagnetic Coulomb forces have negative entropy and should be thermodynamically unstable. This would suggest that the arrow of time for these systems could be non-standard one in ZEO. For charge neutral systems entropy can be positive. It does not matter whether the system consists of matter or antimatter.
2. Second definition differs from the first one only in that T_{gr} is the magnitude of acceleration with a sign factor telling whether repulsion or attraction is in question. In this temperature can have both signs but entropy is always non-negative. For systems consisting dominantly of matter or antimatter with long range Coulomb interactions the temperature would be negative but entropy positive. This would suggest that the arrow of geometric time is non-standard one. Again it does not matter whether the system consists of matter or antimatter.

The obvious idea is that the thermal instability could imply matter antimatter asymmetry. The original argument that antimatter and matter would correspond to opposite arrows of geometric time turned out to be wrong. One can however modify the argument to state that thermal instability leads to a generation of regions consisting preferentially of matter and antimatter and having non-standard arrow of geometric time so that from the point of view of standard arrow of geometric time these region are formed rather than decay as second law would dictate. For definiteness the definition of geometric temperature as trace of the second fundamental form is assumed but the argument can be easily modified to the second case.

1. Does the negative entropy mean that the time evolutions assignable to the systems consisting mostly of matter (or antimatter) obeys opposite arrow of geometric time? From the point of view of observer with standard arrow of time these systems would obey second law in reverse direction of the geometric time. Spontaneous self assembly of biomolecules representes a standard example about this and the interpretation would be in terms of formation of structures consisting preferentially of matter or antimatter. Could this lead to a separation of antimatter to separate domains in the final states identifiable as negative energy parts of zero energy states? If so then matter matter antimatter asymmetry would relate to the purely geometric thermodynamics.
2. The arrow of geometric time emerges [K5] by a not too well-understood mechanism involving arguments from TGD inspired theory of consciousness. Thermodynamics must be involved since the sequence of quantum jumps identified as moments of consciousness induces the arrow of experienced time. Could it be that the arrow of geometric time is opposite for charged particles and antiparticles from thermodynamic stability?
    (a) Quite generally, positive energy parts of at past boundary of CD energy states would have definite particle number and also other quantum numbers whereas the outcome of measurement at future boundary dictated by $M-$matrix would be a superposition of final states at the opposite end of CD. This defines the arrow of geometric time as the direction of geometric time induced by that of thermodynamical time and experienced time defined in terms of a sequence of quantum jumps.
    (b) What mechanism selects the light-like boundary of CD which corresponds to the initial prepared states with second one identified in terms of the outcome of the scattering process expressible as a superpositions of states with well defined particle numbers and also other quantum numbers? The mechanism should relate to the square root of density matrix appearing in $M-$matrix and therefore to entropy of virtual bosons consisting of basic building blocks which are on mass shell massless particles with both signs of energy assignable to what I call wormhole throats to be discussed below.
    (c) The wrong sign of the entropy for systems consisting predominantly of matter or antimatter means they have must have negative energies and second law realized as properties of zero energy states would correspond to opposite direction of geometric time allowed in TGD based generalization of thermodynamics. The arrow would emerge at scales longer than wavelength and would be therefore a macroscopic phenomenon if wavelength is taken as the borderline between microscopic and macroscopic.

How to circumvent the difficulties in generalizing EG to relativistic situation

One argument against EG is that it applies as such to Newtonian gravity only. The general coordinate invariant and Lorentz invariant definitions of T and S have been already considered and are favore in TGD framework and give always non-negative temperature depending only on purely local data. In the following the original definitions of T and S involving equi-potential surfaces in the case of T are considered.

To make the generalization in a coordinate invariant manner two physically preferred coordinates defined modulo diffeomorphism are required: time coordinate t allowing to identify gravitational scalar potential Φ_{gr} as the deviation of g_{tt} from unity and radial coordinate orthogonal to the equipotential surfaces of Φ_{gr} so that Φ_{gr} itself could be regarded as the second preferred coordinate. This requires a slicing of space-time by 2-dimensional surfaces parametrized by (t, r) with remaining space-time coordinates regarded as constant for a given slice. The other two coordinates could define a dual slicing.

Entropy density $s_{gr} = dS_{gr}/dA$ per unit are of flux tube would be proportional to Φ_{gr} and a unique physical identification of the radial coordinate would be as proportional to the entropy $S_{gr}(r) = \int s_{gr} dA \propto \int \Phi_{gr} dA$ proportional to radial coordinate in the Newtonian limit. One should somehow specify what one precisely means with flux tube and here the area could be identified as the area inside which Kähler magnetic flux has definite sign if monopole flux is involved. If not then Kähler flux could take this role.

A possible identification of the preferred coordinates (t, r) is in terms of stringy slicing of the space-time surface by 2-D surfaces required by general consistency conditions. Strings would connect the points of partonic 2-surfaces carrying fermion number and the braids defining the orbits of these points would define string world sheets so that a rather concrete concretization of TGD as almost topological QFT would be obtained.

The physical interpretation of stringy slicing is in terms of integrable distribution of planes of non-physical polarization directions assignable to massless fields and orthogonal dual slicing would correspond to the directions of physical polarizations. The existence of the stringy slicing is motivated also by number theoretical considerations [K88]. The general ansatz for the preferred extremals leads to an identification of time preferred coordinate as a coordinate associated with the flow lines of conserved currents defining Beltrami flow. Second stringy coordinate could correspond to the direction for the gradient of gravitational field $\Phi_{gr} = g_{tt} - 1$ in accordance with the idea that gravitons flow along string like flux tubes and the polarizations of gravitons are orthogonal to the direction of their motion.

The translation of Witten's ideas about knots (see `http://tinyurl.com/y96gbdck`) to TGD framework [K47] lead to the string worlds sheets could correspond to inverse images of geodesic spheres of CP_2 for the embedding map of space-time surface to CP_2. This would conform with the idea that wormhole throats are magnetic monopoles at the ends of stringy flux tubes.

8.6.2 The Conceptual Framework Of TGD

There are several reasons to expect that something analogous to thermodynamics results from quantum TGD. The following summarizes the basic picture, which will be applied to a proposal about how to quantize (or rather de-quantize!) Einstein-Maxwell system with quantum states identified as the modes of classical WCW spinor field with spinors identifiable in terms of Clifford algebra of WCW generated by second quantized induced spinor fields of H.

1. In TGD framework quantum theory can be regarded as a "complex square root" of thermodynamics in the sense that zero energy states can be described in terms of what I call $M-$matrices which are products of hermitian square roots of density matrices and unitary $S-$matrix so that the moduli squared gives rise to a density matrix. The mutually orthogonal Hermitian square roots of density matrices span a Lie algebra of a subgroup of the unitary group and the $M-$matrices define a Kac-Moody type algebra with generators proportional to powers of S assuming that they commute with S. Therefore this algebra acts as symmetries of the theory.

   What is nice that this algebra consists of generators multi-local with respect to partonic 2-surfaces and represents therefore a generalization of Yangian algebra. The algebra of $M-$matrices makes sense if causal diamonds (double light-cones) have sizes coming as integer multiples of CP2 size. $U-$matrix has as its rows the $M-$matrices. One can look how much of this structure could make sense in GRT framework.

2. In TGD framework one is forced to geometrize WCW [K46] consisting of 3-surfaces to which one can assign a unique space-time surfaces as analogs of Bohr orbits and identified as preferred extremals of Kähler action (Maxwell action essentially). The 3-surfaces could be identified as the intersections space-time surface with the future and past light-like boundaries causal diamond (CDs analogous to Penrose diagrams). The preferred extremals associated with the preferred 3-surfaces allow to realize General Coordinate Invariance (GCI) and its natural to assign quantum states with these.

   GCI in strong sense implies even stronger form of holography. Space-time regions with Euclidian signature of metric are unavoidable in TGD framework and have interpretation as particle like structure and are identified as lines of generalized Feynman diagrams. The light-like 3-surfaces at which the signature of the induced metric changes define equally good candidates for 3-surfaces with which to assign quantum numbers. If one accepts both identifications then the intersections of the ends of space-time surfaces with these light-like surfaces should code for physics. In other words, partonic 2-surfaces plus their 4-D tangent space-data would be enough and holography would be more or less what the holography of ordinary visual perception is!

   In the sequel the 3-surfaces at the ends of space-time and and light-like 3-surfaces with degenerate 4-metric will be referred to as *preferred 3-surfaces*.

3. WCW spinor fields are proportional to a real exponent of Kähler function of WCW defined as Kähler action for a preferred extremal so that one has indeed square root of thermodynamics also in this sense with Kähler essential one half of Hamiltonian and Kähler coupling strength playing the role of dimensionless temperature in "vibrational" degrees of freedom. One should be able to identify the counterpart of Kähler function also in General Relativity and if one has Einstein-Maxwell system one could hope that the Kähler function is just the Euclidian part of Maxwell action for a preferred extremal and therefore formally identical with the Kähler function in TGD framework. The phase factor from the Minkowskian contribution emerges naturally as one takes complex square root of the Boltzmann factor. The delicacies of this picture are discussed in [K56].

   Fermionic degrees of freedom correspond to spinor degrees of freedom and are representable in terms of oscillator operators for second quantized induced spinor fields [K104]. This means geometrization of fermionic statistics. There is no quantization at WCW level and everything is classical so that one has "quantum without quantum" as far as quantum states are considered.

4. The dynamics of the theory must be consistent with holography. This means that the Kähler action for preferred extremal must reduce to an integral over 3-surface. Kähler action density decomposes to a sum of two terms. The first term is $j^\alpha A_\alpha$ and second term a boundary term reducing to integral over light-like 3-surfaces and ends of the space-time surface. The first term must vanish and this is achieved if the Kähler current j^α is proportional to Abelian instanton current

$$j^\alpha \propto *j^\alpha = \epsilon^{\alpha\beta\gamma\delta} A_\beta J_{\gamma\delta} \tag{8.6.5}$$

   since the contraction involves A_α twice. This is at least part of the definition of preferred extremal property but not quite enough. Note that in Einstein-Maxwell system without matter j^α vanishes identically so that the action reduces automatically to a surface term.

5. The action would reduce reduce to terms which should make sense at light-like 3-surfaces. This means that only Abelian Chern-Simons term is allowed. This is guaranteed if the weak form of electric-magnetic duality [K104] stating

$$*F^{n\beta} = kF^{n\beta} \tag{8.6.6}$$

   at preferred at light-like throats with degenerate four-metric and at the ends of space-time surface. These conditions reduce the action to Chern-Simons action with a constraint term realizing what I call weak form of electric-magnetic duality. One obtains almost topological QFT since the constraint term depends on metric. This is of course what one wants.

   Here the constant k is integer multiple of basic value which is proportional to g_K^2 from the quantization of Kähler electric charge which corresponds to U(1) part of electromagnetic charge. Fractional charges for quarks require $k = ng_K^2/3$. Physical particles correspond to several Kähler magnetically charged wormhole throats with vanishing net magnetic charge but with non-vanishing Kähler electric proportional to the sum $\sum_i \epsilon_i k_i Q_{m,i}$, with $\epsilon_i = \pm 1$ determined by the direction of the normal component of the magnetic flux for i: th throat.

   The first guess is that the length of magnetic flux tube associated with the particle is of order Compton length or perhaps corresponds to weak length scale as was the original proposal. The screening of weak isospin can be understood as magnetic confinement such that neutrino pair at the second end of magnetic flux tube screens the weak charged leaving only electromagnetic charge. Also color confinement could be understood in terms of flux tubes of length of order hadronic size scales. Compton length hypothesis is enough to understand color confinement and weak screening.

   Note that $1/g_K^2$ factor in Kähler action is compensated by the proportionality of Chern-Simons action to g_K^2. This need not mean the absence of non-perturbative effects coming as powers of $1/g_K^2$ since the constraint expressing electric magnetic duality depends on g_K^2 and might introduce non-analytic dependence on g_K^2.

6. In TGD the space-like regions replace black holes and a concrete model for them is as deformations of CP_2 type vacuum extremals which are just warped embeddings of CP_2 to $M^4 \times CP_2$ with random light-like random curve as M^4 projection: the light-like randomness gives Virasoro conditions. This reflects as a special case the conformal symmetries of light-like 3-surfaces and those assignable to the light-like ends of the CDs.

   One could hope that this picture more or less applies for the GRT limit of quantum TGD.

8.6.3 What One Obtains From Quantum TGD By Replacing Space-Times As Surfaces With Abstract 4-Geometries?

It is interesting to see what one obtains when one applies TGD picture by replacing space-times as 4-surfaces with abstract geometries as in Einstein's theory and assumes holography in the sense that space-times satisfy besides Einstein-Maxwell equations also conditions guaranteeing Bohr orbit like property. The resulting picture could be also regarded as GRT type limit of quantum TGD obtained by dropping the condition that space-times are surfaces.

GRT is a more general theory than TGD in the sense that much more general space-times are allowed than in TGD - this leads also to difficulties - and one could also argue that the mathematical existence of WCW Kähler geometry actually forces the restriction of these geometries to those imbeddable in $M^4 \times CP_2$ so that the quantization of GRT type theory would lead to TGD.

What one wants?

What one wants is at least following.

1. Euclidian regions of the space-time should reduce to metrically deformed pieces of CP_2. Since CP_2 spinor structure does not exist without the coupling of the spinors to Kähler gauge potential of CP_2 one must have Maxwell field. CP_2 is gravitational instanton and constant curvature space so that cosmological constant is non-vanishing unless one adds a constant term to the Maxwell action, which is non-vanishing only in Euclidian regions. It is matter of taste, whether one regards V_0 as term in Maxwell action or as cosmological constant term in gravitational part of the action. CP_2 radius is determined by the value of this term so that it would define a fundamental constant.

   This raises an interesting question. Could one say that one has a small value of cosmological constant defined as the average value of cosmological constant assignable to the Euclidian regions of space-time? The average value would be proportional to the fraction of 3-space populated by Euclidian regions (particles and possibly also macroscopic Euclidian regions). The value of cosmological constant would be positive as is the observed value. In TGD framework the proposed explanation for the apparent cosmological constant is different but one must remain open minded. In fact, I have proposed the description in terms of cosmological constant also as a proper description in the approximation to TGD provided by GRT like theory. The answer to the question is far from obvious since the cosmological constant is associated with Euclidian rather than Minkowskian regions: all depends on the boundary conditions at the wormhole throats where the signature of the metric changes.

2. One can also consider the addition of Higgs term to the action in the hope that this could allow to get rid of constant term which is non-vanishing only in Euclidian regions. It turns turns out that only free action for Higgs field is possible from the condition that the sum of Higgs action and curvature scalar reduces to a surface term and that one must also now add to the action the constant term in Euclidian regions. Conformal invariance requires that Higgs is massless.

   The conceptual problem is that the surface term from Higgs does not correspond to topological action since it is expressible as as flux of $\Phi\nabla\Phi$. Hence the simplest possibility is that Kähler action contains a constant term in Euclidian regions just as in TGD, where curvature scalar is however absent. Einstein-Maxwell field equations however apply that it vanishes and is effectively absent also in GRT quantized like TGD.

3. Reissner-Nordström solutions are obtained as regions exterior to CP_2 type regions. In black hole horizons (when they exist) the 3- metric becomes light-like but 4-metric remains non-degenerate. Hence R-N solution cannot be directly glued to a deformed CP_2 type region

at horizon but a transition region in which the determinant of 4-metric becomes zero must be present. The simplest possibility is that R-N metric is deformed slightly so that one has $g_{tt} = 0$ and $g_{rr} < \infty$ at the horizon. This surface would correspond to a wormhole throat in TGD framework. Most of the blackhole interior would be replaced with CP_2 type region. In TGD black hole solutions indeed fail to be imbeddable at certain radius so that deformed CP_2 type vacuum extremal is much more natural object than black hole. In the recent framework the finite size of CP_2 means that macroscopic size for the Euclidian regions requires large deformation of CP_2 type solution. For masses $M < Q/\sqrt{G}$ R-N metric has no horizons so that in the case of elementary particles the situation is more complex than this.

Remark: In TGD framework large value of $\hbar$ and space-time as 4-surface property changes the situation. The generalization of Nottale's formula for gravitational Planck constant in the case of self gravitating system gives $\hbar_{gr} = GM^2/v_0$, where $v_0/c < 1$ has interpretation as velocity type parameter perhaps identifiable as a rotation velocity of matter in black hole horizon [K79, K66]. This gives for the Compton length associated with mass M the value $L_C = \hbar_{gr}/M = GM/v_0$. For $v_0 = c/2$ one obtains Scwartschild radius as Compton length. The interpretation would be that one has CP_2 type vacuum extremal in the interior up to some macroscopic value of Minkowski distance. One can whether even the large voids containing galaxies at their boundaries could correspond to Euclidian blackhole like regions of space-time surface at the level of dark matter.

4. The geometry of CP_2 allows to understand standard model symmetries when one considers space-times as surfaces [K53]. This is not necessarily the case for GRT limit.
   (a) In the recent case one has different situation color quantum numbers make sense only inside the Euclidian regions and momentum quantum numbers in Minkowskian regions. This is in conflict with the assumption that quarks can carry both momentum and color. On the other, color confinement could be used to argue that this is not a problem.
   (b) One could assume that spinors are actually 8-component $M^4 \times CP_2$ spinors but this would be somewhat ad hoc assumption in general relativistic context. Also the existence of this kind of spinor structure is not obvious for general solutions of Einstein-Maxwell equations unless one just assumes it.
   (c) It is far from clear whether the symplectic transformations of CP_2 could be interpreted as isometries of WCW in general relativity like theory [K46, K25]. These symmetries certainly act in non-trivial manner on Euclidian regions but it is highly questionable whether this could give rise to a genuine symmetry. Same applies to Kac-Moody symmetries assigned to isometries of $M^4 \times CP_2$ in TGD framework. These symmetries are absolutely essential for the existence of WCW Kähler geometry in infinite-D context as already the uniqueness of the loop space Kähler geometries demonstrates [A27] (maximal group of isometries is required by the existence of Riemann connection).

What GRT limit of TGD could mean?

How Einstein's equations and General Relativity in long length scales emerges from TGD has been a long-standing interpretational problem of TGD, whose resolution came from the realization that GRT is only an effective theory obtained by endowing M^4 with effective metric.

1. The replacement of superposition of fields with superposition of their effects means replacing superposition of fields with the set-theoretic union of space-time surfaces. Particle experiences sum of the effects caused by the classical fields at the space-time sheets (see **Fig.** `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg` or **Fig. ??** in the appendix of this book).
2. This is true also for the classical gravitational field defined by the deviation from flat Minkowski metric instandard M^4 coordinates for the space-time sheets. One can define effective metric as sum of M^4 metric and deviations. This effective metric would correspond to that of General Relativity. This resolves long standing issues relating to the interpretation of TGD.
3. Einstein's equations could hold true for the effective metric. They are motivated by the underlying Poincare invariance which cannot be realized as global conservation laws for the effective metric. The conjecture vanishing of divergence of Kähler energy momentum tensor can be seen as the microscopic justification for the claim that Einstein's equations hold true for the effective space-time.

4. The breaking of Poincare invariance could have interpretation as effective breaking in zero energy ontology (ZEO), in which various conserved charges are length dependent and defined separately for each causal diamond (CD).

This picture reduces at classical level Equivalence Principle to its formulation in GRT - Einstein's equations for the effective metric of M^4. Similar description applies to gauge interactions. One can also consider other limits. At elementary particle levels CP_2 would be natural background for GRT type theory with cosmological constant. Also cosmic strings $X^2 \times S^2$, where X^2 is string orbit in M^4 and S^2 a homologically non-trivial geodesic sphere in CP_2 can serve as backgrounds for GRT like theory. Now one has two cosmological constants.

At quantum level the equality of gravitational and inertial masses reduces to quantum classical correspondence [K99]. A further approach reduces EP to strong form of general coordinate invariance (GCI) implying strong form of holography. The equivalence of super-conformal representations associated with space-like and light-like 3-surfaces implies equality of corresponding four-momenta and if they can be identified as inertial and gravitational momenta, EP follows. A more convincing manner to obtain EP is by identifying classical Kähler four-momentum with the eigenvalues of quantal four-momentum associated with Kähler-Dirac action.

It seems that this picture can cope with the most obvious objections.

1. One could argue that GRT limit does not make sense since in Minkowskian regions the theory knows nothing about the color and electroweak quantum numbers: there is only metric and Maxwell field. This is true for single space-time sheet but many-sheetedness changes the situation. The interpretation of GRT limit as effective theory lumping the many-sheeted space-time to M^4 with effective metric and effective gauge fields indeed resolves this problem.

2. One could also argue that Einstein-Maxwell equations are un-realistic since also weak and color gauge fields are present as also particles represented as regions allowing representation as map from $M^4 \rightarrow CP_2$. One can however argue that in TGD one has color confinement and weak screening by magnetic confinement so that in macroscopic scales this approach might be realistic. If the functional integral over Euclidian regions representing generalized Feynman diagrams is enough to construct scattering amplitudes, pure Einstein-Maxwell system in Minkowskian regions might be enough. All experimental data is expressible in terms of classical em and gravitational fields. If Weinberg angle vanishes in Minkowskian regions, electromagnetic field reduces to Kähler form and the interpretation of the Maxwell field as em field should make sense. The very tight empirical constraints on the value of Kähler coupling strength α_K indeed allow its identification as fine structure constant at electron length scale.

3. One can also worry about the almost total disappearance of the metric from the basic TGD. This is not a problem in TGD framework since all elementary particles correspond to many-fermion states. For instance, gauge bosons are identified as pairs of fermion and anti-fermion associated with opposite throats of a wormhole connecting two space-time sheets with Minkowskian signature of the induced metric. Similar picture should make sense also now.

4. TGD possesses also approximate super-symmetries and one can argue that also these symmetries should be possessed by the GRT limit. All modes of induced spinor field generate a badly broken SUSY with rather large value of $\mathcal{N}$ (number of spinor modes) and right-handed neutrino and its antiparticle give rise to $\mathcal{N} = \in$ SUSY with R-parity breaking induced by the mixing of left- and right handed neutrinos induced by the Kähler-Dirac equation. This picture is consistent with the existing data from LHC and there are characteristic signatures -such as the decay of super partner to partner and neutrino- allowing to test it. These supersymmetries might make sense if one replaces ordinary space-time spinors with 8-D spinors.

   Note that the possible inconsistency of Minkowskian and Euclidian 4-D spinor structures might force the use of 8-D Minkowskian spinor structure.

Basic properties of Reissner-Nordström metric

Denote the coordinates of M^4_+ by (m^0, r_M, θ, ϕ) and those of X^4 by (t, r_M, θ, ϕ). The expression for Reissner-Nordström metric reads as

$$\begin{aligned} ds^2 &= Adt^2 - Bdr_M^2 - r_M^2 d\Omega^2 \ , \\ A &= 1 - \frac{r_s}{r_M} + \frac{r_Q^2}{r_M^2} \ , \quad B = \frac{1}{A} \ , \\ r_s &= 2GM \ , \quad r_Q^2 = Q^2 G \ . \end{aligned} \tag{8.6.7}$$

Here the charge $Q^2 = g^2 q^2 = 4\pi\alpha\hbar q^2$ contains gauge coupling g for the Maxwell field. For Kähler field one would have $g = g_K$.

The metric has two horizons for large enough mass values corresponding to the vanishing of function A implying that the sphere at which the vanishing takes place becomes metrically effectively 2-dimensional light-like 3-surface analogous to the boundary of light-cone. Note however that the determinant of the 4-metric is non-vanishing but just the finiteness of the radial component of the metric (something rather natural) would make it vanishing if g_{tt} remains zero. The horizon radii are given by

$$r_\pm = \frac{r_s}{2}\left[1 \pm \sqrt{1 - (\frac{r_Q}{r_s})^2}\right] \ . \tag{8.6.8}$$

$r_\pm$ is real for

$$M \geq M_Q = \frac{Q}{\sqrt{G}} \ . \tag{8.6.9}$$

For smaller masses one has no horizons and naked singularity at origin. The imbeddability condition however implies that the embedding fails below some critical radius.

Some general comments about the relation to TGD are in order [K99].

1. Reissner-Nordström metric has embedding as a vacuum extremal but not as non-vacuum extremal for which induced Kähler field would appear as Maxwell field. Vacuum extremals which are very important in TGD framework have no counterpart in Maxwell-Einstein system, which forces to question the assumption that Einstein-Maxwell system could serve as a GRT type limit of TGD except at macroscopic scales defined by the mass condition.

2. The solution is not expected to describe the exterior metric of objects with $M < M_Q$ at short distances. For elementary particles one expects different space-time correlates of gravitational interaction. One might optimistically guess that this is the realm where TGD replaces General Relativity.

3. The determinant of the four metric is non-vanishing at the horizons so that they cannot correspond to wormhole throats. There must be a transition region within which the determinant of the metric goes to zero at both Euclidian and Minkowskian region. The transition region could be around either horizon of the Reissner-Nordström metric when these exist. For elementary particles the situation is different since R-N metric has no horizon in this case. The critical mass corresponds to a condensed matter blob with size scale of living cell and one can ask whether it might be possible to test experimentally whether something happens in the transition region.

4. Non-vacuum extremals of Kähler action are relevant near wormhole throats and an interesting and the behavior of radially symmetric extremal of Kähler action with induced Kähler form defining the Maxwell field is still an open question. This kind of extremal would serve as the first guess for a model of the exterior space-time of elementary particle but could be quite too simple. In fact, the light-likeness of wormhole throats suggests a more complex zitterbegung like behavior so that stationarity and spherical symmetry would be quite too strong conditions on the metric.

It is interesting to apply the formula for the gravitational Planck constant [K79] to the lower bound for M. The formula reads as

$$\hbar_{gr} = \frac{GMm}{v_0} , \quad \frac{v_0}{c} < 1 \quad . \qquad (8.6.10)$$

The parameter v_0 has dimensions of velocity and for the space-time sheets mediating gravitational interaction between Sun and the 4 inner planets one has $v_0 \simeq 2^{-11}$. By writing the expression for M_Q as $M_Q = q\sqrt{\alpha_K}\hbar_{gr}\sqrt{G}$, where α_K can be assumed to be equal to fine structure constant, one finds that horizons exist only if the condition

$$q \leq \frac{v_0}{\sqrt{G}m\sqrt{\alpha_K}} \quad . \qquad (8.6.11)$$

Therefore solar system would represent a genuine elementary particle like realm in which Reissner-Nordstöm like metric does not apply unless the electromagnetic charge is so small that it vanishes by its quantization, which is of course a non-realistic condition. This idealized argument suggests the smallness of the electric charge as a condition for the applicability of GRT type description and this indeed guarantees that space-time sheets are near vacuum extremals so that small deformation of Scwartschild metric should apply.

Reduction of the quantization of Kähler electric charge to that of electromagnetic charge

The best manner to learn more is to challenge the form of the weak electric-magnetic duality based on the induced Kähler form.

1. Physically it would seem more sensible to pose the duality on electromagnetic charge rather than Kähler charge. This would replace induced Kähler form with electromagnetic field, which is a linear combination of induced Kahler field and classical Z^0 field

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{12} \quad , \\ Z^0 &= 2R_{03} \quad . \end{aligned} \qquad (8.6.12)$$

   Here $Z_0 = 2R_{03}$ is the appropriate component of CP_2 curvature form [L1]. For a vanishing Weinberg angle the condition reduces to that for Kähler form.

2. For the Euclidian space-time regions having interpretation as lines of generalized Feynman diagrams Weinberg angle should be non-vanishing. In Minkowskian regions Weinberg angle could however vanish. If so, the condition guaranteeing that electromagnetic charge of the partonic 2-surfaces equals to the above condition stating that the em charge assignable to the fermion content of the partonic 2-surfaces reduces to the classical Kähler electric flux at the Minkowskian side of the wormhole throat. One can argue that Weinberg angle must increase smoothly from a vanishing value at both sides of wormhole throat to its value in the deep interior of the Euclidian region.

3. The vanishing of the Weinberg angle in Minkowskian regions conforms with the physical intuition. Above elementary particle length scales one sees only the classical electric field reducing to the induced Kähler form and classical Z^0 fields and color gauge fields are effectively absent. Only in phases with a large value of Planck constant classical Z^0 field and other classical weak fields and color gauge field could make themselves visible. Cell membrane could be one such system [K32, K75]. This conforms with the general picture about color confinement and weak massivation.

The GRT limit of TGD suggests a further reason for why Weinberg angle should vanish in Minkowskian regions.

1. The value of the Kähler coupling strength mut be very near to the value of the fine structure constant in electron length scale and these constants can be assumed to be equal.

2. Einstein-Maxwell limit would make sense only for a vanishing Weinberg angle in Minkowskian regions. A non-vanishing Weinberg angle would make sense in the deep interior of the Euclidian regions where the approximation as a small deformation of CP_2 makes sense.

Preferred extremal property for Einstein-Maxwell system

Consider now the preferred extremal property defined to be such that the action reduces to Chern-Simons action at space-like 3-surfaces at the ends of space-time surface and at light-like wormhole throats.

1. In Maxwell-Einstein system the field equations imply

$$j^\alpha = 0 \ . \tag{8.6.13}$$

   so that the Maxwell action for extremals reduces automatically to a surface term assignable to the preferred 3-surfaces. Note that Higgs field could in principle serve as a source of Kähler field but its presence does not look like a good idea since it is not present in the field equations of TGD and because the resulting boundary term is not topological.

2. The condition

$$J = k \times *J \tag{8.6.14}$$

   at preferred 3-surfaces guarantees that the surface term from Kähler action reduces to Abelian Chern-Simons term and one has hopes about almost topological QFT.

   Since CP_2 type regions carry magnetic monopole charge and since the weak form of electric-magnetic duality implies that electric charge is proportional to the magnetic charge, one has electric charge without electric charge as Wheeler would express it. The identification of elementary building blocks as magnetic monopoles leads in TGD context to the picture about particle as Kähler magnetic flux tubes having opposite magnetic charges at their ends. It is not quite clear what the length of the tubes is. One possibility is Compton length and second possibility is weak length scale and the color confinement length scale. Note that in TGD the physical charges reside at the wormhole throats and correspond to massless fermions.

3. CP_2 is constant curvature space and satisfies Einstein equations with cosmological constant. The simplest manner to realize this is to add to the action constant volume term which is non-vanishing only in Euclidian regions. This term could be also interpreted as part of Maxwell action so that it is somewhat a matter of taste whether one speaks about cosmological constant or not. In any case, this would mean that the action contains a constant potential term

$$V = V_0 \times \frac{(1+sign(g))}{2} \ , \tag{8.6.15}$$

   where $sign(g) = -1$ holds true in Minkowskian regions and $sign(g) = 1$ holds true in Euclidian regions.

   Note that for a piece of CP_2 V_0 term can be expressed is proportional to Maxwell action and by self-duality this is proportional to instanton action reducible to a Chern-Simons term so that V_0 is indeed harmless from the point of view of holography.

4. For Einstein-Maxwell system with similar constant potential in Euclidian regions curvature scalar vanishes automatically as a trace of energy momentum tensor so that no interior or surface term results and the only surface term corresponds to a pure Chern-Simons term for Maxwell field. This is exactly the situation also in quantum TGD. The constraint term guaranteeing the weak form of electric-magnetic duality implies that the metric couples to the dynamics and the theory does not reduce to a purely topological QFT.

5. In TGD framework a non-trivial theory is obtained only if one assumes that Kähler function corresponds apart from sign to either the Kähler action in the Euclidian regions or its negative in Minkowskian regions. This is required also by number theoretic vision. This implies a beautiful duality between field descriptions and particle descriptions.

   This also guarantees that the Kähler function reducing to Chern-Simons term is negative definite: this is essential for the existence of the functional integral and unitarity of the theory. This is due to the fact that Kähler action density as a sum of magnetic and electric energy densities is positive definite in Euclidian regions. This duality would be very much analogous to that implied by the possibility to perform Wick rotation in QFTs. Therefore it seems natural to postulate similar duality also in the proposed variant of quantized General Relativity.

6. The Kähler function of the WCW would be given by Chern-Simons term with a constraint expressing the weak form of electric-magnetic duality both in TGD and General Relativity. One should be able regard also in GRT framework WCW as a union of symmetric spaces with Kähler structure possessing therefore a maximal group of isometries. This is an absolutely essential prerequisite for the existence of WCW Kähler geometry. The symmetric spaces in the union are labelled by zero modes which do not contribute to the line element and would represent classical degrees of freedom essential for quantum measurement theory. In TGD the induced CP_2 Kähler form would represents such degrees of freedom and the quantum fluctuating degrees of freedom would correspond to symplectic group of $\delta M^4_\pm \times CP_2$.

   The difference between TGD and GRT would be that light-like 3-surfaces for all possible space-times containing Euclidian and Minkowskian regions would be considered for GRT type theory. In TGD these space-times are representable as surfaces of $M^4 \times CP_2$. In TGD framework the imbeddability assumption is crucial for the mathematical existence of the theory since it eliminates space-times with non-physical characteristics. The problem posed by arbitrarily large values of cosmological constants is one of the basic problems solved by this assumption. Also mass density is sub-critical for cosmologies with infinite duration and critical cosmologies are unique apart from their duration and quantum critical cosmologies replace inflationary cosmologies.

7. Note that one could consider assigning the gravitational analog of Chern-Simons term with the preferred 3-surfaces: this kind of term is discussed by Witten in this classic work about Jones polynomial. This term is a non-abelian version of Chern-Simons term and one must replace curvature tensor with its contraction with sigma matrices so that 4-D spinor structure is necessarily involved. The objection is that this term contains second derivatives. In TGD spinor structure is induced from that of $M^4 \times CP_2$ and this kind of term need not make sense as such since gamma matrices are expressed in terms of embedding space gamma matrices: among other things this resolves the problems caused by the non-existence of spinor structure for generic 4-geometries. The coupling to the metric however results from the constraint term expressing weak form of electric-magnetic duality.

   The difference between TGD and GRT would be basically due to the factor of scattering amplitudes coming from the duality expressing electric-magnetic duality and due to the fact that induced metric in terms of H-coordinates and Maxwell potential is expressible in terms of CP_2 coordinates. The latter implies topological field quantization and many-sheeted space-time crucial for the interpretation of quantum TGD.

Could the action contain also Higgs part?

One could criticize Maxwell-Einstein action with cosmological constant non-vanishing only in Euclidian regions and ask whether a coupling to Higgs field could change the situation. This is not the case.

1. If the action contains also Higgs part, Einstein-Higgs part of the action must reduce to a surface term. The trace $G^{\alpha\beta}$ equals to the trace of the Higgs energy momentum tensor and one obtains

$$-kG = kR = -T \ ,$$

   and

$$T = -(\nabla\Phi)^2 + 4V(\Phi) = -L_H + 2V(\Phi) \ .$$

   This gives

$$L_H + kR = 2L_H - 2V(\Phi) \ .$$

2. The kinetic term of Higgs field can be written as

$$(\nabla\Phi)^2 = \nabla\cdot(\Phi\nabla\Phi) - \Phi\nabla^2\Phi \ .$$

The first term reduces to a surface term and second term can be expressed as

$$\Phi\nabla^2\Phi = -\Phi\frac{\partial V}{\partial\phi} \ .$$

Similar formula applies also if the number of Higgs components is higher than one. The condition that only the surface term remains gives

$$-2V + \Phi\frac{\partial V}{\partial\Phi} = 0$$

giving

$$V(\Phi) = \frac{m^2}{2}\Phi^2 \ . \tag{8.6.16}$$

3. The presence of constant term in V does not matter in field equations for Φ so that one can have

$$V(\Phi) = V_0 + \frac{m^2}{2}\Phi^2 \ . \tag{8.6.17}$$

In order to have both CP_2 like Euclidian regions and Reissner-Nordström type exterior solutions one must allow the Higgs potential to depend on the signature of the metric so that for massless Higgs favored by conformal invariance one would have

$$V(\Phi) = V_0\times\frac{(1+sign(g))}{2} \ , \tag{8.6.18}$$

where one has $sign(g) = -1$ for Minkowskian regions and $sign(g) = 1$ for Euclidian regions. V_0 would be a constant of nature coding for CP_2 radius about 10^4 Planck lengths.

Since the introduction of Higgs field does not allow to circumvent the introduction of a term having interpretation in terms of cosmological constant and since one loses topological QFT property, it seems that the idea about Higgs is not good.

Could ZEO and the notion of CD make sense in GRT framework?

The notion of CD is crucial in ZEO and one can ask whether the notion generalizes to GRT context. In the previous arguments related to EG the notion of ZEO plays a fundamental role since it allows to replace $S-$matrix with $M-$matrix defining "complex square root" of density matrix.

1. In TGD framework CDs are Cartesian products of Minkowskian causal diamonds of M^4 with CP_2. The existence of double light-cones in curved space-time would be required and its is not clear whether this makes sense generally. TGD suggest that the scales of these diamonds defined in terms of the proper time distance between the tips are integer multiples of CP_2 scale defined in terms of the fundamental constant V_0 (the more restrictive assumption allowing only 2^n multiples would explain p-adic length scale hypothesis but would not allow the generalization of Kac-Moody algebra spanned by M-matrices). The difference between boundaries of GRT CDs and wormhole throats would be that four-metric would not be degenerate at CDs.

2. The conformal symmetries of light-cone boundary and light-like wormhole throats generalize also now since they are due to the metric 2-dimensionality of light-like 3-surfaces. It is however far from clear whether one can have anything something analogous to conformal variants of symplectic algebra of $\delta M^4_\pm \times CP_2$ and isometry algebra of $M^4 \times CP_2$.

   Could one perhaps identify four-momenta as parameters associated with the representations of the conformal algebras involved? This hope might be unrealistic in TGD framework: the basic idea behind TGD indeed is that Poincare invariance lost in GRT is retained if space-times are surfaces in $H = M^4 \times CP_2$. The reason is that that super-Kac-Moody symmetries correspond to localized isometries of H whereas the super-conformal algebra associated with the symplectic group is assignable to the light-like boundaries $\delta M^4_\pm \times CP_2$ of CD of H rather than space-time surface.

3. One could of course argue that some physical conditions on GRT -most naturally just the highly non-trivial mathematical existence of WCW Kähler geometry and spinor structure-could force the representability of physically acceptable 4-geometries as surfaces $M^4 \times CP_2$. If so, then also CDs would the same CDs as in TGD and quantization of GRT would lead to TGD and all the huge symmetries would emerge from quantum GRT alone.

   The first objection is that the induced spinor structure in TGD is not consistent with that natural in GRT. Second objection is that in TGD framework Einstein-Maxwell equations are not true in general and Einstein's equations can be assumed only in long length scales for the vacuum extremals of Kähler action. The Einstein tensor would characterize the energy momentum tensor assignable to the topologically condensed matter around these vacuum extremals and neither geometrically nor topologically visible in the resolution defined by very long length scale. If Maxwell field corresponds to em field in Minkowskian regions, the vacuum extremal property would make sense in scales where matter is electromagnetic neutral and em radiation is absent.

8.6.4 What Can One Conclude?

The previous considerations suggest that a surprisingly large piece of TGD can be applied also in GRT framework and raise the possibility about quantization of Einstein-Maxwell system in terms of Kähler geometry of WCW consisting of 3-geometries instead of 3-surfaces. One can even consider a new manner to understand TGD as resulting from the quantization of GRT in terms of WCW Kähler geometry in the space of 3-metrics realizing holography and making classical theory an exact part of quantum theory. Since the space-times allowed by TGD define a subset of those allowed by GRT one can ask whether the quantization of GRT leads to TGD or at least sub-theory of TGD. The arguments represented above however suggest that this is not the case. The generalization of S-matrix to a complex of U-matrix, S-matrix and algebra of M-matrices forced by ZEO gives a natural justification for the modification of EG allowing gravitons and giving up the rather nebulous idea about emergent space-time. Whether ZEO crucial for EG makes sense in GRT picture is not clear. A promising signal is that the generalization of EG to all interactions in TGD framework leads to a concrete interpretation of gravitational entropy and temperature, to a more precise view about how the arrow of geometric time emerges, to a more concrete realization of the old idea that matter antimatter asymmetry could relate to different arrows of geometric time (not however for matter and antimatter but for space-time sheets mediating attractive and repulsive long range interactions), and to the idea that the small value of cosmological constant could correspond to the small fraction of non-Euclidian regions of space-time with cosmological constant characterized by CP_2 size scale.

8.7 Emergent gravity and dark Universe

Eric Verlinde has published article with title *Emergent Gravity and the Dark Universe* [B22] (see `http://tinyurl.com/grwc3fz`). The article represents his recent view about gravitational force as thermodynamical force described earlier in [B12] and suggests an explanation for the constant velocity spectrum of distant stars around galaxies and for the recently reported correlation between the real acceleration of distant stars with corresponding acceleration caused by baryonic matter [E48]. In the following I discuss Verlinde's argument and compare the physical picture with that provided by TGD. I have already earlier discussed Verlinde's entropic gravity from TGD view point [K99].

Before continuing it is good to recall the basic argument against the identification of gravity as entropic force. The point is that neutron diffraction experiments [B10] suggests that gravitational potential appears in the Schrödinger equation. This cannot be the case if gravitational potential has thermodynamic origin and therefore follows from statistical predictions of quantum theory: to my opinion Verlinde mixes apples with oranges.

8.7.1 Verlinden's argument

Consider now Verlinde's argument.

1. Verlinde wants to explain the recent empirical finding that the observed correlation between the acceleration of distant stars around galaxy with that of baryonic matter [E48] (see `http://tinyurl.com/jd2m9ll`) in terms of apparent dark energy assigned with entanglement entropy proportional to volume rather than horizon area as in Bekenstein-Hawking formula. This means giving up the standard holography and introducing entropy proportional to volume.

   To achieve this he replaces anti-de-Sitter space (AdS) to which AdS/CFT duality is usually assigned with de-Sitter space(dS) space with cosmic horizon expressible in terms of Hubble constant and assign it with long range entanglement since in AdS only short range entanglement is believed to be present (area law for entanglement entropy). This would give rise to an additional entropy proportional to the volume rather than area. Dark energy or matter would corresponds to a thermal energy assignable to this long range entanglement. One can of course criticize this assumption as ad hoc hypothesis.

2. Besides this Verlinde introduces tensor nets as justification for the emergence of gravitation: this is just a belief. All arguments that I have seen about this are circular (one introduces 2-D surfaces and thus also 3-space from beginning) and also Verlinde uses dS space. What is to my opinion alarming that there is no fundamental approach really explaining how space-time and gravity emerges. Emergence of space-time should lead also to the emergence of spinor structure of space-time and this seems to me something impossible if one really starts from mere Hilbert space.

3. Verlinde introduces also analogy with the thermodynamics of glass involving both short range crystal structure and amorphous long range behaviour that would correspond to entanglement entropy in long scales long range structure. Above the horizon size the contribution proportional to volume would begin to dominate in entropy. Also analogy with elasticity is introduced. Below Hubble scale the microscopic states do not thermalize below the horizon and display memory effects. Dark gravitational force would be analogous to elastic response due to what he calls entropy displacement.

4. Verlinde admits that this approach does not say much about cosmology or cosmic expansion, and even less about inflation.

8.7.2 The long range correlations of Verlinde correspond to hierarchy of Planck constants in TGD framework

The physical picture has analogies with my own approach [L20] to the explanation of the correlation between baryonic acceleration with observed acceleration of distant stars. In particular, long range entanglement has the identification of dark matter in terms of phases labelled by the hierarchy of Planck constants as TGD counterpart.

1. Concerning the emergence of space and gravitation TGD leads to a different view. It is not 3-space but the experience about 3-space - proprioception -, which would emerge via tensor nets realized in TGD in terms of magnetic flux tubes emerging from 3-surfaces defining the nodes of the tensor net [L17]. This picture leads to a rather attractive view about quantum biology (see for instance `http://tinyurl.com/q4jyoc5`).

2. Twistor lift of TGD has rapidly become a physically convincing formulation of TGD [K38] (see `http://tinyurl.com/zjgmax6`). One replaces space-time surfaces in $M^4 \times CP_2$ with

the 12-D product $T(M^4 \times CP_2)$ of the twistor spaces $T(M^4)$ and $T(CP_2)$ and Kähler action with its 6-D variant. This requires that $T(M^4)$ and $T(CP_2)$ have Kähler structure. This is true but only for M^4 (and its variants E^4 and S^4) and CP_2. Hence TGD is completely unique also mathematically and physically (providing a unique explanation for the standard model symmetries). The preferred extremal property for Kähler action could reduce to the property that the 6-D surface as an extremal of 6-D Kähler action is twistor space of space-time surface and thus has the structure of S^2 bundle. That this is indeed the case for the preferred extremals of dimensionally reduced 4-D action expressible as a sum of Kähler action and volume term remains to be rigorously proven.

3. Long range entanglement even in cosmic scales would be crucial and give the volume term in entropy breaking the holography in the usual sense. In TGD framework hierarchy of Planck constants $h_{eff} = n \times h$ satisfying the additional condition $h_{eff} = h_{gr}$, where $h_{gr} = GMm/v_0$ (M and m are masses and v_0 is a parameter with dimensions of velocity) is the gravitational Planck constant introduced originally by Nottale [E25], and assignable to magnetic flux tubes mediating gravitational interaction makes [K79, K66, **?**]. This makes possible quantum entanglement even in astrophysical and cosmological long length scales since h_{gr} can be extremely large [**?**, K67]. In TGD however most of the galactic dark matter and energy is associated with cosmic strings having galaxies along it (like pearls in necklace) [K80, K26]. Baryonic dark matter could correspond to the ordinary matter which has resulted in the decay of cosmic strings taking the role of inflaton field in very early cosmology. This gives automatically a logarithmic potential giving rise to constant spectrum velocity spectrum modified slightly by baryonic matter and a nice explanation for the correlation, which served as the motivation of Verlinde. In particular, the only parameter of the model is string tension and TGD allows to estimate also this and the value is completely fixed for the ideal cosmic strings and reduces as they thicken.

4. Also glass analogy has TGD counterpart. Kähler action has 4-D spin glass degeneracy giving rise to 4-D spin-glass degeneracy. In twistor lift of TGD cosmological term appears and reduces the degeneracy by allowing only minimal surfaces rather than all vacuum extremals. This removes the non-determinism. Cosmological constant is however extremely small implying non-perturbative behavior in the sense that the volume term for the action is extremely small and depends very weakly on the preferred extremal. This suggests that spin glass in 3-D sense remains as Kähler action with varying sign is added: the space-time regions dominated by electric or magnetic fields give contributions with different sign and one can obtain the characteristic fractal spin glass energy landscape with valleys inside valleys.

5. The mere Kähler action for the Minkowskian (at least) regions of the preferred extremals reduces to a Chern-Simons terms at light-like 3-surfaces at which the signature of the induced metric of the space-time surface changes from Minkowskian to Euclidian. The interpretation could be that TGD is almost topological quantum field theory. Also the interpretation in terms of holography can be considered.

   Volume term proportional to cosmological constant given by the twistor lift of TGD [K38] (see `http://tinyurl.com/zjgmax6`) could mean a small breaking of holography in the sense that it cannot be reduced to a 3-D surface term. One must however be very cautious here because TGD strongly suggests strong form of holography meaning that data at string world sheets and partonic 2-surfaces (or possibly at their metrically 2-D light-like orbits for which only conformal equivalence class matters) fix the 4-D dynamics.

   Certainly volume term means a slight breaking of the flatness of the 3-space in cosmology since 3-D curvature scalar cannot vanish for Robertson-Walker cosmology imbeddable as a minimal surface except at the limit of infinitely large causal diamond (CD) implying that cosmological constant, which is proportional to the inverse of the p-adic length scale squared, vanishes at this limit. Note that the dependence $\Lambda \propto 1/p$, p p-adic prime, allows to solve the problem caused by the large value of cosmological constant in very early cosmology. Quite generally, volume term would describe finite volume effects analogous to those encountered in thermodynamics.

8.7.3 The argument against gravitation as entropic force can be circumvented in zero energy ontology

Could TGD allow to resolve the basic objection against gravitation as entropic force or generalize this notion?

1. In Zero Energy Ontology (ZEO) quantum theory can be interpreted as "complex square root of thermodynamics". Vacuum functional is an exponent of the action determining preferred extremals - Kähler action plus volume term present for twistor lift. This brings in gravitational constant G and cosmological Λ constant as fundamental constants besides CP_2 size scale R and Kähler coupling strength α_K [K38]. Vacuum functional would be analogous to an exponent of $E_c/2$, where E_c would be complexified energy. I have also considered the possibility that vacuum functional is analogous to the exponent of free energy but following argument favors the interpretation as exponent of energy.

2. The variation of Kähler action would give rise to a 4-D analog of TdS term and the variation of cosmological constant term to the analog of $-pdV$ term in $dE = TdS - pdV$. Both T and p would be complex and would receive contributions from both Minkowskian and Euclidian regions. The contributions of Minkowskian and Euclidian regions to the action would differ by a multiplication with imaginary unit and it is possible that Kähler coupling strength is complex as suggested in [K34].

   If the inverse of the Kähler coupling is strength is proportional to the zero of Riemann zeta at critical line, it is complex, and the coefficient of the volume term must have the same phase: otherwise space-time surfaces are extremals of Kähler action and minimal surfaces simultaneously. In fact, the known non-vacuum extremals of Kähler action are surfaces of this kind, and one cannot exclude the possibility that preferred extremals have quite generally this property. The physical picture below does not favor this idea.

   Note: One can consider also the possibility that the values of Kähler coupling strength correspond to the imaginary part for the zero of Riemann zeta.

3. Suppose that both terms in the action are proportional to the same phase factor. The part of the variation of the Kähler action with respect to the embedding space coordinates giving the analog of TdS term would give the analog of entropic force. Since the variation of the entire action vanishes, the variation of Kähler action would be equal to the negative of the variation of the volume term with respect to the induced metric given by $-pdV$. Since the variations of Kähler action and volume term cancel each other, the entropic force would be non-vanishing only for the extremals for which Kähler action density is non-vanishing. The variation of Kähler action contains variation with respect to the induced metric and induced Kähler form so that the sum of gravitational and U(1) force would be equal to the analog of entropic force and Verlinde's proposal would not generalize as such.

   The variation of the volume term gives rise to a term proportional to the trace of the second fundamental form, which is 4-D generalization of ordinary force and vanishes for the vacuum extremals of Kähler action in which case one has analog of geodesic line. More generally, Kähler action gives rise to the generalization of U(1) force on particle so that the field equations give a 4-D generalization of equations of motion for a point like particle in U(1) force having also interpretation as a generalization of entropic force.

4. In Zero Energy Ontology (ZEO) TGD predicts a dimensional hierarchy of basic objects analogous to the brane hierarchy in M-theory: space-time surfaces as 4-D objects, 3-D light-like orbits of partonic 2-surfaces as boundaries of Minkowskian and Euclidian regions plus space-like 3-surfaces defining the ends of space-time surface at the opposite boundaries of CD, 2-D partonic surfaces and string world sheets, and 1-D boundaries of string world sheets. The natural idea is to identify the dynamics D-dimensional objects in terms of action consisting of D-dimensional volume in induced metric and D-dimensional analog of Kähler action. The surfaces at the ends of space-time should be freely choosable apart from the conditions related to super-symplectic algebra realizing strong form of holography since they correspond to initial values.

   For the light-like orbits of partonic 2-surfaces 3-volume vanishes and one has only Chern-Simons type topological term. For string world sheets one has area term and magnetic flux, which is topological term reducing to a mere boundary term so that minimal surface equations are obtained. For the dynamical boundaries of string world sheets one obtains 1-D volume term as the length of string world line and the boundary term from string world sheet. This gives 1-D equation of motion in U(1) force just like in Maxwell's theory but with induced Kähler form defining the U(1) gauge field identifiable as the counterpart of classical U(1) field of standard model. Induced spinor fields couple at boundaries only to the induced em gauge potential since induced classical W-boson gauge fields vanish at string world sheets in order to achieve a well-defined and conserved spinorial em charge (here the absolutely minimal option would be that the W and Z gauge potentials vanish only at the time-like boundaries of string world sheet). Should world-line geometry couple to the induced em gauge field instead of induced Kähler form? The only logical option is however that geometry couples to the U(1) charge perhaps identifiable in terms of fermion number.

5. There however an objection against this picture. All known extremals of Kähler action are minimal surfaces and there are excellent number theoretical arguments suggesting that all preferred extremals of Kähler action are also minimal surfaces so that the original picture would be surprisingly near to the truth. The separate vanishing of variation implies that the solutions do not depend at all on coupling parameters as suggested by number theoretical universality and universality of the dynamics at quantum criticality. The discrete coupling constant evolution makes it however visible via boundary conditions classically. This would however predicts that the analogs to TdS and pdV vanish identically in space-time interior.

   The variations however involve also boundary terms, which need not vanish separately since the actions in Euclidian and Minkowskian regions differ by multiplication with $\sqrt{-1}$! The variations reduce to terms proportional to the normal component of the canonical momentum current contracted with the deformation at light-like 3-surfaces bounding Euclidian and Minkowskian space-time regions. These must vanish. If Kähler coupling strength is real, this implies decoupling of the dynamics due to the volume term and Kähler action also at light-like 3-surfaces and therefore also exchange of charges - in particular four-momentum - becomes impossible. This would be a catastrophe.

   If α_K is complex as quantum TGD as a square root of thermodynamics and the proposal that the spectrum of $1/\alpha_K$ corresponds to the spectrum of zeros of zeta require [K34], the normal component of the canonical momentum current for Kähler action equals to that for the volume term at the other side of the bounding surface. The analog of $dE = TdS - pdV = 0$ would hold true in the non-trivial sense at light-like 3-surfaces and thermodynamical analogy holds true (note that energy is replaced with action). The reduction of variations to boundary terms would also conform with holography. Strong form of holography would even suggest that the 3-D boundary term in turn reduces to 2-D boundary terms.

   A possible problem is caused by the variation of volume term: $\sqrt{g_4}$ vanishes at the boundary and g^{nn} diverges. The overall result should be finite and should be achieved by proper boundary conditions. What I have called weak form of electric-magnetic duality [K6] allows to avoid similar problems for Kähler action, and implies self-duality of the induced Kähler form at the boundary. A weaker form of boundary conditions would state that the sum of the variations of Kähler action and volume term is finite.

Physically this picture is very attractive and makes cosmological constant term emerging from the twistor lift rather compelling. What is nice that this picture follows from the field equations of TGD rather than from mere heuristic arguments without underlying mathematical theory.

Chapter 9

TGD and Possible Gravitational Anomalies

9.1 Introduction

In this chapter the applications of TGD to various real or potential anomalies of GRT approach are discussed.

1. Allais effect represents one of the anomalies associated with gravitational interaction discarded by the average theoretician. In TGD framework this effect could be interpreted as an interference effect made possible by the gigantic value of gravitational Planck constant [K66]. As an interference effect it is extremely sensitive to the parameters of the problem and this explains why the sign and size of the effects varies so much.
2. Gravimagnetism is one of the predictions of GRT which is being tested experimentally. TGD predicts deviations from the predictions of GRT which unfortunately are not seen in the satellite experiment to be discussed below. The claimed discovery of gravimagnetic effect in super-conductors having strength 20 orders of magnitude larger than predicted by GRT raises the question whether TGD might explain the effect. TGD inspired model of gravimagnetism is studied. The claims about strong gravimagnetism are considered in terms large $\hbar$ hypothesis. It turns out that the identification $h_{gr} = h_{eff}$ at elementary particle level and assumption that h_{gr} for superconductor corresponds to h_{gr} for Earth Cooper pair system predicts correctly the amplification factor needed to obtain strong enough gravimagnetic variant of Thomson magnetic field to explaoin the discrepancy motivating the work of Tajmar *et al.* Also a direct connection with hypothesis identifying EEG photons as dark photons decaying to ordinary visible photons emerges.
3. For many-sheeted space-time light velocity is assigned to light-like geodesic of space-time sheet rather than light-like geodesics of embedding space $M^4 \times CP_2$. The effective velocity determined from time to travel from point A to B along different space time sheets is different and therefore also the signal velocity determined in this way. The light-like geodesics of space-time sheet corresponds in the generic case time-like curves of the embedding space so that the light-velocity is reduced from the maximal signal velocity. Space-time sheet is bumpy and wiggled so that the path is longer. Each space-time sheet corresponds to different light velocity as determined from the travel time. The maximal signal velocity is reached only in an ideal situation when the space-time geodesics are geodesics of Minkowski space.The dependence of operationally defined light velocity on space-time sheet distinguishes between the sub-manifold gravity of TGD and the abstract manifold gravity GRT. Possible evidence for the effect is discussed. These effects are discussed in several sections.
4. There exists an infinite number of warped imbeddings of M^4 to $M^4 \times CP_2$, which are metrically equivalent with the canonical imbedding with CP_2 coordinates constant. These imbeddings are characterized by anomalous time dilation due to the warping even when gravitational fields are absent and the dilation can be large. It is conceivable that preferred extremals obtained as deformations of these warped M^4:s are possible.

5. There are also some considerations not strictly related to anomalies such as possible interpretations of Machian Principle in TGD framework.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L8].

9.2 Allais Effect And TGD

Allais effect represents one of the anomalies associated with gravitational interaction discarded by the average theoretician. In TGD framework this effect can be interpreted as an interference effect made possible by the gigantic value of gravitational Planck constant. As an interference effect it is extremely sensitive to the parameters of the problem and this explains why the sign and size of the effects varies so much.

9.2.1 The Effect

Allais effect [E2, E45] is a fascinating gravitational anomaly associated with solar eclipses. It was discovered originally by M. Allais, a Nobelist in the field of economy, and has been reproduced in several experiments but not as a rule. The experimental arrangement uses so called paraconical pendulum, which differs from the Foucault pendulum in that the oscillation plane of the pendulum can rotate in certain limits so that the motion occurs effectively at the surface of sphere.

Experimental findings

Consider first a brief summary of the findings of Allais and others [E45].

1. In the ideal situation (that is in the absence of any other forces than gravitation of Earth) paraconical pendulum should behave like a Foucault pendulum. The oscillation plane of the paraconical pendulum however begins to rotate.
2. Allais concludes from his experimental studies that the orbital plane approach always asymptotically to a limiting plane and the effect is only particularly spectacular during the eclipse. During solar eclipse the limiting plane contains the line connecting Earth, Moon, and Sun. Allais explains this in terms of what he calls the anisotropy of space.
3. Some experiments carried out during eclipse have reproduced the findings of Allais, some experiments not. In the experiment carried out by Jeverdan and collaborators in Romania it was found that the period of oscillation of the pendulum decreases by $\Delta f/f \simeq 5 \times 10^{-4}$ [E2, E42] which happens to correspond to the constant $v_0 = 2^{-11}$ appearing in the formula of the gravitational Planck constant. It must be however emphasized that the overall magnitude of $\Delta f/f$ varies by five orders of magnitude. Even the sign of $\Delta f/f$ varies from experiment to experiment.
4. There is also quite recent finding by Popescu and Olenici, which they interpret as a quantization of the plane of oscillation of paraconical oscillator during solar eclipse [E50]. There is also evidence that the effect is present also before and after the full eclipse. The time scale is 1 hour.

TGD based models for Allais effect

I have already earlier proposed an explanation of the effect in terms of classical Z^0 force [K9]. If the Z^0 charge to mass ratio of pendulum varies and if Earth and Moon are Z^0 conductors, the resulting model is quite flexible and one might hope it could explain the high variation of the experimental results.

The rapid variation of the effect during the eclipse is however a problem for this approach and suggests that gravitational screening or some more general interference effect might be present. Gravitational screening alone cannot however explain Allais effect. Also the combination of gravitational screening and Z^0 force assuming Z^0 conducting structures causing screening fails to explain the discontinuous behavior when massive objects are collinear.

A model based on the idea that gravitational interaction is mediated by topological light rays (MEs) and that gravitons correspond to a gigantic value of the gravitational Planck constant however explains the Allais effect as an interference effect made possible by macroscopic quantum coherence in astrophysical length scales. Equivalence Principle fixes the model to a high degree and one ends up with an explicit formula for the anomalous gravitational acceleration and the general order of magnitude and the large variation of the frequency change as being due to the variation of the distance ratio $r_{S,P}/r_{M,P}$ (S, M, and P refer to Sun, Moon, and pendulum respectively). One can say that the pendulum acts as an interferometer.

9.2.2 Could Gravitational Screening Explain Allais Effect

The basic idea of the screening model is that Moon absorbs some fraction of the gravitational momentum flow of Sun and in this manner partially screens the gravitational force of Sun in a disk like region having the size of Moon's cross subsection. The screening is expected to be strongest in the center of the disk. Screening model happens to explain the findings of Jevardan but fails in the general case. Despite this screening model serves as a useful exercise.

Constant external force as the cause of the effect

The conclusions of Allais motivate the assumption that quite generally there can be additional constant forces affecting the motion of the paraconical pendulum besides Earth's gravitation. This means the replacement $\overline{g} \rightarrow \overline{g}+\Delta\overline{g}$ of the acceleration g due to Earth's gravitation. $\Delta\overline{g}$ can depend on time.

The system obeys still the same simple equations of motion as in the initial situation, the only change being that the direction and magnitude of effective Earth's acceleration have changed so that the definition of vertical is modified. If $\Delta\overline{g}$ is not parallel to the oscillation plane in the original situation, a torque is induced and the oscillation plane begins to rotate. This picture requires that the friction in the rotational degree of freedom is considerably stronger than in oscillatory degree of freedom: unfortunately I do not know what the situation is.

The behavior of the system in absence of friction can be deduced from the conservation laws of energy and angular momentum in the direction of $\overline{g} + \Delta\overline{g}$. The explicit formulas are given by

$$\begin{aligned} E &= \frac{ml^2}{2}(\frac{d\Theta}{dt})^2 + sin^2(\Theta)(\frac{d\Phi}{dt})^2 + mglcos(\Theta) \ , \\ L_z &= ml^2 sin^2(\Theta)\frac{d\Phi}{dt} \ . \end{aligned} \quad (9.2.1)$$

and allow to integrate Θ and Φ from given initial values.

What causes the effect in normal situations?

The gravitational accelerations caused by Sun and Moon come first in mind as causes of the effect. Equivalence Principle implies that only relative accelerations causing analogs of tidal forces can be in question. In GRT picture these accelerations correspond to a geodesic deviation between the surface of Earth and its center. The general form of the tidal acceleration would thus the difference of gravitational accelerations at these points:

$$\Delta\overline{g} = -2GM[\frac{\Delta\overline{r}}{r^3} - 3\frac{\overline{r}\cdot\Delta\overline{r}\overline{r}}{r^5}] \ . \quad (9.2.2)$$

Here $\overline{r}$ denotes the relative position of the pendulum with respect to Sun or Moon. $\Delta\overline{r}$ denotes the position vector of the pendulum measured with respect to the center of Earth defining the geodesic deviation. The contribution in the direction of $\Delta\overline{r}$ does not affect the direction of the Earth's acceleration and therefore does not contribute to the torque. Second contribution corresponds to an acceleration in the direction of $\overline{r}$ connecting the pendulum to Moon or Sun. The direction of this vector changes slowly.

This would suggest that in the normal situation the tidal effect of Moon causes gradually changing force $m\Delta\overline{g}$ creating a torque, which induces a rotation of the oscillation plane. Together with dissipation this leads to a situation in which the orbital plane contains the vector $\Delta\overline{g}$ so that no torque is experienced. The limiting oscillation plane should rotate with same period as Moon around Earth. Of course, if effect is due to some other force than gravitational forces of Sun and Earth, paraconical oscillator would provide a way to make this force visible and quantify its effects.

What would happen during the solar eclipse?

During the solar eclipse something exceptional must happen in order to account for the size of effect. The finding of Allais that the limiting oscillation plane contains the line connecting Earth, Moon, and Sun implies that the anomalous acceleration $\Delta|g$ should be parallel to this line during the solar eclipse.

The simplest hypothesis is based on TGD based view about gravitational force as a flow of gravitational momentum in the radial direction.

1. For stationary states the field equations of TGD for vacuum extremals state that the gravitational momentum flow of this momentum. Newton's equations suggest that planets and moon absorb a fraction of gravitational momentum flow meeting them. The view that gravitation is mediated by gravitons which correspond to enormous values of gravitational Planck constant in turn supports Feynman diagrammatic view in which description as momentum exchange makes sense and is consistent with the idea about absorption. If Moon absorbs part of this momentum, the region of Earth screened by Moon receives reduced amount of gravitational momentum and the gravitational force of Sun on pendulum is reduced in the shadow.

2. Unless the Moon as a coherent whole acts as the absorber of gravitational four momentum, one expects that the screening depends on the distance travelled by the gravitational flux inside Moon. Hence the effect should be strongest in the center of the shadow and weaken as one approaches its boundaries.

3. The opening angle for the shadow cone is given in a good approximation by $\Delta\Theta = R_M/R_E$. Since the distances of Moon and Earth from Sun differ so little, the size of the screened region has same size as Moon. This corresponds roughly to a disk with radius $.27 \times R_E$.

   The corresponding area is 7.3 per cent of total transverse area of Earth. If total absorption occurs in the entire area the total radial gravitational momentum received by Earth is in good approximation 92.7 per cent of normal during the eclipse and the natural question is whether this effective repulsive radial force increases the orbital radius of Earth during the eclipse.

   More precisely, the deviation of the total amount of gravitational momentum absorbed during solar eclipse from its standard value is an integral of the flux of momentum over time:

$$\begin{aligned}\Delta P^k_{gr} &= \int \frac{\Delta P^k_{gr}}{dt}(S(t))dt \ , \\ \frac{\Delta P^k_{gr}}{dt}(S(t)) &= \int_{S(t)} J^k_{gr}(t)dS \ . \end{aligned} \tag{9.2.3}$$

   This prediction could kill the model in classical form at least. If one takes seriously the quantum model for astrophysical systems predicting that planetary orbits correspond to Bohr orbits with gravitational Planck constant equal to $\hbar_{gr} = GMm/v_0$, $v_0 = 2^{-11}$, there should be not effect on the orbital radius. The anomalous radial gravitational four-momentum could go to some other degrees of freedom at the surface of Earth.

4. The rotation of the oscillation plane is largest if the plane of oscillation in the initial situation is as orthogonal as possible to the line connecting Moon, Earth and Sun. The effect vanishes when this line is in the initial plane of oscillation. This testable prediction might explain why some experiments have failed to reproduce the effect.

5. The change of $|\overline{g}|$ to $|\overline{g}+\Delta\overline{g}|$ induces a change of oscillation frequency given by

$$\frac{\Delta f}{f} = \frac{\overline{g}\cdot\Delta\overline{g}}{g^2} = \frac{\Delta g}{g}cos(\theta) \ . \tag{9.2.4}$$

If the gravitational force of the Sun is screened, one has $|\overline{g}+\Delta\overline{g}| > g$ and the oscillation frequency should increase. The upper bound for the effect corresponds to vertical direction is obtained from the gravitational acceleration of Sun at the surface of Earth:

$$\frac{|\Delta f|}{f} \leq \frac{\Delta g}{g} = \frac{v_E^2}{r_E} \simeq 6.0\times 10^{-4} \ . \quad (9.2.5)$$

What kind of tidal effects are predicted?

If the model applies also in the case of Earth itself, new kind of tidal effects are predicted due to the screening of the gravitational effects of Sun and Moon inside Earth. At the night-side the paraconical pendulum should experience the gravitation of Sun as screened. Same would apply to the "night-side" of Earth with respect to Moon.

Consider first the differences of accelerations in the direction of the line connecting Earth to Sun/Moon: these effects are not essential for tidal effects. The estimate for the ratio for the orders of magnitudes of the these accelerations is given by

$$\frac{|\Delta\overline{g}_{\perp}(Moon)|}{|\Delta\overline{g}_{\perp}(Sun)|} = \frac{M_S}{M_M}(\frac{r_M}{r_E})^3 \simeq 2.17 \ . \quad (9.2.6)$$

The order or magnitude follows from $r(Moon) = .0026$ AU and $M_M/M_S = 3.7\times 10^{-8}$. These effects are of same order of magnitude and can be compensated by a variation of the pressure gradients of atmosphere and sea water. The effects caused by Sun are two times stronger. These effects are of same order of magnitude and can be compensated by a variation of the pressure gradients of atmosphere and sea water.

The tangential accelerations are essential for tidal effects. They decompose as

$$\frac{1}{r^3}\left[\Delta\overline{r} - 3|\Delta\overline{r}|cos(\Theta)\frac{\overline{r}}{r}\right] \ .$$

$\pi/4 \leq \Theta \leq \pi/2$ is the angle between $\Delta\overline{r}$ and $\overline{r}$. The above estimate for the ratio of the contributions of Sun and Moon holds true also now and the tidal effects caused by Sun are stronger by a factor of two.

Consider now the new tidal effects caused by the screening.

1. Tangential effects on day-side of Earth are not affected (night-time and night-side are of course different notions in the case of Moon and Sun). At the night-side screening is predicted to reduce tidal effects with a maximum reduction at the equator.
2. Second class of new effects relate to the change of the normal component of the forces and these effects would be compensated by pressure changes corresponding to the change of the effective gravitational acceleration. The night-day variation of the atmospheric and sea pressures would be considerably larger than in Newtonian model.

The intuitive expectation is that the screening is maximum when the gravitational momentum flux travels longest path in the Earth's interior. The maximal difference of radial accelerations associated with opposite sides of Earth along the line of sight to Moon/Sun provides a convenient manner to distinguish between Newtonian and TGD based models:

$$\begin{aligned} |\Delta\overline{g}_{\perp,N}| &= 4GM\times\frac{R_E}{r^3} \ , \\ |\Delta\overline{g}_{\perp,TGD}| &= 4GM\times\frac{1}{r^2} \ . \end{aligned} \quad (9.2.7)$$

The ratio of the effects predicted by TGD and Newtonian models would be

$$\begin{aligned} \frac{|\Delta\overline{g}_{\perp,TGD}|}{|\Delta\overline{g}_{\perp,N}|} &= \frac{r}{R_E} \ , \\ \frac{r_M}{R_E} &= 60.2 \ , \ \frac{r_S}{R_E} = 2.34\times 10^4 \ . \end{aligned} \quad (9.2.8)$$

M_M/M_S	M_E/M_S	R_M/R_E	d_{E-S}/AU	d_{E-M}/AU
3.0×10^{-6}	3.69×10^{-8}	.273	1	.00257
R_E/d_{E-S}	R_E/d_{E-M}	g_S/g	g_M/g	
4.27×10^{-5}	01.7×10^{-7}	6.1×10^{-4}	2.8×10^{-4}	

Table 9.1: Table gives basic data relevant for tidal effects. The subscript E, S, M refers to Earth, Sun, Moon; R refers to radius; d_{X-Y} refers to the distance between X and Y g_S and g_M refer to accelerations induced by Sun and Moon at Earth surface. $g = 9.8$ m/s^2 refers to the acceleration of gravity at surface of Earth. One has also $M_S = 1.99 \times 10^{30}$ kg and $AU = 1.49 \times 10^{11}$ m, $R_E = 6.34 \times 10^6$ m.

The amplitude for the oscillatory variation of the pressure gradient caused by Sun would be

$$\Delta|\nabla p_S| = \frac{v_E^2}{r_E} \simeq 6.1 \times 10^{-4} g$$

and the pressure gradient would be reduced during night-time. The corresponding amplitude in the case of Moon is given by

$$\frac{\Delta|\nabla p_s|}{\Delta|\nabla p_M|} = \frac{M_S}{M_M} \times (\frac{r_M}{r_S})^3 \simeq 2.17 \quad .$$

$\Delta|\nabla p_M$ is in a good approximation smaller by a factor of 1/2 and given by $\Delta|\nabla p_M| = 2.8 \times 10^{-4} g$. Thus the contributions are of same order of magnitude.

One can imagine two simple qualitative killer predictions assuming maximal gravitational screening.

1. Solar eclipse should induce anomalous tidal effects induced by the screening in the shadow of the Moon.
2. The comparison of solar and moon eclipses might kill the scenario. The screening would imply that inside the shadow the tidal effects are of same order of magnitude at both sides of Earth for Sun-Earth-Moon configuration but weaker at night-side for Sun-Moon-Earth situation.

An interesting co-incidence

The value of $\Delta f/f = 5 \times 10^{-4}$ in experiment of Jeverdan is exactly equal to $v_0 = 2^{-11}$, which appears in the formula $\hbar_{gr} = GMm/v_0$ for the favored values of the gravitational Planck constant. The predictions are $\Delta f/f \leq \Delta p/p \simeq 3 \times 10^{-4}$. Powers of $1/v_0$ appear also as favored scalings of Planck constant in the TGD inspired quantum model of bio-systems based on dark matter [K32]. This co-incidence would suggest the quantization formula

$$\frac{g_E}{g_S} = \frac{M_S}{M_E} \times \frac{R_E^2}{r_E^2} = v_0 \tag{9.2.9}$$

for the ratio of the gravitational accelerations caused by Earth and Sun on an object at the surface of Earth.

It must be however admitted that the larger variation in the magnitude and even sign of the effect does not favor this kind of interpretation.

Summary of the predicted new effects

Let us sum up the basic predictions of the model assuming maximal gravitational screening.

1. The first prediction is the gradual increase of the oscillation frequency of the conical pendulum by $\Delta f/f \leq 3 \times 10^{-4}$ to maximum and back during night-time in case that the pendulum has vanishing Z^0 charge. Also a periodic variation of the frequency and a periodic rotation of the oscillation plane with period co-inciding with Moon's rotation period is predicted. Already Allais observed both 24 hour cycle and cycle which is slightly longer and due to the fact that Moon rates around Earth.

2. A paraconical pendulum with initial position, which corresponds to the resting position in the normal situation should begin to oscillate during solar eclipse. This effect is testable by fixing the pendulum to the resting position and releasing it during the eclipse. The amplitude of the oscillation corresponds to the angle between $\overline{g}$ and $\overline{g}+\Delta\overline{g}$ given in a good approximation by

$$sin[\Theta(\overline{g},\overline{g}+\Delta\overline{g})] = \frac{\Delta g}{g} sin[\Theta(\overline{g},\Delta\overline{g})] \ . \tag{9.2.10}$$

An upper bound for the amplitude would be $\Theta \leq 3\times 10^{-4}$, which corresponds to.015 degrees. Z^0 charge of the pendulum would modify this simple picture.

3. Gravitational screening should cause a reduction of tidal effects at the "night-side" of Moon/Sun. The reduction should be maximum at "midnight". This reduction together with the fact that the tidal effects of Moon and Sun at the day side are of same order of magnitude could explain some anomalies know to be associated with the tidal effects [F32]. A further prediction is the day-night variation of the atmospheric and sea pressure gradients with amplitude which is for Sun $3\times 10^{-4}g$ and for Moon $1.3\times 10^{-3}g$.

To sum up, the predicted anomalous tidal effects and the explanation of the limiting oscillation plane in terms of stronger dissipation in rotational degree of freedom could kill the model assuming only gravitational screening.

Comparison with experimental results

The experimental results look mutually contradictory in the context provided by the model assuming only screening. Some experiments find no anomaly at all as one learns from [E2]. There are also measurements supporting the existence of an effect but with varying sign and quite different orders of magnitude. Either the experimental determinations cannot be trusted or the model is too simple.

1. The *increase* (!) of the frequency observed by Jeverdan and collaborators reported in Wikipedia article [E2] for Foucault pendulum is $\Delta f/f \simeq 5\times 10^{-4}$ would support the model even quantitatively since this value is only by a factor 5/3 higher than the maximal effect allowed by the screening model. Unfortunately, I do not have an access to the paper of Jeverdan *et al* to find out the value of $cos(\Theta)$ in the experimental arrangement and whether there is indeed a decrease of the period as claimed in Wikipedia article. In [E34] two experiments supporting an effect $\Delta g/g = x\times 10^{-4}$, $x = 1.5$ or 2.6 but the sign of the effect is different in these experiments.

2. Allais reported an anomaly $\Delta g/g \sim 5\times 10^{-6}$ during 1954 eclipse [E6]. According to measurements by authors of [E34] the period of oscillation increases and one has $\Delta g/g \sim 5\times 10^{-6}$. Popescu and Olenici report a decrease of the oscillation period by $(\Delta g/g)cos(\Theta) \simeq 1.4\times 10^{-5}$.

3. In [E39] a *reduction* of vertical gravitational acceleration $\Delta g/g = (7.0\pm 2.7)\times 10^{-9}$ is reported: this is by a factor 10^{-5} smaller than the result of Jeverdan.

4. Small pressure waves with $\Delta p/p = 2\times 10^{-5}$ are registered by some micro-barometers [E6] and might relate to the effect since pressure gradient and gravitational acceleration should compensate each other. $\Delta g cos(\Theta)/g$ would be about 7 per cent of its maximum value for Earth-Sun system in this case. The knowledge of the sign of pressure variation would tell whether effective gravitational force is screened or amplified by Moon.

9.2.3 Allais Effect As Evidence For Large Values Of Gravitational Planck Constant?

One can represent rather general counter arguments against the models based on Z^0 conductivity and gravitational screening if one takes seriously the puzzling experimental findings concerning frequency change.

1. Allais effect identified as a rotation of oscillation plane seems to be established and seems to be present always and can be understood in terms of torque implying limiting oscillation plane.

2. During solar eclipses Allais effect however becomes much stronger. According to Olenici's experimental work the effect appears always when massive objects form collinear structures.

3. The behavior of the change of oscillation frequency seems puzzling. The sign of the frequency increment varies from experiment to experiment and its magnitude varies within five orders of magnitude.

What one an conclude about general pattern for $\Delta f/f$?

The above findings allow to make some important conclusions about the nature of Allais effect.

1. Some genuinely new dynamical effect should take place when the objects are collinear. If gravitational screening would cause the effect the frequency would always grow but this is not the case.

2. If stellar objects and also ring like dark matter structures possibly assignable to their orbits are Z^0 conductors, one obtains screening effect by polarization and for the ring like structure the resulting effectively 2-D dipole field behaves as $1/\rho^2$ so that there are hopes of obtaining large screening effects and if the Z^0 charge of pendulum is allow to have both signs, one might hope of being to able to explain the effect. It is however difficult to understand why this effect should become so strong in the collinear case.

3. The apparent randomness of the frequency change suggests that interference effect made possible by the gigantic value of gravitational Planck constant is in question. On the other hand, the dependence of $\Delta g/g$ on pendulum suggests a breaking of Equivalence Principle. It however turns out that the variation of the distances of the pendulum to Sun and Moon can explain the experimental findings since the pendulum turns out to act as a sensitive gravitational interferometer. An apparent breaking of Equivalence Principle could result if the effect is partially caused by genuine gauge forces, say dark classical Z^0 force, which can have arbitrarily long range in TGD Universe.

4. If topological light rays (MEs) provide a microscopic description for gravitation and other gauge interactions one can envision these interactions in terms of MEs extending from Sun/Moon radially to pendulum system. What comes in mind that in a collinear configuration the signals along S-P MEs and M-P MEs superpose linearly so that amplitudes are summed and interference terms give rise to an anomalous effect with a very sensitive dependence on the difference of S-P and M-P distances and possible other parameters of the problem. One can imagine several detailed variants of the mechanism. It is possible that signal from Sun combines with a signal from Earth and propagates along Moon-Earth ME or that the interferences of these signals occurs at Earth and pendulum.

5. Interference suggests macroscopic quantum effect in astrophysical length scales and thus gravitational Planck constants given by $\hbar_{gr} = GMm/v_0$, where $v_0 = 2^{-11}$ is the favored value, should appear in the model. Since $\hbar_{gr} = GMm/v_0$ depends on both masses this could give also a sensitive dependence on mass of the pendulum. One expects that the anomalous force is proportional to $\hbar_{gr}$ and is therefore gigantic as compared to the effect predicted for the ordinary value of Planck constant.

Model for interaction via gravitational MEs with large Planck constant

Restricting the consideration for simplicity only gravitational MEs, a concrete model for the situation would be as follows.

1. The picture based on topological light rays suggests that the gravitational force between two objects M and m has the following expression

$$\begin{aligned} F_{M,m} &= \frac{GMm}{r^2} = \int |S(\lambda,r)|^2 p(\lambda) d\lambda \\ p(\lambda) &= \frac{h_{gr}(M,m)2\pi}{\lambda} \;\; , \;\; \hbar_{gr} = \frac{GMm}{v_0(M,m)} \;\; . \end{aligned} \tag{9.2.11}$$

$p(\lambda)$ denotes the momentum of the gravitational wave propagating along ME. v_0 can depend on (M,m) pair. The interpretation is that $|S(\lambda,r)|^2$ gives the rate for the emission of gravitational waves propagating along ME connecting the masses, having wave length λ, and being absorbed by m at distance r.

2. Assume that $S(\lambda,r)$ has the decomposition

$$\begin{aligned} S(\lambda,r) &= R(\lambda) exp\left[i\Phi(\lambda)\right] \frac{exp\left[ik(\lambda)r\right]}{r} \;\; , \\ exp\left[ik(\lambda)r\right] &= exp\left[ip(\lambda)r/\hbar_{gr}(M,m)\right] \;\; , \\ R(\lambda) &= |S(\lambda,r)| \;\; . \end{aligned} \tag{9.2.12}$$

The phases $exp(i\Phi(\lambda))$ might be interpreted in terms of scattering matrix. The simplest assumption is $\Phi(\lambda) = 0$ turns out to be consistent with the experimental findings. The substitution of this expression to the above formula gives the condition

$$\int |R(\lambda)|^2 \frac{d\lambda}{\lambda} = v_0 \;\; . \tag{9.2.13}$$

Consider now a model for the Allais effect based on this picture.

1. In the non-collinear case one obtains just the standard Newtonian prediction for the net forces caused by Sun and Moon on the pendulum since $Z_{S,P}$ and $Z_{M,P}$ correspond to non-parallel MEs and there is no interference.
2. In the collinear case the interference takes place. If interference occurs for identical momenta, the interfering wavelengths are related by the condition

$$p(\lambda_{S,P}) = p(\lambda_{M,P} \;\; . \tag{9.2.14}$$

This gives

$$\frac{\lambda_{M,P}}{\lambda_{S,P}} = \frac{\hbar_{M,P}}{\hbar_{S,P}} = \frac{M_M}{M_S}\frac{v_0(S,P)}{v_0(M,P)} \;\; . \tag{9.2.15}$$

3. The net gravitational force is given by

$$\begin{aligned} F_{gr} &= \int |Z(\lambda, r_{S,P}) + Z(\lambda/x, r_{M,P})|^2 p(\lambda) d\lambda \\ &= F_{gr}(S,P) + F_{gr}(M,P) + \Delta F_{gr} \;\; , \\ \Delta F_{gr} &= 2\int Re\left[S(\lambda, r_{S,P})\overline{S}(\lambda/x, r_{M,P}))\right] \frac{\hbar_{gr}(S,P)2\pi}{\lambda} d\lambda \;\; , \\ x &= \frac{\hbar_{S,P}}{\hbar_{M,P}} = \frac{M_S}{M_M}\frac{v_0(M,P)}{v_0(S,P)} \;\; . \end{aligned} \tag{9.2.16}$$

Here $r_{M,P}$ is the distance between Moon and pendulum. The anomalous term ΔF_{gr} would be responsible for the Allais effect and change of the frequency of the oscillator.

4. The anomalous gravitational acceleration can be written explicitly as

$$\begin{aligned} \Delta a_{gr} &= 2\frac{GM_S}{r_Sr_M}\frac{1}{v_0(S,P)}\times I \ , \\ I &= \int R(\lambda)R(\lambda/x)cos\left[\Phi(\lambda)-\Phi(\lambda/x)+2\pi\frac{(y_Sr_S-xy_Mr_M)}{lambda}\right]\frac{d\lambda}{\lambda} \ , \\ y_M &= \frac{r_{M,P}}{r_M} \ , \ y_S=\frac{r_{S,P}}{r_S} \ . \end{aligned} \quad (9.2.17)$$

Here the parameter y_M (y_S) is used express the distance $r_{M,P}$ ($r_{S,P}$) between pendulum and Moon (Sun) in terms of the semi-major axis r_M (r_S)) of Moon's (Earth's) orbit. The interference term is sensitive to the ratio $2\pi(y_Sr_S-xy_Mr_M)/\lambda$. For short wave lengths the integral is expected to not give a considerable contribution so that the main contribution should come from long wave lengths. The gigantic value of gravitational Planck constant and its dependence on the masses implies that the anomalous force has correct form and can also be large enough.

5. If one poses no boundary conditions on MEs the full continuum of wavelengths is allowed. For very long wave lengths the sign of the cosine terms oscillates so that the value of the integral is very sensitive to the values of various parameters appearing in it. This could explain random looking outcome of experiments measuring $\Delta f/f$. One can also consider the possibility that MEs satisfy periodic boundary conditions so that only wave lengths $\lambda_n=2r_S/n$ are allowed: this implies $sin(2\pi y_Sr_S/\lambda)=0$. Assuming this, one can write the magnitude of the anomalous gravitational acceleration as

$$\begin{aligned} \Delta a_{gr} &= 2\frac{GM_S}{r_{S,P}r_{M,P}}\times\frac{1}{v_0(S,P)}\times I \ , \\ I &= \sum_{n=1}^{\infty}R(\frac{2r_{S,P}}{n})R(\frac{2r_{S,P}}{nx})(-1)^ncos\left[\Phi(n)-\Phi(xn)+n\pi\frac{xy_M}{y_S}\frac{r_M}{r_S}\right] \ . \end{aligned} \quad (9.2.18)$$

If $R(\lambda)$ decreases as λ^k, $k>0$, at short wavelengths, the dominating contribution corresponds to the lowest harmonics. In all terms except cosine terms one can approximate $r_{S,P}$ *resp.* $r_{M,P}$ with r_S *resp.* r_M.

6. The presence of the alternating sum gives hopes for explaining the strong dependence of the anomaly term on the experimental arrangement. The reason is that the value of xyr_M/r_S appearing in the argument of cosine is rather large:

$$\frac{xy_M}{y_S}\frac{r_M}{r_S}=\frac{y_M}{y_S}\frac{M_S}{M_M}\frac{r_M}{r_S}\frac{v_0(M,P)}{v_0(S,P)}\simeq 6.95671837\times 10^4\times\frac{y_M}{y_S}\times\frac{v_0(M,P)}{v_0(S,P)} \ .$$

The values of cosine terms are very sensitive to the exact value of the factor M_Sr_M/M_Mr_S and the above expression is probably not quite accurate value. As a consequence, the values and signs of the cosine terms are very sensitive to the values of y_M/y_S and $\frac{v_0(M,P)}{v_0(S,P)}$.
The value of y_M/y_S varies from experiment to experiment and this alone could explain the high variability of $\Delta f/f$. The experimental arrangement would act like interferometer measuring the distance ratio $r_{M,P}/r_{S,P}$. Hence it seems that the condition

$$\frac{v_0(S,P)}{v_0(M,P)} \neq const. \quad (9.2.19)$$

implying breaking of Equivalence Principle is not necessary to explain the variation of the sign of $\Delta f/f$ and one can assume $v_0(S,P)=v_0(M,P)\equiv v_0$. One can also assume $\Phi(n)=0$.

Scaling law

The assumption of the scaling law

$$R(\lambda) = R_0(\frac{\lambda}{\lambda_0})^k \tag{9.2.20}$$

is very natural in light of conformal invariance and masslessness of gravitons and allows to make the model more explicit. With the choice $\lambda_0 = r_S$ the anomaly term can be expressed in the form

$$\begin{aligned} \Delta a_{gr} &\simeq \frac{GM_S}{r_S r_M}\frac{2^{2k+1}}{v_0}(\frac{M_M}{M_S})^k R_0(S,P)R_0(M,P) \times \sum_{n=1}^{\infty}\frac{(-1)^n}{n^{2k}} cos\left[\Phi(n) - \Phi(xn) + n\pi K\right] \ , \\ K &= x \times \frac{r_M}{r_S} \times \frac{y_M}{y_S} \ . \end{aligned} \tag{9.2.21}$$

The normalization condition of Eq. 9.2.13 reads in this case as

$$R_0^2 = v_0 \times \frac{1}{2\pi\sum_n(\frac{1}{n})^{2k+1}} = \frac{v_0}{\pi\zeta(2k+1)} \ . \tag{9.2.22}$$

Note the shorthand $v_0(S/M,P) = v_0$. The anomalous gravitational acceleration is given by

$$\begin{aligned} \Delta a_{gr} &= \sqrt{\frac{v_0(M,P)}{v_0(S,P)}}\frac{GM_S}{r_S^2} \times XY \times \sum_{n=1}^{\infty}\frac{(-1)^n}{n^{2k}} cos\left[\Phi(n) - \Phi(xn) + n\pi K\right] \ , \\ X &= 2^{2k} \times \frac{r_S}{r_M} \times (\frac{M_M}{M_S})^k \ , \\ Y &= \frac{1}{\pi\sum_n(\frac{1}{n})^{2k+1}} = \frac{1}{\pi\zeta(2k+1)} \ . \end{aligned} \tag{9.2.23}$$

It is clear that a reasonable order of magnitude for the effect can be obtained if k is small enough and that this is essentially due to the gigantic value of gravitational Planck constant.

The simplest model consistent with experimental findings assumes $v_0(M,P) = v_0(S,P)$ and $\Phi(n) = 0$ and gives

$$\begin{aligned} \frac{\Delta a_{gr}}{gcos(\Theta)} &= \frac{GM_S}{r_S^2 g} \times XY \times \sum_{n=1}^{\infty}\frac{(-1)^n}{n^{2k}} cos(n\pi K) \ , \\ X &= 2^{2k} \times \frac{r_S}{r_M} \times (\frac{M_M}{M_S})^k \ , \\ Y &= \frac{1}{\pi\sum_n(\frac{1}{n})^{2k+1}} = \frac{1}{\pi\zeta(2k+1)} \ , \\ K &= x \times \frac{r_M}{r_S} \times \frac{y_M}{y_S} \ , \ x = \frac{M_S}{M_M} \ . \end{aligned} \tag{9.2.24}$$

Numerical estimates

To get a numerical grasp to the situation one can use $M_S/M_M \simeq 2.71 \times 10^7$, $r_S/r_M \simeq 389.1$, and $(M_S r_M/M_M r_S) \simeq 1.74 \times 10^4$. The overall order of magnitude of the effect would be

$$\begin{aligned} \frac{\Delta g}{g} &\sim XY \times \frac{GM_S}{R_S^2 g} cos(\Theta) \ , \\ \frac{GM_S}{R_S^2 g} &\simeq 6 \times 10^{-4} \ . \end{aligned} \tag{9.2.25}$$

The overall magnitude of the effect is determined by the factor XY.

k	1	1/2	1/4
$\frac{\Delta g}{gcos(\Theta)}$	1.1×10^{-9}	4.3×10^{-6}	1.97×10^{-4}

Table 9.2: Table gives overall magnitudes of the effect for $k = 1, 2/2$ and $1/4$ as predicted by the model.

1. For $k = 0$ the normalization factor is proportional to $1/\zeta(1)$ and diverges and it seems that this option cannot work.
2. **Table 9.2** gives the predicted overall magnitudes of the effect for $k = 1, 2/2$ and $1/4$.

   For $k = 1$ the effect is too small to explain even the findings of [E39] since there is also a kinematic reduction factor coming from $cos(\Theta)$. Therefore $k < 1$ suggesting fractal behavior is required. For $k = 1/2$ the effect is of same order of magnitude as observed by Allais. The alternating sum equals in a good approximation to -.693 for $y_S/y_M = 1$ so that it is not possible to explain the finding $\Delta f/f \simeq 5\times 10^{-4}$ of Jeverdan.
3. For $k = 1/4$ the expression for Δa_{gr} reads as

$$\begin{aligned} \frac{\Delta a_{gr}}{gcos(\Theta)} &\simeq 1.97\times 10^{-4}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^{1/2}}cos(n\pi K) \quad , \\ K &= \frac{y_M}{y_S}u \ , \ u = \frac{M_S}{M_M}\frac{r_M}{r_S} \simeq 6.95671837\times 10^4 \ . \end{aligned} \tag{9.2.26}$$

   The sensitivity of cosine terms to the precise value of y_M/y_S gives good hopes of explaining the strong variation of $\Delta f/f$ and also the findings of Jeverdan. Numerical experimentation indeed shows that the cosine sum alternates and increases as y_M/y_S increases in the range $[1,2]$.

The eccentricities of the orbits of Moon *resp.* Earth are $e_M = .0549$ *resp.* $e_E = .017$. Denoting semimajor and semiminor axes by a and b one has $\Delta = (a-b)/a = 1-\sqrt{1-e^2}$. $\Delta_M = 15\times 10^{-4}$ *resp.* $\Delta_E = 1.4\times 10^{-4}$ characterizes the variation of y_M *resp.* y_M due to the non-circularity of the orbits of Moon *resp.* Earth. The ratio $R_E/r_M = .0166$ characterizes the range of the variation of $\Delta y_M = \Delta r_{M,P}/r_M \leq R_E/r_M$ due to the variation of the position of the laboratory. All these numbers are large enough to imply large variation of the argument of cosine term even for $n = 1$ and the variation due to the position at the surface of Earth is especially large.

The duration of full eclipse is of order 8 minutes which corresponds to angle $\phi = \pi/90$ and at equator roughly to a $\Delta y_N = (\sqrt{r_M^2 + R_E^2 sin^2(\pi/90)} - r_M)/r_M \simeq (\pi/90)^2 R_E^2/2r_M^2 \simeq 1.7\times 10^{-7}$. Thus the change of argument of $n = 1$ cosine term during full eclipse is of order $\Delta\Phi = .012\pi$ at equator. The duration of the eclipse itself is of order two 2 hours giving $\Delta y_M \simeq 3.4\times 10^{-5}$ and the change $\Delta\Phi = 2.4\pi$ of the argument of $n = 1$ cosine term.

Other effects

There are also other strange effects involved.

1. One should explain also the recent finding by Popescu and Olenici, which they interpret as a quantization of the plane of oscillation of paraconical oscillator during solar eclipse [E50]. A possible TGD based explanation would be in terms of quantization of $\Delta\overline{g}$ and thus of the limiting oscillation plane. This quantization could reflect the quantization of the angular momentum of the dark gravitons decaying into bunches of ordinary gravitons and providing to the pendulum the angular momentum inducing the change of the oscillation plane. The knowledge of friction coefficients associated with the rotation of the oscillation plane would allow to deduce the value of the gravitational Planck constant if one assumes that each dark gravitons corresponds to its own approach to asymptotic oscillation plane. The flux would be reduced in a stepwise manner during the solar eclipse as the distance traversed by the flux through Moon increases and reduced in a similar manner after the maximum of the eclipse.

2. There is also evidence for the effect before and after the main eclipse [E50]. The time scale is 1 hour. A possible explanation is in terms of a dark matter ring analogous to rings of Jupiter surrounding Moon. From the average orbital velocity $v = 1.022$ km/s of the Moon one obtains that the distance traversed by moon during 1 hour is $R_1 = 3679$ km. The mean radius of moon is $R = 1737.10$ km so that one has $R_1 = 2R$ with 5 per cent accuracy ($2 \times R = 3474$ km). The Bohr quantization of the orbits of the inner planets discussed in [K79] with the value $\hbar_{gr} = GMm/v_0$ of the gravitational Planck constant predicts $r_n \propto n^2GM/v_0^2$ and gives the orbital radius of Mercury correctly for the principal quantum number $n = 3$ and $v_0/c = 4.6\times10^{-4} \simeq 2^{-11}$. From the proportionality $r_n \propto n^2GM/v_0^2$ one can deduce by scaling that in the case of Moon with $M(moon)/M(Sun) = 3.4 \times 10^{-8}$ the prediction for the radius of $n = 1$ Bohr orbit would be $r_1 = (M(Moon)/M(Sun)) \times R_M/9 \simeq .0238$ km for the same value of v_0. This is too small by a factor 6.45×10^{-6}. $r_1 = 3679$ km would require $n \sim 382$ or $n = n(Earth) = 5$ and $v_0(Moon)/v_0(Sun) \simeq 2^{-4}$.

9.2.4 Could Z^0 Force Be Present?

One can understand the experimental results without a breaking of Equivalence Principle if the pendulum acts as a quantum gravitational interferometer. One cannot exclude the possibility that there is also a dependence on pendulum. In this case one would have a breaking of Equivalence Principle, which could be tested using several penduli in the same experimental arrangement. The presence of Z^0 force could induce an apparent breaking of Equivalence Principle. The most plausible option is Z^0 MEs with large Planck constant. One can consider also an alternative purely classical option, which does not involve large values of Planck constant.

Could purely classical Z^0 force allow to understand the variation of $\Delta f/f$?

In the earlier model of the Allais effect (see the Appendix of [K9]) I proposed that the classical Z^0 force could be responsible for the effect. TGD indeed predicts that any object with gravitational mass must have non-vanishing em and Z^0 charges but leaves their magnitude and sign open.

1. If both Sun, Earth, and pendulum have Z^0 charges, one might even hope of understanding why the sign of the outcome of the experiment varies since he ratio of Z^0 charge to gravitational mass and even the sign of Z^0 charge of the pendulum might vary. Constant charge-to-mass ratio is of course the simplest hypothesis so that only an effective scaling of gravitational constant would be in question. A possible test is to use several penduli in the same experiment and find whether they give rise to same effect or not.

2. If Moon and Earth are Z^0 conductors, a Z^0 surface charge canceling the tangential component of Z^0 force at the surface of Earth is generated and affects the vertical component of the force experienced by the pendulum. The vertical component of Z^0 force is $2F_Zcos(\theta)$ and thus proportional to $cos(\Theta)$ as also the effective screening force below the shadow of Moon during solar eclipse. When Sun is in a vertical direction, the induced dipole contribution doubles the radial Z^0 force near surface and the effect due to the gravitational screening would be maximal. For Sun in horizon there would be no Z^0 force and gravitational tidal effect of Sun would vanish in the first order so that over all anomalous effect would be smallest possible: for a full screening $\Delta f/f \simeq \Delta g^2/4g^2 \simeq 4.5 \times 10^{-8}$ would be predicted. One might hope that the opposite sign of gravitational and Z^0 contributions could be enough to explain the varying sign of the overall effect.

3. It seems necessary to have a screening effect associated with gravitational force in order to understand the rapid variation of the effect during the eclipse. The fact that the maximum effect corresponds to a maximum gravitational screening suggests that it is present and determines the general scale of variation for the effect. If the maximal Z^0 charge of the pendulum is such that Z^0 force is of the same order of magnitude as the maximal screening of the gravitational force and of opposite sign (that is attractive), one could perhaps understand the varying sign of the effect but the effect would develop continuously and begin before the main eclipse. If the sign of Z^0 charge of pendulum can vary, there is no difficulty in explaining the varying sign of the effect. An interesting possibility is that Moon, Sun and Earth have dark matter halos so that also gravitational screening could begin before the eclipse. The real test for the

effect would come from tidal effects unless one can guarantee that the pendulum is Z^0 neutral or its Z^0 charge/mass ratio is always the same.

4. As noticed also by Allais, Newtonian theory does not give a satisfactory account of the tidal forces and there is possibility that tides give a quantitative grasp on situation. If Earth is Z^0 conductor tidal effects should be determined mainly by the gravitational force and modified by its screening whereas Z^0 force would contribute mainly to the pressure waves accompanying the shadows of Moon and Sun. The sign and magnitude of pressure waves below Sun and Moon could give a quantitative grasp of Z^0 forces of Sun and Moon. Z^0 surface charge would have opposite signs at the opposite sides of Earth along the line connecting Earth to Moon *resp.* Sun and depending on sign of Z^0 force the screening and Z^0 force would tend to amplify or cancel the net anomalous effect on pressure.

5. A strong counter argument against the model based on Z^0 force is that collinear configurations are reached in continuous manner from non-collinear ones in the case of Z^0 force and the fact that gravitational screening does not conform with the varying sign of the discontinuous effect occurring during the eclipse. It would seem that the effect in question is more general than screening and perhaps more like quantum mechanical interference effect in astrophysical length scale.

Could Z^0 MEs with large Planck constant be present?

The previous line of arguments for gravitational MEs generalizes in a straightforward manner to the case of Z^0 force. Generalizing the expression for the gravitational Planck constant one has $\hbar_{Z^0} = g_Z^2 Q_Z(M) Q_Z(m)/v_0$. Assuming proportionality of Z^0 charge to gravitational mass one obtains formally similar expression for the Z^0 force as in previous case. If Q_Z/M ratio is constant, Equivalence Principle holds true for the effective gravitational interaction if the sign of Z^0 charge is fixed. The breaking of Equivalence Principle would come naturally from the non-constancy of the $v_0(S,P)/v_0(M,P)$ ratio also in the recent case. The variation of the sign of $\Delta f/f$ would be explained in a trivial manner by the variation of the sign of Z^0 charge of pendulum but this explanation is not favored by Occam's razor.

9.3 Gravimagnetism And TGD

Gravimagnetism is one of the predictions of GRT which is being tested experimentally. TGD predicts deviations from the predictions of GRT which unfortunately are not seen in the satellite experiment to be discussed below. The claimed discovery of gravimagnetic effect in super-conductors having strength 20 orders of magnitude larger than predicted by GRT raises the question whether TGD might explain the effect.

9.3.1 Gravity Probe B And TGD

Gravity Probe B experiment tests the predictions o General Relativity related to gravimagnetism. Only the abstract [E24] of the talk C. W. Francis Everitt summarizing the results is available when I am writing this.

The NASA Gravity Probe B (GP-B) orbiting gyroscope test of General Relativity, launched from Vandenberg Air Force Base on 20 April, 2004, tests two consequences of Einstein's theory: 1) the predicted 6.6 arc-s/year geodetic effect due to the motion of the gyroscope through the curved space-time around the Earth; 2) the predicted 0.041 arc-s/year frame-dragging effect due to the rotating Earth. The mission has required the development of cryogenic gyroscopes with drift-rates 7 orders of magnitude better than the best inertial navigation gyroscopes. These and other essential technologies, for an instrument which once launched must work perfectly, have come into being as the result of an intensive collaboration between Stanford physicists and engineers, NASA and industry. GP-B entered its science phase on August 27, 2004 and completed data collection on September 29, 2005. Analysis of the data has been in continuing progress during and since the mission. This paper will describe the main features and challenges of the experiment and announce the first results.

The article [E20] gives an excellent summary of various test of GRT. The predictions tested by GP-B relate to gravimagnetic effects. The field equations of general relativity in post-Newtonian approximation with a choice of a preferred frame can in good approximation ($g_{ij} = -\delta_{ij}$) be written in a form highly reminiscent of Maxwell's questions with g_{tt} component of metric defining the counterpart of the scalar potential giving rise to gravito-electric field and g_{ti} the counterpart of vector potential giving rise to the gravimagnetic field.

Rotating body generates a gravimagnetic field so that bodies moving in the gravimagnetic field of a rotating body experience the analog of Lorentz force and gyroscope suffers a precession similar to that suffered by a magnetic dipole in magnetic field (Thirring-Lense effect or frame-drag). Besides this there is geodetic precession due to the motion of the gyroscope in the gravito-electric field present even in the case of non-rotating source which might be perhaps understood in terms of gravito-Faraday law. Both these effects are tested by GP-B.

In the following I represent some general comments about how TGD and GRT differs and also say something about the predictions of TGD concerning GP-B experiment.

TGD and GRT

Consider first basic differences between TGD and GRT.

1. In TGD local Lorentz invariance is replaced by exact Poincare invariance at the level of the embedding space $H = M^4 \times CP_2$. Hence one can use unique global Minkowski coordinates for the space-time sheets and gets rids of the problems related to the physical identification of the preferred coordinate system.

2. General coordinate invariance holds true in both TGD and GRT.

3. The basic difference between GRT and TGD is that in TGD framework gravitational field is induced from the metric of the embedding space. One important cosmological implication is that the embeddings of the Robertson-Walker metric for which the gravitational mass density is critical or overcritical fail after some value of cosmic time. Also classical gauge potentials are induced from the spinor connection of H so that the geometrization applies to all classical fields. Very strong constraints between fundamental interactions at the classical level are implied since CP_2 are the fundamental dynamical variables at the level of macroscopic space-time.

4. Equivalence Principle (EP) holds in classical sense for the GRT limit of TGD understood as effective theory with effective space-time in long length scales defined as M^4 endowed with an effective metric defined as the sum of Minkowski metric and sum of deviations from Minkowski metric for various space-time sheets involved. Thus GRT space-time lumps together the space-time sheets of many-sheeted spacetime and there is always a length scale resolution involved. EP reduces to Einstein's equations and reflects underlying Poincare invariance.

   In Zero Energy Ontology (ZEO) zero energy states and corresponding space-time surfaces reside always inside some causal diamond (CD) characterized by scale. Therefore the conserved four-momentum assingable to either end of CD is scale dependent quantity, and the apparent non-conservation of four-momentum as scale is changed is not in conflict with Poincare invariance.

   At the quantum level EP would hold true in the sense that classical Noether charges in the Cartan algebra of isometries and associated with Kähler action are equal to the eigenvalues of the quantal charges of Kähler-Dirac action: EP would reduce to Quantum classical correspondence. Holography allows to consider also the possibility that gravitational and inertial charges correspond to those assignable to light-like and space-like 3-surfaces respectively.

TGD and GP-B

There are excellent reasons to expect that Maxwellian picture holds true in a good accuracy if one uses Minkowski coordinates for the space-time surface. In fact, TGD allows a static solutions with 2-D CP_2 projection for which the prerequisites of the Maxwellian interpretation are satisfied (the deviations of the spatial components g_{ij} of the induced metric from $-\delta_{ij}$ are negligible).

Schwartschild and Reissner-Norström metrics allow embeddings as 4-D surfaces in H but Kerr metric [E7] assigned to rotating systems probably not. If this is indeed the case, the gravimagnetic field of a rotating object in TGD Universe cannot be identical with the exact prediction of GRT but could be so in the Post-Newtonian approximation. Scalar and vector potential correspond to four field quantities and the number of CP_2 coordinates is four. Embedding as vacuum extremals with 2-D CP_2 projection guarantees automatically the consistency with the field equations but requires the orthogonality of gravito-electric and -magnetic fields. This holds true in post-Newtonian approximation in the situation considered.

Hence apart from restrictions caused by the failure of the global embedding at short distances it *might* be possible to imbed Post-Newtonian approximations into H in the approximation $g_{ij} = -\delta_{ij}$. If so, the predictions for Thirring-Lense effect would not differ measurably from those of GRT. The predictions for the geodesic precession involving only scalar potential would be identical in any case.

The imbeddability in the post-Newtonian approximation is however questionable if one assumes vacuum extremal property and small deformations of Schwartschild metric indeed predict a gravimagnetic field differing from the dipole form.

1. Simplest candidate for the metric of a rotating star

The simplest situation for the metric of rotating start is obtained by assuming that vacuum extremal imbeddable to $M^4 \times S^2$, where S^2 is the geodesic sphere of CP_2 with vanishing homological charge and induce Kähler form. Use coordinates (Θ, Φ) for S^2 and spherical coordinates (t, r, θ, ϕ) in space-time identifiable as M^4 spherical coordinates apart from scaling and r-dependent shift in the time coordinate.

1. For Schwartschild metric one has

$$\Phi = \omega t \ , \ sin(\Theta) = f(r) \ . \tag{9.3.1}$$

   f is fixed highly uniquely by the embedding of Schwartschild metric and asymptotically one must have

$$f = f_0 + \frac{C}{r}$$

   in order to obtain $g_{tt} = 1 - 2GM/r$ $(\equiv 1 + \Phi_{gr})$ behavior for the induced metric.

2. The small deformation giving rise to the gravimagnetic field and metric of rotating star is given by

$$\Phi = \omega t + n\phi \tag{9.3.2}$$

   There is obvious analogy with the phase of Schödinger amplitude for angular momentum eigenstate with $L_z = n$ which conforms with the quantum classical correspondence.

3. The non-vanishing component of A^g is proportional to gravitational potential Φ_{gr}

$$A^g_\phi = g_{t\phi} = (n/\omega)\Phi_{gr} \ . \tag{9.3.3}$$

4. A little calculation gives for the magnitude of B^θ_g from the curl of A^g the expression

$$B^\theta_g = \frac{n}{\omega} \times \frac{1}{sin(\theta)} \times \frac{2GM}{r^3} \ . \tag{9.3.4}$$

   In the plane $\theta = \pi/2$ one has dipole field and the value of n is fixed by the value of angular momentum of star.

5. Quantization of angular momentum is obtained for a given value of ω. This becomes clear by comparing the field with dipole field in $\theta = \pi/2$ plane. Note that GJ, where J is angular momentum, takes the role of magnetic moment in B_g [E20] appears as a free parameter analogous to energy in the embedding and means that the unit of angular momentum varies. In TGD framework this could be interpreted in terms of dynamical Planck constant having in the most general case any rational value but having a spectrum of number theoretically preferred values. Dark matter is interpreted as phases with large value of Planck constant which means possibility of macroscopic quantum coherence even in astrophysical length scales. Dark matter would induce quantum like effects on visible matter. For instance, the periodicity of small n states might be visible as patterns of visible matter with discrete rotational symmetry.

2. Comparison with the dipole field

The simplest candidate for the gravimagnetic field differs in many respects from a dipole field.

1. Gravitomagnetic field has $1/r^3$ dependence so that the distance dependence is same as in GRT.
2. Gravitomagnetic flux flows along z-axis in opposite directions at different sides of $z = 0$ plane and emanates from z-axis radially and flows along spherical surface. Hence the radial component of B_g would vanish whereas for the dipole field it would be proportional to $cos(\theta)$.
3. The dependence on the angle θ of spherical coordinates is $1/sin(\theta)$ (this conforms with the radial flux from z-axis whereas for the dipole field the magnitude of B_g^θ has the dependence $sin(\theta)$. At $z = 0$ plane the magnitude and direction coincide with those of the dipole field so that satellites moving at the gravimagnetic equator would not distinguish between GRT and TGD since also the radial component of B_g vanishes here.
4. For other orbits effects would be non-trivial and in the vicinity of the flux tube formally arbitrarily large effects are predicted because of $1/sin(\theta)$ behavior whereas GRT predicts $sin(\theta)$ behavior. Therefore TGD could be tested using satellites near gravito-magnetic North pole.
5. The strong gravimagnetic field near poles causes gravi-magnetic Lorentz force and could be responsible for the formation of jets emanating from black hole like structures. This additional force might have also played some role in the formation of planetary systems and the plane in which planets move might correspond to the plane $\theta = \pi/2$ where gravimagnetic force has no component orthogonal to the plane. Same applies in the case of galaxies.

3. Consistency with the model for the asymptotic state of star

In TGD framework natural candidates for the asymptotic states of the star are solutions of field equations for which gravitational four-momentum is locally conserved. Vacuum extremals must therefore satisfy the field equations resulting from the variation of Einstein's action (possibly with cosmological constant) with respect to the induced metric. Quite remarkably, the solution representing asymptotic state of the star is necessarily rotating.

The proposed picture is consistent with the model of the asymptotic state of star. Also the magnetic parts of ordinary gauge fields have essentially similar behavior. This is actually obvious since CP_2 coordinates are fundamental dynamical variables and the field line topologies of induced gauge fields and induced metric are therefore very closely related.

As already discussed, the physicists M. Tajmar and C. J. Matos and their collaborators working in ESA (European Satellite Agency) have made an amazing claim of having detected strong gravimagnetism with gravimagnetic field having a magnitude which is about 20 orders of magnitude higher than predicted by General Relativity [E57, E37, E38]. Hence there are some reasons to think that gravimagnetic fields might have a surprise in store.

When I am writing this (day later than what is above I have learned that the error bars for the frame-dragging effect are still twice the size of the effect as predicted by GRT. Already this information would have killed TGD inspired model unless the satellite would have been at the equator.

9.3.2 Does Horizon Correspond To A Degenerate Four-Metric For The Rotating Counterpart Of Schwartshild Metric?

The metric determinant at Schwartschild radius is non-vanishing. This does not quite conform with the interpretation as an analog of a light-like partonic 3-surface identifiable as a wormhole throat for which the determinant of the induced 4-metric vanishes and at which the signature of the induced metric changes from Minkowskian to Euclidian.

An interesting question is what happens if one makes the vacuum extremal representing embedding of Schwartshild metric a rotating solution by a very simple replacement $\Phi \rightarrow \Phi + n\phi$, where Φ is the angle angle coordinate of homologically trivial geodesic sphere S^2 for the simplest vacuum extremals, and ϕ the angle coordinate of M^4 spherical coordinates. It turns out that Schwartschild horizon is transformed to a surface at which $det(g_4)$ vanishes so that the interpretation as a wormhole throat makes sense.

The modification implies that the components $g_{t\phi}$ and $g_{r\phi}$ of the Schwartschild metric become non-vanishing and $g_{\phi\phi}$ component receives a small modification. Using the notations of the subsection *"Embedding of Reissner-Nordström metric"* , one has

$$\begin{aligned} g_{t\phi} &= \omega_1 n \times \frac{R^2}{4} s^{eff}_{\phi\phi} \ , \\ g_{r\phi} &= \partial_{r_M} f n \times \frac{R^2}{4} s^{eff}_{\phi\phi} \ , \\ \Delta g_{\phi\phi} &= n^2 \times \frac{R^2}{4} s^{eff}_{\phi\phi} \ . \end{aligned} \tag{9.3.5}$$

It is easy to see that $g_{r\phi}/g_{t\phi}$ is of order $\sqrt{r_S/r}$, $r_S = 2GM$, so that in an excellent approximation $g_{r\phi} = 0$ holds true at large distances and previous considerations related to gravimagnetic fields remain true.

The vanishing of the 4-D metric determinant reduces to that for 3-D metric determinant $det(g_3)$ associated with (t, r, ϕ). In the case of the Scwartshild metric this determinant is given by

$$\begin{aligned} det(g_3) &= -g_{\phi\phi} - A g^2_{r\phi} + \frac{g^2_{t\phi}}{A} \ , \\ A &= 1 - \frac{2GM}{r} \equiv 1 - u \ . \end{aligned} \tag{9.3.6}$$

Since A changes sign at Schwartchild radius $r_s = 2GM$, the determinant can indeed vanish near r_s. In a good approximation can neglect the contribution of $g_{r\phi}$ in the equation and put $r = r_S$ in the slowly varying functions. This gives

$$\frac{R^2}{4} \omega_1^2 s^{eff}_{\phi\phi} \simeq \lambda^2 \tag{9.3.7}$$

from the condition $u = r_S/r = 1$ applied to the induced metric. This gives

$$\begin{aligned} g_{\phi\phi} &\simeq -r_S^2 sin^2(\theta) - \frac{n^2}{\omega^2} \lambda^2 \ , \\ g_{t\phi} &\simeq -\frac{n}{\omega} \lambda^2 \ . \end{aligned} \tag{9.3.8}$$

The singular surface for which $det(g_3)$ vanishes satisfies the approximate equation

$$u - 1 = \frac{g^2_{t\phi}}{g_{\phi\phi}} (r = r_S) = \frac{n^2 \lambda^4}{\omega^2 r_S^2 sin^2(\theta) - n^2 \lambda^2} \ . \tag{9.3.9}$$

Since the left hand side can have both signs, the solution certainly exists but it can happen that part of it is inside and part outside Schwartshild radius.

$\theta = 0$ allows solution only for $r > r_S$: hence some portion of the surface is always outside r_S. If the condition

$$\lambda^2 > \frac{\omega^2 r_S^2}{n^2} \tag{9.3.10}$$

is satisfied, the surface belongs belongs as a whole to the region $r > r_S$. The singular surface has a cigar like shape approaching sphere $r = \lambda^2 r_S$, $\lambda > 1$ at large quantum number limit $n \to \infty$. For $n = 0$ no solution is obtained. If one assumes that black hole horizon is analogous to a wormhole contact, only rotating black hole like structures with quantized angular momentum are possible in TGD Universe.

9.3.3 Has Strong Gravimagnetism Been Observed?

Physicists M. Tajmar and C. J. Matos and their collaborators working in ESA (European Satellite Agency) have made an amazing claim of having detected strong gravimagnetism with gravimagnetic field having a magnitude which is about 20 orders of magnitude higher than predicted by General Relativity [E37]. If the findings are replicable they mean a revolution in the science of gravity and, as one might hope, force a long-waited serious reconsideration of the basic assumptions of the dominating super-string approach.

The starting point of the theory is the so called Thomson magnetic moment generated in rotating charged super-conductors adding a constant contribution to the exponentially damped Meissner contribution to the magnetic field. This contribution can be understood as being due to the massivation of photons in super-conductors. The modified Maxwell equations are obtained by just adding scalar potential mass term to Gauss law and vector potential mass term to the equation related the curl of the magnetic field to the em current.

The expression for the Thomson magnetic field is given by

$$B = 2\omega_R n_s \times \lambda_\gamma^2 \ , \tag{9.3.11}$$

where ω_R is the angular velocity of superconductor, n_s is charge density of super-conducting particles and $\lambda_\gamma = \hbar/m_\gamma$ is the wave length of a massive photon at rest. In the case of ordinary superconductor one has $\lambda_\gamma = \sqrt{m^*/q^* n_s}$, where $m^* \simeq 2m_e$ and $q^* = -2e$ are the mass and charge of Cooper pair. Hence one has

$$B = -2\frac{m^*}{2e}\omega_R \ . \tag{9.3.12}$$

Magnetic field extends also outside the super-conductor and by measuring it with a sufficient accuracy outside the super-conductor one can determine the value of the electron mass. Instead of the theoretical value $m^*/2m_e = .999992$ which is smaller than one due to the binding energy of the Cooper pair the value $m^*/2m_e = 1.000084$ was found by Tate [E38]. This inspired the theoretical work generalizing the notion of Thomson field to gravimagnetism and the attempt to explain the anomaly in terms of the effects caused by the gravimagnetic field.

Note that in the case of ordinary matter the equations would lead to an inconsistency at the limit $m_\gamma = 0$ since the value of Thomson magnetic field would become infinite. The resolution of the problem proposed in [E57] is based on the replacement of rotation frequency ω with electron's spin precession frequency $\omega_L = -eB/2m$ so that the consistency equation becomes $B = -B = 0$ for a unique choice $1/\lambda_\gamma^2 = -\frac{q}{m}n$. One could also consider the replacement of ω with electron's cyclotron frequency $\omega_c = 2\omega_L$. To my opinion there is no need to assume that the modified Maxwell's equations hold true in the case of ordinary matter.

Gravimagnetic field

The perturbative approach to the Einstein equations leads to equations which are essentially identical with Maxwell's equations. The g_{tt} component of the metric plays the role of scalar potential and the components g_{ti} define gravitational vector potential. Also the generalization to the superconducting situation in which graviphotons develop a mass is straightforward. Just add the scalar potential mass term to the counterpart of Gauss law and vector potential mass term to the equation relating the curl of the gravimagnetic field to the gravitational mass current.

In the case of a rotating superconductor Thomson magnetic moment is replaced with its gravimagnetic counterpart

$$B_{gr} = -2\omega_R\rho_m\lambda_g^2 \ . \tag{9.3.13}$$

Obviously this formula would give rise to huge gravimagnetic fields in ordinary matter approaching infinite values at the limit of vanishing gravitational mass. Needless to say, these kind of fields have not been observed.

Equivalence Principle however suggests that the gravimagnetic field must be assigned with the rotating coordinate frame of the super-conductor. Equivalence principle would state that seing the things in a rotating reference frame is equivalent of being in a gravimagnetic field $B_{gr} = -2\omega_R$ in the rest frame. This fixes the graviphoton mass to

$$\frac{1}{\lambda_{gr}^2} = (\frac{m_{gr}}{\hbar})^2 = G\rho_m \ . \tag{9.3.14}$$

For a typical condensed matter density parameterized as $\rho_m = Nm_p/a^3$, $a = 10^{-10}$ m this gives the order of magnitude estimate $m_{gr} \sim N^{1/2}10^{-21}/a$ so that graviton mass would be extremely small.

If this is all what is involved, gravimagnetic field should have no special effects. In [E57] it is however proposed that in superconductors a small breaking of Equivalence Principle occurs. The basic assumptions are following.

1. Super-conducting phase and the entire system obey separately their gravitational analogs of Maxwell field equations.
2. The ad hoc assumption is that for super-conducting phase the sign of the gravimagnetic field is opposite to that for the ordinary matter. If purely kinematic effect were in question so that graviphotons were pure gauge degrees of freedom, the value of m_{gr}^2 should should be proportional to ρ_m and $\rho_m - \rho_m^*$ respectively.
3. Graviphoton mass is same for both ordinary and super-conducting matter and corresponds to the net density ρ_m of matter. This is essential for obtaining the breaking of Equivalence Principle.

With these assumptions the gravimagnetic field giving rise to acceleration field detected in the rest system would be given by

$$B_{gr}^* = \frac{\rho_m^*}{\rho} \times 2\omega \tag{9.3.15}$$

This is claimed to give rise to a genuine acceleration field

$$g^* = -\frac{\rho_m^*}{\rho}a \tag{9.3.16}$$

where a is the radial acceleration due to the rotational motion.

Explanation for the too high value of measured electron mass in terms of gravimagnetic field

A possible explanation of the anomalous value of the measured electron mass [E38] is in terms of gravimagnetic field affecting the flux Bohr quantization condition for electrons by adding to the electromagnetic vector potential term q^*A_{em} gravitational vector potential m^*A_{gr}. By requiring that the quantization condition

$$\oint (m^*v + q^*A_{em} + m^*A_{gr})dl = 0 \quad (9.3.17)$$

is satisfied for the superconducting ring, one obtains

$$B = -\frac{2m}{e}\omega - \frac{m}{e}B_{gr} \ . \quad (9.3.18)$$

This means that the magnetic field is slightly stronger than predicted and it has been known that this is indeed the case experimentally.

The higher value of the magnetic field could explain the slightly too high value of electron mass as determined from the magnetic field. This gives

$$B_{gr} = \frac{\Delta m_e}{m_e} \times 2\omega = \frac{\Delta m_e}{m_e} \times em_e \times B \ . \quad (9.3.19)$$

The measurement implies $\Delta m_e/m_e = 9.2\times10^{-5}$. The model discussed in [E57] predicts $\Delta m_e/m_e \sim \rho^*/\rho$. The prediction is about 23 times smaller than the experimental result.

9.3.4 Is The Large Gravimagnetic Field Possible In TGD Framework?

TGD allows top consider several alternative solutions for the claimed effect.

1. TGD predicts the possibility of classical electro-weak fields at larger space-time sheets. If these couple to Cooper pairs generate exotic weak charge at super-conducting space-time sheets the Bohr quantization conditions modify the value of the magnetic field. Exotic weak charge would however mean also exotic electronic em charge so that this option is excluded. It would also require that the Z^0 charge of test bodies used to measure the acceleration field is proportional to their gravitational mass.
2. According to the simplest recent view about Kähler-Dirac action [K104] the modes of Dirac operator are confined to 2-D string world sheets at which classical W boson fields vanish. This guarantees that em charge is well-defined for the modes. The stronger condition that also classical Z^0 field vanishes makes also sense and should hold at least in the length scales in which weak bosons do not appear. This guarantees the absence of axial couplings and parity breaking effects. In living matter parity breaking effects are large and one could consider the possibility that weak length scale is scaled up for $h_{eff} > h$ and that classical Z^0 fields are present below the weak scale.
3. One cannot exclude the possibility that the classical weak fields vanish for entire space-time surface. In this case spinor modes can still be seen as continuous superpositions of 2-D ones.In principle one can consider also other options - such as vanishing of induced Kähler form or classical em field besides that of W fields.

The conservative option is that classical weak fields vanish in the situation considered so that there is room for the strong gravimagnetic field.

1. The formula used by Tajmar *et al* [E57] for the gravimagnetic variant of Thomson magnetic field is direct generalization for the Thomson field for ordinary super-conductor. The gravimagnetic field is proportional to the product $B_g = \omega_R r^2$ of the rotation frequency ω_R of super-conductor and square of the ratio $r = (\lambda_g/\lambda_{g,T})$ where $\lambda_g = \hbar/m_g$ is graviton wave length and $\lambda_{g,T}$ is gravimagnetic peneration length obtained as generalization of the magnetic

penetration length for super-conductors by replacing charge with mass. The latter is purely classical quantity whereas graviton wave length depends on Planck constant. Graviton mass can be argued to result in gravitational Meissner effect and can be estimated from the value of cosmological constant Λ being essentially its square root. The resulting value of B_g is too small by 28 orders of magnitude.

2. Tajmar *et al* [E57] suggests that graviton mass is larger by a factor of order 10^{14} in conflict with the experimental upper bound of order 10^{55} kg for m_g. TGD proposal is that it is Planck constant which should be replaced with effective Planck constant $h_{eff} = nh$ equal to gravitational Planck constant h_{gr} for electron Cooper pair in Earth's gravitational field. The model for planetary orbits as Bohr orbits together with Equivalence Principle implies $\hbar_{gr} = GMm/v_0$ at flux tubes connecting particle with mass m to Sun with mass M. v_0 has dimensions of velocity and has order of magnitude correlating with a typical rotation velocity of planetary orbit by Bohr quantization rules.

3. In the recent case the rotation velocity v_0 is the rotation velocity of Earth at its surface: $v_0(E)/c = 2.16 \times 10^{-6}$ to be compared with $v_0(S)/c \simeq .5 \times 10^{-3}$ for Sun-Earth system. The scaling of λ_g is given by $h_{gr}(E, pair)/h = (h_{gr,S,pair}/h) \times (M_E/M_S) \times v_0(S)/v_0(E)$. This gives

$$r \equiv \frac{h_{gr,S,pair}}{h} = \frac{\lambda(h_{gr,S,pair}}{\lambda(h, pair)} = \frac{\frac{GM}{v_0(S)}}{\lambda_c(pair)} = \frac{\frac{r_S}{v_0(S)}}{\lambda_c(e)} \ .$$

Using $r_S = 3km$ and $\lambda_e = .243 \times 10^{-12}$ m and $v_0(S) \simeq 2^{-11}$, $M_E/M_S = 3.0 \times 10^{-6}$ one obtains $r \simeq 3.6 \times 10^{14}$. This happens to be correct order of magnitude! Maybe the model might have something to do with reality. Even better, also the value of h_{eff} is consistent with its value spectrum appearing in EEG if one requires that the energy of dark EEG photon with frequency of order 10 Hz is that of biophoton with frequency of about 5×10^14 Hz. If this picture is correct the values of $h_{eff} = h_{gr}$ would comes as proportional to the masses of particles and cyclotron energies proportional to heB/m would not depend on the mass of the particle at all.

4. What is nice that the model unifies the notions of gravitational Planck constant and dark Planck constant. The basic observation is that Equivalence Principle allows to understand the effects of h_{gr} by reducing it to elementary particle level interpreted in terms of flux tubes connecting particle to the bigger system. This allows to avoid gigantic values of h_{gr} and gives connection with TGD inspired quantum biology. The new quantum physics associated with gravitation would also become key part of quantum biology.

9.4 Some Differences Between GRT And TGD

In the following some effects possibly differentiating between GRT and TGD are discussed.

9.4.1 Do Neutrinos Travel With Superluminal Speed?

The newest particle physics rumour has been that the CERN OPERA (see `http://tinyurl.com/676bxl8`) team working in Gran Sasso, Italy has reported 6.1 sigma evidence that neutrinos move with a super-luminal speed. The total travel time is measured in milliseconds and the deviation from the speed of the light is nanoseconds meaning $\Delta c/c \simeq 10^{-6}$ which is roughly 10^3 times larger than the uncertainty 4.5×10^{-9} in the measured value of the speed of light (see `http://tinyurl.com/3wdq4`). If the result is true it means a revolution in the fundamental physics. There is now an article by OPERA collaboration [H2] in arXiv (see `http://tinyurl.com/3h44vxw`) so that superluminal neutrinos are not a rumour anymore. Even the finnish tabloid "Iltalehti" reacted to the news and this is really something unheard! Maybe the finding could even stimulate colloquium in physics department of Helsinki University!

The superluminal speed of neutrino has stimulated intense email debates and blog discussions. The reactions to the potential discovery depend on whether the person can imagine some explanation for the finding or not. In the latter case the reaction is denial: most physics bloggers have chosen this option for understandable reasons. What else could they do? Personally I cannot take tachyonic neutrinos seriously but I would not however choose the easy option and argue that the result is due to a bad experimentation as Lubos Motl (see `http://tinyurl.com/3erbfg5`) and Jester (see `http://tinyurl.com/3dll93`) do. The six sigma statistics does not leave much room for objections but there could of course be some very delicate systematical error involved. Lubos Motl (see `http://tinyurl.com/3meymp`) wrote quite an interesting piece about possible errors of this kind and classified the possible errors to timing errors either at CERN or Italy or to errors in distance measurement.

Basic data

The neutrinos used are highly relativistic having average energy 17 GeV much larger than the mass scale of neutrinos of order.1 eV. The distance between CERN and Gran Sasso is roughly 750 km, which corresponds to the time of travel equal to $T = 2.4$ milliseconds. The nasty neutrinos arrived to Gran Sasso $\Delta T = 60.7 \pm 6.9$ (statistical) ± 7.4 (systematic) ns before they should have done so. This time corresponds to a distance $\Delta L = 18$ m. From this is is clear that the distance and timing measurements must be extremely accurate. The claimed distance precision is 20 cm [H2] (see `http://tinyurl.com/3h44vxw`).

Experimentalists tell that they have searched for al possible systematic errors that they are able to imagine. The relative deviation of neutrino speed from the speed of light is

$$\frac{c - v)}{v} = (5.1 \pm 2.9) \times 10^{-5} \quad ,$$

which is much larger than the uncertainty related to the value of the speed of light. The effect does not depend on neutrino energy. 6.1 sigma result is in question so that it can be a statistical fluctuation with probability of 10^{-9} in the case that there is no systematic error.

The result is not the first of this kind and the often proposed interpretation is that neutrinos behave like tachyons. The following is the abstract (see `http://tinyurl.com/ycto3v2z`) of the article [H7] giving a summary about the earlier evidence that neutrinos can move faster than the speed of light.

From a mathematical point of view velocities can be larger than c. It has been shown that Lorentz transformations are easily extended in Minkowski space to address velocities beyond the speed of light. Energy and momentum conservation fixes the relation between masses and velocities larger than c, leading to the possible observation of negative mass squared particles from a standard reference frame. Current data on neutrino mass squared yield negative values, making neutrinos as possible candidates for having speed larger than c. In this paper, an original analysis of the SN1987A supernova data is proposed. It is shown that all the data measured in "87" by all the experiments are consistent with a description of neutrinos as combination of superluminal mass eigenstates. The well known enigma on the arrival times of the neutrino bursts detected at LSD, several hours earlier than at IMB, K2 and Baksan, is explained naturally. It is concluded that experimental evidence for superluminal neutrinos was recorded since the SN1987A explosion, and that data are quantitatively consistent with the introduction of tachyons in Einstein's equation.

TGD inspired model

This kind of effect is actually one of the basic predictions of TGD reflecting the differences between kinematics of relativities based on a view about space-time as abstract manifold and TGD in which one has sub-manifold gravitation and emerged for more than 20 years ago. Also several Hubble constants are predicted and explanation for why the distance between Earth and Moon seems to increasing as an apparent phenomenon emerges. There are many other strange phenomena which find an explanation [K99, K80, K79].

It is sub-manifold geometry which allows to fuse the good aspects of both special relativity (the existence of well-defined conserved quantities due to the isometries of embedding space) and general relativity (geometrization of gravitation in terms of the induced metric). As an additional bonus one obtains a geometrization of the electro-weak and color interactions and of standard model quantum numbers. The choice of the embedding space is unique. The new element is the generalization of the notion of space-time: space-time identified as a four-surface has shape as seen from the perspective of the embedding space $M^4 \times CP_2$. The study of field equations leads among other things to the notion of many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig.** 9 in the appendix of this book).

For many-sheeted space-time light velocity is assigned to light-like geodesic of space-time sheet rather than light-like geodesics of embedding space $M^4 \times CP_2$. The effective velocity determined from time to travel from point A to B along different space time sheets is different and therefore also the signal velocity determined in this manner. The light-like geodesics of space-time sheet corresponds in the generic case time-like curves of the embedding space so that the light-velocity is reduced from the maximal signal velocity. Space-time sheet is bumpy and wiggled so that the path is longer. Each space-time sheet corresponds to different light velocity as determined from the travel time. The maximal signal velocity is reached only in an ideal situation when the space-time geodesics are geodesics of Minkowski space.

1. Estimate fro the light velocity from Robertson-Walker cosmology

Robertson-Walker cosmology imbedded as 4-surface (this is crucial!) in $M^4 \times CP_2$ [K80] gives a good estimate for the light velocity in cosmological scales.

1. One can use the relationship

$$\frac{da}{dt} = g_{aa}^{-1/2}$$

relating the curvature radius a of RW cosmology space (equal to M^4 light-cone proper time, the light-like boundary of the cone corresponds to the moment of Big Bang) and cosmic time t appearing in Robertson-Walker line element

$$ds^2 = dt^2 - a^2 d\sigma_3^2 \ .$$

2. If one believes that Einstein's equations in long scales, one obtains

$$\frac{8\pi G}{3} \times \rho = \frac{(g_{aa}^{-1} - 1)}{a^2} \ .$$

One can solve from this equation g_{aa} and therefore get an estimate the cosmological speed of light -call it $c_\#$ as

$$c_\# = (g_{aa})^{1/2} \ .$$

3. By plugging in the estimates

$$a \simeq t \simeq 13.8 \times Gy \ \text{(the actual value is around 10 Gy)} \ ,$$

$$\rho \simeq \frac{5m_p}{m^3} \ \text{(5 protons per cubic meter)} \ ,$$

$$G = 6.7 \times 10^{-11} m^3 kg^{-1} s^{-2} \ ,$$

one obtains the estimate

$$c_\# = (g_{aa})^{1/2} \simeq .73 \ ,$$

What can one conclude from the estimate?

1. The result leaves a lot of room to explain various anomalies (problems with determination of Hubble constant, apparent growth of the Moon-Earth distance indicated by the measurement of distance by laser signal, ...). The effective velocity can depend on the scale of space-time sheet along which the relativistic particles arrive (and thus on distance distinguishing between OPERA experiment and SN1987A, see `http://tinyurl.com/ycto3v2z`), it can depend on the character of ultra relativistic particle (photon, neutrino, electron, ...), etc. The effect is testable by using other relativistic particles -say electrons.

2. The energy independence of the results fits perfectly with the predictions of the model since the neutrinos are relativistic. There can be dependence on length scale: in other words distance scale and this is needed to explain SN1987A -CERN difference in $\Delta c/c$. For SN1987A neutrinos were also relativistic and travelled a distance is L=cT=168, 000 light years and the neutrinos arrived about $\Delta T = 2-3$ hours earlier than photons (see `http://tinyurl.com/ycto3v2z`). This gives $\Delta c/c = \Delta T/T \simeq .8 - 1.2 \times 10^{-6}$ which is considerably smaller than for the recent experiment. Hence the tachyonic model fails but scale and particle dependent maximal signal velocity can explain the findings easily.

3. The space-time sheet along which particles propagate would most naturally correspond to a small deformation of a "massless extremal" ("topological light ray" [K11]) assignable to the particle in question. Many-sheeted space-time could act like a spectroscope forcing each (free) particle type at its own kind of "massless extremal". The effect is predicted to be present for *any* relativistic particle. A more detailed model requires a model for the propagation of the particles having as basic building bricks wormhole throats at which the induced metric changes its signature from Minkowskian to Euclidian: the Euclidian regions have interpretation in terms of lines of generalized Feynman graphs. The presence of wormhole contact between two space-time sheets implies the presence of two wormhole throats carrying fermionic quantum numbers and the massless extremal is deformed in the regions surrounding the wormhole throat. At this stage I am not able to construct detailed model for deformed MEs carrying photons, neutrinos or some other relativistic particles.

2. Can one understand SN1987A-OPERA difference in TGD framework?

The challenge for sub-manifold gravity approach is to understand the SN1987A-OPERA difference qualitatively. Why neutrino (and any relativistic particle) travels faster in short length scales?

1. Suppose that this space-time sheet is massless extremal topologically condensed on a magnetic flux tube thickened from a string like object $X^2 \times; Y^2$ subset $M^4 \times CP_2$ to a tube of finite thickness. Suppose that this means that the properties of the magnetic flux tube determine the maximal signal velocity. The longer and less straight the tube, the slower the maximal signal velocity since the light-like geodesic along it is longer in the induced metric (time-like curve in $M^4 \times CP_2$). There is also rotation around the flux lines increasing the path length: see below.

2. For a planar cosmic string (X^2 is just plane of M^4) the maximal signal velocity would be as large as it can be but is expected to be reduced as the flux tube develops 4-D M^4 projection. In thickening process flux is conserved so that B scales as $1/S$, S the transversal area of the flux tube. Magnetic energy per unit length scales as 1/S and energy conservation requires that the length of the flux tube scales up like S during cosmic expansion. Flux tubes become longer and thicker as time passes.

3. The particle -even neutrino!- can rotate along the flux lines of electroweak fields inside the flux tube and this makes the path longer. The thicker/longer the flux tube, - the longer the path- the lower the maximal signal velocity. I emphasize that classical Z^0 and W fields (and also gluon fields!) are the basic prediction of TGD distinguishing it from standard model: again the notion of induced gauge field pops up!

4. Classically the cyclotron radius is proportional to the cyclotron energy. For a straight flux tube there is free relativistic motion in longitudinal degrees of freedom and cyclotron motion in transversal degrees of freedom and one obtains essentially harmonic oscillator like states with degeneracy due to the presence of rotation giving rise to angular momentum as an additional quantum number. If the transversal motion is non-relativistic, the radii of cyclotron orbits are proportional to a square root of integer. In Bohr orbitology one has quantization of the neutrino speeds: wave mechanically the same result is obtained in average sense. Fermi statistics implies that the states are filled up to Fermi energy so that several discrete effective light velocities are obtained. In the case of a relativistic electron the velocity spectrum would be of form

$$c_{eff} = \frac{L}{T} = \frac{c_{\#}}{\sqrt{1+n\hbar\frac{eB}{m}}} \quad .$$

Here L denotes the length of the flux tube and T the time taken by a motion along a helical orbit when the longitudinal motion is relativistic and transversal motion non-relativistic. In this case the spectrum for c_{eff} is quasi-continuous. Note that for large values of $\hbar = n\hbar_0$ (in TGD Universe) quasicontinuity is lost and in principle the spectrum might allow to the determination of the value of $\hbar$.

5. Neutrino is a mixture of right-handed and left handed components and right-handed neutrino feels only gravitation where left-handed neutrino feels long range classical Z^0 field. In any case, neutrino as a particle having weakest interactions should travel faster than photon and relativistic electron should move slower than photon. One must be however very cautious here. Also the energy of the relativistic particle matters.

This would be the qualitative mechanism explaining why the neutrinos (and relativistic particles in general) travel faster in short scales. The model can be also made quantitative since the cyclotron motion can be understood quantitatively once the field strength is known.

Here brane-theorists trying to reproduce TGD predictions are in difficulties since the notion of induced gauge field is required besides that of induced metric. Also the geometrization of classical electro-weak gauge fields in terms of the spinor structure of embedding space is needed. It is almost impossible to avoid $M^4 \times CP_2$ and TGD.

3. What about electrons and photons?

If I were a boss at CERN, I would suggest that the experiment should be carried out for relativistic electrons whose detection would be much easier and for which one could use much shorter scale.

1. Could one use both photon and electron signal simultaneously to eliminate the need to measure precisely the distance between points A and B.
2. Can one imagine using mirrors for photons and relativistic electrons and comparing the times for $A \rightarrow B \rightarrow A$?

As a matter fact, there is an old result by electric engineer Obolensky [H5] (see `http://tinyurl.com/y83g3lhj`) that I have mentioned earlier [?], and which states that in circuits signals seem to travel at superluminal speed. The study continues the tradition initiated by Tesla who started the study of what happens when relays are switched on or off in circuits.

1. The experimental arrangement of Obolensky suggest that part of circuit - the base of the so called Obolensky triangle- behaves as a single coherent quantum unit in the sense that the interaction between the relays defining the ends of the base is instantaneous: the switching of the relay induces simultaneously a signal from both ends of the base.
2. There are electromagnetic signals propagating with velocities c_0 (with values $271 \pm 1.8 \times 10^6$ m/s and $278 \pm 2.2 \times 10^6$ m/s) and $c_1(200.110 \times 10^6$ m/s): these velocities are referred to as Maxwellian velocities and they are below light velocity in vacuum equal to $c = 3 \times 10^8$ m/s. c_0 and c_1 would naturally correspond to light velocities affected by the interaction of light with the charges of the circuit.
3. There is also a signal propagating with a velocity c_2 ($(620 \pm 2.7) \times 10^6$ m/s), which is slightly more than twice the light velocity in vacuum. Does the identification $c_2 = c_{max}$, where c_{max} is the maximal signal velocity in $M^4 \times CP_2$, make sense? Could the light velocity c in vacuum correspond to light velocity, which has been reduced from the light velocity $c_{\#} = .73c_{max}$ in cosmic length scales due to the presence of matter to $c_{\#} = .48c_{max}$. Note that this interpretation does not require that electrons propagate with a super-luminal speed.
4. If Obolensky's findings are true and interpreted correctly, simple electric circuits might allow the study of many-sheeted space-time in garage!

If these findings survive they will provide an additional powerful empirical support for the notion of many-sheeted space-time and could be for TGD what Mickelson-Morley was for Special Relativity. It is sad that TGD predictions must still be verified via accidental experimental findings. It would be much easier to do the verification of TGD systematically. In any case, Laws of Nature do not care about science policy, and I dare hope that the mighty powerholders of particle physics are sooner or later forced to accept TGD as the most respectable known candidate for a theory unifying standard model and General Relativity.

4. Additional support for TGD view from ICARUS experiment

Tommaso Dorigo [C2] (see `http://tinyurl.com/3q79xnb`) managed to write the hype of his life about super-luminal neutrinos. This kind of accidents are unavoidable and any blogger sooner or later becomes a victim of such an accident. To my great surprise Tommaso Dorigo described in a completely uncritical and hypeish manner a study by ICARUS group [C8] (see `http://tinyurl.com/y7y3wfaj`) in Gran Sasso and concluded that it definitely refutes OPERA result. This if of course a wrong conclusion and based on the assumption that special and general relativity hold true as such and neutrinos are genuinely superluminal.

Also Sascha Vongehr [C1] (see `http://tinyurl.com/3uk6g7t`) wrote about ICARUS as a reaction to Tommaso Dorigo's surprising posting but this was purposely written half-joking hype claiming that ICARUS proves that neutrinos travel the first 18 meters with a velocity at least 10 times higher than c. Sascha also wrote a strong criticism of the recent science establishment. The continual uncritical hyping is leading to the loss of the respectability of science and I cannot but share his views. Also I have written several times about the ethical and moral decline of the science community down to what resembles the feudal system of middle ages in which Big Boys have first night privilege to new ideas: something which I have myself had to experience many times.

What ICARUS did was to measure the energy distribution of muons detected in Gran Sasso. This result is used to claim that OPERA result is wrong. The measured energy distribution is compared with the distribution predicted assuming that Cohen-Glashow interpretation (see `http://tinyurl.com/6bwzcl3`) [C6] is correct. This is an extremely important ad hoc assumption without which the ICARUS demonstration fails completely.

1. Cohen and Glashow assume a genuine super-luminality and argue that this leads to the analog of Cherenkov radiation leading to a loss of neutrino energy: 28.2 GeV at CERN is reduced to average of 12.1 GeV at Gran Sasso. From this model one can predict the energy distribution of muons in Gran Sasso.
2. The figure 2 in Icarus preprint (see `http://arxiv.org/pdf/1110.3763v1`) demonstrates that the distribution assuming now energy loss fits rather well the measured energy distribution of muons. The figure does not show the predicted distribution but the figure text tells that the super-luminal distribution would be much "leaner", which one can interpret as a poor fit.
3. From this ICARUS concludes that neutrinos cannot have exceeded light velocity. The experimental result of course tells only that neutrinos did not lose energy: about the neutrino velocity it says nothing without additional assumptions.

At the risk of boring the reader I repeat: the fatal assumption is that a genuine super-luminality is in question. The probably correct conclusion from this indeed is that neutrinos would lose their energy during their travel by Cherenkov radiation.

In TGD framework situation is different [L3]. Neutrinos move in excellent approximation velocity which is equal to the maximal signal velocity but slightly below it and without any energy loss. The maximal signal velocity is however higher for a neutrino carrying space-time sheets than those carrying photons- a basic implication sub-manifold gravity. I have explained this in detail in previous postings and in [L3].

The conclusion is that ICARUS experiment supports the TGD based explanation of OPERA result. Note however that at this stage TGD does not predict effective super-luminality but only allows and even slightly suggests it and provides also a possible explanation for its energy independence and dependences on length scale and particle. TGD suggests also new tests using relativistic electrons instead of neutrinos.

It is also important to realize that the apparent neutrino super-luminality -if true- provides only single isolated piece evidence for sub-manifold gravity. The view about space-time as 4-surface permeates the whole physics from Planck scale to cosmology predicting correctly particle spectrum and providing unification of fundamental interactions, it is also in a key role in TGD inspired quantum biology and also in quantum consciousness theory inspired by TGD.

5. OPERA confirms super-luminal velocity of neutrinos

OPERA collaboration has published an eprint Measurement of the neutrino velocity with the OPERA detector in the CNGS beam [C10] (see `http://tinyurl.com/3h44vxw`) providing further support for the claim that neutrinos move faster than photons. Tommaso Dorigo (see `http://tinyurl.com/7te5knu`) describes the improved measurements in this blog. The abstract of the preprint is following.

The OPERA neutrino experiment at the underground Gran Sasso Laboratory has measured the velocity of neutrinos from the CERN CNGS beam over a baseline of about 730 km with much higher accuracy than previous studies conducted with accelerator neutrinos. The measurement is based on high-statistics data taken by OPERA in the years 2009, 2010 and 2011. Dedicated upgrades of the CNGS timing system and of the OPERA detector, as well as a high precision geodesy campaign for the measurement of the neutrino baseline, allowed reaching comparable systematic and statistical accuracies. An early arrival time of CNGS muon neutrinos with respect to the one computed assuming the speed of light in vacuum of (57.8 ± 7.8 (stat.)+8.3-5.9 (sys.)) ns was measured. This anomaly corresponds to a relative difference of the muon neutrino velocity with respect to the speed of light $(v-c)/c = (2.37 \pm 0.32$ *(stat.) (sys.))* $\times 10^{-5}$. *The above result, obtained by comparing the time distributions of neutrino interactions and of protons hitting the CNGS target in 10.5 μs long extractions, was confirmed by a test performed using a beam with a short-bunch time-structure allowing to measure the neutrino time of flight at the single interaction level.*

In the new experiment the spacing between pulses was only 3 ns. This implies that pulse shape and duration cannot explain the earlier OPERA result as a measurement error. Effectively one studies individual neutrinos. Pulse shape and size has provided for the main stream theorist a cheap and fast way to explain the observation out from his mindscape. Certainly this finding also kills a large class of explanations for neutrino super-luminality. Of course, one must still keep mind open for some delicate measurement error. Lubos Motl suggests that there is a systematic error in GPS system, other colleagues have not taken this option seriously.

Second new finding is that there is a "jitter" in travel times: the arrival times vary within 50 ms range which corresponds to a distance about 15 m. The shortening of travel times is not however not less than 40 ns from that when neutrinos move with light velocity as the figure (see `http://tinyurl.com/hogr3fd`) that can be found from the posting of Phil Gibbs (see `http://tinyurl.com/o7868kx`) demonstrates [C4]. Is the determination of the arrival time inaccurate? Or does the neutrino velocity have values above minimum velocity larger than c?

1. In TGD framework this could mean that the space-time sheet along which neutrino arrives would vary from neutrino to neutrino. The simplest possibility is that its length varies and velocity is constant: this does not look totally implausible.

2. Also the state of neutrino inside space-time sheet could vary from neutrino to neutrino. Classical long ranged Z^0 fields are one of the basic predictions of TGD and in the earlier posting I proposed that neutrino feels classical Z^0 magnetic field and arrives along cyclotron orbit. This would give a discrete spectrum of arrival velocities as

$$v = \frac{c_{\#}}{[1 + n \times \hbar \times \frac{Q_Z(\nu) g_Z B_Z}{m_\nu}]^{1/2}}$$

with $n = 0, 1, 2, \ldots$. For some value of n the velocity would become sub-luminal. If $\hbar$ is large enough, the discrete spectrum could be seen in the arrival times. This spectrum does not however look an attractive explanation for the jitter for which spectrum seems to be above minimum value rather than below maximum value.

6. Answers to questions by Eugen Stefanovich

Eugen Stefanovich made in my blog some questions allowing to bring additional details to the overall picture. The answers should reveal what the questions where.

1. There is no energy dependence. There is particle and scale dependence. There is an argument suggesting that the velocity is higher for neutrinos than for photon and for photon higher than for relativistic electron. The difference between neutrino families is expected to be small if the proposed mechanism based on electroweak interactions is correct: this because of the universality/flavor independence of electroweak interactions.
2. The dependence on the length scale of the orbit should be via p-adic length scale and therefore piecewise constant. This kind of jump would come at half octaves of basic length scale and might be therefore observable. Increasing or decreasing the distance between CERN and receiver by a factor of $\sqrt{2}$ could reveal this effect.
3. The distance between CERN and Gran Sasso is 750 km. If I understood correctly, the distance travelled by neutrinos in MINOS experiment is 734 km) [C3] (see `http://tinyurl.com/yc6f6nyo`). 734 km is slightly above p-adic length scale $L(151+2\times 46) = 2^{46}\times L(151) = 2^{46}\times 10^{-8}$ meters= $L(243)$=703 km. If I take p-adic length scale hypothesis seriously then the result should be the same.
4. In cosmic scales one can estimate maximal signal velocity for photon: a very rough estimate using embedding of Roberston-Walker cosmology as Lorentz invariant 4-surface is 73 per cent from absolute maximum (for light-like geodesic of M^4). For SN1987A neutrinos and photons the velocity difference would be much smaller than in shorter scales suggesting that the deviation from absolute maximum approaches to zero at very long distance scales.
   (a) One possibility is
   $$\Delta[\frac{v}{c}(p] \propto L_p^{-n} \propto p^{-n/2} \quad ,$$
   where $L_p \propto p^{1/2}$ is the p-adic length scale. By p-adic length scale hypothesis the p-adic prime p satisfies $p \simeq 2^k$. n is an exponent which need not be an integer.
   (b) Second suggestive possibility is logarithmic dependence on L_p and therefore on p.

Superluminal neutrinos cannot be tachyons

New Scientist (see `http://tinyurl.com/6qtjgny`) reported about the sad fate of the tachyonic explanation of neutrino super-luminality. The argument is extremely simple.

1. One can start by assuming that a tachyon having negative mass squared: $m(\nu)^2 < 0$ and assume that super-luminal velocity is in question. The point is that one knows the value of the superluminal velocity $v(1+\epsilon)c$, $\epsilon \simeq 10^{-5}$. One can calculate the energy of the neutrino as
   $$E = |m(\nu)|[-1 + v^2/(v^2-1)]^{1/2} \quad ,$$
   $|m(\nu)| = (-m(\nu)^2)^{1/2}$ is the absolute value of formally imaginary neutrino mass.
2. In good approximation one can write
   $$E = |m(\nu)|[-1 + (2\epsilon^{-1/2})]^{1/2} \simeq |m(\nu)|(2\epsilon)^{-1/2} \quad .$$
   The order of magnitude of $|m(\nu)|$ is not far from one eV - this irrespective of whether neutrino is tachyonic or not. Therefore the energy of neutrino is very small: not larger than keV. This is in a grave contradiction with what is known: the energy is measured using GeV as a natural unit so that there is discrepancy of 6 orders of magnitude at least. One can also apply energy conservation to the decay of pion to muon and neutrino and this implies that muon has gigantic energy: another contradiction.

What is amusing that this simple kinematic fact was not noticed from beginning. In any case, this finding kills all tachyonic models of neutrino super-luminality assuming energy conservation, and gives additional support for the TGD based explanation in terms of maximal signal velocity, which depends on pair of points of space-time sheet connected by signal and space-time sheet itself characterizing also particular kind of particle.

Even better, one can understand also the jitter (see `http://tinyurl.com/3h44vxw`) [C10] in the spectrum of the arrival times which has width of about 50 ns in terms of an effect caused fluctuations in gravitational fields to the maximal signal velocity expressible in terms of the induced metric [K79]. The jitter could have interpretation in terms of gravitational waves inducing fluctuation of the maximal signal velocity $c_\#$, which in static approximation equals to $c_\# = c(1+\Phi_{gr})^{1/2}$, where Φ_{gr} is gravitational potential.

Surprisingly, effectively super-luminal neutrinos would make possible

gravitational wave detector [K79]! The gravitational waves would be however gravitational waves in TGD sense having fractal structure since they would correspond to bursts of gravitons resulting from the decays of large $\hbar$ gravitons emitted primarily rather than to a continuous flow [K66]. The ordinary detection criteria very probably exclude this kind of bursts as noise. The measurements of Witte (see `http://tinyurl.com/yb98uhvd`) [E53] attempting to detect absolute motion indeed observed this kind of motion identifiable as a motion of Earth with respect to the rest frame of galaxy but superposed with fractal fluctuations proposed to have interpretation in terms of gravitational turbulence - gravitational waves.

Icarus refutes Opera result

Icarus collaboration (see `http://tinyurl.com/y7eqcsma`) [C9] has replicated the measurement of the neutrino velocity. The abstract summarizes the outcome.

The CERN-SPS accelerator has been briefly operated in a new, lower intensity neutrino mode with about 10^{12} *p.o.t. /pulse and with a beam structure made of four LHC-like extractions, each with a narrow width of about 3 ns, separated by 524 ns. This very tightly bunched beam structure represents a substantial progress with respect to the ordinary operation of the CNGS beam, since it allows a very accurate time-of-flight measurement of neutrinos from CERN to LNGS on an event-to-event basis. The ICARUS T600 detector has collected 7 beam-associated events, consistent with the CNGS delivered neutrino flux of* 2.2×10^{16} *p.o.t. and in agreement with the well known characteristics of neutrino events in the LAr-TPC. The time of flight difference between the speed of light and the arriving neutrino LAr-TPC events has been analyzed. The result is compatible with the simultaneous arrival of all events with equal speed, the one of light. This is in a striking difference with the reported result of OPERA that claimed that high energy neutrinos from CERN should arrive at LNGS about 60 ns earlier than expected from luminal speed.*

As explained, Tthe TGD based explanation for the anomaly would not have been superluminality but the dependence of the maximal signal velocity on space-time sheet: the geodesics in induced metric are not geodesics of the 8-D embedding space. In principle the time taken to move from A (say CERN) to point B (say Gran Sasso) depends on space-time sheets involved. One of these space-time sheets would be that assignable to particle beam - a good guess is "massless extremal" [K11]: along this the velocity is in in the simplest case (cylindrical "massless extremals") the maximal signal velocity in $M^4 \times CP_2$.

Other space-space-time sheets involved can be assigned to various systems such as Earth, Sun, Galaxy and they contribute to the effect. It is important to understand how the physics of test particle depends on the presence of parallel space-times sheets. Simultaneous topological condensation to all the sheets is extremely probable so that at classical level forces are summed. Same happens at quantum level. The superposition of various fields assignable to parallel space-time sheets is not possible in TGD framework and is replaced with the superposition of their effects. This allows to resolve one of the strongest objections against the notion induced gauge field.

The outcome of ICARUS experiment is not able to kill this prediction since at this moment I am not able to fix the magnitude of the effect. It is really a pity that such a fantastic possibility to wake up the sleeping colleagues is lost. I feel like a parent in a nightmare seeing his child to drown and being unable to do anything.

There are other well-established effects in which the dependence of maximal signal velocity on space-time sheet is visible: one such effect is the observed slow increase of the time spend by light ray to propagate moon and back. The explanation (see `http://tinyurl.com/yawof8yt`) [K79] is that the effect is not real but due to the change of the unit for velocity defined by the light-velocity assignable to the distant stars. The maximal signal velocity is for Robertson-Walker cosmology gradually increasing and the anomaly emerges as an apparent anomaly when one assumes that the natural coordinate system assignable to the solar system (Minkowski coordinates) is the natural coordinate system in cosmological scales. The size of the effect is predicted correctly. Since the cosmic signal velocity defining the unit increases, the local maximal signal velocity which is constant seems to be reducing and the measured distance to the Moon seems to be increasing.

9.4.2 SN1987A And Many-Sheeted Space-Time

Lubos Motl has written a highly rhetoric, polemic, and adrenaline rich posting (see `http://tinyurl.com/px4hzdc`) about the media buzz related to supernova SN1987A. The target of Lubos Motl is the explanation proposed by James Franson from the University of Maryland for the findings discussed in Physics Archive Blog (see `http://tinyurl.com/mde7jat`). I do not have any strong attitude to Franson's explanation but the buzz is about very real thing: unfortunately Lubos Motl tends to forget the facts in his extreme orthodoxy.

What happened was following. Two separate neutrino bursts arrived from SN 1987 A. At 7.35 AM Kamionakande detected 11 antineutrons, IMB 8 antineutrinos, and Baksan 5 antineutrinos. Approximately 3 hours later Mont Blanc liquid scintillator detected 5 antineutrinos. Optical signal came 4.7 hours later.

The are several very real problems as one can get convinced by going to Wikipedia (`http://tinyurl.com/mglkm4`):

1. If neutrinos and photons are emitted simultaneously and propagate with the same speed, they should arrive simultaneously. I am not specialist enough to try to explain this difference in terms of standard astrophysics. Franson however sees this difference as something not easy to explain and tries to explain it in his own model.
2. There are two neutrino bursts rather than one. A modification of the model of supernova explosion allowing two bursts of neutrinos would be needed but this would suggest also two photon bursts.

These problems have been put under the carpet. Those who are labelled as crackpots often are much more aware about real problems than the academic career builders.

In TGD framework the explanation would be in terms of many-sheeted space-time. In GRT limit of TGD the sheets of the many-sheeted space-time (see **Fig.** `http://tgdtheory.fi/appfigures/manysheeted.jpg` or **Fig. A-6.1** in the appendix of this book) are lumped to single sheet: Minkowski space with effective metric defined by the sum of Minkowski metric and deviations of the metrics of the various sheets from Minkowski metric. The same recipe gives effective gauge potentials in terms of induced gauge potentials.

Different arrival times for neutrinos and photons would be however a direct signature of the many-sheeted space-time since the propagation velocity along space-time sheets depends on the induced metric. The larger the deviation from the flat metric is, the slower the propagation velocity and thus longer the arrival time is. Two neutrino bursts would have explanation as arrivals along two different space-time sheets. Different velocity for photons and neutrinos could be explained if they arrive along different space-time sheets. I proposed for more than two decades ago this mechanism as an explanation for the finding of cosmologists that there are two different Hubble constants: they would correspond to different space-time sheets.

The distance of SN1987A is 168, 000 light- years. This means that the difference between velocities is $\Delta c/c \simeq \Delta T/T \simeq 3hours/168 \times 10^3 \simeq 2 \times 10^{-9}$. The long distance is what makes the effect visible.

I proposed earlier sub-manifold gravity as an explanation for the claimed super-luminality of the neutrinos coming to Gran Sasso from CERN. In this case the effect would have been $\Delta c/c \simeq 2.5 \times 10^{-5}$ and thus four orders of magnitude larger than four supernova neutrinos. It however turned out that the effect was not real.

Towards the end of 2014 Lubos Motl Motl had a posting about galactic blackhole Sagittarius A as neutrino factory (see `http://tinyurl.com/pvzrqoz`). Chandra X-ray observatory (see `http://tinyurl.com/6jdp7es`) and also Nustar (`http://tinyurl.com/89b8r96`) and Swift Gamma-Ray Burst Mission (see `http://tinyurl.com/ybmrpuu6`) detected some X-ray flares from Sagittarius A. 2-3 hours earlier IceCube (see `http://tinyurl.com/lg7mko`) detected high energy neutrinos by IceCube on the South Pole.

Could neutrinos arrive from the galactic center? If they move with the same (actually somewhat lower) velocity than photons, this cannot be the case. The neutrinos did the same trick as

SN1987A neutrinos and arrived 2-3 hours before the X-rays! What if one takes TGD seriously and estimates $\Delta c/c$ for this event? The result is $\Delta c/c \sim (1.25-1.40)\times 10^{-8}$ for 3 hours lapse using the estimate $r = 25,900 \pm 1,400$ light years (see `http://tinyurl.com/5vexvq`). $\Delta c/c$ is by a factor 4 larger than for SN1987A at distance about 168, 000 light years (see `http://tinyurl.com/mglkm4`). This distance is roughly 8 times longer. This would suggests that the smaller the space-time sheets the nearer the velocity of neutrinos is to its maximal value. For photons the reduction from the maximal signal velocity is larger.

9.4.3 Anomalous Time Dilation Effects Due To Warping As Basic Distinction Between TGD And GRT

TGD predicts the possibility of large anomalous time dilation effects due to the warping of space-time surfaces, and the experimental findings of Russian physicist Chernobrov about anomalous changes in the rate of flow of time [J7, J3] provide indirect support for this prediction.

Anomalous time dilation effect due to the warping

Consider first the ordinary gravitational time dilation predicted by GRT. For simplicity consider a stationary spherically symmetric metric $ds^2 = g_{tt}dt^2 - g_{rr}dr^2 - r^2d\Omega^2$ in spherical coordinates. The time dilation is characterized by the difference $\Delta = \sqrt{g_{tt}} - 1$. In the weak field approximation one has $g_{tt} = 1 + 2\Phi_{gr}$, where Φ_{gr} is gravitational potential. The ordinary time dilation is given by $\Delta = \sqrt{g_{tt}} - 1 \simeq 2\Phi_{gr}$. At the Earth's surface the gravitational potential of the Earth is about $\Phi_{gr} = GM/R_E \simeq 10^{-9}$.

Consider next the situation for space-time surfaces. There exists an infinite number of warped embeddings of M^4 to $M^4 \times CP_2$ given by $s^k = s^k(m^0)$, which are metrically equivalent with the canonical embedding with CP_2 coordinates constant. New M^4 time coordinate is related by a diffeomorphism to the standard one. By restricting the embedding to $M^4 \times S^1$, where S^1 a geodesic circle with radius $R/2$ (using the chosen convention for the definition CP_2 radius), the time component of the induced metric is $g_{m^0m^0} = 1-R^2\omega^2/4$. The identification of M^4 coordinates as the preferred natural standard coordinate frame allows to overcome the difficulties related to the identification of the preferred time coordinate in general relativity in the case the metric does not approach asymptotically flat metric. For this choice an anomalous time dilation $\sqrt{1 - R^2\omega^2/4}$ due to the warping results even when gravitational fields are absent. Moreover, the dilation can be large.

The study of the embeddings of Scwhartshild metric as vacuum extremals demonstrates that this vacuum warping is also seen as the degeneracy of the embeddings of stationary spherically symmetric metrics. If m^0 is used as a time coordinate, anomalous time dilation is obtained also at $r_M \to \infty$ and is given by

$$\sqrt{g_{m^0m^0}} = \frac{1}{\lambda} . \tag{9.4.1}$$

This time dilation is seen only if the clocks to be compared are at different space-time sheets. The anomalous time dilation can be quite large since the order of magnitude for the parameter ωR is naturally of order one for the embeddings of R-N metrics.

Mechanisms producing anomalous time dilation

Anomalous time dilation could result in many ways.

1. An adiabatic variation of the parameters λ and ω_1 of the space-time sheet containing the clock could be induced by some physical mechanism. For instance, X^4_c could move "over" a large space-time sheet X^4 and gradually form $\#$ and $\#_B$ contacts with it. Topological light rays (MEs) define a good candidate for X^4. The parameter values λ and ω could change quasi-continuously if X^4_c gradually generates CP_2 sized wormhole contacts or flux tubes connecting it to X^4. This process would not affect the gravitational flux fed to X^4_c.

   For instance, if X^4 is at rest with respect to Earth, this motion would result from the rotation of Earth and the effect should appear periodically from day to day. If it is at rest with respect to Sun, the effect should appear once a year.

The generation of vacuum extremals X^4_{vac} (not gravitational vacua), which is in principle possible even by intentional action since conservation laws are not broken, could induce anomalous time dilation by this mechanism.

2. A phase transition increasing the value of $\hbar$ increases the size of the space-time sheet in the same proportion. This transition could quite well affect also the parameter λ. If this phase transition occurs for the space-time sheet X^4_c at which the clock feeds its gravitational flux, this mechanism could provide a feasible manner to induce an anomalous time dilation.

3. The system containing the clock could suffer a temporary topological condensation to a smaller space-time sheet and thus feed its gravitational flux to this space-time sheet. This would require coherently occurring splitting of # contacts and their regeneration. It is not possible to say anything definite about the probability of this kind of process except that it does not look very feasible.

The findings of Chernobrov

The findings claimed by Russian researcher V. Chernobrov support anomalous time dilation effect [J7, J3]. Chernobrov has studied anomalies in the rate of time flow defined operationally by comparing the readings of clocks enclosed inside a spherical volume with the readings of clocks outside this volume. The experimental apparatus involves a complex Russian doll like structure of electromagnets.

Chernobrov reports a slowing down of time by about 30 seconds per hour inside his experimental apparatus [J7] so that the average dilation factor during hour would be about $\Delta = 1/120$. If the dilation is present all the time, the anomalous contribution to the gravitational potential would be by a factor $\sim 10^7$ larger than that of Earth's gravitational potential and huge gravitational perturbations would be required to produce this kind of effect.

The slowing of the time flow is reported to occur gradually whereas the increase for the rate of time flow is reported to occur discontinuously. Time dilation effects were observed in connection with the cycles of moon, diurnal fluctuations, and even the presence of operator.

Consider now the explanation of the basic qualitative findings of Chernobrov.

1. The gradual slowing of the time flow suggests that the parameter values of λ and ω change adiabatically. This favors option 1) since the formation of # contacts occurs with some finite rate.

2. Also the sudden increase of the rate of time flow is consistent with option 1) since the splitting of # contacts occurs immediately when the sheets X^4_c and X^4 are not "over" each other.

3. The occurrence of the effect in connection with the cycles of moon, diurnal fluctuations, and in the presence of operator support this interpretation. The last observation would support the view that intentional generation of almost vacuum space-time sheets is indeed possible.

Vacuum extremals as means of generating time dilation effects intentionally?

Field equations allow a gigantic family of vacuum extremals: any 4-surface having CP_2 projection, which belongs to a 2-dimensional Lagrange manifold with a vanishing induced Kähler form, is a vacuum extremal. Symplectic transformations and diffeomorphisms of CP_2 produce new vacuum extremals. Vacuum extremals carry non-vanishing classical electro-weak and color fields which are reduced to some $U(1)$ subgroup of the full gauge group and also classical gravitational field. Although the vacuum extremals are not absolute minima, their small deformations could define such. These vacuum extremals, call them X^4_{vac} for brevity, could be generated by intentional action. In the first quantum jump the p-adic variant of the vacuum extremal representing an intention to create X^4_{vac} would appear and in some subsequent quantum jump it would be transformed to a real space-time sheet.

The creation of these almost vacuum extremals could generate time dilation effects. The material system would gradually generate CP_2 sized wormhole contacts and/or flux tubes connecting its space-time sheet to X^4_{vac} and this could change the values of the vacuum parameters λ, ω.

Could warping have something to do with condensed matter physics?

Warping predicts the reduction of the effective light velocity. There is a report [D9] of an experimental study of a condensed-matter system (graphene, a single atomic layer of carbon, in which electron transport is essentially governed by massless Dirac's equation. According to the report, the charge carriers in graphene mimic relativistic particles with zero rest mass and have an effective "speed of light" $c_1 = c/300 = 10^6$ m/s.

The study reveals a variety of unusual phenomena that are characteristic of two-dimensional Dirac fermions. Graphene's conductivity never falls below a minimum value corresponding to the quantum unit of conductance, even when the concentrations of charge carriers tend to zero. The integer quantum Hall effect in graphene is anomalous in that it occurs at half-integer filling factors. The cyclotron mass m_c of massless current carriers in graphene is defined in terms of the energy of current carrier as $E = m_c c_1^2$.

The authors believe that these phenomena can be understood in the framework of the ordinary QED and this might be the case. One can however wonder whether the massless Dirac equation for the 2-dimensional system could correspond to the Kähler-Dirac equation for the induced spinor fields and whether the reduction of the maximal signal velocity to c_1 could have the warping of the space-time sheet as a space-time correlate. In the idealization that the CP_2 projection of the space-time surface is a geodesic circle of CP_2, and using M^4 coordinates for space-time surface, so that one would have $\Phi = \omega t$ for S^2 coordinate Φ, one would have $g_{tt} = c_1^2 = 1 - R^2\omega^2/4 = 10^{-4}/9$.

9.4.4 Evidence For Many-Sheeted Space-Time From Gamma Ray Flares

MAGIC collaboration has found evidence for a gamma ray anomaly. Gamma rays in different energy ranges seem to arrive with different velocities from Mkn 501 [E22]. The delay in arrival times is about 4 minutes. The proposed explanation is in terms of broken Lorentz invariance. TGD allows to explain the finding in terms of many-sheeted space-time and there is no need to invoke breaking of Lorentz invariance.

TGD based explanation at qualitative level

One of the oldest predictions of many-sheeted space-time is that the time for photons to propagate from point A to B along given space-time sheet depends on space-time sheet because photon travels along light-like geodesic of space-time sheet rather than light-like geodesic of the embedding space and thus increases so that the travel time is in general longer than using maximal signal velocity.

Many-sheetedness predicts a spectrum of Hubble constants and gamma ray anomaly might be a demonstration for the many-sheetedness. The spectroscopy of arrival times would give information about how many sheets are involved.

Before one can accept this explanation, one must have a good argument for why the space-time sheet along which gamma rays travel depends on their energy and why higher energy gamma rays would move along space-time sheet along which the distance is longer.

1. Shorter wavelength means that the wave oscillates faster. Space-time sheet should reflect in its geometry the the matter present at it. Could this mean that the space-time sheet is more "wiggly" for higher energy gamma rays and therefore the distance traveled longer? A natural TGD inspired guess is that the p-adic length scales assignable to gamma ray energy defines the p-adic length scale assignable to the space-time sheet of gamma ray connecting two systems so that effective velocities of propagation would correspond to p-adic length scales coming as half octaves. Note that there is no breaking of Lorentz invariance since gamma ray connects the two system and the rest system of receiver defines a unique coordinate system in which the energy of gamma ray has Lorentz invariant physical meaning.

2. One can invent also an objection. In TGD classical radiation field decomposes into topological light rays ("massless extremals", MEs) which could quite well be characterized by a large Planck constant in which case the decay to ordinary photons would take place at the receiving end via de-coherence. Gamma rays could propagate very much like a laser beam along the ME. For the simplest MEs the velocity of propagation corresponds to the maximal signal velocity and there would be no variation of propagation time.

One can imagine two ways to circumvent to the counter argument.

i) Also topological light rays for which light-like geodesics are replaced with light-like curves of M^4 are highly suggestive as solutions of field equations. For these MEs the distance travelled would be in general longer than for the simplest MEs.

ii) The gluing of ME to background space-time by wormhole contacts (actually representation for photons!) could force the classical signal to propagate along a zigzag curve formed by simple MEs with maximal signal velocity. The length of each piece would be of order p-adic length scale. The zigzag character of the path of arrival would increase the distance between source and receiver.

Quantitative argument

A quantitative estimate runs as follows.

1. The source in question is quasar Makarian 501 with redshift $z = .034$. Gamma flares of duration about 2 minutes were observed with energies in bands.25-.6 TeV and 1.2-10 TeV. The gamma rays in the higher energy band were near to its upper end and were delayed by about $\Delta\tau = 4$ min with respect to those in the lower band. Using Hubble law $v = Hct$ with $H = 71$ km/Mparsec/s, one obtains the estimate $\Delta\tau/\tau = 1.6 \times 10^{-14}$.

2. A simple model for the induced metric of the space-time sheet along which gamma rays propagate is as a flat metric associated with the flat embedding $\Phi = \omega t$, where Φ is the angle coordinate of the geodesic circle of CP_2. The time component of the metric is given by

$$g_{tt} = 1 - R^2\omega^2 \quad .$$

ω appears as a parameter in the model. Also the embeddings of Reissner-Norström and Schwartschild metrics contain frequency as free parameter and space-time sheets are quite generally parameterized by frequencies and momentum or angular momentum like vacuum quantum numbers.

3. ω is assumed to be expressible in terms of the p-adic prime characterizing the space-time sheet. The parameterization to assumed in the following is

$$\omega^2 R^2 = Kp^{-r} \quad .$$

It turns out that $r = 1/2$ is the only option consistent with the p-adic length scale hypothesis. The naïve expectation would have been $r = 1$. The result suggests the formula

$$\omega^2 = m_0 m_p \text{ with } m_0 = \frac{K}{R} \quad .$$

ω would be the geometric mean of a slowing varying large p-adic mass scale and p-adic mass scale m_p.

The explanation for the p-adic length scale hypothesis leading also to a generalization of Hawking-Bekenstein formula assumes that for the strong form of p-adic length scale hypothesis stating $p \simeq 2^k$, k prime, there are two p-adic length scales involved with given elementary particle. L_p characterizes particle's Compton length and L_k characterizes the size of the wormhole contact or throat representing the elementary particle. The guess is that ω is proportional to the geometric mean of these two p-adic length scales:

$$\omega^2 R^2 = \frac{x}{2^{k/2}\sqrt{k}} \quad .$$

4. A relatively weak form of the p-adic length scale hypothesis would be $p \simeq 2^k$, k an odd integer. M_{127} corresponds to the mass scale $m_e 5^{-1/2}$ in a reasonable approximation. Using $m_e \simeq .5$ MeV one finds that the mass scales $m(k)$ for $k = 89 - 2n$, $n = 0, 1, 2..., 6$ are $m(k)/TeV = x$, with $x = 0.12, 0.23, 0.47, 0.94, 1.88, 3.76, 7.50$. The lower energy range contains the scales $k = 87$ and 85. The higher energy range contains the scales $k = 83, 81, 79, 77$. In this case the proposed formula does not make sense.

5. The strong form of p-adic length scale hypothesis allows only prime values for k. This would allow Mersenne prime M_{89} (intermediate gauge boson mass scale) for the lower energy range and $k=83$ and $k=79$ for the upper energy range. A rough estimate is obtained by assuming that the two energy ranges correspond to $k_1=89$ and $k_2=79$.
6. The expression for τ reads as $\tau=(g_{tt})^{1/2}t$. $\Delta\tau/\tau$ is given by

$$\frac{\Delta\tau}{\tau} \simeq (g_{tt})^{-1/2}\frac{\Delta g_{tt}}{2} \simeq R^2\Delta\omega^2 = x[(k_2p_2)^{-1/2}-(k_1p_1)^{-1/2}] \simeq x(k_2p_2)^{-1/2}$$

$$= x2^{-79/2}79^{-1/2} \quad .$$

Using the experimental value for $\Delta\tau/\tau$ one obtains $x\simeq .45$. $x=1/2$ is an attractive guess.

9.4.5 Do Ultracold Neutrons Provide Direct Evidence For Many-Sheeted Space-Time?

There was a very interesting article about magnetic anomaly UCN trapping (see `http://tinyurl.com/y8z6kff5`) . UCN is a shorthand for ultra-cold neutrons. The article [C5] had a somewhat provocative title *Magnetic anomaly in UCN trapping: signal for neutrons oscillations to parallel world?*. Perhaps this was explains why I did not bother to look at it at the first time I saw it.

As I saw again the popular story hyping the article, I realized that the anomaly - if real - could provide a direct evidence for the transitions of neutrons between parallel space-time sheets of many-sheeted space-time. TGD of course predicts that this phenomenon is completely general applying to all kinds of particles.

The interpretation of authors is that ultra-cold neutrons oscillate between parallel worlds-albeit in different sense as in TGD. The authors describe this oscillation using same mathematical model as describing neutrino oscillations. What would be observed would be that in statistical sense neutrons in the beam disappear and reappear periodically. The model predicts that the frequency for this is just the Larmor frequency $\omega=\mu\cdot B/2$ for the precession of spin of neutron in magnetic field. The authors claim that just this is observed and the interpretation is somewhat outlandish looking. Standard model gauge group is doubled: all particles have exact mirror copies with same quantum numbers. This of course is extremely inelegant interpretation. Something much more elegant is needed.

TGD based description of the situation

TGD allows to understand the finding in terms of many-sheeted space-time and one ends up with a phenomenological model similar to that of authors. Now however the phenomenon is predicted to be completely general applying to all kinds of particles and does not require the weird doubling of standard model symmetries.

Imagine the presence of two space-time sheets (or even more of them) carrying magnetic fields which decompose to flux tubes.

1. Suppose that neutron is topologically condensed in one of these flux tubes. What happens when the flux tubes are "above each other" in the sense that their Minkowski space projections intersect and the flux tubes are extremely near to each other: the distance is of order CP_2 size of order 10^4 Planck lengths. It took long time to take seriously the obvious: neutrons topologically condense on both space-time sheets and experience the sum of the magnetic fields in these regions. This actually allows to overcome the basic objection against TGD due to the fact that all classical gauge fields are expressible in terms of CP_2 coordinates and their gradients so that enormously powerful constraints between classical gauge fields are satisfied and linear superposition of fields is lost. In many-sheeted space-time this superposition is replaced with the superposition of their effects in multiple topological condensation,
2. In the regions where the intersection of M^4 projections of flux tubes is empty, topological condensation takes place on either space-time sheet.
3. What happens when one has neutrons propagating along flux tube 1 characterized by magnetic field B_1 arrive to a region where flux tube 2 of magnetic field B_2 resides? In the intersection region the neutrons experience the field B_1+B_2 in good approximation. The interaction

energy $E = \mu B \cdot \sigma$, where B is the magnetic field and σ is the spin of neutron. In flux tube 1 has $B = B_1$, in flux tube 2 one has $B = B_2$ and in the intersection region $B = B_1 + B_2$. It can happen that neutron arriving along flux tube 1 continues its travel along flux tube 2.

4. Magnetic fields in question actually consists of large number of nearly parallel flux tubes and the travel of neutron is a series of segments: $B_{i_1} \rightarrow B_1 + B_2 \rightarrow B_{i_2} \rightarrow .....$ As if neutron would make jumps between parallel worlds. Now these worlds are geometrically parallel rather than identifiable as copies in tensor product of standard model gauge groups.

A phenological description predicting the probabilities for the transitions between the parallel worlds assignable to the two magnetic fields could be based on simple Hamiltonian used to describe also neutrino mixing. Hamiltonian is sum of spin Hamiltonians $H_i = \mu B_i \cdot \sigma$ and of non-diagonal mixing term ϵ. $H = H_1 \oplus H_2 + \epsilon$. The diagonal term H_i are non-vanishing in the nonintersecting region i and non-diagonal describing what happens in the intersecting regions. Just this description was used by the authors of the article to parametrize the observed anomaly.

One can test this interpretation by introducing a third magnetic field. The interpretation of authors might force to introduce even third copy of standard model gauge group.

Amusing co-incidence

What is so amusing that the magnetic field used in the experiments was .2 Gauss. It is exactly the nominal value of the endogenous magnetic field needed to explain the strange quantal effects of radiation at cyclotron frequencies of biologically important ions on vertebrate brain. The frequencies are extremely low - in EEG range - and corresponding thermal energies are 10 orders below thermal energy so that standard quantum mechanics predicts no effects. The explanation assumes $B_{end} = .2$ GeV containing dark variants of these ions with so large Planck constants that the cyclotron energies are above thermal energy at physiological temperatures.

Why experimentalists happened to use just this.2 Gauss magnetic field which is 2/5 of the nominal value of the Earth's magnetic field $B_E = .5$ Gauss? If I were a paranoid, I would swear that the experimentalists were well aware of TGD. Of course they were not! One cannot be aware of TGD in a company of respectable scientists and even less in respectable science journals!

9.4.6 Is Gravitational Constant Really Constant?

The most convincing TGD based model for the p-adic coupling constant evolution identified hitherto [L48] predicts that gravitational coupling constant is proportional to the square of p-adic length scale: $G \propto L_p^2$. Together with p-adic length scale hypothesis this would predict that gravitational coupling strength can have values differing from its standard value by a power of 2. $p = M_{127}$ would characterize the space-time sheet mediating ordinary gravitational interactions. In the following possible indications for the variation of G is discussed.

The case of Bullet cluster

The studies of the Bullet cluster [E29, E19], provide the best evidence to date for the existence of dark matter. Bullet cluster [E3] consists of two colliding clusters of galaxies (strictly speaking, the term refers to the smaller one of the two clusters). The major components of the cluster pair, stars, gas and the putative dark matter, behave differently during collision, allowing them to be studied separately.

The stars of the galaxies, observable in visible light, were not greatly affected by the collision, and most passed right through, gravitationally slowed but not otherwise altered. The hot gas of the two colliding components, seen in X-rays, represents about 90 per cent of the mass of the ordinary matter in the cluster pair. The gases interact electromagnetically, so that the velocity change for the gases of clusters is larger than for the stars of clusters. The dominating dark matter component was detected indirectly by its gravitational lensing. The observation that the lensing is strongest in two separated regions near the visible galaxies, confirms with the assumption that most of the mass in the cluster pair is in the form of collisionless dark matter.

Particularly compelling results were inferred from the Chandra observations of the bullet cluster. Those authors report that the cluster is undergoing a high-velocity [around 4500 km/s] merger, evident from the spatial distribution of the hot, X-ray emitting gas, but this gas lags behind the sub-cluster galaxies. Furthermore, the dark matter clump, revealed by the weak-lensing map, is coincident with the collisionless galaxies, but lies ahead of the collisional gas.

Later came the work of Glennys Farrar, Rachel Rosen, and Volker Springler [E56] suggesting that the situation might not be as simple as this (for a popular article see [E59]). The velocity of the bullet of dark matter is higher than it should be in the cold dark matter scenario (CDM). The proposal is that dark matter has its own additional attractive interaction of finite range, "fifth force". Since the finite range of the force is not actually significant in the situation considered, the model is mathematically equivalent with a model assuming that dark gravitational coupling strength. A good fit is obtained by assuming that the net effective gravitational force is by a factor 2 stronger than gravitational force.

The hypothesis is claimed to solve also some other problems of the cold dark matter scenario (CDM). The number of dwarf galaxies around ordinary galaxies is considerably smaller than predicted by CDM. The strong binding of dark matter in dwarfs would make them more compact and this in turn would mean that the binding of visible matter is weaker so that ordinary galaxies would have ripped this matter off and dwarfs would be more difficult to detect. CDM also predicts less galaxy clusters and stronger attraction for dark matter could resolve the problem.

TGD strongly suggests that gravitational constant is proportional to the square of p-adic length scale: $G \propto L_p^2 \equiv L(k)^2$, $p \simeq 2^k$, k integer, in particular power of prime. Ordinary gravitational constant would correspond to $p = M_{127} = 2^{127} - 1$, which is the largest Mersenne prime which is not completely super-astrophysical and corresponds to electron's p-adic length scale. One can however ask whether it might be possible to have situations in which the p-adic length scale assigned to the space-time sheets mediating gravitational interaction differs from M_{127}. $L(k)$ $k = 2^7 = 128$, would would correspond to $G \rightarrow 2G$. The growth of the gravitational coupling strength could be a transient phenomenon taking place only during the collision.

Shrinking kilogram

The definition of kilogram [E8] is not the topics number one in coffee table discussions and definitely not so because it could lead to heated debates. The fact however is that even the behavior of standard kilogram can open up fascinating questions about the structure of space-time.

The 118-year old International Prototype Kilogram is an alloy with 90 per cent Platinum and 10 per cent Iridium by weight (gravitational mass). It is held in an environmentally monitored vault in the basement of the BIPMs House of Breteuil in Sevres on the outskirts of Paris. It has forty copies located around the world which are compared with Sevres copy with a period of 40 years.

The problem is that the Sevres kilogram seems to behave in a way totally in-appropriate taking into account its high age if the behavior of its equal age copies around the world is taken as the norm [C17], [E8]. The unavoidable conclusion from the comparisons is that the weight (gravitational mass) of Sevres kilogram has been reduced by about 50 μg during 118 years which makes about

$$\frac{dlog(m)}{dt} = -4.2 \times 10^{-10}/year \ .$$

for Sevres copy or relative increase of same amount for its copies.

Specialists have not been able to identify any convincing explanation for the strange phenomenon. For instance, there is condensation of matter from the air in the vault which increases the weight and there is periodic cleaning procedure which however should not cause the effect.

1. Could the non-conservation of gravitational energy explain the mystery?

The natural question is whether there could be some new physics mechanism involved. If the copies were much younger than the Sevres copy, one could consider the possibility that gravitational mass of all copies is gradually reduced. This is not the case. One can still however look what this could mean.

Equivalence Principle (EP) holds in classical sense for the GRT limit of TGD understood as effective theory with effective space-time in long length scales defined as M^4 endowed with an effective metric defined as the sum of Minkowski metric and sum of deviations from Minkowski metric for various space-time sheets involved. Thus GRT space-time lumps together the space-time sheets of many-sheeted space-time and there is always a length scale resolution involved. EP reduces to Einstein's equations and reflects underlying Poincare invariance.

In Zero Energy Ontology (ZEO) zero energy states and corresponding space-time surfaces reside always inside some causal diamond (CD) characterized by scale. Therefore the conserved four-momentum assignable to either end of CD is scale dependent quantity, and the apparent non-conservation of four-momentum as scale is changed is not in conflict with Poincare invariance.

At the quantum level EP would hold true in the sense that classical Noether charges in the Cartan algebra of isometries and associated with Kähler action are equal to the eigenvalues of the quantal charges of Kähler-Dirac action: EP would reduce to Quantum classical correspondence. Holography allows to consider also the possibility that gravitational and inertial charges correspond to those assignable to light-like and space-like 3-surfaces respectively.

One cannot expected Einstein's equations and EP for *single space-time sheet* endowed with the induced metric. For the embeddings of metrics (not necessary extremals of Kähler action) for which it is possible to define gravitational energy gravitational energy need not be non-conserved. This occurs even in the case of stationary metrics such as Reissner-Nordström exterior metric and the metrics associated with stationary spherically symmetric star models imbedded as vacuum extremals as has been found.

The basic reason is that Schwartschild time t relates by a scaling and shift to Minkowski time m^0:

$$m^0 = \lambda t + h(r)$$

such that the shift depends on the distance r to the origin. The Minkowski shape of the 3-volume containing the gravitational energy changes with M^4 time but this does not explain the effect. The key observation is that the vacuum extremal of Kähler action is not an extremal of the curvature scalar (these correspond to asymptotic situations). What looks first really paradoxical is that one obtains a constant value of energy inside a fixed constant volume but a non-vanishing flow of energy to the volume. The explanation is that the system simply destroys the gravitational energy flowing inside it! The increase of gravitational binding energy compensating for the feed of gravitational energy gives a more familiar looking articulation for the non-conservation.

Amusingly, the predicted rate for the destruction of the inflowing gravitational energy is of same order of magnitude as in the case of kilogram. Note also that the relative rate is of order $1/a$, a the value of cosmic time of about 10^{10} years. The spherically symmetric star model also predicts a rate of same order.

This approach of course does not allow to understand the behavior of the kilogram since it predicts no change of gravitational mass inside volume and does not even apply in the recent situation since all kilograms are in same age. The co-incidence however suggests that the non-conservation of gravitational energy might be part of the mystery. The point is that if the inflow satisfies Equivalence Principle then the inertial mass of the system would slowly increase whereas gravitational mass measured far enough from gravitational acceleration caused by the *entire* system would remain constant: this would hold true only in steady state.

2. Minimal model assuming EP and many-sheeted space-time?

The minimal model assumes EP and many-sheeted space-time and its description based on GRT. In TGD Universe the increase of the inertial (and gravitational) mass could be due to the flow of matter from larger space-time sheets to the system, now the masses which are brought to Sevres for comparison.

The additional mass would not enter in via the surface of the kilogram but like a Trojan horse from the interior and it would be thus impossible to control using present day technology. The flow would continue until a flow equilibrium would be reached with as much mass leaving the kilogram as entering it.

What would distinguish between Sevres kilogram and its cousins? Could it be that the inertial mass of every kilogram increases gradually until a steady state is achieved? When the system is transferred to another place the saturation situation is changed to a situation in which genuine transfer of inertial and gravitational mass begins again and leads to a more massive steady state. The very process of transferring the comparison masses to Sevres would cause their increase.

As noticed, the in-flow of the gravitational mass per mass unit is of the same order of magnitude as that for the gravitational energy predicted by simple star models. Does this mean that the embedding of Schwartschild metric as vacuum extremal with effective metric identifiable as induced metric is a good approximation? If so then also single-sheetedness would be a good approximation. The hypothesis that GRT space-times correspond to vacuum extremals has indeed been a basic assumption in TGD based models of cosmology and astrophysics [K99, K80].

9.5 Could The Measurements Trying To Detect Absolute Motion Of Earth Allow To Test Sub-Manifold Gravity?

The history of the modern measurements of absolute motion has a long - more than century beginning from Michelson-Morley 1887. The reader can find in web a list of important publications (see `http://tinyurl.com/ycxmc9lb`) [E5] giving an overall view about what has happened. The earliest measurements assumed aether hypothesis. Cahill identifies the velocity as a velocity with respect to some preferred rest frame and uses relativistic kinematics although he misleadingly uses the terms absolute velocity and aether. The preferred frame could galaxy, or the system defining rest system in cosmology. It would be easy to dismiss this kind of experiments as attempts to return to the days before Einstein but this is not the case. It might be possible to gain unexpected information by this kind of measurements. Already the analysis of CMB spectrum demonstrated that Earth is not at rest in the Robertson-Walker coordinate system used to analysis CMB data and similar motion with respect to galaxy is quite possible and might serve as a rich source of information also in GRT based theory.

In TGD framework the situation is especially interesting.

1. Sub-manifold gravity predicts that the effective light-velocity measured in terms of M^4 time taken for a light signal to propagate from point A to B depends on space-time sheet, on points A and B, in particular the distance between A and B. The maximal signal velocity determined in terms of light-like geodesics has this dependence because light-like geodesics for the space-time surface are in general not light-like geodesics for M^4 but light-like like curves. The maximal signal velocity is in general smaller than its absolute maximum obtained light-like geodesics of M^4, depends on particle, and could be larger than for photon space-time sheets. This might explain neutrino super-luminality [C10] [K58].
2. Space-time sheets move with respect to larger space-time sheets and it makes sense to speak about the motion of solar system space-time sheet with respect to galactic space-time sheet and this velocity is in principle measurable. Maximal signal velocity can be defined operationally in terms of time needed to travel from point A to B using Minkowski coordinates of the embedding space as preferred coordinates. It depends on pair of points involved: basically on the direction on and spatial distance along effectively light-like geodesic defined by the sum of the perturbations of the induced metric for the space-time sheets involved. The question is whether one could say something interesting about various experiments carried out to measure the absolute motion interpreted in terms of velocity of space-time sheet with respect to say galactic space-time-sheet.

Also in Special Relativity the motion relative to the rest system of a larger system is a natural notion. In General Relativistic framework situation should be the same but the mathematical description of the situation is somewhat problematic since Minkowski coordinates are not global due to the loss of Poincare invariance as a global symmetry. In practice one must however introduce linear Minkowski coordinates and this makes sense only if one interprets the general relativistic space-time as a perturbation of Minkowski space. This background dependence is in conflict with general coordinate invariance. For sub-manifold gravity the situation is different.

Could the measurements performed already by Michelson-Morley and followers could provide support for the sub-manifold gravity? This might indeed be the case as the purpose of the following arguments demonstrate. The basic results of this analysis are following.

1. The basic formulas for interferometer experiments using relativistic kinematics instead of Galilean one are same as the predictions of Cahill [E51] using different basic assumptions,

and allow to conclude that already the data of Mickelson and Morley show the motion of Earth -not with respect to aether- but with galactic rest system.

2. The only difference is the appearance of the maximal signal velocity $c_\#$ for space-time sheet to which various gravitational fields contribute. In the static approximation sum of gravitational potentials contributes to $c_\#$.
3. This allows to utilize the results of Cahill [E51], who has carried out a re-analysis of experiments trying to detect what he calls absolute motion using these formulas. Cahill has also replicated [E52] the crucial experiments of Witte [E53].
4. The value of the velocity as well as its direction can be determined and the results from various experiments are consistent with each other. The travel time data demonstrate a periodicity due to the rotation of Earth and motion with respect a preferred frame identifiable as a galactic rest frame. The tell-tale signature is the periodicity of sidereal day instead of exact 24 hour periodicity. The travel time for photons shows fluctuations which might be interpreted in terms of gravitational waves having fractal patterns. TGD view about gravitons would suggest that the emission takes place -not as a continuous stream- but in burst-wise manner producing fractal fluctuation spectrum. These fluctuations could show themselves as a jitter also in the neutrino travel times discovered by Opera collaboration [C10].

9.5.1 The Predictions Of TGD For The Local Light-Velocity

An interesting question is what various experiments carried out during more than century could allow to conclude about TGD predictions and what they are.

Theoretical issues

One must answer several questions before one can make predictions.

1. The reduction of light velocity in the case that there are many space-time sheets whose M^4 projections intersect, is described using common M^4 coordinates for the space-time sheets. The induced metric for given space-time sheet is the sum of flat M^4 metric and CP_2 contribution identified as classical gravitational field. The hypothesis is that in good approximation a linear superposition for the effects of the gravitational fields holds true in the sense that a test particle having wormhole throat contacts to these space-time sheets experiences the sum of the gravitational fields of various sheets. Similar description holds for induced gauge fields. From this one can identify the reduced light velocity in the static situation as $c_\# = \sqrt{g_{tt}}$. In a more realistic necessary non-local treatment one calculates the effective light-velocity by assuming that the orbit of massless state n geometric optics approximation is light-like geodesic for the sum of the metric perturbations: this line is not a light-like geodesic of any of the space-time sheets.

   In the general the effective metric defined in this manner is not imbeddable as induced metric. This description of linear super-position allows to circumvent the basic objection against TGD, which is that induced metric and gauge fields are extremely strongly correlated since they are expressible in terms of CP_2 coordinates and their gradients and that the variety of metrics representable as induced metrics is extremely restricted. Same of course applies to gauge fields.
2. How the reduced light-velocity $c_\#$ relates to the reduced light-velocity in medium which is usually described by introducing the notions of free and polarization charges and magnetization and magnetization currents. In the simple situation when polarization tensor is scalar, refractive index n characterizes the reduction of the light velocity: $V = c_\#/n$. Since the reduction of maximal signal velocity due to sub-manifold is purely gravitational and its reduction in medium has an electromagnetic origin, one can argue that the two notions have nothing to do with each other. Hence $c_\#$ should be treated as a local concept possibly depending on direction of motion by taking the limit when light-like geodesic with respect to effective metric becomes infinitesimally short. This dependence can be deduced by comparing light-like geodesics emanating from a point and calculating the maximal signal velocity as a function of direction angles of the light-like geodesic and the spatial distance along it.

3. What happens to the boundary conditions between different media deduced from the structural equations of classical electrodynamics and Maxwell equations? For instance, does the refraction of light take place also when $c_\#$ changes? It might of course be that $c_\#$ changes only in astrophysical scales - maybe at the surfaces of astrophysical objects - and stays constant at the boundaries between two media in laboratory scale but nevertheless this issue should be understood. The safest guess is that at the level of kinematic local Lorentz invariance still holds true so that the tangential wave vectors identifiable in terms of massless momentum components are conserved at boundaries and one obtains law of refraction also now.

4. In TGD Universe space-time sheets can move with respect to each other and the larger space-time sheet defines the analog of absolute reference frame in this kind of situation. Also in cosmology one can assign to CMB radiation a specific frame and Earth indeed moves with respect to it rather than being at rest in the global Robertson-Walker coordinate system. For Earth solar system is one such frame. Galactic rest system is second such preferred reference frame. To both one can assign linear Minkowski coordinates, which play a special physical. The obvious question is whether this kind of motion could be detected and whether the measurements carried out to detect absolute motion could allow to deduce this kind of velocity with respect to galactic rest system.

5. The question is how photons in medium behave when this kind of motion is present. Assume that the medium is characterized by refractive index n so that one has $V = c_\#/n$ and that space-time sheet moves with respect to larger one by velocity v characterized by direction angles and magnitude. Here $c_\# < c_0$ is the maximal signal velocity at the space-time sheet. For definiteness assume that the larger space-time sheet corresponds to galaxy.

   (a) In the measurements of light velocity the light propagates in medium with velocity $V < c_\# < c_0$, and the question is how to describe this mathematically. In his experiments Michelson assumed summation of velocities based on Galilean invariance. This is of course wrong and Special Relativity suggests summation of velocities according to the relativistic formula:

$$\begin{aligned} V &\rightarrow V_1(v,u) \equiv \frac{V+vu}{1+\frac{Vv}{c_\#^2}u)} = \frac{V+vu}{V+\frac{v}{nc_\#}u)} \ , \\ V &= \frac{c_\#}{n} \ , \quad u = cos(\theta) \ . \end{aligned} \quad (9.5.1)$$

   Here θ is the direction of the light signal with respect to the velocity v. This formula might be justified in TGD framework: also photon has very small but non-vanishing mass and summation formula for velocities can be applied. This demands the assumption of local Lorentz invariance made routinely in General Relativity. Also it requires that the complex process of repeated absorption and emission of photons is described by a propagation of photon with the reduced velocity.

   (b) This predicts two effects which might be seen in the experiments trying to measure absolute velocity and its direction. Both solar and galactic gravitational field and also its perturbations - even gravitational waves- can affect the signal velocity via fluctuations in $c_\#$ deduced from the superposition of the perturbative contributions of CP_2 to the effective induced metric. Second effect is due to the change of the propagation time. This change depends on the propagation direction. Note however that also $c_\#$ in general has the directional dependence and only in the situation when the components g_{ti} vanish, this dependence is trivial. In the Newtonian approximation the assumption $g_{ti} \simeq 0$ is made and corresponds to the description of the situation in terms of gravitational potential.

Basic predictions

From the above summarized assumptions one can deduce the basic predictions for what should happen in various experiments measuring $c_\# < c_0$ and v.

1. One can have gravitational reduction or even increase of the light velocity from its standard value which need not corresponds to its absolute maximum. The model for neutrino

super-luminality assumes that $c_\#$ characterizes particle space-time sheets - perhaps massless extremals carrying the small deformations of CP_2 type vacuum extremals - topologically condensed aren magnetic flux tubes characterizing particles. For neutrinos one has $(c_\# - c_0)/c_0 \sim 10^{-5}$ where $c_0 < c$ is what we have used to call light-velocity in vacuum.

2. The variation of the propagation time visible in interferometer experiments as a variation of the position interference fringes with the direction of light signal demonstrates the possible dependence of the light velocity on direction. This dependence is predicted for $n > 1$ only. The motion of Earth around Earth induces the variation of the angle θ even in the situation that the interferometer is not rotated.

It is straightforward to derive a formula for the difference of propagation times along orthogonal arms of the interferometer.

1. What determines the position of interference fringes is the quantity

$$r = \frac{\Delta T(v,\theta)}{T_0} \quad , \ , T_0 = \frac{2L}{V} = \frac{2Ln}{c_\#} \quad . \tag{9.5.2}$$

Here $T(0,0)$ is back and form propagation time for interferometer arm of length L $v = 0$.

2. The time difference ΔT is the difference of times for the propagation back and forth along orthogonal interferometer arms of length L:

$$\begin{aligned} \Delta T &= T(v,\theta) - T(v,\theta+\pi/2) \ , \\ T(v,\theta) &= \frac{L}{V_1(v,u=cos(\theta))} + \frac{L}{V_1(v,-u=cos(\theta+\pi))} \ , \\ V_1(v,u) &= \frac{V+vu}{1+\frac{Vv}{c_\#^2}u} \ , \ u = cos(\theta) \ , \ V = \frac{c_\#}{n} \ . \end{aligned} \tag{9.5.3}$$

Assuming that $\beta = v/c_\#$ is small one obtains

$$\frac{\Delta T}{T_0} \simeq (n^2-1)\beta^2 cos(2\theta) = (n^2-1)(\frac{v}{c_\#})^2 cos(2\theta) \ . \tag{9.5.4}$$

The formula contains also dependence on $c_\#$ and in principle the interferometer could allow to detect gravitational waves via their effect on $c_\#$. The formula is consistent with the formula proposed by Cahill (see `http://tinyurl.com/y9cjp7js`) [E51]. Unfortunately, I am unable to understand the argument of Cahill, who speaks about Lorentz contractions whereas the above arguments assumes just the relativistic addition formula for velocities.

3. For interferometer experiments using gas phase the deviation of n from unity is small: $n = 1+\epsilon$, $\epsilon << 1$ and one can write in good approximation

$$\frac{\Delta T}{T_0} \simeq 2\epsilon\beta^2 cos(2\theta) = 2\epsilon(\frac{v}{c_\#})^2 cos(2\theta) \ . \tag{9.5.5}$$

4. IThe Newtonian picture applied by Michelson-Morley and many followers the basic formula would be

$$\frac{\Delta T}{T_0} \simeq 2\beta^2 n^2 cos(2\theta) \ . \tag{9.5.6}$$

Therefore the value of the velocity deduced by using TGD would be much larger than by using Newtonian kinematics and this means that the small anisotropy of $\beta \simeq 10^{-5}$ reported already by Michelson and Morley is amplified by a factor of order $\sqrt{1/\epsilon} \simeq 10^2/\sqrt{3}$. (one has $\epsilon \simeq 2.9 \times 10^{-4}$ for air and becomes of order $\beta \simeq 10^{-3}$ consistent with the value reported by of Torr and Kolen, De Witte, and Cahill in experiments using propagation of RF light in axial cable. Hence the claim of Cahill that already Michelson and Morley measured the anisotropy of velocity of light would make sense also in TGD framework when appropriately re-interpreted.

Second interesting situation corresponds to one-way and two-way propagation times measured for RF waves propagating along straight co-axial cables.

1. In this case the relevant quantity is

$$\begin{aligned} r &= \frac{\Delta T}{T_0} = \frac{T(v,\theta)-T_0}{T_0} \\ &= \frac{1+\frac{\beta u}{n}}{1+\beta u n} - 1 \simeq \beta(\frac{1-n^2}{n})u - \beta^2 u^2 \ , \\ n &= \frac{c_{\#}}{V} \ , \ u = cos(\theta) \ . \end{aligned} \quad (9.5.7)$$

   For $n = 1+\epsilon$, $\epsilon << 1$ has in good approximation

$$\begin{aligned} r &\simeq -2\epsilon\beta u - \beta^2 u^2 \ , \\ n &= \frac{c_{\#}}{V} \ , \ u = cos(\theta) \ . \end{aligned} \quad (9.5.8)$$

   If one writes $c_{\#} = (1+\epsilon_{\#})c_0$ and $n_0 = 1+\epsilon_0$ (no gravitational perturbations) one obtains in good approximation $\epsilon \simeq \epsilon_{\#} + \epsilon_0$. Again it is essential that r is proportional to the deviation $n-1$.

2. For two-way propagation time the relevant quantity is in the same approximation

$$\begin{aligned} r &= \frac{\Delta T}{2T_0} = \frac{T(v,\theta)+T(v,\theta+\pi)-2T_0}{2T_0} \simeq -\beta^2 u^2 (n^2-1) \simeq -2\epsilon\beta^2 u^2 \ , \\ u &= cos(\theta) \ . \end{aligned} \quad (9.5.9)$$

   The linear term in β is absent in this expression defining the building block of the expression for r interferometer experiments.

All these formulas are consistent with those proposed by Cahill (see `http://tinyurl.com/y9cjp7js`) although the argument leading to them is different. The new element is of course the appearance of $c_{\#}$ bringing in the dependence of the maximal signal velocity on induced space-time metric and therefore gravitational effects.

What can one say about super-luminal neutrinos in this framework?

The proposed framework applies as such to super-luminal neutrinos reported by OPERA collaboration [C10].

1. $n = 1$ is natural for neutrinos so that no directional dependence from the velocity v with respect to the galactic frame is expected. The dependence of $c_{\#}$ on particle type and on the gravitational fields at other parallel space-time sheets could however explain both super-luminality and the observed jitter in the arrival time [C10].
2. The value of $c_{\#}$ depends on the primary space-time sheet along which the neutrinos propagate and could be larger than for the space-time sheets of photons. Massless extremal topologically condensed at the magnetic flux tubes with neutrinos represented by wormhole contacts is a good candidate for neutrino space-time sheet pair. It is also possible that classical Z^0 fields affect the situation by giving rise to a cyclotron orbit [K58].

3. The presence of also other space-time sheets - in particular those assigned to Earth, Sun, and Galaxy - is possible and plausible and they contribute to $c_{\#}$. This contribution is precisely defined if one accepts that in common M^4 coordinates for space-time sheets the sum of CP_2 contributions to the effective metric determines the effective metric and therefore also $c_{\#}$. Also the fluctuations of the gravitational fields suggested by Cahill to have interpretation as gravitational waves affect $c_{\#}$ and therefore maximal signal velocity for neutrinos. The question which does not first come into mind is therefore whether the jitter in the neutrino propagation time is due to gravitational waves!

9.5.2 The Analysis Of Cahill Of The Measurements Trying To Measure Absolute Motion

The primary inspiration for looking various experiments related to the determination of absolute motion came from P. O. Ulianov's proposal described in article *The Witte Effect: The Neutrino Speed and The Anisotropy of the Light Speed, as Defined in the General Theory of Relativity* (see `http://tinyurl.com/ydxhrakx`) [E49].

Ulianov proposed that one could perhaps understand neutrino super-luminality in terms of Witte effect (see `http://tinyurl.com/yb98uhvd`) [E53]. This idea does not to work as such. $n = 1$ is natural for neutrinos and would predict vanishing directional effect. If the directional effect is present it would be oscillatory behavior around a value, which is below c and would not allow super-luminality even momentarily. Fluctuations due to the variations of $c_{\#}$, which itself could be larger than for photon space-time sheets are however possible and could explain the observer jitter in the arrival time [C10].

The reading of this article led to the realization that delicate effects related to the many-sheeted space-time concept might have been observed already by Michelson and Morley, who indeed report a small anisotropy for the magnitude of the light velocity- something that TGD based view about maximal signal velocity indeed suggests. I also found that R. T. Cahill had come into similar thoughts so that I decided to study the articles of Cahill.

1. Cahill describes the history of the experiments trying to detect the absolute motion in his article *Absolute Motion and Gravitational Effects* (see `http://tinyurl.com/y9cjp7js`) [E51]. Cahill has his terminology and own views about the correct interpretation but the open-minded reader should not allow this to disturb too much.

2. A less technical article describing the contribution of De Witte is titled *The Roland De Witte 1991 Experiment (to the Memory of Roland De Witte)* (see `http://tinyurl.com/yb98uhvd`) [E53].

3. The article *A New Light-Speed Anisotropy Experiment: Absolute Motion and Gravitational Waves Detected* (see `http://tinyurl.com/yb98uhvd`) [E52] describes the measurement of Cahill himself using RF waves propagating along co-axial cable. The reader should not take the term "Absolute Motion" too emotionally since it can be replaced with relative motion of a small system with respect to much larger system. The formulas of Witte are also consistent with local Special Relativity although one can disagree about their derivation.

Re-analysis of old experiments by Cahill

There are two basic methods to measure the value of c and detect its possible dependence on the direction of travel. The interferometer experiments were used by Michelson and Morley (see `http://tinyurl.com/y3quue`) [E10] and their followers. The measurements of propagation time for RF signal propagating in co-axial cable were carried out by Torr and Kolen, De Witte and by Cahill. Cahill reports (see `http://tinyurl.com/yb98uhvd`) [E53] that 7 interferometer experiments has been carried out during more than century.

Cahill has re-analyzed (see `http://tinyurl.com/y9cjp7js`) [E51] the earlier interferometer experiments using his theory and concluded that already these experiments reveal the motion with respect to some system - most naturally galactic rest frame - and allow to deduce the magnitude and direction of the velocity of motion. It must be emphasized that all this is consistent with special relativity: the formulas used are just the above formulas obtained by putting $c_{\#} = c$.

Cahill's analysis applies therefore also to TGD predictions. The variability of $c_\#$ gives however additional liberty in interpretation.

1. Cahill analyzes the unpublished experiments of De Witte (1991) (see `http://tinyurl.com/yb98uhvd`) [E53]. RF travel time along co-axial cable was in question. Data was taken over 178 days. The experimental apparatus was already earlier used by Torr and Kolen and is described in detail. The length of the cable was $L = 1.5$ km. The frequency of radio waves was 5 MHz. The refractive index of the cable was $n = 1.5$. The signals were sent between clusters of atomic clocks along RF cable in synchronization purpose.

   The value of the velocity $\beta = v/c$ derived by De Witte and later by Cahill himself, is about 400 km/s corresponding to $\beta \simeq 1.3 \times 10^{-3}$ and surprisingly large. The direction of β coincides with the direction of β given by right ascension ($\alpha = 5.2$ hr, $\delta = 67^0$) deduced by Miller in this interferometer experiments 1932-1933. Cahill interprets β as the velocity of Earth with respect to galactic rest frame. De Witte did not yet realize the possibility of this interpretation. There are also fluctuations in the value of the velocity v deduce in this manner to be discussed later.

   In TGD framework $\Delta T/T$ is proportional $\beta^2 = (vcos(\theta)/c_\#)^2$ and Earth's rotation causes the oscillatory variation of $cos(\theta)$, which is indeed seen: see **Fig.** 1 of the article (see `http://tinyurl.com/yb98uhvd`). Fluctuations in propagation time can be understood as being due to the fluctuations of $c_\#$.

2. Cahill re-analyzes (see `http://tinyurl.com/y9cjp7js`) [E51] the earlier interferometer experiments using what is equivalent with relativistic addition formula for the velocities applied to photons with $V < c$. All interferometer experiments have been regarded to be consistent with Special Relativity. Michelson and Morley (1887) and also Miller (1932-1933) however observed small fringe shifts but interpreted them as measurement errors.

   (a) Miller found $v = 10$ km/s and also deduced the right ascension (see `http://tinyurl.com/476zk5g`) for the velocity as $(\alpha, \delta) = (5.2$ hr, $67^o)$. Cahill obtains $v = 420 \pm 30$ km/s from the re-analysis of Miller experiments and interprets it as a velocity with respect to galactic rest frame. CMB anisotropy corresponds to a motion with respect to "cosmic" rest frame and is 369 km/s in direction characterize by right ascension (α, δ)=(11.20 hr, -7.22^o(, which differs Miller's direction.
   Cahill improves his fit by introducing to velocity field corrections which he calls in-flows and defined from the formula $v^2 = \Phi_{gr}$ for Earth and Sun assuming that v is in radial direction. The corrections are measured using 10 km/s as a natural unit. The first guess is that these corrections might be understood in TGD framework in terms of the effect of the dependence of $c_\# = \sqrt{g_{tt}}$ in static approximation on the gravitational potentials of Earth and Sun.

   (b) The value of v from Michelson-Morley experiments using Galilean kinematics would be about $v = 9$ km/s gives $\beta = v/c \simeq 10^{-5}$. Cahill deduces the value of v using what reduces to relativistic kinematics and obtains $v = 328 \pm 50$ km/s. Cahill also performs a fit using Miller's velocity and direction and obtains what he regards as a good fit.

   (c) Cahill has also repeated the experiments of Witte with improved technology (2006) and reports the results int he article *A New Light-Speed Anisotropy Experiment: Absolute Motion and Gravitational Waves Detected* (see `http://tinyurl.com/y6u9c4yp`) [E52] and obtains results consistent with those of Witte. Unfortunately the terminology of the title and the use of the taboo terms "absolute motion" and "aether" serving as deeply emotional signals for the members of the academic mainstream creates easily misinterpretations. The motion is relative and most naturally relative to the galactic rest system.

Additional observations

Already Witte and later Cahill makes the following additional observations.

1. Already Witte observed that the effective velocity deduced from $\Delta T/T$ for one-way propagation time has an oscillatory behavior with a period consistent with the sidereal day suggesting that the fluctuation is caused by galactic gravitational field rather than being of solar origin.

Hence v would have the most natural interpretation as a velocity for the motion with respect to galactic rest frame.

2. All these experimenters find fluctuations - "turbulence" - in the magnitude of the velocity v deduced using the basic formulas. The fluctuations are illustrated by **Fig.** 2 of [E53] (see `http://tinyurl.com/yb98uhvd`). Cahill reports that the fluctuations have a fractal spectrum (in the sense that no scale is present).

   The model of Cahill forces to assign these fluctuations to the velocity field v. The assumption that the velocity of a solar sized system could fluctuate so rapidly looks non-realistic. Cahill indeed introduces a modification of general relativity in which 3-space is the fundamental object and gravitational field is replaced by a velocity field so that the fluctuations of velocity field would correspond to those of gravitational field. Cahill also suggests the interpretation of the fluctuations as gravitational waves: this looks much more reasonable than the fluctuations in velocity of Earth. Velocity field is assigned to what Cahill calls quantum foam. To me this idea does not look attractive.

   Cahill seems to identify the density of the non-relativistic kinetic energy as gravitational potential: $v^2/2 = \Phi_{gr}$. In Newtonian theory this would correspond to the vanishing of the total energy density. In TGD framework the analog would be the identification of the phase in which Einstein's equations holds true as vacuum extremals for which the induced Kähler field vanishes. Any 4-surface with a CP_2 projection which is Lagrangian and thus at most 2-D sub-manifold of CP_2 satisfies this condition. $c_\# = c$ restriction leaves no other possibility.

   In TGD framework the fluctuations can be assigned to $c_\#$ and therefore to gravitational potential in static approximation so that gravitational waves or their analogs could indeed be in question. Certainly gravitational waves should make themselves visible in $\Delta T/T$. $\Delta c_\#/c_\#$ for the fluctuations would be below 10^{-3}. The amplitude of the fluctuations seem quite large but the idea about the bursts of ordinary gravitons created in the decays of large $\hbar$ gravitons very large energy might produce fractal spectrum.

3. Cahill correctly notices that the interpretation of the interferometer experiments proposed by Michelson and Morley and followers is wrong because a non-relativistic addition formula for the addition of velocities is used. Cahill re-interprets the experiments using formulas which are equivalent with those obtained by replacing Galilean addition of velocities with Lorentzian one, and finds that with his assumptions the findings of the earlier experiments conform with the findings from co-axial cable experiments.

   I must admit that I do not understand the argument of Cahill. Cahill however concludes that ΔT must be proportional to $n(n^2-1)$ rather than n^3 and this implies that the value of β deduced from interferometer experiments is for $n \sim 1$ by a factor $n/\sqrt{n^2-1}$ larger than in Newtonian framework. Cahill also correctly notices that $n > 1$ is essential for a non-trivial effect so that only gas interferometers are capable of observing the motion with respect to galactic rest system. This is obvious from the relativistic additional formula for velocities.

4. Cahill as an honest experimentalist notices also that there is an issue, which is not understood at all in his interpretation. Optical fibers (see `http://tinyurl.com/9632q`) would provide and excellent manner to test the theory. Fiber can be in a form of loop and even 4 meter long fiber could be enough as Cahill notices.

   (a) The amazing finding is that there is no directional effect in this case. Cahill calls this optical fiber effect (see `http://tinyurl.com/yb2pzu8p`) [E52]. Anti-crackpot would of course immediately conclude that the case is closed. As an inhabitant of TGD Universe I have however learned to be very cautious in this kind of situations. There are two ways to reduce the local light velocity.

      i. The standard manner is based on electromagnetic interactions and boils down to refractive index n.

      ii. The new manner relies on gravitational interactions and boils down to deviation of $c_\#$ from c_0. This allows $c_\#$ to depend on condensed matter phase- parameters characterizing the material, to have a slow dependence on position in astrophysical scales, as well as the dependence on the direction of and spatial distance along light-like geodesic in the effective metric (involving sum over CP_2 contributions associated with various space-time sheets involved), and even the dependence on gravitational waves inducing time dependent modification of the effective metric.

(b) The conservative attitude is that $n = 1.5$ for the optical fiber at the static limit is due to electromagnetic interactions but that for the specific frequencies used in IR transmissions $n(f) =\simeq 1$ holds true in excellent approximation. The use of index of refraction at the zero frequency limit would be simply wrong. If I have understood correctly the propagation without absorptions and reflections is the defining property of an ideal optical fibre. This would mean that the light at the frequencies considered propagates without any interactions except the reflections at the boundaries of the optical fiber.

(c) Could the reduction of light velocity from c_0 for optical fiber be mostly due to the reduction of $c_\#$ so that in good approximation one would have $n = 1$? This hypothesis is rather radical and would mean that gravitational physics becomes an essential part of condensed matter physics. What one expects is refraction of gravitational waves and this is expected to take place in astrophysical rather than the scales of the everyday world. This proposal should be also consistent with the meaning of refractive index. In particular, the reduction of light velocity gravitational should give rise to the refraction of light waves also now. For these reasons this proposal does not look realistic.

9.5.3 Cahill's Work In Relation To TGD

Cahill has also introduced a theoretical framework to explain the findings of De Witte and reinterpreted interferometer experiments.

1. Cahill claims that the $v \sim 400$ km/s of Earth with respect to a galactic rest system explains roughly the findings of various experiments. To improve the fit Cahill introduces additional velocities which he interprets as velocities of quantum foam towards Sun and Earth respectively. Cahill seems to interpret gravitational potential as a density of non-relativistic kinetic energy per unit mass: $v^2/2 = \Phi_{gr}$. In TGD framework It might be possible to interpret these additional contributions to the velocity field as counterparts for the contributions of the gravitational potentials of Sun and Earth to the overall gravitational potential and affective $c_\#$ and providing it with a directional dependence.

2. If I have understood correctly Cahill assumes that Lorentz-Fitzgerald contraction occurs but in the Earth's rest system rather than in the rest system with respect to which Earth is moving. The motivation for the assumption is that in the rest system of galaxy time dilation would compensate Lorentz contraction completely. Cahill notices that the deviation of V from c is essential and gives rise to a non-trivial effect for interferometer which is not idealizable as empty space ($n = c/V > 1$). I must admit that I do not understand here Cahill's argument although he ends up with the same formula for $\Delta T/T$ as I do using relativistic addition formula for velocities.

3. Cahill has proposed what he calls quantum flow information theory of gravity (see `http://tinyurl.com/y9zvld5m`) [B45]. Cahill introduces velocity field v, which replaces gravitational potential: $v^2 \propto \Phi_{gr}$, where Φ_{gr} is Newtonian gravitational potential is the basic identification. The motivation is presumably the necessity to introduce radial inward velocities to Sun and Earth in order to improve the interpretation of the various experiments trying to detect absolute motion. Space-time is replaced with 3-space but special relativity is assumed to hold true. This of course makes the theory vulnerable to criticism and D. Martin has criticized Cahill's quantum flow information theory of gravity in *Comments on Cahill's quantum foam inflow theory of gravity* (see `http://tinyurl.com/ya38rgf8`) [B17].

4. The quantum foam in-flow has a physical analogy in TGD framework. Gravitational acceleration involves real four-momentum transfer in TGD Universe. By quantum classical correspondence this transfer should have a space-time counterpart and could be realized in terms of topological field quanta, presumably magnetic flux tubes along which gravitons propagate. The attractiveness of gravitation means inward momentum flux. This picture has been applied to explain Allais effect [K99] in terms of the large Planck constant assignable to space-time sheets mediating gravitational interactions. I have also suggested that the gigantic value of gravitational Planck constant implies that large $\hbar$ gravitons decay to bursts of ordinary

gravitons and instead of a continuous flow of gravitons there would be bursts which would be probably interpreted as noise [K66]. This might even lead to a failure to detect gravitons. The evidence for the fluctuations in the spectrum of $\Delta T/T$ for the travel time in the experiments trying to detect absolute motion might conform with this interpretation.

So: What attitude should one take on Cahill?

1. Anti-crackpot would resolve the irritating cognitive dissonance by claiming that Torr and Kolen, Witte, and Cahill make the same systematic error in their measurements. Experimental apparatus is indeed essentially the same. Also the absence of the directional dependence for optical fibers provides a weapon for easy debunking.
2. The appearance of the sidereal day as a period produces problems for the anti-crackpot. Any systematic effect - say to temperature variations - would have exactly 24 hours period. Anti-crackpot can of course argue that the period is actually this and that sidereal day as period is due to a systematic error or wishful thinking. This is however not very convincing argument. What is also irritating is the fact that the simple formula of Cahill deducible directly from the relativistic formula for the addition of velocities allows to understand satisfactorily all experiments in terms of single velocity β in direction determined by Miller. Could it be that the experiments are right and there is indeed a motion of Earth relative to galaxy causing non-trivial effects?
3. Anti-crackpot might also argue that the model used by Cahill to analyze the experiments is wrong so that the whole issue should be forgotten. Basic formulas are however consistent with special relativity. To my opinion the other notions introduced by Cahill might be seen as an attempt to right direction and could have interpretation in terms of $c_{\#}$ interpreted in terms of a sum of gravitational potentials at the static limit. The genuine new element is that local light velocity can be affected also by gravitation besides electromagnetic effects.

I have nothing personal against theorists but my own conclusion based on experience of decades is that I trust more on experimentalists than theorists. Cahill and his precedessors are excellent experimentalists and might have been able to make discoveries much before the time is ripe for them. These experiments not only give direct support for TGD but could even provide new approach to detect time dependent gravitational perturbations and perhaps even gravitational waves. Although I cannot agree with the theoretical proposals of Cahill, I must admit that they have analogs in TGD framework.

9.6 Miscellaneous Topics

I have collected in this section miscellaneous topics for which I have not found any natural place in preceding sections. They are not necessarily about gravitational anomalies.

9.6.1 Michelson Morley Revisited

The famous Michelson-Morley experiment [E10] carried out for about century ago demonstrated that the velocity of light does not depend on the velocity of the source with respect to the receiver and killed the ether hypothesis. This could have led to the discovery of Special Relativity. Reality is not so logical however: actually Einstein ended up with his Special Relativity from the symmetries of Maxwell's equations. Amusingly, for hundred years later Sampo Pentikäinen told me about a Youtube video reporting a modern version of Michelson-Morley experiment by Martin Grusenick [E46] in which highly non-trivial results challenging the general relativistic view about the nearby gravitational fields of astrophysical objects are reported.

To my best knowledge there is no written document about the experiment of Martin Grusenick in web but the Youtube video [E46] is excellent. The reader willing to learn in more detail how Michelson-Morley interferometer works might find Youtube videos [E1] interesting. The result could be an artefact of the experimental arrangement, and it indeed turned out that the attempt of Frank Pierce to reproduce the effect one year later failed. Pierce also demonstrates in his video [E35] a possible reason for the artefact. I decided however to keep this section as it was since the attempt to explain this probably non-existing effect led to a considerable increase in the understanding of zero energy ontology.

Experimental arrangement and results

Grusenick's interferometer [E46] uses green light (532 nm wavelength) with 5 mW power from a laser powered by a battery. The light from the laser arrives at a beam splitter at angle of 45 degrees with respect to the beam direction and decomposes to two beams in directions orthogonal to each other. These beams are reflected back at mirrors having same distance from the splitter and combine again at beam splitter and travel to what to my best understanding should be a concave mirror magnifying and reflecting the interference pattern to a plywood screen. Also the longer video demonstrates that the mirror must be concave. Grusenick however talks about planar mirror but this cannot be the case if the mirror is orthogonal to the incoming beam as it seems to be (in the German version of the video he uses "einfach" instead of "planar" so that a linguistic lapsus or wrong pattern recognition on my side is probably in question). Video camera records the time development of the interference pattern as the arrangement mounted to a rotating tripod is rotated in plane. The plane is parallel to the Earth's surface in the first experiment and orthogonal to it in the second one.

When the rotation takes place in the plane parallel to the surface of Earth the interference pattern remains stationary during rotation. This is the result that Michelson obtained for 100 years ago. When the plane of rotation is orthogonal to the surface of Earth situation changes dramatically, and there is a clear shift of the interference pattern depending on the angle of rotation. The effect cannot be explained as being due to a motion of Earth with respect to ether since the direction of motion would be orthogonal to the Earth's surface at the measurement point and can be so only at certain measurement times since Earth rotates.

When the rotation plane is orthogonal to the surface of Earth one finds following.

1. The maximum shift of the interference pattern corresponds to 11-11.5 peaks which translates to a distance difference of 22-23 wavelengths from the beam splitter to the two orthogonal reflecting mirrors. The corresponding distance is $x = 11.70 - 12.23$ μm. I do not know the precise distance between the beam splitter and mirror. If it equals to $l = .1$ meters, the difference in distance generated during the travel from the splitter to a mirror and back can be expressed as $x/l = x \times 10^{-5} \sim 10^{-4}$. From the point of view of the failing ether hypothesis this would mean $v/c \sim 10^{-4}$.

2. The shift for the interference pattern becomes stationary and changes sign when the splitter is parallel to the Earth's surface. The vertical distances from the splitter to the mirrors are the same at this point. The maximum shift occurs when the beam splitter forms angle of 45 degrees with the Earth's surface. In this situation the vertical distance difference is maximum since the first mirror is in vertical direction and second mirror in horizontal direction.

3. There is also a slight dependence on time of day.

The result might have a trivial explanation. The changes of the distance are rather small: of order 10 microns. Suppose that contraction is in question. In TGD framework there are two distances involved: M^4 distance and the distance defined by the induced metric. The M^4 distance between mirror and splitter in vertical position might shorten by a contraction due to the weight of the system. Alternatively the contraction (if contraction is in question) could correspond to a shortening of the length in the induced metric leaving Minkowski distance invariant. One must estimate the shortening due to the weight of the system to clarify this issue.

Estimate for the change of distance implied by elasticity

The elasticity of the steel plate at which the system is mounted induces by its own weight a change of the vertical distance between beam splitter and mirror above it. Since only order of magnitude estimate is in question, the effects of the instruments mounted on the steel plate are not taken into account in the model so that system (steel plate) is effectively one-dimensional.

1. Using standard elasticity theory in one-dimensional situation [D1], one can express the counterpart for Newton's equations for an effectively one-dimensional elastic medium in a static equilibrium under its own gravitational force as

$$E\partial_z^2 u + \rho g = 0 \tag{9.6.1}$$

Here u denotes displacement, and E is Young's modulus. ρ is the mass density of the medium and g is the acceleration of gravity at the surface of Earth. The equation states that gravitational force is compensated by the atomic forces modeled using a linear force density $f = E\partial_z^2 u$.

2. From this equation one can solve the displacement $u(z)$ as

$$u(z) = -\frac{\rho g z^2}{2E} + bz + c \ . \quad (9.6.2)$$

At the bottom of the plate one has $u(0) = 0$. If one has also $(du/dz)(0) = b = 0$ implied by the condition that momentum current in the vertical direction vanishes at the upper end, one obtains

$$u(z) = -\frac{\rho g z^2}{2E} \ . \quad (9.6.3)$$

This gives for the change of the distance between beam splitter and mirror in vertical position the estimate

$$\Delta h = -\frac{\rho g (h_2^2 - h_1^2)}{2E} \ . \quad (9.6.4)$$

3. A rough estimate for the relevant parameters of the system are following. The height of the steel plate is $h \simeq .5$ m. The height of the beam splitter is $h \simeq .25$ m and the height of the mirror at maximum height is $h \simeq .35$. The density of steel is $\rho \simeq 8 \times 10^3$ kg/m^3. Young's modulus for steel is $E = 2 \times 10^{11} N/m^2$. This gives the estimate $\Delta h = 1.2 \times 10^{-8}$ m, which is by three orders of magnitude smaller than the estimated distance difference.

Schwartschild metric does not explain the result

In the framework of general relativity the only manner to understand these effects is in terms of distance difference along vertical and horizontal directions. This difference would be due to the deviation of the space-time metric from Minkowski metric in such a way that the distance in radial direction is changed due to the presence of gravitational field of Earth.

1. One can start from the Schwartschild metric as an idealized model for the situation outside the surface of Earth (I use units with $c = 1$).

$$\begin{aligned} ds^2 &= Kdt^2 - \frac{1}{K}dr^2 - r^2(d\theta^2 + sin^2(\theta)d\phi^2) \ , \\ K &= 1 - \frac{r_s}{r} \ , \ r_s = 2GM \ . \end{aligned} \quad (9.6.5)$$

Here M denotes the mass of Earth and r the distance from the center of mass of Earth. r_s is Schwartschild radius. At the Earth's surface ($r = R$) one has $r_s c^2/R^2 \simeq g = 10 \ m/s^2$, the gravitational acceleration at the surface of Earth. For $M = 0$ one obtains Minkowski metric and no effect.

2. The maximum distance difference x would correspond to

$$\begin{aligned} \frac{x}{l} &= 2\frac{\int_R^{R+l}(\sqrt{\frac{1}{K}} - 1)dr}{l} \ , \\ K &= 1 - \frac{r_s}{r} \ , \ r_s = 2GM \ . \end{aligned} \quad (9.6.6)$$

Here r_s denotes the Schwartschild radius and $R \simeq 6371km$ the Earth's radius. $l \sim .1$ m is the Minkowskian distance between beam splitter and mirror. Since the value of K is extremely small, the integral can be evaluated easily and gives (not surprisingly)

$$\frac{x}{l} \simeq \frac{r_s}{R} = \frac{2gR}{c^2} \simeq 1.4 \times 10^{-9} \ . \tag{9.6.7}$$

The predicted value is by a factor of order 10^{-5} too small if one assumes $l = .1$ m.

The modification of Schwartschild metric explaining the result

If the finding of Grusenick is real it means that the value of g_{rr} at the Earth's surface is much larger than for Schwartschild metric. One cannot exclude a large deviation of g_{rr} from the prediction of Schwartschild metric also in the case of stars by what is known about planetary orbits. The point is that for exactly circular orbits g_{rr} does not appear at all in the equations determining the orbits since $dr = 0$ holds true for these orbits. For elliptic orbits the effects are in principle visible and Mercury's perihelion shift poses bounds on the deviation.

1. To see what the needed deviation means quantitatively it is convenient to parameterize the deviation as

$$g_{rr} \rightarrow (1 + \Delta(r))g_{rr} = -(1 + \Delta(r))\frac{1}{K} \ . \tag{9.6.8}$$

   Restricting the consideration to the free fall near the Earth's surface one can perform the approximation $\Delta(r) \simeq \Delta(R)$. The value of Δ is fixed by the results of the experiment of Grusenick to be of order

$$\Delta(R) \simeq 5 \times 10^{-4} \tag{9.6.9}$$

   if the distance between the mirror and beam splitter is taken to be.1 m (the estimate for the distance is by bare eyes from the video).

2. Einstein's equations for a free fall in radial direction give

$$\frac{d^2t}{ds^2} + 2\left\{ \begin{array}{c} t \\ r\ r \end{array} \right\}\frac{dt}{ds} fracdrds = 0 \ ,$$
$$g_{tt}(\frac{dt}{ds})^2 + g_{rr}(\frac{dr}{s})^2 = 1 \ . \tag{9.6.10}$$

3. The first equation can be integrated to give

$$\frac{dt}{ds} = \frac{C}{g_{tt}} = \frac{C}{1-K} \ . \tag{9.6.11}$$

   The result is same as for Scwartshcild metric. The constant C is determined by the initial height in free fall.

4. The second equation expresses the conservation of energy. One can solve dr/ds from it in terms of E and g_{rr}. For Schwartschild metric one obtains

$$(\frac{dr}{ds})^2 - \frac{K}{r} = C^2 - 1 \equiv 2E \ . \tag{9.6.12}$$

   The interpretation in terms of energy conservation is obvious. For the modified metric one obtains

$$(\frac{dr}{ds})^2 - (1 + \Delta)\frac{K}{r} = (1 + \Delta)2E \ . \tag{9.6.13}$$

The results is same as obtained for Schwartschild metric if the value of the Newton's constant Gandenergy E are replaced with effective values given by

$$G_{eff} = (1+\Delta)G \ , \ E_{eff} = (1+\Delta)E \ . \quad (9.6.14)$$

5. The dependence on time of day might reflect a similar contribution of the gravitational field of Sun to the gravitational field if the radial component of Sun's gravitational field has similar behavior. From the $1/r$ dependence of Δ the order of magnitude for the additional contribution assuming $\Delta_S(R_S) = \Delta(R)$ (R_S denotes solar radius and r_E the distance of Earth from Sun in the following formula) would be given by

$$\frac{\Delta_S(r_E)}{\Delta(R)} \sim \frac{R_S}{r_E} \sim 4.6\times 10^{-2} \ . \quad (9.6.15)$$

Therefore the effect of Sun could be visible in the interference pattern and would be maximal when the Sun the measurement point and Sun are at same line and minimal when the normal of Earth is orthogonal to the line connecting Earth with Sun.

For a free fall in a direction orthogonal to the surface of Earth the effect is maximum since g_{rr} is visible in the geodesic equations of motion and means that the effective value of GM estimated in this manner would differ from its actual value. There are several ways, such as Cavendish experiment [E4] to measure G and from the measured value of g to deduce also the value of M. The values of G however vary in surprisingly wide range [E11] with variations up to one per cent. If similar behave holds true also for the gravitational fields of masses used in the experiments determining the value of G, it might be possible to understand these deviations.

What can one conclude about $\Delta(r)$ if the mass density remains zero outside the Earth's surface?

An interesting question is what one can conclude about $\Delta(r)$ by assuming that G^{tt} component for Einstein's tensor remains zero.

1. For a spherically symmetric metric parameterized as

$$ds^2 = Bdt^2 - Adr^2 - r^2 d\Omega^2 \quad (9.6.16)$$

the expressions for the components of Einstein tensor for spherically symmetric stationary metric are deduced in this this chapter and given by

$$\begin{aligned} G^{rr} &= \frac{1}{A^2}(-\frac{\partial_r B}{Br} + \frac{(A-1)}{r^2}) \ , \\ G^{\theta\theta} &= \frac{1}{r^2}[-\frac{\partial_r^2 B}{2BA} + \frac{1}{2Ar}(\frac{\partial_r A}{A} - \frac{\partial_r B}{B}) \\ &+ \frac{\partial_r B}{4AB}(\frac{\partial_r A}{A} + \frac{\partial_r B}{B}] \ , \\ G^{tt} &= \frac{1}{AB}(-\frac{\partial_r A}{Ar} + \frac{(1-A)}{r^2}) \ . \end{aligned} \quad (9.6.17)$$

2. In the recent case one obtains

$$\begin{aligned} G^{tt} &= -\frac{1}{(1+\Delta)r^2}\times(\frac{r\partial_r\Delta}{1+\Delta} + \frac{\Delta}{1-\frac{r_s}{r}}) \ , \\ G^{rr} &= \frac{\Delta}{(1+\Delta)r^2}(1-\frac{r_s}{r}) \ , \\ G^{\theta\theta} &= \frac{\partial_r\Delta}{2(1+\Delta)r^3}\times(2-3\frac{r_s}{r}) \ , \\ r_s &= 2GM \ . \end{aligned} \quad (9.6.18)$$

$r_s = 2GM$ denotes Schwartschild radius.

3. If one requires that the mass density outside the Earth's radius vanishes one obtains $G^{tt}=0$ giving the differential equation

$$\frac{\partial_r \Delta}{\Delta(1+\Delta)} = -\frac{1}{r(1-\frac{r_s}{r}} . \quad (9.6.19)$$

The solution is

$$\Delta(r) = \frac{X}{1-X} , \quad X = \frac{R}{r}\times\frac{1-\frac{r_s}{R}}{1-\frac{r_s}{r}}\times\frac{\Delta(R)}{1+\Delta(R)} . \quad (9.6.20)$$

The deviation approaches zero like $1/r$. The effects on planetary orbits would be negligible in the case that this expression holds true in case of Sun with same order of magnitude for Δ. Only when the orbits is very near to the surface of the star the situation changes. This situation might prevail for some exoplanets.

4. If the deviation is due the interaction of the massive body with the external world, it is dictated in the first approximation by the average density of matter in cosmos and by the geometry of the body meaning that the function $\Delta(r)$ for $r \gg R$ is universal and therefore same for all systems.

Is the proposal consistent with causality?

The general expressions for the Einstein tensor listed above allow to deduce how the "pressure" components G^{rr} and $G^{\theta\theta}$ of the Einstein tensor are affected. $G^{tt}=0$ combined with non-vanishing of "pressure" components of $G^{\alpha\beta}$ seems to break causality. It is also at odds with the general wisdom about the structure of a typical energy momentum tensor of matter. The attempt to understand what is involved induces a series of arguments and counter arguments leading to what seems to provide a deeper understanding of Einstein's equations in zero energy ontology and also of the notion of virtual particle as well as the realization of twistor program in TGD framework.

1. In TGD framework one has sub-manifold gravity for which Einstein equations hold true at long length scale limit with the constraint space-time surfaces are extremals of Kähler action. The Schwartschild coordinates (t,r,θ_M,ϕ_M) for the embedding of Schwartschild solution in $M^4\times CP_2$ are related to Minkowski coordinates (m^0,r_M,θ,ϕ) by the conditions $(m^0 = \Lambda t + h(r_M), r_M = r, \theta_M = \theta, \phi_M = \phi)$. As a consequence, the time component of the energy momentum tensor is non-vanishing in Minkowski coordinates and one might hope that the apparent breaking of causality could be a mere coordinate artefact.

2. A possible general coordinate invariant characterization of the causality would be as the condition $G\geq 0$. In the recent case this condition reads as

$$G = \Delta\left[1-\frac{1}{(1+\Delta)^2}-\frac{x}{2(1-x)}\right]\geq 0 , \quad x=\frac{r_s}{r} . \quad (9.6.21)$$

For small values of Δ the sign of this quantity is determined by the sign of Δ since the first two terms in the brackets cancel each other in good approximation and is positive if Δ is negative. Hence $\Delta\leq 0$ guarantees causality in this sense.

3. In TGD framework one can consider also a stronger form of causality. The vector field $G^{\alpha k} = G^{\alpha r}\partial_r m^k$, where m^k denotes linear coordinates for M^4, is proportional to four-momentum current. It is space-like since $|\partial_r h(r)| < 1$ holds true to guarantee that $t = constant$ 3-surface is space-like so that $G^{\alpha k}$ seems to describe a tachyonic energy momentum current. In quantum context this need not be a catastrophe. Quantum classical correspondence suggests the identification of $G^{\alpha k}$ in the matter free regions as the four-momentum current associated with virtual particles mediating the interactions of the system with the external world. Note that also gravitons must contribute to the energy momentum tensor $T^{\alpha\beta}$ if this is the correct interpretation.

4. It is however very difficult to understand how the energy momentum tensor of matter could behave like $G^{\alpha k}$ does. The resolution of the problem is very simple in zero energy ontology. In zero energy ontology bosons (and their super counterparts) correspond to wormhole contacts carrying fermion and anti-fermion numbers at the light-like wormhole throats and having opposite signs of energy. This allows the possibility that the fermions at the throats are on mass shell and the sum of their momenta gives rise to off mass shell momentum which can be also space-like. In zero energy ontology $G^{\alpha\beta}$ would naturally correspond to the sum of on mass shell energy energy momentum tensors $T_{\pm}^{\alpha\beta}$ associated with positive and negative energy fermions and their super-counterparts. Note that for the energy momentum tensor $T^{\alpha\beta} = (\rho+p)u^\alpha u^\beta - pg^{\alpha\beta}$ of fluid with $u^\alpha u_\alpha = 1$ constraint stating on mass shell condition the allowance of virtual particles would mean giving up the condition $u^\alpha u_\alpha = 1$ for the velocity field.

5. This identification suggests also a nice formulation of the twistor program [K94, ?]. The basic idea is that massive on mass shell states can be regarded as massless states in 8-dimensional sense so that twistor program generalizes to the case of massive on mass shell states associated with the representations of super-conformal algebras. One has however also now off mass shell states and the question is how to describe them in terms of generalized twistors. In the case of wormhole contacts the answer is obvious. Since bosons and their super partners correspond to pairs of positive and negative energy on mass shell states, both on mass shell and of mass shell states can be described using a pair of twistors associated with composite momenta massless in 8-D sense.

6. How can one then interpret virtual fermions and their super-counterparts? Fermions and their super-partners have been assumed to consist of single wormhole throat associated with a deformation of CP_2 vacuum extremal so that the proposed definition would allow only on mass shell states. A possible resolution of the problem is the identification of also virtual fermions and their super-counterparts as wormhole contacts in the sense that the second wormhole throats is fermionic Fock vacuum carrying purely bosonic quantum numbers and corresponds to a state generated by purely bosonic generators of the super-symplectic algebra whose elements are in 1-1 correspondence with Hamiltonians of $\delta M^4_\pm \times CP_2$. Thus the distinction between on mass shell and of mass shell states would be purely topological for fermions and their super partners.

   The concrete physical interpretation would be that particle scattering event involves at least two parallel space-time sheets. Incoming (outgoing) fermion is topologically condensed at positive energy (negative energy) sheet and in the interaction region touches with a high probably the other sheet since the distance between sheets is about 10^4 Planck lengths. The touching (topological sum) generates a second wormhole throat with a spherical topology and carrying no fermion number. Virtual fermions would be fermions in interaction region [K94, K8].

The conclusion would be following. A large deviation of the radial component of the metric from empty space metric near Earth's surface could explain the finding claimed by Grusenick without contradictions with what is known about the metric for planetary orbits assuming that similar deviation occurs quite generally. Michelson-Morley interferometer would provide a very precise method to measure g_{rr} at various heights (say in satellites) so that a very precise testing of the proposed model becomes possible. Also the value of g_{rr} of solar gravitational field at Earth's surface might be deduced from the diurnal variation of the interference pattern.

9.6.2 Various Interpretations Of Machian Principle In TGD Framework

TGD allows several interpretations of Machian Principle and leads also to a generalization of the Principle.

1. Machian Principle is true in the sense that the notion of completely free particle is non-sensible. Free CP_2 type extremal (having random light-like curve as M^4 projection) is a pure vacuum extremal and only its topological condensation creates a wormhole throat (two of them) in the case of fermion (boson). Topological condensation to space-time sheet(s) generates all quantum numbers, not only mass. Both thermal massivation and massivation

via the generation of coherent state of Higgs type wormhole contacts are due to topological condensation.

2. Machian Principle has also interpretation in terms of p-adic physics [K87]. Most points of p-adic space-time sheets have infinite distance from the tip light-cone in the real sense. The discrete algebraic intersection of the p-adic space-time sheet with the real space-time sheet gives rise to effective p-adicity of the topology of the real space-time sheet if the number of these points is large enough. Hence p-adic thermodynamics with given p also assigned to the partonic 3-surface by the Kähler-Dirac operator makes sense. The continuity and smoothness of the dynamics corresponds to the p-adic fractality and long range correlations for the real dynamics and allows to apply p-adic thermodynamics in the real context. p-Adic variant of Machian Principle says that p-adic dynamics of cognition and intentionality in literally infinite scale in the real sense dictates the values of masses among other things.

3. A further interpretation of Machian Principle is in terms of number theoretic Brahman=Atman identity or equivalently, Algebraic Holography [K86]. This principle states that the number theoretic structure of the space-time point is so rich due to the presence of infinite hierarchy of real units obtained as ratios of infinite integers that single space-time point can represent the entire world of classical worlds. This could be generalized also to a criterion for a good mathematics: only those mathematical structures which are representable in the set of real units associated with the coordinates of single space-time point are really fundamental.

9.6.3 Einstein's Equations And Second Variation Of Volume Element

Jacobsen [B47] has derived Einstein's equations from thermodynamical considerations. The argument involves approximate Poincare invariance, Equivalence principle, and proportionality of entropy to area ($dS = kdA$) so that the result is perhaps not a complete surprise.

One starts from an expression for the variation of the area element dA for certain kind of variations in direction of light-like Killing vector field and ends up with Einstein's equations. Ricci tensor creeps in via the variation of dA expressible in terms of the analog of geodesic deviation involving curvature tensor in its expression. Since geodesic equation involves first variation of metric, the equation of geodesic deviation involves its second variation expressible in terms of curvature tensor.

The result raises the question whether it makes sense to quantize Einstein Hilbert action and in light of quantum TGD the worry is justified. In TGD (and also in string models) Einstein's equations result in long length scale approximation whereas in short length scales stringy description would provide the space-time correlate for Equivalence Principle. It has turned out that GRT limit of TGD can be obtained as effective theory in which M^4 is endowed with an effective metric defined as sum of flat Minkowski metric and sum over the deviations of the effective metrics of various space-time sheets from flat metric. Similar description applies to various gauge fields. Classical form of Equivalence Principle reduces to its formulation in GRT. With this interpretation the quantization of the effective metric does not seem sensical.

In the following I will consider different -more than 10 year old - argument implying that empty space vacuum equations state the vanishing of first and second variation of the volume element in freely falling coordinate system and will show how the argument implies empty space vacuum equations in the "world of classical worlds". I also show that empty space Einstein equations at space-time level allow interpretation in terms of criticality of volume element - perhaps serving as a correlate for vacuum criticality of TGD Universe. I also demonstrate how one can derive non-empty space Einstein equations in TGD Universe and consider the interpretation.

Vacuum Einstein's equations from the vanishing of the second variation of volume element in freely falling frame

The argument of Jacobsen leads to interesting considerations related to the second variation of the metric given in terms of Ricci tensor. In TGD framework the challenge is to deduce a good argument for why Einstein's equations hold true in long length scales and one ends up to an idea how one might understand the content of these equations geometrically.

1. The first variation of the metric determinant gives rise to

$$\delta\sqrt{g} = \partial_\mu \sqrt{g} dx^\mu \propto \sqrt{g} \left(\begin{array}{c} \rho \\ \rho \ \mu \end{array} \right) dx^\mu .$$

The possibility to find coordinates for which this variation vanishes at given point of space-time realizes Equivalence Principle locally.

2. Second variation of the metric determinant gives rise to the quantity

$$\delta^2 \sqrt{g} = \partial_\mu \partial_\nu \sqrt{g} dx^\mu dx^\nu = \sqrt{g} R_{\mu\nu} dx^\mu dx^\nu .$$

The vanishing of the second variation gives Einstein's equations in empty space. Einstein's empty space equations state that the second variation of the metric determinant vanishes in freely moving frame. The 4-volume element is critical in this frame.

The world of classical worlds satisfies vacuum Einstein equations

In quantum TGD this observation about second variation of metric led for two decades ago to Einstein's vacuum equations for the Kähler metric for the space of light-like 3-surfaces ("world of classical worlds"), which is deduced to be a union of constant curvature spaces labeled by zero modes of the metric. The argument is very simple. The functional integration over WCW degrees of freedom (union of constant curvature spaces a priori: $R_{ij} = kg_{ij}$) involves second variation of the metric determinant. The functional integral over small deformations of 3-surface involves also second variation of the volume element $\sqrt{g}$. The propagator for small deformations around 3-surface is contravariant metric for Kähler metric and is contracted with $R_{ij} = \lambda g_{ij}$ to give the infinite-dimensional trace $g^{ij} R_{ij} = \lambda D = \lambda \times \infty$. The result is infinite unless $R_{ij} = 0$ holds. Vacuum Einstein's equations must therefore hold true in the world of classical worlds.

Non-vacuum Einstein's equations: light-like projection of four-momentum projection is proportional to second variation of four-volume in that direction

An interesting question is whether Einstein's equations in non-empty space-time could be obtained by generalizing this argument. The question is what interpretation one should give to the quantity

$$\sqrt{g_4} T_{\mu\nu} dx^\mu dx^\nu$$

at a given point of space-time.

1. If one restricts the consideration to variations for which dx^μ is of form $k^\mu \epsilon$, where k is light-like vector, one obtains a situation similar to used by Jacobsen in his argument. In this case one can consider the component dP_k of four-momentum in direction of k associated with 3-dimensional coordinate volume element $dV_3 = d^3x$. It is given by

$$dP_k = \sqrt{g_4} T_{\mu\nu} k^\mu k^\nu dV_3 .$$

2. Assume that dP_k is proportional to the second variation of the volume element in the deformation $dx^\mu = \epsilon k^\mu$, which means pushing of the volume element in the direction of k in second order approximation:

$$\frac{d^2\sqrt{g_4}}{d\epsilon^2} \sqrt{g_4} dV_3 = \frac{d^2\sqrt{g_4}}{\partial x^\mu \partial x^\nu} k^\mu k^\nu \sqrt{g_4} dV_3 = \sqrt{g_4} R_{\mu\nu} k^\mu k^\nu dV_3 .$$

By light-likeness of k^μ one can replace $R_{\mu\nu}$ by $G_{\mu\nu}$ and add also $g_{\mu\nu}$ for light-like vector k^μ to obtained covariant conservation of four-momentum. Einstein's equations with cosmological term are obtained.

That light-like vectors play a key role in these arguments is interesting from TGD point of view since light-like 3-surfaces are fundamental objects of TGD Universe.

The interpretation of non-vacuum Einstein's equations as breaking of maximal quantum criticality in TGD framework

What could be the interpretation of the result in TGD framework.

1. In TGD one assigns to the small deformations of vacuum extremals average four-momentum densities (over ensemble of small deformations), which satisfy Einstein's equations. It looks rather natural to assume that statistical quantities are expressible in terms of the purely geometric gravitational energy momentum tensor of vacuum extremal (which as such is not physical). The question why the projections of four-momentum to light-like directions should be proportional to the second variation of 4-D metric determinant.

2. A possible explanation is the quantum criticality of quantum TGD. For induced spinor fields the Kähler-Dirac equation gives rise to conserved Noether currents only if the second variation of Kähler action vanishes. The reason is that the Kähler-Dirac gamma matrices are contractions of the first variation of Kähler action with ordinary gamma matrices.

3. A weaker condition is that the vanishing occurs only for a subset of deformations representing dynamical symmetries. This would give rise to an infinite hierarchy of increasingly critical systems and generalization of Thom's catastrophe theory would result. The simplest system would live at the V shaped graph of cusp catastrophe: just at the verge of phase transition between the two phases.

4. Vacuum extremals are maximally quantum critical since both the first and second variation of Kähler action vanishes identically. For the small deformations second variation could be non-vanishing and probably is. Could it be that vacuum Einstein equations would give gravitational correlate of the quantum criticality as the criticality of the four-volume element in the local freely falling frame. Non-vacuum Einstein equations would characterize the reduction of the criticality due to the presence of matter implying also the breaking of dynamical symmetries (symplectic transformations of CP_2 and diffeomorphisms of M^4 for vacuum extremals).

Chapter 10

About the Nottale's formula for h_{gr} and the relation between Planck length and CP_2 length

10.1 Introduction

This chapter is about two topics: about the identification of the parameter v_0 with dimensions of velocity appearing in the Nottale's formula for gravitational Planck constant [L34], and about possible TGD explanation for the observed variation of gravitational constant assuming that Planck length l_P is actually CP_2 radius R as the condition that TGD as a TOE has only one fundamental length requires, and that the formula $G = R^2/\hbar_{eff}$ holds true meaning that Newton's constant is different for various levels in dark matter hierarchy [L44].

10.1.1 About the physical interpretation of the velocity parameter in the formula for the gravitational Planck constant

Nottale's formula [E25] for the gravitational Planck constant $\hbar_{gr} = GMm/v_0$ involves parameter v_0 with dimensions of velocity. I have worked with the quantum interpretation of the formula [K79, K66, K67, **?**] but the physical origin of v_0 - or equivalently the dimensionless parameter $\beta_0 = v_0/c$ (to be used in the sequel) appearing in the formula has remained open hitherto. In the following a possible interpretation based on many-sheeted space-time concept, many-sheeted cosmology, and zero energy ontology (ZEO) is discussed.

A generalization of the Hubble formula $\beta = L/L_H$ for the cosmic recession velocity, where $L_H = c/H$ is Hubble length and L is radial distance to the object, is suggestive. This interpretation would suggest that some kind of expansion is present. The fact however is that stars, planetary systems, and planets do *not* seem to participate cosmic expansion. In TGD framework this is interpreted in terms of quantal jerk-wise expansion taking place as relative rapid expansions analogous to atomic transitions or quantum phase transitions. The TGD based variant of Expanding Earth model assumes that during Cambrian explosion the radius of Earth expanded by factor 2 [L40] [L40, L39, L47].

There are two measures for the size of the system. The M^4 size L_{M^4} is identifiable as the maximum of the radial M^4 distance from the tip of CD associated with the center of mass of the system along the light-like geodesic at the boundary of CD. System has also size L_{ind} defined defined in terms of the induced metric of the space-time surface, which is space-like at the boundary of CD. One has $L_{ind} < L_{M^4}$. The identification $\beta_0 = L_{M^4}/L_H < 1$ does not allow the identification $L_H = L_{M^4}$. L_H would however naturally corresponds to the size of the magnetic body of the system in turn identifiable as the size of CD.

One can deduce an estimate for β_0 by approximating the space-time surface near the light-cone boundary as Robertson-Walker cosmology, and expressing the mass density ρ defined as $\rho = M/V_{M^4}$, where $V_{M^4} = (4\pi/3)L_{M^4}^3$ is the M^4 volume of the system. ρ can be expressed as a fraction

ϵ^2 of the critical mass density $\rho_{cr} = 3H^2/8\pi G$. This leads to the formula $\beta_0 = \sqrt{r_S/L_{M^4}} \times (1/\epsilon)$, where r_S is Schwartschild radius.

This formula is tested for planetary system and Earth. The dark matter assignable to Earth can be identified as the innermost part of inner core with volume, which is .01 per cent of the volume of Earth. Also the consistency of the Bohr quantization for dark and ordinary matter is discussed and leads to a number theoretical condition on the ratio of the ordinary and dark masses.

$\beta_0/4\pi$ is analogous to gravitational fine structure constant for $h_{eff} = h_{gr}$. Could one see it as fundamental coupling parameter appearing also in other interactions at quantum criticality in which ordinary perturbation series diverges? Remarkably, the value of G does not appear at all in the perturbative expansion in quantum critical phase! Could G can have several values?

There is also a problem: the twistorialization of TGD [K78] leads to the conclusion that the radius of twistor sphere for M^4 is given by Planck length l_P so that - contrary to the view held for decades - one would have two fundamental lengths - l_P and CP_2 radius R and there is no idea about how they are related. Quantum criticality cannot relate them since they are not coupling parameters.

The formula for $G = l_p^2/\hbar$ however suggests a generalization $G = R^2/h_{eff}$ with h_{eff}/h_0 having value in the range $10^7 - 10^8$: one would have $l_P = R$! Also classical gravitation could tolerate the spectrum of G since Newton's equations in gravitational field is invariant under scaling $h_{eff} \rightarrow xh_{eff}$ inducing $G \rightarrow G/x$ and $t \rightarrow t/x, r \rightarrow r/x$ with scales up the size scale of space-time sheets as the proportionality of Compton length to h_{eff} requires.

10.1.2 Is the hierarchy of Planck constants behind the reported variation of Newton's constant?

Nowadays it is fantastic to be a theoretical physicists with a predictive theory. Every week I get from FB links to fascinating experimental findings crying for explanation (I am grateful for people providing these links). The last link of this kind was to a popular article (see http://tinyurl.com/ya2wekch) telling about the article [E58] (see http://tinyurl.com/yanvzxj6) reporting measurements of Newton's constant G carried out by Chinese physicists Shan-Qing Yang, Cheng-Gang Shao, Jun Luo and colleagues at the Huazhong University of Science and Technology and other institutes in China and Russia. The outcomes of two experiments using different methods differ more than the uncertainties in the experiments, which forces to consider the possibility that G can vary.

The experiments

The experiments use torsion pendulum: this method was introduced by Henry Cavendish in 1978.

Remark: A remark about terminology is in order. Torque $\tau = F \times r$ on particle has dimensions Nm. Torsion (see http://tinyurl.com/q8esymu) in solid is essentially the density of torque per volume and has dimensions N/m^2. Twist angle is induced by torsion in equilibrium. The situation is governed by the theory of elasticity.

Basically one has torsion balance in which the gravitational torque produced by two source masses on masses associated with a torsion pendulum - dumbbell shaped system having identical masses at the ends of a bar and hanging from a thread at the middle point of the bar. As the source masses are rotated a twist of the thread emerges and twist angle corresponds to an equilibrium in which the torsion of the thread compensates the torque produced by gravitational interaction with source masses. Cavendish achieved 1 per cent accuracy in his measurements.

Refined variations of these measurements have been developed during years and the current precision is 47 parts per million (ppm). In some individual experiments the precision is 13.7 ppm. Disagreements larger than 500 ppm are reported, which suggests that new physics might be involved.

The latest experiments were made by the above mentioned research group. Two methods are used. TOS (Time Of Swing) and AAF (Angular Acceleration Feedback). AAF results deviates from the accepted value whereas TOS agrees. The accuracies were 11.64 ppm and 11.61 ppm in TOS and AAF respectively. AAF however gave by 45 ppm larger value of G.

In TOS technique the pendulum oscillates. The frequency of oscillation is determined by the positions of the external masses and G can be deduced by comparing frequencies for two different mass configurations. There are two equilibrium positions. The pendulum is either parallel to the line connecting masses relatively near to each other ("near" position). The pendulum orthogonal to the line connecting masses in "far" position. By measuring the different oscillation frequencies one can deduce the value of G.

Angular-acceleration feedback (AAF) method involves rotating the external masses and the pendulum on two separate turn tables. Twist angle is kept zero by changing the angular velocity of the other turn table: thus feedback is involved. If I have understood correctly, the torsion induced by gravitational torque compensates the torsion created by twisting of the thread around its axis in opposite direction and from the value of torsion for zero twist angle one deduces G. One could perhaps say that in AAF torsion is applied actively whereas in TOS it appears as reaction.

Why the measured value obtained for G would be larger for AAF? Could the active torsion inducing compensating twisting of the torsion pendulum actually increase G?

A possible TGD explanation for the variation of G

In TGD framework the hierarchy of Planck constants $h_{eff} = nh_0$, $h = 6h_0$ together with the condition that theory contains CP_2 size scale R as only fundamental length scale, suggest the possibility that Newtons constant is given by $G = R^2/\hbar_{eff}$, where R replaces Planck length ($l_P = \sqrt{\hbar G} \rightarrow l_P = R$) and $\hbar_{eff}/h$ is in the range $10^6 - 10^7$. The spectrum of Newton' constant is consistent with Newton's equations if the scaling of $\hbar_{eff}$ inducing scaling G is accompanied by opposite scaling of M^4 coordinates in $M^4 \times CP_2$: dark matter hierarchy would correspond to discrete hierarchy of scales given by breaking of a continuous scale invariance to a discrete one.

In the special case $h_{eff} = h_{gr} = GMm/v0$ - gravitational Planck constant originally introduced by Nottale [E25]- assignable to quantum critical dynamics gravitational fine structure constant $\alpha_{gr} = GMm/(4\pi\hbar_{gr}) = (v_0/c)/4\pi$ serves as coupling constant and has no dependence of the value of G or masses M and m in accordance with the universality of quantum critical dynamics.

In this chapter I consider a possible interpretation for the finding of a Chinese research group measuring two different values of G differing by 47 ppm in terms of varying h_{eff}. Also a model for fountain effect of superfluidity as de-localization of wave function and increase of the maximal height of vertical orbit due to the change of the gravitational acceleration g at surface of Earth induced by a change of h_{eff} due to super-fluidity is discussed. Also Podkletnov effect is considered. TGD inspired theory of consciousness allows to speculate about levitation experiences possibly induced by the modification of G_{eff} at the flux tubes for some part of the magnetic body accompanying biological body in TGD based quantum biology.

10.2 About the physical interpretation of the velocity parameter in the formula for the gravitational Planck constant

Nottale's formula [E25] for the gravitational Planck constant $\hbar_{gr} = GMm/v_0$ involves parameter v_0 with dimensions of velocity. I have worked with the quantum interpretation of the formula [K79, K66, K67, **?**] but the physical origin of v_0 - or equivalently the dimensionless parameter $\beta_0 = v_0/c$ (to be used in the sequel) appearing in the formula has remained open hitherto. In the following a possible interpretation based on many-sheeted space-time concept, many-sheeted cosmology, and zero energy ontology (ZEO) is discussed.

A generalization of the Hubble formula $\beta = L/L_H$ for the cosmic recession velocity, where $L_H = c/H$ is Hubble length and L is radial distance to the object, is suggestive. This interpretation would suggest that some kind of expansion is present. The fact however is that stars, planetary systems, and planets do *not* seem to participate cosmic expansion. In TGD framework this is interpreted in terms of quantal jerk-wise expansion taking place as relative rapid expansions analogous to atomic transitions or quantum phase transitions. The TGD based variant of Expanding Earth model assumes that during Cambrian explosion the radius of Earth expanded by factor 2 [L40] [L40, L39, L47].

There are two measures for the size of the system. The M^4 size L_{M^4} is identifiable as the maximum of the radial M^4 distance from the tip of CD associated with the center of mass of the system along the light-like geodesic at the boundary of CD. System has also size L_{ind} defined defined in terms of the induced metric of the space-time surface, which is space-like at the boundary of CD. One has $L_{ind} < L_{M^4}$. The identification $\beta_0 = L_{M^4}/L_H < 1$ does not allow the identification $L_H = L_{M^4}$. L_H would however naturally corresponds to the size of the magnetic body of the system in turn identifiable as the size of CD.

One can deduce an estimate for β_0 by approximating the space-time surface near the light-cone boundary as Robertson-Walker cosmology, and expressing the mass density ρ defined as $\rho = M/V_{M^4}$, where $V_{M^4} = (4\pi/3)L^3_{M^4}$ is the M^4 volume of the system. ρ can be expressed as a fraction ϵ^2 of the critical mass density $\rho_{cr} = 3H^2/8\pi G$. This leads to the formula $\beta_0 = \sqrt{r_S/L_{M^4}} \times (1/\epsilon)$, where r_S is Schwartschild radius.

This formula is tested for planetary system and Earth. The dark matter assignable to Earth can be identified as the innermost part of inner core with volume, which is .01 per cent of the volume of Earth. Also the consistency of the Bohr quantization for dark and ordinary matter is discussed and leads to a number theoretical condition on the ratio of the ordinary and dark masses.

$\beta_0/4\pi$ is analogous to gravitational fine structure constant for $h_{eff} = h_{gr}$. Could one see it as fundamental coupling parameter appearing also in other interactions at quantum criticality in which ordinary perturbation series diverges? Remarkably, the value of G does not appear at all in the perturbative expansion in quantum critical phase! Could G can have several values?

There is also a problem: the twistorialization of TGD [K78] leads to the conclusion that the radius of twistor sphere for M^4 is given by Planck length l_P so that - contrary to the view held for decades - one would have two fundamental lengths - l_P and CP_2 radius R and there is no idea about how they are related. Quantum criticality cannot relate them since they are not coupling parameters.

The formula for $G = l_P^2/\hbar$ however suggests a generalization $G = R^2/h_{eff}$ with h_{eff}/h_0 having value in the range $10^7 - 10^8$: one would have $l_P = R$! Also classical gravitation could tolerate the spectrum of G since Newton's equations in gravitational field is invariant under scaling $h_{eff} \rightarrow xh_{eff}$ inducing $G \rightarrow G/x$ and $t \rightarrow t/x, r \rightarrow r/x$ with scales up the size scale of space-time sheets as the proportionality of Compton length to h_{eff} requires.

10.2.1 Formula for the gravitational Planck constant and some background

The formula

$$\hbar_{gr} = \frac{GMm}{v_0} \tag{10.2.1}$$

for the gravitational Planck constant was originally introduced by Nottale [E25]. Here v_0 is a parameter with dimensions of velocity.

The formula is expected to hold true at the magnetic flux tubes mediating gravitational interaction and obeying also the general formula

$$h_{gr} = h_{eff} \quad , \quad h_{eff} = nh_0 \quad , \quad h = 6h_0 \quad . \tag{10.2.2}$$

The support for the formula $h = 6h_0$ is discussed in [L18, L36]. The value of h_{gr} can be very large unlike the value of h_{eff} associated with say valence bonds.

There are two kinds of flux tubes - homologically non-trivial and trivial ones corresponding to two kinds of geodesic spheres of CP_2, and they seem to correspond to small and large values of h_{eff}.

1. Since the Kähler magnetic energy of homologically non-trivial flux tubes carrying monopole magnetic flux is large, the natural expectation is that gravitation and presumably also other long range interactions mediated by massless particles - with color interactions perhaps forming an exception - correspond to homologically trivial flux tubes for which only the volume

energy due to cosmological constant is non-vanishing. Massive particles would correspond to flux tubes carrying monopole magnetic flux associated with homologically non-trivial flux tubes. Homology could therefore define a key difference between massive and massless bosons at space-time level.

2. One can argue the flux tubes accompanying flux tubes with non-trivial homological charge are relatively short: since the length of the flux tube is expected to be proportional to h_{eff} or its positive power, this would suggest small values of h_{eff} for them. For instance, valence bonds for which non-standard value of h_{eff} is suggestive could correspond to relatively flux tubes carrying monopole flux [L27].

3. Suppose that the value of exponent of Kähler function for the "world of classical worlds" (WCW) is exponent of Kähler function expressible as the 6-D variant of Kähler action for the twistor lift of 4-D Kähler action reducing to the sum of 4-D Kähler action and volume term in the dimensional reduction of the 6-surface to S^2 bundle over space-time surface required by the induction of twistor structure [K94, K78, K8]. If so, the shortness of homologically non-trivial flux tubes could be forced by the large values of Kähler magnetic action and energy making the exponent small.

10.2.2 A formula for β_0 from ZEO

I have made some attempts relate the value of $\beta_0 = v_0/c$ appearing in the formula for h_{gr} to some typical rotation velocity in the system [K79, K66] but although orders of magnitude are reasonable, these attempts have not led to a prediction of v_0. It might be that the explanation is hidden at deeper level and involves many-sheeted space-time and the view about quantum theory based on zero energy ontology (ZEO) in an essential manner.

A generalization of the Hubble formula $\beta = L/L_H$ for the cosmic recession velocity, where $L_H = c/H$ is Hubble length and L is radial distance to the object, is suggestive. Some kind of expansion suggests itself. The fact is however that stars, planetary systems, and planets do *not* seem to participate cosmic expansion. In TGD framework this is interpreted in terms of quantal jerk-wise expansion taking place as relative rapid expansions analogous to atomic transitions or quantum phase transitions. The TGD based variant of Expanding Earth model assumes that during Cambrian explosion the radius of Earth expanded by factor 2 [L40] [L40, L39, L47].

The interpretation of the velocity parameter β_0 to be discussed involves in an essential manner ZEO based quantum measurement theory giving rise to a quantum theory of consciousness [L30]. The causal diamond CD assignable to given conscious entity expands state function reduction by state function and this expansion is very much analogous to cosmic expansion.

In TGD inspired theory of consciousness, which is essentially quantum measurement theory in ZEO [L30], self as a conscious entity corresponds to a sequence of analogs of weak measurements changing the members of state pairs at active boundary of CD and increasing the size of CD by shifting the active boundary farther away from the passive boundary. Passive boundary and the members of state pairs at it remain invariant. This produces a generalized Zeno effect leaving both passive boundary and states at it invariant. This gives the unchanging contribution to the consciousness that one might call "soul". Experienced time corresponds to the increasing distance between the tips of CD and experienced time to the sequence of weak measurements. Active boundary gives rise to changing part in the contents of consciousness. Self dies and reincarnates in opposite time direction when the big state function reduction changing the roles of the boundaries of CD occurs and CD begins to increase in opposite time direction.

To make progress one must consider more precisely what space-time as 4-surface property means in ZEO. The unchanging part of the consciousness corresponds to the passive light-like boundary of CD and various constant parameters should be assigned with the quantum state at it.

There are two measures for the size of the system at the passive boundary and also a measure for the size of its magnetic body mediating gravitational interactions.

1. One can identify M^4 size L_{M^4} as the maximum of the radial M^4 distance from the tip of CD associated with center of mass of the system to the boundary of the system along the light-like geodesic at the passive boundary of CD.

2. System has also size L_{ind} defined as the maximum distance in the induced metric of the space-time surface, which is space-like at the boundary of CD. L_{ind} cannot correspond to Hubble length L_H since this would give $\beta > 0$.

3. A reasonable option is that L_H corresponds to the size scale of the part of the magnetic body of the system responsible for mediation of gravitational interactions. L_H would thus correspond to effective range of gravitational interactions. The simplest guess is that L_H corresponds the maximal radial size of CD given as $L_H = T/2$, where T is the temporal distance between the tips of the CD.

One can deduce an estimate for β_0 by approximating the space-time surface near the passive boundary of CD as Robertson-Walker cosmology. This approximation is indeed natural since space-time surface is small deformation of future/past light-cone near the boundary. The assumption about RW cosmology is *not* needed elsewhere inside CD. This conforms with the holography.

This estimate is only an approximation involving the ratio $\epsilon^2 = \rho/\rho_{cr} < 1$ of the average mass density ρ to the critical mass density

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

besides H. One can consider at least two options.

1. Option I: ρ corresponds to the average density $\rho = M/V_{M^4}$ within M^4 volume $V_{M^4} = (4\pi/3)L_{M^4}^3$ at the passive boundary. The condition $\rho = \epsilon^2\rho_{cr}$ allows to solve $\beta = L/L_H$ as

$$\beta_0 = \frac{L_{M^4}}{L_H} = \frac{1}{\epsilon}\sqrt{\frac{r_S}{L_{M^4}}} \quad , \quad r_S = 2GM \quad . \tag{10.2.3}$$

Here r_S is Schwartschild radius. As noticed, a reasonable identification for L_H would be as the size scale of the gravitational magnetic body given by the size $L_H = T/2$. It turns that this formula is rather reasonable and consistent with earlier results in the case of planetary system and Earth.

2. Option II gives up completely the attempt to interpret the situation in terms of Hubble constant and identifies $\beta_0 = L_{ind}/L_{M^4} < 1$. In this case the expression in terms of mass density in terms of critical mass density does not help to obtain a more detailed formula. If one requires consistency with the previous formula, one obtains L_{ind} as pr $L_{ind} = \sqrt{r_S L_{M^4}}/\epsilon$. For $\epsilon = 1$ one has geometric mean.

10.2.3 Testing the model in the case of Sun and Earth

One can test these equations for Sun and Earth to see whether they could make sense. The restriction to the option I with volume V identified as the volume in the induced metric at the passsive boundary of CD. Option II is obtained at the limit $\epsilon_! = 1$.

Consider first Sun.

1. In the case of Sun the model for the Bohr quantization of planetary orbits was originally proposed by Nottale [E25] and was developed further in TGD framework in [K79, K66] assuming that genuine quantum coherence in astrophysical scales possible for dark matter is in question. The value of β_0 is in a reasonable approximation $\beta_0(inner) = 2^{-11}$ for the 4 inner planets and $\beta_0(out) = \beta_0(inner)/5$ for the outer planets.

2. For the 4 inner planets, the distance of Earth given by astronomical unit $AU = .149 \times 10^9$ km is the natural estimate for L_H so that one has $L_H = AU$. For outer planets the natural choice is of the order of the orbit of the outer planet with largest orbital radius, which is Neptune with distance of 30 AU for Neptune. The prediction of the model for the orbital radius of Neptune is 25 AU so that the estimate looks reasonable. Note that the radii in Bohr model are proportional to $h_{gr}^2 n^2$, n the principal quantum number, so that the scaling $v_0 \to v_0/5$ scales the radius by factor 5^2. This also means that scaling $n \to kn$ and scaling $v_0 \to v_0/k$ produces the same scaled orbital radius.

3. For the 4 inner planets one obtains

$$\beta_0 = \frac{r_S}{L_H} \times \frac{1}{\epsilon} = 1.1 \times 10^{-4} \times \frac{1}{\epsilon} \ .$$

The value co-incides with $\beta_0 = 2^{-11}$ providing a reasonable approximation in Nottale model for $r = 4.55$. This leaves open the fraction $\epsilon^2 = \rho/\rho_{crit}$. One would have $\epsilon^2 = .048$. The size scale of CD would be about $1/\beta = 2^{11}$ using AU as a unit.

Consider next Earth. One can consider two choices for L.

1. Case I: Earth radius $R_E = 6.371 \times 10^3$ km is the first candidate: this choice might be relevant for the applications at Earth's surface such as fountain effect in super-fluidity.
2. Case II: The distance $d_M = 60.3 R_E$ of Moon, is second choice for the scale L. The Schwartschild radius of Earth is $r_S = 9$ mm.

The value of β_0 in these two cases is given by.

$$\beta_0(I) = \sqrt{\frac{r_S}{R_E}}\frac{1}{\epsilon} = .38 \times 10^{-4}\frac{1}{\epsilon} \ ,$$
$$\beta_0(II) = \sqrt{\frac{r_S}{d_M}}\frac{1}{\epsilon} = .04 \times 10^{-4}\frac{1}{\epsilon} \ .$$

The condition $\beta_0(I) = 2^{-11}$ is marginally consistent with the biology related considerations of [L35] and requires $r = 13.16$. The size of the CD would be about $2^{11} R_E$ for option I.

For the same value of r for both I and II one has $\beta(I) = 7.76\beta(II) \simeq 8\beta(II)$ so that option II could be obtained from option I by the scaling $\beta(I) \rightarrow \beta/8$ inducing the scaling $R_E \rightarrow 64 R_E > 60.3 R_E$. By the proportionality of Bohr orbit radius to $1/\beta^2$, the ratio $r(II)/r(I) = \sqrt{64/60.33} = 1.030$ would compensate this error. The mass mass of the moon is $M_M = .012 M_E$ so that the replacement of M_E with the $M_E + M_M$ would produce correction factor 1.012 which is by 2 per cent smaller than the required correction factor.

10.2.4 Under what conditions the models for dark and ordinary Bohr orbits are consistent with each other?

Under what conditions the Bohr orbitologies for dark and ordinary matter are consistent with each other?

1. The condition $v^2 = GM/r$ determines the relationship between velocity and radius in Newtonian theory. The values of v and r cannot therefore change for ordinary matter, which must coupled to all matter - both ordinary and dark matter of the central system.
2. A natural assumption is that dark matter couples only to the dark matter within the volume closed by its orbit. If dark object corresponds to an object modellable as point-like object (the alternative option is that dark matter is along a closed flux tubes along Bohr orbit) then the above condition reads $v_D^2 = GM_D/r$ so that one has

$$\frac{v_D}{v} = \sqrt{\frac{M_D}{M}} \ . \tag{10.2.4}$$

There seems to be no reason why the velocities of dark matter and ordinary matter could not be different. In the case of dark matter there is also Bohr orbit condition giving for gravitational Bohr radius as a generalization of $a_0 = \hbar/\alpha m_e \rightarrow a_{gr} = \hbar_{gr}/\alpha_{gr} m$ with $\alpha = e^2/4\pi\hbar \rightarrow \alpha_{gr} = GMm/4\pi\hbar_{gr} = v_0/4\pi$. This gives

$$a = a_{gr,D} n_D^2 \ , \quad a_{gr} = \frac{4\pi G M_D}{v_0^2} \ . \tag{10.2.5}$$

This formula should be consistent with the formula originally derived for matter and motivated by the idea that ordinary matter forms bound states with dark matter. I have considered also the option that dark matter is delocalized along the flux tube associated with the orbit of the planet.

3. The two formulas make sense simultaneously only if one can interpret the Bohr orbit for M_D as Bohr orbit for M having same radius. This condition gives $M_D n_D^2 = Mn^2$ giving

$$n_D^2 = \frac{M}{M_D} n^2 \ . \tag{10.2.6}$$

Therefore M/M_D should be square of integer, which is rather strong constraint.

One can test this formula in the case of planetary system and for Earth.

1. The first guess is that the inner core of Sun with radius in the range $.2R_S$ and $.25R_S$ corresponds mostly to dark matter. Solar core contains about 34 cent of solar mass (see `http://tinyurl.com/nrcojr2`). This gives in excellent approximation $M/M_D = 3$, which is however not square. $M/M_D = 4$ would satisfy the condition and would have $n_D = 2n$.

   Since dark matter corresponds to extensions of rationals, one can ask whether one could allow for dark matter algebraic integers as values of n_D so that $n_D = \sqrt{3}n$ would be allowed for an extension containing $\sqrt{3}$. This would be a number theoretic generalization of quantization in terms of in terms of integers somewhat analogous to that associated with quantum groups.

2. For Earth the estimate [L35] gives $M/M_D \simeq .5 \times 10^4$ giving $\beta_0 = 4.4 \times 10^{-4}$ rather near to $\beta_0 = 2^{-11} \simeq 5 \times 10^{-4}$. It is enough to find integer sufficiently near to 5000 having the property that it is square. One has $70^2 = 4900$ and $71^2 = 5041$.

   One would have $n_D \simeq 5000 \times n$ and consistency with the formula. Earth has outer core occupying 15 cent of its volume, inner core occupying 1 cent of the volume and innermost inner core with radius 300 km occupying fraction 10^{-4} of the volume (see `http://tinyurl.com/y8vf7vc3`) suggests that the innermost inner core consists of dark mass with density twice the average density.

   Remark: I have considered for M_D a probably too science fictive identification in terms of possibly existing gravitational analog of Dirac monopole. The gravitational flux would emanate radially from the center of the Earth along flux tubes carrying magnetic monopole flux and turn back at certain distance and return back along second space-time sheet and back to the original space-time sheet at wormhole like structure. This field would not be visible at large enough distances.

   If one has $M_D = 2 \times 10^{-4} M_E$, the density of the innermost inner core would be 2ρ, where ρ is the average density of Earth. From Wikipedia (see `http://tinyurl.com/ma6xqnh`) one learns that the average density ρ_E of Earth is $5.52 \times \rho_W$, ρ_W= kg/dm^3 and the density in the inner core varies in the range $\rho/\rho_w \in [12.6 - 13.0]$. The lower limit is approximately $2 \times \rho$. This suggests that the density of the innermost inner core is somewhat larger than 2ρ.

10.2.5 How could Planck length be actually equal to much larger CP_2 radius?!

The following argument stating that Planck length l_P equals to CP_2 radius R: $l_P = R$ and Newton's constant can be identified $G = R^2/\hbar_{eff}$. This idea looking non-sensical at first glance was inspired by an FB discussion with Stephen Paul King.

First some background.

1. I believed for long time that Planck length l_P would be CP$_2$ length scale R squared multiplied by a numerical constant of order $10^{-3.5}$. Quantum criticality would have fixed the value of l_P and therefore $G = l_P^2/\hbar$.

2. Twistor lift of TGD [K94, K8, K78, L50] led to the conclusion that that Planck length l_P is essentially the radius of twistor sphere of M^4 so that in TGD the situation seemed to be settled since l_P would be purely geometric parameter rather than genuine coupling constant. But it is not! One should be able to understand why the ratio l_P/R but here quantum criticality, which should determine only the values of genuine coupling parameters, does not seem to help.

   Remark: M^4 has twistor space as the usual conformal sense with metric determined only apart from a conformal factor and in geometric sense as $M^4 \times S^2$: these two twistor spaces are part of double fibering.

Could CP_2 radius R be the radius of M^4 twistor sphere, and could one say that Planck length l_P is actually equal to R: $l_P = R$? One might get $G = l_P^2/\hbar$ from $G = R^2/\hbar_{eff}$!

1. It is indeed important to notice that one has $G = l_P^2/\hbar$. $\hbar$ is in TGD replaced with a spectrum of $\hbar_{eff} = n\hbar_0$, where $\hbar = 6\hbar_0$ is a good guess [L18, L36]. At flux tubes mediating gravitational interactions one has

$$\hbar_{eff} = \hbar_{gr} = \frac{GMm}{v_0} \quad ,$$

where v_0 is a parameter with dimensions of velocity. I recently proposed a concrete physical interpretation for v_0 [L34] (see `http://tinyurl.com/yclefxb2`). The value $v_0 = 2^{-12}$ is suggestive on basis of the proposed applications but the parameter can in principle depend on the system considered.

2. Could one consider the possibility that twistor sphere radius for M^4 has CP_2 radius R: $l_P = R$ after all? This would allow to circumvent introduction of Planck length as new fundamental length and would mean a partial return to the original picture. One would $l_P = R$ and $G = R^2/\hbar_{eff}$. $\hbar_{eff}/\hbar$ would be of $10^7 - 10^8$!

The problem is that $\hbar_{eff}$ varies in large limits so that also G would vary. This does not seem to make sense at all. Or does it?!

To get some perspective, consider first the phase transition replacing $\hbar$ and more generally $\hbar_{eff,i}$ with $\hbar_{eff,f} = h_{gr}$.

1. Fine structure constant is what matters in electrodynamics. For a pair of interacting systems with charges Z_1 and Z_2 one has coupling strength $Z_1Z_2e^2/4\pi\hbar = Z_1Z_2\alpha$, $\alpha \simeq 1/137$.

2. As shown in [K79, K66, K67, ?] one can also define gravitational fine structure constant α_{gr}. Only α_{gr} should matter in quantum gravitational scattering amplitudes. α_{gr} wold be given by

$$\alpha_{gr} = \frac{GMm}{4\pi\hbar_{gr}} = \frac{v_0}{4\pi} \quad . \tag{10.2.7}$$

$v_0/4\pi$ would appear as a small expansion parameter in the scattering amplitudes. This in fact suggests that v_0 is analogous to α and a universal coupling constant which could however be subject to discrete number theoretic coupling constant evolution.

3. The proposed physical interpretation is that a phase transition $\hbar_{eff,i} \rightarrow \hbar_{eff,f} = h_{gr}$ at the flux tubes mediating gravitational interaction between M and m occurs if the perturbation series in $\alpha_{gr} = GMm/4\pi/\hbar$ fails to converge ($Mm \sim m_{Pl}^2$ is the naïve first guess for this value). Nature would be theoretician friendly and increase h_{eff} and reducing α_{gr} so that perturbation series converges again.

   Number theoretically this means the increase of algebraic complexity as the dimension $n = h_{eff}/h_0$ of the extension of rationals involved increases fron n_i to n_f [L23] and the number n sheets in the covering defined by space-time surfaces increases correspondingly. Also the scale of the sheets would increase by the ratio n_f/n_i.

   This phase transition can also occur for gauge interactions. For electromagnetism the criterion is that $Z_1Z_2\alpha$ is so large that perturbation theory fails. The replacement $\hbar \rightarrow Z_1Z_2e^2/v_0$ makes $v_0/4\pi$ the coupling constant strength. The phase transition could occur for atoms having $Z \geq 137$, which are indeed problematic for Dirac equation. For color interactions the criterion would mean that $v_0/4\pi$ becomes coupling strength of color interactions when α_s is above some critical value. Hadronization would naturally correspond to the emergence of this phase.

   One can raise interesting questions. Is v_0 (presumably depending on the extension of rationals) a completely universal coupling strength characterizing any quantum critical system independent of the interaction making it critical? Can for instance gravitation and electromagnetism are mediated by the same flux tubes? I have assumed that this is not the case. It it could be the case, one could have for $GMm < m_{Pl}^2$ a situation in which effective coupling strength is of form $(GmMm/Z_1Z_2e^2)(v_0/4\pi)$.

The possibility of the proposed phase transition has rather dramatic implications for both quantum and classical gravitation.

1. Consider first quantum gravitation. v_0 does not depend on the value of G at all! The dependence of G on $\hbar_{eff}$ could be therefore allowed and one could have $l_P = R$. At quantum level scattering amplitudes would not depend on G but on v_0. I was of course very happy after having found the small expansion parameter v_0 but did not realize the enormous importance of the independence on G! Quantum gravitation would be like any gauge interaction with dimensionless coupling, which is even small! This might relate closely to the speculated TGD counterpart of AdS/CFT duality between gauge theories and gravitational theories.

2. What about classical gravitation? Here G should appear. What could the proportionality of classical gravitational force on $1/\hbar_{eff}$ mean? The invariance of Newton's equation

$$\frac{d\overline{v}}{dt} = -\frac{GM\overline{r}}{r^3} \tag{10.2.8}$$

under $h_{eff} \rightarrow xh_{eff}$ would be achieved by scaling $\overline{r} \rightarrow \overline{r}/x$ and $t \rightarrow t/x$. Note that these transformations have general coordinate invariant meaning as scalings of Minkowski coordinates of M^4 in $M^4 \times CP_2$. This scaling means the zooming up of size of space-time sheet by x, which indeed is expected to happen in $h_{eff} \rightarrow xh_{eff}$!

What is so intriguing that this connects to an old problem that I pondered a lot during the period 1980-1990 as I attempted to construct to the field equations for Kähler action approximate spherically symmetric stationary solutions [K99]. The naïve arguments based on the asymptotic behavior of the solution ansatz suggested that the one should have $G = R^2/\hbar$. For a long time indeed assumed $R = l_P$ but p-adic mass calculations [K53] and work with cosmic strings [K26] forced to conclude that this cannot be the case. The mystery was how $G = R^2/\hbar$ could be normalized to $G = l_P^2/\hbar$: the solution of the mystery is $\hbar \rightarrow \hbar_{eff}$ as I have now - decades later - realized!

10.3 Is the hierarchy of Planck constants behind the reported variation of Newton's constant?

Nowadays it is fantastic to be a theoretical physicists with a predictive theory. Every week I get from FB links to fascinating experimental findings crying for explanation (I am grateful for people providing these links). The last link of this kind was to a popular article (see `http://tinyurl.com/ya2wekch`) telling about the article [E58] (see `http://tinyurl.com/yanvzxj6`) reporting measurements of Newton's constant G carried out by Chinese physicists Shan-Qing Yang, Cheng-Gang Shao, Jun Luo and colleagues at the Huazhong University of Science and Technology and other institutes in China and Russia. The outcomes of two experiments using different methods differ more than the uncertainties in the experiments, which forces to consider the possibility that G can vary.

In the sequel I consider a possible interpretation for the finding of a Chinese research group measuring two different values of G differing by 47 ppm in terms of varying h_{eff}. Also a model for fountain effect of superfluidity as de-localization of wave function and increase of the maximal height of vertical orbit due to the change of the gravitational acceleration g at surface of Earth induced by a change of h_{eff} due to super-fluidity is discussed. Also Podkletnov effect is considered. TGD inspired theory of consciousness allows to speculate about levitation experiences possibly induced by the modification of G_{eff} at the flux tubes for some part of the magnetic body accompanying biological body in TGD based quantum biology.

10.3.1 The experiments

The experiments use torsion pendulum: this method was introduced by Henry Cavendish in 1978.

Remark: A remark about terminology is in order. Torque $\tau = F \times r$ on particle has dimensions Nm. Torsion (see `http://tinyurl.com/q8esymu`) in solid is essentially the density of torque per volume and has dimensions N/m^2. Twist angle is induced by torsion in equilibrium. The situation is governed by the theory of elasticity.

Basically one has torsion balance in which the gravitational torque produced by two source masses on masses associated with a torsion pendulum - dumbbell shaped system having identical masses at the ends of a bar and hanging from a thread at the middle point of the bar. As the source masses are rotated a twist of the thread emerges and twist angle corresponds to an equilibrium in which the torsion of the thread compensates the torque produced by gravitational interaction with source masses. Cavendish achieved 1 per cent accuracy in his measurements.

Refined variations of these measurements have been developed during years and the current precision is 47 parts per million (ppm). In some individual experiments the precision is 13.7 ppm. Disagreements larger than 500 ppm are reported, which suggests that new physics might be involved.

The latest experiments were made by the above mentioned research group. Two methods are used. TOS (Time Of Swing) and AAF (Angular Acceleration Feedback). AAF results deviates from the accepted value whereas TOS agrees. The accuracies were 11.64 ppm and 11.61 ppm in TOS and AAF respectively. AAF however gave by 45 ppm larger value of G.

In TOS technique the pendulum oscillates. The frequency of oscillation is determined by the positions of the external masses and G can be deduced by comparing frequencies for two different mass configurations. There are two equilibrium positions. The pendulum is either parallel to the line connecting masses relatively near to each other ("near" position). The pendulum orthogonal to the line connecting masses in "far" position. By measuring the different oscillation frequencies one can deduce the value of G.

Angular-acceleration feedback (AAF) method involves rotating the external masses and the pendulum on two separate turn tables. Twist angle is kept zero by changing the angular velocity of the other turn table: thus feedback is involved. If I have understood correctly, the torsion induced by gravitational torque compensates the torsion created by twisting of the thread around its axis in opposite direction and from the value of torsion for zero twist angle one deduces G. One could perhaps say that in AAF torsion is applied actively whereas in TOS it appears as reaction.

Why the measured value obtained for G would be larger for AAF? Could the active torsion inducing compensating twisting of the torsion pendulum actually increase G?

10.3.2 TGD based explanation in terms of hierarchy of Newton's constants

Some time ago I added a piece to an article telling about change in my view about Planck length [L34] (see `http://tinyurl.com/yclefxb2`). In TGD hierarchy of Planck constants is predicted: $\hbar_{eff} = nh_0$ is integer multiple of $h_0 = h/6$. During writing this, it became clear that h_0 need not be minimal value h_{min} of h_{eff} as I have assumed for some time (the first guess was that h is the minimal value).

This suggests also a hierarchy of Newton's constants $G_{eff} = l_P^2/\hbar_{eff}$ as subharmonics of l_P^2, where Planck length l_P is now re-identified as $l_P = R$, where R is CP_2 "radius" which for $G_{eff} = G$ is about $10^{3.5}$ larger than ordinary Planck length $l_P = \sqrt{\hbar G}$. The corresponding value of $\hbar_{eff}$, call it $\hbar_{eff}(gr)$, would be $\hbar_{eff}(gr)/h_{min} \simeq 2^{24}$. $\hbar_{eff}(gr)$ should not be confused with $\hbar_{gr} = GMm/v_0$ proposed by Nottale [E25] which for $M = M_E$ and $m = 2m_p$ is much larger.

Remark: This raises a problem to be discussed in the application to fountain effect. $h_{eff}(gr)$ is by factor of order 2^{24} larger than h, which looks strange since it involves delocalization of wave function to 2^{24} larger scale.

Could the variation of G - or better to call it G_{eff} - correspond to a variation of $h_{eff}/h = n$ in G_{eff}? Newton's constant for dark matter would be different from that for ordinary matter and vary in huge limits.

1. This looks non-sensical at first but would guarantee that one can scale up the solutions to Newton's equations by $h_{eff}/\hbar$ by scaling lengths by $n/n_0 = n/6$ [L18, L36, L37]: one would have thus scaling symmetry scaling also G_{eff} as is natural since it is dimensional parameter. Dark matter would be in rather precise sense zoomed up variants of ordinary matter and n would label the possible zoom ups.

2. h_{eff} has spectrum and as a special case one has $\hbar_{eff} = h_{gr} = GMm/v_0$. Is this case case the gravitational coupling become $G_{eff}Mm = v_0$ and does not depend on masses or G at all. In quantum scattering amplitudes a dimensionless parameter $(1/4\pi)v_0/c$ would appear in the role of gravitational fine structure constant and would be obtained from $\hbar_{eff} = h_{gr} = GMm/v_0$ consistent with Equivalence Principle. The miracle would be that G_{eff} would disappear totally from the perturbative expansion in terms of GMm as one finds by looking what $\alpha_{gr} = GMm/\hbar_{gr}$ is! This picture would work when GMm is larger than perturbative expansion fails to converge. For Mm above Planck mass squared this is expected to be the case. What happens below this limit is yet unclear (n is integer).

   Could v_0 be fundamental coupling constant running only mildly? This does not seem to be the case: Nottale's original work proposing $\hbar_{gr}$ proposes that v_0 for outer planets is by factor 1/5 smaller than for the 4 inner planets [K79, K67].

3. This picture works also for other interactions [?] Quite generally, nature would be theoretician friendly and induce a phase transition increasing $\hbar$ when the coupling strength exceeds the value below which perturbation series converges so that perturbation series converges. In adelic physics this would mean increase of the algebraic complexity since $h_{eff}/h = n$ is the dimension of extension of rationals inducing the extensions of various p-adic number fields and defining the particular level in the adelic hierarchy [L28, L29]. The parameters characterizing space-time surfaces as preferred extremals of the action principle would be numbers in this extension of rationals so that the phase transition would have a well-defined mathematical meaning. In TGD the extensions of rationals would label different quantum critical phases in which coupling constants would not run so that coupling constant evolution would be discrete as function of the extension.

4. This vision allows also to understand discrete coupling constant evolution replacing continuous coupling constant evolution of quantum field theories as being forced by the convergence of perturbation expansion and induced by the evolution defined by the hierarchy of extensions of rationals. When convergence is lost, a phase transition increasing algebraic complexity takes place and increases n. Extensions of rationals have also other characteristics than the dimension n.

   For instance, each extension is characterized by ramified primes and the proposal is that favoured p-adic primes assignable to cognition and also to elementary particles and physics in general correspond to so called ramified primes analogous to multiple zeros of polynomials. Therefore number theoretic evolution would also give rise to p-adic evolution as analog of ordinary coupling constant evolution with length scale.

   At quantum criticality coupling constant evolution is trivial and in QFT context this would mean that loops vanish separately or at least they sum up to zero for the critical values of coupling constants. This argument however seems to make the whole argument about convergence of coupling constant expansion obsolete unless one allows only the quantum critical values of coupling constants guaranteeing that quantum TGD is quantum critical. There are strong reasons to believe that the TGD analog of twistor diagrammatics involves only tree diagrams and there are strong number theoretic argument for this: infinite sum of diagrams does not in general give a number in given extension of rationals. Quantum criticality would be forced by number theory.

5. This would solve a really big conceptual problem, which I did not realize as I discovered the twistor lift of TGD making the choice $M^4 \times CP_2$ unique [K94, K78] [L34]. The usual Planck length $l_P = \sqrt{\hbar G}$ as the radius of the M^4 twistor sphere would separate length scale from CP_2 scale R it is not a coupling constant like parameter and quantum criticality does not allow even in principle its understanding. The presence of two separate fundamental length scales in a theory intended to be unification does simply not make sense.

The variability of G with $\hbar_{eff}$ could explain the variation of G in various experiments since for gravitational flux tubes $\hbar_{eff}/\hbar \sim 10^7$ would be true. The smallest variation would be of order 10^{-7} as n varies by one unit. This is a testable prediction (see `http://tinyurl.com/yclefxb2`).

As already explained, the maximum for the variation of G is 500 ppm $= 5 \times 10^{-4}$. This would correspond to $\Delta n \sim 5 \times 10^3$. The difference between TOS and AAF is 47 ppm and would correspond to $\Delta n \sim 470$. The variation could be also due to a small variation, say $k \to k+1$, for a prime factor k of n. 47 ppm would give $k \simeq 2,128$. For $k = 2^{11} \rightarrow k-1$ in TOS to AAF and favored by number theoretic considerations would give $\Delta k/k = 49$ ppm.

Why small variations for the factors of n would be favored? If one assumes that number theoretical evolution corresponds to the increasing order of the Galois group such that the new Galois group contains the earlier Galois group as a subgroup (this would serve as an analogy for conserved genes in biological evolution). Larger Galois groups would naturally contain the "standard" Galois group associated with N as a sub-group. From number theoretic point of view the proposal $\hbar_{eff}/\hbar = N = 2^{24}$ is perhaps the simplest one since all Galois groups appearing as its sub-groups would have order with is 6×2^k for $h = 6h_0$. Larger values of $h_{eff}/\hbar$ should have N as a factor.

Why the presence of of the feedback torque on the torsion pendulum would reduce the value of $\hbar_{eff}/h = n$ by about 5×10^3 units in AAF for the gravitational flux tubes connecting the source masses to the masses of torsion pendulum from that in TOS? Somehow the value of h_{eff} should be reduced.

10.3.3 A little digression: Galois groups and genes

As found, the question about possible variations of G_{eff}, leads to the idea that subgroups of Galois group could be analogous to conserved genes in that they could be conserved in number theoretic evolution. In small variations such as above variation Galois subgroups as genes would change only a little bit. For instance, the dimension of Galois subgroup would change.

The analogy between subgoups of Galois groups and genes goes also in other direction. I have proposed long time ago that genes (or maybe even DNA codons) could be labelled by $h_{eff}/h = n$. This would mean that genes (or even codons) are labelled by a Galois group of Galois extension (see `http://tinyurl.com/zu5ey96`) of rationals with dimension n defining the number of sheets of space-time surface as covering space. This could give a concrete dynamical and geometric meaning for the notin of gene and it might be possible some day to understand why given gene correlates with particular function. This is of course one of the big problems of biology.

One should have some kind of procedure giving rise to hierarchies of Galois groups assignable to genes. One would also like to assign to letter, codon and gene and extension of rationals and its Galois group. The natural starting point would be a sequence of so called intermediate Galois extensions E^H leading from rationals or some extension K of rationals to the final extension E. Galois extension has the property that if a polynomial with coefficients in K has single root in E, also other roots are in E meaning that the polynomial with coefficients K factorizes into a product of linear polynomials. For Galois extensions the defining polynomials are irreducible so that they do not reduce to a product of polynomials.

Any sub-group $H \subset Gal(E/K))$ leaves the intermediate extension E^H invariant in element-wise manner as a sub-field of E (see `http://tinyurl.com/y958drcy`). Any subgroup $H \subset Gal(E/K))$ defines an intermediate extension E^H and subgroup $H_1 \subset H_2 \subset ...$ define a hierarchy of extensions $E^{H_1} > E^{H_2} > E^{H_3}...$ with decreasing dimension. The subgroups H are normal - in other words $Gal(E)$ leaves them invariant and $Gal(E)/H$ is group. The order $|H|$ is the dimension of E as an extension of E^H. This is a highly non-trivial piece of information. The dimension of E factorizes to a product $\prod_i |H_i|$ of dimensions for a sequence of groups H_i.

Could a sequence of DNA letters/codons somehow define a sequence of extensions? Could one assign to a given letter/codon a definite group H_i so that a sequence of letters/codons would correspond a product of some kind for these groups or should one be satisfied only with the assignment of a standard kind of extension to a letter/codon?

Irreducible polynomials define Galois extensions and one should understand what happens to an irreducible polynomial of an extension E^H in a further extension to E. The degree of E^H increases by a factor, which is dimension of E/E^H and also the dimension of H. Is there a standard manner to construct irreducible extensions of this kind?

1. What comes into mathematically uneducated mind of physicist is the functional decomposition $P^{m+n}(x) = P^m(P^n(x))$ of polynomials assignable to sub-units (letters/codons/genes) with coefficients in K for a algebraic counterpart for the product of sub-units. $P^m(P^n(x))$ would be a polynomial of degree $n+m$ in K and polynomial of degree m in E^H and one could assign to a given gene a fixed polynomial obtained as an iterated function composition.

Intuitively it seems clear that in the generic case $P^m(P^n(x))$ does not decompose to a product of lower order polynomials. One must be however cautious here. It can be shown (see `https://arxiv.org/pdf/1511.06446.pdf`) that the probability that a random polynomial with rational coefficients is irreducible behaves as $O(logN/N)$, where N is upper bound for the magnitude of coefficients. On the other hand, the probability that a random monic polynomial (integer coefficients and unit constant coefficient) is not irreducible (factorizes) goes as $O(1/N)$. It is also shown that by their special properties permutation groups S_n are strongly favoured as Galois groups.

One could use also polynomials assignable to codons or letters as basic units. Also polynomials of genes could be fused in the same manner.

The choice of polynomials P^n is rather free since for given order of Galois group there are only finite number of finite groups and the number of polynomials is infinite. The first cautious guess is that the Galois group depends rather weakly on the rational coefficients regarded as real numbers.

2. If the iteration of polynomial maps indeed gives a Galois extensions, the dimension m of the intermediate extension should be same as the order of its Galois group. Composition would be non-commutative but associative as the physical picture demands. The longer the gene, the higher the algebraic complexity would be. Could functional decomposition define the rule for who extensions and Galois groups correspond to genes? Very naïvely, functional decomposition in mathematical sense would correspond to composition of functions in biological sense.

3. This picture would conform with $M^8-M^4\times CP_2$ correspondence [L23] in which the construction of space-time surface at level of M^8 reduces to the construction of zero loci of polynomials of octonions, with rational coefficients. DNA letters, codons, and genes would correspond to polynomials of this kind.

Could one say anything about the Galois groups of DNA letters?

1. Since $n = h_{eff}/h$ serves as a kind of quantum IQ, and since molecular structures consisting of large number of particles are very complex, one could argue that n for DNA or its dark variant realized as dark proton sequences can be rather large and depend on the evolutionary level of organism and even the type of cell (neuron viz. soma cell). On the other, hand one could argue that in some sense DNA, which is often thought as information processor, could be analogous to an integrable quantum field theory and be solvable in some sense. Notice also that one can start from a background defined by given extension K of rationals and consider polynomials with coefficients in K. Under some conditions situation could be like that for rationals.

2. The simplest guess would be that the 4 DNA letters correspond to 4 non-trivial finite groups with smaller possible orders: the cyclic groups Z_2, Z_3 with orders 2 and 3 plus 2 finite groups of order 4 (see the table of finite groups in `http://tinyurl.com/j8d5uyh`). The groups of order 4 are cyclic group $Z_4 = Z_2 \times Z_2$ and Klein group $Z_2 \oplus Z_2$ acting as a symmetry group of rectangle that is not square - its elements have square equal to unit element. All these 4 groups are Abelian. Polynomial equations of degree not larger than 4 can be solved exactly in the sense that one can write their roots in terms of radicals.

3. Could there exist some kind of connection between the number 4 of DNA letters and 4 polynomials of degree less than 5 for whose roots one an write closed expressions in terms of radicals as Galois found? Could it be that the polynomials obtained by a a repeated functional composition of the polynomials of DNA letters have also this solvability property?

This could be the case! Galois theory states that the roots of polynomial are solvable by radicals if and only if the Galois group is solvable meaning that it can be constructed from abelian groups using Abelian extensions (see `http://tinyurl.com/ybcua92y`).

Solvability translates to a statement that the group allows so called sub-normal series $1 < G_0 < G_1 ... < G_k$ such that G_{j-1} is normal subgroup of G_j and G_j/G_{j-1} is an abelian group. An equivalent condition is that the derived series $G \triangleright G^{(1)} \triangleright G^{(2)} \triangleright ...$ in which $j+1$:th group is commutator group of G_j ends to trivial group. If one constructs the iterated polynomials by using only the 4 polynomials with Abelian Galois groups, the intuition of physicist suggests that the solvability condition is guaranteed!

4. Wikipedia article also informs that for finite groups solvable group is a group whose composition series has only factors which are cyclic groups of prime order. Abelian groups are trivially solvable, nilpotent groups are solvable, p-groups (having order, which is power prime) are solvable and all finite p-groups are nilpotent.

   Every group with order less than 60 elements is solvable. Fourth order polynomials can have at most S_4 with 24 elements as Galois groups and are thus solvable. Fifth order polynomials can have the smallest non-solvable group, which is alternating group A_5 with 60 elements as Galois group and in this case are not solvable. S_n is not solvable for $n > 4$ and by the finding that S_n as Galois group is favored by its special properties (see `http://tinyurl.com/y6wyq9v2`).

   A_5 acts as the group icosahedral orientation preserving isometries (rotations). Icosahedron and tetrahedron glued to it along one triangular face play a key role in TGD inspired model of bio-harmony and of genetic code [L9, L46]. The gluing of tetrahedron increases the number of codons from 60 to 64. The gluing of tetrahedron to icosahedron also reduces the order of isometry group to the rotations leaving the common face fixed and makes it solvable: could this explain why the ugly looking gluing of tetrahedron to icosahedron is needed? Could the smallest solvable groups and smallest non-solvable group be crucial for understanding the number theory of the genetic code.

An interesting question inspired by M^8-H-duality [L23] is whether the solvability could be posed on octonionic polynomials as a condition guaranteeing that TGD is integrable theory in number theoretical sense or perhaps following from the conditions posed on the octonionic polynomials. Space-time surfaces in M^8 would correspond to zero loci of real/imaginary parts (in quaternionic sense) for octonionic polynomials obtained from rational polynomials by analytic continuation. Could solvability relate to the condition guaranteeing M^8 duality boiling down to the condition that the tangent spaces of space-time surface are labelled by points of CP_2. This requires that tangent or normal space is associative (quaternionic) and that it contains fixed complex subspace of octonions or perhaps more generally, there exists an integrable distribution of complex subspaces of octonions defining an analog of string world sheet.

10.3.4 Does fountain effect involve non-standard value of G or delocalization due to a large value of h_{eff}?

Deviations in the value of G are not new, and I have written about several gravitational anomalies. This could mean also anti-gravity effects in a well-defined sense which is however not the same as often thought (negative gravitational masses or repulsive gravitational force).

In particular, in the well-known fountain effect (`http://tinyurl.com/kx3t52r`) of superfluidity, superfluid seems to defy gravitation. I have asked whether $h_{eff}/h_0 = n$ increases at superfluid flux tubes to h_{gr} and this gives to the effect as a de-localization in much longer scale [**?**]. The delocalization could be also due to the reductionof h_{em} or possibly h_Z assignable to long range classical Z^0 force predicted by TGD.

If G is reduced - this means violation of Equivalence Principle in its standard form - the effect would be possible also classically. Since in superfluidity one has h_{eff} larger than usually, this might happen if gravitons travel also along flux tubes at which super fluid flows.

A simple model for the situation discussed in [**?**] would rely on Schrödinger equation at the flux quantum which is locally a thin hollow cylinder turning around at the top of the wall of the container.

1. One obtains 1-dimensional Schrödinger equation

$$(-\frac{\hbar_{eff}^2\partial_z^2}{2m}+mg_{eff}z)\Psi=E\Psi \quad , \quad h_{eff}=nh_0=\frac{nh}{6} \quad . \tag{10.3.1}$$

   It is easy to see that the energy spectrum is invariant under the scaling $h\rightarrow h_{eff}=xh$ and $z\rightarrow z/x$. One has $\Psi_{xh,g_{eff}=g/x}(z)=\Psi_{h,g}(z/x)$ so that simple scaling of the argument z in question. The energy of the solution is same. If the ordinary solution has size scale L, the scaled up solution has size scale xL.

   The height for a trajectory in gravitational field of Earth is scaled up for a given initial vertical velocity v_i is scaled as $h\rightarrow xh$ so quantum behavior corresponds to the classical behavior and de-localization scale is scaled up. Could this happen at various layers of magnetic body for dark particles so that they would be naturally at much higher heights. Cell scale would be scaled to Earth size scale of even larger sizes for the values of $\hbar_{eff}/h = n$ involved.

For classical solution with initial initial vertical velocity $v_i = 1$ m/s the height of the upwards trajectory is $h = v_i^2/2g$ 5 cm. Quantum classical correspondence would be given by $E = mv_i^2/2$ and this allows to look the delocalization scale of a solution.

2. One can introduce the dimensionless variable u (note that one has $g_{eff}/g = 1/x$, $x = h/h_{eff}$) as

$$u = \frac{z - \frac{E}{mg_{eff}}}{z_0} \ , \qquad z_0 = \left[\frac{2m^2 g_{eff}}{\hbar_{eff}^2}\right]^{-1/3} = \frac{h_{eff}}{h}(\frac{m}{m_p})^{2/3} \times (\frac{g}{L_p^2})^{-1/3} \simeq \frac{h_{eff}}{h} \times (\frac{m}{m_p})^{2/3} \times 2.4\ mm \ ,$$

$$L_p = \frac{\hbar c}{m_p} = \simeq 2.1 \times 10^{-16}\ \mathrm{m} \ ,$$

(10.3.2)

Here m_p denotes proton mass and L_p proton Compton length. z_0 scales as $\hbar_{eff}$ as one might expect. z_0 characterizes roughly the scale of the s lution. From the scale of the fountain effect about 1 meter, one can conclude that one should have $h_{eff}/h \sim 2^8$.

This allows to cast the equation to the standard form of the equation for Airy functions encountered in WKB approximation

$$-\frac{d^2\Psi}{du^2} + u\Psi = 0 \ . \tag{10.3.3}$$

Remark: Note that the classical solution depends on m. In central force problem with $1/r$ and $h_{eff} = GMm/v_0$ the binding energy spectrum $E = E_0/n^2$ has scale $E_0 = v_0^2 m$ and is universal.

3. The interesting solutions correspond to Airy functions $Ai(u)$ which approach rapidly zero for the values of $u > 1$ and oscillate for negative values of u. These functions $Ai(u + u_1)$ are orthogonal for different values of u_1. The values of u_1 correspond to different initial kinetic energies for the motion in vertical direction. In the recent situation these energies correspond to the initial vertical velocities of the super-fluid in the film. $u = u_0 = 1$ defines a convenient estimate for the value of z coordinate above which wave function approaches rapidly to zero.

For classical solution with initial initial vertical velocity $v_i = 1$ m/s the height of the upwards trajectory is $h = v_i^2/2g$ 5 cm. Quantum classical correspondence would be given by $E = mv_i^2/2 = E$ and this allows to look the delocalization scale of a solution.

The Airy function $Ai(u)$ approaches rapidly to zero (see the graph of `https://en.wikipedia.org/wiki/Airy_function`) and one can say that above $u_0 = 3$ the function vanishes. Already at $u_0 = 1$ wave function is rather small as compared with its value at $u = 0$. This condition translates to a condition for z as

$$z_0 = z_{cl} + u_0 z_0 \ , \quad z_{cl} = \frac{E}{mg_{eff}} \ , \quad z_0 = \frac{h_{eff}}{h}\left[\frac{\hbar^2}{2m^2 g}\right]^{1/3} \ . \tag{10.3.4}$$

The condition is consistent with the classical picture and the classical height z_{cl} scales like h_{eff}/h. The parameter $u_0 z_0$ defines the de-localization scale consistent with the expectations. Below z_{cl} the wave function oscillates which intuitively corresponds to the sum of waves in upwards and downwards directions.

What can one conclude about the value of $x = h_{eff}/h_0$ in the case of super-fluidity?

(a) Using the previous formula, the condition that z_0 is of order 1 meter fixes its value to $h_{eff}/h_0 \sim 2^8$. Could super-fluidity correspond to the value of $h_{eff} = h_{em} > h$ assignable to electromagnetic flux tubes? The generalization $h_{em} = Ze^2/v_0$ of the Nottale's formula would require that the super fluid phase has a large total em charge Z. The Cooper pairs are however neutral. This leaves under consideration only the old idea that super-fluidity corresponds to Z^0 super-conductivity inspired by the idea that TGD predicts long range Z^0 fields and by the fact that nuclei carry indeed carry non-vanishing Z^0 charge mostly due to neurons.

(b) Both $\hbar_{eff}(gr)/\hbar \simeq 2^{24}$ and $\hbar_{gr} = GMm/v_0$ given by Nottale's hypothesis give quite too large value of z_0.

The gravitational Compton length λ_{gr} is given by $\lambda_{gr} = GM_e/v_0 = r_S/2v_0$ and - in accordance with the Equivalence Principle - does not depend on m. The Schwartschild radius of Earth is $r_S = .9$ cm. One could argue that λ_{gr} is a reasonable lower bound for z_0 if h_{gr} appears in the gravitational Schrödinger equation. For $v_0/c = 2^{-11}$ required by the Bohr orbit model for the inner planets, this would give $\lambda_{gr} = 9$ m. The energy scale of dark cyclotron states comes out correctly if one has $v_0/c = 1/2$ giving the lower bound $z_0 \geq r_S = .9$ cm.

However, the proportionality of z_0 to h_{eff}/h implies that the z_0 is scaled by a factor of order $2GM_Em_p/v_0 \sim 10^{14}$ from its value $z_0 = .2$ mm and would be gigantic. It seems that this option indeed fails.

(c) Could the fountain effect be due to the reduction of g in principle possible if G is prediction and CP_2 length replaces Planck length as fundamental scale? If one assumes $h_{eff} = h$ and scaled down value of g corresponding to $G_{eff} = R^2/\hbar_{gr}$ such that $\hbar_{gr}$ is scaled from its normal value: $\hbar_{gr} \to y\hbar_{gr}$, $G_{eff} \to G_{eff}/y$. This would give the scaling of $z_0 \propto g^{-1/3}$ as $z_0 \to y^{1/3}z_0$ giving $z_0 \simeq .2$ mm should be scaled up to about 1 mm which would give $y \sim 10^9$. This would mean a huge breaking of Equivalence Principle.

10.3.5 Does Podkletnov effect involve non-standard value of G?

Podkletnov observed [H6] at eighties a few percent reduction of gravity: he immediately lost his job in Tampere University in Finland. It was regarded as a scandalous event. Something new might have been discovered in finnish laboratory!

I have considered a possible mechanism explaining the finding of Podkletnov [L12]. One could however ask whether the presence of a superconductor involving also the presence of phase with non-standard value of Planck constant could also affect the value of h_{eff} assignable to the flux tubes of the Kähler magnetic field? The mechanism could be the same as in the fountain effect. The non-standard value of h_{em} could induce delocalization and reduction of g. Now also a small change g from its normal value can be considered and would have been few per cent in this case. This would mean a small breaking of

10.3.6 Did LIGO observe non-standard value of G and are galactic blackholes really supermassive?

Also smaller values of G than the G_N are possible and in fact, in condensed matter scales it is quite possible that $n = R^2/G$ is rather small. Gravitation would be stronger but very difficult to detect in these scales. Neutron in the gravitational field of Earth might provide a possible test. The general rule would be that the smaller the scale of dark matter dynamics, the larger the value of G and maximum value would be $G_{max} = R^2/h_0$, $h = 6h_0$.

Are the blackholes detected by LIGO really so massive?

LIGO (see `http://tinyurl.com/bszfs29`) has hitherto observed 3 fusions of black holes giving rise to gravitational waves. For TGD view about the findings of LIGO see [L19, L16] (see `http://tinyurl.com/y79yqw6q` and `http://tinyurl.com/ya8ctxgc`). The colliding blackholes were deduced to have unexpectedly larger large masses: something like 10-40 solar masses, which is regarded as something rather strange.

Could it be that the masses were actually of the order of solar mass and G was actually larger by this factor and h_{eff} smaller by this factor? The mass of the colliding blackholes could be of order solar mass and G would larger than its normal value - say by a factor in the range (10,50). If so, LIGO observations would represent the first evidence for TGD view about quantum gravitation, which is very different from superstring based view. The fourth fusion was for neutron stars rather than black holes and stars had mass of order solar mass.

This idea works if the physics of gravitating system depends only on $G(M+m)$. That classical dynamics depends on $G(M+m)$ only, follows from Equivalence Principle. But is this true also for gravitational radiation? If the power of gravitational radiation distinguishes between different values of M when GM is kept constant, the idea is dead.

(a) If the power of gravitational radiation distinguishes between different values of M+m, when $G(M+m)$ is kept constant, the idea is dead. This seems to be the case. The dependence on $G(M+m)$ only leads to contradiction at the limit when M+m approaches zero and $G(M+m)$ is fixed. The reason is that the energy emitted per single period of rotation would be larger than M+m. The natural expectation is that the radiated power per cycle and per mass M+m depends on $G(M+m)$ only as a dimensionless quantity.

(b) From arXiv one can find an article (see `http://tinyurl.com/y99j3fpr`) in which the energy per unit solid angled and frequency radiated in collision of blackholes is estimated. The outcome is proportional to $E^2G(M+m)^2$, where E is the energy of the colliding blackhole.

The result is proportional mass squared measured in units of Planck mass squared as one might indeed naïvely expect since $G(M+m)^2$ is analogous to the total gravitational charge squared measured using Planck mass.

The proportionality to E^2 comes from the condition that dimensions come out correctly. Therefore the scaling of G upwards would reduce mass and the power of gravitational radiation would be reduced down like $M+m$. The power per unit mass depends on $G(M+m)$ only. Gravitational radiation allows to distinguish between two systems with the same Schwartschild radius, although the classical dynamics does not allow this.

(c) One can express the classical gravitational energy E as gravitational potential energy proportional to GM/R This gives only dependence on GM as also Equivalence Principle for classical dynamics requires and for the collisions of blackholes R is measured by using $G(M+m)$ as a natural unit.

Remark: The calculation uses the notion of energym which in general relativity is precisely defined only for stationary solutions. Radiation spoils the stationarity. The calculations of the radiation power in GRT is to some degree artwork feeding in the classical conservation laws in post-Newtonian approximation lost in GRT. In TGD framework the conservation laws are not lost and hold true at the level of $M^4 \times CP_2$.

What about supermassive galactic blacholes?

What about supermassive galactic black holes in the centers of galaxies: are they really supermassive or is G super-large! The mass of Milky Way super-massive blackhole is in the range $10^5 - 10^9$ solar masses. Geometric mean is $n = 10^7$ solar masses and of the order of the standard value of $R^2/G_N = n \sim 10^7$. Could one think that this blackhole has actually mass in the range 1-100 solar masses and assignable to an intersection of galactic cosmic string with itself! How galactic blackholes are formed is not well understood. Now this problem would disappear. Galactic blackholes would be there from the beginning!

The general conclusion is that only gravitational radiation allows to distinguish between different masses $M+m$ for given $G(M+m)$ in a system consisting of two masses so that classically scaling the opposite scalings of G and $M+m$ is a symmetry.

10.3.7 Is it possible to determine experimentally whether gravitation is quantal interaction?

Marletto and Vedral have proposed an interesting method for measuring whether gravitation is quantal interaction (see `https://arxiv.org/pdf/1707.06036.pdf`).

I tried to understand what the proposal suggests and how it translates to TGD language.

(a) If gravitational field is quantum it makes possible entanglement between two states. This is the intuitive idea but what it means in TGD picture? Feynman interpreted this as entanglement of gravitational field of an objects with the state of object. If object is in a state, which is superposition of states localized at two different points x_i, the classical gravitational fields ϕ_{gr} are different and one has a superposition of states with different locations

$$|I\rangle = \sum_{i=1,2} |m_i \ at \ x_i > |\phi_{gr,x_i}\rangle \equiv |L\rangle + |R\rangle \ \ .$$

(b) Put two such de-localized states with masses m_i at some distance d to get state $|1\rangle|2\rangle$, $|i\rangle = |L\rangle_i + |R\rangle >_i$. The 4 components pairs of the states interact gravitationally and since there are different gravitational fields between different states the states get different phases, one can obtain entangled state.

Gravitational field would entangle the masses. If one integrates over the degrees of freedom associated with gravitational field one obtains density matrix and the density matrix is not pure if gravitational field is quantum in the sense that it entangles with the particle position.

That gravitation is able to entangle the masses would be a proof for the quantum nature of gravitational field. It is not however easy to detect this. If gravitation only serves as a parameter in the interaction Hamiltonian of the two masses, entanglement can be generated but does not prove that gravitational interaction is quantal. It is required that the only interaction between the systems is gravitational so that other interactions do not generate entanglement. Certainly, one should use masses having no em charges.

(c) In TGD framework the view of Feynman is natural. One has superposition of space-time surfaces representing this situation. Gravitational field of particle is associated with the magnetic body of particle represented as 4-surface and superposition corresponds to a de-localized quantum state in the "world of classical worlds" with x_i representing particular WCW coordinates.

I am not specialist in quantum information theory nor as quantum gravity experimentalist, and hereafter I must proceed keeping fingers crossed and I can only hope that I have understood correctly. To my best understanding, the general idea of the experiment would be to use interferometer to detect phase differences generated by gravitational interaction and inducing the entanglement. Not for photons but for gravitationally interacting masses m_1 and m_2 assumed to be in quantum coherent state and be describable by wave function analogous to em field. It is assumed that gravitational interact can be describe classically and this is also the case in TGD by quantum-classical correspondence.

(a) Authors think quantum information theoretically and reduce everything to qubits. The de-localization of masses to a superposition of two positions correspond to a qubit analogous to spin or a polarization of photon.

(b) One must use and analog of interferometer to measure the phase difference between different values of this "polarization".

In the normal interferometer is a flattened square like arrangement. Photons in superpositions of different spin states enter a beam splitter at the left-lower corner of interferometer dividing the beam to two beams with different polarizations: horizontal (H) and vertical (V). Vertical (horizontal) beam enters to a mirror which reflects it to horizontal (vertical beam). One obtains paths V-H and H-V meeting at a transparent mirror located at the upper right corner of interferometer and interfere.

There is detector D_0 resp. D_1 detecting component of light gone through in vertical resp. horizontal direction of the fourth mirror. Firing of D_1 would select the H-V and the firing of D_0 the V-H path. This thus would tells what path (V-H or H-V) the photon arrived. The interference and thus also the detection probabilities depend on the phases of beams generated during the travel: this is important.

(c) If I have understood correctly, this picture about interferometer must be generalized. Photon is replaced by mass m in quantum state which is superposition of two states with polarizations corresponding to the two different positions. Beam splitting would mean that the components of state of mass m localized at positions x_1 and x_2 travel along different routes. The wave functions must be reflected in the first mirrors at both path and transmitted through the mirror at the upper right corner. The detectors D_i measure which path the mass state arrived and localize the mass state at either position. The probabilities for the positions depend on the phase difference generated during the path. I can only hope that I have understood correctly: in any case the notion of mirror and transparent mirror in principle make sense also for solutions of Schrödinger eequation.

(d) One must however have two interferometers. One for each mass. Masses m_1 and m_2 interact quantum gravitationally and the phases generated for different polarization states differ. The phase is generated by the gravitational interaction. Authors estimate that phases generate along the paths are of form

$$\Phi_i = \frac{m_1 m_2 G}{\hbar d_i}\Delta t \quad .$$

$\Delta t = L/v$ is the time taken to pass through the path of length L with velocity v. d_1 is the smaller distance between upper path for lower $mass m_2$ and lower path for upper mass m_1. d_2 is the distance between upper path for upper mass m_1 and lower m_2. See Figure 1 of the article (see `https://arxiv.org/pdf/1707.06036.pdf`).

What one needs for the experiment?

(a) One should have de-localization of massive objects. In atomic scales this is possible. If one has $h_{eff}/h_0 > h$ one could also have zoomed up scale of de-localization and this might be very relevant. Fountain effect of superfluidity pops up in mind.

(b) The gravitational fields created by atomic objects are extremely weak and this is an obvious problem. Gm_1m_2 for atomic mass scales is extremely small: since Planck mass m_P is something like 10^{19} proton masses and atomic masses are of order 10-100 atomic masses.

One should have objects with masses not far from Planck mass to make Gm_1m_2 large enough. Authors suggest using condensed matter objects having masses of order $m \sim 10^{-12}$ kg, which is about 10^{15} proton masses 10^{-4} Planck masses. Authors claim that recent technology allows de-localization of masses of this scale at two points. The distance d between the objects would be of order micron.

(c) For masses larger than Planck mass one could have difficulties since quantum gravitational perturbation series need not converge for $Gm_1m_2 > 1$ (say). For proposed mass scales this would not be a problem.

What can one say about the situation in TGD framework?

(a) In TGD framework the gravitational Planck $h_{gr} = Gm_1m_2/v_0$ assignable to the flux tubes mediating interaction between m_1 and m_2 as macroscopic quantum systems could enter into the game and could reduce in extreme case the value of gravitational fine structure constant from $Gm_1m_2/4\pi\hbar$ to $Gm_1m_2/4\pi\hbar_{eff} = \beta_0/4\pi$, $\beta_0 = v_0/c < 1$. This would make perturbation series convergent even for macroscopic masses behaving like quantal objects. The physically motivated proposal is $\beta_0 \sim 2^{-11}$. This would zoom up the quantum coherence length scales by $h_{gr}/\hbar$.

(b) What can one say in TGD framework about the values of phases Φ?

i. For $\hbar \rightarrow \hbar_{eff}$ one would have

$$\Phi_i = \frac{Gm_1 m_2}{\hbar_{eff} d_i}\Delta t \quad .$$

For $\hbar \rightarrow \hbar_{eff}$ the phase differences would be reduced for given Δt. On the other hand, quantum gravitational coherence time is expected to increase like h_{eff} so that the values of phase differences would not change if Δt is increased correspondingly. The time of 10^{-6} seconds could be scaled up but this would require the increase of the total length L of interferometer arms and/or slowing down of the velocity v.

ii. For $\hbar_{eff} = \hbar_{gr}$ this would give a universal prediction having no dependence on G or masses m_i

$$\Phi_i = \frac{v_0 \Delta t}{d_i} = \frac{v_0}{v} \frac{L}{d_i} \quad .$$

If Planck length is actually equal to CP_2 length $R \sim 10^{3.5}\sqrt{G_N \hbar}$, one would has $G_N = R^2/\hbar_{eff}$, $\hbar_{eff} \sim 10^7$. One can consider both smaller and larger values of G and for larger values the phase difference would be larger. For this option one would obtain $1/\hbar_{eff}^2$ scaling for Φ. Also for this option the prediction for the phase difference is universal for $h_{eff} = h_{gr}$.

iii. What is important is that the universality could be tested by varying the masses m_i. This would however require that m_i behave as coherent quantum systems gravitationally. It is however possible that the largest quantum systems behaving quantum coherently correspond to much smaller masses.

10.3.8 Fluctuations of Newton's constant in sub-millimeter scales

Sabine Hossenfelder had a post with link to an article "*Hints of Modified Gravity in Cosmos and in the Lab?*" [E44] (see `http://tinyurl.com/y6j8sntw`). Here is the part of abstract that I find the most interesting.

On sub-millimeter scales we show an analysis of the data of the Washington experiment (Kapner et al. (2007) searching for modifications of Newton's Law on sub-millimeter scales and demonstrate that a spatially oscillating signal is hidden in this dataset. We show that even though this signal cannot be explained in the context of standard modified theories (viable scalar tensor and f(R) theories), it is a rather generic prediction of nonlocal gravity theories.

What is interesting from TGD point of view that the effect - if it is indeed real - appears in scale of .085 mm about 10^{-4} μm, which is the scale defined by the density of dark energy in recent universe and thus by cosmological constant. This is also size scale of large neuron.

Findings

Washington group studied gravitational torque on torque pendulum for sub-millimeter distances of masses involved [E31] (see `http://tinyurl.com/y2un6686`). Figure 19 of [E44] (see `http://tinyurl.com/y6j8sntw`) illustrates data points representing the deviation of the gravitational torque from the Newtonian prediction as a function of distance in the range .05-10 mm.

The deviation can parameterized in terms of effective scaling $G \rightarrow kG$ of Newton's constant, which is assumed to be predictable rather than due to fluctuations and depend on the distance only

$$k = 1 + x cos(\frac{2\pi r}{\lambda} + \frac{3\pi}{4}) \quad .$$

x is a numerical parameter. The highly non-trivial assumption is that Newton's potential is modified by an oscillating term, which must go to zero at large distances: its amplitude could approach to zero like $1/r$. The model predicts an anomalous gravitational torque $\Delta\tau$ proportional to $k-1$ and having the form

$$\Delta\tau = a cos(\frac{2\pi r}{\lambda} + \frac{3\pi}{4}) \quad ,$$

where r is the distance between the masses. The parameter $\lambda = \hbar/m$ is formally analogous to Compton length for imaginary mass m.

The finding is that the statistical significance for the best fit to the data is $(a, \lambda) = (0.004 \quad fNm, 65 \quad mm^{-1})$ is more than 3σ, where a is the amplitude of the deviation. The highly non-trivial problem is however that one obtains also other minima of χ^2 measuring the goodness of the fit with different values of the parameter λ.

I am not specialist but while looking at the data, I cannot avoid the feeling that the fit does not make much sense and reflects theoretical prejudices (belief in modified gravity of some kind) rather than reality. My first impression that fluctuations in the value of Newton's constant G are in question. The value of G is indeed known to vary from experiment to experiment and the variation is too large to be explained in terms of measurement inaccuracies [?]see `http://tinyurl.com/yanvzxj6`).

Could it be that the value of G fluctuates, and for some reason in the length scale range around .1 mm the fluctuations are especially large meaning different values of G are large? Could some kind of criticality enhanced rather dramatically below .1 mm be involved?

Could fluctuations in the value of G explain the findings?

Twistor lift of TGD [K94, K78, K8, L50] predicts that cosmological constant is length scale dependent and that Newton's constant G has a spectrum reflecting the spectrum of effective Planck constant $h_{eff} = nh_0$ ($h = 6h_0$ is a good guess [L18]): dark matter would correspond to $h_{eff} = nh_0$ phases of ordinary matter.

p-Adic length scale hypothesis allows to assign to cosmological constant Λ two length scales: the cosmological p-adic scale defined by Λ itself and the short p-adic length scale determined by the density of dark energy so that physics is cosmological scales and physics in microscopic scales reflect each other.

This encourages the idea that one might understand the experimental findings in terms of fluctuations of G induced by quantum fluctuations of h_{eff} at quantum criticality.

(a) TGD suggests a spectrum for the values of G. The starting points is the expression for the effective Planck constant ass $h_{eff} = n \times h_0$. In adelic physics the value of n is identified as the number of sheets for the space-time surface as covering space and would correspond to the order of Galois group of extension of rationals inducing the extensions of p-adic number fields appearing in the adele [L28, L29].

(b) An additional hypothesis is that space-time surface can be regarded as covering of both M^4 and CP_2 with numbers of sheets equal to n_1 and n_2: $n = n_1 n_2$. The number of sheets over M^4 would be n_1 so that CP_2 coordinates would be n_1-valued functions of M^4 coordinates. The number of sheets over CP_2 would be n_2 and one would have effective n_2 copies of n_1 valued regions in M^4.

The gravitational Planck constant $\hbar_{gr} = GMm/v_0$ originally introduced by Nottale [E25] is proposed to correspond to $\hbar_{eff} = \hbar_{gr} = n_1 n_2 \hbar_0$. The real Planck length $l_P(real)$ would correspond to $l_P(real) = R$, the CP_2 size scale identified as geodesic length, and Newton's constant would correspond to

$$G = \frac{R^2}{\hbar_1} = \frac{R^2}{n_1 \hbar_0} \quad .$$

One would have $n_1 \sim 6 \times 10^7$ from $l_P^2/R^2 \sim 10^7$.

(c) The value of n_1 can fluctuate and induce fluctuations of G. The fluctuations could be even large. One can even ask whether the fountain effect of superfluidity involves a large value of n_1 responsible for macroscopic quantum coherence and due to the increase of the value of $\hbar eff$ caused the increase of n_1 in turn reducing the value of G [?].

Could the fluctuations of n_1 explain the findings about the value of G deduced from Washington experiment? The appearance of several values for parameter λ might signal about fluctuations of G rather than modification of the radial dependence of gravitational potential.

Why the fluctuations in the value of G would be so large in sub-millimeter length scales?

(a) Cosmological constant $\Lambda \simeq 1.1 \times 10^{-52}$ m^{-2} has dimension of $1/L^2$, L length scale. The density of dark energy $\rho_{vac} = \Lambda/8\pi G$ has dimensions of $\hbar/L^4$. One can assign to Λ very long p-adic length scale $L(k_1) = 2^{k_1/2}R$ ($p_1 \simeq 2^{k_1}$), and to $\rho_{vac}/\hbar$ rather short p-adic length scale $L(k_2) = 2^{k_2/2}R$. One has

$$\frac{\rho_{vac}}{\hbar} = \frac{x_2}{L(k_2)^4} = \frac{x_1}{8\pi}\frac{1}{l_P^2 L(k_1)^2} \ ,$$

where x_1 and x_2 are numerical constants not far from unity. This would give

$$L(k_2) = (8\pi\frac{x_2}{x_1})^{1/4}(L(k_1)l_P)^{1/2} \ .$$

$L(k_2)$ would be proportional the geometric mean of $L(k_1)$ and l_P. This implies

$$2^{2k_2} = \frac{x_2}{x_1} \times 8\pi \times (\frac{l_P}{R})^2 2^{k_1} \ .$$

Very roughly, $k_1 \sim 2k_2 - 26$ would hold true for $x_2/x_1 \sim 1$. It turns out that k_2 corresponds to a p-adic length scale about 10^{-4} meters, which happens to be the size of large neuron suggesting that quantum gravitation is indeed highly relevant to biology but in manner different from that speculated by Penrose.

(b) p-Adic fractality suggests that cosmological constant is not actually constant or even time varying but depends on p-adic length scales so that the values are indeed extremely large as one approaches CP_2 scale and get very small as one approaches cosmological scales. This would solve the cosmological constant problem. The dependence would be $\Lambda(k) \propto 1/L(k)^2$, where $L(k)$ is the p-adic length scale characterizing the size of the space-time sheet. There would be a sequence of phase transition reducing Λ and these phase transition would involve quantum criticality and long length scale fluctuations possibly assignable to those of h_{eff} and thus of n_2 and G.

If one assumes that k_2 corresponds to preferred p-adic lengths scales assignable to elementary particles, nuclei, atomic physics and biology, one obtains a prediction that the corresponding p-adic length scales correspond to cosmologically important length scales via $k_1 \sim 2k_2$. One could study cosmology by studying gravitation in laboratory scales!

In these scales quantum phase transitions changing cosmological constant could make themselves visible via microscopic physics. Phase transitions involve long length scale fluctuations characteristic for criticality. In TGD these quantum fluctuations correspond to fluctuations of h_{eff} since Compton lengths scale like h_{eff}. The fluctuations of n_1 in $n = \hbar_{eff}/\hbar = n_1 n_2$ would induce fluctuations of G.

(c) Especially interesting are the p-adic length scales which are biologically important. The number theoretical miracle is that there are as many as 4 very closely located Gaussian Mersenne primes $M_{G,n} = (1+i)^m - 1$ in the range of cell membrane thickness and size of cell nucleus corresponding to $k = 151, 157, 163, 167$. The corresponding p-adic length scales $L(k) = 2^{(k-151)/2}L(151)$, $L(151) \simeq 10$ nm could be also gravitationally especially interesting. The hierarchical coiling of DNA might relate to the hierarchy of Gaussian Mersennes and phase transitions changing cosmological constant and the density of magnetic and volume energies assignable to the magnetic flux tubes playing key role in TGD inspired biology. These phase transitions would scale the thickness of the flux tubes determined by p-adic lengths scale.

It should be relatively easy to check whether the p-adic length scale hierarchy up to biological length scales has scaled variant in astrophysical and cosmological scales.

10.3.9 Conscious experiences about antigravity

Conscious experiences about anti-gravitational effects have been also reported and since I have nothing to lose as a happy pensioner and consciousness theorist [L30] I can take the liberty to talk also about these effects, even at personal level.

(a) There are stories about flying yoga masters. I am skeptic but I know from my own experience that out-of-body and levitation experiences - I mean indeed *experiences* - feel very real. I have proposed a model explaining them based on the notion of magnetic body as intentional agent carrying dark matter and using biological body as sensory receptor and motor instrument.

(b) I have indeed spent at younger age many moments in a kind of between away-and-sleep state in the roof of bedroom trying to prove myself that I really am there and then suddenly returned back to normal in wake-up state. Even the matresse behaved how it is expected to behave as some-one falls on it. Maybe part of my magnetic body was out-of-biological body after having experienced $h_{eff}/h = n$ increasing phase transition! Sometimes I have experienced wakeup quite concretely as a kind of contraction in which I have returned to my body: reduction of $h_{eff}/h = n$ for some part of magnetic body would explain this.

(c) I have had also altered states of consciousness between wake-up and sleep in which I felt my body like oscillating and being attracted by refrigerator, whose sound had started to amplify. I experienced the refrigerator as a living being and I was afraid that it intended engulf my consciousness! I had to decide whether I let it go but did not have courage to do it and I returned to the normal state.

(d) In dreams I have been also routinely flying and with somewhat childish narcissism pretended to the other people in dream that this is perfectly normal for me, it just occurred me that it would be fun to fly but honestly: I did not realize that it might make you scared! What was remarkable that I never got above about 10 meters: could this correspond to jumping in air in a reduced gravitational field? As a matter of fact, in dream I was typically going down in stairs and then decided to fly. I often landed at the end of stairs. This would fit with reduced gravity implying weaker downwards gravitational acceleration.

10.4 Three alternative generalizations of Nottale's hypothesis in TGD framework

Gravitational Planck constant $h_{gr} = GMm/v_0$ was originally introduced by [E25] and its form realizes Equivalence Principle (EP) in its Newtonian form (gravitational acceleration does not depend on mass m). The generalization of the idea was formulated in the TGD framework in [K79, K66]. $h_{eff} = nh_0 = h_{gr}$ would characterize the U-shaped flux tube tentacles emanating from M and mediating gravitational interaction.

One implication is that the parameter $v_0/c = \beta_0 < 1$ appears as a natural expansion parameter of the gravitational scattering amplitudes in the perturbative expansion replacing GMm. There is no dependence of GMm. Note that $\hbar_{gr} \geq h$ requires $GMm \geq v_0$.

$v_0 \simeq 2^{-11}$ suggested by the Nottale's Bohr orbit model for the 4 inner planets and is consistent with the model for the fountain effect of superfluidity [K29]. Indeed, the gravitational Compton length of the superfluid particle is $GM/v_0 \simeq 10$ m, which makes sense.

However, the model has a problem. For $M = M_E$, the cyclotron energies $\hbar_{gr}eB_{end}/m$ of dark ions in the endogenous magnetic field $B_{end} = 2/5B_E = .2$ Gauss explaining the findings of Blackman [J2] in terms of the $h_{eff} = nh_0 = h_{gr}$ hypothesis would be given by $E_c = GM_E/v_0 \times ZeB_{end}$ and would not depend on the mass m of the charged particle. For $\beta_0 \simeq 2^{-11}$ E_c would be in keV range and 3-4 orders of magnitude above visible range. Biophoton energies are however in visible and UV range.

10.4.1 Three ways to solve the problem of the too large cyclotron energy scale

One can imagine three ways to solve the too large cyclotron energy scale.

(a) The dark mass M_D is by 3-4 orders of magnitude smaller than the mass M_E of Earth. Here one should be able to understand why dark particles couple only to a part of M_E.

(b) Gravitational constant G_D for dark mass is by 3-4 orders of magnitude smaller than G. This would mean a violation of Equivalence Principle (EP). In the TGD framework, G indeed follows as a prediction and might vary [K7]. This could also provide an alternative explanation for the fountain effect.

(c) The velocity parameter $\beta_0 = v_0/c \leq 1$ has the value $\beta_0 = 1$ or is near to but below this value. For instance $\beta_0 = 1/2$ is enough. This option is favored by the Nottale's Bohr orbit model of the planetary system. The outer planets β_0 indeed varies: one has $\beta_0(outer) = \beta_0(inner)/5$. One has also $M = M_D$ and $G = G_D$.

Can dark mass M_D be smaller than the total mass M?

The model [K71] for the effects of ELF radiation on vertebrate brain [J2] led to a generalization of Nottale's hypothesis by replacing the total mass M in the case of Earth by $M_D \simeq 10^{-4}M_E$ suggesting that in this case the dark particles involved couple only to a part of mass identifiable as dark mass M_D.

A possible interpretation is that at long distance from mass M the flux tubes fused to larger flux tubes and the gravitational mass M_D interacting with the test particle increases to M at large distances. This might be in conflict with known facts.

The dark mass M_D appearing in the gravitational Planck constant $h_{eff} = h_{gr} = GM_Dm/v_0$ must at short distances depend approximately linearly on the distance between the masses M_D and m . In the average sense. M_D would depend linearly on distance r. This is required by the condition that the Bohr radii correspond to the classical radii in the average sense. The actual dependence of M_D on r is expected to be a staircase like function.

At the quantum level, this effectively eliminates the average gravitational force in scales below the critical radius r_{cr} above which $M_D = M$ is true. Indeed, due to the average $M_D \propto r$ dependence, gravitational potential would be constant on the average. Could one regard this effective elimination of the gravitational force as a kind of Quantum EP or as an analog of asymptotic freedom?

Or could the value of G be reduced to $G_D < G$?

The reduction of $\hbar_{gr}$ could be also due to the reduction G to G_D. This is because only the parameter GM appears in the basic formulas.

(a) In the TGD framework Planck length as a fundamental length is replaced by CP_2 length R and Planck length or rather, Newton's constant G follows as a prediction. One can write

$$G = \frac{\hbar_{gr}\beta_0}{Mm} \quad .$$

(b) In the number theoretic vision about h_{eff} one can identify $\hbar_{gr}$ as the dimension of the Galois group of an extension of rationals [L66]. Since one has in the general case extension of extension of ...of rationals, one has a factorization $\hbar_{gr}/h_0 = \prod_i n_i$ where n_i are dimensions of extensions of extensions in the sequence.

This suggests that Mm corresponds to an integer in suitable units, say CP_2 length $R \simeq 10^{3.5}m_{Pl}$ and β_0 could also correspond to inverse of integer. $\hbar_{gr}$ would correspond to an integer and the reduction of G to G_D would correspond to a dropping of integer factor from this integer.

(c) $\hbar_{gr}$ would factorize to integers assignable to M and m and the integer assignable to G would be reduced in $G \rightarrow G_D$. If this integer factorized into a product assignable to M and m characterizing their gravitational couplings, one could understand why the reduction of G occurs only for superfluidity and dark phases in living matter. No additional assumptions about flux tube distribution would be needed.

Does variable β option make sense?

The third option would assume $\beta_0 = 1$ or near to but of order $\beta_0 = 2^{-11}$ for the dark ions in living matter. This conforms with the idea that dark dark matter interacts with all matter and satisfies EP.

The value of β_0 could be seen as the property of the dark matter particle and depend on the particle or on the distance from the central object as in the case of the solar system.

Gravitational Compton length l_{gr} Bohr orbit radius a_{gr} are given by $l_{gr} = GM/v_0 = r_S/2v_0$ and $a_{gr} = 2\pi r_S/v_0^2$. The reduction of β_0 scales up the quantum scale considered. Could this give some idea about how the value of β_0 relates to the size scale of the system considered? For the dark ions at magnetic flux tubes the l_{gr} would be about $r_S/2 \simeq .45$ cm, which is a biological scale. Could it correspond to the size scale of some structure of the vertebrate brain, say pineal gland with radius .37 cm?

In the sequel these options will be considered. I try to not take any of these options as a favorite but I must admit that the last option looks the most plausible one - at least now.

10.4.2 Could $M_D < M$ make sense?

For the generalization of the Nottale hypothesis discussed in the introduction, the gravitational Planck constant $\hbar_{gr} = GM_Dm/v_0$ introduced by Nottale [E25] is proportional to dark mass M_D which is in general would be smaller than the entire mass M.

Remark: As noticed in the introduction, it is GM that appears in h_{gr}, so that an alternative option is that G is reduced G_D. It would naturally characterize mass m rather than flux tube. Violation of Equivalence Principle would be in question.

Dark cyclotron energies $E_c = \hbar_{gr}eB_{end}/m = GM_DeB/v_0$ do not depend on the mass of the particle. The condition that the cyclotron frequencies in EEG range correspond to biophoton energy scale in visible and UV range for $B_{end} = .2$ Gauss, gives the estimate $M_D \simeq 2 \times 10^{-4}M_E < M_E$. One proposal is that M_D corresponds to the mass of the inner-inner core of Earth: see the appendix of [L80].

This raises the question about how the gravitational flux tubes emanating from mass M and connecting it to small masses m - say elementary particles, atoms of ions - are distributed. At short distances, the entire mass would not be connected to a given mass m by this kind of flux tubes. Does the amount M_D of the mass connected to mass m depend on the distance between m and M? How the allowed values of m are distributed and do they depend on distance? For instance, the condition $GM_Dm/v_0 > \hbar$ must be satisfied.

Remark: One can argue that radial magnetic flux tubes are not realistic. One can also consider the possibility that U-shaped flux tubes acting as kind of tentacles in TGD inspired quantum biology, are in question so that magnetic flux would return back. The fusion of flux tubes to larger flux tubes at longer distances makes sense also now.

Some guide lines

There are several hints, which suggest answers to some of these questions.

(a) In the TGD variant [K79] of the Bohr model for the planetary orbits [E25] around Sun, the dark mass M_D for Sun equals to solar mass: $M_D = M_{Sun}$. This suggests that at large enough distances M_D approaches the total mass M of the object. One can imagine that the flux tubes from M fuse to larger flux tubes so that m experiences $h_{gr} \propto M_{Sun}$ at large distances.

(b) In the Bohr orbit model of the planetary system in the gravitational potential of mass M_D, the gravitational binding energy of mass m at the lowest Bohr orbit with $n = 1$ is proportional to $\alpha_{gr}^2 m/2 = mv_0^2/8\pi^2$ ($\alpha_{gr} = v_0/4\pi$) and does not depend on M_D at all. This is true also for higher orbits with $n > 1$.

The consistency with the classical formula for the potential energy $V_{gr}(r) = GM_Dm/r$ suggests that M_D is in average sense proportional to the distance between M and m at small distances.

The radius r_B of the gravitational Bohr orbit is $r_B = \hbar_{gr}/\alpha_{gr}m = 4\pi GM_D/v_0^2$ and does not depend on m at all (note that $2GM_D$ is the Schwartchild radius associated with M_D). The larger the value of M_D, the larger the distance of m to M. This supports $M_D \propto r$ proportionality at small distances in average sense. There is some distance at which the value of M_D reaches M and does not grow anymore.

These arguments suggest that $M_D \propto r$ holds true in a reasonable approximation and that the gravitational flux tubes from smaller parts of M fuse to form larger flux tubes corresponding to the sum of the masses. A particle at a small distance would experience only part of the gravitational force created by M.

M_D/r would be constant on the average sense below the critical radius R_{cr} at which M_D becomes M and the values of M_D would form a linear staircase. At a given step of the staircase, the value of M_D would be constant and M_D/r would decrease. The radial gravitational force averaged over the staircase would vanish. In the average sense, one would have a free particle in a box.

Taking seriously the identification of M_D at the surface of Earth as the mass of the inner-inner core of the Earth, leads to ask whether the gravitational staircase could correlate with the layered structure of the Earth's interior.

Gravitational force is effectively eliminated below R_{cr}. Could this be interpreted in terms of Quantum Equivalence Principle? Asymptotic freedom is another analogy that pops in mind.

Magnetospheric sensory representations as a test of the proposal

This proposal can be tested in the TGD based model for sensory representations realized at magnetosphere [K52, K50].

(a) The proposal is that the magnetosphere of Earth defines sensory representations for the life forms at the surface of Earth. The communication and control would rely on dark photons with energies $E = h_{gr}f$ above thermal energy at physiological temperatures. For energies in visible and UV range dark photons can induce molecular transitions crucial for biochemistry by transforming to ordinary photons identifiable as biophotons [K10, K21].

(b) The energetic condition should be true near the surface of Earth, inside the rotating inner magnetosphere, and also in the outer magnetosphere extending to the distance of order $200R_E$. In plasma sheet, the order of magnitude for B is $B \sim 10-20$ nTesla. One has $B/B_{end} \sim 5\times 10^{-4}$ for $B = 10$ nTesla.

(c) The cyclotron energies are given by $E_c = GM_DeB/v_0$ and do not depend on m. At the surface of Earth one has $M_D \simeq 2\times 10^{-4}M_E$. At large enough distances one has $M_D = M_E$. In the outer magnetosphere this is expected to be true.
This would give $E_c(outer) = (M/M_D)\times(B/B_{end})E_c(Earth) \simeq 2.5E_c(Earth)$. The cyclotron energies would be of the same order of magnitude as required.

(d) Note that the values of v_0 are assumed to the same in inner and outer magnetosphere. In the Nottale's Bohr orbit model for the planetary orbits, outer planets and the 4 inner planets have different value of v_0: $v_0(outer) = v_0(inner)/5$. This would scale down the gravitational binding energy for outer planets by factor 1/25, which is reasonable. Scaling of v_0 in the case of Earth would increase cyclotron energy scale.

Critical summary

It must be admitted that I have not been able to develop the generalization of Nottale's hypothesis in a completely satisfactory form and it is best still to summarize the essentials. There is an excruciating uncertainty about the details related to the hypothesis.

(a) The hypothesis involves two parameters: $M_D \leq M$ and $\beta_0 = v_0/c$. The integer n labelling the Bohr orbit is an additional parameter. The critical question is whether M_D can really differ from M.

(b) Bohr orbit conditions expressing Newton's equation for circular orbit and angular momentum quantization in units of $\hbar_{gr}$ gives for the orbital radius T and velocity v the expressions in terms of the basic parameters.

$$\begin{aligned} R(n) &= n^2\frac{GM_D}{\beta_0^2} = \frac{GM_D}{v^2} \ , \\ v &= \frac{\beta_0}{n} \ , \\ E &= \frac{mv_0^2}{8\pi^2 n^2} \ . \end{aligned} \tag{10.4.1}$$

What is remarkable and perhaps strange looking is that velocity and binding energy are independent of the value of M_D. If one knows the orbital parameters, such as radius and period T one can

One can use various inputs in an attempt to fix the parameters of the model.

(a) In the case of the Sun, the radii and the velocities of the orbits of planets provide the information which allows to determine these parameters. $\beta_0(outer) = \beta_0(inner)/5$ relates the inner and outer planets. The value of n and $\beta_0(inner) \simeq 2^{-11}$ are determined by the planetary velocities. $M_D = M$ is implied by the known orbital radii.

(b) In the case of Earth there is no analog of planetary data available. The situation should look classical so that the values of n involved are large unlike in the case of Sun.

If the orbit of a stationary satellite is regarded as a Bohr orbit, one can get an estimate for n. In this case $v = v_0/n$ can be deduced from the period T and radius $R(n)$ of the orbit. For the stationary orbit, one has $R/R_E \simeq 6.62$. Newton's equation gives $GM_D/R = \beta^2$ so that $M_D = M$ must be true. If M_D depends on distance, $M_D \simeq M_E$ must hold true at distance about $6R_E$.

For $M_D = M$ and $\beta_0 = 2^{-11}$, $\beta = \beta_0/n$ gives $n \simeq 50$. Bohr orbit with Earth radius would have $n \simeq 19$. The reduction of M_D to $2 \times 10^{-4} M_E$ while keeping the radius of the Bohr orbit same, would require $n = 19 \rightarrow 1343$.

The above considerations are consistent with $M_D = M$. The hypothesis $M_D \simeq 2 \times 10^{-4} M_D$ deserves a critical discussion.

(a) The condition that the cyclotron frequencies in the endogenous magnetic field $B_{end} = .2$ Gauss postulated to explain the findings of Blackman and others correspond to $h_{eff} = h_{gr}$ for which the frequencies at EEG frequency range correspond to the energies in the energy range of biophotons. This gives $M_D \sim 2 \times 10^{-4} M_E$ and the proposed identification is as the mass of the inner-inner core of Earth. Its radius is roughly 5 per cent of the radius of Earth. The model for the fountain effect of super-fluidity is consistent with this estimate of M_D.

(b) If M_D really varies, the small masses m cannot couple to the entire mass of (say) Earth: this could be perhaps understood in the flux tube picture in the proposed way.

10.4.3 What about the reduction of G to G_D?

As noticed in the introduction, it is actually the parameter GM_D that appears in Bohr conditions. Could it be G is replaced with G_D and one has $M = M_D$? In TGD the value of G indeed comes out as a prediction. CP_2 length R defines the counterpart of Planck length l_P and Newton's constant G is predicted to be $G\hbar = R^2/n_1$, where $n_1 =\sim 10^7$.

One can also write $G = \frac{\hbar_{gr}\beta_0}{Mm}$. Could the value of n_1 increase so that the value of G is reduced to G_D?

(a) The condition is that also the new value divides $\hbar_{gr}$ or more precisely, the integer assignable to G in the decomposition of $\hbar_{gr}$ to a product of integers.

(b) n_1 has a number theoretic interpretation [K7] as a factor of the order of the Galois group assignable to $\hbar_{gr} = n_{gr} h_0$. The variation of n_1 is in principle possible and there is evidence for small variations of G perhaps assignable to that of n_1.

(c) The increase of n_1 by a factor about $10^4/2$ is in principle possible: one would have $G_D = 2 \times 10^{-4} G$. The new value of n_1 should also divide n_{gr}. This kind of reduction of G for the superfluid phase could also explain the fountain effect as a dramatic weakening of the Earth's gravitation at the gravitational flux tubes connecting Earth to superfluid.

(d) Why would not G be reduced for the ordinary matter? It seems that the superfluid-/dark particle property must change the coupling to gravity? The factorization of

$\hbar_{gr} = G_D Mm/v_0$ would naturally correspond to the factorization of n_{gr} to a product of factors characterizing masses M, m and the flux tube?

If $G\hbar$ - when expressed using CP_2 length as unit - factorizes to product of integers assignable to M and m, then the integer associated with m would be reduced so that the reduction of G would characterize the dark particle with mass m.

Note that also Podkletnov effect [H6, H3] discussed from the TGD point of view in [L12] suggests a few per cent reduction of G.

(e) A geometric interpretation suggests itself [L66] . The basic factorization would correspond to a decomposition to $n_{gr} = n_1 n_2$. n_1 would correspond to the number of sheets of space-time surface as a covering of M^4 and n_2 as covering of CP_2: the interpretation as a quantum coherent flux tube bundle of n_2 tubes is suggestive. The values of n_2 would be larg and correspond to the factor Mm or Mm/v_0. n_1 would be relatively small and could correspond to G or its factorization to a produce of integers assignable to M and m. This makes sense since the coupling of m to gravitational flux tubes is assumed to be by touching.

To sum up, it seems that one should improve the physical understanding of the Galois group of extension, which in general is extension of extension of ... so that its dimension n is the product of dimensions of extensions involved. Do these dimensions correspond to effective Planck constants assignable to various interactions as suggested in [K7]?

10.4.4 The option based on variable value of β_0

The motivations for the model with a variable value of $\beta_0 = v_0/c$ have been already explained. In the sequel I will develop a model for the communications between dark matter phases with $h_{eff} = nh_0$ satisfying $h_{eff} = h_{gr}$. One can consider two options for the communications depending on whether the value of h_{eff} changes as (for instance) in the communications between dark and ordinary matter or whether it is preserved.

(a) If the value of h_{eff} can change, energy conservation for $E = h_{eff} f$ allows energy resonance whereas the frequency changes. The simplest option is that the dark photon transforms to say ordinary photon with the same amplitude

(b) If the value h_{eff} is preserved, one has both energy and frequency resonance. In the case of cyclotron radiation, the simultaneous occurrence of energy and frequency resonances poses strong conditions on the values of the magnetic fields, the values of charged particle masses, and the parameter β_0 at the ends of the communication line.

Conditions for frequency - and energy resonance

The condition that the frequency is the same at both ends implies for cyclotron frequencies $f_c = ZeB/2\pi m$ the condition

$$\frac{Z_1 B_1}{m_1} = \frac{Z_2 B_2}{m_2} \ . \tag{10.4.2}$$

For $h_{eff} = h_{gr}$ the condition that the cyclotron energy $E_c = GMZeB/v_0$ at both ends is same implies

$$\frac{Z_1 B_1}{v_{0,1}} = \frac{Z_2 B_2}{v_{0,2}} \ . \tag{10.4.3}$$

Together these conditions give

$$\frac{m_1}{m_2} = \frac{Z_1 B_1}{Z_2 B_2} = \frac{\beta_{0,1}}{\beta_{0,2}} \ . \tag{10.4.4}$$

4. For instance, if the two particles are proton and electron, one obtains

$$\frac{\beta_{0,1}}{\beta_{0,2}} \simeq \frac{m_e}{m_p} \quad .$$

This ratio is is consistent with the values $\beta_{0,2} = 1$ and $\beta_{0,1} = 2^{-11}$ in the accuracy considered. Is this a mere accident?

Resonance conditions for communications from the Earth's surface to the magnetosphere?

The simplest option is that the interacting particles have the same values of mass and β_0 and magnetic fields are identical. This is achieved if the flux tubes have constant thickness. Whether this is the case is not clear.

However, the idea that the flux tube picture about magnetic fields is locally consistent with the Maxwellian view inspires the question whether also the magnetic field strength at the flux tubes of B_{end} behaves like $B_{end} \propto 1/r^3$ as B_E in dipole approximation behaves.

B_{end} is by flux conservation proportional to $1/S$, where S is the area of the flux tube. sOne would have $S \propto r^3$. The constancy of B_{end}/m would suggest $m \propto 1/r^3$. If the charged particles are ions characterized by the A/Z ratio.

This would suggest that the regions of tubes/sheets in frequency resonance are at distances

$$\frac{r}{r_0} = (\frac{Z}{Z_0})^{-1/3}(\frac{A_0}{A})^{-1/3}$$

for ions Z_0, A_0 at the surface of the Earth. The heaviest ions would be nearest to the surface of Earth. Energy resonance condition

$$\frac{B_{end}(r)}{\beta_{0,2}} = \frac{B_{end}(R_E)}{v_{0,1}}$$

would give the additional condition

$$\frac{\beta_{0,2}}{\beta_{0,1}} = (\frac{R_E}{r})^3 = \frac{Z}{Z_0} \times \frac{A_0}{A} \quad .$$

β_0 would be quantized and would decrease with the distance.

Magnetosphere as sensory canvas

TGD leads to a model of the "personal" magnetic body (MB) as being associated with the Earth's MS. Different regions of the body and brain would be mapped to regions of the MS, which would give rise to sensory representations at the personal MB [K52, K50]. Personal MB, which would have size scale of at least of the Earth's MS, would also control biological body.

1. An interesting finding relates to the values of the magnetic field $B_{end} \simeq 2B_E/5$ (perhaps identifiable as the monopole flux part of B_E) and the value of $B \sim 10$ nT in the magnetotail at the night-side of the Earth.

   One has $B/B_{end} \sim 2^{-11}$ so that for dark proton-dark electron communications between the Earth's surface and this region of outer MS the resonance conditions would be satisfied for $\beta_0 = x$ and $\beta_0 = 2^{-11}x$, where $x < 1$ not far from unity.

2. Could the parameter β_0 characterize particles and act as a tunable control parameter allowing to achieve energy resonance? Also the values of B are tunable by changing the thickness of the flux tubes as a kind of motor action of MB.

This idea can be applied to the h_{eff} preserving communications between biological body and the MS of the Earth.

1. The quantum coherence condition suggests that the communications are optimal when the wavelength of dark photon is larger than the distance considered: $\lambda > r$ or equivalently the frequency satisfies $f \leq c/r$ (one has $c = 1$ in the units used). If the structure of the MS has distances from the Earth's surface below r_{max} then the frequencies $f \leq 1/r_{max}$ are optimal.

2. Given the distance r_{max} and assuming $B = B_{end}$ at the surface of Earth, one obtains for the cyclotron frequencies the condition

$$f_c = \frac{ZeB_{end}}{2\pi m} \leq \frac{1}{r_{max}} \quad .$$

   For instance, EEG frequency 10 Hz corresponds to 3×10^7 m. The cyclotron frequency of DNA sequence does not depend on its length and composition since DNA has constant charge per unit length. One has $f_c \simeq 1$ Hz so that the corresponding distance is $r = 3 \times 10^8$ m, that is $r = 46.9 R_E$.

   Remark: B_{end} probably has a spectrum. Music experiences relies on frequency scale and if the audible frequencies correspond to cyclotron frequencies then eB_{end}/m is variable. This suggests that the spectrum of B_{end} covers at least the range of the audible frequencies spanning roughly 10 octaves [K74].

10.5 Can TGD predict the value of Newton's constant?: the view two years later

Newton's constant G cannot be a fundamental constant in TGD framework . G has dimensions of length squared divided by Planck constant and CP_2 length R is the only fundamental length in TGD Universe. The analog of Newton's constant $G = R^2/\hbar$ is too larger by factor of order $10^7 - 10^8$: the previous estimate gives for this factor the value $2^{24} = 16,777,216 = 1.6777216 \times 10^7$.

The first guess was that one must modify the formula by replacing $\hbar$ with h_{eff} with $h_{eff} = nh_0$, $h = 6h_0$: $G = R^2/\hbar_{eff}$ (see [L18, L36, L69]).

n has however has arbitrarily values and the proposal cannot be correct as such if one accepts the notion of gravitational Planck constant $\hbar_{gr} = GMm/v_0 = h_{eff} = nh_0$: here M and m are masses of systems having gravitational interaction and $v_0 < c$ is velocity parameter having value $v_0/c \simeq 2^{-11}$ for inner planets [E25] [K79, K66, K67]. h_{gr} is assigned with the flux tubes mediating gravitational interaction.

One can assign also to other interactions corresponding effective Planck constants - for instance, $h_{em} = Z_1 Z_2 e^2/\beta_0$, $\beta = v_0/c < 1$ to electromagnetic interactions. The general idea is that when the value of coupling strength $Q_1 Q_2 g^2/\hbar$ for a two particle system becomes so large that perturbation theory fails, Planck constant is replaced with h_{eff} and perturbation theory works again. Topologically this means a phase transition replacing space-time sheets with their n-fold coverings.

10.5.1 Development of ideas

More general formula for G

The more general proposal is that h_{eff} in the formula for G must be replaced with $h_{gr,1} = n_{gr} h_0$, where n_{gr} is closely related to the $n = h_{eff}/h_0$ but not equal to it. The estimate $G\hbar/R^2 \simeq 1.6 \times 107$ and $\hbar = 6\hbar_0$ gives the estimate $n_{gr} = 6 \times 2^{24} \simeq 1.00663296 \times 10^8$.

To make the continuation easier, it is good to express the idea in more detail.

1. CP_2 "radius" R identified in terms of geodesic length $l = 2\pi R$ is the fundamental geometrically realized unit of length measurement, and takes the role of Planck length $l_P^2 = G\hbar$ having only dimensional analytic justification. G is now the prediction and the first guess is $G = R^2/\hbar_{gr,1}$, where $\hbar_{gr,1} = n_{gr} \times \hbar_0$ is effective Planck constant with n_{gr} identified as dimension of "gravitational" extension of rationals.

$n = h_eff/h_0$ is the number of sheets of covering of space-time surface transformed to each other by Galois group. Since l_P/R is in the range $10^{-7} - 10^{-8}$, one must have $\hbar_{gr,1}/\hbar$ in the range $10^7 - 10^8$.

2. In principle $n_{gr}/6$ could have values $10^{-7} - 10^{-8}$ times smaller than the value associated with G. If so, G could be up to factor $10^7 - 10^8$ times larger than the standard value $G = G_N$. The downwards fluctuations of $\hbar_{eff}$ strengthen the gravitational attraction. One cannot exclude even large fluctuations of G.

The first attempt to identify n_{gr} fails

The motivation for this article came from an attempt to understand the value of gravitational constant G as a prediction of TGD - I have already earlier developed a model in which gravitational constant is predicted in terms of CP_2 radius R and a number related to effective Planck constant $h_{eff} = nh_0$.

1. The first proposal was that one can write $h_{eff}/h_0 = n$ as $n = n_1 \times n_2$, where n_1 is the number of sheets of space-time surface as a covering M^4 (space-time points with same CP_2 coordinates) and n_2 is the number of sheets as covering CP_2 (space-time points with same M^4 coordinates). There would be n_1 different space-time sheets for given M^4 projection - this corresponds to the idea about many-sheeted space-time. There would be n_2 different regions of space-time for given region of CP_2 projection. One can imagine n_2 parallel flux tubes in M^4 forming a coherent structure. This intuitive picture could but need not survive in more precise formulation.

2. The improved formula would be $G = R^2/n_{gr}\hbar_0$, where one as either *a)*: $n_{gr} = n_1$ or *b)*: $n_{gr} = n_2$. Which option - if either of them - is the correct one? Note that the option $n_{gr} = n$ is not possible since b can have huge values and G would approach to zero for dark matter in long length scales: with the recent understanding of physics this does not look plausible.

The limit $n_{gr} < n_{nmax} \sim 10^8$ means a bound on the number of space-time sheets over M^4 or CP_2.

1. For option *a)* with $n_{gr} = n_1 < n_{max} \sim 10^8$ one can imagine that the Galois group corresponds to a discrete finite sub-group of $SU(3)$, analogous to the isometry groups of Platonic solids. In the case of $SO(3)$ the order of this group is bounded to the order 60 of isosahedral group unless the group is Abelian. The largest discrete sub-group of $SU(3)$ analogous to icosahedral group has order 1080 and is too small by several orders of magnitude.

   Remark: The number parallel flux tubes could be arbitrarily large for tis option - a possible interpretation would be that gravitational quantum coherence is true in very long length scales.

2. For option *b)* with $n_{gr} = n_2 < n_{max}$ would state that the number of parallel flux tubes forming a coherent structure is bounded. The number of space-time sheets over M^4 could be arbitrarily large. The only natural symmetry group for M^4 is discrete sub-group of $SO(3)$. For the icosahedral group the order is 60 and quite too small.

Both options fail.

A modified formula for G

The failure forces to consider a more general formula for G. The outcome is the following argument.

1. TGD predicts a hierarchy of effective Planck constants $h_{eff}/h_0 = n$, where n is the order of Galois group of Galois extension defining extension of rationals [L28, L29] [**?**, K67]. Dimension n of extension factorizes to a product $n = n_1 n_2 ...$ for extension E_1 of extension E_2 of .... rationals. $M^8 - H$ correspondence allows to associate the Galois group with an irreducible polynomial characterizing space-time surface as an algebraic surface in M^8. The gradual increase of extension by forming a functional composite of a new polynomial with the already existing one ($P \to P_{new} \circ P$) would be analogous to the evolution of genome: earlier extensions would be analogous to conserved genes.

2. The proposal modifying the earlier proposal is $G = R^2/n_{gr}\hbar_0$, where n_{gr} is the order of Galois group G_{gr} "at the bottom" of the hierarchy of extensions, and one has $\hbar = 6h_0$. One would have $n = n_1 n_2 ... n_{gr}$. G_{gr} "at the bottom" is proposed to represent number theoretically geometric information about the embedding space by providing a discretization for the product of maximal finite discrete sub-group of isometries and tangent space rotations of embedding space.

3. By $M^8 - H$ duality these sub-groups should be identical for H and M^8. The prediction is that maximal G_{gr} is product of icosahedral group I with 3 copies of coverings $\overline{I}$. Rather remarkably, the prediction for G is correct if one assumes that the value of R is what p-adic mass calculation for electron mass gives.

Since the hierarchy of Planck constants relates to number theoretical physics proposed to describe the correlates of cognition, the connection with cognition strongly suggests itself. Icosahedral and tetrahedral geometries occur also in the TGD based model of genetic code in terms of bio-harmony [L9], which suggests that genetic code represents geometric information about embedding space symmetries. These connections are discussed in detail.

10.5.2 A formula for G in terms of order of gravitational Galois group and implications

In the sequel the formula $G = R^2/n_{gr}\hbar_0$ will be deduced from number theoretical vision based on adelic physics [L28, L29] and $M^8 - H$ duality [L24, L25, L26, L58]. The prediction allows variation of G - G is indeed known to vary more than expected. These "small" variations and also possible large variations are discussed. The successful prediction forces to consider seriously the connections between quantum gravitation, cognition, and quantum biology, in particular genetic code.

An improved attempt to identify n_{gr}

The original proposal for a formula of $G = R^2/n_{gr}\hbar_0$ failed and one must try something more general.

1. The Galois group of Galois extension has a decomposition in terms of a hierarchy of normal sub-groups. G can be represented as product of maximal normal sub-group H and group G/H. H in turn has similar decomposition and the process can be continued to get a hierarchical decomposition. This leads to a concrete model for "small" state function reduction (SSFR) as a cascade of cognitive measurements. Some special normal sub-group in the hierarchy relevant for gravitation is a good candidate for "gravitational" Galois group G_{gr}, whose order is n_{gr}.

   An attractive assumption is that the Galois group assignable to gravitational interactions is fundamental in the sense that it corresponds to the lowest step of the Galois ladder. The vision about evolution inspired by $M^8 - H$ duality is as an increasing hierarchy of polynomials P with rational coefficients defining space-time surfaces as algebraic surfaces in complexified M^8: their real projections would define 4-D space-time surfaces mapped to $H = M^4 \times CP_2$ by $M^8 - H$ duality [L24, L25, L26].

   Polynomials P would be functional composites as a generalization of abstraction process as statements about statements and evolution would proceed as sequence of abstraction steps $P \rightarrow P_{new} \circ P$. This step preserve the roots of P if the polynomials involved vanish at origin: $P(0) = 0$. Besides Galois groups the roots associated with earlier steps would be evolutionary invariants analogous to conserved genes. If one has decomposition $P = P_1 \circ P_2 ... \circ P_{gr}$, one could understand why why n_{gr} is almost universal constant.

2. Gravitation relates to space-time geometry and a good guess is that G_{gr} provides a representation for a discrete finite sub-group of the isometry group of embedding space and perhaps also for the sub-group $SO(3) \subset SO(3,1)$ acting in $E^3 \subset M^4$ or its lift to $SU(2)$. Octonionic structure in M^8 indeed selects unique rest system and even the spatial origin of the linear coordinate system is fixed. This would reduce the attempt to identify n_{gr} to a study of finite

discrete sub-groups of embedding space isometries and spin covering of its tribein rotation group.

It must be made clear that G_{gr} would be associated with the space-time sheets mediating gravitational interactions: this would include gravitational flux tubes with $\hbar_{gr} = GMm/v_0$. For flux tubes mediating - say - electromagnetic interaction the counterpart of G_{gr} could be much smaller, it would however include the group $Z_2 \times Z_3$, which is center for $SU(2) \times SU(3)$ predicting $h = 6h_0$ suggested by some empirical findings [L18, L36, L69].

3. By $M^8 - H$ duality one must consider the isometry groups and 3-D tangent space-groups of both M^8 and H to see whether n_{gr} could find a natural identification. $M^8 - H$ duality requires that the gravitational sub-group is same for M^8 and H options.

   The group $SO(3) \times U(2)$ is shared by $SO(3) \times SU(3)$ for H and $SO(3) \times SO(4)$. Tangent space group of $E^3 \subset M^4$ is $SU(2)$ and tangent space group of CP_2 is $U(2)$ in the two cases and if only maximally non-Abelian groups are accepted $U(2)$ effectively reduces to $SU(2)$, which can however correspond to a non-trivial sub-group of $U(2)$. This would mean that the maximal finite discrete sub-group of isometries and vielbein groups is direct product of 4 groups, which are icosahedral groups I or their coverings $\overline{I}$.

   The orders of icosahedral group I without reflection *resp.* its covering $\overline{I}$ is 60 *resp.* 120. $SU(2)$ for the tangent space groups is natural hypothesis since one has also fermions. For $I \times \overline{I}^3$ one would have $n_{gr} = 8 \times 6^4 \times 10^4 \simeq 32^{11} \times 15^4 = 1.0368 \times 10^8$. This is to be compared with the rough estimate $n_{gr} = 6 \times 2^{24} \simeq 1.00663296 \times 10^8$. The proposal works amazingly well!

4. Also other Platonic groups assignable to Platonic solids (tetrahedron, cube and octahedron, icosahedron and dodecahedron) are in principle possible: actually all discrete and finite sub-groups of $SU(2)$ can be considered. The non-Platonic groups however act on plane polygons, and might be more naturally assignable other than gravitational interactions. They are also associated with Mac-Kay correspondence [K93] assigning to these finite groups ADE Lie groups/Kac-Moody algebras. This hierarchy is also associated with inclusions of hyperfinite factors of type II_1 (HFFs) proposed in TGD framework to provide a representation for finite measurement resolution [K103, K36].

Could Newton's constant vary and what about formulas for other coupling strengths?

The proposed formula for G forces to consider the possibility of large variations of G due to the variation of n_{gr} as the order of gravitational Galois group G_{gr}. This group is fixed by $M^8 - H$ duality to be a product of finite discrete sub-groups of $SO(3)$ with 3 discrete sub-groups of $SU(2)$. By loosening the conditions one can however think also the possibility of other choices

1. The allowance of only Platonic solids would make possible to understand possible large increases of G but not its reduction. What is interesting is that the increase of G implies increase of gravitational Compton length $\Lambda_{gr} = G(M + m)/v_0$ unless G/v_0 is constant.

2. If one accepts also the non-Platonian finite sub-groups of $SU(2)$ with representations are realized as 2-D polygons, the range of values of G is much larger and both large and small variations of G from the preferred value become possible as variations of n_{gr}.

3. If one wants to explain the reported small but theoretically too large variations of G allowing only Platonic solids, one must allow superpositions of space-time surfaces with different values of n_{gr}. In general $\langle n_{gr} \rangle$ would be smaller than the maximal value and $\langle G \rangle$ would increase. Large variations decreasing G cannot be explained in terms of Platonic solids or their superpositions.

4. If one gives up $M^8 - H$ duality larger variations of G downwards become possible. For instance, $\overline{I}$ in the case of $SU(3)$ isometries could be replaced with $\Sigma(1080)$ with 1080 elements (`http://tinyurl.com/uq3nxko`). This would reduce G by factor $1080/120 = 9$. More generally, in $SU(3)$ there are following analogs of Platonic groups labelled as $\Sigma(n)$, $n \in \{60, 168, 36 \times 3, 72 \times 3, 21 \times 3, 72 \times 3 = 216, 216 \times 3 = 648, 360 \times 3 = 1080\}$. Also the semi-direct products $\Sigma(60) \times Z_3$ and $\Sigma(168) \times Z_3$ belong to the list.

The counterpart of ADE hierarchy for $SU(3)$ is obvious interest from the point of view of color interactions if one allows the breaking of $M8-H$ duality. There is an article by Ludl in arXiv [B39] (`http://tinyurl.com/uq3nxko`) about the finite discrete sub-groups of $SU(3)$. Table 1 of the article provides a summary of the discrete sub-groups.

1. There are 3 series parameterized by several integers with no general formula for the order. The are however infinite series of groups which belong to these series and have unbounded order. These groups are semi-direct products, which makes their representability as Galois groups of Galois extensions possible.

   Could these groups be associated with the flux tubes mediating color interactions? Could colour coupling strength be expressible as $\alpha_s = g_s^2/4\pi\hbar_s$, where $\hbar_s = n_s\hbar_0$? Could the value of g_s^2 be equal to the square g_K^2 of Kähler coupling defining fundamental constant. Could similar expression hold true also for electroweak coupling strengths. Could the breaking of gauge and gravitational symmetries be coded by different values of n_s,$n_{SU(2)_{ew}}$, $n_{U(1)}$, and n_{gr}.

2. There are also following exceptional groups analogous to Platonic groups for $SU(3)$ and labelled as $\Sigma(n)$, $n \in \{60, 168, 36\times 3, 72\times 3, 21\times 3, 72\times 3 = 216, 216\times 3 = 648, 360\times 3 = 1080\}$. Also the semi-direct products $\Sigma(60)\times Z_3$ and $\Sigma(168)\times Z_3$ belong to the list. The largest order for this series is 1080. The smallest order is 60 and corresponds to icosahedral group.

The discrete sub-groups of $SO(4)$ are interesting in M^8 picture and could contain also semi-direct products as sub-groups for products of sub-groups of $SO(3)$ and $SU(2)$. These sub-groups are listed in the appendix of the article by de Medeiros and Figueroa-O'Farrill (`http://tinyurl.com/tyagn3c`).

What do experiments say?

What do experiments say? Various experiments have been already discussed.

1. Several experiments suggests small variations of G, which are however too large theoretically. There are experiments in millimeter scales and also Podkletnov's experiment [H6, H4] [L12].

2. Could the fountain effect of super-fluidity be understood as a large reduction the value of G. It seems that a more elegant explanation is in terms of macroscopic quantum coherence due to the large value of $h_{gr} = GMm/v_0$ for space-time sheets mediating gravitation in the case of super-fluid [**?**].

3. The findings reported by Martin Grusenick [K85] - *if true* - would suggests a huge increase of G by a factor of order 10^5 if the increase of spatial lengths in the direction of the Earth's magnetic field causes the effect. The variation is too large to have an explanation allowing only Platonic solids alone. The effect could be due to the contraction of the measurement apparatus under its own weight.

   Perhaps a more elegant explanation for Grusenick's claim would be in terms of warping of space-time surface possible even in absence of gravitational field predicted by TGD. Warping means that the space-time surface has metric isometric with Minkowski metric but when the M^4 coordinates of $M^4 \subset M^4\times CP_2$ are used, there is a scaling of the metric in various directions since CP_2 projection of the embedding is not a point but geodesic circle. This would modify the propagation velocity in radial direction.

4. One can also ask whether the unexpected mass for the blackhole candidates observed by LIGO could be due to anomalously large value of G. In TGD framework the view about blackhole like entities is much more detailed than in GRT and one could understand them also without variation of G.

Since consciousness, cognition, and gravitation are closely related in TGD Universe, one cannot avoid association with the claims made by meditators about levitation. Could the experience about levitation mean a genuine levitation of dark matter at the level of magnetic body (MB), which corresponds to a higher level cognitive consciousness and naturally gravitational consciousness by huge values of $\hbar_{gr}$.

1. Could G be reduced producing anti-gravitational effect at MB? If one allows only $M^8 - H$ duality and Platonic solids G is smallest possible and cannot be reduced. Allowing also polygons would allow arbitrary small values of G. This option does not look however plausible since one can argue that the experience would reduce from 3-D for Platonic solids to 2-D for regular polygons.

2. Perhaps a more elegant explanation is that levitation experiences and out-of-body experiences [K89] (OBEs, which I have had also myself), are due to the delocalization of particles of "personal" MB due to the large value of h_{gr}. One could perhaps say that the active flux tubes of MB correspond to those mediating gravitational interaction and having $h_{gr} = GMm/v_0$. Ironically, gravitational consciousness would be experience of no having no weight.

A connection gravitation and genetic code?

A deep connection between gravitation and genetic code suggests itself.

1. TGD suggests at least two fundamental representations of genetic code besides the usual chemical representation. The first representation is terms of dark nuclei consisting of sequences dark proton triplets representing codons [L15, L32]. Both DNA, RNA, tRNA and amino-acids have analogs as dark proton sequences. Second representation is in terms of dark photon triplets defining what I call bio-harmony [L9]. Basic objection against emission of 3-dark photons simultaneously is that the process is extremely probable. If one has however Galois confinement in the sense that only Galois singlets appear as asymptotic states, the assumption that dark photons are Z_3 triplets allows only the emission of triplets [L69].

2. What is fascinating that both icosahedral and tetrahedral groups appear in the model for the genetic code in terms of bio-harmony [L9, L46, L51]. Could genes and associated molecules DNA, RNA, tRNA, and amino-acids code for information about the geometry of embedding space in some sense? DNA codons correspond to 20 triangular faces of icosahedron (3 Hamiltonian cycles are used obtain 202+20+20=60 codons) and 4 triangular faces of tetrahedron to get the remaining 4 codons. By icosahedral-dodecahedral duality gene as a sequence of these faces defines a path at dodecahedron - two subsequent codon of gene would not however map to nearest points at dodecahedron. What could this mean if anything?

3. Genes code for information and therefore could relate to cognition, and the proposed representations of genetic code would mean that genes emerge already at the fundamental level: chemical representation would be only mimicry of the dark nuclear code at higher, chemical level. The hierarchy of Planck constants relates also directly to information and h_{eff} can be seen as a kind of "IQ".

   The dependence of G on n_{gr} suggest that also gravitation relates to cognition. This would not be surprising since the long-ranged non-screened character of gravitation could make possible quantum coherence in astrophysical scales: the value $h_{eff}/h_0 = n = h_{gr}/h_0$ is indeed a direct measure of the evolutionary level.

   The connection with cognition could also explain why ancient mathematicians managed to discover the mathematical structures encountered two millenia later in theories trying to unify fundamental interactions.

Could Newton's constant relate to cognition?

After having discovered the above argument fixing n_{gr} from $M^{-}H$ duality, I could have written conclusions of the paper. The emphasis however shifted to TGD based view about evolution and cognition and its connection with gravitation. h_{eff} indeed closely relates to an evolutionary hierarchy of cognition via the idea that gravitational/geometric part of Galois group is fundamental and "at the bottom" of the hierarchy of Galois extensions of rationals. Extensions of rationals would define cognitive representations representing discretizations of spaces of various dimensions as subsets of reals or complex numbers, and also allow to represent discrete sub-groups approximating continuous groups as Galois groups.

The most fundamental physics related groups to be approximated as Galois groups would relate to the isometries and vielbein rotations of embedding space. The maximally compact sub-group would be in question both cases. The important point would be that these groups would act on extension of rationals providing cognitive representation as subset of reals/complex numbers rather than in embedding space. This kind of representation would be analogous to a linguistic, linear representation of geometric object as opposed to concrete geometric representation in embedding space.

10.5.3 Could gravitation and geometric cognition relate?

It has been already demonstrated how one can predict the value G correctly as in TGD framework. The emphasis of this section is on geometric cognition and the possibility that the value of G directly reflects this connection.

Hierarchy of effective Planck constants and Galois extensions of rationals

In adelic vision [L28, L29, L24, L25, L25] about TGD $n = h_{eff}/h_0$ corresponds to the dimension of extension of rationals characterizing space-time surface. n is also the order of Galois group of extension for Galois extensions. Recall that Galois extension has the nice property that the order of Galois group equals to the dimension of the extension. Galois extension can be regarded as extension of extension of....rationals an there is a hierarchy of Galois group such that the included sub-groups are normal sub-groups. One can express n as a product $n = n_1n_2..$ of the dimensions of these extensions.

This leads to the vision about the reduction of evolution to a hierarchy of Galois extensions such that evolution means increase of the extension and therefore number theoretical complexity and of h_{eff} meaning increase of quantum coherence scale.

If the extensions tend to emerge as further extensions preserving the earlier extensions - as is natural to think -, the extension "at the bottom" of the hierarchy of extensions is rather stable. Since the geometric cognitive consciousness can be argued to be fundamental, the dimension of Galois group corresponds to n_{gr} in $n = n_1n_2....n_{gr} = m \times n_{gr}$. n_{gr} would be rather stable factor of n.

n_{gr} would be analogous to the conserved genes of primary life form from which evolution started. The change of genome at this level would induce dramatic changes making the survival of the new life form implausible. This alone would not predict unique value for n_{gr} but only that its value is dynamically rather stable. One must of course understand why this particular value of n_{gr} would be selected. What distinguishes this extension from a general extension? The groups in question allow infinite number of finite discrete sub-groups but $M^8 - H$ duality would select highly unique sub-groups as common to both. Only groups, which are products of 4 isometry groups of Platonic solids or there double coverings and maximal order for the group minimizing G would leave only the icosahedral group I and its coverings into consideration.

$M^8 - H$ duality and representation of space-time surfaces in M^8 as algebraic surfaces assignable to polynomials with rational coefficients

$M^8 - H$ duality [L58] provides a concrete realization of the number theoretic vision in terms of space-time surfaces, and also allows to realize the view about number theoretical evolution in terms of a hierarchy of polynomials obtained by functional composition of polynomials.

The articles [L24, L25, L26] contain a detailed description of $M^8 - H$ duality. The article [L68] described a possible connection with chaos theory and Mandelbrot/Julia fractals based on the possibility that time evolution by "small" state function reductions (SSFRs) correspond in good approximation iteration of polynomial. The article [L71] describes a model of SSFR as a cognitive measurement identified as a reduction cascade in the group algebra of Galois group having a decomposition in terms of normal sub-groups.

Basic vision

Consider first what TGD space-time is.

1. In TGD framework space-times can be regarded 4-surfaces in $H = M^4 \times CP_2$ or in complexifiation of octonionic M^8. Linear Minkowski coordinates or Robertson-Walker coordinates

for light-cone (used in TGD based cosmology) provide highly unique coordinate choice and this problem disappears.

2. The solutions of field equations are preferred extremals satisfying extremely powerful additional conditions giving rise to a huge generalization of the ordinary 2-D conformal symmetry to 4-D context. In fact, twistor twist of TGD predicts that one has minimal surfaces, which are also extremals of 4-D Kähler action apart from 2-D singularities identifiable as string world sheets and partonic 2-surfaces having a number theoretical interpretation. The huge symmetries act as maximal isometry group of "world of classical worlds" (WCW) consisting of preferred extremals connecting pair of 3-surfaces, whose members are located at boundaries of causal diamond (CD). These symmetries strongly suggest that TGD represents completely integrable system and thus non-chaotic and diametrical opposite of a chaotic system. Therefore the chaos - if present - would be something different.

M^8-H duality suggests an analogous picture at the level of M^8. M^8-H duality in itse most restrictive form states that space-time surfaces are characterized by "roots" of rational polynomials extended to complexified octonionic ones by replacing the real coordinate by octonionic coordinate o [L24, L25, L26].

1. One can define the imaginary and real parts $IM(P)$ and $RE(P)$ of $P(o)$ in octonionic sense by using the decomposition of octonions $o = q_1 + I_4 q_2$ to two quaternions so that $IM(P)$ and $RE(P)$ are quaternion valued. For 4-D space-time surfaces one has either $IM(P) = 0$ or $RE(P) = 0$ in the generic case. The curve defined by the vanishing of imaginary or real part of complex function serves as the analog.

2. If the condition $P(0) = 0$ is satisfied, the boundary of δM^8_+ of M^8 light-cone is special. By the light-likeness of δM^8_+ points the polynomial $P(o)$ at δM^8_+ reduces to ordinary real polynomial $P(r)$ of the radial M^4 coordinate r identifiable as linear M^4 time coordinate t: $r = t$.

   Octonionic roots $P(o) = 0$ at M^8 light-cone reduce to roots $t = r_n$ of the real polynomial $P(r)$ and give rise to 6-D exceptional solutions with $IM(P) = RE(P) = 0$ vanish. The solutions are located to δM^8_+ and have topology of 6-sphere S^6 having 3-balls B^3 with $t = r_n$ as of M^4_+ projections. The "fiber" at point of B^3 with radial M^4 coordinate $r_M \leq r_n$ is 3-sphere $S^3 \subset E^4 \subset M^8 = M^4 \times E^4$ contracting to point at the δM^4_+.

   These 6-D objects are analogous to 5-branes in string theory and define "special moments in the life of self". At these surfaces the 4-D "roots" for $IM(P)$ or $RE(P)$ intersect and intersection is 2-D partonic surface having interpretation as a generalization of vertex for particles generalized to 3-D surfaces (instead of strings). In string theory string world sheets have boundaries at branes. Strings are replaced with space-time surfaces and branes with "special moments in the life of self".

   Quite generally, one can consider gluing 4-D "roots" for different polynomials P_1 and P_2 at surface $t = r_n$ when r_n is common root. For instance, P and its iterates $P^{\circ N}$ having r_n and the lower inverse iterates as common roots can be glued in this manner.

3. It is possible complexify M^8 and thus also r. Complexification is natural since the roots of P are in general complex. Also 4- space-time surface is complexified to 8-D surface and real space-time surface can be identified as its real projection.

To sum up, space-time surfaces would be coded a polynomial with rational or at most algebraic coefficients. Essentially the discrete data provided by the roots r_n of P would dictate the space-time surface so that one would have extremely powerful form of holography.

Should one allow also transcendental extensions?

One can consider generalizations of the simplest picture.

1. One can also consider a generalization of polynomials to general analytic functions F of octonions obtained as octonionic continuation of a real function with rational Taylor coefficients: the identification of space-time surfaces as "roots" of $IM(F)$ or $RE(F)$ makes sense.

2. What is intriguing that for space-time surfaces for which $IM(F_1) = 0$ and $IM(F_2) = 0$, one has $IM(F_1F_2) = RE(F_1)IM(F_2) + IM(F_1)RE(F_2) = 0$. One can multiply space-time surfaces by multiplying the polynomials. Multiplication is possible also when one has $RE(F_1) = 0$ and $IM(F_2) = 0$ or $RE(F_2) = 0$ or $IM(F_1) = 0$ since one has $RE(F_1F_2) = RE(F_1)RE(F_2) - IM(F_1)IM(F_2) = 0$.

   For $IM(F) = 0$ type space-time surfaces one can even define polynomials analytic functions of the space-time surface with rational Taylor coefficients. One could speak of functions having space-time surface as argument, space-time surface itself would behave like number.

3. One can also form functional composites $P \circ Q$ (also for analytic functions with complex coefficients). Since $P \circ Q$ at $IM(Q) = 0$ surface is quaternionic, its image by P is quaternionic and satisfies $IM(P \circ Q) = 0$ so that one obtains a new solution. One can iterate space-time surfaces defined by $Im(P) = 0$ condition by iterating these polynomials to give $P, P^{circ2}, ..., P^{\circ N}...$ From $IM(P) = 0$ solutions one obtains a solutions with $RE(Q) = 0$ by multiplying the M^8 coordinates with I_4 appearing in $o = q_1 + I_4 q_2$.

   The $Im(P) = 0$ solutions can be iterated to give $P \to P \circ P \to ..$, which suggests that the sequence of SSFRs could at least approximately correspond to the dynamics of iterations and generalizations of Mandelbrot and Julia sets and other complex fractals and also their space-time counterparts. Chaos (or rather, complexity theory) including also these fractals could be naturally part of TGD!

Evolution of cognition

Polynomials in M^8 obtained as continuation of real polynomials with rational (or perhaps even algebraic) coefficients and vanishing at origin define a concrete representation for the extensions of rationals. There is infinite number of polynomials realizing the same extension. The interpretation is as an evolutionary hierarchy.

Since the number of extensions larger than given extension is larger than those smaller than it, the sequence of BSFRs changing the extension leads unavoidably to evolution as a statistical increase of the dimension of extension. The functional composition of polynomials which vanish at origin gives rise to evolutionary hierarchies for which the number theoretical complexity increases as one climbs up in the hierarchy. Extensions in these hierarchies are analogous to conserved genes if the replacement of extension F in BSFR can only extend F to larger extension E. This might be true in statistical sense.

Extensions could increase statistically also in SSFRs. In [L68] I considered the possibility that the sequence of SSFRs could correspond in reasonable approximation to an iteration of polynomial P. This would give direct connection with the Mandelbrot and Julia fractals.

The basic question is whether the number theoretical vision based on M^8 and adelic physics could be seen as exact dual of the geometric vision based on $H = M^4 \times CP_2$ and the notion of "WCW" (WCW) or does number theoretical view describe cognitive representations as approximate mimicry of actual physics so that the duality would be many-to-1.

The latter option seems to more plausible. Evolution leads to an improved representations but 1-1 correspondence is not reached even at the level of algebraic numbers allowing cognitive representations dense at space-time surface, but might be reached by accepting transcendental extensions replacing polynomials with analytic functions with rational (or even algebraic) coefficients to guarantee the continuation to p-adic number fields. One argument in favor of transcendentals is that exponential functions and trigonometric functions should be possible. Exponential functions would force e which however defines finite-D extension of p-adic numbers. The roots of trigonometric functions would bring in π and is powers.

General ideas about cognition and cognitive representations

Consider first cognitive representations at space-time level.

1. Cognitive representations at the level space-time surfaces would be provided by the points of space-time surface with embedding space coordinates in extension of rationals considered. One the coordinates of embedding space are fixed, these discretization are unique. The selection of coordinates is in the octonionic case highly unique. Only time translation in the rest system defined by the linear octonion coordinates is allowed. Also in case H the

coordinates are unique apart from color rotations. Also vielbein/spin rotation group of 3-surface could have representation as a Galois group.

2. Galois group would act on the cognitive representation at space-time level and in general would not leave it invariant so that one would obtain new space-time surface. The wave functions in the space of space-time surfaces would correspond to wave functions in the space of cognitive representations which would correspond to elements of Galois group or factor space if sub-group of Galois group leaves the representation invariant. Wave functions would be elements of the group algebra of Galois group with possible conditions corresponding of invariance with respect to sub-group restricting the function to coset space effectively. This picture leads to a vision about "small" state function reductions (SSFRs) as cascades of measurements leading to a tensor product of states in the hierarchy of normal sub-groups of Galois group [L71]. The interpretation would be as cognitive measurements.

3. What about fermions? Fermionic Fock states have in TGD framework interpretation in terms of quantum variant Boolean algebra realized in terms of multi-qubits. One can say that the spinor structure of space is kind of square root of metric and describes correlates of logic [L70]. This would apply even at the level of WCW.

   What could finite measurement and cognitive resolution for fermions mean? The natural hypothesis is that the group algebras of Galois groups generated by wave functions in Galois group and having dimension n equal that for extension of rationals describe bosonic degrees of freedom and that fermionic state correspond to the spinors in this algebra- possible restrictions come from chirality restrictions. The dimension of the spinor space would be at most 2^n.

Cognitive representations at space-time level would be rather concrete. But is it possible to realize mathematical imagination, is it possible to imagine higher-D spaces?

1. Cognitive representations would indeed occur already at the level of number system. The extension of rationals can be regarded as n-D space over rationals instead of reals and would be mapped to a dense subset of real variant of n-D space. One can say that subset of real (or complex) numbers represents cognitively the higher-D space. The Galois group would represent discretization for the symmetries of these n-D space and from this one can say something about the possible isometry group of the corresponding real or complex space.

   This ability to imagine real and complex spaces of arbitrary dimension and might be fundamental aspect of mathematical consciousness.

2. If one takes seriously the idea about the connection with Newton's constant G, one can ask whether the evolution of the mathematical cognition proceeded via the gradual increase of the order of G_{gr} and meant gradual reduction of G in rather dramatic steps if only Platonic groups are allowed.

   Remark: Nottale's proposal for h_{gr} implies that gravitational Compton length for two particle system is $G(M+m)/v_0$ and increase with G since h_{gr} increases. If the velocity parameter v_0 and G do not correlate, larger value of G and therefore smaller value of n_{gr} and lower level of space-time consciousness would mean longer gravitational Compton length as a measure for quantum coherence and higher level of consciousness. This looks somewhat strange. Should one conclude that v_0 and G correlate: for instance, could G/v_0 be independent of G_{gr}?

How could mathematical physics as correlation between cognitive/imagined and sensory worlds have emerged?

1. Somehow the idea that we live in Euclidian 3-space emerged and later emerged special relativity, general relativity and its followers. It seems essential that the cognitive representations at the level of number field found counterparts at the level of sensory world represented as 3-space and eventually space-time and embedding space.

   Quaternions and octonions are naturally assignable to M^8, M^4 and H. Quaternions have $SO(3)$ as the analog of Galois group with concrete geometric interpretation. The discovery would be that this group acts on the object of sensory world. Could it be that these two equivalent choices of embedding space are the only ones for which this consciousness about this sensory-cognitive correspondence can evolve? The essential point would be that the symmetry groups of physics would be sub-groups of automorphism groups for octonions and quaternions.

Remark: The extension allowing discrete sub-group of $SO(3)$ as Galois group must be distinguished from much smaller extension needed to represent this sub-group as 3×3 orthogonal matrices.

2. Could the emergence of the idea of Platonic solids - say in mathematics of ancient Greece - correspond to a step in evolution in which this sensory-cognitive correspondence emerged. Cognitive and sensory started to resonate, as one might say.

Could Galois groups provide a representation for the discrete sub-groups of isometries and tangent space rotations of embedding space?

I have already earlier considered the possibility that Galois groups could provide representations for the finite sub-groups of isometry groups of $H = M^4 \times CP_2$ and $M^8 = M^4 \times E^4 = M^2 \times E^2 \times E^4$, see for instance [L71].

1. A natural looking assumption is that only finite discrete sub-groups having a hierarchical decomposition in terms of normal sub-groups characterizing Galois extensions and having thus order equal to dimension of extension would be allowed.

   In case of sub-groups of the rotation group, one can of course consider also sub-group generated as products of discrete sub-groups but they have infinite number of elements, which does not conform with the idea about finiteness of cognition. For instance, one can take Platonic groups and groups C_n and D_{2n} such that there rotation axis does not go through a point of Platonic solid and generate the product group. This group would have the product of Galois groups as Galois group. One could think that also these are allowed if one has finite measurement resolution and cognitive resolution. This brings in the notion of approximation, which might have emerged in cognitive evolution too.

2. In terms of polynomials defining the space-time surface in M^8 as algebraic surface, one would have $P = P_1 \circ ....P_N \circ P_{gr}$. The Galois group associated with gravitational polynomial P_{gr} of degree n_{gr} would be normal sub-group of the entire Galois group and the Galois group of $P_1 \circ ....P_N$ would be factor group. This polynomial would correspond to higher evolutionary level and perhaps consciousness not directly related to embedding space geometry.

   G_{gr} would be sub-group of embedding space isometries and vielbein rotations and therefore have the characteristic decomposition to a direct product. Direct product decomposition could be replaced with sub-direct product decomposition for sub-groups of direct product. Product- or semi-direct product decomposition would correspond to that assumed for the original proposal and interpreted in terms of many-sheetedness over M^4 *resp.* CP_2 (flux tube bundles in M^4.

3. $M^8 - H$ duality forces the identification of the direct product as four-fold product of discrete sub-groups of $SU(2)$ appearing in McKay correspondence and to the special role of icosahedral group and its covering. As found in the introduction, the condition that the Gal_{gr} is discrete finite sub-group of product of M^8 and H isometries leads to a unique identification for this group as $I \times \overline{I} \times \overline{I} \times \overline{I}$, where I is icosahedral group and $\overline{I}$ its covering, and predicts correctly the value of G.

The assumption that the product of discrete isometry groups of the factors of embedding space is representable as Galois group of Galois extension representable in terms of a polynomial can be criticized. Can the Galois group for Galois extension of rationals defined by irreducible polynomial be a direct product of Galois groups for extensions?

1. The answer to the question can be found from web (`http://tinyurl.com/sj26xrc`): it is found that this is possible for Galois extensions if the product of extensions is the extension and the intersection of extensions consists of rationals. This question is physically highly

relevant since Z_6 should have representation as Galois group having interpretation as direct product of centers of $SU(2)$ and $SU(3)$.

2. If this were not the case, one would be in trouble since this would exclude representations of the products $G_1 \times G_2$ of discrete sub-groups associated with isometries H and M^8 as Galois groups. One can of course think of having discrete sub-groups of $G_1 \times G_2$ having a lower order with direct products of sub-groups of G_i excluded. These are possible. $Z_2 \times Z_2$ allows the sub-groups $\{(0,0),(1,0)\}$, $\{(0,0),(0,1)\}$, and$\{(0,0),(1,1)\}$ and these are not products.

3. More generally, one could have a semi-direct product of normal sub-groups of $H_1 \subset G_1$ and $H_2 \subset G_2$ (`http://tinyurl.com/zhx5xpz`). This implies a correlation between the discrete isometries of the factors of embedding space, which would have physical interpretation. Semi-direct product allows surjective projections to $p_i : G_i \rightarrow H_i$ with normal sub-groups N_i as kernels. The product group $G_1/N_1 \times G_2/N_2$ is the graph of isomorphism $G_1/N_1 \equiv G_2/N_2$. This obviously poses strong conditions on the groups. For $G_1 = G_2$ one can would have $N_1 = N_2$. Since Z_2 is always normal sub-group, one would obtain an acceptable group in this manner if both factors have even order, and the order would be reduced by factor $1/4$. The orders of the acceptable sub-groups are factors of $ord(G_1) \times ord(G_2)$.

Remark: One should be of course be very cautious in considering the isometry groups. For instance, could the discrete sub-groups automorphism group G_2 of octonions be relevant in M^8 picture? One can also ask whether the finite discrete sub-groups of $SO(7)$ as maximal compact subgroup of $SO(1,7)$ might be relevant.

Genetic code and geometric consciousness

TGD predict at least two representations of genetic code. The first representation is in terms of dark photon triplets and second representation in terms of dark proton triplets.

TGD based model for genetic code based on bio-harmony realizes genetic code as a code for communications by dark photons. Triplet of dark photons having interpretation as 3-chord of bio-harmony is the basic idea. Icosahedral and tetrahedral geometries connect bio-harmony with geometry [L9, L51].

1. 12-note scale is represented as Hamiltonian cycle at icosahedron having 12 vertices. By assigning to edge of the Hamiltonian cycle quint (scaling of frequency by factor 3/2), the Hamiltonian cycle defines a harmony with 20 3-chords assignable to the triangular faces of the icosahedron. Hamiltonian cycles are characterized by their symmetry group S, which is Z_6, Z_4 and Z_2 (here one has two variants one depending on whether Z_2 represents reflection or rotation by π) or Z_1 (no symmetry, disharmony). By combining 3 Hamiltonian cycles with symmetries Z_6, Z_4, and Z_2 one obtains 60 3-chords.

2. One can assign to given 3-chord DNA codon and the analog amino-acid as the orbit of this chord under the symmetry group of the cycle. One almost obtains vertebrate genetic code with correct number of DNA codons associated with given amino-acid as number of faces at the orbit associated with it. Only 4 amino-acids and 4 DNA codons are missing. Tetrahedral harmony defined by unique Hamilton cycle gives the remaining 4 chords assignable to the triangular faces of tetrahedron. The outcome is vertebrate genetic code.

3. Icosahedron is in a unique position. Icosahedron has 17 Hamiltonian cycles whereas tetrahedron cube and dodecahedron have only 1 and octahedron 2. In case of dodecahedron the Hamiltonian cycle divides the dodecahedron to two identical parts with 6 pentagons suggesting that the the symmetry group is Z_6 and the number of amino-acids is 2.

4. There is large number of bioharmonies obtained by combining unique Z_6 harmony with pairs of Z_4 and Z_2 harmonies. Since music expresses and induces emotions, the identification would be as correlates for fundamental emotion/moods appearing already at molecular level, and perhaps even at deeper levels [L38]. The interpretation of codon as 6-bit would correspond to the standard reductionistic view about information represented as bit sequences. Harmony would code for the holistic aspects of information. These two views would correspond to intelligence in the usual sense and emotional intelligence.

Second representation of genetic code is in terms of dark nuclei consisting of sequences of dark protons triplets [L15, L69]. Codon corresponds to an entangled state of 3 dark protons forming a linear or circular structure with ordering of protons. The dark protons sequences associated with flux tubes parallel to ordinary DNA double strands would provide pairing of dark and ordinary DNA. Also RNA, tRNA, and amino-acids would be represented as dark proton triplets and DNA-amino-acid correspondence has a natural description.

One can raise questions about the interpretation of these two representations of the genetic code (and also about chemical representation).

1. Could genetic code be represented in terms of bio-harmony provide a quantum representation for two Platonic solids: icosahedron and tetrahedron, perhaps their product in $M^4 \times CP_2$. This would answer the question why both icosahedron and tetrahedron. An alternative interpretation is that one has product of isometries and tangent space rotations for M^4 (or CP_2).

   Could genes somehow represent concretely information about embedding space geometry and its symmetries - could one even imagine that genes are kind of statements? Could also dark proton representation have interpretation as a concrete representation in sensory realm.

2. One can raise questions about the bio-harmony. Why just 3 Hamiltonian cycles at icosahedron plus tetrahedral cycle? Could these 4 factors correspond to the 2+2 factors due to the $M^4 \times CP_2$ isometries and tangent space rotations. One would have representation for all these factors. But why one of them would be tetrahedron rather than icosahedron in which case one would have 80 codons? Why the symmetry groups S of Hamiltonian cycles would be Z_6, Z_4 and Z_2?

   Remark: Tetrahedral symmetries and orientation preserving octahedral symmetries are sub-groups of icosahedral symmetries (`http://tinyurl.com/vav2n2r`).

3. What about representation of color symmetries of CP_2 Platonic solid in terms of dark codons? Could one assign to dark codon formed by protons a representation in $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$ to get colored variants of genetic code. Genes would have vanishing total color. Can one consider representation of color as a subgroup of Galois group. Also more general Galois groups can be considered and genes as units would be defined as Galois singlets [L69].

Could the notion of genetic code generalize to the level of more general Galois groups.

1. Could one consider a generalization of the genetic code to cognitive representations based on Galois group and its coset groups. Restrict first the consideration to any finite discrete sub-group of isometries of H or M^8. Represent it physically in M^4 or CP_2 as a discrete structure analogous to Platonic solid. Form all Hamiltonian paths in the discretization and identify the n-D basic cells of this n-D structure as basic entities - analogs of DNA codons/chords. Identify the orbits of these entities under symmetry group of the cycle as analogs of amino-acids. Define the analog of genetic code as in the case of ordinary genetic code.

2. Could one imagine cognitive representation of arbitrary Galois group in terms of wave functions in group or its coset space. Could one consider generalization of bio-harmony in terms of Hamiltonian cycles in this coset space. Could one assign analogs of DNA codons to the faces of the polyhedron and could amino-acids correspond to the orbits of the faces under symmetries of the Hamiltonian cycle? Amino-acid wave functions would be constant at the orbits of the symmetry group of the cycle.

3. The relation to the model of "small" state function reductions (SSFRs) [L71] is interesting. SSFRs would have an interpretation as cognitive measurements in Galois group of extension. Let E be the extension of rationals and F the largest sub-field of E: let the corresponding Galois groups be G and H. The reduction would be a cascade starting with a reduction of the wave function in Galois group of E/F to a product of wave functions in G/H and H. At the next step same would take place for H and after finite number of steps one would have full reduction [L71].

These reduction cascades provide a model for cognitive processing as cognitive quantum measurements. This process brings in mind the translation of DNA to amino-acids. Could map to amino-acid involving transition from I to sub-group I/S, S the symmetry group of bio-harmony, be analogous to a state function reduction.

10.6 TGD inspired solution to three cosmological and astrophysical anomalies

I learned within a period of week about two cosmological anomalies new to me. The first anomaly is 160 minute oscillations discovered by Kotov and associated with a wide range of astrophysical systems. Second anomaly is the ionization of the interstellar gas. There might be a connection between these anomalies.

10.6.1 Could 160 minute oscillation affecting Galaxies and the Solar System correspond to cosmic "alpha rhythm"?

Kotov has discovered that many celestial objects involve 160 minute oscillation, whose origin is not identified. There is an overwhelming evidence that a non-local phenomenon is in question. TGD suggests an explanation as a kind of cosmic alpha rhythm. Fe^{2+} has 10 Hz alpha frequency, which is fundamental biorhythm as cyclotron frequency in .2 Gauss magnetic field assigned as endogenous magnetic field to living matter in TGD based quantum model of living matter. In .2 nT magnetic field which is consistent with empirically estimated values of interstellar magnetic field the cyclotron period is 160 minutes.

This co-incidence suggests that dark cyclotron photons with large value of Planck constant $h_{eff} = nh_0$ assigned with the phases of ordinary matter identifiable as dark matter and residing at magnetic flux tubes - in particular those carrying dark gravitons - induces the oscillations. The quantum coherence of dark matter would induce the coherence of oscillations in astrophysical length scales. The quantum effects on visible matter could be non-trivial since the energy $E = h_{eff}f$ of dark photons can be above thermal threshold. The same mechanism is central in TGD based quantum model for the control of visible bio-matter by dark matter.

Observations

The blog posting in Tallbloke's talkshop titled "*Evidence for a 160 minute oscillation affecting Galaxies and the Solar System*" [L54] (see `http://tinyurl.com/y5en9cxz`) tells about the finding by Valery Kotov that many celestial objects have parameters, which correspond to a fundamental frequency of 160.0101 minutes. There is an overwhelming evidence that a non-local phenomenon is in question. For instance, Earth day is 9 times 160 minutes.

The blog articles [L54, E15] give a long list of links to the works demonstrating the presence of this period: see for instance [E12, E13].

160 minute period occurs in many contexts.

1. Infrasonic oscillations, measured by Doppler effect, on the surface of Sun corresponds to a period of 160,01 minutes. These oscillations were discovered by Severny, Kotov, and Tsapp [E12, E13] and independently by Brookes *et al.* They were later conformed by two other teams - for references see the article "*Solar Activity, Wave of Kotov and Strange Coincidences*" [E15] (see `http://tinyurl.com/y6bfzy4q`). The following properties of Kotov waves are listed.

   (a) These waves are perfectly periodic and regular: no break of phase was observed over more than thirty years of observations

   (b) There are periods when the oscillation becomes blurred for the benefit of it's lobe in 159.956 minutes (modulation in 400 days).

   (c) The mode of vibration is badly identified.

   (d) The mechanism is not understood. V. Kotov proposes the influence of gravitational waves to explain the phenomenon but this explanation seems unrealistic.

2. The 160.0102 ± 0.0002 minutes appears also in solar eruptions.
3. There is a variation of the luminosity of Sun with period about 160/and or 80 minutes of Sun
4. The period of variations of luminosity of Delta Scuti stars has been found to be 162± 4 min and RR Lyrae stars 161.4 ± 1.6 minutes.
5. Kotov waves have been reported to occur even in quasars such as NGC 4151 and 3C 273 (see `http://tinyurl.com/yxcwh4rl`).

A possible TGD based explanation of Kotov waves

This finding relates in an interesting manner to the TGD based model of living systems in which cyclotron frequencies in endogenous magnetic field of $B_{end} = .2$ Gauss $= .2 \times 10^{-4}$ Tesla play a key role. The nominal value for the strength of the magnetic field of Earth varies since the value of B_E depends on position on surface of Earth. I have taken it as $B_E = .5$ Gauss but also $B_E = .3$ Gauss is mentioned. Whether $B_{end} = B_E$ can be assumed, is not clear.

1. For iron the cyclotron frequency of Fe^{2+} ion playing crucial role in oxygen based life is around 10 Hz, which serves as a fundamental biorhythm - alpha rhythm.
2. 160 min cyclotron frequency for Fe would correspond to magnetic field of .2 nT.
3. Interstellar or galactic magnetic field strengths are not far from this strength.
   - 1 nT for galactic magnetic field is claimed (see `http://tinyurl.com/yzesn4k`). This would give 32 min period.
   - For interstellar magnetic field the value 0.1 nTesla for interstellar magnetic field is claimed (see `http://tinyurl.com/y45hq72k`). Also the value .3 nT is claimed (see `http://tinyurl.com/glj8gvu`).

The proposed value .2 nT is half-way between these two values. Maybe there is fundamental biorhythm in cosmic scales! This is more or less predicted by TGD based vision about quantum coherence in all length scales made possible by the hierarchy $h_{eff} = n \times h_0$ of Planck constants predicted to define phases of ordinary matter identifiable as dark matter.

1. For large values of h_{eff} predicted by TGD the energies of the dark cyclotron photons can be above thermal threshold in living matter. This implies that the dark cyclotron radiation can have non-trivial effects on living manner: this kind of effects actually led to the idea about hierarchy of Planck constants. Now it can be deduced from what I call adelic physics [L28] (see `http://tinyurl.com/ycbhse5c`). The proposal is that bio-photons covering at least visible and UV range - the range of molecular transition energies - result as dark photons with say EEG frequencies transform to ordinary photons [K10].
2. In TGD inspired biology the cyclotron frequencies define coordinating rhythms [K71, K70] and the recent proposal [L62] (see `http://tinyurl.com/y4vtcv8u`) is that both sensory perception and motor actions and long term memory rely on a universal mechanism based on formation of holograms and their reading using dark cyclotron photon beam as reference beam. Could this mean that this mechanism is used even in galactic and cosmic scales so that life would be everywhere as TGD based theory of consciousness predicts?
3. If quantum coherence in astrophysical scales is involved, the values of h_{eff} would be very large and given by the Nottale formula $h_{eff} = h_{gr}0GMm/v_0$, where $v_0 < c$ is velocity parameter and M and m are the masses connected by the magnetic flux tubes carrying gravitons [L33]. The dark photons involved would have large energies $E = h_{eff}f$ and could therefore energies in the range of molecular transition energies and have effects on the dynamics of astrophysical system just as they would have on the physiology of brain behavior [J2].

4. Note that the magnetic flux tubes as parts of topologically highly non-trivial space-time surface would have sphere rather than disk as cross section. Although the value of Kähler magnetic and ordinary magnetic fields are non-vanishing at it, the total flux vanishes so that there is no observable magnetic field in the scale of cross section. No current is needed to generate the magnetic field in question. This kind of flux tubes are not possible in Maxwell's theory.

In this framework Kotov waves could be seen as a direct support for the magnetic flux tubes along which gravitons propagate. The control action forcing the synchronous oscillations would be by dark matter at gravitational flux tubes and the large value of h_{gr} would make possible coherent oscillations with 160 minute period to have effect on ordinary matter.

10.6.2 26 second pulsation of Earth: an analog of EEG alpha rhythm?

There is an interesting article in Discover Magazine with title "*The Earth Is Pulsating Every 26 Seconds, and Seismologists Don't Agree Why*" (`https://cutt.ly/ogI6soU`). That mini earthquakes would appear with a period of 26 seconds is a rather fascinating possibility and one can ask what the TGD based explanation for the poorly understood origin of the rhythm might be.

What has been observed?

The pulsations are Rayleigh waves in which the motion of the mass is vertical. The source of these pulsations can be located near the coast of the Gulf of Guinea. The amplitude of pulsations is largest during storms and during summer time, which suggests that ocean waves feed energy to some kind of waves. The first proposal is that deep ocean waves striking at the shore are the source of the pulsations. The problem is that the periods of these waves vary up to 20 s and shorter than the period 26 s of the pulsations.

Second hypothesis suggests that these microseisms are a form of harmonic tremor associated with the magmatic activity beneath the South Atlantic Ocean. The source is located suspiciously near a large volcano on the island of Sao Tome in the Bight of Bonny proposed to be the source. Also some other volcanoes are accompanied by microseism but the problem is why not all volcanoes would serve as sources.

The popular article talks about periodic pulsations and calls them mini earthquakes. What does this imply if one assumes that the author of the article is using the words in precise sense?

1. Stresses in the Earth's crust are involved with seismic waves. There are three basic kinds of stresses. The stress can due to the compression or stretching: in this case one speaks of tension. This could cause an oscillation. Oscillating string is a very simple example. Pulsations would be oscillations in the vertical direction. This phenomenon could be purely classical and involve no quantum jumps.

2. Ordinary earthquakes are however generated by shear stress: in an earthquake two parallel layers of rock touch each other in a fault. Faults need not be non-horizontal. When a large enough external force parallel to the fault acts on the second layer, the friction fails to keep the pieces together, and the layers start to slip. This event would be naturally quantum jump by its discontinuity. A phenomenological description is in terms of catastrophe theory but there is no proper classical description for what really happens when slippage starts.

3. Periodic mini earthquakes result if these slippages are induced by a periodic force acting on the other piece of rock in the direction of the fault. The analog of local pulsation would require a nearly vertical fault. The challenge would be to explain this periodic force. Standard physics might satisfactorily explain the periodic force and provide an estimate for the period but the description of the discontinuous transition might require TGD based quantum theory.

For the purpose of building a simple mental model, consider a 2-D lattice like structure consisting of cylindrical tectonic plates touching each other. At the border of the abyss at which the water depth suddenly increases deep ocean waves would act as an oscillating pressure to a cylinder and force it to oscillate.

If pulsations are indeed in question, the resulting horizontal motion of cylinders should be transformed to vertical motion. How this could be achieved? The pressure of ocean waves causes a compression in the horizontal direction. Since the material in question is incompressible and therefore preserves its volume, the cylinder must stretch in the vertical direction. The non-linearity of the coupling making possible period doubling could be due to the fact that the vertical stretching is a secondary effect. In the situation considered the coupling could be especially strong and make possible period doubling. The nearness of the volcano could increase the strength of coupling.

Could period doubling be involved?

Pulsations represent a special case of microseismic waves.

The microseism spectrum involves two parts: first part the period extends to 15 s as for deep ocean waves and for the second the frequencies are above 30 s and extend to 300s. However, 30 s is rather near to 26 seconds. If there is a coupling of deep ocean waves arriving at shore with microseism waves, one must explain how the almost period doubling results. In general linear coupling between oscillations preserves frequency so that non-linearity suggests itself. What comes in mind is that the system exhibits for frequency around $T = 13$ s a period doubling occurring universally in non-linear systems near chaos. Originally closed orbits in the configuration space of the system with period T are transformed in bifurcation to orbits with period $2T$. Why should $T = 13$ s be so special? In the TGD Universe, magnetic body carrying dark matter as $h_{eff} = nh_0$ phases acts as master controlling ordinary matter. The basic rule is that $h_{eff} \rightarrow nh_{eff}$ scales the energies $E = h_{eff}f$ of say phonons by n. The frequencies for the transitions preserving energy are scaled by $1/n$. Could the period doubling correspond to a transition $h_{eff} \rightarrow 2h_{eff}$ at MB and occur for $T = 13$ s, which could correspond to a cyclotron frequency of 1/13 Hz for MB. Quite generally, the cyclotron frequencies of MB of Earth would couple resonantly to various frequencies appearing in the dynamics of ordinary matter with $h_{eff} = h = 6h_0$. This would make the control possible. For $B = 2^{-7}B_{end}$ with $B_{end}02/B_E/5$, $B_E = .5$ Gauss, the cyclotron period of iron ion would be near 13 s. 25.6 Hz is rather near to 26 Hz and corresponds to 2^8:th sub-harmonic of the alpha rhythm 10 Hz, which suggests period doubling appearing in the approach to chaos as an explanation: 8^{th} period doubling of EEG alpha frequency could be in question!

Trying to understand the pulsation frequency

Could one understand the origin of the frequency 26 s in TGD framework as reflecting the presence of magnetic body (MB)? First some background about TGD.

1. TGD based quantum theory relies on zero energy ontology [L61] (`https://cutt.ly/jgI6du1`) and predicts quantum coherence in all scales being assignable to the magnetic bodies of systems consisting of ordinary matter. MBs would carry dark matter as $h_{eff} = n \times h_0$ macroscopically quantum coherent phases.

2. Ordinary ("big") state function reductions (BSFRs) would change the arrow of time and this implies that they look like deterministic smooth time evolutions leading to the final state of BSFR. The world would be quantum coherent but look classical in all scales! The change of the arrow of time leads to a radically new view about self-organization and about biology and also self-organized quantum criticality emerges naturally and leads to the emergence of "breathing systems" so that the applications to living systems are natural. In fact, evidence for very simple "breathing" systems is emerging [L59] (`https://cutt.ly/QgI6fuE`).

   Earthquakes have some strange features and this led to the proposal that earth quarks could involve BSFR in macroscopic scales at the level of MB of Earth [L56] (`https://cutt.ly/ogI6gc3`). Could also these mini earthquakes involve BSFRs? Could they be interpreted as a sequence of life cycles for a conscious entity with a life time of about 26 seconds assignable to Earth?

3. It is known that electromagnetic activity accompanies Earth quarks and this activity is such that the interpretation in terms of time reversal suggests itself. Could 26 seconds define a period for an analog of alpha rhythm in EEG? There is also another strange rhythm with a period of 160 minutes assignable to astrophysical systems and I have proposed an interpretation as a "cosmic" alpha rhythm [L54] (`https://cutt.ly/SgI6h92`).

This picture leads to ask whether the p-adic length scale hierarchy predicted by TGD could provide some understanding concerning the period of $T = 26$ seconds associated with the pulsations.

1. TGD predicts a hierarchy of p-adic length scales $L_p \propto p^{1/2}$, $p \simeq 2^k$, $k > 0$ preferred integer, coming as half octaves. TGD does not deny the possibility of scaled variants of various particles. For instance, electron could correspond to several integers k with masses proportional to $2^{k/2}$).

2. Secondary p-adic length scales correspond to scales $p^{1/2}L_p \propto p$. There also tertiary etc. time scales forming a fractal hierarchy coming in powers of$p^{1/2}$ and by p-adic length scales as preferred half octaves.

3. For instance, electron corresponds to p-adic prime $p = 2^{127} - 1$ (the largest Mersenne prime, which does not yet correspond to super-astrophysical length scale). Secondary p-adic length scale corresponds to a period $T_e \simeq .1$ seconds. This is a fundamental biorhythm appearing in alpha band of EEG. Also quarks correspond to secondary p-adic length scales which correspond to human time scales.

   $T = 26$ seconds is rather precisely equal to $2^8 \times T_e$, $T_e = .1$ seconds: the relative error is 1/64 or about 2 per cent. A scaled version of electron with mass $m = m_e/2^4 \simeq 32$ keV would correspond to 25.6 seconds. The p-adic prime $p \simeq 2^k$, $k = 127 + 8 = 135$ defining p-adic scale about .4 Angstrom. This is not far from Bohr radius $a_B = .53$ Angstrom for hydrogen atom.

Of course, the new dark particle need not be electron. One can consider more detailed attempts to understand the situation.

Option I:

The first attempt involves the notion of electropion or more generally, leptopion, see [K97] (`http://tgdtheory.fi/pdfpool/leptc.pdf`) for which there is empirical support and empirical evicence that ordinary pion allows p-adically scaled up variants.

1. The scenaro would be based on axion-like states proposed also as candidates for dark matter predicted by TGD. They would be indeed dark also in TGD but in TGD sense being particles having $h_{eff} = n \times h_0 > h$. This would explain why they are not seen in decay widths in particle accelerators (and excluding them).

2. There is evidence for electropion with mass $2 \times m_e$ (already from 1970's) decaying to an electron-positron pair but forgotten since it does not conform with the standard model (it would increase decay widths of weak bosons). TGD provides a model for this state and predicts similar states for muon and tau and evidence also for these states have been found but also forgotten.

TGD also suggest fractally scaled variants of pion states with different p-adic length scales $p \propto 2^k$ and there is empirical evidence for these states with masses both larger and smaller than pion mass.

1. One can also imagine scaled variants of electropion with different p-adic lengths scales. The primary p-adic time scale assignable to electropion scales corresponds to $k \leq 127$. How to estimate k?

If the mass squared (conformal weight is additive in p-adic mass calculations then mass squared of electropion is $m^2 = 2m_e^2$ giving $m = 2^{1/2} \times m_e$ for $k = 127$. Correct mass requires $k_e = 127 \rightarrow 126$. Compton time of electropion would be $T(electropion, 126) = T_c(126, e)/2$, where $T_c(126, e)$ is the Compton time of electron with $k = 126$.

The secondary p-adic time Compton time associated with the scaled variant of $k = 126$ electropion corresponds to $T(electropion, 126 + \Delta k) = 2^{\Delta k} T_e/2$. One must have $\Delta k = 8 + 2 = 10$ and $k = 137$. Amusingly, $k = 137$ corresponds to atomic length scale and to fine structure constant. This co-incidence could be regarded as a cosmic joke.

Why this atomic length scale, or rather the corresponding secondary p-adic length scale of scaled electropion, would be associated with the Earth's pulsations? Electropions should be dark and perhaps form a coherent state as in the model for the production of anomalous electron-positron pairs based on electropion involving in an essential manner non-orthogonal electric and magnetic fields of colliding nuclei?

Option II: The second proposal is based on TGD inspired quantum biology involving Bose-Einstein condensates of Cooper pairs of electrons, protons, and fermionic ions and also of bosonic ions at magnetic flux tubes and characterized by effective Planck constant $h_{eff} = nh_0$, $h = 6h_0$, making possible quantum coherence in length scales longer than Compton length.

1. Consider the Bose-Einstein condensate of electron Cooper pairs. Electron Cooper pairs has Compton length equal to $L_{2e} = L_e/2$, L_e the electronic Compton length. Secondary Compton time equals to $T_{2e}^{2)} = 2^{127/2} T_e/2 = .05$ s. Superconductivity in longer length scales than Compton length requires $h_{eff} > h$. The scaled up Compton scale $L_{n,2e} = nL^{2e}$ gives the coherence length of a superconductor and the secondary Compton time scales to $nT_{2e}^{2)} = .05n$ s. This time equals to $T = 25.6$ s for $n = 2^9$. The interpretation in terms of period doubling can be considered.

2. The general hypothesis [K50] is that there is resonance between dark and p-adic length scales so that this dark scale would correspond to identical p-adic length scale which would correspond to $L(k = 127 + 18 = 145) \sim 1.25$ nm equal to the transversal length scale for DNA.

3. TGD predicts that ordinary dark DNA in aqueous environment is accompanied by dark DNA realized as flux tubes carrying dark proton triplets realizing genetic code. Also amino-acids would be accompanied by these dark proton triplets and electrons would neutralize proteins charge which would be 3 proton charges per amino-acid. This would suggest that this scale relates to dark DNA, RNA, and proteins, which would involve space-time sheets which are electronic super conductors, and that the 26 second rhythm reflects the presence of water.

Option III: This alternative is nearest the idea about 260 Hz rhythm as analog of alpha rhythm. Iron ion has cyclotron frequency 10 Hz in B_{end}. Period doublings could correspond to the scalings of B_{end} by powers 2^{-n} of two scaling the cyclotron frequency by factor 2^{-2n}. The area of the flux tube would be scaled up by 2^n. If h_{eff} is scaled by 2^n, the energies are unaffected. For $n = 8$ the cyclotron frequency of iron ion would be near to 25.6 s. Could also the powers $2^{-n} \times 10$ Hz appear in the microseismic spectrum as period doubled alpha rhythm in the approach to chaos?

Could 26 second rhythm be kind of a bio-rhythm for Earth analogous to heart-beat or breathing? These two rhythms are highly varying and assignable to self-organization. EEG alpha rhythm is however universal. Could the Earthly bio-rhythm be analogous to the alpha band in the analog of EEG of Earth with frequencies scaled down by factor 1/256?

Each period would correspond to a mini earth quake. Also the ordinary EEG would involve similar BSFRs as an analog of sleep-awake rhythms and all bio-rhythms could be this kind of sleep-awake rhythms. One could of course check whether the 26 second rhythm has an electromagnetic analog?

There exists also another analogous rhythm, the 160 minute rhythm assignable to many astrophysical objects. I have proposed an interpretation as a kind of cosmic alpha rhythm.

1. 160 minute period is obtained from 26 second rhythm by scaling by a factor about $369 \simeq 2^{8.5}$ with error of 2 per cent - half octave again.

2. For the electro-pion option, one can think that one scales electropion with $k = 127$ having mass $2^{1/2} \times m_e$ to $k = 127 \rightarrow 127 + 17 = 144$ to get secondary Compton time scale $2^{16+1/2)}T_e = 154.5$ minutes not too far from 160 seconds. The interpretation as 17^{th} period doubling for $k = 127$ electro-pion with $T_c = \sqrt{2}T_e$ could make sense. There is indeed evidence for the period doubling of pion-like state. $f_c = f_e/\sqrt{2} \simeq 7.1$ Hz is lower than the nominal value $f_S = 7.8$ Hz of the lowest Schumann frequency. The cyclotron frequency of K^+ in B_{end} is 7.7 Hz and rather near to f_S.

3. For the Cooper pair option one could argue that since h_{eff} is integer valued, one can allow a value of n near to $2^{17.5} \simeq 185364$: this would give p-adic length scale $L(162)$, $L(163)$, which corresponds to one of the miracle length scales $k \in \{151, 157, 163, 167\}$ defining scales assignable to DNA coiling, would have been a more desired outcome.

10.6.3 Why is intergalactic gas ionized?

I became aware about new-to-me cosmological anomaly (see `http://tinyurl.com/y6ps6tb8`). FB really tests by tolerance threshold but it is also extremely useful. The news is that the sparsely distributed hot gas in the space between galaxies is ionized. This is difficult to understand: as universe cooled below the temperature at which hydrogen atoms became stable, it should neutralized in standard cosmology.

In bio-systems there is similar problem. Why biologically important ions are indeed ions at physiological temperatures? Even the understanding of electrolytes is plagued by a similar problem. It sounds like sacrilege to even mention to a fashionable deeply-reductionistic popular physicist talking fluently about Planck scale physics, multiverses, and landscape about the scandalous possibility that electrolytes might involve new physics! The so called cold fusion is however now more or less an empirical fact [L21] (see `http://tinyurl.com/y7u5v7j4`) and takes place in electrolytes - also living matter is an electrolyte.

TGD explanation is based on the hierarchy of Planck constants $h_{eff} = n \times h_0$ predicted by adelic physics as kind of IQ of the system.

1. The energy of radiation with very low frequencies - such as EEG frequencies - can be in the range of ionisation energies of atoms by $E = h_{eff} \times f$ - typically in UV range. Hence interaction between long and short length scales characterized by different values of h_{eff} becomes possible and in TGD magnetic body (MB) in long scales would indeed control biomatter at short scales in this manner. Cyclotron radiation from magnetic flux tubes of MB carrying dark ions would be used as control tool and Josephson radiation from cell membrane would be utilized to transfer sensory input to MB.

2. TGD variant of Nottale's hypothesis predicts really large values of h_{eff}. One would have $h_{eff} = h_{gr} = GMm/v_0$ at the magnetic flux tubes connecting masses M and m and carrying gravitons ($v_0 < c$ is a parameter with dimensions of velocity) [L33] (see `http://tinyurl.com/y63l7624`). What is important that at gravitational flux tubes cyclotron frequencies would not depend on m being thus universal. For instance, bio-photons with energies in UV and visible range would result from dark photons with large $h_{eff} = h_{gr}$ for frequencies even in EEG range and below.

The ordinary photons resulting from dark photons would ionize biologically important atoms and molecules. In the interstellar space the situation would be the same: dark photons transforming to ordinary higher energy photons would ionize the interstellar gas.

This relates closely to another cosmological mystery.

1. Standard model based cosmology cannot explain the origin of magnetic fields appearing in all scales. Magnetic fields require in Maxwell's theory current and in cosmology thermal equilibrium does not allow any currents in long length scales. In TGD however magnetic flux tubes carrying monopole fluxes are possible by the topology of CP_2. They would have closed 2-surface as cross section rather than disk. They are stable and do not require current

to generate the magnetic field. These flux tubes would be carriers of dark matter generating the dark cyclotron radiation ionizing interstellar gas in the scale of wavelength, which would be astrophysical.

2. There are also another kind of magnetic flux tubes for which cross section is sphere but the flux vanishes since the sphere is contractible. hese flux tubes are not stable against splitting. There would be no magnetic field in the scale of flux tube. Magnetic field is however non-vanishing and ions in it generate dark cyclotron radiation. These flux tubes would naturally carry gravitons and photons. These flux tubes could could mediate gravitational and electromagnetic interactions: gravitons and photons (also dark) would propagate along them.

3. This picture leads to a model for the formation of galaxies as tangles of long monopole flux carrying cosmic strings looking like dipole field in the region of galaxy (for TGD based model of quasars [L49] see `http://tinyurl.com/y2jbru4k`): the energy of these tangle would transform to ordinary matter as the cosmic strings would gradually thicken - this corresponds to cosmic expansion. The process would be the analog of inflation in TGD. Also stars and even planets could be formed in this manner, and thickened cosmic strings would be carriers of dark matter in TGD sense. The model explains the flat galactic rotation curves trivially.

4. Dark ions responsible for the intergalactic ionization could reside at these monopole flux tubes or at the flux tubes which vanishing magnetic flux carrying mediating gravitational interactions. Which option is correct? Or can one consider both options?

   There might be a connection with the $T = 160$ minute period appears in astrophysics in many scales from stars to quasars. The observation is hat dark cyclotron photons created by Fe^{2+} ions in interstellar magnetic field about .2 nT have period of 160 minutes.

   (a) In TGD inspired biology the endogenous magnetic field is about .2 Gauss and now the time scale is $t = .1$ seconds which corresponds to alpha rhythm, the fundamental biorhythm. 160 minutes would correspond to cosmic alpha rhythm! Also cyclotron photons with this frequency could induce ionization of interstellar scales. This would require h_{gr} which is by a factor $T/t = 10^5$ higher. For ordinary alpha frequency M is naturally proportional to the mass of Earth: $M = k_E M_E$. Solar mass is 3.33×10^5 times higher than the solar mass M_S, which suggests that the flux tubes of system with mass of Sun are involved. Could the dark matter in question be associated with the flux tubes connecting Sun to smaller masses m mediating gravitational interaction? The ratio of Planck constants would be

   $$\frac{h_{gr,S}}{h_{gr,E}} = \frac{k_S}{k_E} \times \frac{v_{0,E}}{v_{0,S}} \times \frac{M_S}{M_E} \quad .$$

   This would demand

   $$\frac{k_S}{k_E} \times \frac{v_{0,E}}{v_{0,S}} = \frac{1}{3.33} \simeq 3 \quad .$$

   (b) Note that the 160 minute period was discovered in the dynamics of Sun: no mechanism is not know for an oscillation coherent in so long length scale. Could this mean that the MB of Sun controls dynamics of Sun just as the MB of Earth controls the dynamics of biosphere? Is Sun a conscious, intelligent, entity?

10.7 Fast radio wave bursts: is life a cosmic fractal?

I encountered a highly interesting popular article with title "*Mysterious 'fast radio burst' detected closer to Earth than ever before*" (`https://cutt.ly/QdNX5Xc`)

Fast radio wave bursts (FRBs) arrive from a distance of hundreds of millions of light years - the scale of a large void. If the energy of FRBs is radiated isotropically in all directions - an assumption to be challenged below - the total energy is of the same order of magnitude that the energy of the Sun produced during a century. There are FRBs repeating with a period of 16 days located to a distance of 500 million light years from Earth.

The latest bursts arrive from a distance of only about 30 thousand light years from our own galaxy Milky Way described in the popular article can be assigned with magnetar (see `https://cutt.ly/udNMKRF`), which is a remnant of neutron start and has extremely strong magnetic field of about 10^{11} Tesla.

10.7.1 Basic findings

Below is the abstract of the article [E33] (`https://cutt.ly/sdNX69z`) reporting the discovery.

> *We report on International Gamma-Ray Astrophysics Laboratory (INTEGRAL) observations of the soft γ ray repeater SGR 1935+2154 performed between 2020 April 28 and May 3. Several short bursts with fluence of $\sim 10^{-7}$–10^{-6} erg cm^{-2} were detected by the Imager on-board INTEGRAL (IBIS) instrument in the 20–200 keV range. The burst with the hardest spectrum, discovered and localized in real time by the INTEGRAL Burst Alert System, was spatially and temporally coincident with a short and very bright radio burst detected by the Canadian Hydrogen Intensity Mapping Experiment (CHIME) and Survey for Transient Astronomical Radio Emission 2 (STARE2) radio telescopes at 400–800 MHz and 1.4 GHz, respectively.*
>
> *Its lightcurve shows three narrow peaks separated by ~ 29 ms time intervals, superimposed on a broad pulse lasting $\sim$ 0.6 s. The brightest peak had a delay of 6.5 $\pm$ 1.0 ms with respect to the 1.4 GHz radio pulse (that coincides with the second and brightest component seen at lower frequencies). The burst spectrum, an exponentially cutoff power law with photon index $\Gamma = 0.7^{+0.4}_{-0.2}$ and peak energy $E_p = 65 \pm 5$ keV, is harder than those of the bursts usually observed from this and other magnetars.*
>
> *By the analysis of an expanding dust-scattering ring seen in X-rays with the Neil Gehrels Swift Observatory X-ray Telescope (XRT) instrument, we derived a distance of $4.4^{+2.8}_{-1.3}$ kpc for SGR 1935+2154, independent of its possible association with the supernova remnant G57.2+0.8. At this distance, the burst 20–200 keV fluence of $(6.1 \pm 0.3) \times 10^{-7}$ erg cm^{-2} corresponds to an isotropic emitted energy of $\sim 1.4 \times 10^{39}$ erg. This is the first burst with a radio counterpart observed from a soft γ ray repeater and it strongly supports models based on magnetars that have been proposed for extragalactic fast radio bursts.*

What could be the interpretation of the finding in the TGD framework? The weirdest feature of the FRB is its gigantic total energy assuming that the radiation is isotropic during the burst. This assumption can be challenged in the TGD framework, where the stellar systems are connected to a monopole flux tube network and radiation flows along flux tubes, which can also branch. This brings strongly in mind the analog of a nervous system in cosmic scales and this analogy is used in what follows.

10.7.2 TGD based model for the FRBs

TGD based model is motivated by the fractality of the TGD Universe and zero energy ontology (ZEO) based view about quantum measurement theory predicting that self-organization correspond in all scales corresponds to a formation systems living in at least primitive sense.

An essential element is the hierarchy of effective Planck constants $h_{eff} = nh_0$ implied by adelic physics formulating the number theoretic vision about TGD. h_{eff} labels phases of ordinary particles behaving like dark matter and n corresponds to the dimension of extension of rationals. The first generalization of Nottale's hypothesis $\hbar_{gr} = GMm/v_0$ to be discussed below in more detail was to $h_{eff} = \hbar_{gr}$. The recent form of the hypothesis is that $\hbar_{gr}$ corresponds to a large integer factor of $h_{eff}/h_0 = n$.

The differences between TGD based view about classical fields lead to the notion of magnetic body consisting of flux quanta. Entire Universe would be a fractal network of nodes (say stars, planets, etc... identifiable as flux tube tangles identifiable as spaghetti like structures) connected by flux tubes, which can come in two varieties depending on whether the magnetic flux associated with them vanishes or is monopole flux.

10.7.3 Heuristic picture

With this background in mind one can start the heuristic model building.

1. The duration of pulses is few milliseconds: the duration of nerve pulses is the same. Is this a wink-wink to the Poirots of astrophysics?

2. Bursts can arrive regularly - for instance with a period of $T = 16.35$ days [E23] (https://cutt.ly/xdNMjQK). This brings in the mind of astro-Poirot biorhythm, in particular EEG rhythms. This would not be the only such rhythms: also the period of $T_{alpha} = 160$ minutes, for which have proposed an interpretation as a cosmic analog of alpha rhythm is known [L54]. The ratio $T/T_\alpha = 147.15$ would give for the analogous brain rhythm the value of 14.7 seconds.

3. Let us assume that stellar systems indeed form an analog of neural network connected by flux and assume that the topology of this network is analogous to that defined by axons. In TGD framework neural communications between neurons occur actually by using dark photons with effective Planck constant $h_{eff} = nh_0$ along the flux tubes with the velocity of light so that feedback from brain and even from the magnetic body of brain back to sensory organs as virtual sensory input becomes possible. The function of nerve pulses is to connect the outgoing branch of the flux tube associated with the axon and those associated with dendrites of the post-synaptic neuron to a longer flux tubes by using neurotransmitters as relays.

4. The stellar object as an analog of a neuron would send its dark photon signals along the flux tube assignable to a single axon. Axon would later branch to dendrites arriving to other stellar systems and eventually perhaps to planets as analogs of synaptic contacts. An interesting question is whether also the analogs of nerve pulses and neurotransmitters acting as relays in the synaptic contacts defined by planets could make sense. What could nerve pulses propagating along the flux tube correspond to?

   Remark: In the TGD based model of brain there would be also flux tube network analogous to the meridian system of Eastern medicine and responsible for the holistic and spatial aspects of consciousness since more than one flux tube can emanate from a given node making possibly non-linear networks [L22]. Nervous system with tree- like structure would be responsible for the linear and temporal aspects of conscious experience. Meridian system would be a precedessor of the neural system.

5. The distances of FRBs are of the order of large voids having galaxies at their boundaries and forming lattice-like networks possibly assignable to the tessellations of 3-D hyperbolic space defining cosmic time= constant surfaces. This kind of tessellations could accompany also brain [L67]. In the fractal Universe of TGD one can wonder whether these voids are analogs of cells or even neurons and form cosmic biological organisms with flux tubes forming a network allowing communications.

10.7.4 The total emitted energy if it is analogous to nerve pulse pattern along flux tube directed to solar system

The basic implication is that the energy of the emitted radiation could be dramatically smaller than that predicted by am isotropic radiation burst. It is interesting to look whether the proposed picture survives quantitative modelling.

1. The reduction factor r for the total emitted energy would be essentially $r = S/A$, where S is the area of the "axonal" flux tube and $A = 4\pi R^2$ is the surface area of the magnetar. One must estimate the value of r.

2. Flux quantization for a single sheet of the many-sheeted magnetic flux tube involved would give $eBS = \hbar_0\, h = 6h_0$ [L18, L36]. The general order of magnitude estimate is $eB \sim \hbar_0/S$. If each sheet carries out the same energy, the number of sheets is $n = h_{eff}/h_0$ and the effective area of a flux tube is $S = \hbar_0/eB$. Does the magnetic field assigned with magnetar correspond to a single sheet or to all sheets? If the field is measured from cyclotron energies assuming

$h_{eff} = h$ it would correspond to all sheets and the measured magnetic field would be the effective magnetic field $B_{eff} = nB/6$ for $h = 6h_0$.

3. The branching of the flux tube could correspond to the splitting of the many-sheeted flux tube to tubes with smaller number of sheets and involve reduction of h_{eff}. This would give the estimate $r = \hbar_0/eBA$. Magnetic field of 1 Tesla corresponds to a unit flux quantum with radius - magnetic length - about 2.6×10^{-8} meters. Assuming the estimate $R = 20$ km for the magnetar radius, one has $r \sim 10^{-25}/6$.

4. The estimate for the total emitted energy assuming isotropic radiation is the energy radiated by the Sun during a century. Sun transforms roughly $E_{100} = 1.3 \times 10^{19}$ kg of mass to radiation during a century. This gives for the energy emitted in FRB the estimate $E = rE_{100} \sim 10^{-6}/6$ kg which is roughly 7.5 Planck masses $m_{Pl} \simeq 2.2 \times 10^{-8}$ kg $= 1.2 \times 10^{19}$ GeV. The order of magnitude is Planck mass. The estimate is of course extremely rough.

   In any case, the idea that pulses could have mass of order few Planck masses is attractive. Note that a large neuron with radius about 10^{-4} meters has a mass of order Planck mass [L63].

5. From the total detected energy $dE/dS = 6.1 \times 10^{-7}$ erg m^{-2} =$3.8 \times 10^9 eVm^{-2}$ and total radiated energy $E = 7.5 m_{Planck}$ one can estimate the total area S covered by the branched energy flux if it covers the entire area with a shape of disk of radius R. This gives some idea about how wide the branching is. The total energy is $E = (dE/dS) \times \pi R^2$ giving $R = \sqrt{E/\pi(dE/dS)} \simeq .9 \times 10^9$ m. The equitoral radius of the Sun is $R_{Sun} = .7 \times 10^9$ m. $R_{Sun} \sim .78R$ This conforms with the idea that the radiation arrives along the axon-like flux tube connecting Sun and the magnetar branching so that it covers entire Sun.

10.7.5 Is the ratio $\hbar_{gr}/\hbar$ equal to the ratio of the total emitted energy to the total energy received by Sun?

The ratio h_{eff}/h should be of the same order of magnitude as the ratio $X = E/E_{rad}$, where E_{rad} is the energy of the radio wave photon with frequency 1.4 GHz for $h_{eff} = h$: $X \sim h_{eff}/h$. The ratio $Y = X/(h_{eff}/h)$ should satisfy $Y \sim 1$.

1. To proceed further, one can use the TGD variant of Nottale's hypothesis. The hypothesis states that one can assign to gravitational flux tubes gravitational Planck constant $\hbar_{gr}$. The original hypothesis was $\hbar_{eff} = \hbar_{gr}$ and the more recent form inspired by the adelic vision states that h_{gr} corresponds to a large integer factor of h_{eff}. One has $\hbar_{gr} = GMm/v_0 = r_S m/2v_0$. Here M is the mass of the large object - now that of magnetar. m is the mass of the smaller quantum coherent object in contact with the gravitational flux tube mediating gravitational interaction as dark graviton exchanges.

   v_0 is a velocity parameter, which for Sun would be $\beta_{0,S} = v_0/c \simeq 2^{-11}$ from the model for the inner planets as Bohr orbits [E25] [K79, K66, K67, ?].

2. The Planckian educated guess is $m \sim m_{Pl}$ so that one would have $\hbar_{gr}/\hbar = r_S(M)/(2L_{Pl}\beta_0)$, where L_{Pl} is Planck length and $r_S(M)$ is the Schwartshild radius of the magnetar. This would give $Y = X/(h_{gr}/\hbar) = .4$ if one has $r_S = 3$ km as for the Sun. r_S is probably large but smaller than magnetar radius about 20 km. The masses of the magnetars are in the range 1-2 solar masses. For $M = 2M_S$ one obtains $Y = .8$

   The rough estimate is not far from $Y = 1$ and suggests that the interacting quantum units at the receiving end have mass of order Planck mass. Interestingly, the mass of a large neuron with radius 10^{-4} m is about Planck mass [L63], which supports the view that quantum gravitation in the TGD sense is fundamental for life - even in the cosmic scales.

10.7.6 The parameter v_0 as analog of nerve pulse conduction velocity?

The physical interpretation of the velocity parameter v_0 is one of the key challenges of TGD.

1. The order of magnitude of v_0 is the same as for the rotational velocities in the solar system. I have considered a geometry based interpretation in [L34, L33] [K7].

2. The analogy with the neural system encourages the question whether v_0 could have a concrete interpretation as the analog of the nerve pulse conduction velocity assignable to the dark magnetic flux tubes connecting distant systems.

   In TGD framework nerve pulses [K75] are proposed to be induced by Sine-Gordon solitons for the generalized Josephson junctions assignable to the cell membrane and identifiable as transversal flux tubes assignable to various membrane proteins such as ion channels and pumps. The dark variants of the biologically important ions would give rise to the supra currents.

   Could the gravitational flux tubes analogous to axons have this kind of structure and give rise to generalized Josephson junctions with ions serving also in this case as current carriers?

To sum up, the proposed interpretation as cosmic neural networks conforms with the basic assumptions of TGD. Most importantly, quantitative predictions are correct. The picture is of course not deduce from axioms: this is pattern recognition with basic principles predicting a lot of new physics.

10.8 Appendix: About the dependence of scattering amplitudes on $\hbar_{eff}$

In TGD $\hbar$ is replaced with $\hbar_{eff} = nh_0 = nh/6$ [L18, L36, L37], and it is important to know the general dependence of scattering amplitudes on $\hbar_{eff}$. In QFT formalism the standard choice of units is $\hbar = 1, c = 1$ so that it requires some work to deduce the general dependence of the scattering amplitudes and rate on $\hbar_{eff}$. One must also check whether this dependence is consistent TGD with view about coupling constant evolution as a discrete sequence of phase transitions between quantum critical states.

10.8.1 General observations about the dependence of n-particle scattering amplitudes on $\hbar$

The "*Quantum Field Theory*" by Itzykson and Zuber [B31] provides the information about the general dependence of scattering amplitudes on $\hbar$ albeit in implicit form since units $\hbar = 1, c = 1$ are used.

1. Since putting $\hbar = 1$ is not possible in TGD framework, one must carefully check how the scattering amplitudes and rates depend on $\hbar$. In this respect tree scattering amplitudes in Abelian gauge theory like QED are characterized by the number of vertices. Each vertex involves g. Besides this there are delta functions expressing on mass shell conditions and momentum conservation.

   The amplitude involving n gauge boson-fermion vertices is proportional to g^n and scattering rate is proportional to g^{2n}. g^2 has dimension of $\hbar$ so that the condition that the coupling parameters give dimensionless factor requires additional power of $\hbar$ giving rise to α^{2n} factor, where $\alpha = g^2/4\pi$ is the analog of fine structure constant.

2. The general rule must be that gFF vertex involves factor $g/\sqrt{4\pi\hbar}$. The origin of $1/\sqrt{4\pi\hbar}$ factor can be traced out to the dimensions $[\sqrt{\hbar}/L]$ of scalar and vector boson fields, and the dimension $[\sqrt{\hbar}/L^{3/2}]$ spinor fields following from the condition that Hamiltonian for free fields has dimension $[\hbar/L]$ of energy. This implies that in gauge boson-fermion vertex one has $g/\sqrt{\hbar}$ and in a gauge theory having no dimensional couplings $g/\sqrt{\hbar}$ appears as coupling constant quite generally. In non-abelian gauge theory 3-boson vertices involving g and 4-boson vertices involving g^2 are also present and this rule gives power α^n, $n = n_3 + 2n_4$, where n_3 is the number of 3-vertices (BBB and BBF) and n_4 is the number of bosonic 4-vertices.

   This is however gauge theory limit at which particles become points-like and the flux tubes giving rise to a tensor network are neglected. In this framework one could interpret $g^2/4\pi\hbar$ as coupling parameter assignable to the flux tube connecting particles and this is indeed more natural number theoretically since $\hbar_{eff}/h_0$ is integer. In case of gravitation this seems to be the only possibility.

3. The density of states factor appearing in the rate does not depend on $\hbar$. In particle-in-the box quantization momenta are given by $p = n\hbar/L$ and density of states is $d^3n = Vd^3p/\hbar^3$. When one scales up $\hbar$ also V is scaled so that d^3n remains invariant.

One can now look the scattering amplitudes and rates in more detail. The "*Quantum Field Theory*" by Itzykson and Zuber [B31] provides examples of practical calculations and allows to deduce simple rules for $\hbar$ dependence of scattering amplitudes and rates.

1. For fermion-fermion scattering in Abelian gauge theories in the lowest order $2 \to 2$ scattering $\hbar$ disappears from the scattering cross section, and one obtains just the classical result. For instance, electrodynamics lowest order scattering cross sections - say for Compton scattering or electro-electron scattering - are proportional to α^2/m^2 in units $\hbar = 1$, $c = 1$. Putting in $\hbar$ one obtains $\alpha^2\hbar^2/m^2$. $\alpha = e^2/4\pi\hbar$ implies that $\hbar$ disappears so that its value does not matter. Therefore there is strong dependence on $\hbar_{eff}$ for fermion-fermion in gauge theory in tree approximation. For the radiative corrections to 2-2 scattering coming in powers of α the value of $\hbar$ matters and the larger its value the smaller the corrections are and this gives hopes about the convergence of the perturbation theory. The theoretician friendly Nature would induce a phase transition increasing h_{eff} to guarantee the convergence of perturbation series.

2. For a gauge theory scattering of type $2 \to n > 2$ via tree diagrams there are n vertices and the total scattering cross section is proportional to α^n/m^2 and thus depends on $\hbar$ for $n > 2$. The rate for production of states with higher particle number decrease with $\hbar_{eff}$. Hence $\hbar$ is measurable also in this manner.

3. For particle decays the rate is proportional to $1/\hbar_{eff}$: $\alpha^2 m$ is the basic dependence from dimensional analysis. Increase of $\hbar_{eff}$ scales up life-time as one might expect. For the decay of positronium non-perturbative effects due to bound state nature bring in additional power of α and the life time scales like a higher power of $\hbar_{eff}$.

4. It is often sloppily argued that classical limit corresponds to the limit $\hbar = 0$. This limit however completely fails as an approximation in situations in which $\hbar \to 0$ limit does not make sense. For instance, for atoms bound state energies are proportional to $1/\hbar^2$ and approach to infinite value as $\hbar$ goes to zero.

Clearly, $2 \to 2$ scattering for massive particles is very special in that for tree diagrams in QED and gauge theories the outcome does not depend on $\hbar_{eff}$ at all. It is intriguing that $2 \to 2$ scattering is main provider of information. This leaves room for the possibility of $\hbar_{eff}$ hierarchy.

10.8.2 Photon-photon scattering as objection against TGD view about discrete coupling constant evolution

Twistor approach suggests in TGD framework that perturbative corrections for a given extension of rationals vanish altogether [K38, K94, K78].

1. The weak form of the proposal is that this occurs only for critical values of coupling constants so that the sum over loop diagrams would vanish in these cases. Coupling constants would depend on extension of rationals and coupling constant evolution would be induced by the hierarchy of these extensions and coupling constant evolution would be discrete. This picture follows if space-time surfaces correspond to zero loci for real or imaginary parts of octonionic polynomials at M^8 side of $M^8 - H$ duality [L23].

   One could argue that the hierarchy of extensions of rationals defines a hierarchy of cognitive resolutions obtained by approximation analytic functions of octonions at M^8 side of $M^8 - H$ duality with polynomials. For space-time surfaces represented as zero loci of real or imaginary part of an *arbitrary* analytic function, the radiative corrections would not vanish.

2. Strong form of the proposal would mean that individual loop corrections vanish identically.

An objection against vanishing of loops is photon-photon scattering, which occurs via box diagram at QFT limit of TGD. This gives for sigma the behavior α^4/E^2 by dimensional argument. The rate is proportional to $1/\hbar_{eff}^2$. Photon-photon scattering is observed and QED predictions are correct.

What the vanishing of loops - in particular box diagrams - at QFT limit TGD could mean for photon-photon scattering? Does this kill the idea about the reduction of scattering amplitudes to tree level?

1. TGD description is based on many-sheeted space-time and the fundamental scattering events in twistor diagrams are for fermions. It is this level at which one would have only the analogs of tree diagrams. QFT limit is only an effective description, and the action is expected to be standard model action in a good approximation. If so, the problem disappears.

2. How photon-photon scattering could emerge at the fundamental level? TGD picture relies on twistor diagrams rather than Feynman diagrams. The proposal is that at fundamental level twistor diagrams at $M^4 \times CP_2$ side of $M^8 - H$ duality involve only fermions and their bound states.

   At M^8 side of $M^8 - H$ duality the geometric variant of approach would be realized. Components of super field would correspond to components of super-octonion and polynomial of super-octonion would be analogs of super-field. The vanishing of the real or imaginary part (in quaternionic sense) for the component polynomials would assign to each component of this super-polynomial a space-time surface in M^8.

   For twistor diagrams the analogs of virtual particles are possible but they would have on-mass-shell complex momenta. Photon-photon scattering could occur as on-mass-shell process in this sense and involve the decay of photon to fermion antifermion pair with complex momenta. Second incoming photon would absorb the antifermion with complex momentum. The reaction would proceed in the similar manner in the remaining two vertices.

10.8.3 What about quantum gravitation for dark matter with large enough h_{eff}?

It is interesting to look what h_{gr} hypothesis implies for quantum gravitation for dark matter. Does the QFT type description for quantum gravitation of dark matter make sense in TGD framework?

1. One can consider two identifications for the fundamental parameter as either G or l_P^2. These identifications lead to same predictions as far the dependence of scattering amplitudes on $\hbar_{eff}$ is considered.

   (a) G is the fundamental parameter GMm has same dimension $[hbar]$ as $Z_1Z_2e^2$ and thus one can define the analog of gravitational fine structure constant as GM_P^2. The 2-2 scattering cross section is completely analogous to that for Coulomb scattering and does not depend on $\hbar_{eff}$ at all. This result is rather satisfactory.

   (b) Second option is that Planck length l_P defines fundamental length and G is identified as $G = l_p^2/\hbar_{eff}$. This gives $GMm = l_P^2 Mm/\hbar_{eff}$ with Plack length identified as CP_2 radius R: $l_P = R$ [L44]. The independence of the cross section or $2 \rightarrow 2$ scattering on $\hbar_{eff}$ in lowest order holds true also now. $\sqrt{\hbar_{eff}}M/M_P = \sqrt{G}M = Ml_P/\sqrt{\hbar}$ would serve as analog of e now.

2. In the lowest order the scattering amplitude for $2 \rightarrow 2$ scattering by graviton exchange should be essentially Fourier transform of Newton's gravitational potential at the static limit. The independence of $2 \rightarrow 2$ scattering cross section on h_{eff} looks a natural condition since in the lowest order the scattering would not depend at all on the value of $\hbar_{eff}$. Coupling strength GMm is analogous to $Z_1Z_2e^2$ and both have dimension $[\hbar]$. Therefore the cross section for $2 \rightarrow 2$ scattering does not depend on $\hbar$ if one expresses $G = l_P^2/\hbar_{eff}$, $l_P = R$. This implies that QFT type description with point-like particles can serve as an approximate description of gravitational interaction.

This and Nottale's proposal [E25] would require that $GMm/\hbar_{eff}$ serves as dimensionless coupling parameter. Coupling strength α_{gr} would characterize pair of interacting particles rather than particle and would be naturally associated with flux tube mediating the interaction as graviton exchange and has an interpretation as generalization of string model picture. This picture makes sense also for gauge bosons.

3. Does the description of two-particle system with masses M and m make sense using Schrödinger equation? De-localization might cause problems and TGD proposal is that only the de-localization of dark matter occurs and also this takes place only on flux tubes along the orbits of planets [K79, K66, K67].

   The first observation is that the parameter $GMm/\hbar$ is for planetary systems so huge so that perturbation series fails. $Mm = m_P^2 = \hbar/l_P^2$ serves as an estimate for the upper bound of Mm. For $\hbar_{gr}$ situation changes and one can write the gravitational analog of Schrödinger equation as

$$(-\frac{\nabla_u^2}{2} + \frac{\beta_0^2}{u})\Psi = e\Psi \ , \quad e = \frac{E\beta_0^2}{m} \ , \quad u = GM = \frac{r_S}{2} \ . \tag{10.8.1}$$

   $\beta_0 = v_0/c = v_0$ for $c = 1$ clearly occurs in the role of e and the scaling $E = me/\beta_0^2$.

4. If gravitational Shrödinger equation makes sense, the gravitational analogs of atomic transitions should also make sense. For h_{gr} huge pulses of gravitational radiation would accompany the transitions of the gravitational analog of hydrogen atom since binding energies are proportional to mv_0^2/n^2, m the mass of the planet. What would happen would be emission of dark graviton with energy equal to say energy difference of initial and final states (planetary Bohr orbits), which would then decay to a bunch of ordinary gravitons [K66].

   One could estimate the rate of transitions using the existing results from atomic physics. One can also try to estimate the transition rate from a generalization of Uncertainty Principle (UP): $\Delta T = \hbar_{gr}/\Delta E$. Order of magnitude is about GMn^2/v_0^3 ($c = 1$). This gives $10^5 n^2$ seconds for $v_0/c = 2^{-11}$. This time is of order 30 hours! The transition would be associated with dark matter. This looks totally unrealistic. This estimate makes sense only if there is de-localization of dark matter to analogs of hydrogen orbitals.

   A better estimate should include the interaction with dark graviton field rather than mere UP. Here one can use Fermi's Golden Rule (see `http://tinyurl.com/yblec2on`). The change of energy would be huge and therefore also graviton's energy and momentum. Wave vector however matters and would be give by $k = p/\hbar_{gr}$ and de Broglie wavelength would be of order of planetary orbit so that the analog of dipole approximation $exp(ik.x) = 1 + ik \cdot x$ would make sense. The time for transition would be about $\Delta T = \hbar_{gr}\Delta E/E^2$ and of the same order of magnitude as previous estimate. This does not make sense. De-localization of dark parts of planets in the scale of solar system would lead to surreal effects.

5. In TGD picture the dark matter is assumed to be de-localized only at the flux tubes associated with planetary orbits. TGD approach relies on zero energy ontology (ZEO) in which quantum states correspond to quantum superpositions of preferred extremals of action (sum of Kähler action and volume term proportional to cosmological constant). The transition would involve classical orbits transforming to each other by dark graviton emission. The transition would occur as a replacement of flux tube trajectory with given energy with a trajectory having lower energy. If one assumes Bohr quantization for the trajectories, the energy liberated as dark graviton in the transition is huge using normal standards for quantum transitions.

   The basic condition is that the trajectories intersect. For instance, if the original trajectory is circle, the final trajectory could be ellipsoidal trajectory with a lower energy and located inside the circular trajectory and touching it at diametrically opposite points. A natural expectation is that the transition rate is proportional to $P = (V_{12}/\sqrt{V_1V_2})^2$, where V_{12} is the volume shared by the two flux tubes V_i are flux tube volumes. The square roots $\sqrt{V_i}$ of the flux tube volumes would correspond to normalization factors for dark matter wave functions at flux tubes. The square of this factor would give a very small coefficient and make the transition very slow despite the factor that the dimensionless coupling analogous to α would be $\beta_0/4\pi$.

One would have $V_{12} \sim d^3$, where d is flux tube thickness. Flux tube volume would be $2\pi^2 Rd^2$ so that one would have orer of magnitude estimate $P \sim (1/4\pi^4)(d/R)^2$ determined by the ratio of the thickness of the flux tube to the area of the orbit determined by it. If the thickness of the flux tube is of the order of planet radius, P for Earth has order of magnitude 10^{-11}. By multiplying the estimate about 30 hours given by Uncertainty Principle would would obtain a rough estimate 10^9 years for the lifetime of the flux tube orbit of Earth.

This kind of transitions should correspond to "big" state function reductions analogous to ordinary quantum measurements rather than "small" state function reductions having so called weak measurements (see `http://tinyurl.com/zt36hpb`) as analogs. In "big" state function reductions the arrow of geometric time changes in the sense that the roles of passive and active boundary of causal diamond (CD) change and the sequence of weak measurements occurs at opposite boundary of CD shifting farther away from the passive boundary, which was active boundary before the "big" state function reduction. Note that the temporal distance between the tips of CD increases and gives rise to clock time as a counterpart of experienced time defined by the sequence of "small" state function reductions)

6. For QFT description of quantum gravitation $\sqrt{\hbar}E/M_P = El_P/\sqrt{\hbar} = E\sqrt{G}$ would serve the role of the coupling parameter analogous to e. To get some idea what happens one can look graviton-graviton scattering amplitude for 4 gravitons having all 2 positive 2 negative helicities and known as M^{--++}. Lowest order calculations without loops at Minkowski limit (tree diagrams, see `http://tinyurl.com/y82rsw9y`) give an expression as a sum of terms proportional to x^2, where the dimensionless variable x is $x = El_P/\sqrt{\hbar_{eff}}$: E is energy scale. Amplitude is proportional $1/\hbar_{eff}$ and the scattering amplitude approaches zero for large values of h_{eff}.

10.8.4 A little sidetrack: How a finite number of terms in perturbation expansion can give a good approximation although perturbation series fails to converge?

The perturbative expansion of electrodynamics does not converge. This looks paradoxical since the predictions of QED are extremely accurate. This statement is of course somewhat sloppy since there are many notions of convergence. For instance, converge could occur in some kinematical regions and fail to do so in some other regions.

If convergence does not occur in kinematically important regions, how can then apply the perturbative expansion at all? Part of the explanation is certainly that in $2 \rightarrow 2$ scattering the lowest order does not depend on $\hbar$ at all so that it could be calculated by using so large a value of $\hbar$ that convergence occurs. Could one take the convergent result cut to a finite number of powers of α in convergence region and continue it by replacing α with its actual value to region where the convergence fails? Finite cutoffs would not deviate much from the correct result but the remainder would be infinite.

Chapter 11

Expanding Earth Model and Pre-Cambrian Evolution of Continents, Climate, and Life

11.1 Introduction

TGD inspired quantum cosmology [K80, K79] predicts that astrophysical objects do not follow cosmic expansion except in jerk-wise quantum leaps increasing the gigantic value of the gravitational Planck constant characterizing space-time mediating gravitational interactions between two masses or gravitational self interactions. This assumption provides explanation for the apparent cosmological constant.

Also planets are predicted to expand in a stepwise way. This provides a new version of Expanding Earth theory originally postulated to explain the intriguing findings suggesting that continents have once formed a connected continent covering almost the entire surface of Earth but with radius which was one half of the recent one [K79].

This leads also to a rather fascinating vision about biology. The mysterious Cambrian Explosion [I1] in which a large number of new species emerged suddenly (realized already Darwin as the strongest objection against his theory) could be understood if the life would have gone to underground lakes and seas formed during the expansion period as fractures were formed and the underground cavities expanded and were filled with water. This would have allowed the life to escape cosmic radiation, meteoric bombardment, and the extremely cold climate during Proterozoic period preceding the Cambrian Explosion and migrate back as highly developed life forms as the period of glaciations ended.

Before the Proterozoic era the radius of Earth would have been one half of its recent value and started to grow with gradually accelerating rate. This forces to rewrite the entire geological and climate history of Earth during the Proterozoic period.

1. The postulated physically implausible cyclic appearance of single connected super-continent containing all land mass can be given up and replaced with a single continent containing large inland seas. There is no need to postulate the existence of series of super-oceans whose ocean floor would have subduced totally so that no direct information about them would exist nowadays. It is also possible that the underground oceans have burst into the surface during the phase transition.

   What is amusing that this kind of sea with water volume three times that in ordinary seas has been discovered quite recently (`http://time.com/2868283/subterranean-ocean-reservoir-core-ringwoodite/`) at depth of about 600 km to be compared to the depth of core which is about 2900 km. Water is associated with a mineral known as ringwoodite and ordinary sea water could have originated from this water.

2. The dominating model for pre-Cambrian climate is so called Snowball Earth model [F29] inspired by the finding that signatures of glaciations have been found at regions of Earth,

which should have been near Equator during the Proterozoic. Snowball model has several difficulties: in particular, there is a lot of evidence that a series of ordinary glaciations was in question. For $R/2$ option the regions located to Equator would have actually been near North Pole so that the glaciations would have indeed been ordinary glaciations proceeding from the poles. A killer prediction is the existence of non-glaciated regions at apparent southern latitudes around about 45 degrees and there is evidence for these indeed exists [F47]! The model makes also testable paleomagnetic killer predictions. In particular, during periods when the magnetic dipole in the direction of rotation axis the directions of the magnetic fields for $R/2$ model are predicted to be same at South Pole and apparent Equator and opposite for the standard option.

The appendix of the book gives a summary about basic concepts of TGD with illustrations. Pdf representation of same files serving as a kind of glossary can be found at `http://tgdtheory.fi/tgdglossary.pdf` [L8].

11.2 Experimental Evidence For Accelerated Expansion Is Consistent With TGD based model

There are several pieces of evidence for accelerated expansion, which need not mean cosmological constant, although this is the interpretation adopted in [E14]. It is interesting to see whether this evidence is indeed consistent with TGD based interpretation.

11.2.1 The Four Pieces Of Evidence For Accelerated Expansion

Supernovas of type Ia

Supernovas of type Ia define standard candles since their luminosity varies in an oscillatory manner and the period is proportional to the luminosity. The period gives luminosity and from this the distance can be deduced by using Hubble's law: $d = cz/H_0$, H_0 Hubble's constant. The observation was that the farther the supernova was the more dimmer it was as it should have been. In other words, Hubble's constant increased with distance and the cosmic expansion was accelerating rather than decelerating as predicted by the standard matter dominated and radiation dominated cosmologies.

Mass density is critical and 3-space is flat

It is known that the contribution of ordinary and dark matter explaining the constant velocity of distance stars rotating around galaxy is about 25 per cent from the critical density. Could it be that total mass density is critical?

From the anisotropy of cosmic microwave background one can deduce that this is the case. What criticality means geometrically is that 3-space defined as surface with constant value of cosmic time is flat. This reflects in the spectrum of microwave radiation. The spots representing small anisotropies in the microwave background temperature is 1 degree and this correspond to flat 3-space. If one had dark matter instead of dark energy the size of spot would be.5 degrees!

Thus in a cosmology based on general relativity cosmological constant remains the only viable option. The situation is different in TGD based quantum cosmology based on sub-manifold gravity and hierarchy of gravitational Planck constants.

The energy density of vacuum is constant in the size scale of big voids

It was observed that the density of dark energy would be constant in the scale of 10^8 light years. This length scale corresponds to the size of big voids containing galaxies at their boundaries.

Integrated Sachs-Wolf effect

Also so called integrated Integrated Sachs-Wolf effect supports accelerated expansion. Very slow variations of mass density are considered. These correspond to gravitational potentials. Cosmic expansion tends to flatten them but mass accretion to form structures compensates this effect so that gravitational potentials are unaffected and there is no effect of CMB. Situation changes if dark matter is replaced with dark energy the accelerated expansion flattening the gravitational potentials wins the tendency of mass accretion to make them deeper. Hence if photon passes by an over-dense region, it receives a little energy. Similarly, photon loses energy when passign by an under-dense region. This effect has been observed.

11.2.2 Comparison With TGD

The minimum TGD based explanation for accelerated expansion involves only the fact that the embeddings of critical cosmologies correspond to accelerated expansion. A more detailed model allows to understand why the critical cosmology appears during some periods.

Accelerated expansion in classical TGD

The first observation is that critical cosmologies (flat 3-space) imbeddable to 8-D embedding space H correspond to negative pressure cosmologies and thus to accelerating expansion. The negativity of the counterpart of pressure in Einstein tensor is due to the fact that space-time sheet is forced to be a 4-D surface in 8-D embedding space. This condition is analogous to a force forcing a particle at the surface of 2-sphere and gives rise to what could be called constraint force. Gravitation in TGD is sub-manifold gravitation whereas in GRT it is manifold gravitation. This would be minimum interpretation involving no assumptions about what mechanism gives rise to the critical periods.

Accelerated expansion and hierarchy of Planck constants

One can go one step further and introduce the hierarchy of Planck constants. The basic difference between TGD and GRT based cosmologies is that TGD cosmology is quantum cosmology. Smooth cosmic expansion is replaced by an expansion occurring in discrete jerks corresponding to the increase of gravitational Planck constant. At space-time level this means the replacement of 8-D embedding space H with a book like structure containing almost-copies of H with various values of Planck constant as pages glued together along critical manifold through which space-time sheet can leak between sectors with different values of $\hbar$. This process is the geometric correlate for the phase transition changing the value of Planck constant.

During these phase transition periods critical cosmology applies and predicts automatically accelerated expansion. Neither genuine negative pressure due to "quintessence" nor cosmological constant is needed. Note that quantum criticality replaces inflationary cosmology and predicts a unique cosmology apart from single parameter. Criticality also explains the fluctuations in microwave temperature as long range fluctuations characterizing criticality.

Accelerated expansion and flatness of 3-cosmology

Observations 1) and 2) about super-novae and critical cosmology (flat 3-space) are consistent with this cosmology. In TGD dark energy must be replaced with dark matter because the mass density is critical during the phase transition. This does not lead to wrong sized spots since it is the increase of Planck constant which induces the accelerated expansion understandable also as a constraint force due to embedding to H.

The size of large voids is the characteristic scale

The TGD based model in its simplest form model assigns the critical periods of expansion to large voids of size 10^8 ly. Also larger and smaller regions can express similar periods and dark space-time sheets are expected to obey same universal "cosmology" apart from a parameter characterizing the duration of the phase transition. Observation 3) that just this length scale defines the scale below which dark energy density is constant is consistent with TGD based model.

The basic prediction is jerk-wise cosmic expansion with jerks analogous to quantum transitions between states of atom increasing the size of atom. The discovery of large voids with size of order 10^8 ly but age much longer than the age of galactic large voids conforms with this prediction.

One the other hand, it is known that the size of galactic clusters has not remained constant in very long time scale so that jerk-wise expansion indeed seems to occur.

Do cosmic strings with negative gravitational mass cause the phase transition inducing accelerated expansion

Quantum classical correspondence is the basic principle of quantum TGD and suggest that the effective antigravity manifested by accelerated expansion might have some kind of concrete space-time correlate. A possible correlate is super heavy cosmic string like objects at the center of large voids which have negative gravitational mass under very general assumptions. The repulsive gravitational force created by these objects would drive galaxies to the boundaries of large voids. At some state the pressure of galaxies would become too strong and induce a quantum phase transition forcing the increase of gravitational Planck constant and expansion of the void taking place much faster than the outward drift of the galaxies. This process would repeat itself. In the average sense the cosmic expansion would not be accelerating.

11.3 Quantum Version Of Expanding Earth Theory

TGD predicts that cosmic expansion at the level of individual astrophysical systems does not take place continuously as in classical gravitation but through discrete quantum phase transitions increasing gravitational Planck constant and thus various quantum length and time scales. The reason would be that stationary quantum states for dark matter in astrophysical length scales cannot expand. One would have the analog of atomic physics in cosmic scales. Increases of $\hbar$ by a power of two are favored in these transitions but also other scalings are possible.

This has quite far reaching implications.

1. These periods have a highly unique description in terms of a critical cosmology for the expanding space-time sheet. The expansion is accelerating. The accelerating cosmic expansion can be assigned to this kind of phase transition in some length scale (TGD Universe is fractal). There is no need to introduce cosmological constant and dark energy would be actually dark matter.

2. The recently observed void which has same size of about 10^8 light years as large voids having galaxies near their boundaries but having an age which is much higher than that of the large voids, would represent one example of jerk-wise expansion.

3. This picture applies also to solar system and planets might be perhaps seen as having once been parts of a more or less connected system, the primordial Sun. The Bohr orbits for inner and outer planets correspond to gravitational Planck constant which is 5 times larger for outer planets. This suggests that the space-time sheet of outer planets has suffered a phase transition increasing the size scale by a factor of 5. Earth can be regarded either as n=1 orbit for Planck constant associated with outer planets or n= 5 orbit for inner planetary system. This might have something to do with the very special position of Earth in planetary system. One could even consider the possibility that both orbits are present as dark matter structures. The phase transition would also explain why n=1 and n=2 Bohr orbits are absent and one only n=3, 4, and 5 are present.

4. Also planets should have experienced this kind of phase transitions increasing the radius: the increase by a factor two would be the simplest situation.

The obvious question - that I did not ask - is whether this kind of phase transition might have occurred for Earth and led from a completely granite covered Earth - Pangeia without seas - to the recent Earth. Neither it did not occur to me to check whether there is any support for a rapid expansion of Earth during some period of its history.

Situation changed when my son visited me and told me about a Youtube video [F46] by Neal Adams, an American comic book and commercial artist who has also produced animations for geologists. We looked the amazing video a couple of times and I looked it again yesterday. The video is very impressive artwork but in the lack of references skeptic probably cannot avoid the feeling that Neal Adams might use his highly developed animation skills to cheat you. I found also a polemic article [F1] of Adams but again the references were lacking. Perhaps the reason of polemic tone was that the concrete animation models make the expanding Earth hypothesis very convincing but geologists refuse to consider seriously arguments by a layman without a formal academic background.

11.3.1 The Claims Of Adams

The basic claims of Adams were following.

1. The radius of Earth has increased during last 185 million years (dinosaurs [I2] appeared for about 230 million years ago) by about factor 2. If this is assumed all continents have formed at that time a single super-continent, Pangeia, filling the entire Earth surface rather than only 1/4 of it since the total area would have grown by a factor of 4. The basic argument was that it is very difficult to imagine Earth with 1/4 of surface containing granite and 3/4 covered by basalt. If the initial situation was covering by mere granite -as would look natural- it is very difficult for a believer in thermodynamics to imagine how the granite would have gathered to a single connected continent.

2. Adams claims that Earth has grown by keeping its density constant, rather than expanded, so that the mass of Earth has grown linearly with radius. Gravitational acceleration would have thus doubled and could provide a partial explanation for the disappearance of dinosaurs: it is difficult to cope in evolving environment when you get slower all the time.

3. Most of the sea floor is very young and the areas covered by the youngest basalt are the largest ones. This Adams interprets this by saying that the expansion of Earth is accelerating. The alternative interpretation is that the flow rate of the magma slows down as it recedes from the ridge where it erupts. The upper bound of 185 million years for the age of sea floor requires that the expansion period - if it is already over - lasted about 185 million years after which the flow increasing the area of the sea floor transformed to a convective flow with subduction so that the area is not increasing anymore.

4. The fact that the continents fit together - not only at the Atlantic side - but also at the Pacific side gives strong support for the idea that the entire planet was once covered by the super-continent. After the emergence of subduction theory this evidence as been dismissed.

5. I am not sure whether Adams mentions the following objections [F5]. Subduction only occurs on the other side of the subduction zone so that the other side should show evidence of being much older in the case that oceanic subduction zones are in question. This is definitely not the case. This is explained in plate tectonics as a change of the subduction direction. My explanation would be that by the symmetry of the situation both oceanic plates bend down so that this would represent new type of boundary not assumed in the tectonic plate theory.

6. As a master visualizer Adams notices that Africa and South-America do not actually fit together in absence of expansion unless one assumes that these continents have suffered a deformation. Continents are not easily deformable stuff. The assumption of expansion implies a perfect fit of *all* continents without deformation.

Knowing that the devil is in the details, I must admit that these arguments look rather convincing to me and what I learned from Wikipedia articles supports this picture.

11.3.2 The Critic Of Adams Of The Subduction Mechanism

The prevailing tectonic plate theory [F26] has been compared to the Copernican revolution in geology. The theory explains the young age of the seafloor in terms of the decomposition of the litosphere to tectonic plates and the convective flow of magma to which oceanic tectonic plates participate. The magma emerges from the crests of the mid ocean ridges representing a boundary of two plates and leads to the expansion of sea floor. The variations of the polarity of Earth's magnetic field coded in sea floor provide a strong support for the hypothesis that magma emerges from the crests.

The flow back to would take place at so called oceanic trenches [F19] near continents which represent the deepest parts of ocean. This process is known as subduction. In subduction oceanic tectonic plate bends and penetrates below the continental tectonic plate, the material in the oceanic plate gets denser and sinks into the magma. In this manner the oceanic tectonic plate suffers a metamorphosis returning back to the magma: everything which comes from Earth's interior returns back. Subduction mechanism explains elegantly formation of mountains [F20] (orogeny), earth quake zones, and associated zones of volcanic activity [F36] .

Adams is very polemic about the notion of subduction, in particular about the assumption that it generates steady convective cycle. The basic objections of Adams against subduction are following.

1. There are not enough subduction zones to allow a steady situation. According to Adams, the situation resembles that for a flow in a tube which becomes narrower. In a steady situation the flow should accelerate as it approaches subduction zones rather than slow down. Subduction zones should be surrounded by large areas of sea floor with constant age. Just the opposite is suggested by the fact that the youngest portion of sea-floor near the ridges is largest. The presence of zones at which both ocean plates bend down could improve the situation. Also jamming of the flow could occur so that the thickness of oceanic plate increases with the distance from the eruption ridge. Jamming could increase also the density of the oceanic plate and thus the effectiveness of subduction.

2. There is no clear evidence that subduction has occurred at other planets. The usual defense is that the presence of sea is essential for the subduction mechanism.

3. One can also wonder what is the mechanism that led to the formation of single super continent Pangeia covering 1/4 of Earth's surface. How probable the gathering of all separate continents to form single cluster is? The later events would suggest that just the opposite should have occurred from the beginning.

11.3.3 Expanding Earth Theories Are Not New

After I had decided to check the claims of Adams, the first thing that I learned is that Expanding Earth theory [F5], whose existence Adams actually mentions, is by no means new. There are actually many of them.

The general reason why these theories were rejected by the main stream community was the absence of a convincing physical mechanism of expansion or of growth in which the density of Earth remains constant.

1. 1888 Yarkovski postulated some sort of aether absorbed by Earth and transforming to chemical elements (TGD version of aether could be dark matter). 1909 Mantovani postulated thermal expansion but no growth of the Earth's mass [F45].

2. Paul Dirac's idea about changing Planck constant led Pascual Jordan in 1964 to a modification of general relativity predicting slow expansion of planets. The recent measurement of the gravitational constant imply that the upper bound for the relative change of gravitational constant is 10 time too small to produce large enough rate of expansion. Also many other theories have been proposed but they are in general conflict with modern physics.

3. The most modern version of Expanding Earth theory is by Australian geologist Samuel W. Carey. He calculated that in Cambrian period (about 500 million years ago) all continents were stuck together and covered the entire Earth. Deep seas began to evolve then.

11.3.4 Summary Of TGD Based Theory Of Expanding Earth

TGD based model differs from the tectonic plate model but allows subduction which cannot imply considerable back-flow of magma. Let us sum up the basic assumptions and implications.

1. The expansion is or was due to a quantum phase transition increasing the value of gravitational Planck constant and forced by the cosmic expansion in the average sense.

2. Tectonic plates do not participate to the expansion and therefore new plate must be formed and the flow of magma from the crests of mid ocean ridges is needed. The decomposition of a single plate covering the entire planet to plates to create the mid ocean ridges is necessary for the generation of new tectonic plate. The decomposition into tectonic plates is thus prediction rather than assumption.

3. The expansion forced the decomposition of Pangeia super-continent covering entire Earth for about 530 million years ago to split into tectonic plates which began to recede as new non-expanding tectonic plate was generated at the ridges creating expanding sea floor. The initiation of the phase transition generated formation of deep seas.

4. The eruption of plasma from the crests of ocean ridges generated oceanic tectonic plates which did not participate to the expansion by density reduction but by growing in size. This led to a reduction of density in the interior of the Earth roughly by a factor 1/8. From the upper bound for the age of the seafloor one can conclude that the period lasted for about 185 million years after which it transformed to convective flow in which the material returned back to the Earth interior. Subduction at continent-ocean floor boundaries and downwards double bending of tectonic plates at the boundaries between two ocean floors were the mechanisms. Thus tectonic plate theory would be more or less the correct description for the recent situation.

5. One can consider the possibility that the subducted tectonic plate does not transform to magma but is fused to the tectonic layer below continent so that it grows to an iceberg like structure. This need not lead to a loss of the successful predictions of plate tectonics explaining the generation of mountains, earthquake zones, zones of volcanic activity, etc...

6. From the video of Adams it becomes clear that the tectonic flow is East-West asymmetric in the sense that the western side is more irregular at large distances from the ocean ridge at the western side. If the magma rotates with slightly lower velocity than the surface of Earth (like liquid in a rotating vessel), the erupting magma would rotate slightly slower than the tectonic plate and asymmetry would be generated.

7. If the planet has not experienced a phase transition increasing the value of Planck constant, there is no need for the decomposition to tectonic plates and one can understand why there is no clear evidence for tectonic plates and subduction in other planets. The conductive flow of magma could occur below this plate and remain invisible.

The biological implications might provide a possibility to test the hypothesis.

1. Great steps of progress in biological evolution are associated with catastrophic geological events generating new evolutionary pressures forcing new solutions to cope in the new situation. Cambrian explosion indeed occurred about 530 years ago (the book "Wonderful Life" of Stephen Gould [I16] explains this revolution in detail) and led to the emergence of multicellular creatures, and generated huge number of new life forms living in seas. Later most of them suffered extinction: large number of phylae and groups emerged which are not present nowadays.

   Thus Cambrian explosion is completely exceptional as compared to all other dramatic events in the evolution in the sense that it created something totally new rather than only making more complex something which already existed. Gould also emphasizes the failure to identify any great change in the environment as a fundamental puzzle of Cambrian explosion. Cambrian explosion is also regarded in many quantum theories of consciousness (including TGD) as a revolution in the evolution of consciousness: for instance, micro-tubuli emerged at this time. The periods of expansion might be necessary for the emergence of multicellular life forms on planets and the fact that they unavoidably occur sooner or later suggests that also life develops unavoidably.

2. TGD predicts a decrease of the surface gravity by a factor 1/4 during this period. The reduction of the surface gravity would have naturally led to the emergence of dinosaurs 230 million years ago as a response coming 45 million years after the accelerated expansion

ceased. Other reasons led then to the decline and eventual catastrophic disappearance of the dinosaurs. The reduction of gravity might have had some gradually increasing effects on the shape of organisms also at microscopic level and manifest itself in the evolution of genome during expansion period.

3. A possibly testable prediction following from angular momentum conservation ($\omega R^2 = constant$) is that the duration of day has increased gradually and was four times shorter during the Cambrian era. For instance, genetically coded bio-clocks of simple organisms during the expansion period could have followed the increase of the length of day with certain lag or failed to follow it completely. The simplest known circadian clock is that of the prokaryotic cyanobacteria. Recent research has demonstrated that the circadian clock of Synechococcus elongatus can be reconstituted in vitro with just the three proteins of their central oscillator. This clock has been shown to sustain a 22 hour rhythm over several days upon the addition of ATP: the rhythm is indeed faster than the circadian rhythm. For humans the average innate circadian rhythm is however 24 hours 11 minutes and thus conforms with the fact that human genome has evolved much later than the expansion ceased.

4. Scientists have found a fossil of a sea scorpion with size of 2.5 meters [I6], which has lived for about 10 million years for 400 million years ago in Germany. The gigantic size would conform nicely with the much smaller value of surface gravity at that time. The finding would conform nicely with the much smaller value of surface gravity at that time. Also the emergence of trees could be understood in terms of a gradual growth of the maximum plant size as the surface gravity was reduced. The fact that the oldest known tree fossil is 385 million years old [I12] conforms with this picture.

11.3.5 Did Intra-Terrestrial Life Burst To The Surface Of Earth During Cambrian Expansion?

Intra-terrestrial hypothesis [?] is one of the craziest TGD inspired ideas about the evolution of life and it is quite possible that in its strongest form the hypothesis is unrealistic. One can however try to find what one obtains from the combination of the IT hypothesis with the idea of pre-Cambrian granite Earth. Could the harsh pre-Cambrian conditions have allowed only intra-terrestrial multicellular life? Could the Cambrian explosion correspond to the moment of birth for this life in the very concrete sense that the magma flow brought it into the day-light?

1. Gould emphasizes the mysterious fact that very many life forms of Cambrian explosion looked like final products of a long evolutionary process. Could the eruption of magma from the Earth interior have induced a burst of intra-terrestrial life forms to the Earth's surface? This might make sense: the life forms living at the bottom of sea do not need direct solar light so that they could have had intra-terrestrial origin. It is quite possible that Earth's mantle contained low temperature water pockets, where the complex life forms might have evolved in an environment shielded from meteoric bombardment and UV radiation.

2. Sea water is salty. It is often claimed that the average salt concentration inside cell is that of the primordial sea: I do not know whether this claim can be really justified. If the claim is true, the cellular salt concentration should reflect the salt concentration of the water inside the pockets. The water inside water pockets could have been salty due to the diffusion of the salt from ground but need not have been same as that for the ocean water (higher than for cell interior and for obvious reasons). Indeed, the water in the underground reservoirs in arid regions such as Sahara is salty, which is the reason for why agriculture is absent in these regions. Note also that the cells of marine invertebrates are osmoconformers able to cope with the changing salinity of the environment so that the Cambrian revolutionaries could have survived the change in the salt concentration of environment.

3. What applies to Earth should apply also to other similar planets and Mars [E9] is very similar to Earth. The radius is .533 times that for Earth so that after quantum leap doubling the radius and thus Schumann frequency scale (7.8 Hz would be the lowest Schumann frequency) would be essentially same as for Earth now. Mass is.131 times that for Earth so that surface

gravity would be.532 of that for Earth now and would be reduced to.131 meaning quite big dinosaurs! have learned that Mars probably contains large water reservoirs in it's interior and that there is an un-identified source of methane gas usually assigned with the presence of life. Could it be that Mother Mars is pregnant and just waiting for the great quantum leap when it starts to expand and gives rise to a birth of multicellular life forms. Or expressing freely how Bible describes the moment of birth: in the beginning there was only darkness and water and then God saidLet the light come!

To sum up, TGD would not only provide the long sought mechanism of expansion of Earth but also a possible connection with the biological evolution. It would be indeed fascinating if Planck constant changing quantum phase transitions in planetary scale would have profoundly affected the biosphere.

11.4 Implications Of Expanding Earth Model For The Pre-Cambrian Evolution Of Continents, Of Climate, And Of Life

Expanding Earth hypothesis is by no means not new. It was proposed by Mantovani and I learned about it from the video animations of [F46, F1] demonstrating that the continents fit nicely to form a single continent covering entire Earth if the radius is one half of the recent radius. What TGD has to give is a new physics justification for Expanding Earth hypothesis: cosmic expansion is replaced with a sequence of fast expansion periods increasing the value of Planck constant and these transitions occur in all scales.

If Expanding Earth hypothesis is correct it forces to modify dramatically the view about pre-Cambrian period. The super-continent theory could be replaced by much simpler theory and it might be possible to give up the assumption about hypothetical super continents and super oceans. The view about glaciations [F7] must be modified dramatically. Concerning the evolution of life the natural hypothesis is that it escaped to the underground seas formed as a consequence of expansion during pre-Cambrian era and returned back to the surface in Cambrian Explosion. In this section super-continent and super-ocean theory is discussed from TGD point of view. A model for glaciations based on the assumption that the radius of Earth was in good approximation one half of the recent radius during pre-Cambrian era is developed and shown to reduce to a sequence of ordinary glaciations initiated at pole caps. Snowball theory serves as a convenient reference. Expanding Earth theory is discussed also from paleomagnetic point of view and some experimental signatures of $R/2$ scenario differentiating it from standard scenarios are developed. Finally the hypothesis about underground evolution is discussed.

11.4.1 Super-Continent Theory

Super-continent theory assumes a cyclic formation of hypothetical super continents [F30]. Rodinia [F28], Pannotia [F24], and Pangea [F23] might have preceded by earlier super-continents. The period would be roughly 250 Myr.

1. The super-continent Rodinia [F28] is assumed to have existed during interval: 1100-750 Myr. 750 Myr ago Rodinia rifted into three continents: Proto-Laurasia which broke up and eventually reformed to form Laurasia (North America and Asia), the continental craton of Congo (part of Africa), and Gondwana (now southern hemisphere plus India).

2. Pannotia [F24] existed during time interval 600-540 Myr. Pannotia rifted in the beginning of Cambrian era to Laurentia (North America), Baltica, Siberia and Gondwana. See the illustration of Pannotia at [F13].

3. Wegener [F2] ended up to postulate that super-continent Pangea should have existed about 250 Myr ago [F23]. The support for its existence is rather strong since tectonic plate model and paleo-magnetic methods allows to trace the drift of the tectonic plates.

One can criticize the cyclic model. The concentration of land mass to Southern Hemisphere during Rodinia period does not look very probable event. The cyclically occurring formation of connected land mass surrounded by much larger ocean looks even less probable unless one can develop some very good physical mechanism forcing this. The basic motivation for super-continent theory are various correlations between distant parts of Earth which would cannot be understood otherwise. In $R/2$ model the continents would have been quite near to each other during the expansion and the notion of cyclic formation of super-continents becomes un-necessary since land bridges between the continents could explain the correlations. There would have been just single super-continent all the time.

11.4.2 Standard View About Oceans

In the standard model the total area covered by oceans has reduced since pre-Cambrian era due to the increase of the continental cover, which is nowadays 29 per cent. Oceans cover the remaining 71 per cent with Antarctica and Arctica included. The evolution of Oceans in standard model requires the introduction of hypothetical oceans which left no trace about their existence (subduction mechanism provides perhaps too convenient trash bin for hypothetical theoretical constructs).

1. Proto-Atlantic Ocean was introduced to explain some contradictions with Wegener's Pangea model allowing to conclude which parts at opposite sides of Atlantic Ocean had been in contact. Proto-Atlantic Ocean closed as Pangea formed and opened again in slightly different manner to form Atlantic Ocean. This process implied mixing of older pieces of the continents and explained the contradictions. Large inland sea is a natural counterpart of the Proto-Atlantic Ocean in $R/2$ option.

2. Mirovia [F17] was the super-ocean surrounding Rodinia. It transformed to Pan-African Ocean surrounding Pannotia. Pan-African ocean was then closed so that the ocean floor of Mirovia disappeared by subduction and left no signs about its existence.

3. In the rifting [F27] of Pannotia Panthalassic ocean [F25] emerged and was the precedessor of the Pacific ocean.

The presence of super-oceans is forced by the assumption that the radius of Earth was the recent one during the pre-Cambrian era plus the local data related to the evolution of continents. The questionable aspect is that these oceans did not leave any direct trace about their existence. In $R/2$ model there is no need for these super-oceans except possibly the counterpart of Panthalassic Ocean [F25].

11.4.3 Glaciations During Neoproterozoic Period

Glaciations dominated the Neoproterozoic period [F18] between 1-.542 billion years. The period is divided into Tonian [F35], Cryogenian [F3], and Ediacaran periods [F4]. The most severe glaciations occurred during Cryogenian period.

It is believed that during Cryogenian period [F3] two worldwide glaciations -Sturtian and Marinoan glaciations- took place. This involves extrapolation of continental drift model and plate tectonics theory. Also hypothesis about hypothetical super-continents is needed so that one must take these beliefs with some skepticism. In $R/2$ model the world wide glaciations are replaced with ordinary glaciations proceeding from poles.

1. Sturtian glaciation occurred 750-700 Myr. The breakup of Rodinia is believed to have occurred at this time. One can wonder whether there is a correlation between these events. $R/2$ model suggest that the energy needed to compensate the reduction of gravitational energy in expansion could have caused the cooling.

2. Marinoan (Varanger) glaciation ended around 635 Myr ago.

Deposits of glacial tillites [F33] at low latitudes serve as support for the claim that these glaciations were world wide. In $R/2$ model Equator corresponds to North pole in TGD framework where Rodinia covered entire Earth and the interpretation would as ordinary glaciations.

After the end of Marinoan glaciation followed Ediacaran period during 635-542 Myr [F4]. The first multicellular fossils appeared at this time. Their relationship to Cambrian fossils is unclear. The standard interpretation for the small number of fossils in pre-Cambrian period is that hard shells needed for fossilization were not yet developed. The problem is that these shells should have developed almost instantaneously in Cambrian explosion.

11.4.4 Snowball Earth Model For The Glaciation During Pre-Cambrian Era

Snowball Earth [F44, F39, F29] is recently the leading model for the glaciations [F8] during Proterozoic era. The term is actually somewhat misleading: Iceball Earth would more to the point. Slushball earth [F41] is a variant of Snowball Earth which does not assume total freezing near equator.

The history behind the Snowball Earth concept is roughly following [F29].

1. Mawson studied the Neoproterozoic stratigraphy of South Australia and identified extensive glacial sediments and speculated with the possibility of global glaciation. He did not know anything about continental drift hypothesis and plate tectonic theory and thought that the ancient position of Australia was the same as it is today. Continent drifting hypothesis however explained the finding as sediments deposited at the higher latitudes the hypothesis was forgotten.

2. Later Harland suggested on basis of geomagnetic data that glacial tillites [F33] in Svalbard and Greenland were deposited at tropical latitudes. In TGD framework with with $R \to R/2$ these tillites would have been at higher latitudes towards North Pole.

3. The facts are that Sun was 6 per cent fainter at that time and glaciations are known to occur. The question is whether they were global and long-lasting or a sequence of short-lasting possibly local glaciations. The Russian climatologist Budyko constructed a model based on energy balance and found that it is possible to have a global glaciation if the ice sheets proceeded enough from polar regions (to about 30 degree latitude). The model was based on the increased reflectiveness (albedo) of the Earth's surface due to the ice covering giving rise to positive feedback loop. Budyko did not believe that global glaciation had occurred since the model offered no way to escape eternal glaciation.

4. Kirschwink introduced the term Snowball Earth, which is actually misleading. Iceball Earth would be more to the point. He found that the so called banded iron formations are consistent with a global glaciation. He also proposed a mechanism for melting the snowball. The accumulation of CO_2 from volcanoes would have caused ultra-greenhouse effect causing warming of the atmosphere and melting of the ice.

5. Slushball Earth [F41] differs from Snowball Earth in that only a thin ice cover or even its absence near equator is assumed. The model allows to explain various findings in conflict with Snowball Earth, such as the evidence for the presence of melt-water basins.

6. Zipper rift model [F40] assumes that there was a sequence of glaciations rather similar to the glaciations that have occurred later. The model assumes that the rifts [F27] of the super-continent Rodinia occurred simultaneously with glaciations. The associated tectonic uplift led to the formation of high plateaus hosting the glaciers. The iron band formation can be assigned with inland seas allowing complex chemistries and anoxicity near the sea floor.

The basic ideas of the Snowball Earth model

Snowball Earth [F44, F39, F29] differs from ordinary glaciations in that only oceans are frozen whereas in the ordinary glaciation land mass is covered by ice. The basic ideas of the snowball Earth relate to the mechanism initiating the global freezing and melting.

1. The glaciation would have been initiated by some event, say a creation of super-volcano. Also astrophysical mechanism might be involved. Somewhat paradoxically, tropical continents

during cryogenian period [F3] are needed for the initiation because they reflect the solar radiation more effectively than tropical oceans.

2. The positive ice-albedo feedback is an essential concept: the more ice the larger the fraction of the radiation reflected back so that the more ice is generated. If the glaciation proceeds over a critical latitude about 30 degrees positive feedback forces a global glaciation.

3. The problem of the model is how to get rid of the glaciation. The proposal of Kirschvink was that the accumulation of CO_2 from volcanoes could have led to a global super-warming. The time scale for CO_2 emissions is measured in millions of years. The needed atmospheric concentration of CO_2 is by a factor 350 higher than the recent concentration. Due the ice cover the CO_2 could not be absorbed to the siliceous rocks and concentration would increase. The melting of the ice meant higher absorbtion of heat by uncovered land. Positive feedback loop was at work again but in different direction.

Evidence for and objections against Snowball Earth

Wikipedia article about Snowball Earth [F29] discusses both evidence for and objections against Snowball Earth. Low latitude sediments at tropical latitudes and tropical tillites at Equatorial latitudes provide strong piece of evidence for Snowball Earth. Calcium carbonate deposits having ^{13}C signature (per cent for the depletion of ^{13} isotope and large for organic material) consistent with that for mantle meaning abiotic origin is second evidence. Iridium anomaly located at the base of Calcium Carbonate deposits is third piece of evidence. The evidence for Snowball Earth will be discussed in more detail later since it is convenient to relate the evidence to $R/2$ model for glaciations.

1. Paleomagnetic data [F22] used to the dating of sediments assuming tectonic plane theory and super-continent drifting might be misleading. No pole wandering maps exist and the polarity of the magnetic field must be deduced by statistical methods. The primary magnetization could have been reset and the orientation of the magnetic minerals could have changed from the original one. It is also possible that magnetic field patterns were not dipolar. Also the assumption of hypothetical super-continents and oceans brings in uncertainties. In $R/2$ model of course the determination of the positions changes completely.

2. Carbon isotope ratios are not what they should be. There are rapid variations of $^{12}C/^{13}C$ ratio with organic origin. Suggests that freezing and melting followed each other in rapid succession. In standard framework this would suggest Slushball Earth meaning ice-free and ice-thin regions around the equator and hydrological cycles. In $R/2$ model the regions at Equator are near North Pole and the explanation would be in terms of ordinary glaciations.

3. The distribution of isotopes of element Boron suggest variations of pH of oceans. The explanation is in terms of buildup of carbon dioxide in atmosphere dissolved into oceans/seas. In $R/2$ model a sequence of glaciations would explain the findings.

4. Banded iron formations providing support for the model are actually rather rare and absent during Marinoan glaciation.

5. Wave-formed ripples, far-traveled ice-rafted debris and indicators of photosynthetic activity, can be found throughout sediments dating from the "Snowball Earth" periods. This serves a evidence open-water deposits. In snow-ball model these could be "oases" of melt-water but computer simulations suggest that large areas of oceans would have left ice-free. in $R/2$ model these would be signatures of ordinary glaciations.

6. Paleomagnetic data have led to the conclusion that Australia was at Equator. In $R/2$ model it would have been near North Pole. Namibia was also thought to be near Equator [F31]. Indirect arguments forced the conclusion that it at 75 degree Southern latitude. In $R/2$ model this corresponds to 60 degrees Southern latitude and ordinary glaciation proceeding from South Pole is a natural explanation and ordinary glaciation would be in question in both cases.

7. There is evidence for the continental ice cover does not fit with Snowball Earth predicts that there should be no continental ice-cover. The reason is that freezing of the ocean means that there is no evaporation from oceans and no water circulation so that ice-cover cannot develop on continents. There is considerable evidence that continents were covered by thick ice [F29]. This suggests ordinary glaciations possible in $R/2$ model.

11.4.5 TGD Point Of View About Pre-Cambrian Period

What is new in TGD based view about pre-Cambrian period is basically due to the $R/2$ hypothesis.

TGD view about evolution of continents

The hypothesis about the existence of the super-continent Pangea [F23] was inspired by the work of Wegener [F2]. The hypothesis about the existence of former super-continents were forced by the correlations with fossil records suggesting connected continent. This is not necessary if the gigantic ocean was absent during $R/2$ era. The continent Rodinia [F28] could look much like the Rodinia of standard geology except that they formed single connected region with radius $R/2$.

1. It is possible that there was only single super-continent with widening inland seas all the time until 250 billion Myr. The first option is R increased slowly and that inland lake formed. Rifts could have got wider gradually during this era. If there were land bridges between the continents there would be no need for postulating the cyclic re-formation of super-continent.

2. One can pose many questions about the character of the expansion.

    (a) What was the duration of the expansion? Could the expansion have occurred in the time period 750-100 Myr (100 Myr corresponds to the age of dinosaurs with large body size made possible by the reduced gravitation and oxygenation of the atmosphere)? Duration would have been about 650 Myr in this case. Or did it began already at the beginning of Neoproterozoic period [F18] when super-continent Rodinia began to break up? In this case the duration would be about 1 Myr. The estimate based on the quantum model of gravitational radiation predicts that the transition lasted for about 1.1 Gy so that the latter option would be more plausible in this framework.

    (b) Did the expansion accelerate as does also cosmic expansion in TGD based universal model for the expansion periods containing only the duration of the expansion period as a parameter [K80] and applying in alls scales? It seems that accelerated expansion is the only sensible option since around 540 Myr the size of Earth should have been rather near to $R/2$ (perhaps so even at the period of Pangea around 250 My) unless one assumes that super-continent re-formed again.

3. One can also consider the possibility that the continents indeed broke up and reformed again during Cambrian era. One should however have a good physical reason for why this happened. Something must have connected the pieces together and created correlations. Gravitational magnetic flux tubes and phase transitions increasing and reducing Planck constant? Or could it be that the bridges connecting the continents acted like strings inducing oscillation of the distance between continents so that Pangea was surrounded by a large ocean?

4. The formation of the rift [F27] feeding magma from core to the surface would be due to the expansion leading to the formation of fractures. The induced local elevations would be like mountains. As in zipper-rift model ice could have covered these plateaus because the temperature was lower. This is not however essential for TGD based model of glaciations.

5. TGD based variant of Expanding Earth allows subduction but its role could have been small before the Pangeia period if the expansion was accelerating and led only to a relatively small increase of the radius before the Mesozoic period [F16] and continued with an accelerating rate during Mesozoic from 250 Myr to 65 Myr. It is interesting that Mesozoic period begins with the most intensive known extinction of history- so called Permian-Triassic extinction event [I4] - known as Great Dying. About 95 of marine species and 70 percent of terrestrial species became extinct. Maybe genetically determined bio-rhythms could not follow the

rapidly changing circadian rhythm. Another explanation for the extinction is the warming of the climate. For this there is indeed support: there is evidence that Antarctica was climate refuge during the extinction [I14]. Perhaps both factors were involved and were not independent of each other since rapid expansion might have generated massive methane leakages from underground seas and lakes.

TGD based view about evolution of oceans

Continents would have covered most of the area during $R/2$ era and the covered fraction was slightly smaller than 1/4 of the recent area of Earth. This depends on the area taken by inland seas and polar caps. Nowadays the area covered by continents and inland seas is about 31 per cent so that continental area has increased and would be due to the expansion in vertical direction and deepening of the oceans. The area covered by oceans has increased from a small value to about 70 per cent. Only a small fraction of ocean floor would be subduced in Expanding Earth model. The Proto-Atlantic would have been only a small inland sea. Panthalassic Ocean was inland sea, which expanded to Pacific Ocean during expansion. Pacific Ocean could contain data about ancient ice ages if it was frozen. It however seems that data are consistent with the absence of global glaciation.

Model for glaciations

In TGD framework single super continent covering most of Earth becomes the counterpart of Rodinia [F28]. The hypothetical oceans are replaced with inland seas and polar caps. The super-continent covering most of Earth absorbs less solar heat than tropical oceans so that glaciations become more probable. Snowball Earth is replaced with a series of ordinary glaciations proceeding from poles since the places at Equator were near North Pole. There is no need for the glaciations to progress to the equator. The rifting for the counterpart of Rodinia is consistent with the formation of fractures due to the expansion of Earth. The reduction of gravitational binding energy due to the increase of the radius requires feed of energy and this could be one reason for the cooling and initiation of the glaciation.

There are several questions which must be answered if one wants to gain a more detailed understanding.

1. How does $R/2$ model modify the view about glaciations? Very probably there was a frozen polar cap. Snowball Earth could be replaced with ordinary glaciations proceeding from North and South Pole.
2. How does the predicted 3+3 hour diurnal cycle modify the ordinary picture? Certainly 3-hour day reduces the amplitude of the diurnal temperature variations. Could this period have left genetic traces to the mono-cellulars, say biological clocks with this period?
3. How does the predicted four times stronger surface gravity affect the glaciation process? Could strong gravity leave detectable signatures such as anomalously strong effects on the shape of surface of Earth or deeper signatures about the motion of ice.

There are also questions related to the energetics of the expansion.

1. The expansion required energy and could have induce glaciations in this manner. Energy conservation would hold for the total mechanical and gravitational energy of Earth given by

$$E = \frac{L^2}{2I} - k\frac{GM^2}{R} < 0 \ . \tag{11.4.1}$$

Here L is the conserved angular momentum of order $L \simeq I\omega$ and ω increases from $1/4\omega_{now}$ to ω_{now} during the expansion. The moment of inertia I is of order of magnitude $I \sim MR^2$ and k is a numerical constant not too far from unity. The kinetic energy is actually negligible as compared to the gravitational potential energy. The reduction of the gravitational binding energy requires a compensating energy, which could come both from Earth interior or from the Earth's surface. Both effects would induce a cooling possibly inducing glaciations.

2. One expects that in the initial stages of the expansion there was just an expansion. This meant stretching requiring also energy. The formation of rifts leading to the formation of oceans as magma flowed out would have started already in the beginning of Proterozoic period. Eventually fractures were formed and in TGD framework one might expect that the distribution of fractures could have been fractal. A considerable fraction of fractures was probably volcanoes so that CO_2 begun to leak to the atmosphere and local "oasis" were formed. Also hot springs liberating heat energy from Earth crust could have been formed as in Island. The pockets inside Earth increased in size and were filled with water. Life started to escaped to the walls of the fractures and to the water pockets. Also the recent oceans can be seen as widened cracks which transformed to the expanding sea floors whereas continents did not expand. As the continental crust ceased to expand no heat was needed for the expansion and this together with increased CO_2 content of atmosphere would explain why there was no further glaciations and heating of the Earth. At this period the flow of the magma from Earth core provided the energy needed to compensate the reduction of gravitational energy.

3. It must be emphasized that TGD variant of Expanding Earth theory is not in conflict with tectonic plate theory. It explains the formation of tectonic plates and the formation of magma flow from rifts giving also rise to subduction and is therefore a natural extension of the tectonic plate theory to times before the expansion ceased.

Estimate for the duration of the transition changing gravitational Planck constant

The reader without background in quantum physics and TGD can skip this section developing an estimate for the duration of the transition changing Planck constant and inducing the scaling of the radius of Earth by a factor two. The estimate is about 1.1 Gy. It must be emphasized that the estimate is not first principle calculation and relies strongly on quantum classical correspondence.

The duration of the quantum transition inducing the expansion of the gravitational space-time sheet of Earth and thus of Earth itself by a factor two can be estimated by using the same general formula as used to estimate the power of gravitational radiation emitted in a transition in which gravitational Planck constant assignable to star-planet system is reduced [K66].

1. The value of gravitational Planck constant characterizing the gravitational field body of Earth is GM^2/v_0, where the velocity parameter $v_0 < 1$ ($c = 1$) is expected to be larger than $v_0 \simeq 2^{-11}$ characterizing Sun-Earth system.

2. Assuming a constant mass density for Earth the gravitational potential energy of Earth is given by

$$V = \frac{M}{2}\omega^2 r^2 \ , \ \omega = \sqrt{\frac{6GM}{R^3}} \ . \tag{11.4.2}$$

   As far as radial oscillations are considered, the system is mathematically equivalent with a harmonic oscillator with mass M. The energies for the radial oscillations are quantized as $E = (n + 1/2)\hbar_{gr}\omega$.

3. The radii of Bohr quantized orbits for the harmonic oscillator scale like $\sqrt{\hbar}$ so that $\hbar \to 4\hbar$ is needed to obtain $R \to 2R$ rather than $\hbar \to 2\hbar$ as the naïve Compton length argument would suggest. This requires the scaling $v_0 \to v_0/4$. The change of the ground state energy in this quantum transition is

$$\begin{aligned} \Delta E &= \frac{1}{2}(\hbar_{gr,f}\omega_f - \hbar_{gr,i}\omega_i) \ , \\ \hbar_{gr,f} &= 4\hbar_{gr,i} = \frac{4GMm}{v_{0,i}} \ , \\ \omega_i &= 2^{3/2}\omega_f = 2^{3/2}\sqrt{\frac{6GM}{R_f^3}} \ . \end{aligned} \tag{11.4.3}$$

$R_f = R$ denotes the recent radius of Earth.

4. From the estimate for the power of gravitational radiation in similar transition the estimate for the duration τ of the quantum transition is

$$\begin{aligned} \tau &= a(v_{0,i}v_{0f})^{-k/2} \times \frac{(\hbar_{gr,i} + \hbar_{gr,f})}{2\Delta_E} \ , \\ &= a2^{-k}v_{0f}^{-k} \times \frac{1+r}{r\omega_f - \omega_i} \ , \ r = \frac{\hbar_f}{\hbar_i} = 4 \ . \end{aligned} \quad (11.4.4)$$

The average of Planck constants associated with the initial and final states and geometric mean of the parameters v_{0i} and v_{0f} is dictated by time reversal invariance. The exponent k is chosen to be same as that obtained for from the condition that the ratio of the power to the classical radiation power emitted in the transition between planetary Bohr orbits does not depend on v_0 (quantum classical correspondence). This gives $k = 5$. The condition that the power of gravitational radiation from Hulse-Taylor binary is same as the power predicted by the classical formula (quantum classical correspondence) gives $a = .75$.

5. The explicit expression for τ reads as

$$\begin{aligned} \tau &= K \times av_{0,f}^{-5} \times (\frac{R}{2GM})^{1/2} \times \frac{R}{c} \ , \\ K &= \frac{5 \times 2^{-7} \times (2 + 2^{1/2})}{3^{1/2}} \ . \end{aligned} \quad (11.4.5)$$

6. The basic data are $M_{Sun} = 332900M$ (mass of Sun using Earth's mass as unit) and the mnemonic $r_{S,Sun} = 2GM_{Sun} = 2.95 \times 10^3$ m: together with $R = 6371 \times 10^3$ m these data allow a convenient estimation of $R/2GM$. For $k = 10$ and $a = .75$ this gives $\tau = 1.17$ Gyr. This is twice the estimate obtained by requiring that the transition begins at about 750 Myr (the beginning of Sturtian glaciation) and ends around 100 My (the age of gigantic animals whose evolution would be favored by the reduction of surface gravity). The estimate would suggest that the quantum transition began already around 1.1 Gyr, which in the accuracy used corresponds to the beginning of Neoproterozoic at 1 Gyr [F18]. The breaking of super-continent Rodinia indeed began already at this time.

7. Note that the value of v_{0f} for the gravitational field body of Earth as it is now would be $v_{0f} = 2^{-10}$ to be compared with $v_0 \simeq 2^{-11}$ for Sun-Earth gravitational field body.

Snowball Earth from TGD point of view

In TGD framework the main justification for Snowball Earth disappears since the samples believed to be from Equator would be from North pole and glaciation could be initiated from pole caps. Consider next in more detail the evidence for Snowball Earth from TGD point of view.

1. Low latitude glacial deposits, glacial sediments at tropical latitudes, tropical tillites, etc. providing support for snowball Earth [F29] would be near North pole of at Northern latitudes. Ordinary glaciations proceeding from poles would explain the findings [F10]. If total glaciations were present, a rough scaling suggests that the evidence from them should be found from southern latitudes around 45 degrees in the standard model framework.

   The testable prediction is that the evidence for glaciations in ice-ball Earth framework should be found only below Equator and near South Pole. This finding would be of course extremely weird and would strongly favor $R/2$ option. Interestingly, in Southern Brasil all indicators for glaciations are absent (see [F47] and references therein). This region belonged to Godwana continent and there is evidence that its location was at middle latitudes at Southern Hemisphere.

2. Banded iron formations [F29] are regarded as evidence for Snowball Earth and occur at tropical levels (near North Pole in $R/2$ model). Iron dissolved in anoxic ocean would have become in a contact with photosynthetically produced oxygen and implied the formation of iron-oxide. The iron formation would have been produced at the tipping points of anoxic and oxygenated ocean. One can consider also an explanation in terms of deep inland seas, which become stagnant and anoxic near the sea floor.

   In TGD framework sea floor near North Pole could contain banded iron formations. This would explain also why the banded iron formations are rather rare. The oxygen could have come also from underground after the formation of cracks and led to the oxygenation of inland seas from bottom. The assumption that oxygenation took place already during the first glaciation, could explain why banded iron formations are absent during the second glaciation.

3. Calcium carbonate deposits [F29] have ^{13}C signature (per cent for the depletion of ^{13} isotope and large for organic material) is consistent with that for mantle meaning abiotic origin. The explanation of Calcium carbonate deposits in TGD framework could be the same as in Snowball Earth model. Atmospheric CO_2 could come from the volcanoes and react with the silicates during the ice-free periods to form calcium carbonate which then formed the deposits. CO_2 could have also biological origin and come from the underground life at the walls of the expanding fractures/volcanoes or in underground seas or lakes. In this case also methane is expected. This option would predict ^{13}C signature characteristic for organic matter. Also this kind of signatures have been observed and support ordinary glaciations. Also rapid fluctuations of the signature from positive to negative take place and might have signatures of temporary melting induced organic contribution to the calcium carbonate.

4. Iridium anomaly [F29] is located at the base of Calcium Carbonate deposits. In Snowball Earth model Iridium deposits derive from the Iridium of cosmic rays arriving at the frozen ice surface. As the ice melts, Iridium deposits are formed. In $R/2$ model the condensation of Iridium would proceed through the same mechanism. The possible problem is whether the time is long enough for the development of noticeable deposits. Near poles (Equator and South pole in standard model) this could be the case.

11.4.6 Paleo-Magnetic Data And Expanding Earth Model

Paleomagnetic data from pre-Cambrian period might allow to test $R/2$ hypothesis. This data could in principle help to trace out the time development $R(t)$ from $R/2$ to R if the non-dipole contribution to magnetic field depends on $R(t)$.

About paleo-magnetism

Paleomagnetism [F22] provides quantitative methods to determine the latitude at which the sample of sedimentary rock was originally. Magnetic longitude cannot be determined because of rotational symmetry so that other information sources must be used. There are several methods allowing to deduce the direction and also the magnitude of the local magnetic field and from this the position of the sample during the time the sample was formed.

1. Below the Curie point thermal remanent magnetization is preserved in basalts of the ocean crust and not affected by the later magnetic fields unless they are too strong. This allows to deduced detail maps from continental drifting and polar wander maps after 250 Myr (Pangea period). During pre-Cambrian period the ocean floors of hypothetical oceans would have disappeared by subduction. In $R/2$ model there are no oceans: only inland seas.

2. In the second process magnetic grains in sediments may align with the magnetic field during or soon after deposition; this is known as detrital remnant magnetization (DRM). If the magnetization is acquired as the grains are deposited, the result is a depositional detrital remnant magnetization (dDRM); if it is acquired soon after deposition, it is a post-depositional detrital remnant magnetization (pDRM).

3. In the third process magnetic grains may be deposited from a circulating solution, or be formed during chemical reactions, and may record the direction of the magnetic field at the time of mineral formation. The field is said to be recorded by chemical remnant magnetization (CRM). The mineral recording the field commonly is hematite, another iron oxide. Red-beds, clastic sedimentary rocks (such as sandstones) that are red primarily because of hematite formation during or after sedimentary diagenesis, may have useful CRM signatures, and magnetostratigraphy [F15] can be based on such signatures. Snowball model predicts that nothing came to the bottoms of big oceans! How can we know that they existed at all!

During pre-Cambrian era the application of paleomagnetic methods [F22] is much more difficult.

1. Reliable paleomagnetic data range up to 250 My, the period of Pangaea, and magnetization direction serves as a reliable information carrier allowing detailed polar wander maps. During pre-Cambrian era one cannot use polar wander maps and the polarity of the magnetic field is unknown. Therefore theoretical assumptions are needed including hypothetical super-continents, hypothetical oceans, and continental drift and plate tectonics. All this is on shaky grounds since no direct information about super-continents and ancient oceans exists. $R/2$ model suggests that continental drift and plate tectonics have not been significant factors before the expansion period when only inland seas and polar ice caps were present. Measurements have been however carried out about magnetization for pre-Cambrian sediments at continents recently and gives information about the strength of the magnetic field [F14]: the overall magnitude of the magnetic field is same as nowadays.

2. At Precambrian period the orientation of iron rich materials can serve as a record. The original records can be destroyed by various mechanisms (diagenesis). Also the orientations of the sediments can change in geological time scales.

3. Tens of thousands of reversals of the magnetic polarity [F6] have occurred during Earth's history. There have been long periods of stability and periods with a high frequency of reversals. The average duration of glaciation is around one Myr. The determination of the polarity of B possible by using samples from different points.

4. Mountain building orogeny [F21] releases hot water as a byproduct. This water can circulate in rocks thousands of kilometers and can reset the magnetic signature. The formation of fractures during the expansion of Earth could have released hot water having the same effect.

Could paleomagnetic data kill or prove $R/2$ model?

The first question is how one might kill $R/2$ model using data from pre-Cambrian era. Paleomagnetic data could do the job.

1. Remanent magnetization is proportional to the value of magnetic field causing it in weak magnetic fields. Therefore the magnetization in principle gives information about the magnetic fields that prevailed in early times.

2. Suppose that the currents generating the magnetic field can be idealized to conserved surface currents K around cylindrical surfaces of radius r and height h scaled down to to $r/2$ and $h/2$ and that the value of K is not affected in the process. With this assumptions the magnetic moment behaves $\mu \sim Ir^2h \rightarrow \mu/8$. A continuous current vortices with $j = k/\rho$, which is ir-rotational outside the symmetry axis, produce a similar result if the radius of the vortices scales as $r \rightarrow r/2$. Since dipole magnetic field scales as $1/r^3$ and is scaled up by a factor 8 in $R \rightarrow R/2$, the scalings compensate and the dipole magnetic fields at surface do not allow to distinguish between the two options. Non-dipole contributions might allow to make the distinction.

3. The group led by Lauri J. Pesonen in Helsinki University [F14] has studied paleomagnetic fields at pre-Cambrian era. The summary of results is a curve at the home page of the group

and shows that the scale of the magnetic during pre-Cambrian era is same as nowadays. On the other hand, the recent thesis by Johanna Salminen- one of the group members- reports abnormally high values of magnetization in Pre-Cambrian intrusions and impact structures in both Fennoscandia and South Africa [F43]. No explanation for these values has been found but it is probably not the large value of primary magnetization.

Another manner to do test the $R/2$ model is by comparing the signs of the magnetizations at magnetic equator and poles. They should be of opposite sign for dipole field. The polarity of magnetic field varies and there are no pre-Cambrian polar wander maps. One can deduce from the condition $B_r/rB_\theta = 2cot(\theta)$ holding true for dipole field the azimutal distance $\Delta\theta$ along the direction of the measured magnetic field to the pole along geodesic circle in the direction of the tangential component of B. One cannot however tell the sign of $\Delta\theta$, in other words whether a given pre-Cambrian sample belongs to Norther or Southern magnetic hemisphere. There are however statistical methods allowing to estimate the actual pole position using samples from several positions (for an excellent summary see [F43]).

For instance, if the magnetic field is in North-South direction during Rodinian period [F28], standard model would predict that the sign at the Equator is opposite to that at South Pole. In $R/2$ model the sample would be actually near North Pole and polarizations would have same sign. The sign of magnetization at apparent southern latitude around 45 degrees would have been opposite to that at South pole which is in conflict with dipole field character. Maybe the global study of magnetization directions when magnetic field was approximately in North-South direction could allow to find which option is correct. Also the dependence of the strength of the magnetic field as function of θ could reveal whether $R/2$ model works or not. The testing requires precise dating and position determination of the samples and a detailed model for the TGD counterpart of Rodinia and its construction requires a specialist.

If the expansion continued after 250 Myr with an accelerating rate and Earth radius was still considerably below its recent value, the comparison of pole wandering charts deduced from ocean floor paleomagnetic data at faraway locations might allow to show that the hypothesis about dipole field is not globally consistent for R option. Even information about the time evolution of the radius could be deduced from the requirement of global consistency.

11.4.7 Did Life Go Underground During Pre-Cambrian Glaciations?

The basic idea of Expanding Earth model is that the life developed in underground seas and emerged to the surface of Earth in Cambrian explosion. The series of pre-Cambrian glaciations explains why the life escaped underground and how the underground seas were formed.

1. If one believes that the reduction of gravitational binding energy was responsible the cooling, then the expansion of Earth could have begun at the same time as Sturtian glaciation [F3] . On the other hand, the TGD estimate for the duration of the expansion period giving 1.1 Gyr, suggests that the breakup of the Rodinia, which began in the beginning of Proterozoic period corresponds to the beginning of the expansion. The simplest assumption is that the radius of R at the beginning of Cambrian period was not yet much larger than $R/2$ and continued to increase during Cambrian period and ended up around 100 My, when dinosaurs and other big animals had emerged (possibly as a response to the reduction of gravity). This means that there were land bridges connecting the separate continents.

2. One must explain the scarcity of fossils during pre-Cambrian era. If the more primitive life forms at the surface of Earth did not have hard cells and left no fossils one can understand the absence of highly evolved fossils before Cambrian explosion [I1]. If life-forms emerged cracks and underground seas there would be no fossils at the surface of Earth. In the case of volcanoes dead organisms would have ended to gone to the bottom of the water containing volcano and burned away.

3. The expansion had formed the underground pockets and fractures made possible for the water to flow from the surface to the pockets. Life would have evolved in fractures and pockets. The first multicellular fossils appeared during Ediacaran period (segmented worms, fronds, disks, or immobile bags) [F4] and have little resemblance to recent life forms and

their relationship with Cambrian life forms is also unclear. Ediacaran life forms could have migrated from the fractures and Cambrian fossils from from the underground seas and lakes. The highly evolved life-forms in Cambrian explosion could have emerged from underground seas through fractures.

One can make also questions about the underground life.

1. The obvious question concerns the sources of metabolic energy in underground seas. In absence of solar radiation photosynthesis was not possible plants were absent. The lowest levels in the metabolic hierarchy would have received their metabolic energy from the thermal or chemical energy of Earth crust or from volcanoes. The basic distinction between plants and animals might be that the primitive forms of plants developed at the surface of Earth and those of animals in underground seas.

2. At first it seems strange that the Cambrian life-forms had eyes although there was no solar radiation in the underground seas. This is actually not a problem. These life-forms had excellent reasons for possessing eyes and in absence of sun-light the life forms had to invent lamp. Indeed, many life forms in deep sea and sea trenches produce their own light [I3]. It would be interesting to try to identify from Cambrian fossils the body parts which could have served as the light source.

11.4.8 Great Unconformity As A New Piece Of Support For Expanding Earth Model

I hope that this chapter demonstrates convincingly that single hypothesis - a sudden phase transition increasing the radius of Earth by a factor 2 natural in the many-sheeted space-time of TGD - explains Cambrian explosion in biology (a sudden emergence of huge number of life forms after very slow Precambrian evolution), and also provides a model for Precambrian evolution of continents, climate and life.

Already Darwin realized that the absence of fossils from Precambrian era (see `http://tinyurl.com/65zeh5`) is a deep problem for his theory and assumed that this is an artefact due to the incomplete fossil record. Fossils of Precambrian origin have been indeed found after Darwin's time but they are simple and very rare, and the conclusion is that Cambrian explosion (see `http://tinyurl.com/3flhcw`) [I1] meaning a huge diversification was real. Two mysteries therefore remain. Why the development of life was so slow during Precambrian era? Why the diversification was so incredibly fast during Cambrian explosion? Various explanations have been proposed. Did the oxygen content of the atmosphere reach a critical value and lead to the diversification? Or did predation pose the evolutionary pressure making the pace of evolution dramatically faster?

In New Scientist (see `http://tinyurl.com/nenk8nq`) [F42] geologists Robert Gaines and Shanan Peters describe a geological finding perhaps related to the Cambrian Explosion: the mysterious "Great Unconformity" (see `http://tinyurl.com/bqm9ndz`) [F9], which is a juxtaposition of two different types of rock of very different geological ages along a prominent surface of erosion. This surface represents a very long span of "missing" time. More than 1 billion years of geological record is missing in many places! From the figure (see `http://tinyurl.com/y8tnbneb`) of the Wikipedia article [F9] about Great Unconformity visible in Grand Canyon the thickness of the missing layer can be estimated to be about 12.6 km. Somehow before the Cambrian the uppermost rocks of the continents were stripped away exposing the underlying crystalline basement rocks. The cause of this gap remains a complete mystery so that we have three mysteries! Plus the mysteries related to the evolution of climate (problems of Snowball Earth model).

The authors suggest that the formation of Great Unconformity relates to the Cambrian explosion. Large scale erosion and chemical weathering of the exposed crystalline rock caused mineralization of the sea water. The hypothesis is that this led to bio-mineralization: animal groups possessing mineral skeletons - such as silica shells and calcium carbonate shells - emerged. This hypothesis looks rather plausible but does not solve the three great mysteries.

The authors indeed leave open the question about the origin of Great Unconformity and of Cambrian explosion. The TGD based explanation of Cambrian explosion comes from the model realizing the old idea about Expanding Earth in terms of TGD inspired new physics. Already Wegener observed that continents can be fit together nicely and this led to the recent view about plate tectonics. Wegener's model however fits only "half" of the continent boundaries together. One could however do much better: the observation is that the continents would fit nicely to cover the entire surface of Earth if the radius of Earth were 1/2 of its recent value! Expanding Earth model postulates that the radius of Earth grows slowly. Geologists have not taken Expanding Earth model seriously: one good reason is that there is no physics allowing it.

As has been found, TGD predicts a candidate for the needed new physics.

1. At given sheet of the many-sheeted space-time cosmic expansion is predicted to take place as sudden phase transitions in which the size of some space-time sheet suddenly increases. By p-adic length scale hypothesis the preferred scaling factors are powers of 2 and the most favored scaling factor is just two. The proposal is that during the Precambrian era life resided in underground seas being thus shielded from meteor bombardment and cosmic rays. This explains the scarcity of the fossil records and the simplicity of the fossils found. The sudden phase transition was a very violent process increasing the area of the Earth's surface by a factor of 4. The area of continents is 29.1 per cent from the recent area of the Earth's surface - not too far from the naïvely predicted fraction 1/4.

2. It is easy to imagine that the uppermost rocks of the continent covering the entire Earth were stripped away and correspond nowadays to 100 km thick continental tectonic plates consisting of mainly silicon and aluminium). This expansion created split first the topmost layer as continental plates and regions between them giving rise to oceans. The magma which was uncovered by the process cooled down and solidified and the continued expansion gave rise to ocean plates with different composition (mainly silicon and magnesium).

3. The expansion phase corresponds to criticality so that fractality of the expansion is expected. At least for continental plates this process could have been fractal occurring in various length scales characterizing the thickness and the area of the sub-plates generated in the process. p-Adic length scale hypothesis suggests that the scales involved should appear as powers of $\sqrt{2}$ or 2. Generation of Great Unconformity as a process in which the underlying crystalline basement rocks were uncovered could correspond to a splitting of a layer of the continental plates to pieces. The length scale characterizing the thickness is 12.6 km from the above estimate and with 1 per cent accuracy by a factor 1/8 shorter than 100 km length scale for tectonic plates. This conforms with p-adic fractality. If the process of expansion involved a cascade of scalings by factor 2, one can wonder whether it proceeded from long to short length scales or vice versa. In other words: did continental and oceanic tectonic plates form first and after than the smaller structures such as the Great Unconformity or vice versa?

4. Note that the Compton scale $L_e(237)$ corresponding $p \simeq 2^{237}$ is 88 km - ten per cent smaller than 100 km. Maybe thermal expansion could account the discrepancy if the original thickness was $L(237)$. Second interpretation could be that besides electron Compton scale $L_e(239)$ the p-adic scale $L(239) = L_e(239)/\sqrt{5} \simeq 78.7$ km matters. The importance of $L(k)$ does not implicate that of scaled up electron, and the following argument suggests that it is p-adic length scale rather than corresponding electron Compton scale that matters now. Remarkably, also M_{241} is Gaussian Mersenne and corresponding electronic Compton scale is $L_e(241) = 154.7$ km.

   Note that 88 km is rather precisely the thickness of the atmosphere above which there is ionosphere (see `http://tinyurl.com/lqr85j`) [F11]. The thickness of Kennelly–Heaviside layer (see `http://tinyurl.com/25ur2tl`) [F12] inside which radio waves used in terrestrial radio communications propagate, has thickness about 150 km which roughly corresponds to $L(239)$. Note that Continental litosphere (see `http://tinyurl.com/d96kw`) [F26] has typical thickness of 200 km ($L(239)$) whereas oceanic litosphere is 100 km thick ($L(237)$). This fits at least qualitatively with the proposed formation mechanism of continental tectonic plates.

   There is a nice fractal analogy with cell membrane and connection with Gaussian Mersennes (see `http://tinyurl.com/pptxe9c`) [A4] expected to be of special importance in TGD Universe. The scales $L(239)$ and $L(241)$ would be in the same relation as the thickness $L_e(149)$ of the lipid layer of cell membrane to the cell membrane thickness $L_e(151)$ characterized by Gaussian Mersenne $M_{151,G}$. The two kinds of tectonic plates (continental and oceanic) would be analogous to the lipid layers of cell membrane.

5. The rapid expansion process could have also brought in daylight the underground seas and the highly developed life in them so that Cambrian diversification would have been only apparent. Skeptic can of course ask whether it is necessary to assume that life resided in underground seas during Precambrian era. Could just the violent geological process be enough to induce extremely fast diversification? This might of course be true.

6. There is one further argument in favor of the Expanding Earth model. The fact that the solar constant was during proto Earth period (see `http://tinyurl.com/pc83uvt`) [F34] only 73 per cent from its recent value, is a problem for the models of the very early evolution of life. If the radius of Earth was 1/2 of its recent value the duration of day and night was from conservation of angular momentum only 1/4: th of the recent value and thus 3 hours. This could have made the environment much more favorable for the evolution of life even at the surface of the Earth since the range for the temperature variation would have been much narrower.

11.4.9 Where Did The Oceans Come From?

TGD based vision about life has been developing rapidly thanks to the realization that hierarchy of Planck constants and dark matter could relate directly to criticality: consider only long range correlations, phase separation, and classical non-determinism near critical point as common aspects [?]. The article "Half of the Earth's water formed before the sun was born" (`http://news.sciencemag.org/earth/2014/09/half-earths-water-formed-sun-was-born`) describes research results proving additional support for the TGD inspired idea about the occurrence of prebiotic evolution in underground water reservoirs shielded from meteorites and cosmic rays. The idea relies on TGD inspired variant of Expanding Earth hypothesis [K66, L40].

1. Article represents first a standard argument in favor of late formation of oceans. The collisions by astreroids and meteorites could have evaporated the water or blown off it in to space. Hence surface water at Earth should have emerged much later. Note that one can replace "water" with "life" in the argument.

2. The researchers end up to propose that the water emerged already before Sun, and also oceans did so rather early. Carbonaceous chondrites (`http://tinyurl.com/75fh74p`), which formed at the same time as Sun and well before the planets, could have served as a source of water. These meteorites were formed very early, already earlier than Sun. Their composition resembles that of bulk solar system composition. By studying basaltic meteorites from asteroid Vesta, which is known to be formed in the same region as Earth, the reaserachers found that they contain same hydrogen isotopic composition as carbonaeous chondrites.

   This motivates the proposal that chondrites contained the water. A further proposal is that the water reservoirs formed at the surface of Earth as it formed. Here I beg to disagree: the objection represented in the beginning is difficult to circumvent!

The article stimulates several interestig questions in TGD based conceptual framework.

1. Why not to assume formation of underground water reservoir? Here meteorites and UV radiation did not form a problem. And there is indeed recent evidence for the previous existence of large underground reservoirs (`http://tinyurl.com/k2d2ttj`). The formation process for Earth could have naturally led to the evaporation of of chondrite water from the interior of Earth and its transfer nearer to surface and getting caught inside reservoirs.

   Also prebiotic life could have evolved in the underground water reservoirs and already in chondrites (DNA, RNA, aminoacids, tRNA represented as dark proton sequences at flux tubes) and transformed to the life as we know. Mother Gaia's womb was nice place: no meteorite bombardment, no cosmic rays, and metabolic energy provided by Mother Gaia as dark photons. Cambrian explosion as Earth's radius increased by a factor of two was the birthday of the life as we identify it, the (child) water burst to the surface and seas were formed and life began to evolve at the surface of Earth.

   Recall that in TGD continous cosmological expansion at level of space-time sheets is at quantum level replaced with a sequence of phase transitions increasing $h_{eff} = n \times h$ and/or p-adic length scale of the space-time sheet - by p-adic length scale hypothesis most naturally by a factor of two. This kind of transition explains why the continents of Earth fit nicely together to cover entire Earth if the radius is half of its recent value, the emergence of gigantic life forms, etc... [L40].

2. The basic objection relates to the basic mechanisms of metabolism. What replaced plants receiving metabolic energy from solar light as source of metabolic energy? What replaced Sun? Did the dark photon radiation generated by Earth - or maybe also Sun - and penetrating ordinary matter as dark radiation, replace sun light? Any critical system could generate this radiation and it should not be difficult to identify this kind of system: the boundary between core and mantle is the most obvious candidate for a critical system as also for a rapid self-organization process). I proposed for more than decade ago this option half-jokingly as metabolic sources of IT (intraterrestrial) life as I called it.

3. Dark photon radiation would have had a universal energy spectrum - the spectrum of biophotons in visible and UV range. Part of it would have transformed to biophotons (http://tinyurl.com/yb9hnmu7) taking the role of solar radiation as a metabolic energy source. An interesting question is whether the life at the bottom of oceans could give some hints about the counterpart of photosynthesis based on bio-photons? The discovery that the metabolic reactions thought to require complex catalytic maschinery can take place in the environment simulating ocean bottom (http://tinyurl.com/ydc8g7r4) supports the idea about the evolution of life from prebiotic life forms in the womb of Mother Gaia. In TGD framework these prebiotic life forms could correspond to dark proton sequences (dark nuclei) at magnetic flux tubes associated with the negatively charged exclusion zones discovered by Pollack [L10] (http://tinyurl.com/oyhstc2).

11.5 What about other planets?

11.5.1 How Was Ancient Mars Warm Enough for Liquid Water?

The popular article "Mars Mystery: How Was Ancient Red Planet Warm Enough for Liquid Water?" (see http://tinyurl.com/gsbwyhe) tells about a mystery related to the ancient presence of water at the surface of Mars. It is now known that the surface of Mars was once covered with rivers, streams, ponds, lakes and perhaps even seas and oceans. This forces to consider the possibility there was once also life in Mars and might be still. There is however a problem. The atmosphere probably contained hundreds of times less carbon dioxide than needed to keep it warm enought for liquid water to last. There are how these signature of flowing water there. Here is one more mystery to resolve.

The TGD version of Expanding Earth Hypothesis states that Earth has experienced a geologically fast expansion period in its past. The radius of the Earth's space-time sheet would have increased by a factor of two from its earlier value. Either the p-adic length scale or effective value of Planck constant $h_{eff}/h = n$ for the space-time sheet of Earth or both would have increased by factor 2.

This violent event led to the burst of underground seas of Earth to the surface with the consequence that the rather highly developed lifeforms evolved in these reservoirs shielded from cosmic rays and UV radiation burst to the surface: the outcome was what is known as Cambrian explosion. This apparent popping of advanced lifeforms out of nowhere explains why the earlier less developed forms of these complex organisms have not been found as fossile. I have discussed the model for how life could have evolved in underground water reservoirs [L14].

The geologically fast weakening of the gravitational force by factor 1/4 at surface explains the emergence of gigantic life forms like sauri and even ciant crabs. Continents were formed: before this the crust was like the surface of Mars now. The original motivation of EEH indeed was that the observation that the continents of recent Earth seem to fit nicely together if the radius were smaller by factor 1/2. This is just a step further than Wegener went at his time. The model explains many other difficult to understand facts and forces to give up the Snowball Earth model. The recent view about Earth before Cambrian Explosion is very different from that provided by EEH. The period of rotation of Earth was 4 times shorter than now - 6 hours - and this would be visible of physiology of organisms of that time. Whether it could have left remnants to the physiology and behavior of recently living organisms is an interesting question.

What about Mars? Mars now is very similar to Earth before expansion. The radius is one half of Earth now and therefore same as the radius of Earth before the Cambrian Explosion! Mars is near Earth so that its distance from Sun is not very different. Could also recent Mars contain complex life forms in water reservoirs in its interior. Could Mother Mars (or perhaps Martina, if the red planet is not the masculine warrior but pregnant mother) give rise to their birth? The water that has appeared at the surface of Mars could have been a temporarily leakage. An interesting question is whether the appearance of water might correspond to the same event that increased the radius of Earth by factor two.

Magnetism is important for life in TGD based quantum biology. A possible problem is posed by the very weak recent value of the magnetic field of Mars. The value of the dark magnetic field $B_{end} = .2$ Gauss of Earth deduced from the findings of Blackman about effects of ELF em fields on vertebrate brain has strength, which is 2/5 of the nominal value of B_E. Hence the dark MBs of living organisms perhaps integrating to dark MB of Earth seem to be entities distinct from MB of Earth. Could also Mars have dark magnetic fields?

Schumann resonances might be important for collective aspects of consciousness. In the simplest model for Schumann resonances the frequencies are determined solely by the radius of Mars and would be 2 times those in Earth now. The frequency of the lowest Schumann resonance would be 15.6 Hz.

11.5.2 New Horizons About Pluto

ew Horizons (see `http://tinyurl.com/cjdzsk9`) is a space probe that has just been passing by Pluto and has taken pictures about the surface of Pluto and its Moon Kharon. The accuracy of the pictures is at best measured in tens of meters. Pluto has lost its status as a genuine planet and is now regarded as dwarf planet in the Kuiper belt - a ring of bodies beyond Neptune. Using Earthly unis its radius, mass (from New Horizons data), and distance from Sun are $R = .18R_E$, $M = .0022 \times M_E$ and $d = 40d_E$.

Pictures have yielded a lot of surprises. Pluto is not the geologically dead planet it was though to be. The following summarizes what I learned by reading a nice popular article by Markku Hotakainen in finnish weekly journal ("Suomen Kuvalehti") and also represents a TGD based interpretation of the findings.

1. Surprisingly, the surface of the Pluto is geologically young: the youngest surface shapes have age about 10^8 that is .1 billion years. This is strange since the temperature is about -240 °C at the cold side and it receives from Sun only 1/1000 of the energy received by Earth. Textbook wisdom tells that everything should have been geologically totally frozen for billions of years.

2. There is a large champaign - one guess is that it has born as an asteroid or comet has collided with the surface of Pluto. The region is now officially called Tombaugh Regio. The reader can Google the reason for this. The flat region does not seem to have any craters so that it should be rather young. The boundary of this lowland area is surrounded by high (up to 3.5 km) mountains. Also these formations seem to be young. Nitrogen, methane and CO-ice cannot form so high formations.

   Several explanations have been imagined for the absence of craters: maybe there are active processes destroying the craters very effectively. Maybe there is tectonic activity. This however requires energy source. Radioactivity inside Pluto? Underground seas liberating heat? Or maybe tidal forces: the motions of Pluto and its moon Kharon are locked and they turn always the same side towards each other. There is a small variation in the distance of Kharon causing tidal forces. Could this libration deform Pluto and force the liberation of heat produced by frictional forces?

3. The flat region decomposes to large polygons with diameter of 20-30 km. The mechanism producing the polygons is a mystery. Also their presence tells that the surface is geologically young: at some places only .1 billion years old.

4. The atmosphere of Pluto has also yielded a surprise. About 90 per cent of atmosphere (78 per cent at Earth) is nitrogen but it is estimated to leak with a rate of 500 tons per hour since the small gravitational acceleration (6 per cent of that on Earth) cannot prevent the gas molecules from leaking out. How Pluto manages to keep so much nitrogen in its atmosphere?

5. Kharon - the largest moon of Pluto - has radius which is half of that for Pluto. Also the surface texture of Kharon exhibits signs about upheavals and has similarities to that in Pluto. Craters seem to be lacking. North Pole has great dark region - maybe crater. Equator is surrounded by precipices with depths of hundreds of meters, maybe up to kilometers. If they are torn away so should have been also the precipices.

Can one understand the surface texture of Pluto and Kharon? For years I proposed a model for the finding that the continents of Earth seem to fit nicely to form a single supercontinent if the radius of Earth is taken to be one half of its recent radius. This led to a TGD variant of Expanding Earth theory [L40].

1. It is known that cosmic expansion does not occur locally. In many-sheeted space-time of TGD this could mean that the space-time sheets of astrophysical objects comove at the the large space-time sheet representing expanding background but do not themselves expand. Another possibility is that they expand in rapid jerks by phase transitions increasing the radius. p-Adic length scale hypothesis suggests that scaling of the radius by two is the simplest possibility.

2. If this kind of quantum phase transition occurred for the space-time sheet of Earth about .54 billion years ago it can explain the weird things associated with Cambrian explosion (see `http://tinyurl.com/ntvx38e`). Suddenly totally new life forms appeared as from nowhere to only disappear soon in fight for survival. Could highly evolved life in underground seas shielded from UV radiation and meteoric bombardment have burst to the surface. The process would have also reduced the value of the gravitational acceleration by factor 1/4 and increased the length of the day by factor 4. The reduction of the surface gravity might have led to emergence of various gigantic lifeforms such as dinosauri, which later lost the evolutionary battle because of their small brains. Climate would have changed dramatically also and the Snowball Earth model is replaced by a new view.

If these sudden quantum phase transitions at the level of dark matter ($h_{eff} = n \times h$ phases of ordinary matter) is the manner how cosmic expansion universally happens then also Pluto might so the signs of this mechanism.

1. The surface of Pluto is indeed geologically young: the age is measured in hundreds of millions of years. Could the sudden jerkwise expansion have occurred - not only for Earth but - for objects in some region surrounding Earth and containing also Pluto?

2. The polygonal structure could be understood as a ripping of the surface of Pluto in the sudden expansion involving also cooling of magma and its compression (the analogy is what happens to the wet clay as it dries and becomes solid). The lowland region could correspond to the magma burst out from the interior of Pluto being analogous to the magma at the bottom of oceans at Earth. The young geological age of this region would explain the absence of craters. Also the surface texture of Kharon could be understood in the similar manner.

Could one understand the presence of nitrogen?

1. If the gravitational acceleration was 4 times larger (24 percent of that in Earth) before the explosion, the leakage would have been slower before it. Could this make it easier to understand why Pluto has so much nitrogen? Could the burst of material from the interior have increased the amount of nitrogen in the atmosphere? Geochemist could probably answer these questions.

2. A more radical explanation is that primitive life forms have prevented the leakage by binding the nitrogen to organic compounds like methane. If underground oceans indeed existed (and

maybe still exist) in Pluto as they seem to exist in Mars, one can wonder whether life has been evolving as an underground phenomenon also in Pluto - as so many nice things in this Universe must do;-). Could these lifeforms have erupted to the surface of Pluto in the sudden expansion from underground seas and could some of them - maybe primitive bacteria - have survived. Nitrogen (see http://tinyurl.com/yb3yexsu) is essential for life and binds the nitrogen to heavier chemical compounds so that its leakage slows down. Could there exist an analog of nitrogen cycle (see http://tinyurl.com/yc4r39o8) meaning that underground life bind the nitrogen from the atmosphere of Pluto and slow down its leakage?

11.6 Expanding Earth hypothesis, Platonic solids, and plate tectonics as symplectic flow

A FB discussion inspired by the evidence reported by Nasa for the existence of life in Mars coming from a generation of methane (see http://tinyurl.com/y735g9kn) (thanks to Nikolina Bedenikovic for the link). It seems that it must originate below the surface of Mars - possibly from underground oceans. The emission of methane is periodic having the year of Mars as a period and has maximum during summer time. This suggests that solar radiation somehow serves as a source of metabolic energy. The TGD based explanation might be in terms of dark photons able to propagate through the crust to the underground oceans.

The finding provides support for TGD based Expanding Earth model [L40] explaining Cambrian explosion, which is one of the mysteries of recent day biology. According to this model life would have evolved in underground oceans where it was shielded from UV light, cosmic rays, and meteor bombardment, and burst to the surface of Earth during the period when Earth expanded and the crust developed cracks.

One can wonder whether Expanding Earth model is consistent with plate tectonics and with the motivating claim of Adams that the continents fit together nicely to cover the entire surface of Earth if its radius were one half of the recent radius. The outcome was what one might call Platonic plate tectonics.

1. The expansion would have started from or generated decomposition of the Earth's crust to an icosahedral lattice with 20 faces, which contain analogs of what is known as cratons and having a total area equal to that of Earth before expansion. The prediction for the recent land area fraction is 25 per cent is 4.1 per cent too low. The cause could be sedimentation or expansion continuing still very slowly.

2. Craton like objects (in the sequence briefly cratons) would move like 2-D rigid bodies and would fuse to form continents.

3. The memory about the initial state should be preserved: otherwise there would exist no simple manner to reproduce the observation of Adams by simple motions of continents combined with downwards scaling. This might be achieved if cratons are connected by flux tubes to form a network. For maximal connectivity given triangular face is connected by flux tube to to all 3 nearest neighbour faces. Minimal connectivity corresponds to an essentially unique dodecahedral Hamiltonian cycle connecting cratons to single closed string. At least for maximal connectivity this memory would allow to understand the claim of Adams stating that the reduction of radius by factor 1/2 plus simple motions for the continents allow to transform the continents to single continent covering the entire surface of the scaled down Earth.

4. The dynamics in scales longer than that of craton would be naturally a generalization of an incompressible liquid flow to area preserving dynamics defined by symplectic flow. The assumption that Hamilton satisfies Laplace equation and is thus a real or imaginary part of analytic function implies additional symmetry: the area preserving flow has dual. The flow has vanishing divergence and curl. Sources and sinks and rotation are however possible in topological sense if the tectonic plate has holes.

11.6.1 Summary of the model

Expanding Earth hypothesis in TGD framework

The TGD variant of Expanding Earth hypothesis [L40] (see `http://tinyurl.com/y75hku4x`) can be motivated by both cosmological and biological considerations.

1. The basic observation is that astrophysical objects seem to not take part of cosmic expansion but only to co-move. This leads to the idea that the corresponding space-time sheets experience cosmic expansion as relatively rapid jerks and have constant size between these jerks. Second motivation comes from the claim of Adams [F1] (see `http://tinyurl.com/fxsve`) that the continents would fit nicely together to form a single continent covering the entire surface of Earth if the radius of Earth were 1/2 its recent radius.

2. There is also a connection with biology. Cambrian explosion (see `http://tinyurl.com/ntvx38e`) is a poorly understood period in the history of life at Earth. Suddenly a burst of highly developed life forms emerged from some unknown source. TGD explanation would be in terms of rather rapid increase of the radius of Earth by factor of two from the recent size $R_{Mars} \simeq R_E/2$ of Mars to the recent size R_E of Earth with the consequences that the stretching developed cracks. Since the radial scaling caused similar stretching everywhere, the decomposition to a lattice at some critical value of the scale parameter λ would have generated the cracks. The generation of a lattice in drying clay serves as an analogy.

   The relatively highly developed underground life would had evolved below the surface of Earth, where it was shielded from the bombardment by meteors, cosmic rays, and UV radiation and was burst to the surface as the oceans were formed on the cracks.

   The increase of the radius of Earth by factor 2 increased the duration of day by factor 4 and reduced the surface gravity by a factor 1/4. The genetically conserved features preceding the expansion would be still seen in biology. For instance, there might exist a 3 hour bio-rhythm if the underground life received solar radiation somehow. The reduction of gravity could explain the emergence of giant sized organisms such as dinosaurs.

   Underground life must have some source of metabolic energy and photosynthesis should have developed already before the Cambrian expansion. This suggests that visible light from some source must have been present. I have considered possible sources in [L14]. The most science fictive proposal is that part of the photons of solar radiation transform to dark photons identified as a phase of ordinary photons residing at magnetic flux tubes. They would have had a non-standard value of Planck constant $h_{eff} = n \times h_0$ and in absence of direct interactions with the ordinary manner would have managed to penetrate through the crust to the underground oceans.

   In the recent biology bio-photons with energies in visible and UV range would emerge as energy conserving transformations of large h_{eff} photons to ordinary photons. The value of h_{eff} for charged particle of mass m would be by a generalization of Nottale's proposal equal to $\hbar_{eff} = n \times \hbar_0 = h_{gr} GMm/v_0$, where M could correspond to a dark mass assignable to Earth and v_0 is a parameter having dimensions of velocity. This hypothesis implies that cyclotron energies of charged particles do not depend at all on the mass of the charged particle so that cyclotron photons can induce transitions of bio-molecules [**?**, K67].

   Remark: h_0 is the minimal value of h_{eff}: the best guess for the ordinary Planck constant corresponds to $n = 6$ [L18, L36].

   This mechanism for the transfer of solar energy under the surface of Mars could explain the annual periodicity of the methane production in Mars. Magnetic fields serve as a shield against UV radiation and cosmic rays in the case of Earth. Mars has only weak and local magnetic fields above its surface. This gives a good reason why for the Martian life to stay below the surface. The strengthening of the Earth's magnetic field might have preceded or accompanied the proposed expansion of Earth.

3. This vision profoundly modifies the ideas about what happened before Cambrian explosion. In particular, Snowball Earth hypothesis (see `http://tinyurl.com/prem7nj`) about the climate evolution must be given up. The magnetic history of Earth allows to test the model.

Basic ideas of Platonic plate tectonics

The FB discussion raised the question whether the TGD based Expanding Earth model [L40] is consistent with plate tectonics and with the motivating claim of Adams that the continents fit nicely to cover the entire surface of Earth if its radius were one half of the recent radius. The outcome was what one might call Platonic plate tectonics.

1. The expansion would have started from or generated decomposition of the Earth's crust to an icosahedral lattice with 20 faces, which contain what could be identified as cratons (see http://tinyurl.com/y8juty2q) having a total area equal to that of Earth before expansion. Cratons represent the stable part of the continental litosphere and are found in the interiors of the tectonic plates. They consist of ancient crystalline basement rock and maybe be covered by younger sedimentary rock. They have a thick crust and deep litospheric roots. The prediction 25 per cent for the recent land area is 4.1 per cent too low. The simplest explanation is that expansion still continues but very slowly. Also the formation of sedimentary rocks could have increased the area.

2. The cratons would move like 2-D rigid bodies and would fuse to form continents.

3. The memory about the initial state should be preserved: otherwise there would exist no simple manner to reproduce the observation of Adams by simple motions of continents combined with downwards scaling. This could be achieved if cratons are connected by flux tubes to form a network (for tensor networks in TGD Universe see [L17]). For maximal connectivity given triangular face is connected by flux tube to to all 3 nearest neighbour faces. Minimal connectivity corresponds to an essentially unique dodecahedral Hamilton's cycle [A22] (see http://tinyurl.com/pf33vkt) connecting cratons to single closed string. At least for maximal connectivity this memory would allow to understand the claim of Adams stating that the reduction of radius by factor 1/2 plus simple motions for the continents allow to transform the continents to single continent covering the entire surface of the scaled down Earth.

4. The dynamics in scales longer than that of craton would be naturally a generalization of an incompressible liquid flow to area preserving dynamics defined by symplectic flow. The assumption that Hamilton satisfies Laplace equation and is thus a real or imaginary part of analytic function implies additional symmetry: the area preserving flow has dual. The flow has vanishing divergence and curl. Sources and sinks and rotation are however possible in topological sense if the tectonic plate has holes. This would suggest conformal invariance.

The proposal is that the expansion of Earth taking place as discrete jerkes is basically a quantum phenomenon in astrophysical scales.

1. In TGD framework magnetic flux tubes are carriers of dark matter identified as phases of ordinary matter with non-standard value of Planck constant. As explained, the value of gravitational Planck constant h_{gr} would be enormous and imply quantum coherence in the size scale of Earth at the magnetic body forcing coherence at the level of ordinary matter [K67]. The transitions changing the value of h_{eff} would change the length of flux tubes and these transitions would be crucial for the dynamics of water [L45] (see http://tinyurl.com/ydhknc2c).

2. Also the ability of biomolecules to find each other in molecular soup would rely on the same mechanism. In biology also the formation of organs and organelles from cells would involve the shortening of flux tubes [L43] (see http://tinyurl.com/y9pxr9dx). In brain synchronously firing neuron groups would form dynamical networks. An interesting question inspired by the huge value of h_{gr} is whether cratons could be seen as analogs of cells and continents as analogs of organs of Mother Gaia. Note that the magnetic bodies of living systems with EEG would have layers with size scale of Earth [K32].

What happened in the expansion of Earth and after that?

One can try to imagine what happened during and after the expansion of Earth.

1. The spherical crust developed at least one hole as the radius increased by factor 2: $R_f = 2R_i$. The crust free regions became frozen magma covered by ocean. The total area of crust was preserved. A stronger condition is that only some minimal stretching required by the increase of the radius occurred. Too large a stretching would have generated the cracks.

   The experimentation with toy models leads to the conclusion that minimal stretching is achieved if the crust decomposes into a spherical lattice - regular tessellation- having maximal number of cells. Platonic solids are the only regular tessellations of sphere. The dual P_D of platonic solid P has as its vertices the faces of P and vice versa. The list of Platonic solids (see `http://tinyurl.com/p4rwc76`) is short.

   - Self-dual tetrahedron (4 faces and 4 vertices).
   - Cube with 6 faces and 8 vertices faces and its dual octahedron.
   - Icosahedron and its dual dodecahedron with 20 and 12 faces respectively. For icosahedron the number of faces is maximal and the size of the face minimal and the local stretching is therefore minimal. The faces of icosahedron correspond to the vertices of the dual dodecahedron and icosahedral tessellation is the best candidate to begin with. Note however that the 6 faces of cube could correspond to the 6 continents. One can of course image that the moving cratons later evolved to form an approximate cubical tessselation.

   Remark: Surfaces with flat metric (plane and cylinder) allow warpings (see `http://tinyurl.com/ycyregve`) for which the induced metric remains flat so that the deformation can be regarded as an isometry with no stretching but non-trivial bending. For instance, for the surface $z(x,y) = z_0$ one can have warping $z = z_0 + f(x)$. The dynamics for the page of book provides a good example of this kind of warping. Could this kind of warpings leading to one-dimensional deformations of the surface of Earth happen for continents in sufficiently short scales?

2. During subsequent evolution radius R_f remains (approxmately) constant and the pieces of crust move along the surface of Earth. No stretching condition prevents the change of shape. If changes of shape are allowed, the first guess is that this evolution was area preserving and thus generated as by a Hamiltonian flow. This would be just classical Hamiltonian mechanics in 2-D phase phase associated with the piece of crust.

   If distances inside cratons were preserved (no stretching and change of shapce), the dynamics for small enough plates would reduce in a reasonable approximation to a rigid body rotation in the tangent plane at the center of mass of the plate and movement along a geodesic line along the Earth's surface plus collisions. If one accepts that the initial state was a tessellation defined by a Platonic solid, in particular icosahedron, the symplectic evolution trivializes in this manner. The faces contain cratons with area scaled down by factor 1/4. If craton like object is a disk with radius d one would have $d = (1/2\sqrt{20})R_E \simeq .11R_E$. Using $R_E = 6371$ km this gives $d = 1425$ km.

3. The first guess is that the expansion period is over now and one has $R_f = 2 \times R_i$ exactly. As found, the predicted fraction of land area for $R_f = 2 \times R_i$ is 4.1 per cent smaller than the actual value about 29.1 per cent. A possible explanation for 4.1 per cent is the generation of sedimentary rocks. This would give a probably testable prediction for the fractional area due to sedimentation. Subduction would increase this estimate.

   One can also ask whether the expansion still continues slowly so that the radius is not yet quite equal to $R_f = 2 \times R_i$ so that the fraction of land area is larger than 25 per cent. One would have $R_f = 2xR_i$, $x = .93$. Subduction tends to increase and sedimentation to reduce the value of x. The separation of expansion period from the period during, which R_f stays constant would be a good approximation if the time scales for tectonics are considerably shorter than for the expansion.

Could flux tube network reproduce the claims of Adams?

The triangular faces can move around and can scale down their size scale by factor 1/2 to the size of craton so that a fusion of cratons to larger units forming continents becomes possible. If one takes the claim of Adams [F1] (see `http://tinyurl.com/fxsve`) seriously, the subsequent dynamics for the faces containing the cratons must be such that it is easy to see how to move continents in the scaling down of the radius of Earth to achieve the gluing together without overlaps and holes (the mere scaling down does not allow to achieve this since the distances between scaled down continents would be 1/2 of the recent distances).

The dynamics must remember the initial regular icosahedral tessellation at S^2_i. In the ideal situation every face must "remember" its former nearest neighbours at S^2_i even when some of them can be faraway at S^2_f. This requires a network connecting the faces. If the faces are connected by a large enough number of flux tubes able to change their lengths this can be realized and as the radius is imagined to decrease by a factor 1/2, all faces combine to form a spherical crust without overlaps. One can consider two extreme situations.

1. Maximal connectedness requires that every face of icosahedron is connected to each of its 3 nearest neighbours. In this case the dynamics can only involve condensation of the cratons/faces of the network to form continents and for this option the claim of Adams seems trivial.

2. The minimally connected network would correspond to a string connecting the 20 faces to single non-self-intersecting closed string identifiable as a Hamiltonian cycle at dodecahedron. One identifies cycles differing only by an isometry of dodecahedron and already Hamilton discovered that dodecahedron allows only single cycle if one identifies cycles differing only by an isometry of dodecahedron. Given triangle would be connected by flux tube to 2 (rather than 3) nearest neighbors.

   Remark: Hamilton's cycles at icosahedron [A22, A8, A19, A6, A17] with 12 vertices play fundamental role in TGD inspired model for music harmony lead to a model of genetic code and of bio-harmony. In this case there is large number of harmonies [L9] [L46].

   Whether this option is consistent with the claim of Adams is not clear. One can argue that without additional assumptions the dynamics of the Hamiltonian cycle can destroy the information about the initial icosahedral tessellation by permuting the faces. Could the condition that no self intersections of the flux tubes (strings) of the cycle take place, be enough to preserve the information about initial configuration? The (unique apart from isometries) Hamiltonian cycle can have a fold so that it turns back. The cratons of the antiparallel nearby portions of string can fuse together. The pairing induced by the folding can take place in several ways: say ...(1,6)-(2,5)-(3,4) or ...(-1,6)-(0,5)-(1,4)-(2,3). Here (a,b) corresponding fusion of cratons and - for the Hamiltonian link between neighbouring faces. The increase of the land area by 4.1 percent forces some overlap in the final state if the expansion period has ceased.

11.6.2 Plate tectonics as a symplectic flow in scales longer than the size of craton?

For the icosahedral model the short scale dynamics reduces to much simpler dynamics of 2-D rigid bodies at S^2 having collisions leading to subductions. Cratons however fuse together to form continents having plate tectonics as their dynamics. Tectonic dynamics applies in length scales longer than craton size and cratons could be idealized as point like objects analogous to lipids in cell membrane.

The first guess for the dynamics after the expansion period is symplectic flow preserving the signed area of the continent defining an area preserving map for each value of the time parameter. The area preserving flow is analogous to an incompressible liquid flow in 3 dimensions and serves as a natural model for liquid crystals. For instance, cell membrane is liquid crystal. In this case lipids are idealized as point like objects with symplectic dynamics making sense in length scales longer than the thickness of lipid.

Symplectic flow would be therefore a natural model for plate tectonics (see `http://tinyurl.com/hmby9d4`), and the idealization of cratons as pointlike entities would allow to overcome the objection due to stretching. Symplectic flows could be also used to model the emergence of cracks using Hamiltonians discontinuous along cuts and to model "self-subductions" as flows, which become non-injective and generate mountains.

Remark: Symplectic flows could also be used to model the liquid magma in the outer core idealized as 2-D layer analogous to liquid crystal.

What conditions could one pose on the Hamiltonian defining the symplectic flow? The observation that Hamiltonians identified as real or imaginary parts of analytic functions have additional symmetry implying the existence of a dual flow for which flow lines are orthogonal to those for the flow. A good guess therefore that the local tectonics for a continent is defined by a Hamiltonian satisfying Laplace equation. There would be a nice connection between analytic functions and symplectic flows.

A model for the continuous time evolution of tectonic plate

The simplest model for a continuous local evolution of given tectonic plate in length scales longer than the size of craton after the expansion period and formation of continents assumes the conservation of signed area meaning that the evolution is symplectic flow generated by some Hamiltonian defined in the region defined by the continent. The symplectic flow would be a 2-D variant of incompressible hydrodynamics.

1. The dynamics would be dictated by the conservation of signed area element $dS = R^2 sin(\theta) d\theta \wedge d\phi$ defined by the symplectic form of $J = J_{kl} ds^k \wedge ds^l$ of S^2. Symplectic transformations preserve the local area form and are generated by the exponentiation of Hamiltonian function H giving models for time evolutions as exponentiation of H defining a flow along the continent.

2. A model for the generation of cracks could be based on Hamiltonian function, which has line discontinuities completely analogous to discontinuities of imaginary or real part of an analytic function. The Hamiltonian flow would take the two sides of the cut to opposite directions in the Hamiltonian flow and crack would develop. The cracks would be filled with water and become oceans.

3. Hamiltonian time evolution defines symplectic map for each value of the time parameter t, which can cease to be injection at some moment of time at some point and give rise to growing regions into which two different regions of the continent are mapped. Cusp catastrophe with 3 sheets gives a standard topological description for what would have happened. The folding would have 3 plates above each other in the fold region. This "self-subduction" would produce regions analogous to those formed in subduction in which two continents drifting at the surface of magma collide and subduce. Also this process can generate mountains.

   The signed area of the middle sheet of the cusp is negative if the area of the other sheets is positive. The formation of the cusp seems therefore to reduce the land area since the middle sheet and lowest sheet of the cusp are invisible. When plate subduces another plate visible land area is also lost. One can imagine two explanations for the missing 4.1 per cent: sedimentation has generated new land area or the expansion period has not yet ended.

One can formulate this picture in more detail as follows.

1. The area preserving symplectic time evolution obeys in general coordinates s^k for S^2 the formula

$$\frac{ds^k}{dt} = j^k = J^{kl}\partial_l H \quad , \quad J_k{}^r J^r{}_l = -s_{kl} \quad . \tag{11.6.1}$$

   where J_{kl} and s_{kl} are the symplectic form and standard metric of S^2. In spherical coordinates (θ, ϕ) one has $J_{\theta\phi} = -J_{\phi\theta} = sin(\theta)$. $H = H(\theta, \phi)$ is the function defining the Hamiltonian and subject to physical constraints. j^k has vanishing divergence:

$$D_k j^k = 0 \ . \tag{11.6.2}$$

This equation codes for the local conservation of area.

2. The real or imaginary part of an analytic function having cut along curve can serve as a Hamiltonian in this case. Analyticity would give strong additional constraints on the discontinuity since Laplace equation would be satisfied meaning that not only the current j^k but also the dual current $j_D = g^{kl}H_l$ is conserved:

$$D_k j_D^k = 0 \ . \tag{11.6.3}$$

j_D^k and j^k are orthogonal and correspond to real and imaginary parts of an analytic function. Also j_D^k defines an area preserving flow. This connection between conformal symmetries and symplectic symmetries for Hamiltonians satisfying Laplace equation does not seem to be very familiar to physicists. As a consequence the flow has vanishing divergence and curl. Sources and sinks and global rotation are possible in topological sense if the tectonic plate has holes. This would suggest conformal invariance in some sense.

The absence of sinks implies that one can express j_D^k as a curl of vector field orthogonal to S^2. A possible interpretation is as induced Kähler magnetic field or Z^0 magnetic field. One of the first ideas related to the applications of TGD to condensed matter was that hydrodynamic flow could give rise to Z^0 magnetic fields just like em currents give rise to magnetic fields and that vortices of the flow correspond to magnetic flux tubes. This picture makes sense for Kähler magnetic field as well - an option that seems more natural now. The different directions of rotational axis and magnetic dipole axis of Earth would correspond to different directions of the ordinary magnetic field and Z^0 or Kähler magnetic field. These magnetic fields would be effective magnetic fields identified as sums of magnetic fields considered at different space-time sheets at quantum field theory limit of TGD. The flow dynamics could be essentially that of induced Kähler magnetic field orthogonal to S^2.

Remark: At fundamental level only the effects of classical fields on test particle touching several space-time sheets sum up, not the fields. At QFT limit induced fields from different space-time sheets sum up.

The equation for the flow can be integrated for a given flow line as

$$s^k(t) = exp(tj^r\partial_r)s^k(0) \ . \tag{11.6.4}$$

3. The model for the emergence of a crack requires Hamiltonian discontinuous along a 1-D cut. One has $H = H_\pm$ at the two sides of the cut. The expression of $s^k(t)$ for the flow lines beginning from the point $s^k(0) = s_\pm^k(0)$ of the cut and continuing to the side $\pm$ is given by

$$s_\pm^k(t) = exp(tJ^{rl}\partial_l H_\pm)\partial_r)s^k(0) \ . \tag{11.6.5}$$

The model for the emergence of "self-subductions" and generation of mountains can be constructed using non-injective Hamiltonian evolutions in which regions having as pre-images two regions appear. These regions correspond to two continent plates above each other. Both self-subduction and subduction reduce the land area.

11.6.3 Appendix: Some mathematical details

The icosahedral model for the generation of continents was an outcome of experimentation. I started with a model inspired by the idea that an analog of super-continent Gondwana was generated as single cap during the expansion period but realized soon that it requires quite too large stretching unless one allows generation of cracks. Also a model with two gaps seemed non-realistic. Homogenous upwards scaling of the Earth's radius suggests strongly lattice like structure and the minimization of stretching led to icosahedral model. I however decided to include these attempts as Appendix - a kind of confession. Hasty reader can skip these parts of the Appendix.

Generation of one or two caps requires too much stretching

The basic objection against single cap model is that the proposed model for expansion requires quite much stretching, which requires large energy. It is also clear that too much stretching leads to a generation of cracks. The following argument is more precise formulation of this observation in terms of a toy model.

1. The first option is that supercontinent analogous to Gondwana (see `http://tinyurl.com/hcgjnrb`) was generated as an expanding hole hole in the crust of S_i^2 emerged somewhere in what became Pasific Ocean - call this place "South pole". Gondwana hypothesis is consistent with Wegener's construction.

2. This period corresponds to a total area preserving map taking the spherical surface (crust) of S_i^2 to a cap of S_f^2 with the same area. The area of the cap should have been thus fraction $S_f/S_i = R_i^2/R_f^2 = 1/4$ of the total area: this corresponds to 25 per cent of the area of Earth. The actual portion of continents from total area is 29.1 per cent. 4 per cent of new land area should have been generated later by some mechanism.

3. The expansion would take the crust covering entire S_i^2 to a supercontinent covering part of S_f^2. The simplest map of this kind maps the surface of S_i^2 to a cap of S_f^2 defined by the condition $\theta_f \in [0, \pi/3]$: this corresponds to [0, 60] degrees. $\theta_f = 0$ would correspond to the "North Pole". This model is certainly non-realistic since it requires large stretching at the bottom of the gap. The stretching is expected to cause cracks mainly in the direction of the coordinate lines of θ_f.

   For the cap at "North pole" the stretching along the coordinate circles of ϕ_f would be very large near the bottom of the cap. One possibility is that cracks in direction of θ_f were generated or that the boundary of cap or that the boundary was "wavy".

A slightly more plausible option reducing the stretching along coordinate circles of ϕ_f would assume generation of 2 caps located at "South pole" and "North pole" as a crack along equator was generated. Also now a wavy crack would allow to minimize the stretching along the coordinate circles of ϕ_f. There would be also stretching along coordinate lines of θ_f. In this case one would have two separate super-continents from the beginning and fitting together along their boundaries of the gaps.

Cap models for the expansion period

The expansion period as generation of one or two caps is unrealistic since it produces too much stretching. In the following however the details of the model are given.

1. There exists no isometry between the crust associated with S_i^2 and connected crust associated with S_f^2. Isometry would require that curvature scalars are same and this is impossible since the radii of S_i^2 and S_f^2 are different.

2. The conservation of total area in the map $S_i^2 \rightarrow S_f^2$ taking spherical crust to cap $0 \leq \theta_f \leq \theta_{max}$ with same area: $S_f = S_i$.

3. If the expansion begins from an icosahedral lattice the dynamics of expansion period could reduces to simple scaling in a reasonable approximation. The fraction of land area is however

29.1 per cent rather than 25 per cent however that the expansion is still occurring albeit very slowly. Therefore one cannot separate expansion period completely from the tectonic dynamics. One can however think of time dependent scaling combined with the motion and collisions of cratons leading to their fusion.

Consider a more detailed definition of the cap models.

1. In the case of single-cap model the simplest manner to guarantee this is to require $cos(\theta_{f,max}) = cos(\theta_{i,max})/4 + 3/4 = 1/2$ giving $cos(\theta_{f,max}) = 1/2$ and $\theta_{f,max} = \pi/3$, which corresponds to 60 degrees. As mentioned the large strength in ϕ_f direction requires either a wavy boundary of generations of cracks in θ_f direction.

2. For the two-cap model the hemispheres $\theta_i < \pi/2$ and $\theta_i > \pi/2$ are contracted to caps when the crack at $\theta_i = \pi/2$ is generated. The condition that no stretching occurs along the coordinate circles of ϕ_f is guaranteed if one has

$$2sin(\theta_f) = sin(\theta_i) \ . \tag{11.6.6}$$

For small values of $sin(\theta_f)$ near poles this condition reduces approximately to the condition $2\theta_f = \theta_i$, which guarantees that the distances along coordinate lines of θ_f are same as along those of θ_i so that stretching is minimal also along this direction near poles.

This correspondence is well-defined only for $sin(\theta_f) \leq 1/2$, which corresponds to $|cos(\theta_f)| \geq \sqrt{3}/2$. On the other hand, the condition that the sum of the areas of the caps equals the area of S_i^2 gives $|cos(\theta_f)| \geq 3/4 < \sqrt{3}/2$ so that one must have larger gaps than allowed by no-stretching condition along coordinate circles of ϕ_f. A possible manner to solve the problem is to assume that the boundaries of the gaps are wave or that cracks are generated mainly in θ_f direction.

One can model the expansion period $t = (0, T)$ as a homotopy $R = R(t)$, $[R(0) = R_i = R, R(T) = R_f = 2R]$. During this period the cap develops and $\theta_{f,max}$ satisfies the formulas guaranteeing the conservation of distances along coordinate circles of ϕ_i and of total area.

1. For single-cap case one has

$$\frac{R(t)}{R_i} sin(\theta_f) = sin(\theta_i) \ , \quad (\frac{R(t)}{R_i})^2 (1 - cos(\theta_{f,max}) = 2 \ . \tag{11.6.7}$$

The first condition can be satisfied only for $cos(\theta_f) \geq \sqrt{1 - (R_i/R(t))^2}$. This lower limit should be smaller than the limit given by the latter condition: $R_i/R(t) \leq \sqrt{7}/4$. For $R(t)/R_i > 4/\sqrt{7} < 2$ the conditions are consistent with each other.

2. The 2-gap case gives

$$\frac{R(t)}{R_i} sin(\theta_f) = sin(\theta_i) \ , \quad (\frac{R(t)}{R_i})^2 (1 - cos(\theta_{f,max})) = 1 \ . \tag{11.6.8}$$

Also for this option one must have $cos(\theta_f) \geq \sqrt{1 - (R_i/R(t))^2}$. The condition $cos(\theta f, max) = 1 - (R_i/R(t))^2$ implies that the first condition cannot be satisfied for all values of $cos(\theta_f)$.

11.7 New support for the view about Cambrian explosion being caused by rapid increase of Earth radius

There was an interesting popular article in Quanta Magazine titled "*Oxygen and Stem Cells May Have Reshaped Early Complex Animals*" (see `http://tinyurl.com/y86ta45l`).

The article dicusses the work of geobiologist Emma Hammarlund and tumor biologist Sven Påhlman: their interdisciplinary hypothesis is published as article in Nature [I11] with title "*Refined control of cell stemness allowed animal evolution in the oxic realm*" (see `http://tinyurl.com/y85ufngz`).

Here is the abstract of their article.

Animal diversification on Earth has long been presumed to be associated with the increasing extent of oxic niches. Here, we challenge that view. We start with the fact that hypoxia ($\leq 1-3$ per cent O_2) maintains cellular immaturity (stemness), whereas adult stem cells continuously - and paradoxically- regenerate animal tissue in oxygenated settings. Novel insights from tumour biology illuminate how cell stemness nevertheless can be achieved through the action of oxygen-sensing transcription factors in oxygenated, regenerating tissue. We suggest that these hypoxia-inducible transcription factors provided animals with unprecedented control over cell stemness that allowed them to cope with fluctuating oxygen concentrations. Thus, a refinement of the cellular hypoxia-response machinery enabled cell stemness at oxic conditions and, then, animals to evolve into the oxic realm. This view on the onset of animal diversification is consistent with geological evidence and provides a new perspective on the challenges and evolution of multicellular life.

11.7.1 The proposal of Hammarlund and Påhlman

Cambrian explosion (see `http://tinyurl.com/ntvx38e`) during which highly advanced lifeforms suddenly emerged - prolifiration and diversification of animal life are the terms used about this - is one of the mysteries of biology. For most of its 4.5-billion-year history, Earth has sustained life — but that life was largely limited to microbial organisms: bacteria, plankton, algae. For about 540 million years ago did larger, more complex species are assumed to dominate the oceans, but within just a few tens of millions of years (very short time on the evolutionary timescale), the planet had filled up with all kinds of animals. The fossil record from that period shows the beginnings of almost all modern animal lineages: animals with shells and animals with spines, animals that swam and animals that burrowed, animals that could hunt and animals that could defend themselves from predators. Also many lineages that disappeared were present as one learns from the book of Stephen Jay Gould describing in detail the Burgess Shale finding that revolutionized the picture about evolutionary biology and remains still a puzzle (see `http://tinyurl.com/y9orfy43`).

The belief is that the environment became considerable more oxic - that is contained oxygen - and lifeforms had to cope with this change. Before the change the animals in seas (believed to exist!) were anaerobic. The shifting to aerobic respiration was however an enormous metabolic advantage since the effectiveness of metabolic energy gain become roughly 20-fold. Increased metabolic feed in turn made possible the emergence of complexity during Cambrian period.

1. The proposal of the authors is that the evolution of the capacity to maintain stem cells even in an oxic environment allowed the animals to keep stocks of stem cells needed for tissue growth and repair for that this required at gene level new genes coding for so called HIFs.

2. Stem cells require low oxygen levels to preserve their stemness. Heightened oxygen levels cause them to differentiate abruptly. This explains why stems cells are often located in hypoxic regions of the body (say bone marrow) having low oxygen levels. There are however exceptions to this rule: stem cells can also survive in ocix regions such as skin or retina. Cancers also utilize stem cells to achieve growth.

3. Hammarlund and Påhlman turned their attention to HIFs (hypoxia-inducible transcription factors), which are proteins, which for hypoxic environment shift the metabolism from aerobic to an-aerobic. For oxic environment they are not needed.

   HIF-2α remains however active also in oxic environment and make the cells behave as if the environment were hypoxic. This would allow the stem cells to survive. HIF-2α would however keep the stem cells in immature state also in the case of cancer. The hypothesis of Hammarlund and Påhlman was that HIF-2α functions similarly in normal animal tissues. They have seen some preliminary evidence for the hypothesis but further work is needed.

4. HIFs could have helped the animals to survive in oxic environment. Consider an organism as a blob of cells. Before the oxygenation the stem cells would have been forced to the deep interior of the blob, where oxygen concentration was especially low. When oxygenation took place, and oxygen level varied, this trick did not work anymore and HIFs had to be invented.

5. Hammarlund and Påhman postulate what they call HIF-1, which would have helped stem cells to behave as if the environment were hypoxic. Later HIF-2α unique to vertebrates emerged and improved the situation further. Vertebrates are bigger and have longer time spans that invertebrates and they can live in oxygenated environments. Inveratebrates such as insects live most of their life as larvae under low-oxygen conditions and they cannot regenerate tissues as vertebrates can.

6. Cancer would be the price paid for this evolutionary advance since cancer cells can proliferate because HIF-2 keeps the stem cells alive. OH present in oxygen rich environment is an oxidant causing cancer.

What caused the oxygenation? So called Great Oxygenation Event (GOE,see http://tinyurl.com/q7qfd55) is believed to have occurred about about 2.25 billion years ago and thus preceded Cambrian explosion that occured about .5 billion years ago.The time lapse between these events is about 1.75 billion years and much longer than the duration of Cambrian period, which was only tens of millions years. Thus GOE was not the reason for the Cambrian explosion. What caused a further oxygenation or were the effects of GOE somehow postponed (wink-wink!)?

11.7.2 TGD view

My own proposal is that life evolved in underground oceans and entered to the surface of Earth in Cambrian explosion (see http://tinyurl.com/ntvx38e) when oceans were formed at the surface of Eearth from cracks formed when Earth expanded rapidly in geological time scale. Before the explosion Earth did not have oceans and continents and was like Mars nowadays: even its radius was that of Mars. This picture follows from TGD based variant of Expanding Earth hypothesis [L40, L39] (see http://tinyurl.com/yc4rgkco and http://tinyurl.com/yb68uo3y) .

The habitat changed in the rapid expansion of Earth from hypoxic to oxic and the emergence of the hypothetical HIF-1 transcription factor would have been forced by this evolutionary pressure and made it possible for the lifeforms to adapt oxygen based metabolism. This would have led to a rapid evolution of animals and emergence of vertebrates. One can of course think that oxygenation developed already in the underground oceans as cracks caused in the crust by the expansion of Earth began to develop and provided oxygen. The alternative - not so plausible sounding - option is that the highly developed organisms developed underground slowly and only bursted to the surface of Earth in the explosion.

1. Chemical markers (see http://tinyurl.com/ntvx38e) indeed indicate dramatic change in the environment at the start of the Cambrian period. The markers are consistent with a massive warming due to the release of methane ice (clathrate hydrate, see http://tinyurl.com/peq9gmw) trapped within the crystal structure of water. Methane clathrate is found deep under the sediments at the ocean floors. Methane hydrates are believed to form by migration of gas from deep along geological faults (the cracks produced by rapid expansion of Earth [L39]!).

2. During the period before Cambrian explosion Earth would have been very much like in recent Mars. Even its radius would have been that of recent Mars! One can ask whether GOE forced the existing primitive lifeforms underground or saved only those already living underground. Situation would have been be very much like in the recent Mars, which also seems to possess undergound life.

The development of HIF proteins (hypoxia inducing factor) making possible for stem cells to survive in environments with varying and thus temporarily higher oxygen content would have been a natural reaction to the dramatic changes in habitat.

What can one say about the emergence of animal life in TGD framework?

1. The rapid evolution leading to the emergence of animals - if it was present - would relate to the quantum criticality associated with the increase of the effective Planck constant $h_{eff}/h_0 = n$ by factor 2 increasing the size scale of Earth. The increase of $h_{eff}/h_0 = n$ might have occurred at several levels of dark matter hierarchy, also at biological relevant scales and led to an increase of biological "IQ" (note that evolution corresponds in TGD to gradual increase of number theoretical complexity and n characterizes the dimension of extension of rationals characterizing the complexity [L29, L28]).

2. Animals use oxygen for breathing and are multicellular eukaryotes having cell membrane enclosing nucleus and other membrane bound organelles. The quantum critical period could have led to the emergence of a kind of symbiosis of various kind of organelles within cell membrane bounded volume. The p-adic length scale $L(k)$ determined by the value of n assignable to the outer membrane of organelles could correspond to the prime $k = 163$ (or 167). Inside plant cells having no cell membrane these organelles correspond to vacuoles (see `http://tinyurl.com/yd879b2d`). The outer membrane that emerged in the transition increasing h_{eff}/h_0 meant increase of the scale of quantum coherence to a longer p-adic length scale - say $k = 167$ (or $k = 169 = 13^2$ if doubling took place).

3. Mitochondria would have emerged and made possible oxygen based respiration whereas plant like organisms preceding them utilized anaerobic respiration. Methanogenesis (see `http://tinyurl.com/y97gkym8`) utilizing carbon instead of oxygen and producing carbon-dioxide and methane CH_4(water in O_2 based respiration) is the most natural option. The large methane storages underground would be due to methanogenesis.

   The recent findings (see `http://tinyurl.com/y735g9kn`) indicate that there is life in Mars: methane emissions occurring periodically with a period of Martian year have been detected. This suggests that solar radiation is somehow able to enter to the interior of Mars or that it heats the underground Oceans. In TGD one can consider also the possibility that some part of solar photons transforms to dark photons and is able to propagate to the underground oceans through the Martian crust [L39].

4. What was the primary source of metabolic energy? Direct solar radiation was absent in underground oceans. The immediate source of metabolic energy for the plant like organisms might have been dark nuclei consisting of dark proton sequences and liberating energy in the transitions reducing of $h_{eff}/h_0 = n$. Dark proton triplets give rise to dark variants of DNA, RNA, tRNA, and amino-acids [L15, L14, L42]. These dark proton sequences could have formed by Pollack effect at the surface of Earth possibly containing some water and could have propagated along dark flux tubes to the interior: also in "cold fusion" dark nuclei would be formed. Some fraction of them would transform to ordinary nuclei and liberate practically all the nuclear binding energy. Also transitions to dark nuclei with a smaller value of h_{eff}/h_0 is possible and liberates energy usable as metabolic energy. Most dark nuclei could leak out along magnetic flux tubes [L21]. The hen-egg problem - which came first, metabolism or genetic code - would trivialize in this framework.

   For p-adic length scale $L(k = 149) = 5$ nm - thickness of cell membrane - the typical dark nuclear excitation energy was about .5 eV, the nominal value of metabolic energy quantum. For $L(151) = 10$ nm (thickness of neuronal membrane and DNA double strand its value is .25 eV. These estimates are based on the scaling of the typical nuclear excitation energy taken to be 1 MeV and are uncertain by a factor of 2 at least. One of course expects also higher excitation energies - even so high that they correspond to visible ordinary photons. Metabolic energy could have been liberated as dark photons in dark nuclear transitions transforming to ordinary photons and absorbed by the photosynthetic machinery.

   The (rough) estimate for the typical value of the dark photon energy is considerably lower than in ordinary photosynthesis. Pollack effect [L10] occurring in presence of gel phase bounding water volume suggests that for $k = 149$ the transformation of dark proton sequences to ordinary ones: this mechanism would liberate energy per proton ~ 1.5 eV [L31], which corresponds to infrared photon. The small value of the metabolic energy quantum need not be a problem: there is recent evidence that IR light with energy 1.76 eV can be used in photosynthesis (see `http://tinyurl.com/yc6pqjed`).

11.7.3 Could Mars have intra-martial life?

A popular article in National Geographic (see `http://tinyurl.com/y5unt6y7`) tells about unexpected findings made by the first robotic geophysicist, the Insight lander revealed in the European Planetary Science Congress and in the American Astronomical Society. There are odd magnetic pulsations with frequency around 10 mHz [E28] (see `http://tinyurl.com/y3l18kcg`) occurring at Martian night-time: for Earth these pulsations occur in frequency range 1 mHz to 1 Hz. Mars has much stronger magnetic field as expected. The magnetic field was detected at heights 96-400 km.

Besides this there is evidence for the existence for a global electrically conductive layer about 6 km below the surface, which suggest an underground reservoir of water. This has enormous implications for potential existence of life in Mars. There is also earlier evidence for the existence of salty, liquid water measuring about 19 km across (see `http://tinyurl.com/ycjaky5g`).

The strange findings about Mars can be understood in the framework provided by TGD based model for expanding Earth providing also explanation for the mysterious Cambrian explosion assuming that the life developed in Earth's interior, TGD based notion of magnetic field, and dark matter identified as phases with nonstandard value $h_{eff} = nh_0$ of Planck constant.

Connection with the model of Expanding Earth

These findings bring in mind TGD based model for expanding Earth [L40, L39] (see `http://tinyurl.com/yc4rgkco`, `http://tinyurl.com/yb68uo3y`, and `http://tinyurl.com/ya68nggs`).

1. The observation is that if Earth has radius one half of itse recent radius the continents fit nicely together to cover entire surface of Earth. This lead to the proposal that during Cambrian explosion in which highly developed life formed mysteriously emerged, the Earth radius grew by factor 2 in a relatively short time. The life would have evolved in Mother Gaia's womb, underground oceans perhaps between crust and astenosphere at depth not larger than 80 km, shielded from cosmic rays and meteoric bombardment.

2. The sudden expansion can be modelled in TGD inspired new physics as a phase transition increasing the p-adic length scale of Earth and reducing the scale dependent cosmological constant assignable to Earth by factor 1/4: these kind of phase transitions replace smooth cosmological expansion in TGD inspired cosmology.

   This led to the splitting of the continuous crust to continents and oceans emerged as the water from underground oceans containing the highly developed life forms bursted to the surface.

3. The intriguing coincidence is that Mars has radius which is 1/2 of Earth's recent radius. Could also Mars have underground ocean with rather developed life forms waiting for the moment of birth? Magnetic field is necessary in TGD based model of life and the article tells that Mars has unexpectedly strong magnetic field. It also tells about underground ocean at death about 100 km! The boundary between Earth's core and astenosphere, where the ancient oceans might have been is at dept of about 80 km.

There is something weird in the magnetic field of Mars

The assumption that magnetic field of Mars can be approximated as a dipole field leads to a paradoxical situation in Maxwellian framework.

1. Wikipedia article about Earth's magnetosphere (see `http://tinyurl.com/y3t78oka`) gives a criterion for the height below which magnetic field can survive under the pressure caused by solar wind. The criterion reads

$$\frac{R_{CF}}{R_P} = (\frac{B^2}{\rho_{sw} v_{sw}^2})^{1/6} \quad .$$

Here R_P is planet radius, B is the strength of the magnetic field at its surface, and ρ_{sw} and v_{sw} are the mass density and velocity of solar wind. The ratio R_{CF}/R_P is essentially the ratio of the density of magnetic energy and density of kinetic energy. This implies that the strength of B is about 10 times higher than the strength of the Earth's magnetic field at surface about .5 Gauss. The recent findings should increase the earlier estimate $R_{CF}/R_P \sim 1$ given in Wikipedia. For Earth the thickness of magnetosphere is about 10 times Earth radius giving $R_{CF}/R_P \sim 11$.

2. The strength of magnetic field behaves like $1/r^3$ in dipole approximation and scaling R_P by factor 2 would reduce magnetic field strength at surface down by factor 1/8, which is near to value of the Earth's magnetic field strength B_E. Could one think that also Earth had similar magnetic field before the expansion an that the expansion of Earth radius by factor 2 gave rise to the recent magnetic field? $B_{Mars} \sim 10B_E$ however suggests that the magnetic field of Mars in dipole approximation should actually extend equally far as the Earth's magnetic field! This does not seem to make sense.

   Could one think that the matter at the flux tubes of Martian magnetic field is dark matter as $h_{eff} = nh_0$ phases and is not visible in the ordinary sense. For instance, cyclotron energies proportional to $h_{eff}eB/m$ would be much higher than expected. Another option is that the magnetic field corresponds carries monopole fluxes at its flux tubes carrying dark particles.

What looks mysterious is that if Martian magnetic field is dipole field in reasonable approximation, it should be more or less like Earth's magnetic field! One would expect cyclotron radiation and van Allen belts. Why they are not seen? The answer could be simple.

1. Also Earth's magnetic field would decompose to stable part for which flux tubes carry quantized monopole flux and ordinary part. Monopole part does not need current to sustain it and this has been used to explain why Earth's magnetic field has not disappeared long time ago. The varying part of the Earth's magnetic field would be created by convection currents in the solar. Since Mars does not have outer core, it would not have this part of magnetic field. I have proposed this model for the maintenance of Earth's magnetic field at [L13] (see `http://tinyurl.com/y5anawyk`).

2. I have assumed that dark matter as $h_{eff} = nh_0$ phases of ordinary matter essential for life resides at the flux tubes of this field having strength which is 2/5 of the Earth's ordinary magnetic field. I have called this field endogenous magnetic field and its existence and existence of h_{eff} hierarchy was deduced from the explanation of quantal effects of ELF em fields on vertebrate brain. If Mars has only dark magnetic field, the magnetic field of Mars could be invisible! The ordinary part of this magnetic field should appear in the analog of Cambrian explosion as the radius of Mars increases to that of Earth and core radius increase by factor 2 and the core becomes unstable against division to two layers.

3. It has been thought that Martian magnetic field is so weak because the outer core of Mars has been seized up in distant past leading to a collapse of the magnetic field. Could one think that the reverse of this process took place for Earth in the expansion and created the outer core, perhaps by splitting of the core to outer and inner core? This picture would fit nicely with the p-adic length scale hypothesis suggesting layered structures with thickness of layer coming as some power of 2: the thickness of core would have double and core would have divided to two layers. If the strength of the Earth's magnetic field has been stronger by factor 8 before Cambrian explosion, this should be seen in magnetic records.

   The rotation of the outer core would create ordinary magnetic field after the expansion. Before that various ions from solar wind would have entered to the dark flux tubes and entered to the interior of Mars. Same would have happened also in Earth and would explain how oxygen atmosphere emerged in Cambrian explosion and life could burst safely to the surface of Mars.

4. Intriguingly, Mars has its own version of Northern lights (see http://tinyurl.com/y5z7jlkb). Without magnetic field auroras should not exist! Could it be that they are dark auroras associated with dark magnetic field of Mars. In reconnections of the magnetic field o Martian magnetic field and those associated with solar wind dark ions would transform to ordinary ones and create Norther and Souther lights. Van Allen belts are in the height range .6-58 Mm (Earth radius is 6,4 Mm). Mars should have dark van Allen belts along which ions of solar wind would end down to the interior of Mars.

5. What about the pulsed oscillations of Martian magnetic field at frequency around 10 ms, which corresponds to a period of 3.33... minutes detected at the night-side of Mars?

   The pulsations could correspond to a biorhythm. Also Earth's magnetic field has pulsations with frequencies varying between 1 mHz and 1 Hz. 1 mHz corresponds to 3/3.6 minutes and 1 Hz to average DNA cyclotron frequency in endogenous magnetic field $B_{end} = .2$ Gauss identifiable as dark magnetic field.

   Could these pulsations correspond to a heartbeat or breathing of Martian magnetic Mother Gaia - rather concrete pulsation of its magnetic body made from flux tubes and/or sheets? Why the pulsations appear only at the dark side? Could the pressure of the solar wind prevent the pulsations at the day-side?

One can wonder what the measured magnetic field is. Is it the sum of dark and ordinary part or only ordinary part. If test particles touch all space-time sheets involved, they experience the sum of the magnetic fields so that the usual measurements should give the sum. If it is only the ordinary part, one would still have the problem why the field having strength near to Earth's magnetic field is not visible as van Allen belts for instance. The QFT limit of TGD indeed corresponds to the replacement of space-times sheets with single region of Minkowski space and the identification of fields as the sums of the induced fields from various space-time sheets.

Intraplanetary life

The new observations allow to make the existing model for intra-planetary life much more detailed. The following applies to both Earth and Mars.

1. At Earth the multicellular life forms would have emerged in Cambrian explosion suddenly from the Earth interior as its size increased by factor 2. The expansion would be one stepwise cosmic expansion and associated with the decrease of length scale dependent cosmological constant associated with Earth. Same should happen in Mars sooner or later. So that there is no reason to worry. If we destroy our species and many other at the same time, intelligent life forms will develop in Mars.

2. If the multicellular life forms represented intraterrestrial life, photosynhesis and even oxygen based life should have evolved in underground ocean. The breathing animals would be like fishes using the oxygen in water.

3. The dark magnetic flux tubes of planet would served as channels for solar photons propagating as dark photons to the ocean in the interior of the planet. Dark photons would have transformed to ordinary photons (that is bio-photons) and used in photosynthesis making possible chemical energy storage. Photosythesis would have produced oxygen O_2, which would not have been lost to outer space now: a good reason for intraplanetary life when oxygen atmosphere is missing.

   Thus breathing animals would have become possible besides plants like organisms performing the photosynthesis. Also animal-plants doing photosynthesis themselves can be considered. Even we could use the metabolic energy stored chemically in manner analogous to photosynthesis. The machinery is very similar and there is evidence that even humans can use sunlight as metabolic energy. Pollack effect [L10] would be key element here. Pollack effect generates charge separation and thus voltage and this gives rise to a battery.

4. The evolution of life inside planets could solve Fermi paradox. Universe is full of planetary systems. Life would be everywhere but inside the planets in planetary wombs. We might

be pioneers. An alternative solution is that we are already in telepathic contact with higher life forms at dark magnetic bodies but do not realize it. We ourselves would have magnetic bodies with Earth sized and perhaps even galaxy sized layers.

This is not the only possible TGD inspired solution of Fermi paradox. Our own magnetic body would have layers with size of Earth scale and perhaps even galactic scale. We could be in continual contact with the magnetic bodies of members of other civilizations without knowing it - say during dream states. We could have even neural machinery activating these flux tube contacts. DMT is the only psychelic produced by body itself, and is assigned with pineal gland, which Descartes identified as the seat seat of soul. In zero energy ontology light velocity would not be a problem for communications with distant civlization since signals could propagate in both directions and time reflection would make communications forth and back in time possible [L11].

The newest news from Mars tells that scientists have measured the seasonal variation of methane CO_2 and oxygen O_2 in the Gale Crater of Mars.

1. The largest amount of methane CO_2 hitherto, 21 parts per billion volume units has been measured in Mars by Curiosity Mars rover (see `http://tinyurl.com/w2p4uh8`). The presence of high level of methane discovered by Curiosity has been known for years. Methane is associated with living organisms. At Earth it is produced by microbes but can be also created through interactions between rocks and water. Curiosity cannot determine whether the source is local or is the methane present everywhere and whether the source is biology or geology. The recent measurement gave an increased level of methane but it is not known how long the transient lasts and whether there is a seasonal variation. The transient however suggests that the source of methane is local. Chemically reactive soil containing sources and sinks of methane has been considered.

2. Also oxygen was observed to behave in an unexpected manner (see `http://tinyurl.com/wlu4xq5`). The knowledge of the surface chemistry allows to estimate the yearly variation of oxygen, and the predictions confirm with measurements almost all the year. At spring time of Mars the rules of chemistry are however broken, and the concentration of oxygen rises as much as 30 per cent during spring and summer and the returns to the levels predicted by the chemistry. Something gives oxygen and then takes it back.

What smoothed out Earth's surface for 600 million years ago - or was it already smooth?

I learned about new fascinating finding (see `http://tinyurl.com/y339u6qo`) related to the geological history of Earth. During a geologically very brief period Earth's surface would have somehow lost its surface details such as rivers and lakes. This would have happened for 600 million years ago. Before this there would have been period of snowball Earth.

TGD provides different view about the renovation of the surface of Earth by loss of details and about the period before this change: snowball Earth hypothesis seems un-necessary [L40, L39]. There were no oceans. The situation would have been the same as in Mars now. Water was in underground oceans as has been observed recently in Mars. One could deduce the story of Earth from what we know about Mars on basis of latest discoveries.

Cambrian explosion, in which complex multicellulars suddenly and mysteriously emerged took place about 512 million years whereas the loss of details is claimed to have happened about 600 million years ago. TGD based theory of expanding Earth assumes that Cambrian explosion happened because of geologically very fast expansion of Earth so that the radius of Earth increased by a factor 2. Amusingly, the radius of Earth would have been before the expansion same as that of Mars now!

Expansion would correspond to one fast jerk in the sequence of jerks, which replaces smooth cosmic expansion in TGD Universe: it is indeed known that astrophysical objects co-move in expansion but do not expand themselves (except by jerks). These jerks would be induced by reduction of length scale dependent cosmological constant by factor 1/4, or more generally, negative power of 2. The findings suggest that the expansion started about 600 million years ago and happened geologically very fast. Note that length scale dependent cosmological constant solves the basic problem of standard cosmology, which has killed many theories, also superstring theory.

Before this life would have evolved in underground oceans - the womb of Mother Gaia - shielded from cosmic rays and meteors. Oxygen and other important molecules could not leak out so that oxygen based life could evolve. In the expansion the core of Earth split into two parts (Mars has same radius as Earth and only single core) and the rotation of the outer core created ordinary magnetic field preventing oxygen and other important molecules to leak out. Otherwise the life at surface would not have survived. The surface of Earth split into pieces giving rise to continents and underground oceans gave rise to the oceans.

Consider now the mystery of lost details.

1. The surface of Earth before expansion should have looked very much the same as that of Mars now. Since there was very little water, rivers, lakes and this kind of features would have been practically absent. I do not know whether this is enough to explain the findings.

2. If the surface of Earth was stretched in the sudden expansion, the details of Earth's surface would have tended to disappear since gradients are reduced in the stretching. Whether the stretching really occured is however not clear: one could argue that the surface split into pieces like clay soil as it dries and formed continents.

3. The bottoms of oceans consisted of the magma from the interior of Earth and they should have been rather smooth. Also this might help to understand the findings.

11.7.4 Earhquakes and volcanic eruptions as macroscopic quantum jumps in zero energy ontology

In ZEO the signature of "big" (ordinary) state function reduction is the change of the arrow of time at some level of the hierarchy of space-time sheets (selves) and one could start to search evidence for this effect. Also "small" state function reductions are possible and correspond to "weak" measurements. I did not however have the change of the arrow of time in mind when I encountered a highly interesting article "*Cosmic-solar radiation as the cause of earthquakes and volcanic eruptions*" by Jamal Shrair (see `http://tinyurl.com/y3g3khtd`) telling about the findings related to earthquakes and volcanic eruptions challenging the rational mind making its deductions in standard ontology.

1. The occurrence of earthquakes up to 34 kilometers below the surface of Earth and volcanic eruptions up to 9 km below the surface has strong correlation with the sunspot minima (solar activity) and cosmic ray flux. One could think that the system consisting of tectonic plates or magma is critical and sensitive to small perturbations. But how do the cosmic rays get so deep in Earth interior without losing their energy?

   TGD based answer is simple. During sunspot minimum the dark monopole part of the magnetic field of Sun is strong and the charged particles of solar wind arrive along the flux tubes and by reconnection end up to the flux tubes of the Earth's dark magnetic field (van Allen belts) and along them to the interior of Earth, where they end up to quantum critical system formed by magma or tectonic plates and induces the eruption of earthquake.

2. This however requires that the number of dark monopole flux tubes is large during sunspot minima. Sunspots would be formed in reconnections of very long U-shaped monopole flux tubes coming from Sun and carrying solar wind as dark particles. This would reduce the number of monopole flux tubes but generate ordinary magnetic field by creating currents creating them - monopole flux tubes do not need any current. Therefore the number of monopole flux tubes would be maximal during sunspot minima.

   Quite generally, cosmic rays would arrive to Sun along monopole flux tubes of flux tube network [L60] connecting galaxies and having flux tubes of stellar objects as sub-tangles and continue from Sun to Earth. The highly energetic dark cosmic rays preserving their energy as dark particles could end up to the Earth interior along monopole flux tubes and could induce eruptions and earthquakes. This mechanism would also take dark ions of solar wind to underground oceans in Earth interior in the model of prebiotic life [L40].

Consider now the observations in this framework.

1. In the model of Japanese researchers led by Toshikazu Ebisuzaki cosmic muons are assumed to induce volcanic eruptions. The assumption is that solar magnetic field repulses cosmic rays. When it is weak as believed to be during solar minima, the cosmic rays can arrive to Earth. Volcano would act as a volcanic bubble chamber in which the cosmic rays induce a phase transition (see `http://tinyurl.com/y3d52r7c`). The model however considered only the eruptions not deeper that 10 m below surface rather whereas most eruptions occur at depths up to 10 km. The objection is obvious: for the cosmic muons as ordinary particles it is difficult to get so deep into the interior.

2. NASA researchers reported that earthquakes are preceded by large fluctuations of densities of electrons and other charged particles in the upper part of atmosphere. Perturbations are detected at at heights 100-600 km above Earth's surface. For Earth quakes the depths vary down to 35 km. If cosmic rays induce the earth quakes, one would expect that the time order as indeed proposed by NASA researchers in their model. The problem is that electric perturbations precede the earthquakes rather than vice versa.

   Here ZEO comes in rescue: The time order was indeed opposite. Macroscopic quantum jump of a quantum critical system took place changing the direction of time. There is precise analogy with the findings of Minev *et al* in atomic systems showing that a deterministic and smooth time evolution seems to lead to the final state of quantum jump [L52] [L52]. The time evolution however has opposite arrow of time and starts from the final state. Libet's findings [J1] have the same explanation in terms of act of free will realized as state function reduction. Now the "big" state function reduction would correspond to the earthquake/volcanic eruption and would be induced by cosmic rays serving as stimulus. The bad news is that when the electromagnetic fluctuation are detected, the quantum jumps has already occurred and nothing can be done to prevent the catastrophe.

3. In Maxwellian picture one expects that the magnetic pressure of solar magnetic field is minimum during sunspot minimum: just the opposite is true as experiments show (see `http://tinyurl.com/y3g3khtd`! The stronger the solar minimum the stronger the magnetic pressure. This is indeed the case in TGD picture if the detected magnetic field corresponds to the sum of magnetic field associated with monopole flux tubes and ordinary flux tubes! This is what the QFT limit of TGD predicts since spacetime at this limit carries the sum of induced fields associated with the sheets of the many-sheeted space-time.

   These findings inspired the proposal of the article that motivated these comments (see `http://tinyurl.com/y3g3khtd`): the magnetic pressure of solar wind could induce the earthquake/volcanic eruption somehow but leaves the detailed mechanism open. In TGD this assumption is not needed. The dark cosmic rays from the monopole flux tubes of solar magnetic field reconnected to with similar flux tubes of the Earth's magnetic field would travel along them to the interior of Earth.

4. The article of Shrair also mentions earth lights, which are luminous phenomena associated with the lines of tectonic activity. I have proposed already earlier an explanation in terms of dark photons liberated from the regions with high tectonic stresses. These dark photons could be phase conjugate photons with non-standard arrow of time accompanying mini earthquakes already occurred with respect to subjective time. Even bigger earthquakes could be in question if the irradiation of phase conjugate dark photons with non-standard time direction continues for a long time after the earthquake, which will happen in our geometric future.

11.7.5 Correlation between earthquakes and volcanic eruptions with the spin dynamics of Earth

Wes Johnson send a link (`http://tinyurl.com/ydqhngkq`) telling about te correlation between the dynamics of Earth's spin and earthquakes and volcanic eruptions. There are two directions involved corresponding to geographic axis and rotation axis. The direction of Earth's magnetic field defines the geographic axis. These events tend to occur and are largest when the angle between

Earth's rotation axis and geographic (magnetic) axis is largest. This is is an excellent benchmark test for TGD based view about magnetic fields.

The new findings might have a connection with the TGD inspired solution of several other mysteries.

1. Quantal effects of radiation at ELF frequencies on vertebrate brain discovered by Blackman and others [J2]. Photon energies are ridiculously small: there should be no effects.
2. Maintenance problem of Earth's magnetic field.
3. Why the direction of Earth's magnetic field is different from that for the rotation axis which is a natural direction for convective plasma currents?
4. What causes the precession of Earth's rotation axis? The explanation in terms of gravitational effects fails.
5. There are time anomalies associated with earthquakes and volcanic eruptions. Cause and effect seem to be in wrong order.

Earth's magnetic field should have disappeared long time ago. TGD based solution relies on difference between magnetic fields in Maxwellian theory and TGD:

1. TGD provides a solution to the maintenance problem [L13] (`http://tinyurl.com/yjstfvg3`.

   In TGD framework magnetic field has two parts.

   (a) Monopole flux tube part with strength B_{end} = about .2 Gauss ($B_E$has nominal value of .5 Gauss). The existence of B_{end} is deduced from the effects of radiation at ELF frequencies on vertebrate brain (Problem 1). It would carry dark matter in TGD sense ($h_{eff} = n \times h_0$ phases of ordinary matter) and be crucial in biology. This part needs no current to maintain it and this solves the maintenance problem for Earth's B_E having nominal value $B_E = .5$ Gauss (Problem 2).

   (b) Second part B_o is the ordinary Maxwellian part and currents are needed to maintain it since it decays exponentially due to the dissipation of the currents. The change of the direction of monopole flux induces currents refreshing B_o. Just now monopole flux part is changing direction and this causes the direction of B_o part to change: magnetic North Pole is moving towards Siberia rather fast. A good first guess for the direction of B_o is the rotation axis of Earth.

   It would not be surprising if the difference between directions of B_{end} and B_o would have physical effects and that the effects occur when Θ becomes large enough. The size of the effects would naturally increase as Θ increases. Earthquakes and volcanic eruptions could be these effects.

   Remark: The direction of monopole flux part is not that of geographic axis since it represents direction of the entire magnetic field having nominal value $B_E = 0.5$ Gauss. The angle Θ between B_{end} and rotation axis is larger than that of geographic and rotational axis.

2. Monopole flux tubes provide also a solution to the precession problem [L4] (`http://tinyurl.com/ybezl7tj`).

   The change of the direction of monopole part B_{end} inducing change of the rotation axis could be due to the change direction of flux tubes in much longer length scale than that of Earth. Precession of the rotation axis could be the outcome and precession would not be caused by gravitational effects in solar system. TGD explanation involves magnetic flux tubes and dark matter in TGD sense in much larger scale than that of Earth.

3. TGD suggests also a solution to the time anomalies associated with earthquakes and volcanic eruptions [L56] (`http://tinyurl.com/yjpjgozk`).

   Zero energy ontology (ZEO) is the corner stone of TGD based quantum measurement theory solving its basic paradox and allowing to extend it to a theory of consciousness. ZEO changes profoundly the views about the relationship between experienced time and geometric time. The arrow of time changes in the counterpart of the ordinary state function reduction and is preserved in the counterpart of weak measurement.

(a) Earthquakes and volcanic eruptions would be induced by macroscopic quantum jumps - ordinary state function reductions in ZEO - involving large value of $h_{eff} = nh_0$ and its change at the level of magnetic body (MB) of the system. MB would correspond to flux tubes of B_{end}.

(b) In ZEO these quantum jumps change the arrow of time temporarily at the level of MB involved and induce effects in "wrong" direction of time also at the level of ordinary matter. Indeed, ELF radiation has been observed *before* the earthquake as it would cause the earthquake it - not very realistic sounding idea - and could therefore used to predict the Earth quakes.

The original model however predicted that ELF should occur after the earthquake as is indeed very natural. The ZEO based explanation of the paradox is that the arrow of time changes at MB. This would be the effect of Minev *et al* [L52] [L52] (http://tinyurl.com/yj9prkho) observed recently in atomic systems but in macroscopic scale. Also Libet's finding [J1] that neural activity seems to precede conscious decision would be similar illusion and at the same time proof the idea that act of free will corresponds to macroscopic quantum jump in ZEO.

This picture suggests an understanding of the correlation between earthquakes and volcanic eruptions and the dynamics of Earth's spin. As noticed, the macroscopic quantum jumps associated with changes of directions of B_{end} and B_o are expected to cause more dramatic effects when the deviation between the B_{end} and B_o (rotation axis) is largest. The angle Θ would change in these events. If B_{end} flux tubes change direction, a current is induced. This would change the direction of rotation axis if it is same as the direction of convective current generating B_o.

Question: Could the precession of the rotation axis consist of small steps changing the directions of B_{end} and B_o and their relative direction and be associated to earthquakes and volcanic eruptions?

11.7.6 No continents before Cambrian Explosion

I learned about highly interesting finding by geobiologists Benjamin Johnson and Boswell Wing. One can find a a popular article about the work with title "*Earth Could Have Once Been a Waterworld Covered by a Global Ocean, Study Suggests*" (http://tinyurl.com/uwjgfew). The research article with title "*Limited Archaean continental emergence reflected in an early Archaean 18O-enriched ocean* " is published in Nature Geoscience (http://tinyurl.com/rq7o4t9).

The finding is very interesting from the point of view of TGD based Expanding-Earth model [L40] (http://tinyurl.com/yc4rgkco) allowing to understand Cambrian Explosion (CE) (http://tinyurl.com/ntvx38e) that happened about .54 billion years ago leading to a sudden and rather mysterious emergence of multi-cellular life in a rather short time scale (13-25 million years).

TGD inspired cosmology predicts that cosmic expansion occurs for various astrophysical objects as relatively rapid jerks rather than smoothly. This allows to understand the paradoxical finding that astrophysical objects participate in cosmic expansion but do not seem to expand themselves. The expansion occurs in jerks in which the value of scale dependent cosmological constant characterizing the system decreases.

1. The radius of Earth would have expanded by a factor 2 in relatively short time scale from a value near to the radius of recent Mars and led to the formation of continents as the Earth's crust ripped. Multi-cellular life would had evolved in underground oceans shielded from meteoric bombardment and cosmic rays and bursted to the newly formed oceans at the surface.

2. The basic prediction of the model is that before CE there were no continents. This also kills the so called Snowball Earth model (http://tinyurl.com/prem7nj) for the climate before CE.

The findings give a direct support for the absence of continents before CE. What the researchers did was following.

1. The goal was to understand the temperature ancient Earth about 3.2 billion years ago, and the researchers studied what they believed to be a bottom of an ancient sea. The isotope ratio of ^{16}O to ^{18}O decreases with temperature. The researchers constructed the temperature profile of the ancient Earth, and the surprising finding was that there was 3.2 per cent more ^{18}O than expected. This is 4 per cent more than in today's relatively ice-free oceans and much higher than the previous estimates.

2. The isotope ratio is sensitive to landmass. The conclusion of researchers is that the unexpectedly low ratio could be due to the lack of landmasses. The Earth's surface could have been wet but there is no need to assume oceans. TGD prediction does not exclude wet surface but just the existence of continents. The high wetness could have been due to the percolation of water from underground oceans preceding the great burst - note that 3.2 billion years is by factor about 6 longer time than .54 billion years.

This picture however poses difficult questions. When and how did the continents emerge? How did life emerge if there were no landmasses so that standard model must be given up?

As already explained, TGD based model for Expanding Earth solving basic mystery of standard cosmology provides an answer to these questions and also to the mystery of Cambrian Explosion.

11.8 Updated version of Expanding Earth model

This section was motivated by several articles. The first popular article "Was ancient Earth a water world?" (https://cutt.ly/fbFqydU tells that Earth could have been covered by water for billions of years ago. As the Earth cooled, this water later sinked in the interior of Earth as water of crystallization forming underground "oceans". The amount of salty crystal water inside the Earth is estimated to be of the same order of magnitude as in the recent oceans.

The article "Constraining the Volume of Earth's Early Oceans With a Temperature-Dependent Mantle Water Storage Capacity Model" [F37] (https://cutt.ly/wbFqekI) The model of the article assumes that the water in the mantle and crust is water of crystallization https://cutt.ly/nbD65jZ. The water bound on on salt and metal crystals is not chemically bound but forms hydrogen bonds. In heating this water is liberated. For proteins the amount of crystal water can per 50 per cent. Heating leads to a loss of water of crystallization. Cooling induces opposite process and this would have led to the leakage of the water at the surface to the interior, even to mantle, where it bonded with crystals.

The water of crystallization does not however seem to be the only form of water inside Earth. The popular article "Pockets of water may lie deep below Earth's surface" (https://cutt.ly/4bFqi8M) told about pockets of exotic solid water - ice VII - in the mantle, which corresponds to the range of depths 610-800 km. The temperature in this range varies between 1300-4000 K and the corresponding thermal is in the range .13-.4 eV. The solid form is made possible by the large pressure.

The article also tells about the presence of freely flowing liquid water in the transition zone between Earth's crust and mantle. This corresponds to the depth range 410-660 km (https://cutt.ly/4bDOJIr) and temperatures below 1300 K. Underground life is known to exist at surprisingly large depths although high temperature and pressure pose strong constraints.

The popular article "Life Thrives Within the Earth's Crust" published in TheScientist provides a nice summary about what is known (https://cutt.ly/qbDOAON. From the article one learns that bacteria found at depths reaching 6 km. Fungi (multicellulars differing from animals in that they have chitin in their cell walls) and even animals are found at depths 700-800 m. The article "Anaerobic consortia of fungi and sulfate reducing bacteria in deep granite fractures" [I8] (https://cutt.ly/VbDOGvb)tells about sulfate reducing bacteria and fungi found at at granite fractures at depth 740 m.

The reason why these articles are so interesting from the TGD point of view, is that they lead to a more detailed version of the TGD inspired Expanding Earth model (EEM) [L40, L39].

EEM predicts that Earth suffered in the Cambrian Explosion (CE) about half billion years ago a relatively rapid expansion during which the radius of Earth increased by factor 2. There are however several objections against this model and the article provides insights allowing to circumvent these objections and supports the original vision.

In the sequel motivations for and objections against EEM are summarized. Also a resolution of objections based on a more precise model for EEMis discussed.

11.8.1 Motivations for EEM

There are three basic motivations for EEM.

1. The theoretical motivation is that the occurrence of this rapid expansion of Earth fits with the TGD view about cosmological expansion [L53] as rapid phase transitions replacing smooth cosmological expansion of GRT and solve the problem of GRT caused by the fact that astrophysical objects are not found to expand themselves although they participate to expansion by comoving with it.

2. The geological motivation is that the continents seem to fit nicely together to cover the entire Earth if the radius of Earth is 1/2 of its recent radius.

3. Cambrian explosion (CE) (`https://cutt.ly/AbFljuv`) serves as a biological motivation. CE started roughly 541 million years and lasted about 13 – 25 million years. During this relatively short period highly advanced multicellular life forms emerged. From the times before this there are only monocellular fossils.

4. Fermi paradox summarizes the empirical fact that there is no evidence for life as we understand it anywhere in the known Universe. One can imagine several reasons for this. A possible TGD based explanation is that life is present in the interiors of planets as it was in the interior of Earth before CE.

The rapid expansion would have broken the crust of Earth to pieces creating continents and the water from the interior of Earth containing multicellular life would have bursted to the surface and created oceans, absent before CE. The rapid evolution would have occurred during CE or already before CE in the "womb of Mother Gaia" in water pockets or even underground oceans shielded from cosmic rays and meteor bombardments.

The effects of Cambrian Explosion in deep mantle

Roughly a year after writing the original version of this article I learned about a possibe new piece of evidence for the TGD view about Cambrian Explosion. The popular article titled "Traces of life in the Earth's deep mantle" in Phys-Org (`https://cutt.ly/AAIj7Ss`) told about the work of Giuliani *et al*l about discussed in the article "Perturbation of the deep-Earth carbon cycle in response to the Cambrian Explosion" [F38] (`https://cutt.ly/wAIko6S`).

The sudden emergence of advanced multicellular lifeforms in the Cambrian Explosion (CE) about 540 Ma ago is still one of the great mysteries of mainstream biology. The team led by ETH researcher Andrea Giuliani found in rocks from deep mantle what can be regarded as traces of CE. The proposal is that partly organic material would have been subducted to the deep mantle after CE and changed the isotopic compositions of Carbon and other elements. Also other elements, for instance strontium and hafnium showed a pattern similar to carbon.

The group of Giuliani examined rare diamond-containing volcanic rocks known as kimberlites from different epochs of the Earth's history. These special rocks originate from the lowest regions of the Earth's mantle. The isotopic composition of carbon in about 150 samples of these special rocks was determined. The composition of younger kimberlites, which are less than 250 million years old, was found to vary considerably from that of older rocks. In many of the younger samples, the composition of the carbon isotopes differs significantly from that expected for typical rocks from the mantle.

The isotope ratio $R = {}^13C/{}^12C$ for Carbon in the deep mantle is considerably lower for the kimberlites younger than 250 Ma whereas the value for kimberlites older than 250 Ma is typical mantle value. The values of younger kimberlites are also more variable. More quantitatively, $\delta^{13}C = (R_{sample}/R_{standard} - 1) \times 10^3$ serves as a parameter. For a typical sample from the mantle, the value is $\delta \simeq -5 \pm 1$ per mille whereas for the studied samples δ is in the range [-20.-30] per mille.

The increased subduction caused by plate tectonics of the material from the surface of Earth could explain this. The estimate is that it takes 200 Ma for the material from the surface to reach the lower mantle. In the standard geology, the natural interpretation is that the dramatic increase in the generation of organic matter in CE has reduced the carbon isotope ratio. One must however assume that the partly organic material from the surface should have ended down to the deep mantle along specific routes by subduction.

Is the TGD inspired hypothesis for Cambrian Explosion life consistent with these findings?

1. The proposal is that life evolved in underground oceans ("womb of Mother Gaia") and suddenly emerged to the surface in the CE as highly developed multicellular organisms. CE was caused by a rapid increase of Earth radius by factor 2, which generated bursts of the underground water reservoirs to the surface and created the oceans. The expansion broke the Earth's crust to pieces and led to the emergence of plate tectonics, subduction, and oceans. Note that in Mars this tectonics is not present and the radius of Mars is actually roughly 1/2 of the radius of Earth.

2. The rapid increase of the Earth radius is the TGD counterpart for a smooth increase of Earth radius in cosmic expansion. All astrophysical objects look as if they would not participate in cosmic expansion: this is a mystery in general relativity. In TGD this mystery is resolved by replacing smooth cosmological expansion with a sequence of rapid expansions followed by stationary periods [L40] [L81].

Is the TGD view consistent with the findings of Giuliani *et al*?

1. The conclusion of Giuliani *et al* seems undeniable: the isotope decomposition in the mantle changed 200 Ma ago and was caused by the transport of the material affected by CE to the lower mantle.

2. In the TGD framework these findings do not however force the conclusion that life emerged suddenly in CE. Rather, multicellular life was present in the underground oceans before CE but plate tectonics and subduction were absent.

   The increase of the radius of Earth reduced the average density of Earth dramatically, and created the proposed subduction routes to the lower mantle, which dramatically increased the rate of transport of the organic material to the deep mantle.

3. Interestingly, the process analogous to CE appears to have occurred episodically throughout most of Earth's history, with the oldest diamonds that contain eclogitic inclusions forming at roughly 3 billion years (Ga) [F38]. In the TGD framework this suggests that the rapid expansions of Earth have occurred episodically and have led to the emergence of life forms from the interior to the surface and the transport of the material containing organic carbon to the mantle.

11.8.2 Objections against EEM and their resolution

There are several objections against EEM.

The reduction of density by factor 1/8 is impossible in standard physics

New physics is needed to make so dramatic a reduction of the density possible. The natural assumption is that the thickness of flux tubes of the magnetic body basically responsible for the density of condensed matter increased by a factor 2 and induced the increase of the radius of atomic volume. One can consider two options for what happened at the atomic level.

Option I. The value of h_{eff} labelling dark matter as phases of ordinary matter increased by factor 2, which led to scaling of atomic sizes by this factor and induced the reduction of the density by factor 1/8. The findings of Randel Mills can be explained if one has $h_{eff} = 6h_0$, where h_0 is the minimum value of $h_{eff} = nh_0$. The problem is that the total binding energy of electrons must have been compensated in the transition and it is not clear whether the energy liberated in the thickening of the flux tubes can provide the needed energy.

For Fe, which is also biologically important, the needed energy is for for $h_{eff} = h/2$ about 52 keV and seems quite too large if the string tension of atomic flux tubes is scaled to atomic scale from the hadronic string tension giving energy of order 10 keV per atomic length $L(137)$. The phase transition should have been entropy driven.

Note that in the TGD framework, the second law is implied by the negentropy maximization principle (NMP) for the sum of non-positive entanglement negentropy in the real sector, and the non-negative p-adic entanglement negentropy assignable to cognition. NMP forces the increase of the total entanglement negentropy and its positivity. This also forces the increase of entanglement entropy of ordinary matter. Thus the entropy driven phase transition allowing the emergence of Fe essential for oxygen based life would have meant an increase in negentropy and an evolutionary leap as it indeed meant. $h/2$ option is of course non-realistic but the argument applies also to the second option.

$h/2$ phase could have prevailed only during the period when the temperature was so high that atoms were unstable against ionization for the ordinary value of h. One can argue that as atomic physics with $h_{eff} = h$ became thermodynamically possible, it emerged in a phase transition. The electronic binding energies in question are of order $(Z/n)^2 E_H$, $E_H = 13.6$ eV. For instance, for Ca this temperature is about 2.7×10^8 K and corresponds to the temperature at which nuclear reactions become possible. The possible $h/2$ atomic physics might make itself visible at these temperatures.

Option II. Chemistry, and therefore the density of the condensed matter, is believed to be determined by chemical bonds, in particular by valence electrons. The TGD based view of valence bonds is discussed in [L27, L94]. Could the thickening of flux tubes by a factor 2 have induced the increase of h_{eff} of the valence electrons by a factor 2.

In the sequel these two options will be considered.

How could photosynthesis emerge in the Earth interior?

The animals that emerged in EEM performed photosynthesis. How could photosynthesis emerge inside Earth where ordinary solar light cannot get? I have proposed that dark photons with non-standard value of h_{eff} travelling along magnetic flux tubes managed to reach the evolving life inside Earth.

The recent proposals allow variants of this explanation.

Option I. The solar radiation with $h_{eff}/h_0 = 6$ was dark relative to the environment surrounding the water pockets. Life could have evolved already before or during CE in the water pockets with $h_{eff} = h = 6h_0$ larger than $h_{eff} = 3h_0$ in the environment. Solar photons with $h_{eff} = h = 6h_0$ did not "see" the presence of the environment because via direct interactions describable as Feynman graphs. Only the transformation $h_{eff} = 6h_0 \rightarrow 3h_0$ of the solar photon made scattering and absorption possible.

Option II. If the values of h_{eff} for the valence electrons were scaled up by a factor 2 inside the water blobs with flux tubes having thickness twice of those in the environment, and if valence electrons indeed determine what atoms are chemically, the water blobs would have behaved like dark matter relative to environment, and could have survived inside Earth.

The temperatures in crust and mantle are too high to allow the emergence of multi-cellular life

One can argue that the temperature in the crust and mantle is too high to allow the presence of multicellular or even monocellular life. However, if the pockets and environment were dark relative to each other, the situation changes. There would have been a very low rate of transfer of energy between these phases. The temperatures of pockets could have been much lower than that of the environment.

The gradual approach to thermal equilibrium characterized at magnetic body (MB) by Hagedorn temperature would have led to "death" of these primitive life forms but the occurrences of ordinary ("big") state function reduction reversing the arrow of time would have meant reincarnation with opposite arrow of time. This would conform with the TGD based view about life based on zero energy ontology (ZEO) [L93].

The original proposal that multicellular life evolved in the Earth's interior already before CE, is consistent with both options. The water at the surface of Earth was present already billions of years ago, and could have been dark in either of the proposed senses so that it could have leaked to the Earth interior and formed pockets with low temperature and low pressure. Note that solar light must have had $h_{eff} = 6$ already at time and for Option I could have induced the h_{eff} changing phase transition for the water molecules and perhaps also of the other molecules at the surface of Earth.

The proposed explanation of the Fermi paradox in terms of intra-planetary life would be nice but the obvious objection is that the circumstances in the interior make chemical life (as we know it) impossible unless some new physics, which allows the thermo-dynamical conditions prevailing at the surface of Earth, is involved.

Option I. Could the dark planets with $h_{eff} = h/2$ contain intra-planetary life as Earth did before CE, so that both the darkness of these planets and the lower evolutionary level of this life would be the reason for the failure to observe this life.

Option II. Also the scaling $h_{eff} \rightarrow 2h_{eff}$ for valence electrons could allow dark water blobs inside all planets if one believes that valence electrons dictate chemistry.

11.8.3 How the reduction of the density of Earth was possible?

The increase of the radius of Earth by a factor of 2 means that the average density is reduced by a factor 1/8 (in the case the system is genuinely 3-D; one can consider also effectively 1-D flux tube spaghetti). In standard condensed matter physics this kind of change in the local density is impossible.

The reason is that the density ρ - and thus the number density $n = \rho/mp$ of nucleons of condensed matter increases slowly with the mass number A (`https://cutt.ly/LbGMu9u`. Only very high pressures and chemical composition involving heavy elements can affect n significantly. For instance, the density of Earth varies from 2.2 g/cm^3 in the crust to 13 g/cm^3 in the inner core (`https://cutt.ly/4bD0JIr`) and therefore roughly by a factor 6.

Interestingly, the density of water is 1 g/cm^3 and by a factor $r < 1/2$ lower than the densities of the components of the crust. The low mass density of water might make it somehow special. Could water be seen as a mixture of phases with varying densities corresponding to varying radii for the flux tubes as suggested in [L45] to explain the numerous thermodynamic anomalies of water. The phases would correspond to different values of $h_{eff} = nh_0$ for flux tubes. The thickness of the flux tube would correspond to the p-adic length scale determined by the p-adic prime identified as a ramified prime associated with the extension.

If the local density at least in the mantle and crust contributing roughly one half to the total mass of Earth remained unchanged, a kind porous structure with pores filling 7/8 of the volume would have been formed. This looks implausible.

It seems that the expansion - if it happens at all - involves new physics.

Does the TGD view about dark matter allow to understand the reduction of the density

The basic prediction of the adelic physics [L29, L28, L6] is the identification of dark matter as hierarchy of phases of ordinary matter with effective Planck constant $h_{eff} = nh_0$.

1. In adelic physics n is interpreted as a degree of polynomial determining the space-time regions which corresponds to the particle. n measures the algebraic complexity of space-time region and serves as a kind of IQ and measure of the scale of quantum coherence. Evolution would corresponds to the increase of algebraic complexity and therefore also to the increase of n.

2. Functional composition of polynomials would give rise to evolutionary hierarchies in which the degree of polynomial at a given level divides the degrees at higher levels [L64, L65]. For instance $n = 3 \rightarrow n = 6$ conforms with this picture.

3. $n = 1$ would correspond to the simplest form of matter: the roots of polynomials defining the the space-time region would be rational and if the polynomial is irreducible, it is linear polynomials with rational coefficients. It is not clear whether $n = 1$ phase does exist.

   The phases with different values of n would not have direct couplings with ordinary matter describable in terms of Feynman diagrams. The transformation of particles, say photons, with different values of h_{eff} to each other are however possible and would occur for photons. Biophotons would be ordinary photons resulting from $h_{eff} > h$ dark photons by this kind of transition.

There are two guidelines available.

1. The scaling by factor 2 suggests a transition $h_{eff} \rightarrow 2h_{eff}$. Option I and II are possible.

2. The findings of Randel Mills [D7] can be understood if $h_{eff} = h = 6h_0$ holds true for ordinary matter [L18] and in the experiments of Milss a phase with $h_{eff} = 3h_0$ was formed. This would support Option I.

Although it turns out that Option I is not plausible model for CE, the phase transition $h_{eff} = 3 \rightarrow h_{eff} = 6$ is interesting as such.

1. This transition could have preceded by a transition $h_{eff} = h_0 \rightarrow 3h_0$ of $n = 1$ phase is possible at all. One could imagine a hierarchy in which cosmic strings correspond to $n = 1$ and flux tubes obtained by their thickening correspond to $n > 1$ phases. n cannot however directly relate to the value p-adic length scale characterizing the string like object.

2. Fine structure constant is proportional to $1/h_{eff}$ and would have decreased by a factor $1/2$ from its value before the transition. The atomic binding energy scale would have been 4 times larger.

   If $h_{eff} = 3$ is possible for stars, the radiation from them has an energy spectrum scaled up by factor 4. $h_{eff} = 3$ photons should transform to ordinary $h_{eff} = 6$ photons before interaction with the ordinary matter. The rate for this transformation could be low so that this kind of stars are difficult to observe. Dark matter could therefore be partially also $h_{eff} < 6$ matter.

3. One can ask whether the $h_{eff} = 3 \rightarrow 6$ transformation of the planetary matter near the planetary surface was induced by the interaction with solar radiation. The second question is whether it took place for each planet independently or whether a collective phase transition in cosmological scales occurred. The minimal assumption is that this transition is part of the evolution of the astrophysical object and those objects for which it has not occurred are dark relative to us.

   $h_{eff} < h$ phase would represent only one form of dark matter when darkness is regarded as a relative notion. Valence electrons would also represent dark matter with $h_{eff} = h_{em} > h$ as also dark protons assignable to hydrogen bonds. Another form would be the Kähler magnetic and volume energy and possibly dark particles at cosmic strings transformed to flux tubes. This includes the dark matter satisfying Nottale hypothesis $h_{eff} = h_{gr} = GMm/v_0$ and associated with gravitational flux tubes [E25].

4. Both planets and observed exoplanets must have $h_{eff} = 6$ since the reflection of solar light from the surface is expected to be occur only if the $h_{eff} = 6$ stellar photons transform to $h_{eff} = 3$ photons. Note that the known exoplanets belong to the Milky Way whose size is about 50,000 ly and much shorter scale than the 500 million ly defined by the time of CE.

5. Planet 9 (`https://en.wikipedia.org/wiki/Planet_Nine`, whose existence has been proposed because its gravitational field could explain the unusual clustering of orbits for a group of extreme trans-Neptunian objects (ETNOs), bodies beyond Neptune that orbit the Sun at

distances averaging more than 250 times that of the Earth. Planet 9 is too distant to be seen directly. Witten has proposed an interpretation as a blackhole [E27]. An alternative identification would be as an $h_{eff}/h_0 < h$ object.

6. The $h_{eff} = 6h_0$ life in the interiors of $h_{eff} = 3$ planets could be considered as a possible solution of the Fermi paradox. In the proposed model, the life below the surface of Mars would be possible only near its surface and mono-cellular as most of the life in the Earth's crust.

Cambrian explosion as a quantum jump in a planetary scale?

In ZEO [L61, L75] based quantum measurement theory, there are two kinds of state function reductions (SFRs): "big" (ordinary) state function reductions (BSFR), which involve time reversal and "small" SFRs, which correspond to "weak" measurements in which the arrow of time is preserved. The sequence of SSFRs defines a conscious entity and aBSFR can be regarded as death in a universal sense.

In biology BSFR [K49] [L93, L92] corresponds to the death of subsystem and its reincarnation with an opposite arrow of time occurring at the level of magnetic body (MB) of the system controlling it because of this higher IQ due to the much larger value of h_{eff}. h_{eff} hierarchy predicts that quantum coherence and SFRs are possible in all scales at the level of MB.

Although BSFR is discontinuous, it looks for an observer with a standard arrow of time (briefly, "outsider") like average over deterministic time evolutions leading to the final state of BSFR. In the ZEO framework, the Universe looks therefore classical in all scales.

Could CE correspond to BSFR, or actually two BSFRs to achieve original arrow of time - at the level of MB? The duration of the average deterministic classical time evolution of this BSFR seen by an outsider would be about 13-25 million years.

What CE as BSFR could look like for an outsider?

1. Water has a unique role in biology since living matter consists mostly of water. In TGD inspired quantum biology it is seen as a primitive life form preceding chemical life. For instance, water memory would be behind the immune system [K44].

   Pollack effect [L10, I17, I13], associated with water irradiated in the presence of gel phase and leading to the formation of negatively charged exclusion zones (EZs), would be behind charge separation associated with cell, DNA, and microtubules. Part of protons would become dark and would be transferred to magnetic flux tubes where they could give rise to a fundamental representation of genetic code [L15, L51]. An attractive assumption is that the phase transition thickening the flux tubes by factor of 2 occurred first for the water phase. The density of water is one half of that from that for the density of the crust and this could be understood if water consists of flux tube-like structures.

2. Since solar radiation was present already billions of years ago and also Earth was covered by water, it is possible that the solar radiation induced the phase transition for water at the surface of Earth. Call this water activated water.

   For both Options I and II, the electromagnetic interactions of the activated water with the matter of primordial Earth were very weak and it could leak to the Earth interior - not as a crystal water but as pockets with much lower temperature and pressure inside Earth. The solar radiation also reached the interior of the Earth so that an evolution leading to photosynthesis and metabolic machinery could have indeed occurred in the interior in the womb of Mother Gaia.

   Note that the TGD based model for valence bonds [L27] requires that solar radiation corresponds to several values of h_{eff} or that the transitions of $h_{eff} = 6 \rightarrow h_{eff} > h$ are possible.

3. Pollack effect could have led to the formation of the basic structures of the chemical life inside these pockets. The flux tube connections with large h_{eff} between pockets could have formed and made possible larger structures consisting of separate units and controlled by its MB. Even underground oceans can be imagined.

4. One can consider two options for the evolution of multicellulars. According to the original option, multicellular life evolved already before CE. The standard view about CE is that it occurred during CE. In the TGD framework, the original option looks more plausible. The emergence of life would mean a scaling $h_{eff} \rightarrow 2h_{eff}$ for valence electrons (Option II) $h_{eff} = 3 < h$. Scaling $h/2 \rightarrow 2h$ for all electrons (Option I) looks implausible. Maybe the liquid phase and low density could allow to understand why.

11.8.4 The transition increasing flux tube thickness as a phase transition induced by magnetic body

There are two options to consider: I and II. Option I assuming that the thickening of flux tubes by factor 2 induces the phase transition $h/2 \rightarrow h$ does not seem realistic. It is however possible that this phase transition has occurred much earlier at temperatures allowing nuclear fusion and could also occur in the laboratory in these circumstances.

The following discussion applies to both options: the only difference is that the total electron binding energy E_B is replaced with that for valence electrons.

The best manner to proceed is to develop objections against the proposals. It is easy to develop a rather scaring objection.

Minimization of free energy as basic principle

The minimization of free energy F can be taken as a basic principle since temperature is expected to remain constant during the phase transition.

1. If the temperature stays constant in the transition,the basic thermodynamic condition is that free energy decreases

$$\Delta F = \Delta E - T\delta S \leq 0 \ . \tag{11.8.1}$$

2. One can express ΔE as

$$\Delta E = \Delta E_B + \Delta E_{MB} \ . \tag{11.8.2}$$

The subscript "B" refers to the binding energy which for Option I is the total binding energy $E_B = E_{B,atom}$ and for Option II the total binding energy $E_{B,val}$ of valence electrons. The subscript "MB" refers to the magnetic body assignable in the TGD framework to magnetic flux tubes, whose thickening by a factor 2 would liberate energy and kick the atoms to new ground states.

3. From the $1/h^2_{eff}$ proportionaly of the binding energies, the reduction of the binding energy in the transition is given by

$$\Delta E_B = \frac{3E_B}{4} \ . \tag{11.8.3}$$

The thickness of the flux tube is expected to correspond to the atomic length scale of order Angstrom so that atomic physics would involve a new length scale relevant for the density of condensed matter.

$$\Delta E = \Delta E_{B,tot} + \Delta E_{MB} \ . \tag{11.8.4}$$

4. One can express ΔS in an analogous manner

$$\Delta S = \Delta S_{atom} + \Delta S_{MB} \quad . \tag{11.8.5}$$

The subscript "atom" refers to entropy assignable to scaledup atoms and "MB" to magnetic body.

One can consider two ways to satisfy this condition depending on whether the transition is energy or entropy driven.

1. For $\Delta S = 0$, one has $\Delta E \leq 0$. This would mean that the energy needed to kick electrons to new states with binding energy reduced by factor 1/4 must come somewhere. The fundamental quantum phase transition at MB should provide it, most naturally as energy liberated when the string tension of flux tubes is reduced as they thicken.

2. The alternative option is that that the transition develops a lot of entropy

$$\Delta S \geq \frac{\Delta E}{T} \quad . \tag{11.8.6}$$

It is important to note that in the TGD framework negentropy maximization principle (NMP) is the basic principle and implies second law for ordinary matter.

The p-adic contribution to entanglement negentropy coming from cognition ispositive unlike real contribution, which is non-positive. NMP implies that in adelic physics p-adic contribution to negentropy exceeds in general real contribution. The generation of p-adic negentropy however forces also a generation of real entropy and this conforms with the paradoxical proposal of Jeremy England that living systems produce entropy.

In the recent case the generation of large entropy at the level of visible matter would correspond to a generation of large p-adic negentropy assignable to the MBs in question. Hence the Cambrian phase transition would mean a cognitive revolution of some kind.

Estimates for the total binding energy

The following rough estimates assume Bohr model. In Bohr model, the atomic energies at a given shell n, which corresponds to the row of the Periodic Table. The energy shell contains n^2 states with angular momenta $l = 0, .., n-1$ are given by $E_n = (Z^2/n^2)E_H$. The number of states in full shell is $2n^2$. Full shells are realized only for $n = 1, 2$. The total binding energy in a full shell is nZ^2E_H, $E_H \simeq 13.6$ eV.

One could naively argue that the filling of all sub-shells l is energetically more favorable since the total binding energy would be maximized in this manner. However, already for $n > 3$ only the 8 states at s and p subshells are realized and d shell is missing so that the $n = 3$ shell contains the same angular momentum eigenstates as $n = 2$ shell. For $n = 3$ shell Argon corresponds to configuration $[Ne]s^2p^6$. K does not correspond to$[Ne]s^2p^6d$ but to [Ar]4s. The reason for this is not clear to me and one can of course ask whether the $h/2 \rightarrow h$ could have favored smaller binding energies and even led to the increase of n instead of full shell.

The expression for the total binding energy is given by

$$E_B = \sum_n N_n \frac{Z^2}{n^2} E_H \quad , \quad E_H = 13.6\ eV \quad . \tag{11.8.7}$$

For full shells (the rows $n = 1, and 2$) $N_n = 2n^2$ and the energy is $2Z^2$.

The following equations represent the total binding energies in Bohr model for some important ions in biology.

atom	Z	configuration	E_B/E_H	E_B/keV	$\lambda/\AA$
O	8	$[He]2s^22p^4$	$8^2(2+3/2) = 224$	3.05	4.1
P	15	$[Ne]3s^23p^3$	$15^2(2+2+5/9) = 1025$	13.9	.9 (11.8.8)
Ca	20	$[Ar]4s^2 = [Ne]3s^2p^64s^2$	$20^2(2+2+8/9+1/8) = 2050-400/9 \simeq 2005.6$	27.3	.45
Fe	26	$[Ar]3d^64s^2$	$26^2(2+2+14/9+1/8) = 3840.1$	52.2	.22

Note that Bohr radius is .53 Å so that for Ca the wavelength defined λ defined formally by the total binding energy is rather near to Bohr radius. The energies are rather high. For valence electrons the total binding energies are much lower and for Fe one has $E_{val}(Fe) = 1.15keV$.

These results support the view that Option II is more realistic than Option I.

1. The flux tubes are characterized by a p-adic length scale L_p, where p-adic length scale hypothesis states $p \simeq 2^k$. One has $L_p \equiv L(k) \simeq 2^{(k-151)/2}L(151)$, $p \simeq 2^k$, $L(151) \simeq 10$ nm. p-adic length scale $L(137) = 2^{-7}L(151)$, corresponds to .78 Å and $L(139)$ to 1.56 Å.

2. The magnetic flux tubes assignable to condensed matter and determining the density of the condensed matter should have thickened by a factor 2 in the transition. The phase transition $k = 137 \rightarrow 139$ is a natural candidate. Note that this pair defines twin primes.

   An estimate for the value of string tension follows from the hadronic string tension $T_H \simeq 1$ GeV^2, which corresponds to $k = 107$: this gives $T(137) = 2^{-137-127}T_H = 2^{-30}T_H \simeq 1eV^2$. The energy of a string portion with length $L(137)$ is $E(137) \simeq 10$ keV. An energy of this order would would be been liberated in the transition $k = 137 \rightarrow 139$.

3. Option I does not look realistic. From **Eqs.** 11.8.8 one finds that for Ca the total binding energy is 27.3 keV. For Fe the energy is 46.1 keV. These energies are too large: the transition in the case of Fe should be strongly entropy driven.

4. Option II looks more reasonable. For Fe the total binding energy assignable to the valence bonds is $E_{val} = 1.2$ keV. The maximal binding energy of valence electrons is $Z^2(n^2-1)/n^2E_H \simeq Z^2E_H$. Not all angular momentum subshells are however filled and this energy is maximum for atoms towards the right end of the row and for Krypton one has $n = 3$ is $D_{val} = 1.1$ keV. 1.24 keV corresponds to the energy of a photon with a wavelength of 1 Angstrom, qhixh looks also reasonable. Since the energy liberated from the flux tube portion is considerably higher than E_{val}, it would have induced expansion.

What could one say about the phase transition at the level of MB?

The phase transition at the level of MB induces the phase transition at the lower levels. Can one say anything about the phase transition at the level of MB?

1. The twistor lift of TGD predicts that energy of the magnetic flux tube is a sum $E = E_1 + E_2$ of two terms.

   The first term is a volume term proportional to the TGD counterpart of cosmological constant Λ predicted to be length scale dependent and by dimensional considerations proportional to $1/L_p^2G$. Λ would be reduced by a factor 1/4 in the proposed transition transitions. This terms gives a contribution

   $$E_1 = aSL \ ,$$

   where S is the transversal area of the flux tube and L its lengthThe scaling $a \rightarrow a/2$ would occur in the transition.

   The energy also contains Kähler magnetic energy. If the flux tube carriesmonopole flux, the contribution is of form

   $$E_2 = \frac{b}{S}L \ ,$$

   Assume that the scaling $b \rightarrow b/2$ occurs in the transition.

2. The minimization of the total energy

$$E = E_1 + E_2 = aS + \frac{b}{S} \quad (11.8.9)$$

with respect to S is assumed and gives

$$E = 2\sqrt{ab} \ . \quad (11.8.10)$$

In the scaling the energy transforms as $E \rightarrow E/2$. The liberated energy $E/2$ could be used to reduce the binding energy of the atoms by $3E_B/4$.

3. The natural expectation is that the total energy for a flux tube portion of length L_p is of order of photon with a wavelength L_p. This energy is given by $E = \hbar/L_p = 1.24/L_p/\mu m$. For $k = 137$ this gives energy $E \simeq 16$ keV. For Ca one has $3E_B/4 = 20.5$ keV and for P one has $3E_B/4 = 10.3$ keV.

   This suggests that the first phase transition could take place only for the biologically important atoms and molecules formed from them - in particular water molecules - and would not produce much entropy. Second phase transitions identifiable as Cambrian explosion would take place for heavier atoms and require large ΔS in turn requiring large generation of negentropy at the level of MB. This would accompany a rapid evolution of life at the surface of Earth.

11.8.5 Cambrian explosion, the Great Oxidation Event, and Expanding Earth hypothesis

I encountered two interesting articles related to the Great Oxidation Event that started long before the Cambrian Explosion (CE) and reached its climax during CE (about 541 million years ago) leading to the oxygen based multicellular life in a very rapid time scale.

The standard view is that oceans before CE had very low oxygen content. The emergence of photosynthesizing cyanobacteria producing oxygen as a side product led to the oxygenation of the atmosphere and to mysteriously rapid evolution of life. How this is possible at all is not understood.

The first popular article (`https://cutt.ly/UQWZA31`) discusses the proposal [I10] that the slowing down of the spinning of Earth was somehow related to this. The idea is that the lengthening of the day made photosynthesis by cyanobacteria more effective since their reaction to the dawn of the day was slow. The second article in Quanta Magazine (`https://cutt.ly/PQWZDzD`) tells about the finding [I9] that during the Cambrian Explosion (`https://cutt.ly/1QWZF4E`) the oxygen content of the studied shallow ocean show fluctuations with with about 4-5 peaks. The reduction/increase of the oxygen content was even 40 per cent, which is a huge number. The reduction of oxygen content caused extinctions and its increase was accompanied by the emergence of new species. The mystery is how this could happen so fast and which caused the fluctuations.

Expanding Earth hypothesis

Expanding Earth theory hypothesis is not originally TGD based but TGD provides its realization. The proposal is that the Cambrian Explosion was caused by a rapid increase of the radius of Earth by factor 2 [L40, L81].

This hypothesis also solves one of the basic mysteries of cosmology. Astrophysical objects participate in cosmological expansion by comoving with it but do not expand themselves. Why? The prediction that the expansion of the astrophysical objects did not occur smoothly but as rapid phase transitions and the expansion was very slow in the intermediate states. Cambrian Explosion would correspond to one particular jerk of this kind in which the radius of Earth grew by a factor

2 (p-adic length scale hypothesis). The length of the day increased by factor 4 from conservation of angular momentum. This might relate to the conjecture of the first article.

The rapid expansion led to the breakage of the Earth crust and to the birth of plate tectonics. It also led to the burst of underground oceans to the surface of the Earth. The photosynthesizing multicellular life had developed in these oceans and emerged almost instantaneously and led to a rapid oxygenation of the atmosphere. One can say that life evolved in the womb of Mother Gaia shielded from meteorites and cosmic rays. No superfast evolution was needed. Already Charles Darwin realized that the sudden appearance of trilobites was a heavy objection against the theory of natural selection.

Possible scenarios for the phase transition are discussed in [L81]. The thickening of magnetic flux tubes for water blobs at the surface of Earth led to the increase of the volume of water blob and induced the increase of h_{eff} a factor 2 for valence electrons but not for the inner electrons. Since valence electrons are responsible for chemistry, atoms became effectively dark and the water blobs could leak to the interior of Earth. By their darkness they could have much lower temperature and pressure than the matter around them and the life could evolve.

How photosynthesis was possible underground?

What made photosynthesis possible in the underground oceans? One possible explanation is that the photons from the Sun propagated along flux tubes of the "endogenous" part of the Earth's magnetic field as dark photons with $h_{eff} = nh_0 > h$. Endogenous part would be the part of Earth's magnetic field with a strength about 2/5 of the Earth's magnetic field for which flux tubes carry monopole flux: this is possible in TGD but not in Maxwell's theory.

Since these photons behave like dark matter with respect to the ordinary matter, they were not absorbed considerably and reached the water blobs (or actually their magnetic bodies consisting of flux tubes) in underground oceans having a portion with the same value of $h_{eff} \geq h$. Of course, several values of h_{eff} were possible since this is the case in quantum critical system (large values of h_{eff} characterize the quantum scales of long range fluctuations). One can also consider other variants of the model. The ordinary matter in Earth's crust had $h_{eff} = h/2$ and photons with $h_{eff} = h$ propagated to the interior and reached the water blobs with $h_{eff} = h$.

The sudden emergence of multicellulars and oxygen fluctuations

Before the expansion period was much like the surface of Mars now and contained no oceans, perhaps some ponds allowing primitive monocellular lifeforms. As the ground of Earth broke here and there during the rapid expansion period, lakes and oceans were formed at the surface of Earth. The multicellulars bursted to these oceans and oxygenation of the atmosphere started locally.

Since the oxygen rich water was mixed with the water in the shallow oceans, the local oxygen content of the burst water was reduced and this led to an eventual extinction of many multicellulars in the burst. Burgess Shale fauna contained entire classes, which suffered extinction. In the average sense the oxygen concentration increased and led to the apparent very rapid evolution of multicellulars, which had actually already occurred underground. Of course, also evolution at the surface of Earth took place.

11.9 Has venus turned itself inside-out and why its magnetic field vanishes?

News about unexpected findings relating to the physics of astrophysical objects emerge on an almost daily basis. The most recent news (`https://cutt.ly/YQSZgpv`) told about the lack of craters and volcanic activity in Venus (`https://cutt.ly/wQSZzaS`). The findings are actually not new. The resurfacing history of Venus was summarized 1979 by Schaber *et al* [E36] . Turcotte and Rome have proposed cyclic global catastrophic events as an analog of the plate tectonics allowing a heat transfer from the interior of Venus and effectively turning Venus inside out [E41].

The Venus does not have appreciable magnetic field although dynamo mechanism suggests magnetic field as in the case of Earth, has been also known.

11.9.1 Has Venus turned itself inside-out?

The surface of Venus was expected to have craters, just like the surface of Earth, Moon, and Mars but the number of craters is very small. The surface of Venus also has weird features and many volcanoes. Also trace signs of erosion and tectonic shifts were found. The impression is that the surface of Venus had been turned inside out in a catastrophic event that occurred about 750 million years ago.

Since Venus is our sister planet with almost the same mass and radius, it is interesting to notice that the biology of Earth experienced the Cambrian explosion 541 million years ago.

1. The TGD explanation for Cambrian Explosion relies on Expanding Earth Model (EEM) [L40, L39, L81]. The model assumes that there was a relatively fast increase of the Earth's radius by factor, which led to the burst of underground oceans to the surface of the Earth and led to the formation of oceans. Standard cosmology predicts a continuous smooth expansion of astrophysical objects. Contrary to this prediction, astrophysical objects do not seem to expand smoothly. In the TGD Universe, the smooth expansion is replaced by rapid jerks and the Cambrian Explosion would be associated with this kind of phase transitions.

2. In this expansion the multicellular photosynthesizing life burst to the surface. This explains the sudden emergence of highly evolved life forms during the Cambrian Explosion that Darwin realized to be a heavy objection against his theory.

3. There are many objections to be circumvented. For instance, how photosynthesis could evolve in the underground ocean. Here TGD views dark matter as $h_{eff} = nh_0$ phases of ordinary matter, which are relatively dark with respect to each other, come in rescue. Dark water blobs could leak into the interior of Earth and the solar light possessing a dark portion could do the same so that photosynthesis became possible [L81].

4. Did Venus experience a similar rapid expansion 200 million years earlier, about 750 million years ago (or maybe roughly at the same time). Venus does not have water at its surface. This can be understood in terms of heat from solar radiation forcing the evaporation of water and subsequent loss. This also prevented the leakage of the water to the interior of Venus. If there were no water reservoirs inside Venus, no oceans were formed. The cracks of the crust created expanding areas of magma, which were like the bottoms of the oceans at Earth. Also at Earth a fraction about 2/3 of the Earth's surface is sea bottom.

11.9.2 Why does Venus not possess a magnetic field?

Venus also offers a second puzzle. Venus does not have an appreciable magnetic field although it has been speculated that it has had it (`https://cutt.ly/VQSZt9m`). The solar dynamo mechanism would suggest its presence.

1. TGD predicts that there are two kinds of flux tubes carrying Earth's magnetic field B_E with a nominal value of .5 Gauss. This applies quite generally. The flux tubes have a closed cross section - this is possible only in TGD Universe, where the space-time is 4-surface in $M^4 \times CP_2$. The flux tubes can have a vanishing Kähler magnetic flux or non-vanishing quantized monopole flux: this has no counterpart in Maxwellian electrodynamics.

   For Earth, the monopole part would correspond to about .2 Gauss - 2/5 of the full strength of B_E.

2. Monopole part needs no currents to maintain it and this makes it possible to understand how the Earth's magnetic field has not disappeared a long time ago. This also explains the existence of magnetic fields in cosmological scales.

   The orientation of the Earth's magnetic field is varying. In the TGD based model the monopole part plays the role of master. When the non-monopole part becomes too weak, the magnetic body defined by the monopole part changes its orientation. This induced currents refresh the non-monopole part [L13]. The standard dynamo model is part of this model.

3. There is an interesting (perhaps more than) analogy with the standard phenomenological description of magnetism in condensed matter. One has $B = H + M$. H field is analogous to the monopole part and the non-monopole part is analogous to the magnetization M induced by H. $B = H + M$ would represent the total field. If this description corresponds to the presence of two kinds of flux tubes, the TGD view about magnetic fields would have been part of electromagnetism from the beginning!

   Flux tubes can also carry electric fields and also for them this kind of decomposition makes sense. Could also the fields D, P, and E have a similar interpretation?

   In the linear model of magnetism, one has $M = \chi H$ and $B = \mu H = (1+\chi)H$. For diamagnets one has $\chi \leq 0$ and for paramagnets $\chi \geq 0$. Earth would be paramagnetic with $\chi \simeq 3/2$ if the linear model works. χ is a tensor in the general case so that B and H can have different directions.

4. All stars and planets, also Venus, correspond to flux tube tangles formed from monopole flux tubes. This leaves only one possibility. Venus behaves like a super-conductor and is an ideal diamagnet with $\chi = -1$ so that B vanishes. The monopole part would be present however.

   This could provide a totally new insight to the Meissner effect and loss of superconductivity. In TGD the based model [L73], monopole flux tubes carry supracurrent. The BCS model however requires the absence of a magnetic field. Could the induced non-monopole field cancelling the monopole part. Venus would indeed be a superconductor!

5. The TGD based model of superconductivity [L73] also predicts superconductivity driven by an external energy feed would be also above critical temperature. The energy feed would increase the value of h_{eff} and below the critical temperature it would be provided by the energy liberated in the formation of Cooper pairs, which need not actually be the current carriers since dark electrons can carry the current without dissipation. In TGD inspired biology and quite universally, the basic role of metabolic energy feed is to prevent the reductions of the values of h_{eff}.

   Superconductivity means in the TGD framework large h_{eff} and therefore complexity, intelligence, and long quantum coherence length [L94]. Could Venus be alive but in a very different sense than Earth?

6. Could the superconductivity be forced by the thermal energy feed from the interior of Venus? The tilt of the rotation axis relative to the plane of rotation around the Sun is very small for Venus, about 3 degrees and much smaller than for the Earth. This implies that the surface temperature of Venus is roughly constant. At Earth plate tectonics makes possible the heat transfer from the interior to the surface and its leakage to outer space. For Venus this is not possible. Could the energy flow from the interior of Venus force the superconductivity by increasing the values of h_{eff}. This would in turn force the vanishing of the magnetic field of Venus.

7. Sun has an enormous feed of metabolic energy from the core: could it be alive? Also in the case of Earth, the energy feed from the interior could have been crucial for the development of life in the interior of Earth and made possible even the development of photosynthesis.

   The possibility that life actually appears in cosmic scales and is associated with quantum coherent flux tube networks associated with the active galactic nuclei usually identified as supermassive blackholes containing stellar and planetary systems as tangles is suggested by the TGD based model of galactic jets [L79] explaining also ultrahigh energy cosmic rays. The model inspires the proposal that active galactic nuclei having typically sizes 1-2 AU (!) involve gravitationally quantum coherent regions of radius at most of the Schwartschild radius defining a minimal gravitational Compton length [L79].

8. Also Mars lacks the global magnetic field although it has auroras assigned with local fields. Could also Mars be alive in the same same sense as Venus? Note that the recent radius of Mars is about 1/2 of Earth's radius. If Venus expanded by factor 2, all these 3 planets would have had roughly the same radius for about 750 million years ago. Mars would be waiting for the moment of expansion.

11.9.3 Could superionic phase of water give rise to planetary superconductivity and Meissner effect?

A superionic ice-like phase of water at high temperature and pressure (20 GPa but much less than the expected pressure, which is higher than 50 GPa) has been discovered. Inside Earth, 20-25 GPa pressure exists in the transition zone between upper and lower mantle. The new phases, bcc and fcc cubic lattices emerge at T=2000 K. See the popular article "*Scientists find strange black 'superionic ice' that could exist inside other planets*" (`https://www.eurekalert.org/news-releases/933099`) and the article "*Structure and properties of two superionic ice phases*" of Prakapenka *et al* [D10] (`https://cutt.ly/7TPvYlL`).

The bonds between hydrogen atoms and oxygen ions are broken in this phase and ionized hydrogen atoms form a fluid, a kind of proton ocean in which the oxygen lattice floats.

In the TGD framework dark proton sequences with effective Planck constant $h_{eff} \geq h$ at monopole magnetic tubes play a key role in quantum biology. Dark DNA codons would be 3-proton triplets at monopole flux tubes parallel to DNA strands and would give rise to a fundamental realization of the genetic code.

One can wonder whether the protons of this superionic could be dark in the TGD sense and reside in monopole flux tubes. Could they form a superfluid-like or superconductor-like phase by a universal mechanism which I call Galois confinement, which requires that the total momenta of composites of dark protons with are algebraic integer valued momenta are ordinary integers in suitable units (periodic boundary conditions) [L77, L78].

It is conjectured that this kind phase could reside in the interiors of Neptune and Uranus perhaps even deep inside the Earth. Could superionic phases of water in the interior of planets like Mars and Venus give rise to the speculated super-conductivity implying the vanishing of large scale magnetic field via the TGD variant of the Meissner effect?

Could superionic ice appear in the interior of Earth? Could one consider the following scenario?

Primordial Earth had a vanishing magnetic field by the Meissner effect caused by superionic ice. Part of the superconducting superionic water melted and formed ordinary water at lower temperature and pressure and gave rise to underground oceans. Superconductivity was lost in the Earth scale but the monopole flux based magnetic field and the ordinary magnetic field induced by the currents that it generated remained but did not cancel each other anymore. In the transition increasing the radius of Earth by factor 2 during the Cambrian explosion the water in these oceans bursted to the surface of Earth.

Earthquakes that should not occur

There is an interesting finding, which seems to relate to the superionic ice. It has been discovered that there are earthquakes much deeper in the interior of Earth than expected (`https://cutt.ly/VTSEe5j`). These earthquakes are in the transition zone between upper and lower mantle and (the depth range 410-620 km) even below it (750 km). The pressure range is 20-25 GPa. The temperature at the base of the transition zone is estimated to be about 1900 K (`https://cutt.ly/jTSWxbA`). This parameter range inspires the question whether superionic could emerge at the base of the transition zone and whether the appearance of hydrogen as liquid in pores could make possible the earthquakes below the transition zone just as the presence of ordinary liquid in pores is believed to make them possible above the transition zone.

In the crust above 20 km depth the rocks are cold and brittle and prone to breaking and most earthquakes occur in this region. At deeper the rocks deform under high pressures and no breaking occurs. Deeper in the crust the matter is hotter and pressure higher and breaking does not occur easily.

Around a depth of 400 km, just above the transition zone, the upper mantle of the rock consists of olivine, which is brittle. In the transition zone olivine is believed to transform to wadsleyite and at deeper depth ringwoodite. At 680 km, where the upper mantle ends, ringwoodite would transform to bridgmanite and periclase. The higher pressure phases are analogous to graphite, which deforms easily under pressure and does not break whereas olivine is analogous to diamond and is brittle.

One can understand the earthquakes down to 400 km near the upper boundary of the transition zone in terms of the model in which water in the proposed upper mantle is pushed away from the pores by pressure, which leads to breaking. Below this depth water is believed to be totally squeezed out from the pores so that mechanism does not work. The deepest reported earthquake occurs at a depth 750 km and looks mysterious. There are several proposals for its origin.

The area of Bonin island is a subduction zone and it has been proposed that the boundary between upper and lower mantle is at a larger depth than thought. The cold Earth crust could allow a lower temperature so that matter would remain brittle since the transition to high pressure forms of rock would not occur. Another proposal is that the region considered is not homogenous and different forms of rock are present. Even direct transition of olivine to ringwoodite is possible and it has been suggested that this could make the earthquakes possible.

Could superionic ice and earthquakes relate?

TGD allows us to consider the situation from a new perspective by bringing in the notions of magnetic flux tubes carrying dark matter. Also the zero energy ontology (ZEO) might be highly relevant. The following represents innocent and naive questions of a layman at the general level.

1. ZEO inspires the proposal that earthquakes correspond to "big" state function reductions (BSFRs) in which the arrow of time at the magnetic body of the system changes. This would explain the generation of ELF radiation before the earthquake although one would expect it after the earthquake [L56].

   The BSFRs would occur at quantum criticality and the question is what this quantum criticality corresponds to. Could the BSFR correspond to the occurrence of a phase transition in which the superionic ice becomes ordinary water? If this is the case, the transition zone, and also a region below it, would be near quantum criticality and prone to earthquakes.

2. The dark magnetic flux tubes are 1-D objects and possess Hagedorn temperature T_H as a limiting temperature. The heat capacity increases without limit as T_H is approached. Could a considerable part of thermal energy go to the flux tube degrees of freedom so that the temperature of the ordinary matter would remain lower than expected and the material could remain in a brittle olivine form.

3. Could the energy liberated in the earthquake correspond to the dark magnetic energy (for large enough value of h_{eff} assignable to gravitational magnetic flux tubes) assignable to the flux tubes rather than to the elastic energy of the rock material? Could the liberated energy be dark energy liberated as h_{eff} decreases and flux tubes suddenly shorten? Could this correspond to a phase transition in which superionic ice transforms to an ordinary phase of water?

One can also ask more concrete questions.

1. Suppose that water below the transition zone (P$\geq$ 20 GPa and T $\geq$ 1900 K) can exist in superionic ice containing hydrogen ions in liquid form. Could the high pressure force the superionic liquid out from the pores and induce the breaking?

2. In the range 350-655 km, the temperature varies in the range 1700-1900 K (`https://cutt.ly/jTSWxbA`). The temperature at the top of transition zones would be slightly above 1700 K. Could regions of superionic ice appear already at 1700 K, which is below T=2000 K?

3. Could the transition zone be at criticality against the phase transition to superionic water? This idea would conform with the proposal that the region in question is not homogenous.

Chapter i

Appendix

A-1 Introduction

Originally this appendix was meant to be a purely technical summary of basic facts but in its recent form it tries to briefly summarize those basic visions about TGD which I dare to regarded stabilized. I have added illustrations making it easier to build mental images about what is involved and represented briefly the key arguments. This chapter is hoped to help the reader to get fast grasp about the concepts of TGD.

The basic properties of embedding space and related spaces are discussed and the relationship of CP_2 to the standard model is summarized. The basic vision is simple: the geometry of the embedding space $H = M^4 \times CP_2$ geometrizes standard model symmetries and quantum numbers. The assumption that space-time surfaces are basic objects, brings in dynamics as dynamics of 3-D surfaces based on the induced geometry. Second quantization of free spinor fields of H induces quantization at the level of H, which means a dramatic simplification.

The notions of induction of metric and spinor connection, and of spinor structure are discussed. Many-sheeted space-time and related notions such as topological field quantization and the relationship many-sheeted space-time to that of GRT space-time are discussed as well as the recent view about induced spinor fields and the emergence of fermionic strings. Also the relationship to string models is discussed briefly.

Various topics related to p-adic numbers are summarized with a brief definition of p-adic manifold and the idea about generalization of the number concept by gluing real and p-adic number fields to a larger book like structure analogous to adele [L29, L28]. In the recent view of quantum TGD [L88], both notions reduce to physics as number theory vision, which relies on $M^8 - H$ duality [L64, L65] and is complementary to the physics as geometry vision.

Zero energy ontology (ZEO) [L61] [K107] has become a central part of quantum TGD and leads to a TGD inspired theory of consciousness as a generalization of quantum measurement theory having quantum biology as an application. Also these aspects of TGD are briefly discussed.

A-2 Embedding space $M^4 \times CP_2$

Space-times are regarded as 4-surfaces in $H = M^4 \times CP_2$ the Cartesian product of empty Minkowski space - the space-time of special relativity - and compact 4-D space CP_2 with size scale of order 10^4 Planck lengths. One can say that embedding space is obtained by replacing each point m of empty Minkowski space with 4-D tiny CP_2. The space-time of general relativity is replaced by a 4-D surface in H which has very complex topology. The notion of many-sheeted space-time gives an idea about what is involved.

Fig. 1. Embedding space $H = M^4 \times CP_2$ as Cartesian product of Minkowski space M^4 and complex projective space CP_2. `http://tgdtheory.fi/appfigures/Hoo.jpg`

Denote by M^4_+ and M^4_- the future and past directed lightcones of M^4. Denote their intersection, which is not unique, by CD. In zero energy ontology (ZEO) [L61, L76] [K107] causal diamond

(CD) is defined as cartesian product $CD \times CP_2$. Often I use CD to refer just to $CD \times CP_2$ since CP_2 factor is relevant from the point of view of ZEO.

Fig. 2. Future and past light-cones M^4_+ and M^4_-. Causal diamonds (CD) are defined as their intersections. `http://tgdtheory.fi/appfigures/futurepast.jpg`

Fig. 3. Causal diamond (CD) is highly analogous to Penrose diagram but simpler. `http://tgdtheory.fi/appfigures/penrose.jpg`

A rather recent discovery was that CP_2 is the only compact 4-manifold with Euclidian signature of metric allowing twistor space with Kähler structure. M^4 is in turn is the only 4-D space with Minkowskian signature of metric allowing twistor space with Kähler structure [A35] so that $H = M^4 \times CP_2$ is twistorially unique.

One can loosely say that quantum states in a given sector of "world of classical worlds" (WCW) are superpositions of space-time surfaces inside CDs and that positive and negative energy parts of zero energy states are localized and past and future boundaries of CDs. CDs form a hierarchy. One can have CDs within CDs and CDs can also overlap. The size of CD is characterized by the proper time distance between its two tips. One can perform both translations and also Lorentz boosts of CD leaving either boundary invariant. Therefore one can assign to CDs a moduli space and speak about wave function in this moduli space.

In number theoretic approach it is natural to restrict the allowed Lorentz boosts to some discrete subgroup of Lorentz group and also the distances between the tips of CDs to multiples of CP_2 radius defined by the length of its geodesic. Therefore the moduli space of CDs discretizes. The quantization of cosmic recession velocities for which there are indications, could relate to this quantization.

A-2.1 Basic facts about CP_2

CP_2 as a four-manifold is very special. The following arguments demonstrate that it codes for the symmetries of standard models via its isometries and holonomies.

CP_2 as a manifold

CP_2, the complex projective space of two complex dimensions, is obtained by identifying the points of complex 3-space C^3 under the projective equivalence

$$(z^1, z^2, z^3) \equiv \lambda(z^1, z^2, z^3) \ . \tag{A-2.1}$$

Here λ is any non-zero complex number. Note that CP_2 can be also regarded as the coset space $SU(3)/U(2)$. The pair z^i/z^j for fixed j and $z^i \neq 0$ defines a complex coordinate chart for CP_2. As j runs from 1 to 3 one obtains an atlas of three coordinate charts covering CP_2, the charts being holomorphically related to each other (e.g. CP_2 is a complex manifold). The points $z^3 \neq 0$ form a subset of CP_2 homoeomorphic to R^4 and the points with $z^3 = 0$ a set homeomorphic to S^2. Therefore CP_2 is obtained by "adding the 2-sphere at infinity to R^4".

Besides the standard complex coordinates $\xi^i = z^i/z^3$, $i = 1, 2$ the coordinates of Eguchi and Freund [A30] will be used and their relation to the complex coordinates is given by

$$\begin{aligned} \xi^1 &= z + it \ , \\ \xi^2 &= x + iy \ . \end{aligned} \tag{A-2.2}$$

These are related to the "spherical coordinates" via the equations

$$\begin{aligned} \xi^1 &= rexp(i\frac{(\Psi + \Phi)}{2})cos(\frac{\Theta}{2}) \ , \\ \xi^2 &= rexp(i\frac{(\Psi - \Phi)}{2})sin(\frac{\Theta}{2}) \ . \end{aligned} \tag{A-2.3}$$

The ranges of the variables r, Θ, Φ, Ψ are $[0, \infty], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively.

Considered as a real four-manifold CP_2 is compact and simply connected, with Euler number Euler number 3, Pontryagin number 3 and second $b = 1$.

Fig. 4. CP_2 as manifold. `http://tgdtheory.fi/appfigures/cp2.jpg`

Metric and Kähler structure of CP_2

In order to obtain a natural metric for CP_2, observe that CP_2 can be thought of as a set of the orbits of the isometries $z^i \rightarrow exp(i\alpha)z^i$ on the sphere S^5: $\sum z^i\bar{z}^i = R^2$. The metric of CP_2 is obtained by projecting the metric of S^5 orthogonally to the orbits of the isometries. Therefore the distance between the points of CP_2 is that between the representative orbits on S^5.

The line element has the following form in the complex coordinates

$$ds^2 = g_{a\bar{b}}d\xi^a d\bar{\xi}^b \ , \tag{A-2.4}$$

where the Hermitian, in fact Kähler metric $g_{a\bar{b}}$ is defined by

$$g_{a\bar{b}} = R^2\partial_a\partial_{\bar{b}}K \ , \tag{A-2.5}$$

where the function K, Kähler function, is defined as

$$\begin{aligned} K &= log(F) \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-2.6}$$

The Kähler function for S^2 has the same form. It gives the S^2 metric $dzd\bar{z}/(1+r^2)^2$ related to its standard form in spherical coordinates by the coordinate transformation $(r, \phi) = (tan(\theta/2), \phi)$.

The representation of the CP_2 metric is deducible from S^5 metric is obtained by putting the angle coordinate of a geodesic sphere constant in it and is given

$$\frac{ds^2}{R^2} = \frac{(dr^2 + r^2\sigma_3^2)}{F^2} + \frac{r^2(\sigma_1^2 + \sigma_2^2)}{F} \ , \tag{A-2.7}$$

where the quantities σ_i are defined as

$$\begin{aligned} r^2\sigma_1 &= Im(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_2 &= -Re(\xi^1 d\xi^2 - \xi^2 d\xi^1) \ , \\ r^2\sigma_3 &= -Im(\xi^1 d\bar{\xi}^1 + \xi^2 d\bar{\xi}^2) \ . \end{aligned} \tag{A-2.8}$$

R denotes the radius of the geodesic circle of CP_2. The vierbein forms, which satisfy the defining relation

$$s_{kl} = R^2 \sum_A e_k^A e_l^A \ , \tag{A-2.9}$$

are given by

$$\begin{aligned} e^0 &= \frac{dr}{F} \ , \quad e^1 = \frac{r\sigma_1}{\sqrt{F}} \ , \\ e^2 &= \frac{r\sigma_2}{\sqrt{F}} \ , \quad e^3 = \frac{r\sigma_3}{F} \ . \end{aligned} \tag{A-2.10}$$

The explicit representations of vierbein vectors are given by

$$\begin{array}{rclrcl} e^0 & = & \frac{dr}{F} \ , & e^1 & = & \frac{r(sin\Theta cos\Psi d\Phi + sin\Psi d\Theta)}{2\sqrt{F}} \ , \\ e^2 & = & \frac{r(sin\Theta sin\Psi d\Phi - cos\Psi d\Theta)}{2\sqrt{F}} \ , & e^3 & = & \frac{r(d\Psi + cos\Theta d\Phi)}{2F} \ . \end{array} \tag{A-2.11}$$

The explicit representation of the line element is given by the expression

$$ds^2/R^2 = \frac{dr^2}{F^2} + \frac{r^2}{4F^2}(d\Psi + cos\Theta d\Phi)^2 + \frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2) \ . \tag{A-2.12}$$

From this expression one finds that at coordinate infinity $r = \infty$ line element reduces to $\frac{r^2}{4F}(d\Theta^2 + sin^2\Theta d\Phi^2)$ of S^2 meaning that 3-sphere degenerates metrically to 2-sphere and one can say that CP_2 is obtained by adding to R^4 a 2-sphere at infinity.

The vierbein connection satisfying the defining relation

$$de^A = -V^A_B \wedge e^B \ , \tag{A-2.13}$$

is given by

$$\begin{array}{rclrcl} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r - \frac{1}{r})e^3 \ , & V_{12} & = & (2r + \frac{1}{r})e^3 \ . \end{array} \tag{A-2.14}$$

The representation of the covariantly constant curvature tensor is given by

$$\begin{array}{rclrcl} R_{01} & = & e^0 \wedge e^1 - e^2 \wedge e^3 \ , & R_{23} & = & e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ R_{02} & = & e^0 \wedge e^2 - e^3 \wedge e^1 \ , & R_{31} & = & -e^0 \wedge e^2 + e^3 \wedge e^1 \ , \\ R_{03} & = & 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ , & R_{12} & = & 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ . \end{array} \tag{A-2.15}$$

Metric defines a real, covariantly constant, and therefore closed 2-form J

$$J = -is_{a\bar{b}} d\xi^a d\bar{\xi}^b \ , \tag{A-2.16}$$

the so called Kähler form. Kähler form J defines in CP_2 a symplectic structure because it satisfies the condition

$$J^k_{\ r} J^{rl} = -s^{kl} \ . \tag{A-2.17}$$

The condition states that J and g give representations of real unit and imaginary units related by the formula $i^2 = -1$.

Kähler form is expressible locally in terms of Kähler gauge potential

$$J = dB \ , \tag{A-2.18}$$

where B is the so called Kähler potential, which is not defined globally since J describes homological magnetic monopole.

$dJ = ddB = 0$ gives the topological half of Maxwell equations (vanishing of magnetic charges and Faraday's induction law) and self-duality $*J = J$ reduces the remaining equations to $dJ = 0$. Hence the Kähler form can be regarded as a curvature form of a $U(1)$ gauge potential B carrying a magnetic charge of unit $1/2g$ (g denotes the gauge coupling).

The magnetic flux of J through a 2-surface in CP_2 is proportional to its homology equivalence class, which is integer valued. The explicit representations of J and B are given by

$$\begin{aligned} B &= 2re^3 \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) = \frac{r}{F^2} dr \wedge (d\Psi + cos\Theta d\Phi) + \frac{r^2}{2F} sin\Theta d\Theta \wedge d\Phi \ . \end{aligned} \tag{A-2.19}$$

The vierbein curvature form and Kähler form are covariantly constant and have in the complex coordinates only components of type (1, 1).

Useful coordinates for CP_2 are the so called canonical (or symplectic or Darboux) coordinates in which the Kähler potential and Kähler form have very simple expressions

$$\begin{aligned} B &= \sum_{k=1,2} P_k dQ_k \ , \\ J &= \sum_{k=1,2} dP_k \wedge dQ_k \ . \end{aligned} \tag{A-2.20}$$

The relationship of the canonical coordinates to the "spherical" coordinates is given by the equations

$$\begin{aligned} P_1 &= -\frac{1}{1+r^2} \ , \\ P_2 &= -\frac{r^2 cos\Theta}{2(1+r^2)} \ , \\ Q_1 &= \Psi \ , \\ Q_2 &= \Phi \ . \end{aligned} \tag{A-2.21}$$

Spinors In CP_2

CP_2 doesn't allow spinor structure in the conventional sense [A25]. However, the coupling of the spinors to a half odd multiple of the Kähler potential leads to a respectable spinor structure. Because the delicacies associated with the spinor structure of CP_2 play a fundamental role in TGD, the arguments of Hawking are repeated here.

To see how the space can fail to have an ordinary spinor structure consider the parallel transport of the vierbein in a simply connected space M. The parallel propagation around a closed curve with a base point x leads to a rotated vierbein at x: $e^A = R^A_B e^B$ and one can associate to each closed path an element of $SO(4)$.

Consider now a one-parameter family of closed curves $\gamma(v) : v \in (0, 1)$ with the same base point x and $\gamma(0)$ and $\gamma(1)$ trivial paths. Clearly these paths define a sphere S^2 in M and the element $R^A_B(v)$ defines a closed path in $SO(4)$. When the sphere S^2 is contractible to a point e.g., homologically trivial, the path in $SO(4)$ is also contractible to a point and therefore represents a trivial element of the homotopy group $\Pi_1(SO(4)) = Z_2$.

For a homologically nontrivial 2-surface S^2 the associated path in $SO(4)$ can be homotopically nontrivial and therefore corresponds to a nonclosed path in the covering group Spin(4) (leading from the matrix 1 to -1 in the matrix representation). Assume this is the case.

Assume now that the space allows spinor structure. Then one can parallel propagate also spinors and by the above construction associate a closed path of Spin(4) to the surface S^2. Now, however this path corresponds to a lift of the corresponding $SO(4)$ path and cannot be closed. Thus one ends up with a contradiction.

From the preceding argument it is clear that one could compensate the non-allowed -1-factor associated with the parallel transport of the spinor around the sphere S^2 by coupling it to a gauge potential in such a way that in the parallel transport the gauge potential introduces a compensating -1-factor. For a $U(1)$ gauge potential this factor is given by the exponential

$exp(i2\Phi)$, where Φ is the magnetic flux through the surface. This factor has the value -1 provided the $U(1)$ potential carries half odd multiple of Dirac charge $1/2g$. In case of CP_2 the required gauge potential is half odd multiple of the Kähler potential B defined previously. In the case of $M^4 \times CP_2$ one can in addition couple the spinor components with different chiralities independently to an odd multiple of $B/2$.

Geodesic sub-manifolds of CP_2

Geodesic sub-manifolds are defined as sub-manifolds having common geodesic lines with the embedding space. As a consequence the second fundamental form of the geodesic manifold vanishes, which means that the tangent vectors h^k_α (understood as vectors of H) are covariantly constant quantities with respect to the covariant derivative taking into account that the tangent vectors are vectors both with respect to H and X^4.

In [A43] a general characterization of the geodesic sub-manifolds for an arbitrary symmetric space G/H is given. Geodesic sub-manifolds are in 1-1-correspondence with the so called Lie triple systems of the Lie-algebra g of the group G. The Lie triple system t is defined as a subspace of g characterized by the closedness property with respect to double commutation

$$[X,[Y,Z]] \in t \ \text{ for } \ X,Y,Z \in t \ . \qquad \text{(A-2.22)}$$

$SU(3)$ allows, besides geodesic lines, two nonequivalent (not isometry related) geodesic spheres. This is understood by observing that $SU(3)$ allows two nonequivalent $SU(2)$ algebras corresponding to subgroups $SO(3)$ (orthogonal 3×3 matrices) and the usual isospin group $SU(2)$. By taking any subset of two generators from these algebras, one obtains a Lie triple system and by exponentiating this system, one obtains a 2-dimensional geodesic sub-manifold of CP_2.

Standard representatives for the geodesic spheres of CP_2 are given by the equations

$$S^2_I \ : \ \xi^1 = \bar{\xi}^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Psi = 0) \ ,$$

$$S^2_{II} \ : \ \xi^1 = \xi^2 \ \text{ or equivalently } \ (\Theta = \pi/2, \Phi = 0) \ .$$

The non-equivalence of these sub-manifolds is clear from the fact that isometries act as holomorphic transformations in CP_2. The vanishing of the second fundamental form is also easy to verify. The first geodesic manifold is homologically trivial: in fact, the induced Kähler form vanishes identically for S^2_I. S^2_{II} is homologically nontrivial and the flux of the Kähler form gives its homology equivalence class.

A-2.2 CP_2 geometry and Standard Model symmetries

Identification of the electro-weak couplings

The delicacies of the spinor structure of CP_2 make it a unique candidate for space S. First, the coupling of the spinors to the $U(1)$ gauge potential defined by the Kähler structure provides the missing $U(1)$ factor in the gauge group. Secondly, it is possible to couple different H-chiralities independently to a half odd multiple of the Kähler potential. Thus the hopes of obtaining a correct spectrum for the electromagnetic charge are considerable. In the following it will be demonstrated that the couplings of the induced spinor connection are indeed those of the GWS model [B32] and in particular that the right handed neutrinos decouple completely from the electro-weak interactions.

To begin with, recall that the space H allows to define three different chiralities for spinors. Spinors with fixed H-chirality $e = \pm 1$, CP_2-chirality l, r and M^4-chirality L, R are defined by the condition

$$\begin{aligned} \Gamma\Psi &= e\Psi \ , \\ e &= \pm 1 \ , \end{aligned} \qquad \text{(A-2.23)}$$

where Γ denotes the matrix $\Gamma_9 = \gamma_5 \otimes \gamma_5$, $1 \otimes \gamma_5$ and $\gamma_5 \otimes 1$ respectively. Clearly, for a fixed H-chirality CP_2- and M^4-chiralities are correlated.

The spinors with H-chirality $e = \pm 1$ can be identified as quark and lepton like spinors respectively. The separate conservation of baryon and lepton numbers can be understood as a consequence of generalized chiral invariance if this identification is accepted. For the spinors with a definite H-chirality one can identify the vielbein group of CP_2 as the electro-weak group: $SO(4)$ having as its covering group $SU(2)_L \times SU(2)_R$.

The covariant derivatives are defined by the spinorial connection

$$A = V + \frac{B}{2}(n_+ 1_+ + n_- 1_-) \ . \tag{A-2.24}$$

Here V and B denote the projections of the vielbein and Kähler gauge potentials respectively and $1_{+(-)}$ projects to the spinor H-chirality $+(-)$. The integers $n_\pm$ are odd from the requirement of a respectable spinor structure.

The explicit representation of the vielbein connection V and of B are given by the equations

$$\begin{array}{lllllll} V_{01} & = & -\frac{e^1}{r} \ , & V_{23} & = & \frac{e^1}{r} \ , \\ V_{02} & = & -\frac{e^2}{r} \ , & V_{31} & = & \frac{e^2}{r} \ , \\ V_{03} & = & (r - \frac{1}{r})e^3 \ , & V_{12} & = & (2r + \frac{1}{r})e^3 \ , \end{array} \tag{A-2.25}$$

and

$$B = 2re^3 \ , \tag{A-2.26}$$

respectively. The explicit representation of the vielbein is not needed here.

Let us first show that the charged part of the spinor connection couples purely left handedly. Identifying Σ_3^0 and Σ_2^1 as the diagonal (neutral) Lie-algebra generators of $SO(4)$, one finds that the charged part of the spinor connection is given by

$$A_{ch} = 2V_{23}I_L^1 + 2V_{13}I_L^2 \ , \tag{A-2.27}$$

where one have defined

$$\begin{array}{lll} I_L^1 & = & \frac{(\Sigma_{01} - \Sigma_{23})}{2} \ , \\ I_L^2 & = & \frac{(\Sigma_{02} - \Sigma_{13})}{2} \ . \end{array} \tag{A-2.28}$$

A_{ch} is clearly left handed so that one can perform the identification of the gauge potential as

$$W^\pm = \frac{2(e^1 \pm ie^2)}{r} \ , \tag{A-2.29}$$

where $W^\pm$ denotes the charged intermediate vector boson.

The covariantly constant curvature tensor is given by

$$\begin{array}{l} R_{01} = -R_{23} = e^0 \wedge e^1 - e^2 \wedge e^3 \ , \\ R_{02} = -R_{31} = e^0 \wedge e^2 - e^3 \wedge e^1 \ , \\ R_{03} = 4e^0 \wedge e^3 + 2e^1 \wedge e^2 \ , \\ R_{12} = 2e^0 \wedge e^3 + 4e^1 \wedge e^2 \ . \end{array} \tag{A-2.30}$$

The charged part of the curvature tensor is left handed.

This is to be compared with the Weyl tensor, which defines a representation of quaternionic imaginary units.

$$
\begin{aligned}
W_{03} = W_{12} &\equiv 2I_3 = 2(e^0\wedge e^3 + e^1\wedge e^2) \ , \\
W_{01} = W_{23} &\equiv I_1 = -e^0\wedge e^1 - e^2\wedge e^3 \ , \\
W_{02} = W_{31} &\equiv I_2 = -e^0\wedge e^2 - e^3\wedge e^1 \ .
\end{aligned} \tag{A-2.31}
$$

The charged part of the Weyl tensor is right-handed and that the relative sign of the two terms in the curvature tensor and Weyl tensor are opposite.

Consider next the identification of the neutral gauge bosons γ and Z^0 as appropriate linear combinations of the two functionally independent quantities

$$
\begin{aligned}
X &= re^3 \ , \\
Y &= \frac{e^3}{r} \ ,
\end{aligned} \tag{A-2.32}
$$

appearing in the neutral part of the spinor connection. We show first that the mere requirement that photon couples vectorially implies the basic coupling structure of the GWS model leaving only the value of Weinberg angle undetermined.

To begin with let us define

$$
\begin{aligned}
\bar{\gamma} &= aX + bY \ , \\
\bar{Z}^0 &= cX + dY \ ,
\end{aligned} \tag{A-2.33}
$$

where the normalization condition

$$ad - bc = 1 \ ,$$

is satisfied. The physical fields γ and Z^0 are related to $\bar{\gamma}$ and $\bar{Z}^0$ by simple normalization factors.

Expressing the neutral part of the spinor connection in term of these fields one obtains

$$
\begin{aligned}
A_{nc} &= [(c+d)2\Sigma_{03} + (2d-c)2\Sigma_{12} + d(n_+1_+ + n_-1_-)]\bar{\gamma} \\
&+ [(a-b)2\Sigma_{03} + (a-2b)2\Sigma_{12} - b(n_+1_+ + n_-1_-)]\bar{Z}^0 \ .
\end{aligned} \tag{A-2.34}
$$

Identifying Σ_{12} and $\Sigma_{03} = 1\times\gamma_5\Sigma_{12}$ as vectorial and axial Lie-algebra generators, respectively, the requirement that γ couples vectorially leads to the condition

$$c = -d \ . \tag{A-2.35}$$

Using this result plus previous equations, one obtains for the neutral part of the connection the expression

$$A_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \tag{A-2.36}$$

Here the electromagnetic charge Q_{em} and the weak isospin are defined by

$$
\begin{aligned}
Q_{em} &= \Sigma^{12} + \frac{(n_+1_+ + n_-1_-)}{6} \ , \\
I_L^3 &= \frac{(\Sigma^{12} - \Sigma^{03})}{2} \ .
\end{aligned} \tag{A-2.37}
$$

The fields γ and Z^0 are defined via the relations

$$
\begin{aligned}
\gamma &= 6d\bar{\gamma} = \frac{6}{(a+b)}(aX+bY) \ , \\
Z^0 &= 4(a+b)\bar{Z}^0 = 4(X-Y) \ .
\end{aligned} \tag{A-2.38}
$$

The value of the Weinberg angle is given by

$$sin^2\theta_W = \frac{3b}{2(a+b)} \ , \quad \text{(A-2.39)}$$

and is not fixed completely. Observe that right handed neutrinos decouple completely from the electro-weak interactions.

The determination of the value of the Weinberg angle is a dynamical problem. The original approach was based on the assumption that it makes sense to talk about electroweak action defined at fundamental level and introduce a symmetry breaking by adding an additional term proportional to Kähler action. The recent view is that Kähler action plus volume term defines the fundamental action.

The Weinberg angle is completely fixed if one requires that the electroweak action contains no cross term of type γZ^0. This leads to a definite value for the Weinberg angle.

One can however add a symmetry breaking term proportional to Kähler action and this changes the value of the Weinberg angle. As a matter fact, color gauge action identifying color gauge field as proportional to $H^A J_{\alpha\beta}$ is proportional to Kähler action. A possible interpretation would be as a sum of electroweak and color gauge interactions.

To evaluate the value of the Weinberg angle one can express the neutral part F_{nc} of the induced gauge field as

$$F_{nc} = 2R_{03}\Sigma^{03} + 2R_{12}\Sigma^{12} + J(n_+ 1_+ + n_- 1_-) \ , \quad \text{(A-2.40)}$$

where one has

$$\begin{aligned} R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\ R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) \ , \\ J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \end{aligned} \quad \text{(A-2.41)}$$

in terms of the fields γ and Z^0 (photon and Z- boson)

$$F_{nc} = \gamma Q_{em} + Z^0(I_L^3 - sin^2\theta_W Q_{em}) \ . \quad \text{(A-2.42)}$$

Evaluating the expressions above, one obtains for γ and Z^0 the expressions

$$\begin{aligned} \gamma &= 3J - sin^2\theta_W R_{12} \ , \\ Z^0 &= 2R_{03} \ . \end{aligned} \quad \text{(A-2.43)}$$

For the Kähler field one obtains

$$J = \frac{1}{3}(\gamma + sin^2\theta_W Z^0) \ . \quad \text{(A-2.44)}$$

Expressing the neutral part of the symmetry broken YM action

$$\begin{aligned} L_{ew} &= L_{sym} + fJ^{\alpha\beta}J_{\alpha\beta} \ , \\ L_{sym} &= \frac{1}{4g^2}Tr(F^{\alpha\beta}F_{\alpha\beta}) \ , \end{aligned} \quad \text{(A-2.45)}$$

where the trace is taken in spinor representation, in terms of γ and Z^0 one obtains for the coefficient X of the γZ^0 cross term (this coefficient must vanish) the expression

$$
\begin{aligned}
X &= -\frac{K}{2g^2}+\frac{fp}{18} \ , \\
K &= Tr\left[Q_{em}(I_L^3 - sin^2\theta_W Q_{em})\right] \ ,
\end{aligned}
\quad \text{(A-2.46)}
$$

This parameter can be calculated by substituting the values of quark and lepton charges and weak isospins.

In the general case the value of the coefficient K is given by

$$
K = \sum_i \left[-\frac{(18+2n_i^2)sin^2\theta_W}{9}\right] \ , \quad \text{(A-2.47)}
$$

where the sum is over the spinor chiralities, which appear as elementary fermions and n_i is the integer describing the coupling of the spinor field to the Kähler potential. The cross term vanishes provided the value of the Weinberg angle is given by

$$
sin^2\theta_W = \frac{9\sum_i 1}{(fg^2+2\sum_i(18+n_i^2))} \ . \quad \text{(A-2.48)}
$$

In the scenario where both leptons and quarks are elementary fermions the value of the Weinberg angle is given by

$$
sin^2\theta_W = \frac{9}{(\frac{fg^2}{2}+28)} \ . \quad \text{(A-2.49)}
$$

The bare value of the Weinberg angle is 9/28 in this scenario, which is not far from the typical value 9/24 of GUTs at high energies [B13]. The experimental value at the scale length scale of the electron can be deduced from the ratio of W and Z boson masses as $sin^2\theta_W = 1-(m_W/m_Z)^2 \simeq .22290$. This ratio and also the weak boson masses depend on the length scale.

If one interprets the additional term proportional to J as color action, one could perhaps interpret the value of Weinberg angle as expressing a connection between strong and weak coupling constant evolution. The limit $f \rightarrow 0$ should correspond to an infinite value of color coupling strength and at this limit one would have $sin^2\theta_W = \frac{9}{28}$ for $f/g^2 \rightarrow 0$. This does not make sense since the Weinberg angle is in the standard model much smaller in QCD scale Λ corresponding roughly to pion mass scale. The Weinberg angle is in principle predicted by the p-adic coupling constant evolution fixed by the number theoretical vision of TGD.

One could however have a sum of electroweak action, correction terms changing the value of Weinberg angle, and color action and coupling constant evolution could be understood in terms of the coupling parameters involved.

Electroweak symmetry breaking

One of the hardest challenges in the development of the TGD based view of weak symmetry breaking was the fact that classical field equations allow space-time surfaces with finite but arbitrarily large size. For a fixed space-time surface, the induced gauge fields, including classical weak fields, are long ranged. On the other hand, the large mass for weak bosons would require a short correlation length. How can one understand this together with the fact that a photon has a long correlation length?

In zero energy ontology quantum states are superpositions of space-time surfaces as analogs of almost unique Bohr orbits of particles identified as 3-D surfaces. For some reason the superposition should be such that the quantum averages of weak gauge boson fields vanish below the weak scale whereas the quantum average of electromagnetic fields is non-vanishing.

This is indeed the case.

1. The supersymplectic symmetries form isometries of the world of classical worlds (WCW) and they act in CP_2 degrees of freedom as symplectic transformations leaving the CP_2 symplectic form J invariant and therefore also its contribution to the electromagnetic field since this part is the same for all space-time surfaces in the superposition of space-time surfaces as a representation of supersymplectic isometry group (as a special case a representation of color group).

2. In TGD, color and electroweak symmetries acting as holonomies are not independent and for the $SU(2)_L$ part of induced spinor connection the symplectic transformations induces $SU(2)_L \times U(1)_R$ gauge transformation. This suggests that the quantum expectations of the induced weak fields over the space-time surfaces vanish above the quantum coherence scale. The averages of W and of the left handed part of Z^0 should therefore vanish.

3. $\langle Z^0 \rangle$ should vanish. For $U(1)_R$ part of Z^0, the action of gauge transformation is trivial in gauge theory. Now however the space-time surface changes under symplectic transformations and this could make the average of the right-handed part of Z^0 vanishing. The vanishing of the average of the axial part of the Z^0 is suggested by the partially conserved axial current hypothesis.

One can formulate this picture quantitatively.

1. The electromagnetic field [L91] contains, besides the induced Kähler form, also the induced curvature form R_{12}, which couples vectorially. Conserved vector current hypothesis suggests that the average of R_{12} is non-vanishing. One can express the neutral part of the induced gauge field in terms of induced spinor curvature and Kähler form J as

$$
\begin{aligned}
R_{03} &= 2(2e^0 \wedge e^3 + e^1 \wedge e^2) = J + 2e^0 \wedge e^3 \ , \\
J &= 2(e^0 \wedge e^3 + e^1 \wedge e^2) \ , \\
R_{12} &= 2(e^0 \wedge e^3 + 2e^1 \wedge e^2) = 3J - 2e^0 \wedge e^3 \ , \qquad \text{(A-2.50)}
\end{aligned}
$$

2. The induced fields γ and Z^0 (photon and Z- boson) can be expressed as

$$
\begin{aligned}
\gamma &= 3J - sin^2\theta_W R_{12} \ , \\
Z^0 &= 2R_{03} = 2(J + 2e^0 \wedge e^3) \qquad \text{(A-2.51)} \\
per. & \qquad \text{(A-2.52)}
\end{aligned}
$$

The condition $\langle Z^0 \rangle = 0$ gives $2\langle e^0 \wedge e^3 \rangle = -2J$ and this in turn gives $\langle R_{12} \rangle = 4J$. The average over γ would be

$$
\langle \gamma \rangle = (3 - 4sin^2\theta_W)J \ .
$$

For $sin^2\theta_W = 3/4 \ langle\gamma\rangle$ would vanish.

The quantum averages of classical weak fields quite generally vanish. What about correlation functions?

1. One expects that the correlators of classical weak fields as color invariants, and perhaps even symplectic invariants, are non-vanishing below the Compton length since in this kind of situation the points in the correlation function belong to the same 3-surface representing particle, such as hadron.

2. The intuitive picture is that in longer length scales one has disjoint 3-surfaces with a size scale of Compton length. If the states associated with two disjoint 3-surfaces are separately color invariant there are no correlations in color degrees of freedom and correlators reduce to the products of expectations of classical weak fields and vanish. This could also hold when the 3-surfaces are connected by flux tube bonds.

   Below the Compton length weak bosons would thus behave as correlated massless fields. The Compton lengths of weak bosons are proportional to the value of effective Planck constant h_{eff} and in living systems the Compton lengths are proposed to be even of the order of cell size. This would explain the mysterious chiral selection in living systems requiring large parity violation.

3. What about the averages and correlators of color gauge fields? Classical color gauge fields are proportional to the products of Hamiltonians of color isometries induced Kähler form and the expectations of color Hamiltonians give vanishing average above Compton length and therefore vanishing average. Correlators are non-vanishing below the hadron scale. Gluons do not propagate in long scales for the same reason as weak bosons. This is implied by color confinement, which has also classical description in the sense that 3-surfaces have necessarily a finite size.

   A large value of h_{eff} allows colored states even in biological scales below the Compton length since in this kind of situation the points in the correlation function belong to the same 3-surface representing particle, such as dark hadron.

Discrete symmetries

The treatment of discrete symmetries C, P, and T is based on the following requirements:

1. Symmetries must be realized as purely geometric transformations.

2. Transformation properties of the field variables should be essentially the same as in the conventional quantum field theories [B15] .

The action of the reflection P on spinors of is given by

$$\Psi \quad \rightarrow \quad P\Psi = \gamma^0 \otimes \gamma^0 \Psi \ . \qquad \text{(A-2.53)}$$

in the representation of the gamma matrices for which γ^0 is diagonal. It should be noticed that W and Z^0 bosons break parity symmetry as they should since their charge matrices do not commute with the matrix of P.

The guess that a complex conjugation in CP_2 is associated with T transformation of the physicist turns out to be correct. One can verify by a direct calculation that pure Dirac action is invariant under T realized according to

$$\begin{aligned} m^k &\rightarrow T(M^k) \ , \\ \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \gamma^1\gamma^3 \otimes 1\Psi \ . \end{aligned} \qquad \text{(A-2.54)}$$

The operation bearing closest resemblance to the ordinary charge conjugation corresponds geometrically to complex conjugation in CP_2:

$$\begin{aligned} \xi^k &\rightarrow \bar{\xi}^k \ , \\ \Psi &\rightarrow \Psi^\dagger\gamma^2\gamma^0 \otimes 1 \ . \end{aligned} \qquad \text{(A-2.55)}$$

As one might have expected symmetries CP and T are exact symmetries of the pure Dirac action.

A-3 Induction procedure and many-sheeted space-time

Since the classical gauge fields are closely related in TGD framework, it is not possible to have space-time sheets carrying only single kind of gauge field. For instance, em fields are accompanied by Z^0 fields for extremals of Kähler action.

Classical em fields are always accompanied by Z^0 field and some components of color gauge field. For extremals having homologically non-trivial sphere as a CP_2 projection em and Z^0 fields are the only non-vanishing electroweak gauge fields. For homologically trivial sphere only W fields are non-vanishing. Color rotations does not affect the situation.

For vacuum extremals all electro-weak gauge fields are in general non-vanishing although the net gauge field has U(1) holonomy by 2-dimensionality of the CP_2 projection. Color gauge field has $U(1)$ holonomy for all space-time surfaces and quantum classical correspondence suggest a weak form of color confinement meaning that physical states correspond to color neutral members of color multiplets.

A-3.1 Induction procedure for gauge fields and spinor connection

Induction procedure for gauge potentials and spinor structure is a standard procedure of bundle theory. If one has embedding of some manifold to the base space of a bundle, the bundle structure can be induced so that it has as a base space the imbedded manifold, whose points have as fiber the fiber if embedding space at their image points. In the recent case the embedding of space-time surface to embedding space defines the induction procedure. The induced gauge potentials and gauge fields are projections of the spinor connection of the embedding space to the space-time surface (see `http://tgdtheory.fi/appfigures/induct.jpg`).

Induction procedure makes sense also for the spinor fields of embedding space and one obtains geometrization of both electroweak gauge potentials and of spinors. The new element is induction of gamma matrices which gives their projections at space-time surface.

As a matter fact, the induced gamma matrices cannot appear in the counterpart of massless Dirac equation. To achieve super-symmetry, Dirac action must be replaced with Kähler-Dirac action for which gamma matrices are contractions of the canonical momentum currents of Kähler action with embedding space gamma matrices. Induced gamma matrices in Dirac action would correspond to 4-volume as action.

Fig. 9. Induction of spinor connection and metric as projection to the space-time surface. `http://tgdtheory.fi/appfigures/induct.jpg`.

A-3.2 Induced gauge fields for space-times for which CP_2 projection is a geodesic sphere

If one requires that space-time surface is an extremal of Kähler action and has a 2-dimensional CP_2 projection, only vacuum extremals and space-time surfaces for which CP_2 projection is a geodesic sphere, are allowed. Homologically non-trivial geodesic sphere correspond to vanishing W fields and homologically non-trivial sphere to non-vanishing W fields but vanishing γ and Z^0. This can be verified by explicit examples.

$r=\infty$ surface gives rise to a homologically non-trivial geodesic sphere for which e_0 and e_3 vanish imply the vanishing of W field. For space-time sheets for which CP_2 projection is $r=\infty$ homologically non-trivial geodesic sphere of CP_2 one has

$$\gamma = (\frac{3}{4} - \frac{sin^2(\theta_W)}{2})Z^0 \simeq \frac{5Z^0}{8} \ .$$

The induced W fields vanish in this case and they vanish also for all geodesic sphere obtained by $SU(3)$ rotation.

$Im(\xi^1) = Im(\xi^2) = 0$ corresponds to homologically trivial geodesic sphere. A more general representative is obtained by using for the phase angles of standard complex CP_2 coordinates constant values. In this case e^1 and e^3 vanish so that the induced em, Z^0, and Kähler fields vanish but induced W fields are non-vanishing. This holds also for surfaces obtained by color rotation. Hence one can say that for non-vacuum extremals with 2-D CP_2 projection color rotations and weak symmetries commute.

A-3.3 Many-sheeted space-time

TGD space-time is many-sheeted: in other words, there are in general several space-sheets which have projection to the same M^4 region. Second manner to say this is that CP_2 coordinates are many-valued functions of M^4 coordinates. The original physical interpretation of many-sheeted space-time time was not correct: it was assumed that single sheet corresponds to GRT space-time and this obviously leads to difficulties since the induced gauge fields are expressible in terms of only four embedding space coordinates.

Fig. 10. Illustration of many-sheeted space-time of TGD. `http://tgdtheory.fi/appfigures/manysheeted.jpg`

Superposition of effects instead of superposition of fields

The first objection against TGD is that superposition is not possible for induced gauge fields and induced metric. The resolution of the problem is that it is effects which need to superpose, not the fields.

Test particle topologically condenses simultaneously to all space-time sheets having a projection to same region of M^4 (that is touches them). The superposition of effects of fields at various space-time sheets replaces the superposition of fields.This is crucial for the understanding also how GRT space-time relates to TGD space-time, which is also in the appendix of this book).

Wormhole contacts

Wormhole contacts are key element of many-sheeted space-time. One does not expect them to be stable unless there is non-trivial Kähler magnetic flux flowing through then so that the throats look like Kähler magnetic monopoles.

Fig. 11. Wormhole contact. `http://tgdtheory.fi/appfigures/wormholecontact.jpg`

Since the flow lines of Kähler magnetic field must be closed this requires the presence of another wormhole contact so that one obtains closed monopole flux tube decomposing to two Minkowskian pieces at the two space-time sheets involved and two wormhole contacts with Euclidian signature of the induced metric. These objects are identified as space-time correlates of elementary particles and are clearly analogous to string like objects.

The relationship between the many-sheeted space-time of TGD and of GRT space-time

The space-time of general relativity is single-sheeted and there is no need to regard it as surface in H although the assumption about representability as vacuum extremal gives very powerful constraints in cosmology and astrophysics and might make sense in simple situations.

The space-time of GRT can be regarded as a long length scale approximation obtained by lumping together the sheets of the many-sheeted space-time to a region of M^4 and providing it with an effective metric obtained as sum of M^4 metric and deviations of the induced metrics of various space-time sheets from M^4 metric. Also induced gauge potentials sum up in the similar manner so that also the gauge fields of gauge theories would not be fundamental fields.

Fig. 12. The superposition of fields is replaced with the superposition of their effects in many-sheeted space-time. `http://tgdtheory.fi/appfigures/fieldsuperpose.jpg`

Space-time surfaces of TGD are considerably simpler objects that the space-times of general relativity and relate to GRT space-time like elementary particles to systems of condensed matter physics. Same can be said about fields since all fields are expressible in terms of embedding space coordinates and their gradients, and general coordinate invariance means that the number of bosonic field degrees is reduced locally to 4. TGD space-time can be said to be a microscopic description whereas GRT space-time a macroscopic description. In TGD complexity of space-time topology replaces the complexity due to large number of fields in quantum field theory.

Topological field quantization and the notion of magnetic body

Topological field quantization also TGD from Maxwell's theory. TGD predicts topological light rays ("massless extremals (MEs)") as space-time sheets carrying waves or arbitrary shape propagating with maximal signal velocity in single direction only and analogous to laser beams and carrying light-like gauge currents in the generi case. There are also magnetic flux quanta and electric flux quanta. The deformations of cosmic strings with 2-D string orbit as M^4 projection gives rise to magnetic flux tubes carrying monopole flux made possible by CP_2 topology allowing homological Kähler magnetic monopoles.

Fig. 13. Topological quantization for magnetic fields replaces magnetic fields with bundles of them defining flux tubes as topological field quanta. `http://tgdtheory.fi/appfigures/field.jpg`

The imbeddability condition for say magnetic field means that the region containing constant magnetic field splits into flux quanta, say tubes and sheets carrying constant magnetic field. Unless one assumes a separate boundary term in Kähler action, boundaries in the usual sense are forbidden except as ends of space-time surfaces at the boundaries of causal diamonds. One obtains typically pairs of sheets glued together along their boundaries giving rise to flux tubes with closed cross section possibly carrying monopole flux.

These kind of flux tubes might make possible magnetic fields in cosmic scales already during primordial period of cosmology since no currents are needed to generate these magnetic fields: cosmic string would be indeed this kind of objects and would dominated during the primordial period. Even superconductors and maybe even ferromagnets could involve this kind of monopole flux tubes.

A-3.4 Embedding space spinors and induced spinors

One can geometrize also fermionic degrees of freedom by inducing the spinor structure of $M^4 \times CP_2$.

CP_2 does not allow spinor structure in the ordinary sense but one can couple the opposite H-chiralities of H-spinors to an $n = 1$ ($n = 3$) integer multiple of Kähler gauge potential to obtain a respectable modified spinor structure. The em charges of resulting spinors are fractional (integer valued) and the interpretation as quarks (leptons) makes sense since the couplings to the induced spinor connection having interpretation in terms electro-weak gauge potential are identical to those assumed in standard model.

The notion of quark color differs from that of standard model.

1. Spinors do not couple to color gauge potential although the identification of color gauge potential as projection of $SU(3)$ Killing vector fields is possible. This coupling must emerge only at the effective gauge theory limit of TGD.

2. Spinor harmonics of embedding space correspond to triality $t = 1$ ($t = 0$) partial waves. The detailed correspondence between color and electroweak quantum numbers is however not correct as such and the interpretation of spinor harmonics of embedding space is as representations for ground states of super-conformal representations. The wormhole pairs associated with physical quarks and leptons must carry also neutrino pair to neutralize weak quantum numbers above the length scale of flux tube (weak scale or Compton length). The total color quantum numbers or these states must be those of standard model. For instance, the color quantum numbers of fundamental left-hand neutrino and lepton can compensate each other for the physical lepton. For fundamental quark-lepton pair they could sum up to those of physical quark.

The well-definedness of em charge is crucial condition.

1. Although the embedding space spinor connection carries W gauge potentials one can say that the embedding space spinor modes have well-defined em charge. One expects that this is true for induced spinor fields inside wormhole contacts with 4-D CP_2 projection and Euclidian signature of the induced metric.

2. The situation is not the same for the modes of induced spinor fields inside Minkowskian region and one must require that the CP_2 projection of the regions carrying induced spinor field is such that the induced W fields and above weak scale also the induced Z^0 fields vanish in order to avoid large parity breaking effects. This condition forces the CP_2 projection to be 2-dimensional. For a generic Minkowskian space-time region this is achieved only if the

spinor modes are localized at 2-D surfaces of space-time surface - string world sheets and possibly also partonic 2-surfaces.

3. Also the Kähler-Dirac gamma matrices appearing in the modified Dirac equation must vanish in the directions normal to the 2-D surface in order that Kähler-Dirac equation can be satisfied. This does not seem plausible for space-time regions with 4-D CP_2 projection.

4. One can thus say that strings emerge from TGD in Minkowskian space-time regions. In particular, elementary particles are accompanied by a pair of fermionic strings at the opposite space-time sheets and connecting wormhole contacts. Quite generally, fundamental fermions would propagate at the boundaries of string world sheets as massless particles and wormhole contacts would define the stringy vertices of generalized Feynman diagrams. One obtains geometrized diagrammatics, which brings looks like a combination of stringy and Feynman diagrammatics.

5. This is what happens in the the generic situation. Cosmic strings could serve as examples about surfaces with 2-D CP_2 projection and carrying only em fields and allowing delocalization of spinor modes to the entire space-time surfaces.

A-3.5 About induced gauge fields

In the following the induced gauge fields are studied for general space-time surface without assuming the preferred extremal property (Bohr orbit property). Therefore the following arguments are somewhat obsolete in their generality.

Space-times with vanishing em, Z^0, or Kähler fields

The following considerations apply to a more general situation in which the homologically trivial geodesic sphere and extremal property are not assumed. It must be emphasized that this case is possible in TGD framework only for a vanishing Kähler field.

Using spherical coordinates (r, Θ, Ψ, Φ) for CP_2, the expression of Kähler form reads as

$$\begin{aligned} J &= \frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta) d\Phi) + \frac{r^2}{2F} sin(\Theta) d\Theta \wedge d\Phi \ , \\ F &= 1 + r^2 \ . \end{aligned} \tag{A-3.1}$$

The general expression of electromagnetic field reads as

$$\begin{aligned} F_{em} &= (3+2p)\frac{r}{F^2} dr \wedge (d\Psi + cos(\Theta) d\Phi) + (3+p)\frac{r^2}{2F} sin(\Theta) d\Theta \wedge d\Phi \ , \\ p &= sin^2(\Theta_W) \ , \end{aligned} \tag{A-3.2}$$

where Θ_W denotes Weinberg angle.

1. The vanishing of the electromagnetic fields is guaranteed, when the conditions

$$\begin{aligned} \Psi &= k\Phi \ , \\ (3+2p)\frac{1}{r^2 F}(d(r^2)/d\Theta)(k + cos(\Theta)) &+ (3+p) sin(\Theta) = 0 \ , \end{aligned} \tag{A-3.3}$$

hold true. The conditions imply that CP_2 projection of the electromagnetically neutral space-time is 2-dimensional. Solving the differential equation one obtains

$$
\begin{aligned}
r &= \sqrt{\frac{X}{1-X}} \ , \\
X &= D\left[|\frac{k+u}{C}|\right]^{\epsilon} \ , \\
u &\equiv cos(\Theta) \ , \ C = k + cos(\Theta_0) \ , \ D = \frac{r_0^2}{1+r_0^2} \ , \ \epsilon = \frac{3+p}{3+2p} \ ,
\end{aligned} \tag{A-3.4}
$$

where C and D are integration constants. $0 \leq X \leq 1$ is required by the reality of r. $r = 0$ would correspond to $X = 0$ giving $u = -k$ achieved only for $|k| \leq 1$ and $r = \infty$ to $X = 1$ giving $|u + k| = [(1 + r_0^2)/r_0^2)]^{(3+2p)/(3+p)}$ achieved only for

$$
sign(u+k) \times [\frac{1+r_0^2}{r_0^2}]^{\frac{3+2p}{3+p}} \leq k+1 \ ,
$$

where $sign(x)$ denotes the sign of x.

The expressions for Kähler form and Z^0 field are given by

$$
\begin{aligned}
J &= -\frac{p}{3+2p} X du \wedge d\Phi \ , \\
Z^0 &= -\frac{6}{p} J \ .
\end{aligned} \tag{A-3.5}
$$

The components of the electromagnetic field generated by varying vacuum parameters are proportional to the components of the Kähler field: in particular, the magnetic field is parallel to the Kähler magnetic field. The generation of a long range Z^0 vacuum field is a purely TGD based feature not encountered in the standard gauge theories.

2. The vanishing of Z^0 fields is achieved by the replacement of the parameter ϵ with $\epsilon = 1/2$ as becomes clear by considering the condition stating that Z^0 field vanishes identically. Also the relationship $F_{em} = 3J = -\frac{3}{4}\frac{r^2}{F} du \wedge d\Phi$ is useful.

3. The vanishing Kähler field corresponds to $\epsilon = 1, p = 0$ in the formula for em neutral space-times. In this case classical em and Z^0 fields are proportional to each other:

$$
\begin{aligned}
Z^0 &= 2e^0 \wedge e^3 = \frac{r}{F^2}(k+u)\frac{\partial r}{\partial u} du \wedge d\Phi = (k+u) du \wedge d\Phi \ , \\
r &= \sqrt{\frac{X}{1-X}} \ , \ X = D|k+u| \ , \\
\gamma &= -\frac{p}{2} Z^0 \ .
\end{aligned} \tag{A-3.6}
$$

For a vanishing value of Weinberg angle ($p = 0$) em field vanishes and only Z^0 field remains as a long range gauge field. Vacuum extremals for which long range Z^0 field vanishes but em field is non-vanishing are not possible.

The effective form of CP_2 metric for surfaces with 2-dimensional CP_2 projection

The effective form of the CP_2 metric for a space-time having vanishing em,Z^0, or Kähler field is of practical value in the case of vacuum extremals and is given by

$$\begin{aligned} ds^2_{eff} &= (s_{rr}(\frac{dr}{d\Theta})^2 + s_{\Theta\Theta})d\Theta^2 + (s_{\Phi\Phi} + 2ks_{\Phi\Psi})d\Phi^2 = \frac{R^2}{4}[s^{eff}_{\Theta\Theta}d\Theta^2 + s^{eff}_{\Phi\Phi}d\Phi^2] \ , \\ s^{eff}_{\Theta\Theta} &= X \times \left[\frac{\epsilon^2(1-u^2)}{(k+u)^2} \times \frac{1}{1-X} + 1 - X\right] \ , \\ s^{eff}_{\Phi\Phi} &= X \times \left[(1-X)(k+u)^2 + 1 - u^2\right] \ , \end{aligned} \tag{A-3.7}$$

and is useful in the construction of vacuum embedding of, say Schwartchild metric.

Topological quantum numbers

Space-times for which either em, Z^0, or Kähler field vanishes decompose into regions characterized by six vacuum parameters: two of these quantum numbers (ω_1 and ω_2) are frequency type parameters, two (k_1 and k_2) are wave vector like quantum numbers, two of the quantum numbers (n_1 and n_2) are integers. The parameters ω_i and n_i will be referred as electric and magnetic quantum numbers. The existence of these quantum numbers is not a feature of these solutions alone but represents a much more general phenomenon differentiating in a clear cut manner between TGD and Maxwell's electrodynamics.

The simplest manner to avoid surface Kähler charges and discontinuities or infinities in the derivatives of CP_2 coordinates on the common boundary of two neighboring regions with different vacuum quantum numbers is topological field quantization, 3-space decomposes into disjoint topological field quanta, 3-surfaces having outer boundaries with possibly macroscopic size.

Under rather general conditions the coordinates Ψ and Φ can be written in the form

$$\begin{aligned} \Psi &= \omega_2 m^0 + k_2 m^3 + n_2\phi + \text{Fourier expansion} \ , \\ \Phi &= \omega_1 m^0 + k_1 m^3 + n_1\phi + \text{Fourier expansion} \ . \end{aligned} \tag{A-3.8}$$

m^0,m^3 and ϕ denote the coordinate variables of the cylindrical M^4 coordinates) so that one has $k = \omega_2/\omega_1 = n_2/n_1 = k_2/k_1$. The regions of the space-time surface with given values of the vacuum parameters ω_i,k_i and n_i and m and C are bounded by the surfaces at which space-time surface becomes ill-defined, say by $r > 0$ or $r < \infty$ surfaces.

The space-time surface decomposes into regions characterized by different values of the vacuum parameters r_0 and Θ_0. At $r = \infty$ surfaces n_2,ω_2 and m can change since all values of Ψ correspond to the same point of CP_2: at $r = 0$ surfaces also n_1 and ω_1 can change since all values of Φ correspond to same point of CP_2, too. If $r = 0$ or $r = \infty$ is not in the allowed range space-time surface develops a boundary.

This implies what might be called topological quantization since in general it is not possible to find a smooth global embedding for, say a constant magnetic field. Although global embedding exists it decomposes into regions with different values of the vacuum parameters and the coordinate u in general possesses discontinuous derivative at $r = 0$ and $r = \infty$ surfaces. A possible manner to avoid edges of space-time is to allow field quantization so that 3-space (and field) decomposes into disjoint quanta, which can be regarded as structurally stable units a 3-space (and of the gauge field). This doesn't exclude partial join along boundaries for neighboring field quanta provided some additional conditions guaranteeing the absence of edges are satisfied.

For instance, the vanishing of the electromagnetic fields implies that the condition

$$\Omega \equiv \frac{\omega_2}{n_2} - \frac{\omega_1}{n_1} = 0 \ , \tag{A-3.9}$$

is satisfied. In particular, the ratio ω_2/ω_1 is rational number for the electromagnetically neutral regions of space-time surface. The change of the parameter n_1 and n_2 (ω_1 and ω_2) in general generates magnetic field and therefore these integers will be referred to as magnetic (electric) quantum numbers.

A-4 The relationship of TGD to QFT and string models

The recent view of the relationship of TGD to QFT and string models has developed slowly during years and it seems that in a certain sense TGD means a return to roots: instead of QFT like description involving path integral one would have wave mechanics for 3-surfaces.

A-4.1 TGD as a generalization of wave mechanism obtained by replacing point-like particles with 3-surfaces

The first vision of TGD was as a generalization of quantum field theory (string models) obtained by replacing pointlike particles (strings) as fundamental objects with 3-surfaces.

The later work has revealed that TGD could be seen as a generalization of the wave mechanism based on the replacement of a point-like particle with 3-D surface. This is due to holography implied by general coordinate invariance. The definition of the metric of the "world of classical worlds" (WCW) must assign a unique or at least almost unique space-time surface to a given 3-surface. This 4-surface is analogous to Bohr orbit so that also Bohr orbitology becomes an exact part of quantum physics. The failure of strict determinism forces to replace 3-surfaces with 4-surfaces and this leads to zero energy ontology (ZEO) in which quantum states are superpositions of space-time surfaces [K46, K25, K77] [L78, L88].

Fig. 5. TGD replaces point-like particles with 3-surfaces. `http://tgdtheory.fi/appfigures/particletgd.jpg`

A-4.2 Extension of superconformal invariance

The fact that light-like 3-surfaces are effectively metrically 2-dimensional and thus possess generalization of 2-dimensional conformal symmetries with light-like radial coordinate defining the analog of second complex coordinate suggests that this generalization could work and extend the super-conformal symmetries to their 4-D analogs.

The boundary $\delta M^4_+ = S^2 \times R_+$- of 4-D light-cone M^4_+ is also metrically 2-dimensional and allows extended conformal invariance. Also the group of isometries of light-cone boundary and of light-like 3-surfaces is infinite-dimensional since the conformal scalings of S^2 can be compensated by S^2-local scaling of the light-like radial coordinate of R_+. These simple facts mean that 4-dimensional Minkowski space and 4-dimensional space-time surfaces are in a completely unique position as far as symmetries are considered.

In fact, this leads to a generalization of the Kac-Moody type symmetries of string models. $\delta M^4_+ \times CP_2$ allows huge supersymplectic symmetries for which the radial light-like coordinate of δM^4_+ plays the role of complex string coordinate in string models. These symmetries are assumed to act as isometries of WCW.

A-4.3 String-like objects and strings

String like objects obtained as deformations of cosmic strings $X^2 \times Y^2$, where X^2 is minimal surface in M^4 and Y^2 a holomorphic surface of CP_2 are fundamental extremals of Kähler action having string world sheet as M^4 projections. Cosmic strings dominate the primordial cosmology of the TGD Universe and the inflationary period corresponds to the transition to radiation dominated cosmology for which space-time sheets with 4-D M^4 projection dominate.

Also genuine string-like objects emerge from TGD. The conditions that the em charge of modes of induces spinor fields is well-defined requires in the generic case the localization of the modes at 2-D surfaces -string world sheets and possibly also partonic 2-surfaces. This in Minkowskian space-time regions.

Fig. 6. Well-definedness of em charge forces the localization of induced spinor modes to 2-D surfaces in generic situations in Minkowskian regions of space-time surface. `http://tgdtheory.fi/appfigures/fermistring.jpg`

A-4.4 TGD view of elementary particles

The TGD based view about elementary particles has two key aspects.

1. The space-time correlates of elementary particles are identified as pairs of wormhole contacts with Euclidean signature of metric and having 4-D CP_2 projection. Their throats behave effectively as Kähler magnetic monopoles so that wormhole throats must be connected by Kähler magnetic flux tubes with monopole flux so that closed flux tubes are obtained.

2. At the level of H Fermion number is carried by the modes of the induced spinor field. In space-time regions with Minkowski signature the modes are localized at string world sheets connecting the wormhole contacts.

Fig. 7. TGD view about elementary particles. a) Particle orbit corresponds to a 4-D generalization of a world line or b) with its light-like 3-D boundary (holography). c) Particle world lines have Euclidean signature of the induced metric. d) They can be identified as wormhole contacts. e) The throats of wormhole contacts carry effective Kähler magnetic charges so that wormhole contacts must appear as pairs in order to obtain closed flux tubes. f) Wormhole contacts are accompanied by fermionic strings connecting the throats at the same sheet: the strings do not extend inside the wormhole contacts. `http://tgdtheory.fi/appfigures/elparticletgd.jpg`

Particle interactions involve both stringy and QFT aspects.

1. The boundaries of string world sheets correspond to fundamental fermions. This gives rise to massless propagator lines in generalized Feynman diagrammatics. One can speak of "long" string connecting wormhole contacts and having a hadronic string as a physical counterpart. Long strings should be distinguished from wormhole contacts which due to their superconformal invariance behave like "short" strings with length scale given by CP_2 size, which is 10^4 times longer than Planck scale characterizing strings in string models.

2. Wormhole contact defines basic stringy interaction vertex for fermion-fermion scattering. The propagator is essentially the inverse of the superconformal scaling generator L_0. Wormhole contacts containing fermion and antifermion at its opposite throats behave like virtual bosons so that one has BFF type vertices typically.

3. In topological sense one has 3-vertices serving as generalizations of 3-vertices of Feynman diagrams. In these vertices 4-D "lines" of generalized Feynman diagrams meet along their 3-D ends. One obtains also the analogs of stringy diagrams but stringy vertices do not have the usual interpretation in terms of particle decays but in terms of propagation of particles along two different routes.

Fig. 8. a) TGD analogs of Feynman and string diagrammatics at the level of space-time topology. b) The 4-D analogs of both string diagrams and QFT diagrams appear but the interpretation of the analogs stringy diagrams is different. `http://tgdtheory.fi/appfigures/tgdgraphs.jpg`

A-5 About the selection of the action defining the Kähler function of the "world of classical worlds" (WCW)

The proposal is that space-time surfaces correspond to preferred extremals of some action principle, being analogous to Bohr orbits, so that they are almost deterministic. The action for the preferred extremal would define the Kähler function of WCW [K46, K77].

How unique is the choice of the action defining WCW Kähler metric? The problem is that twistor lift strongly suggests the identification of the preferred extremals as 4-D surfaces having 4-D generalization of complex structure and that a large number of general coordinate invariant actions constructible in terms of the induced geometry have the same preferred extremals.

A-5.1 Could twistor lift fix the choice of the action uniquely?

The twistor lift of TGD [K78] [L78, L83, L84] generalizes the notion of induction to the level of twistor fields and leads to a proposal that the action is obtained by dimensional reduction of the action having as its preferred extremals the counterpart of twistor space of the space-time surface identified as 6-D surface in the product $T(M^4) \times T(CP_2)$ twistor spaces of $T(M^4)$ and $T(CP_2)$

of M^4 and CP_2. Only M^4 and CP_2 allow a twistor space with Kähler structure [A35] so that TGD would be unique. Dimensional reduction is forced by the condition that the 6-surface has S^2-bundle structure characterizing twistor spaces and the base space would be the space-time surface.

1. Dimensional reduction of 6-D Kähler action implies that at the space-time level the fundamental action can be identified as the sum of Kähler action and volume term (cosmological constant). Other choices of the action do not look natural in this picture although they would have the same preferred extremals.

2. Preferred extremals are proposed to correspond to minimal surfaces with singularities such that they are also extremals of 4-D Kähler action outside the singularities. The physical analogue are soap films spanned by frames and one can localize the violation of the strict determinism and of strict holography to the frames.

3. The preferred extremal property is realized as the holomorphicity characterizing string world sheets, which generalizes to the 4-D situation. This in turn implies that the preferred extremals are the same for any general coordinate invariant action defined on the induced gauge fields and induced metric apart from possible extremals with vanishing CP_2 Kähler action.

   For instance, 4-D Kähler action and Weyl action as the sum of the tensor squares of the components of the Weyl tensor of CP_2 representing quaternionic imaginary units constructed from the Weyl tensor of CP_2 as an analog of gauge field would have the same preferred extremals and only the definition of Kähler function and therefore Kähler metric of WCW would change. One can even consider the possibility that the volume term in the 4-D action could be assigned to the tensor square of the induced metric representing a quaternionic or octonionic real unit.

Action principle does not seem to be unique. On the other hand, the WCW Kähler form and metric should be unique since its existence requires maximal isometries.

Unique action is not the only way to achieve this. One cannot exclude the possibility that the Kähler gauge potential of WCW in the complex coordinates of WCW differs only by a complex gradient of a holomorphic function for different actions so that they would give the same Kähler form for WCW. This gradient is induced by a symplectic transformation of WCW inducing a U(1) gauge transformation. The Kähler metric is the same if the symplectic transformation is an isometry.

Symplectic transformations of WCW could give rise to inequivalent representations of the theory in terms of action at space-time level. Maybe the length scale dependent coupling parameters of an effective action could be interpreted in terms of a choice of WCW Kähler function, which maximally simplifies the computations at a given scale.

1. The 6-D analogues of electroweak action and color action reducing to Kähler action in 4-D case exist. The 6-D analog of Weyl action based on the tensor representation of quaternionic imaginary units does not however exist. One could however consider the possibility that only the base space of twistor space $T(M^4)$ and $T(CP_2)$ have quaternionic structure.

2. Kähler action has a huge vacuum degeneracy, which clearly distinguishes it from other actions. The presence of the volume term removes this degeneracy. However, for minimal surfaces having CP_2 projections, which are Lagrangian manifolds and therefore have a vanishing induced Kähler form, would be preferred extremals according to the proposed definition. For these 4-surfaces, the existence of the generalized complex structure is dubious.

   For the electroweak action, the terms corresponding to charged weak bosons eliminate these extremals and one could argue that electroweak action or its sum with the analogue of color action, also proportional Kähler action, defines the more plausible choice. Interestingly, also the neutral part of electroweak action is proportional to Kähler action.

Twistor lift strongly suggests that also M^4 has the analog of Kähler structure. M^8 must be complexified by adding a commuting imaginary unit i. In the E^8 subspace, the Kähler structure of E^4 is defined in the standard sense and it is proposed that this generalizes to M^4 allowing also generalization of the quaternionic structure. M^4 Kähler structure violates Lorentz invariance but could be realized at the level of moduli space of these structures.

The minimal possibility is that the M^4 Kähler form vanishes: one can have a different representation of the Kähler gauge potential for it obtained as generalization of symplectic transformations acting non-trivially in M^4. The recent picture about the second quantization of spinors of $M^4 \times CP_2$ assumes however non-trivial Kähler structure in M^4.

A-5.2 Two paradoxes

TGD view leads to two apparent paradoxes.

1. If the preferred extremals satisfy 4-D generalization of holomorphicity, a very large set of actions gives rise to the same preferred extremals unless there are some additional conditions restricting the number of preferred extremals for a given action.

2. WCW metric has an infinite number of zero modes, which appear as parameters of the metric but do not contribute to the line element. The induced Kähler form depends on these degrees of freedom. The existence of the Kähler metric requires maximal isometries, which suggests that the Kähler metric is uniquely fixed apart from a conformal scaling factor Ω depending on zero modes. This cannot be true: galaxy and elementary particle cannot correspond to the same Kähler metric.

Number theoretical vision and the hierarchy of inclusions of HFFs associated with supersymplectic algebra actings as isometries of WcW provide equivalent realizations of the measurement resolution. This solves these paradoxes and predicts that WCW decomposes into sectors for which Kähler metrics of WCW differ in a natural way.

The hierarchy subalgebras of supersymplectic algebra implies the decomposition of WCW into sectors with different actions

Supersymplectic algebra of $\delta M^4_+ \times CP_2$ is assumed to act as isometries of WCW [L88]. There are also other important algebras but these will not be discussed now.

1. The symplectic algebra A of $\delta M^4_+ \times CP_2$ has the structure of a conformal algebra in the sense that the radial conformal weights with non-negative real part, which is half integer, label the elements of the algebra have an interpretation as conformal weights.

   The super symplectic algebra A has an infinite hierarchy of sub-algebras [L88] such that the conformal weights of sub-algebras $A_{n(SS)}$ are integer multiples of the conformal weights of the entire algebra. The superconformal gauge conditions are weakened. Only the subalgebra $A_{n(SS)}$ and the commutator $[A_{n(SS)}, A]$ annihilate the physical states. Also the corresponding classical Noether charges vanish for allowed space-time surfaces.

   This weakening makes sense also for ordinary superconformal algebras and associated Kac-Moody algebras. This hierarchy can be interpreted as a hierarchy symmetry breakings, meaning that sub-algebra $A_{n(SS)}$ acts as genuine dynamical symmetries rather than mere gauge symmetries. It is natural to assume that the super-symplectic algebra A does not affect the coupling parameters of the action.

2. The generators of A correspond to the dynamical quantum degrees of freedom and leave the induced Kähler form invariant. They affect the induced space-time metric but this effect is gravitational and very small for Einsteinian space-time surfaces with 4-D M^4 projection.

   The number of dynamical degrees of freedom increases with $n(SS)$. Therefore WCW decomposes into sectors labelled by $n(SS)$ with different numbers of dynamical degrees of freedom so that their Kähler metrics cannot be equivalent and cannot be related by a symplectic isometry. They can correspond to different actions.

Number theoretic vision implies the decomposition of WCW into sectors with different actions

The number theoretical vision leads to the same conclusion as the hierarchy of HFFs. The number theoretic vision of TGD based on M^8-H duality [L88] predicts a hierarchy with levels labelled by the degrees $n(P)$ of rational polynomials P and corresponding extensions of rationals characterized by Galois groups and by ramified primes defining p-adic length scales.

These sequences allow us to imagine several discrete coupling constant evolutions realized at the level H in terms of action whose coupling parameters depend on the number theoretic parameters.

1. Coupling constant evolution with respect to $n(P)$

The first coupling constant evolution would be with respect to $n(P)$.

1. The coupling constants characterizing action could depend on the degree $n(P)$ of the polynomial defining the space-time region by M^8-H duality. The complexity of the space-time surface would increase with $n(P)$ and new degrees of freedom would emerge as the number of the rational coefficients of P.

2. This coupling constant evolution could naturally correspond to that assignable to the inclusion hierarchy of hyperfinite factors of type II_1 (HFFs). I have indeed proposed [L88] that the degree $n(P)$ equals to the number $n(braid)$ of braids assignable to HFF for which super symplectic algebra subalgebra $A_{n(SS)}$ with radial conformal weights coming as $n(SS)$-multiples of those of entire algebra A. One would have $n(P)=n(braid)=n(SS)$. The number of dynamical degrees of freedom increases with n which just as it increases with $n(P)$ and $n(SS)$.

3. The actions related to different values of $n(P)=n(braid)=n(SS)$ cannot define the same Kähler metric since the number of allowed space-time surfaces depends on $n(SS)$.

   WCW could decompose to sub-WCWs corresponding to different actions, a kind of theory space. These theories would not be equivalent. A possible interpretation would be as a hierarchy of effective field theories.

4. Hierarchies of composite polynomials define sequences of polynomials with increasing values of $n(P)$ such that the order of a polynomial at a given level is divided by those at the lower levels. The proposal is that the inclusion sequences of extensions are realized at quantum level as inclusion hierarchies of hyperfinite factors of type II_1.

   A given inclusion hierarchy corresponds to a sequence $n(SS)_i$ such that $n(SS)_i$ divides $n(SS)_{i+1}$. Therefore the degree of the composite polynomials increases very rapidly. The values of $n(SS)_i$ can be chosen to be primes and these primes correspond to the degrees of so called prime polynomials [L86] so that the decompositions correspond to prime factorizations of integers. The "densest" sequence of this kind would come in powers of 2 as $n(SS)_i=2^i$. The corresponding p-adic length scales (assignable to maximal ramified primes for given $n(SS)_i$) are expected to increase roughly exponentially, say as 2^{r2^i}. $r=1/2$ would give a subset of scales $2^{r/2}$ allowed by the p-adic length scale hypothesis. These transitions would be very rare.

   A theory corresponding to a given composite polynomial would contain as sub-theories the theories corresponding to lower polynomial composites. The evolution with respect to $n(SS)$ would correspond to a sequence of phase transitions in which the action genuinely changes. For instance, color confinement could be seen as an example of this phase transition.

5. A subset of p-adic primes allowed by the p-adic length scale hypothesis $p\simeq 2^k$ defining the proposed p-adic length scale hierarchy could relate to n_S changing phase transition. TGD suggests a hierarchy of hadron physics corresponding to a scale hierarchy defined by Mersenne primes and their Gaussian counterparts [K58, K59]). Each of them would be characterized by a confinement phase transition in which n_S and therefore also the action changes.

2. Coupling constant evolutions with respect to ramified primes for a given value of $n(P)$

For a given value of $n(P)$, one could have coupling constant sub-evolutions with respect to the set of ramified primes of P and dimensions $n = h_{eff}/h_0$ of algebraic extensions. The action would only change by U(1) gauge transformation induced by a symplectic isometry of WCW. Coupling parameters could change but the actions would be equivalent.

The choice of the action in an optimal manner in a given scale could be seen as a choice of the most appropriate effective field theory in which radiative corrections would be taken into account. One can interpret the possibility to use a single choice of coupling parameters in terms of quantum criticality.

The range of the p-adic length scales labelled by ramified primes and effective Planck constants h_{eff}/h_0 is finite for a given value of $n(SS)$.

The first coupling constant evolution of this kind corresponds to ramified primes defining p-adic length scales for given $n(SS)$.

1. Ramified primes are factors of the discriminant $D(P)$ of P, which is expressible as a product of non-vanishing root differents and reduces to a polynomial of the n coefficients of P. Ramified primes define p-adic length scales assignable to the particles in the amplitudes scattering amplitudes defined by zero energy states.

   P would represent the space-time surface defining an interaction region in $N-$-particle scattering. The N ramified primes dividing $D(P)$ would characterize the p-adic length scales assignable to these particles. If $D(P)$ reduces to a single ramified prime, one has elementary particle [L86], and the forward scattering amplitude corresponds to the propagator.

   This would give rise to a multi-scale p-adic length scale evolution of the amplitudes analogous to the ordinary continuous coupling constant evolution of n-point scattering amplitudes with respect to momentum scales of the particles. This kind of evolutions extend also to evolutions with respect to $n(SS)$.

2. According to [L86], physical constraints require that $n(P)$ and the maximum size of the ramified prime of P correlate.

   A given rational polynomial of degree $n(P)$ can be always transformed to a polynomial with integer coefficients. If the integer coefficients are smaller than $n(P)$, there is an upper bound for the ramified primes. This assumption also implies that finite fields become fundamental number fields in number theoretical vision [L86].

3. p-Adic length scale hypothesis [L89] in its basic form states that there exist preferred primes $p \simeq 2^k$ near some powers of 2. A more general hypothesis states that also primes near some powers of 3 possibly also other small primes are preferred physically. The challenge is to understand the origin of these preferred scales.

   For polynomials P with a given degree $n(P)$ for which discriminant $D(P)$ is prime, there exists a maximal ramified prime. Numerical calculations suggest that the upper bound depends exponentially on $n(P)$.

   Could these maximal ramified primes satisfy the p-adic length scale hypothesis or its generalization? The maximal prime defines a fixed point of coupling constant evolution in accordance with the earlier proposal. For instance, could one think that one has $p \simeq 2^k$, $k = n(SS)$? Each p-adic prime would correspond to a p-adic coupling constant sub-evolution representable in terms of symplectic isometries.

Also the dimension n of the algebraic extension associated with P, which is identified in terms of effective Planck constant $h_{eff}/h_0 = n$ labelling different phases of the ordinary matter behaving like dark matter, could give rise to coupling constant evolution for given $n(SS)$. The range of allowed values of n is finite. Note however that several polynomials of a given degree can correspond to the same dimension of extension.

Number theoretic discretization of WCW and maxima of WCW Kähler function

Number theoretic approach involves a unique discretization of space-time surface and also of WCW. The question is how the points of the discretized WCW correspond to the preferred extremals.

1. The exponents of Kähler function for the maxima of Kähler function, which correspond to the universal preferred extremals, appear in the scattering amplitudes. The number theoretical approach involves a unique discretization of space-time surfaces defining the WCW coordinates of the space-time surface regarded as a point of WCW.

   In [L88] it is assumed that these WCW points appearing in the number theoretical discretization correspond to the maxima of the Kähler function. The maxima would depend on the action and would differ for ghd maxima associated with different actions unless they are not related by symplectic WCW isometry.

2. The symplectic transformations of WCW acting as isometries are assumed to be induced by the symplectic transformations of $\delta M^4_+ \times CP_2$ [K46, K25]. As isometries they would naturally permute the maxima with each other.

A-6 Number theoretic vision of TGD

Physics as number theory vision is complementary to the physics as geometry vision and has developed gradually since 1993. Langlands program is the counterpart of this vision in mathematics [L87].

The notion of p-adic number fields emerged with the motivation coming from the observation that elementary particle mass scales and mass ratios could be understood in terms of the so-called p-adic length scale hypothesis [K61, K53, K22]. The fusion of the various p-adic physics leads to what I call adelic physics [L29, L28]. Later the hypothesis about hierarchy of Planck constants labelling phases of ordinary matter behaving like dark matter emerged [K28, K29, K30, K30].

Eventually this led to that the values of effective Planck constant could be identified as the dimension of an algebraic extension of rationals assignable to polynomials with rational coefficients. This led to the number theoretic vision in which so-called M^8-H duality [L64, L65] plays a key role. M^8 (actually a complexification of real M^8) is analogous to momentum space so that the duality generalizes momentum position duality for point-like particles. M^8 has an interpretation as complexified octonions.

The dynamics of 4-surfaces in M^8 is coded by polynomials with rational coefficients, whose roots define mass shells H^3 of $M^4 \subset M^8$. It has turned out that the polynomials satisfy stringent additional conditions and one can speak of number theoretic holography [L86, L87]. Also the ordinary 3→ 4 holography is needed to assign 4-surfaces with these 3-D mass shells. The number theoretic dynamics is based on the condition that the normal space of the 4-surface in M^8 is associative (quaternionic) and contains a commutative complex sub-space. This makes it possible to assign to this surface space-time surface in $H = M^4 \times CP_2$.

At the level of H the space-time surfaces are by holography preferred extremals and are assumed to be determined by the twistor lift of TGD [K78] giving rise to an action which is sum of the Kähler action and volume term. The preferred extremals would be minimal surfaces analogous to soap films spanned by frames. Outside frames they would be simultaneous extremals of the Kähler action, which requires a generalization of the holomorphy characterizing string world sheets.

In the following only p-adic numbers and hierarchy of Planck constants will be discussed.

A-6.1 p-Adic numbers and TGD

p-Adic number fields

p-Adic numbers (p is prime: 2, 3, 5, ...) can be regarded as a completion of the rational numbers using a norm, which is different from the ordinary norm of real numbers [A24]. p-Adic numbers are representable as power expansion of the prime number p of form

$$x = \sum_{k\geq k_0} x(k)p^k ,\ x(k) = 0, ..., p-1 \ . \tag{A-6.1}$$

The norm of a p-adic number is given by

$$|x| = p^{-k_0(x)} \ . \tag{A-6.2}$$

Here $k_0(x)$ is the lowest power in the expansion of the p-adic number. The norm differs drastically from the norm of the ordinary real numbers since it depends on the lowest pinary digit of the p-adic number only. Arbitrarily high powers in the expansion are possible since the norm of the p-adic number is finite also for numbers, which are infinite with respect to the ordinary norm. A convenient representation for p-adic numbers is in the form

$$x = p^{k_0}\varepsilon(x) \ , \tag{A-6.3}$$

where $\varepsilon(x) = k + ...$ with $0 < k < p$, is p-adic number with unit norm and analogous to the phase factor $exp(i\phi)$ of a complex number.

The distance function $d(x,y) = |x-y|_p$ defined by the p-adic norm possesses a very general property called ultra-metricity:

$$d(x,z) \leq max\{d(x,y), d(y,z)\} \ . \tag{A-6.4}$$

The properties of the distance function make it possible to decompose R_p into a union of disjoint sets using the criterion that x and y belong to same class if the distance between x and y satisfies the condition

$$d(x,y) \leq D \ . \tag{A-6.5}$$

This division of the metric space into classes has following properties:

1. Distances between the members of two different classes X and Y do not depend on the choice of points x and y inside classes. One can therefore speak about distance function between classes.

2. Distances of points x and y inside single class are smaller than distances between different classes.

3. Classes form a hierarchical tree.

Notice that the concept of the ultra-metricity emerged in physics from the models for spin glasses and is believed to have also applications in biology [B26]. The emergence of p-adic topology as the topology of the effective space-time would make ultra-metricity property basic feature of physics.

Canonical correspondence between p-adic and real numbers

The basic challenge encountered by p-adic physicist is how to map the predictions of the p-adic physics to real numbers. p-Adic probabilities provide a basic example in this respect. Identification via common rationals and canonical identification and its variants have turned out to play a key role in this respect.

1. Basic form of the canonical identification

There exists a natural continuous map $I : R_p \rightarrow R_+$ from p-adic numbers to non-negative real numbers given by the "pinary" expansion of the real number for $x \in R$ and $y \in R_p$ this correspondence reads

$$\begin{aligned} y &= \sum_{k>N} y_k p^k \rightarrow x = \sum_{k<N} y_k p^{-k} \ , \\ y_k &\in \{0,1,..,p-1\} \ . \end{aligned} \tag{A-6.6}$$

This map is continuous as one easily finds out. There is however a little difficulty associated with the definition of the inverse map since the pinary expansion like also decimal expansion is not unique (1 = 0.999...) for the real numbers x, which allow pinary expansion with finite number of pinary digits

$$\begin{aligned} x &= \sum_{k=N_0}^{N} x_k p^{-k} \ , \\ x &= \sum_{k=N_0}^{N-1} x_k p^{-k} + (x_N - 1)p^{-N} + (p-1)p^{-N-1} \sum_{k=0,..} p^{-k} \ . \end{aligned} \tag{A-6.7}$$

The p-adic images associated with these expansions are different

$$\begin{aligned} y_1 &= \sum_{k=N_0}^{N} x_k p^{k} \ , \\ y_2 &= \sum_{k=N_0}^{N-1} x_k p^{k} + (x_N - 1)p^{N} + (p-1)p^{N+1} \sum_{k=0,..} p^{k} \\ &= y_1 + (x_N - 1)p^N - p^{N+1} \ , \end{aligned} \tag{A-6.8}$$

so that the inverse map is either two-valued for p-adic numbers having expansion with finite pinary digits or single valued and discontinuous and non-surjective if one makes pinary expansion unique by choosing the one with finite pinary digits. The finite pinary digit expansion is a natural choice since in the numerical work one always must use a pinary cutoff on the real axis.

2. The topology induced by canonical identification

The topology induced by the canonical identification in the set of positive real numbers differs from the ordinary topology. The difference is easily understood by interpreting the p-adic norm as a norm in the set of the real numbers. The norm is constant in each interval $[p^k, p^{k+1})$ (see **Fig. A-6.1**) and is equal to the usual real norm at the points $x = p^k$: the usual linear norm is replaced with a piecewise constant norm. This means that p-adic topology is coarser than the usual real topology and the higher the value of p is, the coarser the resulting topology is above a given length scale. This hierarchical ordering of the p-adic topologies will be a central feature as far as the proposed applications of the p-adic numbers are considered.

Ordinary continuity implies p-adic continuity since the norm induced from the p-adic topology is rougher than the ordinary norm. p-Adic continuity implies ordinary continuity from right as is clear already from the properties of the p-adic norm (the graph of the norm is indeed continuous from right). This feature is one clear signature of the p-adic topology.

Fig. 14. The real norm induced by canonical identification from 2-adic norm. `http://tgdtheory.fi/appfigures/norm.png`

The linear structure of the p-adic numbers induces a corresponding structure in the set of the non-negative real numbers and p-adic linearity in general differs from the ordinary concept of linearity. For example, p-adic sum is equal to real sum only provided the summands have no common pinary digits. Furthermore, the condition $x +_p y < max\{x, y\}$ holds in general for the p-adic sum of the real numbers. p-Adic multiplication is equivalent with the ordinary multiplication only provided that either of the members of the product is power of p. Moreover one has $x \times_p y < x \times y$ in general. The p-Adic negative -1_p associated with p-adic unit 1 is given by $(-1)_p = \sum_k (p-1)p^k$ and defines p-adic negative for each real number x. An interesting possibility is that p-adic linearity might replace the ordinary linearity in some strongly nonlinear systems so these systems would look simple in the p-adic topology.

These results suggest that canonical identification is involved with some deeper mathematical structure. The following inequalities hold true:

$$\begin{aligned} (x+y)_R &\leq x_R+y_R \ , \\ |x|_p|y|_R \leq (xy)_R &\leq x_R y_R \ , \end{aligned} \quad \text{(A-6.9)}$$

where $|x|_p$ denotes p-adic norm. These inequalities can be generalized to the case of $(R_p)^n$ (a linear vector space over the p-adic numbers).

$$\begin{aligned} (x+y)_R &\leq x_R+y_R \ , \\ |\lambda|_p|y|_R \leq (\lambda y)_R &\leq \lambda_R y_R \ , \end{aligned} \quad \text{(A-6.10)}$$

where the norm of the vector $x \in T_p^n$ is defined in some manner. The case of Euclidian space suggests the definition

$$(x_R)^2 = (\sum_n x_n^2)_R \ . \quad \text{(A-6.11)}$$

These inequalities resemble those satisfied by the vector norm. The only difference is the failure of linearity in the sense that the norm of a scaled vector is not obtained by scaling the norm of the original vector. Ordinary situation prevails only if the scaling corresponds to a power of p.

These observations suggests that the concept of a normed space or Banach space might have a generalization and physically the generalization might apply to the description of some non-linear systems. The nonlinearity would be concentrated in the nonlinear behavior of the norm under scaling.

3. Modified form of the canonical identification

The original form of the canonical identification is continuous but does not respect symmetries even approximately. This led to a search of variants which would do better in this respect. The modification of the canonical identification applying to rationals only and given by

$$I_Q(q = p^k \times \frac{r}{s}) = p^k \times \frac{I(r)}{I(s)} \quad \text{(A-6.12)}$$

is uniquely defined for rationals, maps rationals to rationals, has also a symmetry under exchange of target and domain. This map reduces to a direct identification of rationals for $0 \leq r < p$ and $0 \leq s < p$. It has turned out that it is this map which most naturally appears in the applications. The map is obviously continuous locally since p-adically small modifications of r and s mean small modifications of the real counterparts.

Canonical identification is in a key role in the successful predictions of the elementary particle masses. The predictions for the light elementary particle masses are within extreme accuracy same for I and I_Q but I_Q is theoretically preferred since the real probabilities obtained from p-adic ones by I_Q sum up to one in p-adic thermodynamics.

4. Generalization of number concept and notion of embedding space

TGD forces an extension of number concept: roughly a fusion of reals and various p-adic number fields along common rationals is in question. This induces a similar fusion of real and p-adic embedding spaces. Since finite p-adic numbers correspond always to non-negative reals n-dimensional space R^n must be covered by 2^n copies of the p-adic variant R_p^n of R^n each of which projects to a copy of R_+^n (four quadrants in the case of plane). The common points of p-adic and real embedding spaces are rational points and most p-adic points are at real infinity.

Real numbers and various algebraic extensions of p-adic number fields are thus glued together along common rationals and also numbers in algebraic extension of rationals whose number belong to the algebraic extension of p-adic numbers. This gives rise to a book like structure with rationals and various algebraic extensions of rationals taking the role of the back of the book. Note that Neper number is exceptional in the sense that it is algebraic number in p-adic number field Q_p satisfying $e^p \ mod \ p = 1$.

Fig. 15. Various number fields combine to form a book like structure. `http://tgdtheory.fi/appfigures/book.jpg`

For a given p-adic space-time sheet most points are literally infinite as real points and the projection to the real embedding space consists of a discrete set of rational points: the interpretation in terms of the unavoidable discreteness of the physical representations of cognition is natural. Purely local p-adic physics implies real p-adic fractality and thus long range correlations for the real space-time surfaces having enough common points with this projection.

p-Adic fractality means that M^4 projections for the rational points of space-time surface X^4 are related by a direct identification whereas CP_2 coordinates of X^4 at these points are related by I, I_Q or some of its variants implying long range correlates for CP_2 coordinates. Since only a discrete set of points are related in this manner, both real and p-adic field equations can be satisfied and there are no problems with symmetries. p-Adic effective topology is expected to be a good approximation only within some length scale range which means infrared and UV cutoffs. Also multi-p-fractality is possible.

The notion of p-adic manifold

The notion of p-adic manifold is needed in order to fuse real physics and various p-adic physics to a larger structure which suggests that real and p-adic number fields should be glued together along common rationals bringing in mind adeles. The notion is problematic because p-adic topology is totally disconnected implying that p-adic balls are either disjoint or nested so that ordinary definition of manifold using p-adic chart maps fails. A cure is suggested to be based on chart maps from p-adics to reals rather than to p-adics (see the appendix of the book)

The chart maps are interpreted as cognitive maps, "thought bubbles".

Fig. 16. The basic idea between p-adic manifold. `http://tgdtheory.fi/appfigures/padmanifold.jpg`

There are some problems.

1. Canonical identification does not respect symmetries since it does not commute with second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map arithmetic operations which requires pinary cutoff below which chart map takes rationals to rationals so that commutativity with arithmetics and symmetries is achieved in finite resolution: above the cutoff canonical identification is used

2. Canonical identification is continuous but does not map smooth p-adic surfaces to smooth real surfaces requiring second pinary cutoff so that only a discrete set of rational points is mapped to their real counterparts by chart map requiring completion of the image to smooth preferred extremal of Kähler action so that chart map is not unique in accordance with finite measurement resolution

3. Canonical identification violates general coordinate invariance of chart map: (cognition-induced symmetry breaking) minimized if p-adic manifold structure is induced from that for p-adic embedding space with chart maps to real embedding space and assuming preferred coordinates made possible by isometries of embedding space: one however obtains several inequivalent p-adic manifold structures depending on the choice of coordinates: these cognitive representations are not equivalent.

A-6.2 Hierarchy of Planck constants and dark matter hierarchy

Hierarchy of Planck constants was motivated by the "impossible" quantal effects of ELF em fields on vertebrate cyclotron energies $E = hf = \hbar \times eB/m$ are above thermal energy is possible only if $\hbar$ has value much larger than its standard value. Also Nottale's finding that planetary orbits migh be understood as Bohr orbits for a gigantic gravitational Planck constant.

Hierachy of Planck constant would mean that the values of Planck constant come as integer multiples of ordinary Planck constant: $h_{eff} = n \times h$. The particles at magnetic flux tubes characterized by h_{eff} would correspond to dark matter which would be invisible in the sense that only particle with same value of h_{eff} appear in the same vertex of Feynman diagram.

Hierarchy of Planck constants would be due to the non-determism of the Kähler action predicting huge vacuum degeneracy allowing all space-time surfaces which are sub-manfolds of any $M^4 \times Y^2$, where Y^2 is Lagrangian sub-manifold of CP_2. For agiven Y^2 one obtains new manifolds Y^2 by applying symplectic transformations of CP_2.

Non-determinism would mean that the 3-surface at the ends of causal diamond (CD) can be connected by several space-time surfaces carrying same conserved Kähler charges and having same values of Kähler action. Conformal symmetries defined by Kac-Moody algebra associated with the embedding space isometries could act as gauge transformations and respect the light-likeness property of partonic orbits at which the signature of the induced metric changes from Minkowskian to Euclidian (Minkowskianb space-time region transforms to wormhole contact say). The number of conformal equivalence classes of these surfaces could be finite number n and define discrete physical degree of freedom and one would have $h_{eff} = n \times h$. This degeneracy would mean "second quantization" for the sheets of n-furcation: not only one but several sheets can be realized.

This relates also to quantum criticality postulated to be the basic characteristics of the dynamics of quantum TGD. Quantum criticalities would correspond to an infinite fractal hierarchy of broken conformal symmetries defined by sub-algebras of conformal algebra with conformal weights coming as integer multiples of n. This leads also to connections with quantum criticality and hierarchy of broken conformal symmetries, p-adicity, and negentropic entanglement which by consistency with standard quantum measurement theory would be described in terms of density matrix proportional $n \times n$ identity matrix and being due to unitary entanglement coefficients (typical for quantum computing systems).

Formally the situation could be described by regarding space-time surfaces as surfaces in singular n-fold singular coverings of embedding space. A stronger assumption would be that they are expressible as as products of n_1 -fold covering of M^4 and n_2-fold covering of CP_2 meaning analogy with multi-sheeted Riemann surfaces and that M^4 coordinates are n_1-valued functions and CP_2 coordinates n_2 -valued functions of space-time coordinates for $n = n_1 \times n_2$. These singular coverings of embedding space form a book like structure with singularities of the coverings localizable at the boundaries of causal diamonds defining the back of the book like structure.

Fig. 17. Hierarchy of Planck constants. `http://tgdtheory.fi/appfigures/planckhierarchy.jpg`

A-6.3 $M^8 - H$ duality as it is towards the end of 2021

The view of $M^8 - H$ duality (see Appendix **??**) has changed considerably towards the end 2021 [L78] after the realization that this duality is the TGD counterpart of momentum position duality of wave mechanics, which is lost in QFTs. Therefore M^8 and also space-time surface is analogous to momentum space. This forced us to give up the original simple identification of the points $M^4 \subset M^4 \times E^4 = M^8$ and of $M^4 \times CP_2$ so that it respects Uncertainty Principle (UP).

The first improved guess for the duality map was the replacement with the inversion $p^k \rightarrow m^k = \hbar_{eff} p^k/p^2$ conforming in spirit with UP but turned out to be too naive.

The improved form [L78] of the $M^8 - H$ duality map takes mass shells $p^2 = m^2$ of $M^4 \subset M^8$ to cds with size $L(m) = \hbar_{eff}/m$ with a common center. The slicing by mass shells is mapped to a Russian doll like slicing by cds. Therefore would be no CDs in M^8 contrary to what I believed first.

Quantum classical correspondence (QCC) inspires the proposal that the point $p^k \in M^8$ is mapped to a geodesic line corresponding to momentum p^k starting from the common center of cds. Its intersection with the opposite boundary of cd with size $L(m)$ defines the image point. This is not yet quite enough to satisfy UP but the additional details [L78] are not needed in the sequel.

The 6-D brane-like special solutions in M^8 are of special interest in the TGD inspired theory of consciousness. They have an M^4 projection which is $E = E_n$ 3-ball. Here E_n is a root of the real polynomial P defining $X^4 \subset M^8_c$ (M^8 is complexified to M^8_c) as a "root" of its octonionic continuation [L64, L65]. E_n has an interpretation as energy, which can be complex. The original interpretation was as moment of time. For this interpretation, $M^8 - H$ duality would be a linear identification and these hyper planes would be mapped to hyperplanes in $M^4 \subset H$.

This motivated the term "very special moment in the life of self" for the image of the $E = E_n$ section of $X^4 \subset M^8$ [L57]. This notion does not make sense at the level M^8 anymore.

The modified M^8-H duality forces us to modify the original interpretation [L78]. The point $(E_n, p = 0)$ is mapped $(t_n = \hbar_{eff}/E_n, 0)$. The momenta (E_n, p) in $E = E_n$ plane are mapped to the boundary of cd and correspond to a continuous time interval at the boundary of CD: "very special moment" becomes a "very special time interval".

The quantum state however corresponds to a set of points corresponding to quark momenta, which belong to a cognitive representation and are therefore algebraic integers in the extension determined by the polynomial. These active points in E_n are mapped to a discrete set at the boundary of cd(m). A "very special moment" is replaced with a sequence of "very special moments".

So called Galois confinement [L72] forces the total momenta for bound states of quarks and antiquarks to be rational integers invariant under Galois group of extension of rationals determined by the polynomial P [L78]. These states correspond to states at boundaries of sub-CDs so that one obtains a hierarchy. Galois confinement provides a universal number theoretic mechanism for the formation of bound states.

A-7 Zero energy ontology (ZEO)

ZEO is implied by the holography forced in the TGD framework by general coordinate invariance.

A-7.1 Basic motivations and ideas of ZEO

The following gives a brief summary of ZEO [L61] [K107].

1. In ZEO quantum states are not 3-dimensional but superpositions of 4-dimensional deterministic time evolutions connecting ordinary initial 3-dimensional states. By holography they are equivalent to pairs of ordinary 3-D states identified as initial and final states of time evolution. One can say that in the TGD framework general coordinate invariance implies holography and the slight failure of its determinism in turn forces ZEO.

   Quantum jumps replace this state with a new one: a superposition of deterministic time evolutions is replaced with a new superposition. Classical determinism of individual time evolution is not violated and this solves the basic paradox of quantum measurement theory. There are two kinds of quantum jumps: ordinary ("big") state function reductions (BSFRs) changing the arrow of time and "small" state function reductions (SSFRs) (weak measurements) preserving it and giving rise to the analog of Zeno effect [L61].

2. To avoid getting totally confused it is good to emphasize some aspects of ZEO.

   (a) ZEO does not mean that physical states in the usual 3-D sense as snapshots of time evolution would have zero energy state pairs defining zero energy states as initial and final states have same conserved quantities such as energy. Conservation implies that one can adopt the conventions that the values of conserved quantities are opposite for these states so that their sum vanishes: one can think that incoming and outgoing particles come from geometric past and future is the picture used in quantum field theories.

   (b) ZEO means two times: subjective time as sequence of quantum jumps and geometric time as space-time coordinate. These times are identifiable but are strongly correlated.

3. In BSFRs the arrow of time is changed and the time evolution in the final state occurs backwards with respect to the time of the external observer. BSFRs can occur in all scales since TGD predicts a hierarchy of effective Planck constants with arbitrarily large values. There is empirical support for BSFRs.

   (a) The findings of Minev et al [L52] in atomic scale can be explained by the same mechanism [L52]. In BSFR a final zero energy state as a superposition of classical deterministic time evolutions emerges and for an observer with a standard arrow of time looks like a superposition of deterministic smooth time evolutions leading to the final state. Interestingly, once this evolution has started, it cannot be stopped unless one changes the stimulus signal inducing the evolution in which case the process does not lead to anywhere: the interpretation would be that BSFR back to the initial state occurs!

(b) Libets' experiments about active aspects of consciousness [J1] can be understood. Subject person raises his finger and neural activity starts before the conscious decision to do so. In the physicalistic framework it is thought to lead to raising of the finger. The problem with the explanation is that the activity beginning .5 seconds earlier seems to be dissipation with a reversed arrow of time: from chaotic and disordered to ordered at around .15 seconds. ZEO explanation is that macroscopic quantum jump occurred and generated a signal proceeding backwards in time and generated neural activity and dissipated to randomness.

(c) Earthquakes involve a strange anomaly: they are preceded by ELF radiation. One would expect that they generate ELF radiation. The identification as BSFR would explain the anomaly [L56]. In biology the reversal of the arrow of time would occur routinely and be a central element of biological self-organization, in particular self-organized quantum criticality (see [L59, L93].

A-7.2 Some implications of ZEO

ZEO has profound implications for understanding self-organization and self-organized quantum criticality in terms of dissipation with non-standard arrow of time looking like generation of structures [L59, L93]. ZEO could also allow understanding of what planned actions - like realizing the experiment under consideration - could be.

1. Second law in the standard sense does not favor - perhaps even not allow - realization of planned actions. ZEO forces a generalization of thermodynamics: dissipation with a non-standard arrow of time for a subsystem would look like self-organization and planned action and its realization.

   Could most if not all planned action be like this - induced by BSFR in the geometric future and only apparently planned? There would be however the experience of planning and realizing induced by the signals from geometric future by a higher level in the hierarchy of conscious entities predicted by TGD! In long time scales we would be realizing our fates or wishes of higher level conscious entities rather than agents with completely free will.

2. The notion of magnetic body (MB) serving as a boss of ordinary matter would be central. MB carries dark matter as $h_{eff} = nh_0$ phases of ordinary matter with n serving as a measure for algebraic complexity of extension of rationals as its dimension and defining a kind of universal IQ. There is a hierarchy of these phases and MBs labelled by extension of rationals and the value of n.

   MBs would form a hierarchy of bosses - a realization for master slave hierarchy. Ordinary matter would be at the bottom and its coherent behavior would be induced from quantum coherence at higher levels. BSFR for higher level MB would give rise to what looks like planned actions and experienced as planned action at the lower levels of hierarchy. One could speak of planned actions inducing a cascade of planned actions in shorter time scales and eventually proceeding to atomic level.

A-8 Some notions relevant to TGD inspired consciousness and quantum biology

Below some notions relevant to TGD inspired theory of consciousness and quantum biology.

A-8.1 The notion of magnetic body

Topological field quantization inspires the notion of field body about which magnetic body is especially important example and plays key role in TGD inspired quantum biology and consciousness theory. This is a crucial departure fromt the Maxwellian view. Magnetic body brings in third level to the description of living system as a system interacting strongly with environment. Magnetic body would serve as an intentional agent using biological body as a motor instrument and sensory receptor. EEG would communicated the information from biological body to magnetic body and Libet's findings from time delays of consciousness support this view.

The following pictures illustrate the notion of magnetic body and its dynamics relevant for quantum biology in TGD Universe.

Fig. 18. Magnetic body associated with dipole field. http://tgdtheory.fi/appfigures/fluxquant.jpg

Fig. 19. Illustration of the reconnection by magnetic flux loops. http://tgdtheory.fi/appfigures/reconnect1.jpg

Fig. 20. Illustration of the reconnection by flux tubes connecting pairs of molecules. http://tgdtheory.fi/appfigures/reconect2.jpg

Fig. 21. Flux tube dynamics. a) Reconnection making possible magnetic body to "recognize" the presence of another magnetic body, b) braiding, knotting and linking of flux tubes making possible topological quantum computation, c) contraction of flux tube in phase transition reducing the value of h_{eff} allowing two molecules to find each other in dense molecular soup. http://tgdtheory.fi/appfigures/fluxtubedynamics.jpg

A-8.2 Number theoretic entropy and negentropic entanglement

TGD inspired theory of consciousness relies heavily p-Adic norm allows an to define the notion of Shannon entropy for rational probabilities (and even those in algebraic extension of rationals) by replacing the argument of logarithm of probability with its p-adic norm. The resulting entropy can be negative and the interpretation is that number theoretic entanglement entropy defined by this formula for the p-adic prime minimizing its value serves as a measure for conscious information. This negentropy characterizes two-particle system and has nothing to do with the formal negative negentropy assignable to thermodynamic entropy characterizing single particle. Negentropy Maximization Principle (NMP) implies that number theoretic negentropy increases during evolution by quantum jumps. The condition that NMP is consistent with the standard quantum measurement theory requires that negentropic entanglement has a density matrix proportional to unit matrix so that in 2-particle case the entanglement matrix is unitary.

Fig. 22. Schrödinger cat is neither dead or alive. For negentropic entanglement this state would be stable. http://tgdtheory.fi/appfigures/cat.jpg

A-8.3 Life as something residing in the intersection of reality and p-adicities

In TGD inspired theory of consciousness p-adic space-time sheets correspond to space-time correlates for thoughts and intentions. The intersections of real and p-adic preferred extremals consist of points whose coordinates are rational or belong to some extension of rational numbers in preferred embedding space coordinates. They would correspond to the intersection of reality and various p-adicities representing the "mind stuff" of Descartes. There is temptation to assign life to the intersection of realities and p-adicities. The discretization of the chart map assigning to real space-time surface its p-adic counterpart would reflect finite cognitive resolution.

At the level of "world of classical worlds" (WCW) the intersection of reality and various p-adicities would correspond to space-time surfaces (or possibly partonic 2-surfaces) representable in terms of rational functions with polynomial coefficients with are rational or belong to algebraic extension of rationals.

The quantum jump replacing real space-time sheet with p-adic one (vice versa) would correspond to a buildup of cognitive representation (realization of intentional action).

Fig. 23. The quantum jump replacing real space-time surface with corresponding p-adic manifold can be interpreted as formation of though, cognitive representation. Its reversal would correspond to a transformation of intention to action. http://tgdtheory.fi/appfigures/padictoreal.jpg

A-8.4 Sharing of mental images

The 3-surfaces serving as correlates for sub-selves can topologically condense to disjoint large space-time sheets representing selves. These 3-surfaces can also have flux tube connections and this makes possible entanglement of sub-selves, which unentangled in the resolution defined by the size of sub-selves. The interpretation for this negentropic entanglement would be in terms of sharing of mental images. This would mean that contents of consciousness are not completely private as assumed in neuroscience.

Fig. 24. Sharing of mental images by entanglement of subselves made possible by flux tube connections between topologically condensed space-time sheets associated with mental images. http://tgdtheory.fi/appfigures/sharing.jpg

A-8.5 Time mirror mechanism

Zero energy ontology (ZEO) is crucial part of both TGD and TGD inspired consciousness and leads to the understanding of the relationship between geometric time and experience time and how the arrow of psychological time emerges. One of the basic predictions is the possibiity of negative energy signals propagating backwards in geometric time and having the property that entropy basically associated with subjective time grows in reversed direction of geometric time. Negative energy signals inspire time mirror mechanism (see **Fig.** http://tgdtheory.fi/appfigures/timemirror.jpg or **Fig.** 24 in the appendix of this book) providing mechanisms of both memory recall, realization of intentational action initiating action already in geometric past, and remote metabolism. What happens that negative energy signal travels to past and is reflected as positive energy signal and returns to the sender. This process works also in the reverse time direction.

Fig. 25. Zero energy ontology allows time mirror mechanism as a mechanism of memory recall. Essentially "seeing" in time direction is in question. http://tgdtheory.fi/appfigures/timemirror.jpg

Acknowledgements: I am grateful for Dainis Zeps for enlightnening discussions concerning CP_2 geometry.

REFERENCES

Mathematics

[A1] Atyiah-Singer index-theorem. Available at: https://en.wikipedia.org/wiki/Atiyah-Singer_index_theorem.

[A2] Einstein manifold. Available at: https://en.wikipedia.org/wiki/Einstein_manifold.

[A3] Frobenius theorem (differential topology). Available at: https://en.wikipedia.org/wiki/Frobenius_theorem_(differential_topology).

[A4] Gaussian Mersenne. Available at: https://primes.utm.edu/glossary/xpage/GaussianMersenne.html.

[A5] Groupoid. Available at: https://en.wikipedia.org/wiki/Groupoid.

[A6] Hamiltonian cycles on icosahedron? Available at: https://cs.smith.edu/~orourke/MathOverflow/hpaths.html.

[A7] Hyperbolic manifold. Available at: https://en.wikipedia.org/wiki/Hyperbolic_manifold.

[A8] Icosahedral graph. Wolfram MathWorld. Available at: https://mathworld.wolfram.com/IcosahedralGraph.html.

[A9] Kac-Moody algebra. Available at: https://en.wikipedia.org/wiki/Kac\OT1\textendashMoody_algebra.

[A10] Mostow Rigidity Theorem. Available at: https://en.wikipedia.org/wiki/Mostow_rigidity_theorem.

[A11] Octonions. Available at: https://en.wikipedia.org/wiki/Octonions.

[A12] Pfaffian system. Available at: https://en.wikipedia.org/wiki/Pfaffian_system.

[A13] Quaternions. Available at: https://en.wikipedia.org/wiki/Quaternion.

[A14] Ricci flow. Available at: https://en.wikipedia.org/wiki/Ricci_flow.

[A15] Riemann-Hilbert problem. Available at: https://en.wikipedia.org/wiki/Riemann-Hilbert_problem.

[A16] Super Virasoro algebra. Available at: https://en.wikipedia.org/wiki/Super_Virasoro_algebra.

[A17] Symmetrical Icosahedral Hamiltonians. Available at: https://www.flickr.com/photos/edwynn/sets/72157625709580605/.

[A18] Thurston's theorem. Available at: https://en.wikipedia.org/wiki/Thurston's_theorem.

[A19] Why are there 1024 Hamiltonian cycles on an icosahedron? Available at: https://tinyurl.com/pmghcwd.

[A20] Yangian symmetry. Available at: https://en.wikipedia.org/wiki/Yangian.

[A21] Connes A. Heart bit #1, 2007.Available at: https://noncommutativegeometry.blogspot.com/2007/10/heart-bit-1.html.

[A22] Hopkins B. Hamiltonian paths on Platonic graphs. *IJMMS*, 30:1613–1616, 2004. Available at: https://tinyurl.com/o84ahk6.

[A23] Mandelbrot B. *The Fractal Geometry of Nature* . Freeman, New York, 1977.

[A24] Shafarevich IR Borevich ZI. *Number Theory.* Academic Press, 1966.

[A25] Pope CN. Eigenfunctions and $Spin^c$ Structures on CP_2, 1980.

[A26] Kreimer D Connes A. *Hofp algebras, renormalization, and non-commutative geometry*, volume 1999. Kluwer, 1998. Available at: https://arxiv.org/abs/hep-th/9912092.

[A27] Freed DS. The Geometry of Loop Groups, 1985.

[A28] Witten E. Quantum field theory and the Jones polynomial. *Comm Math Phys*, 121:351–399, 1989.

[A29] Zeeman EC. *Catastrophe Theory.* Addison-Wessley Publishing Company, 1977.

[A30] Hanson J Eguchi T, Gilkey B. *Phys Rep*, 66, 1980.

[A31] Eisenhart. *Riemannian Geometry.* Princeton University Press, 1964.

[A32] Jones FR. *Braid groups, Hecke algebras and type II_1 factors.* 1983.

[A33] Pope CN Gibbons GW. CP_2 as gravitational instanton. *Comm Math Phys*, 55, 1977.

[A34] Pope CN Hawking SW. Generalized Spin Structures in Quantum Gravity. *Phys Lett*, (1), 1978.

[A35] N. Hitchin. Kählerian twistor spaces. *Proc London Math Soc*, 8(43):133–151, 1981.. Available at: https://tinyurl.com/pb8zpqo.

[A36] Besse AL. Arthur L (Besse is a pseudonym for a group of mathematicians mostly in France). *Einstein Manifolds.* Springer Verlag, 1987.

[A37] Dixmier J. *Von Neumann Algebras.* North-Holland, Amsterdam, 1981. First published in French in 1957: Les Algebres d'Operateurs dans l'Espace Hilbertien, Paris: Gauthier-Villars.

[A38] Milnor J. *Topology form Differential Point of View.* The University Press of Virginia, Virginia, 1965.

[A39] Baez JC. The Octonions. *Bull Amer Math Soc*, 39(2002), 2001. Available at: https://math.ucr.edu/home/baez/Octonions/octonions.html.

[A40] Gunaydin M. $N = 4$ superconformal algebras and gauges Wess-Zumino-Witten models. *Phys Rev D*, 47(8), 1993.

[A41] Spivak M. *Differential Geometry I,II,III,IV.* Publish or Perish, Boston, 1970.

[A42] Thom R. *Comm Math Helvet*, 28, 1954.

[A43] Helgason S. *Differential Geometry and Symmetric Spaces.* Academic Press, New York, 1962.

[A44] Sawin S. Links, Quantum Groups, and TQFT's, 1995.Available at: https://arxiv.org/abs/q-alg/9506002.

[A45] Tao T. 2006 ICM: Etienne Ghys, Knots and dynamics, 2007. Available at: https://tinyurl.com/omdo949.

[A46] Jones V. In and around the origin of quantum groups, 2003. Available at: https://arxiv.org/abs/math/0309199.

[A47] Wallace. *Differential Topology.* W. A. Benjamin, New York, 1968.

Theoretical Physics

[B1] Black hole thermodynamics. Available at: https://en.wikipedia.org/wiki/Black_hole_thermodynamics.

[B2] Chern-Simons theory. Available at: https://en.wikipedia.org/wiki/Chern\OT1\textendashSimons_theory.

[B3] Korteweg-de Vries equation. Available at: https://en.wikipedia.org/wiki/Korteweg\OT1\textendashde_Vries_equation.

[B4] Lax pair. Available at: https://en.wikipedia.org/wiki/Lax_pair.

[B5] Montonen Olive Duality. Available at: https://en.wikipedia.org/wiki/Montonen-Olive_duality.

[B6] Non-linear Schrödinger equation. Available at: https://en.wikipedia.org/wiki/Non-linear_Schr\unhbox\voidb@x\bgroup\let\unhbox\voidb@x\setbox\@tempboxa\hbox{o\global\mathchardef\accent@spacefactor\spacefactor}\let\begingroup\endgroup\relax\let\ignorespaces\relax\accent127o\egroup\spacefactor\accent@spacefactordinger_equation.

[B7] Rayleigh-Taylor instability. Available at: https://en.wikipedia.org/wiki/Rayleigh\OT1\textendashTaylor_instability.

[B8] Sine-Gordon equation. Available at: https://en.wikipedia.org/wiki/Sine-Gordon.

[B9] Israel Gelfand, 2009. Available at: https://terrytao.wordpress.com/2009/10/07/israel-gelfand/#more-2860.

[B10] Kobakhidze A. Gravity is not an entropic force, 2010. Available at: https://arxiv.org/abs/1009.5414.

[B11] Lakthakia A. *Beltrami Fields in Chiral Media*, volume 2. World Scientific, Singapore, 1994.

[B12] Verlinde A. On the Origin of Gravity and the Laws of Newton, 2010. Available at: https://arxiv.org/abs/1001.0785.

[B13] Zee A. *The Unity of Forces in the Universe.* World Sci Press, Singapore, 1982.

[B14] Trnka Y Arkani-Hamed N. The Amplituhedron, 2013. Available at: https://arxiv.org/abs/1312.2007.

[B15] Drell S Björken J. *Relativistic Quantum Fields.* Mc Graw-Hill, New York, 1965.

[B16] Tureanu A Chaichian M, Oksanen M. On gravity as an entropic force, 2011. Available at: https://arxiv.org/abs/1104.4650.

[B17] Martin D. Comments on Cahill's quantum foam inflow theory of gravity, 2004. Available at: https://arxiv.org/pdf/gr-qc/0407059v4.

[B18] Rapoport D. Stochastic processes in conformal Riemann-Cartan-Weyl gravitation, 1991. Available at: https://link.springer.com/article/10.1007/BF00675614.

[B19] Goodstein DL. *States of Matter.* Prentice-Hall Inc., 1975.

[B20] Witten E Dolan L, Nappi CR. Yangian Symmetry in $D = 4$ superconformal Yang-Mills theory, 2004. Available at: https://arxiv.org/abs/hep-th/0401243.

[B21] Plefka J Drummond J, Henn J. Yangian symmetry of scattering amplitudes in $\mathcal{N} = 4$ super Yang-Mills theory, 2009. Available at: https://cdsweb.cern.ch/record/1162372/files/jhep052009046.pdf.

[B22] Verlinde E. Emergent Gravity and the Dark Universe, 2016. Available at:https://tinyurl.com/grwc3fz.

[B23] Arkani-Hamed N et al. The All-Loop Integrand For Scattering Amplitudes in Planar N=4 SYM, 2010. Available at: https://arxiv.org/abs/1008.2958.

[B24] Minev ZK et al. To catch and reverse a quantum jump mid-flight, 2019. Available at: https://arxiv.org/abs/1803.00545.

[B25] Ghrist G Etnyre J. Generic hydrodynamic instability of curl eigen fields, 2003. Available at: https://arxiv.org/abs/math/0306310.

[B26] Parisi G. *Field Theory, Disorder and Simulations.* World Scientific, 1992.

[B27] Marsh GE. *Helicity and Electromagnetic Field Topology.* World Scientific, 1995.

[B28] Marsh GE. *Helicity and Electromagnetic Field Topology.* World Scientific, 1995.

[B29] Julien R Guinier A. *The Solid State: From Superconductors to Superalloys.* Oxford University Press, Oxford, 1987.

[B30] Haken H. *Information and Self-Organization.* Springer Verlag, Berlin, 1988.

[B31] Zuber J-B Itzykson C. *Quantum Field Theory.* Mc Graw-Hill, New York, 1980.

[B32] Huang K. *Quarks,Leptons & Gauge Fields.* World Scientific, 1982.

[B33] Pitaevski LP Lifshitz EM. *Relativistic Quantum Theory: Part 2.* Pergamon Press, 1974.

[B34] Stanisic MM. *The Mathematical Theory of Turbulence.* Springer Verlag, 1985.

[B35] O'Connell D Monteiro R. The Kinematic Algebra From the Self-Dual Sector, 2011. Available at: https://arxiv.org/abs/1105.2565.

[B36] Bogoyavlenskij OI. Exact unsteady solutions to the Navier-Stokes equations and viscous MHD equations. *Phys Lett A*, pages 281–286, 2003.

[B37] Dorey P. Exact S-matrices, 1998. Available at: https://arxiv.org/abs/hep-th/9810026.

[B38] Shaffman PG. *Topics in Nonlinear Physics.* Springer Verlag, 1968.

[B39] Ludl PO. Comments on the classification of the finite subgroups of SU(3), 2011. Available at: https://arxiv.org/pdf/1101.2308.pdf.

[B40] Kiehn R. Electromagnetic waves in vacuum with torsion and spin. Available at: https://www22.pair.com/csdc/pdf/helical6.pdf.

[B41] Kiehn R. Hydrodynamics wakes and minimal surfaces with fractal boundaries. Available at: https://www22.pair.com/csdc/pdf/barcelon.pdf.

[B42] Kiehn R. Instability patterns, wakes and topological limit sets. Available at: https://arxiv.org/pdf/physics/0102005.pdf.

[B43] Kiehn R. Topological Torsion and Thermodynamic Irreversibility, 2001. Available at: https://www22.pair.com/csdc/pdf/irevtors.pdf.

[B44] Cutkosky RE. *J Math Phys*, (1):429–433, 1960.

[B45] Cahill RT. Gravity as Quantum Foam In-Flow, 2003. Available at: https://arXiv.org/abs/physics/0307003.

[B46] Hossenfelder S. Comments on and comments on comments on verlinde's paper "on the origin of gravity and the laws of newton", 2011. Available at: https://tinyurl.com/ondxowl.

[B47] Jacobson T. Thermodynamics of Spacetime: The Einstein Equation of State, 1995. Available at: https://arxiv.org/abs/gr-qc/9504004.

Particle and Nuclear Physics

[C1] ICARUS Proves Neutrinos More Than 10 Times Faster Than Light. Available at: https://tinyurl.com/3uk6g7t.

[C2] ICARUS Refutes Opera's Superluminal Neutrinos. Available at: https://tinyurl.com/3q79xnb.

[C3] MINOS, 2011. Available at: https://en.wikipedia.org/wiki/MINOS.

[C4] OPERA fail to find error in Faster Than Light Measurement, 2011. Available at: https://tinyurl.com/o7868kx.

[C5] Nesti F Berezhiani Z. Magnetic anomaly in UCN trapping: signal for neutron oscillations to parallel world? *Eur Phys J C*, 72(1974), 2012. Available at: https://www.springerlink.com/content/h68g501352t57011/fulltext.pdf.

[C6] Glashow SL Cohen AG. New Constraints on Neutrino Velocities, 2011. Available at: https://arxiv.org/abs/1109.6562.

[C7] Alice Collaboration. Charged-particle multiplicity density at mid-rapidity in central Pb-Pb collisions at $\sqrt{s_{NN}}$= 2. 76 TeV, 2010. Available at: https://arxiv.org/abs/1011.3916.

[C8] ICARUS collaboration. A search for the analogue to Cherenkov radiation by high energy neutrinos at superluminal speeds in ICARUS, 2011. Available at: https://arxiv.org/abs/1110.3763.

[C9] Icarus Collaboration. Measurement of the neutrino velocity with the ICARUS detector at the CNGS beam, 2012. Available at: https://arxiv.org/pdf/1203.3433.pdf.

[C10] OPERA collaboration. Measurement of the neutrino velocity with the OPERA detector in the CNGS beam, 2011. Available at: https://arxiv.org/abs/1109.4897.

[C11] Back BB et al. *Phys Rev Lett* .. Available at: https://arxiv.org/abs/hep-ex/0208004, 89(22), 2002.

[C12] Barber BR et al. *Phys Rev*, 72(9):1380, 1994.

[C13] Brandt A et al. *Phys Lett B*, 297:417, 1992.

[C14] Derrick M et al. *Phys Lett B*, 315:481, 1993.

[C15] Nesvizhevsky VV et al. Measurement of quantum states of neutrons in the Earth's gravitational field, 2003. Available at: https://arxiv.org/abs/hep-ph/0306198.

[C16] Smith AM et al. *Phys Lett B*, 163:267, 1985.

[C17] McAlpine K. Incredible shrinking proton raises eyebrows, 2010. Available at: https://tinyurl.com/3yf4n77.

[C18] Violini G Queen NM. *Dispersion Theory in High Energy Physics*. The Macmillan Press Limited, 1974.

Condensed Matter Physics

[D1] Linear elasticity. Available at: https://en.wikipedia.org/wiki/Linear_elasticity.

[D2] Viscosity. Available at: https://en.wikipedia.org/wiki/Viscosity.

[D3] Pietiläinen P Chakrabarty T. *The Fractional Quantum Hall Efect*. Springer Verlag, 1988.

[D4] Ginsberg DM(ed). *Physical Properties of High Temperature Super Conductors, Vol I*, volume II. 1989.

[D5] Barber BR et al. *Phys Rev*, 72(9):1380, 1994.

[D6] Cao C et al. Universal Quantum Viscosity in a Unitary Fermi Gas, 2010. Available at: https://www.sciencemag.org/content/early/2010/12/08/science.1195219.

[D7] Mills R et al. Spectroscopic and NMR identification of novel hybrid ions in fractional quantum energy states formed by an exothermic reaction of atomic hydrogen with certain catalysts, 2003. Available at: https://www.blacklightpower.com/techpapers.html.

[D8] Morris DJ et al. Dirac Strings and Magnetic Monopoles in Spin Ice Dy2Ti2O7. *Physics World*, 326(5951):411–414, 2009.Available at: https://physicsworld.com/cws/article/news/40302.

[D9] Novoselov KS et al. Two-dimensional gas of massless Dirac fermions in graphene. *Nature*, 438, November 2005.

[D10] Prakapenka VB et al. Structure and properties of two superionic ice phases. *Nature Physics*, 17(11):1–6, 2021. Available at: https://www.researchgate.net/publication/355212207_Structure_and_properties_of_two_superionic_ice_phases.

[D11] Wilczek F Mackenzie R. *Rev Mod Phys . A*, 3:2827, 1988.

[D12] Anderson PW. *Rev Mod Phys*, 38, 1966.

[D13] Sachdev S. Quantum phase transitions (summary). *Physics World*, pages 33–38, April 1999.

[D14] Glashow SL. Can Science Save the World?, 1999.Available at: https://www.hypothesis.it/nobel/nobel99/eng/pro/pro_2.htm.

[D15] Tilley K Tilley DR. *Super Fluidity and Super Conductivity*. Adam Hilger Ltd, 1986.

[D16] Brown WF. *Micromagnetics*. John Wiley, 1963.

Cosmology and Astro-Physics

[E1] A brief video demonstrating the basic idea of Michelson-Morley experiment. Available at: https://www.youtube.com/watch?v=7TOd7o8X2-E. See also https://www.youtube.com/watch?v=VyePASErr5Q&NR=1.

[E2] Allais effect. Available at: https://en.wikipedia.org/wiki/Allais_effect.

[E3] Bullet Cluster. Available at: https://en.wikipedia.org/wiki/Bullet_cluster.

[E4] Cavendish experiment. Available at: https://en.wikipedia.org/wiki/Cavendish_experiment.

[E5] Cosmic Ether-Drift and Dynamic Energy in Space. Bibliography and Resources. Available at: https://www.orgonelab.org/energyinspace.htm.

[E6] French Nobel Laureate turns back clock: Marshall's global experiment, von Braun memories evoked during August 11 solar eclipse. Available at: https://science.nasa.gov/newhome/headlines/ast12oct99_1.htm.

[E7] Kerr metric. Available at: https://en.wikipedia.org/wiki/Kerr_metric.

[E8] Kilogram. Available at: https://en.wikipedia.org/wiki/Kilogram.

[E9] Mars. Available at: https://en.wikipedia.org/wiki/Mars.

[E10] Michelson-Morley experiment. Available at: https://en.wikipedia.org/wiki/Michelson-Morley.

[E11] The Controversy over Newton's Gravitational Constant. Available at: https://www.npl.washington.edu/eotwash/experiments/bigG/bigG.html.

[E12] *Nat. V.*, 259:87, 1996.

[E13] *Astron. Nachr.*, 318:121–128, 1997.

[E14] Dark energy. *Physics World*, 2004. Available at: https://physicsworld.com/cws/article/print/19419.

[E15] Solar Activity, Wave of Kotov and Strange Coincidences, 2012. Available at:https://lempel.pagesperso-orange.fr/soleil_onde_kotov_uk.htm.

[E16] Evidence for a 160 minute oscillation affecting Galaxies and the Solar System, 2012. Available at:https://tinyurl.com/y5en9cxz.

[E17] Middleditch A. A White Dwarf Merger Paradigm for Supernovae and Gamma -Ray Bursts, 2003. Available at: https://arxiv.org/abs/astro-ph/0311484.

[E18] Durrant CJ. *The Atmosphere of the Sun.* IOP Publishing Ltd., 1988.

[E19] Markevitch M Clowe D, Gonzales A. Weak lensing mass reconstruction of the interacting cluster 1E0657-558: Direct evidence for the existence of dark matter, 2003. Available at: https://arxiv.org/astro-ph/0312273.

[E20] Will CM. Confrontation between General Relativity and Experiment, 2006.Available at: https://arxiv.org/PS_cache/gr-qc/pdf/0510/0510072v2.pdf.

[E21] Boggs SE Coburn W. Polarization of Prompt gamma-ray emission from the gamma-ray burst of 6 December 2002. *Nature*, 2003. Available at: https://arxiv.org/abs/astro-ph/0305377. See also *Twisted secrets of gamma-ray bursts*. New Scientist Vol. 178 No. 2398 -07 June 2003, page 25..

[E22] MAGIC collaboration. Probing Quantum Gravity using Photons from a Mkn 501 Flare observed in MAGIC, 2007Available at: https://arxiv.org/abs/0708.2889.

[E23] The CHIME/FRB Collaboration. Periodic activity from a fast radio burst source. *Nature*, 582:351–355, 2020. Available at: https://www.nature.com/articles/s41586-020-2398-2. .

[E24] Everitt CWF. In *Results from Gravity Probe B*, 2007. Available at: https://meetings.aps.org/Meeting/APR07/Event/64567.

[E25] Nottale L Da Rocha D. Gravitational Structure Formation in Scale Relativity, 2003. Available at: https://arxiv.org/abs/astro-ph/0310036.

[E26] Rujula De A Dado S, Dar A. The Supernova associated with GRB030329, 2003. Available at: https://arxiv.org/abs/astro-ph/0304106.

[E27] Wittem E. Searching for a Black Hole in the Outer Solar System, 2020.Available at: https://arxiv.org/abs/2004.14192.

[E28] Chi PJ et al. InSight Observations of Magnetic Pulsations on Martian Surface: Initial Findings and Implications. *EPSJ abstracts, EPSC-DPS2019-838-1*, 13, 2019. Available at:https://tinyurl.com/y3ll8kcg.

[E29] Clove D et al. A direct empirical proof of the existence of dark matter, 2006. Available at: https://arxiv.org/abs/astro-ph/0608407.

[E30] Hjorth J et al. A very energetic supernova associated with the gamma-ray burst of 29 March 2003. *Nature*, 423, 2003.

[E31] Kapner DJ et al. Tests of the Gravitational Inverse-Square Law below the Dark-Energy Length Scale. *Phys Rev Lett*, 98(021101), 2007.Available at: https://arxiv.org/abs/hep-ph/0611184.

[E32] Meikle W et al. *Nature*, 329, 1987.

[E33] Mereghetti S et al. INTEGRAL Discovery of a Burst with Associated Radio Emission from the Magnetar SGR 1935+2154. *The Astrophysical Journal Letters*, 2020. Available at: https://iopscience.iop.org/article/10.3847/2041-8213/aba2cf. .

[E34] Mihaila I et al. A new confirmation of the Allais effect during the solar eclipse of 31 2003 May, 2003. Available at: https://home.t01.itscom.net/allais/blackprior/mihaila/2003/mihaila2003.pdf.

[E35] Pierce JM et al. *Nature*, 371, 1994.

[E36] Schaber R et al. The Resurfacing History of Venus, 1997. Available at: https://www.researchgate.net/publication/234304451_The_Resurfacing_History_of_Venus.

[E37] Tajmar M et al. Experimental Detection of Gravimagnetic London Moment, 2006. Available at: https://arxiv.org/abs/gr-gc0603033. See also the popular article "Towards new tests of general relativity", at https://www.esa.int/SPECIALS/GSP/SEM0L6OVGJE_0.html.

[E38] Tate J et al. *Phys Rev*, 62(8), 1989.

[E39] Wang Q-S et al. Precise measurement of gravity variations during a total solar eclipse. *Phys Rev D*, 62, 2000. Available at: https://home.t01.itscom.net/allais/blackprior/wang/wangetal.pdf.

[E40] Zirin H. *Astrophysics of the Sun.* Cambridge University Press, Cambridge, 1988.

[E41] Romeo I and Turcotte DL. Resurfacing on Venus . *Planetary and Space Science*, 58(10):1374–1380, 2010. Available at: https://doi.org/10.1016/j.pss.2010.05.022.

[E42] Antonescu V Jeverdan GT, Rusu GI. Experiments using the Foucault pendulum during the solar eclipse of 15 February, 1961. *Bibl Astr*, 1(55), 1981.

[E43] Katz JI. *High Energy Astrophysics.* Addison Wesley Publishing Company, 1987.

[E44] Perivolaropoulos L and Kazantzidis L. Hints of Modified Gravity in Cosmos and in the Lab?, 2019. Available at:https://arxiv.org/pdf/1904.09462.pdf.

[E45] Allais M. Should the Laws of Gravitation Be Reconsidered: Part I,II,III?, 1959. Available at: https://home.t01.itscom.net/allais/blackprior/allais/lawgravit/lawgrav-one.pdf.

[E46] Grusenick M. Extended Michelson-Morley interferometer experiment, 2009. Available at: https://www.youtube.com/watch?v=7T0d7o8X2-E.

[E47] Moshina M. The surface ferrite layer of Sun, 2005. Available at: https://www.thesurfaceofthesun.com/TheSurfaceOfTheSun.pdf.

[E48] Lelli F McGaugh SS and Schombert JM. The radial acceleration relation in rotationally supported galaxies, 2016. Available at:https://arxiv.org/pdf/1609.05917v1.pdf.

[E49] Ulianov PO. The Witte Effect: The Neutrino Speed and The Anisotropy of the Light Speed, as Defined in the General Theory of Relativity, 2011. Available at: https://www.atomlig.com.br/poli/WitteEffect-IG.pdf.

[E50] Olenici D Popescu VA. A confirmation of the Allais and Jeverdan-Rusu-Antonescu effects during the solar eclipse from 22 September 2006, and the quantization behavior of pendulum, 2007. Available at: https://www.hessdalen.org/sse/program/Articol.pdf.

[E51] Cahill RT. Absolute Motion and Gravitational Effects. *Apeiron*, 11:53–111, 2004. Available at: https://redshift.vif.com/JournalFiles/V11NO1PDF/V11N1CA2.pdf.

[E52] Cahill RT. A New Light-Speed Anisotropy Experiment: Absolute Motion and Gravitational Waves Detected. *Progr Phys*, 25:73–92, 2006. Available at: https://arxiv.org/abs/physics/0610076.

[E53] Cahill RT. The Roland De Witte 1991 Experiment (to the Memory of Roland De Witte). *Progr Phys*, 3:60–65, 2006. Available at: https://www.orgonelab.org/EtherDrift/CahillDeWitte2006.pdf.

[E54] Weinberg S. *Gravitation and Cosmology.* Wiley, New York, 1967.

[E55] Teukolsky SA Shapiro SL. *Black Holes, White Dwarfs and Neutron Stars.* John Wiley, 1986.

[E56] Farrar G Springel V. The speed of the 'bullet' in the merging galaxy cluster 1E0657-56, 2007.Available at: https://arxiv.org/abs/astro-ph/0703232.

[E57] Matos de CJ Tajmar M. Local Photon and Graviton Mass and Its Consequences, 2006. Available at: https://arxiv.org/abs/gr-gc0603032.

[E58] Sao CG et al Yang SQ. Measurements of the gravitational constant using two independent methods. *Nature*, 560:582–588, 2018. Available at:https://www.nature.com/articles/s41586-018-0431-5.

[E59] Merali Z. Dark Matter gets its own dark force. *New Scientist*, (2587), 2007.

Physics of Earth

[F1] A challenge to all geologists of Earth. Available at: https://www.nealadams.com/challenge.html.

[F2] Continental drift. Available at: https://en.wikipedia.org/wiki/Continental_drift.

[F3] Cryogenian. Available at: https://en.wikipedia.org/wiki/Cryogenian.

[F4] Ediacaran. Available at: https://en.wikipedia.org/wiki/Ediacaran.

[F5] Expanding Earth Theory. Available at: https://en.wikipedia.org/wiki/Expanding_earth_theory.

[F6] Geomagnetic reversal. Available at: https://en.wikipedia.org/wiki/Geomagnetic_reversal.

[F7] Glacial. Available at: https://en.wikipedia.org/wiki/Glacial.

[F8] Glaciation. Available at: https://en.wikipedia.org/wiki/Glaciation.

[F9] Great Unconformity. Available at: https://en.wikipedia.org/wiki/Great_Unconformity.

[F10] Ice age. Available at: https://en.wikipedia.org/wiki/Ice_age.

[F11] Ionosphere. Available at: https://en.wikipedia.org/wiki/Ionosphere.

[F12] Kennelly-Heaviside layer. Available at: https://en.wikipedia.org/wiki/Kennelly-Heaviside_layer.

[F13] Late precambrian supercontinent and ice house world. Available at: https://scotese.com/precambr.htm.

[F14] Magnetic field intensity. Available at: https://www.geo.physics.helsinki.fi/english/eng_magnetic.html.

[F15] Magnetostratigraphy. Available at: https://en.wikipedia.org/wiki/Magnetostratigraphy.

[F16] Mesozoic. Available at: https://en.wikipedia.org/wiki/Mesozoic.

[F17] Mirovia. Available at: https://en.wikipedia.org/wiki/Mirovia.

[F18] Neoproterozoic. Available at: https://en.wikipedia.org/wiki/Neoproterozoic.

[F19] Oceanic trench. Available at: https://en.wikipedia.org/wiki/Oceanic_trench.

[F20] Orogenies. Available at: https://en.wikipedia.org/wiki/Orogenies.

[F21] Orogeny. Available at: https://en.wikipedia.org/wiki/Orogeny.

[F22] Paleomagnetism. Available at: https://en.wikipedia.org/wiki/Paleomagnetism.

[F23] Pangea. Available at: https://en.wikipedia.org/wiki/Pangea.

[F24] Pannotia. Available at: https://en.wikipedia.org/wiki/Pannotia.

[F25] Panthalassic. Available at: https://en.wikipedia.org/wiki/Panthalassic_Ocean.

[F26] Plate tectonics. Available at: https://en.wikipedia.org/wiki/Plate_tectonics.

[F27] Rift. Available at: https://en.wikipedia.org/wiki/Rift.

[F28] Rodinia. Available at: https://en.wikipedia.org/wiki/Rodinia.

[F29] Snowball Earth. Available at: https://en.wikipedia.org/wiki/Snowball_earth.

[F30] Supercontinent cycle. Available at: https://en.wikipedia.org/wiki/Supercontinent_cycle.

[F31] The Cryogenian Period. Available at: https://www.palaeos.com/Proterozoic/Neoproterozoic/Cryogenian/Cryogenian.html.

[F32] Tides. Available at: https://en.wikipedia.org/wiki/Tides.

[F33] Tillite. Available at: https://en.wikipedia.org/wiki/Tillite.

[F34] Timeline of natural history. Available at: https://en.wikipedia.org/wiki/Timeline_of_natural_history.

[F35] Tonian. Available at: https://en.wikipedia.org/wiki/Tonian.

[F36] Volcano. Available at: https://en.wikipedia.org/wiki/Volcano.

[F37] Dong J et al. Constraining the Volume of Earth's Early Oceans With a Temperature-Dependent Mantle Water Storage Capacity Model. *AGU Advances*, 2(1), 2021. Available at: https://doi.org/10.1029/2020AV000323.

[F38] Giuliani A et al. Perturbation of the deep-Earth carbon cycle in response to the Cambrian Explosion. *Science Advances*, 8(9), 2022. Available at: https://www.science.org/doi/10.1126/sciadv.abj1325.

[F39] Hoffman PF et al. A Neoproterozoic Snowball Earth. *Science*, 281:1342, 1998. Available at: https://www.snowballearth.org/pdf/Hoffman_Science1998.pdf.

[F40] Januszczak N Eyles N. "Zipper-rift": A tectonic model for Neoproterozoic glaciations during the breakup of Rodinia after 750 Ma. *Earth-Sci Rev*, 65:1–73, 2004. Available at: https://tinyurl.com/pqfvu28.

[F41] Kennedy MJ Fairchild IJ. Neoproterozoic glaciations in the Earth System. *J Geol Soc*, 164:895–921, 2007.

[F42] Peters S Gaines E. *New Scientist*, 218(2921), 2013. Available at: https://tinyurl.com/nenk8nq.

[F43] Salminen J. Paleomagnetic and rock magnetic study with emphasis on the Precambrian intrusions and impact structures in Fennoscandia and South Africa, 2009. Available at: https://oa.doria.fi/handle/10024/44628. University of Helsinki Department Of Physics Report Series In Geophysics No. 59 Paleomagnetic.

[F44] Kirschvink JL. When All of the Oceans Were Frozen. *Recherche*, 355:26–30, 2002. Available at: https://www.gps.caltech.edu/~jkirschvink/pdfs/laRechercheEnglish.pdf.

[F45] Mantovani R. Available at: https://en.wikipedia.org/wiki/Roberto_Mantovani.

[F46] Adams N. Conspiracy of Sci, Earth is in fact growing, 2006. Available at: https://tinyurl.com/pqfvu28.

[F47] Eerola T. "A tropical paradise"? Neoproterozoic glaciations from the southern Brazilian perspective. 2002. Available at: https://www.geologinenseura.fi/geologi-lehti/2-2007/eerola.pdf.

Fringe Physics

[H1] Selvam AM. Quantum-like Chaos in Prime Number Distribution and in Turbulent Fluid Flows, 2000. Available at: https://arxiv.org/abs/physics/0005067.

[H2] OPERA Collaboration. Measurement of the neutrino velocity with the OPERA detector in the CNOS beam, 2011. Available at: https://arxiv.org/abs/1109.4897.

[H3] Modanese G. On the theoretical interpration of E. Podkletnov's experiment, 1996. Available at: https://www.gravity.org/ont.html.

[H4] Schnurer J. Demonstration of transient weak gravitational shielding by a YBCO LEVHEX at the super-conducting transition, 1999. Available at: https://www.gravity.org/exp.htm.

[H5] Pappas PT Obolensky A. Thirty six seconds faster than light. *Electronics and Wireless World*, 1988. Available at: https://wbabin.net/historical/pappas.pdf.

[H6] Nieminen R Podkletnov E. Weak gravitational shielding properties of composite bulk YBa2Cu3O7-x super-conductor below 70 K under electro-magnetic field, 1992. Available at: https://arxiv.org/abs/cond-mat/9701074. Report MSU-chem 95, improved version at https://arxiv.org/abs/cond-mat/9701074.

[H7] Giani S. On velocities beyond the speed of light c, 1998. Available at: https://arxiv.org/abs/hep-ph/9712265.

Biology

[I1] Cambrian explosion. Available at: https://en.wikipedia.org/wiki/Cambrian_explosion.

[I2] Dinosaurs. Available at: https://en.wikipedia.org/wiki/Dinosaur.

[I3] Marine biology. Available at: https://en.wikipedia.org/wiki/Marine_biology#Deep_sea_and_trenches.

[I4] Permian-Triassic extinction event. Available at: https://en.wikipedia.org/wiki/Permian-Triassic_extinction_event.

[I5] The Fourth Phase of Water: Dr. Gerald Pollack at TEDxGuelphU, 2014. Available at: https://www.youtube.com/watch?v=i-T7tCMUDXU.

[I6] Tetlie OE Braddy SJ, Poschmann M. Giant claw reveals the largest ever arthropod. *Biol Lett*, 2007. Available at: https://www.journals.royalsoc.ac.uk/content/t15r2588mn27n0w1.

[I7] Murogoki P Comorosan S, Hristea M. On a new symmetry in biological systems. *Bull Math Biol*, page 107, 1980.

[I8] Drake H et al. Anaerobic consortia of fungi and sulfate reducing bacteria in deep granite fractures. *Nature Communications*, 8(55), 2017. Available at: https://www.nature.com/articles/s41467-017-00094-6.

[I9] He T et al. Possible links between extreme oxygen perturbations and the Cambrian radiation of animals. *Nature Geoscience*, 12:468–474, 2019. Available at: https://www.nature.com/articles/s41561-019-0357-z.

[I10] Klatt JM et al. Possible link between Earth's rotation rate and oxygenation. *Nature Geoscience*, 14:564–570, 2021. Available at: https://www.nature.com/articles/s41561-021-00784-3.

[I11] von Stedingk K Hammarlund EU and På;hlman S. Refined control of cell stemness allowed animal evolution in the oxic realm. *Nature Ecology & Evolution*, 2:220–228, 2018. Available at:https://www.nature.com/articles/s41559-017-0410-5.

[I12] O′Donoghue J. How trees changed the world? *New Scientist*, 2631, 2007. Available at: https://tinyurl.com/our2ldk.

[I13] Zhao Q Pollack GH, Figueroa X. Molecules, water, and radiant energy: new clues for the origin of life. *Int J Mol Sci*, 10:1419–1429, 2009. Available at: https://tinyurl.com/ntkfhlc.

[I14] Barley S. Antarctica was climate refuge during great extinction. *New Scientist*, 2009. Available at: https://tinyurl.com/ykk7txl.

[I15] Comorosan S. On a possible biological spectroscopy. *Bull Math Biol*, page 419, 1975.

[I16] Gould SJ. *Wonderful Life.* Penguin Books, 1991.

[I17] Pollack GH Zheng J-M. Long-range forces extending from polymer-gel surfaces. *Phys Rev E*, 68:031408–, 2003. Available at: https://tinyurl.com/ntkfhlc.

Neuroscience and Consciousness

[J1] Libet B. Readiness potentials preceding unrestricted spontaneous and preplanned voluntary acts, 1982. Available at: https://tinyurl.com/jqp1. See also the article *Libet's Research on Timing of Conscious Intention to Act: A Commentary* of Stanley Klein at https://tinyurl.com/jqp1.

[J2] Blackman CF. *Effect of Electrical and Magnetic Fields on the Nervous System*, pages 331–355. Plenum, New York, 1994.

[J3] Reed D. Conceptual Hurdles to New Millennium Physics. *Explore!*, (1), 2003.

[J4] Abdelmeik H et al. Changes in electrical properties of frog sciatic nerves at low temperature: superconductor-like behaviour, 2003. Available at: https://tgdtheory.com/public_html/articles/hafpaper.pdf.

[J5] Penrose R. *Shadows of the Mind.* Vintage, 1994.

[J6] Hameroff S. Quantum Coherence in Microtubules: a Neural Basis for Emergent Consciousness? *J Consc Studies*, 1(1), 1994. Available at: https://en.wikipedia.org/wiki/Orch-OR.

[J7] Chernobrov V. Experiments on the Change of Direction and Rate of Time Motion. In *Proc. of III Int. Conf. On Problems of Space, Time, and Gravitation. Russian Academy of Sci s, St. Petersburg, Russia*, 1996.

Books related to TGD

[K1] Pitkänen M. *Topological Geometrodynamics.* 1983. Thesis in Helsinki University 1983.

[K2] Pitkänen M. DNA as Topological Quantum Computer. In *Quantum - and Classical Computation in TGD Universe. https://tgdtheory.fi/tgdhtml/Btgdcomp.html*. Available at: https://tgdtheory.fi/pdfpool/dnatqc.pdf, 2015.

[K3] Pitkänen M. Topological Quantum Computation in TGD Universe. In *Quantum - and Classical Computation in TGD Universe. https://tgdtheory.fi/tgdhtml/Btgdcomp.html*. Available at: https://tgdtheory.fi/pdfpool/tqc.pdf, 2015.

[K4] Pitkänen M. Philosophy of Adelic Physics. In *TGD as a Generalized Number Theory: Part I. https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/adelephysics.pdf, 2017.

[K5] Pitkänen M. About Nature of Time. In *TGD Inspired Theory of Consciousness: Part I. https://tgdtheory.fi/tgdhtml/Btgdconsc1.html*. Available at: https://tgdtheory.fi/pdfpool/timenature.pdf, 2023.

[K6] Pitkänen M. About Preferred Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time: Part I. https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/prext.pdf, 2023.

[K7] Pitkänen M. About the Nottale's formula for h_{gr} and the possibility that Planck length l_P and CP_2 length R are related. In *Dark Matter and TGD: https://tgdtheory.fi/tgdhtml/Bdark.html*. Available at: https://tgdtheory.fi/pdfpool/vzerovariableG.pdf, 2023.

[K8] Pitkänen M. About twistor lift of TGD? In *Quantum TGD: Part III. https://tgdtheory.fi/tgdhtml/Btgdquantum3.html*. Available at: https://tgdtheory.fi/pdfpool/hgrtwistor.pdf, 2023.

[K9] Pitkänen M. Anomalies Related to the Classical Z^0 Force and Gravitation. In *TGD and Fringe Physics. https://tgdtheory.fi/tgdhtml/Bfreenergies.html*. Available at: https://tgdtheory.fi/pdfpool/Zanom.pdf, 2023.

[K10] Pitkänen M. Are dark photons behind biophotons? In *TGD and Quantum Biology: Part I. https://tgdtheory.fi/tgdhtml/Bqbio1.html*. Available at: https://tgdtheory.fi/pdfpool/biophotonslian.pdf, 2023.

[K11] Pitkänen M. Basic Extremals of Kähler Action. In *Physics in Many-Sheeted Space-Time: Part I. https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/class.pdf, 2023.

[K12] Pitkänen M. Basic Properties of CP_2 and Elementary Facts about p-Adic Numbers. In *Quantum TGD: Part I. https://tgdtheory.fi/tgdhtml/Btgdquantum1.html*. Available at: https://www.tgdtheory.fi/public_html/pdfpool/append.pdf, 2023.

[K13] Pitkänen M. *Bio-Systems as Conscious Holograms.* Online book. Available at: https://www.tgdtheory.fi/tgdhtml/holography.html., 2023.

[K14] Pitkänen M. *Bio-Systems as Self-Organizing Quantum Systems.* Online book. Available at: https://www.tgdtheory.fi/tgdhtml/bioselforg.html., 2023.

[K15] Pitkänen M. Bio-Systems as Super-Conductors: Part I. In *Bio-Systems as Self-Organizing Quantum Systems. https://tgdtheory.fi/tgdhtml/BbioSO.html*. Available at: https://tgdtheory.fi/pdfpool/superc1.pdf, 2023.

[K16] Pitkänen M. Bio-Systems as Super-Conductors: part II. In *Bio-Systems as Self-Organizing Quantum Systems. https://tgdtheory.fi/tgdhtml/BbioSO.html*. Available at: https://tgdtheory.fi/pdfpool/superc2.pdf, 2023.

[K17] Pitkänen M. Biological Realization of Self Hierarchy. In *Bio-Systems as Self-Organizing Quantum Systems. https://tgdtheory.fi/tgdhtml/BbioSO.html*. Available at: https://tgdtheory.fi/pdfpool/bioselfc.pdf, 2023.

[K18] Pitkänen M. Breaking of CP, P, and T in cosmological scales in TGD Universe. In *Physics in Many-Sheeted Space-Time: Part II. https://tgdtheory.fi/tgdhtml/Btgdclass2.html*. Available at: https://tgdtheory.fi/pdfpool/rotuniverse.pdf, 2023.

[K19] Pitkänen M. Category Theory and Quantum TGD. In *TGD and Hyper-finite Factors. https://tgdtheory.fi/tgdhtml/BHFF.html*. Available at: https://tgdtheory.fi/pdfpool/categorynew.pdf, 2023.

[K20] Pitkänen M. Classical TGD. In *Topological Geometrodynamics: Overview: Part I:https://tgdtheory.fi/tgdhtml/Btgdview1.html*. Available at: https://tgdtheory.fi/pdfpool/tgdclass.pdf, 2023.

[K21] Pitkänen M. Comments on the recent experiments by the group of Michael Persinger. In *TGD and EEG: Part I. https://tgdtheory.fi/tgdhtml/Btgdeeg1.html*. Available at: https://tgdtheory.fi/pdfpool/persconsc.pdf, 2023.

[K22] Pitkänen M. Construction of elementary particle vacuum functionals. In *p-Adic Physics. https://tgdtheory.fi/tgdhtml/Bpadphys.html*. Available at: https://tgdtheory.fi/pdfpool/elvafu.pdf, 2023.

[K23] Pitkänen M. Construction of Quantum Theory: M-matrix. In *Quantum TGD: Part I. https://tgdtheory.fi/tgdhtml/Btgdquantum1.html*. Available at: https://tgdtheory.fi/pdfpool/towards.pdf, 2023.

[K24] Pitkänen M. Construction of Quantum Theory: Symmetries. In *Quantum TGD: Part I. https://tgdtheory.fi/tgdhtml/Btgdquantum1.html*. Available at: https://tgdtheory.fi/pdfpool/quthe.pdf, 2023.

[K25] Pitkänen M. Construction of WCW Kähler Geometry from Symmetry Principles. In *Quantum Physics as Infinite-Dimensional Geometry. https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/compl1.pdf, 2023.

[K26] Pitkänen M. Cosmic Strings. In *Physics in Many-Sheeted Space-Time: Part II. https://tgdtheory.fi/tgdhtml/Btgdclass2.html*. Available at: https://tgdtheory.fi/pdfpool/cstrings.pdf, 2023.

[K27] Pitkänen M. Could categories, tensor networks, and Yangians provide the tools for handling the complexity of TGD? In *TGD and Hyper-finite Factors. https://tgdtheory.fi/tgdhtml/BHFF.html*. Available at: https://tgdtheory.fi/pdfpool/Yangianagain.pdf, 2023.

[K28] Pitkänen M. Criticality and dark matter: part I. In *Dark Matter and TGD: https://tgdtheory.fi/tgdhtml/Bdark.html*. Available at: https://tgdtheory.fi/pdfpool/qcritdark1.pdf, 2023.

[K29] Pitkänen M. Criticality and dark matter: part II. In *Dark Matter and TGD: https://tgdtheory.fi/tgdhtml/Bdark.html*. Available at: https://tgdtheory.fi/pdfpool/qcritdark2.pdf, 2023.

[K30] Pitkänen M. Criticality and dark matter: part III. In *Dark Matter and TGD: https://tgdtheory.fi/tgdhtml/Bdark.html*. Available at: https://tgdtheory.fi/pdfpool/qcritdark3.pdf, 2023.

[K31] Pitkänen M. Dark Forces and Living Matter. In *Bio-Systems as Self-Organizing Quantum Systems. https://tgdtheory.fi/tgdhtml/BbioSO.html*. Available at: https://tgdtheory.fi/pdfpool/darkforces.pdf, 2023.

[K32] Pitkänen M. Dark Matter Hierarchy and Hierarchy of EEGs. In *TGD and EEG: Part I. https://tgdtheory.fi/tgdhtml/Btgdeeg1.html*. Available at: https://tgdtheory.fi/pdfpool/eegdark.pdf, 2023.

[K33] Pitkänen M. Dark Nuclear Physics and Condensed Matter. In *TGD and Nuclear Physics. https://tgdtheory.fi/tgdhtml/Bnucl.html*. Available at: https://tgdtheory.fi/pdfpool/exonuclear.pdf, 2023.

[K34] Pitkänen M. Does Riemann Zeta Code for Generic Coupling Constant Evolution? In *TGD as a Generalized Number Theory: Part III. https://tgdtheory.fi/tgdhtml/Btgdnumber3.html*. Available at: https://tgdtheory.fi/pdfpool/fermizeta.pdf, 2023.

[K35] Pitkänen M. Does TGD Predict a Spectrum of Planck Constants? In *Dark Matter and TGD: https://tgdtheory.fi/tgdhtml/Bdark.html*. Available at: https://tgdtheory.fi/pdfpool/Planck, 2023.

[K36] Pitkänen M. Evolution of Ideas about Hyper-finite Factors in TGD. In *Topological Geometrodynamics: Overview: Part II. https://tgdtheory.fi/tgdhtml/Btgdoverview2*. Available at: https://tgdtheory.fi/pdfpool/vNeumannnew, 2023.

[K37] Pitkänen M. Expanding Earth Model and Pre-Cambrian Evolution of Continents, Climate, and Life. In *Physics in Many-Sheeted Space-Time: Part I. https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/expearth.pdf., 2023.

[K38] Pitkänen M. From Principles to Diagrams. In *Quantum TGD: Part III. https://tgdtheory.fi/tgdhtml/Btgdquantum3.html*. Available at: https://tgdtheory.fi/pdfpool/diagrams.pdf, 2023.

[K39] Pitkänen M. Fusion of p-Adic and Real Variants of Quantum TGD to a More General Theory. In *TGD as a Generalized Number Theory: Part I. https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/mblocks.pdf, 2023.

[K40] Pitkänen M. General Ideas about Many-Sheeted Space-Time: Part II. In *Physics in Many-Sheeted Space-Time: Part I. https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/newviews.pdf, 2023.

[K41] Pitkänen M. *Genes and Memes*. Online book. Available at: https://www.tgdtheory.fi/tgdhtml/genememe.html., 2023.

[K42] Pitkänen M. Geometric Theory of Bio-Harmony. In *Genes and Memes: Part II. https://tgdtheory.fi/tgdhtml/Bgenememe2.html*. Available at: https://tgdtheory.fi/pdfpool/harmonytheory.pdf, 2023.

[K43] Pitkänen M. Holography and Hamilton-Jacobi Structure as a 4-D Generalization of 2-D Complex Structure. In *Physics in Many-Sheeted Space-Time: Part I. https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/HJ.pdf, 2023.

[K44] Pitkänen M. Homeopathy in Many-Sheeted Space-Time. In *TGD Universe as a Conscious Hologram. https://tgdtheory.fi/tgdhtml/Bholography.html*. Available at: https://tgdtheory.fi/pdfpool/homeoc.pdf, 2023.

[K45] Pitkänen M. *Hyper-finite Factors and Dark Matter Hierarchy.* Online book. Available at: https://www.tgdtheory.fi/tgdhtml/neuplanck.html., 2023.

[K46] Pitkänen M. Identification of the WCW Kähler Function. In *Quantum Physics as Infinite-Dimensional Geometry.* *https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/kahler.pdf., 2023.

[K47] Pitkänen M. Knots and TGD. In *Quantum Physics as Infinite-Dimensional Geometry.* *https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/knotstgd.pdf, 2023.

[K48] Pitkänen M. Langlands Program and TGD. In *TGD as a Generalized Number Theory: Part II.* *https://tgdtheory.fi/tgdhtml/Btgdnumber2.html*. Available at: https://tgdtheory.fi/pdfpool/Langlands.pdf, 2023.

[K49] Pitkänen M. Life and Death, and Consciousness. In *TGD Inspired Theory of Consciousness: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdconsc1.html*. Available at: https://tgdtheory.fi/pdfpool/lifedeath.pdf, 2023.

[K50] Pitkänen M. Magnetic Sensory Canvas Hypothesis. In *TGD and Quantum Biology: Part I.* *https://tgdtheory.fi/tgdhtml/Bqbio1.html*. Available at: https://tgdtheory.fi/pdfpool/mec.pdf, 2023.

[K51] Pitkänen M. *Magnetospheric Consciousness.* Online book. Available at: https://www.tgdtheory.fi/tgdhtml/magnconsc.html., 2023.

[K52] Pitkänen M. Magnetospheric Sensory Representations. In *Magnetospheric Consciousness.* *https://tgdtheory.fi/tgdhtml/Bmagnconsc.html*. Available at: https://tgdtheory.fi/pdfpool/srepres.pdf, 2023.

[K53] Pitkänen M. Massless states and particle massivation. In *p-Adic Physics.* *https://tgdtheory.fi/tgdhtml/Bpadphys.html*. Available at: https://tgdtheory.fi/pdfpool/mless.pdf, 2023.

[K54] Pitkänen M. *Mathematical Aspects of Consciousness Theory.* Online book. Available at: https://www.tgdtheory.fi/tgdhtml/mathconsc.html, 2023.

[K55] Pitkänen M. More Precise TGD View about Quantum Biology and Prebiotic Evolution. In *Evolution in TGD Universe.* *https://tgdtheory.fi/tgdhtml/Btgdevolution.html*. Available at: https://tgdtheory.fi/pdfpool/geesink.pdf, 2023.

[K56] Pitkänen M. Motives and Infinite Primes. In *TGD as a Generalized Number Theory: Part III.* *https://tgdtheory.fi/tgdhtml/Btgdnumber3.html*. Available at: https://tgdtheory.fi/pdfpool/infmotives.pdf, 2023.

[K57] Pitkänen M. Negentropy Maximization Principle. In *TGD Inspired Theory of Consciousness: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdconsc1.html*. Available at: https://tgdtheory.fi/pdfpool/nmpc.pdf, 2023.

[K58] Pitkänen M. New Physics Predicted by TGD: Part I. In *p-Adic Physics.* *https://tgdtheory.fi/tgdhtml/Bpadphys.html*. Available at: https://tgdtheory.fi/pdfpool/TGDnewphys1.pdf, 2023.

[K59] Pitkänen M. New Physics Predicted by TGD: Part II. In *p-Adic Physics.* *https://tgdtheory.fi/tgdhtml/Bpadphys.html*. Available at: https://tgdtheory.fi/pdfpool/TGDnewphys2.pdf, 2023.

[K60] Pitkänen M. Number theoretic vision, Hyper-finite Factors and S-matrix. In *Quantum TGD: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdquantum1.html*. Available at: https://tgdtheory.fi/pdfpool/UandM.pdf, 2023.

[K61] Pitkänen M. *p-Adic length Scale Hypothesis.* Online book. Available at: https://www.tgdtheory.fi/tgdhtml/padphys.html., 2023.

[K62] Pitkänen M. p-Adic Particle Massivation: Hadron Masses. In *p-Adic Physics.* *https://tgdtheory.fi/tgdhtml/Bpadphys.html*. Available at: https://tgdtheory.fi/pdfpool/mass3.pdf, 2023.

[K63] Pitkänen M. p-Adic Physics: Physical Ideas. In *TGD as a Generalized Number Theory: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/phblocks.pdf, 2023.

[K64] Pitkänen M. Physics as a Generalized Number Theory. In *Topological Geometrodynamics: Overview: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdview1.html*. Available at: https://tgdtheory.fi/pdfpool/tgdnumber.pdf, 2023.

[K65] Pitkänen M. Quantum Antenna Hypothesis. In *Bio-Systems as Self-Organizing Quantum Systems.* *https://tgdtheory.fi/tgdhtml/BbioSO.html*. Available at: https://tgdtheory.fi/pdfpool/tubuc.pdf, 2023.

[K66] Pitkänen M. Quantum Astrophysics. In *Physics in Many-Sheeted Space-Time: Part II.* *https://tgdtheory.fi/tgdhtml/Btgdclass2.html*. Available at: https://tgdtheory.fi/pdfpool/qastro.pdf, 2023.

[K67] Pitkänen M. Quantum gravity, dark matter, and prebiotic evolution. In *Evolution in TGD Universe.* *https://tgdtheory.fi/tgdhtml/Btgdevolution.html*. Available at: https://tgdtheory.fi/pdfpool/hgrprebio.pdf, 2023.

[K68] Pitkänen M. Quantum Hall effect and Hierarchy of Planck Constants. In *TGD and Condensed Matter.* *https://tgdtheory.fi/tgdhtml/BTGDcondmat.html*. Available at: https://tgdtheory.fi/pdfpool/anyontgd.pdf, 2023.

[K69] Pitkänen M. *Quantum Hardware of Living Matter.* Online book. Available at: https://www.tgdtheory.fi/tgdhtml/bioware.html., 2023.

[K70] Pitkänen M. Quantum Mind and Neuroscience. In *TGD and EEG: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdeeg1.html*. Available at: https://tgdtheory.fi/pdfpool/lianPN.pdf, 2023.

[K71] Pitkänen M. Quantum Mind, Magnetic Body, and Biological Body. In *TGD and Quantum Biology: Part I.* *https://tgdtheory.fi/tgdhtml/Bqbio1.html*. Available at: https://tgdtheory.fi/pdfpool/lianPB.pdf, 2023.

[K72] Pitkänen M. Quantum Model for Bio-Superconductivity: I. In *TGD and Quantum Biology: Part I.* *https://tgdtheory.fi/tgdhtml/Bqbio1.html*. Available at: https://tgdtheory.fi/pdfpool/biosupercondI.pdf, 2023.

[K73] Pitkänen M. Quantum Model for Bio-Superconductivity: II. In *TGD and Quantum Biology: Part I.* *https://tgdtheory.fi/tgdhtml/Bqbio1.html*. Available at: https://tgdtheory.fi/pdfpool/biosupercondII.pdf, 2023.

[K74] Pitkänen M. Quantum Model for Hearing. In *TGD and EEG: Part II.* *https://tgdtheory.fi/tgdhtml/Btgdeeg2.html*. Available at: https://tgdtheory.fi/pdfpool/hearing.pdf, 2023.

[K75] Pitkänen M. Quantum Model for Nerve Pulse. In *TGD and EEG: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdeeg1.html*. Available at: https://tgdtheory.fi/pdfpool/nervepulse.pdf, 2023.

[K76] Pitkänen M. *Quantum TGD.* Online book. Available at: https://www.tgdtheory.fi/tgdhtml/tgdquantum.html., 2023.

[K77] Pitkänen M. Recent View about Kähler Geometry and Spin Structure of WCW . In *Quantum Physics as Infinite-Dimensional Geometry.* *https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/wcwnew.pdf, 2023.

[K78] Pitkänen M. Some questions related to the twistor lift of TGD. In *Quantum TGD: Part III.* *https://tgdtheory.fi/tgdhtml/Btgdquantum3.html*. Available at: https://tgdtheory.fi/pdfpool/twistquestions.pdf, 2023.

[K79] Pitkänen M. TGD and Astrophysics. In *Physics in Many-Sheeted Space-Time: Part II.* *https://tgdtheory.fi/tgdhtml/Btgdclass2.html*. Available at: https://tgdtheory.fi/pdfpool/astro.pdf, 2023.

[K80] Pitkänen M. TGD and Cosmology. In *Physics in Many-Sheeted Space-Time: Part II.* *https://tgdtheory.fi/tgdhtml/Btgdclass2.html*. Available at: https://tgdtheory.fi/pdfpool/cosmo.pdf, 2023.

[K81] Pitkänen M. *TGD and EEG.* Online book. Available at: https://www.tgdtheory.fi/tgdhtml/tgdeeg.html, 2023.

[K82] Pitkänen M. *TGD and Fringe Physics.* Online book. Available at: https://www.tgdtheory.fi/tgdhtml/freenergy.html., 2023.

[K83] Pitkänen M. TGD and M-Theory. In *Topological Geometrodynamics: Overview: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdview1.html*. Available at: https://tgdtheory.fi/pdfpool/MTGD.pdf, 2023.

[K84] Pitkänen M. TGD and Nuclear Physics. In *TGD and Nuclear Physics.* *https://tgdtheory.fi/tgdhtml/Bnucl.html*. Available at: https://tgdtheory.fi/pdfpool/padnucl.pdf, 2023.

[K85] Pitkänen M. TGD and Potential Anomalies of GRT. In *Physics in Many-Sheeted Space-Time: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/granomalies.pdf., 2023.

[K86] Pitkänen M. TGD as a Generalized Number Theory: Infinite Primes. In *TGD as a Generalized Number Theory: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/visionc.pdf, 2023.

[K87] Pitkänen M. TGD as a Generalized Number Theory: p-Adicization Program. In *Quantum Physics as Number Theory: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/visiona.pdf, 2023.

[K88] Pitkänen M. TGD as a Generalized Number Theory: Quaternions, Octonions, and their Hyper Counterparts. In *TGD as a Generalized Number Theory: Part I.* *https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/visionb.pdf, 2023.

[K89] Pitkänen M. TGD Based Model for OBEs. In *TGD Inspired Theory of Consciousness: Part III.* *https://tgdtheory.fi/tgdhtml/Btgdconsc3.html*. Available at: https://tgdtheory.fi/pdfpool/OBE.pdf, 2023.

[K90] Pitkänen M. *TGD Based View About Living Matter and Remote Mental Interactions.* Online book. Available at: https://www.tgdtheory.fi/tgdhtml/tgdlian.html, 2023.

[K91] Pitkänen M. *TGD Inspired Theory of Consciousness.* Online book. Available at: https://www.tgdtheory.fi/tgdhtml/tgdconsc.html., 2023.

[K92] Pitkänen M. TGD View about Coupling Constant Evolution. In *Quantum TGD: Part II.* *https://tgdtheory.fi/tgdhtml/Btgdquantum2.html*. Available at: https://tgdtheory.fi/pdfpool/ccevolution.pdf, 2023.

[K93] Pitkänen M. TGD view about McKay Correspondence, ADE Hierarchy, Inclusions of Hyperfinite Factors, $M^8 - H$ Duality, SUSY, and Twistors. In *TGD and Hyper-finite Factors.* *https://tgdtheory.fi/tgdhtml/BHFF.html*. Available at: https://tgdtheory.fi/pdfpool/McKaygeneral, 2023.

[K94] Pitkänen M. The classical part of the twistor story. In *Quantum TGD: Part III. https://tgdtheory.fi/tgdhtml/Btgdquantum3.html*. Available at: https://tgdtheory.fi/pdfpool/twistorstory.pdf, 2023.

[K95] Pitkänen M. The Geometry of the World of the Classical Worlds. In *Topological Geometrodynamics: Overview: Part I:https://tgdtheory.fi/tgdhtml/Btgdview1.html*. Available at: https://tgdtheory.fi/pdfpool/tgdgeom.pdf, 2023.

[K96] Pitkänen M. The Notion of Free Energy and Many-Sheeted Space-Time Concept. In *TGD and Fringe Physics. https://tgdtheory.fi/tgdhtml/Bfreenergies.html*. Available at: https://tgdtheory.fi/pdfpool/freenergy.pdf, 2023.

[K97] Pitkänen M. The Recent Status of Lepto-hadron Hypothesis. In *p-Adic Physics. https://tgdtheory.fi/tgdhtml/Bpadphys.html*. Available at: https://tgdtheory.fi/pdfpool/leptc.pdf, 2023.

[K98] Pitkänen M. The Recent View about Twistorialization in TGD Framework. In *Quantum TGD: Part III. https://tgdtheory.fi/tgdhtml/Btgdquantum3.html*. Available at: https://tgdtheory.fi/pdfpool/smatrix.pdf, 2023.

[K99] Pitkänen M. The Relationship Between TGD and GRT. In *Physics in Many-Sheeted Space-Time: Part I. https://tgdtheory.fi/tgdhtml/Btgdclass1.html*. Available at: https://tgdtheory.fi/pdfpool/tgdgrt.pdf., 2023.

[K100] Pitkänen M. *Topological Geometrodynamics: an Overview.* Online book.Available at: https://www.tgdtheory.fi/tgdhtml/tgdview.html., 2023.

[K101] Pitkänen M. Topological Geometrodynamics: Basic Visions. In *TGD Inspired Theory of Consciousness: Part I. https://tgdtheory.fi/tgdhtml/Btgdconsc1.html*. Available at: https://tgdtheory.fi/pdfpool/lianPTGD.pdf, 2023.

[K102] Pitkänen M. Unified Number Theoretical Vision. In *TGD as a Generalized Number Theory: Part I. https://tgdtheory.fi/tgdhtml/Btgdnumber1.html*. Available at: https://tgdtheory.fi/pdfpool/numbervision.pdf, 2023.

[K103] Pitkänen M. Was von Neumann Right After All? In *TGD and Hyper-finite Factors. https://tgdtheory.fi/tgdhtml/BHFF.html*. Available at: https://tgdtheory.fi/pdfpool/vNeumann.pdf, 2023.

[K104] Pitkänen M. WCW Spinor Structure. In *Quantum Physics as Infinite-Dimensional Geometry. https://tgdtheory.fi/tgdhtml/Btgdgeom.html*. Available at: https://tgdtheory.fi/pdfpool/cspin.pdf, 2023.

[K105] Pitkänen M. Why TGD and What TGD is? In *Topological Geometrodynamics: an Overview. https://tgdtheory.fi/tgdhtml/Btgdview1.html*. Available at: https://tgdtheory.fi/pdfpool/WhyTGD.pdf, 2023.

[K106] Pitkänen M. Wormhole Magnetic Fields. In *Bio-Systems as Self-Organizing Quantum Systems. https://tgdtheory.fi/tgdhtml/BbioSO.html*. Available at: https://tgdtheory.fi/pdfpool/wormc.pdf, 2023.

[K107] Pitkänen M. Zero Energy Ontology. In *Quantum TGD: Part I. https://tgdtheory.fi/tgdhtml/Btgdquantum1.html*. Available at: https://tgdtheory.fi/pdfpool/ZEO.pdf, 2023.

Articles about TGD

[L1] Pitkänen M. Basic Properties of CP_2 and Elementary Facts about p-Adic Numbers. Available at: https://tgdtheory.fi/pdfpool/append.pdf., 2006.

[L2] Pitkänen M. About the Nature of Time. Available at: https://tgdtheory.fi/public_html/articles/time.pdf., 2008.

[L3] Pitkänen M. Are neutrinos superluminal? Available at: https://tgdtheory.fi/public_html/articles/speedofneutrino.pdf., 2011.

[L4] Pitkänen M. Do we really understand the solar system? Available at: https://tgdtheory.fi/public_html/articles/precession.pdf., 2011.

[L5] Pitkänen M. TGD inspired vision about entropic gravitation. Available at: https://tgdtheory.fi/public_html/articles/egtgd.pdf., 2011.

[L6] Pitkänen M. Are dark photons behind biophotons? *Journal of Non-Locality*, 2(1), 2013. See also https://tgdtheory.fi/pdfpool/biophotonslian.pdf.

[L7] Pitkänen M. CMAP representations about TGD. Available at: https://www.tgdtheory.fi/cmaphtml.html, 2014.

[L8] Pitkänen M. CMAP representations about TGD, and TGD inspired theory of consciousness and quantum biology. Available at: https://www.tgdtheory.fi/tgdglossary.pdf., 2014.

[L9] Pitkänen M. Geometric theory of harmony. Available at: https://tgdtheory.fi/public_html/articles/harmonytheory.pdf., 2014.

[L10] Pitkänen M. Pollack's Findings about Fourth phase of Water : TGD View. Available at: https://tgdtheory.fi/public_html/articles/PollackYoutube.pdf., 2014.

[L11] Pitkänen M. Psychedelic induced experiences as key to the understanding of the connection between magnetic body and information molecules? Available at: https://tgdtheory.fi/public_html/articles/psychedelics.pdf., 2014.

[L12] Pitkänen M. Could Podkletnov effect be understood using $h_{gr} = h_{eff}$ hypothesis? Available at: https://tgdtheory.fi/public_html/articles/Podkletnovagain.pdf., 2015.

[L13] Pitkänen M. Maintenance problem for Earth's magnetic field. Available at: https://tgdtheory.fi/public_html/articles/Bmaintenance.pdf., 2015.

[L14] Pitkänen M. More Precise TGD Based View about Quantum Biology and Prebiotic Evolution. Available at: https://tgdtheory.fi/public_html/articles/geesink.pdf., 2015.

[L15] Pitkänen M. About Physical Representations of Genetic Code in Terms of Dark Nuclear Strings. Available at: https://tgdtheory.fi/public_html/articles/genecodemodels.pdf., 2016.

[L16] Pitkänen M. Can one apply Occam's razor as a general purpose debunking argument to TGD? Available at: https://tgdtheory.fi/public_html/articles/simplicity.pdf., 2016.

[L17] Pitkänen M. Holography and Quantum Error Correcting Codes: TGD View. Available at: https://tgdtheory.fi/public_html/articles/tensornet.pdf., 2016.

[L18] Pitkänen M. Hydrinos again. Available at: https://tgdtheory.fi/public_html/articles/Millsagain.pdf., 2016.

[L19] Pitkänen M. LIGO and TGD. Available at: https://tgdtheory.fi/public_html/articles/ligotgd.pdf., 2016.

[L20] Pitkänen M. TGD interpretation for the new discovery about galactic dark matter. Available at: https://tgdtheory.fi/public_html/articles/darknew.pdf., 2016.

[L21] Pitkänen M. Cold fusion, low energy nuclear reactions, or dark nuclear synthesis? Available at: https://tgdtheory.fi/public_html/articles/krivit.pdf., 2017.

[L22] Pitkänen M. DMT, pineal gland, and the new view about sensory perception. Available at: https://tgdtheory.fi/public_html/articles/dmtpineal.pdf., 2017.

[L23] Pitkänen M. Does M^8-H duality reduce classical TGD to octonionic algebraic geometry? Available at: https://tgdtheory.fi/public_html/articles/ratpoints.pdf., 2017.

[L24] Pitkänen M. Does M^8-H duality reduce classical TGD to octonionic algebraic geometry?: part I. Available at: https://tgdtheory.fi/public_html/articles/ratpoints1.pdf., 2017.

[L25] Pitkänen M. Does M^8-H duality reduce classical TGD to octonionic algebraic geometry?: part II. Available at: https://tgdtheory.fi/public_html/articles/ratpoints2.pdf., 2017.

[L26] Pitkänen M. Does M^8-H duality reduce classical TGD to octonionic algebraic geometry?: part III. Available at: https://tgdtheory.fi/public_html/articles/ratpoints3.pdf., 2017.

[L27] Pitkänen M. Does valence bond theory relate to the hierarchy of Planck constants? Available at: https://tgdtheory.fi/public_html/articles/valenceheff.pdf., 2017.

[L28] Pitkänen M. Philosophy of Adelic Physics. Available at: https://tgdtheory.fi/public_html/articles/adelephysics.pdf., 2017.

[L29] Pitkänen M. Philosophy of Adelic Physics. In *Trends and Mathematical Methods in Interdisciplinary Mathematical Sciences*, pages 241–319. Springer.Available at: https://link.springer.com/chapter/10.1007/978-3-319-55612-3_11, 2017.

[L30] Pitkänen M. Re-examination of the basic notions of TGD inspired theory of consciousness. Available at: https://tgdtheory.fi/public_html/articles/conscrit.pdf., 2017.

[L31] Pitkänen M. About dark variants of DNA, RNA, and amino-acids. Available at: https://tgdtheory.fi/public_html/articles/darkvariants.pdf., 2018.

[L32] Pitkänen M. About the Correspondence of Dark Nuclear Genetic Code and Ordinary Genetic Code. Available at: https://tgdtheory.fi/public_html/articles/codedarkcode.pdf., 2018.

[L33] Pitkänen M. About the physical interpretation of the velocity parameter in the formula for the gravitational Planck constant. Available at: https://tgdtheory.fi/public_html/articles/vzeronew.pdf., 2018.

[L34] Pitkänen M. About the physical interpretation of the velocity parameter in the formula for the gravitational Planck constant . Available at: https://tgdtheory.fi/public_html/articles/vzero.pdf., 2018.

[L35] Pitkänen M. Clustering of RNA polymerase molecules and Comorosan effect. Available at: https://tgdtheory.fi/public_html/articles/clusterRNA.pdf., 2018.

[L36] Pitkänen M. Dark valence electrons and color vision. Available at: https://tgdtheory.fi/public_html/articles/colorvision.pdf., 2018.

[L37] Pitkänen M. Dark valence electrons, dark photons, bio-photons, and carcinogens. Available at: https://tgdtheory.fi/public_html/articles/carcinogen.pdf., 2018.

[L38] Pitkänen M. Emotions as sensory percepts about the state of magnetic body? Available at: https://tgdtheory.fi/public_html/articles/emotions.pdf., 2018.

[L39] Pitkänen M. Expanding Earth hypothesis, Platonic solids, and plate tectonics as a symplectic flow. Available at: https://tgdtheory.fi/public_html/articles/platoplate.pdf., 2018.

[L40] Pitkänen M. Expanding Earth Model and Pre-Cambrian Evolution of Continents, Climate, and Life. Available at: https://tgdtheory.fi/public_html/articles/expearth.pdf., 2018.

[L41] Pitkänen M. Further comments about classical field equations in TGD framework . Available at: https://tgdtheory.fi/public_html/articles/morecomments.pdf., 2018.

[L42] Pitkänen M. Homonymy of the genetic code from TGD point of view. Available at: https://tgdtheory.fi/public_html/articles/homonymy.pdf., 2018.

[L43] Pitkänen M. How molecules in cells "find" one another and organize into structures? Available at: https://tgdtheory.fi/public_html/articles/moleculefind.pdf., 2018.

[L44] Pitkänen M. Is the hierarchy of Planck constants behind the reported variation of Newton's constant? Available at: https://tgdtheory.fi/public_html/articles/variableG.pdf., 2018.

[L45] Pitkänen M. Maxwell's lever rule and expansion of water in freezing: two poorly understood phenomena. Available at: https://tgdtheory.fi/public_html/articles/leverule.pdf., 2018.

[L46] Pitkänen M. New results in the model of bio-harmony. Available at: https://tgdtheory.fi/public_html/articles/harmonynew.pdf., 2018.

[L47] Pitkänen M. New support for the view about Cambrian explosion being caused by rapid increase of Earth radius. Available at: https://tgdtheory.fi/public_html/articles/HIFs.pdf., 2018.

[L48] Pitkänen M. TGD view about coupling constant evolution. Available at: https://tgdtheory.fi/public_html/articles/ccevolution.pdf., 2018.

[L49] Pitkänen M. TGD view about quasars. Available at: https://tgdtheory.fi/public_html/articles/meco.pdf., 2018.

[L50] Pitkänen M. The Recent View about Twistorialization in TGD Framework. Available at: https://tgdtheory.fi/public_html/articles/smatrix.pdf., 2018.

[L51] Pitkänen M. An overall view about models of genetic code and bio-harmony. Available at: https://tgdtheory.fi/public_html/articles/gcharm.pdf., 2019.

[L52] Pitkänen M. Copenhagen interpretation dead: long live ZEO based quantum measurement theory! Available at: https://tgdtheory.fi/public_html/articles/Bohrdead.pdf., 2019.

[L53] Pitkänen M. Cosmic string model for the formation of galaxies and stars. Available at: https://tgdtheory.fi/public_html/articles/galaxystars.pdf., 2019.

[L54] Pitkänen M. Could 160 minute oscillation affecting Galaxies and the Solar System correspond to cosmic "alpha rhythm"? Available at: https://tgdtheory.fi/public_html/articles/cosmoalpha.pdf., 2019.

[L55] Pitkänen M. Does coupling constant evolution reduce to that of cosmological constant? Available at: https://tgdtheory.fi/public_html/articles/ccevoTGD.pdf., 2019.

[L56] Pitkänen M. Earhquakes and volcanic eruptions as macroscopic quantum jumps in zero energy ontology. Available at: https://tgdtheory.fi/public_html/articles/earthquakes.pdf., 2019.

[L57] Pitkänen M. $M^8 - H$ duality and consciousness. Available at: https://tgdtheory.fi/public_html/articles/M8Hconsc.pdf., 2019.

[L58] Pitkänen M. New results related to $M^8 - H$ duality. Available at: https://tgdtheory.fi/public_html/articles/M8Hduality.pdf., 2019.

[L59] Pitkänen M. Quantum self-organization by h_{eff} changing phase transitions. Available at: https://tgdtheory.fi/public_html/articles/heffselforg.pdf., 2019.

[L60] Pitkänen M. Solar Surprise. Available at: https://tgdtheory.fi/public_html/articles/solarsycle.pdf., 2019.

[L61] Pitkänen M. Some comments related to Zero Energy Ontology (ZEO). Available at: https://tgdtheory.fi/public_html/articles/zeoquestions.pdf., 2019.

[L62] Pitkänen M. Three findings about memory recall and TGD based view about memory retrieval. Available at: https://tgdtheory.fi/public_html/articles/memoryrecall.pdf., 2019.

[L63] Pitkänen M. Waterbridge experiment from TGD point of view. Available at: https://tgdtheory.fi/public_html/articles/waterbridge.pdf., 2019.

[L64] Pitkänen M. A critical re-examination of $M^8 - H$ duality hypothesis: part I. Available at: https://tgdtheory.fi/public_html/articles/M8H1.pdf., 2020.

[L65] Pitkänen M. A critical re-examination of $M^8 - H$ duality hypothesis: part II. Available at: https://tgdtheory.fi/public_html/articles/M8H2.pdf., 2020.

[L66] Pitkänen M. Can TGD predict the value of Newton's constant? Available at: https://tgdtheory.fi/public_html/articles/Gagain.pdf., 2020.

[L67] Pitkänen M. Could brain be represented as a hyperbolic geometry? Available at: https://tgdtheory.fi/public_html/articles/hyperbolicbrain.pdf., 2020.

[L68] Pitkänen M. Could quantum randomness have something to do with classical chaos? Available at: https://tgdtheory.fi/public_html/articles/chaostgd.pdf., 2020.

[L69] Pitkänen M. New results about dark DNA inspired by the model for remote DNA replication. Available at: https://tgdtheory.fi/public_html/articles/darkdnanew.pdf., 2020.

[L70] Pitkänen M. Still about quantum measurement theory in ZEO. Available at: https://tgdtheory.fi/public_html/articles/qmeasuretgd.pdf., 2020.

[L71] Pitkänen M. The dynamics of SSFRs as quantum measurement cascades in the group algebra of Galois group. Available at: https://tgdtheory.fi/public_html/articles/SSFRGalois.pdf., 2020.

[L72] Pitkänen M. About the role of Galois groups in TGD framework. https://tgdtheory.fi/public_html/articles/GaloisTGD.pdf., 2021.

[L73] Pitkänen M. Comparing the Berry phase model of super-conductivity with the TGD based model. https://tgdtheory.fi/public_html/articles/SCBerryTGD.pdf., 2021.

[L74] Pitkänen M. Is genetic code part of fundamental physics in TGD framework? Available at: https://tgdtheory.fi/public_html/articles/TIH.pdf., 2021.

[L75] Pitkänen M. Negentropy Maximization Principle and Second Law. Available at: https://tgdtheory.fi/public_html/articles/nmpsecondlaw.pdf., 2021.

[L76] Pitkänen M. Some questions concerning zero energy ontology. https://tgdtheory.fi/public_html/articles/zeonew.pdf., 2021.

[L77] Pitkänen M. TGD and Condensed Matter. https://tgdtheory.fi/public_html/articles/TGDcondmatshort.pdf., 2021.

[L78] Pitkänen M. TGD as it is towards the end of 2021. https://tgdtheory.fi/public_html/articles/TGD2021.pdf., 2021.

[L79] Pitkänen M. TGD view of the engine powering jets from active galactic nuclei. https://tgdtheory.fi/public_html/articles/galjets.pdf., 2021.

[L80] Pitkänen M. Time reversal and the anomalies of rotating magnetic systems. Available at: https://tgdtheory.fi/public_html/articles/freereverse.pdf., 2021.

[L81] Pitkänen M. Updated version of Expanding Earth model. https://tgdtheory.fi/public_html/articles/expearth2021.pdf., 2021.

[L82] Pitkänen M. What could 2-D minimal surfaces teach about TGD? https://tgdtheory.fi/public_html/articles/minimal.pdf., 2021.

[L83] Pitkänen M. About TGD counterparts of twistor amplitudes: part I. https://tgdtheory.fi/public_html/articles/twisttgd1.pdf., 2022.

[L84] Pitkänen M. About TGD counterparts of twistor amplitudes: part II. https://tgdtheory.fi/public_html/articles/twisttgd2.pdf., 2022.

[L85] Pitkänen M. Comparison of Orch-OR hypothesis with the TGD point of view. https://tgdtheory.fi/public_html/articles/penrose.pdf., 2022.

[L86] Pitkänen M. Finite Fields and TGD. https://tgdtheory.fi/public_html/articles/finitefieldsTGD.pdf., 2022.

[L87] Pitkänen M. Some New Ideas Related to Langlands Program *viz.* TGD. https://tgdtheory.fi/public_html/articles/Langlands2022.pdf., 2022.

[L88] Pitkänen M. Trying to fuse the basic mathematical ideas of quantum TGD to a single coherent whole. https://tgdtheory.fi/public_html/articles/fusionTGD.pdf., 2022.

[L89] Pitkänen M. Two objections against p-adic thermodynamics and their resolution. https://tgdtheory.fi/public_html/articles/padmass2022.pdf., 2022.

[L90] Pitkänen M. New findings related to the number theoretical view of TGD. https://tgdtheory.fi/public_html/articles/M8Hagain.pdf., 2023.

[L91] Pitkänen M. Reduction of standard model structure to CP_2 geometry and other key ideas of TGD. https://tgdtheory.fi/public_html/articles/cp2etc.pdf., 2023.

[L92] Pitkänen M and Rastmanesh R. DNA and Time Reversal. Available at: https://tgdtheory.fi/public_html/articles/DNAtimereversal., 2020.

[L93] Pitkänen M and Rastmanesh R. Homeostasis as self-organized quantum criticality. Available at: https://tgdtheory.fi/public_html/articles/SP.pdf., 2020.

[L94] Pitkänen M and Rastmanesh R. The based view about dark matter at the level of molecular biology. Available at: https://tgdtheory.fi/public_html/articles/darkchemi.pdf., 2020.

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